Effect of Madden Julian oscillation on convectively coupled equatorial waves

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SINGAPORE

**Effect of Madden Julian oscillation on convectively
coupled equatorial waves**

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**SCHOOL OF PHYSICAL AND MATHEMATICAL
SCIENCES**

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for the
degree of Doctor of Philosophy

2019

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Abstract

Effect of Madden Julian oscillation (MJO) on convectively coupled equatorial waves (CCEW) is studied using TRMM-3B42 precipitation dataset. The investigation is carried out for the two seasons of DJFM (December January February March) and JJAS (June July August September). Key finding for both seasons is that when MJO enhances precipitation, wave activity becomes stronger, while when MJO suppresses convection wave activity is weakened. Greater effect of MJO on CCEW is noticed over the zonal extent in which the convective component of MJO is strong, ranging from the Indian Ocean to western Pacific Ocean (nearly 60°E to 180°). MJO is also found to affect CCEW over Africa and near South America if wave activity peaks in these regions, however the effect is nearly half as strong or weaker.

In order to understand the impact of MJO on CCEW, a simplified theoretical model of convectively coupled waves is employed. Since MJO has a larger spatial structure and greater time period as compared to CCEW, therefore the theoretical model is modified by using MJO as a background. Fluctuations in barotropic wind, baroclinic shear and diabatic heating due to MJO are considered in the background. Linear analysis of the theoretical model reveals that out of the three factors considered, fluctuations in diabatic heating due to MJO is the dominant factor affecting CCEW. Effect of diabatic heating on the instability mechanism of the model, affecting the evolution of convectively coupled waves is explored.

Acronyms

MJO Madden Julian Oscillation

CCEW Convectively Coupled Equatorial Waves

TRMM Tropical Rainfall Measuring Mission

DJFM December January February March

JJAS June July August September

GCM General Circulation Model

CRM Cloud Resolving Model

CQE Convective Quasi Equilibrium

SST Sea Surface Temperature

WIG Westward Inertia Gravity

MRG Mixed Rossby Gravity

CIN Convective Inhibition

CISK Convective Instability of Second Kind

Chapter 1

Introduction

1.1 Atmospheric dynamics

Atmospheric dynamics is the study of motions in the atmosphere with its causes and effects. It is a matter of observation that the scales and velocity of motion in the earth's atmosphere are within the realms of applicability of classical mechanics and thermodynamics. Also, the scales of motions are much larger than the mean free path of molecules (at least in the lower 100km of the atmosphere) so that the discrete molecular nature of gases can be ignored. Under these conditions, the atmospheric motions can be successfully approximated by the laws of fluid mechanics. Fluid mechanics (a sub-discipline of continuum mechanics) assumes that the fluid is continuous (continuum assumption) and obeys the law of conservation of mass, momentum, and energy. These laws are expressed as a set of partial differential equations (from hereon-wards PDEs) which express the dependent flow variables (describing fluid motion) as functions of independent variables of space and time. The PDEs expressing these laws in a reference frame attached to the earth are given below

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \quad (1.1)$$

$$\frac{D\mathbf{U}}{Dt} = -2\boldsymbol{\Omega} \times \mathbf{U} - \frac{1}{\rho} \nabla p + \mathbf{g} + \mathbf{F}, \quad (1.2)$$

$$\rho \frac{De}{Dt} = -p \nabla \cdot \mathbf{U} + \rho J. \quad (1.3)$$

Here ρ , e and p stand for the fluid density, internal energy and pressure respectively. $\mathbf{U}$ is the velocity vector, J is the rate of heating of the fluid and $\mathbf{F}$ represents frictional forces in the fluid. The first term on the right hand side of the momentum equation 1.2 accounts for the Coriolis force where $\boldsymbol{\Omega}$ is the angular velocity of earth's rotation. The centrifugal force is combined with the gravitational force in the term $\mathbf{g}$ (we need to consider these forces in the momentum equation since the reference frame attached to earth is non-inertial). A complete derivation of these PDEs can be found in texts like, Batchelor (2000) and Kundu et al. (1990).

1.2 Tropical atmosphere

Geographically, tropics refer to the region of earth between the latitudes of Tropic of Cancer (lat. 23.5°N) and Tropic of Capricorn (lat. 23.5°S). Meteorologically, however, tropics refer to those parts of the earth where the atmospheric dynamics differ from that observed at high latitudes. The latitudinal range thus can vary with longitude and also with the season as the sun migrates over the latitudes. For most practical purposes, tropics can be considered to stretch from a latitudinal range of 30°N to 30°S .

Due to the curvature of earth, Sun rays strike the tropics at a lower zenith angle than near the poles, this results in higher radiation intensity per unit area in the tropics than near the poles. In spite of tropics getting higher radiative energy, the tropical atmosphere has a net radiative cooling. The surplus radiative energy received by the tropics is used to heat the surfaces of land and oceans. This energy eventually finds its way up in the troposphere as sensible heat or latent heat. Sensible heat refers to the heat transfer due to thermal conduction, while latent heat refers to the heat release associated with change of phase of water which occurs in phenomenon such

as convective clouds, snow formation, etc.

Figure 1.1 shows the zonally averaged precipitation as a function of latitude. It can be noted from the figure that precipitation is significantly higher in the tropics than at higher latitudes, with the peak a few degrees north of equator. In fact, the latent heat released during precipitation is a very important effect in the tropics and is the primary source of energy for large scale circulation features of the tropical atmosphere. The zonally averaged large scale circulation in the tropics exhibits the following features. Moist air converges near the surface near equator. Radiative cooling of the troposphere destabilises it and results in formation of convective clouds which cause heavy precipitation. Release of latent heat makes the air buoyant and results in strong upward motion which rises up to the tropopause, at which point the rising air is no longer buoyant. Unable to rise, the air starts moving polewards (both northwards and southwards) and finally starts sinking near latitudes of 30° . As the air sinks, it warms up adiabatically and experiences a decrease in its relative humidity. It then starts moving towards the equator and picks up moisture along the way, due the Coriolis force it also gains a westward component, thus creating the trade winds. This complete circulation pattern of the tropics is known as the Hadley cell and is diagrammatically depicted in figure 1.2.

Other than the Hadley cell, convection (as latent heat release happens in association with the convective clouds) interact with the large scale flow at different temporal scales. An important variability of tropical atmosphere at interannual time scale is El nino, which is a global scale phenomenon involving variations in wind pattern, precipitation and sea level pressure. Detailed information on El nino can be found in L'Heureux (2014).

In addition to the interannual variability due to El nino, tropical convection also interacts with large scale flow at an intraseasonal time scale of 30 to 90 days and produces a phenomenon known as the Madden Julian oscillation (MJO), named after the meteorologist who discovered it (Madden & Julian, 1971). During a strong period

Figure 1.1: Zonal mean precipitation in mm/day over a period from 1979 to 2010, with the thin horizontal line depicting global climatological mean, Pendergrass and for Atmospheric Research Staff (Eds) (2016).

Figure 1.2: Diagrammatic representation of Hadley cell circulation with air rising near equator which is a region of heavy precipitation and sinking near 30° north and south.

of MJO, vigorous convection develops over Indian Ocean which strengthens and moves eastward at a speed of nearly 5m/sec, and finally dies near mid Pacific Ocean. This movement of convection is depicted in figure 1.3 F to C.

Convection in MJO is also coupled to a global wind circulation pattern. Wind anomalies associated with MJO are depicted as loops with arrows in figure 1.3. It should be noted that in the convectively active region, wind converges at lower levels,

risers up, and diverges at higher levels. On the east of convectively active regions in figure 1.3 F to A, there are regions of sinking wind, where wind converges at high levels, sinks, and diverges at lower levels. For figure 1.3 B and C the regions of sinking winds are on the west of the convective regions. These regions of sinking wind are associated with suppressed convection, so MJO not only enhances convection over certain regions but also suppresses convection over certain other regions.

Even if the convection associated with MJO dies over mid Pacific Ocean, the wind anomalies continue to move eastward and may encircle the whole globe, figure 1.3 D and E.

Figure 1.3: Schematic representation of convection and wind circulation pattern associated with MJO. Region of enhanced convection is represented by cloud. The top of each figure shows variations in tropopause height and bottom of each figure shows variations in sea level pressure associated with MJO (Madden & Julian, 1972).

1.2.1 Waves in tropical atmosphere

Presence of large scale waves in the atmosphere is an important aspect of its dynamics.

In the previous section, a set of equations is given which is used to study the dynamics

of the atmosphere. This set of equations can also be used to examine important wave motions in the tropical atmosphere, known as equatorial waves. These are called equatorial waves as these disturbances move eastward or westward along the equator and their amplitude decay north or south from the equator. Reader familiar with equatorial waves may skip directly to section 1.2.2.

To study the dynamics of these small amplitude waves, the equations derived using Boussinesq approximation in appendix A are linearised about a basic state of rest.

The linearised set of equations, is given below

$$\frac{\partial u}{\partial t} = fv - \frac{1}{\rho_m} \frac{\partial p}{\partial x}, \quad (1.4)$$

$$\frac{\partial v}{\partial t} = -fu - \frac{1}{\rho_m} \frac{\partial p}{\partial y}, \quad (1.5)$$

$$\frac{1}{\rho_m} \frac{\partial p}{\partial z} = \frac{T}{T_m} g, \quad (1.6)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1.7)$$

$$\frac{\partial T}{\partial t} + wS = 0, \quad (1.8)$$

where $\left(\frac{dT_0}{dz} + \frac{g}{c_p}\right)$ is written as S and tilde are omitted for convenience of writing. J is set to zero as no heating or cooling is considered in the present case.

Using the method of separation of variables the solution to the set of PDEs can be written as

$$[u, v, p] = [u', v', p']\psi(z), \quad (1.9)$$

from the mass conservation equation 1.7

$$w = w' \int_{-H}^z \psi(z) dz, \quad (1.10)$$

from the hydrostatic equation 1.6

$$T = T' \frac{d\psi(z)}{dz}, \quad (1.11)$$

where the amplitudes u', v', p', w' and T' are functions of (x, y, t) and $z = -H$ represents the bottom of the atmosphere.

substituting these variables in the energy equation 1.8 gives

$$\frac{\partial T'}{\partial t} \frac{d\psi(z)}{dz} + w' \int_{-H}^z \psi(z) dz S = 0. \quad (1.12)$$

Approximating S to be constant with height (which is a good approximation for the troposphere), differentiating the equation w.r.t. z and dividing by S gives

$$\frac{\partial T'}{\partial t} \frac{1}{S} \frac{d^2\psi(z)}{dz^2} + w' \psi(z) = 0, \quad (1.13)$$

for this equation to be valid description of the vertical structure of the solution, the vertical part of the equation should have the form

$$\frac{1}{S} \frac{d^2\psi(z)}{dz^2} = \lambda \psi(z), \quad (1.14)$$

where λ is a constant. This equation has the so called Stürm-Liouville form for which the various solutions are orthogonal. This equation must be supplied with appropriate boundary conditions so that its solutions may be obtained. Assuming a rigid bottom at the ground and a rigid upper lid at the top of the atmosphere, the boundary conditions are

$$w = 0 \text{ at } z = -H \text{ and } z = 0$$

from equation (1.6) and equation (1.8), w can be written in terms of $\psi(z)$ as

$$w = -\frac{T_m}{\rho_m S g} \frac{\partial p'}{\partial t} \frac{d\psi(z)}{dz}, \quad (1.15)$$

which gives the boundary condition as $\frac{d\psi(z)}{dz} = 0$ at $z = -H$ and $z = 0$. With these boundary conditions equation (1.14) has the solution

$$\psi(z) = \cos(m\pi z/H), \quad (1.16)$$

where $m = 0, 1, 2, 3, \dots$ and each m corresponds to a different vertical mode. This implies that u, v, p has a vertical structure in the form $\cos(m\pi z/H)$ and w, T have a vertical structure of the form $\sin(m\pi z/H)$. Making these substitutions in the linearized set of equations for m th vertical mode, replacing p by T using equation 1.6 and w by using equation 1.7 we get

$$\frac{\partial u}{\partial t} = fv + \frac{Hg}{m\pi T_m} \frac{\partial T}{\partial x}, \quad (1.17)$$

$$\frac{\partial v}{\partial t} = -fu + \frac{Hg}{m\pi T_m} \frac{\partial T}{\partial y}, \quad (1.18)$$

$$\frac{Hg}{m\pi T_m} \frac{\partial T}{\partial t} - \frac{H^2 g S}{m^2 \pi^2 T_m} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0, \quad (1.19)$$

rescaling temperature by $\frac{Hg}{m\pi T_m}$ and writing $\frac{H^2 g S}{m^2 \pi^2 T_m}$ as c_m^2 where $m = 1, 2, 3, \dots$, the system of equations can be written as

$$\frac{\partial u}{\partial t} = fv + \frac{\partial T}{\partial x}, \quad (1.20)$$

$$\frac{\partial v}{\partial t} = -fu + \frac{\partial T}{\partial y}, \quad (1.21)$$

$$\frac{\partial T}{\partial t} - c_m^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0. \quad (1.22)$$

This system from equation (1.20) to equation (1.22) describes the dynamics of m th vertical mode. The mode corresponding to $m = 0$ is called the *barotropic mode*, this mode does not feel the stratification of the fluid. The remaining modes corresponding to $m \geq 1$ are known as *baroclinic modes*. If the dynamical equations can be solved for one vertical mode then they can be solved for any another vertical mode using the same procedure. The total solution can then be written as a sum of individual solutions by using the superposition principle (as the equations are linear). The dynamical system described by equations (1.20) to (1.22) is equivalent to that of a fluid having a constant density and a free surface of mean height H from the bottom, as shown in figure 1.4. Comparing to this, $c_m \equiv gH$ and $T \equiv h$ (fluctuation in mean height). H is known as the equivalent height or equivalent depth and h is the fluctuation in height of the free surface. For detailed calculations refer to Kundu et al. (1990) chapter 14 section 8.

Figure 1.4: A fluid of constant density with the free surface at a mean height H from an impenetrable bottom.

For tropical atmosphere, this system of PDE is examined using the beta-plane approximation for a channel centered at equator, expanding f as $f_0 + \beta y$. As the channel is centred at equator, $f_0 = 0$ since $f_0 = 2\Omega \sin \phi_0$ where ϕ_0 is the latitude which is zero at equator. We seek the solutions of this system of PDEs in terms of

zonally propagating waves having the form $\hat{a}(y)e^{i(kx-\omega t)}$, where $\hat{a}(y)$ is the amplitude of variables u, v and T . Under the condition $\omega^2 \neq c_m^2 k^2$, the system of equations can be reduced to a single second order ordinary differential equation in variable $\hat{v}$ as.

$$\frac{\partial \hat{v}^2}{\partial y^2} + \left[\left(\frac{\omega^2}{c_m^2} - k^2 - \frac{k}{\omega} \beta \right) - \frac{\beta^2 y^2}{c_m^2} \right] \hat{v} = 0. \quad (1.23)$$

We assume that $\hat{v}$ decays as $|y| \rightarrow \infty$. This boundary condition is necessary since we are looking for equatorially trapped wave motion and the the beta-plane approximation is not valid for latitudes beyond 30° north and south. Equation (1.23) is an eigenvalue problem same as the Schrödinger equation for a simple harmonic oscillator. Solutions to this equation with the given boundary conditions are possible when the constant part in the square brackets in the equation satisfies the condition.

$$\frac{c_n}{\beta} \left(\frac{\omega^2}{c_m^2} - k^2 - \frac{k}{\omega} \beta \right) = 2n + 1; \quad n = 0, 1, 2, \dots \quad (1.24)$$

For each n and wavenumber k this relation results in a cubic equation in ω with three solutions for ω . Analytical expressions for approximate solution of ω are given as (see equations 10 and 11 of Matsuno, 1966)

$$\omega_{1,2} = \pm c_m \sqrt{k^2 + \frac{\beta}{c_m} (2n + 1)}, \quad (1.25a)$$

$$\omega_3 = \frac{-\beta k}{\left(k^2 + \frac{\beta}{c_m} (2n + 1) \right)}. \quad (1.25b)$$

First two of these solutions $\omega_{1,2}$ have higher frequency and are termed as eastward and westward moving inertia-gravity waves (EIG and WIG waves) and the third solution ω_3 has a lower frequency and is known as a westward moving equatorial Rossby wave (ER waves). Replacing y as a nondimensional meridian coordinate $\xi = (\beta/c_m)^{1/2} y$ the solutions can be written as

$$\hat{v} = v_0 H_n(\xi) e^{-\xi^2/2}, \quad (1.26)$$

where $H_n(\xi)$ is the n th Hermite polynomial. A few Hermite polynomials are given below.

$$H_0 = 1, H_1(\xi) = 2\xi, H_2(\xi) = 4\xi^2 - 2, H_3(\xi) = 8\xi^3 - 12\xi$$

It can be seen that the Hermite polynomials are either symmetric or antisymmetric about the equator. Therefore the equatorial waves have either a symmetric or an antisymmetric structure about the equator. The dispersion relation for inertia-gravity waves and equatorial Rossby waves are shown as black curves in figure 1.5 (a) for $n = 1$ and 3 (symmetric waves) and (b) for $n = 2$ and 4 (antisymmetric waves), where wavenumber is the number of waves around the earth. Complete structure of the various wave modes can be seen in figure 1.6.

