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2009

Zhang, B., Wu, B. I., & Chen, H. (2009). Optical delay of a signal through a dispersive invisibility cloak. *Optics Express*, 17(8), 6721-6726.

<https://hdl.handle.net/10356/95037>

<https://doi.org/10.1364/OE.17.006721>

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Optical delay of a signal through a dispersive invisibility cloak

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Abstract: We present a full-wave analysis method on the transmission of a Gaussian light pulse through a spherical invisibility cloak with causal dispersions. The spatial energy distribution of the Gaussian light pulse is distorted after the transmission. A volcano-shaped spatial time-delay distribution of the transmitted light pulse is demonstrated as a concrete example in our physical model. Both the time-delay and the energy transport depend on the polarization of light waves. This study helps to provide a complete picture of energy propagation through an invisibility cloak.

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OCIS codes: (230.3205) Invisibility cloaks; (290.5839) Scattering, invisibility; (160.1190) Anisotropic optical materials.

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1. Introduction

How a light signal propagates through a physical dispersive invisibility cloak is one of the fundamental problems in studying transformation based invisibility cloaking. The interaction of light with an invisibility cloak at its cloaking frequency was first illustrated via the use of ray theory which ascribes the invisibility of an ideal cloak to the perfect guiding of light around the concealed region along predetermined trajectories [1-6]. Extending the use of ray theory to a physical light signal going through a dispersive cloak, however, has lead to some concerns about causality [7] and energy gluing at the inner boundary [8]. These problems must be carefully examined before we get a clear picture of energy propagation through a dispersive invisibility cloak.

To study the light signal propagation through a physical dispersive cloak, we consider several key quantities: group velocity, energy transport velocity, time-delay, and distortion of energy distribution. Previous single frequency analyses [1-6, 9-12] did not provide results of the group velocity and the energy transport velocity. Some studied the group velocity and the energy transport velocity inside a cloak from the view of ray theory [7, 8]. Ray theory gives good demonstrations for a dispersionless case, because it can show the paths of rays inside the cloak. However, when it is applied to a dispersive cloak, the analysis in [7] raised the issue that such a cloak may violate causality, since the group velocity at the inner boundary diverges. In addition, [8] stated that a ray would take nearly infinite time to reach from one side of the cloak to another side if it points at the origin of the cloak, since the energy transport velocity approaches zero at the inner boundary. Furthermore, the ray theory approach applied in previous studies did not address the issues of scattering and distortion of energy distribution during the process of energy transport.

We study these problems by using a real signal (a Gaussian light pulse) together with full-wave analysis. The instantaneous field and Poynting vector at each point in space and time can be calculated, from which we show that inside the cloak, the local group velocity and the energy transport velocity may no longer be clearly defined due to the serious local distortion of the signal. However, if the bandwidth is sufficiently narrow, we can still define the time-delay of the signal on a target plane behind the cloak, and examine the spatial distortion of the transmitted pulse/energy.

2. Setup of the physical model and definition of signal time-delay

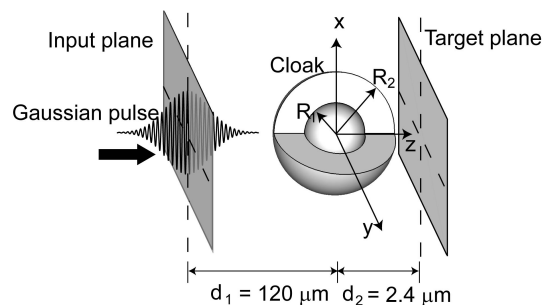


Fig. 1. Propagation of a narrow-band Gaussian pulse through a frequency-dispersive spherical invisibility cloak with a target plane set up at $z = 2.4 \mu\text{m}$. The pulse peak is at $z = -120 \mu\text{m}$ when $t = 0$. $R_2 = 2R_1 = 1.8 \mu\text{m}$.

