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Abstract

We consider the inventory control problem of an independent supplier in a continuous review system. The supplier faces demand from a single customer who in turn faces Poisson demand and follows a continuous review (R, Q) policy. If no information about the inventory levels at the customer is available, reviews and ordering are usually carried out by the supplier only at points in time when a customer demand occurs. It is common to apply an installation stock reorder point policy. However, as the demand faced by the supplier is not Markovian, this policy can be improved by allowing placement of orders at any point in time. We develop a time delay policy for the supplier, wherein the supplier waits until time t after occurrence of the customer demand to place his next order. If the next customer demand occurs before this time delay, then the supplier places an order immediately. We develop an algorithm to determine the optimal time delay policy. We then evaluate the value of information about the customer's inventory level. Our numerical study shows that if the supplier were to use the optimal time delay policy instead of the installation stock policy then the value of the customer's inventory information is not very significant.

Keywords: *Supply chain management, Erlangian demand, Continuous review system, Optimal no-information policy, Value of information*

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1. Introduction

With increased focus on supply chain management and the tremendous improvements in the IT and communications infrastructure in the last decade, there has been a greater interest in formation of strategic alliances and exchange of information among suppliers and customers in a supply chain. Software solutions and concepts such as web-based Enterprise Resource Planning and Collaborative Planning, Forecasting and Replenishment (CPFR) have made this easily possible. This has led to initiatives for collaboration and greater visibility of demand and inventory information in the supply chain.

While technology exists to share information among the entities in a supply chain, whether it is being effectively done and/or is beneficial to all parties is a question that still requires considerable research. Beyond the well publicized cases of successful information sharing and collaboration efforts, there have only been limited success stories (Larsen, Thernøe, and Andresen 2003). One reason for the apparent lack of success in collaboration and sharing information is the reluctance of companies to share what is perceived as sensitive data (Scheraga 2002, Danese 2006). A specific issue that has attracted the attention of researchers is the benefit of sharing information about inventory/demand in a supply chain.

In this paper, we consider the inventory control problem of a supplier facing demand from a single customer who in turn faces Poisson demand and follows a continuous review (R, Q) policy. The supplier therefore faces batch demands of fixed size Q, whose inter-arrival times are Erlangian. The supplier is an independent entity and not part of an integrated supply chain.

In an integrated system, the inventory at the supplier would be referred to as *installation stock* (of the supplier) and the sum of the inventory at the supplier and the customer together would be referred to as *echelon stock* (of the supplier). As the customer is the last echelon, for

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him the echelon stock and the installation stock (of the customer) refer to the same thing which is his own inventory. Even though we are not dealing with an integrated system in this paper, we still refer to the installation stock for the supplier as his own inventory and the echelon stock as the sum of the inventory at his installation as well as that at the customer.

The supplier follows a continuous review system and his objective is to minimize his long run average inventory costs. Traditionally, reviews are carried out by the supplier only at points in time when a customer demand occurs. If no information about the inventory levels at the customer is available, it is then common for the supplier to adopt an installation stock policy, where both the reorder point and the order quantity are multiples of the demand batch size Q. In an installation stock policy, a replenishment order is triggered at the supplier when the inventory position at his location (inventory on hand + outstanding orders - backorders) drops to or below the reorder point. However, as the demand faced by the supplier is not Markovian, this policy can be improved by allowing placement of orders at any point in time. We develop a time delay policy for the supplier, and also develop an algorithm to find the optimal time delay policy. If the information on inventory levels of the customer were available to the supplier, then the optimal policy would be an echelon stock policy, where an order is placed when the echelon inventory position (sum of the installation inventory positions at the customer and the supplier) reaches the reorder point. We evaluate the value of information about the customer's inventory level for the supplier.

There are a number of related papers that have considered the value of information sharing in supply chains. These papers, in general, adopt the perspective of an integrated multi-echelon supply chain, rather than that of an independent supplier. Examples of such papers are Chen (1998), Moinzadeh (2002), Marklund (2002), and Axsäter and Marklund (2008).

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The idea of a time delay policy for non-Markovian demands (where orders are allowed to be placed at any time) was earlier proposed by Moinzadeh and Zhou (2008), as well as by Katircioglu (1996) (see also Schultz 1989). However, in this paper, we derive the time delay policy for our setting in a simpler way. Moreover, we also develop an efficient method to determine the optimal time delay policy when the supplier has non-zero set-up costs. The other main contribution of the paper is the finding that when the optimal time delay policy is used, the value of information about the customer's inventory level is quite negligible for the supplier.

The paper is organized as follows. We provide a review of the related literature in the rest of this section. In the next section, we develop our model and analyze the problem for the traditional installation stock policy, the echelon stock policy (which requires sharing of the customer's inventory information) as well as our time delay policy with no information sharing. Algorithms to determine the optimal policy under each of these scenarios are also developed. In Section 3, we extend the analysis to the case with positive set-up costs at the supplier. In Section 4, we present the results of a computational study. Finally some concluding remarks and possible issues for future research are discussed in Section 5.

1.1. Literature review

When no information is shared across the supply chain and each echelon acts to optimize its own costs, the demand variability in the supply chain gets amplified as it moves upstream in the supply chain. This phenomenon, known as the *bullwhip effect* was first observed by Forrester (1961), and has been studied further by Lee, Padmanabhan and Whang (1997) and later by Chen, Drezner, Ryan and Simchi-Levi (2000). One of the solutions proposed to counter the bullwhip effect is to have information sharing across the supply chain.

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The literature on the value of inventory or demand information in this context can be classified based on the type of external demand pattern (from the end-customer) and type of the review system used. Lee, So and Tang (2000) considered a manufacturer facing periodic demands from a retailer. The external demand at the retailer follows a non-stationary AR(1) process. As there are no set-up costs, orders are placed in every period in this periodic review system. Lee et al. quantified the value of the demand information and showed that there is value for the manufacturer to have information about external demand. Raghunathan (2001) studied the same model, and showed that if the manufacturer knows the form of the external demand process, and if the autocorrelation coefficient for the AR(1) process is non-negative, there is no additional value in knowing the actual external demand in each period. Gaur, Giloni and Seshadhri (2005) extended Lee et al.'s model to consider a more general ARMA process for the demand. They identify conditions when demand information is valuable for the manufacturer. They also show that for the AR(1) process with autocorrelation coefficient less than -0.5, there is value in information sharing for the manufacturer.

The rest of the related papers in the literature have mainly considered the value of information for the entire, integrated supply chain as opposed to an independent supplier. Gavirneni, Kapuscinski and Tayur (1999) consider a two-echelon, capacitated supply chain under a periodic review system. The supplier has no ordering cost, but has constraints on the production capacity in each period. The retailer faces independent and identically distributed (i.i.d.) demands in each period and follows an (s, S) policy. Gavirneni et al. consider three different scenarios. In the first scenario (M0), the supplier has no information about the external demand, but by using the history of past orders from the retailers, assumes an i.i.d. demand from the retailer, and correspondingly adopts a stationary modified base stock policy in each period.

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(In their modified base stock policy, the quantity ordered is either the quantity based on the specified base stock level or the maximum capacity available in the period, whichever is lower.) In the second scenario (M1), the supplier monitors the number of periods since the previous order from the retailer, and also the retailer's policy values (*s*, *S*). He uses this information to determine a different modified base stock policy for each period. In the final scenario (M2) the supplier has full information about the retailer's demand and inventory position and determines the optimal modified base stock policy in each period according to this information. The lead-time for production and shipment is assumed to be zero and to make the model tractable, it is also assumed that the retailer obtains the necessary stock from another external supplier, in case the supplier is not able to satisfy the demand. Gavirneni et al. (1999) quantify the value of information by comparing the inventory cost for the two-echelon supply chain under the three scenarios. They find that there is a significant value of information even when comparing the scenarios M1 and M2.

Cachon and Fisher (2000) considered a one supplier, multi-retailer supply chain with stationary and stochastic external demand at each retailer in a periodic review setting. In a traditional no information policy, the supplier uses a (*R*, *nQ*) policy which is not necessarily optimal for this case. A lower bound on the total supply chain cost under any scenario is developed using simulation. The difference between this and the cost of the traditional, no-information policy is the upper bound on the value of information. While there is some benefit in sharing inventory/demand information, Cachon and Fisher found that the benefit of other technology improvements such as lead-time reduction was much more significant than the benefit from information sharing.

