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Evaluation of Shear Strength Design Methodologies for Slender Shear-Critical RC Beams

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Abstract:

This paper seeks to examine the concrete contribution to shear strength, and determine the inclination of the compressive strut within the variable truss model for slender reinforced concrete (RC) shear-critical beams with stirrups. Utilizing the Modified Compression Field Theory (MCFT) in place of the conventional statistical regression of experimental data, the expression for the concrete contribution to shear strength was derived and the inclination of compressive struts determined. A simplified explicit expression for shear strength was then provided, with which shear strength can be calculated without extensive iterative computations. This method was then verified using the available experimental data of 209 RC rectangular beams with stirrups and compared to the current methods in ACI 318R-08 and CSA-04. The theoretical results are shown to be consistent with the experimentally observed behavior of shear-critical RC beams.

CE Database subject headings: Shear strength; concrete contribution to shear strength; inclination of strut; modified compression field theory; evaluation.

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Introduction

While the flexural behavior of RC beams is generally well understood, the explanation of shear mechanisms is relatively inadequate. Over the last century, many researchers have managed to develop semi-empirical theories based on extensive experimental data (ACI-ASCE Committee 426 1973; ASCE-ACI Committee 445 1998). Representative models include the limit equilibrium theory, the truss model, the strut and tie model, the plastic theory, the shear friction theory, *etc.* However, given the complexity of shear failure mechanisms, none of the aforementioned theories can offer a complete explanation and as such, there has been no unanimously accepted theory. Recent years have seen renewed efforts to develop a theoretical model that is verified by experimental data.

Many truss analogy models such as the traditional 45° truss model, constant or variable angle truss model, and MCFT (Vecchio and Collins 1986) are widely used as the basis of most shear design methodologies for RC beams. The general methods in AASHTO LRFD-04 (2004) and CSA-04 (2004) are both based on MCFT. Using the method in AASHTO LRFD-04, for beams with stirrups, the two factors, β and θ , need to be looked up in the data charts. On the other hand, in CSA-04, it is necessary to determine the longitudinal strain at the mid-depth of the member using extensive iterative computations and a rough gauge of its initial value. While the proposed approach in this paper is based on MCFT, rendering it unnecessary for iterative calculations or reference to data tables. The results of the proposed approach are verified using the experimental data of 209 RC beams with stirrups and compared with the results obtained through the methods mentioned in ACI 318R-08 and CSA-04.

Shear Strength for Slender Shear-Critical RC Beams

It is worthwhile to note that for the beams with a small λ or the deep beams, the hypothesis that plane sections remain plane is not satisfied, and parts of the shear are directly transmitted to the supports by arch action. If the sectional shear design method is utilized, the results may be conservative without consideration of arch action. For RC beams with stirrups, when $\lambda \geq 2.5$, the arch action could be

considered small (ASCE-ACI Committee 445 1998). In this paper, the present approach for shear strength based on MCFT is aimed mainly at the slender beams, which means λ of the beam is equal to or more than 2.5, because the most practical RC beams are slender, with λ ranging from approximately 2.5 to 6 (Kassian 1990, Li and Tran 2008, 2012).

Formulas for shear strength in many codes for RC beams take into account the contribution of concrete, V_c and the contribution of stirrups, V_s . The MCFT has made an attempt to simplify the transmitting mechanism of concrete using average stresses, average strains, and local variations (Collins and Mitchell 1991). In the theory, the cracked concrete beam must be capable of resisting the effects of the shear, or the beam will fail before the breakdown of the aggregate interlock mechanism, in order to develop the capacity of a rough and interlocked crack interface for shear transfer. Derived by Collins *et al.* (Collins and Mitchell 1991), the contribution of concrete to shear is:

$$V_c = \min \left(\frac{0.18\sqrt{f'_c}bd_v}{0.31 + \frac{24\varepsilon_1}{(a+16)\left(\frac{\sin\theta}{s} + \frac{\cos\theta}{s_x}\right)}}, \frac{0.33\alpha_1\alpha_2bd_v\sqrt{f'_c}\cot\theta}{1 + \sqrt{500\varepsilon_1}} \right) \quad (1)$$

From Eq. (1), it can be seen that there are two unknowns to calculate the shear strength, namely the crack angle θ and the principal tensile strain ε_1 .

