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ESSAYS ON THE DYNAMICS OF THE U.S. REAL EXCHANGE RATE

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Executive Summary

The existing new open-economy macroeconomic literature is almost entirely developed based on the sticky price model. However, the sticky price framework has received both theoretical and empirical criticism over the years. Recently, the sticky information model is introduced as an alternative by addressing some of the concerns with the sticky price model. While developed within a closed economy framework, such model has not yet been extended to the research in international macroeconomics. Therefore, this thesis primarily aims to explore the power of the sticky information model in replicating and explaining some stylized facts of the U.S. real exchange rate dynamics.

A structural vector autoregressive (VAR) analysis in Chapter 2 detects (1) declining and insignificant impact of monetary policy on the U.S. real exchange rate and other variables and (2) disappearance of hump-shape real exchange rate response to monetary policy shock during the great moderation (starting from the mid-1980s). Given this background, a two-country sticky information dynamic stochastic general equilibrium (DSGE) model with public misperception on the true nature of monetary policy shock is constructed to explore the possible sources of such structural change. Sensitivity tests show that less persistent monetary policy shock and smaller price stickiness are two key factors contributing to the weaker real exchange rate response, while the hump-shape response (i.e. delayed overshooting) is a phenomenon associated with public misperception.

Historically, the fluctuations in the real exchange rate are highly persistent, commonly known as the purchasing power parity (PPP) puzzle. This puzzle is investigated in Chapter 3 using a two-country sticky information DSGE model with tradable and non-tradable goods. Highly persistent PPP deviations are reproduced in the benchmark model with persistent productivity shocks, nonpersistent monetary policy shock and fairly flexible prices. Sensitivity tests show that persistent monetary policy shock

can also generate such persistent deviations with conventionally used price stickiness in existing literature, which is fairly high. It is shown analytically that the sticky information model can generate high persistence in the variables without much difficulty as all agents are allowed to make adjustments in each period and thus can fully respond to shocks in their information sets. Therefore, given a persistent shock, the pass-through of the shock and its persistence to various macroeconomic variables can be fairly high in the sticky information model.

In Chapter 4, structural vector error-correction model (SVECM) is used to identify the sources of the U.S. real effective exchange rate fluctuations during the great moderation. Time series of other variables are constructed against an aggregate of industrialized countries. Three long-run relationships are found in the data, one of which states that the U.S. real exchange rate appreciates with higher relative output. Subsequent structural form analysis shows that only relative productivity shock has a significant long-run impact on the U.S. real exchange rate. Forecast error variance decomposition also shows that while transitory shocks and relative fiscal policy shock account for more than two thirds of short-run movements in the U.S. real exchange rate, relative productivity shock dominates its long-run fluctuations. Again, the effect of relative monetary policy shock on the U.S. real exchange rate is found to be insignificant in this period.

Chapter 1

Introduction

Research on real exchange rate dynamics has been active in the field of international macroeconomics for decades.[1] Generally, a few empirical stylized facts have been documented in the literature. First, real exchange rates are much more volatile than many other key macroeconomic variables such as output and inflation. Second, real exchange rates deviate from their purchasing power parity (PPP) values persistently. Third, the currencies of more productive countries are usually overvalued in terms of PPP value.[2] As a result, most studies on real exchange rate fluctuations aim to (1) explain why the real exchange rate is relatively much more volatile and seek empirical support, (2) replicate the documented persistent real exchange rate fluctuations and identify the possible causes and 3) identify the underlying propagation mechanism of productivity shock and its impact on the real exchange rate. Against this background and newly found empirical evidence, this thesis contributes both theoretically and empirically to the literature from the following aspects: (1) what factors make the impact of monetary policy on the U.S. real exchange rate weaker and insignificant since the mid-1980s? (2) why does the real exchange rate exhibit so persistent PPP deviations? and (3) which factors contribute most to the U.S. real exchange rate

[1] Most real exchange rates analyzed in the literature are expressed in terms of U.S. dollar and thus can be treated as bilateral U.S. real exchange rates.

[2] See, for example, Rogoff (1996) and Chari et al. (2002) for a survey.

fluctuations in the short run and long run?

Two similar but distinct sticky information dynamic stochastic general equilibrium (DSGE) models are developed in Chapter 2 and Chapter 3 to emphasize different aspects of the real exchange rate. Particularly, a two-country setting is adopted to match the status of the U.S. economy. The reasons for choosing the sticky information model instead of the sticky price model in existing new open-economy macroeconomic literature are twofold. First, within the sticky price framework, high degree of price stickiness is usually required to reproduce the real exchange rate fluctuations. On average, the prices are assumed to be fixed for at least one year or longer. However, empirical evidence shows that prices are getting less sticky over time. For example, using the U.S. retail price data for 1995-1997, Bils and Klenow (2004) document that the mean frequency of price adjustment is 23.6% per month. Dhyne et al. (2005) report that 15.1% of prices in the Euro Area countries are adjusted per month on average using individual price data for 1989-2004.[3] As a result, the success of some amended sticky price models in reproducing real exchange rate dynamics under floating exchange rate regimes is not convincing. Second, even though the sticky information model is introduced as a competent candidate to replace the sticky price model, the power of the sticky information model in replicating and explaining some stylized facts in the international macroeconomics has not yet been explored.

Within the sticky information framework, nominal rigidity is introduced by incorporating pervasive stickiness in firms, workers and consumers through sticky information. The idea of sticky information was first introduced by Mankiw and Reis (2002) where it is assumed that information about macroeconomic conditions spreads only slowly as a result of costly acquisition, absorption and processing of information. In such model, all agents are allowed to make adjustments in each period based on their own information. In each period, only a random fraction of agents are able to receive new information. They are known as "attentive" agents. The attentive agents will

[3] The price data of some countries may cover shorter periods within this range.

plan a new path of optimal decisions as soon as they obtain updated information, while the other inattentive agents continue to follow their old plans made in previous periods. For example, all firms can re-set prices in each period. Attentive firms re-set optimal price based on newly received information, while the remaining firms re-set prices to expected optimal levels based on outdated information. Generally, the degrees of stickiness in prices, wages and consumption depend significantly on the fractions of firms, workers and consumers receiving new information in each period. A higher fraction implies lower stickiness, and vice versa. In recent years, a growing body of evidence on inattentiveness is found based on micro-data.[4] More importantly, the estimates of the fraction of attentive firms in each period are consistent with the findings of Bils and Klenow (2004) and Dhyne et al. (2005).[5]

In a typical three-agent sticky information model, there are consumers, workers and firms. First, a consumer plays two independent roles in each period without any information exchange: a shopper and a planner. The decision-maker as a shopper has full information of prices and chooses the best composite of all goods, taking the total consumption as given. The planner decides how much to consume based on information updated k periods ago. In each period, only a random fraction, λ_c, of planners can receive new information, but nobody can obtain any new information before the next update. Thus, with different degrees of sticky information, consumers are divided into many heterogeneous groups. Second, within a firm, two departments are making decisions independently: a hiring department and a sales department. The hiring department has full information on nominal wages and hires the best combination of labor inputs, taking the total production as given. The sales department sets prices in the Home market and the Foreign market based on information updated k periods ago. Accordingly, it produces an output to clear the market. In each period, a random fraction, λ_f, of sales departments receive new information. In this setup, again,

[4] See, for example, Mankiw and Reis (2001), Carroll (2003), Mankiw et al. (2003), Reis (2006a,b), Klenow and Willis (2007), Knotek (2010).

[5] See, for example, Mankiw and Reis (2007) and Reis (2009).

there are many heterogeneous groups of firms. Third, a worker has to set wage for his distinctive labor service based on information received k periods ago, and accordingly provide enough labor supply to meet the demand for his labor service. In each period, a random fraction, λ_w, of workers can update new information. Correspondingly, workers are also divided into a number of heterogeneous groups.

This thesis contributes to the international macroeconomics in several aspects. First, using sticky information leads to better understanding of the US real exchange rate dynamics. In Chapter 2, it is shown that lower degree of sticky information in the Federal Reserve and the public can explain well the change in the response of the U.S. real exchange rate to monetary policy shocks prior to and post mid-1980s. In Chapter 3, it is found that highly persistent real exchange rate can be simulated easily and without unrealistically high price stickiness using a sticky information framework. Second, the theoretical implication of Chapter 2 may also explain why industrial countries are 2.5 times less volatile relative to developing countries.[6] The explanation based on sticky information is: compared with the monetary authorities in industrial countries, the monetary authorities in developing countries are less experienced and skilled in acquiring, absorbing and processing information. Without sufficient updated data and information, they are more skeptical in acting decisively, which makes monetary policy shock more persistent. Therefore, their countries experience more volatile real exchange rates. Third, the empirical analysis on the U.S. data in Chapter 4 provides additional support for the argument that productivity innovations in the tradable sector will appreciate the real exchange rate. The debate on appreciation or depreciation is firmly related with the phenomenon of overvalued currencies in rich countries, as most productivity innovations have occurred in the tradable sector.[7]

The theoretical analysis in Chapter 2 is initially motivated by some empirical findings. In the literature, it is shown that the impact of monetary policy on the U.S economy

[6] Hausmann et al. (2006) show that the difference in volatility cannot be explained by the assumption that developing countries face larger shocks, or that they are more sensitive to these shocks.

[7] See, for example, De Gregorio et al. (1994) and Rogoff (1996).

omy is much smaller than before.[8] A structural vector autoregressive (VAR) analysis similar to that of Boivin and Giannoni (2006) is applied to the U.S. real exchange rate and other macroeconomic variables for two subsample periods: 1975Q1-1986Q2 and 1986Q3-2006Q1 (the great moderation period). The most interesting changes found in the second subsample period are: (1) besides output and inflation, the response of the U.S. real exchange rate to monetary policy shock also declines substantially and becomes insignificant and (2) delayed overshooting in the real exchange rate is not found. Given this background, a sticky information DSGE model is developed to explore the weaker impact of the U.S. monetary policy. To replicate the delayed overshooting found in the first subsample period, public misperception on the true nature of monetary policy shock (persistent or transitory) is added into the framework following Gourinchas and Tornell (2004) and Hoffmann et al. (2011). The sensitivity tests find that monetary policy shock persistence and price stickiness are key potential factors responsible for the change in the real exchange rate response. By setting key parameter values that fit the characteristics of the great moderation period, the benchmark model is capable of replicating the response of the U.S. real exchange rate in this period. With more persistent monetary policy shock and fewer attentive firms in each period, the simulated impulse response replicates exactly the response of the U.S. real exchange rate before the mid-1980s, including the delayed overshooting pattern. The two parameter values changed in this simulation are just in line with historical evidence. A more detailed description is provided in Chapter 2.

The issue of highly persistent real exchange rates, which is commonly known as the PPP puzzle in the international macroeconomic literature, is revisited in Chapter 3. As mentioned earlier, the simulations of sticky price models usually depend much on high degrees of price stickiness. For example, Bergin and Feenstra (2001) show that for monetary shock to generate the observed persistence, long-lasting and unrealistic

[8] See, for example, Kuttner and Mosser (2002), Barth and Ramey (2002), and Boivin and Giannoni (2002, 2006).

contracts of 8 to 12 quarters are needed. Chari et al. (2002) assume that prices are fixed for one year, but the simulated real exchange rate remains less persistent compared to the data. A sticky information DSGE model with both tradable and non-tradable sectors is constructed in this chapter. The benchmark model is capable of generating real exchange rate that is as persistent as in historical data with persistent productivity shocks, nonpersistent monetary policy shock and fairly flexible prices. It is analytically shown that, in contrast to the sticky price model, agents in the sticky information model can fully respond to shocks in their information sets, as they are allowed to make adjustments in each period. Therefore, given persistent shocks, the pass-through of shocks and their persistence to the variables is higher in the sticky information model than in the sticky price model. One supporting evidence is that persistent monetary policy shock in the model can also reproduce the observed persistence in the real exchange rate with high price stickiness conventionally used in existing literature. It is also noted that nominal rigidity in prices and wages makes the real exchange rate less persistent when productivity shocks dominate. A plausible explanation is provided based on the rather different propagation mechanisms of productivity and monetary policy shocks.

In Chapter 4, a comprehensive empirical investigation for the U.S. real effective exchange rate is conducted using a structural vector error-correction model (VECM). According to Alexius (2005), information contained in the levels of data is not fully utilized if only the changes in variables (such as first difference) are analyzed. The misspecification may lead to biased estimate if any cointegration relation exists. Again, the dataset in this chapter mainly covers the great moderation period. Except for the real exchange rate, all the variables are constructed in relative terms against an aggregate of industrialized countries. Three cointegration relations are detected amongst the variables, one of which states that the U.S. real exchange rate appreciates with higher relative output in the long run. Structural form analysis shows that the U.S. real exchange rate is exclusively affected by relative productivity shock in the long run, which is in support of the finding in Chapter 3. With parameter values set in line with empirical estimates, highly persistent real exchange rate is reproduced in the benchmark economy of Chapter 3, and the generated persistence is found completely due to persistent productivity shocks. Relative monetary policy shock, on the other hand, has negligible and insignificant effect on the U.S. real exchange rate during this sample period. This finding is again consistent with the empirical finding in Chapter 2. Forecast error variance decompositions draw a similar conclusion. While transitory shocks and relative fiscal policy shocks account for more than two thirds of short-run movements in the U.S. real exchange rate, relative productivity shock dominates its long-run fluctuations.

Chapter 5 summarizes the findings and policy implications of the thesis and discusses possible extensions for future research.

Chapter 2

Declining impact of monetary policy on the U.S. real exchange rate

2.1 Introduction

The volatilities of the U.S. inflation and output have declined dramatically since the mid-1980s.[1] Improved monetary policies, smaller external shocks (i.e. "good luck"), and changed economic structures are the main explanations provided for this phenomenon, which is dubbed "the Great Moderation".[2] Beyond the debate over which is the most important contributor, there is a large body of evidence showing that the impact of monetary policy on various macroeconomic variables is much smaller than before.[3] This change receives two opposite interpretations. One is that monetary policy has become less powerful in influencing the economy, and the other is that weaker monetary policy implies increased policy effectiveness. Boivin and Giannoni (2006) argue

[1] See, for example, McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Stock and Watson (2003b,a).

[2] See Bernanke (February 20, 2004) for a survey.

[3] See, for example, Kuttner and Mosser (2002), Barth and Ramey (2002), and Boivin and Giannoni (2002, 2006).

that if the monetary policy is extremely effective in offsetting the effects of external shocks on the economy, macroeconomic variables should have negligible responses to monetary policy shock.[4] In the context of the U.S. economy, monetary policy shock is defined as deliberately introduced deviation of the federal funds rate from its target level (i.e. "policy spread").[5] The latter interpretation is overall more acceptable as economists generally agree that monetary policies have performed better during the great moderation period than before.

In fact, the U.S. real exchange rate also becomes more stable during the great moderation time, although its volatility declines less than other macroeconomic variables.[6] There are two possible reasons for the smaller decline. First, many countries started to implement more flexible exchange rate policy from the middle 1980s. [7] Besides, the status of the U.S. dollar as an anchor currency is also weakened over the past three decades.[8] Theoretically, these two movements should have dampened the decrease in the U.S. real exchange rate volatility. Second, the responses of the real exchange rate and other variables to the improved monetary policy may be asymmetric. So, is the real exchange rate one of the variables moderated by the more effective monetary policy? To explore this issue, a similar structural VAR analysis as in Boivin and Giannoni (2006) is applied with the real exchange rate added and for two subsample periods: 1975Q1-1986Q2 and 1986Q3-2006Q1. Several interesting changes are found in the great moderation period. First, the standard deviations of structural shocks become smaller, which supports the good luck argument. Second, similar to inflation

[4]Boivin and Giannoni (2006) credit the change to stronger responses of the Fed to inflation expectations.

[5]Some interpret the spread as the Federal Reserve fails in keeping the federal funds rate at its target level, but more evidence shows it to be intentional deviation. See, for example, Clarida et al. (1998), Rudebusch (2002) and Romer and Romer (2004).

[6]According to IFS database, the standard deviation of H-P filtered U.S. real effective real exchange rate based on relative labor costs drops from 5.33 for 1975Q1-1986Q2 to 4.39 for 1986Q3-2006Q1, but its standard deviation ratio relative to real output increases from 2.43 to 4.72 for the same time periods.

[7]This trend is documented under both the *de jure* and the *de facto* classification. See, for example, Reinhart and Rogoff (2004), Shambaugh (2004), and Hoffmann (2007).

[8]See Bracke and Bunda (2011).

and real output, the response of the U.S. real exchange rate to monetary policy shock declines substantially and becomes insignificant, even after taking into consideration the impact of reduced shock size. The third interesting finding is that delayed overshooting pattern, that the real exchange rate does not overshoot immediately, is not found in this period.

A two-country sticky information DSGE model is developed to investigate why the U.S. real exchange rate responds so differently to monetary policy shocks in two subsamples. To reproduce the delayed overshooting phenomenon in the U.S. real exchange rate before the mid-1980s, public misperception on the true nature of monetary policy shock is included into the model. Gourinchas and Tornell (2004) introduce misperception in investors on the persistence of interest rate innovations and show that it is possible for their model to generate delayed overshooting in nominal exchange rate. Hoffmann et al. (2011) incorporate similar public misperception into a small open economy model and succeed in generating delayed overshooting in the real exchange rate. I follow this strand of literature, but model public misperception differently. In my setup, monetary policy shock follows an autoregressive process of order 1, but attentive agents are not able to identify whether a new monetary policy shock is going to be persistent or transitory immediately. These agents form their expectations based on a simple probability-weighted average rule and respond moderately to the shock. Only agents who update information in later periods are able to observe the shock's true nature. As the true process of monetary policy shock is persistent, the smaller the probability assigned to the possibility of "persistent", the higher the degree of public misperception.

One unexpected advantage of the sticky information model is that the degree of monetary policy shock persistence is associated with the degree of sticky information in the monetary authority, which makes the model an integrated framework. Generally, the degree of monetary policy shock persistence is positively correlated with that of external shocks. If the Federal Reserve has updated information about the economic environment and thus fights against external disturbances decisively and precisely in each period, monetary policy shock should be as persistent as external disturbances. If the Federal Reserve is not fully certain about the current economic situation and thus fights back conservatively and gradually, monetary policy shock will turn out to be more persistent than exogenous disturbances. As a result, if the persistence level of monetary policy shock declines, the most probable cause will be that the Federal Reserve is more updated and informed than before. For instance, the Federal Open Market Committee (FOMC) has steadily required more banking institutions to report their reserve positions daily. The updated information allows the Federal Reserve to better understand and predict the need and preference of the banking sector and then conduct open market operations accordingly.

Sensitivity tests based on empirical changes during the great moderation find that less persistent monetary policy shock and lower degree of price stickiness are able to account for the weaker response of the U.S real exchange rate to monetary policy shock since the mid-1980s. With parameter values estimated from the period of 1986Q3-2006Q1, the benchmark model replicates well the response of the real exchange rate in the same period. As empirical evidence shows that the persistence level of monetary policy shock and degree of price stickiness were much higher before the great moderation, the parameter values are changed accordingly in replicating real exchange rate response in that period. With more persistent monetary policy shock and fewer attentive firms in each period, the simulated impulse response replicates exactly the empirical response of the U.S. real exchange rate before the mid-1980s, including the delayed overshooting pattern. The sensitivity tests also re-confirm that severe public misperception and sufficiently persistent monetary policy shock are important in stimulating delayed overshooting in the real exchange rate. This finding is consistent with the historical evidence of more transparent monetary policy over time and explains why the structural impulse response in the structural VAR analysis does not exhibit delayed overshooting pattern after the mid-1980s.

The rest of this chapter is organized as follows. Section 2 describes the structural VAR model and displays the change in the responses of the real exchange rate before and after the middle 1980s. Section 3 constructs a fully specified DSGE model with sticky information in an open economy framework. Section 4 provides the simulation results from the benchmark model and sensitivity tests. Some conclusions then follow in Section 5.

2.2 Empirical Evidence

In this section, a Boivin and Giannoni (2006) version of structural VAR model is formally constructed to investigate the transmission mechanism of monetary policy on the real exchange rate. By imposing just enough restrictions to identify an exogenous shock, a complete specification of the economy is not required. The structural VAR model consists of 5 variables as follows:

$$\mathbf{Z}_t = \mathbf{c} + \mathbf{A}(\mathbf{L})\mathbf{Z}_{t-1} + \mathbf{u}_t$$

where $\mathbf{Z}_t = (\widehat{y}_t,\ \pi_t,\ \pi_t^s,\ i_t,\ \widehat{q}_t)'$ is a 5×1 vector of endogenous variables, $\mathbf{c}$ is a constant term, $\mathbf{A}(\mathbf{L})$ is the lag operator, and $\mathbf{u}_t = (u_t^y,\ u_t^\pi,\ u_t^{\pi^s},\ u_t^i,\ u_t^q)'$ is the error term. More specifically, $\widehat{y}_t$ is output gap, π_t is the inflation rate, π_t^s is the commodity price inflation, i_t is the effective Federal funds rate and $\widehat{q}_t$ is the natural log of real effective exchange rate. Following Boivin and Giannoni (2006), output and prices are assumed to respond with a lag to monetary policy shock. As nominal exchange rates can change very quickly, the response of the real exchange rate is assumed to be instantaneous. Therefore, $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$ where $\mathbf{B}$ is a lower triangular matrix and $\boldsymbol{\varepsilon}_t = (\varepsilon_t^y,\ \varepsilon_t^\pi,\ \varepsilon_t^{\pi^s},\ \varepsilon_t^i,\ \varepsilon_t^q)'$ is the vector of mutually uncorrelated structural shocks.

The output gap is the natural log deviation of quarterly real GDP from a linear deterministic trend. The inflation rate is the annualized rate of change in price measured by GDP deflator between two consecutive quarters. The commodity price inflation rate is the annualized rate of change in aggregated Commodity Research Bureau (CRB) spot market commodity price index between two consecutive quarters. It is conventionally added to limit the extent of a price puzzle since Sims (1992). Real effective exchange rate is measured based on relative unit labor cost. The sign of the logged data has been reversed so that an increase of $\widehat{q}_t$ implies a depreciation. All series are taken from the IMF International Financial Statistics (IFS) database except for the monthly CRB spot market commodity price index series from Standard & Poor's DRI database. The quarterly CRB spot market commodity price index is obtained by averaging the original monthly data of each quarter.

The changes in the effects of monetary policy are assessed by comparing the impulse responses of output gap, inflation, the federal funds rate and the real effective exchange rate to monetary policy shock over two subsamples. As most of the parameters used in the benchmark economy simulation are estimated by Reis (2009) using quarterly U.S. data for 1986Q3-2006Q1, the sample period is split in the way that subsample 1 corresponds to 1975Q1-1986Q2 and subsample 2 corresponds to 1986Q3-2006Q1. Even though the selection of 1986Q3 is later than 1984Q1, the more conventionally used starting date for the change in the volatility of the U.S. economy, this cut-off point is still within the confidence interval of the great moderation period. Based on various residual analyses, 3 lags are included in both subsamples. Specially, monetary policy shock ε_t^i is identified as unexpected change to the federal funds rate.

Estimation shows that external shocks (ε_t^y, ε_t^π, and $\varepsilon_t^{\pi^s}$) become smaller in subsample 2, which is in support of the "good luck" argument for the great moderation. Accordingly, the standard deviation of monetary shock also drops from 1.02 to 0.3. Figure 2.1 displays the impulse responses and the associated 95% confidence intervals of the real exchange rate to an unexpected one standard deviation decrease in the federal funds rate. The impulse responses are notably different in the two subsamples. Consistent with the estimation of Boivin and Giannoni (2006), the estimation shows

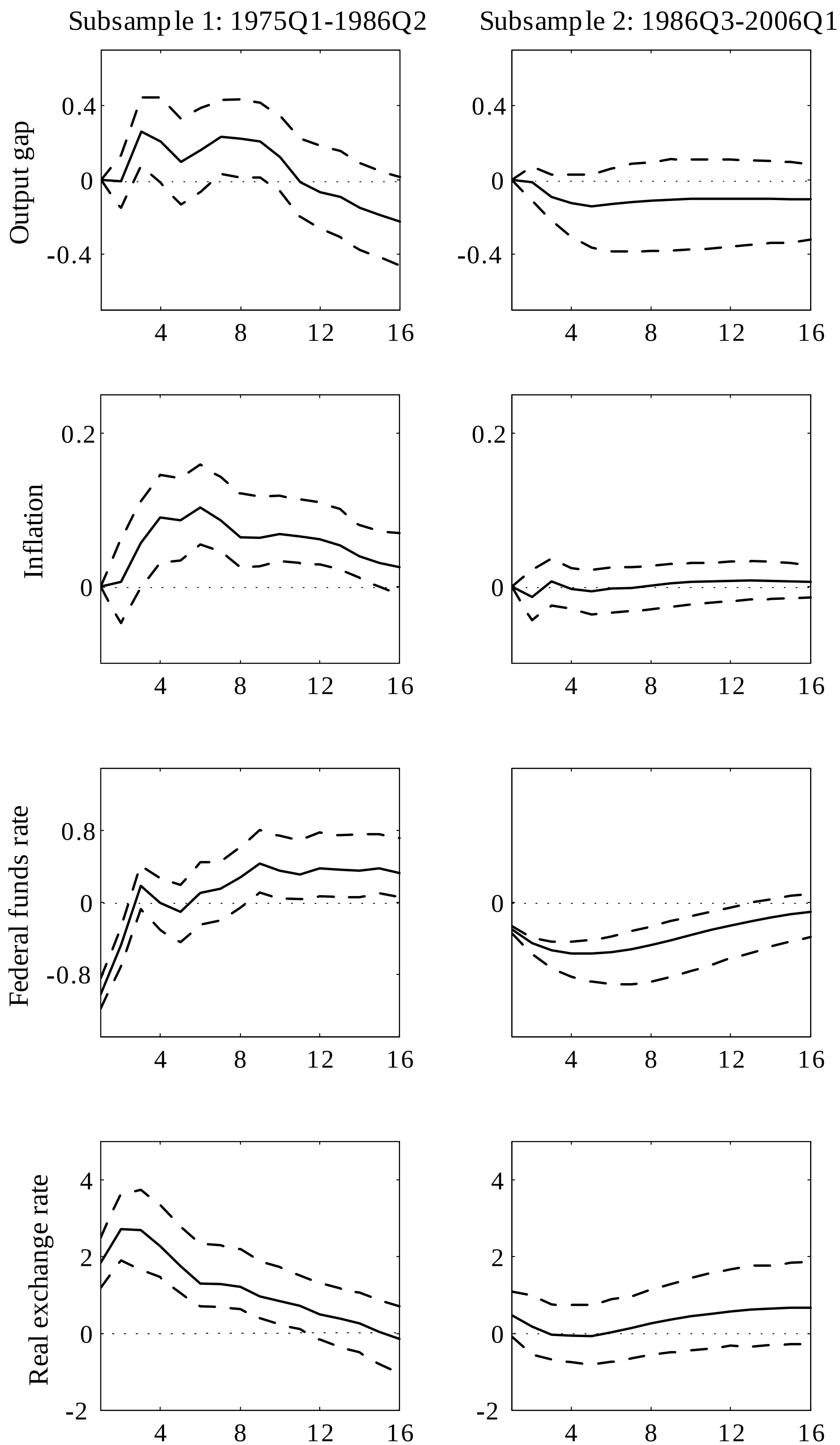

Figure 2.1: Structural impulse responses to expansionary monetary policy shocks

that the responses of output and inflation rate are much less pronounced in subsample 2, even after taking into consideration the impact of reduced shock size. For the variable of interest, the real effective exchange rate also responds more weakly in subsample 2. Its response is significant for 12 quarters in subsample 1 but completely insignificant in subsample 2. Besides, the well-known delayed overshooting in the real exchange rate is only found in subsample 1. These empirical findings in turn supports the argument that the effect of monetary policy shock has changed greatly since the mid-1980s.[9]

2.3 Structure of the benchmark model

In this section, I aim to explore the factors that can account for the change in the behavior of the real exchange rate and other key macroeconomic variables since the mid-1980s using a sticky information DSGE model. The model is basically an open-economy version of Mankiw and Reis (2006) general equilibrium model with public misperception. Consider a two-country world economy consisting of a Home country (denoted by a subscript H) and a Foreign country (denoted by a subscript F) which are similar in all aspects except size. Each country is populated by a large number of identical, infinitely lived consumers, where n and $1 - n$ represent the home country and foreign country population, respectively. There are three types of agents in the economy: consumers, workers and firms. In each country, the monetary authority can enforce the use of a unit of account (denoted as "currency" in the model) and governs the economy by adjusting nominal interest rate. In the goods market, each monopolistic firm produces and sells one differentiated good to consumer; in the labor

[9] In order to check the robustness of the results, we replace q_t with CPI-based real effective real exchange rates from the Bank of International Settlements and the Main Economic Indicators of the OECD, respectively. The responses of the two alternative real exchange rates are very similar to those in Figure 2.1: strong, significant and hump-shaped response in subsample 1 but weak, insignificant and quick response in subsample 2. Their responses are not completely identical to those of q_t in terms of magnitude, which may be because that the three indexes are averaged across different numbers of countries and with different weights assigned to each country.

market, each monopolistic worker sells his distinctive labor service to local firms; and in the incomplete but competitive international bonds market, consumers trade two uncontingent nominal bonds among one another and earn nominal interests. One bond is denominated in Home currency and the other in Foreign currency. The monetary authority intervenes in the bonds market to affect the nominal interest rate of its country. The array of differentiated goods produced by Home firms are indexed by the interval $[0, n]$ and symmetrically, those produced by Foreign firms are indexed by $(n, 1]$. All goods are assumed to be tradable because the fluctuations in the relative prices of non-tradable to tradable goods are found to contribute negligibly to the movements of the real exchange rate in the short run.[10] Two shocks are introduced to the economy: productivity shock and monetary policy shock.

2.3.1 Consumers

The Home country is populated by a large number of identical and infinitely lived consumers, each of which is endowed with a specific labor type. A typical consumer j obtains the utility in each period according to:

$$U(C_t^j, L_t^j) = \ln C_t^j - \frac{\varkappa L_t^{j1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \tag{2.1}$$

where C_t^j is the constant elasticity of substitution (CES) index for consumption of consumer j, and L_t^j is the labor supplied. φ is the Frisch elasticity of labor supply, and $\varkappa$ captures the relative preferences for consumption and leisure.