Figure 1 shows a modulated Gaussian pulse incident upon a PEC (perfect electric conductor) sphere coated with a frequency dispersive cloak. The carrier frequency of the pulse

is f_0 (with period T_0 and wavelength λ_0) and the Gaussian envelope is characterized by $C(t) = e^{-t^2/2\sigma^2}$ in time. Thus the complete signal sent at the input plane at $z = -d_1$ is $\bar{E}(t, -d_1) = \hat{x}C(t)\cos(2\pi f_0 t)$ with the polarization in \hat{x} direction. The spherical cloak is able to conceal arbitrary objects perfectly only at the carrier-frequency f_0 [1]. The cloak layer within $R_1 < r < R_2$ is an anisotropic and inhomogeneous medium with permittivity tensor $\bar{\epsilon} = \epsilon_r \hat{r}\hat{r} + \epsilon_t \hat{\theta}\hat{\theta} + \epsilon_t \hat{\phi}\hat{\phi}$ and permeability tensor $\bar{\mu} = \mu_r \hat{r}\hat{r} + \mu_t \hat{\theta}\hat{\theta} + \mu_t \hat{\phi}\hat{\phi}$. It is chosen such that at f_0 , $\epsilon_t/\epsilon_0 = \mu_t/\mu_0 = R_2/(R_2 - R_1)$ and $\epsilon_r/\epsilon_t = \mu_r/\mu_t = (r - R_1)^2/r^2$ [1]. We adopt the same dispersion types as in [13], i.e.

$$\epsilon_r(r, f) = \epsilon_t \left(1 - \frac{f_p^2}{f(f + i\gamma_1)}\right), \quad (1)$$

$$\mu_r(r, f) = \mu_t \left(1 - \frac{F}{1 + i\gamma_2/f - f_0^2/f^2}\right), \quad (2)$$

where ϵ_t and μ_t are assumed to be constants over the frequency range of interest, f_p and f_0 are parameters for Drude model and Lorentz model respectively, $F = 0.78$, and γ_1 and γ_2 are set to be zero. The background is free space. We set a target plane at $z = d_2$. The signal propagation through each point in this target plane is monitored.

The original Gaussian pulse can be expressed by a Fourier integral as follows

$$\bar{E}_{inc}(\bar{r}, t) = \hat{x} \Re \left\{ \int_0^\infty E_{inc}(\omega) e^{ik(z+d_1) - i\omega t} d\omega \right\}, \quad (3)$$

where \Re means taking the real part, $E_{inc}(\omega)$ is the Fourier transform of $E(t, -d_1)$ and $k = \omega\sqrt{\mu_0\epsilon_0}$. To facilitate calculation, we truncate the original signal from $-T$ to T and let it be periodic with a period of $2T$. A very large period of $2T$ will not affect our treatment of a single Gaussian pulse significantly. The Fourier integral can then be written as a Fourier series

$$\bar{E}_{inc}(\bar{r}, t) = \hat{x} \Re \left\{ \sum_{n=0}^\infty E_n e^{ink_0(z+d_1) - in\omega_0 t} \right\}, \quad (4)$$

where E_n is the coefficient of the n th order item, $\omega_0 = \pi/T$ and $k_0 = \omega_0\sqrt{\mu_0\epsilon_0}$. Due to the narrow band property of the original signal, we can discard the items in the Fourier series except those from $n = N_1$ to $n = N_2$ with frequencies close to f_0 . Thus the total electrical field in presence of the dispersive spherical cloak can be expressed as

$$\bar{E}(\bar{r}, t) = \Re \left\{ \sum_{n=N_1}^{N_2} \left[\hat{x} E_n e^{ink_0(z+d_1)} + \bar{E}_{scat}(\bar{r}) \right] e^{-in\omega_0 t} \right\}, \quad (5)$$

where \bar{E}_{scat} is the scattering field for n th order item with frequency $n\omega_0$, which can be solved by using a multi-layer algorithm [13]. After obtaining $\bar{E}(\bar{r}, t)$ in time-domain, we are able to predict the signal or the energy propagation.