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Other examples of papers dealing with control rules that are based on the echelon stock (or extended information concerning the inventory situation at different echelons or installations in the supply chain) are Chen (1998), Moinzadeh (2002), Marklund (2002), and Axsäter and Marklund (2008). The cost reduction (compared to the traditional installation stock policy) obtained by using such methods is commonly seen as the value of information. In general, these papers find that there is significant value to be obtained by using the inventory information at different echelons. For example, a computational study by Moinzadeh (2002) revealed that the savings from information sharing can be as high as 10 to 20%. Our numerical study shows that if the supplier were to use the optimal time delay policy instead of the installation stock policy as the baseline (or base case) then the value of the customer's inventory information is not very significant.

2. Model and analysis

2.1. Problem formulation

We consider a continuous review inventory control system at a supplier. The supplier has a single customer facing Poisson demand. If we include the customer we have a two-echelon system. However, our purpose is exclusively to minimize the costs at the supplier. The costs at the customer are disregarded. This is a more realistic scenario when the supplier and customer are independent and separate entities in the supply chain. The replenishment lead-time for the supplier is constant. It is known that the customer applies a continuous review (R, Q) policy, so the demands at the considered supplier are in batches of size Q. No partial deliveries are allowed. Furthermore, the intensity of the Poisson demand at the customer is known[1]. However, the

[1] Even if the demand information is not shared by the customer, the supplier can statistically estimate this based on the historical data of the customer orders over long periods of time. The expected value of the customer orders per unit time should be equal to the expected demand per unit time over long time intervals.

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inventory position and the demands at the customer can not be observed. The supplier can only observe the orders from the customer. Note that the reorder point R used by the customer does not affect the steady state situation at the supplier. This is because the stochastic flow of orders is independent of the reorder point.

There are standard holding and backorder costs per unit of time at the considered supplier. The backorder cost may, for example, be a penalty cost that is paid to the customer. Initially we assume that there are no ordering or set-up costs at the supplier, but this assumption is relaxed in Section 3. This assumption means that the supplier, like its customer, will order in customer batches (of size Q), which will meet the batch demands from its customer. The problem is to optimize the timing of these orders under the given limited information. In other words, we wish to optimize the "safety time" for each batch.

For comparison, we shall also consider a standard installation stock policy and an echelon stock policy. The latter policy requires information about the demands at the customer. Let us introduce the following notation:

L	=	lead-time,
h	=	holding cost per unit, per unit time,
p	=	backorder cost per unit, per unit time,
λ	=	known intensity of Poisson demand,
Q	=	known batch quantity, $Q > 1$,
R^i	=	installation stock reorder point at the supplier,
R^e	=	echelon stock reorder point at the supplier,
IP^i	=	installation stock inventory position,
IP^e	=	echelon stock inventory position,
IL	=	inventory level,
$D(L)$	=	stochastic lead-time demand,

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S = order-up-to inventory position (of installation stock),

C = expected cost rate,

$po(i, \lambda)$ = Poisson probability mass function,

$Po(i, \lambda)$ = Poisson cumulative distribution function.

2.2. Poisson demand at the supplier

Let us first assume that the demand at the supplier is Poisson. (This is the case if $Q = 1$. The results with $Q = 1$ are used later when optimizing the policy in the considered case with $Q > 1$.) It is well known (see e.g., Axsäter 2006) that the optimal policy is then an S policy, or equivalently an $(S - 1, S)$ policy. This means that when the installation stock inventory position (inventory on hand + outstanding orders – backorders) declines to $S - 1$, an order for one unit is triggered so that the inventory position is raised to S. For the rest of the discussion in the paper, when we just say "inventory position" or "inventory level", we mean the inventory position (level) of the installation stock at the supplier.

The inventory position is always S. We consider some arbitrary time t. Let us first evaluate a certain S in a standard way. All outstanding orders at time t would have been delivered by time $t + L$. New orders that have been triggered between t and $t + L$ would not be delivered at time $t + L$. We then get the inventory level (inventory on hand – backorders) at time $t + L$, $IL(t + L)$ as

$$IL(t+L) = S - D(L), \tag{1}$$

where $D(L)$ is the Poisson demand during the lead-time. This means that it is easy to determine the distribution of the inventory level. Given the inventory level, we get the cost rate as

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$$C(S) = h\,E(IL^{+}) + p\,E(IL^{-}) = (h+p)E(IL^{+}) - p\,E(IL)$$

$$= (h+p)\sum_{k=0}^{S-1}\frac{(\lambda L)^{k}}{k!}\,e^{-\lambda L}(S-k) - p(S-\lambda L)$$

$$= (h+p)\left(\sum_{k=0}^{S}\frac{(\lambda L)^{k}}{k!}\,e^{-\lambda L}S - \sum_{k=1}^{S}\frac{(\lambda L)^{k}}{k!}\,k\,e^{-\lambda L}\right) - p(S-\lambda L) \tag{2}$$

$$= (h+p)\big(S\,Po(S,\lambda) - \lambda L\,Po(S-1,\lambda)\big) - p(S-\lambda L).$$

In (2) x^{+} = max (x, 0) and x^{-} = max ($-x$, 0). Note that $x = x^{+} - x^{-}$, and hence $p\,E(IL^{-}) = pE(IL^{+}) - p\,E(IL)$. The corresponding cost per unit is $C(S)/\lambda$. It is easy to show that $C(S)$ is convex in S (see Axsäter 2006), and that the optimal S is always nonnegative. Consequently, it is also easy to optimize S. Starting with $S = 0$ we simply need to look for a local optimum. Let the optimal S be denoted by S^{*}.

It is also well known (see Axsäter 1990) that an ordered item will meet the S-th future demand for a unit, and that we can see $C(S)/\lambda$ as the expected cost rate for a unit that is ordered at some time and will meet the S-th forthcoming Poisson demand. What determines the costs is the stochastic time until an ordered unit is demanded. In our case it is the time of an Erlang distribution with S stages and intensity λ. Let

$g^{k}(t)$ = $\lambda^{k} t^{k-1} e^{-\lambda t}/(k-1)!$ = density of an Erlang distribution with k stages and intensity λ,

$G^{k}(t)$ = $1 - Po(k-1, \lambda t)$ = corresponding cumulative distribution function of an Erlang distribution with k stages and intensity λ.

As an alternative to (2) we can therefore equivalently express the costs as

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$$C(S)=\lambda\left(p\int_0^L g^S(t)(L-t)dt+h\int_L^\infty g^S(t)(t-L)dt\right)$$

$$=\lambda\left((h+p)\int_L^\infty g^S(t)(t-L)dt+p\int_0^\infty g^S(t)(L-t)dt\right)=$$

$$=(h+p)\left(S(1-G^{S+1}(L))-\lambda L(1-G^s(L))\right)-p(S-\lambda L) \tag{3}$$

$$=(h+p)\left(S\,Po(S,\lambda)-\lambda L\,Po(S-1,\lambda)\right)-p(S-\lambda L).$$

In (3) we multiply with λ to get the costs per unit of time. If the demand comes after the end of the lead-time, we get a holding cost and if the demand comes before the end of the lead-time, we get a backorder cost. We use that $g^S(t)t=g^{S+1}(t)S/\lambda$.

In this paper we shall use the view in (3) and assume that a demand occurs after a certain Erlang distributed time. Using this view it is also evident that $C(S^*)$ is a lower bound on the cost of any policy.

2.3. Batch demands – installation stock policy

Now we turn to the considered case with batch demands of size $Q > 1$ at the supplier and constrained information. Because no partial deliveries to the customer are allowed, it is obvious that the inventory level at the supplier should be a multiple of Q at all times. Because all demands and all replenishments at the supplier are for batches of size Q, the initial stock at the supplier should also be a multiple of Q. We assume that this is the case.

Assume that we have already determined S^* and $C(S^*)$. As mentioned, $C(S^*)$ is a lower bound on the cost of any policy.