The CES consumption function is defined as:

$$C_t^j = [n^{\frac{1}{\zeta}} C_{H,t}^{j\frac{\zeta-1}{\zeta}} + (1-n)^{\frac{1}{\zeta}} C_{F,t}^{j\frac{\zeta-1}{\zeta}}]^{\frac{\zeta}{\zeta-1}}, \tag{2.2}$$

where $C_{H,t}^j$ is Home consumption of the Home-produced goods and and $C_{F,t}^j$ is Home

[10] See, for example, Engel (1993, 1999), and Chari et al. (2002).

consumption of Foreign-produced goods. ζ is the constant elasticity of substitution between the Home-produced goods and the Foreign-produced goods. Specifically,

$$C^j_{H,t} = [(\frac{1}{n})^{\frac{1}{\upsilon}} \int_0^n C^{j\,\frac{\upsilon-1}{\upsilon}}_{H,t,i}\, di]^{\frac{\upsilon}{\upsilon-1}}, \tag{2.3}$$

$$C^j_{F,t} = [(\frac{1}{1-n})^{\frac{1}{\upsilon}} \int_n^1 C^{j\,\frac{\upsilon-1}{\upsilon}}_{F,t,i^*} di^*]^{\frac{\upsilon}{\upsilon-1}}, \tag{2.4}$$

where $\upsilon > 1$ is the constant elasticity of substitution for goods produced within a country, $C^j_{H,t,i}$ and C^j_{F,t,i^*} are consumer j's demand functions for Home-produced good i and Foreign-produced good i^* ($*$ denote Foreign variables throughout), respectively. The structure of the preferences implies:

$$C^j_{H,t,i} = (\frac{P_{t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_t})^{-\zeta} C^j_t, \tag{2.5}$$

$$C^j_{F,t,i^*} = (\frac{P_{t,i^*}}{P_{F,t}})^{-\upsilon}(\frac{P_{F,t}}{P_t})^{-\zeta} C^j_t, \tag{2.6}$$

The aggregate price P_t is given by:

$$P_t = [nP^{1-\zeta}_{H,t} + (1-n)P^{1-\zeta}_{F,t}]^{\frac{1}{1-\zeta}}, \tag{2.7}$$

where $P_{H,t}$ is the price index for Home goods and $P_{F,t}$ is the price index for Foreign goods. Both are denominated in Home currency:

$$P_{H,t} = [\frac{1}{n}\int_0^n P^{1-\upsilon}_{H,t,i} di]^{\frac{1}{1-\upsilon}}, \tag{2.8}$$

$$P_{F,t} = [\frac{1}{1-n}\int_n^1 P^{1-\upsilon}_{F,t,i^*} di^*]^{\frac{1}{1-\upsilon}}, \tag{2.9}$$

Consumer j chooses consumption, labor and holding of bonds to maximize lifetime utility subject to the following budget constraint:

$$P_t C^j_t + B^j_t + S_t B^{*j}_t = W^j_t L^j_t + (1+i_{t-1})B^j_{t-1} + (1+i^*_{t-1})S_t B^{*j}_{t-1} + T^j_t, \tag{2.10}$$

where B_t^j and B_t^{*j} are the holdings of Home-currency and Foreign-currency denominated one-period bonds in $t+1$, respectively, nominal exchange rate S_t is defined as the Home-currency price of Foreign currency, W_t^j is nominal wage, i_{t-1} and i_{t-1}^* are nominal interest rates on B_{t-1}^j and B_{t-1}^{*j}, respectively, and finally, T_t^j are the lump-sum nominal transfers received. The transfers come either from firm profits, which are equally owned by all households, or from an insurance contract signed at the beginning of each period. The payment from the contract allows every household within a country to start with the same wealth at each period.

Let us denote $F_t^j \equiv [W_t^j L_t^j + (1+i_{t-1})B_{t-1}^j + (1+i_{t-1}^*)S_t B_{t-1}^{*j} + T_t^j]/P_t$ as the real resources with which Home consumer j enters period t. The assumption of perfect insurance implies that $F_t^j = F_t$, identical for all consumers. $V(F_t,.)$ refers to the value function for consumers that update information at period t, taking other state variables in the second argument as given. An attentive consumer at period t solves:

$$
\begin{aligned}
V(F_t) = \max_{\{C_{t+m,m}\}} & E_t[\sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m U(C_{t+m,m},.) \\
& + \beta\lambda_c \sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m V(F_{t+m+1})] \\
\text{s.t. } F_{t+m+1} = {} & R_{t+m+1}(F_{t+m} - C_{t+m,.}) + \frac{W_{t+m+1,.}L_{t+m+1,.} + T_{t+m+1}}{P_{t+m+1}}, \\
& m \geq 0 \text{ and a no-Ponzi condition}
\end{aligned}
$$

where $\beta \in (0,1)$ is discounting factor and R_{t+1} is the real return on nominal bonds.[11] The first term in the equation is the expected discounted utility if the consumer never receive new information again. The second term is the sum of the continuation values that the consumer updates new information again at a certain period in the future, each occurring with a probability $\lambda_c(1-\lambda_c)^m$.

The firs-order conditions show that an attentive consumer plans his consumption

[11] With competitive international bonds market, expected real return is identical on Home and Foreign bonds.

according to the standard Euler equation:

$$C_{t,0}^{-1} = \beta E_t[R_{t+1} C_{t+1,0}^{-1}], \tag{2.11}$$

If a consumer last updated information k periods ago, the total consumption $C_{t,k}$ is decided based on the optimality condition:

$$C_{t,k}^{-1} = E_{t-k}[C_{t,0}^{-1}], \tag{2.12}$$

where the marginal utility of consumption ($C_{t,k}^{-1}$) is set equal to his expectation of the marginal utility of an attentive consumer ($C_{t,0}^{-1}$).[12] With symmetrical setting, similar first-order optimal conditions apply to foreign consumers.

2.3.2 Consumption risk sharing

International bonds market is competitive. Domestic and foreign households can trade in their nominal bonds denominated in both Home currency and Foreign currency. Uncovered interest rate parity holds:

$$1 + i_t = \frac{E_t[S_{t+1}]}{S_t}(1 + i_t^*), \tag{2.13}$$

Consequently, the following consumption risk-sharing equation is obtained:

$$\frac{C_{t,0}}{C_{t,0}^*} = \frac{E_t[Q_t C_{t+1,0}^{*-1}]}{E_t[Q_{t+1} C_{t+1,0}^{-1}]}, \tag{2.14}$$

where Q_t is the real exchange rate defined as $S_t P_t^*/P_t$. Intuitively, an increase in Q_t implies a real depreciation of Home currency. Equation 2.14 looks slightly different from standard international consumption risk-sharing condition when the bonds are complete. However, up to a first-order log-linear approximation, the conventional

[12] See Mankiw and Reis (2006) or the appendix for more algebra details.

expression would be obtained after iteration. The assumption of competitive international bonds market and social insurance contract insures that the real exchange rate and consumption are stationary in the model, as long as shocks are stationary.

2.3.3 Workers

The composite of inputs $L_{t,i}$, labor input of firm i, is defined as:

$$L_{t,i} = [(\frac{1}{n})^{\frac{1}{\gamma}} \int_0^n L_{t,i}^{j\,\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}}, \tag{2.15}$$

where $L_{t,i}^j$ is labor service of worker j hired, and γ is the constant elasticity of substitution within various labor services.

Summing over all firms, the labor market for labor j thus clears as:

$$L_t^j = \frac{1}{n}(\frac{W_t^j}{W_t})^{-\gamma} L_t, \tag{2.16}$$

where $L_t = \int_0^n L_{t,i} di = [(\frac{1}{n})^{\frac{1}{\gamma}} \int_0^n L_t^{j\,\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}}$ is the total labor inputs of all Home firms and the total labor supplied in equilibrium. The corresponding aggregate wage index is given by:

$$W_t = [\frac{1}{n} \int_0^n W_t^{j\,1-\gamma} dj]^{\frac{1}{1-\gamma}}, \tag{2.17}$$

Similar to consumers, an attentive worker at period t solves the dynamic program:

$$\begin{aligned} V(F_t) = \max_{\{W_{t+m,m}\}} E_t[&\sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m U(L_{t+m,m}, .) \\ &+ \beta \lambda_w \sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m V(F_{t+m+1})] \end{aligned}$$

$$
\begin{aligned}
\text{s.t. } F_{t+m+1} &= R_{t+m+1}(F_{t+m} - C_{t+m,.}) + \frac{W_{t+m+1,.}L_{t+m+1,.} + T_{t+m+1}}{P_{t+m+1}}, \\
F_{t+m} &= \frac{W_{t+m,.}L_{t+m,.} + (1+i_{t+m-1})B_{t+m-1} + (1+i^*_{t+m-1})S_t B^*_{t+m-1} + T_{t+m}}{P_{t+m}}, \\
L_{t+m,.} &= \frac{1}{n}(\frac{W_{t+m,.}}{W_{t+m}})^{-\gamma} L_{t+m}, \\
& m \geq 0 \text{ and a no-Ponzi condition.}
\end{aligned}
$$

The first-order conditions show that an attentive worker charges for his labor service with some monopolistic power:

$$
W_{t,0} = \frac{\gamma}{\gamma - 1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{C_{t,0}^{-1}/P_t}, \tag{2.18}
$$

where the wage is set with a fixed markup ($\frac{\gamma}{\gamma-1}$) over the marginal opportunity cost of labor, which equals to the marginal disutility of working ($\varkappa L_{t,0}^{\frac{1}{\varphi}}$) divided by the marginal utility per dollar of consumption ($C_{t,0}^{-1}/P_t$).

The usual Euler equation also holds in the wage setting of an attentive worker:

$$
\frac{L_{t,0}^{\frac{1}{\varphi}}}{W_{t,0}/P_t} = \beta E_t[\frac{R_{t+1} L_{t+1,0}^{\frac{1}{\varphi}}}{W_{t+1,0}/P_{t+1}}], \tag{2.19}
$$

A worker with information received k periods ago sets his wage $W_{t,k}$ according to:

$$
W_{t,k} = \frac{E_{t-k}[L_{t,k}^{1+\frac{1}{\varphi}}]}{E_{t-k}[L_{t,0}^{\frac{1}{\varphi}} L_{t,k}/W_{t,0}]}, \tag{2.20}
$$

which states that the worker sets wages so that his marginal disutility per dollar of working ($\varkappa L_{t,k}^{\frac{1}{\varphi}}/W_{t,k}$) equals to his expected marginal disutility per dollar of working of an attentive worker ($\varkappa L_{t,0}^{\frac{1}{\varphi}}/W_{t,0}$).

2.3.4 Firms

It is assumed that all Home firms share the same technology. The production function for a representative Home firm i is:

$$Y_{t,i} = A_t L_{t,i}^{\xi}, \tag{2.21}$$

where A_t is total factor productivity of the Home country. Its log form ($a_t = \ln A_t$) is denoted as productivity shock and follow an autoregressive process as $a_t = \rho_a a_{t-1} + \mu_t$, where ρ_a is the autocorrelation measuring the persistence level of productivity shock and μ_t is white noise with mean zero and standard deviation σ_a. $\xi \in (0,1)$ measures the labor share of income. Correspondingly, Home aggregate output is given by $Y_t = \int_0^n Y_{t,i} di$.[13]

The objective of a sales department with information received k periods ago is to:

$$\max E_{t-k}[P_{t,k,i} Y_{t,k,i} - W_t L_{t,k,i}]$$

subject to $Y_{t,k,i} = A_t L_{t,k,i}^{\xi}$,

$$Y_{t,k,i} = (\frac{P_{t,k,i}}{P_{H,t}})^{-\upsilon} (\frac{P_{H,t}}{P_t})^{-\zeta} C_t + (\frac{P_{t,k,i}}{S_{t,k} P_{H,t}^{*}})^{-\upsilon} (\frac{P_{H,t}^{*}}{P_t^{*}})^{-\zeta} C_t^{*},$$

where $C_t = \int_0^n C_t^j dj$ and $C_t^* = \int_n^1 C_t^{j^*} dj^*$ are the aggregate consumption of Home country and Foreign country, respectively, and $S_{t,k} = E_{t-k}[S_t]$. The first order condition with respect to $P_{t,k,i}$ is:

$$P_{t,k,i} = \frac{\upsilon}{\upsilon - 1} \frac{E_{t-k}[A_t^{-\frac{1}{\xi}} Y_{t,k,i}^{\frac{1}{\xi}} W_t]}{E_{t-k}(\xi Y_{t,k,i})}, \tag{2.22}$$

With iso-elastic preferences, nominal prices are set with a fixed markup $(\frac{\upsilon}{\upsilon-1})$ over nominal marginal costs, which equal to nominal wage (W_t) divided by the marginal

[13]Defining aggregate output instead as $Y_t = [(\frac{1}{n})^{\frac{1}{\upsilon}} \int_0^n Y_{t,i}^{\frac{\upsilon-1}{\upsilon}} di]^{\frac{\upsilon}{\upsilon-1}}$ leads to the same results, up to a first-order log-linear approximation.

product of the composite labor input ($\xi A_t^{\frac{1}{\xi}} Y_{t,k,i}^{1-\frac{1}{\xi}}$). It is assumed that producers aim to set identical prices in both markets *ex ante*, but some of them fail to do so *ex post* due to sticky information. That is, an attentive firm is able to set the Foreign market price of its product $P^*_{t,0,i}$ equal to $P_{t,0,i}/S_t$, which implies perfect exchange rate pass-through. Otherwise, the Foreign market price $P^*_{t,k,i}$ is actually equal to $P_{t,k,i}/S_{t,k}$ rather than $P_{t,k,i}/S_t$, which implies imperfect exchange rate pass-through. Similarly, the Home market price of Foreign goods P_{t,k,i^*} equals to $S_{t,k}P_{t,k,i^*}$ instead of $S_t P_{t,k,i^*}$ unless $k = 0$. Therefore, in an open economy model with sticky information, the pricing rule is a composite of pricing-to-market and pricing-to-producer.

2.3.5 Monetary policy

In this cashless framework, the monetary authority is assumed to be able to enforce the use of a unit of account and issue nominal bonds denominated in its domestic currency. These bonds are substitutable with the domestic bonds that consumers trade among themselves, so the monetary authority can always clear the market for bonds denominated in its domestic currency at a preferred level through intervention.[14] The monetary authority follows the Taylor rule:

$$i_t = \phi_\pi \ln(\frac{P_t}{P_{t-1}}) + \phi_y \ln(\frac{Y_t}{\overline{Y}_t}) + \varepsilon_t, \tag{2.23}$$

where ϕ_π is the inflation targeting coefficient, ϕ_y is the output gap targeting coefficients and $\overline{Y}_t$ is Home output in absence of information stickiness. Empirically, it is hard to differentiate whether the interest rate rule is in the form of immediate adjustment with persistent shock or in the form of partial adjustment with serially uncorrelated shock. However, the consensus in the short-term interest rate smoothing literature is that there is no interest rate smoothing beyond horizons longer than two months.[15]

[14]See Woodford (1998, 2003) for an exposition of how to implement monetary policy in cashless economy.

[15]See, for example, Mankiw and Miron (1986), Goodfriend (1991), and Rudebusch (1998, 2002)

As a result, the form with persistent monetary policy shock is adopted. The monetary policy shock ε_t is assumed to follow an autoregressive process as:

$$\varepsilon_t = \rho_m \varepsilon_{t-1} + e_t, \tag{2.24}$$

where the autocorrelation ρ_m measures the degree of monetary policy shock persistence and e_t is white noise with mean zero and standard deviation σ_m.

Despite the true process defined in Equation 2.24, when a new monetary policy shock occurs, agents who receive new information in this period actually are not able to determine whether the shock is going to be persistent or transitory. That is, agents have the following perceptions about the monetary policy shock process:

$$\varepsilon_t = m_t + \nu_t, \tag{2.25}$$

$$m_t = \rho_m m_{t-1} + e_t, \tag{2.26}$$

where m_t and ν_t denote the persistent and the transitory components of monetary policy shock, respectively. ν_t is white noise with mean zero and standard deviation σ_ν. For a new shock occurs in period $t = \tau$, attentive agents perceive it to be persistent (e_τ) with a probability of η and to be transitory (ν_τ) with a probability of $1 - \eta$. By assumption, they respond according to a simple probability-weighted average rule.[16] Only agents who update information in later periods $(t > \tau)$ are able to identify the shock is persistent and respond accordingly, taking into account that there remains a fraction of agents who last update information in period $t = \tau$. As the true value of η equals to 1, η is thus a natural measure of public misperception. The lower the value of η assigned by attentive agents in period $t = \tau$, the more severe the extent of public misperception. Similar interest rate rule and assumptions apply to Foreign country.

[16] They do not consider the impact of their current responses on the decisions of agents who update information in later periods.

2.3.6 Aggregate fluctuations

In the absence of shocks and sticky information, the economy reaches its steady state. The variables are log-linearized around the properly normalized steady state up to a first-order. In terms of notation, small letters denote the log-linear deviation of the respective variables from the steady state, with the exception of r_t and r_t^*, which denote the log-linear deviation of $E_t[R_{t+1}]$ and $E_t[R_{t+1}^*]$, respectively.

2.3.6.1 Aggregate supply

Different degrees of sticky information divide firms into many heterogeneous groups. The size of each group equals to $\lambda_f(1-\lambda_f)^k$, where k implies that the group last updated information k periods ago. Therefore, the aggregate supply relations in Home and Foreign country are given by:

$$p_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[np_{H,t} + (1-n)(p_{H,t}^* + s_t) + mc_t], \tag{2.27}$$

$$p_{F,t}^* = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[n(p_{F,t} - s_t) + (1-n)p_{F,t}^* + mc_t^*], \tag{2.28}$$

with real marginal costs mc_t in Home country and mc_t^* in Foreign country defined as:

$$mc_t = \frac{(1-\xi)y_t + \xi[w_t - np_{H,t} - (1-n)(p_{H,t}^* + s_t)] - a_t}{\xi + (1-\xi)\upsilon}, \tag{2.29}$$

$$mc_t^* = \frac{(1-\xi)y_t^* + \xi[w_t^* - n(p_{F,t} - s_t) - (1-n)p_{F,t}^*)] - a_t^*}{\xi + (1-\xi)\upsilon}, \tag{2.30}$$

Consistent with conventional wisdom, output increases mc_t through decreasing returns to scale. In such non-capital setting, wage paid to workers is the only production cost. Accordingly, mc_t increases with real production wage but decreases following productivity innovations as less labor input is needed for any given level of output.[17] Intuitively, $p_{H,t}$ ($p_{F,t}^*$) depends positively on different groups of firms' expectations on

[17] Real production wage is adjusted for inflation on production while real consumption wage in following part is adjusted for inflation on consumption.

aggregate price of Home (Foreign) goods in both countries, nominal exchange rate and real marginal cost in Home (Foreign) country. However, when these factors are increased or decreased by an unexpected shock, only a random fraction (λ_f) of firms are attentive in this period and adjust their prices based on new information. The aggregate price index in Home and Foreign country are $p_t = np_{H,t} + (1-n)p_{F,t}$ and $p_t^* = np_{H,t}^* + (1-n)p_{F,t}^*$, respectively.[18]

2.3.6.2 Aggregate demand

Different degrees of sticky information also divide consumers into many groups, each with the size of $\lambda_c(1-\lambda_c)^k$. With some iterations, aggregate consumptions in both countries are:

$$c_t = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}\overline{R}_t, \tag{2.31}$$

$$c_t^* = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}\overline{R}_t^*, \tag{2.32}$$

where $\overline{R}_t = E_t(\sum_{n=0}^{\infty} r_{t+n})$ and $\overline{R}_t^* = E_t(\sum_{n=0}^{\infty} r_{t+n}^*)$ are the long-term real interest rates in Home country and Foreign country, respectively. The negative relationship implies that consumers will choose to consume less and save more when real interest rate increases. Therefore, the aggregate demand for Home country and Foreign country can be written as:

$$y_t = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}[n\overline{R}_t + (1-n)\overline{R}_t^*] - (1-n)\zeta(p_{H,t} - p_{F,t}), \tag{2.33}$$

$$y_t^* = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}[n\overline{R}_t + (1-n)\overline{R}_t^*] + n\zeta(p_{H,t}^* - p_{F,t}^*), \tag{2.34}$$

Unsurprisingly, the aggregate demand for y_t (y_t^*) is negatively related to consumers' expected cumulative future real interest rates. When consumers expect higher future

[18] $p_{H,t}^* = p_{H,t} - \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k s_{t,k}$ and $p_{F,t} = p_{F,t}^* + \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k s_{t,k}$.

real returns, they reduce consumption and increase savings. Besides, the relative price matters. When the price of Home goods is relatively higher than that of Foreign goods, the demand for Home goods decreases, and vice versa. Since only a random fraction (λ_c) of consumers have up-to-date information, an unexpected shock affects the contemporaneous aggregate demand partially.

2.3.6.3 Wage curve

Similarly, there are many heterogenous groups of workers, each with size of $\lambda_w(1-\lambda_w)^k$. Nominal wages are jointly determined by price level, output, real wages and long real interest rates:

$$w_t = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t + \frac{y_t - a_t}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t - p_t) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t], \quad (2.35)$$

$$w_t^* = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t^* + \frac{y_t^* - a_t^*}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t^* - p_t^*) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t^*], \quad (2.36)$$

We can see the workers care about their real consumption wages much, reflected by the one-for-one increase of nominal wages with expected price level. Higher output, which implies higher demand for labor input, drives nominal wages up as well. In contrast, higher expected future returns encourage workers to save and work more now. The increased labor supply lowers nominal wages. Again, only a fraction of workers, λ_w, set their wages to expectations based on updated information, so the immediate impacts of a shock on wages is limited.

2.3.6.4 Interest rate and consumption risk-sharing condition

Finally, the monetary authorities of the Home country and the Foreign country follow:

$$i_t = \phi_\pi(p_t - p_{t-1}) + \phi_y(y_t - \overline{y}_t) + \varepsilon_t, \quad (2.37)$$

$$i_t^* = \phi_\pi(p_t^* - p_{t-1}^*) + \phi_y(y_t^* - \overline{y}_t^*) + \varepsilon_t^*, \quad (2.38)$$

with $\overline{y}_t = na_t + (1-n)a_t^* + (1-n)\frac{\frac{1}{\varphi}+1}{\frac{1}{\varphi}+1-\xi+\frac{\xi}{\zeta}}(a_t - a_t^*)$ and $\overline{y}_t^* = na_t + (1-n)a_t^* - n\frac{\frac{1}{\varphi}+1}{\frac{1}{\varphi}+1-\xi+\frac{\xi}{\zeta}}(a_t - a_t^*)$.

The consumption risk-sharing condition is fairly conventional:

$$c_{t,0} - c_{t,0}^* = s_t + p_t^* - p_t = q_t, \tag{2.39}$$

2.4 Model analysis

2.4.1 Empirical parameters

Table 2.1 summarizes the parameters used in the simulation of the benchmark economy. Without any empirical reference, the probability of agents expect a new monetary policy shock to be persistent (η) is set at 0.5. Based on Chari et al. (2002), the elasticity of substitution between Home and Foreign goods (ζ) for the U.S. falls within $[1, 2]$. The median value of 1.5 is used in the simulation. For the relative size of Home country (n), 0.25 is set following Obstfeld and Rogoff (2007). Consistent with the literature, the persistence of productivity shock (ρ_a) is set to be 0.9.

The remaining parameters are set according to Reis (2009) which estimates the parameter values using the U.S. data from 1986Q3 to 2006Q1. The estimated persistence of monetary policy shock (ρ_m) in this period equals to 0.29, lower than conventionally used interest rate smoothing coefficients from quarterly postwar data.[19] This is another piece of evidence that monetary policy shock is less persistent over time. For the fraction of attentive consumers in each period (λ_c), the estimate is as low as 0.08. Reis (2009) explains the low value with the fact that 20% to 50% of Americans live hand-to-mouth, which means almost inattentive forever.[20] The fraction of attentive firms in

[19] Most studies on interest rate smoothing using post-war data suggest that the implied value of ρ_m is between 0.7 and 0.9. See Rudebusch (2002) for further discussion.

[20] In an earlier work, Reis (2006a) studies the consumption patterns of agents who face costly acquisition, absorption and processing of information, and predicts that the cost of planning makes some individuals choose to not plan and live hand-to-mouth. This prediction is generally supported by evidence using U.S. aggregate and microeconomic data.

Table 2.1: Benchmark parameter values

Parameters	Value	Description
η	0.50	Probability of a new monetary policy shock to be persistent
ζ	1.50	Elasticity of substitution between Home and Foreign goods
n	0.25	Relative size of Home country
υ	10.09	Elasticity of substitution across Home/Foreign goods
ρ_a	0.90	One-quarter autocorrelation of productivity shock
ρ_m	0.29	One-quarter autocorrelation of monetary policy shock
λ_c	0.08	Fraction of attentive consumers in each period
λ_f	0.52	Fraction of attentive firms in each period
λ_w	0.74	Fraction of attentive workers in each period
γ	9.09	Elasticity of substitution across labor services
φ	5.15	Elasticity of labor supply
ξ	0.67	Labor share of income
ϕ_π	1.17	Inflation targeting coefficient
ϕ_y	0.06	Output gap targeting coefficient

each period (λ_f) is 0.52, which implies a fairly low level of price stickiness. However, the estimate is quite consistent with the empirical findings of Bils and Klenow (2004) and Dhyne et al. (2005). The fraction of attentive workers in each period (λ_w) is 0.74, much higher than conventional values. One possibility is that total compensation are used in estimation, the nonwage component of which may actually be updated quite often.

The rest of the estimates by Reis (2009) are rather conventional. The elasticity of substitution across Home and Foreign goods (υ) equals to 10.09; the elasticity of substitution across labor services (γ) equals to 9.09; the elasticity of labor supply (φ) equals to 5.15; labor share of income (ξ) equals to 0.67; inflation targeting coefficient (ϕ_π) equals to 1.17; and output gap targeting coefficient (ϕ_y) equals to 0.06. All these estimates are in line with the empirical literature.[21]

[21] See, for example, Backus et al. (1994), Basu and Fernald (1995a,b), Basu (1996), Basu and Kimball (1997), Rudebusch (2002).

2.4.2 The benchmark economy

In this symmetrical setting, the impacts of Home country shocks on Home country are of interest. Figure 2.2 shows the responses of the real exchange rate and other macroeconomic variables such as nominal interest rate, output gap and inflation to one unit of persistent expansionary monetary policy shock and one unit of productivity innovation in the benchmark economy.[22] An unexpected expansionary monetary policy shock decreases nominal as well as real interest rates in Home country abruptly, making attentive Home consumers to consume more and attentive Home workers to charge higher nominal wage. Meanwhile, nominal exchange rate depreciates sharply. With perfect exchange rate pass-through in the prices set by attentive firms, Foreign goods as a whole become more expensive than Home goods. Based on the aggregation demand relation in Equation 2.33, higher consumption of attentive consumers and consumption toward cheaper Home goods simultaneously induce attentive Home firms to produce more, opening up a positive output gap. With great concern on real consumption wage, attentive Home workers also immediately consider the imported inflation in wage setting, which in effect increases real production wage. Therefore, real marginal cost of Home country in Equation 2.29 goes up with higher production and higher real production wage. As attentive Home firms respond to this change in price setting, the aggregate price of Home goods also increase in Home country, although much less than imported goods. Note, inattentive firms are unaware of the shock and do not respond with the resulting changes in their price setting. Specifically, the prices of their products set for overseas markets do not reflect the change in nominal exchange rate. With this particular type of price stickiness, the real exchange rate depreciates following nominal exchange rate, as explained by Dornbusch (1976). The responses of the real exchange rate and other variables are rather modest, which matches the structural VAR analysis in the same period (i.e. subsample 2).

In the second test, a positive productivity shock is considered. When there is an

[22] All the simulated impulse responses in this paper are generated by Matlab Version 7.10.

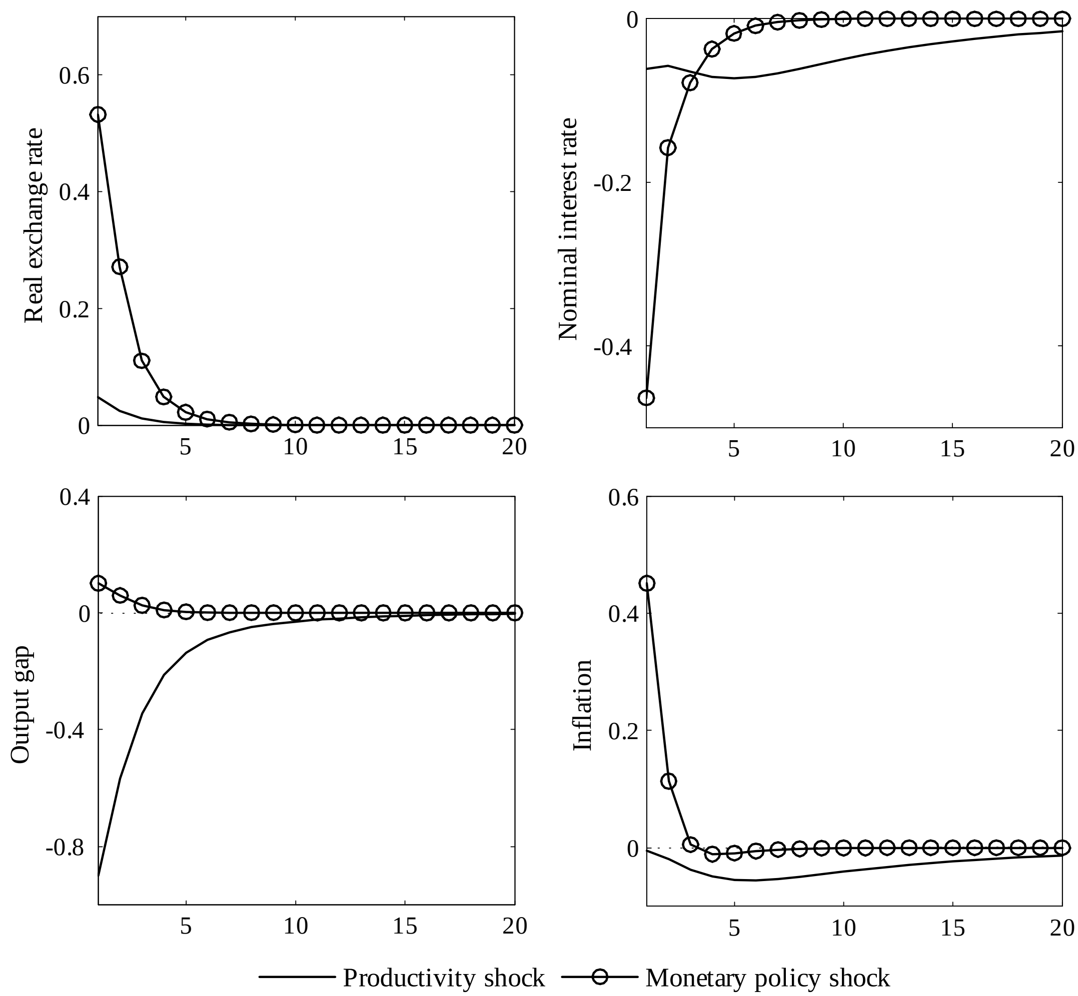

Figure 2.2: The benchmark economy

increase in productivity, less labor input is required in production for any given output level. The lower demand for labor service forces attentive Home workers to charge lower nominal wage. Both less labor input and lower nominal wage contribute to lower level of real marginal costs. Accordingly, attentive Home firms respond with charging lower price for their products. As a result, consumption switches towards cheaper products of attentive Home firms, inducing these firms to produce more. However, there is a fraction of inattentive firms who do not realize the innovation. As the production of these firms are not affected by the innovation, the aggregate output is in fact below its natural level, which means a negative output gap. This, together with deflation, leads to a fall in nominal interest rate. The efficient consumption risk-sharing as described in Equation 2.39 ensures that Foreign consumers benefit from a productivity innovation in Home country through the real exchange rate depreciation.

