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Reliability Assessment of Damaged RC Moment-Resisting Frame against Progressive Collapse under Dynamic Loading Conditions

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Abstract: Reinforced concrete (RC) structures when subjected to sudden destruction of a column by blast pressure, experience dynamic effects in its response. Hence, reliability assessment of the damaged ductile frame against progressive collapse under dynamic loading conditions is conducted in this research. This paper aims at establishing three performance functions, two of which will consider structural collapse due to lack of strength and deformation capacity respectively while the third will incorporate the shear response of structural components for the weakest collapse mechanism. Since any of the performance functions may lead to structural collapse, a global performance function of the damaged structure is developed by considering the minimum of the functions. The results are then used in conjunction with Monte Carlo Simulation (MCS) to estimate the reliability of the damaged structure. A numerical example of a four storey RC frame is presented to address the applicability of the proposed approach and the effects of different structural parameters on its reliability against progressive collapse after the sudden column loss are investigated thereafter.

Key words: reliability assessment, progressive collapse, dynamic loading, performance function.

1. INTRODUCTION

Over past 20 years, accidental or deliberate explosive incidents have revealed the vulnerability of structures to the short durational high intensity dynamic loading caused by detonations from short standoff distances. If the structure is not adequately robust and resilient, progressive collapse is triggered by the blast loading resulting in significant loss of lives. In spite of low probability of such occurrences, considering their damage potential, this has become one of the major concerns of current research. Significant research delving into prevention of progressive collapse has been conducted till date of which Ellingwood's seminal works (1978) laid foundation for the modern studies into progressive collapse. Later, Pretlove et al. (1991)

discussed the influence of dynamic effects of a sudden member failure on the progressive failure of a tension structure. Morris (1993), Malla and Nalluri (1995) studied the dynamic response of truss-type structures after a member failure. Kaewkulchai and Williamson (2003, 2004) developed a beam element formulation and solution procedure for dynamic progressive collapse analysis of planar frames to address the significance of dynamic load redistribution after failure of one or more elements. Hakuno and Meguro (1993) performed collapse simulation of concrete frames by horizontal seismic oscillation using extended distinct-element method (EDEM), modified version of distinct-element method (DEM) proposed by Cundall and Strack (1979). Buscemi and Marjanishvili (2005) developed a

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single-degree-of-freedom (SDOF) model to determine structural potential to progressive collapse using dynamic analysis.

However, majority of the previous studies address this issue in a deterministic manner while random variations of parameters like applied loads, construction material properties and structural geometries are inevitable and have significant influence on structural safety. Therefore, structural reliability assessment based on a probabilistic theory should be more sensible to evaluate structural safety. Three performance functions for the damaged structure are proposed in this paper which can reflect the structural failure due to lack of strength or deformation capacity and structural shear response and calculate their occurrence possibility. Reliability of the damaged structure is estimated based on the global performance function using Monte Carlo Simulation method and a numerical example of a four storey reinforced concrete (RC) frame structure is deduced thereafter.

2. STRUCTURAL MODELING

There are two mass matrices that can be used to represent the continuous distribution of mass over the element length in structural dynamic response analysis: Consistent Mass Matrix which depends on the shape function of the element to generate the stiffness matrix and Lumped Mass Matrix which is generated by concentrating the continuous mass of each element on its ends, with about half of the total element mass on each end. Cook *et al.* (1989) asserted that the consistent mass matrix usually yields precise results in Eigen value problems while the lumped mass matrix is simpler and suitable for inelastic dynamic analysis of structures. Hence, the latter has been widely employed by numerous finite element analysis softwares to simulate the structural inelastic dynamic responses. With the lumped mass matrix, the structural model is schematically plotted in Figure 1 where $m_{i,j}$ indicates the lumped mass concentrated on the j^{th} node of the i^{th} storey and consists of halves of the total mass of the left and the right connected beams ($m_{i,j}^{bl}$ and $m_{i,j}^{br}$) and halves of the total mass of the upper and the lower connected columns ($m_{i,j}^{cu}$ and $m_{i,j}^{cl}$). Therefore, $m_{i,j}$ is expressed as:

$$m_{i,j} = \frac{1}{2} \left(m_{i,j}^{bl} + m_{i,j}^{br} + m_{i,j}^{cu} + m_{i,j}^{cl} \right) \quad (1)$$

The remaining structure will experience dynamic response when subjected to sudden loss of a column due to close-in detonation. For reliability assessment, the structural model of the damaged structure against

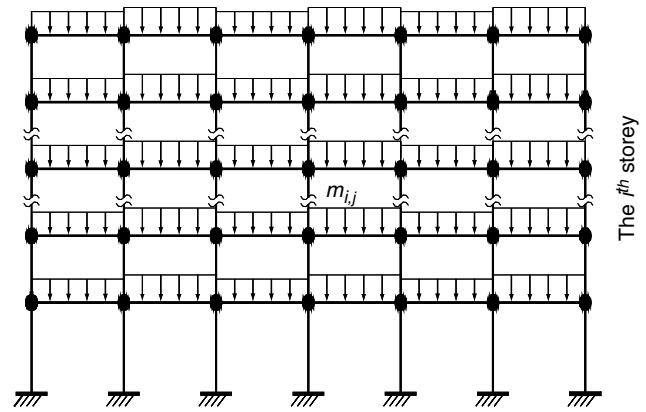


Figure 1. Structural model for dynamic analysis

dynamic progressive collapse is developed based on following assumptions:

- Dynamic response of the damaged structure induced by initial column loss can be calculated in three steps: i) Removal of the failed column from the structural model, ii) Application of service loadings in spans other than the damaged spans statically and iii) Application of service loadings in the damaged spans as rectangular dynamic forces, as substantiated by Kaewkulchai (2003).
- Dynamic response of the damaged structure can be of two types: global structural dynamic response (GSDR) and local dynamic vibrations of structural elements (LDVOE). GSDR, whose natural period is generally in the order of seconds, is characterized by large vertical deformations of the damaged span. Excessive GSDR will directly lead to progressive collapse of the damaged structure. Since LDVOE induced by the changes of elemental internal force, is usually of small amplitude and has higher frequency than GSDR, LDVOE is not considered in this research.
- Based on the load-displacement curve shown in Figure 2, it can be well perceived that an elastic, perfectly-plastic model for the plastic hinges have been adopted in the analysis. However, only planar frames are considered for analysis with an assumption of connection failure to be based on their rotational capacity. Moreover, geometrically nonlinear effects like arching action and catenary action are neglected here.
- Effect of damping on structural response is quite significant for a damaged structure against progressive collapse. During progressive collapse, large plastic deformation occurs and the structural stiffness keeps changing with increasing plastic deformation. Thus, 5% critical

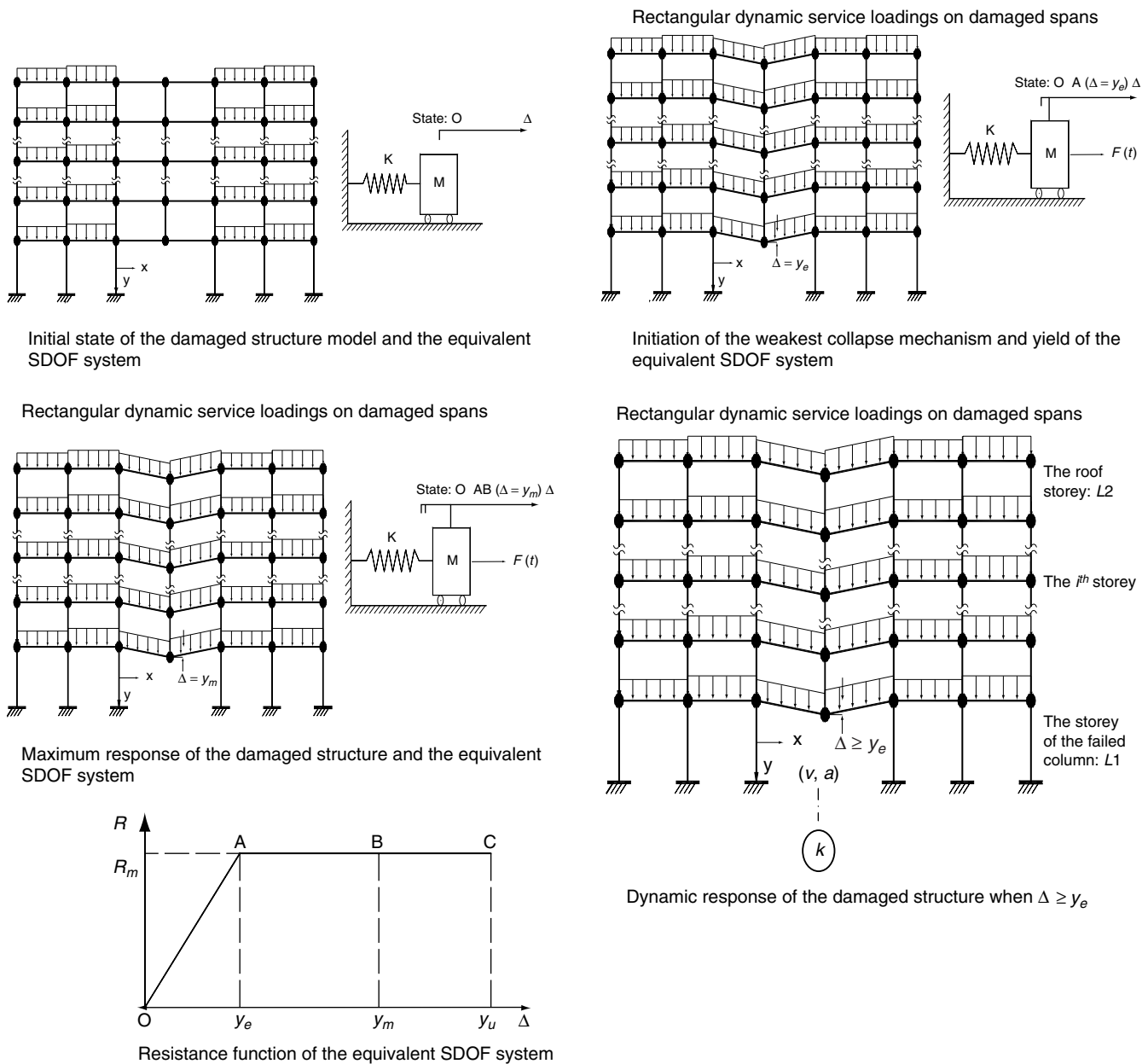


Figure 2. Damaged structure model and the equivalent SDOF system

damping may not be applicable here and more experimental study is required for determining its reasonable magnitude. Hence, damping effect is ignored in this research for simplification, resulting in a conservative outcome. Further research will be conducted on determining the reasonable magnitude of damping and its influence on structural response during progressive collapse.