Determination of Crack Angle, θ

There are three types of shear failure modes for RC beams with stirrups, namely crushing of concrete strut (due to the dominant arch action), shear compression, and diagonal tension. This paper is based on the premise that the stirrups yield when shear failure occurs in the slender RC beams, and the advantages of this assumption are threefold. Firstly, when a slender beam fails in the mode of diagonal tension, the shear force at stirrups yielding is approximately equal to the actual shear strength. Secondly, when a slender beam fails in the mode of shear compression, after stirrups yielding, failure occurs by concrete crushing above the crack. The shear force at stirrups yielding, which is a little lower than the actual

shear strength, is taken as the calculated shear strength, which is a little conservative for design but does not sacrifice accuracy. Lastly, there are calculation methods for the flexural crack width in the codes to control the flexural crack width. However, there are no methods in the codes to control the shear crack width. If the peak compressive strain is taken as the identification of shear failure, the yielding of stirrups will induce a larger crack width, while the shear crack width can be controlled if the yielding of stirrups is considered as the identification of shear failure for a slender shear-critical beam. From Mohr's circle of strain of the web, the following equation can be obtained (Collins and Mitchell 1991):

$$\tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_z - \varepsilon_2} \quad (2)$$

When stirrups yield, $f_1 = v_c \tan \theta$ (Collins and Mitchell 1991), and it can be assumed that $\varepsilon_2 = f_2/E_c$, $f_{sx} = nE_c \varepsilon_x$ and $f_{sz} = nE_c \varepsilon_z$ for simplification. By substituting above equations into the Mohr's circle for the average concrete stresses, combined with Eq. (2), we can get:

$$\tan^2 \theta = \frac{\frac{v_c \cot \theta - v_c \tan \theta}{n\rho_s} + v(\tan \theta + \cot \theta) - v_c \tan \theta}{\frac{v \tan \theta - v_c \tan \theta}{n\rho_v} + v(\tan \theta + \cot \theta) - v_c \tan \theta} \quad (3)$$

To determine θ , therefore, iterative computations are needed to solve for v_c in Eq. (3). For beams with a proper amount of stirrups, the value of V_c/V ranges from 20% to 60%. Here, it is assumed that $v_c = 0.4v$, and this hypothesis has little effect on the final value of θ . Substituting $v_c = 0.4v$ into Eq. (3),

$$\left(0.6 + \frac{0.6}{n\rho_v}\right) \tan^4 \theta = 1 + \frac{1}{n\rho_s} - \left(0.4 + \frac{0.4}{n\rho_s}\right) \tan^2 \theta \quad (4)$$

It is known that θ usually varies from 25° to 45° , and the value of θ in the right side of Eq. (4) is assumed to be equal to 35° , and this hypothesis has little influence on the final value of θ .

$$\theta = \arctan \left[\left(\frac{4}{3} \times \frac{1 + \frac{1}{n\rho_s}}{1 + \frac{1}{n\rho_v}} \right)^{0.25} \right] \quad (5)$$

Rationality of Simplified Equation of Calculating θ

Eq. (5) is derived based on the assumption of $v_c = 0.4v$, and the influence of the v_c/v value on θ is discussed here. Substituting $v_c = 0.25v$ into Eq. (3), we can get $\theta_{(v_c = 0.25v)} / \theta_{(v_c = 0.4v)} = 1.07$; therefore, there is little difference. Similarly, if $v_c = 0.6v$, the final value of θ has little change. In addition, Eq. (5) is derived based on the assumption of the value of θ in the right side of Eq. (4) being 35° . Fig. 1 shows the differences between the calculated values of Eq. (4) and Eq. (5), in which, $n = E_s/E_c = 6.0$, and the calculated value of Eq. (5) is very close to that of Eq. (4). Hence, Eq. (5) can be utilized to calculate θ .

Solution Algorithm for Shear Strength

At shear failure when stirrups yield, the Mohr's circle of concrete average strain can be used to calculate the tensile strain ε_1 , as given below:

$$\varepsilon_1 = \frac{2(\varepsilon_y + \varepsilon_2)}{|\cos 2\theta| + 1} - \varepsilon_2 \quad (6)$$

The principal compressive strain in concrete ε_2 is related to both the principal compressive stress f_2 and the principal tensile strain ε_1 in the manner (Vecchio and Collins 1986). The step-by-step solution process is summarized in the flowchart shown in Fig. 2, and the formulas for calculating $V_s, f_1, f_2, \omega, v_{ci,max}, f_{l,max}$ can be found in the MCFT (Collins and Mitchell 1991).