2.4.3 Sensitivity tests

2.4.3.1 Monetary policy shock persistence

As the interest is on monetary policy, sensitivity tests are performed on monetary policy shock only. As mentioned earlier, the empirical literature finds that the persistence level of monetary policy shock is found to be declining over time. For example, with daily federal funds rate data, Hilton (2005) shows that the average persistence value of policy spreads fell from 0.75 in 1987 to about 0.32 in 1996 and remained almost constant ever since. Nautz and Scheithauer (2010) declare long memory of daily policy spread of federal funds rate for 1984-1994, while short-memory cannot be rejected since 1994. To reflect this characteristic of monetary policy prior to the great moderation, a higher persistence level of monetary policy shock (ρ_m) is chosen in the sensitivity test.[23] As displayed in the first row of Figure 2.3, when ρ_m increases to 0.85, the responses of all variables are amplified. The conventional hump-shape in

[23] Other parameters are identical to those in the benchmark economy. This applies to all sensitivity tests.

the response of the real exchange rate also appears. The reason behind the noticeable difference is intuitive. For an initial shock of same size, higher persistence level of monetary policy shock implies the shock will diminish more slowly. Since the size of remaining shock in each period is larger, the response of informed agents will be stronger. Long-lasting strong response together with increasing fraction of informed agents over time makes it possible for response curves to display a hump-shape pattern.[24] This sensitivity test suggests that higher persistence level of monetary policy shock can be a contributor to stronger response and thus larger volatilities in many macroeconomic variables before the mid-1980s.

2.4.3.2 Sticky information in agents

Row 2, Row 3 and Row 4 of Figure 2.3 display the simulated impulse responses for fewer attentive firms ($\lambda_f = 0.25$ compared to the benchmark value 0.52), fewer attentive workers ($\lambda_w = 0.25$ compared to the benchmark value 0.74), and more attentive consumers ($\lambda_c = 1$ compared to the benchmark value 0.08) in each period, respectively. These parameter values are in line with earlier works of these types of models. Consistent with conventional wisdom, sticky information in firms, which measures the speed of nominal prices adjusting to new information, serves as an important factor in determining the response of the real exchange rate. In contrast, information stickiness in workers and consumers has negligible impact on the response of the real exchange rate. As the fraction of attentive firms (λ_f) in each period is varied from the benchmark 0.52 to 0.25, all responses display more visible changes. Intuitively, higher λ_f implies more attentive firms in each period and thus more prices set for overseas markets take into account the nominal exchange rate depreciation. Foreign goods as a whole thus become further more expensive relative to Home goods, leading to a larger increase in demand for cheaper Home goods. The induced larger output gap in Home

[24] The hump-shape pattern in the real exchange rate cannot be generated solely by increasing monetary policy shock persistence, which will be discussed later.

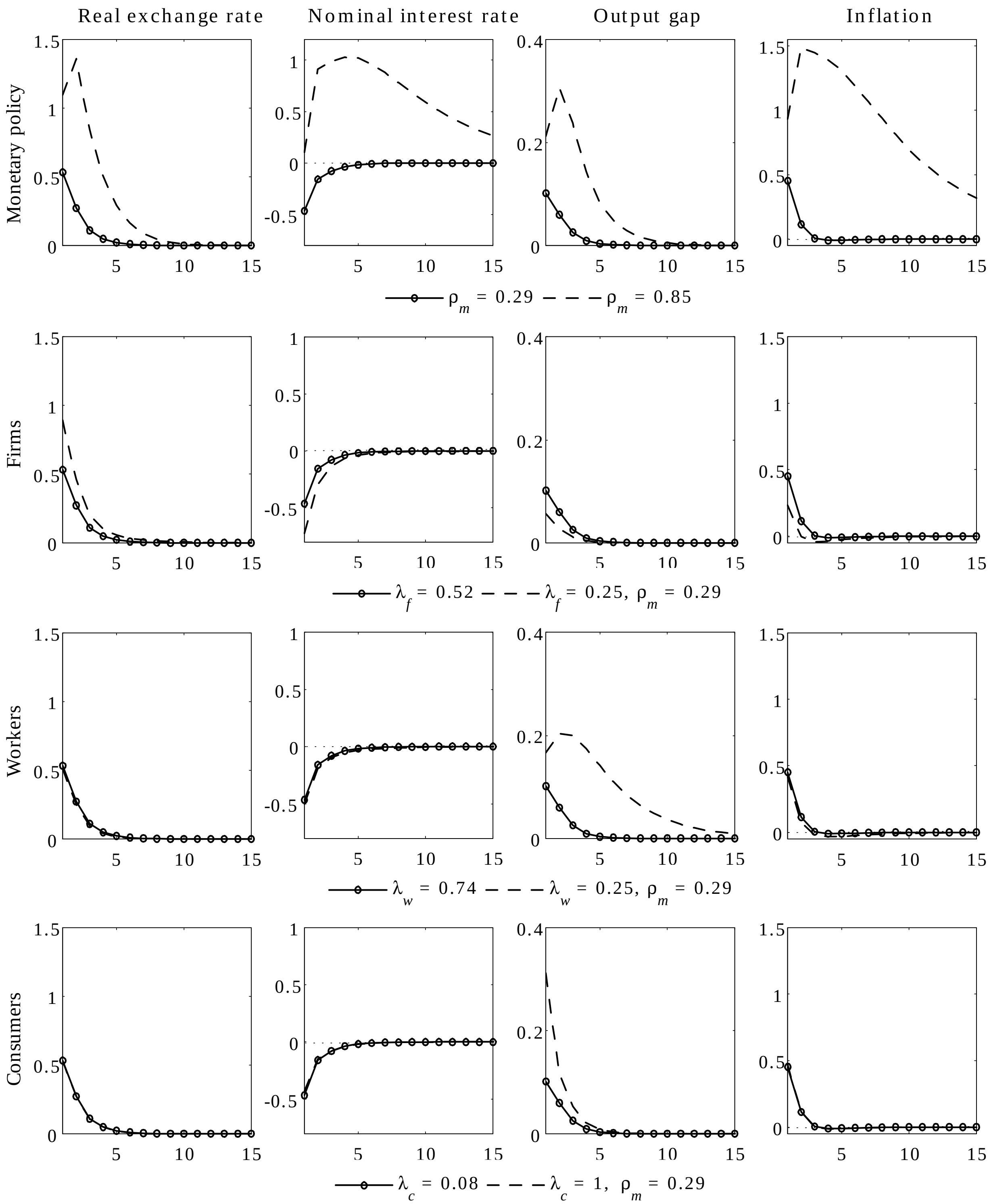

Figure 2.3: Sensitivity tests on the persistence of monetary policy shock (ρ_m) and the fractions of attentive firms (λ_f), workers (λ_w) and consumers (λ_c) in each period.

country subsequently leads to higher inflation for Home goods. With the prices of both Home goods and Foreign goods shooting up further, Home country experiences higher inflation. Particularly, with a higher degree of exchange rate pass-through in the prices, the real exchange rate accordingly depreciates less. Therefore, the documented smaller price stickiness over time may be a second possible contributor to the smaller response of the U.S. real exchange since the mid-1980s.

When the fraction of workers receiving new information (λ_w) drops to 0.25, output gap is the only distinctly affected variable. Intuitively, when λ_w is higher, more Home workers are informed about the shock and then charge higher nominal wages. The further increased real marginal cost is immediately considered by attentive Home firms in price setting. Subsequently ,deteriorated price advantage of Home goods against Foreign goods results in reduction in Home country output gap. However, inflation in Home country does not increase much, which is mainly because imported inflation due to currency depreciation dominates in current setting. With constant degree of price stickiness in the both countries, the change in the response of the real exchange rate is negligible, which is consistent with the finding of Chari et al. (2002). Similarly, when consumers are perfectly attentive, $\lambda_c = 1$, the response of the real exchange rate is essentially unchanged and only the response of output gap demonstrates visible change. Higher λ_c means that more consumers are aware of the shock in each period and adjust to consume more, inducing attentive Home firms to further increase production. Again, as imported inflation dominates, the higher inflation resulting from higher output can hardly be detected.

2.4.3.3 Public misperception

According to Gourinchas and Tornell (2004) and Hoffmann et al. (2011), that agents misperceive the true nature of monetary policy shock contributes greatly to the delayed overshooting phenomenon. Besides, less persistent monetary policy shock can dampen the delay. In this subsection, I re-examine the phenomenon by changing the parameter value of η and ρ_m correspondingly. As noted earlier, when a new monetary policy shock occurs at $t = \tau$, attentive agents are not able to identify the true nature of the shock, persistent or transitory. Therefore, these agents form their expectations by assuming the shock to be persistent with a probability of η. As the true value of η equals to 1, the value assigned to η by attentive agents in period τ is negatively correlated with the extent of public misperception. Figure 2.4 shows real exchange rate responses to one unit of expansionary monetary policy shock under different scenarios. Generally, for the same persistence level of monetary policy shock, it is easier for delayed overshooting to appear when the extent of public misperception is relatively higher (i.e. lower η). When $\eta = 0.75$, which means that the agents' expectation is fairly close to the reality, no hump-shape pattern emerges. Meanwhile, the monetary policy shock persistence is shown to be important as well. When monetary policy shock is not persistent enough (i.e. low ρ_m), delayed overshooting cannot be generated even with serious public misperception (i.e. low η).

Now I am able to explain why the real exchange rate does not respond with a hump shape in subsample 2. Besides the already discussed less persistent monetary policy shock, another possible reason is that the public are more informed than before because the Federal Reserve has improve its policy transparency over the last three decades. For example, the FOMC has released all changes in the federal funds target rate immediately to the general public since February 1994. Before that, the target rate was either unannounced or announced with a delay. Later, starting 2000, the FOMC begun to impart information about its assessment on the economic outlook. From 2003, the FOMC started to inform general public about the implications of the economic outlook on the future target federal funds rate. Such moves make monetary policy more transparent and inevitably reduce public misperception.

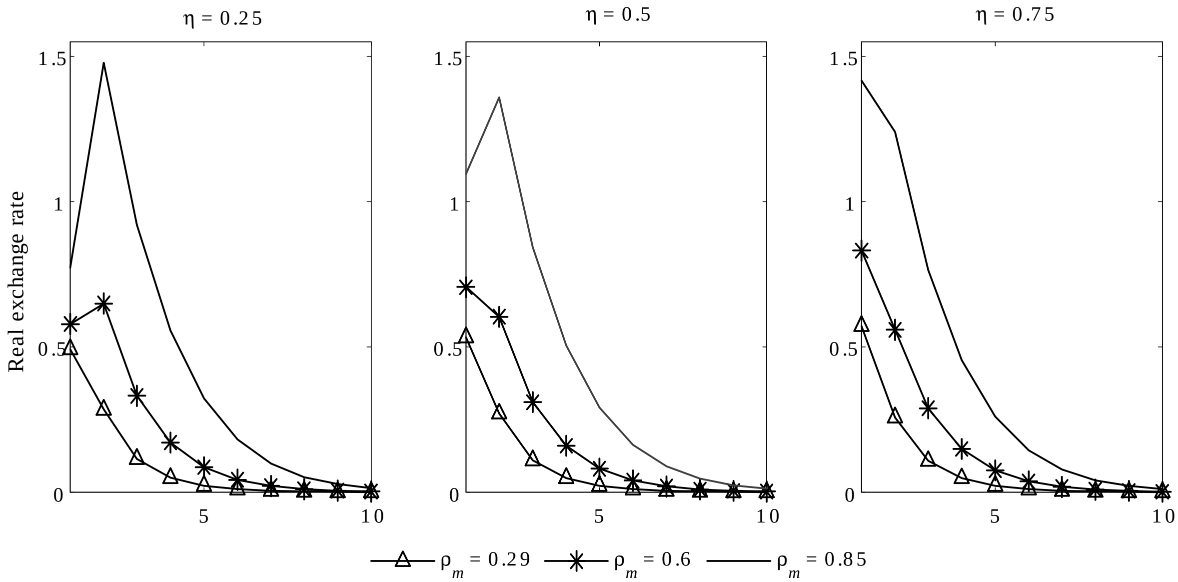

Figure 2.4: Sensitivity tests on the probability for a new monetary policy shock to be persistent (η) and the persistence of monetary policy shocks (ρ_m)

2.4.3.4 Empirical and simulated impulse responses

The sensitivity test results for changes during the great moderation period are summarized in Table 2.2, which visually displays that the weaker impact of monetary policy on the U.S. inflation and output are entirely due to less persistent monetary policy shock whereas less sticky prices also contribute to the weaker response of the U.S. real exchange rate. In addition, more transparent monetary policy, along with smaller monetary policy shock persistence, contributes to the disappear of the delayed overshooting pattern in the U.S. real exchange rate during the great moderation period. Based on these results, I try to qualitatively replicate the empirical responses of the real exchange rate in both subsamples, similar to what have been done in Boivin and Giannoni (2006). Basically, the settings in the empirical model and the theoretical model are different in a few aspects. For example, in contrast to the setting of lagged response in the structural VAR model, all the variables are allowed to respond to shocks immediately in the DSGE model. Therefore, comparing the simulated and empirical responses of the real exchange rate is relatively more meaningful, as it responds

Table 2.2: Sensitivity Test Results

	Impacts of changes during the great moderation period		
Variables/Phenomenon	Less persistent monetary policy shock	Less sticky prices	More transparent monetary policy
Inflation	↓	↑	–
Output	↓	↑	–
Real exchange rate	↓	↓	–
Delayed overshooting	↓	–	↓

Note: ↑ (↓) refers to the impact of monetary policy shock on certain variable is enhanced (weakened), whereas – means negligible change.

to shocks immediately in both frameworks.

Figure 2.5 compares simulated responses of the real exchange rate with its structural responses and associated 95% confidence intervals to an decrease of one standard deviation in interest rate/federal funds rate. The size of the impulse equals to 1.02 in subsample 1 and 0.3 in subsample 2, identical to the size of shocks identified in the structural VAR analysis. All parameter values in subsample 2 from 1986Q3 to 2006Q1 are set identical to those in the benchmark economy. The two important parameters, one-quarter autocorrelation of monetary policy shock ($\rho_m = 0.29$) and the fraction of attentive firms in each period ($\lambda_f = 0.52$), are estimated from data in the same time span. However, no estimates of the two parameters are available for subsample 1 from 1975Q1 to 1986Q2. Monetary policy shock is more persistent in the past, but how persistent? Without other options, the value of ρ_m is set arbitrarily in the simulation of subsample 1 and the value of 0.85 seems to give a good match. λ_f is set to be 0.25 in subsample 1, the implied price stickiness of which is very close to the estimate of Kashyap (1995) for the period of 1953-1987. To reflect the increasing policy transparency over time, η is decreased slightly from 0.5 to 0.4 in the subsample 1, which means higher public misperception in this period.

The right panel of Figure 2.5 shows that simulated response in subsample 2 fits the structural response of the real exchange rate well, and the left panel of Figure 2.5

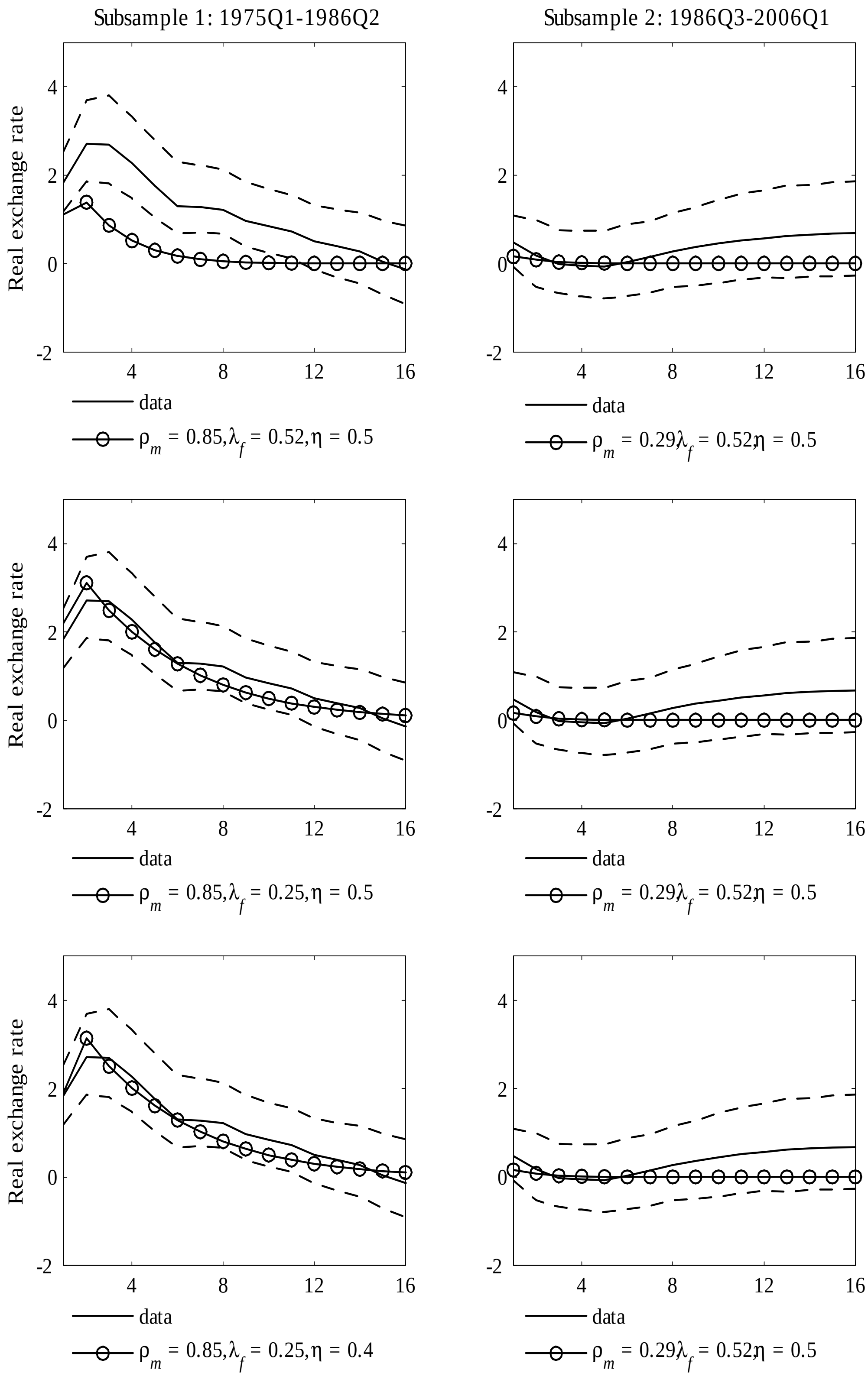

Figure 2.5: Simulated v.s. structural impulse responses, with associated 95% confidence intervals (dash line).

shows how structural response of the real exchange rate in subsample 1 is perfectly matched step by step. In the first step, the monetary policy shock persistence is increased to 0.85, with other parameters identical to the benchmark economy. Clearly, the simulated response (left of row 1) displays a delayed overshooting pattern, but is much smaller than the empirical response. In the second step, the fraction of attentive firms in each period is further decreased to 0.25. Now the simulated response (left of row 2) matches the empirical response both in terms of magnitude and response pattern. After increasing the level of public misperception by reducing η to 0.4, the two curves almost overlap with each other.

2.5 Conclusions

Many studies find that the influence of monetary policy on the U.S. economy is weaker than before. For example, Boivin and Giannoni (2002, 2006) report weaker impacts of monetary policy on inflation and output gap during the great moderation period. A structural VAR analysis is applied to a similar set of macroeconomic variables with the real exchange rate added. In line with the "good luck" hypothesis, the empirical finding shows that shocks are smaller in period from 1986Q3 to 2006Q1 (subsample 1) than in period from 1975Q1 to 1986Q2 (subsample 2). In addition, three other results from the structural VAR analysis are particularly noteworthy. First, after taking into account the fact of smaller shock size, the impact of monetary policy on the real exchange rate and other variables is found to be weaker in subsample 2. Second, the impact in subsample 2 is insignificant. Last, delayed overshooting in the real exchange rate is only found in subsample 1. These changes direct to the fact of different responses to monetary policy shock during the great moderation.

Based on empirical evidence of changes in monetary policy and price stickiness, several sensitivity tests are performed. Monetary policy shock persistence is found to be the major source of the changing monetary policy impact between the two subsamples, especially for output and inflation. Smaller price stickiness makes the real exchange rate more stable but output and inflation slightly more volatile. The benchmark sticky information DSGE model replicates well the responses of the real exchange rate in subsample 2. With more persistent monetary policy shock and fewer attentive firms in each period, the model replicates exactly the U.S. real exchange rate's response in subsample 1. These findings overall suggest that when monetary authorities are better informed and more agents are able to receive new information in each period, the volatility of the real exchange rate and other macroeconomic variables can be stabilized. The success of reproducing delayed overshooting in the real exchange rate is completely due to the introduction of public misperception. It is re-confirmed that delayed overshooting can be generated when public misperception about monetary policy shock persistence is severe and monetary policy shock is sufficiently persistent.

Chapter 3

Productivity shocks and persistent real exchange rate fluctuations

3.1 Introduction

It is a consensus in empirical literature that the revisions of real exchange rates back to their PPP values take an extremely long time, known as the PPP puzzle in international macroeconomics literature. According to Rogoff (1996), the deviations damp out around 15% per year, which suggests a half-life of 3 to 5 years. After using the Hodrick-Prescott (HP) filter, the one-quarter autocorrelation of the deviations is still around 0.8, e.g., Bergin and Feenstra (2001) and Chari et al. (2002). Table 3.1 reports the properties of CPI-adjusted bilateral U.S. real exchange rates against major OECD countries (excluding those in the Euro Area) in the period of 1986Q3-2006Q1.[1] The statistics are based on logged and HP filtered quarterly data. Obviously, the bilateral real exchange rates are as persistent as before and more volatile than output over the same period.

An enormous body of literature attempts to investigate and replicate the persis-

[1] All the data used in the calculation of the bilateral real exchange rates are obtained from International Financial Statistics (IFS) database. The sample period is chosen to be consistent with later simulations, where the parameter values used in the benchmark economy are estimated by Reis (2009) using quarterly U.S. data from 1986Q3 to 2006Q1.

Table 3.1: Properties of real exchange rates (RER)

RER	ρ_{RER}	$\sigma_{RER}/\sigma_{Y^{U.S.}}$
U.S./Australia	0.84	7.31
U.S./Canada	0.81	3.51
U.S./Denmark	0.77	7.18
U.S./Japan	0.78	7.94
U.S./Norway	0.75	6.44
U.S./Sweden	0.82	8.50
U.S./Switzerland	0.76	7.44
U.S./U.K.	0.75	6.46

Note: ρ_{RER} refers to one-quarter autocorrelation of the real exchange rate. $\sigma_{RER}/\sigma_{Y^{U.S.}}$ represents the standard deviation of the real exchange rate relative to the standard deviation of the U.S. output.

tence of the real exchange rate. Most of the studies use the sticky price model as the theoretical framework, with slow adjustment of nominal prices playing an essential role. Bergin and Feenstra (2001) show how long-lasting contracts (8 to 12 quarters) are needed for monetary shock to generate the observed persistence. Chari et al. (2002) assume that prices are fixed for one year, but the simulated real exchange rate is still less persistent than historical data, even after including highly persistent technology shock and fairly sticky wages. A few modifications are then introduced to standard sticky price model, such as high monetary policy inertia and relative price stickiness across countries in Benigno (2004), an aggregate of persistent Phillips curve shocks in Steinsson (2008), and imperfect information in Hoffmann et al. (2011). These amended models generally perform very well by assuming quite sticky prices such that 25% or less of the firms can reset prices in each quarter. However, while real exchange rates are as persistent as before, prices are found to be much less sticky in recent decades, such as in Bils and Klenow (2004) for the U.S. and Dhyne et al. (2005) for the Euro area.

The aim of this chapter is to explore whether a two-country sticky information DSGE model can generate highly persistent real exchange rate as in the data. I extend the sticky information general equilibrium model of Mankiw and Reis (2006) into a two-country framework and further categorize the goods into tradable and non-tradable. The benchmark model manages to reproduce the observed real exchange rate persistence with persistent productivity shock, nonpersistent monetary policy shock and fairly flexible prices. Subsequent sensitivity tests show that the generated persistence in the benchmark economy originates from persistent productivity shocks. Given persistent productivity shocks, higher stickiness in prices and wages makes the real exchange rate less persistent, which is quite surprising. When productivity shocks are not persistent, monetary policy shock becomes dominant. In this case, the level of the real exchange rate persistence is positively associated with the degree of price stickiness as conventionally believed, whereas the impact of wage stickiness is ambiguous in direction but negligible in magnitude. In all the tests, stickiness in consumption has no obvious and clear impact on the level of the real exchange rate persistence. The finding is reasonable as consumption only indirectly influences the real exchange rate by affecting prices and wages in the goods market.

Analytical analysis suggests that the success of the sticky information model in reproducing persistent real exchange rate may mainly attribute to its price-setting and wage-setting assumption. As introduced earlier, firms in the sticky information model reset prices to expected optimal levels in each period, either with updated information or outdated information. With this presumption, firms adequately consider shocks in their information sets when adjusting prices. Apparently, inattentive firms would not consider in pricing shocks that occurred after their last update, as they are not in their information sets. In the sticky price model, prices are either fixed or expected to be fixed for N periods. When firms have the opportunity to adjust price, they reset price to a weighted average of current and expected future optimal levels based on information of that period. The weight assigned to expected future optimal price declines exponentially toward further future. Firms who do not receive any re-pricing signal have to keep prices set in previous periods constant. Similar to prices set by inattentive firms in the sticky information model, these "constant" prices cannot reflect the impact of shocks occurring after the price-setting period. Moreover, they cannot even fully reflect the impact of expected shocks based on information of the price-setting period. Generally, the more remote the price was set, the more different the price is from its expected optimal level, and thus the more dampened the impact of shocks. The difference in price-setting rules also applies to wage-setting rules in two models. Therefore, with other settings identical, the impact of shocks and the pass-through of their persistence to the real exchange rate and other variables is higher in the sticky information model. One strong supporting evidence of this argument is that the sticky information model in this paper can simulate highly persistent real exchange rate without productivity shocks when the one-quarter autocorrelation of monetary policy shock and price stickiness are set in line with those used in standard sticky price model, whereas standard sticky model cannot.

The unconventional finding that higher stickiness in prices and wages makes the real exchange rate less persistent when productivity shocks dominate is also explored. One plausible interpretation is that productivity shocks and monetary policy shocks affect the economy through different propagation mechanisms. When productivity shock occurs, price and wage are affected directly. If all the prices and wages can be adjusted against emerging productivity shocks in each period, the real exchange rate will be almost as persistent as productivity shock.[2] Therefore, price stickiness and wage stickiness in fact dampen the pass-through of productivity shock and its persistence to the economy. That is, more sticky prices and wages lead to less persistent real exchange rate. In contrast, monetary policy shock affects the nominal exchange rate and wage first and then price changes subsequently. As the dampening effect of price stickiness is indirect and negligible, it serves an important source of persistence in the economy. Wage stickiness enhances price stickiness on one hand, but directly limits the pass-through of monetary policy shocks to the economy on the other hand.

[2] The dampening impact of sticky consumption is indirect and thus negligible.

Therefore, its impact on the real exchange rate persistence is not clear-cut. This interpretation suggests, besides the price-setting rule, highly sticky prices and wages may be a second reason for the failure of persistent productivity shock to reproduce observed real exchange rate persistence in standard sticky price model.

The Chapter is organized as follows: Section 2 discusses the structure of the theoretical model; Section 3 displays and interprets the simulated impulse responses in the benchmark economy and simulation results of sensitivity tests; Section 4 concludes.

3.2 The Model

The world consists of two countries, Home and Foreign. They have equal size and identical households who lived infinitely. Therefore, the settings are symmetric in the two countries, including home bias in consumption of domestic tradable goods. In each country, the monetary authority can enforce the use of a unit of account, denoted as "currency" in the model. Consumers, firms, and workers keep the economy running, and the monetary authority governs the economy by affecting nominal interest rate. In the labor markets, each worker is endowed with distinctive labor service and sells it to local firms; in the goods markets, each monopolistic firm produces and sells one differentiated good to consumers; in the incomplete but competitive international bonds market, consumers trade two uncontingent one-period nominal bonds among one another and earn nominal interests. One bond is denominated in Home currency and the other in Foreign currency. The monetary authority adjusts nominal interest rate of its country by intervening in the bonds market. ζ of Home firms produce an array of differentiated tradable goods indexed by the interval $[0, \zeta]$, and the rest produce an array of differentiated non-tradable goods indexed by the interval $(\zeta, 1]$. Similarly, Foreign's tradable goods are indexed by the interval $(1, 1 + \zeta]$, and the remaining Foreign non-tradable goods are indexed by the interval $(1 + \zeta, 2]$. Nominal exchange rate S_t is defined as the Home-currency price of Foreign currency. And the real exchange rate Q_t equals to $S_t P_t^*/P_t$, where P_t and P_t^* are consumer price index in Home country and Foreign country ($*$ denotes Foreign variables throughout), respectively. Terms of trade is conventionally defined as $\tau_t = P_{F,t}/P_{H,t}$.

