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TRANSIENT CONJUGATE FREE CONVECTION FROM A VERTICAL FLAT PLATE IN A POROUS MEDIUM SUBJECTED TO A SUDDEN CHANGE IN SURFACE HEAT FLUX

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Abstract—The paper presents a theoretical study using Kármán–Pohlhausen method for describing the transient heat exchange between the boundary-layer free convection and a vertical flat plate embedded in a porous medium. The unsteady behaviour is developed after generation of an impulsive heat flux step at the right-hand side of the plate. Two cases are considered according to whether the plate has a finite thickness or has no thickness. The time and space evolution of the interface temperature is evidenced.

1. INTRODUCTION

The importance of heat transfer phenomena associated with free convection in porous media is well known. Interest in this phenomena has been motivated by such diverse engineering problems as geothermal energy extraction, storage of nuclear waste material, ground water flows, pollutant dispersion in aquifers and packed-bed reactors, to mention just a few applications. The archival publications on this topic were excellently reviewed by Nield and Bejan [1].

Owing to its fundamental and practical importance, the conjugate coupling heat transfer between a free convection flow and a vertical flat plate of finite thickness embedded in a porous medium has received particular attention [2–7]. Various approaches were used to deal with the difficulties associated with the simultaneous solution of the flow and thermal boundary layers and the longitudinal and transversal heat conduction in the solid plate. Some aspects of this coupling phenomena and bibliography on the topic can be found in the most recent reviews by Kimura *et al.* [8], and Pop and Nakayama [9]. However, despite the existing results in the open literature they do not yet provide a complete description of this important problem, which has a bearing on many practical applications, particularly those related to energy conservation in buildings [10].

The point we wish to take up here is that of the transient conjugate free convection due to a vertical flat plate embedded in a porous medium. We assume that at a given time ($t > 0$) the right-hand side of the plate is suddenly subjected to a uniform heat flux, while the left-hand side of the plate is thermally insulated. The present study was conducted in two phases: with finite thickness or without thickness of the plate, respectively. Analytical and numerical solutions are presented for all possible values of time and space evolution of the interface temperature.

Finally, it is worth mentioning that Vynnycky and Kimura [6] have considered the case when the temperature of the left-hand side of the plate is suddenly raised to, and held at, a uniform temperature.
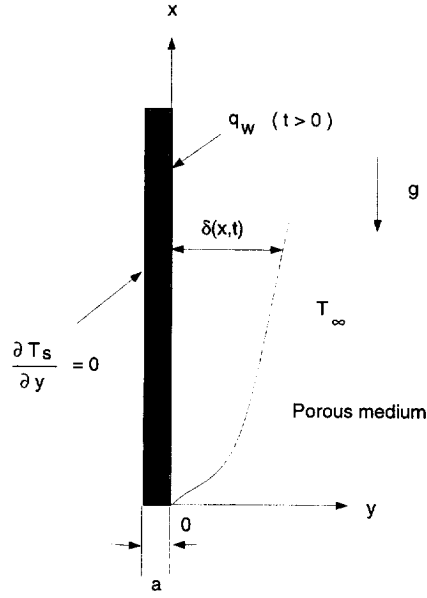
Fig. 1. Physical model and coordinate system.

2. BASIC EQUATIONS

Consider unsteady free convection flow due to a semi-infinite vertical flat plate of finite thickness a adjacent to a semi-infinite fluid-saturated porous medium. Initially, the whole system is at a temperature T_∞, but subsequently the left-hand side of the plate is suddenly raised to, and held at a uniform heat flux q_w. The physical model and coordinate system is shown in Fig. 1. Assuming that the porous medium is isotropic and homogeneous and that the fluid is incompressible, we invoke the boundary layer and the Boussinesq approximations to obtain the following equations.

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Darcy's Law

$$u = \frac{gK\beta}{\nu}(T_f - T_\infty) \tag{2}$$

The equation of energy in the fluid–porous medium

$$\sigma \frac{\partial T_f}{\partial t} + u \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} = \alpha_f \frac{\partial^2 T_f}{\partial y^2} \tag{3}$$

and the equation of the heat transfer inside the solid plate

$$\frac{\partial T_s}{\partial t} = \alpha_s \frac{\partial^2 T_s}{\partial y^2} \tag{4}$$

where (x,y) are cartesian coordinates along and normal to the plate, (u,v) are the velocity components in the (x,y) directions, t is the time, T_f and T_s are the temperatures of the fluid-saturated porous medium and the solid plate, respectively, and the physical constants g, β, ν, K, α_f, α_s and σ are as given in the Nomenclature. Equations (1)–(4) are subject to the following initial and boundary conditions.

