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ABSTRACT

In this paper, a simple method is described for the measurement of small angles of rotation of a flat surface using a circular optical grating, which may be generated and projected onto the test surface using either a standard Michelson interferometer or a computer-controlled LCD projector. In view of its availability in most laboratories, a Michelson interferometer is used in this paper. The diameter of the grating that is generated can be easily magnified or be reduced to suit the size of the test surface. With circular grating, the angular rotation of both diffuse and specularly reflective surfaces about any axis of rotation can be measured from the distortion of the grating. The distorted grating that is diffracted from a specularly reflective surface may be recorded in two different ways using a CCD camera. In the first method, the distorted grating is recorded off the mirror surface as though it were a diffuse surface. In the second method, the distorted grating is specularly reflected onto an opaque screen and the CCD camera subsequently records the grating image off this screen.

Keywords: circular optical grating, LCD projector, Michelson interferometer, small angles of rotation, CCD camera

1. INTRODUCTION

The measurement of angle of rotation of a surface about its axis can be realised in a number of ways using methods based on optical interferometry, internal-reflection of optical elements and projection grating [1-11]. With the rapid development of micro-systems in recent years, small movable mirrors are found in various commercial applications as in multiple-mirror devices for projection displays in HDTV and in beam deflection devices with mirror sizes ranging from about 100 μm to about 3 mm [12-14]. The angular rotation of the micro-mirror about its axis is often controlled through the application of electrical voltage. The existing methods for measuring the rotation angle have thus been modified for such applications. Each method has its advantages and limitations. For example, the method of point-of-light triangulation is easy to perform, requiring only a simple optical arrangement; but if the illuminating point lies on the axis of rotation, the angle of rotation of a diffuse surface cannot be determined unless three non-collinear illuminating points are used. In the method of line-of-light triangulation, the angle of rotation cannot be determined if the direction of the grating lines is perpendicular to the axis of rotation.

In this paper, an alternative method is described for measuring small angles of rotation of a flat surface using a circular optical grating. The main advantage of this method lies in the ability of determining the angular rotation of the test surface about any axis of rotation directly from the distortion of the grating. The optical grating can be generated and projected onto diffuse and specularly reflective surfaces with a standard Michelson interferometer or a computer-controlled LCD projector. In view of its availability in most laboratories, a Michelson interferometer is used in this paper. Furthermore, by placing an additional lens between the beam-splitting cube and the screen (test surface), the diameter of the optical grating may either be magnified or be reduced to suit the size of the test surface.

2. GENERATING CIRCULAR GRATING WITH MICHELSON INTERFEROMETER

Figure 1 illustrates the optical arrangement of a Michelson interferometer for the generation and projection of optical grating. By carefully adjusting the mirrors so that the two light beams exiting from the beam-splitting cube are collinear (but with optical path difference) and normal to a flat screen, a circular grating will be generated and projected onto the screen [15]. Suppose the two light wavefronts exiting from the beam-splitting cube are emanated from two point-light sources

$S(0,0,D)$ and $S'(0,0,D+d_z)$ located on the reference z-axis (Fig. 1). It can easily be shown that the typical point $P_0(x_0, y, 0)$ on a locus of constant phase-difference Δ (which is also on a visible fringe-circle of order N) is described by the following expression [15].

$$x^2 + y^2 = \frac{\left(d_z^2 - \Delta^2\right)^2 + 4D\left(d_z^2 - \Delta^2\right)(D+d_z)}{4\Delta^2} \tag{1}$$

where Δ is related to the fringe-order N via the well-known relation $\Delta = \dfrac{\lambda}{2}N$, with λ denoting the wavelength of the laser source.

If the outward normal of the flat screen makes an angle with the exiting light beams, the circular grating will be distorted in accordance with the angle of inclination (Fig. 2). The following describes a simple method in which the angle of rotation of a flat surface, diffuse as well as specularly reflective, is measured from the distorted circular grating.

