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# Digital in-line holography for dynamic micro-metrology

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## ABSTRACT

In this paper in-line digital holography has been explored for dynamic micro metrological applications. In in-line digital holography, full CCD sensor area is utilized for real image reconstruction of the objects with less speckle noise. Numerical evaluation of the amplitude and phase information during reconstruction process finds promising applications in optical micro-metrology. Vibration analysis of the smaller object has been performed by combining the time average principle with in-line digital holographic methods. A double exposure method has been explored for measurements, which is simultaneously used to suppress the overlapping of zero-order and twin image wave with real image wave. The vibration amplitude and mean static state deformation of the harmonically excited object are analysed separately from time average in-line digital holograms. The experimental results are presented for a thin aluminium membrane of 5mm diameter.

*Keywords:* Digital holography, in-line configuration, CCD, time average method, numerical reconstruction.

## 1. INTRODUCTION

Digital holography (DH) [1] is a rapidly growing opto-digital method for imaging, microscopy and metrology. In DH, the hologram is recorded in digital form using CCD or CMOS sensors and then numerically reconstructed. The ability of numerical evaluation of both amplitude and phase information is its main advantage over other optical imaging methods. The limited pixel size of the commercially available digital detectors restricts the angle between object and reference beams to a few degrees for hologram recording, thus in-line geometry proposed by Gabor [2] becomes the preferred choice for digital holography. Also in-line DH makes the efficient use of full sensor area for hologram recording in comparison to off-axis geometry. The main problem with in-line geometry is the overlapping of zero-order and twin image waves with real image wave during reconstruction process. Various digital methods have been proposed to suppress these unwanted waves [3,4]. The applications of in-line DH in areas such as micro-particles imaging [5], microscopy [6], 3-D imaging [7] and interferometry [8] have been reported.

DH has also been explored for dynamic metrology applications. The instantaneous dynamic deformation of any object can be measured using the pulsed digital holography [9]. The alternative approach is using the high speed CCD cameras for hologram recording for such measurements. For vibration analysis of the objects, time average holography can be used. Powell and Stetson first explained the principle of time average holographic interferometry for vibration analysis [10]. For time average recording of a vibrating object, the exposure time must be larger than the period of vibration. In this case, it can be shown that the amplitude distribution of the object is modulated by the zero order Bessel function  $J_0$ . Electronic Speckle Pattern Interferometry (ESPI) referred to as TV Holography used opto-electronic methods for real-time visualization of time average fringes [11]. These ESPI fringes rely on amplitude correlation and hence the phase information of the beam is lost. In this paper in-line digital holographic interferometry is presented for vibration analysis using time average method. The versatility of digital time average holograms to reconstruct different parameters of interest from the same hologram is demonstrated.

Time average DH to study vibrating objects was first demonstrated by P. Picart et al [12], who showed that the numerically reconstructed object is modulated by the  $J_0$  fringes as in conventional time average holography. The

various capabilities of numerical reconstruction methods of digital holography for vibration analysis have been demonstrated [13-15]. All these methods used either off axis geometry or quasi Fourier set-up and used for vibration analysis of relatively bigger objects (>30mm in dia.). More recently we have demonstrated time average in-line digital holographic interferometry for vibration analysis for smaller objects (10mm in size) [16]. A double exposure method has been established which is useful to separate the mixing of vibration and mean deformation fringes with simultaneous suppression of zero-order and twin image wave from real image wave. In this paper the vibration analysis of 5mm size membrane is further investigated by time average in-line digital holography. A diverging beam is used to illuminate the object in order to get magnified object beam without using any lens. The holograms are recorded in different conditions i.e. for different frequencies and amplitudes and results are presented.

## 2. THEORY

### 2.1 Recording of time average in-line digital holograms

Let the instantaneous displacement of a harmonically excited object placed in  $(x, y)$  plane be written as  $z(x, y, t) = z(x, y) \cos \omega t$ , where  $\omega$  is the angular frequency of vibration. Then instantaneous object wave  $O'(x, y, t)$  scattered from the vibrating object can be written as,

$$O'(x, y, t) = O_0(x, y) e^{i\phi_0(x, y)} e^{i\{\vec{K} \cdot \vec{z}(x, y, t)\}} \quad (1)$$

where  $O_0(x, y)$  is the amplitude of the scattered wave and  $\phi_0(x, y)$  is the phase due to mean static state of the vibration of the object, and  $\vec{K}$  is the sensitivity vector depending upon direction of illumination and observation of the object as defined in fig. (1).