The solution with $n = 0$ is a special case. Under this condition equation (1.24) can be factorised as

$$\left(\frac{\omega}{c_m} - \frac{\beta}{\omega} - k \right) \left(\frac{\omega}{c_m} + k \right) = 0, \quad (1.27)$$

the root $\omega = -c_m k$ is not a permitted solution as we have assumed $\omega^2 \neq c_m^2 k^2$ before deriving equation (1.23). The roots of the first term of equation (1.27) are

$$\omega = k c_m \left[\frac{1}{2} \pm \frac{1}{2} \left(1 + \frac{4\beta}{k^2 c_m} \right)^{1/2} \right], \quad (1.28)$$

the positive root corresponds to a eastward moving inertia-gravity wave, while the negative root corresponds to a westward moving wave which resembles equatorial Rossby wave for large wavenumber. Therefore these waves are called Mixed Rossby gravity waves (MRG waves). The dispersion relation for these waves is shown in figure 1.5 (b) as the red curve. From the figure it can be seen that the dispersion relation for eastward moving waves approaches to that of eastward inertia-gravity waves and

dispersion relation for westward moving waves approaches to that of equatorial Rossby waves for large wavenumber.

Other than the wave modes discussed till now, which were coming from equation (1.23), there is another important equatorially trapped wave mode known as the equatorial Kelvin wave. For this mode, the meridional velocity perturbations vanish. Again taking the solutions of the form $\hat{a}(y)e^{i(kx-\omega t)}$, the equation set (1.20) to (1.22) with $\hat{v} = 0$ reduces to

$$-i\omega\hat{u} = ik\hat{T}, \quad (1.29)$$

$$\beta y\hat{u} = \frac{\partial\hat{T}}{\partial y}, \quad (1.30)$$

$$i\omega\hat{T} + c_m^2(ik\hat{u}) = 0. \quad (1.31)$$

From these equations, the dispersion relation for Kelvin waves can be calculated as

$$\omega^2 = c_m^2 k^2, \quad (1.32)$$

this allows for both eastward and westward moving waves, however, for the westward moving waves $\hat{u}$ and $\hat{T}$ increases exponentially away from the equator, hence this solution is physically unrealistic. The dispersion relation for Kelvin waves is the same as the dispersion relation of a gravity wave without the effects of rotation. Details of calculations of equatorial wave modes can be found in Matsuno (1966).

An equatorial wave can be excited by diabatic heating from tropical convection and can convey the effects of convective thunderstorms over long distances. It has been observed that the tropical convection can couple with the equatorial wave modes and result in a phenomenon known as convectively coupled equatorial waves (CCEW)

Figure 1.5: Figure (a) shows the dispersion relation for meridionally symmetric waves and figure (b) for meridionally antisymmetric waves. In both figures eastward moving waves have positive wavenumber k and westward moving waves have negative wavenumber $-k$. Dispersion relation for Inertia-gravity waves and Equatorial Rossby waves are shown in black for $n=1,2,3$ and 4. Kelvin waves are meridionally symmetric and the dispersion relation is shown in figure (a) as the blue curve. Mixed-Rossby gravity waves have an antisymmetric structure and is shown in figure (b) as the red curve.

which modulate a substantial part of tropical convection. Interestingly, these CCEW have similar dispersion characteristics and horizontal structure as the linear equatorial wave modes, however the vertical structure is more complex. Observational studies (Straub & Kiladis, 2003b, Haertel & Kiladis, 2004) indicate that the vertical structure of CCEW comprises mainly of first and second baroclinic modes (modes $m = 1$ and 2, as discussed before). The CCEW can organize convection over synoptic scales (1000s of kilometers) and have a time period of 1 to 30 days, which is shorter than the time period of MJO. For more details on convectively coupled waves refer to Kiladis et al. (2009).

Figure 1.6: Structure of the equatorially trapped linear wave modes. The arrows represents the flow directions. Dark and lightly shaded regions represent the regions of flow convergence and divergence respectively. The letters H and L represent the regions of high and low temperatures or h (Kiladis et al., 2009).

1.2.2 Aims of the thesis

A large fraction of tropical weather variability at intraseasonal time scale is controlled by Madden Julian oscillation and convectively coupled equatorial waves (CCEW). The Madden Julian oscillation (MJO) is the dominant mode of this variability, having a

planetary scale structure and comprises of a slow eastward moving convective envelope with a period of 30 days to 60 days. On the other hand, CCEW can range from the planetary scale and slowly propagating waves such as Equatorial Rossby waves which have time periods comparable to that of MJO, to the synoptic scale and fast propagating inertia gravity waves which can have periods as short as one day.

MJO's region of enhanced convection is composed of sub-planetary scale convective processes such as mesoscale convective systems (MCS), CCEW and even tropical cyclones. Hence MJO is considered to be a multiscale phenomenon. New theories attempting to explain MJO propose that multiscale interaction is an essential aspect of its dynamics and CCEW may even be the building blocks of MJO. Therefore it is interesting to investigate if MJO has any relation with the strength of CCEW. Previous observational studies indicate MJO's convective envelope can favor or inhibit certain convective wave modes, a detailed review of these studies is given at the beginning of chapter 2. In spite of all these studies, the results are often inconsistent and ambiguities remain. Therefore, the first aim of this thesis is to explore if MJO has any effect on the strength of CCEW. If MJO does have an effect, the second aim of the thesis is to investigate it further by employing a simplified theoretical model of convectively coupled wave, in order that the factors responsible for MJO's effect on CCEW may be understood.

The focus of the present thesis is to understand the effect of slower moving MJO on faster moving CCEWs. In the first part of the thesis, CCEWs are studied in the strong and weak periods of MJO. The second part is concerned with understanding their evolution using a theoretical model in which the MJO is a stationary background. Therefore, only those CCEWs which have time periods shorter than the MJO are only considered in this study. Since convectively coupled equatorial Rossby waves have time periods comparable to that of MJO they are not a part of the current thesis.

Chapter 2

MJO's impact on convectively coupled waves

As discussed in the previous chapter, the variability of convection in the tropics at intraseasonal time scales, and synoptic to planetary spatial scales is mainly controlled by the Madden Julian Oscillation (MJO) and Convectively Coupled Equatorial waves (CCEW). MJO comprises of a convective envelope consisting of a large collection of mesoscale convective systems (MCS) moving eastward at a speed of about 5m/sec. This convective envelope originates in the Indian Ocean and moves towards the Pacific Ocean, where it eventually weakens and dies (Zhang, 2005). During boreal winter, the MJO signals are largely confined to the Indian Ocean and western Pacific Ocean while during boreal summer they exhibit a northward propagation from Bay of Bengal to the South China Sea and a separate convective center is also formed in the eastern Pacific Ocean (Zhang & Dong, 2004). A mesoscale convective system is an amalgamation of individual thunderstorm having a spatial extent of ~ 100 km in at least one direction. These MCS have a time scale of 2 to 20 hours and are responsible for most of the rainfall in the tropics (Houze Jr, 2004). The convectively coupled equatorial waves (CCEW) also comprise of such convective envelopes which move eastwards or westwards along the equator ranging in size from planetary to synoptic scale (M. Wheeler & Kiladis, 1999, Kiladis et al., 2009).

The large convective envelope of MJO is not spatially uniform (as depicted figure

1.3 by a single cloud) , it contains organizations of convection at smaller scales within itself. The study by Nakazawa (1988) identified a hierarchical structure of cloud clusters within the large scale convective envelope of the MJO. According to this study, MJO's convective region contains super clusters which have a scale of several thousand kilometers that moved eastwards with a time period of about 10 days, and cloud clusters which are structures of several hundred kilometers, embedded within the super clusters, moving westwards with a period of one to two days. Figure 2.1 shows the hierarchy of cloud clusters and super clusters as envisioned by Nakazawa.

Figure 2.1: Schematic representation of cloud hierarchy in the convectively active region of MJO. The diagram on the left has distance on horizontal axis (in km) and time on vertical axis (in days) increasing downwards. Diagram on the right shows the details of one of the super cloud cluster (Nakazawa, 1988).

The super cloud clusters have been identified as convectively coupled Kelvin waves (Takayabu & Murakami, 1991) and the westward moving clusters as convectively coupled westward inertia gravity waves (Takayabu et al., 1996).

An extension to the study by Nakazawa was conducted by Dunkerton and Crum (1995) which used OLR (Outgoing Longwave Radiation) dataset to study the movement of cloud clusters. [OLR is the thermal radiation emitted by the earth and its atmosphere into the outer space having a wavelength between $4\mu m$ and $100\mu m$ and is measured from at the top of the atmosphere using satellites mounted sensors

(Liebmann & Smith, 1996). OLR is used as a proxy indicator for convection as the higher cloud tops are cooler than the exposed surface (Arkin & Ardanuy, 1989).]

The study divided the tropical OLR variance into high frequency variability, time period of 2-15 days (Kelvin waves), and low frequency variability with time period of 30 to 60 days (corresponding to MJO). The active phase of MJO was interpreted not as an enhancement of large continuous region of convection but as an enhancement of convective activity at a smaller spatial and temporal scales for a period of 30 to 60 days. It was observed that the high frequency convective activity (time period 2-15 days) was usually enhanced in the convectively active phase of the low frequency activity(MJO), however, enhanced high frequency activity was not limited to the convectively active phase and could also be found in the inactive phase. It was also recognized that the hierarchical structures of cloud clusters as suggested by Nakazawa (1988) may not be applicable to all MJO events.

In another study by Hendon and Liebmann (1994), it was found that the cloud super clusters (Nakazawa, 1988) can be present in the convectively active or suppressed regions of MJO, and were not a unique feature of the convective envelope of MJO. However, the convectively active and suppressed regions of MJO were more effective in modulating precipitation at the mesoscale.

Straub and Kiladis (2003a) examined the relation between MJO and two modes of CCEW (Kelvin waves and MRG waves) for 22 summers (June July August (JJA)) from 1979 to 2000. They noted that the activity of super clusters (Kelvin waves) was more enhanced to the east of MJO's convective envelop than within it. However in their study, the activity of MRG wave was enhanced within the MJO's convective envelope which could be responsible for a significant proportion of convection in it. Similar conclusions were also drawn by Kikuchi and Wang (2010) regarding the activity of Kelvin waves in relation to MJO. In addition, they also found that MJO has a strong influence on modulating the activity of WIG waves. It was noticed that WIG waves were enhanced to the east of most convective region of MJO.

On the other hand, there have been studies that indicate the enhancement of convectively coupled Kelvin waves in the MJO convective envelope (Roundy, 2008, Roundy, 2012). Masunaga et al. (2006) also showed that the convective envelope of MJO does enhance the activity of Kelvin waves and to some extent the activity of Equatorial Rossby waves. They hypothesized that the convective envelope of MJO may be due to a series of convection as a result of a group of Kelvin waves. They also found that interaction of Kelvin and ER waves result in intense convection and may trigger and help in maintaining the MJO's convective envelope.

Yasunaga (2011) studied the modulation of CCEW amplitudes by MJO over the Indian Ocean (60°E-90°E) and western and central Pacific Ocean (120°E-180°W). It was found that all CCEW were enhanced in the convective envelope of MJO in both ocean basins. However, enhancement in the strength of MRG was observed to be less as compared to other waves. The question of modulation of CCEW by MJO convective envelope was further investigated by Dias et al. (2013). In their results, the Kelvin wave enhancement was not specifically related to the convective envelope of MJO and may occur in convectively active or suppressed phase of MJO, while $n = 1$ EIG were stronger within the convective envelope.

In the light of such studies, one can see that MJO does seem to have some influence on the strength of CCEW. However a clear description of such modulation is not sketched or is limited to regions such as Indian ocean or Pacific Ocean. This study attempts to provide as complete as possible a description of the modulation of the strength of CCEW with MJO to see whether the convective envelope of MJO has a preference for a particular CCEW or if the MJO has any effect on the strength of CCEW in regions other than Indian and Pacific Ocean basins. Such results might have important implications for the recent multi-scale theories of MJO which are based on two-way interaction between MJO and smaller scale convective clusters (Majda & Biello, 2004, Majda & Stechmann, 2009, Wang & Liu, 2011, Yang & Ingersoll, 2013).

2.1 Data and Methodology

This study uses the dataset TRMM (Tropical Rainfall Measuring Mission)-3B42 version 7. The dataset contains precipitation rates in mm/hr at a temporal resolution of three hours and spatial resolution of $0.25^\circ \times 0.25^\circ$. The time period for the analysis ranges over a period of 17 years from 1998 to 2014. Both CCEW and MJO have been previously detected in TRMM-3B42 v6 in Yasunaga and Mapes (2012a). More information on the dataset can be found in Huffman et al. (2007).

This study uses the precipitation dataset since precipitation is an indicator of latent heat of condensation released into the atmosphere due to condensing water. Higher precipitation corresponds to a higher latent heat release which corresponds to stronger convection. Therefore, the precipitation field gives a measure of the strength of activities like CCEW and MJO. Figure 2.2 shows three instances of precipitation field taken from the months of January and February of the year 2001. It can be seen that most of the precipitation is over Indian Ocean and Pacific Ocean and some over South America. The three sub-figures of figure 2.2 also show the eastward movement of convection due to a prevailing MJO. In figure 2.2 (a), on 28th of January high convective activity can be seen over Indian Ocean, southeast of Indian peninsula. By 9th February, convection has almost disappeared from the Indian Ocean and shifted eastward towards Maritime continents as seen in figure 2.2 (b) and by 26th February, convection over the Maritime continents has reduced and shifted eastward into the Pacific Ocean, figure 2.2 (c).

Figure 2.3 shows a time-longitude plot (also called Hovmöller plot) for TRMM-3B42 data averaged over a latitudinal range of 15.125°N to 15.125°S for the year 2001. Most of the convective activity is between the longitudes 60.125°E to 0.125°W which includes the region from Indian Ocean to Pacific Ocean, while some convective activity can be seen near 120.125°W (South America). The MJO event discussed in figure 2.2 can also be seen in figure 2.3 within the red ellipse. Other than the MJO,

there are other smaller scale eastward or westward moving disturbances, such as those within the black ellipse, which could be due to CCEW.

Figure 2.2: Visualization of TRMM 3B42 precipitation field as a contour plot for three instances of time for a latitudinal range of 20°N to 20°S with the minimum contour level at 1mm/hr and contour interval of 1mm/hr .

Since the present work is concerned with MJO and CCEW which have time periods of at most 90 days, therefore the annual mean rainfall and first three harmonics of the annual cycle are removed from the data. To get the annual mean rainfall, a climatological year is first computed by taking the average of the 17 years of data present in the record. Annual mean rainfall at a location is then calculated by taking the mean of the rainfall at that location for the climatological year. The harmonics of the annual cycle at a location can be calculated by taking the Fourier transform of the year long time series of rainfall for that location for the climatological year. Figure 2.4 shows the annual mean rainfall and the strength of the first three cycles. From figure 2.4 (a) it can be seen that most of the mean rainfall is in the Indian Ocean and Pacific Ocean, while the rainfall over South America is also comparable. This is in accord with the features seen in figure 2.2 and 2.3. There is also a continuous thin line of mean precipitation stretching from the Maritime continents to South America and

Figure 2.3: Time-longitude plot for TRMM-3B42 for the year 2001, averaged between 15.125°N to 15.125°S .

again from South America to Africa. This narrow region of precipitation is known as the ITCZ (Inter Tropical Convergence Zone), as it is where the trade winds converge. Figure 2.4 (b) shows the strength of yearly rainfall cycle, which is strongest near the west cost of Myanmar and west coast of India. This is due to the South Asian monsoon which causes strong yearly variability in rainfall in this region. Even the region over South America shows considerable strength in yearly cycle due to the South American monsoon system (Raia & Cavalcanti, 2008). Figure 2.4 (c) and (d) show the strength of half-yearly and four-monthly cycles which are weaker compared to the yearly cycle.

CCEW are detected in the data by using space-time spectral analysis. This tech-

Figure 2.4: Annual mean rainfall for TRMM 3B42 and the strength of first three harmonic cycle in the climatological year.

nique is very useful in analysing zonally propagating disturbances as it decomposes the time-longitude data into eastward and westward moving waves (Hayashi, 1971). The data is split into symmetric and antisymmetric components about the equator as $[P(\phi)+P(-\phi)]/2$ and $[P(\phi)-P(-\phi)]/2$ respectively, where $P(\phi)$ and $P(-\phi)$ is the precipitation data at the northern and southern hemisphere respectively at latitude ϕ from equator. This splitting is in accord with the shape of the eigenfunctions of equatorial waves which are either symmetric or antisymmetric about the equator.

The width in latitude is limited from 15.125°N to 15.125°S since the equatorial Rossby radius of deformation, which governs the rate of decay wave modes with distance away from the equator, is on the order of 10° of latitude for CCEWs (see

table 1 of Kiladis et al., 2009), therefore the chosen latitudinal range is wide enough to contain these wave modes. It has also been reported by M. Wheeler and Kiladis (1999) that nearly all of the variance due to MJO and CCEW is contained within these latitudes. A choice of wider latitudinal range can include midlatitude variability. This latitudinal range is also consistent with the previous studies which have used TRMM data to investigate CCEWs (Yasunaga & Mapes, 2012a, Yasunaga, 2011) and with the computation of RMM index, used later in this chapter. Since CCEWs are well contained within this latitudinal range the results in this chapter are not sensitive to this range. Data is averaged over this latitudinal range for both symmetric and antisymmetric parts. The missing data points are filled using linear interpolation in time.

Individual spectra are computed for a period of 100 days with an overlap of 20 days between them, which resulted in 78 spectra for the period of 17 years. Before computing each spectra the mean and linear trend in time at each longitude are removed by a least square fit and data is then tapered at the ends by multiplying it with a window function (a rectangular window with the first and last 10% of data being tapered by a cosine function) to reduce spectral leakage. Composite of 78 spectra for symmetric and antisymmetric data is shown in figure 2.5. Both the spectra have high power for lower wavenumber and frequency. Both the spectra have a bright horizontal lines at 1 cpd (cycles per day) and fainter lines at 2 and 3 cpd. They are due to the diurnal cycle of rainfall and its higher harmonics caused by mechanisms such as land and sea breeze.