- e) Both excessive flexural and shear responses of the damaged frame are needed to be considered in reliability assessment of ductile reinforced concrete (RC) frame structures. For flexural response, the weakest collapse mechanism

consisting of a number of plastic hinges with sufficient rotational capacity will control the progressive collapse event. The damaged structure can be simplified into an equivalent SDOF system as shown in Figure 2. With the rectangular dynamic service loading acting on the damaged spans, the system will respond from its initial state (State "O" ($\Delta = 0$)), through State "A" ($\Delta = Y_e$) where the weakest collapse mechanism occurs, till the maximum response State "B" ($\Delta = Y_m$) reaches. It is obvious that if State "B" is within the deformation capacity of the weakest collapse mechanism (State "C" ($\Delta = Y_u$)), the structure will be safe from progressive

collapse induced by excessive flexural response. The resistance function of the equivalent SDOF system is plotted in Figure 2 by the lines connecting three points “O”, “A” and “C” where the energy dissipation capacity of the damaged structure is represented by the area under the curve OAC. Here, R_m is the strength of the equivalent SDOF system with a constant magnitude from Y_e to Y_u , representing the displacements corresponding to the initiation of the final plastic hinge of the weakest mechanism and the first exceedance of the rotational deformation capacity of the plastic hinges respectively. Further response will induce a radical drop down to the resistance of the weakest mechanism and the SDOF system. Since the plasticity of the damaged structure will gradually develop from its initial state “O” to “A” before formation of the weakest mechanism, constructing the resistance function of the SDOF system in the above way is conservative. For shear response, it is assumed that brittle shear failure of the damaged frame will occur prematurely if the member shear strength is exceeded by the shear force during the structural dynamic response. To simplify the computation, the maximum member shear force can be evaluated based on the weakest collapse mechanism leading to a conservative result when the frame is designed sufficiently strong to sustain more than approximately twice of the static service loading on the damaged spans. However, as the structural collapse event for ductile frames is primarily induced by lack of strength or deformation capacity of structural weakest collapse mechanism, the above conservative error will be negligible.

The initial state of the damaged structure, in Figure 2, is important in its reliability assessment against progressive collapse. Hence, a prior structural analysis is needed to carry out to record the parameters related to the initial state of the damaged structure, like location of the node above the removed column, beam bending moments in the damaged spans etc. For convenience of subsequent discussions, some definitions have been given at this juncture:

- 1) Three types of critical zones Type-I, Type-II or Type-III indicated by superscripts *I*, *II* or *III* can be observed in Figure 3, where the plastic hinges after formation possibly contribute to the

structural collapse mechanism. The total distance between Type-I and Type-III critical zones is named as damaged span with the spans between Type-I and Type-II critical zones and Type-II and Type-III critical zones named as left damaged span l and right damaged span r , respectively.

- 2) Structural members in a critical zone are designated as Beam-1 $b1$ (left beam), Beam-2 $b2$ (left beam), Column-1 $c1$ (bottom column) and Column-2 $c2$ (top column) as shown in Figure 4. Columns directly connected to critical zones are termed as directly connected columns whereas all others are defined as indirectly connected columns.
- 3) In each critical zone, springs are located at both ends of the structural elements with their initial strength similar to the related members' strength. It may induce error in structural reliability assessment as the real plastic hinges on structural beams may not form at the end sections. In order to distinguish between the real plastic hinge and the plastic hinge represented by the yielded end spring, they are named as RPH and ESPH, respectively.
- 4) Strength of the real structural members and the end springs are denoted by M_{ur} and M_u , respectively. If bottom fiber (for beam) or left fiber (for column) of a structural member is in tension, related strength is considered as positive strength and indicated by M_{+ur} or M_{+u} . On the other hand, M_{-ur} and M_{-u} are used to represent negative strength. Plastic hinges corresponding to M_{+ur} , M_{-ur} , M_{+u} and M_{-u} are termed as positive RPH, negative RPH, positive ESPH and negative ESPH, respectively.

3. PERFORMANCE FUNCTIONS

3.1. Construction of Performance Functions

Three performance functions based on structural flexural and shear responses are constructed for the reliability assessment. As the basic requirement for the damaged structure to be safe from progressive collapse, its weakest collapse mechanism must provide sufficient strength to support the service loadings. Considering the occurrence possibility of the structural failure event P_f as a measurement, structural reliability can be evaluated based on the virtual work principle:

$$P_f = P(z < 0) \quad (2)$$

where

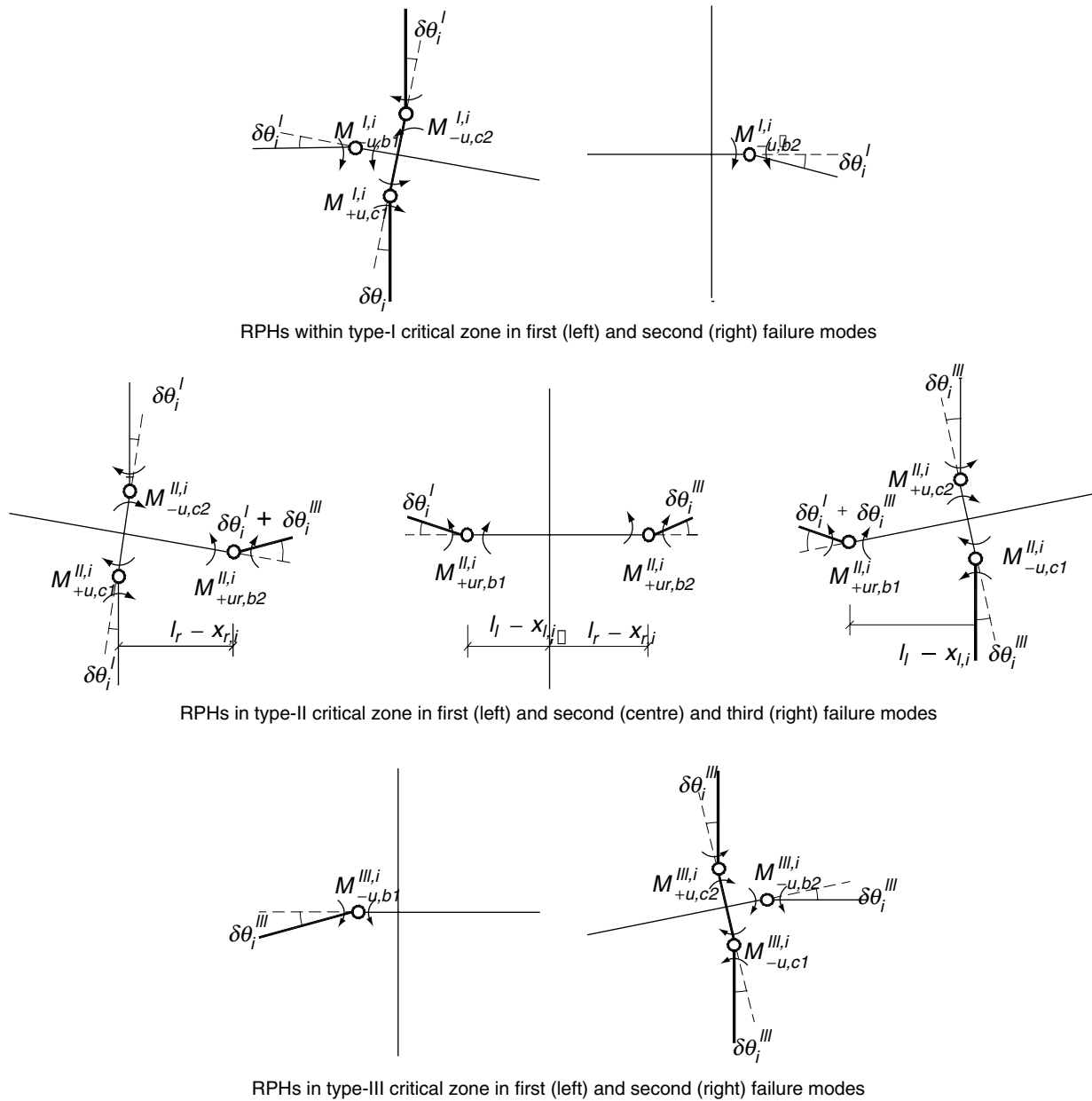


Figure 4. Rotations of the RPHs under $\delta\Delta = 1$

$$z = \delta W_{\text{int}} - \delta W_{\text{ext}} \quad (3)$$

Eqn 3 is structural performance function, where δW_{int} and δW_{ext} indicate the internal and the external virtual work done. For $z < 0$, the structural failure event occur, otherwise the structure remains safe. In addition, $z = 0$ indicates that the structure reaches its ultimate state. The critical collapse mechanism criterion states that the damaged frame structure will fail if its weakest collapse mechanism, which produces the minimum value of the performance function with a given virtual displacement $\delta\Delta$, cannot pledge the safety of the structure. Mathematically, the first performance function z_1 can be expressed as:

$$z_1 = \min(\delta W_{\text{int}} - \delta W_{\text{ext}}) = \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \quad (4)$$

To prevent progressive collapse, the damaged frame must satisfy the second condition i.e. the deformation capacity of the weakest collapse mechanism y_u needs to be adequate to sustain the maximum dynamic response y_m under the rectangular dynamic force as shown in Figure 2. If expressed in terms of energy, it indicates that the energy dissipation capacity of the mechanism ($W_{\text{int},u}$) shall be greater than the external work ($W_{\text{ext},u}$) done by the service loadings on the damaged spans at the deformation level of y_u . Therefore the second performance function z_2 is:

$$z_2 = W_{\text{int},u} - W_{\text{ext},u} \quad (5)$$

Obviously, for $z_2 > 0$, the structural response will rebound before y_u and the structure will be safe against the collapse induced by large deformation whereas for $z_2 < 0$, y_m will exceed y_u and structural collapse occurs. In addition, the structure should not be subjected to the brittle shear failure. Thus, the third performance function z_3 is expressed by:

$$z_3 = \min\left(\left\{V_{r,k}\right\} - \left\{V_{f,k}\right\}\right) k = 1 \cdots nc \quad (6)$$

where $\{V_{r,k}\}$ indicates the total shear strength of the end sections of all members; $\{V_{f,k}\}$ is the total shear force calculated based on structural weakest mechanism; nc denotes the number of cross sections.

Since any of the three performance functions being less than zero can lead to structure collapse, a global performance function z for the damaged structure is employed, which considers the minimum value among z_1 , z_2 and z_3 :

$$z = \min(z_1, z_2, z_3) \quad (7)$$

It can be seen that the performance functions (z_1 , z_2 and z_3) involved in the calculation of z depend on structural weakest collapse mechanism. Hence, identification of this mechanism under dynamic loading conditions is important for structural reliability assessment.