3.2.1 Consumers

A typical Home country household j has the following utility function in each period:

$$U(C_t^j, L_t^j) = \ln C_t^j - \frac{\varkappa L_t^{j1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \tag{3.1}$$

where C_t^j is total consumption of household j, $\varkappa$ captures relative preferences for leisure, φ is the Frisch elasticity of labor supply, and L_t^j is the labor supplied by household j. The consumption index is defined as:

$$C_t^j = [\zeta^{\frac{1}{\eta}} C_{T,t}^{j\frac{\eta-1}{\eta}} + (1-\zeta)^{\frac{1}{\eta}} C_{N,t}^{j\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}, \tag{3.2}$$

where $C_{T,t}^j$ and $C_{N,t}^j$ are consumptions of tradable goods and non-tradable goods, respectively, and η is the constant elasticity of substitution between tradable goods and non-tradable goods. The consumption of tradable goods follows:

$$C_{T,t}^j = [\omega^{\frac{1}{\theta}} C_{H,t}^{j\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} C_{F,t}^{j\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \tag{3.3}$$

where $C_{H,t}^j = [(\frac{1}{\zeta})^{\frac{1}{\upsilon}} \int_0^\zeta C_{H,t,i}^{j\frac{\upsilon-1}{\upsilon}} di]^{\frac{\upsilon}{\upsilon-1}}$ and $C_{F,t}^j = [(\frac{1}{\zeta})^{\frac{1}{\upsilon}} \int_1^{1+\zeta} C_{F,t,i^*}^{j\frac{\upsilon-1}{\upsilon}} di^*]^{\frac{\upsilon}{\upsilon-1}}$, ω is the share of domestic tradable goods, and θ is the constant elasticity of substitution between Home tradable goods and imported goods. $C_{H,t,i}^j$ is the consumption of Home goods i, C_{F,t,i^*}^j is the consumption of Foreign goods i^*, and υ is the constant elasticity of substitution amongst Home tradable goods and imported goods. The consumers have home bias in consumption of tradable goods when $\omega > 1/2$.[3] The consumption of

[3]Foreign consumers have analogous preference in the consumption of tradable goods as $C_{T,t}^{j^*} = [(1-\omega)^{\frac{1}{\theta}} C_{H,t}^{j^*\frac{\theta-1}{\theta}} + \omega^{\frac{1}{\theta}} C_{F,t}^{j^*\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}$ where $C_{H,t}^{j^*} = [(\frac{1}{\zeta})^{\frac{1}{\upsilon}} \int_0^\zeta C_{H,t,i}^{j^*\frac{\upsilon-1}{\upsilon}} di]^{\frac{\upsilon}{\upsilon-1}}$ and $C_{F,t}^{j^*} =$

tradable goods is defined as:

$$C^j_{N,t} = [(\frac{1}{1-\zeta})^{\frac{1}{\upsilon}} \int_{\zeta}^{1} C^{j\,\frac{\upsilon-1}{\upsilon}}_{N,t,i}\, di]^{\frac{\upsilon}{\upsilon-1}}, \tag{3.4}$$

where υ also serves as the constant elasticity of substitution amongst Home non-tradable goods. The structure of the consumption preferences implies:

$$C^j_{H,t,i} = \omega(\frac{P_{H,t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_{T,t}})^{-\theta}(\frac{P_{T,t}}{P_t})^{-\eta}C^j_t, \tag{3.5}$$

$$C^j_{F,t,i^*} = (1-\omega)(\frac{P_{F,t,i^*}}{P_{F,t}})^{-\upsilon}(\frac{P_{F,t}}{P_{T,t}})^{-\theta}(\frac{P_{T,t}}{P_t})^{-\eta}C^j_t, \tag{3.6}$$

$$C^j_{N,t,i} = (\frac{P_{N,t,i}}{P_{N,t}})^{-\upsilon}(\frac{P_{N,t}}{P_t})^{-\eta}C^j_t, \tag{3.7}$$

The consumption preferences also imply that P_t should follow the form of:

$$P_t = [\zeta P^{1-\eta}_{T,t} + (1-\zeta)P^{1-\eta}_{N,t}]^{\frac{1}{1-\eta}}, \tag{3.8}$$

with the Home goods price index $P_{H,t}$ and Foreign goods price index $P_{F,t}$ given by:

$$P_{T,t} = [\omega P^{1-\theta}_{H,t} + (1-\omega)P^{1-\theta}_{F,t}]^{\frac{1}{1-\theta}}, \tag{3.9}$$

$$P_{N,t} = [\frac{1}{1-\zeta}\int_{\zeta}^{1} P^{1-\upsilon}_{N,t,i} di]^{\frac{1}{1-\upsilon}}, \tag{3.10}$$

where $P_{H,t} = [\frac{1}{\zeta}\int_0^{\zeta} P^{1-\upsilon}_{H,t,i} di]^{\frac{1}{1-\upsilon}}$, $P_{F,t} = [\frac{1}{\zeta}\int_1^{1+\zeta} P^{1-\upsilon}_{F,t,i^*} di^*]^{\frac{1}{1-\upsilon}}$, $P_{H,t,i}$ is Home-currency price of Home goods i, and P_{F,t,i^*} is Home-currency price of Foreign goods i^*.

Given the utility function, consumer j make decisions subject to budget constraint:

$$P_t C^j_t + B^j_t + S_t B^{*j}_t = W^j_t L^j_t + (1+i_{t-1})B^j_{t-1} + (1+i^*_{t-1})S_t B^{*j}_{t-1} + T^j_t, \tag{3.11}$$

where B^j_t and B^{*j}_t are holdings of the Home-currency and Foreign-currency denominated bonds entering $t+1$, respectively, i_{t-1} and i^*_{t-1} are nominal interest rates earned

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$[(\frac{1}{\zeta})^{\frac{1}{\upsilon}} \int_1^{1+\zeta} C^{j^*\,\frac{\upsilon-1}{\upsilon}}_{F,t,i^*}\, di^*]^{\frac{\upsilon}{\upsilon-1}}$.

on B^j_{t-1} and B^{*j}_{t-1} at date t, respectively, and finally, T^j_t are lump-sum nominal transfers received by household j. Sources of the transfers include firm profits equally distributed to all households, and insurance contracts signed by all households at the beginning of each period. The payment from the insurance contract makes everybody start each period with the same wealth.

Let us denote $F^j_t \equiv [W^j_t L^j_t + (1+i_{t-1})B^j_{t-1} + (1+i^*_{t-1})S_t B^{*j}_{t-1} + T^j_t]/P_t$ as the real resources with which Home consumer j enters period t. The assumption of perfect insurance implies that $F^j_t = F_t$, identical for all consumers. $V(F_t, .)$ refers to the value function for consumers that update information at period t, taking other state variables in the second argument as given. An attentive consumer at period t solves:

$$\begin{aligned} V(F_t) = & \max_{\{C_{t+m,m}\}} E_t[\sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m U(C_{t+m,m}, .) \\ & + \beta\lambda_c \sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m V(F_{t+m+1})] \\ \text{s.t. } & F_{t+m+1} = R_{t+m+1}(F_{t+m} - C_{t+m,.}) + \frac{W_{t+m+1,.}L_{t+m+1,.} + T_{t+m+1}}{P_{t+m+1}}, \\ & m \geq 0 \text{ and a no-Ponzi condition} \end{aligned}$$

where $\beta \in (0,1)$ is discounting factor and R_{t+1} is the real return on nominal bonds[4]. The first term in the equation is the expected discounted utility if the consumer never receive new information again. The second term is the sum of the continuation values that the consumer updates new information again at a certain future period, each occurring with a probability $\lambda_c(1-\lambda_c)^m$.

According to the first-order conditions, an attentive consumer plans his consumption following the standard Euler equation:

$$C^{-1}_{t,0} = \beta E_t[R_{t+1} C^{-1}_{t+1,0}], \tag{3.12}$$

[4] Competitive international bonds market leads to identical expected real return identical on Home currency and Foreign currency bonds.

For a consumer with information updated k periods ago, consumption $C_{t,k}$ is determined by the condition:

$$C_{t,k}^{-1} = E_{t-k}[C_{t,0}^{-1}], \tag{3.13}$$

which shows that an inattentive consumer sets the marginal utility of consumption ($C_{t,k}^{-1}$) equal to his expectation of the marginal utility of an attentive consumer ($C_{t,0}^{-1}$).[5] With symmetrical setting, foreign consumers decide their consumption similarly.

3.2.2 Workers

Labor inputs for any firm in the Home country, $L_{H,t,i}$ or $L_{N,t,i}$, consist of various labor inputs, $L_{H,t,i}^{j}$, with an elasticity of substitution γ:

$$L_{H,t,i} = [\int_0^1 L_{H,t,i}^{j\,\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}},\ L_{N,t,i} = [\int_0^1 L_{N,t,i}^{j\,\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}}, \tag{3.14}$$

Summing over all firms, the labor market for labor j thus clears as:

$$L_t^j = (\frac{W_t^j}{W_t})^{-\gamma} L_{H,t} + (\frac{W_t^j}{W_t})^{-\gamma} L_{N,t}, \tag{3.15}$$

where $L_{H,t} = \int_0^{\zeta} L_{H,t,i} di$ and $L_{N,t} = \int_{\zeta}^1 L_{N,t,i} di$ are Home country's aggregate demand for labor inputs in the tradable sector and non-tradable sector, respectively. In equilibrium, $L_t = [\int_0^1 L_t^{j\,\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}} = L_{H,t} + L_{N,t}$.

With the hiring rule above, aggregate wage index is given by:

$$W_t = [\int_0^1 W_t^{j\,1-\gamma} dj]^{\frac{1}{1-\gamma}}, \tag{3.16}$$

Similar to an attentive consumer, an attentive worker at period t solves the dynamic

[5] See Mankiw and Reis (2006) or the appendix for more algebra details.

program:

$$
\begin{aligned}
V(F_t) = & \max_{\{W_{t+m,m}\}} E_t[\sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m U(L_{t+m,m}, .) \\
& + \beta\lambda_w \sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m V(F_{t+m+1})] \\
\text{s.t. } F_{t+m+1} = & R_{t+m+1}(F_{t+m} - C_{t+m,.}) + \frac{W_{t+m+1,.}L_{t+m+1,.} + T_{t+m+1}}{P_{t+m+1}}, \\
F_{t+m} = & \frac{W_{t+m,.}L_{t+m,.} + (1+i_{t+m-1})B_{t+m-1} + (1+i^*_{t+m-1})S_t B^*_{t+m-1} + T_{t+m}}{P_{t+m}}, \\
L_{t+m,.} = & \frac{1}{n}(\frac{W_{t+m,.}}{W_{t+m}})^{-\gamma} L_{t+m}, \\
& m \geq 0 \text{ and a no-Ponzi condition.}
\end{aligned}
$$

The first-order optimality conditions for workers are obtained as below:

$$\frac{L_{t,0}^{\frac{1}{\varphi}} P_t}{W_{t,0}} = E_t[\frac{\beta R_{t+1} L_{t+1,0}^{\frac{1}{\varphi}} P_{t+1}}{W_{t+1,0}}], \tag{3.17}$$

$$W_{t,0} = \frac{\gamma}{\gamma-1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{C_{t,0}^{-1}/P_t}, \tag{3.18}$$

$$W_{t,k} = \frac{E_{t-k}[L_{t,k}^{1+\frac{1}{\varphi}}]}{E_{t-k}[L_{t,0}^{\frac{1}{\varphi}} L_{t,k}/W_{t,0}]}, \tag{3.19}$$

Equation 3.17 is the standard Euler equation for an attentive worker. Equation 3.18 implies that an attentive worker, with some monopolistic power, set the wage with a fixed markup ($\frac{\gamma}{\gamma-1}$) over the marginal opportunity cost of labor, which equals to the marginal disutility of working ($\varkappa L_{t,0}^{\frac{1}{\varphi}}$) divided by the marginal utility per dollar of consumption ($C_{t,0}^{-1}/P_t$). Equation 3.19 states that an inattentive worker sets wages so that his marginal disutility of working ($\varkappa L_{t,k}^{\frac{1}{\varphi}}/W_{t,k}$) equals to his expected marginal disutility of working of an attentive worker ($\varkappa L_{t,0}^{\frac{1}{\varphi}}/W_{t,0}$).

3.2.3 Firms

Firms in the tradable sector are assumed to have identical production functions. For a Home firm i in the tradable sector, the production function is:

$$Y_{H,t,i} = A_{H,t} L^{\xi}_{H,t,i}, \tag{3.20}$$

where $A_{H,t}$ is aggregate productivity in the tradable sector, and the corresponding productivity shock, $a_{H,t} = \ln A_{H,t}$, follows an $AR(1)$ process $a_{H,t} = \rho_T a_{H,t-1} + \mu_{H,t}$ where $\mu_{H,t}$ is white noise with mean zero and standard deviation σ_H. $\xi \in (0,1)$ measures the labor share of income.

Similarly, the production function for a Home firm i in non-tradable sector is:

$$Y_{N,t,i} = A_{N,t} L^{\xi}_{N,t,i}, \tag{3.21}$$

where $A_{N,t}$ is aggregate productivity in the non-tradable sector, and the corresponding productivity shock, $a_{N,t} = \ln A_{N,t}$, follows an $AR(1)$ process $a_{N,t} = \rho_N a_{N,t-1} + \mu_{N,t}$ where $\mu_{N,t}$ is white noise with mean zero and standard deviation σ_N. Correspondingly, the aggregate output in the tradable sector ($Y_{H,t}$) and non-tradable sector ($Y_{N,t}$) are given by $Y_{H,t} = \int_0^{\zeta} Y_{H,t,i} di$ and $Y_{N,t} = \int_{\zeta}^1 Y_{N,t,i} di$, respectively.[6]

For a sale department last updating its information k periods ago, the objective is to maximize expected profits. If the firm produces a tradable good, it will:

$$\max E_{t-k}[P_{H,t,k,i} Y_{H,t,k,i} - W_t L_{H,t,k,i}],$$

$$\begin{aligned} \text{s.t.} Y_{H,t,k,i} &= A_{H,t} L^{\xi}_{H,t,k,i}, \\ Y_{H,t,k,i} &= \omega \left(\frac{P_{H,t,k,i}}{P_{H,t}}\right)^{-\upsilon} \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{-\eta} C_t \end{aligned}$$

[6] Defining aggregate output in both sectors as $Y_{H,t} = [(\frac{1}{\zeta})^{\frac{1}{\upsilon}} \int_0^{\zeta} Y_{H,t,i}^{\frac{\upsilon-1}{\upsilon}} di]^{\frac{\upsilon}{\upsilon-1}}$ and $Y_{N,t} = [(\frac{1}{1-\zeta})^{\frac{1}{\upsilon}} \int_{\zeta}^1 Y_{N,t,i}^{\frac{\upsilon-1}{\upsilon}} di]^{\frac{\upsilon}{\upsilon-1}}$ instead leads to the same results, up to a first-order log-linear approximation. The same logic applies to the expression of aggregate output Y_t.

$$+(1-\omega)(\frac{P_{H,t,k,i}}{S_{t,k}P^*_{H,t}})^{-\upsilon}(\frac{P^*_{H,t}}{P^*_{T,t}})^{-\theta}(\frac{P^*_{T,t}}{P^*_t})^{-\eta}C^*_t,$$

where $C_t = \int_0^1 C_t^j dj$ and $C_t^* = \int_0^1 C_t^{j^*} dj^*$ are the aggregate consumption of Home country and Foreign country, respectively, and $S_{t,k} = E_{t-k}[S_t]$.

The first-order condition with respect to $P_{H,t,k,i}$ is:

$$P_{H,t,k,i} = \frac{\upsilon}{\upsilon-1}\frac{E_{t-k}[A_{H,t}^{-\frac{1}{\xi}}Y_{H,t,k,i}^{\frac{1}{\xi}}W_t]}{E_{t-k}[\xi Y_{H,t,k}]}, \tag{3.22}$$

Analogously, prices of non-tradable goods are set according to:

$$P_{N,t,k,i} = \frac{\upsilon}{\upsilon-1}\frac{E_{t-k}[A_{N,t}^{-\frac{1}{\xi}}Y_{N,t,k,i}^{\frac{1}{\xi}}W_t]}{E_{t-k}[\xi Y_{N,t,k,i}]}, \tag{3.23}$$

We can see that monopolistic firms set nominal prices with a fixed markup ($\frac{\upsilon}{\upsilon-1}$) over nominal marginal costs, which equal to nominal wage (W_t) divided by the marginal product of the composite labor input ($\xi A_{H,t}^{\frac{1}{\xi}}Y_{H,t,k,i}^{1-\frac{1}{\xi}}$ or $\xi A_{N,t}^{\frac{1}{\xi}}Y_{N,t,k,i}^{1-\frac{1}{\xi}}$). Similar production functions and price-setting rules apply in Foreign country.

3.2.4 Monetary policy and consumption risk sharing

Although monetary policy is not the focus of this chapter, the model is closed by assuming a cashless economy where the monetary authority adjusts its nominal interest rate by issuing nominal bonds and intervening in the bonds market.[7] Monetary policy is assumed to follow the Taylor rule in both counties:

$$i_t = \phi_y \ln(\frac{Y_t}{\overline{Y}_t}) + \phi_\pi \ln(\frac{P_t}{P_{t-1}}) + \varepsilon_t, \tag{3.24}$$

where ϕ_π is the inflation targeting coefficient, ϕ_y is the output gap targeting coefficients, $Y_t = Y_{H,t} + Y_{N,t}$ is Home country aggregate output, $\overline{Y}_t$ is Home country

[7] For details, see Woodford (1998, 2003).

aggregate output in absence of information stickiness, ε_t is monetary policy shock that follow an $AR(1)$ process: $\varepsilon_t = \rho_m \varepsilon_{t-1} + e_t$ where e_t is white noise with mean zero and standard deviation σ_m. Similar interest rule applies to Foreign country.

In the incomplete but competitive international bonds market, Home and Foreign consumers can exchange their nominal bonds freely, which makes the expected real return, $E_t[R_{t+1}]$, identical for both bonds. Therefore, the uncovered interest rate parity (UIP) must hold:

$$1 + i_t = \frac{E_t[S_{t+1}]}{S_t}(1 + i_t^*), \tag{3.25}$$

With UIP condition and Euler equations in the Home country and the Foreign country, the optimal risk-sharing condition holds as follows:

$$\frac{C_{t,0}}{C_{t,0}^*} = \frac{E_t[Q_t C_{t+1,0}^{*-1}]}{E_t[Q_{t+1} C_{t+1,0}^{-1}]}, \tag{3.26}$$

This equation is identical to standard international consumption risk-sharing condition with complete bonds, up to a first-order log-linear approximation and iteration.

3.2.5 The log-linearized model

The model is log-linearized around a properly normalized steady state, up to a first-order. Small letters denote log-linear deviations of respective variables, with the exception of r_t and r_t^* denoting log-linear deviations of $E_t[R_{t+1}]$ and $E_t[R_{t+1}^*]$, respectively.

3.2.5.1 Aggregate supply

As sticky information divides the firms into many groups, the Home aggregate supply relations in the tradable sector and non-tradable sector are given by:

$$p_{H,t} = \lambda_f \sum_{k=0}^{\infty} (1 - \lambda_f)^k E_{t-k}[\omega p_{H,t} + (1 - \omega) p_{H,t}^* + +(1 - \omega) s_t + mc_{H,t}], \tag{3.27}$$

$$p_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}(p_{N,t} + mc_{N,t}), \tag{3.28}$$

with real marginal costs in Home tradable sector and non-tradable sector defined as:

$$mc_{H,t} = \frac{\xi[w_t - \omega p_{H,t} - (1-\omega)(p^*_{H,t} + s_t)] - a_{H,t} + (1-\xi)y_{H,t}}{\xi + (1-\xi)\upsilon}, \tag{3.29}$$

$$mc_{N,t} = \frac{\xi(w_t - p_{N,t}) - a_{N,t} + (1-\xi)y_{N,t}}{\xi + (1-\xi)\upsilon}, \tag{3.30}$$

In this capital free model, expenditure on labor input serves as the only production cost. Hence, mc_t goes up with sector real production wage but falls with productivity innovations as less labor input is required for any given output level. As usual, mc_t increases with output because of decreasing returns to scale.[8] Equation 3.27 shows that $p_{H,t}$ depends positively on different groups' expectations on $p_{H,t}$ in Home country and Foreign country, nominal exchange rate and real marginal cost in the tradable sector. Similarly but more simply, $p_{N,t}$ is only determined by different groups' expectations on itself and real marginal cost in the non-tradable sector. Note, only the fraction λ_f of attentive firms respond to new shocks immediately.

Foreign country have similar equations:

$$p^*_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[(1-\omega)(p_{F,t} - s_t) + \omega p^*_{F,t} + +mc_{F,t}], \tag{3.31}$$

$$p^*_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}(p^*_{N,t} + mc^*_{N,t}), \tag{3.32}$$

with Foreign real marginal costs in the tradable and non-tradable sectors equal to:

$$mc^*_{F,t} = \frac{\xi[w^*_t - (1-\omega)(p_{F,t} - s_t) - \omega p^*_{F,t}] + (1-\xi)y^*_{F,t} - a^*_{F,t}}{\xi + (1-\xi)\upsilon}, \tag{3.33}$$

[8]Sector real production wage is adjusted for inflation on sector production while real consumption wage in later part is adjusted for inflation on aggregate consumption.

$$mc^*_{N,t} = \frac{\xi(w^*_t - p^*_{N,t}) + (1-\xi)y^*_{N,t} - a^*_{N,t}}{\xi + (1-\xi)\upsilon}, \tag{3.34}$$

Accordingly, aggregate prices in Home country and Foreign country are defined as:

$$p_t = \zeta[\omega p_{H,t} + (1-\omega)p_{F,t}] + (1-\zeta)p_{N,t}, \tag{3.35}$$

$$p^*_t = \zeta[(1-\omega)p^*_{H,t} + \omega p^*_{F,t}] + (1-\zeta)p^*_{N,t}, \tag{3.36}$$

where $p_{F,t} = p^*_{F,t} + \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k S_{t,k}$ and $p^*_{H,t} = p_{H,t} - \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k S_{t,k}$.

3.2.5.2 Aggregate demand

With different groups of consumers, aggregate consumptions in Home country and Foreign country are:

$$c_t = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}\overline{R}_t, \tag{3.37}$$

$$c^*_t = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}\overline{R}^*_t, \tag{3.38}$$

where $\overline{R}_t = E_t[\sum_{n=0}^{\infty} r_{t+n}]$ and $\overline{R}^*_t = E_t[\sum_{n=0}^{\infty} r^*_{t+n}]$, are long-term real interest rates in Home country and Foreign country, respectively.

Consequently, aggregate demands for Home tradable goods and non-tradable goods, respectively, are given by:

$$y_{H,t} = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}[\omega\overline{R}_t + (1-\omega)\overline{R}^*_t] - 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[\omega(p_{T,t} - p_t) + (1-\omega)(p^*_{T,t} - p^*_t)], \tag{3.39}$$

$$y_{N,t} = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}(\overline{R}_t) - \eta(p_{N,t} - p_t), \tag{3.40}$$

In line with standard theory, the demands are negatively related with expected future real interest rates and relative price to other goods. Intuitively, when consumers expect higher future real returns, they reduce current consumption and save more for future consumption. When some kind of goods becomes relatively more expensive, demand for it drops. Since only a share λ_c of consumers have up-to-date information, a shock affects λ_c of contemporaneous aggregate demand.

Aggregate demand for Foreign tradable goods and non-tradable goods follow analogous relations:

$$y_{F,t}^* = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}[(1-\omega)\overline{R}_t + \omega\overline{R}_t^*]$$
$$+ 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[(1-\omega)(p_{T,t} - p_t) + \omega(p_{T,t}^* - p_t^*)], \quad (3.41)$$
$$y_{N,t}^* = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k E_{t-k}(\overline{R}_t^*) - \eta(p_{N,t}^* - p_t^*), \quad (3.42)$$

Given aggregate demand relations for Home goods and Foreign goods above, aggregate outputs in Home country and Foreign country are determined by $y_t = \zeta y_{H,t} + (1-\zeta)y_{N,t}$ and $y_t^* = \zeta y_{F,t}^* + (1-\zeta)y_{N,t}^*$, respectively.

3.2.5.3 Wage curve

With different groups of workers, nominal wage curves in two countries are:

$$w_t = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t + \frac{y_t - \zeta a_{H,t} - (1-\zeta)a_{N,t}}{\xi(\varphi+\gamma)} + \frac{\gamma(w_t - p_t)}{\varphi+\gamma} - \frac{\varphi\overline{R}_t}{\varphi+\gamma}], \quad (3.43)$$
$$w_t^* = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t^* + \frac{y_t^* - \zeta a_{F,t}^* - (1-\zeta)a_{N,t}^*}{\xi(\varphi+\gamma)} + \frac{\gamma(w_t^* - p_t^*)}{\varphi+\gamma} - \frac{\varphi\overline{R}_t^*}{\varphi+\gamma}], \quad (3.44)$$

The equations show that nominal wage fully adjusts to changes in the expected aggregate price level, which implies that real consumption wage is very important for workers. Nominal wage goes up with higher output as the demand for labor increases. In contrast, higher expected future returns encourage workers to save more by working more, and the increased labor supply lowers nominal wages. Again, only a faction λ_w of workers set their wages according to updated information, so the immediate impacts of shocks on wage level are limited.

3.2.5.4 Interest rate and consumption risk-sharing condition

Interest rates of Home country and Foreign country in log-linear form are:

$$i_t = \phi_y(y_t - \overline{y}_t) + \phi_\pi(p_t - p_{t-1}) + \varepsilon_t, \tag{3.45}$$

$$i_t^* = \phi_y(y_t^* - \overline{y}_t^*) + \phi_\pi(p_t^* - p_{t-1}^*) + \varepsilon_t^*, \tag{3.46}$$

where $\overline{y}_t$ and $\overline{y}_t^*$ have complicated expression in parameters and productivity shocks.[9]

The consumption risk-sharing condition has the following standard form:

$$c_{t,0} - c_{t,0}^* = s_t + p_t^* - p_t = q_t. \tag{3.47}$$

3.3 Model analysis

3.3.1 Parameter values

Table 3.2 summarizes the parameters used in the benchmark economy. To compare with earlier works, the parameter values used in the benchmark model correspond closely to values used in the recent literature. The first eleven parameters are estimated by Reis (2009) with quarterly U.S. data from 1986Q3 to 2006Q1. Almost all of them are conventional, except for ρ_m. As discussed in Chapter 2, low level of ρ_m is consistent with a number of studies documenting less persistent monetary policy shock in the U.S. since the mid-1980s. Particularly, $\lambda_f = 0.52$ imply the prices are quite flexible.

[9] See the appendix for more details.

Table 3.2: Benchmark parameter values

Parameters	Value	
υ	10.09	Elasticity of substitution across Home/Foreign tradables, non-tradables
γ	9.09	Elasticity of substitution across labor services
φ	5.15	Elasticity of labor supply
ξ	0.67	Labor share of income
ϕ_π	1.17	Inflation targeting coefficient
ϕ_y	0.06	Output gap targeting coefficient
ρ_m	0.29	One-quarter autocorrelation of monetary policy shock
σ_m	0.44	Standard deviation of monetary policy shock
λ_c	0.08	Fraction of consumers receiving new information in each period
λ_w	0.74	Fraction of workers receiving new information in each period
λ_f	0.52	Fraction of firms receiving new information in each period
η	0.74	Elasticity of substitution between tradable and non-tradable goods
θ	1.50	Elasticity of substitution between Home and Foreign tradable goods
ρ_T	0.90	One-quarter autocorrelation of productivity shock in tradable sectors
σ_T	0.90	Standard deviation of productivity shock in tradable sectors
ρ_N	0.90	One-quarter autocorrelation of productivity shock in non-tradable sectors
σ_N	0.60	Standard deviation of productivity shock in non-tradable sectors
ζ	0.25	Relative size of tradable sectors
ω	0.70	Share of domestic tradable goods

For η, the estimate by Mendoza (1991) from a sample of industrialized countries is adopted, consistent with the empirical literature that support a value that is lower than unity. The elasticity of substitution between Home and Foreign goods θ is set at 1.5, the value used in the benchmark economy of Chari et al. (2002). For productivity shock related coefficients, ρ_T, ρ_N, σ_T, and σ_N, conventional values are used, taking into account that productivity shocks in the tradable sector are more pronounced. Following Obstfeld and Rogoff (2007), the relative size of the tradable sector ζ is set to be 0.25 and the share of domestic tradable goods ω to be 0.7.

3.3.2 Impulse responses in the benchmark economy

With symmetrical setting, the analysis focuses on the impacts of Home country productivity shocks and monetary policy shock. Figure 3.1 shows the impulse responses of various variables to one unit of productivity innovations and expansionary monetary policy shock, respectively.[10] The economic intuitions of the responses are in line with standard theories. An unexpected expansionary monetary policy shock decreases Home nominal and real interest rates immediately. In the meantime, nominal exchange rate depreciates. Facing lower real interest rate, attentive Home consumers choose to consume more whereas attentive Home workers choose to charge higher nominal wage. As attentive firms in the tradable sector fully consider the nominal exchange rate depreciation in price setting, terms of trade deteriorates and Foreign tradable goods overall become more expensive than Home goods. Based on the aggregate demand relations, demand for Home tradable and non-tradable goods both increase, contributed by increased consumption of attentive consumers and consumption towards cheaper goods. Subsequently, attentive Home firms are induced to produce more, opening up a positive output gap. With great concern on real consumption wage, attentive Home workers also take into account the imported inflation in setting nominal wage, which in effect increases real production wages. Therefore, sector real marginal costs of Home country go up with higher productions and higher real production wages. Such increases are reflected in prices set by attentive Home firms, and thus the aggregate price of Home tradable and non-tradable goods also increase in Home country, although much less than Foreign tradable goods. The real exchange rate depreciates as predicted by Dornbusch (1976), because inattentive firms do not respond to the nominal exchange rate depreciation in price. The resulting imported inflation in the tradable sector from nominal depreciation also makes Home non-tradable goods cheaper relative to tradable goods as a whole, defined as $p_{N,t} - p_{T,t}$ in log form. As

[10] All the simulations in this paper are generated by Dynare Version 4.2.4. The code is revised on the work of Verona and Wolters (2012) for a closed economy.

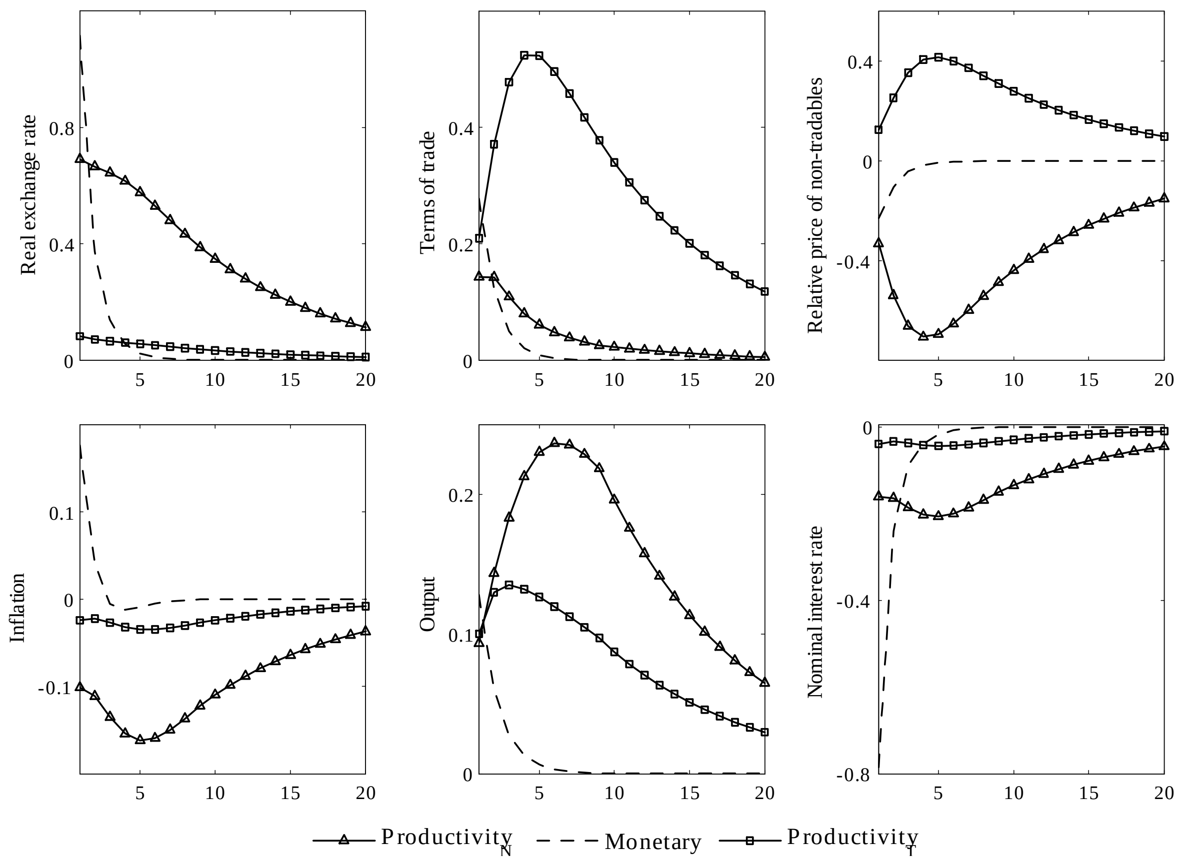

Figure 3.1: The benchmark economy

monetary policy shock is not persistent, all the responses return back to their respective steady states fairly quickly.