For the fluid–porous medium $(y \geq 0)$

$$u = v = 0, \ T_f = T_\infty \text{ at } t = 0 \text{ or } x = 0 \tag{5}$$

$$v = 0 \text{ on } y = 0 \tag{6}$$

$$u = 0, \ T_f = T_\infty \text{ as } y \longrightarrow \infty \tag{7}$$

for the solid $(-a \le y \le 0)$

$$T_s = T_\infty \text{ at } t = 0 \text{ or } x = 0 \tag{8}$$

$$\frac{\partial T_s}{\partial y} = 0 \text{ on } y = -a \tag{9}$$

for the fluid–solid interface

$$T_f = T_s = T_p \text{ on } y = 0 \text{ and } t > 0 \tag{10}$$

$$q_w = |q_s| + |q_f| = k_s \frac{\partial T_s}{\partial y} - k_f \frac{\partial T_f}{\partial y} \text{ on } y = 0 \text{ and } t > 0 \tag{11}$$

where T_p is the interface temperature and k_f and k_s are the thermal conductivities of the fluid and solid, respectively.

We further define the following dimensionless variables

$$x^* = \frac{x}{aR_a}, \ y^* = \frac{y}{a}, \ t^* = \frac{\alpha_s t}{a^2},$$

$$\delta^* = \frac{\delta}{a}, \ u^* = \frac{u}{U_c R_a}, \ v^* = \frac{v}{U_c},$$

$$\theta_f^* = \frac{T_f - T_\infty}{\Delta T}, \ \theta_s^* = \frac{T_s - T_\infty}{\Delta T}, \ \theta_p^* = \frac{T_p - T_\infty}{\Delta T} \tag{12}$$

where δ is the boundary-layer thickness and U_c, ΔT and R_a are the velocity characteristic, temperature characteristic and Rayleigh number which are defined as

$$U_c = \frac{\alpha_f}{a}, \ \Delta T = \frac{aq_w}{k_f}, \ R_a = \frac{gK\beta q_w a^2}{k_f \alpha_f v} \tag{13}$$

equations (1)–(4) then become, subsequently dropping the asterisks

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{14}$$

$$u = \theta_f \tag{15}$$

$$\Gamma \frac{\partial \theta_f}{\partial t} + u \frac{\partial \theta_f}{\partial x} + v \frac{\partial \theta_f}{\partial y} = \frac{\partial^2 \theta_f}{\partial y^2} \tag{16}$$

$$\frac{\partial \theta_s}{\partial t} = \frac{\partial^2 \theta_s}{\partial y^2} \tag{17}$$

subject to

$$u = v = 0, \ \theta_f = \theta_s = 0 \text{ at } t = 0 \text{ or } x = 0 \tag{18}$$

$$v = 0, \ \theta_f = \theta_s = \theta_p \text{ on } y = 0 \text{ and } t > 0 \tag{19}$$

$$u = 0, \theta_f = 0 \text{ as } y \longrightarrow \infty \tag{20}$$

$$\frac{\partial \theta_s}{\partial y} = 0 \text{ on } y = -1 \tag{21}$$

$$k\frac{\partial \theta_s}{\partial y} - \frac{\partial \theta_f}{\partial y} = 1 \text{ on } y = 0 \text{ and } t > 0 \tag{22}$$

where $\Gamma = \sigma\alpha_s/\alpha_f$ and $k = k_s/k_f$.

To obtain the integral form of the governing equations for transient conjugate free convection in a vertical porous layer, we integrate equation (16) in combination with equation (14) and equation (15) across the boundary layer and equation (17) is integrated from -1 to 0, to yield

$$\Gamma\int_0^\delta \frac{\partial \theta_f}{\partial t}dy + \frac{\partial}{\partial x}\int_0^\delta \theta_f^2 dy = -\frac{\partial \theta_f}{\partial y}|_{y=0} \tag{23}$$

$$\int_{-1}^0 \frac{\partial \theta_s}{\partial t}dy = \frac{\partial \theta_s}{\partial y}|_{y=0} \tag{24}$$