3.2. Structural Weakest Collapse Mechanism and Solution of the First Performance Function

With the column removed from the structural model and the service loadings instantaneously applied on the damaged spans, the dynamic response will be evoked to the damaged structure from its initial state. The equation of motion can be written as:

$$F_I + F_R = F_E \quad (8)$$

where F_I , F_R and F_E denote the inertia force, the restoring force and the external force respectively. The equation indicates that the structure will maintain balance if an additional force equal to F_I is applied in the opposite direction of the structure. Thus, given a virtual deformation, Eqn 8 can be expressed based on virtual work principle in another form as:

$$\delta W_I + \delta W_R = \delta W_E \quad (9)$$

where δW_I , δW_R and δW_E represent the virtual works by

F_I , F_R and F_E under given virtual deformation respectively. With the response of the damaged structure exceeding State "A" in Figure 2 ($\Delta \leq y_e$), structural weakest collapse mechanism is formed. For further structural deformation, its dynamic response has been schematically plotted in Figure 2 where v and a are the velocity and the acceleration of the global response of the structure at the axis k of the failed column. Within this structural response stage where Δ changes from y_e to y_m , the virtual works δW_R and δW_E in Eqn 9 become the internal virtual work by the plastic hinges of the weakest mechanism and the external virtual work by the service loadings within the damaged spans under $\delta\Delta$, which are denoted by (δW_{int}) and δW_{ext} respectively. In addition, δW_I can be calculated as:

$$\begin{aligned} \delta W_I &= \delta\Delta \times \sum_{i=L1}^{L2} (m_{i,k} a) = \delta\Delta \times m_{L1,k} \\ &\times a \sum_{i=L1}^{L2} (m_{i,k} / m_{L1,k}) \end{aligned} \quad (10)$$

where $m_{i,k}$ indicates the concentrated nodal lumped mass on axis k and i^{th} storey. Substituting Eqn 10 into Eqn 9 and replacing δW_R and δW_E by $\min(\delta W_{\text{int}})$ and δW_{ext} produces:

$$\begin{aligned} &\delta\Delta \times m_{L1,k} \times (-a) \sum_{i=L1}^{L2} (m_{i,k} / m_{L1,k}) \\ &= \min(\delta W_{\text{int}}) - \delta W_{\text{ext}} \end{aligned} \quad (11)$$

3.3. Solution of the Second Performance Function

The second performance function in Eqn 5 is constructed based on the relationship between the energy dissipation capacity of the weakest collapse mechanism $W_{\text{int},u}$ and the external work $W_{\text{ext},u}$ by the service loadings on the damaged span at the response level y_u . This performance function is presented to assess whether the displacement capacity of the weakest collapse mechanism y_u can accommodate the maximum dynamic response y_m . The flexural response of the damaged structure can be simplified into an equivalent SDOF system by the area under its resistance function curve shown in Figure 2 to conservatively represent $W_{\text{int},u}$. Accordingly, Eqn 5 can be rewritten as:

$$z_2 = W_{\text{int},u} - W_{\text{ext},u} = R_m y_u - \frac{1}{2} R_m y_e - W_{\text{ext},u} \quad (12)$$

where R_m , y_e and y_u denote strength, yielding displacement and displacement capacity of the SDOF system, respectively. It is obvious that R_m , y_e and y_u depend on the weakest collapse mechanism of the damaged structure. However, the weakest collapse mechanism is idealized by fixing all of the plastic hinges at the end sections of structural members and using the reduced strength $M_{+ur,b1}^{II,i}$ or $M_{+ur,b2}^{II,i}$ for the end springs on beams within Type-II critical zones to compensate the probable induced error. In order to distinguish the idealized weakest mechanism and the real weakest collapse mechanism, they are denoted by IWCM and RWCM, respectively. In addition, the real plastic hinge in structural RWCM and the plastic hinge represented by the yielded spring at the end section of a structural member are named RPH and ESPH respectively. In fact, RWCM can be deduced from IWCM by moving the ESPHs on beams within Type-II critical zones (if applicable) to the locations of the corresponding RPHs and applying the flexural strengths of real structural members $M_{+ur,b1}^{II,i}$ or $M_{+ur,b2}^{II,i}$ to these hinges.

For reasonable evaluation of z_2 , parameters R_m , y_e , y_u and $W_{ext,u}$ in Eqn 12 should be calculated based on RWCM. The strength R_m of the SDOF system is equal to the energy dissipated by the RPHs of RWCM under unit displacement $\delta\Delta = 1$.

$$R_m = \sum_{i=L1}^{L2} \delta W_{int,i}^{I,real} + \sum_{i=L1}^{L2} \delta W_{int,i}^{II,real} + \sum_{i=L1}^{L2} \delta W_{int,i}^{III,real} \quad (13)$$

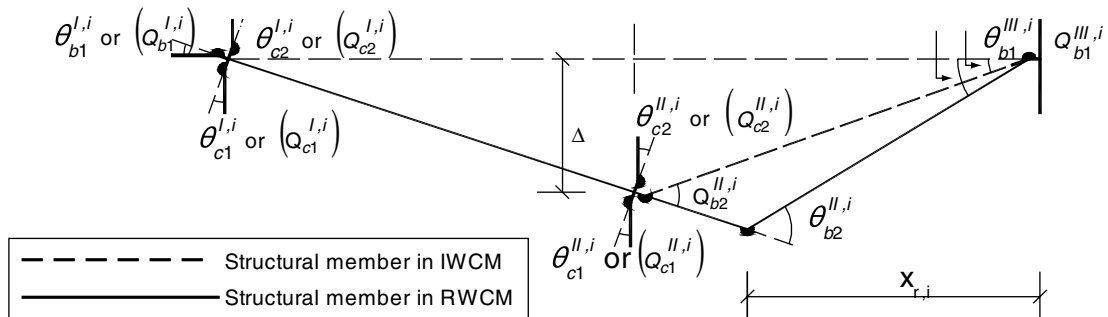


Figure 5. Rotation of the ESPHs and RPHs on the beams in the i^{th} Storey

where $\delta W_{int,i}^{I \sim III,real}$ is the internal energy provided by the RPHs within Type-I ~ Type-III critical zones in the i^{th} storey. Deformation of the damaged structure and the circled part in the i^{th} storey under $\delta\Delta = 1$ as well as the corresponding rotations of the RPHs within different critical zones are plotted in Figures 4 and 5, respectively. Here, $\delta\theta_i^I$ and $\delta\theta_i^{III}$ are the rotations of the RPHs in Type-I and Type-III critical zones in the i^{th} storey under $\delta\Delta = 1$, respectively. Considering further deformation on the RPHs after formation of RWCM, $\delta\theta_i^I$ and $\delta\theta_i^{III}$ can be determined by $x_{l,i}$, $x_{r,i}$ and the failure modes of Type-II critical zones based on the formulae from Table 1. In addition, rotations of the RPHs in Type-II critical zone are different combinations of $\delta\theta_i^I$ and $\delta\theta_i^{III}$ with respect to the failure modes as demonstrated in Figure 4. Based on $\delta\theta_i^I$ and $\delta\theta_i^{III}$, $\delta W_{int,i}^{I \sim III,real}$ can be calculated using following expressions.

Type-I critical zone:

$$\delta W_{int,i}^{I,real} = \min \left[\left(\frac{M_{-u,b1}^{I,i} + M_{+u,c1}^{I,i} + M_{-u,c2}^{I,i}}{M_{-u,b2}^{I,i}} \right), M_{-u,b2}^{I,i} \right] \times \delta\theta_i^I \quad (14)$$

Type-II critical zone in the first failure mode:

$$\delta W_{int,i}^{II,real} = \left(M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i} \right) \times \delta\theta_i^I + M_{+ur,b2}^{II,i} \times \left(\delta\theta_i^I + \delta\theta_i^{III} \right) \quad (15)$$

Type-II critical zone in the second failure mode:

Table 1. Rotations of RPHs in Type-I and Type-III critical zones ($\delta\theta_i^I$ and $\delta\theta_i^{III}$)

	Failure mode of Type-II critical zone		
	First mode	Second mode	Third mode
$\delta\theta_i^I$	$1/l_l$	$1/x_{l,i}$	$(l_l + l_r - x_{l,i})/(l_l \times x_{l,i})$
$\delta\theta_i^{III}$	$(l_l + l_r - x_{r,i})/(l_l \times x_{r,i})$	$1/x_{r,i}$	$1/l_r$

$$\delta W_{\text{int},i}^{\text{II},\text{real}} = M_{+ur,b1}^{\text{II},i} \times \delta \theta_i^{\text{I}} + M_{+ur,b2}^{\text{II},i} \times \delta \theta_i^{\text{III}} \quad (16)$$

Type-II critical zone in the third failure mode:

$$\begin{aligned} \delta W_{\text{int},i}^{\text{II},\text{real}} &= M_{+ur,b1}^{\text{II},i} \times (\delta \theta_i^{\text{I}} + \delta \theta_i^{\text{III}}) \\ &+ (M_{-u,c1}^{\text{II},i} + M_{+u,c2}^{\text{II},i}) \times \delta \theta_i^{\text{III}} \end{aligned} \quad (17)$$

Type-III critical zone:

$$\begin{aligned} \delta W_{\text{int},i}^{\text{III},\text{real}} &= \min \\ &\left[M_{-u,b1}^{\text{III},i}, \left(M_{-u,b2}^{\text{III},i} + M_{-u,c1}^{\text{III},i} + M_{+u,c2}^{\text{III},i} \right) \right] \times \delta \theta_i^{\text{III}} \end{aligned} \quad (18)$$

Substituting Eqns 14 ~ 18 into Eqn 13, R_m can be obtained. Yield displacement y_e of the SDOF system is considered as the displacement corresponding to initiation of the final RPH of RWCM. If the plastic rotations of the RPHs are denoted with a vector $\theta = \{\theta_j\}$ where $j = 1 \dots np$ (np is the number of the RPHs), θ_j will be function of structural global response Δ and can be symbolized as $\theta_j(\Delta)$. Thus, y_e can be mathematically expressed as:

$$\min \left\{ \theta_j \left(y_e \right) \right\} = \varepsilon \quad j = 1 \dots np \quad \varepsilon \rightarrow 0 \quad (19)$$

where ε indicates an arbitrarily small positive value. As, the solution of Eqn 19 is dependent on $\theta_j(\Delta)$, relationship between the plastic rotations of the ESPHs represented by $Q = \{Q_j\}$ and the structural global response Δ should be established (Huang 2008, 2013). Given a large vertical displacement Δ to the additional supports, the corresponding $Q_j(\Delta)$ is listed in Table 2 where $Q_{[b1,b2,c1,c2]}^{[I,II,III],i}$ are the plastic rotations of the ESPHs at the end sections of Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zone on the i^{th} storey; $Q_{[b1,b2,c1,c2],o}^{[I,II,III],i}$ are the plastic rotations of corresponding ESPHs obtained from the structural analysis and moving the additional vertical support to its initial position. In addition, director coefficients $D_{[b1,b2,c1,c2]}^{[I,II,III],i}$ indicate the direction of the ESPHs of the

IWCM with two available values for $D_{[b1,b2,c1,c2]}^{[I,II,III],i}$: -1 for the negative ESPH and 1 for the positive ESPH. Based on Table 2, it can be observed that Q_j will increase linearly with the increase of Δ after the formation of the IWCM.