Comparison with experimental results

The validation of the proposed truss approach is demonstrated by comparison with published experimental results from previous investigations with respect to the shear strength at this state. There are 209 rectangular beams with stirrups with $\lambda \geq 2.4$ (Kim 2004; El-Metwally 2004; Lu 2007). These beams encompass a wide range of sizes and material properties, and all the selected beams are shear-

critical flexural members. The experimental database based on the 209 beams with stirrups was used to evaluate the proposed method. The calculated shear strengths by the proposed method and experimental results are compared as shown in Fig. 3. The mean ratio of the experimental to predicted strength and its coefficient of variation are 1.204 and 0.207 for the proposed iterative method, showing a good correlation between the proposed method and the experimental data. Most importantly, the analytical results based on the proposed method are on the safer side as illustrated in Fig. 3, as the dowel action and shear carried by the compression zone in the concrete contribution were not taken into account.

Determination of Principal Tensile Strain, ε_1

The calculation method described above for shear strength requires a program to resolve the iterative computations, which is complex for practical use. Eq. (1) can be used to calculate V_c explicitly; however, there is still an unknown, ε_1 . The method to resolve the inclined crack width provides us with the information that there is a relation between ε_1 and ε_z . There are many methods to calculate the inclined crack width, and all are related to the strains of stirrups. Sudhira (2008) compared the predicted inclined crack width with the experimental data, and concluded that:

$$\omega = k_w s_{m\theta} \varepsilon_z \quad (7)$$

where k_w is the coefficient for the effect of the web reinforcement angle, and is equal to 1.2 for vertical stirrups. Comparing Eq. (7) with $\omega = \varepsilon_1 s_{m\theta}$ (Collins and Mitchell 1991), we get $\varepsilon_1 = 1.2\varepsilon_z$. Eq. (1) indicates that it will be unsafe for design if ε_1 is taken to be small. For this reason, using the program shown in Fig. 2, based on the database of 209 rectangular beams with stirrups, the calculated mean value of $\varepsilon_1/\varepsilon_y$ is equal to 1.34, and its coefficient of variation is 0.08. The relation, $\varepsilon_1/\varepsilon_y=1.35$, is used in this paper to simplify calculation. Therefore, Eq. (1) is changed to:

$$V_c = \min \left(\frac{0.18\sqrt{f'_c}bd_v}{0.31 + \frac{32.4f_y}{(a_g + 16)E_s} \frac{1}{\frac{\sin \theta}{s} + \frac{\cos \theta}{s_x}}}, \frac{0.33\alpha_1\alpha_2bd_v\sqrt{f'_c} \cot \theta}{1 + \sqrt{675 f_y/E_s}} \right) \quad (8)$$

Evaluations of Shear Methodologies based on Shear Database

Based on the database of 209 rectangular beams with stirrups, the mean ratio of the experimental to predicted strength and its coefficient of variation are 1.204 and 0.207, 1.213 and 0.214, 1.405 and 0.256, and 1.394 and 0.228 for the proposed iterative method and simplified method, and the sectional design methods in ACI 318R-08 and CSA-04 respectively. Comparison of the available models with experimental data indicates that the proposed approach produces better mean ratios of the experimental to predicted strength than others. The methods in ACI 318R-08 and CSA-04 lead to very conservative results when compared with experimental tests of shear-critical RC beams. Also, good correlation between the experimental and predicted strengths across the range of f'_c , d_v , λ , ρ_s , and $\rho_v f_{yv}$ is found, which indicates that the proposed approach represents the effects of these key parameters very well.

Conclusions

In this study, a theoretical method to compute the inclination of struts and predict the shear strength of RC beams is proposed based on MCFT. First, the expression of θ is rationally derived as shown by Eq. (5), accounting for the contribution of the tensile stresses in the concrete between the cracks based on MCFT. A program is then developed to calculate the shear strength. However, the iterative computations are complicated and time-consuming. Associating the calculation of shear crack width with the relationship between the principal tensile strain ε_1 and the strain of stirrups ε_z , a simplified explicit expression for V_c (shown by Eq. (8)) is given which does not require reference to a table or iterative computations.

A shear database is compiled for slender shear-critical beams with $\lambda \geq 2.4$, which is utilized to evaluate the present approach, and the methods in ACI 318R-08, CSA-04. There is a good correlation

between the shear strengths obtained by the proposed simplified method and the published experimental data, with the average ratio of experimental to predicted shear strength of the 209 RC rectangular beams and its coefficient of variation being 1.213 and 0.214. The proposed method therefore provides a potential alternative to the existing techniques.