A simple and straight-forward way to control the inventory under the limited availability of information is to apply an installation stock (R^i, Q) policy. The inventory position at the

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supplier is denoted IP^i. Recall that the supplier only knows Q and the intensity of the Poisson demand at the customer. Furthermore, we can without lack of generality assume that R^i is also a multiple of Q. (See e.g., Axsäter and Rosling 1993.) Because all demands are for batches of size Q, the inventory position is kept at $R^i + Q = jQ$ all the time (where j is an integer). Assume that a batch demand and the corresponding order have just occurred at the supplier. The next such batch demand will evidently appear after Q additional Poisson demands at the customer. Similarly, the j-th future batch demand will occur after jQ Poisson demands at the customer. This is the demand for the batch that just has been ordered. The whole batch will be demanded at the same time. Recall (3) and the discussion immediately before that expression. The costs per batch are $QC(jQ)/\lambda$ and there are λ/Q batches per unit of time. Consequently, we get the costs per unit time as $C(jQ)$. Recall now that $C(S)$ is minimized by S^*. We can consequently state the following simple proposition:

Proposition 1 If S^* is an integer multiple of Q, the best installation stock policy provides the optimal solution (with or without information on the customer's inventory position). The optimal reorder point is then $R^i = S^* - Q$.

Assume that S^* is not a multiple of Q. Let $\hat{j}$ be the smallest integer such that $S^* < \hat{j}Q$. Due to the convexity of the cost rate function, it is obvious that for the installation stock policy, it is optimal to use either $j = \hat{j}-1$ or $j = \hat{j}$, and the resulting costs are obtained as $C^i = \min\{C((\hat{j}-1)Q), C(\hat{j}Q)\}$.

Example: We now provide a numerical example to illustrate the method to determine the optimal installation stock policy. The same example is used throughout this section to illustrate the various policies. The problem parameters for this example are as follows: $\lambda = 50$, $L = 4$, $h = 1$, $p = 20$, $Q = 200$. For this problem, $S^* = 224$ so it is not an integer multiple of 200. It is obvious

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that $\hat{j}$ = 2. We get C (200) = 118.43, and C(400) = 200.00. Therefore the optimal installation stock policy is to keep a base stock level of 200. i.e., as soon the installation inventory position of the supplier goes down to the reorder point 0, an order of size 200 is placed by the supplier to bring the inventory position back to the base stock level $R + Q = 200$. The resulting cost according to (2) or (3) is $118.43.

2.4. Batch demands – echelon stock policy

If we do not have the considered restrictions on the available information, the supplier can apply an echelon stock policy. The echelon stock inventory position IP^e is the sum of the installation stock inventory positions at the supplier and the customer. The reorder point at the customer does not affect the steady state costs at the supplier. Let us therefore for simplicity set this reorder point equal to zero. This means that the inventory position at the customer is uniformly distributed on the interval [1, Q].

Consider first the case in Proposition 1 when S^* is an integer multiple of Q. Let the echelon stock reorder point at the supplier be $R^e = (R^i + Q) = jQ = S^*$. Assume that we start at some time when both the customer and the supplier have just ordered. The echelon stock inventory position at the supplier is then $jQ+Q$ and correspondingly, the installation stock inventory position at the supplier is jQ. A batch that has just been ordered by the supplier will then always be demanded after jQ customer demands and the costs per unit of time are $C(S^*)$, i.e., the same as with the installation stock policy.

Consider then the case where $S^* = jQ + x$ where $0 < x < Q$ and j is an integer. Note that j is unique. Let the echelon stock reorder point at the supplier be $R^e = jQ + x$. Assume that the supplier has just ordered and that the inventory positions just after the order are x at the customer, and $IP^i = (j+1)Q$ and $IP^e = (j+1)Q + x$, at the supplier. The batch that has just been

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ordered will evidently be demanded at customer demand $x + jQ = S^*$. See Table 1 for an example.

Table 1: Development of inventory positions for $j = 1$, Q = 2, $x = 1$, i.e., $S^* = 3$.

Supplier IP^e	5	4	5	4	5
Supplier IP^i	4	2	4	2	4
Customer	1	2	1	2	1
Demands	Initial state	Demand 1	Demand 2	Demand 3	Demand 4

The expected costs per unit of time for this policy will therefore be $C(S^*)$. Recall that this is a lower bound for the costs of any policy. It is easy to see that the expected costs are the same also for future orders. We can state

Proposition 2 Using an echelon stock policy, we can always get the optimal solution.

We note again that an echelon stock policy requires information about the inventory position at the customer, i.e., it is not a feasible solution with our assumptions. The ordering at the supplier and the customer are generally not nested, i.e., they will order at different times.

Example: Using the same example as before (i.e. $\lambda = 50$, $L = 4$, $h = 1$, $p = 20$, $Q = 200$), $S^* = 224$, and the corresponding cost $C(224) = 30.02$. We assume as above, without any loss of generality, that the reorder point at the customer is equal to zero. Evidently $j = 1$ and $x = 24$. Therefore the optimal echelon stock policy is to place an order when the echelon stock inventory position (the sum of the inventory positions at the supplier and the customer) reaches 224. That

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is, when the inventory position of the supplier is 200, and the inventory position of the customer reaches 24, an order of size 200 is placed by the supplier. The resulting cost is \$30.02.

2.5. Batch demands – installation stock policy with a possible time delay

We shall now determine an optimal time delay policy for the supplier under the considered information constraint. The supplier can only see the batch orders from its customer, but not the Poisson demands at the customer. However, he knows the intensity of this Poisson demand.

Assume that a batch demand has just occurred. Assume also that the supplier wants to order the batch that will meet the j-th forthcoming batch demand. We shall determine the optimal j later. The next such batch demand will evidently appear after Q additional Poisson demands at the customer. Similarly, the j-th future batch demand will occur after jQ Poisson demands at the customer. If Q is large there will in general be relatively long times between the batch demands. One possibility is to order the considered batch directly. However, there is also another possibility that must be considered. The supplier can also decide to wait a certain time t. If another batch demand is triggered during this time the considered batch demand will now satisfy the $(j - 1)$-th future batch demand from that time. But it may also happen that no batch demand is triggered, i.e., if there are $0 \leq i < Q$ customer demands during t. In that case the supplier orders when the delay time is over. The batch demand is then triggered by the $(jQ - i)$-th forthcoming customer demand. It is clear that the considered policy is a very general policy under the given information constraint. The time since the last batch demand is the only information we use. Especially, we note that $t = 0$ is equivalent to ordering immediately, or equivalently, apply an

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installation stock policy. Furthermore, $t = \infty$ means that we always will order when the next batch demand occurs, i.e., the ordered batch will meet the $(j - 1)$-th future batch demand instead of the j-th. So this can be seen as using $j - 1$ and $t = 0$ instead of j and $t = \infty$.

Consider first the case where S^* is an integer multiple of Q. According to Proposition 1 it is then optimal (even without any information on the customer's inventory position) to apply an installation stock policy with $R^i = (S^* - Q)$. In terms of our general policy it is then optimal to choose j such that $jQ = S^*$ and order with delay $t^* = 0$. We then obtain the optimal costs $C(S^*)$.

Let us now consider the case when S^* is not an integer multiple of Q. Assume that we have $(\hat{j} - 1)Q < S^* < \hat{j}\,Q$. Assume, furthermore, that we use $j = \hat{j} - 1$ and $t = 0$. This means that an ordered batch will be demanded after $(\hat{j} - 1)Q$ customer demands. Compared to the optimal solution this means that we order too late. It is also evident that it is no advantage to use $t > 0$ because that means that we order even later. Consider then $j = \hat{j}$ and $t = 0$. This means that we order too early. However, it is then possible to delay the order by using $t > 0$. Recall that $j = \hat{j}$ and $t = \infty$ is equivalent to $j = \hat{j} - 1$ and $t = 0$.

We need the following simple lemma:

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Lemma 1 (a) $[po(2, \lambda t) + po(3, \lambda t) + \ldots po(Q - 1, \lambda t)]/ po(1, \lambda t) \to 0$ as $t \to 0$,

(b) $[po(0, \lambda t) + po(1, \lambda t) + \ldots po(Q - 2, \lambda t)]/ po(Q - 1, \lambda t) \to 0$ as $t \to \infty$.

