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Phase extraction from electronic speckle patterns by statistical analysis

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Abstract

In electronic speckle pattern interferometry (ESPI), speckles are information carriers as well as noise that hinders the extraction of high quality phase. This paper presents a phase extraction method based on the statistical property of speckles. Assuming that speckle related phase is a random variable having a uniform distribution, the grey level variance of a number of pixels is found to be related to the modulation intensity of a speckle pattern. The relation is used to establish a connection between the phase to be measured and the variance of grey level difference between two speckle patterns. Subsequently, a phase map wrapped in $[0, \pi)$ is extracted. In order to obtain a standard 2π wrapped phase map, an initial one step phase shift is introduced. The phase value of a pixel under consideration is obtained from the grey levels of its $N \times N$ neighbouring pixels. The optimal value of N is obtained based on a qualitative analysis of the initial results. With an appropriate value of N , an accuracy of 1% can be achieved.

Keywords: Phase extraction; Electronic speckle pattern; Statistical analysis

1. Introduction

Electronic speckle pattern interferometry (ESPI) [1] is a full field measurement technique for the deformation pattern on an object surface. Random speckles are used as carriers from which phase information is extracted. However, during the extraction process the random speckles are a source of noise, which hinders the extraction of high quality phase. In recent years, numerous methods have been developed to extract phase information from electronic speckle patterns. These can be generally categorized into three approaches: phase-shifting technique, Fourier transform method

and image processing algorithms.

A classical phase-shifting technique has been proposed by Creath [2] where four phase shifted speckle patterns are captured before a test object is deformed and a further four after the object is deformed. Carré's method [3] is then used to obtain wrapped phase maps before and after deformation. By subtracting one phase map from another, speckle noise is removed and deformation related information is obtained. Although, the method is theoretically accurate, it suffers from noise caused by uncorrelated pixels in the speckle patterns and non-linear phase shift also affects its accuracy. Creath's method both in implementation and noise immunity has subsequently been improved [4].

Fourier transform method was initially used to measure an object surface profile [5]. It transforms a fringe pattern with carrier fringes into frequency domain and applies a band-pass filter to extract a first order frequency component. It then inversely transforms the filtered spectrum into spatial domain, where a phase value is obtained from the real and imaginary parts of the inverse Fourier transform. When Fourier transform is applied to speckle patterns, it functions as a low pass filter that removes high frequency noise [6,7]. However, a low pass filter also masks the high frequency structural information in a deformation pattern.

Image processing algorithms [8–11] are proposed in an attempt to remove noise and at the same time keep the structural information unchanged. There is a certain degree of difference with the first two approaches. Firstly, image processing algorithms are not limited to speckle patterns and can be used on noise corrupted phase maps as well [10]. Secondly, image processing algorithms are often applied to a speckle pattern obtained by subtraction or addition of two speckle patterns (one before and another after deformation), while phase-shifting technique deals with speckle patterns before any mathematical operation is applied. Thirdly, image processing algorithms are essentially used to facilitate the phase extraction process by removing noise from source data; while they are not frequently used to extract phase directly. Hence, image processing is more of a preprocessing procedure than a phase extraction technique.

In this paper, a phase extraction method is proposed, which extracts phase information from correlated electronic speckle patterns. The principle of the method is based on the statistical property of speckles [12]. Statistical analysis has been shown to be an effective tool for fringe optimization in many algorithms, such as scale-space filter [8,9] and correlation method [11]. The proposed method is different from those in that deformation related phase map is directly extracted in a range of $[0, \pi)$ and an initial $\pi/2$ phase shift can be used to obtain a standard 2π wrapped phase map. As will be seen, the phase range extension process in the proposed method does not require a band-pass filter [7] or a normalizing interferogram averaged over π phase shifted frames [13] to remove background or noise. Experimental and simulated results are presented.

2. Principle of the method

2.1. Phase extraction in the range $[0, \pi)$

The light intensity on an electronic speckle pattern recorded before and after the object deformation can be expressed as

$$I_1 = a + b \cos \theta, \quad (1)$$

$$I_2 = a + b \cos(\theta + \delta), \quad (2)$$

where a is the background intensity, b is the modulation intensity, θ is the speckle related phase and δ is the deformation related phase change. Generally, the background intensity a and modulation intensity b do not change significantly before and after deformation. They are considered to remain unchanged in the following derivations; however, for completeness an expression based on different values of a and b is also given at the end of this section.

Two assumptions are made in the derivation. Firstly, the background intensity a , modulation intensity b and phase δ are slow-varying variables and can be considered constant in a small region of a speckle pattern. Secondly, the speckle related phase θ is assumed to be a random variable uniformly distributed in $[-\pi, \pi)$ [14]. The probability density function of θ is:

$$f(\theta) = \begin{cases} \frac{1}{2\pi} & \theta \in [-\pi, \pi), \\ 0 & \text{other values of } \theta. \end{cases} \quad (3)$$

Based on the theory of probability [15], it is possible to derive the probability density of $\sin \theta$ and $\cos \theta$ from Eq. (3). The derivation is omitted here but it is found that $\sin \theta$ and $\cos \theta$ share a same probability density function. Let Y be $\sin \theta$ or $\cos \theta$. The probability density function of Y is:

$$f(Y) = \begin{cases} \frac{1}{\pi\sqrt{1-Y^2}} & Y \in [-1, +1], \\ 0 & \text{other values of } Y. \end{cases} \quad (4)$$

The expected value and variance of Y are

$$EY = \int_{-1}^{+1} \frac{Y}{\pi\sqrt{1-Y^2}} dY, \quad (5)$$

$$DY = \int_{-1}^{+1} \frac{(Y - EY)^2}{\pi\sqrt{1-Y^2}} dY. \quad (6)$$

It is easy to show that $EY = 0$ and $DY = 0.5$. The intensity of a speckle pattern in Eq. (1) can be expressed in terms of Y as

$$I_1 = a + b \cdot Y. \quad (7)$$

Since a and b are assumed constant, the variance of I_1 is

$$DI_1 = Da + b^2 \cdot DY = \frac{b^2}{2}. \quad (8)$$

Eq. (8) indicates that a number of pixels grey level variance in a small region of a speckle pattern is related to the modulation intensity of that region.

