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Comparison of formulas for drag coefficient and settling velocity of spherical particles

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Abstract

Two formulas are proposed for explicitly evaluating drag coefficient and settling velocity of spherical particles, respectively, in the entire subcritical region. Comparisons with fourteen previously-developed formulas show that the present study gives the best representation of a complete set of historical data reported in the literature for Reynolds numbers up to 2×10^5.

Keywords: drag coefficient, Reynolds number, settling velocity, sphere

1. Introduction

Evaluation of drag coefficient (C_D) and settling velocity (w) is essential for various theoretical analyses and engineering applications. Many empirical or semiempirical formulas are available in the literature for performing such evaluations. Some are simple but only used for limited Reynolds numbers (Re). The others, though applicable for a wide range of Re, may involve tedious application procedure. For example, the correlation presented by Clift et al [1], which has been considered the best approximation, consists of ten piecewise functions applicable for different *Re*.

In this note, two formulas are proposed, one for quantifying the relationship of C_D and *Re*, and the other for explicitly evaluating the terminal velocity of settling particle. Comparisons are also made with fourteen similar formulas available in the literature, which were developed for *Re* from the Stokes regime to about 2×10^5. The results show that the function proposed here, despite its simple form, gives the best approximation of experimental data for *Re* in the subcritical region.

2. Dimensionless parameters

Several dimensionless parameters are used in this study. They include (1) $Re = wd/\nu$, where w is the terminal velocity of settling particle, d is the particle diameter, and ν is the kinematic viscosity of fluid; (2) $C_D = 4\Delta g d/(3w^2)$, where $\Delta = (\rho_s - \rho)/\rho$, ρ_s is the particle density, ρ is the fluid density and g is the gravitational acceleration; (3) $d_* = \left(\Delta g/\nu^2\right)^{1/3} d$ referred to as dimensionless grain diameter; and (4) $w_* = \left(\Delta g \nu\right)^{-1/3} w$ referred to as dimensionless settling velocity. It can be shown that both *Re* and C_D can be expressed as a function of d_* and w_*, i.e. $Re = w_* d_*$ and $C_D = 4d_*/(3w_*^2)$, and d_* and w_* can be also written in terms of *Re* and C_D, i.e. $d_* = \left(3C_D Re^2/4\right)^{1/3}$ and $w_* = \left[4Re/(3C_D)\right]^{1/3}$.

3. Drag coefficient formula proposed in this study

The following five-parameter correlation is proposed to describe the C_D-*Re* relationship,

$$C_D = \frac{24}{Re}\left(1+0.27\,Re\right)^{0.43} + 0.47\left[1-exp\left(-0.04\,Re^{0.38}\right)\right] \qquad (1)$$

In Eq. (1), C_D is predicted with two terms. The first term on the RHS can be considered as an extended Stokes' law applicable approximately for Re < 100, and the second term is an exponential function accounting for slight deviations from the Newton's law for high Re. The sum of the two terms is used to predict C_D for any Re over the entire subcritical region. Altogether six constants are included in Eq. (1). The first constant is taken as 24 following the Stokes' law for very low Re. The other five constants were evaluated by minimizing the deviation when comparing Eq. (1) with the experimental data by Brown and Lawler [2]. After conducting a critical review of historical experimental data on sphere drag, Brown and Lawler produced a high-quality raw data set of 480 points for $Re = 2\times10^{-3} - 2\times10^{5}$. Using this data set, Brown and Lawler also recommended two correlations, i.e. Eqs. (4) and (11), for computing C_D and w, respectively.

Fig. 1 shows the C_D-Re curve plotted using Eq. (1), together with the data provided by Brown and Lawler [2]. The two asymptotes, i.e. the two individual terms on the RHS of Eq. (1), and the Stokes' law are also plotted in the figure.