When a positive productivity shock occurs in Home tradable sector or non-tradable sector, less labor input is needed for any given level of production in that sector. The lower demand for labor service causes attentive Home workers to charge lower nominal wage. Reduced labor input and lower nominal wage together lead to a fall in the sector real marginal costs, and thus attentive Home firms lower the price of their products at any given output level. Recall that substitution exists between Home tradable and Foreign tradable goods and also between tradable and non-tradable goods. The increased demand for cheaper products of attentive Home firms (in the tradable or non-tradable sector) due to substitution induces these firms to produce more. However, a fraction of firms are inattentive and do not respond to the shock with lower price and higher production. Therefore, the aggregate output remains lower than its natural level when all the firms are attentive, which implies a negative output gap. Negative output gap together with deflation causes nominal interest rate to fall. The resulting lower real interest rate subsequently induces attentive Home consumers to consume more, which further enhances the increase in output. Consistent with standard theory, the efficient consumption risk-sharing as described in Equation 3.47 guarantees positive international consumption spillover of productivity innovations. Specifically, the increase in Home country productivity depreciates the real exchange rate, benefiting Foreign consumers. When the productivity innovation takes place in the tradable sector, the aggregate price of Home tradable goods drops first, which deteriorates terms of trade and makes Home non-tradable goods more expensive. A productivity innovation in non-tradable sector decreases the aggregate price of Home non-tradable goods and its relative price to tradable goods first. As consumers switch to consume more Home non-tradable goods, attentive firms in Home and Foreign tradable sectors respond to the reduced demand with lower prices. Due to home bias, the aggregate price of Home tradable goods drops more than that of Foreign tradable goods and thus terms of trade depreciates.

3.3.3 Properties of simulated real exchange rates and sensitivity tests

In this section, whether the sticky information DSGE model can replicate the stylized facts about the real exchange rate is examined. As shown in Table 3.3, the one-quarter autocorrelation of simulated real exchange rate, ρ_q, equals to 0.82 in the benchmark economy, which is perfectly consistent with empirical estimates in Table 3.1. The ratio of the standard deviations of the real exchange rate and of output ("standard deviation ratio", for short), σ_q/σ_y, equals to 3.05, smaller than statistics based on bilateral real exchange rates. However, the value is still reasonable as the two-country model setting makes q more similar to multilateral real exchange rate, which is less volatile

Table 3.3: Behavior of simulated real exchange rates

	ρ_q	σ_q/σ_y
Benchmark economy		
$\rho_T = \rho_N = 0.9$, $\rho_m = 0.29$, $\lambda_f = 0.52$, $\lambda_w = 0.74$, $\lambda_c = 0.08$	0.82	3.05
Information stickiness		
Less attentive firms ($\lambda_f = 0.25$)	0.62	3.43
Less attentive workers ($\lambda_w = 0.25$)	0.76	3.12
Perfectly attentive firms ($\lambda_f = 1$)	0.82	3.22
Perfectly attentive workers ($\lambda_w = 1$)	0.84	3.03
Perfectly attentive consumers ($\lambda_c = 1$)	0.81	1.34
Perfectly attentive agents ($\lambda_f = \lambda_w = \lambda_c = 1$)	0.90	1.32
Other parameters		
High openness ($\zeta = 0.5$)	0.79	1.50
Low trade elasticity ($\theta = 0.25$)	0.74	3.37
Less persistent productivity shocks ($\rho_T = \rho_N = 0.5$)	0.36	5.34
Persistent monetary policy shocks ($\rho_m = 0.8$)	0.71	4.27

Notes: ρ_q refers to one-quarter autocorrelation of simulated real exchange rate. σ_q/σ_y is standard deviation ratio of simulated real exchange rate to simulated output. λ_f, λ_w and λ_c are the fractions of attentive firms, workers and consumers, respectively. θ is trade elasticity, ω is share of domestic tradable goods, and ζ is relative size of tradable sector. ρ_T, ρ_N and ρ_m are one-quarter autocorrelations of productivity and monetary policy shocks, respectively. Parameter values in parentheses indicate differences to the benchmark economy. All the statistics are computed across 2000 simulations.

than bilateral exchange rate. For instance, the CPI-based U.S. real effective exchange rate reported by IFS has a standard deviation ratio of 3.08 in 1986Q3-2006Q1. Intuitively, the impact of some country-specific shocks is substantially reduced as the trade-weighted real effective exchange rate averages across around 50 countries.

Table 3.3 also summarizes how the behavior of simulated real exchange rate varies when one parameter or some parameters change (with the remaining parameter values held constant). In the first step, sensitivity tests are performed on the information stickiness in agents. When the fraction of attentive firms or workers in each period is reduced to 0.25, which implies a higher degree of nominal stickiness in prices or wages, ρ_q becomes smaller than in the benchmark economy.[11] When one type of the agents are perfectly attentive, ρ_q is almost unaffected if compared to benchmark value. When all the agents are assumed to be perfectly attentive, ρ_q jumps up to 0.9, which equals to the one-quarter autocorrelation coefficient of productivity shocks. These test results suggest a pattern that nominal stickiness in prices and wages can make the real exchange rate less persistent, which is at odds with conventional belief. The standard deviation ratio only displays obvious drop when consumers are perfectly attentive. Intuitively, when all the consumers are aware of the shocks and adjust their consumption accordingly, output would have much larger responses than in the benchmark economy. With more volatile output, the value of σ_q/σ_y must become smaller.

Tests are also performed on other parameters, such as high openness ($\zeta = 0.5$), low trade elasticity ($\theta = 0.25$), persistent monetary policy shocks ($\rho_m = 0.8$), and less persistent productivity shocks ($\rho_T = \rho_N = 0.5$). It is shown that the real exchange rate persistence is not very sensitive to change in economy openness, but declines obviously when the trade elasticity is set in line with low value from macro-estimates. The standard deviation ratio drops a lot when the economy becomes more open, which is consistent with the literature.[12] When productivity shocks are less persistent, ρ_q drops from the benchmark value of 0.82 to 0.36 but the standard deviation increases a lot. When monetary policy shocks are more persistent, the real exchange rate also becomes slightly less persistent but much more volatile. Explanation to these results can be found from Figure 3.1, which shows that the responses of the real exchange rate are persistent to productivity innovations but damp out fairly quickly to monetary policy shock. Besides, it is also noted that the real exchange rate responds most to monetary policy shock but output responds least, which implies a large standard deviation ratio.

[11] As the fraction of attentive consumers in each period is already as low as 0.08, we do not attempt to test on an even lower value.

[12] For example, Hau (2002) shows that real exchange rates of more-open economies are less volatile both theoretically and empirically.

Therefore, when productivity shocks dominate, more persistent productivity shocks make the real exchange rate more persistent but the standard deviation smaller. On the contrary, more persistent monetary policy shocks lead to a decrease in ρ_q but an increase in σ_q/σ_y.[13]

The sensitivity tests in Table 3.3 suggest that the real exchange rate persistence generated in the benchmark economy comes essentially from persistent productivity shocks. Besides, the implication of some test results seems to be against existing literature. For example, the literature generally considers nominal stickiness in prices and wages as an important factor generating persistence in the economy. However, in the constructed sticky information model, lower nominal stickiness in prices and wages makes real exchange rate more persistent when productivity shocks dominate. Therefore, more sensitivity tests are performed to obtain more information, the results of which are given in Table 3.4. When ρ_T and ρ_N are reduced from 0.9 to 0.5 with the remaining parameter values held identical to the benchmark economy, the one-quarter autocorrelation of simulated real exchange rate drops from 0.82 to 0.36 (denoted as Scenario 1). Fixing ρ_T and ρ_N at 0.5, the real exchange rate becomes less persistent with less attentive firms in each period but more persistent when firms are perfectly attentive. Unexpectedly, the real exchange rate becomes more persistent in both cases, either with less or perfectly attentive workers in each period. When all the agents are perfectly attentive, ρ_q jumps up to 0.51, which is almost identical to the current level of ρ_T and ρ_N. Therefore, the pattern that nominal stickiness in prices and wages makes the real exchange rate less persistent still exists. When the one-quarter autocorrelation of monetary policy shock increases from 0.29 to 0.8, the real exchange rate accordingly becomes more persistent, with ρ_q rising from 0.36 to 0.61. Note, this persistence value is higher than $\rho_T = \rho_N = 0.5$, which clearly indicates that monetary policy shock has a dominant impact on the economy now.

[13] The situation is slightly different when monetary policy shock dominates. The standard deviation ratio does not increase monotonically with more persistent monetary policy shock becaue the changes in the responses of the real exchange rate and output are asymetric.

Table 3.4: Extended Sensitivity Tests

	ρ_q	σ_q/σ_y
Scenario 1		
$\rho_T = \rho_N = 0.5$, $\rho_m = 0.29$, $\lambda_f = 0.52$, $\lambda_w = 0.74$, $\lambda_c = 0.08$	0.36	5.34
Less attentive firms ($\lambda_f = 0.25$)	0.33	9.15
Less attentive workers ($\lambda_w = 0.25$)	0.37	5.23
Perfectly attentive firms ($\lambda_f = 1$)	0.43	4.43
Perfectly attentive workers ($\lambda_w = 1$)	0.40	5.33
Perfectly attentive consumers ($\lambda_c = 1$)	0.37	1.35
Perfectly attentive agents ($\lambda_f = \lambda_w = \lambda_c = 1$)	0.51	1.26
Persistent monetary policy shocks ($\rho_m = 0.8$)	0.61	6.76
Scenario 2		
$\rho_T = \rho_N = 0.25$, $\rho_m = 0.29$, $\lambda_f = 0.52$, $\lambda_w = 0.74$, $\lambda_c = 0.08$	0.31	6.30
Less attentive firms ($\lambda_f = 0.25$)	0.32	10.7
Less attentive workers ($\lambda_w = 0.25$)	0.34	5.89
Perfectly attentive firms ($\lambda_f = 1$)	0.20	4.61
Perfectly attentive workers ($\lambda_w = 1$)	0.34	6.43
Perfectly attentive consumers ($\lambda_c = 1$)	0.31	1.37
Perfectly attentive agents ($\lambda_f = \lambda_w = \lambda_c = 1$)	0.29	1.26
Persistent monetary policy shocks ($\rho_m = 0.8$)	0.61	6.97
Scenario 3 (No productivity shocks)		
$\rho_m = 0.29$, $\lambda_f = 0.52$, $\lambda_w = 0.74$, $\lambda_c = 0.08$	0.32	10.1
$\rho_m = 0.8$, $\lambda_f = 0.25$, $\lambda_w = 0.74$, $\lambda_c = 0.08$	0.79	5.26

Notes: ρ_q refers to one-quarter autocorrelation of simulated real exchange rate. σ_q/σ_y is standard deviation ratio of simulated real exchange rate to simulated output. ρ_T, ρ_N and ρ_m are one-quarter autocorrelations of productivity and monetary policy shocks, respectively. λ_f, λ_w and λ_c are the fractions of attentive firms, workers and consumers, respectively. Parameter values in parentheses indicate differences to corresponding alternative economies. All the statistics are computed across 2000 simulations.

When ρ_T and ρ_N are further reduced to 0.25 with the remaining parameter values held identical to the benchmark economy (denoted as Scenario 2), the dominant impact of monetary policy shock becomes more obvious. All the simulated real exchange rates have higher than 0.25 one-quarter autocorrelations and also larger standard deviation ratios than previous scenarios. The impact of nominal rigidity also changes to be more consistent with our conventional belief. The real exchange rate becomes slightly more persistent with less attentive firms in each period and much less persistent with perfectly attentive firms. The impact of nominal stickiness in wages and consumptions is inconclusive but very small. When all the agents are perfectly attentive, ρ_q drops back to 0.29, exactly the one-quarter autocorrelation of monetary policy shock. When ρ_m increases from 0.29 to 0.8, the real exchange rate becomes more persistent, although with ρ_q less than 0.8. Subsequently, productivity shocks are excluded from the simulation in Scenario 3. With the remaining parameter values held identical to the benchmark economy, ρ_q of simulated real exchange rate drops from the benchmark 0.82 to 0.32, with σ_q/σ_y increasing from 3.05 to 10.1. Subsequently, with more persistent monetary policy shocks ($\rho_m = 0.8$) and fewer attentive firms in each period ($\lambda_f = 0.25$, i.e. higher price stickiness), the one-quarter autocorrelation of simulated real exchange rate is as high as 0.79, which cannot be generated by standard sticky price model with similar parameter values.

Up to now, some features of the sticky information model can be summarized based on extensive test results in Table 3.3 and Table 3.4. First, when productivity shocks are sufficiently persistent and thus dominate, the generated persistence in the real exchange rate is completely from productivity shocks. In this case, the higher the degree of price stickiness and wage stickiness, the less persistent the simulated real exchange rate. Second, when productivity shocks are nonpersistent and monetary policy shock has a dominant impact on the economy, higher price stickiness helps to generate persistent real exchange rate as expected, with the impact of wage stickiness not clear-cut but negligible. In both cases, stickiness in consumption only affects the standard deviation ratio obviously by affecting the standard deviation of output. In the extreme case where productivity shocks are completely excluded from the economy, the real exchange rate is still as high as observed, even though monetary policy shock is more persistent and prices are more sticky than in the benchmark economy. Therefore, two questions arise naturally: (1) why is the sticky information model able to simulate highly persistent real exchange rate whereas standard sticky price model cannot? and (2) why are the effects of price stickiness and wage stickiness on the real exchange rate persistence so different when different shocks dominate?

One potential explanation for the first question is that two models have very different assumptions on price-setting and wage-setting. In the sticky information model, all the agents are allowed to make adjustments in each period. For example, in a certain period t, attentive firms (workers) adjust price (wage) to optimal level based on updated information and inattentive firms (workers) adjust prices (wages) to expected optimal levels based on outdated information of different degrees. Despite shocks that occurred after the latest updates of firms and workers, the reset prices and wages have fully reflected the impacts of expected shocks based on their information. On the contrary, settings in the sticky price model are quite different. Take the widely used Calvo-style sticky price model for example.[14] In each period, only firms receiving a random price-change signal are allowed to reset price. Specifically, the new price is set equal to a probability-weighted average of the current and all future optimal prices. For example, when a firm i receives the signal in period t, the new price p_i will be set equal to $\lambda_p \sum_{m=0}^{\infty} (1-\lambda_p)^m E_t p^*_{t+m,i}$, where λ_p is the probability for a firm to be the receiver in each period and $E_t p^*_{t+m,i}$ are optimal price in current period ($m = 0$) and expected optimal prices in future periods ($m > 0$). Obviously, the weight assigned to expected future optimal price declines exponentially toward the further future. So if p_i is still alive in period $t + m$, the impact of expected shocks based on information of period t is not sufficiently reflected in the price. However, by default setting, the

[14]See Calvo (1983) or recent Mankiw and Reis (2002) for more details.

price set in period t would remain unchanged in $\lambda_p(1-\lambda_p)^m$ of firms in period $t+m$. Similarly, wages in the sticky price model are also determined in this way if wage stickiness is introduced. With other settings identical, the pass-through of various shocks and their persistence to the economy is thus lower in the sticky price model than in the sticky information model. This difference may largely explain why monetary policy shocks alone can generate persistent real exchange rate in the constructed sticky information model, but cannot in sticky price model.

The second question is probably associated with different transmission mechanisms of productivity shocks and monetary policy shocks. Generally, productivity shock affects price and wage primarily whereas monetary policy shock influences the nominal exchange rate and wage first. If all the firms and workers can realize productivity shock immediately and adjust prices and wages accordingly, perfect pass-through allows the real exchange rate to be almost as persistent as productivity shock. As a result, nominal stickiness in prices and wages in fact decreases the pass-through of productivity shock to the economy and thus the real exchange rate becomes less persistent with more sticky prices and wages. In contrast, monetary policy shock affects price indirectly through goods market and labor market. Without reducing the pass-through of monetary policy shocks directly, price stickiness thus becomes an important source of persistence in the economy when monetary policy shock dominates. As wage is primarily affected by monetary policy shock, wage stickiness does limit the pass-through of monetary policy shocks, similar to the case of productivity shock. However, wage stickiness can enhance price stickiness in the economy and thus makes the real exchange rate more persistent. The net impact of wage stickiness is thus ambiguous. This interpretation suggests, besides the price-setting rule, highly sticky prices and wages may be a second reason for the failure of persistent productivity shock to reproduce observed real exchange rate persistence in standard sticky price model.

3.4 Conclusions

Generally, standard sticky price model cannot generate persistent real exchange rate deviations with persistent monetary shock alone, unless unrealistically sticky prices are introduced. In this chapter, the benchmark sticky information model reproduces the empirically documented persistence without highly sticky prices. It is found that this generated persistence is completely due to persistent productivity shocks. Sensitivity tests also provide a few other interesting findings. When productivity shocks dominate, nominal stickiness in prices and wages prevents the real exchange rate to be as persistent as productivity shocks. However, when monetary policy shock dominates, price stickiness contributes to the real exchange rate persistence as conventionally believed, while wage stickiness has an ambiguous impact. As consumption only affects the economy by adjusting price and wage level in the goods market, stickiness in consumptions also has fairly small impact.

The different behaviors of price stickiness and wage stickiness can be explained by the fact that productivity shocks and monetary policy shock influence the economy differently. When a productivity shock occurs, price and wage are primarily affected. Therefore, endogenous nominal stickiness in prices and wages dampens the pass-through of productivity shocks to the economy and the real exchange rate turns out to be less persistent than productivity shocks. In contrast, a monetary policy shock affects nominal exchange rate and wage directly. As it only reduces the pass-through of monetary policy shock indirectly, price stickiness serves an important factor for monetary policy shock to generate persistence in the real exchange rate. Wage stickiness limits the pass-through of monetary policy shock to the economy but meanwhile increases price stickiness, making its impact on the real exchange rate persistence not clear-cut.

It is shown analytically that the excellent performance of the sticky information model is associated with allowing high pass-through of shocks to the economy. In the sticky information model, agents can fully respond to shocks they know by making adjustment in each period. On the contrary, the sticky price model only allows random fractions of firms and workers to re-set prices and wages in each period. Hence, the new price or wage is set equal to a weighted average of the current and future optimal levels, responding to shocks insufficiently. The feature of higher pass-through of shocks even enables the model to simulate highly persistent real exchange rate with monetary policy shock only.

Chapter 4

Sources of the U.S. real exchange rate fluctuations

4.1 Introduction

As discussed earlier, the revisions of real exchange rates back to their PPP values are fairly slow. Overwhelming empirical evidence shows that PPP does not hold under modern floating regimes. By using longer datasets including gold standard period, more studies find real exchange rates converge to PPP values in the long run. In the literature, the failure of PPP in the short run is usually attributed to sluggish adjustment of nominal prices and wages to monetary shock and financial factors. However, standard sticky price model with pure nominal shocks cannot replicate the persistent PPP deviations in the data. Therefore, there emerges a large body of empirical literature investigating potential long-run relationship between the real exchange rate and economic fundamentals. Besides, researchers are also interested in ranking the factors that deviate real exchange rates from their PPP values.

The most famous theory that points out the importance of supply side factor on the real exchange rate is the Balassa-Samuelson hypothesis. It predicts that, with perfect labor mobility, productivity innovations in the tradable sector will increase the wage rate of that economy. As tradable goods are priced in international markets, the relative price of non-tradable goods thus increases and leads to a real exchange rate appreciation. One accompanying view of this hypothesis is that most productivity innovations have occurred in the tradable sector, which accords with historical data.[1] As technology spillover takes time, deviations from PPP damp out slowly.[2] Empirically, there are substantial analyses supporting this hypothesis. Recently, Enders and Muller (2009), and Enders et al. (2011) provide new evidence of a real exchange rate appreciation following relative productivity innovations. However, there is an opposite strand of theory. Stockman (1987) and Clarida and Gali (1994), among others, argue that a productivity innovation in the tradable sector should depreciate the real exchange rate as the resulting increased supply of domestic tradable goods deteriorates its international relative price. Using conventional calibrated parameters, model simulations of this strand accordingly obtain a real exchange rate depreciation.[3] Therefore, the impact of relative supply shock on the real exchange rate is theoretically ambiguous.

Aggregate demand shocks are found to be influential on real exchange rate dynamics by affecting the relative price of non-tradable goods as well. Compared with shock to households' preference, fiscal policy shock is more frequently analyzed. The underlying theoretical intuition is that government consumption falls more heavily on non-tradable goods and thus has more obvious impact on real exchange rates.[4] Froot and Rogoff (1991) and De Gregorio et al. (1994) find that real exchange rates of the European Monetary System (EMS) countries appreciate when government consumption increase. However, it is also argued that government consumption may have an opposite effect on the real exchange rate. Habermeier and Mesquita (1999) argue that if government consumption is financed by distortionary taxes, high tax burden would depreciate the real exchange rate via slowing down aggregate productivity growth.

[1] See, for example, De Gregorio et al. (1994) and Rogoff (1996).

[2] As in Gali (1999), changes in technology are identified as the only determinant of productivity in the long run.

[3] See, for example, de Walque et al. (2005) and Monacelli (2005).

[4] See, for example, Rogoff (1992).

Besides, fiscal deficits and private investments usually cause current account deficits alternately. Empirically, current account deficits are associated with real exchange rate depreciations over 5 to 10 year horizons.[5] Therefore, the impact of fiscal policy shock on the real exchange rate is not clear cut as well.

There is also no consensus in the empirical literature on which factors affect the real exchange rate most. Clarida and Gali (1994), Weber (1997), and Chadha and Prasad (1997) document that shock from the demand side gives rise to most of the bilateral real exchange rate movements. Rogers (1999) and Eichenbaum and Evans (1995) find that monetary shock is relatively more important. All these studies are based on structural SVAR models using differenced data, and document negligible impact of supply side shock on real exchange rate fluctuations at any horizon. Alexius (2005) argues that information contained in the levels of data is missed if only the changes in the real exchange rate and other variables are analyzed. The misspecification is especially serious if real exchange rates are indeed cointegrated with some fundamentals in the long run. Therefore, he extends the model of Clarida and Gali (1994) to allow for cointegration relations between the levels of the variables. For the sample period of 1960Q1-1998Q4, it is found that bilateral real exchange rate cointegrates with relative output and relative government consumption in the long run. In contrast to previous findings, the structural VECM analysis shows that relative productivity shock dominates long-run fluctuations of bilateral real exchange rates.

In Chapter 3, the benchmark economy suggests persistent productivity shocks as the essential source of high persistence in the real exchange rate, which corresponds to the finding of Alexius (2005). As a result, I am motivated to perform a comprehensive empirical analysis on the U.S. real exchange rate. This analysis fills a gap in the empirical literature from many aspects. First, instead of bilateral real exchange rates conventionally analyzed in previous works, the multilateral U.S. real effective exchange rate is chosen. Correspondingly, the remaining data series for the U.S. are also in

[5] See, for example, Rogoff (1996).

relative terms against an aggregate of industrialized countries comprising Canada, Japan, the U.K. and the Euro area. Compared to analyzing bilateral relative variables, using multilateral data avoids some potential problems. For example, the change in a certain bilateral real exchange rate may originate from a third country which has asymmetric trade relations with the two countries. Therefore, the analysis is more in line with traditional theories that are basically discussed based on two-country setting. Second, the dataset used covers the great moderation period and thus is entirely within the modern floating rate era, which helps to avoid the potential criticism that the econometric implications of mixing data from both the gold standard period and the floating rate era may be unclear. Third, information in the levels of the variables is fully utilized by explicitly taking into account their long-run equilibrium relationships as in Alexius (2005). What needs to be highlighted is, the model distinguishes with that of Alexius (2005) in many ways. Besides the methods for constructing data and the sample periods mentioned earlier, the variables in two models are also different. For example, relative government bond rate is included in my model to account for shock from international financial markets. Exclusion tests reject its exclusion with zero p-value at any cointegration rank. In addition, relative interest rate, rather than relative price level in Alexius (2005), is used to capture relative monetary policy shock.

According to the statistics of Johansen likelihood ratio trace test, there are three long-run equilibrium relationships in my data. Reduced form analysis shows that the long-run equilibrium relationships are among (1) the real exchange rate and relative output, (2) relative interest rate and relative output as well as relative government consumption, and (3) relative government bond rate and relative output, respectively. Each cointegration relation can find its roots in existing economic theories. With proper short-run and long-run restrictions put on the structural form, structural impulse analysis reveals more short-run and median-run responses of the real exchange rate. For example, the impact of relative monetary policy shock is insignificant all the time while the appreciation caused by shock from international financial markets is only borderline significant over the second year. The significant appreciation by relative fiscal policy shock can sustain as long as around three years. The effect of relative productivity shock is insignificant in the short and medium run, but becomes significant in the long run. Similar to Alexius (2005), variance decompositions demonstrate that relative productivity shock dominates long-run fluctuations in real effective exchange rate when cointegration relations are considered, which supports the Balassa-Samuelson hypothesis. In general, the findings prove that the theoretical implication of Chapter 3 that the high persistence in the real exchange rate originates from persistent productivity shocks is in line with empirical data.

The rest of the chapter is organized as follows. Section 2 describes the data used in this chapter. Section 3 shows the reduced form analysis including determining cointegration ranks, just-identifying and over-identifying cointegration relations. Section 4 provides the structural form analysis with proper short-run and long-run restrictions. Section 5 concludes the chapter.

4.2 The data

Data are constructed completely following the method of Enders et al. (2011). The quarterly time series data for the U.S. are in relative terms against the "rest of the world" (hereafter ROW). The ROW comprises the majority of industrialized countries including Canada, Japan, the U.K. and the Euro Area. The model includes the following variables: the real effective exchange rate (q),[6] relative short-term interest rate (ir), relative long-term government bond rate (br), relative government consumption (g), and relative output (y). All the variables are in real and log form except for the interest rate and the government bond rate. ir, br, g, and y are included to capture the conditions of relative monetary policy, international financial markets, relative fiscal

[6] An increase of q implies an appreciation.

policy and relative productivity, respectively.[7] All data series for the U.S., Canada, Japan and the U.K. are obtained from the OECD Economic Outlook database except that the U.S. CPI-based real effective exchange rate is from the Main Economic Indicators of the OECD. All data series for the Euro Area are taken from the ECB's AWM database.[8] The ROW aggregate data are constructed by first calculating each currency area's quarterly growth rates and then combining these series according to each currency area's GDP share in the ROW group. The weights are based on the annual purchasing power parity (PPP) values in the year 2000 reported by the International Monetary Fund (2012). Next, the aggregated data in log form are obtained by cumulating the aggregated growth rates from the normalized base year.

Time series of the datasets are plotted in Figure 4.1. As no longer strictly restricted by the estimation period of 1986Q3-2006Q1 by Reis (2009), the more conventionally used 1984Q1 is chosen as the starting date of the great moderation to include more observations. The ending date of the sample period, 2005Q1, is determined by the data availability of the ECB's AWM database. All variables seem to have a unit root by visual inspection, and output has a trending behavior. Formal unit root tests have been applied on all series and their first differences, which are reported in Table 4.1. Overall all the variables are accepted to be integrated of order 1. When seasonal dummies are included, test statistics change slightly but the conclusion remains.

4.3 Reduced form analysis

The general model of the analysis is a VECM of the form

$$\mathbf{\Delta X}_t = \boldsymbol{\alpha\beta}' \begin{bmatrix} \mathbf{X}_{t-1} \\ \mathbf{Trend}_{t-1} \end{bmatrix} + \mathbf{\Gamma}_1 \mathbf{\Delta X}_{t-1} + ... + \mathbf{\Gamma}_{p-1} \mathbf{\Delta X}_{t-p+1} + \mathbf{const} + \mathbf{\Gamma}_s \mathbf{S}_t + \mathbf{u}_t \quad (4.1)$$

[7]Several studies document that the productivity shocks identified by the VAR models are positively correlated with classic and refined Solow residuals at a reasonably high degree. See, for example, Alexius (2005) and Alexius and Carlsson (2005).

[8]See Fagan et al. (2001) for details.

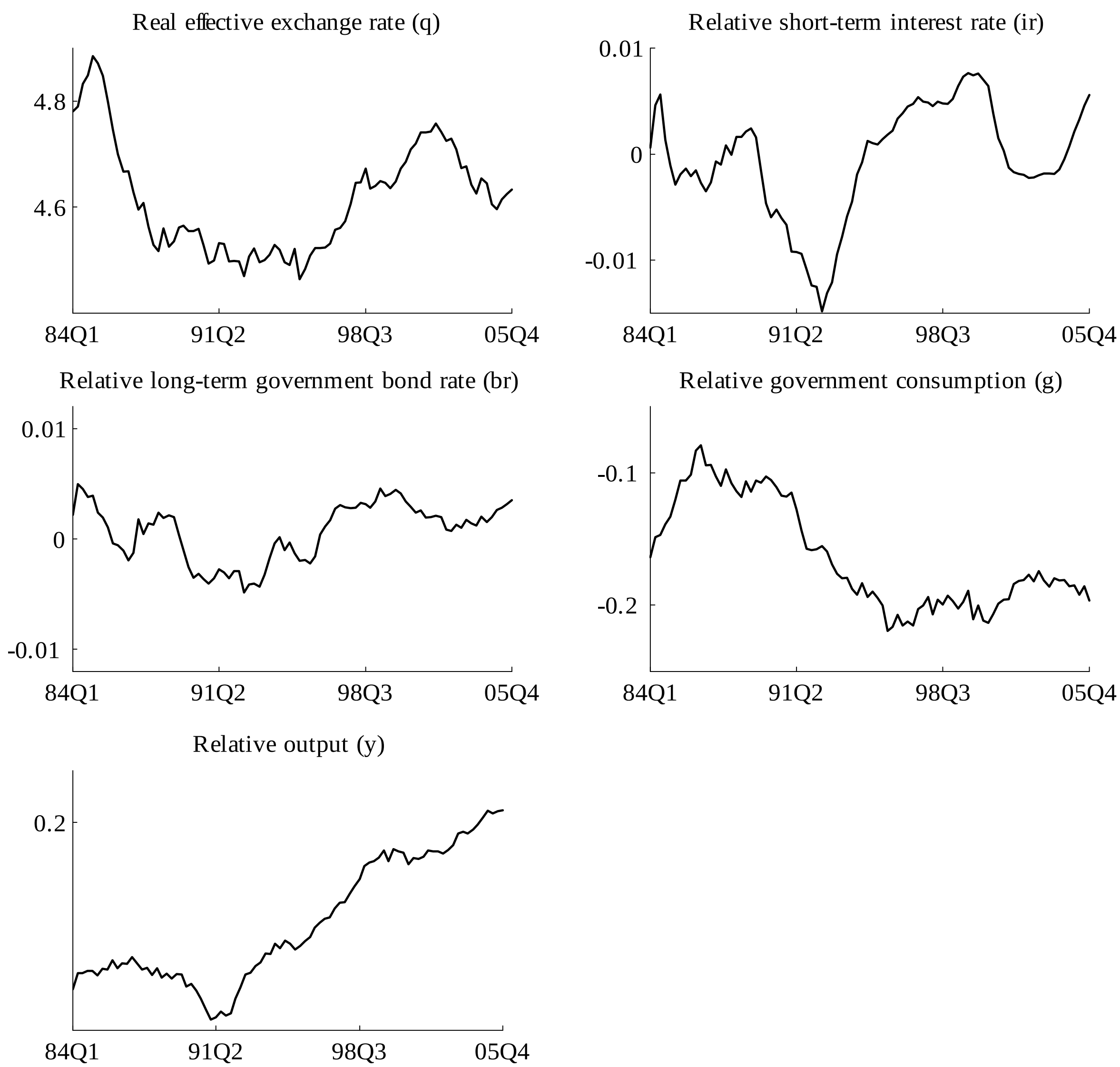

Figure 4.1: Time series of real effective exchange rate (q), relative short-term interest rate (ir), relative long-term government bond rate (br), relative government consumption (g) and relative output (y).