The intensity difference between the speckle patterns in Eqs. (1) and (2) is

$$I_2 - I_1 = -2b \cdot \sin \frac{\delta}{2} \cdot \sin \left(\theta + \frac{\delta}{2} \right).$$

Since δ is assumed constant, it would not affect the probability density of θ . Hence $\sin(\theta + \delta/2)$ has the same probability density as $\sin \theta$ and the intensity difference $I_2 - I_1$ can also be expressed in terms of Y

$$I_2 - I_1 = -2b \cdot \sin \frac{\delta}{2} \cdot Y. \quad (9)$$

Since b and δ are constants, the variance of $I_2 - I_1$ can be written as

$$\begin{aligned} D(I_2 - I_1) &= \left(-2b \cdot \sin \frac{\delta}{2} \right)^2 \cdot DY \\ &= 2b^2 \cdot \sin^2 \frac{\delta}{2}. \end{aligned} \quad (10)$$

From Eqs. (8) and (10), the deformation related phase δ is expressed in the range $[0, \pi)$ as.

$$\delta_{1,2} = 2 \cdot \arcsin \left[\frac{1}{2} \cdot \sqrt{\frac{D(I_2 - I_1)}{DI_1}} \right], \quad (11)$$

where $\delta_{1,2}$ is used to show that δ is obtained from the speckle patterns I_1 and I_2 .

In some situations where the background intensity a and modulation intensity b vary before and after deformation, the corresponding expression for δ is given by

$$\delta_{1,2} = \arccos \left[\frac{E(I_1 \cdot I_2) - EI_1 \cdot EI_2}{\sqrt{DI_1 \cdot DI_2}} \right]. \quad (12)$$

2.2. Phase extraction in the range $[0, 2\pi)$

Phase values obtained using Eq. (11) are wrapped in modules of π instead of 2π . Furthermore, in Eq. (10) all values of $\sin(\delta/2)$ are squared, which removes the original sign

information and would result in a phase map without any appreciable “phase jumps”. There should be phase jumps from π to $-\pi$ or $-\pi$ to π but the negative sign is not retrievable from Eq. (10). Hence standard phase unwrapping algorithms cannot be applied to retrieve an unwrapped phase map. Madjarova [16] proposed an unwrapping algorithm to process such a phase map. However, Madjarova’s algorithm relies on the assumption that phase values are monotonically increasing or decreasing and hence it is not applicable to closed-fringe patterns.

In this section a one step phase-shifting method to extract phase values in the range of $[0, 2\pi)$ is presented. Two speckle patterns with a $\pi/2$ phase shift are recorded before a test object is deformed and another speckle pattern is recorded after the object is deformed. The respective speckle intensities are given by

$$\begin{aligned} I_1 &= a + b \cos \theta, \\ I_2 &= a + b \cos \left(\theta - \frac{\pi}{2} \right), \\ I_3 &= a + b \cos \left(\theta - \frac{\pi}{2} + \delta \right). \end{aligned} \quad (13)$$

Using the procedure as described in Section 2.1, the corresponding deformation related phase δ is given by

$$\delta_{1,3} - \frac{\pi}{2} = 2 \cdot \arcsin \left[\frac{1}{2} \cdot \sqrt{\frac{D(I_3 - I_1)}{DI_1}} \right], \quad (14)$$

$$\delta_{2,3} = 2 \cdot \arcsin \left[\frac{1}{2} \cdot \sqrt{\frac{D(I_3 - I_2)}{DI_2}} \right], \quad (15)$$

where both $\delta_{1,3} - \pi/2$ and $\delta_{2,3}$ range from 0 to π . Since $\cos(\delta_{1,3} - \pi/2) = \sin \delta_{1,3}$, examining the sign of $\sin \delta_{1,3}$ and $\cos \delta_{2,3}$, a phase value φ can be obtained in the range $[0, 2\pi)$. First, compute φ using

$$\varphi = \arctan \left(\frac{\sin \delta_{1,3}}{\cos \delta_{2,3}} \right). \quad (16)$$

If both $\sin \delta_{1,3}$ and $\cos \delta_{2,3}$ are positive, φ would be in the first quadrant. If $\sin \delta_{1,3} > 0$ and $\cos \delta_{2,3} < 0$ the computed φ would have values between $-\pi/2$ and 0 but π should be added to convert it to the second quadrant. If $\sin \delta_{1,3} < 0$ and $\cos \delta_{2,3} < 0$ the computed φ would have values between 0, $\pi/2$ and π should be added to convert it to the third quadrant. If $\sin \delta_{1,3} < 0$ and $\cos \delta_{2,3} > 0$ the computed φ would have values between $-\pi/2$ and 0 and 2π should be added to convert it to the fourth quadrant. Finally, if $\cos \delta_{2,3} = 0$, then $\varphi = \pi/2$ for $\sin \delta_{1,3} > 0$, otherwise $3\pi/2$.

3. Results and discussion

3.1. Simulation and error analysis

Fig. 1 shows computer-simulated speckle patterns of a centrally loaded circular plate before (I_1 & I_2) and after (I_3) loading. A $\pi/2$ phase shift is also introduced between I_1 and I_2 . A theoretical wrapped phase map and the corresponding 3D plot of an unwrapped phase map are shown in Fig. 2. A block-path unwrapping algorithm proposed by Goldstein et al. [17] is used in the unwrapping process.