To derive the relationship between $\theta_j(\Delta)$ and $Q_j(\Delta)$, an example of the circled part on the i^{th} storey of the

damaged structure shown in Figure 3 is discussed. Assuming that the first failure mode occurs to three critical zones included in the selected part, the ESPHs of IWCM, the RPHs of RWCM and their corresponding plastic rotations under Δ are plotted in Figure 5. Here $x_{r,i}$ denotes the location of the RPH on the beam within Type-II critical zone. Plastic rotations of the RPHs are equal to those of the ESPHs except $\theta_{b2}^{\text{II},i} > Q_{b2}^{\text{II},i}$ and $\theta_{b1}^{\text{III},i} > Q_{b1}^{\text{III},i}$. The relationships

between $\theta_{b2}^{\text{II},i}$, $\theta_{b1}^{\text{III},i}$ and $Q_{b2}^{\text{II},i}$, $Q_{b1}^{\text{III},i}$ are deduced based on Figure 5 and expressed respectively as:

$$\theta_{b2}^{\text{II},i} = Q_{b2}^{\text{II},i} \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) \quad (20)$$

$$\theta_{b1}^{\text{III},i} = Q_{b1}^{\text{III},i} + Q_{b2}^{\text{II},i} \times \frac{l_r - x_{r,i}}{x_{r,i}} \quad (21)$$

Substituting $Q_{b2}^{\text{II},i}(\Delta)$ and $Q_{b1}^{\text{III},i}(\Delta)$ listed in Table 2 into Eqns 20 and 21 produces:

$$\begin{aligned} \theta_{b2}^{\text{II},i} &= D_{b2}^{\text{II},i} \times Q_{b2,o}^{\text{II},i} \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) \\ &+ \Delta \times \left(\frac{1}{l_l} + \frac{1}{l_r} \right) \times \left(1 + \frac{l_r - x_{r,i}}{x_{r,i}} \right) \end{aligned} \quad (22)$$

$$\begin{aligned} \theta_{b1}^{\text{III},i} &= D_{b2}^{\text{II},i} \times Q_{b2,o}^{\text{II},i} \times \left(\frac{l_r - x_{r,i}}{x_{r,i}} \right) \\ &+ D_{b1}^{\text{III},i} \times Q_{b1,o}^{\text{III},i} + \Delta \times \frac{l_r - x_{r,i}}{x_{r,i}} \times \left(\frac{1}{l_l} + \frac{1}{l_r} \right) + \frac{\Delta}{l_r} \end{aligned} \quad (23)$$

$\theta_{b2}^{\text{II},i}(\Delta)$ and $\theta_{b1}^{\text{III},i}(\Delta)$ are linearly increasing functions of Δ . Rotations of other RPHs ($\theta_{b1}^{\text{I},i}, \theta_{c1}^{\text{I},i}, \theta_{c2}^{\text{I},i}, \theta_{c1}^{\text{II},i}, \theta_{c2}^{\text{II},i}$) are equal to the rotations of corresponding ESPHs ($Q_{b1}^{\text{I},i}, Q_{c1}^{\text{I},i}, Q_{c2}^{\text{I},i}, Q_{c1}^{\text{II},i}, Q_{c2}^{\text{II},i}$) as shown in Figure 5 and are linearly related to Δ . Similarly, when other failure modes are identified for the critical zones, relationship between θ_j and Q_j can be deduced and listed in Table 3. By substituting $Q_j(\Delta)$ from Table 2 into the equations in Table 3, $\theta_j(\Delta)$ for all RPHs is obtained with respect to the failure mode of related critical zones. $\theta_j(\Delta)$ is computed for $\Delta = \Delta_1$ and $\Delta = \Delta_2$, two arbitrarily values respectively. Then, displacement $y_{e,j}$ is determined to

Table 2. Rotations of the ESPHs $Q_i(\Delta)$ and sectional moments at both ends of beams in the damaged span
 $(M_{l,i}^{leftend}, M_{l,i}^{rightend}, M_{r,i}^{leftend} \text{ and } M_{r,i}^{rightend})$

Type I	Rotations of the ESPHs in critical zones on the i^{th} storey		
	First failure mode	Second failure mode	Third failure mode
ESPH on Beam-1	$D_{b1}^{I,i} = -1$ $Q_{b1}^{I,i} = D_{b1}^{I,i} \times Q_{b1,o}^{I,i} + \frac{\Delta}{l_l}$		
ESPH on Beam-2		$D_{b2}^{I,i} = -1$ $Q_{b2}^{I,i} = D_{b2}^{I,i} \times Q_{b2,o}^{I,i} + \frac{\Delta}{l_l}$	
ESPH on Column-1	$D_{c1}^{I,i} = 1$ $Q_{c1}^{I,i} = D_{c1}^{I,i} \times Q_{c1,o}^{I,i} + \frac{\Delta}{l_l}$		
ESPH on Column-2	$D_{c2}^{I,i} = -1$ $Q_{c2}^{I,i} = D_{c2}^{I,i} \times Q_{c2,o}^{I,i} + \frac{\Delta}{l_l}$		
$M_{l,i}^{leftend}$	$-M_{-u,b1}^{I,i} - M_{+u,c1}^{I,i} - M_{-u,c2}^{I,i} - M_{-u,b2}^{I,i}$		
Type II	Rotations of the ESPHs in Type II critical zones on the i^{th} storey		
	First failure mode	Second failure mode	Third failure mode
ESPH on Beam-1		$D_{b1}^{II,i} = 1$ $Q_{b1}^{II,i} = D_{b1}^{II,i} \times Q_{b1,o}^{II,i} + \frac{\Delta}{l_l}$	$D_{b1}^{II,i} = 1$ $Q_{b1}^{II,i} = D_{b1}^{II,i} \times Q_{b1,o}^{II,i} + \frac{\Delta}{l_l} + \frac{\Delta}{l_r}$
ESPH on Beam-2	$D_{b2}^{II,i} = 1$ $Q_{b2}^{II,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} + \frac{\Delta}{l_l} + \frac{\Delta}{l_r}$	$D_{b2}^{II,i} = 1$ $Q_{b2}^{II,i} = D_{b2}^{II,i} \times Q_{b2,o}^{II,i} + \frac{\Delta}{l_r}$	

Table 2. (Continued)

Table 2. Rotations of the ESPHs $Q_i(\Delta)$ and sectional moments at both ends of beams in the damaged span

$$\left(M_{l,i}^{leftend}, M_{l,i}^{rightend}, M_{r,i}^{leftend} \text{ and } M_{r,i}^{rightend} \right) \text{ (Continued)}$$

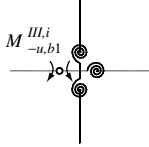
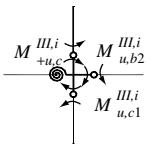
Type II Rotations of the ESPHs in critical zones on the i^{th} storey			
	First failure mode	Second failure mode	Third failure mode
ESPH on Column-1	$D_{c1}^{II,i} = 1$ $Q_{c1}^{II,i} = D_{c1}^{II,i} \times Q_{c1,o}^{II,i} + \frac{\Delta}{l_l}$		$D_{c1}^{II,i} = -1$ $Q_{c1}^{II,i} = D_{c1}^{II,i} \times Q_{c1,o}^{II,i} + \frac{\Delta}{l_r}$
ESPH on Column-2	$D_{c2}^{II,i} = -1$ $Q_{c2}^{II,i} = D_{c2}^{II,i} \times Q_{c2,o}^{II,i} + \frac{\Delta}{l_l}$		$D_{c2}^{II,i} = 1$ $Q_{c2}^{II,i} = D_{c2}^{II,i} \times Q_{c2,o}^{II,i} + \frac{\Delta}{l_r}$
$M_{l,i}^{rightend}$	$M_{+u,b2}^{II,i} + M_{+u,c1}^{II,i} + M_{-u,c2}^{II,i}$	$M_{+u,b1}^{II,i}$	$M_{+u,b1}^{II,i}$
$M_{r,i}^{leftend}$	$M_{+u,b2}^{II,i}$	$M_{+u,b2}^{II,i}$	$M_{+u,b1}^{II,i} + M_{-u,c1}^{II,i} + M_{+u,c2}^{II,i}$
Type III Rotations of the ESPHs in Type-III critical zones on the i^{th} storey			
			
ESPH on Beam-1	$D_{b1}^{III,i} = -1$ $Q_{b1}^{III,i} = D_{b1}^{III,i} \times Q_{b1,o}^{III,i} + \frac{\Delta}{l_r}$		
ESPH on Beam-2		$D_{b2}^{III,i} = -1$ $Q_{b2}^{III,i} = D_{b2}^{III,i} \times Q_{b2,o}^{III,i} + \frac{\Delta}{l_r}$	
ESPH on Column-1		$D_{c1}^{III,i} = -1$ $Q_{c1}^{III,i} = D_{c1}^{III,i} \times Q_{c1,o}^{III,i} + \frac{\Delta}{l_r}$	
ESPH on Column-2		$D_{c2}^{III,i} = 1$ $Q_{c2}^{III,i} = D_{c2}^{III,i} \times Q_{c2,o}^{III,i} + \frac{\Delta}{l_r}$	
$M_{r,i}^{rightend}$	$-M_{-u,b1}^{III,i}$	$-M_{-u,b2}^{III,i} - M_{-u,c1}^{III,i} - M_{+u,c2}^{III,i}$	

Table 3. Relationships between rotations of RPHs and ESPHs*

Type-I critical zone	Type-II critical zone	Type-III critical zone	The relationship between θ_j and Q_j
1 st mode	1 st mode	1 st mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + (l_r - x_{r,i}) / x_{r,i}$
1 st mode	1 st mode	2 nd mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{II,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$, $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{c2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
1 st mode	2 nd mode	1 st mode	$\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$ $\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
1 st mode	2 nd mode	2 nd mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$, $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$
1 st mode	3 rd mode	1 st mode	$\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b1}^{I,i} = Q_{b1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{c1}^{I,i} = Q_{c1}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$
1 st mode	3 rd mode	2 nd mode	$\theta_{c2}^{I,i} = Q_{c2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
2 nd mode	1 st mode	1 st mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
2 nd mode	1 st mode	2 nd mode	$\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c1}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$, $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
2 nd mode	2 nd mode	1 st mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b1}^{III,i} = Q_{b1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
2 nd mode	2 nd mode	2 nd mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$ $\theta_{b2}^{II,i} = Q_{b2}^{II,i} \times [1 + (l_r - x_{r,i}) / x_{r,i}]$, $\theta_{b2}^{III,i} = Q_{b2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$ $\theta_{c2}^{III,i} = Q_{c1}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$, $\theta_{c2}^{III,i} = Q_{c2}^{III,i} + Q_{b2}^{II,i} \times (l_r - x_{r,i}) / x_{r,i}$
2 nd mode	3 rd mode	1 st mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$
2 nd mode	3 rd mode	2 nd mode	$\theta_{b2}^{I,i} = Q_{b2}^{I,i} + Q_{b1}^{II,i} \times (l_l - x_{l,i}) / x_{l,i}$, $\theta_{b1}^{II,i} = Q_{b1}^{II,i} \times [1 + (l_l - x_{l,i}) / x_{l,i}]$

*: Rotation of the RPHs (θ_j) not given in Table 3 is equal to the rotation of the ESPHs (Q_j).

satisfy $\theta_j(y_{e,j})$. Since $\theta_j(\Delta)$ is a linearly increasing function, $y_{e,j}$ can be solved with:

$$y_{e,j} = \Delta_1 - \frac{\theta_j(\Delta_1)}{\theta_j(\Delta_2) - \theta_j(\Delta_1)} \times (\Delta_2 - \Delta_1) \quad (24)$$

The above calculations for all RPHs ($j = 1 \dots np$) are repeated to gain a vector $\{y_{e,j}\}$ and finally, y_e is acquired by finding the maximum element in $\{y_{e,j}\}$, that is $y_e = \max\{y_{e,j}\}$.