Further, we assume a second-order Kármán–Pohlhausen temperature profile in the fluid–porous medium and in the solid with the constraints such that the boundary conditions, equations (18)–(22) hold. We then have [11, 12]

$$\theta_f = \theta_p\left(1 - \frac{y}{\delta}\right)^2 \tag{25}$$

$$\theta_s = \frac{1}{2k}\left(1 - \frac{2\theta_p}{\delta}\right)y^2 + \frac{1}{k}\left(1 - \frac{2\theta_p}{\delta}\right)y + \theta_p \tag{26}$$

Substituting these expressions into equation (23) and equation (24), we obtain

$$\frac{\Gamma}{3}\frac{\partial}{\partial t}(\delta\theta_p) + \frac{1}{5}\frac{\partial}{\partial x}(\delta\theta_p^2) = \frac{2\theta_p}{\delta} \tag{27}$$

$$\frac{\partial}{\partial t}\left(\theta_p + \frac{2}{3k}\frac{\theta_p}{\delta}\right) = \frac{1}{k}\left(1 - \frac{2\theta_p}{\delta}\right) \tag{28}$$

subject to

$$\delta = \theta_p = 0 \text{ at } t = 0 \text{ or } x = 0 \tag{29}$$

For the steady case ($\partial/\partial t = 0$), equation (27) and equation (28) give

$$\theta_{p0} = \left(\frac{5x}{2}\right)^{\frac{1}{3}}, \ \delta_0 = (20x)^{\frac{1}{3}} \tag{30}$$

To solve equation (27) and equation (28) the method of characteristics has been used. They are hyperbolic sets of partial quasi-linear differential equations, which have two characteristic curves. The equations of direction of the characteristics are

$$dx = 0 \text{ and } \frac{\Gamma}{3}\left(\frac{4}{3} + k\delta\right)dx = \frac{1}{5}\theta_p(2 + k\delta)dt \tag{31}$$

so that the wave speed in the porous medium is

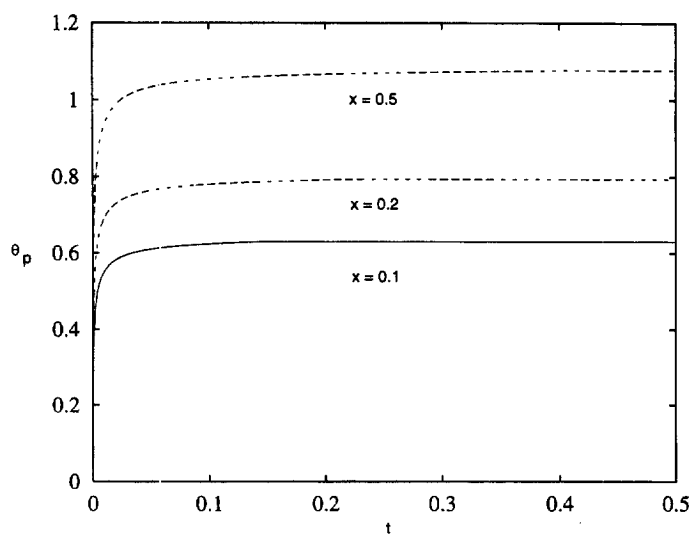$$\frac{9(2 + k\delta)\theta_p}{5\Gamma(4 + 3k\delta)} \tag{32}$$

Fig. 2. Profiles of the interface temperature for $\Gamma = 1$ and $k = 10$.
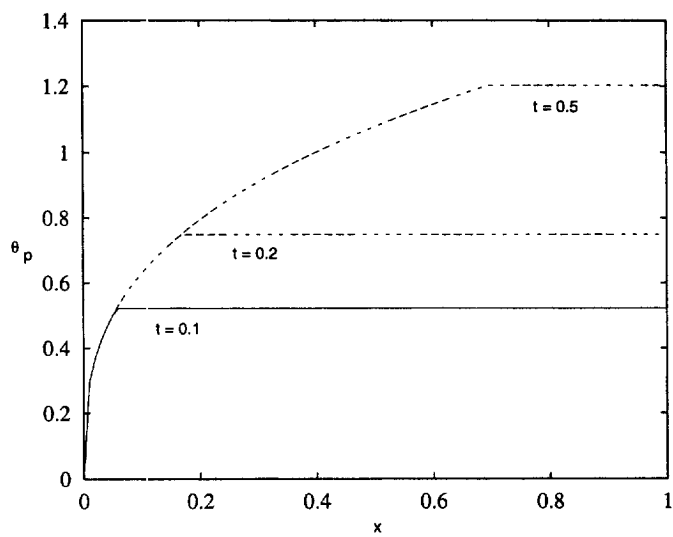

