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Reliability Assessment of Ultimate and Serviceability Limit States of Underground Rock Caverns

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ABSTRACT: Both the stress-induced and structurally controlled instabilities are investigated by means of the Distinct Element program UDEC. The Factor of Safety is used as the criterion for the Ultimate Limit State and the volume of rock mass displaced into the cavern was adopted as the Serviceability Limit State criterion. Numerical experimentations are performed in accordance with the methodology of 2^k factorial design, from which two polynomial regression models are developed for reliability analysis. The First Order Reliability Method (FORM) was used to determine the probability of failure for the limit states.

1 INTRODUCTIONS

One of the major considerations in the design of an underground rock cavern is the evaluation of its stability since the excavation causes a redistribution of the stresses in the proximity of the underground opening. Common numerical methods used to evaluate cavern stability can be categorized as continuum methods such as the Finite Element Method (Meguid & Rowe 2006, Xia et al. 2007) and Finite Difference Method (Roth et al. 2001, Wang & Zhu 2006), and discontinuum methods such as the Distinct Element Method (Cundall 1976) and the Discontinuous Deformation Analysis (Shi 1988). There are no universal quantitative guidelines to determine when one method should be used instead of the other (Bobet et al. 2009).

Conventional deterministic evaluation of stability of geotechnical structures and underground openings involves the calculation of Factor of Safety (FS), the use of which can neither predict the state of system with absolute certainty nor meet the serviceability limit design requirement. For serviceability limit considerations, with regard to the continuous methods, usually a critical location (e.g. cavern roof) and a critical displacement is identified which must not be exceeded. However, for discontinuous numerical methods, no one really knows where the largest displacement will happen around the opening beforehand.

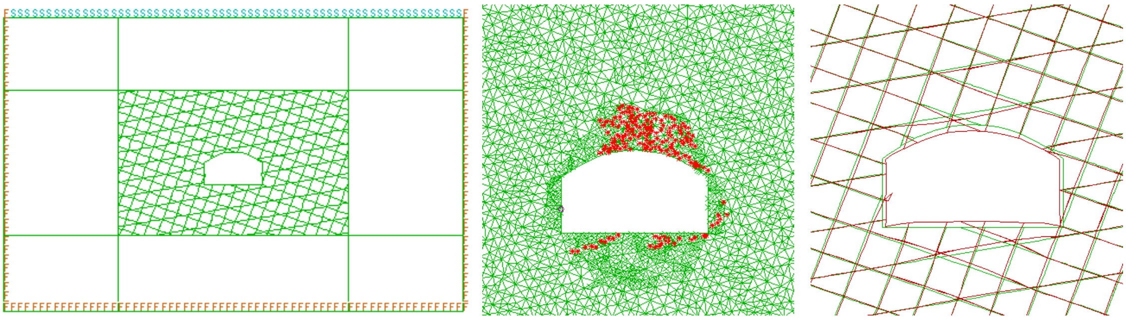
Since underground rock caverns are usually built in a complicated geological environment and may be subjected to different loading conditions, it is hard to define what a satisfactory performance is. The alternative is to assess the reliability index β or the probability of 'failure' P_f of the ultimate and serviceability limit state functions. This involves the determination of the joint probability distribution of resistance R and load S and the integration of the Probability Density Function (PDF) over the failure domain. Typical β and P_f for representative geotechnical components and systems and their expected performance levels have been proposed (USA Army Corps of Engineers, 1997).

This paper presents a new and practical approach to underground rock cavern stability reliability analysis that meets the ultimate and serviceability limit state design requirements. Both the stress-induced and structurally controlled instabilities are investigated by means of the Distinct Element program UDEC. FS is used as the criterion for the Ultimate Limit State and the displaced volume is adopted as the Serviceability Limit State criterion. The following stochastic variables are considered: the in-situ stress ratio, the elastic modulus of the rock mass, the cohesion strength and the friction angle of the rock. Numerical experimentations are performed in accordance with the methodology of 2^k factorial design (Sivakumar & Singh 2010), from which two polynomial regression models are developed for reliability analysis. The displaced volume can be related to the yielding zone volume, from which the threshold value can be determined. The First Order Reliability Method (FORM) is used to determine the probability of failure for the limit states.