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Notations

a_g	Maximum aggregate size
b	Web width
d_v	Effective shear depth taken as flexural lever arm which needs not be taken less than 0.9d
E_c	Modulus of elasticity of concrete
f_1	Principal tensile stress in cracked concrete
f_2	Principal compressive stress in cracked concrete
f'_c	Cylinder strength of concrete
f_{ci}	Compressive stress on crack surface (assumed as zero in this model)
f_{sx}	Tensile stress in the longitudinal steels
f_{sz}	Tensile stress in the transverse steels
f_y	Yielding stress of longitudinal reinforcing steels
f_{vy}	Yielding stress of transverse reinforcing steels
n	ratio of modulus of elasticity of reinforcing steels to modulus of elasticity of concrete, $= E_s/E_c$
s	Spacing of stirrups
s_x	Vertical spacing of longitudinal bars distributed in the web
V_c	Contribution of concrete to shear
V_s	Contribution of stirrups to shear
α_1	Factor accounting for bond characteristics of reinforcement, $\alpha_1=1.0$ for deformed bars, $\alpha_1=0.7$ for plain bars, wires or bonded strands, $\alpha_1=0$ for unbonded reinforcement
α_2	Factor accounting for sustained or repeated loading, $\alpha_2=1.0$ for short-term monotonic loading, $\alpha_2=0.7$ for sustained and/or repeated loads
ϵ_1	Principal tensile strain in cracked concrete
ϵ_2	Principal compressive strain in cracked concrete
ϵ_x	Tensile strain in the longitudinal steels
ϵ_z	Tensile strain in the transverse steels
θ	Angle of the inclined strut in cracked concrete with respect to longitudinal axis of member in variable truss model

λ	Shear span to effective depth of section
v	Applied shear stress
v_c	Contribution of concrete to shear stress, equals to $V_c/(bd_v)$
$v_{ci,max}$	Maximum shear stress on a crack given width can resist
ρ_s	Ratio of area of longitudinal reinforcement to beam effective sectional area
ρ_v	Ratio of volume of shear reinforcement to volume of concrete core measured to outside of Stirrups
ω	Crack width

Captions to Figures

Fig. 1. Difference between Eq. (4) and Eq. (5)

Fig. 2. Flowchart showing solution algorithm for shear strength

Fig. 3. Correlation of experimental and predicted shear strength based on proposed method

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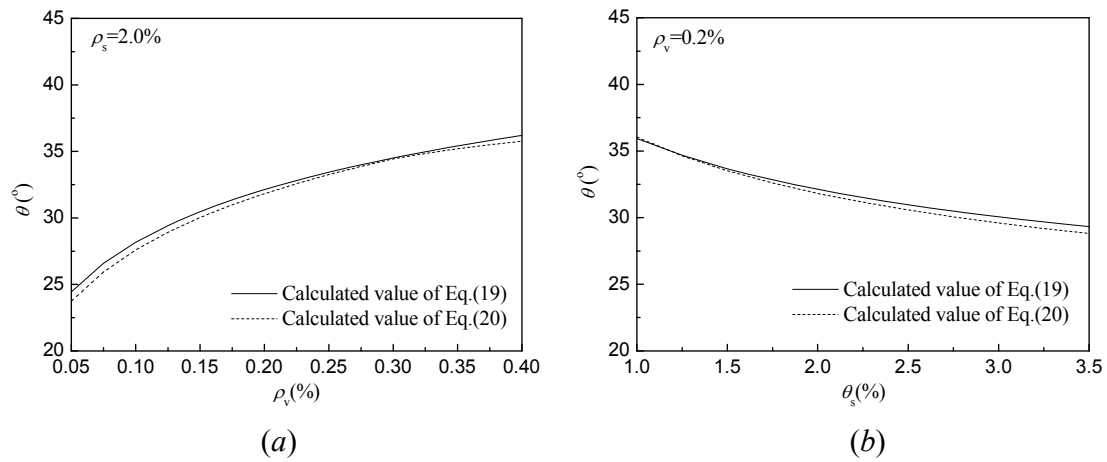


Fig. 1. Difference between Eq. (4) and Eq. (5)