We omit the proof that follows directly from the Poisson probability mass function.

Let us now formulate the following proposition.

Proposition 3 The optimal time delay policy is to use $j = \hat{j}$ and a certain positive and finite delay time. If a new batch demand is triggered during the delay time the considered batch should be ordered immediately.

Proof It follows from the convexity that it is not optimal to use $j > \hat{j}$. Recall that j and $t = \infty$ is equivalent to $j - 1$ and t = 0. So $j > \hat{j}$ means always that we order too early. This demonstrates that $j = \hat{j}$ is optimal. The convexity implies also that it is optimal to order the considered batch immediately in case a new batch demand is triggered. Recall that $j = \hat{j} - 1$ and $t = 0$ means that we order too late.

Assume then that it is optimal to have delay $t^* = 0$. Consider a very small t. Due to Lemma 1 (a) the probability for a single demand during t is λt, while the probabilities for larger demands can be disregarded. Clearly $C(\hat{j}\,Q - 1) < C(\hat{j}\,Q)$, so the delay will reduce the expected costs. Similarly we can rule out $t^* = \infty$ compared to a finite large t. When t becomes larger, Lemma 1 (b) shows that the probabilities for demand sizes 0, 1, ..., $Q - 2$ can be disregarded compared to the probability for demand $Q - 1$. Because $C((\hat{j} - 1)Q + 1) < C((\hat{j} - 1)Q)$, we can conclude that a finite but large t will give lower expected costs. □

It remains to determine the optimal delay and the expected optimal costs C^*. We obtain

$$C^* = \min_t \left[(1 - Po(Q-1, \lambda t)C((\hat{j}-1)Q) + \sum_{i=0}^{Q-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t} C(\hat{j}Q - i) \right]. \tag{4}$$

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The first term in the parenthesis covers the case when another batch demand is triggered during t, and the second term corresponds to the case with less than Q customer demands during the considered delay time. The one-dimensional optimization of t is easy to carry out by a search procedure. For large values of t we can stop when $Po(Q - 1, \lambda t) < \varepsilon$, where ε is a small positive number. The cost in (4) for a given t is then approaching $C((\hat{j} - 1)Q)$. Although, this has not been proved, the cost function seems to be unimodal. It is not convex, though.

It is obvious that the installation stock policy in Section 2.3 provides an upper bound for C^*, while the optimal costs with the echelon stock policy in Section 2.4 gives a lower bound. The difference between C^* and the echelon stock costs can be interpreted as the value of detailed customer information.

Example: For the same example used before (i.e. $\lambda = 50$, $L = 4$, $h = 1$, $p = 20$, $Q = 200$), the optimal time delay is $t^* = 3.364$. We have $\hat{j} = 2$ and get t^* by carrying out the minimization in (4). The optimal time delay policy works as follows: After the inventory position of the supplier reaches 400, place the next order either after a time delay of 3.364 time units or when the next batch demand from the customer occurs, whichever is earlier. The expected cost of this policy is \$40.09. Note that for this example the cost of the optimal echelon stock policy was \$30.02, and the cost of the optimal installation stock policy was \$118.43. If we think of the installation stock policy as the default no-information policy; then the total cost saving potential in this case is \$88.41 (118.43-30.02). However, with the use of the optimal time delay policy, a saving of \$78.34 (118.43-40.09) is obtained. Therefore with the optimal time delay policy, 88.60% (78.34/88.41) of the potential saving is achieved. The value of information in this particular example is only \$10.07.

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3. Analysis of the model for non-zero set-up costs: Ordering larger batches

So far we have assumed that the considered supplier has no ordering or set-up costs. Because all customer orders are for batches of size Q and no partial deliveries are allowed, it is then obvious that the supplier will also order in batches of size Q. However, if there is an ordering or set-up cost it may be more economical to order larger batches that are integer multiples of Q. Because partial deliveries are not allowed, we do not need to consider other batch sizes. In this section we shall therefore consider a batch size nQ where n is a positive integer. Let

$C_n^* =$ expected optimal holding and backorder costs per unit of time for batch size nQ.

Consider a supplier batch of size nQ that is ordered at some time. Assume that the sub-batch of size Q that will be demanded first by the customer is triggered by the k-th Poisson demand at the customer. Clearly, the second sub-batch will be triggered by the $(k + Q)$-th Poisson demand at the customer etc. Consequently, we obtain the corresponding expected costs per unit of time as

$$C_n(k) = \frac{1}{n}\sum_{i=0}^{n-1} C(k + iQ). \qquad (5)$$

Obviously, $C_n(k)$ is a convex function of k. Furthermore, the optimal $k^*(n)$ is less than or equal to S^*. For $n = 1$ we have $k^*(1) = S^*$. $C_1(S^*)$ is evidently also a lower bound for C_n^*. Also, unlike the case for $n = 1$, k^* can take negative values, and this needs to be taken into account in the algorithm for determining the optimal k^*. (If $S \leq 0$, $C(S)$ in (3) degenerates to $C(S) = -p(S - \lambda L)$.) It is evident that $k^*(n)$ is non-increasing in n.

We shall now provide a simple recursive algorithm for determination of $k^*(n)$. Let us first prove the following simple proposition.

Proposition 4 $k^*(n) \geq k^*(n-1) - Q$ for $n > 1$.

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Proof Note that we can express $C_n(k)$ as

$$C_n(k) = C(k)\frac{1}{n} + C_{n-1}(k+Q)\frac{(n-1)}{n}.$$

We know that the second term is minimized for $k + Q = k^*(n-1)$, i.e. for $k = k^*(n-1) - Q$. The first term is minimized for $k^*(1) = S^* \geq k^*(n-1)$. So we can conclude that both terms are non-increasing with k for $k \leq k^*(n-1) - Q$. This proves the proposition. □

Using Proposition 4 we can determine $k^*(n)$ by the following simple procedure. Assume that we know $k^*(n-1)$. Recall that $k^*(n) \leq k^*(n-1)$ and that $k^*(1) = S^*$. We evaluate $k^*(n-1)$, $k^*(n-1)-1$, $k^*(n-1)-2$, ... as potential values of $k^*(n)$ and stop when we reach a local minimum. Due to the convexity we then have the optimal $k^*(n)$. Furthermore, due to Proposition 4 the local minimum will at the latest occur at $k^*(n-1) - Q$, so we never need to consider more than $Q+1$ values for each unit increase in n.

Let us now consider an installation stock policy like in Section 2.3. Again we can without lack of generality assume that the reorder point R^i is a multiple of Q, i.e., $R^i = jQ$. In complete analogy with Proposition 1, we will reach the lower bound if $k^*(n)$ is a multiple of Q. The corresponding optimal installation stock reorder point is then $R^i = k^*(n) - Q$. Consider then the case where $k^*(n)$ is not a multiple of Q. Let $\hat{j}$ be the smallest integer such that $k^*(n) < \hat{j}\,Q$. The convexity implies that it is optimal to use either $j = \hat{j}-1$ or $j = \hat{j}$, and the resulting costs are obtained as $C_n^i = \min\{C_n((\hat{j}-1)Q), C_n(\hat{j}\,Q)\}$.

Next we turn to an echelon stock policy like in Section 2.4. We can always obtain the lower bound $C_n(k^*(n))$. If $k^*(n)$ is a multiple of Q, this is obviously the optimal installation stock as well as echelon stock policy (i.e. $R^i = k^*(n) - Q$, and $R^e = k^*(n)$). Otherwise we can in analogy with Section 2.4 assume that $k^*(n) = jQ + x$ where $0 < x < Q$ and j is an integer. Let the echelon

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stock reorder point at the supplier be $R^e = jQ + x$. Assume that the supplier has just ordered and that the inventory positions just after the order are x at the customer, respectively $IP^i = (j+n)Q$ and $IP^e = (j+n)Q + x$ at the supplier. The first sub-batch of the larger batch that has just been ordered will then evidently be demanded at customer demand $x + jQ$. The expected costs per unit of time for this policy is therefore $C_n(k^*(n))$.