3.1. Comparison with other C_D-Re relationships

Dozens of C_D-Re formulas, empirical or semiempirical, have been published in the literature; some examples are reported by Clift et al [1] and Heiskanen [3]. Seven of them are selected here for comparisons, as listed in Table 1. The selected formulas are different from the others in that they were not only considered of high accuracy but also applicable for the entire subcritical region (e.g., $Re < 2 \times 10^5$). Among the seven correlations, Eqs. (3), (5), (7) and (8), appear complicated while the rest three are given in the same function but with different constants.

To assess the goodness of fit of the correlations when compared with data, three statistical parameters are used. They are (1) average relative error given by $r = \frac{1}{n}\sum\left(\left|C_{D\,cal} - C_{D\exp}\right| / C_{D\exp}\right) \times 100(\%)$, where n is the total number of data points used; (2) sum of squared errors defined as $s_1 = \sum\left(C_{D\,cal} - C_{D\exp}\right)^2 / C_{D\exp}^2$; and (3) sum of the deviation defined using the logarithmic drag coefficient, $s_2 = \sum\left(\log C_{D\,cal} - \log C_{D\exp}\right)^2$. The assessment results are summarized in Table 2, where the formulas are ranked in the increasing order of the average relative error. It can be seen that the proposed formula, Eq. (1), has an average relative error of 2.47%, which gives the best representation of the experimental measurements. In particular, in spite of its simpler form, Eq. (1) performs even better than the formula by Clift et al[1], which has been considered the best approximation, and that derived from the same data set by Brown and Lawler[2]. The average error for the other formulas varies from 3.02% to 4.76%. The values of s_1 and s_2 also clearly indicate the best performance of Eq. (1), by noting that those associated with the other equations could be four times higher.

4. Explicit formula for evaluation of settling velocity

A trial procedure is required if the settling velocity is computed using one of the abovementioned C_D-Re correlations. In this section, an explicit equation is suggested for computing C_D and thus w_* when d_* is given.

Similar to the dependence of C_D on Re, C_D can also be expressed as a function of d_*. This is shown in Fig. 2 with the data by Brown and Lawler [2]. It can be seen that C_D varies with d_* in a similar fashion to that of C_D-Re. In particular, at very low Re, the Stokes' law applies and thus C_D is related to d_* as $C_D = 432/d_*^3$. With this consideration, a general form of C_D-d_* relation, similar to Eq. (1), for the entire subcritical region is proposed here,

$$C_D = \frac{432}{d_*^3}\left(1+0.022d_*^3\right)^{0.54} + 0.47\left[\ 1-exp\left(-0.15d_*^{0.45}\right)\ \right] \tag{2}$$

where the five constants, 0.022, 0.54, 0.47, 0.15 and 0.45, were obtained by minimizing the deviation from the data. Eq. (2) and its two individual terms, and the Stokes' law are superimposed in Fig. 2. With C_D computed from Eq. (2) for d_* given, w_* can be obtained using $w_* = \sqrt{4d_*/(3C_D)}$.

4.1. Comparison with other explicit formulas

Seven explicit formulas for computing *w*, as listed in Table 3, are selected from previous studies for comparisons. All of them were considered applicable for the entire subcritical region. The comparison results are summarized in Table 4, where the formulas are ranked in the increasing order of the average relative error. When evaluating the three statistical parameters (*r*, s_1 and s_2) defined earlier, C_D is replaced here with w_*. It can be seen that the proposed formula, Eq. (2), offers the best fit to the experimental measurements, and the associated average error is 1.66%. For the other formulas, the average error ranges from 2.09% to 7.37%. The good performance can also be observed more clearly from the larger differences in the values of s_1 and s_2. For example, the s_2-value for Eq. (13) is about 18 times greater than that associated with the formula proposed here.

5. Conclusions

In spite of its simple form, the proposed function gives the best representation of experimental data available in the literature, not only for quantifying the standard drag coefficient function but also for explicitly evaluating the terminal velocity of settling particle. The functional relationship of C_D-Re remains similar if the conventional standard drag curve is recast by relating C_D to the dimensionless particle diameter. This similarity may simplify the procedure in conducting alike correlations, as demonstrated in this study.

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Table 1 Previous drag coefficient formulas applicable for the entire subcritical region.