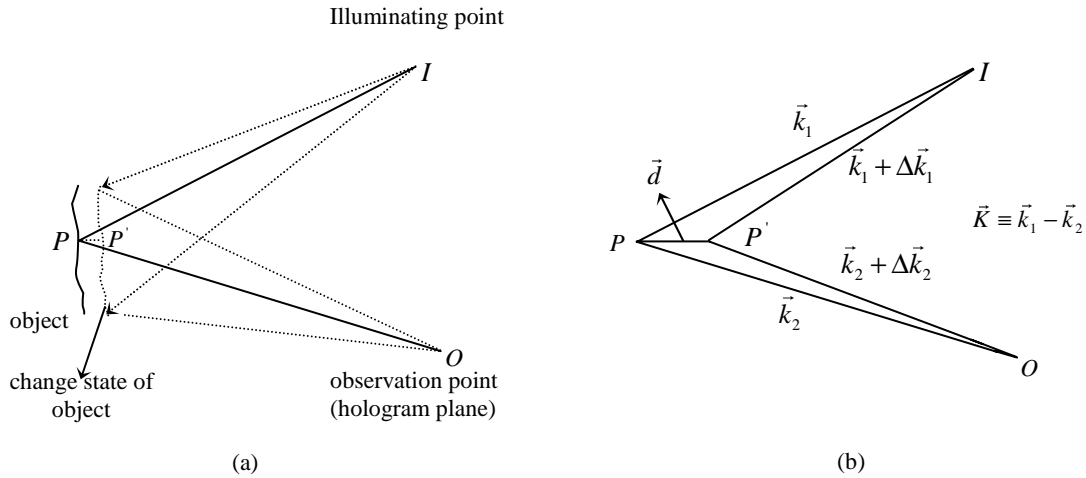


Fig. 1(a) Observation geometry and (b) Sensitivity vector representation.

As mentioned before, for time average recording of digital holograms of vibrating object, the recording time should be much larger than the period of vibration. For time average recording the object wave becomes:

$$O(x, y, t) = \frac{1}{T} \int_0^T O'(x, y, t) dt = O_0 e^{i\phi_0(x, y)} J_0\{\vec{K} \cdot \vec{z}(x, y)\} \quad (2)$$

where  $J_0$  is the zero-order Bessel function.

For the formation of time average in-line digital holograms, this object wave interferes with the in-line plane reference wave. Then the interference pattern is recorded at the hologram plane (i.e. CCD plane) say  $(\xi, \eta)$ , which can be written as:

$$H(\xi, \eta) = |O(\xi, \eta) + R(\xi, \eta)|^2 \quad (3)$$

Let  $M \times N$  be the total number of pixels of the CCD with pixel size  $\Delta\xi$  and  $\Delta\eta$ , then the digitally sampled holograms  $H(m, n)$  can be defined as the convolution of eqn (3) with each pixel of CCD sensor which is limited by the CCD sensor area [17], it can be mathematically represented as:

$$H(m, n) = [H(\xi, \eta) \otimes \text{rect}(\frac{\xi}{\alpha\Delta\xi}, \frac{\eta}{\beta\Delta\eta})] \times \text{rect}(\frac{\xi}{M\Delta\xi}, \frac{\eta}{N\Delta\eta}) \text{comb}(\frac{\xi}{\Delta\xi}, \frac{\eta}{\Delta\eta}) \quad (4)$$

Here  $\otimes$  represents the two dimensional convolution.  $\alpha$  and  $\beta$  are the fill factors of CCD. *rect* and *comb* are the rectangle and comb functions respectively.

## 2.2 Numerical Reconstruction

The coordinate system of recording and reconstruction in digital holography is shown in figure 2. The reconstruction process is performed using the diffraction theory.