Significant peaks in these spectra can be seen by comparing them with a spectra representing background noise using a procedure similar to Masunaga et al. (2006) and M. Wheeler and Kiladis (1999). The background noise spectra is a smooth spectra representing a red noise background in both wavenumber and frequency (i.e. the spectral power falls with both wavenumber and frequency). The statistically significant spectral peaks (99% significant based on Chi-square test for 2x78 dof)

Figure 2.5: Composite symmetric and antisymmetric spectra with wavenumber on x-axis and frequency on y-axis in cycles per day (cpd).

in the symmetric and antisymmetric composite spectra, along with the background spectra is shown in figure 2.6. The dispersion curves for the equatorial wave modes are superimposed upon the spectral peaks. From the figure it can be seen that the significant spectral peaks do lie along the theoretical dispersion curves of different equatorial wave modes (except MJO). These features have been previously discovered in OLR by M. Wheeler and Kiladis (1999) and in TRMM-3B42 v6 by Yasunaga and Mapes (2012a). It can be seen that activity for all CCWE seem to have a lower bound of equivalent depth as 6m, while from the figure, the upper bound for Kelvin waves (refer to eq. (1.32)) is 70m, $n=1$ WIG waves (refer to eq. (1.25) (a)) is 60m and ER waves (refer to eq. (1.25) (b)) as 90m. For the antisymmetric spectra the upper bound for both $n=2$ WIG and MRG waves (refer to eq. (1.25) (a) and eq. (1.28)) is seen to be 50m. The statistically significant peaks for Kelvin waves were seen even for frequencies greater than 1 cpd and wavenumber upto 50. While for $n=1$ and 2 WIG waves for frequencies greater than 2 cpd and extending upto wavenumber of 100 (not shown). Such a broad range of wavenumber and frequencies for the Kelvin and WIG waves in TRMM-3B42 data is consistent with the findings of Tulich and Kiladis (2012). Composite spectra as shown in figure 2.5 and significant spectral peaks as shown in figure 2.6 have also been computed separately for DJFM and JJAS and are

given in appendix D.

Figure 2.6: Sub-figure (a) and (b) show the contour plot of significant peaks in symmetric and antisymmetric spectra with the lowest contour at .1072. Superimposed on these figures are the dispersion curves of equatorial wave modes for different equivalent depths so as to contain the significant peaks between them. The filters for the different wave modes are also highlighted with thick dashed boxes. Sub-figure (c) zooms into sub-figure (a) to clearly show the filters for MJO and Kelvin waves. Sub-figure (d) shows the background spectra relative to which the significance of spectral peaks is estimated.

Precipitation anomalies due to individual wave modes can be filtered out from the data by computing the inverse of space-time spectra. For this, the spectra is calculated for the complete record of 17 years (mean and linear trends are removed and tapering window is applied as before) then the time longitude records for a particular mode can be produced by selecting only those wavenumbers and frequencies which lie within the filter of interest. This process of filtering wave modes is the same as used by M. Wheeler and Kiladis (1999), M. Wheeler et al. (2000), Straub and Kiladis (2003a),

Masunaga et al. (2006). The filters containing the frequency and wavenumbers for different wave modes are also shown in figure 2.6 with a dashed thick black box. Although the significant peaks were seen upto a frequency of 2cpd, for designing the filters however we have chosen an upper bound of 0.8 cpd for WIG waves and 0.5 cpd for Kelvin waves, and the upper limit for wavenumber is 25. Within the bounds of these frequency and wavenumbers the filters follow the dispersion curves which contain the significant spectral peaks. These filters agree well with the filters used in previous studies (M. Wheeler & Kiladis, 1999, M. Wheeler et al., 2000, Yasunaga & Mapes, 2012a). The filters are designed such that the strong spectral peaks for different wave modes lies well within the filter. The major precipitation anomalies in the filtered results are not sensitive to the filter range as long as the strong spectral peaks are contained within the filter. Figure 2.8 shows the filtered precipitation anomalies for Kelvin waves using a modified filter with a cut-off at 0.35 cpd. It can be seen by comparing figure 2.7 (c) and figure 2.8 (b) that the strong wave activities are present in both the figures. A few such cases are a Kelvin wave originating near the dateline in the beginning of March and moving to the western Pacific, Strong wave activity in the months of April, May and June from Maritime continents to the western Pacific, A strong Kelvin wave in the month of August and then October.

In the present results, there are bright red spots both in the symmetric and anti-symmetric spectra in the center bottom (corresponding to MJO) but there is a continuum of significant spectral peaks from Kelvin waves to MJO. Both Kelvin waves and MJO are recognized as different dynamical modes (Straub & Kiladis, 2003a, Roundy, 2008), and in this study we proceed by considering them different. For MJO, the filter spans the wavenumber from 1 to 7 and frequencies from 0.033 to 0.0167 cpd (corresponding to time periods of 30 to 60 days), and for Kelvin waves the lowest frequency in the filter is set to 0.04 cpd which corresponds to a period of nearly 25 days. Using these filters, MJO and Kelvin waves can be separated as seen in figure 2.7. MJO is seen as a large scale disturbance (mostly wavenumber 1 and 2) moving

eastward at approximately 5m/sec while Kelvin waves are seen as fast moving streaks of convection moving eastward at smaller spatial scales. Such propagation speeds for MJO are in agreement with previous studies (Zhang & Dong, 2004, Hendon & Salby, 1994). Note that although the MJO filter is shown in the symmetric spectra the splitting of data into symmetric and antisymmetric components was not used for filtering out MJO since there is no dynamical reason to do it for MJO and it has component in both symmetric and antisymmetric data.

It is also known that there is a seasonal cycle to the strength of MJO. The strength of MJO events is greatest in the months of DJFM (December January February March), while there is also a second peak in the strength in JJAS (June July August September) with a northward latitudinal migration likely related to the summer monsoon (Zhang & Dong, 2004, Lawrence & Webster, 2002, Salby & Hendon, 1994). Therefore, the investigation for the modulation of the strength of CCEW by MJO is studied separately for the months for DJFM and JJAS.

MJO is defined on the basis of real-time multivariate (RMM) index, M. C. Wheeler and Hendon (2004). This index is based on empirical orthogonal functions (EOFs) of the combined fields of zonal (East-West) wind at upper and lower level of troposphere (850 hPa, 200 hPa) and OLR, averaged between the latitudinal range of 15°N to 15°S. The movement of MJO along the equator is described on the basis of the leading pair of EOFs, known as RMM1 and RMM2. State of MJO at a time can be diagnosed by projecting the atmospheric data on RMM1 and RMM2 and can be represented as point on the phase space diagram defined by RMM1 and RMM2. Figure 2.9 shows the activity of MJO for the period of three months (from October 2017 to December 2017) on the phase space diagram.

Amplitude of MJO is defined as $\sqrt{RMM1^2 + RMM2^2}$; an MJO is strong when the amplitude ≥ 1 and weak otherwise. A strong period of MJO is further divided into 8 phase which correspond to the location of enhanced convection along the equator. In figure 2.9, there is a strong MJO event in the month of October (red curve) moving

(a)

(b)

(c)

Figure 2.7: The time longitude plots for (a) TRMM-3B42 intraseasonal anomalies, (b) filtered MJO and (c) filtered Kelvin waves for the year 2001. The filtered MJO successfully captured the strong MJO events starting at February, August and mid December and shows a propagation speed of nearly 5m/sec. The filtered Kelvin waves capture the faster eastward moving convective anomalies.

Figure 2.8: Sub-figure (a) shows the two filters for Kelvin waves and sub-figure (b) shows the filtered results for Kelvin waves obtained by using the second filter having the cut-off at 0.35 cpd.

anticlockwise from phase 4 to 7, and the convection moves from Maritime continents to Western Pacific Ocean. During the month of November, MJO is weak, and is seen as random activity within the circle (green curve), while in December MJO gets strong again.

In the precipitation data, which ranges from 1998 to 2014, there are 16 DJFMs and each might contain several MJO events. We determined the strength of an MJO event by the amplitude calculated from RMM1 and RMM2 indices. Time within each DJFM is divided into periods of strong and weak MJO, where a period of strong MJO is defined as a period when $RMM \geq 1$, and a period of weak MJO is when $RMM < 1$. RMM data for this study is downloaded from (*RMM data*, n.d.). The downloaded data has a resolution of one day, however for the present study it is converted into 3-hourly by repeating every data point in the original data eight times (for 3-hourly resolution there are eight data points for one day).

For all 16 DJFMs there are a total of 10,280 instances of $RMM \geq 1$ and 5,240

Figure 2.9: RMM1 RMM2 phase plot from October to December 2017. The region for weak MJO is within the circle at the center, while strong MJO is divided into 8 phase as labeled in the figure. The regions where MJO causes enhanced precipitation are also labeled, *MJO pahse diagram* (n.d.).

instances of $RMM < 1$. Some of the strong periods can be as short as 2 days and cannot possibly mean an MJO event. Therefore, only periods of strong MJO when the amplitude ≥ 1 for 20 days or longer and periods of weak MJO when amplitude is < 1 for a period of 10 days or longer are considered. This gives 6,760 instances of strong MJOs and 3,368 instances of weak MJOs for all 16 DJFMs.

Similarly for the 17 JJASs (June July August September) in record, there are a total of 9,720 instances of $RMM \geq 1$ and 6,872 instances of $RMM < 1$. After keeping a limit of 20 days for strong MJO and 10 days for weak MJO, there are 4,968 instances for strong MJO and 4,670 instances of weak MJO.

Although MJO is usually associated with a minimum time period of 30 days, a minimum period of 20 days is considered for strong MJO, since RMM data showed a large number of strong MJO events with period between 20 to 30 days and by doing

so as much data as possible is retained for analysis. Figure 2.10 (a) and (b) show the 10,280 instances of $RMM \geq 1$ and 9,720 instances of $RMM \geq 1$ for DJFM and JJAS respectively with the vertical lines separating individual strong MJO events. Figure 2.10 (a) and (b) contain a total of 94 and 113 strong MJO events for DJFM and JJAS respectively. By considering a minimum time period of 20 days for strong MJO, there are 6,760 instances with $RMM \geq 1$ and 27 strong MJO events for DJFM and 4,968 instances with $RMM \geq 1$ and 22 strong MJO events for JJAS, see figure 2.10 (c) and (d). Upon increasing the time period to 30 days, instances with $RMM \geq 1$ reduce to 3,360 and strong MJO events reduce to 9 for DJFM. For JJAS, instances with $RMM \geq 1$ reduce to 1,848 and strong MJO events reduce to 6, see figure 2.10 (e) and (f).

Therefore, it can be seen that a large number of strong MJO events have a time period between 20 to 30 days and the minimum time period for strong MJO event is selected to be 20 days. Strength of CCEW is studied in the strong and weak periods of MJO separately for DJFM and JJAS and then compared to find any change which may be brought about by MJO.

Composite of precipitation anomalies due to MJO for different phase as a function of longitude is given in figure 2.11 for DJFM and JJAS. Effect of MJO on precipitation seems to be contained mainly from a longitudinal range of 60°E to 180° . Enhancement of precipitation in phase 1 is observed east of 60°E . With the increase in phase, this region of enhanced precipitation moves eastward and reaches a maximum in phase 4 near 120°E . Beyond phase 4, enhanced precipitation moves further east and dies near 180° . MJO also suppresses precipitation on the east of the region of enhanced precipitation for phase 1-4 and on the west of the region of enhanced precipitation for phase 5-8. It should be noted that this figure is constructed as a function of longitude and therefore hides the northward propagation of MJO observed during boreal summer.

Figure 2.10: Sub-figure (a) and (b) give the number of instances with $RMM \geq 1$ for DJFM and JJAS respectively with the vertical lines separating individual strong MJO events. Sub-figure (c) and (d) are same as same as sub-figure (a) and (b) when a minimum time period of 20 days for strong MJO events is considered. Sub-figure (e) and (f) are same as same as sub-figure (c) and (d) when a minimum time period of 30 days for strong MJO events is considered.

Figure 2.11: composite MJO precipitation anomalies for different phase as a function of longitude, over a period of 17 years from 1999 to 2014 for (a) DJFM and (b) JJAS.

2.2 Results

If the MJO does affect the strength of a CCEW, there should be a difference in the precipitation due to that particular wave mode between the strong and weak periods of MJO. In order to look for such a difference, a composite of wave power is made for the two different periods and then compared. The procedure for making the composite is as follows.

During filtering of CCEW with the previously defined filters, the inclusion of background noise is inevitable. In the filtered data, it can be seen that there is a background of low amplitude noise spreading nearly uniformly in space and time upon which the CCEW are seen as activities of high amplitude precipitation, figure 2.7. Therefore, the data filtered for a CCEW mode contains both noise and wave activities.

The strength of this background noise is estimated separately in the filtered data for each CCEW for both DJFM and JJAS. To calculate the strength of background noise for a CCEW, a mean of square is calculated of all data points in the time longitude plot which contains the filtered data for that CCEW. This is done for each DJFM and JJAS. This gives 16 variances for 16 DJFMs and 17 variance for 17 JJASs since in the record (considered from 1998 to 2014) there are 16 DJFMs and 17 JJASs. Mean of these variances is calculated and square root of it gives an estimate of the strength of the background noise.

$$\text{Background noise} = \sqrt{\frac{var_1 + var_2 + \dots + var_m + \dots + var_n}{n}}, \quad (2.1)$$

where var_m is the variance for the m^{th} DJFM or JJAS and $n = 16$ or 17 for DJFM or JJAS respectively.

Using this estimate of the strength of background noise, only those points in a filtered data are considered significant which exceed the background noise by a certain

amount. This is shown in figure 2.12, where the filtered Kelvin wave data is shown in the top sub-figure (a) with a vertical line marking the longitude for which the time series is plotted in the bottom sub-figure (b). The bottom sub-figure (b) has two horizontal lines at $\pm$ twice the background noise level. Only those data points which exceed these horizontal lines are considered significant. Strength of a wave is compared for the strong and weak periods of MJO by comparing its wave power, where wave power is defined as the square of absolute values of significant data points (given in red colour in figure 2.12 (b)). Hence wave activity for the strong and weak periods of MJO is compared by computing the mean wave power over these periods.

(a)

(b)

Figure 2.12: Sub-figure (a) shows time longitude plots for filtered Kelvin waves for the year 2001 with the vertical line marking the longitude 0.125W from a period of 1st January to 1st April 2001. Sub-figure (b) shows the time series of the filtered Kelvin wave data for the same longitude and time interval. Black horizontal lines mark $\pm$ twice the background noise level. Significant data points that exceed the background noise are shown in red, while other data points are shown in black.

2.2.1 Kelvin waves

The composite Kelvin wave power for the months of DJFM and JJAS over strong and weak periods of MJO is shown in figure 2.13. The red line refer to strong MJO ($RMM \geq 1$) and the blue line refers to weak MJO ($RMM < 1$). The distribution of Kelvin wave power over the longitude agrees well with the results previously seen in M. Wheeler and Kiladis (1999) and M. Wheeler et al. (2000). There are three major peaks in the power of Kelvin waves in DJFM, one in the Indian Ocean, another broad peak in the central Pacific Ocean and a third narrow peak in the continent of South America. For JJAS, there are two strong peaks in the Indian Ocean and Pacific Ocean, and a third small peak near the western coast of South America. It can be clearly seen from the figure 2.13 (a) that for DJFM in the strong period of MJO, Kelvin wave power is enhanced in the regions of first two peaks i.e. over the Indian Ocean and the Pacific Ocean, while there is a slight reduction in wave power in the region of third peak over South America. For JJAS, a strong enhancement in Kelvin wave power is seen over the Maritime continents (near 120°E) instead of being over the Indian Ocean or the Pacific Ocean, where the wave power peaks. There is reduction in Kelvin wave power from the central to the eastern Pacific Ocean during the strong period of MJO. There is also a slight reduction in wave power during the strong periods of MJO just east of the International Dateline.

To see the differences more clearly, the difference in wave power between the strong and weak periods of MJO for DJFM and JJAS is plotted in figure 2.13 (c),(d). Horizontal lines in the figure are at plus minus one standard deviation of the wave power difference over the whole longitudinal range. Difference in composite wave power produced by MJO is considered significant if it exceeds this threshold. Vertical lines in figure 2.13 (c) and (d) demarcate such longitudinal range. Exact value of these longitudinal range is given in table 2.1 and 2.2.

Figure 2.13 reveals that strong MJO does have an effect on the Kelvin wave

Figure 2.13: Sub-figure (a) and (b) show Kelvin wave power in the strong and weak periods of MJO for DJFM and JJAS. Sub-figure (c) and (d) show the difference in wave power between strong and weak periods of MJO for DJFM and JJAS

strength. The next step in the line of inquiry is to see in which phase of strong MJO does the amplification or attenuation of wave power occurs. A composite of Kelvin wave power for different phases of MJO for DJFM is given in figure 2.14. Composite wave power for strong MJO over different phase is shown in red while the composite wave power for periods of weak MJO is shown by the dotted blue line. The error bars represent the standard error of mean. It can be seen from the figure 2.14 (a) that Kelvin waves are amplified in the Indian Ocean during the phase 2, 3 and 4 and attenuated in the phases 5 to 8, with a sudden drop in wave power from phase 4 to 5. The amplification of wave power over Indian Ocean is stronger than attenuation,

giving an overall increase in wave power as seen in figure 2.13(a).

For the second longitudinal range, covering regions from Maritime continents to the central Pacific Ocean, it can be seen that the attenuation effect is very weak and present only in phase 2 and 8 of MJO when its convective center is over the Indian Ocean and central Pacific ocean respectively. Overall, there is an amplification produced by the MJO from phase 3 to 7 with a maxima in phase 5, during which the convective center of MJO moves from the Indian Ocean to the western Pacific Ocean, see figure 2.14(b). Finally, for the region over South America there is wave amplification for phase 1 and 2 when the convective envelop of MJO moves from Africa to the Indian Ocean. During these two phase there is a negative precipitation anomaly in this region due to the MJO, see figure 2.11 (a). Perhaps it is due to this slightly enhanced precipitation that an increase in wave power is seen. Attenuation is seen from phase 4 to 7 during which the MJO's convective center moves from Maritime continents to western Pacific Ocean and there is a negative precipitation anomaly in this region. Since attenuation in wave power occurs over more phases, this gives an overall reduction in wave power as seen in figure 2.13.