No.	Investigator	C_D-Re relationship	Eq.
1	Almedeij [4]	$C_D=\left[\frac{1}{(\varphi_1+\varphi_2)^{-1}+\varphi_3^{-1}}+\varphi_4\right]^{1/10}$ where $\varphi_1=(24/\mathrm{Re})^{10}+(21/\mathrm{Re}^{0.67})^{10}+(4/\mathrm{Re}^{0.33})^{10}+0.4^{10}$, $\varphi_2=1/\left[(0.148\,Re^{0.11})^{-10}+0.5^{-10}\right]$, $\varphi_3=(1.57\times10^8/Re^{1.625})^{10}$, and $\varphi_4=1/\left[(6\times10^{-17}Re^{2.63})^{-10}+0.2^{-10}\right]$ for $Re<10^6$	(3)
2	Brown and Lawler [2]	$C_D=\frac{24}{Re}(1+0.15\,Re^{0.681})+\frac{0.407}{1+8710\,Re^{-1}}$ for $Re<2\times10^5$	(4)
3	Clift et al. [1]	$C_D=\begin{cases}24/Re+3/16 & \text{for } Re<0.01\\ (24/Re)(1+0.1315\,Re^{0.82-0.05\log Re}) & \text{for } 0.01<Re\le20\\ (24/Re)(1+0.1935\,Re^{0.6305}) & \text{for } 20<Re\le260\\ 10^{1.6435-1.1242\log Re+0.1558\log^2 Re} & \text{for } 260<Re\le1500\\ 10^{-2.4571+2.5558\log Re-0.9295\log^2 Re+0.1049\log^3 Re} & \text{for } 1500<Re\le1.2\times10^4\\ 10^{-1.9181+0.637\log Re-0.0636\log^2 Re} & \text{for } 1.2\times10^4<Re\le4.4\times10^4\end{cases}$	(5)

		$10^{-4.339+1.5809\log Re-0.1546\log^2 Re}$ for $4.4\times10^4 < Re \le 3.38\times10^5$ $29.78-5.3\log Re$ for $3.38\times10^5 < Re \le 4\times10^5$ $0.1\log Re-0.49$ for $4\times10^5 < Re \le 10^6$ $0.19-8\times10^4/Re$ for $10^6 < Re$	
4	Clift and Gauvin [1]	$C_D=\frac{24}{Re}\left(1+0.15\,Re^{0.687}\right)+\frac{0.42}{1+42500\,Re^{-1.16}}$	(6)
5	Concha and Brrientos [5]	$C_D=\frac{0.284153}{Re^2}\left(1+\frac{9.04}{\sqrt{Re}}\right)^2\left(0.9620833\,Re^2+2.736461\times10^{-5}\,Re^3-3.938611\times10^{-10}\,Re^4+\right.$ $\left.+2.476861\times10^{-15}\,Re^5-7.159345\times10^{-21}\,Re^6+7.437237\times10^{-27}\,Re^7\right)$ for $Re < 3\times10^6$	(7)
6	Flemmer and Banks [6]	$C_D=\frac{24}{\mathrm{Re}}10^{\alpha}$ for $Re < 3\times10^5$, where $\alpha=0.261\,Re^{0.369}-0.105\,Re^{0.431}-\frac{0.124}{1+\log^2 Re}$.	(8)
7	Turton and Levenspiel [7]	$C_D=\frac{24}{Re}\left(1+0.173\,Re^{0.657}\right)+\frac{0.413}{1+16300\,Re^{-1.09}}$ for $Re < 2\times10^5$	(9)

Table 2 Comparisons of C_D-*Re* relationships with experimental data

No.	Investigator	Eq.	Prediction error		
			Average relative error, r (%)	Sum of squared relative errors, s_1	Sum of logarithmic deviations, s_2
1	This study	(1)	2.469	0.576	0.105
2	Clift et al. [1]	(5)	3.019	0.816	0.146
3	Brown and Lawler [2]	(4)	3.236	0.810	0.151
4	Turton and Levenspiel [7]	(9)	3.742	1.042	0.188
5	Clift and Gauvin [1]	(6)	4.001	1.198	0.215
6	Concha and Brrientos [5]	(7)	4.222	1.411	0.271
7	Flemmer and Banks [6]	(8)	4.647	2.456	0.578
8	Almedeij [4]	(3)	4.762	1.599	0.316

Table 3 Previous explicit formulas for settling velocity applicable for entire subcritical region.