In the implementation of the algorithm, each phase value is calculated by a random series consisting of $N \times N$ pixels grey levels. These pixels are taken from an $N \times N$ pixel window in the targeted speckle patterns with the point under consideration at the center. Within the window, the background intensity a , modulation intensity b and deformation related phase δ are assumed constant. Although the assumption of a and b to be constants will introduce negligible errors, errors caused by the assumption of δ as a constant are not negligible, especially when N is large or δ is fast changing.

In phase extraction using Eq. (11), if the actual phase value of the point under consideration is larger than the average phase value in an $N \times N$ pixel window, the resultant phase value would be underestimated and if the actual phase value is smaller, the resultant phase value would be overestimated. Hence the resultant phase values would fall into a range smaller than 0 to π . The phase range error is shown graphically in Fig. 3. The graphs show the connection between processing window size N and global maximum and minimum phase values obtained using Eq. (11). As can be seen, the phase range narrows with increase in processing window size.

The following procedure is used to correct the phase range errors.

1. The global maximum and minimum phase values in a phase map are detected. As mentioned above, these values would fall in a range which is less than 0 to π .
2. From these extreme values, a linear equation is obtained as follows:

$$y = \frac{\pi}{p_2 - p_1} \cdot x - \frac{\pi \cdot p_1}{p_2 - p_1}, \quad (17)$$

where p_1 and p_2 represent the global minimum and maximum phase values, respectively, x is the input phase value, and y is the corrected phase value.

3. All phase values in the phase map are corrected using Eq. (17) and the resultant phase map would have a range of 0 to π .

The corrected phase map is subsequently used to extend the range of the phase values to $[0, 2\pi)$ using Eqs. (14)–(16).

Using a processing window size of 3×3 , a resulting phase map with an accuracy of 2.5% is obtained as shown in Fig. 4(a). The corresponding 3D plot of the unwrapped phase is shown in Fig. 4(b). When the window size is increased to 7×7 pixels, a phase map with an improved accuracy of 1% is shown in Fig. 5. The improvement is due to an

increase in the number of pixels used. Since the proposed method assumed a random speckle phase distribution θ , a larger number of elements in a series would result in a better approximation to its theoretical random distribution (Law of Large Numbers [12]). However, it should be noted that a larger window would increase the computation time. For example, in a Pentium 4, 2.0 GHz, personal computer, the computation time to process three 512×512 speckle patterns using a 3×3 pixel window is 6 s, while the corresponding time for a 7×7 pixel window is 18 s and for a 17×17 pixel window is 96 s. Besides an increase in computation time, a larger window may cause the deformation related phase to vary within the window. This problem is further discussed in the following section.

3.2. Experimental results

Fig. 6 shows the experimental set-up, a Michelson-type interferometer, for a centrally loaded plate. The coherent light source is a He-Ne laser, wavelength 632.8 nm. The test specimen is a thin circular plate, radius 4 cm, fully clamped at the boundary. The loading device is a micrometer head. A reference plate is mounted on a PZT stage, connected to a controller. To introduce a phase shift, control commands are initiated by a computer which sends a voltage signal through the controller to the PZT stage. Three speckle patterns (Fig. 7) are recorded in the experiment. The first two patterns, I_1 and I_2 , are recorded before deformation with a $\pi/2$ phase shift in between. The third pattern I_3 is recorded after loading. Figs. 8(a) and (b) show, respectively, the correlation fringe patterns obtained by subtracting I_1 and I_2 from I_3 .

As the experiment aims at studying a static event, acquisition rate is not an important factor. For dynamic events, a high speed CCD camera is preferable and the experiment procedure to be adopted is similar to that described above. Firstly, two phase-shifted speckle patterns are recorded before deformation. When the specimen is deforming, a series of speckle patterns are recorded at different time instants. A 2π wrapped phase map can be extracted from each speckle pattern together with the speckle patterns before deformation. After phase unwrapping, the phase distribution at each time instant can be obtained.

As the speckle phase θ obtained by simulation shows a more random distribution than that obtained by experiment, a larger processing window with more pixels should be used for speckle patterns obtained experimentally to improve the randomness of θ . It is found that for reliable results a minimum processing window size of 7×7 pixels is required.

Using a processing window of 7×7 pixels, a deformation related phase map is obtained using Eqs. (14) and (15), as shown in Fig. 9. The phase angles are wrapped in modules of π and do not have the “phase jumps”, as mentioned in Section 2.2. A subsequent 2π wrapped phase map obtained by Eq. (16) and the corresponding 3D unwrapped phase plot are shown in Figs. 10(a) and (b), respectively. As can be seen, the extracted phase quality is relatively poor. Fig. 11 shows wrapped (π) phase maps using a 17×17 pixel window. Compare with those obtained by a 7×7 pixel window (Fig. 9), the results are significantly improved. Similarly the corresponding wrapped (2π) phase map (Fig. 12(a)), and 3D plot (Fig. 12(b)) show significant improvement. Although a larger window can improve the resultant phase quality, errors would be introduced when

the window size is increased beyond a threshold limit. An erroneous wrapped $2(\pi)$ phase map and a 3D plot using a 31×31 pixel window is shown in Fig. 13. The errors are due to the variation of deformation phase δ , which is no longer a constant within a 31×31 pixel window. Hence an optimal window size for a particular deformation pattern should be used due to the randomness of the speckle phase θ and the variation of the deformation phase δ .

However, if a deformation pattern contains high fringe density, it is difficult to find an optimal processing window size and the proposed method would encounter similar difficulty as the Fourier transform method [7]. In the Fourier transform method, when a low-pass filter for noise removal is applied on the frequency spectrum of a fringe pattern, frequency components of the whole image beyond the filter's threshold would be filtered out. Since Fourier transform performs a global operation on an image, it is necessary to balance noise removal and signal protection over the whole image. If the threshold is too large, noise would not be sufficiently removed. If the threshold is too small, useful signals, especially those in high fringe density area, would be lost.