Table 4.1: Unit root tests

Variable	Deterministic terms	Lagged differences	Test statistic	Critical value		
				10%	5%	1%
q	Constant	$SC, HQ: 0$	-1.65	-2.57	-2.86	-3.43
		$AIC: 4$	-3.65			
Δq	None	$SC, HQ, 0$	-7.45	-1.62	-1.94	-2.56
		$AIC: 3$	-3.36			
ir	Constant	$SC, HQ: 2$	-2.13	-2.57	-2.86	-3.43
		$AIC: 7$	-1.89			
Δir	None	$SC, HQ: 1$	-4.50	-1.62	-1.94	-2.56
		$AIC: 6$	-3.02			
br	Constant	$SC, HQ: 0$	-1.45	-2.57	-2.86	-3.43
		$AIC: 5$	-1.62			
Δbr	None	$SC, HQ: 0$	-8.77	-1.62	-1.94	-2.56
		$AIC: 5$	-4.73			
g	Constant	$SC, HQ, AIC: 1$	-0.71	-2.57	-2.86	-3.43
Δg	None	$SC, HQ, AIC: 0$	-9.98	-1.62	-1.94	-2.56
y	Constant, trend	$SC, HQ, AIC: 2$	-2.06	-3.13	-3.41	-3.96
Δy	Constant	$SC, HQ, AIC: 1$	-4.45	-2.57	-2.86	-3.43

Note: SC=Schwarz Criterion, HQ=Hannan-Quinn Criterion, AIC=Akaike Information Criterion

Table 4.2: Diagnostic tests of residual autocorrelation, normality, and ARCH effect

a. **Multivariate tests**						
Tests for Autocorrelation:	$LM(1)$:	0.819				
	$LM(2)$:	0.229				
Test for Normality:		0.024				
Test for ARCH:	$LM(1)$:	0.353				
	$LM(2)$:	0.383				
b. **Univariate tests**		Δq	Δir	Δbr	Δg	Δy
ARCH(4)		0.097	0.807	0.011	0.200	0.224
Jarque-Bera Test		0.726	0.022	0.418	0.459	0.531
R^2		0.561	0.663	0.551	0.424	0.370

Note: all reported numbers are p-values.

where $\mathbf{X}_t = (\mathbf{q_t}, \mathbf{ir_t}, \mathbf{br_t}, \mathbf{g_t}, \mathbf{y_t})'$ is the vector of endogenous variables, and $\mathbf{\Delta X}_t$ is its first difference vector. Based on the characteristics of all the variables, constant is unrestricted while trend is restricted in the cointegration vectors. Seasonal dummies ($\mathbf{S}_t$) are also added. The loading matrix $\boldsymbol{\alpha}$ contains the weights attached to the cointegration relations in each individual equation, and the cointegration matrix $\boldsymbol{\beta}$ defines the cointegration relations. Both of them are of rank r, which is the number of cointegration vectors. $\mathbf{\Gamma}_i$ $(i = 1, ..., p-1, s)$ are parameter matrices of lagged difference terms and seasonal dummies. $\mathbf{u}_t = (u_t^q, u_t^{ir}, u_t^{br}, u_t^g, u_t^y,)'$ is an unobserved error term with zero mean.

The number of lags, p, is jointly determined by various residual analyses. Both Schwarz Criterion and Hannan-Quinn Criterion suggest a VAR(1) model, but the resulting residuals turn out to be seriously autocorrelated. The likelihood ratio lag reduction tests and residual autocorrelation analysis support a choice of four lags. The main diagnostic test results for the unrestricted VAR(4) model are reported in Table 4.2. The multivariate likelihood maximum test for residual autocorrelation is accepted with a p-value of 0.819 for first order and 0.229 for second order. The multivariate normality test is rejected with a p-value of 0.024. The univariate Jarque-

Table 4.3: Johansen likelihood ratio trace test

r	Eigenvalue	Trace test	95% quantile	p-value
0	0.484	141.720	88.554	0.000
1	0.374	86.074	63.659	0.000
2	0.241	46.710	42.770	0.018
3	0.155	23.553	25.731	0.094
4	0.106	9.372	12.448	0.162

Bera tests for normality indicate that the non-normality is exclusively in the equation for relative interest rate. The multivariate residual ARCH test is accepted with a p-value of 0.353 for first order and 0.383 for second order. Univariate residual ARCH tests accept no ARCH effect for all individual equations except for that of relative government bond rate. The tests are generally satisfactory for artificial data series, so four lags are considered in the analysis.

4.3.1 Cointegration rank test and weak exogeneity test

Johansen likelihood ratio trace test is used to investigate the cointegration rank of the five variables, with detailed results given in Table 4.3.[9] Based on a p-value of 0.05, the statistics suggest a rank of 3. The recursively calculated trace tests in Figure 4.2, scaled by corresponding 95% quantile of the asymptotic distribution, reveals more information. The trace tests for $r = 1, 2, 3$ display quite smooth linear growth, which implies the eigenvalues of the first three cointegration relations are non-zero and have been rather stable over time. The graphs for the two smallest eigenvalues are much less steep and lie below the 5% critical value line all the time, which indicates unit roots or near unit roots. The graphs also exhibit that the power and the size of the trace test depends largely on the sample size. In this study, it is not until 1997 that the second relation can be accepted to be stationary. Similarly, $r = 3$ can only be accepted as late as in the last few quarters. So some stationary relations characterized by slow

[9] Short-run dynamics have been concentrated out in the test and subsequent recursively calculated test.

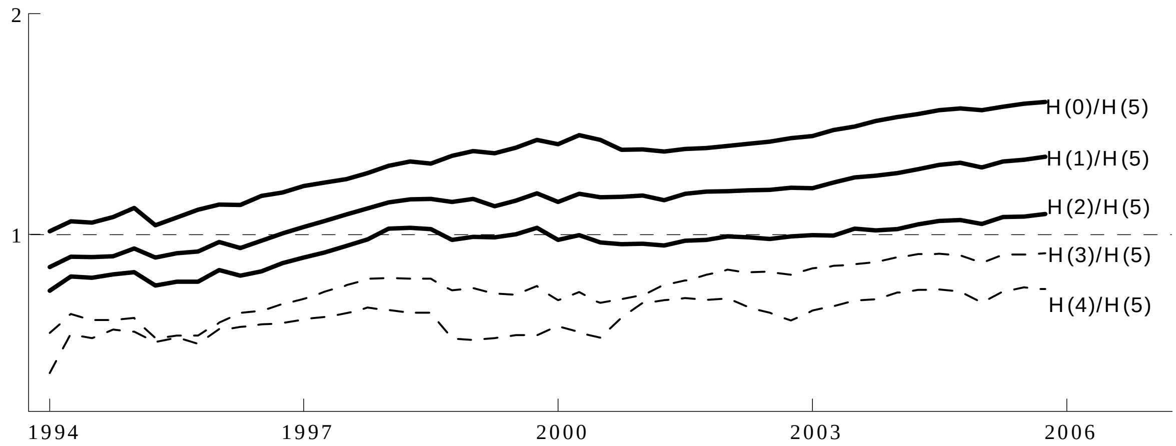

Figure 4.2: The recursive trace test. H(r)/H(5) refers to the hypothesis of rank=r (r=0, 1, 2, 3, 4) against the null hypothesis of rank=5. The test statistics are scaled by the 95% quantile of the asymptotic distribution. Hence, rank=r is rejected at 5% significance level if the test statistic of H(r)/H(5) is beyond the dash line, and vice versa. Computations are performed with CATS in RATS Version 2.01.

but nevertheless significant adjustment may be excluded wrongly in a small sample.[10] Besides the trace test, we can look at the cointegration rank determination problem from a second angle. Based on the weak exogeneity tests reported in Table 4.4, relative output is accepted to be weakly exogenous with a p-value of 0.274 at $r = 3$ whereas relative government consumption is rejected with a p-value of 0.026 and the rest three variables are rejected with zero p-values. The test results are compatible with the underlying assumption of two common stochastic trends.

4.3.2 The cointegration relations

The $r = 3$ VECM is estimated with a two-stage procedure, in which the cointegration vectors are pre-estimated by the simple two step (S2S) estimator proposed by Ahn and Reinsel (1990). Bruggemann and Lutkepohl (2005) find that the S2S estimator outperforms Johansen maximum likelihood estimator when the sample size is relatively small. In the first stage, the individual equations are estimated by OLS with a specified cointegration matrix. In the second stage, the GLS estimator is computed

[10] See Juselius (2006), Chapter 8 and Chapter 9 for more details.

Table 4.4: Test of weak exogeneity, p-values in square brackets

r	5% critical value	q	ir	br	g	y
1	3.841	9.154	0.345	2.290	1.865	3.464
		[0.002]	[0.557]	[0.130]	[0.172]	[0.063]
2	5.991	15.952	9.496	17.010	8.30	3.845
		[0.000]	[0.009]	[0.000]	[0.016]	[0.146]
3	7.815	23.846	17.228	20.962	9.25	3.889
		[0.000]	[0.001]	[0.000]	[0.026]	[0.274]
4	9.488	27.887	19.275	23.314	13.83	4.342
		[0.000]	[0.001]	[0.000]	[0.008]	[0.362]

using the white noise covariance matrix estimated from the residuals of the first stage. The two-stage procedure is advantageous in the sense that further subset restrictions can be accounted for in the second stage. Detailed results for just-identifying cointegration vectors and loading vectors are given in Table 4.5.[11] Different orderings of the variables are checked to make sure the normalized parameters significantly belong to the cointegration vectors. We can see that the first cointegration relation is a real exchange rate relationship, the second one looks like an interest rate rule, while the third one relates relative government bond rate with real terms. Recursively calculated eigenvalues and recursively calculated fluctuation tests, with short-run dynamics concentrated out, show that the vectors of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ matrices are very stable over the entire sample period, which is consistent with the implication of the recursively calculated trace tests in Figure 4.2.[12]

As a weakly exogenous variable, relative output theoretically should respond to all cointegration relations insignificantly (i.e. $\alpha_{1,y} = \alpha_{2,y} = \alpha_{3,y} = 0$). Therefore, with $\alpha_{3,y} = 1.75$ significantly different from 0 at 5% level, the weak exogeneity test result seems to be challenged. However, this puzzling result is due to the rotation of the $\boldsymbol{\beta}$ vectors which causes a corresponding rotation of the α vectors. To ensure a constant

[11] All the computations are performed with JMulTi, Version 4.23 (see Lutkepohl and Kratzig (2004)) unless otherwise stated.

[12] Graphs for recursively calculated eigenvalues and recursively calculated fluctuation tests are available upon request.

Table 4.5: Just-identifying cointegration vectors and loading vectors

	Cointegration vectors			Loadings		
	$\boldsymbol{\beta}_1$	$\boldsymbol{\beta}_2$	$\boldsymbol{\beta}_3$	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$	$\boldsymbol{\alpha}_3$
q	1	.	.	-0.28^{**}	-2.33	5.98^{*}
				(-4.86)	(-1.76)	(2.19)
ir	.	1	.	-0.01^{**}	-0.21^{**}	-0.10
				(-4.30)	(-3.49)	(-0.80)
br	.	.	1	-0.00	0.07	-0.48^{**}
				(-0.78)	(1.57)	(-4.92)
g	0.48	0.08^{*}	0.01	0.07^{**}	0.37	0.30
	(0.90)	(2.35)	(0.75)	(2.87)	(0.69)	(0.28)
y	-1.85^{**}	-0.19^{**}	-0.09^{**}	-0.02	-0.68	1.75^{*}
	(-4.23)	(-6.46)	(-12.05)	(-1.16)	(-1.65)	(2.07)
$Trend$	0.00	0.00	0.00			
	(2.43)	(5.64)	(8.03)			

Notes: Cointegration vectors $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$ show the long-run equilibrium relationships among the variables. Loading vectors $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and $\boldsymbol{\alpha}_3$ indicate how various variables respond to $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$, respectively. The coefficients with * and ** are significant at 5% and 1% level, respectively. Significance level is not labeled for $Trend$ as the t-values for its parameters may not be asymptotically normal.

$\boldsymbol{\alpha}\boldsymbol{\beta}'$ matrix, any changes of the $\boldsymbol{\beta}$ vectors have to be offset by the $\boldsymbol{\alpha}$ coefficients.[13] Therefore, a significant α coefficient for relative output is not at odds with its weak exogeneity. We can see almost all the significant coefficients in $\boldsymbol{\alpha}$ matrix (with * and **) indicate error corrections of the variables to corresponding cointegration relations except for relative government consumption (g) to $\boldsymbol{\beta}_1$.[14] The loading coefficient shows that relative government consumption responds to $\boldsymbol{\beta}_1$ positively ($\alpha_{1,g} = 0.07$) at 1% significance level. However, the coefficient of relative government consumption is also positive in $\boldsymbol{\beta}_1$ with $\beta_{1,g} = 0.48$, even if not significant.

Nevertheless, the insignificant t-values in cointegration vectors (specifically, $\beta_{1,g}$ and $\beta_{3,g}$) suggest further simplifications of the structure. Besides, the trend in $\boldsymbol{\beta}_1$ has a relatively small t-value of 2.43 and may be insignificant. This is because the underlying assumption that the t-values are asymptotically normal is not necessarily true for the parameters of deterministic terms.[15] When only relative government consumption is restricted to be zero in $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_3$, the over-identifying model is accepted with a p-value of 0.172. When relative government consumption is restricted to be zero in $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_3$ and the trend is restricted to be zero in $\boldsymbol{\beta}_1$, the over-identifying model is accepted with a higher p-value of 0.257. Therefore, I proceed with the second over-identifying scheme. Corresponding estimates are displayed in Table 4.6, and the three cointegration relations with short-run dynamics concentrated out are plotted in Figure 4.3. Diagnostic tests for constancy of $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}_i$ ($i = 1, 2, 3, s$) matrices are carried out as well. Despite the initial instability due to small sample size, recursively estimated coefficients do not detect any structural break.[16] Compared to estimates in Table 4.5, the three cointegration vectors in Table 4.6 make more economical sense and the loading matrix $\boldsymbol{\alpha}$ is almost unaffected.

[13] See Juselius (2006), Chapter 13 for more details.

[14] A certain variable j is error-correcting to a cointegration relation β_i if $\alpha_{i,j}$ has an opposite sign to $\beta_{i,j}$. See Chapter 7 of Juselius (2006) for more details.

[15] See Lutkepohl and Kratzig (2004), Chapter 3 for more details.

[16] Graphs for recursively estimated coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}_i$ ($i = 1, 2, 3, s$) matrices are available upon request.

Table 4.6: Over-identifying cointegration vectors and loading vectors

	Cointegration vectors			Loadings		
	$\boldsymbol{\beta}_1$	$\boldsymbol{\beta}_2$	$\boldsymbol{\beta}_3$	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$	$\boldsymbol{\alpha}_3$
q	1	.	.	-0.24^{**}	-2.51	6.19^{*}
				(-4.16)	(-1.86)	(2.24)
ir	.	1	.	-0.01^{**}	-0.20^{**}	-0.07
				(-4.31)	(-3.39)	(-0.59)
br	.	.	1	-0.00	0.08	-0.45^{**}
				(-0.42)	(1.58)	(-4.59)
g	.	0.09^{**}	.	0.07^{**}	0.30	0.69
		(7.77)		(2.92)	(0.58)	(0.65)
y	-0.82^{**}	-0.24^{**}	-0.09^{**}	-0.02	-0.55	1.75^{*}
	(-4.97)	(-13.43)	(-14.21)	(-1.17)	(-1.35)	(2.09)
$Trend$	.	0.00	0.00			
		(15.53)	(11.61)			

Notes: Cointegration vectors $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$ show the long-run equilibrium relationships among the variables. Loading vectors $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$ and $\boldsymbol{\alpha}_3$ indicate how various variables respond to $\boldsymbol{\beta}_1$, $\boldsymbol{\beta}_2$ and $\boldsymbol{\beta}_3$, respectively. The coefficients with * and ** are significant at 5% and 1% level, respectively. Significance level is not labeled for $Trend$ as the t-values for its parameters may not be asymptotically normal.

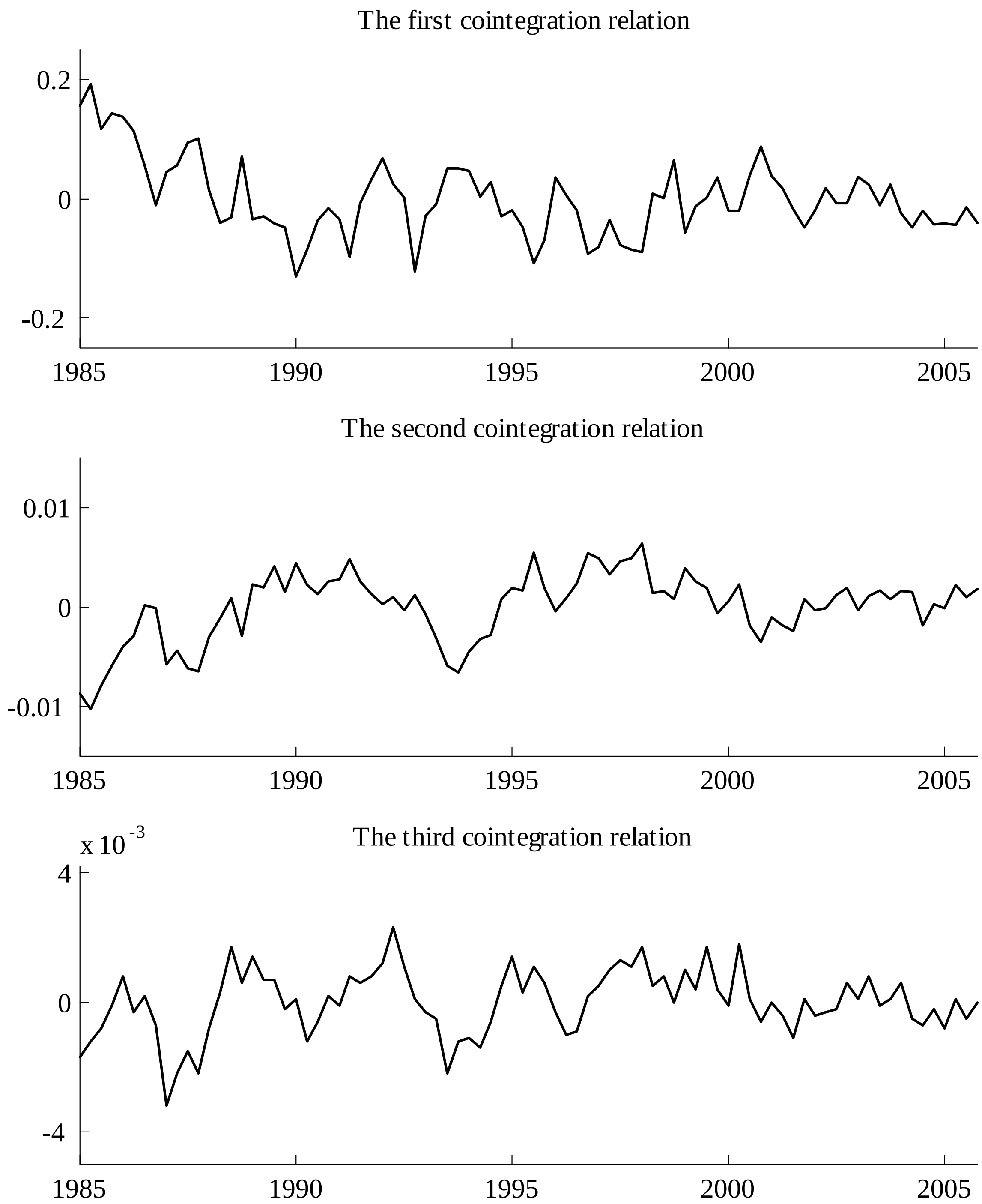

Figure 4.3: Cointegration relations of VECM, with short-run effects concentrated out.

Cointegration relations in Table 4.6 are rewritten into a more interpretable form:

$$q_t = 0.82y_t + const + ec_1$$

$$ir_t = 0.24y_t - 0.09g_t + Trend + const + ec_2$$

$$br_t = 0.09y_t + Trend + const + ec_3$$

where ec_i $(i = 1, 2, 3)$ are deviations from equilibriums. The first relation states that the real effective exchange rate (q) appreciates with higher relative output (y) in the long run, and even trend adjustment is not necessary. This finding strongly supports the Balassa-Samuelson hypothesis as non-tradable goods are also included in the output. The second relation says that in the long run relative interest rate (ir) increases with relative output (y) but decreases with relative government consumption (g). The first part is consistent with the Taylor's rule, which finds that monetary authorities will increase the interest rate to cool down the economy when output is above its potential level and lower the interest rate to stimulate the economy when output falls below its potential level. The second part may be associated with the fact that the increase in budget deficit resulting from increased government consumption is usually monetarized by monetary authorities. The monetization injects money into the economy (i.e. the LM-curve shifts to the right), causing the interest rate to drop in the short run. Theoretically, the short-run decrease should be offset in the long run, as the expansionary fiscal policy will shift the IS-curve to the right. However, many empirical evidence shows that the impact of fiscal policy on output is very short-lived, i.e. the right-ward shift of IS-curve is not permanent as expected.[17] Therefore, the long-run result of expansionary fiscal policy turns out to be a decrease in the interest rate. The third relation relates relative government bond rate (br) with relative output (y), which have two potential interpretations. The first interpretation is that higher interest rate decreases individuals' demand for government bonds and thus lowers the

[17] See, for example, Perotti (2005), Mountford and Uhlig (2009), and Enders et al. (2011).

price of government bond. Therefore, government bond rate is positively correlated with output via interest rate. The second interpretation is that individuals' demand for "risk-free" government bonds is not strong when the economy is booming and the prosperity is expected to continue. In such situation, the price of government bonds drops and government bond rate increases.

4.4 Structural form analysis

The residuals of some equations in the reduced form analysis exhibit some degree of contemporaneous correlation, as displayed in Table 4.7. For example, residuals of relative interest rate equation and relative government bond rate equation have a contemporaneous correlation of 0.64. Therefore, analysis of a structural form can tell us more. The autoregressive form in Equation 4.1 has the following moving average representation:

$$\mathbf{X}_t = \boldsymbol{\Xi} \sum_{s=1}^{t} \mathbf{u}_s + \boldsymbol{\Xi}^*(L)\mathbf{u}_t + \widetilde{\mathbf{D}}_t + \widetilde{\mathbf{X}}_0 \tag{4.2}$$

where $\boldsymbol{\Xi} = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp^{'}(\mathbf{I} - \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i)\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}_\perp^{'}$, $\boldsymbol{\Xi}^*(L) = \sum_{j=0}^{\infty} L^j \boldsymbol{\Xi}_j^*$ with the lag operator L and coefficient matrices $\boldsymbol{\Xi}_j^*$ that diminish to 0 as $j \to \infty$, $\mathbf{u}_t$ is the residual vector from the estimation of the reduced form, $\widetilde{\mathbf{D}}_t$ is a shorthand notation for deterministic components, and $\widetilde{\mathbf{X}}_0$ contains all initial values.[18] Let define the vector of structural shocks $\mathbf{e}_t = (e_t^q, e_t^{ir}, e_t^{br}, e_t^g, e_t^y)'$ as $\mathbf{e}_t = \mathbf{B}^{-1}\mathbf{u}_t$, where $\mathbf{B}$ is the contemporaneous impact matrix. By substituting $\mathbf{B}\mathbf{e}_t$ into Equation 4.2, the following expression is obtained:

$$\mathbf{X}_t = \boldsymbol{\Xi}\mathbf{B} \sum_{s=1}^{t} \mathbf{e}_s + \boldsymbol{\Xi}^*(L)\mathbf{B}\mathbf{e}_t + \widetilde{\mathbf{D}}_t + \widetilde{\mathbf{X}}_0$$

with all the long-run effects of structural shocks in $\boldsymbol{\Xi}\mathbf{B}$ matrix.

[18] See Johansen (1995), Chapter 4; Lutkepohl and Kratzig (2004), Chapter 4; Juselius (2006), Chapter 6 for more details.

Table 4.7: Contemporaneous Correlation

	q	ir	br	g	y
q	1	.	.	.	.
ir	−0.02	1	.	.	.
br	0.26	0.64	1	.	.
g	−0.16	−0.10	−0.07	1	.
y	0.00	0.42	0.41	0.04	1

To identify the structural form, six restrictions are required to separate transitory and permanent shocks and one more exclusion restriction in the long-run matrix. For the contemporaneous impact matrix, three restrictions on the transitory shocks are compulsory. Based on the weakly exogeneity test results and also economic theories, the two common stochastic trends come from relative government consumption and relative output, identified as relative fiscal policy shock (e_t^g) and relative productivity shock (e_t^y). The remaining shocks are labeled as transitory, which are assumed to have no permanent effect on the real exchange rate and other variables. The additional exclusion restriction in the long-run matrix is set as relative fiscal policy shock cannot affect relative output in the long run, which has been discussed already. For the restrictions in contemporaneous impact matrix, the contemporaneous correlation of residuals and economic theories are taken into account. Accordingly, the restrictions are set as (1) structural shock from foreign exchange markets (e_t^q) does not have an instantaneous impact on relative interest rate and relative government bond rate and (2) structural shock from international financial markets (e_t^{br}) does not affect relative interest rate immediately either. The impacts of relative monetary policy shock (e_t^{ir}) are left unrestricted.

The just-identifying structural form estimates are given in Table 4.8, with t-values calculated from 10000 bootstrap replications. Here we mainly look at those significant coefficients (with * and **). For the contemporaneous effects in matrix **B**, 1% increase in the real exchange rate reduce relative government consumption instantly by 0.33%, which should be related with increasing domestic price level. 1% positive

Table 4.8: Just-indentifying SVECM

	Contemporaneous matrix $\mathbf{B}$ $(\times 10^2)$						Long-run matrix $\Xi\mathbf{B}$ $(\times 10^2)$	
	e^q	e^{ir}	e^{br}	e^g	e^y		e^g	e^y
q	$\mathbf{1.37}^{**}$	-0.03	0.37	$\mathbf{0.82}^{*}$	-0.22	. . .	0.00	$\mathbf{0.64}^{**}$
	(4.10)	(-0.10)	(1.27)	(2.50)	(-0.67)		(-0.19)	(3.15)
ir	.	$\mathbf{0.06}^{**}$	.	0.01	$\mathbf{0.03}^{**}$	. . .	$\mathbf{-0.06}^{**}$	$\mathbf{0.19}^{**}$
		(4.46)		(0.52)	(2.48)		(-3.20)	(2.83)
br	.	$\mathbf{0.02}^{**}$	$\mathbf{0.04}^{**}$	$\mathbf{0.02}^{*}$	$\mathbf{0.03}^{**}$	. . .	0.00	$\mathbf{0.07}^{**}$
		(2.92)	(3.98)	(2.07)	(2.68)		(0.02)	(3.15)
g	$\mathbf{-0.33}^{**}$	-0.12	$\mathbf{-0.27}^{**}$	$\mathbf{0.46}^{**}$	-0.01	. . .	$\mathbf{0.71}^{**}$	-0.05
	(-2.84)	(-1.25)	(-2.81)	(3.29)	(-0.09)		(3.20)	(-0.18)
y	0.08	-0.02	-0.10	0.04	$\mathbf{0.48}^{**}$	. . .	.	$\mathbf{0.78}^{**}$
	(1.09)	(-0.33)	(-1.19)	(0.39)	(4.23)			(3.15)

Notes: e^q, e^{ir}, e^{br}, e^g and e^y refer to structural shocks from foreign exchange markets, relative monetary policy, international financial markets, relative fiscal policy and relative output, respectively. The coefficients with * and ** are significant at 5% and 1% level, respectively.

monetary policy shock increases relative government bond rate instantaneously by 0.02%, consistent with standard theory. Not surprisingly, 1% increase in relative government bond rate immediately decreases relative government consumption by 0.27% through higher costs for borrowing. 1% positive relative fiscal policy shock appreciates the real exchange rate by 0.82% through increasing the relative price of non-tradable goods and increases the relative government bond rate by 0.02%. One explanation for the latter phenomenon is that higher government consumption is usually followed by issuing more government bonds, which makes its price drop. 1% productivity innovation not only contemporarily increases relative government bond rate by 0.03% but also increases relative interest rate by 0.03%, which implies the monetary authorities respond to external shocks quickly.

The long-run effects contained in matrix $\Xi\mathbf{B}$ are highly consistent with cointegration relations discussed earlier. The real exchange rate and relative government bond rate are exclusively determined by productivity shock in the long run. For 1% productivity innovation, the former appreciates by 0.64% and the latter increases by

0.07%. In contrast, relative interest rate is influenced by both shock in the long run, decreasing by 0.06% for 1% increase in relative government consumption and increasing 0.29% for 1% productivity innovation. Roughly consistent with the prediction of the weak exogeneity test statistics, relative government consumption is only affected by relative fiscal policy shock and relative output is only affected by relative productivity shock in the long run. The robustness of the results are checked with different exclusion restrictions in the long-run matrix: (1) relative productivity shock cannot affect relative government consumption in the long run and (2) relative fiscal policy shock cannot affect relative output in the long run and relative productivity shock cannot affect relative government consumption in the long run either. Estimated parameters of the two alternative exclusion restriction schemes are almost identical to those in Table 4.8, with the second over-identifying scheme accepted with a p-value of 0.541.