No.	Investigator	w_* - d_* relationship	Eq.
1	Almedeij [4]	$w_* = \left[\dfrac{1}{\left(\dfrac{1}{\psi_1^{-1} + \psi_2^{-1}} + \psi_3 \right)^{-1} + \psi_4^{-1}} \right]^{1/10}$ for $Re < 10^6$ where $\psi_1 = 1/\left[\left(0.055 d_*^2\right)^{-10} + \left(0.126 d_*^{1.256}\right)^{-10} + \left(0.518 d_*^{0.8}\right)^{-10} + \left(1.826 d_*^{0.5}\right)^{-10} \right]$, $\psi_2 = \left(2.834 d_*^{0.422}\right)^{10} + \left(1.633 d_*^{0.5}\right)^{10}$, $\psi_3 = \left(3 \times 10^{-22} d_*^7\right)^{10}$, and $\psi_4 = \left(3393 d_*^{-0.352}\right)^{10} + \left(2.582 d_*^{0.5}\right)^{10}$.	(10)
2	Brown and Lawler [2]	$w_* = \left[\left(\dfrac{18}{d_*^2} \right)^{0.898\left(\frac{0.936 d_* + 1}{d_* + 1}\right)} + \left(\dfrac{0.317}{d_*} \right)^{0.449} \right]^{-1.114}$ for $Re < 2\times10^5$	(11)
3	Clift et al [1]	$w_* = \begin{cases} \dfrac{1}{d_*}\left(\dfrac{N}{24} - 1.7569\times10^{-4} N^2 + 6.9252\times10^{-7} N^3 - 2.3027\times10^{-10} N^4 \right) & \text{for } N \le 73 \\ \dfrac{1}{d_*}\left(-1.7095 + 1.33438 L - 0.11591 L^2 \right) & \text{for } 73 < N \le 580 \end{cases}$	(12)

		$\frac{1}{d_*}\left(-1.81391+1.34671L-0.12427L^2+0.006344L^3\right)$ for $580 < N \le 1.55\times10^7$ $\frac{1}{d_*}\left(5.33283-1.21728L+0.19007L^2-0.007005L^3\right)$ for $1.55\times10^7 < N \le 5\times10^{10}$ where $N=4d_*^3/3$, and $L=\log N$.	
4	Kan and Richardson [8]	$w_* = \frac{\left(2.33d_*^{0.054}-1.53d_*^{-0.048}\right)^{13.3}}{d_*}$ for $Re < 3\times10^5$	(13)
5	Turian et al [9]	$w_* = 10^{\lambda}/d_*$ for $Re < 1.5\times10^5$ where $\lambda = -1.38+1.94Z-0.086Z^2-0.0252Z^3+9.19\times10^{-4}Z^4+5.35\times10^{-4}Z^5$ and $Z = log\sqrt{4d_*^3/3}$.	(14)
6	Turton and Clark [10]	$w_* = \left[\left(\frac{18}{d_*^2}\right)^{0.824}+\left(\frac{0.321}{d_*}\right)^{0.412}\right]^{-1.214}$ for $Re < 2\times10^5$	(15)
7	Zigrang and Sylvester [11]	$w_* = \frac{\left(\sqrt{14.51+1.83d_*^{3/2}}-3.81\right)^2}{d_*}$ for $Re < 3\times10^5$	(16)