As a concrete example, an input Gaussian pulse is modulated at $f_0 = 500$ THz with $\sigma = 20T_0 = 40$ fs. We truncate this signal from -400 fs to 400 fs ($T = 400$ fs). Then the frequency spectrum is discretized by the sampling gap $\delta f = 1.25$ THz. We choose the time-harmonic plane wave items covering the range from 481.25 THz to 518.75 THz. In other words, we reconstruct the original Gaussian pulse by using 31 time-harmonic plane waves with different frequencies. Amplitude of each plane wave can be determined with straightforward calculation. The outer radius of the spherical cloak R_2 is $1.8 \mu\text{m}$ ($3\lambda_0$) and the inner radius R_1 is $0.9 \mu\text{m}$ ($1.5\lambda_0$). In the multi-layer algorithm [13], the number of layers N is set to be 200. We set $d_1 = 200\lambda_0 = 120 \mu\text{m}$ and $d_2 = 4\lambda_0 = 2.4 \mu\text{m}$.

To be consistent with [14] about the definition of the arrival moment as *the exact moment when the main part of the signal arrives*, we first calculate the instantaneous Poynting power transferred through the target plane as

$$S_z(t, x, y, d_2) = \bar{E}(t, x, y, d_2) \times \bar{H}(t, x, y, d_2) \cdot \hat{z}, \quad (6)$$

where (x, y, d_2) is a point in the target plane. Then the total accumulated energy that would transport through a given point in the target plane can be calculated as $W = \int_{4T_0}^{4T_0+2T} S_z dt$. In our numerical integration, the integration step length is set as $\delta t = T_0/20 = 0.1$ fs. Based on that, we can pick the exact moment when half of the total accumulated energy has been transferred as the arrival moment t_a of the signal reaching the target plane. Time-delay of the signal is defined as $t_a - t_{inc}$, where t_{inc} is the arrival moment of the incident signal in absence of the cloak and the PEC core.

3. Numerical results and discussion

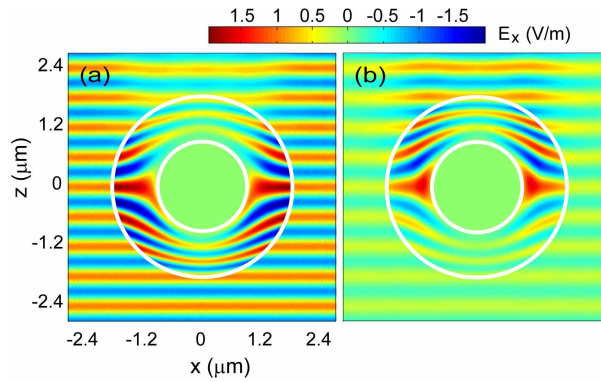


Fig. 2. E_x field distribution when the Gaussian pulse is passing through the dispersive spherical cloak at (a) $t = 400$ fs and (b) $t = 460$ fs.

Now let us discuss the results. Figures 2(a) and 2(b) show the E_x field distribution at the moments of $t = 400$ fs and $t = 460$ fs, respectively. At $t = 400$ fs, the peak of the incident Gaussian pulse is passing the center of the cloak. It is seen that the energy, or the wave envelope, is slowed down inside the cloak, which is similar to the dynamics of a 2D cloak [15]. However, the incident wave in [15] is not a finite signal and thus is not sufficient to describe the whole transport process of a finite signal. It is seen that when the pulse is passing by, some energy is stored inside the cloak. After the pulse leaves, the energy is released. It is worth mentioning that during the whole process, the *phase* of the signal exhibits minimal disturbance. The reason is that, although the cloak is created from the coordinate transformation theory, the theory itself does not provide any direct information about the energy propagation but only shows the perfect transmission of the wave's phase through an invisibility cloak. Therefore, the causality of the cloak cannot be judged by analyzing the behavior of an infinitely long sine wave at a single frequency but should be based on tracing the motion of the wave envelope which represents the energy's location. We can see from Fig. 2(b) that the transmission of the pulse through the cloak is delayed due to a longer physical distance when compared to a free space case. Vividly, the ray's true path in physical space has been elongated as shown in [1-4]. However, is it true that a longer ray path must lead to a larger delay of the wave signal and energy propagation?