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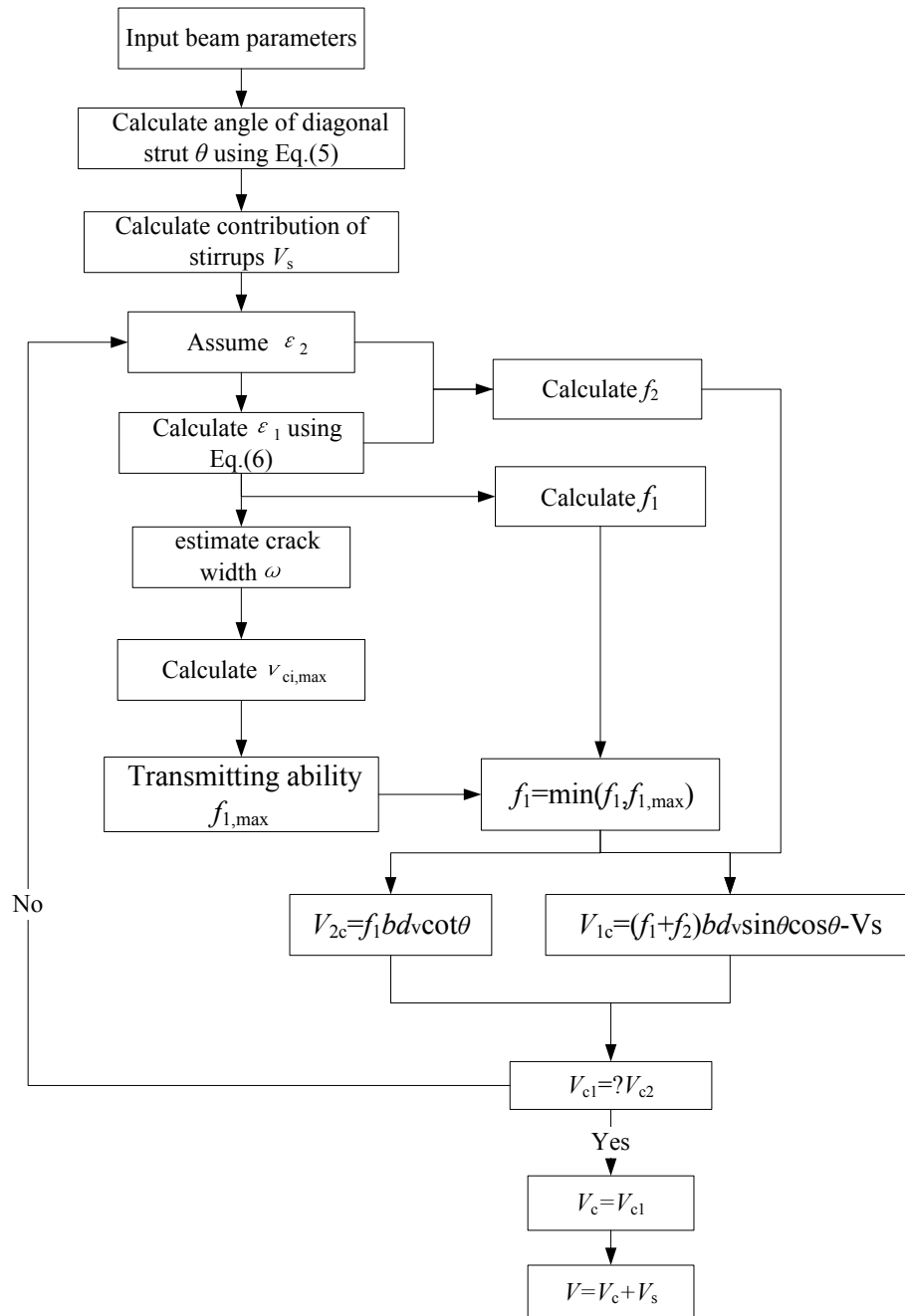


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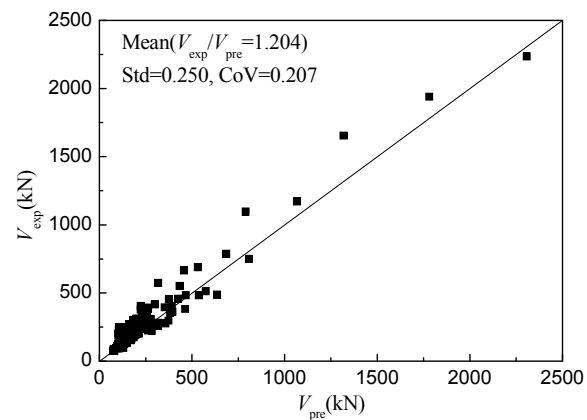


Fig. 3. Correlation of experimental and predicted shear strength based on proposed method

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