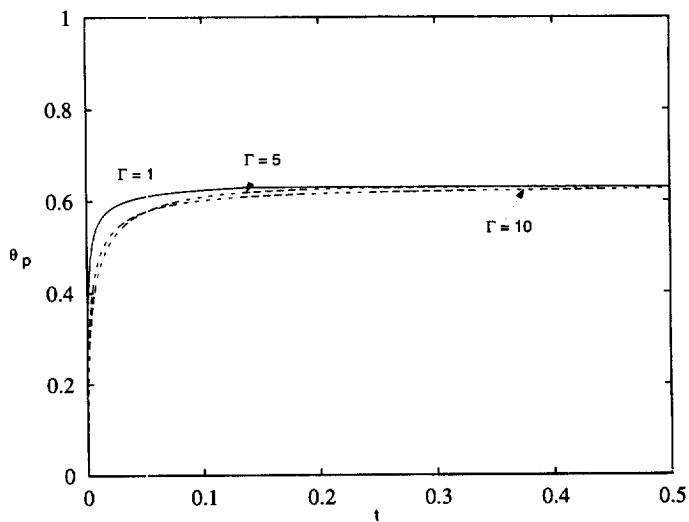Fig. 3. Profiles of the interface temperature for $\Gamma = 1$ and $k = 10$.

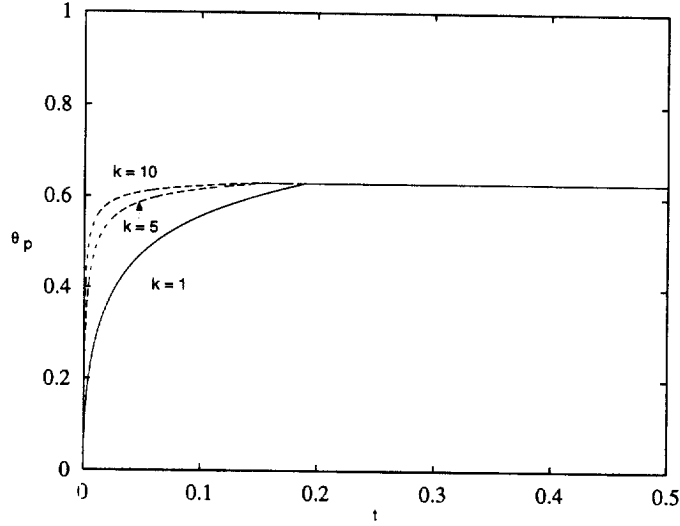Fig. 4. Profiles of the interface temperature for $x = 0.1$ and $k = 10$.

Fig. 5. Profiles of the interface temperature for $\Gamma = 1$ and $x = 0.1$.

The interface temperature distribution θ_p is illustrated in Figs 2–5 for $\Gamma = 1,5$ and 10 with $k = 1,5$ and 10. These figures show that although the value of θ_p increases continuously with both in t and x, its slope exhibits a discontinuity at t_{ss} whose value depends on x, Γ and k. This discontinuity suggests a sudden change in the heat transfer characteristics that can be attributed to the presence of an essential singularity in the governing equations [13–16]. We also notice that θ_p increases continuously with time and approaches for large time the corresponding steady state value θ_{p0}.

3. PLATE WITH NO THICKNESS

We consider the same assumptions and restrictions as in the previous section, but with a semi-infinite flat plate without thickness ($a = 0$). However, instead of a in equation (12) and equation (13), we take now L, where L is the characteristic length of the plate. In this configuration, there are no conduction phenomena.