The displacement capacity y_u of the SDOF system is defined to be the displacement corresponding to first exceedance of the rotational capacity of the RPHs of the RWCM. The main consideration behind this is the strength of the RWCM will quickly drop as the structural global response Δ surpasses y_u with initiation of structural collapse when Δ reaches y_u . However, considering the complicity of the progressive collapse event of the damaged frame, it can be well perceived that $\Delta = y_u$ can only indicate the initiation of such a disaster. Nevertheless, it is adopted as a criterion here for assessing the possibility of structural progressive collapse since it is simple and conservative. According to the definition, y_u is mathematically expressed as:

$$\min \left\{ \theta_{u,j} - \theta_j(y_u) \right\} = \varepsilon, j = 1 \dots np \quad \varepsilon \rightarrow 0 \quad (25)$$

where $\theta_{u,j}$ is the rotational capacity of the j^{th} RPH [14]. Based on the $\theta_j(\Delta)$ obtained, Eqn 25 can be solved with the same method as that for Eqn 19. The external work $W_{ext,u}$ by the service loadings on the damaged spans at the response level of y_u should be the sum of those on the j^{th} structural storey (i ranging from L_1 to L_2) as:

$$W_{ext,u} = \sum_{i=L_1}^{L_2} (W_{ext,u}^{l,i} + W_{ext,u}^{r,i}) = \sum_{i=L_1}^{L_2} \left[\int_0^{l_l} q_{l,i}(x) \Delta_{l,i}^u(x) dx + \int_0^{l_r} q_{r,i}(x) \Delta_{r,i}^u(x) dx \right] \quad (26)$$

where $W_{ext,u}^{l,i}$ and $W_{ext,u}^{r,i}$ are the external work by the service loadings on left and right damaged spans, l_l and l_r , respectively; $q_{l,i}(x)$ and $q_{r,i}(x)$ denote the distribution of the service loadings; $\Delta_{l,i}^u(x)$ and $\Delta_{r,i}^u(x)$ represent the beam displacement functions in the damaged spans when the global response (Δ) of the RWCM reaches y_u . Basically, displacement function of any beam in either

left or right damaged span ($\Delta_{l,i}^u(x)$ or $\Delta_{r,i}^u(x)$) is combination of three items, expressed as:

$$\Delta_{l,i}^u(x) = \Delta_{l,i}^{u,1}(x) + \Delta_{l,i}^{u,2}(x) + \Delta_{l,i}^{u,3}(x) \quad (27)$$

$$\Delta_{r,i}^u(x) = \Delta_{r,i}^{u,1}(x) + \Delta_{r,i}^{u,2}(x) + \Delta_{r,i}^{u,3}(x) \quad (28)$$

$\Delta_{l,i}^{u,1}(x)$, $\Delta_{l,i}^{u,2}(x)$ and $\Delta_{l,i}^{u,3}(x)$ as well as

$\Delta_{r,i}^{u,1}(x)$, $\Delta_{r,i}^{u,2}(x)$ and $\Delta_{r,i}^{u,3}(x)$ are plotted in Figure 6.

The first item of the displacement functions $\Delta_{l,i}^{u,1}(x)$ and $\Delta_{r,i}^{u,1}(x)$ indicates the shape of the beams in the respective damaged spans due to elastic deformation under the end section moments $(M_{l,i}^{leftend} - M_{l,i,o}^{leftend}, M_{l,i}^{rightend} - M_{l,i,o}^{rightend})$, $(M_{r,i}^{leftend} - M_{r,i,o}^{leftend}, M_{r,i}^{rightend} - M_{r,i,o}^{rightend})$ and the service loadings $q_{l,i}(x)$, $q_{r,i}(x)$. If beam moment distributions are denoted by $M_{l,i}(x)$ and $M_{r,i}(x)$, and $\Delta_{l,i}^{i,1}(x)$ and $\Delta_{r,i}^{u,1}(x)$ can be obtained from:

$$\Delta_{l,i}^{u,1}(x) = \int \left[\int \frac{-M_{l,i}(x)}{EI} dx \right] dx$$

$$\text{with } \Delta_{l,i}^{u,1}(0) = 0 \quad \text{and} \quad \Delta_{l,i}^{u,1}(l_l) = 0 \quad (29)$$

$$\Delta_{r,i}^{u,1}(x) = \int \left[\int \frac{-M_{r,i}(x)}{EI} dx \right] dx$$

$$\text{with } \Delta_{r,i}^{u,1}(0) = 0 \quad \text{and} \quad \Delta_{r,i}^{u,1}(l_r) = 0 \quad (30)$$

The second item of the displacement functions $\Delta_{l,i}^{u,2}(x)$ and $\Delta_{r,i}^{u,2}(x)$ shows rigid rotation of the beams due to the global response of $\Delta = y_u$, which can be written respectively as:

$$\Delta_{l,i}^{u,2}(x) = y_u \times \frac{x}{l_l} \quad (31)$$

$$\Delta_{r,i}^{u,2}(x) = y_u \times \left(1 - \frac{x}{l_r}\right) \quad (32)$$

The third item of $\Delta_{l,i}^{u,3}(x)$ and $\Delta_{r,i}^{u,3}(x)$ is associated with $\theta_{b1}^{II,i}(y_u)$ and $\theta_{b2}^{II,i}(y_u)$, plastic rotations of the RPHs on Beam-1 and Beam-2 in Type-II critical zone at the response level y_u and are deduced as:

$$\begin{cases} \Delta_{l,i}^{u,3}(x) = \frac{x \times (l_l - x_{l,i}) \times \theta_{b1}^{II,i}(y_u)}{l_l} & (\text{for } x = 0 \rightarrow x_{l,i}) \\ \Delta_{l,i}^{u,3}(x) = \frac{x_{l,i} \times (l_l - x_{l,i}) \times \theta_{b1}^{II,i}(y_u)}{l_l} & \\ - \frac{(x - x_{l,i}) \times x_{l,i} \times \theta_{b1}^{II,i}(y_u)}{l_l} & (\text{for } x = x_{l,i} \rightarrow l_l) \end{cases} \quad (33)$$

$$\begin{cases} \Delta_{r,i}^{u,3}(x) = \frac{x \times x_{r,i} \times \theta_{b2}^{II,i}(y_u)}{l_r} & (\text{for } x = 0 \rightarrow l_{r,i} - x_{r,i}) \\ \Delta_{r,i}^{u,3}(x) = (l_r - x_{r,i}) \times \theta_{b2}^{II,i}(y_u) & \\ - \frac{x \times (l_r - x_{r,i}) \times \theta_{b2}^{II,i}(y_u)}{l_r} & (\text{for } x = x_{r,i} \rightarrow l_r) \end{cases} \quad (34)$$

Here $x_{l,i}$ and $x_{r,i}$ represent the location of the RPHs along the beam. $\theta_{b1}^{II,i}(y_u)$ and $\theta_{b2}^{II,i}(y_u)$, can be calculated by substituting $\Delta = y_u$ into the functions in Tables 2 and 3. Substituting Eqns 27 ~ 34 into Eqn 26, the external work $W_{ext,u}$ corresponding to y_u can be obtained. For the frame structure under uniformly distributed service loadings within each span, the external work on the beams in the left and the right damaged span ($W_{ext,u}^{l,i}$ and $W_{ext,u}^{r,i}$) can be calculated by:

$$\begin{aligned} W_{ext,u}^{l,i} &= q_{l,i} \times \left[\frac{q_{l,i} \times l_l^5}{120EI} + \frac{(M_{l,i}^{leftend} + M_{l,i}^{rightend} - M_{l,i,o}^{leftend} - M_{l,i,o}^{rightend}) \times l_l^3}{24EI} \right. \\ &\quad \left. + \frac{y_u \times l_l}{2} + \frac{\theta_{b1}^{II,i}(y_u) \times x_{l,i} \times (l_l - x_{l,i})}{2} \right] \end{aligned} \quad (35)$$

$$\begin{aligned} W_{ext,u}^{r,i} &= q_{r,i} \times \left[\frac{q_{r,i} \times l_r^5}{120EI} + \frac{(M_{r,i}^{leftend} + M_{r,i}^{rightend} - M_{r,i,o}^{leftend} - M_{r,i,o}^{rightend}) \times l_r^3}{24EI} \right. \\ &\quad \left. + \frac{y_u \times l_r}{2} + \frac{\theta_{b2}^{II,i}(y_u) \times x_{r,i} \times (l_r - x_{r,i})}{2} \right] \end{aligned} \quad (36)$$

where $M_{l,i}^{leftend}$, $M_{l,i}^{rightend}$, $M_{r,i}^{leftend}$ and $M_{r,i}^{rightend}$ indicate the beam end section moments in the left and right damaged spans respectively; $M_{l,i,o}^{leftend}$, $M_{l,i,o}^{rightend}$, $M_{r,i,o}^{leftend}$

and $M_{r,i,o}^{rightend}$ are their initial values obtained from the analysis of the initial state of the damaged structure shown in Figure 2. After determining the parameters ($R_m, y_e, y_u, W_{ext,u}$), the second performance function z_2 is calculated by substituting their values in Eqn 12.

3.4. Solution of the Third Performance Function

In order to consider the structural collapse due to brittle shear failure in the reliability assessment, the third performance function needs to be evaluated based on shear strength of structural members and the shear force after formation of the structural weakest collapse mechanism. "Unified Facilities Criteria (UFC) – Design of Buildings to Resist Progressive Collapse" (2009) recommended the shear strength to be calculated based on ACI 318-02 (2002). Thus, $V_{r,k}$ in Eqn 4 can be written as:

$$V_{r,k} = V_{c,k} + V_{s,k} \quad (37)$$

where $V_{r,k}$ is the shear strength of the cross section k ; $V_{c,k}$ denotes the shear resistance provided by concrete which is determined by the cross sectional dimensions, the compressive strength of concrete, the shear span-to-depth ratio and the member axial force. $V_{s,k}$ represents the contribution of the shear reinforcement and can be calculated by:

$$V_{s,k} = \frac{A_{v,k} f_{vy,k} (\sin \alpha_k + \cos \alpha_k) d_k}{s_k} \quad (38)$$

where $A_{v,k}$ and $f_{vy,k}$ indicate the area and the yield strength of the shear reinforcement at cross section k ; d_k