4.4.1 Impulse responses

Structural impulse responses of the real exchange rate to positive structural shocks of size one standard deviation and corresponding 95% confidence intervals based on 1000 bootstrap replications are computed and displayed in Figure 4.4. Generally, relative monetary policy shock does not have any significant influence on the real exchange rate, which is in line with the finding in Chapter 2 and many other empirical findings for the great moderation period.[19] The influence from international financial markets is only borderline significant around the second year. The appreciation stimulated by relative fiscal policy shock can stay significant for nearly three years, but damps out at longer horizon. The impact of relative productivity shock is negative and insignificant in the first few years, but finally becomes positive and significant. This finding may be related with the fact that most productivity shocks occur in the tradable sector. As mentioned in the introduction of Chapter 4, a relative productivity innovation in

[19] See, for example, Boivin and Giannoni (2002, 2006).

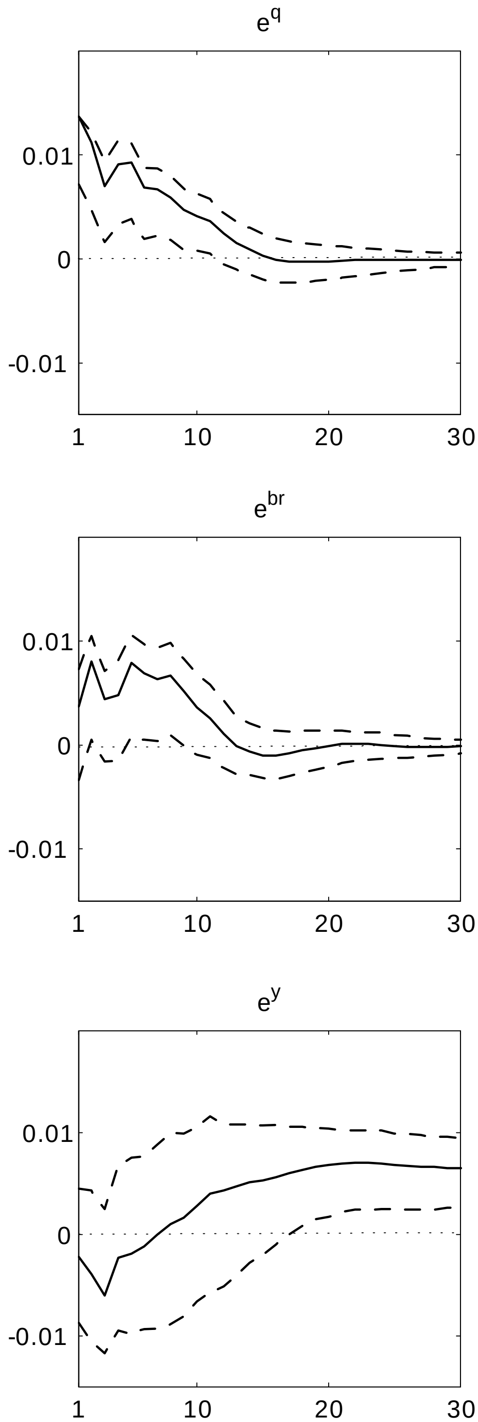

Figure 4.4: Impulse responses of the U.S. real effective exchange rate to one standard deviation structural shocks from foreign exchange markets (e^q), relative monetary policy (e^{ir}), international financial markets (e^{br}), relative fiscal policy (e^g) and relative output (e^y).

Table 4.9: Forecast error variance decompositions

Horizon		Transitory shocks			Permanent shocks	
		e^q	e^{ir}	e^{br}	e^g	e^y
1	quarter	0.69	0.00	0.05	0.24	0.02
8	quarters	0.36	0.05	0.17	0.38	0.04
16	quarters	0.30	0.04	0.15	0.41	0.09
40	quarters	0.21	0.03	0.11	0.29	0.37
60	quarters	0.17	0.02	0.09	0.23	0.49
80	quarters	0.14	0.02	0.07	0.19	0.58
100	quarters	0.12	0.02	0.06	0.16	0.64

the tradable sector increases the relative price of non-tradable goods but decreases the international relative price of tradable goods. The two opposite effects may offset each other in the short run, but the Balassa-Samuelson effect gradually dominates.

4.4.2 Variance decomposition

Forecast error variance decompositions for the real exchange rate are reported in Table 4.9 as the horizon is increased from 1 to 100 quarters. In the short run, transitory shock from foreign exchange markets (e^q) accounts for around 50% of the forecast error variance during the first four quarters. But as expected, their relative influence keeps declining at longer forecasting horizons. Based on the computation, relative monetary policy shock (e^{ir}) has very limited impact on the real exchange rate movements throughout the time. The influence from international financial markets (e^{br}) is slightly larger and achieves its peak at the end of the second year. Relative fiscal policy shock (e^g) is another source of short-run real exchange rate fluctuations, the impact of which also displays a parabola pattern. The effect of relative productivity shock (e^y) is not obvious in the first few years, but begins to dominate the forecast error variance of the real exchange rate since the 9th year. We can see that 64% of the 25-year fluctuations is due to relative productivity shock.

4.5 Conclusions

With the development of time series econometric techniques, one key interest of research on real exchange rate dynamics is to empirically identify the macroeconomic shocks that fluctuate the real exchange rate. Earlier research, such as Clarida and Gali (1994), Weber (1997) and Chadha and Prasad (1997), uses structural VAR model and documents that the impact of supply side shock is negligible on real exchange rate fluctuations. Later studies start to explicitly consider cointegration relations in the data and get quite opposite conclusions. For example, Alexius (2005) finds relative productivity shock to be the most important source of long-run fluctuations in a few bilateral real exchange rates. This chapter uses the latter statistical approach to analyze the U.S. real effective exchange rate, rather than the conventional bilateral rates. Other variables are accordingly constructed relative to the "rest of the world" comprising of an aggregate of industrialized countries. The dataset mainly covers the great moderation period, completely within the modern floating rate era.

Reduced form estimation detects three stable long-run relationships in the data, which are (1) the real exchange rate appreciates when relative output increases, (2) relative interest rate increases with relative output but decreases with relative government consumption, and (3) relative government bond rate increases with relative output. Different from Alexius (2005), no long-run relationship between the real exchange rate and relative government consumption is detected in the data. Subsequent structural analysis shows that while the real exchange rate mainly responds to shock from relative government bond rate and relative government consumption in the short run, its long-run dynamics is exclusively affected by relative productivity shock. Again, during the great moderation, relative monetary policy shock exhibits negligible impact on the real exchange rate at any time horizon. Forecast error variance decompositions for the real exchange rate obtain similar results. In the first few years, structural shocks from foreign exchange markets, international financial markets and relative fiscal policy account for more than two thirds of the fluctuations in the real exchange rate. However, relative productivity shock begins to dominate long-run movements in the real exchange rate. In summary, the analysis provides another piece of evidence to the Balassa-Samuelson hypothesis as the U.S. real effective exchange rate is found to appreciate exclusively with higher relative output in the long run.

Chapter 5

Conclusions

Historically, the real exchange rate displays highly volatile and persistent fluctuations, and usually appreciates toward more productive countries. The existing literature explains the volatility with monetary shocks and attributes the persistence to real shocks, subject to an important underlying assumption that prices are highly sticky. Specifically for the great moderation period, the U.S. real exchange rate becomes less volatile in absolute value, accompanied by weaker impact of monetary policy on the economy. Besides, even if prices are found to be becoming less sticky, the U.S. real exchange rate is also as persistent as before. Overall, the thesis proves that the sticky information model is very powerful in replicating the characteristics of the U.S. real exchange rate in this period, with a reasonably low degree of price stickiness set correspondingly. The empirical analysis of the U.S. real exchange rate fluctuations also proves that the settings and the implication of my theoretical frameworks are in line with the data.

5.1 Findings and policy implications

In Chapter 2, it is empirically found that the U.S. real exchange rate responds to monetary policy shock quite differently before and after the mid of 1980s. The response in the great moderation becomes weaker and insignificant with no delayed overshooting. The sticky information DSGE model identifies monetary policy shock persistence and price stickiness as the possible sources of the change. The benchmark model replicates well the weak and quick response of the real exchange rate during the great moderation. With higher monetary policy shock persistence and higher price stickiness, which is consistent with historical evidence, the simulated impulse response replicates exactly the strong and hump-shaped response of the U.S. real exchange rate before the mid-1980s. The sensitivity tests also re-confirm that delayed overshooting in the real exchange rate is the outcome of sufficiently high public misperception and sufficiently persistent monetary policy shock. These findings suggest that when monetary authorities are better informed and more agents are able to receive new information in each period, the real exchange rate and other macroeconomic variables will be stabilized. Therefore, monetary authorities need to establish a more efficient mechanism for efficient acquisition, absorption and processing of information and also make the monetary policies more transparent to the public. With the fast development of information technology, requiring targeted firms or institutions to report their daily operation status through electronic systems is feasible. Publishing and explaining any change in monetary policy to the public is technologically feasible as well.

The benchmark sticky information model in Chapter 3 is capable of generating highly persistent real exchange rate as in the data with (1) persistent productivity shock and low level of price stickiness set in line with recent estimates or (2) persistent monetary policy shock and high level of price stickiness widely used in existing literature. As standard sticky price model cannot do so with similar parameter values in (1) or (2), the settings of the sticky price model and the sticky information model are compared analytically. In the sticky price model, only random fractions of firms and workers can make adjustments in each period. Therefore, they set prices or wages equal to a weighted average of the current and all future optimal levels once they are allowed to. On the contrary, all agents in the sticky information model are allowed to make adjustments in each period and thus can fully respond to shocks they know in each adjustment. Given persistent shocks, the pass-through of shocks and their persistence to the variables is thus higher in the sticky information model. Therefore, the sticky information model can generate high persistence in the macroeconomic variables more easily compared to the sticky price model.

The U.S. real effective exchange rate is analyzed in Chapter 4 for the great moderation period. Three long-run equilibrium relations are found in the data, one of which states that the real exchange rate is positively cointegrated with relative output. The structural form analysis shows that the impact of relative productivity shock on the U.S. real exchange rate is insignificant in the short run but becomes significant in the long run, which is in support of the finding in Chapter 3 that the high persistence in the real exchange rate is due to persistent productivity shocks. Forecast error variance decompositions for the real exchange rate draw a similar conclusion that relative productivity shock dominates the long-run fluctuations of the U.S. real exchange rate but have negligible impact in the first few years. Specifically, relative monetary policy shock in this period has negligible and insignificant influence on the real exchange rate at any horizon, which re-confirms the empirical finding in Chapter 2.

5.2 Future research

It has been shown that the sticky information model is very powerful in reproducing some stylized facts of the real exchange rate. In the future, I will continue to explore whether the sticky information model can help to better explain other interesting but controversial issues in the research field of international macroeconomics. For example, as mentioned in Chapter 4, standard economic theory predicts productivity innovations in the tradable sector should depreciate the real exchange rate but empirical data show otherwise. Similarly, the international consumption risk sharing condition is conventionally set in open economy models but can hardly find any empirical support from the data (also known as "Backus–Smith puzzle"). The impacts of institutional factors such as the Plaza accord on the real exchange rate and trade is also interesting and worth of investigating.

One limitation of the thesis is that the analysis mainly focuses on the great moderation period, whereas many countries are still caught in recession now. For practical purpose, it is challenging but meaningful to investigate whether the sticky information model can replicate the current situation well and provide promising policy implications. A second limitation of the current framework is that there is no financial sector in the real sense. Mishkin (2011) points out that one of the lessons learned from the financial crisis is that the impact on the economy of developments in the financial sector has been underestimated in existing macroeconomics literature and monetary policy. In future research, a financial sector will be incorporated into the framework so that the influence of the financial sector on the economy can be analyzed.

Appendix I. Detailed Algorithm of the Model in Chapter 2

AI.1 Market clearing conditions

AI.1.1 Goods market

Consumption preferences specified in the chapter imply that:

$$C^{j}_{H,t,i} = (\frac{P_{t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_t})^{-\zeta}C^{j}_{t},$$
$$C^{j*}_{H,t,i} = (\frac{P^*_{t,i}}{P^*_{H,t}})^{-\upsilon}(\frac{P^*_{H,t}}{P^*_t})^{-\zeta}C^{j*}_{t},$$

where $P_t = [nP^{1-\zeta}_{H,t} + (1-n)P^{1-\zeta}_{F,t}]^{\frac{1}{1-\zeta}}$ and $P^*_t = [nP^{*1-\zeta}_{H,t} + (1-n)P^{*1-\zeta}_{F,t}]^{\frac{1}{1-\zeta}}$.

Summing over all consumers, the goods market for variety i clears as:

$$Y_{t,i} = (\frac{P_{t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_t})^{-\zeta}C_t + (\frac{P^*_{t,i}}{P^*_{H,t}})^{-\upsilon}(\frac{P^*_{H,t}}{P^*_t})^{-\zeta}C^*_t,$$

AI.1.2 Labor market

The total labor input of each producer is a Dixit-Stiglitz aggregator of various labor inputs hired, so that:

$$L^j_{t,i} = \frac{1}{n}(\frac{W^j_t}{W_t})^{-\gamma}L_{t,i},$$

Summing over all firms, the labor market for labor j clears as:

$$L^j_t = \frac{1}{n}(\frac{W^j_t}{W_t})^{-\gamma}L_t,$$

where $L_t = \int_0^n L_{t,i}di = [(\frac{1}{n})^{\frac{1}{\gamma}}\int_0^n L^{j\,\frac{\gamma-1}{\gamma}}_t dj]^{\frac{\gamma}{\gamma-1}}$ is the total labor inputs of all Home firms and the total labor supplied in equilibrium. Analogous conditions hold for Foreign goods and Foreign labor market.

AI.2 Consumer behavior as a planner

Note that all the consumers are identical aside from how long they last planned. For the Home country, let denote $F_t^j \equiv [W_t^j L_t^j + (1+i_{t-1})B_{t-1}^j + (1+i_{t-1}^*)S_t B_{t-1}^{*j} + T_t^j]/P_t$ as the real resources with which planner j enters period t. The assumption of perfect insurance implies that $F_t^j = F_t$, identical for all planners. $V(F_t, .)$ refers to the value function for planners that plan at period t, taking other state variables in the second argument as given. For a newly updated planner, he solves:

$$V(F_t) = \max_{\{C_{t+m,m}\}} \{\sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m U(C_{t+m,m}, .) + \beta\lambda_c \sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m E_t[V(F_{t+m+1})]\}$$

$$\text{s.t. } F_{t+m+1} = R_{t+m+1}(F_{t+m} - C_{t+m,.}) + (W_{t+m+1,.}L_{t+m+1,.} + T_{t+m+1})/P_{t+m+1},$$

$$m \geq 0 \text{ and a no-Ponzi condition}$$

where $\beta \in (0,1)$ is the discounting factor, and $R_{t+1} = (1+i_t)P_t/P_{t+1}$ is the real return on nominal bonds.[1] The first-order condition with respect to $C_{t+m,m}$ are:

$$\beta^m (1-\lambda_c)^m C_{t+m,m}^{-1} = \beta\lambda_c \sum_{k=m}^{\infty} \beta^k (1-\lambda_c)^k E_t[V'(F_{t+k+1})R_{t+m,t+k+1}]$$

where $R_{t+m,t+k+1} = \prod_{z=t+m}^{t+k} R_{z+1}$, the compound return between two dates. The envelope theorem condition is:

$$V'(F_t) = \beta\lambda_c \sum_{k=0}^{\infty} \beta^k (1-\lambda_c)^k E_t[V'(F_{t+k+1})R_{t,t+k+1}]$$

By setting $m = 0$ in the first condition, together with the second condition, $V'(F_t) = C_{t,0}^{-1}$ is obtained. Together with the substitution condition between Home

[1] With competitive international bonds market, expected real return is identical on Home and Foreign bonds.

goods and Foreign goods, optimality conditions can be summarized as:

$$C_{t,0}^{-1} = \beta E_t[R_{t+1} C_{t+1,0}^{-1}],$$
$$C_{t,k}^{-1} = E_{t-k}[C_{t,0}^{-1}]$$

Similar conditions apply in Foreign country.

AI.3 Worker behavior

Similar to consumers, workers solve the dynamic program:

$$V(F_t) = \max_{\{W_{t+m,m}\}} E_t[\sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m U(L_{t+m,m},.) + \beta\lambda_w \sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m V(F_{t+m+1})]$$

$$\text{s.t. } F_{t+m+1} = R_{t+m+1}(F_{t+m} - C_{t+m,.}) + (W_{t+m+1,.} L_{t+m+1,.} + T_{t+m+1})/P_{t+m+1},$$
$$F_{t+m} = [W_{t+m,.} L_{t+m,.} + (1+i_{t+m-1})B_{t+m-1} + (1+i_{t+m-1}^*)S_t B_{t+m-1}^* + T_{t+m}]/P_{t+m},$$
$$L_{t+m,.} = \frac{1}{n}(\frac{W_{t+m,.}}{W_{t+m}})^{-\gamma} L_{t+m},$$
$$m \geq 0 \text{and a no-Ponzi condition.}$$

The first-order condition with respect to $W_{t+m,m}$ is:

$$\beta^m (1-\lambda_w)^m \frac{\gamma}{\gamma-1} \varkappa E_t[L_{t+m,m}^{1+\frac{1}{\varphi}}] W_{t+m,m}^{-1} = \beta\lambda_w \sum_{k=m}^{\infty} \beta^k (1-\lambda_w)^k E_t[\frac{V'(F_{t+1+k}) R_{t+m,t+k+1} L_{t+m,m}}{P_{t+m}}]$$

The envelope theorem condition:

$$V'(F_t) = \beta\lambda_w \sum_{k=0}^{\infty} \beta^k (1-\lambda_w)^k E_t[V'(F_{t+1+k}) R_{t,t+k+1}]$$

Setting $m = 0$ in the first condition, $W_{t,0} = \frac{\gamma}{\gamma-1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{V'(F_t)/P_t}$is obtained. Using this result, I can get:

$$W_{t,0} = \frac{\gamma}{\gamma-1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{C_{t,0}^{-1}/P_t},$$

$$\frac{L_{t,0}^{\frac{1}{\varphi}} P_t}{W_{t,0}} = E_t[\frac{\beta R_{t+1} L_{t+1,0}^{\frac{1}{\varphi}} P_{t+1}}{W_{t+1,0}}],$$

$$W_{t,k} = \frac{E_t[L_{t,k}^{1+\frac{1}{\varphi}}]}{E_{t-k}[L_{t,0}^{\frac{1}{\varphi}} L_{t,k}/W_{t,0}]},$$

Foreign workers make decision following similar conditions.

AI.4 Firm behavior as a sale department

A sale department last updating its information k periods ago needs to:

$$\max E_{t-k}[P_{t,k,i} Y_{t,k,i} - W_t L_{t,k,i}]$$

$$\text{s.t. } Y_{t,k,i} = A_t L_{t,k,i}^{\xi},$$

$$Y_{t,k,i} = (\frac{P_{t,k,i}}{P_{H,t}})^{-\upsilon} (\frac{P_{H,t}}{P_t})^{-\zeta} C_t + (\frac{P_{t,k,i}}{S_{t,k} P_{H,t}^*})^{-\upsilon} (\frac{P_{H,t}^*}{P_t^*})^{-\zeta} C_t^*,$$

where $S_{t,k} = E_{t-k}[S_t]$.

The first-order condition with respect to $P_{t,k,i}$ is:

$$P_{t,k,i} = \frac{\upsilon}{\upsilon - 1} \frac{E_{t-k}[A_t^{-\frac{1}{\xi}} Y_{t,k,i}^{\frac{1}{\xi}} W_t]}{E_{t-k}[\xi Y_{t,k,i}]}$$

Producers in Foreign country will set prices similarly.

AI.5 Monetary Policy

In this cashless economy, the monetary authority sets nominal interest rates according to the Taylor rule:

$$i_t = \phi_\pi \ln(\frac{P_t}{P_{t-1}}) + \phi_y \ln(\frac{Y_t}{Y_t^N}) + \varepsilon_t$$

where ϕ_π is the inflation targeting coefficient, ϕ_y is the output gap targeting coefficients, Y_t is Home output, $\overline{Y}_t$ is Home output with full information, ε_t is monetary policy shock that follows an $AR(1)$ process. Situation is similar in Foreign country.

With competitive international bond market, the uncovered interest rate parity holds:

$$1 + i_t = \frac{E_t[S_{t+1}]}{S_t}(1 + i_t^*)$$

which implies that an optimal risk-sharing condition holds:

$$\frac{C_{t,0}}{C_{t,0}^*} = \frac{E_t[Q_t C_{t+1,0}^{*-1}]}{E_t[Q_{t+1} C_{t+1,0}^{-1}]},$$

where Q_t is the real exchange rate defined as $S_t P_t^* / P_t$.

AI.6 Log-linearization

Small letters denote log-linear deviations of respective variables from steady state, except that r_t and r_t^* denote log-linear deviations of $E_t[R_{t+1}]$ and $E_t[R_{t+1}^*]$, respectively. The log-linearized optimality conditions are summarized below.

AI.6.1 Goods market

The goods market clear condition for Home goods in Section AI.1.1 implies:

$$\begin{aligned} y_{t,k,i} &= n[c_t - \upsilon(p_{t,k,i} - p_{H,t}) - \zeta(p_{H,t} - p_t)] \\ &\quad +(1-n)[c_t^* - \upsilon(p_{t,k,i}^* - p_{H,t}^*) - \zeta(p_{H,t}^* - p_t^*)], \qquad \text{(AI.1)} \\ y_t &= nc_t + (1-n)c_t^* - \zeta(p_{H,t} - p_t), \qquad \text{(AI.2)} \end{aligned}$$

Log-linearizing and iterating the optimality conditions of Home consumers in Section AI.2 gives:

$$c_{t,k} = E_{t-k}[c_{t+1,0} - r_t], \qquad \text{(AI.3)}$$

With many heterogenous groups of consumers, Home aggregate consumption index has the following log form:

$$c_t = \lambda_c \sum_{k=0}^{\infty} (1 - \lambda_c)^k c_{t,k}, \qquad \text{(AI.4)}$$

The pricing rule in Section AI.5 implies:

$$p_{t,k,i} = E_{t-k}[w_t + (\frac{1}{\xi} - 1)y_{t,k,i} - \frac{1}{\xi}a_t], \tag{AI.5}$$

With many heterogenous groups of firms, the aggregate price index for Home goods have the following log form:

$$p_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k p_{t,k,i}, \tag{AI.6}$$

$$p^*_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k (p_{t,k,i} - s_{t,k}), \tag{AI.7}$$

Similarly, Foreign country has:

$$\begin{aligned} y^*_{t,k,i^*} &= n[c_t - \upsilon(p_{t,k,i^*} - p_{F,t}) - \zeta(p_{F,t} - p_t)] \\ &\quad + (1-n)[c^*_t - \upsilon(p^*_{t,k,i^*} - p^*_{F,t}) - \zeta(p^*_{F,t} - p^*_t)], \end{aligned} \tag{AI.8}$$

$$y^*_t = nc_t + (1-n)c^*_t - \zeta(p^*_{F,t} - p^*_t), \tag{AI.9}$$

$$c^*_{t,k} = E_{t-k}[c^*_{t+1,0} - r^*_t], \tag{AI.10}$$

$$c^*_t = \lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k c^*_{t,k}, \tag{AI.11}$$

$$p^*_{t,k,i^*} = E_{t-k}[w^*_t + (\frac{1}{\xi} - 1)y^*_{t,k,i^*} - \frac{1}{\xi}a^*_t], \tag{AI.12}$$

$$p_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k (p^*_{t,k,i^*} + s_{t,k}), \tag{AI.13}$$

$$p^*_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k p^*_{t,k,i^*}, \tag{AI.14}$$

Accordingly, log form aggregate price index is $p_t = np_{H,t} + (1-n)p_{F,t}$ in Home country and $p^*_t = np^*_{H,t} + (1-n)p^*_{F,t}$ in Foreign country, respectively.

AI.6.2 Labor market

The log form Home labor market clear condition in Section AI.1.2 and Home production function are given by:

$$l_{t,k} = l_t - \gamma(w_{t,k} - w_t), \tag{AI.15}$$

$$y_t = a_t + \xi l_t, \tag{AI.16}$$

Log-linearizing and iterating the optimality conditions of Home worker in Section AI.3 gives:

$$w_{t,k} = E_{t-k}[p_t + \frac{1}{\varphi}l_{t,k} - r_t + w_{t+1,0} - p_{t+1} - \frac{1}{\varphi}l_{t+1,0}], \tag{AI.17}$$

$$w_{t,0} = p_t + \frac{1}{\varphi}l_{t,0} + c_{t,0}, \tag{AI.18}$$

With many heterogenous groups of workers, Home aggregate wage index equals:

$$w_t = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k w_{t,k}, \tag{AI.19}$$

Similarly, Foreign country has:

$$l^*_{t,k} = l^*_t - \gamma(w^*_{t,k} - w^*_t), \tag{AI.20}$$

$$y^*_t = a^*_t + \xi l^*_t, \tag{AI.21}$$

$$w^*_{t,k} = E_{t-k}[p^*_t + \frac{1}{\varphi}l^*_{t,k} - r^*_t + w^*_{t+1,0} - p^*_{t+1} - \frac{1}{\varphi}l^*_{t+1,0}], \tag{AI.22}$$

$$w^*_{t,0} = p^*_t + \frac{1}{\varphi}l^*_{t,0} + c^*_{t,0}, \tag{AI.23}$$

$$w^*_t = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k w^*_{t,k}, \tag{AI.24}$$

AI.6.3 Interest rates and consumption risk-sharing condition

With $y^N_t = na_t + (1-n)a^*_t + (1-n)\frac{\frac{1}{\varphi}+1}{\frac{1}{\varphi}+1-\xi+\frac{\xi}{\zeta}}(a_t - a^*_t)$ and $y^{*N}_t = na_t + (1-n)a^*_t - n\frac{\frac{1}{\varphi}+1}{\frac{1}{\varphi}+1-\xi+\frac{\xi}{\zeta}}(a_t - a^*_t)$, log-linearized Taylor rule in Home country follows:

$$i_t = \phi_\pi(p_t - p_{t-1}) + \phi_y(y_t - y^N_t) + \varepsilon_t, \tag{AI.25}$$

The expected real interest rates in Home country thus equals to:

$$r_t = i_t - E_t[\triangle p_{t+1}], \tag{AI.26}$$

Similarly, Foreign country has:

$$i_t^* = \phi_\pi(p_t^* - p_{t-1}^*) + \phi_y(y_t^* - y_t^{*N}) + \varepsilon_t^*, \tag{AI.27}$$

$$r_t^* = i_t^* - E_t[\triangle p_{t+1}^*], \tag{AI.28}$$

Log-linearizing and iterating the consumption risk-sharing condition in Section AI.5 gives:

$$c_{t,k} - c_{t,k}^* = E_{t-k}[s_t + p_t^* - p_t] = E_{t-k}[q_t], \tag{AI.29}$$

AI.7 Equilibrium

Combining Eq. (AI.1), (AI.2), and (AI.5) and substituting into Eq. (AI.6) gives the price index for Home goods:

$$p_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[np_{H,t} + (1-n)(p_{H,t}^* + s_t) + mc_t],$$

with $mc_t = \{(1-\beta)y_t + \beta[w_t - np_{H,t} - (1-n)(p_{H,t}^* + s_t)]\}/[\beta + (1-\beta)\upsilon]$. Eq. (AI.7) implies that:

$$p_{H,t}^* = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[n(p_{H,t} - s_t) + (1-n)p_{H,t}^* + mc_t],$$

Combining Eq. (AI.8), (AI.9), and (AI.12) and substituting into Eq. (AI.14) gives the price index for Foreign goods:

$$p_{F,t}^* = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[n(p_{F,t} - s_t) + (1-n)p_{F,t}^* + mc_t^*],$$

with $mc_t^* = \{(1-\beta)y_t^* + \beta[w_t^* - n(p_{F,t} - s_t) - (1-n)p_{F,t}^*)]\}/[\beta + (1-\beta)\upsilon]$. Similarly, Eq. (AI.13) implies that:

$$p_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[np_{F,t} + (1-n)(p_{F,t}^* + s_t) + mc_t^*],$$

Therefore, the aggregate supply in Home country and Foreign country, respectively, as:

$$\begin{aligned} p_t &= \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[np_t + (1-n)p_t^* + (1-n)s_t + nmc_t + (1-n)mc_t^*], \\ p_t^* &= \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[np_t + (1-n)p_t^* - ns_t + nmc_t + (1-n)mc_t^*], \end{aligned}$$

Iterating forward on Eq. (AI.3) gives:

$$c_{t,k} = -\sum_{i=0}^{K} E_{t-k}[r_{t+i}] + E_{t-k}[c_{t+K+1,0}],$$

When $K \to \infty$, all planners are informed, so that $\lim_{i\to\infty} E_t[r_{t+i}] = \lim_{i\to\infty} E_t[r_{t+i}^N] = 0$, and $\lim_{i\to\infty} E_t[c_{t+i,0}] = \lim_{i\to\infty} E_t[c_{t+i}^N] = 0$. Since the fraction of inattentive planners falls exponentially as time passes by, this limit can be approached quickly to ensure that $\sum_{i=0}^{K} E_{t-k}(r_{t+i})$ converges. Therefore,

$$c_{t,k} = -E_{t-k}[\overline{R}_t], \tag{AI.30}$$

where $\overline{R}_t = E_t[\sum_{i=0}^{\infty} r_{t+i}]$ is the long-term real interest rates in Home country. Similarly,

$$c_{t,k}^* = -E_{t-k}[\overline{R}_t^*], \tag{AI.31}$$

where $\overline{R}_t^* = E_t[\sum_{i=0}^{\infty} r_{t+i}^*]$ is the long-term real interest rates in Foreign country. Replacing for $c_{t,k}$ and $c_{t,k}^*$ in Eq. (AI.4) and (AI.11) and substituting them into Eq.

(AI.2) and (AI.9), respectively, gives the IS curve for both countries:

$$y_t = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k[n\overline{R}_t + (1-n)\overline{R}_t^*] - \zeta(p_{H,t} - p_t),$$
$$y_t^* = -\lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k[nR_t + (1-n)R_t^*] - \zeta(p_{F,t}^* - p_t^*),$$

Using Eq. (AI.18) and (AI.30) and Eq. (AI.17) gives:

$$w_{t,k} = E_{t-k}[p_t + \frac{1}{\varphi}l_{t,k} - \overline{R}_t],$$

Using this result as well as (AI.15), (AI.16) and substituting into Eq. (AI.19) gives the Home wage curve:

$$w_t = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t + \frac{y_t - a_t}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t - p_t) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t],$$

Similarly, the wage curve in Foreign country follows:

$$w_t^* = \lambda_w \sum_{k=0}^{\infty}(1-\lambda_w)^k E_{t-k}[p_t^* + \frac{y_t^* - a_t^*}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t^* - p_t^*) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t^*],$$

Finally, using Eq. (AI.29), (AI.30), and (AI.31), the optimal consumption-sharing condition is obtained as:

$$\overline{R}_t^* - \overline{R}_t = s_t + p_t^* - p_t = q_t.$$

Appendix. Detailed Algorithm of the Model in Chapter 3

AII.1 Market clearing conditions

AII.1.1 Goods market

Consumption preferences specified in the chapter implies:

$$C^j_{H,t,i} = \omega(\frac{P_{H,t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_{T,t}})^{-\theta}(\frac{P_{T,t}}{P_t})^{-\eta}C^j_t,$$
$$C^{j^*}_{H,t,i} = (1-\omega)(\frac{P^*_{H,t,i}}{P^*_{H,t}})^{-\upsilon}(\frac{P^*_{H,t}}{P^*_{T,t}})^{-\theta}(\frac{P^*_{T,t}}{P^*_t})^{-\eta}C^{j^*}_t,$$
$$C^j_{N,t,i} = (\frac{P_{N,t,i}}{P_{N,t}})^{-\upsilon}(\frac{P_{N,t}}{P_t})^{-\eta}C^j_t,$$

where $P_{T,t} = [\omega P^{1-\theta}_{H,t} + (1-\omega)P^{1-\theta}_{F,t}]^{\frac{1}{1-\theta}}$, $P^*_{T,t} = [(1-\omega)P^{*1-\theta}_{H,t} + \omega P^{*1-\theta}_{F,t}]^{\frac{1}{1-\theta}}$, $P_t = [\zeta P^{1-\eta}_{T,t} + (1-\zeta)P^{1-\eta}_{N,t}]^{\frac{1}{1-\eta}}$ and $P^*_t = [\zeta P^{*1-\eta}_{T,t} + (1-\zeta)P^{*1-\eta}_{N,t}]^{\frac{1}{1-\eta}}$.