A similar MJO phase wise breakdown of Kelvin wave power is also performed for JJAS for the marked longitudinal ranges and is given in figure 2.15. For the region over Maritime continents to Pacific Ocean, there is a strong amplification in wave power in phase 4 and 5 and a weak amplification in phase 7. There is some attenuation in phase 2, although weak. It seems that the amplification produced in phase 4 and 5 is responsible for the overall increase in wave power seen in figure 2.13. For the region near the western coast of South America, there is attenuation of wave power at all phases with the least attenuation for phase 1 and 6. This can be contrasted with figure 2.14(c), where there is amplification produced in phase 1 and 2. Strong reduction in wave power is likely due to negative precipitation anomalies related to MJO in this region from phase 2 to 5. For the region near the International Dateline, there is a slight reduction in wave power during the strong periods of MJO.

The reduction in power is contributed from phase 1, 2, 3, 7 and 8 as seen in figure 2.15 (c). There is negative precipitation anomaly due to MJO for phase 1, 2 and 3 in this region which likely results in the reduction of wave power, see figure 2.11 (b). Highest enhancement in wave power can be seen for phase 6 when MJO results in a strong positive precipitation anomaly in this region. A slight enhancement of wave power can also be seen for Phase 4. However, the wave power enhancement is not sufficient to cancel the attenuation contributed by the before mentioned phases, resulting in an overall slight reduction in wave power.

In summary, usually strong periods of MJO enhance Kelvin waves for longitudinal ranges covering regions from the Indian Ocean to the central Pacific Ocean. Enhancement in wave strength during the convectively active phase of MJO is greater than the reduction during the convectively suppressed phase, resulting in an overall increase in wave power. For regions near South America, Kelvin waves are suppressed during strong periods of MJO.

Figure 2.14: Kelvin waves modulation in different phase of MJO in DJFM (a) over Indian Ocean, (b) Maritime continents to Pacific Ocean (c) near South America. The error bars represent twice the standard error of mean.

Figure 2.15: Kelvin waves modulation in the different phase of MJO in JJAS (a) over Maritime continents (b) near South America and (c) in the western Pacific Ocean.

2.2.2 WIG waves

The composite wave power of westward inertia gravity waves for $n = 1$ and 2 , for strong and weak periods for MJO is shown in figure 2.16 for $n=1$ and in figure 2.17 for $n=2$. A striking difference between these two set of figures and the figures showing the composite power for Kelvin waves is that for WIG waves MJO is able to produce a strong reduction in wave power, generally in the region over the Pacific Ocean. For WIG $n=1$, strong periods of MJO also attenuates the waves over Maritime continents in DJFM, near 120°E . For WIG $n=2$, wave power is attenuated in the Indian Ocean in DJFM, although the reduction of wave power is not as strong as seen over the Pacific Ocean.

Strong MJO also results in enhancement of WIG wave power over certain regions. Two such significant regions of enhancement can be seen for WIG $n=2$ waves. The first region stretches from the Indian Ocean to Maritime continents for DJFM and another region lies near international date line for JJAS, see figure 2.17 (c) and (d). The regions of enhancement of wave power can be seen for WIG $n=1$ waves also, over the Indian Ocean, near International date line, South America and Western Africa. However, the enhancement of wave power in these regions is not as prominent as the reduction of wave power seen over the Pacific Ocean, figure 2.16 (d).

Similar to the analysis done in the previous section for Kelvin waves, the horizontal lines in figures 2.16 and 2.17 (c) and (d) are at plus minus one standard deviation of the wave power difference over the whole longitudinal range. The exact values for the position of vertical lines marking the longitudinal ranges for significant wave power deviation are given in table 2.1 and 2.2.

To understand the effect of MJO on WIG waves better, a breakdown of wave power for different MJO phase is done for the selected longitudinal ranges. Figure 2.18 shows the variation of WIG $n=1$ wave power with different phases of MJO in DJFM over different longitudinal ranges. Figure 2.18 (a) reveals that there is a slight

amplification in waves in the phase 4 and 5 when the MJO's convective center is over Maritime continents, while there is reduction in wave power for all the other phase of MJO. The reduction in power suffered by the wave in other phases is stronger than the weak amplification produced in phase 4 and 5, hence there is an overall reduction in power as seen in figure 2.16 (c) near 120°E. Variation of WIG n=1 wave power with MJO phase over the Pacific Ocean (near 180°) in DJFM is given in figure 2.18 (b). The amplification of wave power shifts to phase 7 when the MJO's convective center is over the central Pacific Ocean. The attenuation of wave power is greatest from phase 2 to 5, while the attenuation is weak in phase 1, 6 and 8. It can be observed from figure 2.18 (b) that the attenuation produced from phase 2 to 5 (during which the MJO's convective center moves from the Indian Ocean to Maritime continents) is stronger than the weak amplification produced in phase 7 giving an overall reduction in wave power over the Pacific Ocean as seen in figure 2.16 (c). MJO also results in a reduction in WIG n=1 wave power near the western coast of Africa as seen in figure 2.16 (c). An MJO phase wise breakdown of this variation in wave power is given in figure 2.18 (c), from which it can be seen that the reduction in wave power can be attributed to phase 4 and 5 of MJO, as the attenuation of wave power in the other phases of MJO is relatively small.

MJO phase wise breakdown of WIG n=1 wave power for JJAS for various longitudinal ranges is given in figure 2.19. As seen in figure 2.19 (a), variation of wave power with MJO phase over Maritime continents for JJAS is similar to as seen for the wave power variation over Maritime continents in the season of DJFM as seen in figure 2.18 (a). However, the wave power enhancement in JJAS stretches from phase 4 to 6 (when MJO's convective center moves from the Indian Ocean to the western Pacific Center) with the minimum amplification in phase 5. Also, the attenuation in wave power produced in phase 1 to 3 is stronger than the attenuation in phase 7 and 8 when the MJO's convective center is in the Indian Ocean (for phase 1 to 3) and the eastern Pacific Ocean (for phase 7 and 8).

As opposed to the observation in DJFM, there is an overall increase in wave power due to the influence of MJO in the regions of Western Africa in JJAS. An MJO phase wise breakdown of this variation is given in figure 2.19 (b). An increase in wave power is seen in phase 1 and 2, while there is also an increase in wave power in phase 4 and 5 which is in contrast to what is seen in figure 2.18 (c). The enhancement of wave power in phase 4 and 5 instead of attenuation is the main contributor to the observed overall increase in wave power, as phase 6 to 8 produce slight attenuation.

Slight increase in WIG $n=1$ wave power due to strong MJO in JJAS is also seen over narrow longitudinal range, one in the Pacific Ocean (near 180°) and another near South America (near 60°W), figure 2.16 (d). Figure 2.19 (c) reveals that the increase in wave power occurs over a range of phase, from 2 to 7, in the Pacific Ocean. However, uncertainty in phase 2 and 3 is greater and wave power enhancement can be attributed to phase 4 to 7. Near South America, the trend is nearly reversed with wave power attenuation from phase 3 to 6 and slight enhancement in phase 1, 2, 7 and 8, figure 2.19 (d).

MJO phase wise breakdown of WIG $n=2$ wave power for the four marked longitudinal range for DJFM is given in figure 2.20. In figure 2.17 (c), MJO has an overall attenuating effect on wave power in the Indian Ocean over the longitudinal range centered at nearly 80°E , while in the neighboring longitudinal range extending from Indian Ocean to Maritime continents there is an overall increase in power. Variation of wave power over these two longitudinal range with MJO phase is revealed in figure 2.20 (a) and (b). The basic variation of power with MJO phase seen in these two figures is similar to the variation in wave power seen for Kelvin waves over the Indian Ocean in figure 2.14 (a). In the first longitudinal range over the Indian Ocean, wave power enhancement is weak and occurs only in phase 2 and 3, while wave power attenuation is strong and stretches from phase 4 to 8 with a minima in phase 5, resulting in an overall reduction in wave power. For the second longitudinal range, wave power amplification is strong and occurs over a range of phase from 1

to 5. Wave power attenuation is however weak and is seen from phase 6 to 8, thus giving an overall increase in wave power.

For region over the Pacific Ocean, there is an overall reduction in WIG $n=2$ wave power by MJO. The MJO phase wise variation of wave power over the Pacific Ocean is given in figure 2.20 (c), which shows slight enhancement in wave power for phase 7 and attenuation for all other phase. This is similar to the variation of WIG $n=1$ wave power in the Pacific Ocean, figure 2.18 (b). Like WIG $n=1$, there is also a reduction in wave power seen for WIG $n=2$ over Western Africa. Figure 2.20 (d) indicates that reduction in power stretches from phase 3 to 7. Reduction in wave power in phase 4 and 5 is maximum, however attenuation at other phases is comparable as opposed to WIG $n=1$, where reduction in wave power is dominated by phase 4 and 5. A slight increase in wave power is also seen at phase 2 and 8.

MJO phase wise breakdown of WIG $n=2$ wave power for the three marked longitudinal range for JJAS is given in figure 2.21. The variation in wave power over Maritime continents is similar to WIG $n = 1$ waves in JJAS, figure 2.21 (a). However, wave power enhancement from phase 4 to 6 is more uniform. Another minor point of difference is that attenuation is stronger in phase 7 and 8 as compared to phase 1 to 3.

Over the Pacific Ocean, WIG $n=2$ waves experience an increase of strength in the strong periods of MJO. Figure 2.21 (b) shows that there is an increase in wave power for all phase of MJO except phase 1 with maximum phase amplification occurring in phase 5 and 6. In region over Central Africa, WIG $n=2$ waves show an increase in power in strong periods of MJO. Figure 2.21 (c) shows that the increase in power is present over all phase except phase 4, 6 and 7 with maximum power for phase 5. Wave power attenuation is weaker in phase 4, 6 and 7 as compared to the power amplification in phase 1, 2, 3 and 5, therefore an overall increase in power is observed.

In summary, it can be seen that MJO is able to influence WIG waves over multiple locations such as Africa, western coast of south America other than the Indian Ocean,

Maritime continents and the Pacific Ocean. Enhancement of WIG waves is seen during the convectively active phase of MJO at most of the locations, this is similar to the results of Nakazawa (1988) and Kikuchi and Wang (2010). However, the suppression of waves in the convective suppressed phase is greater, resulting in an overall reduction in wave power unlike the results for Kelvin waves. Although WIG waves are suppressed during strong periods of MJO at most of the locations, however slight increase in wave power is seen over certain longitudinal ranges in the Indian Ocean, central Pacific Ocean and near the western coast of South America.

Figure 2.16: Sub-figure (a) and (b) show WIG $n=1$ wave power in the strong and weak periods of MJO for DJFM and JJAS. Sub-figure (c) and (d) show the difference in wave power between strong and weak periods of MJO for DJFM and JJAS respectively.

Figure 2.17: Sub-figure (a) and (b) show WIG $n=2$ wave power in the strong and weak periods of MJO for DJFM and JJAS. Sub-figure (c) and (d) show the difference in wave power between strong and weak periods of MJO for DJFM and JJAS respectively.

(a)

(b)

(c)

Figure 2.18: WIG $n=1$ waves modulation in the different phase of MJO in DJFM (a) over Maritime continents, (b) over Pacific Ocean and (c) near Western Africa.

(a)

(b)

(c)

(d)

Figure 2.19: WIG $n=1$ waves modulation in the different phase of MJO in JJAS (a) over Maritime continents, (b) near Western Africa, (c) over Pacific Ocean and (d) near South America.

(a)

(b)

(c)

(d)

Figure 2.20: WIG $n=2$ waves modulation in the different phase of MJO in DJFM, (a) and (b) over Indian Ocean, (c) over Pacific Ocean and (d) near Western Africa.

(a)

(b)

(c)

Figure 2.21: WIG $n=2$ waves modulation in the different phase of MJO in JJAS, (a) over Maritime continents, (b) over Pacific Ocean and (c) near Central Africa.

2.2.3 MRG westward moving waves

The composite wave power for westward moving Mixed Rossby Gravity waves for the strong and weak periods of MJO is given in figure 2.22. From figure 2.22 it can be seen that the main impact of MJO on MRG waves is to reduce their strength in the Indian Ocean and the Pacific Ocean during its strong periods, which is similar to what is seen in case of WIG waves. As in the previous two sections, the horizontal lines in the figure 2.22 (c) and (d) are at plus minus one standard deviation of the wave power difference and the vertical lines mark the longitudinal range for significant wave power deviation between strong and weak periods of MJO.

Although the most prominent impact of MJO is the reduction in MRG wave power, however strong periods of MJO also result in slight increase in wave power over certain regions such as the Indian Ocean for DJFM, Maritime continents and Pacific Ocean for JJAS.

The MJO phase wise breakdown of MRG wave power for DJFM over the marked longitudinal ranges is given in figure 2.23. Figure 2.23 (a) shows that MRG waves undergo a decrease in wave power for all phase of MJO over the Maritime continents except for phase 5 where there is a slight increase in power. Minimum power is observed in phase 1 and it increases almost linearly from phase 1 to 5. Even the decrease in wave power from phase 5 to 7 is linear and wave power in phase 7 and 8 nearly the same. Figure 2.23 (b) shows the MJO phase wise variation of MRG wave power over the Pacific Ocean. An attenuation of wave power is observed over all MJO phase. The decrease in wave power as compared to wave power in weak periods of MJO is almost constant from phase 1 to 5, which are the main contributors to the observed overall reduction in power. There is attenuation in wave power for phase 7 and 8 also, however it is slightly weaker than the attenuation in phase 1 to 5. Minimum attenuation is seen for phase 6, which even appears as a peak compared to the wave power at other phases.

The breakdown of MRG wave power over different MJO phases for JJAS is given in figure 2.24. The range of longitude covers regions from the Indian Ocean to Maritime continents. Wave power steadily rises from phase 1 to 4 and is attenuated from phase 1 to 3, whereas at phase 4 there is a slight amplification. At phase 5 there is a strong reduction in wave power while again at phase 6 there is a slight amplification. With further increase in phase decrease in wave power can be seen from phase 6 to 8.

MRG signal does not show a decrease during high amplitude MJO events in the west Pacific during boreal summer unlike during boreal winter. This is because a strong increase in wave power is seen in phase 7 when the MJO's convective center is in the western Pacific, see figure E.3 (b) (also the attenuation in wave power during JJAS is weak for phase 1 to 4 as compared to DJFM). Enhancement of MRG waves in MJO's convective envelope in the western Pacific during boreal summer has also been observed by Straub and Kiladis (2003a). During boreal summer the MJO's convective envelope moves both eastward and northwards, also a northwestward movement of the MRG waves in the western Pacific is indicated in the results of Straub and Kiladis (2003a). This could likely result in the MRG waves to be present within the MJO's convective envelope in the western Pacific and get enhanced.

In summary, the results for westward moving MRG waves are similar to the results seen for WIG waves with strong suppression of waves seen in regions from the Indian Ocean to the Pacific Ocean during strong periods of MJO. The overall suppression is due to greater reduction in wave power during the convectively suppresses phases of MJO than the increase in wave power during convectively active phase of MJO.

Figure 2.22: Sub-figure (a) and (b) show westward moving MRG wave power in the strong and weak periods of MJO for DJFM and JJAS. Sub-figure (c) and (d) show the difference in wave power between strong and weak periods of MJO for DJFM and JJAS respectively.

Figure 2.23: MJO phase wise variation of westward moving MRG wave power for DJFM over (a) Maritime continents and (b) Pacific Ocean.

(a)

Figure 2.24: MJO phase wise variation of MRG wave power for JJAS over (a) Maritime continents.

2.2.4 MRG eastward moving waves

The composite wave power for eastward moving MRG waves for the strong and weak periods of MJO for DJFM and JJAS is given in figure 2.25. Figure 2.25 (a) and (c) show that strong periods of MJO produce an increase in wave power in the regions of Indian Ocean and regions stretching from Maritime continents to Pacific Ocean in DJFM. This behaviour is similar to the change in wave power seen for Kelvin waves in figure 2.13 (a) and (c). However, two differences can be pointed out. First, for MRG waves there is a decrease in wave power in the regions of Pacific Ocean near 180°W longitude, while for Kelvin waves an increase of wave power can be seen. Although, the increase of Kelvin wave power over this region of Pacific Ocean is not strong as compared to the increase seen over Indian Ocean and Maritime continents. Second difference is that, there is an increase in wave power over the continent of South America which is comparable to the increase in wave power over Maritime continents, figure 2.25 (c). Figure 2.13 (c) also shows a slight increase in wave power over South America, however it is located at the western coast near the longitude of 80°W .

For JJAS, it can be seen that the region of wave enhancement has shifted westwards over the Pacific Ocean near 180°W where there is a strong enhancement in wave power during strong periods of MJO. A weak enhancement of wave power over the Indian Ocean is also present (east of 60°E), however it is not comparable to the one over Pacific Ocean, figure 2.25 (b) and (d). A clear difference between figure 2.25 (a) and (b) is that eastward moving MRG waves seem not to be active in JJAS in South America and MJO is not able to produce any significant change in the wave power in this region, figure 2.25 (d).

As done for the other wave modes in the previous sections, an MJO phase wise breakdown of eastward moving MRG wave power variation for DJFM over the different marked longitudinal range is given in figure 2.26. In figure 2.26 (a), it can

be seen that there is amplification in wave power from the phase 1 to 4, with the greatest amplification in phase 3 and 4. There is a sharp decrease in wave power from phase 4 to 5, similar to Kelvin waves over Indian Ocean in DJFM. An attenuation in wave power is present from phase 5 to 7 with slight wave amplification in phase 8. A stronger amplification of wave power from phase 1 to 4 as compared to the attenuation in phase 5 to 7 results in an overall increase wave power over Indian Ocean, as seen in figure 2.25 (c). Figure 2.25 (c) also indicates a greater increase in wave power over the Maritime continents. This can be attributed to the fact that an increase in wave power over Maritime continents is observed for all phase of MJO except phase 8, figure 2.26 (b). Amplification of wave power falls slowly from phase 1 to 3 and goes to maximum at phase 4. Wave power amplification is also seen for phase 5 to 7, however, it is not as strong as the amplification present from phase 1 to 3.