3. MEASURING THE ANGLE OF ROTATION OF A DIFFUSE SURFACE

As shown in Fig. 3, the circular grating (Eq. (1)) is projected onto a planar diffuse surface that is inclined at the angle $\theta = \theta_1$. It can easily be derived that the expression for the distorted grating that is observed on the test surface is as follows.

$$x^2 + y^2 = \frac{\left(d_z^2 - \Delta^2\right)^2 + 4D\left(d_z^2 - \Delta^2\right)(D+d_z)}{4\Delta^2} + \left[\frac{d_z^2 - \Delta^2}{\Delta^2}\right]Z^2 - \left[\frac{\left(d_z^2 - \Delta^2\right)(2D+d_z)}{\Delta^2}\right]Z \tag{2}$$

where Z, the z-coordinate of any point $P(x, y, Z)$ on an interference fringe that is observed on the inclined surface, is defined as $Z = x\tan\theta$.

For simplicity, the image of the grating on the test surface is recorded with unit magnification using a camera whose direction of recording is parallel with the reference z-axis. The length of a fringe-circle on the xy-plane ($\theta = 0$) is denoted by $X = 2x_0$ in a direction parallel with the x-axis, and by Y in a direction parallel with the y-axis. Suppose the test surface is rotated slightly about the y-axis from the initial angle θ_1 to the angle θ_2. Then, as shown in Fig. 3, the value of X is changed from X_1 to X_2 whereas the value of Y is unchanged throughout the angular change. For two distantly located but closely spaced point-light sources $S(0,0,D)$ and $S'(0,0,D+d_z)$, and for small angle of rotation of a relatively small object surface, it can easily be shown that the angle of rotation (from θ_1 to θ_2) may be determined from the distortion of the circular grating (from X_1 to X_2) using the following expression.

$$\theta_2 - \theta_1 \approx -\frac{K}{2}\left(\frac{X_2 - X_1}{Y}\right) = -\frac{K}{2}\psi \tag{3}$$

where $K \equiv \dfrac{Y}{D\xi}$ is defined as a sensitivity factor related to the optical arrangement; and ξ, which is related to the phase-difference Δ (or fringe order N), is defined as $\xi \equiv \left(\dfrac{d_c^2 - \Delta^2}{\Delta^2}\right)$.

4. MEASURING THE ANGLE OF ROTATION OF A SPECULARLY REFLECTIVE SURFACE

The angle of rotation of a specularly reflective surface can be measured in one of the following two ways, depending on the manner in which the distorted grating is recorded. In the first method, the distorted grating is recorded directly off the reflective surface as if the surface were diffuse; hence Eq. (3) is used in the measurement procedure. In the second method, the image of the grating that is projected onto the surface is specularly reflected onto an opaque screen, after which the distorted image is recorded off this screen. As illustrated in Fig. 4, the optical grating is projected along the reference z-axis onto a specularly reflective surface that is inclined at the angle θ_1 with the reference x-axis. A fringe-circle that is cast on the inclined surface is specularly reflected so that the length $P_1 P_1'(\equiv X_1)$ of the distorted fringe-circle on the camera image plane (parallel with the x-axis) is now recorded as $P_3 P_3'(\equiv X_1')$ on the camera image plane. For simplicity in the derivation, it is assumed that the reflection angles at P_1 and P_1' are equal to $2\theta_1$ as shown in Fig. 4, and that the size Y of the fringe-circle on the y-axis is unchanged. As before, unit magnification factor is considered and the recording direction is assumed to be parallel with the reference z-axis. As the inclination angle θ of the surface changes from θ_1 to θ_2, the value of X is changed from X_1' to X_2' whereas the value of Y is unchanged. For small angle of rotation of a relatively small test surface, it can be shown that the angular change $(\theta_2 - \theta_1)$ can be estimated from the measured values $(X_2' - X_1')$ using the following expression.

$$\theta_2 - \theta_1 \approx -\frac{K'}{2}\left(\frac{X_2' - X_1'}{Y}\right) = -\frac{K'}{2}\psi \qquad (4)$$

where $K' \equiv \dfrac{Y}{D_c \xi}$ denotes the optical sensitivity, with ξ defined as $\xi \equiv \left(\dfrac{d_c^2 - \Delta^2}{\Delta^2}\right)$.