The mathematical approach is based on the energy semi-integral equation (27) with $\Gamma = 1$, since for this configuration $t^* = \alpha_f t/(\sigma a^2)$, and using also the second-order Kármán–Pohlhausen temperature profile (equation (23)). The boundary conditions remain identical as in the previous case, while the initial conditions are

$$\theta_f = 0 \text{ for } t \leq 0$$

$$\left(\frac{\partial \theta_f}{\partial y}\right)_{y=0} = -1 \text{ for } t > 0 \tag{33}$$

The thermal boundary-layer thickness δ as deduced from equation (33), where θ_f is replaced by its expression (equation (25)), is given by

$$\delta = 2\theta_p \tag{34}$$

Substituting δ given by equation (34) into equation (27), where $\Gamma = 1$, leads to the following partial differential equation

$$\frac{4}{3}\theta_p\frac{\partial \theta_p}{\partial t} + \frac{6}{5}\theta_p^2\frac{\partial \theta_p}{\partial x} = 1 \tag{35}$$

subject to

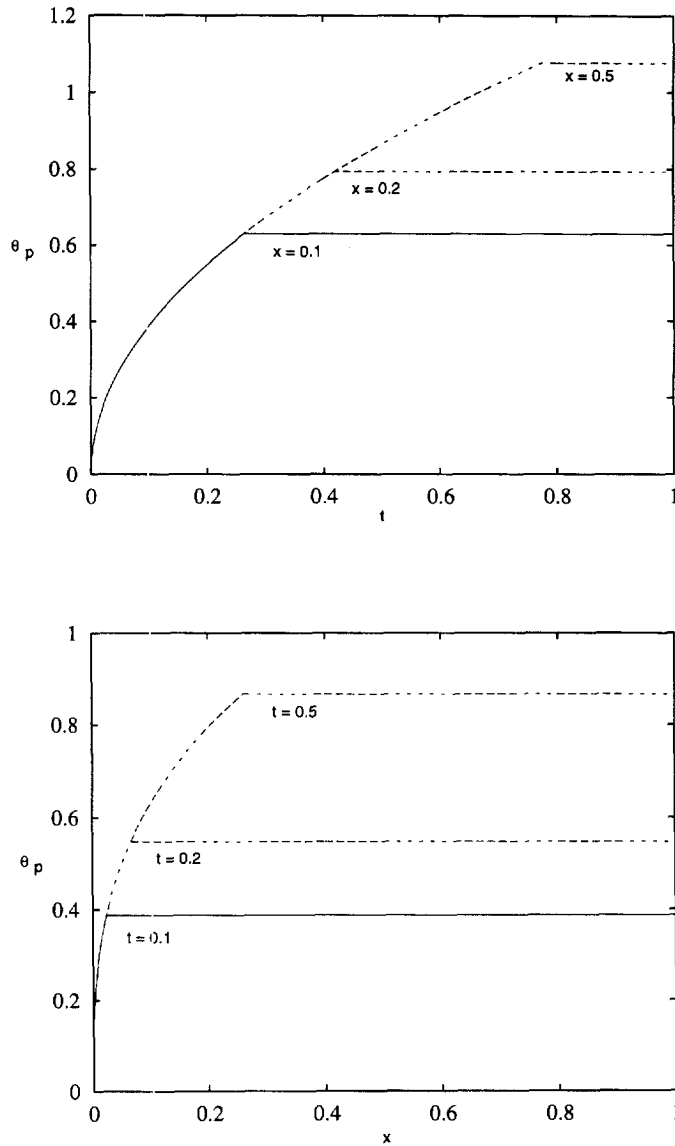$$\theta_p = 0 \text{ at } t = 0 \text{ or } x = 0 \tag{36}$$

Fig. 7. Profiles of the interface temperature for a thin ($a = 0$) flat plate.

It is noticed that equation (35) has only one characteristic curve and the wave speed now is $\frac{9\theta_p}{10}$. The temperature distribution θ_p are also shown in Fig. 6 and 7. The same behaviour of θ_p can also be seen in Figs 2–5.

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NOMENCLATURE

a thickness of the conducting plate
g acceleration due to gravity
K permeability of the porous medium
k thermal conductivity ratio
k_f effective thermal conductivity of the porous medium
k_s thermal conductivity of the plate
q_w heat flux at the plate
R_a Rayleigh number
T temperature
t time
t_{ss} time to reach the steady state
U_c characteristic velocity
u,v velocity components along (x,y) axes
x,y Cartesian coordinates

Greek symbols

α_f effective thermal diffusivity of the porous medium
α_s thermal diffusivity of the solid plate
β coefficient of thermal expansion
Γ dimensionless parameter
ΔT temperature characteristic
δ boundary-layer thickness
θ dimensionless temperature
v kinematic viscosity
σ ratio of heat capacity of the porous medium to that of fluid

Subscripts

0   steady state condition
f fluid
p interface condition
s solid
w wall condition
∞ ambient condition

Superscript

*   dimensionless variables