The reduction of wave power over the Pacific Ocean with MJO can be interpreted from figure 2.26 (c). Wave power is attenuated at all the phases of MJO except phase 1 giving an overall reduction in power. The attenuation in power is strongest from phase 4 to 6. In phase 2 and 3, the wave power is comparable to the power in the weak periods of MJO. For Phase 7 the attenuation is weak as compared to that seen for phase 6 and 8, and therefore it even appears like a peak in comparison.

For JJAS, there is only one marked longitudinal range for significant deviation in wave power covering region over the Pacific Ocean. An MJO phase wise breakdown of the MRG wave power for this longitudinal range is given in figure 2.27. As expected, the figure shows that the overall wave power amplification produced over different phases of MJO exceeds the attenuation. Strong amplification in wave power is seen in phase 2, 3 and 6. Amplification in phase 7 is comparatively not so strong and wave power in phase 4, 5 and 8 can be compared to wave power during weak periods of MJO.

Unlike for the westward moving MRG waves, strength of eastward moving MRG waves is increased during the strong periods of MJO over most of the locations con-

sidered. The MJO phase wise distribution of wave power over the Indian Ocean is similar to the results seen for Kelvin waves with the increase in wave power from phase 2 to 4 and decrease in wave power in phase 5, 6 and 7. However unlike Kelvin waves there is a reduction in wave power in the Pacific Ocean and enhancement near South America for DJFM.

Figure 2.25: Sub-figure (a) and (b) show eastward moving MRG wave power in the strong and weak periods of MJO for DJFM and JJAS. Sub-figure (c) and (d) show the difference in wave power between strong and weak periods of MJO for DJFM and JJAS respectively.

(a)

(b)

(c)

(d)

Figure 2.26: MJO phase wise variation of eastward moving MRG wave power in DJFM, over (a) Indian Ocean, (b) Maritime continents, (c) Pacific Ocean and (d) South America.

(a)

Figure 2.27: MJO phase wise variation of eastward moving MRG wave power over (a) Pacific Ocean.

Table 2.1: Longitudinal range for significant wave power deviation for DJFM

Wave mode	Indian Ocean	Maritime continents	Pacific Ocean	South America	Africa
Kelvin	85.875°E-110.875°E	144.375°E-181.375°W	-	48.625°W-29.375°W	-
WIG n=1	-	117.675°E-135.825°E	158.625°E-173.125°W	-	2.125°W-11.375°E
WIG n=2	71.125°E-82.375°E	-	156.875°E-166.875°W	-	2.125°W-11.375°E
	90.125°E-111.375°E				
MRG westward	-	96.625°E-135.625°E	148.625°E-165.875°W	-	-
MRG eastward	83.875°E-108.625°E	133.125°E-163.375°E	173.375°E-157.375°W	-	-

Table 2.2: Longitudinal range for significant wave power deviation for JJAS

Wave mode	Indian Ocean	Maritime continents	Pacific Ocean	South America	Africa
Kelvin	-	103.375°E-150.625°E	164.375°E-171.625°E	100.125°W-82.625°W	-
WIG n=1	-	139.375°E-165.375°E	171.625°E-178.125°W	79.625°W-70.125°W	12.375°W-2.625°E
WIG n=2	-	138.125°E-165.375°E	173.375°E-170.875°W	-	10.125°E-19.175°E
MRG westward	-	75.875°E-119.375°E	-	-	-
MRG eastward	-	-	161.375°E-166.375°W	-	-

2.3 Synthesis and discussion

Modulation of CCEW by MJO at intraseasonal time scales is studied using TRMM 3B42 v7 (rainfall data). Based on the 17 years of data used, it is seen that MJO does have the capacity to influence the strength of all CCEW modes under study (convectively coupled ER waves are not a part of this study therefore no comments are made on it) for both DJFM and JJAS. MJO may cause either increase or decrease in their strength based on its phase and region under consideration. The effect of MJO on CCEW wave power is usually strongest in regions where the wave activity peaks. For all of the waves under consideration, these include regions of Indian Ocean, Maritime continents and Pacific Ocean. Effect of MJO on CCEW is not only limited to the ocean basins, significant influence on wave power due to MJO is also present near Africa and South America, if the wave activity peaks in these regions. However, the effect of MJO on CCEW could be nearly half as strong or weaker. Modulation of wave power by MJO is similar for both DJFM and JJAS except that the longitudinal range over which significant wave power deviation occurs may change.

For wave modes showing significant enhancement in power over the Indian Ocean during strong periods of MJO, the increase in wave power occurs in phase 2, 3 and 4 when the convective envelope of MJO is over the Indian Ocean. Moving eastwards,

for Maritime continents, wave power enhancement is concentrated mainly in phase 4 and 5. Further east, for regions over the Pacific Ocean, increase in wave power occurs in phase 5, 6 or 7 depending upon the longitudinal range. Since MJO's convective envelop moves eastward from the Indian Ocean to the Pacific Ocean with increasing phase, this indicates that increase in wave power over any region roughly coincides with the location of MJO's convective envelop. Even decrease in wave power over a region occurs when MJO suppress convection over that region. For the Indian Ocean, decrease in wave power is observed for phase 5, 6 and 7 while for regions over the Pacific Ocean, decrease in wave power mainly occurs in phase 2, 3, and 4. In summary, convectively active (inactive) regions of MJO enhance (reduce) the strength of CCEW over majority of the longitudinal ranges considered. Over some regions the enhancement in wave power is greater than the reduction resulting in an overall increase in wave power, while over other regions reduction in wave power is greater than the enhancement.

MJO's convective envelop does not seem to have a preference for any specific wave mode as all waves experience an increase in power when precipitation is enhanced by MJO. Study by Straub and Kiladis (2003a) indicated that westward moving MRG waves were enhanced in the convectively active region of MJO in the Pacific Ocean, however in the present study it found that MJO is more effective in suppressing MRG waves than enhancing them. This is in line with the conclusion drawn by Yasunaga (2011).

Enhancement in the strength of Kelvin waves is greater than that for other wave modes. This must be contrasted with WIG $n=1,2$ for which MJO is more effective in reducing the wave strength than increasing it. It is also seen that Kelvin waves are not specific to MJO's convective envelop, but are only enhanced in the strong periods of MJO. Figure 2.28 shows the TRMM data filtered for Kelvin waves overlaid with precipitation anomalies due to MJO, for the year 2007. Kelvin waves are strong in the convectively enhanced regions of MJO as can be seen in the months of June and

December and strong wave activity is not seen in the convectively suppressed regions of MJO. However, strong wave activity can also occur outside the convectively active region of MJO as seen in the month of May and September. This is similar to the conclusions of Dunkerton and Crum (1995), Hendon and Liebmann (1994) and Dias et al. (2013). Although, enhancement of Kelvin waves does indicate that MJO changes the environment in such a way as to make it more conducive for the growth of Kelvin waves.

The large scale changes in the tropical atmosphere due to MJO which might affect the growth of convectively coupled waves are explored in the next chapter.

Figure 2.28: TRMM data filtered for Kelvin waves for the year 2007. Black solid (black dashed) contours indicate convectively active (suppressed) regions of MJO. Solid (dashed) contours indicate precipitation anomalies greater (less) than 1.5 times the standard deviation of the TRMM data filtered for MJO.

Chapter 3

Dynamical fields associated with MJO

MJO has a planetary scale structure with its convective signal mainly in wavenumber 1 and 2 and time period of 30 to 90 days. In comparison, CCEW are smaller scale disturbances with their signal dominant from wavenumber 5 to 10 and have a shorter time scale of 1 to 20 days (except convectively coupled ER waves). With such difference in spatial and temporal scales, modulation of CCEW by MJO may be understood by considering MJO as a background on which faster moving CCEW evolve.

Other than modulation in convection, MJO also has large scale fluctuations in diabatic heating associated with convection (among other processes), wind circulation, temperature and humidity linked with the MJO. Coupled to convection in an MJO, is a strong zonal (east-west) wind circulation pattern. Convectively active region of MJO involves wind converging at lower levels (near 850 hPa), rising upwards through the entire depth of the troposphere and diverging at higher level (near 200 hPa). In the convectively suppressed regions of MJO, wind directions are reverse. Winds converge at higher level, sink down and diverge at lower level. Vertical structure of Zonal wind with respect to MJO's convective center can be seen in figure 3 of Kiladis et al. (2005). Meridional (north-south) winds have a structure of forced Kelvin wave and Equatorial Rossby wave (refer to eq. (1.32) and (1.25)). To the east of convective center, the wind pattern resembles that of a Kelvin wave while to the west the low level (high level) winds form a pair of cyclone (anticyclone) which resemble Equatorial

Rossby wave, Rui and Wang (1990).

Large scale variations in humidity have also been recognized to be a part of MJO cycle, and even hypothesized to be important for its dynamics, Kemball-Cook and Weare (2001). Convectively active (inactive) regions of MJO have been associated with anomalously high (low) humidity for MJO events during northern hemisphere winter (November-April), Myers and Waliser (2003). Even variation in moisture in the upper troposphere (between 200 and 100 hPa) due to MJO have been observed, Mote et al. (2003). Kiladis et al. (2005) showed that the lower troposphere is anomalously moist to the east of MJO's convective envelope, while to the west it is anomalously dry.

Diabatic heating due to MJO's convective envelope was studied by J. Lin et al. (2004), where the vertical diabatic heating profile of MJO's convective envelope was recognized to be top heavy, i.e. there is more heating in the upper troposphere than in the lower troposphere. Similar results were also produced by Kiladis et al. (2005). Diabatic heating fluctuations due to MJO for Northern hemisphere winter (November-April) for different MJO phase was studied by Ling and Zhang (2011) and Jiang et al. (2011). Regions of enhanced (reduced) precipitation were found to overlap with the regions of enhanced (reduced) diabatic heating. Enhanced diabatic heating also moved eastwards from Indian Ocean to Pacific Ocean from phase 1 to 8 along with the precipitation anomaly.

Although large scale fluctuations in dynamical fields associated with MJO have been investigated before, however, most of the previous research do not include MJO phase wise evolution of such dynamical fields in the vertical plane over the whole longitudinal range for both seasons of DJFM and JJAS. The objective of current chapter is to explore the relation of these fields with MJO for different MJO phase over the whole longitudinal range, for seasons of DJFM and JJAS. The results from this chapter can be used to understand how the MJO affects the background on which CCEW evolve.

3.1 Data and methodology

Data used in this study is the National Centers for Environmental Predictions-National Center for Atmospheric Research (NCEP-NCAR) reanalysis, Kalnay et al. (1996), for a period of 17 years from 1998 to 2014. Data fields have a spatial resolution of $2.5^\circ \times 2.5^\circ$ and temporal resolution of one day. Data for zonal wind, meridional wind, vertical wind and air temperature is used for 12 vertical pressure levels from 1000 hPa to 100 hPa. Data for relative humidity is available for only eight vertical pressure levels upto 300 hPa. All data is considered for latitudinal range extending from 15°N to 15°S which is consistent with the latitudinal range considered in chapter 2 for TRMM 3B42 dataset.

Data is averaged over the latitudinal range and filtered for eastward moving waves with wavenumber 1 to 7 and time period of 30 to 90 days for all vertical pressure levels, in order to extract the fluctuations associated with MJO. Annual mean and first three harmonics of the annual cycle are removed from all the data fields prior to filtering, as done for TRMM 3B42 dataset in the previous chapter. Composite data fields for every MJO phase are calculated from the filtered data for DJFM and JJAS, for strong periods of MJO, over the time period of 17 years. For weak periods of MJO, composite are created for DJFM and JJAS by keeping the annual mean and the annual cycles in the data fields. Therefore, data field composite for weak periods represent the state of atmosphere during a season (DJFM or JJAS) in absence of MJO, while filtered data field composite for a MJO phase represent the fluctuation in the state of atmosphere due to MJO during that phase.

Total diabatic heating $Q1$, is calculated for latitudinal range from 15°N to 15°S using the thermodynamic equation

$$Q1 = \frac{C_p T}{\theta} \left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} + \omega \frac{\partial \theta}{\partial p} \right), \quad (3.1)$$

where u, v and ω are zonal, meridional and vertical winds respectively; T is the temperature; $\theta = T(p_s/p)^{R/C_p}$ is the potential temperature; p is the pressure and p_s is the surface pressure; C_p is the specific heat capacity and R is the specific heat constant of dry air. This procedure has been previously used by Yanai et al. (1973), Hagos et al. (2010), Nigam et al. (2000) and Ling and Zhang (2011).

Similar to other data fields, composite for diabatic heating are created for different MJO phase and also for weak periods of MJO for both DJFM and JJAS.

3.2 Results

3.2.1 Zonal wind

Zonal wind anomalies due to MJO, for phase 1 to 8, for different pressure levels as a function of longitude is shown in figure 3.1 and figure 3.2 for DJFM and JJAS respectively. The wind anomalies have first baroclinic mode structure in the vertical. In figure 3.1 for phase 1 near 60°E, in the lower half of the troposphere, zonal winds show positive anomalies to the West of 60°E while to the east there is a negative anomaly. This indicates convergence in the lower troposphere. Situation in the upper half of the troposphere is reverse with negative anomalies on the west and positive anomalies to the east, indicating divergence. Convergence in the lower troposphere and divergence in the upper troposphere also indicates upward motion of wind in the region. To the east (just west of 180°W) there is a region of sinking wind with convergence in the top half of the troposphere and divergence in the lower half, completing the zonal wind circulation pattern due to MJO in the vertical plane. This circulation pattern moves eastwards with increasing phase. Region of sinking wind reaches near 60°E for phase 6 (in both DJFM and JJAS) and shifts further eastwards in phase 7 and 8.

It should also be noted that the zonal winds anomalies due to MJO also vary

significantly in the meridional direction, although it is not captured in the present results. This can be seen in the results of Zhang and Dong (2004). There is strong meridional shear in the zonal wind anomalies due to the presence of cyclonic and anti-cyclonic gyres which are an integral part of the wind anomalies associated with MJO (Rui & Wang, 1990, Hendon & Salby, 1994). During the position of MJO's convective center between 80°E to 90°E , strong westerly winds result in strong meridional shear to the west of convective center and accompanying cyclonic circulation to the north of equator and a weaker circulation in the southern hemisphere, as reported by Maloney and Hartmann (1998). The study also identifies that positive precipitation anomalies due to MJO near Central America are coincident with cyclonic shear of zonal wind at 850 hPa, while in the later phase, during negative precipitation anomalies over this region there is anticyclonic shear of 850 hPa zonal wind.

During the boreal summer there are strong meridional shifts in the location of MJO's zonal wind anomalies. This will result in strong meridional shear in the zonal wind anomalies. It has been suggested that the structure of equatorial waves most affected by the meridional shear are the westward moving MRG waves and equatorial Rossby waves (Zhang & Webster, 1989). For CCEW, the meridional shear does not seem to have a significant effect on Kelvin waves, however the instability of westward moving MRG waves and equatorial Rossby waves is enhanced (Han & Khouider, 2010).

Zonal wind pattern for weak periods of MJO for DJFM and JJAS is given in figure 3.3. Convergence (divergence) in lower (upper) troposphere can be seen to the east of 120°E in the maritime continents, indicating rising winds and heavy precipitation, which can be seen in figure 2.4 (a). The strong lower (upper) tropospheric easterlies (westerlies) to the east of 180°W in figure 3.3 (a) are due the returning monsoon in DJFM, while strong lower (upper) westerlies (easterlies) to the west of 120°E in figure 3.3 (b) are due to the oncoming monsoon in JJAS.

Figure 3.1: Composite zonal wind anomalies from phase 1 to 8 for DJFM.

Figure 3.2: Composite zonal wind anomalies from phase 1 to 8 for JJAS.

(a)

(b)

Figure 3.3: Composite zonal wind pattern in the troposphere for weak periods of MJO for (a) DJFM and (b) JJAS; white region in (b) denotes values less than -15 m/sec.

3.2.2 Relative humidity

Relative humidity anomalies associated with MJO, for phase 1 to 8, for different pressure levels and as a function of longitude is shown in figure 3.4 and figure 3.5 for DJFM and JJAS respectively. Superimposed on the relative humidity anomalies are precipitation anomalies due to MJO. MJO seems to modulate the relative humidity from a pressure level of 700 hPa to 300 hPa with maximum modulation near 500 hPa. Positive anomaly in relative humidity can be seen from a longitudinal range of 0°E to 60°E in phase 1 for both DJFM and JJAS. With increase in phase, the positive anomalies move eastward and reach the Pacific Ocean by phase 8. Regions of positive (negative) anomalies of relative humidity seems to be well co-located with the regions of enhanced (suppressed) convection. Moving from phase 1 to 4, positive anomaly in both relative humidity and precipitation gets stronger as they move eastwards, while from phase 5 onward they progressively weaken. Such correlation is however not so strong for phase 7 and 8 when the region of enhanced relative humidity is in the central Pacific Ocean to the east of 180° .

Composite relative humidity in the troposphere for weak periods of MJO for DJFM and JJAS is given in figure 3.6. High relative humidity can be seen at lower levels, since ocean surface is the source of moisture in the atmosphere. Higher relative humidity above 700 hPa can be seen near Maritime continents and South America (near 120°E and 60°W), as these regions are dominated by strong convection and convection lifts the air to saturation, thus resulting in higher relative humidity. In comparison to figure 3.6 (a), higher relative humidity can be seen in figure 3.6 (b) for JJAS over a longitudinal range of 60°E to 90°E and near 0°E . This can be attributed to convection due to the monsoon cycle as seen in North et al. (2014).