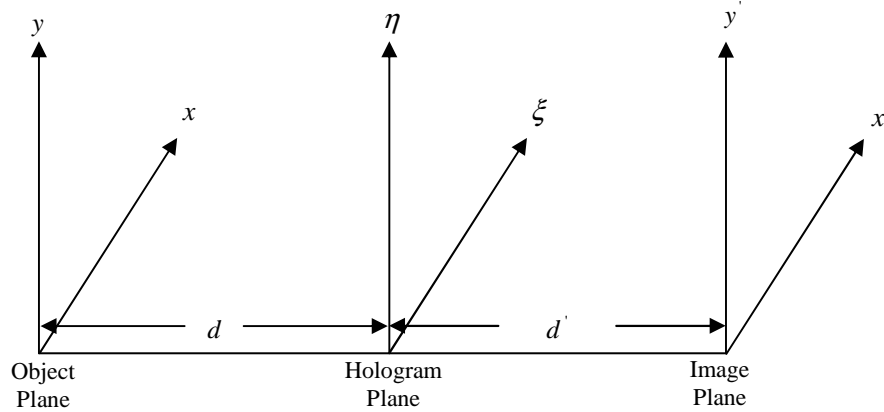


Fig. 2 Coordinate system for digital holography

The reconstructed wavefield at the image plane, placed at the distance  $d'$  from the hologram plane can be obtained by using the Fresnel diffraction integral [18]:

$$U(x', y') = \frac{e^{ikd'}}{i\lambda d'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\xi, \eta) R(\xi, \eta) \exp\left[\frac{i\pi}{\lambda d'} \{(x' - \xi)^2 + (y' - \eta)^2\}\right] d\xi d\eta \quad (5)$$

For numerical reconstruction process the above equation can be converted into discrete form and the wavefield is obtained by using the discrete Fresnel transformation. For the hologram recorded with CCD contains  $M \times N$  pixels with pixel size  $\Delta\xi$  and  $\Delta\eta$  respectively, the reconstructed field defined by eqn. (5) can be converted to finite sums as follows [1]:

$$U(k, l) = \frac{e^{ikd'}}{i\lambda d'} e^{i\pi\lambda d' \left(\frac{k^2}{M^2\Delta\xi^2} + \frac{l^2}{N^2\Delta\eta^2}\right)} \times \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} H(m, n) R(m, n) e^{\frac{i\pi}{\lambda d'} (m^2\Delta\xi^2 + n^2\Delta\eta^2)} e^{-2\pi i \left(\frac{mk}{M} + \frac{nl}{N}\right)} \quad (6)$$

The eqn. (6) is the discrete Fresnel transformation. The matrix  $U(k, l)$  is the discrete Fourier transform of the product of  $H(m, n)$ ,  $R(m, n)$  and  $\exp\{\frac{i\pi}{\lambda d'}(m^2 \Delta \xi^2 + n^2 \Delta \eta^2)\}$ . Thus calculation of reconstructed wavefield can be done effectively by using the fast Fourier transform (FFT) algorithm. For such reconstruction, the pixel size of the numerically reconstructed image varies with the reconstruction distance, wavelength and CCD array size, given as:

$$\Delta x' = \frac{\lambda d'}{M \Delta \xi}, \text{ and } \Delta y' = \frac{\lambda d'}{N \Delta \eta} \quad (7)$$

The real image wave is reconstructed when the numerically defined reconstruction wave is an exact replica of the complex conjugate of the reference wave that was used during recording process. The numerically reconstructed real image wave is convolved by a filtering function of the discrete Fresnel transform and becomes spatially enlarged. Hence, it is not strictly same as the initial object wave during recording. However, its fundamental form remains same, the reconstructed wave at the image plane can be written as,

$$U_{real}(x', y') = O_0(x', y') e^{i\phi_0(x', y')} J_0\{\vec{K} \cdot \vec{z}(x', y')\} \quad (8)$$

The intensity  $I$  and the phase  $\phi_0$  at the reconstructed image plane can be calculated from the reconstructed wavefield as follows,

$$I = |U_{real}(x', y')|^2 = O_0^2(x', y') J_0^2\{\vec{K} \cdot \vec{z}(x', y')\} \quad (9)$$

$$\phi_0(x', y') = \arctan \left[ \frac{\text{Im}\{U_{real}(x', y')\}}{\text{Re}\{U_{real}(x', y')\}} \right] \quad (10)$$

Thus the reconstructed intensity shows the familiar  $J_0^2$  vibration fringes and more importantly, the reconstructed phase contains information about the mean static state of the object.