Summing over all consumers, the goods market for variety i, either tradable or non-tradable, clears as:

$$\begin{aligned} Y_{H,t,i} &= \omega(\frac{P_{H,t,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_{T,t}})^{-\theta}(\frac{P_{T,t}}{P_t})^{-\eta}C_t + (1-\omega)(\frac{P^*_{H,t,i}}{P^*_{H,t}})^{-\upsilon}(\frac{P^*_{H,t}}{P^*_{T,t}})^{-\theta}(\frac{P^*_{T,t}}{P^*_t})^{-\eta}C^*_t, \\ Y_{N,t,i} &= (\frac{P_{D,t,i}}{P_{D,t}})^{-\upsilon}(\frac{P_{D,t}}{P_t})^{-\eta}C_t, \end{aligned}$$

AII.1.2 Labor market

The total labor input of each producer is a Dixit-Stiglitz aggregator of various labor inputs hired, so that:

$$\begin{aligned} L^j_{H,t,i} &= (\frac{W^j_t}{W_t})^{-\gamma}L_{H,t,i}, \\ L^j_{N,t,i} &= (\frac{W^j_t}{W_t})^{-\gamma}L_{N,t,i}, \end{aligned}$$

Summing over all firms, the labor market for labor j clears as:

$$L^j_t = \zeta(\frac{W^j_t}{W_t})^{-\gamma}L_{H,t} + (1-\zeta)(\frac{W^j_t}{W_t})^{-\gamma}L_{N,t},$$

where $L_{H,t} = \int_0^{\zeta} L_{H,t,i} di$ and $L_{N,t} = \int_{\zeta}^{1} L_{N,t,i} di$ are the total labor inputs in Home tradable sector and non-tradable sector, respectively. In equilibrium, $L_t = \int_0^1 L_t^{j\frac{\gamma-1}{\gamma}} dj]^{\frac{\gamma}{\gamma-1}} = L_{H,t} + L_{N,t}$. Analogous conditions hold for Foreign country.

AII.2 Consumer behavior as a planner

Note that all the consumers are identical aside from how long they last planned. For the Home country, let us denote $F_t^j \equiv [W_t^j L_t^j + (1+i_{t-1})B_{t-1}^j + (1+i_{t-1}^*)S_t B_{t-1}^{*j} + T_t^j]/P_t$ as the real resources with which planner j enters period t. The assumption of perfect insurance implies that $F_t^j = F_t$, identical for all planners. $V(F_t, .)$ refers to the value function for planners that plan at period t, taking other state variables in the second argument as given. For a newly updated planner, he solves:

$$V(F_t) = \max_{\{C_{t+m,m}\}} \{\sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m U(C_{t+m,m}, .) + \beta\lambda_c \sum_{m=0}^{\infty} \beta^m (1-\lambda_c)^m E_t[V(F_{t+m+1})]$$

$$\text{s.t. } F_{t+m+1} = R_{t+m+1}(F_{t+m} - C_{t+m,.}) + (W_{t+m+1,.}L_{t+m+1,.} + T_{t+1+m})/P_{t+m+1},$$

$$m \geq 0 \text{ and a no-Ponzi condition}$$

where $\beta \in (0,1)$ is the discounting factor, and $R_{t+1} = (1+i_t)P_t/P_{t+1}$ is the real return on nominal bonds.[2] The first-order condition with respect to $C_{t+m,m}$ are:

$$\beta^m (1-\lambda_c)^m C_{t+m,m}^{-1} = \beta\lambda_c \sum_{k=m}^{\infty} \beta^k (1-\lambda_c)^k E_t[V'(F_{t+1+k}) R_{t+m,t+k+1}]$$

where $R_{t+m,t+k+1} = \prod_{z=t+m}^{t+k} R_{z+1}$, the compound return between two dates.

The envelope theorem condition is:

$$V'(F_t) = \beta\lambda_c \sum_{k=0}^{\infty} \beta^k (1-\lambda_c)^k E_t[V'(F_{t+k+1}) R_{t,t+k+1}]$$

[2] With competitive international bonds market, expected real return is identical on Home and Foreign bonds.

By setting $m = 0$ in the first condition, together with the second condition, $V'(F_t) = C_{t,0}^{-1}$ is obtained. Together with the substitution condition between Home goods and Foreign goods, optimality conditions can be summarized as:

$$C_{t,0}^{-1} = \beta E_t[R_{t+1} C_{t+1,0}^{-1}],$$
$$C_{t,k}^{-1} = E_{t-k}[C_{t,0}^{-1}]$$

Similar conditions hold in Foreign country.

AII.3 Worker behavior

Similar to consumers, workers solve the dynamic program:

$$V(F_t) = \max_{\{W_{t+m,m}\}} E_t[\sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m U(L_{t+m,m},.) + \beta\lambda_w \sum_{m=0}^{\infty} \beta^m (1-\lambda_w)^m V(F_{t+m+1})]$$

$$\text{s.t. } F_{t+m+1} = R_{t+m+1}(F_{t+m} - C_{t+m,.}) + (W_{t+m+1,.} L_{t+m+1,.} + T_{t+m+1})/P_{t+m+1},$$
$$F_{t+m} = [W_{t+m,.} L_{t+m,.} + (1+i_{t+m-1})B_{t+m-1} + (1+i^*_{t+m-1})S_t B^*_{t+m-1} + T_{t+m}]/P_{t+m},$$
$$L_{t+m,.} = (\frac{W_{t+m,.}}{W_{t+m}})^{-\gamma} L_{t+m},$$
$$m \geq 0 \text{ and a no-Ponzi condition}$$

The first-order condition with respect to $W_{t+m,m}$ is:

$$\beta^m(1-\lambda_w)^m \frac{\gamma}{\gamma-1} \varkappa E_t[L_{t+m,m}^{\frac{1}{\varphi}}] W_{t+m,m}^{-1} = \beta\lambda_w \sum_{k=m}^{\infty} \beta^k (1-\lambda_w)^k E_t[\frac{V'(F_{t+1+k}) R_{t+m,t+k+1}}{P_{t+m}}]$$

The envelope theorem condition is:

$$V'(F_t) = \beta\lambda_w \sum_{k=0}^{\infty} \beta^k (1-\lambda_w)^k E_t[V'(F_{t+1+k}) R_{t,t+k+1}]$$

Setting $m = 0$ in the first condition, $W_{t,0} = \frac{\gamma}{\gamma-1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{V'(F_t)/P_t}$ is derived. Using this result, I can get:

$$W_{t,0} = \frac{\gamma}{\gamma - 1} \frac{\varkappa L_{t,0}^{\frac{1}{\varphi}}}{C_{t,0}^{-1}/P_t}$$

$$\frac{L_{t,0}^{\frac{1}{\varphi}} P_t}{W_{t,0}} = E_t[\frac{\beta R_{t+1} L_{t+1,0}^{\frac{1}{\varphi}} P_{t+1}}{W_{t+1,0}}]$$

$$W_{t,k} = \frac{E_{t-k}[L_{t,k}^{1+\frac{1}{\varphi}}]}{E_{t-k}[L_{t,0}^{\frac{1}{\varphi}} L_{t,k}/W_{t,0}]}$$

Foreign workers make decision following similar conditions.

AII.4 Firm behavior as a sale department

For a Home country firm producing tradable goods and last updating its information k periods ago, the sale department needs to:

$$\max E_{t-k}[P_{H,t,k,i} Y_{H,t,k,i} - W_t L_{H,t,k,i}],$$

$$\text{s.t. } Y_{H,t,k,i} = A_{H,t} L_{H,t,k,i}^{\xi},$$

$$Y_{H,t,k,i} = \omega(\frac{P_{H,t,k,i}}{P_{H,t}})^{-\upsilon}(\frac{P_{H,t}}{P_{T,t}})^{-\theta}(\frac{P_{T,t}}{P_t})^{-\eta} C_t + (1-\omega)(\frac{P_{H,t,k,i}}{S_{t,k} P_{H,t}^*})^{-\upsilon}(\frac{P_{H,t}^*}{P_{T,t}^*})^{-\theta}(\frac{P_{T,t}^*}{P_t^*})^{-\eta} C_t^*,$$

where $S_{t,k} = E_{t-k}[S_t]$. The first-order condition with respect to $P_{H,t,k,i}$ is:

$$P_{H,t,k,i} = \frac{\upsilon}{\upsilon - 1} \frac{E_{t-k}[A_{H,t}^{-\frac{1}{\xi}} Y_{H,t,k,i}^{\frac{1}{\xi}} W_t]}{E_{t-k}[\xi Y_{H,t,k,i}]}$$

Similarly, Home country firms in the non-tradable sector set price according to:

$$P_{N,t,k,i} = \frac{\upsilon}{\upsilon - 1} \frac{E_{t-k}[A_{N,t}^{-\frac{1}{\xi}} Y_{N,t,k,i}^{\frac{1}{\xi}} W_t]}{E_{t-k}[\xi Y_{N,t,k,i}]},$$

Firms in Foreign country will set prices in a similar way.

AII.5 Monetary Policy

In this cashless economy, the monetary authority sets nominal interest rates according to Taylor rule:

$$i_t = \phi_\pi \ln(\frac{P_t}{P_{t-1}}) + \phi_y \ln(\frac{Y_t}{\overline{Y}_t}) + \varepsilon_t$$

where ϕ_π is the inflation targeting coefficient, ϕ_y is the output gap targeting coefficients, Y_t is Home output, $\overline{Y}_t$ is Home output with full information, ε_t is monetary policy shock that follows $AR(1)$ process. Situation is similar in Foreign country. With the existence of competitive international bond market, the uncovered interest rate parity holds:

$$1 + i_t = \frac{E_t[S_{t+1}]}{S_t}(1 + i_t^*)$$

which implies that an optimal risk-sharing condition holds:

$$\frac{C_{t,0}}{C_{t,0}^*} = \frac{E_t[Q_t C_{t+1,0}^{*-1}]}{E_t[Q_{t+1} C_{t+1,0}^{-1}]},$$

where Q_t is the real exchange rate defined as $S_t P_t^* / P_t$.

AII.6 Log-linearization

The variables are log-linearized around steady state. Small letters denote log-linear deviations of respective variables, except that r_t and r_t^* denote log-linear deviations of $E_t[R_{t+1}]$ and $E_t[R_{t+1}^*]$, respectively. The set of log-linearized optimality conditions is summarized below.

AII.6.1 Goods market

The goods market clear condition for Home goods in Section AII.1.1 has the following log form:

$$
\begin{aligned}
y_{H,t,k,i} &= \omega[c_t - \upsilon(p_{H,t,k,i} - p_{H,t}) - \theta(p_{H,t} - p_{T,t}) - \eta(p_{T,t} - p_t)] \\
&\quad + (1-\omega)[c_t^* - \upsilon(p_{H,t,k,i}^* - p_{H,t}^*) - \theta(p_{H,t}^* - p_{T,t}^*) - \eta(p_{T,t}^* - p_t^*)], \qquad \text{(AII.1)}
\end{aligned}
$$

$$
y_{H,t} = \omega c_t + (1-\omega)c_t^* - 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[\omega(p_{T,t} - p_t) + (1-\omega)(p_{T,t}^* - p_t^*)], \qquad \text{(AII.2)}
$$

$$
y_{N,t,k,i} = c_t - \upsilon(p_{N,t,k,i} - p_{N,t}) - \eta(p_{N,t} - p_t), \qquad \text{(AII.3)}
$$

$$
y_{N,t} = c_t - \eta(p_{N,t} - p_t), \qquad \text{(AII.4)}
$$

Log-linearizing and iterating the optimality conditions of Home consumers in Section AII.2 gives:

$$
c_{t,k} = E_{t-k}[c_{t+1,0} - r_t], \qquad \text{(AII.5)}
$$

With many heterogenous groups of consumers, Home aggregate consumption index has the following log form:

$$
c_t = \lambda_c \sum_{k=0}^{\infty} (1-\lambda_c)^k c_{t,k}, \qquad \text{(AII.6)}
$$

The pricing rule in Section AII.5 implies:

$$
\begin{aligned}
p_{H,t,k,i} &= E_{t-k}[w_t + (\frac{1}{\xi} - 1)y_{H,t,k,i} - \frac{1}{\xi}a_{H,t}], \qquad \text{(AII.7)} \\
p_{N,t,k,i} &= E_{t-k}[w_t + (\frac{1}{\xi} - 1)y_{N,t,k,i} - \frac{1}{\xi}a_{N,t}], \qquad \text{(AII.8)}
\end{aligned}
$$

With many heterogenous groups of firms, the aggregate price index for Home goods have the following log form :

$$
p_{H,t} = \lambda_f \sum_{k=0}^{\infty} (1-\lambda_f)^k p_{H,t,k,i}, \qquad \text{(AII.9)}
$$

$$p^*_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k(p_{H,t,k,i} - s_{t,k}), \tag{AII.10}$$

$$p_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k p_{N,t,k,i}, \tag{AII.11}$$

Similarly, Foreign country has:

$$\begin{aligned} y^*_{F,t,k,i^*} = {} & (1-\omega)[c_t - \upsilon(p_{F,t,k,i^*} - p_{F,t}) - \theta(p_{F,t} - p_{T,t}) - \eta(p_{T,t} - p_t)] \\ & + \omega[c^*_t - \upsilon(p^*_{F,t,k,i^*} - p^*_{F,t}) - \theta(p^*_{F,t} - p^*_{T,t}) - \eta(p^*_{T,t} - p^*_t)], \end{aligned} \tag{AII.12}$$

$$y^*_{F,t} = (1-\omega)c_t + \omega c^*_t + 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[(1-\omega)(p_{T,t} - p_t) + \omega(p^*_{T,t} - p^*_t)], \tag{AII.13}$$

$$y^*_{N,t,k,i^*} = c^*_t - \upsilon(p^*_{N,t,k,i^*} - p^*_{N,t}) - \eta(p^*_{N,t} - p^*_t), \tag{AII.14}$$

$$y^*_{N,t} = c^*_t - \eta(p^*_{N,t} - p^*_t), \tag{AII.15}$$

$$c^*_{t,k} = E_{t-k}[c^*_{t+1,0} - r^*_t], \tag{AII.16}$$

$$c^*_t = \lambda_c \sum_{k=0}^{\infty}(1-\lambda_c)^k c^*_{t,k}, \tag{AII.17}$$

$$p^*_{F,t,k,i^*} = E_{t-k}[w^*_t + (\frac{1}{\xi} - 1)y^*_{F,t,k,i^*} - \frac{1}{\xi}a^*_{F,t}], \tag{AII.18}$$

$$p^*_{N,t,k,i^*} = E_{t-k}[w^*_t + (\frac{1}{\xi} - 1)y^*_{N,t,k,i^*} - \frac{1}{\xi}a^*_{N,t}], \tag{AII.19}$$

$$p_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)(p^*_{F,t,k,i^*} + s_{t,k}), \tag{AII.20}$$

$$p^*_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k p^*_{t,k,i^*}, \tag{AII.21}$$

$$p^*_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k p^*_{N,t,k,i^*}, \tag{AII.22}$$

Accordingly, log form aggregate price index is $p_t = \zeta p_{T,t} + (1-\zeta)p_{N,t}$ in Home country where $p_{T,t} = \omega p_{H,t} + (1-\omega)p_{F,t}$ and $p^*_t = \zeta p^*_{T,t} + (1-\zeta)p^*_{N,t}$ in Foreign country where $p^*_{T,t} = (1-\omega)p^*_{H,t} + \omega p^*_{F,t}$, respectively. Similarly, log form aggregate output is $y_t = \zeta y_{H,t} + (1-\zeta)y_{N,t}$ in Home country and $y^*_t = \zeta y^*_{H,t} + (1-\zeta)y^*_{N,t}$ in Foreign country.

AII.6.2 Labor market

The log form Home labor market clear condition in Section AII.1.2 and Home production function are given by:

$$l_{t,k} = \zeta[l_{H,t} - \gamma(w_{t,k} - w_t)] + (1-\zeta)[l_{N,t} - \gamma(w_{t,k} - w_t)], \tag{AII.23}$$

$$y_{H,t} = a_{H,t} + \xi l_{H,t}, \tag{AII.24}$$

$$y_{N,t} = a_{H,t} + \xi l_{N,t}, \tag{AII.25}$$

Log-linearizing and iterating the optimality conditions of Home worker in Section AII.3 gives:

$$w_{t,k} = E_{t-k}[p_t + \frac{1}{\varphi} l_{t,k} - r_t + w_{t+1,0} - p_{t+1} - \frac{1}{\varphi} l_{t+1,0}], \tag{AII.26}$$

$$w_{t,0} = p_t + \frac{1}{\varphi} l_{t,0} + c_{t,0}, \tag{AII.27}$$

With many heterogenous groups of workers, Home aggregate wage index has the following log form:

$$w_t = \lambda_w \sum_{k=0}^{\infty} (1-\lambda_w)^k w_{t,k}, \tag{AII.28}$$

Similarly, Foreign country has:

$$l^*_{t,k} = \zeta[l^*_{F,t} - \gamma(w^*_{t,k} - w^*_t)] + (1-\zeta)[l^*_{N,t} - \gamma(w^*_{t,k} - w^*_t)], \tag{AII.29}$$

$$y^*_{F,t} = a^*_{F,t} + \xi l^*_{F,t}, \tag{AII.30}$$

$$y^*_{N,t} = a^*_{N,t} + \xi l^*_{N,t}, \tag{AII.31}$$

$$w^*_{t,k} = E_{t-k}[p^*_t + \frac{1}{\varphi} l^*_{t,k} - r^*_t + w^*_{t+1,0} - p^*_{t+1} - \frac{1}{\varphi} l^*_{t+1,0}], \tag{AII.32}$$

$$w^*_{t,0} = p^*_t + \frac{1}{\varphi} l^*_{t,0} + c^*_{t,0}, \tag{AII.33}$$

$$w^*_t = \lambda_w \sum_{k=0}^{\infty} (1-\lambda_w)^k w^*_{t,k}, \tag{AII.34}$$

AII.6.3 Interest rates and consumption risk-sharing condition

If $\lambda = \delta = \omega = 1$, agents have full information. In such economy, all agents within a country have identical responses, and real variables behave in the following pattern:

$$\overline{y}_{N,t} + \overline{y}^*_{N,t} = -\frac{\zeta(1-\frac{1}{\eta})}{\frac{1}{\xi} - 1 + \frac{1}{\eta}}(a_{H,t} + a^*_{F,t}) + \frac{\frac{1}{\xi} - (1-\zeta)(1-\frac{1}{\eta})}{\frac{1}{\xi} - 1 + \frac{1}{\eta}}(a_{N,t} + a^*_{N,t}),$$

$$\overline{y}_{H,t} + \overline{y}^*_{F,t} = \frac{\frac{1}{\xi} - \zeta(1-\frac{1}{\eta})}{\frac{1}{\xi} - 1 + \frac{1}{\eta}}(a_{H,t} + a^*_{F,t}) - \frac{(1-\zeta)(1-\frac{1}{\eta})}{\frac{1}{\xi} - 1 + \frac{1}{\eta}}(a_{N,t} + a^*_{N,t}),$$

$$A_1(\overline{y}_{H,t} - \overline{y}^*_{F,t}) = B_1(\overline{y}_{N,t} - \overline{y}^*_{N,t}) + C_1(a_{H,t} - a^*_{F,t}) + D_1(a_{N,t} - a^*_{N,t}),$$

$$A_2(\overline{y}_{H,t} - \overline{y}^*_{F,t}) = B_2(\overline{y}_{N,t} - \overline{y}^*_{N,t}) + C_2(a_{H,t} - a^*_{F,t}) + D_2(a_{N,t} - a^*_{N,t}),$$

where A_1, B_1, C_1, D_1, A_2, B_2, C_2 and D_2 equal to:

$$\begin{aligned}
A_1 &= \frac{(2\omega-1)\eta}{(1-\eta)\theta\varphi\xi} - 4\omega(1-\omega)(\frac{\zeta}{\varphi\xi} + \frac{1}{\xi} - 1) - \frac{1}{\theta},\\
B_1 &= 4\omega(1-\omega)\frac{1-\zeta}{\varphi\xi} - \frac{(2\omega-1)(1-\eta+\eta\frac{1-\zeta}{\zeta\varphi\xi} + \frac{\eta}{\zeta\xi})}{(1-\eta)\theta},\\
C_1 &= \frac{(2\omega-1)\eta}{(1-\eta)\theta\varphi\xi} - 4\omega(1-\omega)(\frac{\zeta}{\varphi\xi} + \frac{1}{\xi}),\\
D_1 &= \frac{(2\omega-1)\eta(\frac{1-\zeta}{\zeta\varphi\xi} + \frac{1}{\zeta\xi})}{(1-\eta)\theta} - 4\omega(1-\omega)\frac{1-\zeta}{\varphi\xi},\\
A_2 &= (2\omega-1)(\frac{\zeta}{\varphi\xi} + \frac{1}{\xi} - 1) - \frac{\zeta}{\varphi\xi} + \frac{1}{(1-\eta)\varphi\xi},\\
B_2 &= -2(\omega-1)\frac{1-\zeta}{\varphi\xi} + \frac{1}{\xi} - 1 - \frac{1}{1-\eta}(\frac{1-\zeta}{\zeta\varphi\xi} + \frac{1}{\zeta\xi}),\\
C_2 &= (2\omega-1)(\frac{\zeta}{\varphi\xi} + \frac{1}{\xi}) - \frac{\zeta}{\varphi\xi} + \frac{1}{(1-\eta)\varphi\xi},\\
D_2 &= 2(\omega-1)\frac{1-\zeta}{\varphi\xi} - \frac{1}{\xi} + \frac{1}{1-\eta}(\frac{1-\zeta}{\zeta\varphi\xi} + \frac{1}{\zeta\xi}),
\end{aligned}$$

With $\overline{y}_t = \zeta\overline{y}_{H,t} + (1-\zeta)\overline{y}_{N,t}$ and $\overline{y}^*_t = \zeta\overline{y}^*_{F,t} + (1-\zeta)\overline{y}^*_{N,t}$, log-linear Taylor rule in Home country follows:

$$i_t = \phi_\pi(p_t - p_{t-1}) + \phi_y(y_t - \overline{y}_t) + \varepsilon_t, \tag{AII.35}$$

The expected real interest rates in Home country thus equals to:

$$r_t = i_t - E_t[\triangle p_{t+1}], \tag{AII.36}$$

Similarly, Foreign country has:

$$i_t^* = \phi_\pi(p_t^* - p_{t-1}^*) + \phi_y(y_t^* - \overline{y}_t^*) + \varepsilon_t^*, \tag{AII.37}$$

$$r_t^* = i_t^* - E_t[\triangle p_{t+1}^*], \tag{AII.38}$$

Log-linearizing and iterating the consumption risk-sharing condition in Section AII.5 gives:

$$c_{t,k} - c_{t,k}^* = E_{t-k}[s_t + p_t^* - p_t] = E_{t-k}[q_t], \tag{AII.39}$$

AII.7 Equilibrium

Combining Eq. (AII.1), (AII.2), and (AII.7) and substituting into Eq. (AII.9) gives the price index for Home tradable goods:

$$p_{H,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[\omega p_{H,t} + (1-\omega)p_{H,t}^* + (1-\omega)s_t + mc_{H,t}],$$

with $mc_{H,t} = \{(1-\xi)y_t + \xi[w_t - np_{H,t} - (1-n)(p_{H,t}^* + s_t)] - a_{H,t}\}/[\xi + (1-\xi)\upsilon]$. Eq. (AII.10) implies that:

$$p_{H,t}^* = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[\omega p_{H,t} + (1-\omega)p_{H,t}^* - \omega s_t + mc_{H,t}],$$

Combining Eq. (AII.3), (AII.4), and (AII.8) and substituting into Eq. (AII.11) gives the price index for Home non-tradable goods:

$$p_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}(p_{N,t} + mc_{N,t}),$$

with $mc_{N,t} = \{\xi(w_t - p_{N,t}) + (1-\xi)y_{N,t} - a_{N,t}\}/[\xi + (1-\xi)\upsilon]$.

Combining Eq. (AII.12), (AII.13), and (AII.18) and substituting into Eq. (AII.21) gives the price index for Foreign tradable goods:

$$p^*_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[(1-\omega)p_{F,t} + \omega p^*_{F,t} - (1-\omega)s_t + mc^*_{F,t}],$$

with $mc^*_{F,t} = \{\xi[w^*_t - (1-\omega)(p_{F,t} - s_t) - \omega p^*_{F,t}] + (1-\xi)y^*_{F,t} - a^*_{F,t}\}/[\xi + (1-\xi)\upsilon]$. Similarly, Eq. (AII.20) imply that:

$$p_{F,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}[(1-\omega)p_{F,t} + \omega p^*_{F,t} + \omega s_t + mc^*_{F,t}],$$

Combining Eq. (AII.14), (AII.15), and (AII.19) and substituting into Eq. (AII.22) gives the price index for Foreign non-tradable goods:

$$p^*_{N,t} = \lambda_f \sum_{k=0}^{\infty}(1-\lambda_f)^k E_{t-k}(p^*_{N,t} + mc^*_{N,t}),$$

with $mc^*_{N,t} = \{\xi(w^*_t - p^*_{N,t}) + (1-\xi)y^*_{N,t} - a^*_{N,t}\}/[\xi + (1-\xi)\upsilon]$.

Therefore, the aggregate supply in Home country and Foreign country, respectively, are obtained as:

$$\begin{aligned} p_t &= \zeta[\omega p_{H,t} + (1-\omega)p_{F,t}] + (1-\zeta)p_{N,t} \\ p^*_t &= \zeta[(1-\omega)p^*_{H,t} + \omega p^*_{F,t}] + (1-\zeta)p^*_{N,t}, \end{aligned}$$

Iterating forward on Eq. (AII.5) gives:

$$c_{t,k} = -\sum_{i=0}^{K} E_{t-k}[r_{t+i}] + E_{t-k}[c_{t+K+1,0}],$$

When $K \to \infty$, all planners are informed, so that $\lim_{i\to\infty} E_t[r_{t+i}] = \lim_{i\to\infty} E_t[r_{t+i}^N] = 0$, and $\lim_{i\to\infty} E_t[c_{t+i,0}] = \lim_{i\to\infty} E_t[c_{t+i}^N] = 0$. Since the fraction of inattentive planners falls exponentially along the time, this limit can be approached quickly enough to ensure that $\sum_{i=0}^{K} E_{t-k}(r_{t+i})$ converges. Therefore,

$$c_{t,k} = -E_{t-k}[\overline{R}_t], \tag{AII.40}$$

where $\overline{R}_t = E_t[\sum_{i=0}^{\infty} r_{t+i}]$ is the long-term real interest rates in Home country. Similarly,

$$c_{t,k}^* = -E_{t-k}[\overline{R}_t^*], \tag{AII.41}$$

where $\overline{R}_t^* = E_t[\sum_{i=0}^{\infty} r_{t+i}^*]$ is the long-term real interest rates in Foreign country. Replacing for $c_{t,k}$ and $c_{t,k}^*$ in (AII.6) and (AII.17) and substituting them into Eq. (AII.2), (AII.4), (AII.13) and (AII.15), respectively, gives:

$$\begin{aligned}
y_{H,t} &= -\lambda_c \sum_{k=0}^{\infty} (1-\lambda_c)^k E_{t-k}[\omega \overline{R}_t + (1-\omega)\overline{R}_t^*] \\
&\quad - 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[\omega(p_{T,t} - p_t) + (1-\omega)(p_{T,t}^* - p_t^*)], \\
y_{N,t} &= -\lambda_c \sum_{k=0}^{\infty} (1-\lambda_c)^k E_{t-k}(\overline{R}_t) - \eta(p_{N,t} - p_t), \\
y_{F,t}^* &= -\lambda_c \sum_{k=0}^{\infty} (1-\lambda_c)^k E_{t-k}[(1-\omega)\overline{R}_t + \omega \overline{R}_t^*] \\
&\quad + 2\omega(1-\omega)\theta(p_{H,t} - p_{F,t}) - \eta[(1-\omega)(p_{T,t} - p_t) + \omega(p_{T,t}^* - p_t^*)], \\
y_{N,t}^* &= -\lambda_c \sum_{k=0}^{\infty} (1-\lambda_c)^k E_{t-k}(\overline{R}_t^*) - \eta(p_{N,t}^* - p_t^*),
\end{aligned}$$

Accordingly, aggregate demand relations in Home country and Foreign country are obtained as:

$$\begin{aligned} y_t &= \zeta y_{H,t} + (1-\zeta) y_{N,t}, \\ y_t^* &= \zeta y_{F,t}^* + (1-\zeta) y_{N,t}^*, \end{aligned}$$

Using Eq. (AII.27) and (AII.40) and Eq. (AII.26) gives:

$$w_{t,k} = E_{t-k}[p_t + \frac{1}{\varphi} l_{t,k} - \overline{R}_t],$$

Using this result as well as (AII.23), (AII.24), (AII.25) and substituting into Eq. (AII.28) gives the Home wage curve:

$$w_t = \lambda_w \sum_{k=0}^{\infty} (1-\lambda_w)^k E_{t-k}[p_t + \frac{y_t - [\zeta a_{H,t} + (1-\zeta) a_{N,t}]}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t - p_t) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t],$$

Similarly, the wage curve in Foreign country follows:

$$w_t^* = \lambda_w \sum_{k=0}^{\infty} (1-\lambda_w)^k E_{t-k}[p_t^* + \frac{y_t^* - [\zeta a_{F,t}^* + (1-\zeta) a_{N,t}^*]}{\xi(\varphi+\gamma)} + \frac{\gamma}{\varphi+\gamma}(w_t^* - p_t^*) - \frac{\varphi}{\varphi+\gamma}\overline{R}_t^*],$$

Finally, using Eq. (AII.39), (AII.40), and (AII.41), the optimal consumption-sharing condition is obtained as:

$$\overline{R}_t^* - \overline{R}_t = s_t + p_t^* - p_t = q_t.$$

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