Relative humidity is a relevant variable to consider as previous studies have pointed out that it can be an important control on convection. Correlation between rainfall and mid to upper-level relative humidity has been pointed out by Brown and

Zhang (1997). Dependence of convection on relative humidity has also been suggested by Takemi et al. (2004) and Roca et al. (2005). Saturation deficit as a control on convection has been used in the theoretical study by Zehnder (2001) and also in the theoretical model of convective coupled equatorial waves developed by Khouider and Majda (2006). Since the results of the present section might be used to study CCEWs in MJO's background, variation in relative humidity due to MJO can prove to be important in modifying the theoretical model of CCEW.

Figure 3.4: Composite relative humidity anomalies superimposed with precipitation anomalies (mm/hr) from phase 1 to 8 for DJFM.

Figure 3.5: Composite relative humidity anomalies superimposed with precipitation anomalies (mm/hr) from phase 1 to 8 for JJAS.

(a)

(b)

Figure 3.6: Composite relative humidity pattern in the troposphere for weak periods of MJO for (a) DJFM and (b) JJAS.

3.2.3 Total diabatic heating

Total diabatic heating (Q_1) anomalies associated with MJO, superimposed with precipitation anomalies for phase 1 to 8, for different pressure levels and as a function of longitude is shown in figure 3.7 and figure 3.8 for DJFM and JJAS respectively. MJO modulates Q_1 heating almost through the entire depth of troposphere from 850 hPa to 150 hPa. The heating is top heavy with maximum heating above 500 hPa.

As in the case for relative humidity, precipitation anomalies are well co-located with Q_1 anomalies, also positive anomalies in Q_1 get stronger from phase 1 to 4 as they move eastward and weaken from phase 5 onward. The peaks and valleys in precipitation anomaly match up with positive and negative Q_1 anomalies over the entire longitudinal range, unlike for relative humidity in phase 7 and 8.

Composite total diabatic heating in the troposphere for weak periods of MJO for DJFM and JJAS is given in figure 3.9. Main features can be compared to those seen for relative humidity in figure 3.6. These include large Q_1 heating over Maritime continents and South America, increased Q_1 heating from 60°E to 90°E in JJAS due to the Indian monsoon and increased heating over South America in DJFM due to South American monsoon.

A westward tilt in the heating profile of MJO's convective center has been pointed out previously (J. Lin et al., 2004, Ling & Zhang, 2011 and Jiang et al., 2011). This is due to lower level heating (due to shallow cumulus clouds) just east of convective center and upper level heating (due to stratiform precipitation) just west of it. Similar westward tilt in heating can be seen in phase 3 to 6 for JJAS, even westward tilt in relative humidity anomalies can be observed for the corresponding phase. Westward tilt in relative humidity anomalies for DJFM can also be seen, however such tilt in the heating profiles for DJFM are not so clear. Although there is a difference in the westward tilt in heating profiles between JJAS and DJFM, not much of such difference in the zonal wind composites is observed. The westward tilt in heating is due to the

shallow cumulus and stratiform precipitation which effect the second baroclinic mode, while the MJO zonal wind circulation is dominant in the first baroclinic mode (see section 6 of Haertel et al., 2008). This could likely be the reason that the MJO zonal wind circulation does not show a strong westward tilt. Slight westward tilt in the MJO zonal wind anomalies have been observed by Sperber (2003). Even the results of Kiladis et al. (2005) show a slight westward tilt in MJO zonal wind anomalies, although this tilt is not present at all the locations considered. Since there is little westward tilt in the MJO zonal wind anomalies it could be so that its seasonal variation is also small and hence can not be observed in the current figures.

It has been noted in previous studies that relative phase between low level zonal wind convergence and convective center of MJO varies during its movement eastwards from the Indian Ocean (Rui & Wang, 1990; Hendon & Salby, 1994; Sperber, 2003). To check whether this feature is present in the MJO phase wise processed data, zonal wind anomalies are superimposed with $Q1$ anomalies and are plotted in figure 3.10 and 3.11 for DJFM and JJAS. Region of positive $Q1$ anomaly is considered to represent MJO's convective center. From phase 1 to 3, MJO's convective center coincides with lower level zonal wind convergence for both DJFM and JJAS while the convective center lies in the Indian Ocean. This corresponds to mode I of Zhang and Anderson (2003). From phase 4 onward, the convective center lags behind the zonal wind pattern and coincides with low level westerlies. This corresponds to mode II of Zhang and Anderson (2003), and is more pronounced in DJFM as compared to JJAS. It can also be noticed that negative anomalies of $Q1$ lie to the west of low level divergence for Phase 1 to 3 in both DJFM and JJAS. This can also be seen for phase 4 and 5 for JJAS. From phase 6 to 8, negative anomalies of $Q1$ coincide with low level divergence in both DJFM and JJAS.

Figure 3.7: Composite total diabatic heating anomalies superimposed with precipitation anomalies (mm/hr) from phase 1 to 8 for DJFM.

Figure 3.8: Composite total diabatic heating anomalies superimposed with precipitation anomalies (mm/hr) from phase 1 to 8 for JJAS.

(a)

(b)

Figure 3.9: Composite total diabatic heating pattern in the troposphere for weak periods of MJO for (a) DJFM and (b) JJAS.

Figure 3.10: Composite zonal wind anomalies superimposed with $Q1$ anomalies with thick black contour for 0.2 K/day and dotted black contour for -0.2 K/day, from phase 1 to 8 for DJFM.

Figure 3.11: Composite zonal wind anomalies superimposed with $Q1$ anomalies with thick black contour for 0.2 K/day and dotted black contour for -0.2 K/day, from phase 1 to 8 for JJAS.

3.3 Discussion

Fluctuations in zonal wind, relative humidity and total diabatic heating associated with MJO have been considered. MJO signal in these data fields is extracted by filtering them for eastward moving disturbances having wavenumber 1 to 7 and time period of 30 to 90 days as this is the range of wavenumber and time period for MJO peaks seen in TRMM and OLR spectra (M. Wheeler & Kiladis, 1999). Filtered data fields composited MJO phase wise, show features that move eastward with increasing MJO phase for both seasons of DJFM and JJAS, similar to MJO precipitation anomalies. Although MJO signal in OLR and TRMM is dominant from wavenumber 1-3 the associated dynamical fields seem to be dominant in wavenumber 1. This has been noticed before for 850 hPa zonal wind by Xie and Arkin (1997).

Zonal wind anomalies show lower (higher) level convergence (divergence) in regions close to MJO's convective center. This structure of zonal wind anomalies is similar to that reported by Kiladis et al. (2005) for MJO's convective center crossing the Pacific Ocean at 155°E . Phase relation of zonal wind anomalies with respect to MJO's convective center changes from model I to model II as it moves from the India Ocean to the Pacific Ocean. This is in accord with the results from previous studies (Zhang & Anderson, 2003; Sperber, 2003). In model I, the convective center lies in between surface easterlies and westerlies, while in model II the convective center is colocated with the surface westerly winds. It can be noticed that model II is more prominent in the season DJFM since the convective center is colocated with the westward wind anomalies in the lower troposphere (see figure 3.10, especially phase 3 to 8), while for JJAS model I seems to be dominant. This could have important implications for air-ocean interactions associated with MJO. Fluctuations in sea surface temperature are greater for model II since fluctuations in solar radiation flux and fluctuations in latent heat flux enhance each other (Zhang & Anderson, 2003). A greater SST fluctuation could have a stronger feedback to the MJO by inducing stronger anomalies in surface

heat fluxes. Therefore, air-ocean interaction in case of MJO could be stronger for DJFM as compared to JJAS.

Anomalies in relative humidity and total diabatic heating (Q_1) seem to be well co-located with MJO precipitation anomalies. Even the strength of anomalies in these data fields rise and fall with the corresponding strength of precipitation anomalies, at least over a longitudinal range covering regions from Indian Ocean to Pacific Ocean. Considering such match, it can safely be concluded that the fluctuations observed in the filtered data fields can be associated with MJO.

The composite for weak periods of MJO represent the state of tropical atmosphere when MJO activity is weak or absent. Signal for Indian monsoon and South American monsoon is visible in them and is consistent among all three data fields in DJFM and JJAS. This is expected since Monsoon has a strong seasonal cycle.

In conclusion, the data fields computed both for MJO anomalies and weak periods of MJO appear to be sound. They are consistent with each other and contain features which have been recognized in previous studies. Although these features have been seen in previous studies, the new information presented in the results of this chapter is that the various data fields associated with MJO have been computed over the whole longitudinal range separately for the seasons of DJFM and JJAS. With these results any seasonal differences in MJO related fields can be seen, such as in the case for model I and II. The results of this chapter are important for the next chapter since these data fields are considered as background on which CCEW evolve.

Chapter 4

Theoretical study

Theoretical models aimed at explaining the mechanism of CCEW have been developed. Correctly formulating the coupling between convection and large scale circulation has been a major challenge yet these simple models are able to capture the basic features of CCEW. A brief review of the proposed mechanisms of CCEW is given below.

Efforts have been made in developing theoretical models to explain the coupling between convection and large scale wavelike disturbances in the tropics. Yamasaki (1969) and Hayashi (1970) came up with the concept of wave-CISK which is based on CISK (convective instability of second kind, developed by Charney and Eliassen (1964) to explain the growth mechanism of tropical cyclones). In this, convective heating due to clouds is assumed to be proportional to the low level upward motion as a result of large scale convergence due to wave motion. However, wave-CISK has largely lost its acceptance, mainly as wave-CISK based models show the highest growth rate at smallest resolvable spatial scale, giving rise to the so called "CISK catastrophe" (Crum & Dunkerton, 1992; Matthews & Lander, 1999).

Another concept, of convective quasi equilibrium (CQE) has also been put forward to understand the interaction between convection and large scale circulation. CQE assumes that over large scale, the creation of convective instabilities due to large scale processes (such as radiation, heat fluxes) is countered by adjustment in convection. Convection may adjust to the large scale forcing instantaneously or over a finite time

period. This scheme was first formulated by Arakawa and Schubert (1974), a detailed description of it can be found in the review article by Yano and Plant (2012).

Earlier models for CCEW involving CQE considered only the first baroclinic mode (Emanuel, 1987, Neelin et al., 1987, Neelin & Yu, 1994, Yano et al., 1995). This assumes that convective heating has one sign over the full depth of the troposphere with maximum heating near mid-troposphere (like a half sine curve). This approach was used since the latent heat released in deep convective towers have such vertical profile. Observational studies have revealed the presence of second baroclinic mode in the vertical structure of CCEW (Straub & Kiladis, 2003b, Haertel & Kiladis, 2004).

A model for CCEW using two baroclinic modes was developed by B. E. Mapes (2000). The first mode was excited by deep convective heating and the second mode by stratiform and congestus heating. Stratiform heating involves heating in the upper and cooling in the lower troposphere, while congestus heating involves heating in the lower troposphere (Houze Jr, 2004). Instead of CQE, convective inhibition (CIN, energy barrier which must be overcome to trigger convection) was recognized as important. CIN was controlled by the second mode due to its effect on lower troposphere. The model did produce waves with phase speeds and vertical structure similar to those observed in the atmosphere.

Models involving two vertical baroclinic modes were further developed by Majda & Shefter, 2001, Majda et al., 2004. In these models, deep convection was modulated by the temperature anomalies due to the second mode. Effect of moisture on convection was considered by Khouider and Majda (2006) and Khouider and Majda (2008) with second mode playing an important role in modulating the moisture from lower to mid-troposphere. Deep convection could only occur if the mid-troposphere was sufficiently moist. Effect of mid-tropospheric moisture on the strength of deep convection is also supported by observational and numerical studies (Brown & Zhang, 1997; Sherwood, 1999; Kuang & Bretherton, 2006).

Kuang (2008a) used a cloud system-resolving model (CSRM) to simulate the cou-

pling between convection and linear gravity waves for a single horizontal wavenumber at a time. Convectively coupled waves developed spontaneously in these simulations. The cloud system-resolving model was not limited to two baroclinic modes yet the spontaneously developed convectively coupled waves showed vertical structure containing mainly the first two modes. Convectively coupled waves in this study could develop without including radiative effects, surface fluxes and radiation of wave energy into the stratosphere, however, waves did not develop if advection of moisture by wave motion was disabled. Using the results of this study, a two mode model explaining the instability resulting in the generation of convectively coupled waves including the effects of mid-tropospheric moisture was developed in Kuang (2008b). This model was based on the concept of CQE, the vertical structure of waves and the phase relation between deep convective and stratiform precipitation was in accord with observations, B. Mapes et al. (2006).

Since previous studies point in the direction that CCEW are composed of two vertical modes and simple models based on this approach do produce the wave structures comparable with observations, therefore the aim of the present chapter is to investigate if a simple theoretical model can shed light on the modulation of CCEW by MJO. For this purpose, the model developed by Kuang (2008b) is used due to its conceptually simpler treatment of convective parameterization and mid-tropospheric moisture.

4.1 Theoretical model

Equations governing the dynamics of the first two baroclinic modes (neglecting rotation of earth) are given below. First mode is excited by deep convective heating while

the second mode is excited by congestus and stratiform heating.

$$\frac{\partial u_1}{\partial t} + U_0 \frac{\partial u_1}{\partial x} + \frac{3}{4} U_1 \frac{\partial u_2}{\partial x} = \frac{\partial T_1}{\partial x} \quad (4.1a)$$

$$\frac{\partial T_1}{\partial t} + U_0 \frac{\partial T_1}{\partial x} + U_1 \frac{\partial T_2}{\partial x} - c_1^2 \frac{\partial u_1}{\partial x} = j_1 \quad (4.1b)$$

$$\frac{\partial u_2}{\partial t} + U_0 \frac{\partial u_2}{\partial x} = \frac{\partial T_2}{\partial x} \quad (4.1c)$$

$$\frac{\partial T_2}{\partial t} + U_0 \frac{\partial T_2}{\partial x} + \frac{1}{4} U_1 \frac{\partial T_1}{\partial x} - c_2^2 \frac{\partial u_2}{\partial x} = j_2, \quad (4.1d)$$

here $u_{1,2}$ and $T_{1,2}$ are the zonal wind and scaled temperature perturbations associated with the first and second baroclinic modes respectively. U_0 and U_1 are the background barotropic wind and baroclinic shear, and $c_{1,2}$ is the wave speed of the two modes. The above set of equations is derived following the approach given in section 2 of Stechmann et al. (2008), considering only first two baroclinic modes with rigid lid at both the top and bottom of the troposphere. Effect of earth's rotation is neglected in these equations for simplicity. Even without including the effect of earth's rotation the model still contains the gravity wave dynamics and is a good approximation for Kelvin and WIG waves and is useful enough to compare the effect of MJO's barotropic wind, baroclinic wind and diabatic heating on convectively coupled waves.

For convective heating, simplified model of Kuang (2008b) is used. Heating for the two vertical modes ($j_{1,2}$) according to this model is given as a function of flow variables as

$$\frac{\partial j_1}{\partial t} = \frac{1}{\tau_j} (j_{1eq} - j_1), \quad (4.2)$$

where

$$j_{1eq} = \frac{1}{b_1 + b_2 \gamma_0} \left(b_2 \gamma_q q - F \frac{\partial}{\partial t} [f T_1 + (1 - f) T_2] \right), \quad (4.3)$$

and

$$j_2 = \gamma_0 j_1 - \gamma_q q, \quad (4.4)$$

where

$$\gamma_0 = \frac{1 - r_0}{1 + r_0} \quad (4.5a)$$

$$\gamma_q = \frac{2r_q}{1 + r_0}, \quad (4.5b)$$

q is the mid-tropospheric humidity, r_0 is the ratio of upper to lower heating of the troposphere, r_q is the sensitivity of convection to mid-tropospheric humidity and j_{1eq} , physically, is the convective heating required for strict quasi equilibrium (strict means that the adjustment is instantaneous).

These equations are accompanied by another equations for mid-tropospheric humidity and subcloud layer moist static energy h_b , given as

$$\frac{\partial q}{\partial t} = m_1 j_1 + m_2 j_2, \quad (4.6)$$

where m_1 and m_2 are the moistening effects for j_1 and j_2 , and

$$\frac{\partial h_b}{\partial t} = E - b_1 j_1 - b_2 j_2, \quad (4.7)$$

where b_1 and b_2 represent the change in moist static energy due to j_1 and j_2 . E represent the heating due to the surface flux which in the present model is set to zero. Convective parameterization of this model is based on the concept of convective quasi equilibrium (CQE), by assuming that the moist static energy of subcloud layer is in quasi equilibrium with the saturation moist static energy of a layer above it (see appendix C). This is given below as

$$\frac{\partial h_b}{\partial t} = \left\langle \frac{\partial h^*}{\partial t} \right\rangle, \quad (4.8)$$

where the brackets on the right hand side of the equation denotes averaging over a layer above the subcloud layer and h^* represent the saturation moist static energy.

Equation 4.8 can be written as

$$\frac{\partial h_b}{\partial t} = F \frac{\partial}{\partial t} [fT_1 + (1 - f)T_2], \quad (4.9)$$

where $F = \frac{\partial h^*}{\partial t}$, while f and $(1 - f)$ represent relative weights of temperature fluctuation of the two modes. An f close to 1 implies that the subcloud layer is in CQE with the whole depth of the troposphere and the first baroclinic mode is dominant in maintaining the quasi-equilibrium, while a value close to 0 implies that the second baroclinic mode is dominant and the CQE should be considered between the subcloud layer and the shallow depth of troposphere above it.