Table 4 Comparisons of w_*-d_* relationships with experimental data

No.	Investigator	Eq.	Prediction error		
			Average relative error, r (%)	Sum of squared relative errors, s_1	Sum of logarithmic deviations, s_2
1	This study	(2)	1.660	0.218	0.041
2	Clift et al. [1]	(12)	2.094	0.360	0.070
3	Almedeij [4]	(10)	3.284	1.409	0.211
4	Brown and Lawler [2]	(11)	3.770	0.972	0.184
5	Kan and Richardson [8]	(13)	5.364	3.299	0.781
6	Turton and Clark [10]	(15)	5.777	2.236	0.461
7	Turian et al [9]	(14)	6.029	3.536	0.613
8	Zigrang and Sylvester [11]	(16)	7.366	3.416	0.720

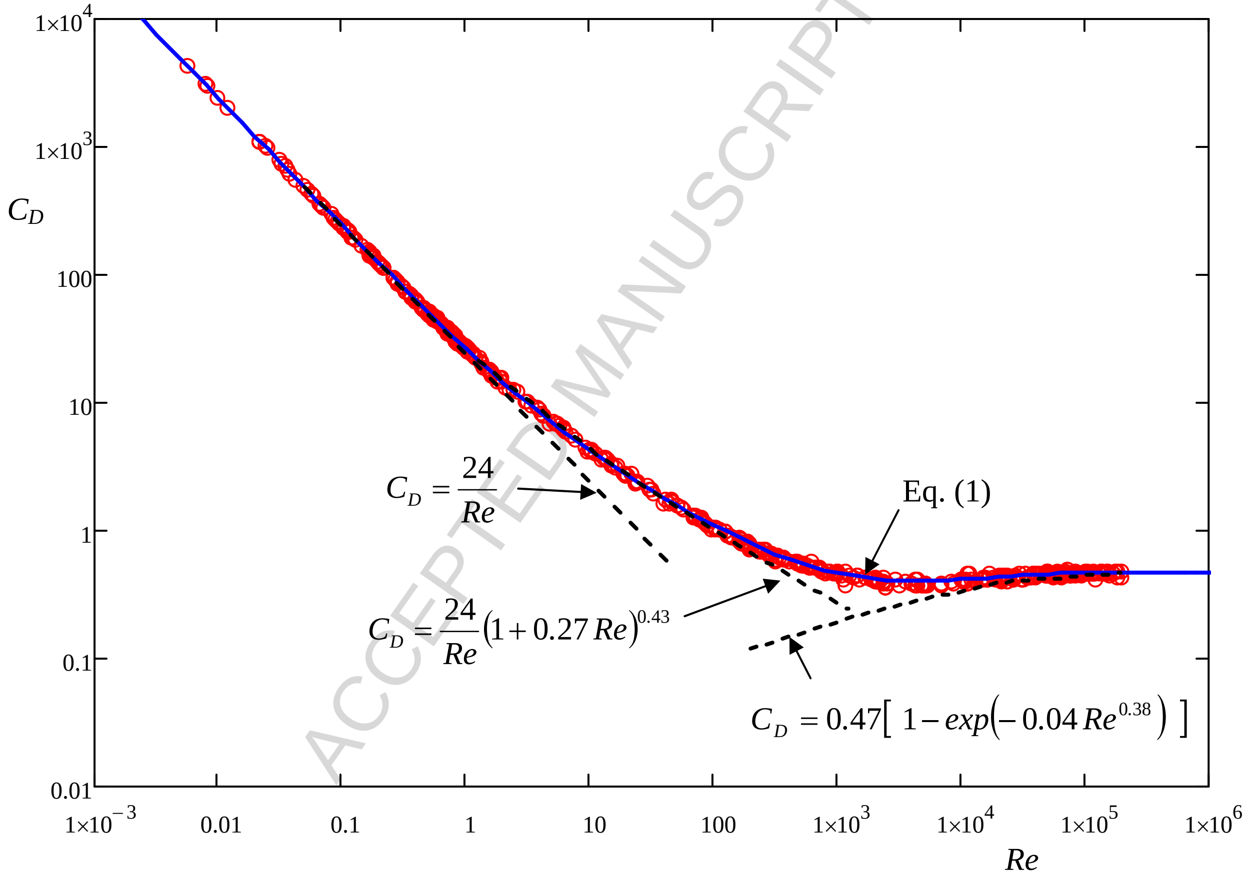

Fig. 1. Standard drag curve represented by Eq. (1) in comparison with experimental data[2].