2 NUMERICAL METHOD FOR ROCK CAVERN

The Universal Distinct Element Code (UDEC) was adopted to carry out the stability analyses using the Shear Strength Reduction (SSR) technique to solve for FS (Griffiths & Lane 1999). In conventional SSR analysis, FS is obtained by systematically reducing (or increasing) the shear strengths of soil or rock materials until stress-induced failure occurs. It has been demonstrated that the SSR method works for a wide range of problems, including stability problems in blocky rock masses (Hammah et al. 2007a) and cavern problems (Hammah et al. 2007b).

Both stress-controlled and structure-controlled instabilities were considered. For simplicity, three sets of joints angled 70°, -20°and 10° respectively, were considered as the only structural features in the model. The cross-section of the underground cavern configuration and the boundary fixity conditions are shown in Figure 1a. The cavern roof arc is elliptical and the cavern dimension is deterministic with the height of roof arc 6 m, wall height 12 m and cavern span 30 m. The cross section area of the cavern is 479.114 m^2. The pressure from the 30 m thick surface residual soil is 0.549MPa. The overburden height D is fixed at D=80m. Full-face excavation was considered. Table 1 lists the values and distributions of input variables (deterministic or probabilistic) considered (D for deterministic). Figure 1b and 1c show a typical plot of the plastic state and the magnified deformation of the model respectively.



a. Cavern configuration and boundary fixity b. Plastic zone state c. Magnified deformation

Figure 1. Numerical model and calculation results.

Table 1. Values and distributions of input variables.

Variables	*Statistics*	*Variables*	*Statistics*
Intact rock properties		*Joint properties*	
Unit weight γ (kN/m^3)	D, 26.5	Orient. of joint sets 1/2/3 (°)	D, 70 / -20 / 10
Young's modulus E (GPa)	Normal, (40,8)	Spacing of joint sets 1/2 (m)	D, 6 / 6 / 8
Poisson's ratio υ	D, 0.24	Normal/Shear stiffness k_n / k_s (GPa/m)	D, 10 / 10
Cohesion *c* (MPa)	Normal, (0.6, 0.12)	Cohesion c_j (MPa)	D, 0.258
Friction angle ϕ (°)	Normal, (35, 4.2)	Friction angle ϕ_j (°)	D, 45.6
Tensile strength σ_t (MPa)	D, 7	*Other inputs*	*Statistics*
Dilation angle φ (°)	D, 0	In situ stress ratio k_0	Normal, (2.5, 0.3)

For each random variable, two design combinations were considered and denoted by the “+” and “-” notations to represent the high and low values of each uncertain input variable. A high value is $x_h=\mu+1.645\sigma$ ($\sigma=\mu\times Cov$) where μ is the mean value and a low value is $x_l=\mu-1.645\sigma$. Table 2 summarizes four design factors considered. Factor combinations for these 4 uncertain input variables are assumed as in the order of in situ stress ratio k_0, Young’s modulus E, cohesion *c* and friction angle ϕ.

Standard notations are followed to provide clarity with regard to the various terms involved in factorial designs. Therefore, in the present study, in situ stress ratio k_0, Young's modulus E, cohesion c and friction angle ϕ are represented as the A, B, C and D respectively. 16 design runs for the 2^4 design using the "+" and "-" notation to represent the low and high levels of the factors are shown in Table 3. In Table 3 column 'Run label' indicates the standard order of sixteen experimental run labels for different factor combinations as (1) *a, b, c, d, ab, ac, ad, bc, bd, cd, abc, abd, acd, bcd* and *abcd*. Numerical experiments are performed for each design combination and the observations of the response quantities, namely FS and the displaced volume into the opening are tabulated in Table 3. For each numerical simulation, FS is solved using SSR technique and the displaced volume V together with the observed plastic zone volume are calculated using the self-compiled FISH language of UDEC.

Table 2. Input parameters for the 2^4 factorial design.

Design notation	Mean	COV	Standard deviation	Low level value	High level value
A	2.5	12%	0.3	2.0	3.0
B	40	20%	8	26.8	53.2
C	0.6	20%	0.12	0.4	0.8
D	35	12%	4.2	28.1	41.9

Table 3. Numerical experimentation results.