Figure 3 shows the instantaneous Poynting power S_z at the points $(0.9, 0, 2.4)$, $(0.45, 0, 2.4)$ and $(0, 0, 2.4)$ μm in the target plane compared to the incident signal's instan-

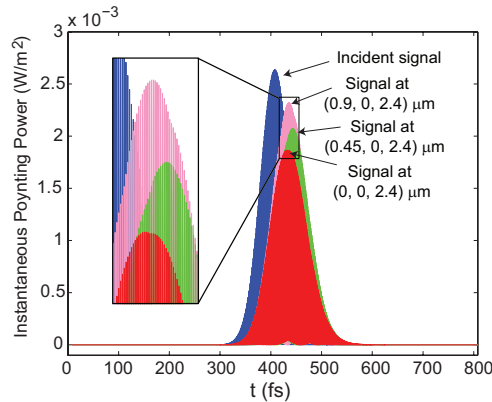


Fig. 3. Instantaneous Poynting power S_z at $(0.9, 0, 2.4)$, $(0.45, 0, 2.4)$ and $(0, 0, 2.4)$ μm in the target plane compared to the incident signal in absence of the cloak and the PEC core.

taneous Poynting power in absence of the cloak and the PEC core. It should be pointed out that the instantaneous Poynting power has fast oscillations inside the modulation envelope. We can see that the signals arriving at the target plane by passing through the cloak have well-defined arrival moments and are indeed delayed when compared to the incident signal. However, it is very interesting to note that the signal coming along the center of the cloak, i.e. passing through $(0, 0, 2.4)$ μm in the target plane, is faster than signals at some other points. In Fig. 4(a), we plot the time-delay distribution using the previously defined arrival moment in the target plane, and it exhibits a volcano-like shape with a pit at the center. This result is quite counterintuitive, since according to the geometrical description, the rays traveling closest to the z axis should have the longest distance to go. In fact, calculation based on geometric optics shows that the ray pointing at the center of the cloak would take nearly infinite time to reach from one side to the other of the cloak [8]. However, the energy propagation is governed by Maxwell's equations and it may not satisfy the ray approximation. Historically, it has been shown that the geometrical model of energy propagation is inadequate in certain cases, where a well-known example is the Poisson's spot [16]. Similarly, in the case of a dispersive spherical cloak, when the wave energy is approaching the inner boundary of the cloak, it cannot be approximated by a geometrical ray anymore. We have shown in [13] that, for the dispersive model we are using, only the frequency components with frequencies above the cloaking frequency can reach the inner boundary, while all the other will be expelled completely. Therefore, no matter how narrow the bandwidth of the signal is, when it approaches the inner boundary, the signal will be seriously distorted and the precise location or arrival moment of the signal or energy is not well-defined in the regions near the inner boundary. Full-wave calculation strictly from Maxwell's equations is necessary in this case. In other words, there is no precise path for the "ray" traveling along the center of the cloak to follow.