But because of in-line geometry, the zero-order wave and the twin image overlap with this real image wave. This results as a background noise in the final reconstructed real image of the object. In order to remove this unwanted noise a double exposure method can be used. In this double exposure approach first we record a reference hologram and then its numerically reconstructed amplitude and phase information are subtracted from amplitude and phase information of the desired time average holograms respectively. Since zero-order wave and out-of-focus twin image wave does not change for the same in-line geometry in different deformation states of the same object. Thus the unwanted noise by zero-order and twin image wave is automatically suppressed during this subtraction process. Eventually the amplitude subtracted pattern used for the study of vibration amplitude information and phase subtraction provides the information about the mean static deformation of the object during vibration cycle. Since the reconstructed wavefields from the double exposure holograms contains both amplitude and phase information of the object wave so their subtraction provides a kind of mixing of fringes representing both vibration and mean deformations of the object. Fig (3) summarizes the reconstruction methodology for the double exposure method used for vibration analysis of the object.

### 3. EXPERIMENT

Experiment has been performed with a thin circular aluminium membrane with 5mm in dia. The membrane is stuck on a metal ring as shown in Fig. 4(a). This membrane is harmonically excited with the help of earphone diaphragm which is placed near to it and connected to a frequency generator. The frequency generator can be tuned to different frequencies and amplitudes and thus provides different harmonic excitations to the membrane.