If an adaptive instead of a constant processing window size is selected, the algorithm would have potential to overcome the problem mentioned above. In areas where deformation phase δ is slow-varying (low fringe density), a large processing window can be used to achieve high noise removal capability. In areas where δ is fast-varying (high fringe density), a small processing window can be used to minimize the variation of δ . In this way, the noise removal capability is compromised only in high fringe density areas. This would be an advantage over the Fourier transform method. In order to select a window size adaptively, a robust algorithm for measuring fringe density with high noise level would need to be developed.

4. Concluding remarks

A phase extraction method using an electronic speckle pattern is proposed. The method is based on the statistical property of speckles, which provides a connection between the phase to be measured and the intensity variance of a speckle pattern. Deformation related phase maps can be directly extracted in the range of $[0, \pi)$ and standard 2π wrapped phase map can be obtained by a one step phase shifting technique. Since phase values are separated from the background and speckle noise, no band-pass filter or other preprocessing procedure is needed to extend the phase range from π to 2π . As system error has an effect in reducing the phase range, an error correction procedure is proposed to improve the accuracy. The proposed method only performs a local operation with a fixed window size on a speckle pattern. A more flexible algorithm which could adaptively alter the window size would be useful in future work.

References

- [1] G.L. Cloud, Optical Methods of Engineering Analysis, Cambridge University Press, Cambridge, 1995.
- [2] K. Creath, Appl. Opt. 24 (1985) 3053.
- [3] P. Carré, Metrologia 2 (1966) 13.
- [4] S.-H. Baik, S.-K. Park, C.-J. Kim, S.-Y. Kim, Opt. Commun. 192 (2001) 205.
- [5] M. Takeda, K. Mutoh, Appl. Opt. 22 (1983) 3977.
- [6] T. Kreis, J. Opt. Soc. Am. A 2 (1986) 847.
- [7] D. Kerr, F.M. Santoyo, J.R. Tyrer, J. Mod. Opt. 36 (1989) 195.
- [8] A. Davila, G.H. Kaufmann, D. Kerr, Opt. Eng. 35 (1996) 3549.
- [9] P.D. Ruiz, G.H. Kaufmann, Opt. Eng. 37 (1998) 2395.
- [10] H.A. Aebischer, S. Waldner, Opt. Commun. 162 (1999) 205.
- [11] D.R. Schmitt, R.W. Hunt, Appl. Opt. 36 (1997) 8848.
- [12] B.R. Frieden, Probability, Statistical Optics, and Data Testing: A Problem Solving Approach, Springer, New York, 2001.
- [13] P.L. Wizinowich, Appl. Opt. 29 (1990) 3271.
- [14] J.C. Dainty, Laser Speckle and Related Phenomena, Springer, Berlin, 1975.
- [15] H. Jeffreys, Theory of Probability, Oxford University Press, New York, 1983.
- [16] V. Madjarova, S. Toyooka, R. Widiastuti, H. Dadono, Opt. Commun. 212 (2002) 35.
- [17] R.M. Goldstein, H.A. Zebker, C.L. Werner, Radio Sci. 23 (1988) 713.

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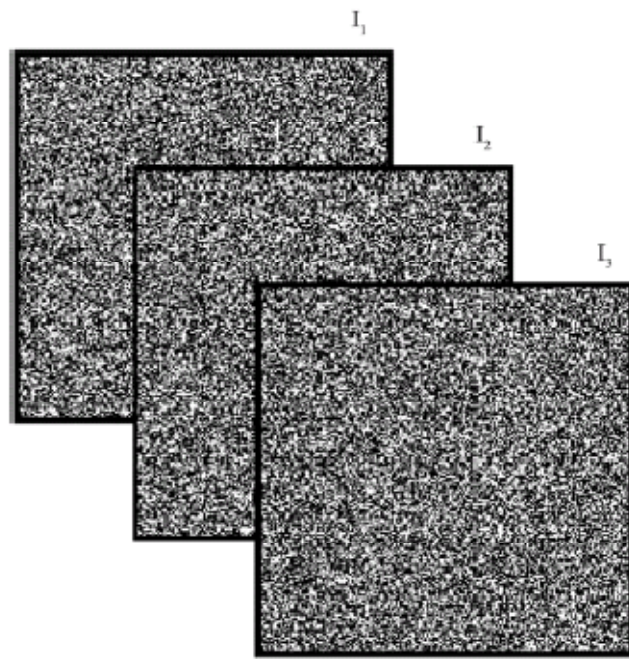