Run No.	A	B	C	D	Run label	FS	Plastic zone volume (m^3)	Displaced Volume (m^3)
1	-	-	-	-	(1)	1.64	698.2	0.668
2	+	-	-	-	a	1.61	1323.9	1.257
3	-	+	-	-	b	1.63	706.2	0.420
4	-	-	+	-	c	2.65	396.6	0.409
5	-	-	-	+	d	1.96	200.9	0.402
6	+	+	-	-	ab	1.60	1591.4	0.893
7	+	-	+	-	ac	2.62	808.8	0.778
8	+	-	-	+	ad	1.93	337.6	0.817
9	-	+	+	-	bc	2.61	370.8	0.255
10	-	+	-	+	bd	1.96	149.6	0.281
11	-	-	+	+	cd	3.06	64.3	0.279
12	+	+	+	-	abc	2.61	744.9	0.473
13	+	+	-	+	abd	1.93	251.0	0.629
14	+	-	+	+	acd	3.02	205.1	0.478
15	-	+	+	+	bcd	3.06	88.1	0.176
16	+	+	+	+	abcd	3.01	112.8	0.349

3 METHODS TO OBTAIN THE REGRESSION MODELS FOR FS AND V

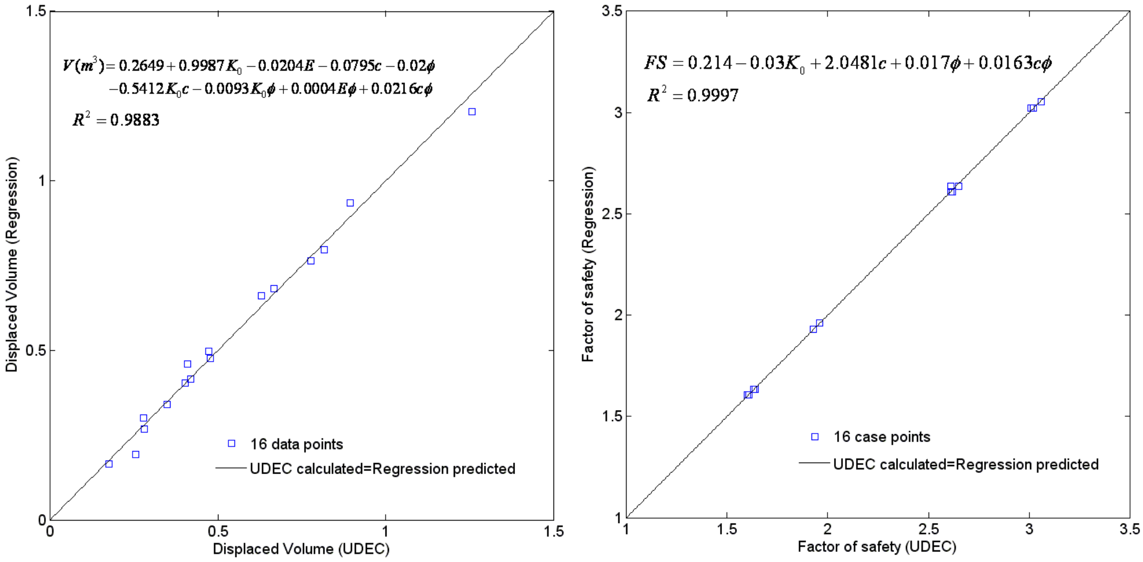
Using 2^4 factorial design method, factor effect estimates and percent contribution values (Sivakumar & Singh 2010) for both regression models are summarized in Table 4.

It can be seen that the main factors/interaction factors of FS include *A, C, D* and *CD*, representing K_0, c, ϕ and cϕ terms accordingly while those for V include *A, B, C, D, AC, AD, BD* and *CD*, representing K_0, E, c, ϕ, K_0c, $K_0\phi$, Eϕ and cϕ respectively. Polynomial regressions are used to relate the main factors/interaction factors to the expression of the performance functions of Factor of safety and displaced volume, as shown in Figure 2.

Table 4. Factor effect estimates and percent contribution.