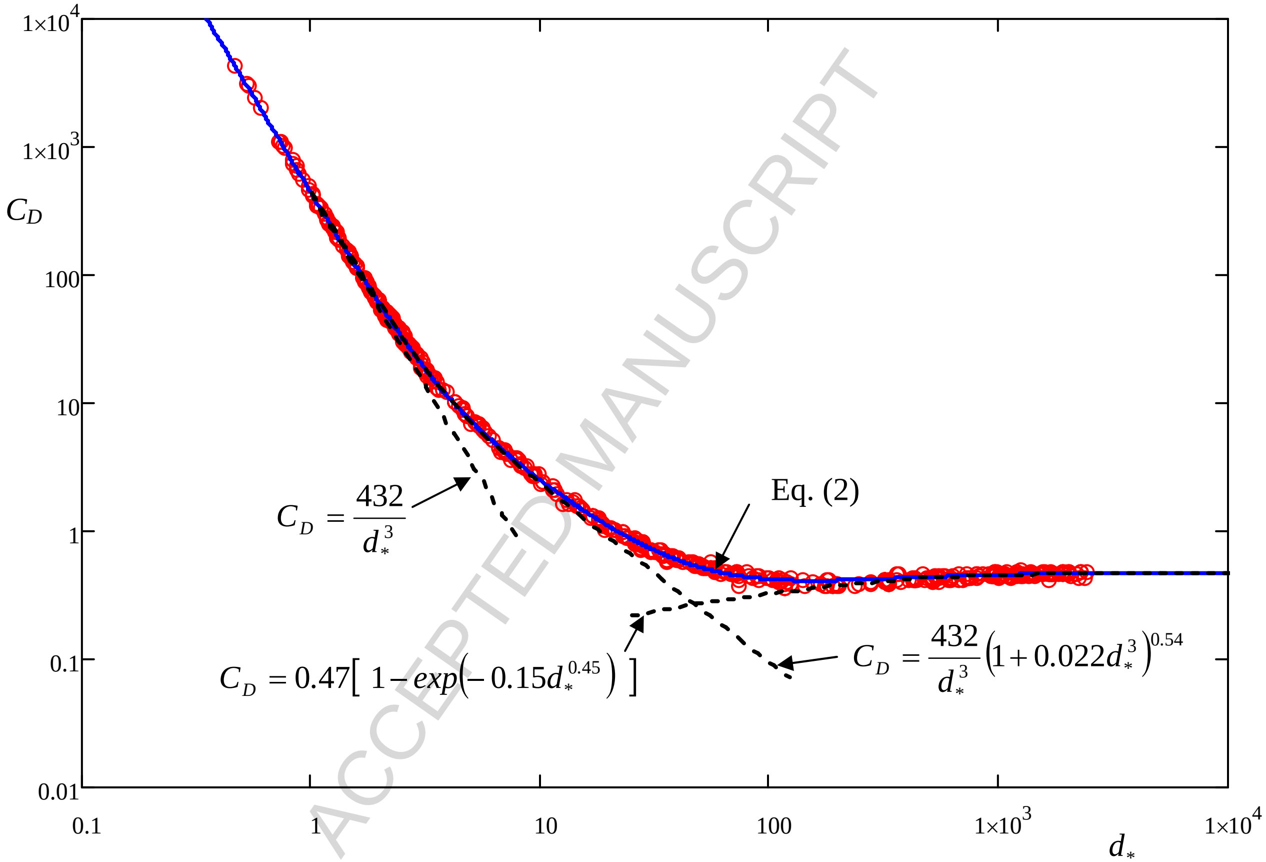

Fig. 2. Variation of C_D with d_* predicted by Eq. (2) in comparison with experimental data[2]