Fig. 1

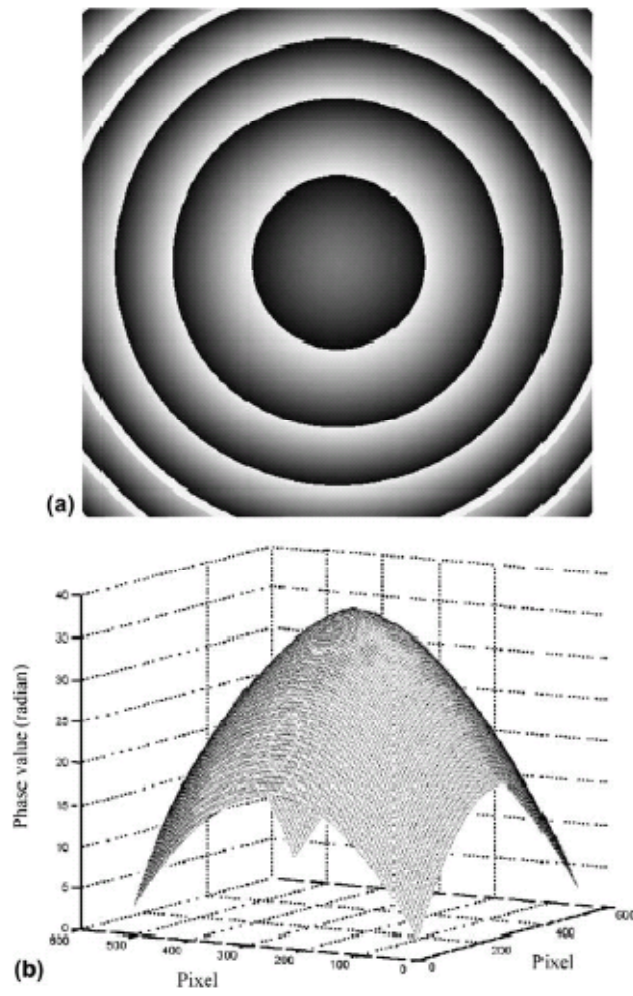


Fig. 2

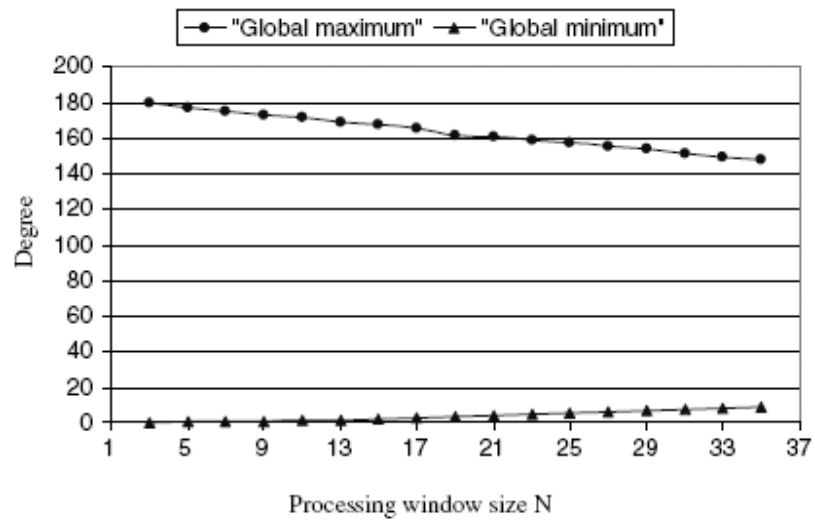


Fig. 3

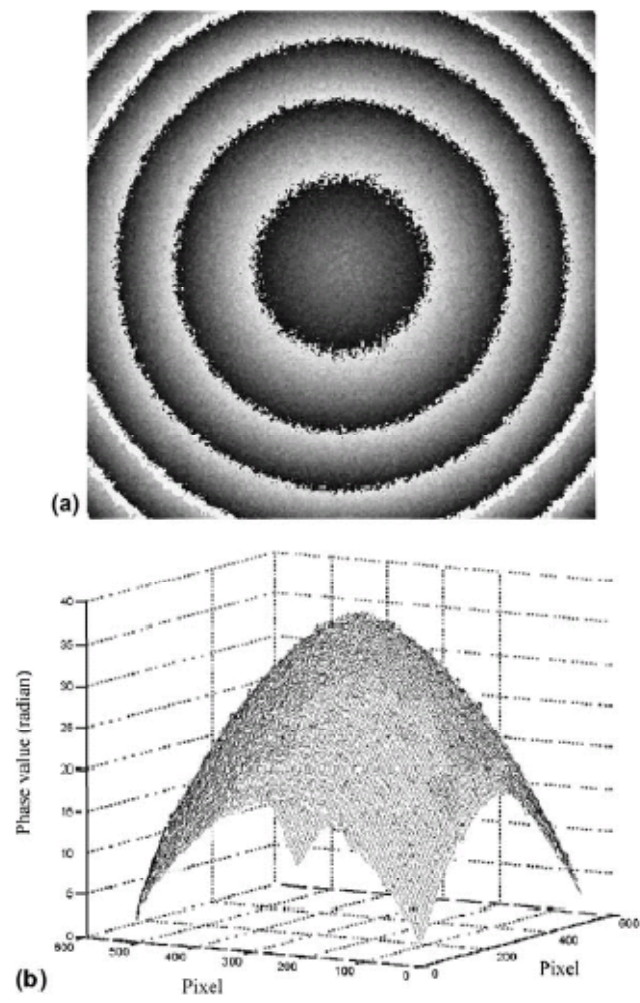