Model term	Design factor	Displaced volume model			FS model		
		Effect estimate	Sum of squares	Percent contribution	Effect estimate	Sum of squares	Percent contribution
A	K_0	0.696	0.969	*39.1*	-0.060	0.007	*0.1*
B	E	-0.403	0.325	*13.1*	-0.020	0.001	0.0
C	c	-0.543	0.589	*23.7*	2.095	8.778	*88.7*
D	ϕ	-0.436	0.379	*15.3*	0.740	1.095	*11.1*
AB	K_0E	-0.090	0.016	0.7	0.005	0.000	0.0
AC	K_0c	-0.217	0.094	*3.8*	0.000	0.000	0.0
AD	$K_0\phi$	-0.129	0.033	*1.3*	-0.015	0.000	0.0
BC	Ec	0.058	0.007	0.3	-0.010	0.000	0.0
BD	Eϕ	0.133	0.035	*1.4*	0.015	0.000	0.0
CD	cϕ	0.119	0.028	*1.1*	0.090	0.016	*0.2*

ABC	K_0Ec	0.002	0.000	0.0	0.005	0.000	0.0
ABD	$K_0E\phi$	0.044	0.004	0.2	-0.010	0.000	0.0
ACD	$K_0c\phi$	0.021	0.001	0.0	-0.015	0.000	0.0
BCD	$Ec\phi$	-0.019	0.001	0.0	0.005	0.000	0.0
ABCD	$K_0Ec\phi$	0.019	0.001	0.0	-0.010	0.000	0.0



a. Factor of Safety b. Displaced Volume

Figure 2. Polynomial expressions of random inputs for FS and Displaced Volume.

4 PROBABILISTIC ASSESSMENT ON THE LIMIT STATES

The reliability index β and the probability of failure P_f for both the ultimate and the serviceability limit states can be calculated using the FORM Spreadsheet method, as shown in Figure 3.

Distribution	Parameters	Para1	Para2	Para3	Para4	x_i^*	Correlation matrix [R]				n_i
Normal	In situ stress ratio K_0	2.5	0.3			2.49831	1	0	0	0	-0.006
Normal	Deformation modulus E (GPa)	40	8			40	0	1	0	0	-5E-08
Normal	Cohesion force c (MPa)	0.6	0.12			0.64604	0	0	1	0	0.3836
Normal	Friction angle ϕ (°)	35	4.2			35.0183	0	0	0	1	0.0044

$$g(\underline{x}) = FS - 1 = 0.214 - 0.03K_0 + 2.0481c + 0.017\phi + 0.0163c\phi - 1$$

g(x)	β	P_f (%)
-2E-10	0.3837	35.06

a. Calculation on β and (or) P_f for Ultimate limit state

Distribution	Parameters	Para1	Para2	Para3	Para4	x_i^*	Correlation matrix [R]				n_i
Normal	In situ stress ratio K_0	2.5	0.3			2.57608	1	0	0	0	0.2536
Normal	Deformation modulus E (GPa)	40	8			39.9597	0	1	0	0	-0.005
Normal	Cohesion force c (MPa)	0.6	0.12			0.59358	0	0	1	0	-0.054
Normal	Friction angle ϕ (°)	35	4.2			34.9756	0	0	0	1	-0.006

g(x)	β	P_f (%)
-7E-10	0.2593	39.77

$$g(\underline{x}) = V_{all} - V = V_{all} - (0.2649 + 0.9987K_0 - 0.0204E - 0.0795c - 0.02\phi - 0.5412K_0c - 0.0093K_0\phi + 0.0004E\phi + 0.0216c\phi)$$

b. Calculation on β and (or) P_f for Serviceability limit state

Figure 3. Calculation on β and (or) P_f using FORM Spreadsheet.

In Figure 3b, the critical displaced volume V_{all} is derived by relating the allowable displaced volume to the ratio of plastic zone volume to the cavern cross section area S_{cavern}. The Polynomial regression for relating displaced volume V_{disp} to plastic zone volume V_{plast} is shown below.

$$V_{disp} = 0.2709 + 0.2706\left(\frac{V_{plast}}{S_{cavern}}\right) - 0.0103\left(\frac{V_{plast}}{S_{cavern}}\right)^2 \qquad (1)$$

Further, the influence on the probability of failure from the proper choice of V_{plas}/S_{cavern} value is investigated in Table 5 below. It is logic that smaller threshold value be exceeded easily, resulting in a relatively higher probability of failure.