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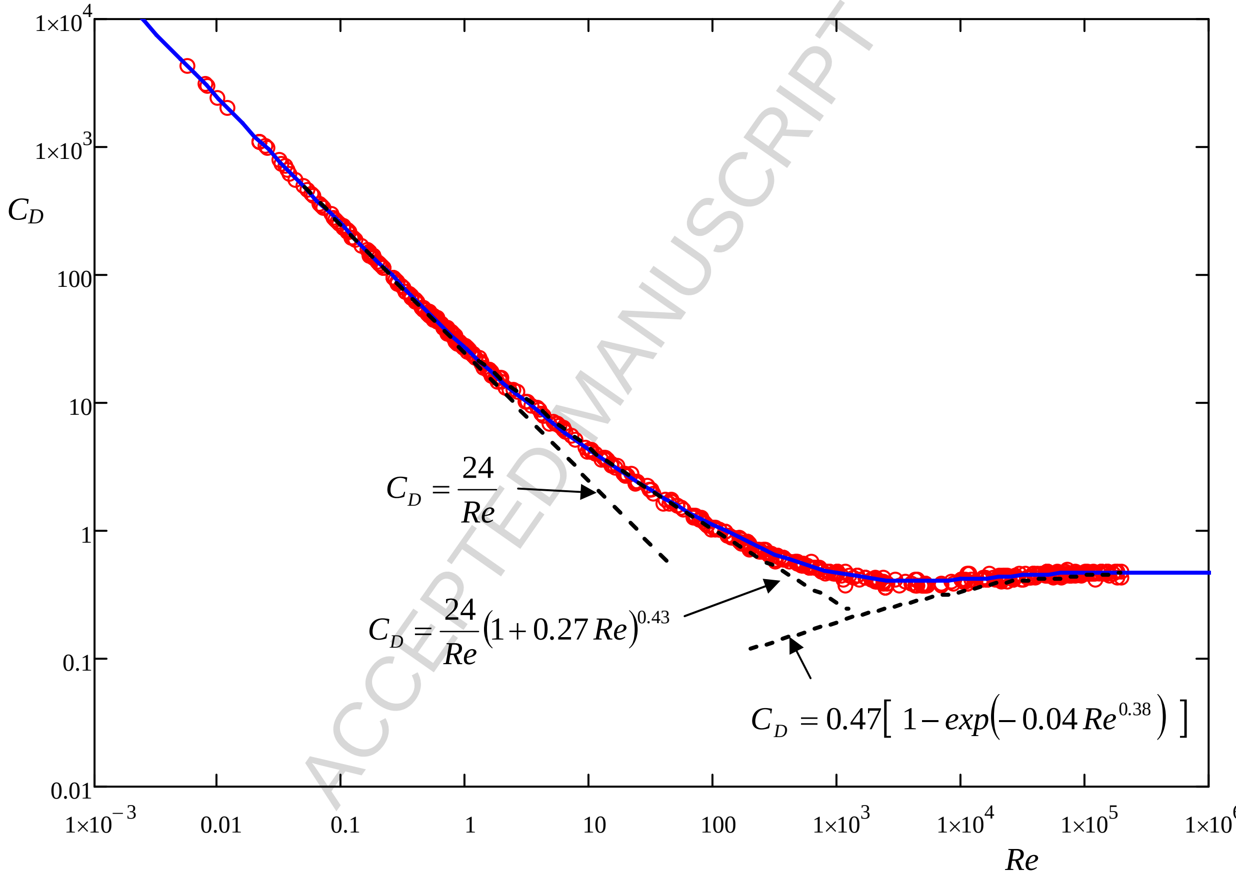

C_D
1×10^4
1×10^3
100
10
1
0.1
0.01
1×10^{-3}
0.01
0.1
1
10
100
1×10^3
1×10^4
1×10^5
1×10^6
Re
$C_D = \frac{24}{Re}$
$C_D = \frac{24}{Re}\left(1+0.27\,Re\right)^{0.43}$
$C_D = 0.47\left[\,1-exp\left(-0.04\,Re^{0.38}\right)\,\right]$