Fig. 4

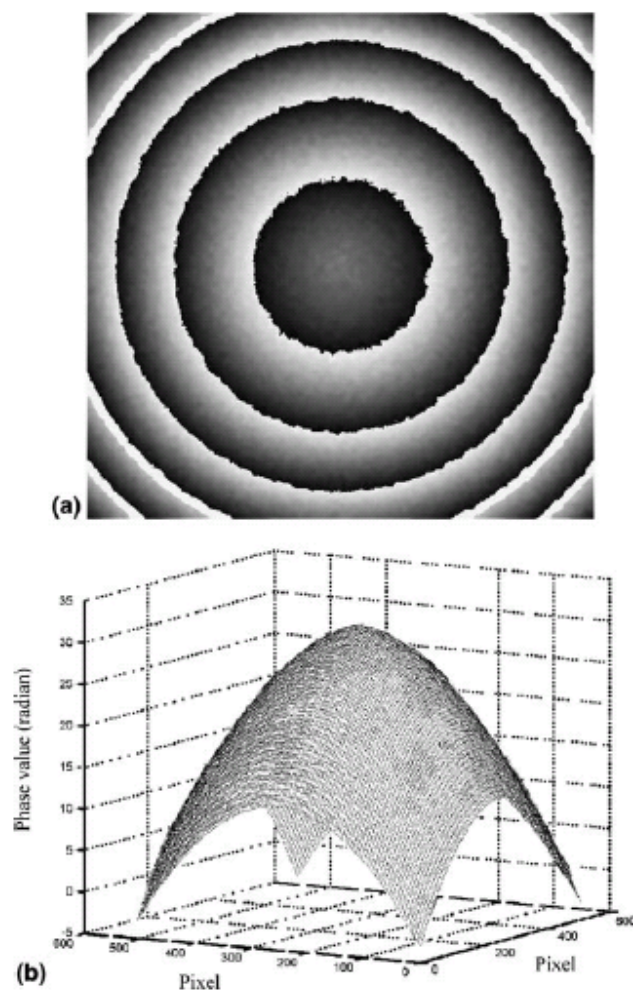


Fig. 5

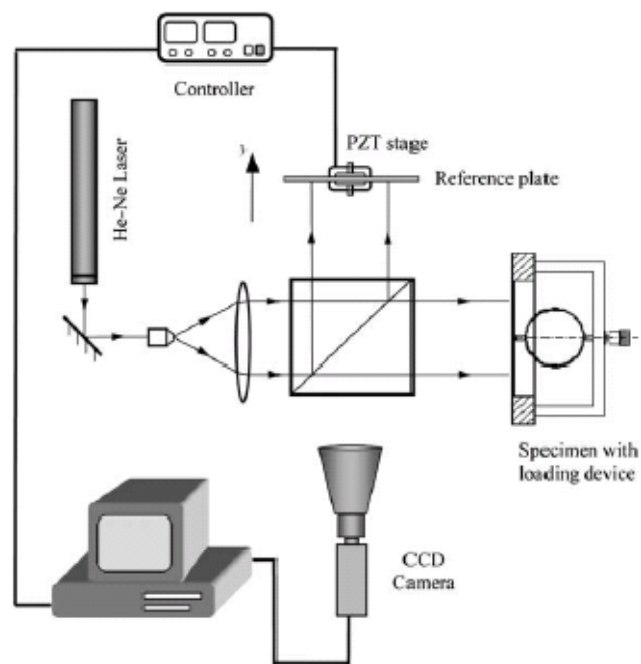


Fig. 6

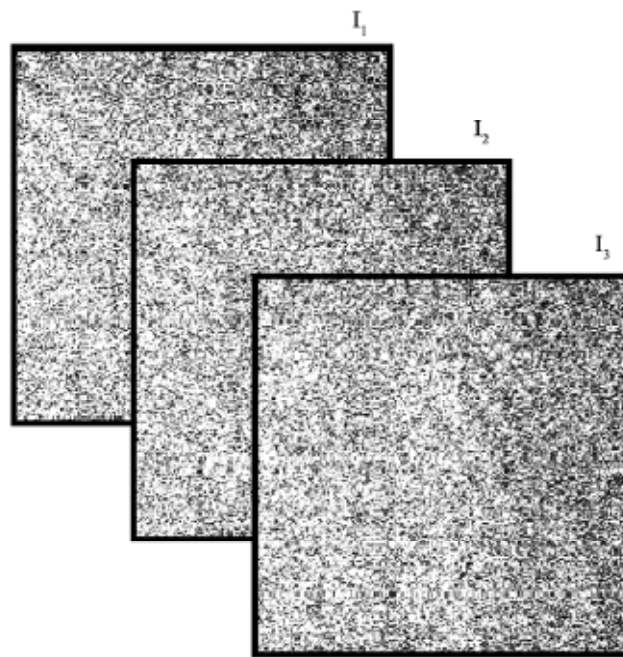


Fig. 7