The model is linearized about a state of radiative-convective equilibrium (REC) and all the flow variables represent departure from this state. Convectively coupled waves correspond to the unstable modes of this model. The solution is assumed to have a form $\exp[i(kx - \omega t)]$, i.e. horizontally traveling wave with a single Fourier component in time and space. Wavenumber k is real while ω can be complex, real part of ω gives the frequency of the wave and its imaginary part gives the growth rate. The phase speed and growth rate for the unstable waves in case of no background flow is given in figure 4.1. Both eastward and westward moving waves move at a phase speed of nearly 25m/sec and have the same growth rate (for a given wavenumber). The numerical values of the parameters used is given in table 4.1. The phase speed of convectively coupled waves in the model is near 25m/sec while the observed phase speed is reported to be around 12-18 m/sec (Nakazawa, 1988 and Kiladis et al., 2009). The phase speed in the model can be adjusted to match the observed phase speed by adjusting c_1 and c_2 (phase speed for the dry gravity waves for first and second baroclinic mode) and τ_j (finite adjustment time to achieve quasi-equilibrium). The current phase speed corresponds to the default values of c_1 , c_2 and τ_j . Tuning these values, however, has no effect on the instability mechanism leading to the formation of convectively coupled waves or on the results in section in 4.2.3 and 4.2.4 showing

variation in growth rates of convectively coupled waves with MJO phase.

Figure 4.1: Phase speed and growth rate for the unstable waves using the convective parameterization of Kuang (2008b) with no background barotropic wind and background shear. Positive (negative) phase speed correspond to eastward (westward) moving waves.

Table 4.1: Numerical values of the parameters used

Symbol	Value	Units
b_1, b_2	1, 2	-
m_1, m_2	0.3, 1	-
r_0	1	-
γ_0	0.7	day^{-1}
γ_q	0.7	day^{-1}
F	4	-
f	0.5	-
τ_j	0	day
c_1, c_2	1, 0.5	50m/sec

4.1.1 Effect of MJO's heating on the evolution of convectively coupled waves

r_0 is defined as the ratio of heating in the upper troposphere to the heating in the lower troposphere. Deep convective clouds results in heating throughout the depth of the troposphere while stratiform clouds causes heating in the upper troposphere while cooling the lower troposphere. On the other hand congestus clouds result in

heating in the lower troposphere. Therefore higher (lower) stratiform heating can increase (decrease) r_0 while higher (lower) congestus heating can decrease (increase) r_0 .

The vertical profile of heating in MJO's convective envelope is known to be top heavy (more heating in the upper troposphere than in lower), J. Lin et al. (2004), Jiang et al. (2011). This feature was also noticed in figure 3.7 and 3.8 of the previous chapter. Other than this, in the convectively suppressed regions of MJO, there is anomalous cooling which is greater in the upper half of the troposphere. Therefore heating (cooling) due to MJO can increase (decrease) r_0 . Figure 4.2 shows the diabatic heating anomalies due to MJO superimposed with r_0 . It can be seen that r_0 is higher (lower) in the convectively enhanced (suppressed) region of MJO corresponding to positive (negative) Q1 anomalies.

Figure 4.2: Diabatic heating anomalies for MJO for phase 7 DJFM superimposed with r_0 . r_0 is calculated by considering the mid-troposphere at 500 hPa.

Effect of r_0 on the evolution of CCEW is evaluated using one way wave model expressed by eq. 26 to eq. 28 of Kuang (2008b). Figure 4.3 shows the evolution of CCEW when r_0 has a uniform value of 1 over space, while figure 4.4 shows evolution of CCEW when r_0 varies sinusoidally over space ($r_0 = 1 - \sin(\frac{2\pi x}{L} + \pi/2)$). The simulations are carried out using finite difference scheme of Lax Friedrichs with

random initial conditions.

In the first case (when r_0 is uniform in space), it can be seen that waves develop uniformly as they propagate from west to east. In contrast, when r_0 varies over space, development of the waves is enhanced near 20,000 km where r_0 is highest. To the regions east and west of it, development of waves is suppressed where r_0 gets less than one. Such behavior of the model is in accord with the results of chapter 2 where it was noticed that all CCEW were enhanced in the convectively active region of MJO and got suppressed in the convectively suppressed region of MJO.

Figure 4.3: Evolution of anomalies of T_1 and J_1 in figure (a) and (b) respectively for r_0 uniform over space.

Figure 4.4: Evolution of anomalies of T_1 and J_1 in figure (a) and (b) respectively for r_0 varying over space. Figure (c) shows variation of r_0 with space.

4.2 Comparison with results from data analysis

As seen in the previous section, according to the theoretical model, CCWE evolve faster in the region of higher r_0 and vice versa. In this section the exploration using the theoretical model is extended by including the effect of barotropic wind and baroclinic shear (U_0 and U_1) in addition to the effect of r_0 produced by MJO.

For the sake of simplicity, effect of rotation of earth is omitted from the theoretical model. Disturbances in the tropical atmosphere can be separated into two broad classes. First, fast moving disturbances such as Kelvin waves and Inertia Grav-

ity waves (refer to eq. (1.32) and (1.25)(a)), for which gravity waves dynamics are important. Second are disturbances such as equatorial Rossby waves (refer to eq. (1.25)(b)), MJO and tropical cyclones for which rotation of earth plays an important part, Raymond et al. (2015), Yasunaga and Mapes (2012b). Due to the lack of beta plane the model is unable to represent MRG waves and equatorial Rossby waves and unable to analyse the effect of meridional variation of MJO wind anomalies and diabatic heating on CCEW.

Even without the effect of rotation of earth, the theoretical model still retains the gravity wave dynamics and can be used for Kelvin waves and WIG waves (WIG waves behave as gravity waves for high wavenumbers). Near equator the structure of Kelvin wave is that of an eastward propagating gravity wave, Straub and Kiladis (2003b). Since the aim of the study is to explore the basic mechanism by which MJO might affect convectively coupled waves, therefore the current model is sufficient for the purpose.

4.2.1 Estimating r_0 , U_0 and U_1

Estimate of r_0 can be made using the heating fields computed for MJO in chapter 3 (see figure 3.7 and 3.8). However, the vertical heating profiles of MJO have been noticed to show differences for different reanalysis datasets, Jiang et al. (2011). It can be seen in figure 7 of Ling and Zhang (2011) that the vertical heating profiles for different datasets are qualitatively similar yet there are minor disagreement among them, in addition there is no reason to consider one reanalysis dataset as superior or more reliable than the others. Therefore, heating calculated from reanalysis datasets is not a quantitatively good estimate of the atmosphere.

To overcome this issue, r_0 is first calculated from the heating field computed in chapter 3 and then correlated with TRMM precipitation data filtered for MJO. To calculate r_0 , heating in the upper troposphere is divided by the heating in lower tro-

posphere over the required longitudinal range. Lower troposphere is considered from pressure level of 1000 hPa to 500 hPa while upper troposphere from 500 hPa to 100 hPa, this is in accord with Kuang (2008b) where mid-troposphere is considered to be at 500 hPa. Correlations between r_0 thus calculated and TRMM precipitation data are given in appendix B. Good correlations are obtained between r_0 and precipitation, which indicates that regions of higher precipitation are associated with higher r_0 . This is due to the fact that nearly two-thirds of the precipitation in MJO's convective envelope is due to stratiform rain, which results in higher heating in the upper troposphere, as can be seen in figure 6 of J. Lin et al. (2004).

The barotropic component of zonal wind (U_0) is estimated by taking the mean of zonal wind over the whole depth of troposphere while the baroclinic component (U_1) is estimated by projecting the zonal wind data on the first baroclinic mode as given in figure 2 of Haertel et al. (2008).

For feeding the data in the theoretical model, background values of U_0 and U_1 (referring to weak periods of MJO) are calculated using the data given in figure 3.3 of chapter 3. To this, fluctuations in U_0 and U_1 due to MJO for a given phase (calculated from data in figure 3.1 and 3.2) are added. Similarly, for r_0 a background value of 0.9 is chosen (for weak periods of MJO) to which the fluctuations due to MJO for different phase are added. A background value of 0.9 is chosen according the vertical profile of latent heating over the longitudinal range from 60°E to 180° as given in figure 2 of Ling and Zhang (2011). This heating profile shows greater latent heating in the lower half of the troposphere, indicating a value of r_0 less than 1. Results in section 4.2.3 and 4.2.4 do not show any appreciable change when the background r_0 value is varied from 0.8 to 1.

4.2.2 Tuning the parameters of theoretical model

Since the theoretical model of Kuang (2008b) is a simplification of the CSRM simulations of Kuang (2008a), parameters of the model were estimated in Kuang (2008b) by comparing the performance of the model against CSRM results. Details of the estimation process can be found in appendix A of Kuang (2008b). It was also recognized that the uncertainties in the estimation of r_q (sensitivity of convection to mid-tropospheric moisture) were large and a default value of 0.7 was chosen. Other than this, as discussed previously in this chapter, the model also contains a parameter f which controls the contribution of the two baroclinic modes in quasi-equilibrium with the subcloud layer. This parameter may have any value between 0 and 1 and a default value of 0.5 was assigned in the original paper.

In the present study, values of the parameters are taken to be the same as in Kuang (2008b). For r_q and f however, an estimate is needed in order to adapt the theoretical model to the real tropical atmosphere. For this purpose, values of r_0 , U_0 and U_1 are fed into the theoretical model for different phase of MJO and the output from mode (i.e, growth rates) is correlated with the data analysis results obtained in chapter 2 over the eight phase of MJO. This is done only for Kelvin waves for the season JJAS as shown in figure 4.5. In this figure, both f and r_q vary from 0.1 to 0.9 with step of 0.1. Correlation vary strongly with f as compared to r_q . It can also be seen that best correlations are obtained for lower values of f and fall rapidly for f beyond 0.7. Results of correlation for WIG waves and at different locations were found to be qualitatively similar.

Considering this result, values of f is chosen to be 0.4. Since the correlation does not vary strongly with r_q , the default value of 0.7 is used. These values are used to compare the output from the theoretical model with results from data analysis as discussed in the next section.

Figure 4.5: Correlation between results from theoretical model and data analysis for (a) Kelvin waves over Maritime continents for JJAS and (b) for Kelvin waves over Pacific Ocean for JJAS, with f and r_q varying from 0.1 to 0.9 with step of 0.1.

4.2.3 Kelvin waves

Growth rates for the eastward moving wave mode of the theoretical model are compared with results of TRMM precipitation data analysis for Kelvin waves. Wavenumber of 7 is used for calculating the growth rates from theoretical model since Kelvin waves show maximum power at this wavenumber, this can be seen in figure 2.6 (a). It was also noticed that there was no appreciable change in the variation of model output with MJO phase over a range of wavenumber from 3 to 10. Comparison of the results for Kelvin waves over different phase of MJO is given in figure 4.6 for DJFM and in figure 4.7 for JJAS.

Correlations between model output and data are high over the Indian Ocean for DJFM and Maritime continents for JJAS. Over the Indian Ocean, model shows higher growth rates for phase 2, 3 and 4 and a steep drop from phase 4 to 5, similar to the data. Even from phase 6 to 8 both model and data show little variation. However for phase 2, the growth rate is lower than for phase 3, while the data shows higher wave power for phase 2 than for phase 3. Over Maritime continents, both the model and data show sharp rise in values for phase 4 and 5 as compared to the rest of phase.

Correlations are comparatively lower over other regions. Over Maritime continents for DJFM, model output does show rise from phase 1 to 5 similar to data. However, the highest values are obtained for phase 6 and 7 while the data shows the highest values in phase 5. For region near South America in DJFM, the model output follows data points as it falls from phase 2 to 4 and rises from phase 5 to 8, similar to the data. The association between model output and data points near South America and western Pacific Ocean for JJAS however is not so good. For region near South America, model output varies smoothly with MJO phase and does not show any peak at phase 6 or valley at phase 2 as seen in the data. Over the western Pacific Ocean the model does show a peak at phase 6 similar to the data, however, valleys at phase 5 and 7 and a peak at phase 4 are lacking.

Figure 4.6: Kelvin waves result comparison for different phase of MJO in DJFM (a) over Indian Ocean, (b) Maritime continents (c) near South America with correlation of 0.79, 0.43 and 0.55 respectively.

Figure 4.7: Kelvin waves result comparison in the different phase of MJO in JJAS (a) over Maritime continents (b) near South America (c) over western Pacific Ocean (near international dateline) with correlations of 0.86, 0.46 and 0.36 respectively.

4.2.4 WIG waves

Growth rates for the westward moving wave mode of the theoretical model are compared with results of TRMM precipitation data analysis for WIG waves. Wavenumber of 15 is used for calculating the growth rates from the theoretical model since WIG waves show maximum power at this wavenumber, this can be seen in figure 2.6 (b) and (c). Comparison of the results for WIG $n=1$ waves over different phase of MJO is given in figure 4.8 for DJFM and in figure 4.9 for JJAS. The corresponding data for WIG $n=2$ waves is given in figure 4.10 and figure 4.11.

The agreement between model output and data is high over Maritime continents and Indian Ocean for both WIG $n=1$ and 2. In case of WIG $n=1$ over Maritime continents in DJFM, the model output follows the data closely showing high values only for phase 4 and 5, while for other phase both the model output and data show little variation. In JJAS, over Maritime continents, data analysis results reveal enhanced wave power from phase 4 to 6 for both modes of WIG waves. Model output also shows similar behavior, where there are high growth rate for phase 4, 5 and 6. Similar to data, model output remains nearly constant from phase 1 to 3, while a sharp increase can be seen moving from phase 3 to 4. Data also shows a steep fall from phase 6 to 7, however the decrease in model output is not so sharp.

Model output closely follow the data points over Indian Ocean in DJFM for WIG $n=2$, showing high output for phase 2, 3 and 4. Over Pacific Ocean in DJFM, both modes of WIG have similar behavior with MJO phase showing a prominent peak at phase 7. Such behavior is well captured by the model output in both cases. The model output does not agree so well over the Pacific Ocean in JJAS and the correlations are lower. However, the model output still follow the data points from phase 5 to 8 for WIG $n=1$ and from phase 2 to 6 for WIG $n=2$.

The agreement between model output and data points for both modes of WIG waves is less over Africa in comparison with the regions previously discussed. Al-

though the model output broadly follows the data points yet it varies smoothly with MJO phase and does not follow the peaks and valleys of the data points. This can be seen in figure 4.9 (b) for phase 4 and 5, and figure 4.11 (c) for phase 4, 5 and 7. Interestingly for WIG $n=1$ in JJAS, both the data and model output varies smoothly with MJO phase and the correlation is very high.

Figure 4.8: WIG $n=1$ waves result comparison in the different phase of MJO in DJFM (a) over Maritime continents, (b) over Pacific Ocean and (c) near Western Africa with correlation of 0.96, 0.86 and 0.82 respectively.

Figure 4.9: WIG $n=1$ waves result comparison in the different phase of MJO in JJAS (a) over Maritime continents, (b) near Western Africa, (c) over Pacific Ocean and (d) near South America with correlation of 0.87, 0.28, 0.55 and 0.91 respectively.

Figure 4.10: WIG $n=2$ waves result comparison in the different phase of MJO in DJFM, (a) and (b) over Indian Ocean, (c) over Pacific Ocean and (d) near Western Africa with correlation of 0.80, 0.75, 0.84 and 0.70 respectively.

Figure 4.11: WIG n=2 waves result comparison in the different phase of MJO in JJAS, (a) over Maritime continents, (b) over Pacific Ocean and (c) near Central Africa with correlation of 0.76, 0.48 and 0.25 respectively.

4.3 Discussion

As seen in the previous section, variation of growth rates calculated from the theoretical model with MJO phase are more or less in agreement with the results obtained in chapter 2 from TRMM data analysis. Correlation between model output and data analysis results is higher over the Indian Ocean and Maritime continents. For WIG waves, correlations over western Pacific Ocean are also high. It can also be seen from the figures in previous section that for regions near South America and Africa, correlation between model output and data are lower in comparison.

As mentioned before, higher correlations between model output and data are seen from the Indian Ocean to the western Pacific Ocean. This is also the longitudinal range over which the variation in precipitation due to MJO is strongest (see figure 2.11 (a) and (b)). In these regions, MJO's heating is found to be the dominant influence on the theoretical model. At other locations MJO's diabatic heating signal is weak and overwhelmed by noise. This could be the likely reason why the model output does not correlate well with the data analysis results over certain locations. Figure 4.12 shows the variation in r_0 with MJO phase along with the model output as shown in the figures of the previous section and the model output calculated using variation in r_0 only (excluding zonal wind variations). It can be seen that there is little difference in the model output calculated by excluding or including zonal wind variations. This indicates that r_0 has the strongest effect on the theoretical model. The accompanying figure 4.13 compares the model output as shown in the figures of the previous section and model output calculated by considering only the changes in U_0 and U_1 due to MJO. It can be seen that model output varies little with MJO phase if the effect of r_0 is removed, confirming that r_0 is the dominant factor in controlling the growth rates. In figure 4.12 (d), model output follows the variations in r_0 near western Africa. Even though near western Africa variations in r_0 are weak, variations in zonal wind are also weak and r_0 remains the dominant effect. Results over other

regions for both WIG and Kelvin waves were found to be similar.

Figure 4.12: Variation of r_0 , model output and model output calculated using only variations in r_0 , with MJO phase, (a) for Kelvin waves over Indian Ocean in DJFM, (b) for WIG n=1 waves over Maritime continents in DJFM, (c) for WIG n=2 waves over Indian Ocean in DJFM and (d) for WIG n=1 waves near Western Africa in JJAS.

Figure 4.13: Comparison of model output as calculated in previous section and model output calculated using only variations in U_0 and U_1 , (a) for Kelvin waves over Indian Ocean in DJFM, (b) for WIG n=1 waves over Maritime continents in DJFM, (c) for WIG n=2 waves over Indian Ocean in DJFM and (d) for WIG n=1 waves near Western Africa in JJAS.