It is obvious from Eqs. (3) and (4) that the angle of rotation about the y-axis of diffuse and specularly reflective surfaces may be determined from the amount of distortion of the fringe-circle $(X_2 - X_1)$ or $(X_2' - X_1')$ that is measured off the image-plane of the CCD camera; the optical sensitivity factor K or K' may be obtained by calibration. It is worth noting that in the actual experiment, the center of the projected circles may not coincide with the center of rotation of the test surface, thereby giving an effect that is equivalent to combined translation and rotation. This effect, therefore, necessitates the inclusion of a correction term in Eqs. (3) and (4). For simplicity, the linear relationship between angular change $(\theta_2 - \theta_1)$ and grating distortion ψ (Eqs. (3) and (4)) may be re-written as follows.

$$\theta_2 - \theta_1 = \Gamma\psi + \Omega \qquad (5)$$

where Γ is related to the optical arrangement; and Ω is a correction term. These parameters are readily determined by calibrating the optical system. During calibration, the test surface is mounted on a rotating stage so that the distortion ψ of the grating corresponding to each angle of rotation $(\theta_2 - \theta_1)$ is measured; the constants Γ and Ω in Eq. (5) are subsequently calculated from the slope and intercept of the $(\theta_2 - \theta_1) \sim \psi$ graph.

5. EXPERIMENTAL ILLUSTRATION

Using the optical set-up shown in Fig. 1, a flat surface (of size 10 mm by 10 mm) in the form of a front-coated mirror is mounted on a rotating stage. Fig. 2 shows a typical circular grating that is projected onto the surface; the innermost fringe-circle (outside the central dark-patch) on the mirror surface has been shrunk to approximately 400 μm in diameter. The linear relationships between grating distortion and rotation angle that are constructed using the two methods of recording are shown in Figs. 5 and 6.

6. CONCLUSION

In this paper, a simple method is described for measuring small angles of rotation of a flat surface that is either diffuse or specularly reflective. A circular grating is generated and projected onto the test surface using a standard Michelson interferometer. The diameter of the optical grating, which governs the minimum size of the test surface that can be used for measurement, can be easily adjusted using a simple lens placed between the beam-splitting cube and the test surface. Experimental results have shown a linear relationship between the amount of distortion of the circular grating and the angle of rotation; this trend is in agreement with the linear relationship predicted by theory. Thus, after calibration, the optical system may be used for measuring small angles of rotation of flat surfaces.