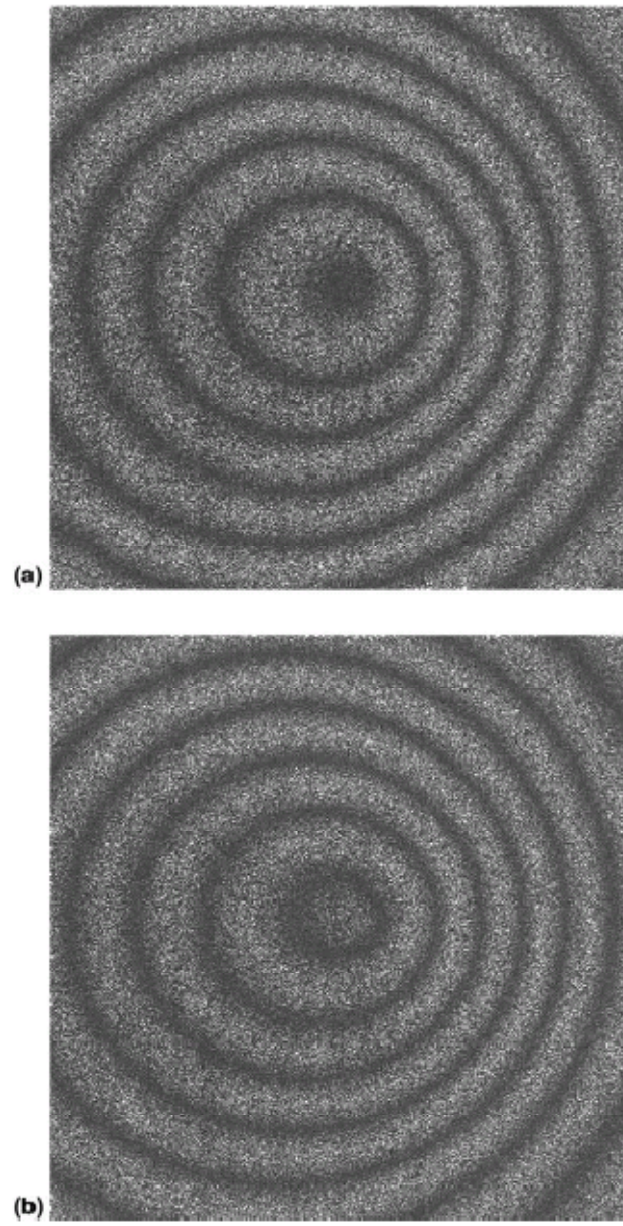


Fig. 8

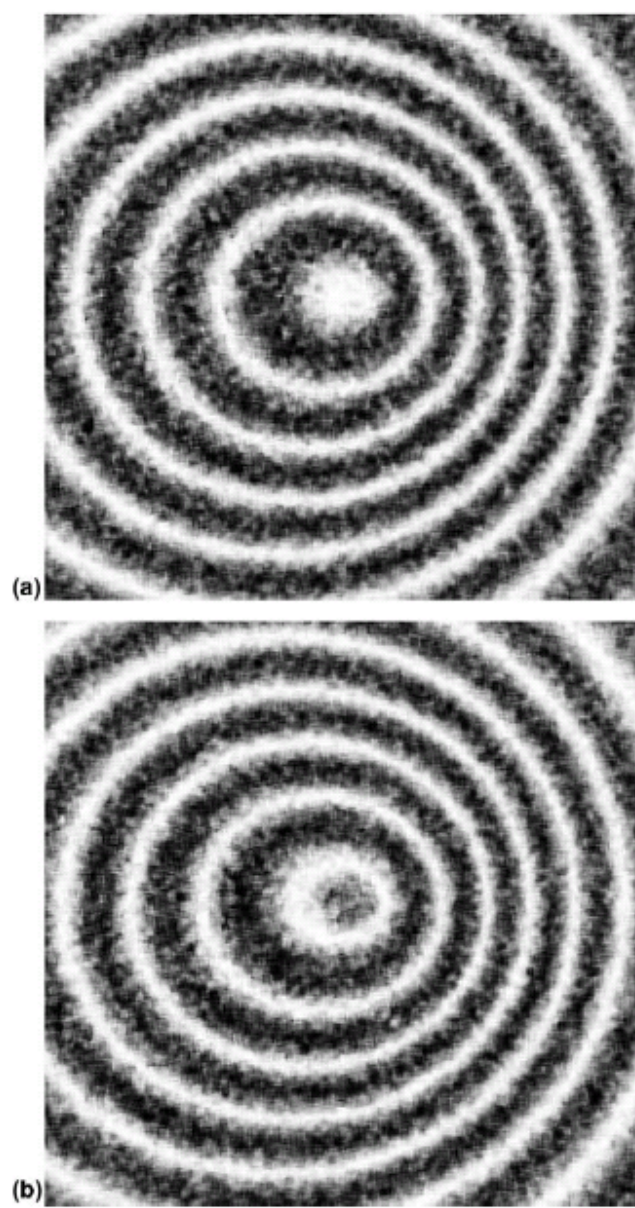


Fig. 9

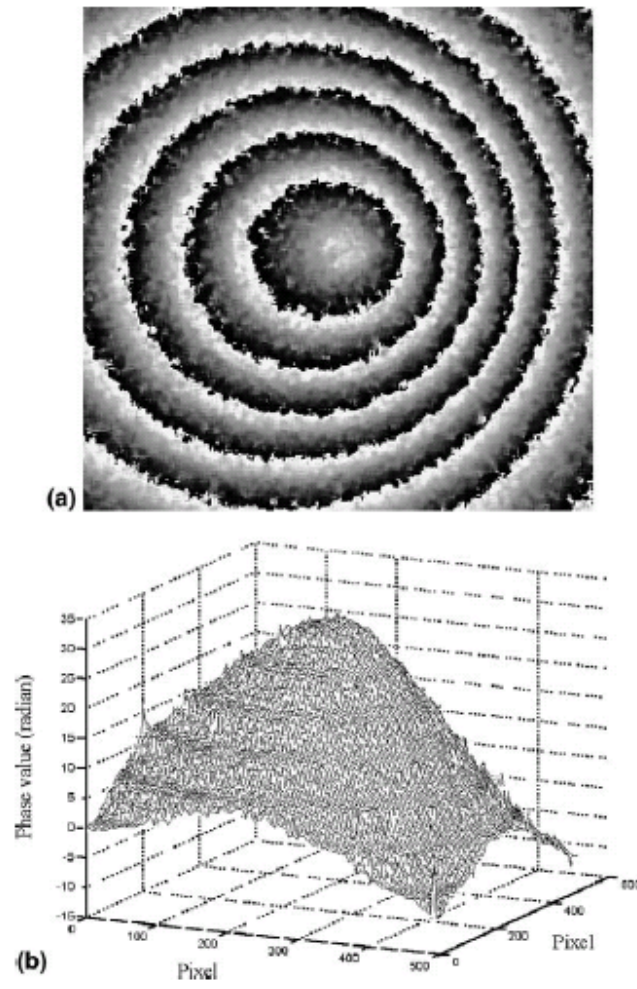


Fig. 10