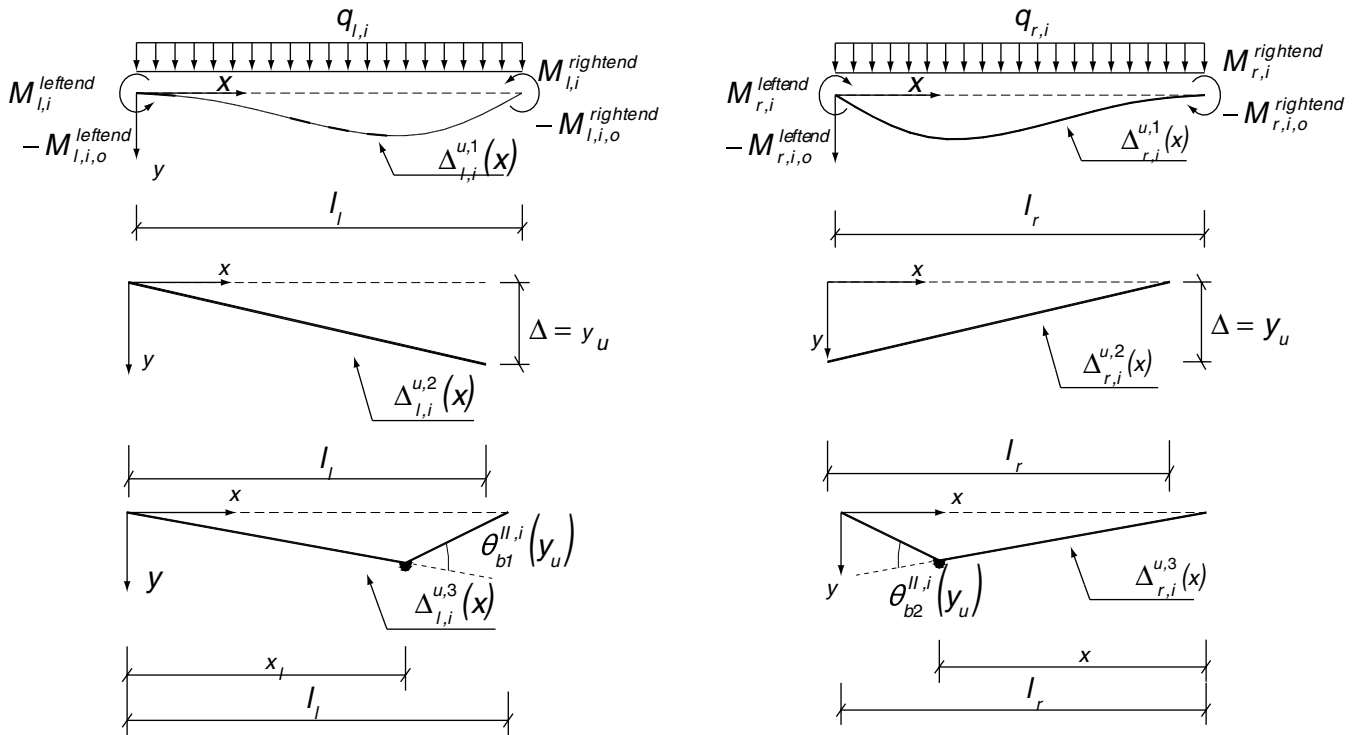


Figure 6. Three items of displacement functions for beams in left and right damaged spans

is the effective depth i.e. the distance from the extreme compression fibre to the centroid of the longitudinal tension bars; s_k is the spacing of the shear reinforcement; α_k is the angle between the shear reinforcement and the member axis. Since it is assumed that the weakest collapse mechanism has occurred to the damaged structure, shear force $V_{f,k}$ in Eqn 4 can be obtained from the structural analysis. Although the structural model is based on IWCM instead of RWCM, it does not result in any additional error in $V_{f,k}$ as the reduced strength of the ESPH on Beam-2 in Type-II critical zone is computed based on the real member moment diagram which further leads to the moment and shear force distributions of the damaged structure with IWCM same as those of the damaged structure with RWCM.

4. NUMERICAL EXAMPLE

A four-storey three-bay RC frame structure presented in Figure 7 and detailed in Table 4 is selected to illustrate the implementation of the approach addressed in the previous sections to assess the reliability of the damaged frame structure against progressive collapse accounting for the dynamic effects of the sudden loss of a column and the structural brittle shear failure. Closed stirrups with area $A_{v,k} = 200\text{mm}^2$, characteristic yield strength

300 MPa are used in all structure members as the shear reinforcement at a spacing of $s_k = 200\text{mm}$. The mean strength of the shear reinforcement is 345 MPa and the characteristic compressive strength of concrete f_{ck} is 30 MPa. The structure is subjected to uniformly distributed service loading with the characteristic values of dead load DL and live load LL as $DL_k = 27\text{kN/m}$ and $LL_k = 30\text{kN/m}$. The structure is assumed to require a High Level of Protection, thus based on “Unified Facilities Criteria (UFC) – Design of Buildings to Resist Progressive Collapse” (2009), the rotational capacities of the real plastic hinges $\theta_{u,j}$ are taken to be 4 degree and both initially failed external and internal columns in close-in blast conditions are considered. With Monte Carlo Simulation (MCS), probabilities of the four performance functions (z_1 , z_2 , z_3 and z) less than zero, $p_f(z_1 < 0)$, $p_f(z_2 < 0)$, $p_f(z_3 < 0)$ and $p_f(z < 0)$, are shown in Figure 8 with respect to different structural samples numbers N_s by Curves 1 ~ 4, respectively. Curves 1 ~ 3 demonstrate the probabilities of structural failure by lack of strength of the weakest collapse mechanism to resist the external service loadings ($z_1 < 0$); lack of deformation capacity of the weakest collapse mechanism to sustain the maximum structural dynamic response ($z_2 < 0$) and lack of shear strength of structural members ($z_3 < 0$) whereas, Curve 4 reflects the

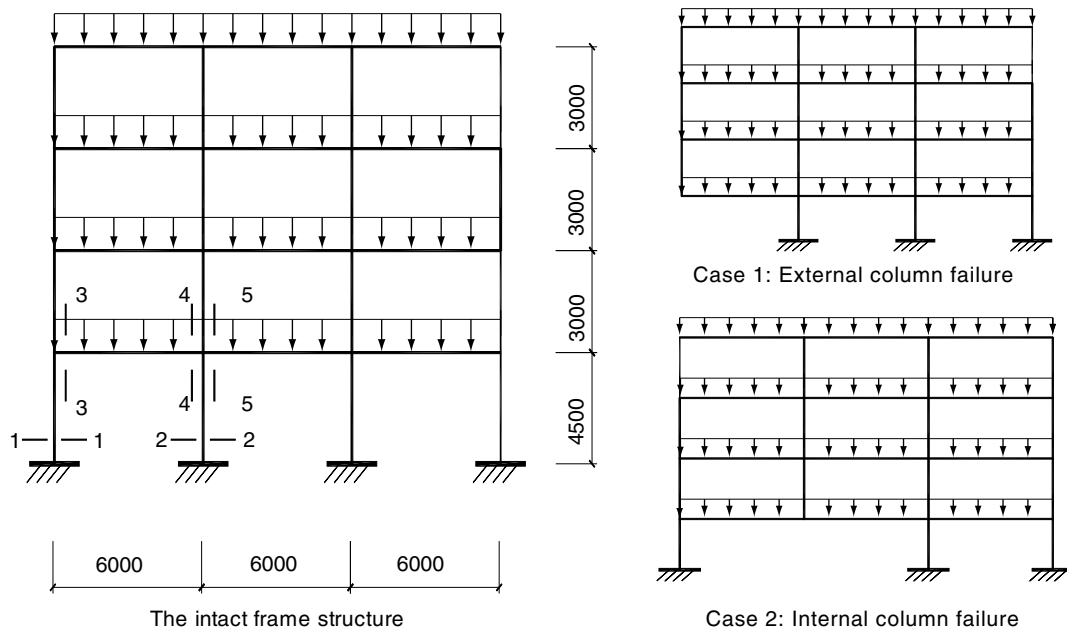
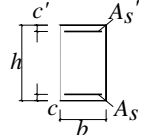
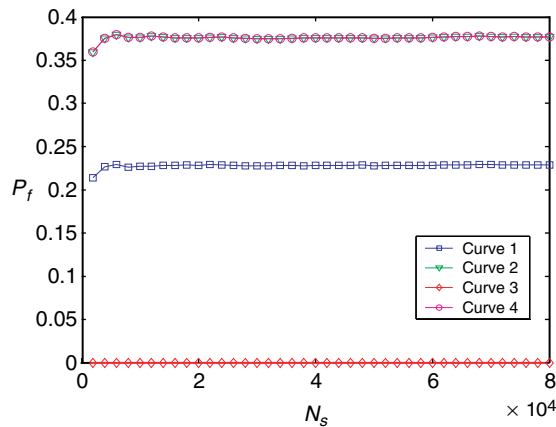


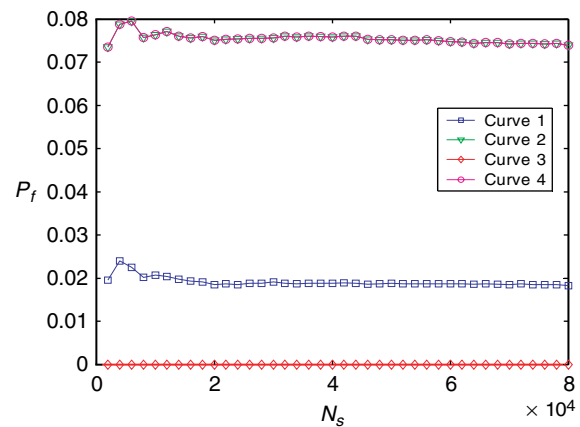
Figure 7. A four storey RC frame Structure for numerical example

Table 4. Dimension and reinforcement for different cross sections

Cross Sections		h (mm)	b (mm)	c' (mm)	c (mm)	A_s' (mm ²)	A_s (mm ²)
	1-1	500	500	30	30	2462	2462
	2-2	500	500	30	30	2826	2826
	3-3	600	300	30	30	1661	1245
	4-4	600	300	30	30	1963	1471
	5-5	600	300	30	30	1963	1471



Case 1: External column failure initially



Case 2: Internal column failure initially

 Figure 8. $P_f(z_1 < 0)$, $P_f(z_2 < 0)$, $P_f(z_3 < 0)$ and $P_f(z < 0)$ versus structural sample numbers

reliability of the damaged frame structure against progressive collapse in a comprehensive manner, where the global performance function z considers the minimum of three performance functions (z_1 , z_2 , and z_3). Based on Figure 8, several observations can be made as follows.

- 1) Since Curve 3 is located near the horizontal axis with $P_f(z_3 < 0) \approx 0$ unlike Curves 1 and 2, it can be understood that structural collapse due to lack of member shear strength is negligible in reliability assessment of the example structure after one column removal.

- 2) Curve 2 is significantly higher than Curve 1, which indicates a significant impact of the dynamic effect induced by the sudden loss of a column on the reliability of the damaged frame structure against progressive collapse. As Curve 2 is quite close to Curve 4 as presented in Figure 8, it can be concluded that the second performance function plays a dominant role in the structural reliability assessment.

In addition, the distributions of the performance functions (z_1 , z_2 , z_3 and z) are plotted in Figure 9 in the form of histograms obtained from MCS with 80000 samples. The normal probability density functions according to the mean values and the CoVs of these parameters are also exhibited in Figure 9. It can be observed that the CoVs of z_2 is the largest among z_1 , z_2 , and z_3 , which are 3.7072 and 0.68133 respectively for Case 1 (external column failure initially) and Case 2 (internal column failure initially). It is reasonable since the dynamic flexural response of the damaged structure controls the structural collapse events. Moreover, it is monitored that the CoVs of z i.e. 8.4028 and 0.69069 respectively for Case 1 and 2, are higher than those of z_2 and the distribution of z has a significant difference from the normal probability density function.

5. PARAMETRIC STUDY

Extensive parametric studies have been conducted in three sub-groups based on effects of random properties of different parameters, the most efficient ways to improve structural safety and effects of storey numbers on structural reliability of the damaged four-storey frame structure.