Finally, we consider the case corresponding to Section 2.5 where $(\hat{j}-1)Q < k^*(n) < \hat{j}Q$. Assume that the supplier wants to order a large batch so that the j-th forthcoming demand for a batch of size Q will be a demand for the first sub-batch. The optimal policy is then to use $j = \hat{j}$ and a time delay t. The optimal delay and the corresponding costs can be determined in complete analogy with (4)

$$C_n^* = \min_t \left[(1 - Po(Q-1, \lambda t) C_n((\hat{j}-1)Q) + \sum_{i=0}^{Q-1} \frac{(\lambda t)^i}{i!} e^{-\lambda t} C_n(\hat{j}Q - i) \right]. \tag{6}$$

If another batch is triggered during t, $j = \hat{j}$ is replaced by $j = \hat{j}-1$ and there should be no additional time delay. This case is covered by the first term in (6). The second term covers the case when no additional customer order is triggered during t.

Finally we determine the optimal time delayed installation stock policy for a given ordering or set-up cost. Let

A = ordering or set-up cost,

C_{tot} = expected total cost rate per unit of time.

We then have

$$C_{tot} = \frac{A\lambda}{nQ} + C_n^*. \tag{7}$$

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We note that the first term in (7) is decreasing with n and that C_n^* obviously is increasing with n. Furthermore, C_n^* is unbounded. This is the case because $C(S) = -p(S - \lambda L)$ for $S < 0$, and because $C(S)$ is asymptotically approaching $h(S–\lambda L)$ as $S \rightarrow \infty$. In order to optimize (7) we can therefore simply evaluate $n = 1, 2, ...$ and stop when C_n^* exceeds the lowest total costs so far.

If we want to determine the optimal installation stock policy without a time delay or the optimal echelon stock policy we replace C_n^* in (7) by C_n^i and $C_n(k^*(n))$ respectively. The optimal value of n is not necessarily the same in these three cases.

4. Computational results

To compare the effectiveness of the three policies namely, installation stock policy (no information sharing), echelon stock policy (with information sharing), and installation stock policy with time delay, we carried out a numerical study. In the first instance, we assumed that the set-up cost $A = 0$. The various parameter values used were $\lambda = 10, 30, 50$; $L = 2, 4, 8$; $h = 1$; $p = 5, 10, 20$; and $Q = (\lambda L/4)$, $(\lambda L/2)$ and λL. Later we ran the tests for the same set of problem parameters, with $A = 50, 100, 200$ and 300 respectively.

The computational results for $\lambda = 10$, $h = 1$, and $A = 0$ are reported in Table 2. Similar results for $\lambda = 30$, and $\lambda = 50$ are available from the authors (included as appendix for the benefit of the referees only). For each value of the parameters L, p, and Q, we report the cost of installation stock policy (*CI*), echelon stock policy (*CE*) and cost of the installation stock policy with time delay (*CT*). The total savings potential in dollars (*CI–CE*), as well as in percentage [(*CI–CE*)/*CI*] is also reported. Traditionally and in earlier papers in the literature (under a continuous review framework) the value of information is represented by (*CI–CE*) or in

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percentage as (*CI*–*CE*)/*CI*. Table 2 also reports the value of information in percentage [(*CT*–*CE*)/*CT*] if the time delay policy is used as the base line.

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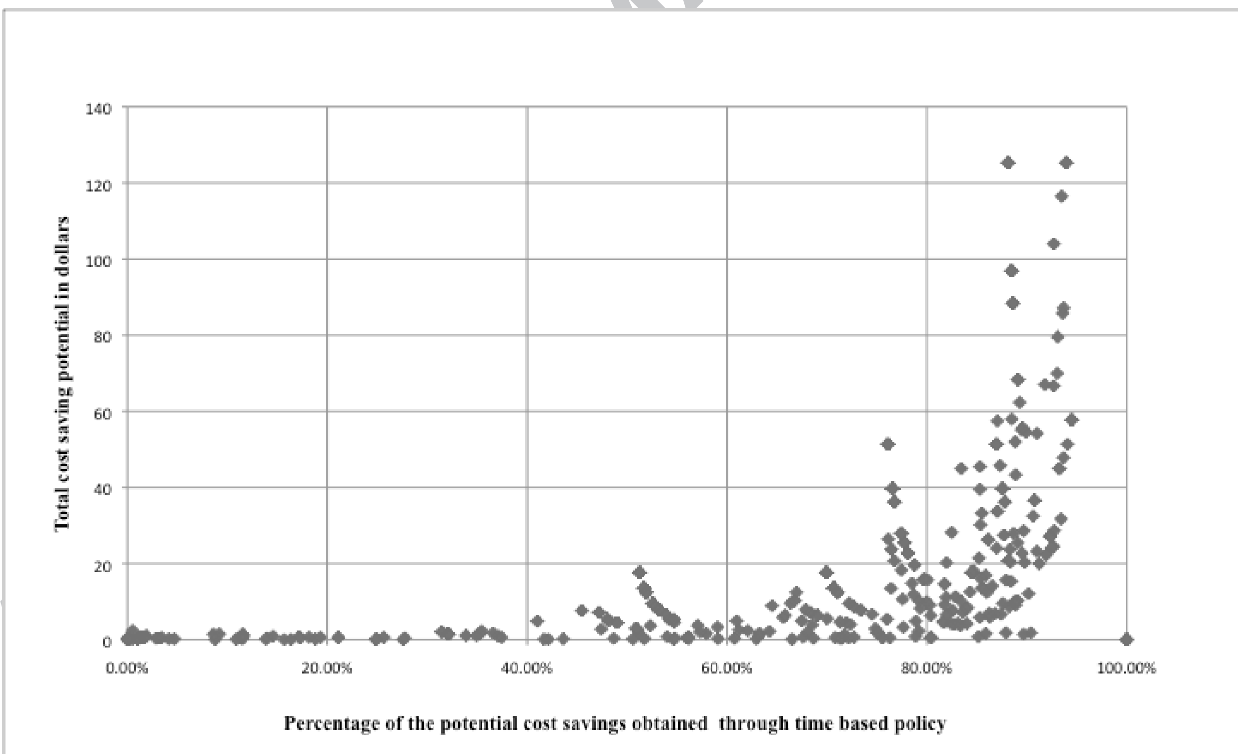
Table 2: Computational results for $\lambda = 10$, $h = 1$, $A = 0$

Lead-time L	Backorder cost p	Batch quantity Q	Cost of installation stock policy CI	Cost of echelon stock policy CE	Cost of installation stock policy with time delay CT	Total cost savings potential in \$ CI-CE	Total cost savings potential in percentage (CI-CE)/CI (Value of information as per traditional definition)	Value of information when base-line is time delay policy (CT-CE)/CT	Percentage of potential savings obtained by time delay policy (CI-CT)/(CI-CE)
2	5	5	6.98	6.93	6.98	0.06	0.85%	0.72%	8.78%
2	5	10	10.19	6.93	7.66	3.27	32.05%	9.53%	77.65%
2	5	20	10.66	6.93	8.53	3.73	35.03%	18.76%	57.07%
2	10	5	8.64	8.41	8.63	0.23	2.71%	2.55%	4.71%
2	10	10	10.35	8.41	9.03	1.95	18.82%	6.87%	68.06%
2	10	20	19.54	8.41	10.42	11.14	56.99%	19.29%	81.93%
2	20	5	10.67	9.85	10.21	0.82	7.69%	3.53%	56.14%
2	20	10	10.67	9.85	10.23	0.82	7.69%	3.71%	54.02%
2	20	20	20.00	9.85	11.96	10.15	50.73%	17.64%	79.23%
4	5	10	11.09	9.70	10.09	1.39	12.56%	3.87%	71.77%
4	5	20	15.11	9.70	11.00	5.41	35.80%	11.82%	76.01%
4	5	40	15.11	9.70	12.15	5.41	35.80%	20.16%	54.66%
4	10	10	12.00	11.78	11.90	0.23	1.89%	1.01%	43.62%
4	10	20	20.03	11.78	13.11	8.26	41.22%	10.14%	83.90%
4	10	40	27.70	11.78	14.99	15.92	57.48%	21.41%	79.78%
4	20	10	13.82	13.72	13.82	0.11	0.76%	0.72%	0.00%
4	20	20	20.07	13.72	14.96	6.35	31.64%	8.29%	80.40%
4	20	40	40.00	13.72	17.36	26.28	65.71%	20.97%	86.13%
8	5	20	20.31	13.64	14.48	6.67	32.83%	5.80%	87.43%
8	5	40	21.39	13.64	15.70	7.74	36.21%	13.12%	73.39%
8	5	80	21.39	13.64	17.27	7.74	36.21%	21.02%	53.23%
8	10	20	20.57	16.47	17.18	4.1	19.91%	4.13%	82.76%
8	10	40	39.21	16.47	18.86	22.74	57.98%	12.67%	89.52%
8	10	80	39.21	16.47	21.45	22.74	57.98%	23.22%	78.10%
8	20	20	21.09	19.18	19.66	1.9	9.02%	2.44%	75.02%
8	20	40	40.00	19.18	21.64	20.82	52.04%	11.37%	88.19%
8	20	80	74.86	19.18	24.99	55.67	74.37%	23.25%	89.58%