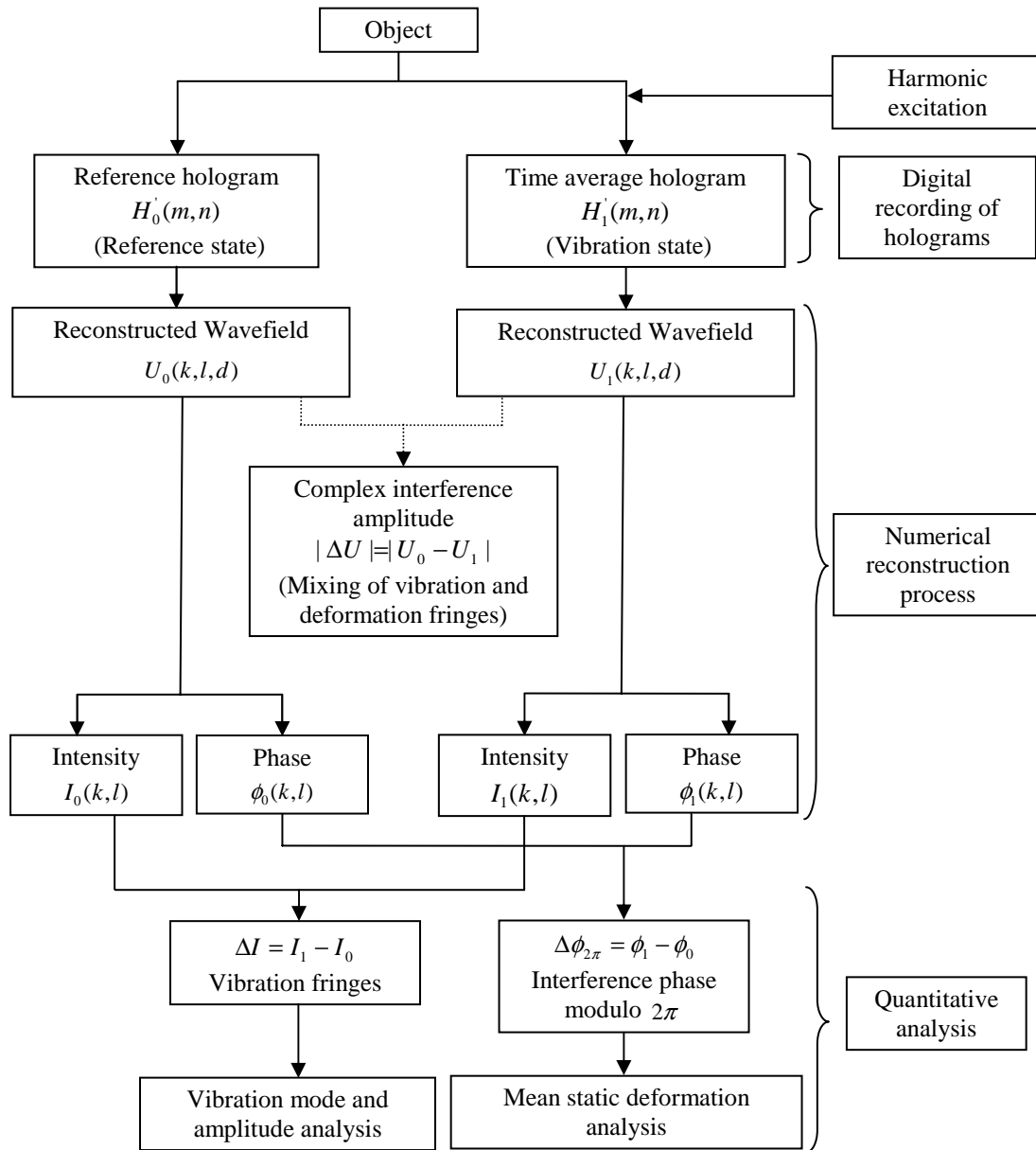


Fig. 3 Reconstruction methodology

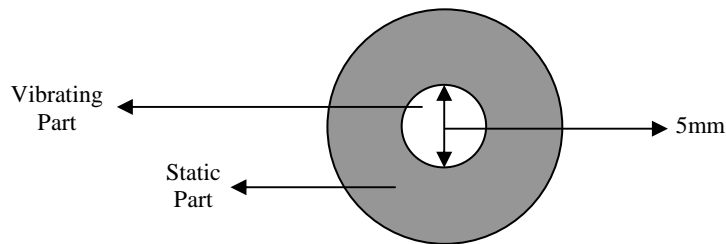


Fig. 4 (a) Object

The experimental arrangement of time average in-line digital holographic interferometry is shown in Fig. 4(b). A beam of frequency doubled Nd-YAG laser (532nm) is attached with a fiber Y-coupler which divides equally it into two parts. The diverging beam coming out from one fiber directly illuminates the vibrating object, thereby magnifying the object wave which interferes with the collimated reference wave coming out from the other fiber. The illumination of the object with a diverging beam is useful to get magnification of the recorded hologram without using any lens. This geometry is more useful for in-line digital holography because it reduces the effect of twin image wave in real image wave during reconstruction process. The twin image wave creates a poor background for such type of geometry which is significantly small in comparison to real image wave. The interference of the object and reference beam is recorded by a CCD camera called the time average digital hologram. The CCD is a Kodak mega-plus camera which contains 2024x2044 square pixels with size  $9\text{ }\mu\text{m}$  and records the holograms with 30 frames per second. In order to satisfy the good sampling requirement of digital recording of the holograms, the object is placed at a distance 350mm.