5.1. Group I of Parametric Studies

Structural behavior exhibits random characteristics due to random properties of structural parameters and external loadings. Hence, significance of CoVs of different parameters, like strength and elastic modulus of concrete and reinforcing bars, member cross-sectional dimensions and external loads including dead load (DL) and live load (LL) on the reliability of the damaged frame structures against progressive collapse is examined. CoVs of different parameters are modified independently as listed in Table 5 and the corresponding $P_f(z < 0)$ of the reference structure is computed after removing the initially failed column. In Table 5, CoV_{sg} , CoV_{dg} , CoV_{DLg} , CoV_{LLg} , CoV_{cg} , CoV_{ecg} and CoV_{wg} denote the generally accepted values of CoVs of related parameters and CoV_{sa} , CoV_{da} , CoV_{DLa} , CoV_{LLa} , CoV_{ca} , CoV_{cca} and CoV_{wa} signify the modified values adopted in the parametric studies with r_{cs} , r_{cd} , r_{cdl} , r_{cll} , r_{cc} , r_{cec}

and r_{cw} being parameter modification ratio (PMR), the ratio between the modified CoVs used in analyses and their generally accepted values.

5.2. Group II of Parametric Studies

To increase the structural reliability post failure of a column, determining an efficient design procedure is pertinent. For this purpose, effects of reinforcement area, mean values of member cross sectional effective depth and concrete strength based on the reference frame structure are explored and modified independently as listed in Table 6 where, A_r and μ_{dr} denote the reinforcement area and mean value of cross sectional effective beam depth; A_a and μ_{da} indicate their modified values adopted in Group II of parametric studies; μ_{cr} and μ_{ca} denote mean value of concrete strength and its modified value employed in the related case respectively.

5.3. Group III of Parametric Studies

In order to focus on the effect of number of storey on resistance against progressive collapse, a five-storey frame structure is built up with its first four storey to be exactly same as the reference structure. However, reinforcement areas of the beams in the fifth storey A_{a5} are varied from 0.9 to 1.1 times of those of the lower storey A_{a4} since all structural members in different storey of the frame structure are rarely identically reinforced. The studied cases are tabulated in Table 7.

Reliability of the damaged structure from the parametric studies listed in Tables 5 ~ 7 is assessed with Monte Carlo Simulation (MCS) method using 80000 samples to achieve a convergent simulation. To quantify the effect of different parameters on structural reliability explicitly, average sensitivity factor (ASF) and average variation ratio (AVR) of $p_f(z < 0)$ are evaluated as follows.

$$ASF = \frac{|P_f(z < 0, PMR = x_2) - P_f(z < 0, PMR = x_1)|}{x_2 - x_1} \quad (39)$$

$$AVR = \frac{P_f(z < 0, PMR = x_2) / P_f(z < 0, PMR = x_1) - 1}{x_2 - x_1} \quad (40)$$

Here, $x_1 = 0, 0.95$ and 0.9 while $x_2 = 1, 1.05$ and 1 respectively in Groups I, II and III of parametric studies. $p_f(z_2 < 0, PMR = x)$ denotes $p_f(z_2 < 0)$ under the related value of PMR. It is clear that the larger the ASF or the greater the distance between AVR and zero, the greater the effect of the parameter to the structural reliability against progressive collapse as exhibited in Tables 8 ~ 10.

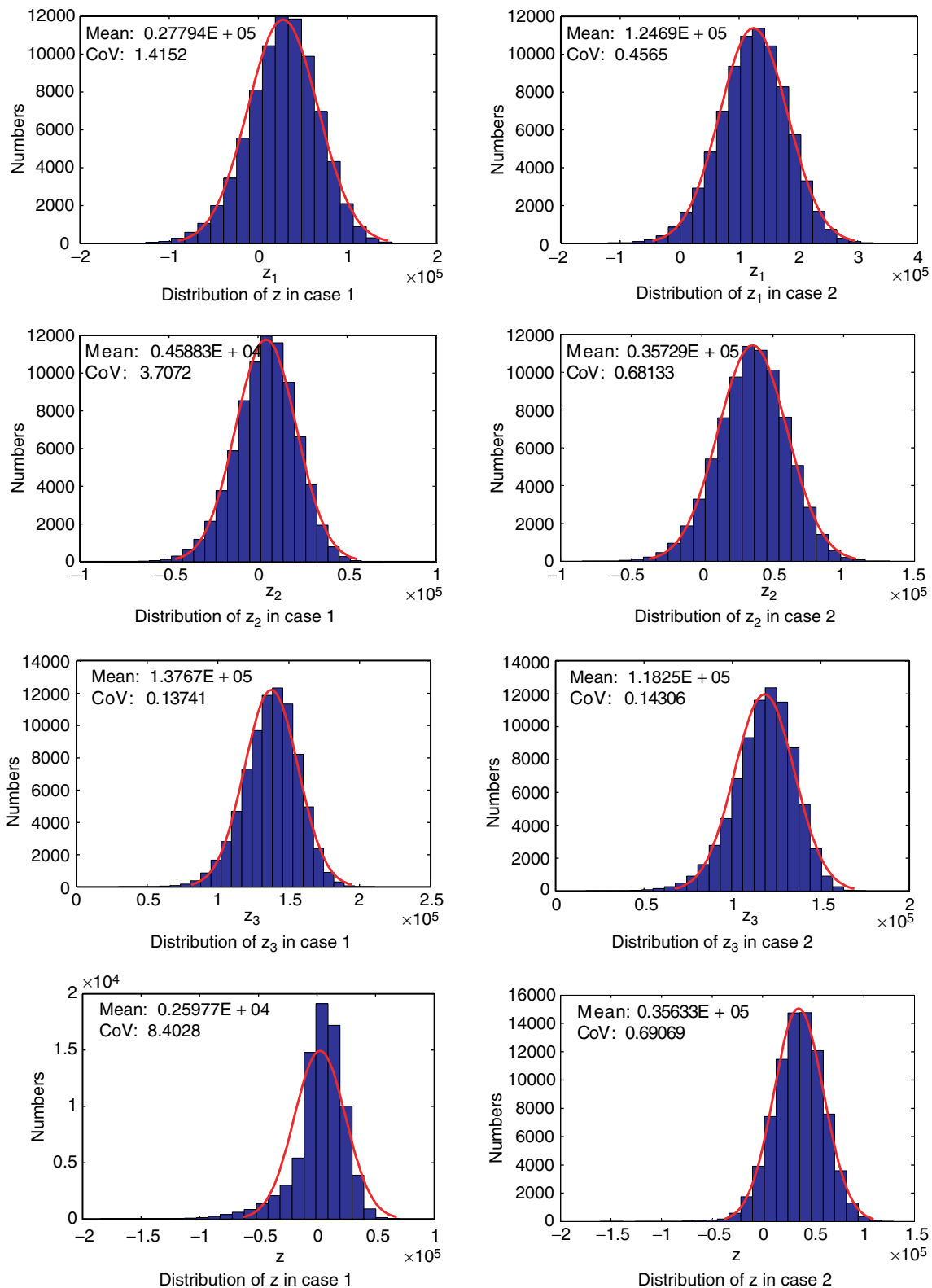


Figure 9. Distributions of the performance functions (z_1 , z_2 , z_3 , and z_4 .)

According to Group I parametric studies, if the time consuming structural reliability assessment is not available for civil engineers and only the deterministic design method is applicable, conservative factors shall be considered at least for the structural parameters

including the strength of the reinforcing bars, effective depth of structural members, the DL and the LL, in the design of the frame structure so as to obtain a reasonable level of confidence about its structural safety after a certain column has initially failed. As for the CoVs of

Table 5. Group I of parameters

Initially failed column			External column			Internal column																																																																																																	
CoV of Strength of Reinforcing Bar	CoV_{sg}	0.08	0.08	0.08	0.08	0.08	0.08																																																																																																
	CoV_{sa}	0.0	0.04	0.08	0.0	0.04	0.08																																																																																																
	$r_{cs} = \text{CoV}_{sa} / \text{CoV}_{sg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																																																
CoV of Structural Members' Cross Sectional Effective Depth	CoV_{dg}	0.05	0.05	0.05	0.05	0.05	0.05																																																																																																
	CoV_{da}	0.0	0.025	0.05	0.0	0.025	0.05																																																																																																
	$r_{cd} = \text{CoV}_{da} / \text{CoV}_{dg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																																																
	CoV_{DLg}	0.05	0.05	0.05	0.05	0.05	0.05																																																																																																
	$r_{cdl} = \text{CoV}_{DLa} / \text{CoV}_{DLg}$	0.0	0.5	1.0	0.0	0.5	1.0		CoV_{LLg}	0.4	0.4	0.4	0.4	0.4	0.4	CoV of Live Load (LL)	CoV_{LLa}	0.0	0.2	0.4	0.0	0.2	0.4		$r_{cll} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0		CoV_{cg}	0.13	0.13	0.13	0.13	0.13	0.13	CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0
	CoV_{LLg}	0.4	0.4	0.4	0.4	0.4	0.4	CoV of Live Load (LL)	CoV_{LLa}	0.0	0.2	0.4	0.0	0.2	0.4		$r_{cll} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0		CoV_{cg}	0.13	0.13	0.13	0.13	0.13	0.13	CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0								
CoV of Live Load (LL)	CoV_{LLa}	0.0	0.2	0.4	0.0	0.2	0.4		$r_{cll} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0		CoV_{cg}	0.13	0.13	0.13	0.13	0.13	0.13	CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																
	$r_{cll} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0		CoV_{cg}	0.13	0.13	0.13	0.13	0.13	0.13	CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																								
	CoV_{cg}	0.13	0.13	0.13	0.13	0.13	0.13	CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																
CoV of Concrete Strength	CoV_{ca}	0.0	0.065	0.13	0.0	0.065	0.13		$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																								
	$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																
CoV of Elastic Modulus Coefficient of Concrete	CoV_{ecg}	0.15	0.15	0.15	0.15	0.15	0.15		CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																								
	CoV_{eca}	0.0	0.075	0.15	0.0	0.075	0.15		$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																
	$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0	CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																								
CoV of Structural Members' Cross Sectional Width	CoV_{wg}	0.03	0.03	0.03	0.03	0.03	0.03		CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																																
	CoV_{wa}	0.0	0.015	0.03	0.0	0.015	0.03		$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																																								
	$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0																																																																																																

Table 6. Group II of parameters

Initially failed column			External column			Internal column	
Reinforcement Areas of Structural Beams	A_r	Table 4	Table 4	Table 4	Table 4	Table 4	Table 4
	A_a	$0.95 \times A_r$	$1.0 \times A_r$	$1.05 \times A_r$	$0.95 \times A_r$	$1.0 \times A_r$	$1.05 \times A_r$
	$r_{ma} = A_a / A_r$	0.95	1.0	1.05	0.95	1.0	1.05
Mean Values of Beam Cross Sectional Effective Depths	μ_{dr}	Table 4	Table 4	Table 4	Table 4	Table 4	Table 4
	μ_{da}	$0.95 \times \mu_{dr}$	$1.0 \times \mu_{dr}$	$1.05 \times \mu_{dr}$	$0.95 \times \mu_{dr}$	$1.0 \times \mu_{dr}$	$1.05 \times \mu_{dr}$
	$r_{md} = \mu_{da} / \mu_{dr}$	0.95	1.0	1.05	0.95	1.0	1.05
	μ_{cr}	38	38	38	38	38	38
Mean Value of Concrete Strength	μ_{ca}	36.1	38	39.9	36.1	38	39.9
	$r_{mc} = \mu_{ca} / \mu_{cr}$	0.95	1.0	1.05	0.95	1.0	1.05