In the final column in Table 2, the savings obtained by the installation stock policy with time delay CI–CT is given as a percentage of the total potential savings CI–CE. If this percentage is very high, then it implies that the value of information is quite low as the smart, no-information policy captures most of the potential savings. This percentage seems to be problem dependent and it varies from 0 to 90% in Table 2. This result might appear very inconclusive. However, a closer examination of the results revealed that whenever the percentage of the potential savings obtained by the time delay policy was low, the potential savings (*CI*–CE) itself was very low. This is shown in Figures 1 and 2. In Figure 1, the potential savings in dollars are plotted against the percentage of potential savings obtained by the installation stock policy with time delay. Figure 2 plots the potential savings in percentage against the percentage of potential savings obtained by the installation stock policy with time delay.

FIGURE 1: Cost saving potential in dollars versus percentage of savings obtained through installation stock policy with time delay



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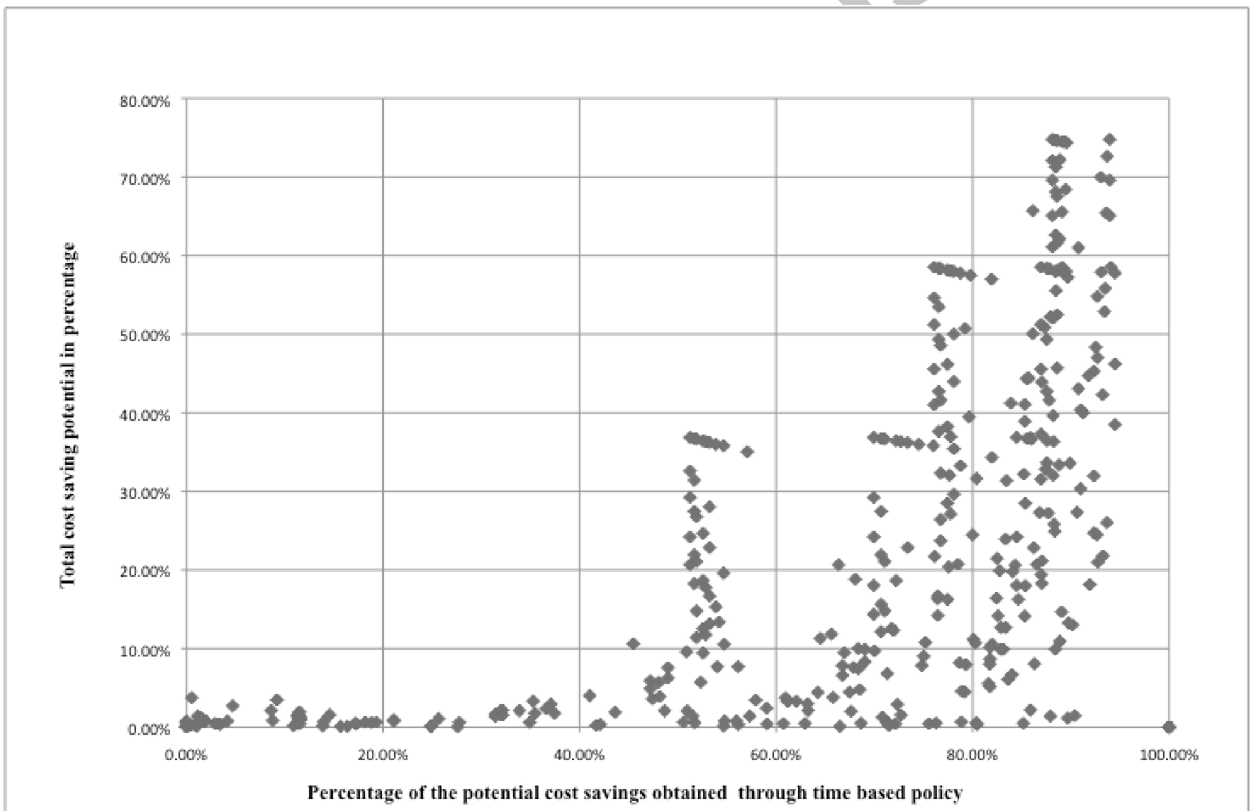
As can be seen from Figure 1 and Figure 2, there are many data points with low values of (*CI*–*CT*)/(*CI*–*CE*), but all these correspond to situations where the potential savings (*CI*–*CE*) itself was low. In other words, whenever the potential savings between *CI* and *CE* was high, the smart no information policy was able to capture most of this. For the problems with $A = 0$, the real value of information given by (*CT*–*CE*)/*CT* is only about 14.6%, whereas based on traditional measures (*CI*–*CE*)/*CI*, this appears to be 54.6%.

FIGURE 2: Cost saving potential in percentage versus percentage of savings obtained through installation stock policy with time delay



We conducted further computational tests for A = 50, 100, 200 and 300. The detailed results for these tests are provided in the online appendix. The set-up costs did not seem to have any discernable relationship with the real value of information, or the percentage of potential

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savings captured by the time delay policy. It was a combination of parameters that influenced this. However, as in the earlier cases, whenever the percentage of potential savings captured by the time delay policy was low, the potential savings itself was low. Figure 1 and Figure 2 actually plots the graph for the entire set of problem data including higher values of *A*. For the entire set of problem data used in the numerical tests, the real value of information given by (*CT*–*CE*)/*CT* is about 4.7%, and the value of information based on traditional measures (*CI*–*CE*)/*CI*, is 23.4%.

5. Summary and conclusions

We have considered the inventory control problem faced by an independent supplier in a continuous review system. The supplier faces batch demands with Erlangian inter-arrival times from a single customer. When the supplier has no information on the customer's inventory position, traditional policies such as (*R*, *Q*) or (*s*, *S*) are optimal only when ordering is restricted to the points in time when the customer demands arrive. We derive and develop an optimal time delay policy for the supplier when no information sharing takes place. We show that if this policy is used, then the value of information sharing is significantly reduced. When there is no information sharing, the supplier can use the time elapsed since the previous demand occurrence as an effective proxy for the inventory position at the customer. Future research could possibly address the problem of determining a corresponding (no information) policy for a supplier with multiple customers.

References

Axsäter, S. 1990. Simple solution procedures for a class of two-echelon inventory problems. *Operations Research*, **38**, 64-69.

Axsäter, S. 2006. *Inventory Control*, Second Edition, Springer, New York.

ACCEPTED MANUSCRIPT

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Axsäter, S. and J. Marklund. 2008. Optimal position-based warehouse ordering in divergent two-echelon inventory systems. *Operations Research,* **56**, 976-991, 1044-1045.

Axsäter, S. and K. Rosling. 1993. Installation vs. echelon stock policies for multilevel inventory control. *Management Science,* 39, 1274-1280.

Cachon, G. P. and M. Fisher. 2000. Supply chain inventory management and the value of shared information. *Management Science*, **46**, 1032-1048.

Chen, F. 1998. Echelon reorder points, installation reorder points, and the value of centralized demand information. *Management Science*, **44,** S221–S234.

Chen, F., Z. Drezner, J. K. Ryan, and D. Simchi-Levi. 2000. Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science*, **46**, 436-443.

Danese, P. 2006. Collaboration forms, information and communication technologies and coordination mechanisms in CPFR. *International Journal of Production Research*, **44**, 3207-3226.