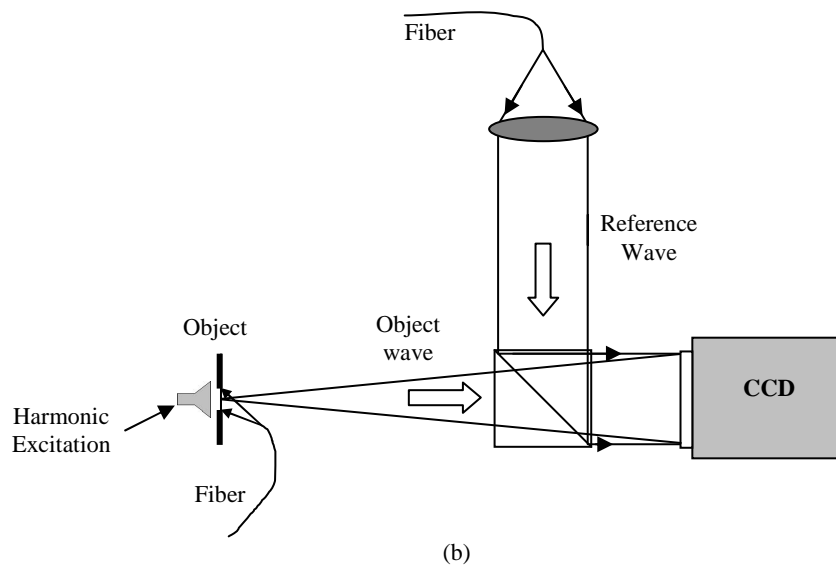


Fig. 4 (b) Experimental set-up of in-line digital holography

#### 4. RESULTS AND DISCUSSION

The vibration behaviour of the membrane is studied with time average in-line digital holographic interferometry. Fig.5 shows the reconstructed intensity from the time average holograms recorded at the resonant frequency 2.6 kHz with increasing amplitudes. The excitation amplitude is increased from 5.5 Volts (Fig. 5(a)) to 9.5 Volts (Fig. 5(i)) in the steps of 0.5 volts. As discussed previously the reconstructed amplitude from time average holograms is modulated by the zero order Bessel function which provides the vibrating amplitude information of the object. For the suppression of the effect of zero order and twin image waves, pre-processing of the holograms are required with one approach is simply subtracting the intensities of object and reference beams from the digitally recorded holograms before the reconstruction process. However for the double exposure method (Fig. 3) there is no such pre-processing of holograms is required. It can be clearly observed from Fig. (5), as excitation voltage increases the number of Bessel fringes also increases which represents the vibration amplitude that can be calculated using eqn. (9).

If the mean static state of the membrane changes with change in vibration amplitudes, then it can be reconstructed with the help of phase information. This mean static deformation occur either by providing the offset voltage by frequency generator or can also be possible due the non-linear mechanical effects between the changes in the excitation harmonic amplitudes. It has been shown previously that the reconstructed phase information from time average hologram also contains information called zero crossing phase along with the mean static deformation, which can be used for measurement of zero order Bessel fringes in case of pure sinusoidal vibrations [14]. We have observed the change in the