Table 5. Influence on the P_f from the proper choice of V_{plas}/S_{cavern}.

V_{plas}/S_{cavern}	Allowable displaced volume V_{all} (m^3)	P_f for serviceability limit state (%)
1	0.53	39.77
2	0.77	31.55
3	0.99	25.15

For the ultimate limit state assessment as performed above, we assume the critical FS value is 1.0. Considering the fact that the deterministic global factors of safety provide a hedge against uncertainties in calculation and that it is never possible to compute with perfect accuracy, through experience, conventions have developed with regard to what FS values are suitable for various geotechnical structures and design situations. However, there are limited guidelines on the choice of a proper FS value for underground rock cavern.

In underground rock cavern design, it is required that both ultimate limit state and serviceability limit state should be met simultaneously for the satisfactory performance of a whole structure. The reliability assessment is performed on the serviceability of displaced volume with the global factor of safety FS set as constraints which must be satisfied during the search for the design point. Reliability indices β corresponding to different mean FS and different allowable displaced volume V_{all} can then be calculated. To illustrate the relationship between the mean FS, β of serviceability and V_{all}, Figure 4 was compiled. It can be seen that a target β = 3.0 (Target performance level: Above average) is not achievable until a mean FS=3.4 or so is required for V_{all} of 0.53 m^3. For an even larger V_{all} of 0.77 m^3, an above average performance level is achievable for a mean FS not smaller than 3.0. The same performance level can also be achieved for V_{all} of 0.99 m^3 with a mean FS larger than 2.75. It can be concluded that to achieve the same performance level, for the case in which V_{all} is smaller, a higher value of mean FS is needed.

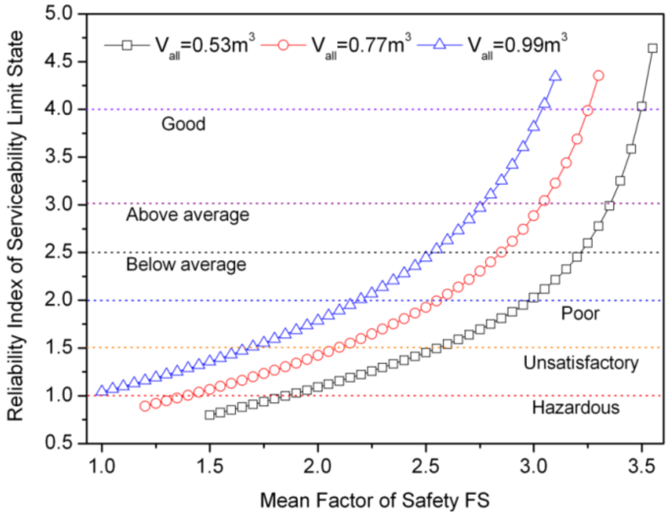


Figure 4. Relationship between reliability index and mean factor of safety for underground rock caverns.

5 SUMMARY AND CONCLUSIONS

It is proposed that to adopt the displaced volume of rock mass into an excavation to determine the serviceability limit of an underground excavation. Calculations of P_f on the serviceability limit state shows that the probability of failure is influenced by the proper choice of allowable displaced volume value. It is logic that smaller threshold value of FS or V_{all} be exceeded easily, resulting in a relatively higher probability of failure. The reliability index and the probability of failure for both the ultimate and the serviceability limit states can be calculated using the FORM Spreadsheet method.

The system reliability state is defined as such a state in which both the ultimate limit state and serviceability limit state should be met as a system for the satisfactory performance of a whole structure. The reliability assessment is performed on the serviceability of the displaced volume together with the global factor of safety is set as constraints which must be satisfied during the search for the design point. Reliability indices corresponding to different mean FS and different V_{all} can be calculated based on the FORM Spreadsheet method. The conclusion is that to achieve the same performance level, for the case in which the allowable displaced volume is smaller, a higher value of mean FS is needed.

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