7. REFERENCES

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Screen or flat surface

O

X-axis

Z-axis

Converging
Lens

Mirror

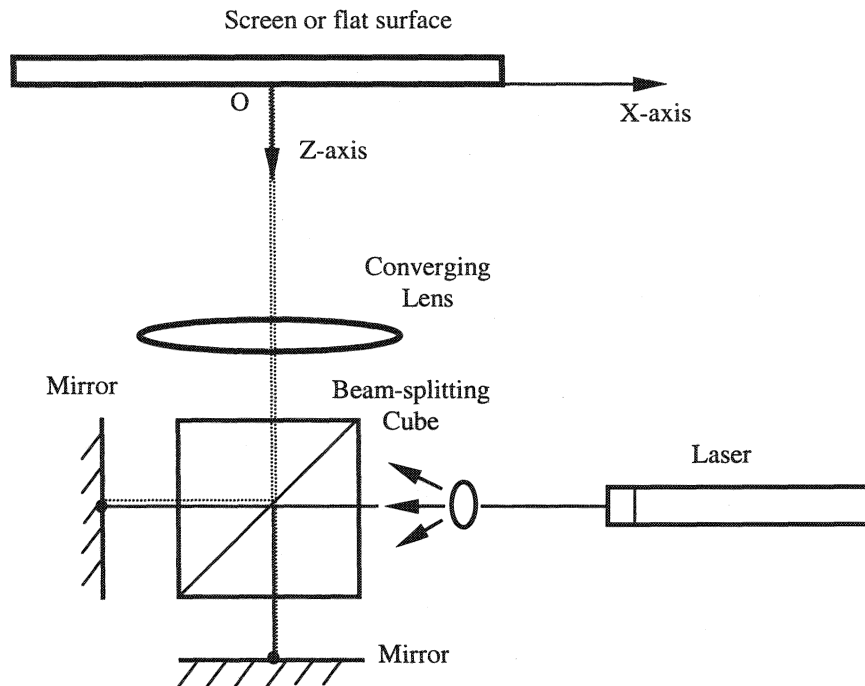Beam-splitting
Cube

Laser

Mirror

Fig. 1.   The use of Michelson interferometer for generating and
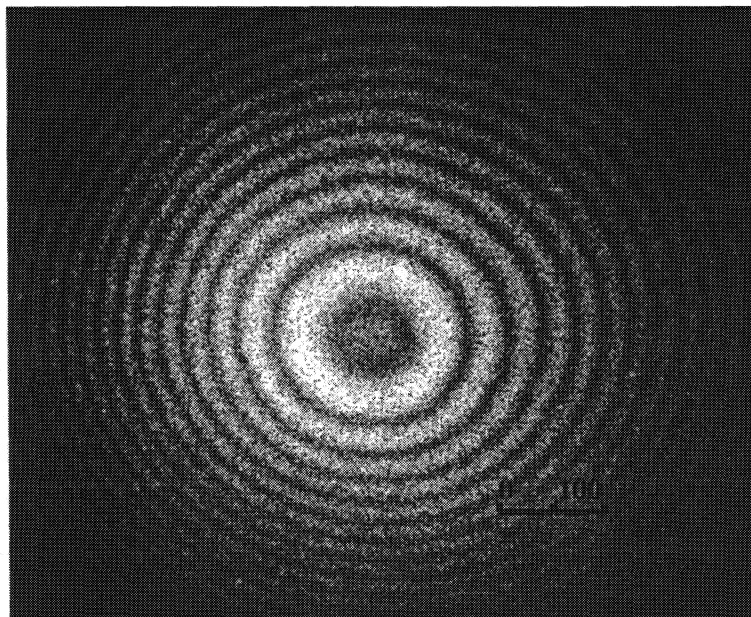projecting optical grating.

Fig. 2.   Distortion of a circular grating observed from a slightly
rotated surface, diameter of innermost circle of grating being
approximately 400 µm.