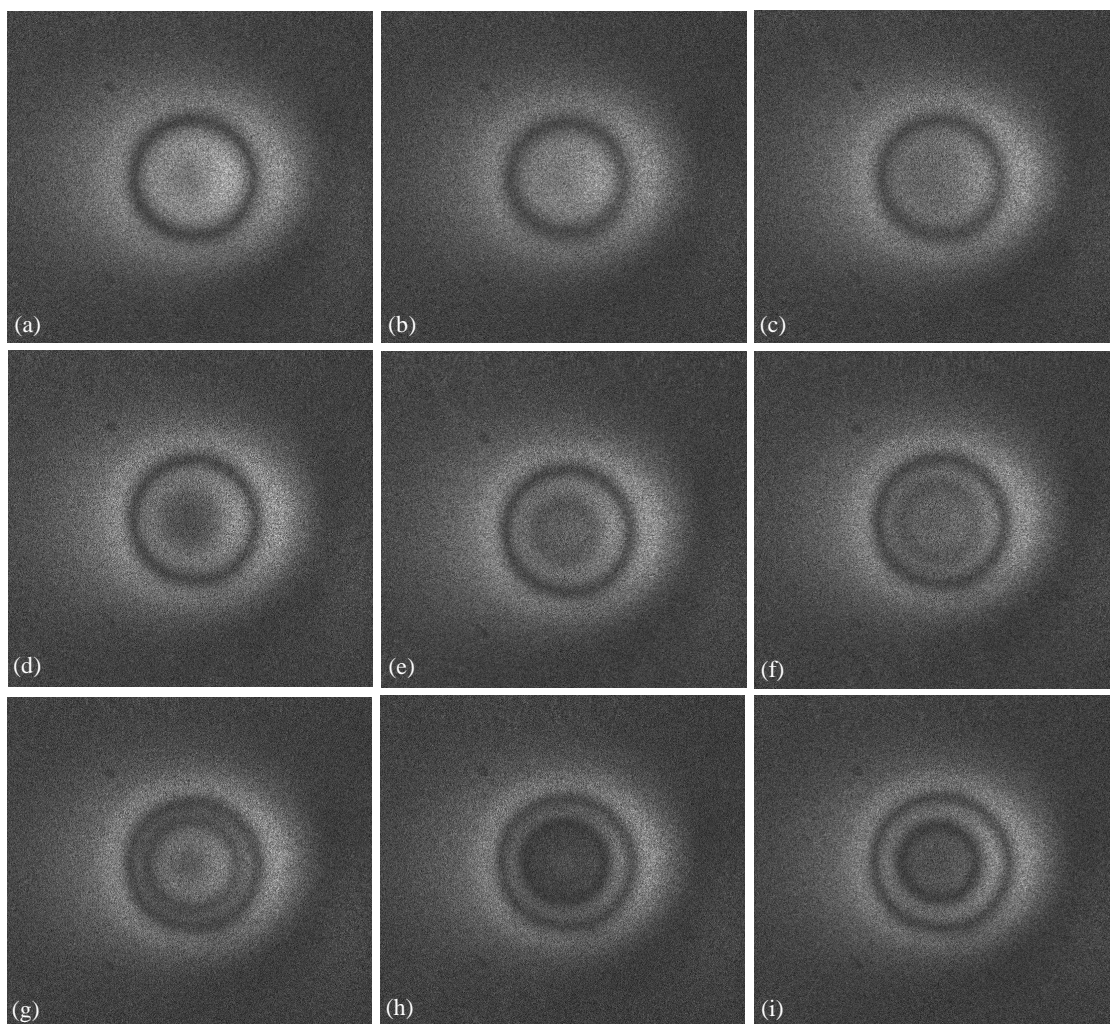


Fig. 5 Vibration behaviour of the membrane at excitation frequency 2.6 kHz with increasing amplitudes from 5.5 Volts to 9.5 volts

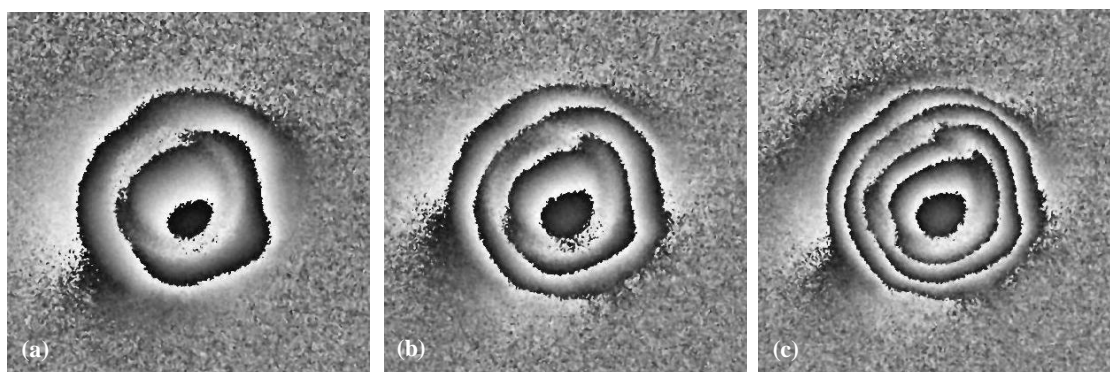


Fig. 6 Phase subtraction shows the mean static deformation of the membrane during the vibration cycle



mean static state of the membrane with change in the excitation voltages. Fig. 6 shows the mean static deformation fringes of the same membrane which are obtained by subtracting the reconstructed phase information from two time average holograms recorded at the same frequency but different amplitudes. Fig 6(a) is the subtraction of phase from the holograms recorded at frequency 2.6 kHz at the excitation voltages 5 volts and 7.5 volts. The increase in the amplitude difference is given in Fig. 6(b) & (c) which corresponding to voltages 8 & 8.5 volts respectively and subtracted from 5 volts. These fringes are non symmetric across the centre which may be attributed to the non-linear mechanical effect that occurs in the membrane during different harmonic excitations.