Forrester, J.W. 1961. *Industrial Dynamics*, MIT Press, Cambridge, MA.

Gaur, V., A. Giloni, and S. Seshadri. 2005. Information sharing in a supply chain under ARMA demand. *Management Science*, **51**, 961–969.

Gavirneni, S., R. Kapuscinski, and S. Tayur. 1999. Value of information in capacitated supply chains. *Management Science*, **45**, 16–24.

Katircioglu, K. 1996. *Essays in Inventory Control*. Ph.D. Dissertation, University of British Columbia, Vancouver, British Columbia, Canada.

Larsen, T. S., C. Thernøe, and C. Andresen. 2003. Supply chain collaboration: Theoretical perspective and empirical evidence. *International Journal of Physical Distribution & Logistics Management*, **33**, 531-549.

Lee, H. L., K. C. So, and C. S. Tang. 2000. The value of information sharing in a two-level supply chain. *Management Science*, **46**, 626–643.

Lee, H. L., P. Padmanabhan, and S. Whang. 1997. Information distortion in a supply chain: The bullwhip effect. *Management Science*, **43**, 546-558.

Marklund, J. 2002. Centralized inventory control in a two-level distribution system with Poisson demand. *Naval Research Logistics*, **49**, 798-822.

Moinzadeh, K. 2002. A multi-echelon inventory system with information exchange. *Management Science*, **48**, 414–426.

Moinzadeh, K. and Y. P. Zhou. 2008. Incorporating a delay mechanism in ordering policies into multi-echelon distribution systems. *IIE Transactions*, **40**, 445-458.

Raghunathan, S. 2001. Information sharing in a supply chain: A note on its value when demand is nonstationary. *Management Science* **47**, 605-610.

Scheraga, D. 2002. Disappointment reigns. *Chain Store Age*, August, 83–85.

ACCEPTED MANUSCRIPT

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Schultz, C. R. 1989. Replenishment delays for expensive slow-moving items. *Management Science*, **35**, 1454–1462.

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Appendix (included for the benefit of the referees only)

Table 3: Computational results for $\lambda = 30$, $h = 1$, $A = 0$

Lead-time L	Backorder cost p	Batch quantity Q	Cost of installation stock policy CI	Cost of echelon stock policy CE	Cost of installation stock policy with time delay CT	Total cost savings potential in \$ CI-CE	Total cost savings potential in percentage (CI-CE)/CI (Value of information as per traditional definition)	Value of information when base-line is time delay policy (CT-CE)/CT	Percentage of potential savings obtained by time delay policy (CI-CT)/(CI-CE)
2	5	15	15.58	11.85	12.47	3.73	23.93%	4.97%	83.36%
2	5	30	18.52	11.85	13.55	6.66	35.98%	12.55%	74.52%
2	5	60	18.52	11.85	14.93	6.66	35.98%	20.63%	53.88%
2	10	15	16.07	14.34	14.77	1.73	10.78%	2.91%	75.19%
2	10	30	30.00	14.34	16.23	15.67	52.21%	11.65%	87.93%
2	10	60	33.94	14.34	18.50	19.61	57.76%	22.49%	78.77%
2	20	15	17.04	16.69	16.87	0.36	2.09%	1.07%	48.67%
2	20	30	30.01	16.69	18.59	13.32	44.40%	10.22%	85.72%
2	20	60	60.00	16.69	21.50	43.31	72.19%	22.37%	88.88%
4	5	30	26.20	16.66	17.84	9.54	36.42%	6.61%	87.60%
4	5	60	26.20	16.66	19.31	9.54	36.42%	13.72%	72.23%
4	5	120	26.20	16.66	21.19	9.54	36.42%	21.38%	52.57%
4	10	30	30.16	20.10	21.23	10.06	33.36%	5.32%	88.80%
4	10	60	48.04	20.10	23.27	27.94	58.16%	13.62%	88.67%
4	10	120	48.04	20.10	26.40	27.94	58.16%	23.86%	77.44%
4	20	30	30.31	23.39	24.34	6.92	22.83%	3.90%	86.23%
4	20	60	60.00	23.39	26.77	36.61	61.02%	12.63%	90.77%
4	20	120	91.71	23.39	30.83	68.32	74.50%	24.13%	89.11%
8	5	60	37.07	23.46	25.43	13.61	36.73%	7.75%	85.52%
8	5	120	37.07	23.46	27.45	13.61	36.73%	14.54%	70.69%
8	5	240	37.07	23.46	30.03	13.61	36.73%	21.88%	51.68%
8	10	60	60.00	28.27	30.36	31.73	52.89%	6.88%	93.42%
8	10	120	67.96	28.27	33.21	39.69	58.40%	14.88%	87.55%
8	10	240	67.96	28.27	37.57	39.69	58.40%	24.75%	76.56%
8	20	60	60.01	32.83	34.91	27.18	45.29%	5.96%	92.34%
8	20	120	120.00	32.83	38.33	87.17	72.64%	14.35%	93.69%
8	20	240	129.74	32.83	44.02	96.91	74.69%	25.42%	88.46%

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Appendix (included for the benefit of the referees only)

Table 4: Computational results for $\lambda = 50$, $h = 1$, $A = 0$

Lead-time *L*	Backorder cost *p*	Batch quantity *Q*	Cost of installation stock policy *CI*	Cost of echelon stock policy *CE*	Cost of installation stock policy with time delay *CT*	Total cost savings potential in $ *CI-CE*	Total cost savings potential in percentage (*CI-CE*)/*CI* (Value of information-as per traditional definition)	Value of information when base-line is time delay policy (*CT-CE*)/*CT*	Percentage of potential savings obtained by time delay policy (*CI-CT*)/(*CI-CE*)
2	5	25	23.92	15.23	16.25	8.69	36.34%	6.28%	88.24%
2	5	50	23.92	15.23	17.60	8.69	36.34%	13.47%	72.70%
2	5	100	23.92	15.23	19.32	8.69	36.34%	21.17%	52.83%
2	10	25	25.30	18.40	19.31	6.91	27.30%	4.71%	86.82%
2	10	50	43.85	18.40	21.17	25.45	58.05%	13.08%	89.09%
2	10	100	43.85	18.40	24.05	25.45	58.05%	23.49%	77.77%
2	20	25	25.58	21.38	22.12	4.2	16.41%	3.35%	82.44%
2	20	50	50.00	21.38	24.34	28.62	57.23%	12.16%	89.68%
2	20	100	83.71	21.38	28.06	62.33	74.46%	23.81%	89.29%
4	5	50	33.84	21.44	23.17	12.4	36.65%	7.47%	86.01%
4	5	100	33.84	21.44	25.03	12.4	36.65%	14.34%	71.06%
4	5	200	33.84	21.44	27.40	12.4	36.65%	21.75%	51.90%
4	10	50	50.01	25.84	27.64	24.18	48.34%	6.51%	92.53%
4	10	100	62.04	25.84	30.25	36.2	58.35%	14.58%	87.80%
4	10	200	62.04	25.84	34.25	36.2	58.35%	24.55%	76.76%
4	20	50	50.03	30.02	31.77	20.01	39.99%	5.51%	91.25%
4	20	100	100.00	30.02	34.89	69.98	69.98%	13.96%	93.04%
4	20	200	118.43	30.02	40.10	88.41	74.65%	25.14%	88.60%
8	5	100	47.86	30.22	32.96	17.64	36.86%	8.31%	84.49%
8	5	200	47.86	30.22	35.52	17.64	36.86%	14.92%	69.94%
8	5	400	47.86	30.22	38.82	17.64	36.86%	22.15%	51.25%
8	10	100	87.75	36.38	39.43	51.36	58.54%	7.74%	94.07%
8	10	200	87.75	36.38	43.08	51.36	58.54%	15.55%	86.96%
8	10	400	87.75	36.38	48.66	51.36	58.54%	25.24%	76.09%
8	20	100	100.00	42.23	45.41	57.77	57.77%	7.00%	94.49%
8	20	200	167.52	42.23	49.82	125.29	74.79%	15.23%	93.95%
8	20	400	167.52	42.23	57.11	125.29	74.79%	26.05%	88.12%

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Appendix (included for the benefit of the referees only)