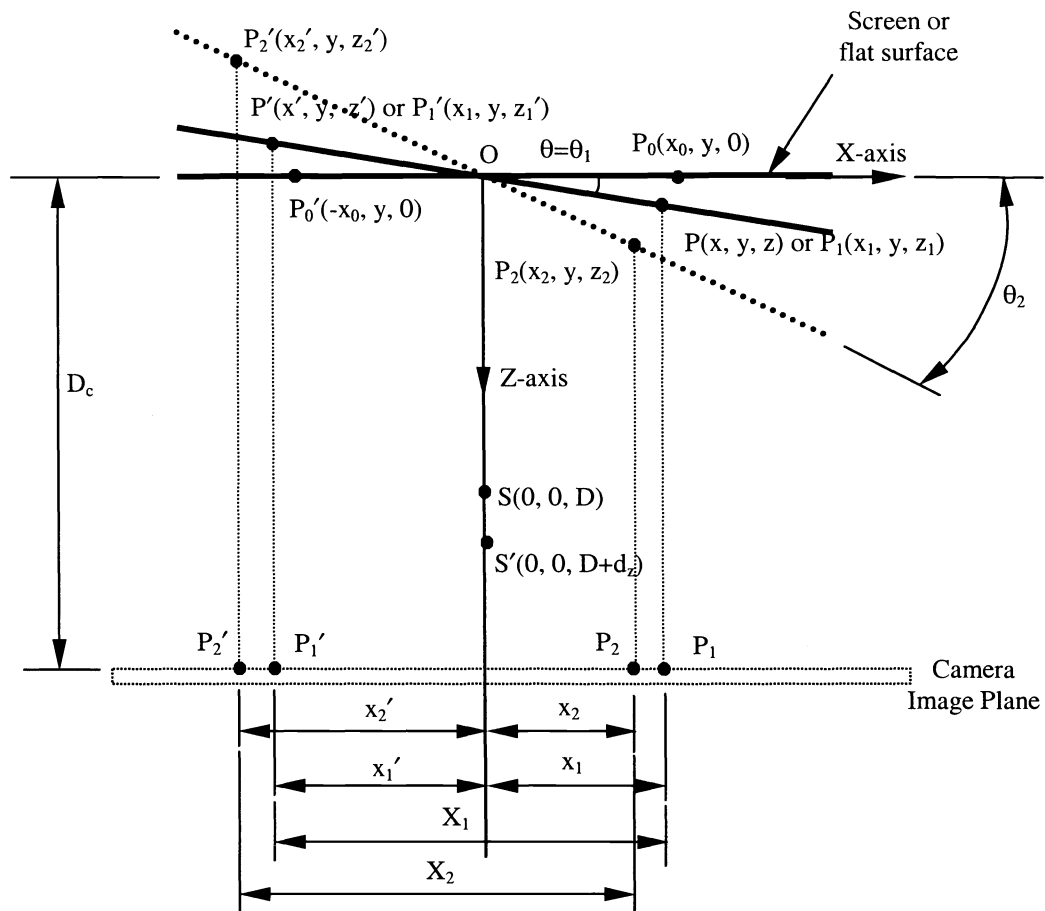Fig. 3. Measurement of the angle of rotation of a diffuse/specularly reflective surface.
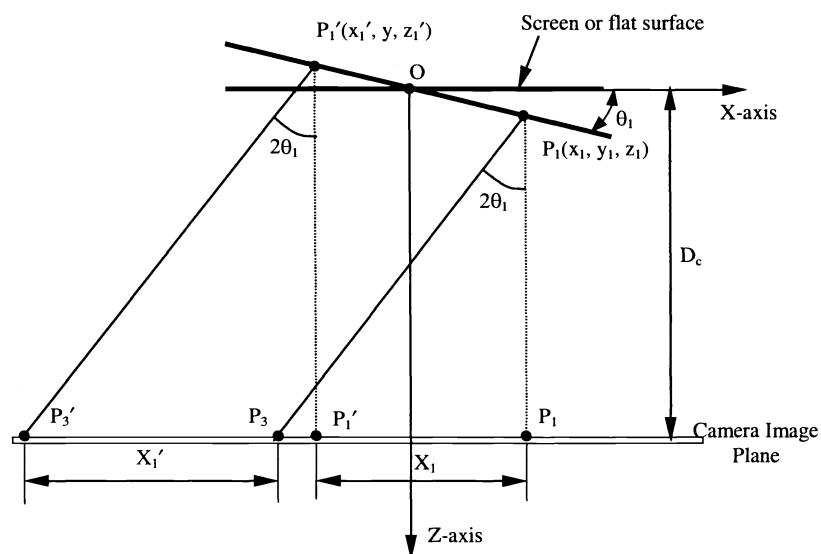