Table 7. Group III of parameters

Initially failed column			External column			Internal column	
Reinforcement Areas of Structural Beams in the Fifth Storey	A_{r4}	Table 4	Table 4	Table 4	Table 4	Table 4	Table 4
	A_{a5}	$0.9 \times A_{r4}$	$1.0 \times A_{r4}$	$1.1 \times A_{r4}$	$0.9 \times A_{r4}$	$1.0 \times A_{r4}$	$1.1 \times A_{r4}$
	$r_{5ma} = A_{a5} / A_{r4}$	0.9	1.0	1.1	0.9	1.0	1.1

the strength and the elastic modulus of concrete, the width of the structural members, their effect on the damaged ductile frame structures against progressive collapse is almost ignorable. The results of Group II parametric studies show that the most efficient method to improve the structural reliability against progressive collapse is to increase the structural member's reinforcement area A_a or its effective depth μ_{da} . Though

the structural reliability can also be improved by the increase of concrete strength, compared to A_a and μ_{da} , the same percentage increase in concrete strength μ_{ca} will lead to a far less significant drop in $p_f(z < 0)$ as denoted by the corresponding smaller absolute values of ASF and AVR. For the adopted ductile frame structures, Group III parametric studies demonstrate that an increase in the number of structural storeys from 4 to 5

Table 8. $P_f(z < 0)$ for group I of parametric studies

Initially failed column		External column			Internal column		
CoV of Strength of Reinforcing Bar	$r_{cs} = \text{CoV}_{sa} / \text{CoV}_{sg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3486	0.3565	0.3740	0.0486	0.0545	0.0736
	ASF		0.0254			0.0250	
	AVR		0.0729			0.5144	
CoV of Structural Members' Cross Sectional Effective Depth	$r_{cd} = \text{CoV}_{da} / \text{CoV}_{dg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3512	0.3575	0.3771	0.0519	0.0583	0.0739
	ASF		0.0259			0.0220	
	AVR		0.0737			0.4239	
CoV of Dead Load (DL)	$r_{cdl} = \text{CoV}_{DLa} / \text{CoV}_{DLg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3687	0.3720	0.3748	0.0685	0.0696	0.0738
	ASF					0.0061	0.0053
	AVR					0.0165	0.0774
CoV of Live Load (LL)	$r_{cdl} = \text{CoV}_{LLa} / \text{CoV}_{LLg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3281	0.3470	0.3743	0.0136	0.0281	0.0734
	ASF		0.0462			0.0598	
	AVR		0.1408			4.3971	
CoV of Concrete Strength	$r_{cc} = \text{CoV}_{ca} / \text{CoV}_{cg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3738	0.3739	0.3746	0.0728	0.0730	0.0739
	ASF		0.0008			0.0011	
	AVR		0.0021			0.0151	
CoV of Elastic Modulus Coefficient of Concrete	$r_{cec} = \text{CoV}_{eca} / \text{CoV}_{ecg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3734	0.3740	0.3756	0.0724	0.0729	0.0736
	ASF		0.0022			0.0012	
	AVR		0.0059			0.0166	
CoV of Structural Members' Cross Sectional Width	$r_{cw} = \text{CoV}_{wa} / \text{CoV}_{wg}$	0.0	0.5	1.0	0.0	0.5	1.0
	$P_f(z < 0)$	0.3748	0.3751	0.3758	0.0729	0.0730	0.0735
	ASF		0.0010			0.0006	
	AVR		0.0027			0.0082	

Table 9. $P_f(z < 0)$ for group II of parametric studies

Initially failed column		External column			Internal column		
Reinforcement Areas of Structural Beams	$r_{ma} = A_a / A_r$	0.95	1.0	1.05	0.95	1.0	1.05
	$P_f(z < 0)$	0.6266	0.3746	0.1844	0.2739	0.0739	0.0135
	ASF		4.422			2.604	
	AVR		-7.057			-9.507	
Mean Values of Structural Beams' Cross Sectional Effective Depths	$r_{md} = \mu_{da} / \mu_{dr}$	0.95	1.0	1.05	0.95	1.0	1.05
	$P_f(z < 0)$	0.6705	0.3738	0.1591	0.3413	0.0736	0.0086
	ASF		5.114			3.327	
	AVR		-7.627			-9.748	
Mean Value of Concrete Strength	$r_{mc} = \mu_{ca} / \mu_{cr}$	0.95	1.0	1.05	0.95	1.0	1.05
	$P_f(z < 0)$	0.3816	0.3756	0.3723	0.0765	0.0735	0.0696
	ASF		0.093			0.069	
	AVR		-0.2437			-0.902	

Table 10. $P_f(z < 0)$ for group III of parametric studies

Initially failed column		External column			Internal column		
Reinforcement Areas of Structural Beams in the Fifth Storey	$r_{5ma} = A_{a5} / A_{r4}$	0.9	1.0	1.1	0.9	1.0	1.1
	$P_f(z < 0)$	0.4140	0.3134	0.2317	0.1106	0.0552	0.0256
	ASF		0.9115			0.4250	
	AVR		-2.2017			-3.8427	

results in a decrease of $p_f(z < 0)$, when the members in the 5th storey are identically reinforced as those in the first four storeys. In addition, the occurrence probability of structural collapse event $p_f(z < 0)$ increases remarkably with decrease of A_{d5} .

6. CONCLUSIONS

This paper focuses on development of three performance functions due to lack of strength, deformation capacity and shear strength of structural members for the weakest collapse mechanism respectively. Reliability of the damaged structure is assessed for a four-storey reinforced concrete (RC) frame against progressive collapse based on combination of the Monte Carlo Simulation method and calculation of the global performance function, minimum of three performance functions. The results confirm the significance of dynamic effects induced by sudden loss of a column during structural progressive collapse with revelation of utmost importance of the second performance function for the reliability assessment. Three groups of parametric studies are accomplished to investigate the effects of random properties of different structural parameters, various efficient ways to improve structural safety and the storey numbers on structural reliability against progressive collapse respectively. Group I parametric studies state that conservative factors must be considered for the reinforcing bar strength, member effective depth, dead and live loads during structural design to obtain a reasonable confidence level about the structural safety after a certain column loss. Group II parametric studies emphasize on increasing the member reinforcement area or effective depth rather than concrete strength to improve structural reliability against progressive collapse. Group III parametric studies indicate that increase in storey number from 4 to 5 decreases the occurrence probability of the structural collapse event when the structural members in the 5th storey are designed identically as those in the first four storeys. However, the occurrence probability of the structural collapse event increases remarkably with decrease in the reinforcement area of the 5th storey.

REFERENCES

- ACI (2002). *Building Code Requirements for Structural Concrete and Commentary (ACI 318-02)*, American Concrete Institute, Farmington Hills, Michigan, USA.
- Buscemi, N. and Marjanishvili, S. (2005). "SDOF model for progressive collapse analysis", *Proceedings of the 2005 Structures Congress and the 2005 Forensic Engineering Symposium*, ASCE, New York, USA.
- Cook, R.D., Malkus, D.S. and Plesha, M.E. (1989). *Concepts and Applications of Finite Element Analysis*, 3rd Edition, John Wiley & Sons, New York.
- Cundall, P.A. and Strack, O.D.L. (1979). "Discrete numerical model for granular assemblies", *Geotechnique*, Vol. 29, No. 1, pp. 47–65.
- DoD (2009). *Unified Facilities Criteria - Design of Buildings to Resist Progressive Collapse*, Department of Defense, Washington, DC, USA.
- Ellingwood, B.R. (1978). "Approaches for design against progressive collapse", *Journal of Structural Engineering*, ASCE, Vol. 104, No. 3, pp. 413–423.
- Hakuno, M. and Meguro, K. (1993). "Simulation of concrete-frame collapse due to dynamic loading", *Journal of Engineering Mechanics*, ASCE, Vol. 119, No. 9, pp. 1709–1723.
- Huang, Z.W., Li, B. and Sengupta, P. (2012). "Reliability assessment of damaged RC moment-resisting frame against progressive collapse under static loading conditions", *Journal of Engineering Mechanics*, ASCE, Vol. 139, No. 1, pp. 1–17.
- Huang, Z.W. (2008). *Reliability Assessment of Damaged Ductile RC Frame Structures against Progressive Collapse in Close-In Detonation Conditions*, PhD Thesis, Nanyang Technological University, Singapore.
- Kaewkulchai, G. and Williamson, E.B. (2003). "Dynamic behaviour of planar frames during progressive collapse", *Proceedings of the 16th ASCE Engineering Mechanics Conference*, Seattle, Washington, USA.
- Kaewkulchai, G. (2003). *Dynamic Progressive Collapse of Frame Structures*, PhD Thesis, University of Texas at Austin, Austin, TX, USA.
- Kaewkulchai, G. and Williamson, E.B. (2004). "Beam element formulation and solution procedure for dynamic progressive collapse analysis", *Computers and Structures*, Vol. 82, No. 7–8, pp. 639–651.
- MacGregor, J.G. and Wight, J.K. (2004). *Reinforced Concrete: Mechanics and Design*, Prentice Hall.
- Malla, R.B. and Nalluri, B. (1995). "Dynamic effects of member failure on response of truss-type space structures", *Journal of Spacecraft and Rockets*, Vol. 32, No. 3, pp. 545–551.
- Morris, N.F. (1993). "Effect of member snap on space truss collapse", *Journal of Engineering Mechanics*, ASCE, Vol. 119, No. 4, pp. 870–886.
- Pretlove, A.J., Ramsden, M. and Atkins, A.G. (1991). "Dynamic effects in progressive failure of structures", *International Journal of Impact Engineering*, Vol. 11, No. 4, pp. 539–546.

NOTATION

A_r	reinforcement area of beams of the reference structure
A_d	reinforcement area of beams adopted in parametric studies
A_{r4}	reinforcement area of beams in first four storeys of the five-storey frame structure

A_{a5}	reinforcement area of beams in the fifth storey of the five-storey frame structure	q_i	service loadings in the damaged span and the i^{th} storey
ASF	average sensitivity factor	R_m	strength of the equivalent SDOF system
AVR	average variation ratio	$V_{r,k}$	shear strength of k^{th} section
b	width of structural member	δW_{int}	internal virtual work done by the damaged structure
c and c'	concrete covers	δW_{ext}	external virtual work done by service loadings
CoV	coefficient of variation	$x_{l,i}$ or $x_{r,i}$	position of positive RPH on beam in left or right damaged span and i^{th} storey
d	effective depth of structural members	y_e	yield displacement of the equivalent SDOF system
DL_k	characteristic value of dead load	y_m	maximum displacement of the damaged structure
$D_{[I,II,III],i}^{[b1,b2,c1,c2]}$	director coefficients of ESPHs on Beam-1, Beam-2, Column-1 and Column-2 within Type-I ~ Type-III critical zones in the i^{th} storey	y_u	displacement capacity of the equivalent SDOF system
E_c	concrete elastic modulus	z	performance function of the damaged structure
E_f	structural failure event	Δ	displacement at the upper node of the failed column
f_c	concrete compressive strength	$\delta\Delta$	virtual displacement at the upper node of the failed column
f_y	yielding strength of reinforcing steel	r	parameter modification ratio
l_l	length of the left damaged span	θ_j	plastic rotations of the j^{th} RPH
l_r	length of the right damaged span	μ	mean value
LL_k	characteristic value of live load		
N_s	number of structural samples used in MCS		
np	number of the RPHs		
P_f	occurrence probability of structural failure event		
Q_j	plastic rotation of the j^{th} ESPH		