Table 5A: Impact of setup cost on percentage of potential savings obtained by time based installation stock policy

Lambda λ	Lead-time L	h	p	Q	Percentage of potential savings obtained by time based policy $A = 0$	$A = 50$	$A = 100$	$A = 200$	$A = 300$
10	2	1	5	5	8.78%	18.81%	15.69%	0.07%	54.66%
10	2	1	5	10	77.65%	50.59%	72.71%	54.77%	100.00%
10	2	1	5	20	57.07%	1.14%	0.00%	100.00%	59.09%
10	2	1	10	5	4.71%	100.00%	51.72%	10.90%	100.00%
10	2	1	10	10	68.06%	4.16%	100.00%	41.70%	60.75%
10	2	1	10	20	81.93%	50.89%	9.21%	14.57%	0.05%
10	2	1	20	5	56.14%	67.63%	11.51%	100.00%	16.35%
10	2	1	20	10	54.02%	74.84%	57.93%	13.98%	1.05%
10	2	1	20	20	79.23%	79.99%	65.64%	52.33%	47.42%
10	4	1	5	10	71.77%	42.12%	70.82%	55.98%	100.00%
10	4	1	5	20	76.01%	11.50%	100.00%	100.00%	68.64%
10	4	1	5	40	54.66%	54.66%	54.20%	1.58%	1.58%
10	4	1	10	10	43.62%	37.45%	0.52%	24.85%	56.15%
10	4	1	10	20	83.90%	71.96%	35.27%	1.31%	2.88%
10	4	1	10	40	79.78%	79.65%	66.37%	49.00%	49.00%
10	4	1	20	10	0.00%	63.21%	79.20%	51.49%	27.76%
10	4	1	20	20	80.40%	84.08%	82.59%	71.32%	68.51%
10	4	1	20	40	86.13%	86.13%	81.97%	78.52%	76.43%
10	8	1	5	20	87.43%	37.09%	100.00%	71.35%	71.35%
10	8	1	5	40	73.39%	73.39%	54.72%	11.41%	11.41%
10	8	1	5	80	53.23%	53.23%	53.23%	53.23%	53.23%
10	8	1	10	20	82.76%	85.31%	64.25%	21.10%	21.10%
10	8	1	10	40	89.52%	85.88%	77.58%	69.03%	69.03%
10	8	1	10	80	78.10%	78.10%	78.10%	78.10%	78.10%
10	8	1	20	20	75.02%	83.12%	81.74%	86.30%	81.61%
10	8	1	20	40	88.19%	88.19%	88.19%	88.36%	85.37%
10	8	1	20	80	89.58%	89.47%	88.84%	87.32%	85.31%

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Appendix (included for the benefit of the referees only)

Table 5B: Impact of setup cost on percentage of potential savings obtained by time based installation stock policy

Lambda λ	Lead-time L	h	p	Q	Percentage of potential savings obtained by time based policy				
					$A = 0$	$A = 50$	$A = 100$	$A = 200$	$A = 300$
30	2	1	5	15	83.36%	62.94%	85.89%	13.87%	66.51%
30	2	1	5	30	74.52%	11.30%	100.00%	75.56%	89.68%
30	2	1	5	60	53.88%	53.88%	0.53%	1.73%	1.73%
30	2	1	10	15	75.19%	25.66%	100.00%	72.16%	78.87%
30	2	1	10	30	87.93%	70.02%	36.61%	3.20%	0.00%
30	2	1	10	60	78.77%	78.77%	45.48%	48.05%	41.03%
30	2	1	20	15	48.67%	84.01%	63.25%	17.28%	1.04%
30	2	1	20	30	85.72%	86.54%	80.29%	67.49%	50.96%
30	2	1	20	60	88.88%	85.50%	76.76%	77.45%	66.95%
30	4	1	5	30	87.60%	33.88%	100.00%	80.38%	90.40%
30	4	1	5	60	72.23%	72.23%	8.62%	11.55%	11.55%
30	4	1	5	120	52.57%	52.57%	52.57%	52.57%	52.57%
30	4	1	10	30	88.80%	81.99%	61.20%	18.17%	0.00%
30	4	1	10	60	88.67%	85.22%	64.51%	67.88%	48.16%
30	4	1	10	120	77.44%	77.44%	77.44%	77.44%	76.14%
30	4	1	20	30	86.23%	89.07%	90.17%	83.62%	72.35%
30	4	1	20	60	90.77%	90.77%	87.70%	86.98%	80.08%
30	4	1	20	120	89.11%	89.11%	89.11%	87.05%	83.45%
30	8	1	5	60	85.52%	84.33%	32.04%	32.04%	0.00%
30	8	1	5	120	70.69%	70.69%	70.69%	70.69%	70.69%
30	8	1	5	240	51.68%	51.68%	51.68%	51.68%	51.68%
30	8	1	10	60	93.42%	91.00%	82.88%	82.88%	60.97%
30	8	1	10	120	87.55%	87.55%	87.55%	87.55%	82.49%
30	8	1	10	240	76.56%	76.56%	76.56%	76.56%	76.56%
30	8	1	20	60	92.34%	92.34%	92.34%	92.74%	89.77%
30	8	1	20	120	93.69%	93.59%	93.09%	91.80%	89.92%
30	8	1	20	240	88.46%	88.46%	88.46%	88.46%	88.46%

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Appendix (included for the benefit of the referees only)

Table 5C: Impact of setup cost on percentage of potential savings obtained by time based installation stock policy

Lambda λ	Lead-time L	h	p	Q	Percentage of potential savings obtained by time based policy				
					$A = 0$	$A = 50$	$A = 100$	$A = 200$	$A = 300$
50	2	1	5	25	88.24%	100.00%	76.29%	87.92%	24.93%
50	2	1	5	50	72.70%	11.43%	11.43%	100.00%	80.46%
50	2	1	5	100	52.83%	52.83%	52.83%	1.89%	1.89%
50	2	1	10	25	86.82%	62.07%	19.32%	27.60%	71.50%
50	2	1	10	50	89.09%	68.36%	68.36%	35.45%	3.45%
50	2	1	10	100	77.77%	77.77%	77.77%	47.21%	47.21%
50	2	1	20	25	82.44%	88.88%	78.90%	57.32%	34.93%
50	2	1	20	50	89.68%	88.29%	84.66%	78.68%	65.81%
50	2	1	20	100	89.29%	88.49%	85.32%	76.46%	76.46%
50	4	1	5	50	86.01%	32.15%	32.15%	100.00%	85.16%
50	4	1	5	100	71.06%	71.06%	71.06%	11.54%	11.54%
50	4	1	5	200	51.90%	51.90%	51.90%	51.90%	51.90%
50	4	1	10	50	92.53%	83.37%	79.33%	59.04%	17.18%
50	4	1	10	100	87.80%	87.80%	85.37%	66.76%	66.76%
50	4	1	10	200	76.76%	76.76%	76.76%	76.76%	76.76%
50	4	1	20	50	91.25%	92.66%	91.93%	88.42%	81.76%
50	4	1	20	100	93.04%	92.69%	91.01%	87.05%	87.05%
50	4	1	20	200	88.60%	88.60%	88.60%	88.60%	88.60%
50	8	1	5	100	84.49%	84.49%	84.49%	31.45%	31.45%
50	8	1	5	200	69.94%	69.94%	69.94%	69.94%	69.94%
50	8	1	5	400	51.25%	51.25%	51.25%	51.25%	51.25%
50	8	1	10	100	94.07%	93.23%	90.63%	81.77%	81.77%
50	8	1	10	200	86.96%	86.96%	86.96%	86.96%	86.96%
50	8	1	10	400	76.09%	76.09%	76.09%	76.09%	76.09%
50	8	1	20	100	94.49%	94.49%	94.49%	93.67%	93.26%
50	8	1	20	200	93.95%	93.95%	93.95%	93.49%	92.71%
50	8	1	20	400	88.12%	88.12%	88.12%	88.12%	88.12%

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Research highlights EJOR D11-00226

- We consider the inventory control problem for an independent supplier in a continuous review inventory system
- The supplier has a single batch-ordering customer
- The paper develops an optimal time delay policy for the supplier
- We evaluate the value of information about the customer's inventory level

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