Fig. 4. Measurement of the angle of rotation of a specularly reflective surface.
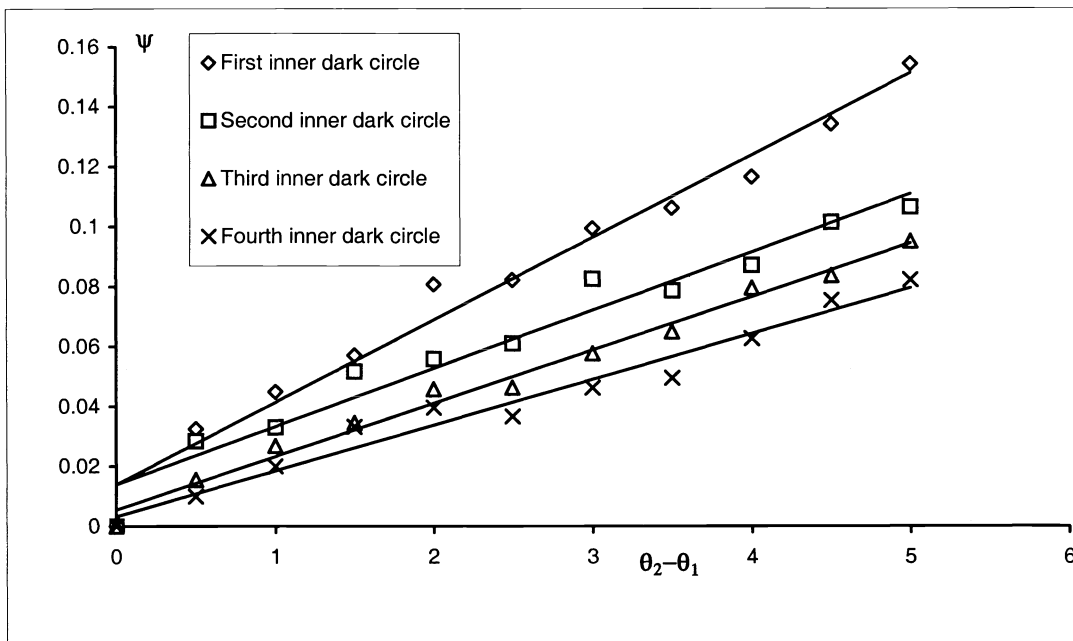
Fig. 5. Experimental relationship between grating distortion and rotation angle of a flat surface from direct measurement of grating on the test surface.
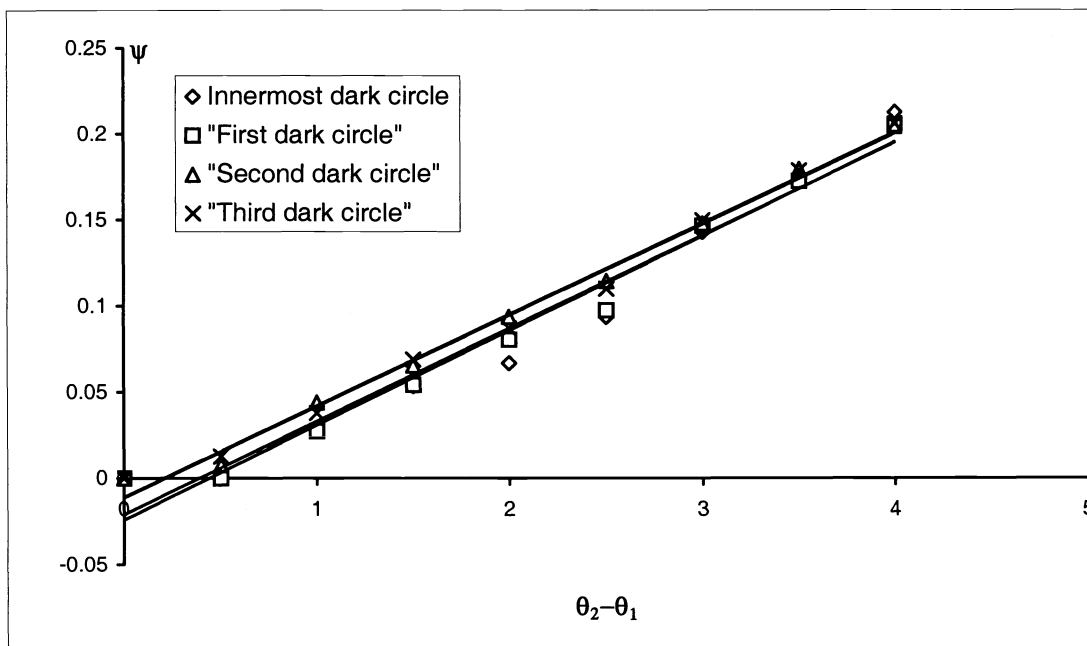

Fig. 6. Experimental relationship between grating distortion and rotation angle of a flat surface from measurement of the reflected grating on an opaque surface.