Regarding the issue of the group velocity [7] and the energy transport velocity [8], it has been shown that they are practically equal to the signal velocity except in anomalous dispersion with high absorption [14]. However, this conclusion has not taken into account the effect of anisotropy. The radial constitutive parameters ϵ_r and μ_r of zero value at the inner boundary enable the complete reflection of all incident waves [11]. Therefore the anisotropy ($\epsilon_r \approx 0$ and $\mu_r \approx 0$) near the inner boundary combined with dispersion causes the response of the material to vary even with an extremely small frequency change: an extremely small frequency change which causes the sign changes of ϵ_r and μ_r will result in the expulsion of field due to $\epsilon_r = 0$ and

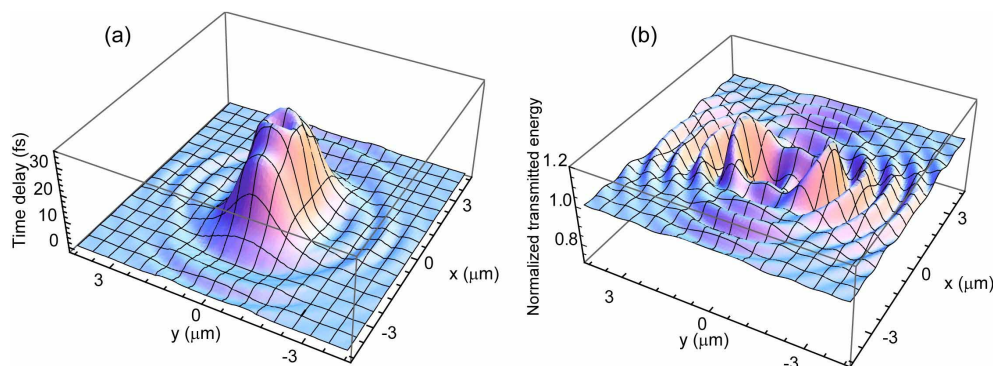


Fig. 4. (a) Time delay distribution and (b) normalized accumulated energy distribution of the signal arriving at the target plane after passing through a dispersive spherical invisibility cloak. In (a), maximum delay is 35.1 fs while the delay at the center is 27.9 fs. In (b), the minimum is 0.7638 while it is 0.9075 at the center.

$\mu_r = 0$ within the small frequency range. Hence the response is strongly frequency sensitive but the material has normal dispersion. Therefore, besides anomalous dispersion with high absorption, in this particular case of a spherical dispersive cloak, the group velocity and the energy transport velocity are no longer equivalent to the signal velocity. Even when the cloak is embedded in a medium with a large refractive index [1], it does not help to change this nonequivalence. As a result, the strong distortion still cannot be avoided.

In Fig. 4(b), we plot the accumulated energy transmission through each point of the target plane. It can be seen that the energy of the signal after traveling through the cloak is redistributed in the target plane due to diffraction, and the energy transported through the center in the target plane is neither the smallest nor the largest in this particular case. In other words, the cloak illuminated by the Gaussian pulse casts a shadow over the target plane with the center being neither the darkest nor the brightest as shown in Fig. 4(b).

From the viewpoint of the ray model used in [1-3, 8], the problem we have is a symmetric one and thus any response of the cloak should be rotationally symmetric with respect to the z axis. However, it is interesting to see from Fig. 4 that neither the time-delay nor the redistributed energy at the target plane is rotationally symmetric. The variation along x axis is different from the variation along y axis. If the polarization of the incident light wave rotates by an angle, the time-delay and the energy redistribution patterns will be rotated by the same angle.

4. Conclusion

The signal propagation of a modulated Gaussian light pulse through a dispersive spherical cloak is studied. With the cloak and PEC core we use here, the time-delay of the transmitted Gaussian light pulse exhibits a volcano-shaped spatial distribution with the signal transmitted along the center of the cloak not being the slowest. In general, the spatial energy distribution of the pulse is distorted after the transmission. The polarization of light waves strongly affects both the time-delay and the energy distortion of the transmitted signals.

Acknowledgments

This work is sponsored by the ONR under Contract No. N00014-06-1-0001, the Department of the Air Force under Air Force Contract No.F19628-00-C-0002, the ZJNSF (R1080320), and the Chinese National Science Foundation under Grant Nos. 60801005 and 60531020.