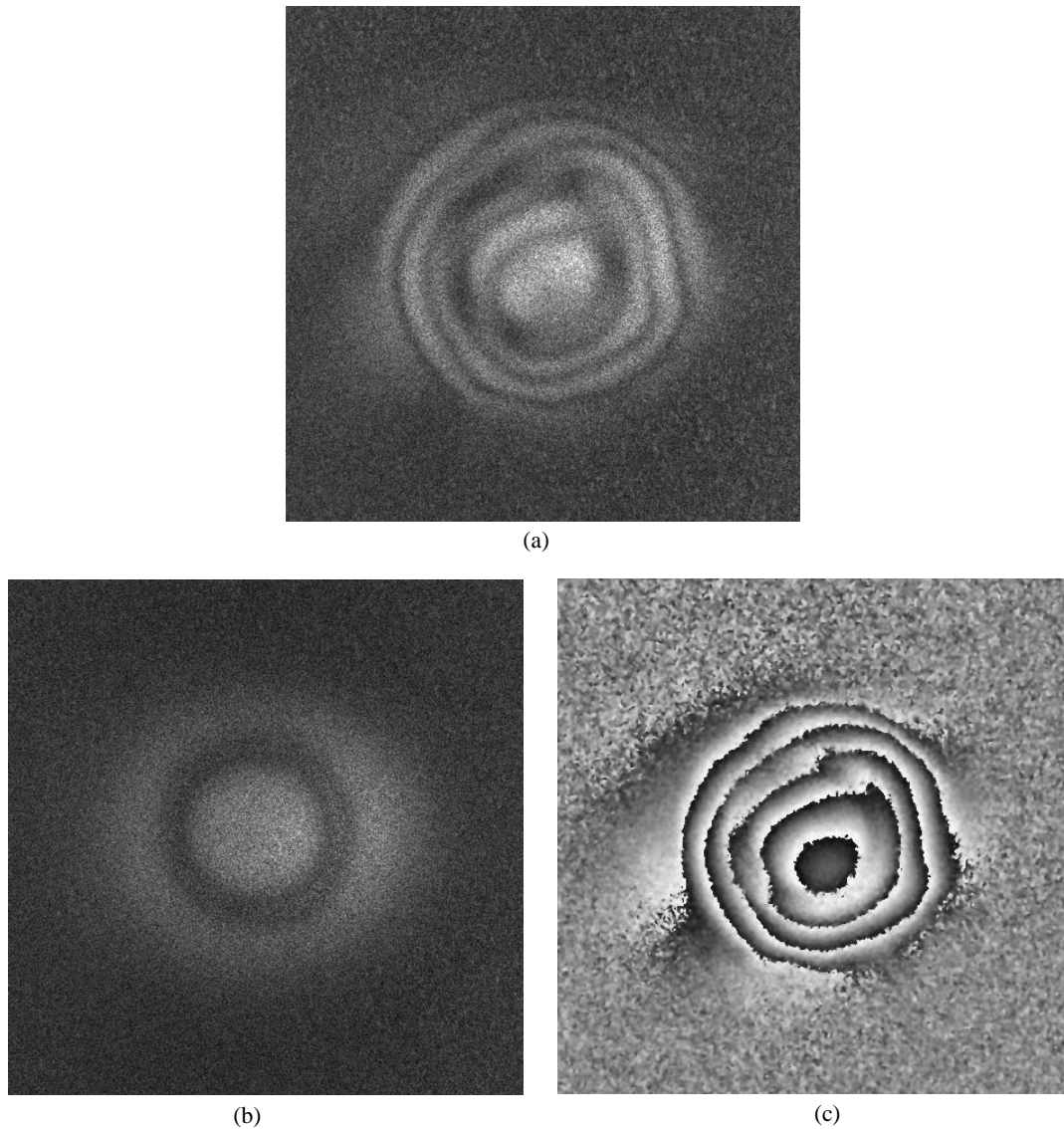


Fig. 7 (a) Mixing of vibration and means deformation fringes, (b) Amplitude subtraction represents the vibration fringes, and (c) Phase subtraction shown the mean deformation fringes.

Since the membrane contains both vibration and mean deformation amplitudes which can be reconstructed from the reconstructed wavefields. Thus the direct subtraction of the wavefields results as a complex pattern, which shows the mixing of fringes representing both vibration and mean deformation fringes. This is clearly shown in Fig. 7(a), which shows the subtraction of reconstructed wavefields of two time average holograms recorded at 5 and 8.5 volts at the vibration frequency 2.6 kHz. This mixing of the fringes can be separated by calculating the amplitude and phase of the

wavefields and then subtract them. The subtraction of amplitudes represents vibration fringes and is shown in Fig. 7(b), while and phase subtraction gives deformation fringes (Fig. 7(c)).

## 5. CONCLUSION

In this paper we have presented time average in-line digital holographic interferometry for the study of vibration analysis of a 5mm size membrane. A double exposure method has been used to suppress the zero-order and twin image waves from real image waves, caused by in-line geometry. The numerical reconstruction of amplitude and phase information from time average in-line digital holograms is used for vibration and mean deformation amplitude analysis. The numerical reconstruction of the phase information is the additional advantage with digital holography in comparison to other existing methods like electronic speckle pattern interferometry (ESPI). Using this approach it is envisaged that this technique can be used for dynamic characterization of micro devices, and can be compatible with the recent challenges in dynamic optical metrology.

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