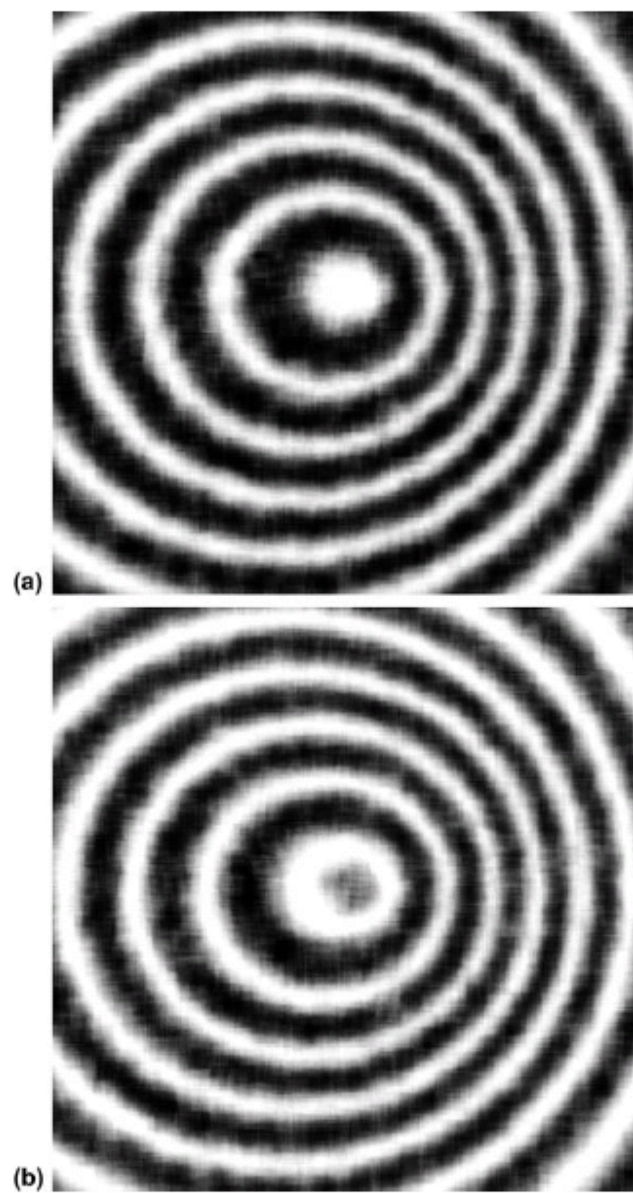


Fig. 11

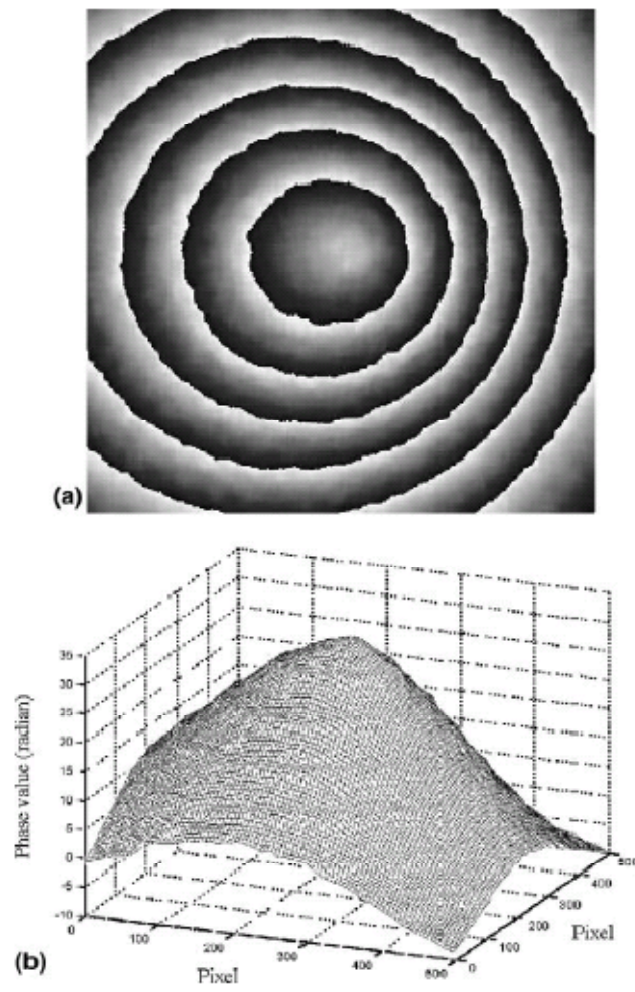


Fig. 12

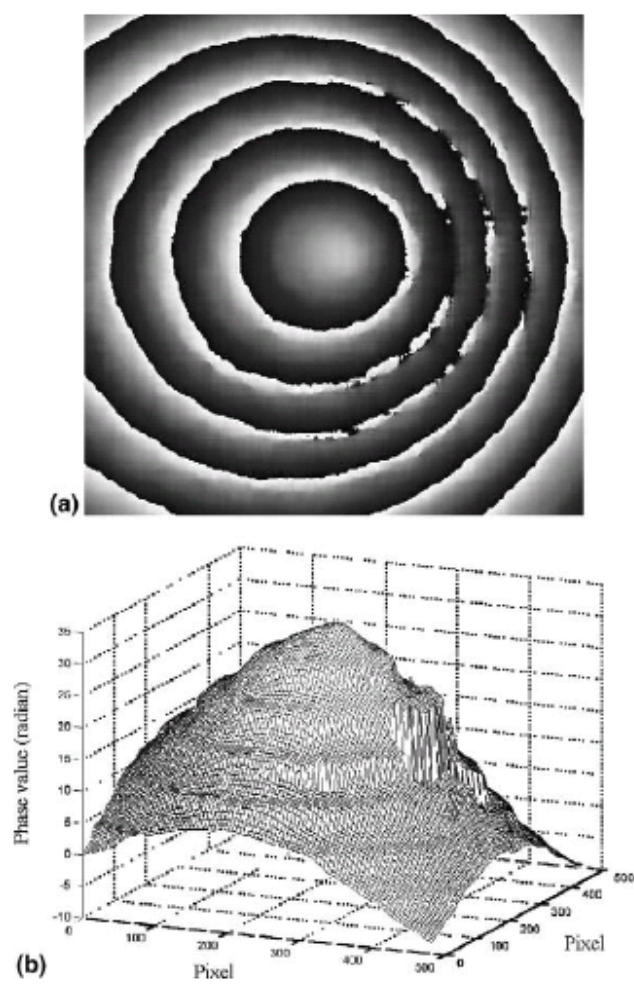


Fig. 13