4.3.1 Effect of r_0 on stability

To understand the effect of r_0 on the stability of convectively coupled waves, the theoretical model is simplified by assuming $b_2 = m_2 = 0$, and considering a limiting

case of $f = 0$. Physically, setting $b_2 = 0$ means that convection in the upper or lower troposphere (due to stratiform and congestus clouds respectively) has no effect on the sub-cloud layer moist static energy and setting $m_2 = 0$ means that the mid-tropospheric humidity is not affected by the heating of the second baroclinic mode (due to congestus and stratiform clouds). Using limiting case of $f = 0$ simplifies the model by decoupling the two baroclinic modes, yet retains the basic instability mechanism. Such assumptions are in accord with section 3 of Kuang (2008b).

Consider eq. F.23 (c) and (d) without convective heating and background zonal wind (i.e. excluding j_2 , U_0 and U_1). The equations are given below again for convenience.

$$\frac{\partial u_2}{\partial t} = \frac{\partial T_2}{\partial x} \quad (4.10a)$$

$$\frac{\partial T_2}{\partial t} - c_2^2 \frac{\partial u_2}{\partial x} = 0. \quad (4.10b)$$

Differentiating eq. 4.10 (a) w.r.t x and eq. 4.10 (b) w.r.t t , u_2 can be eliminated and the system can be written as a single equation for T_2

$$\frac{\partial^2 T_2}{\partial t^2} = c_2^2 \frac{\partial^2 T_2}{\partial x^2}, \quad (4.11)$$

considering the solution of the form e^{ikx} , i.e. having only one Fourier component in space, eq. 4.11 becomes

$$\frac{d^2 T_2}{dt^2} + c_2^2 k^2 T_2 = 0, \quad (4.12)$$

total derivatives are used since now T_2 is a function of time only. This equation can be compared with the equation describing simple harmonic oscillator given as

$$\frac{d^2y}{dt^2} + c\frac{dy}{dt} + sy = 0, \quad (4.13)$$

where y is the displacement from equilibrium position, c and s are damping and spring stiffness per unit mass respectively. Comparing eq. 4.12 and eq. 4.13 it can be seen that $c_2^2k^2$ in eq. 4.12 corresponds to stiffness and there is no damping. For a simple harmonic oscillator the angular frequency is given by square root of stiffness per unit mass ($\sqrt{s}$), similarly waves described by system of eq. 4.10 have an angular frequency of c_2k with c_2 as the phase speed. Due to absence of damping the waves do not grow or decay.

If the dependence of stratiform heating (j_2) on mid-tropospheric moisture is neglected i.e. γ_q in eq. 4.4 is set to 0, then the system of equations 4.10 can be reduced to

$$\frac{d^2T_2}{dt^2} + \frac{c_2^2k^2}{1 + \gamma_0 f/b_1} T_2 = 0, \quad (4.14)$$

by considering solutions of the form e^{ikx} and using eq. 4.4 for j_2 by neglecting γ_q in it. It can be seen by comparing eq. 4.12 and eq. 4.14 that only the stiffness is modified and no damping is added. As a result waves experience a change in frequency with no growth or decay in time.

If γ_0 is set to zero in eq. 4.4, while retaining γ_q , then the system of equations 4.10 becomes

$$\frac{\partial u_2}{\partial t} = \frac{\partial T_2}{\partial x} \quad (4.15a)$$

$$\frac{\partial T_2}{\partial t} - c_2^2 \frac{\partial u_2}{\partial x} = -\gamma_q q, \quad (4.15b)$$

which, following the procedure as before can be reduced to

$$\frac{d^2 T_2}{dt^2} - \frac{\gamma_q m_1 F}{b_1} \frac{dT_2}{dt} + c_2^2 k^2 T_2 = 0, \quad (4.16)$$

comparing with eq. 4.13, it can be seen that there is a negative damping proportional to γ_q . This negative damping results in growth of waves. Hence the dependence of stratiform heating on mid-tropospheric moisture is essential for the instability to occur. It was also recognized in section 3 of Kuang (2008b).

The mechanism of instability for $r_0 = 1$ can be understood as follows. A propagating T_2 anomaly at wavenumber k modulates convective heating j_1 according to eq. 4.17

$$j_1 = -\frac{F}{b_1} \frac{\partial T_2}{\partial t}, \quad (4.17)$$

this is a simplified form of eq. 4.3 under the assumptions used in the present section. If T_2 anomaly has the form $\sin(kx - \omega t)$ then according to eq. 4.17, j_1 will be proportional to $\cos(kx - \omega t)$ and hence lag T_2 by 90° (since $\sin$ and $\cosine$ functions have a phase difference of 90°). Anomalies in mid-tropospheric moisture can be calculated using eq. 4.6 and they lag behind j_1 by another 90° and therefore lag behind T_2 anomalies by 180° . Since $j_2 = -\gamma_q q$, stratiform heating is in phase with T_2 anomalies and hence results in growth. In the previous case when $j_2 = \gamma_0 j_1$, the j_2 anomalies are 90° out of phase with T_2 anomalies and no growth results. An illustration of this instability can be seen in figure 11 of Kuang (2008b).

Considering both γ_0 and γ_q in eq. 4.4, the system of equations 4.10 can be written as

$$\frac{\partial u_2}{\partial t} = \frac{\partial T_2}{\partial x} \quad (4.18a)$$

$$\frac{\partial T_2}{\partial t} - \frac{c_2^2}{1 + \gamma_0 F/b_1} \frac{\partial u_2}{\partial x} = \frac{-\gamma_q}{1 + \gamma_0 F/b_1} q, \quad (4.18b)$$

comparing with eq. 4.15 it can be seen that the wave speed and γ_q are modified by a factor of $1/(1 + \gamma_0 F/b_1)$. Following the procedure as before, the system of equation

4.18 can be written as

$$\frac{d^2 T_2}{dt^2} - \frac{\gamma_q m_1 F}{b_1(1 + \gamma_0 F/b_1)} \frac{dT_2}{dt} + \frac{c_2^2 k^2}{1 + \gamma_0 F/b_1} T_2 = 0. \quad (4.19)$$

Both stiffness and damping are modified by γ_0 . In order to compare with eq. 4.16, eq. 4.19 can be written as

$$\frac{d^2 T_2}{dt^2} - \frac{\gamma_{q-eff} m_1 F}{b_1} \frac{dT_2}{dt} + c_{2-eff}^2 k^2 T_2 = 0, \quad (4.20)$$

where γ_{q-eff} is the effective γ_q and c_{2-eff} is the effective c_2 defined as

$$\gamma_{q-eff} = \frac{\gamma_q}{1 + \gamma_0 F/b_1}, \quad (4.21)$$

and

$$c_{2-eff}^2 = \frac{c_2^2}{1 + \gamma_0 F/b_1}. \quad (4.22)$$

As discussed previously, instability occurs since the response of stratiform heating (j_2) to mid-tropospheric anomalies is in phase with T_2 and results in a positive feedback in the wave equation, as seen in eq. 4.15 as $-\gamma_q q$. The dependence of stratiform heating on deep convective heating (j_1) through γ_0 (see eq. 4.4) results in modification of both the wave speed and the positive feedback in the wave equation, see eq. 4.18. This modulation of the feedback in the wave equation results in the variation in growth rates with γ_0 or r_0 , since γ_0 is a function of r_0 (see eq. 4.5).

Variation of γ_{q-eff} and c_{2-eff} with r_0 are shown in figure 4.14. It can be seen that both γ_{q-eff} and c_{2-eff} rise with r_0 .

Figure 4.15 shows variation in growth rate with r_0 for different values of f ranging from 0 to 0.9 calculated numerically from the system of equations F.23 for $U_0 = U_1 = 0$. Change in growth rate with r_0 is steepest for values of f from 0.2 to 0.5 and reduces for increasing values of f . For $f = 0.9$, growth rate is nearly constant with r_0 . This explains greater correlations between model output and data points for values of f

close to 0.4 as seen in figure 4.5.

In this chapter, with the help of a simple theoretical model the effect of MJO's barotropic wind, baroclinic shear and diabatic heating on CCEW is evaluated. The output from the model do correlate with the data analysis results for most of the locations. The model shows that out of the three factors considered the diabatic heating is the dominant one through its effect on r_0 . It is also found that the dependence of stratiform heating on deep convective heating (through γ_0) modulates the feedback in the wave equation in such a way that it results in the variation of growth rates with r_0 .

Figure 4.14: Variation of c_{2_eff} in km/day and γ_{q_eff} with r_0 .

Figure 4.15: Variation of growth rate with r_0 for different values of f .

Chapter 5

Conclusion

The tropics cover nearly half of the area of the earth, thus motion of the tropical atmosphere is a major part of the dynamics of the whole planetary atmosphere. In the mid-latitudes, horizontal winds are in approximate geostrophic balance and the motion in these regions can be approximately analyzed through quasi-geostrophic theory. For the tropics, no such simplified dynamical approach is yet available.

Diabatic heating due to latent heat release is the primary energy source in the tropics. Most of the latent heat gets released in convective cloud systems. The convective cloud systems can organize themselves from mesoscale superclusters to synoptic and planetary scale structures. The interaction of convective scale to mesoscale to driving of large scale circulation patterns is of primary importance in understanding the dynamics of tropical atmosphere.

The strongest component of intraseasonal variability in the tropics is the Madden Julian oscillation (MJO). Later convectively coupled equatorial waves were discovered to be other dominant modes at intraseasonal time scales (Takayabu, 1994 and M. Wheeler & Kiladis, 1999), though not as strong as MJO.

General circulation models (GCMs) have been used to study MJO and CCEWs, however they have not been able to produce the intraseasonal tropical modes accurately J.-L. Lin et al. (2006). It is thought that the incapability of GCM in accurately producing MJO and CCEW is due to improper parameterization of convection (J.-L. Lin et al. (2006) and Moncrieff et al. (2007)). Since the grid size of a GCM

$\sim 100\text{km}$, therefore individual convective elements (order of few kilometers) cannot be resolved and their effect on the large scale flow is parameterized.

Another approach to study the interaction between convection and large scale flow is by using cloud resolving models (CRMs). CRMs are able to resolve individual convective clouds although fluid turbulence, cloud microphysics and radiation are still parameterized. CRMs have very high spatial resolution of a few kilometers, therefore using CRMs on the whole tropical channel is very expensive and they are used over limited region. Convectively coupled waves using CRM were successfully simulated by Tulich et al. (2007) and Tulich and Mapes (2008) in a 2D domain. The waves had phase speeds and vertical structure in agreement with the observations. Kuang (2008a) also experimented with CRM by running it over a limited domain and coupling it with linear gravity waves for a single wavenumber at a time. Convectively coupled waves developed spontaneously with realistic phase speed and vertical structure. Based on these results a simplified model of convectively coupled waves was developed in Kuang (2008b).

This thesis has made an attempt to explore the effect of MJO on CCEW. As discussed in the beginning of chapter 2, such attempts have been made before, however they have been limited in their approach and no clear conclusion have come forward. Investigations made in chapter 2 indicate that MJO can influence the strength of CCEWs. MJO's effect of CCEWs was found to be strong over a longitudinal range stretching from the Indian Ocean to western Pacific Ocean, where the variation in convection due to MJO is greatest. MJO was also found to influence CCEW over Africa and near South America if the wave activity peaked in these regions. The effect however, was generally weaker as compared to the effect observed over the Indian Ocean and Maritime continents. For all wave modes under study, convectively enhanced (suppressed) regions of MJO enhance (suppress) CCEWs. However, it was noticed that MJO was more effective in enhancing Kelvin waves than suppressing them, while for WIG and MRG waves MJO was more effective in suppressing the

waves.

In chapter 3, the large scale effect of MJO on tropics was explored in terms of fluctuations in zonal wind, relative humidity and diabatic heating associated with it. Large scale dynamical fields computed in chapter 3 had vertical structure in agreement with the previous studies. MJO associated anomalies in all these fields were found to be coherent with the MJO signal in precipitation and showed an eastward propagation with increasing MJO phase. These dynamical fields were computed to be used as a background for the theoretical model.

In order to understand the variation of wave power with MJO phase, obtained in chapter 2, a simplified model of convectively coupled waves developed by Kuang (2008b) was employed. A GCM or CRM could be used for the purpose, however these models are very expensive to run and are so complex that it is a challenge to discern the mechanism even if they produce favourable results. A simple coupled wave convection model offers the advantage of being conceptually simpler and inexpensive to run. Other than this, the instability mechanism leading to the formation of convectively coupled waves is clearly identified in the theoretical model.

The theoretical model was used with the same values of parameters as estimated in Kuang (2008b). For parameters f and r_q (which represent the depth of troposphere in quasi-equilibrium with sub-cloud layer and dependence of stratiform heating on mid-tropospheric moisture respectively) estimation was done using the data only for Kelvin waves and only in JJAS. Data for both Kelvin and WIG waves for both DJFM and JJAS was not used in the estimation process in order to prevent over-fitting of the model to the data. Finally the value of r_q was kept same as the default value in Kuang (2008b) while for f value of 0.4 was used which is close to the default value of 0.5.

Fluctuations due to MJO in barotropic and baroclinic wind, and in diabatic heating were applied as a background to the theoretical model in chapter 4. The output from the model could be matched with the results of data analysis. The model output

correlated well with the data analysis results at least from the Indian Ocean to western Pacific Ocean. For regions near Africa and South America, the situation was difficult to analyze since the heating (r_0) signal due to MJO was weak and overwhelmed by noise.

Among the three factors considered in the background, diabatic heating effect of MJO (on r_0 , which is the ratio of diabatic heating in the upper half of the troposphere to the lower half) was identified to be the dominant factor. Model output could still be correlated with the data analysis results by considering only the diabatic heating effect of MJO in the background. The effect of MJO's diabatic heating (on r_0) on the instability mechanism, resulting in the variation of growth rate with MJO phase was further explored. It was found that the dependence of stratiform heating on deep convective heating (through γ_0) modulates the feedback in the wave equation in such a way that it results in the variation of growth rates with r_0 .

It was also seen in chapter 4 that the effect of r_0 on the instability of convectively coupled waves was higher for lower values of f (below 0.5) and almost vanish as f approaches 0.9. A lower value of f means that the second baroclinic mode is dominant in quasi equilibrium, indicating that sub-cloud layer is in quasi equilibrium with lower half of the troposphere. A higher value would mean the dominance of first baroclinic mode and a quasi equilibrium between sub-cloud layer and whole depth of troposphere. Since the results chapter 4 do not favor higher values of f , the sub-cloud layer may not be in quasi equilibrium with the whole depth of troposphere as considered in some of the previous simplified models of convectively coupled waves Emanuel et al. (1994).

Since the outputs from the theoretical model do correlate with results of data analysis to a good extent. Future models attempting to explain the interaction of convection with the large scale flow may use CQE and consider the effect of free tropospheric moisture as an important control on convection.

5.1 Future work

Kuang's theoretical model does not take into account the effect of surface fluxes unlike the models developed by Majda and Shefter (2001), Khouider and Majda (2006) and Khouider and Majda (2008). Considering the surface fluxes helps in incorporating the effect of sea surface temperature (SST) variation due to MJO, Zhang and Dong (2004). Effect of SST on convectively coupled waves can be seen in the numerical solution in section 5 of Majda and Shefter (2001) and in Khouider and Majda (2007).

The model may be extended to equatorial beta plane. Since the rotation of earth plays an important role in the dynamics of MRG and ER waves, the effect of MJO on these wave modes can be understood by using the equatorial beta plane, as these waves have a different wind and temperature structure than the kelvin-like waves already discussed in this thesis (Andersen & Kuang, 2008). MJO also has significant meridional shear in its wind anomalies, with the beta plane effect of meridional shear on CCEW can be evaluated. Other than this, the precipitation anomalies associated with MJO are concentrated south of equator during DJFM and north of equator during JJAS (due to the northward propagation), Zhang and Dong (2004). With the beta plane, the effect of the accompanied meridional variation in MJO's diabatic heating on all the wave modes can also be evaluated. Extending the model to equatorial beta plane would also help investigate if the rotation of earth has any effect on the instability mechanism.

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Appendix A

Boussinesq approximation

Meteorological phenomenon occur at time scales which are much longer than the time scales for acoustic waves. Therefore it is convenient to devise a set of approximations so as to filter out the sound waves from the set of governing equations in order to avoid complications. Here we discuss the *Boussinesq approximation* which eliminates acoustic waves, but yet retain the gravity waves which are of meteorological interest.

The system of equations governing the large scale flow of the atmosphere is given as

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_x, \quad (\text{A.1})$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_y, \quad (\text{A.2})$$

$$\frac{1}{\rho} \frac{\partial p}{\partial z} = -g, \quad (\text{A.3})$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{U} = 0, \quad (\text{A.4})$$

$$\rho \frac{De}{Dt} = -p \nabla \cdot \mathbf{U} + \rho J, \quad (\text{A.5})$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}, \quad (\text{A.6})$$

x , y , u and v are the zonal and meridional directions and flow velocities receptively, ρ is density, p is pressure, e is the internal energy of air, g is the gravitational constant, J is heating rate and f is the Coriolis parameter. A derivation of this system of equations from primitive equations can be seen in chapter 1 of Holton and Hakim (2012).

We assume that there is a steady background state with the density pressure and temperature given by

$$\begin{aligned} \bar{\rho}(z) &= \rho_m + \rho_0(z), \\ \bar{p}(z) &= p_m + p_0(z), \\ \bar{T}(z) &= T_m + T_0(z). \end{aligned} \quad (\text{A.7})$$

This background state is also assumed to be in hydrostatic balance and obeys the equation of state of the perfect gas

$$\frac{d\bar{p}}{dz} = -\bar{\rho}g, \quad (\text{A.8})$$

$$\bar{p} = R\bar{\rho}\bar{T}. \quad (\text{A.9})$$

The flow induces perturbations in density, temperature and pressure on the background state, therefore these variables can be written as a function of space and time as

$$\begin{aligned}
\rho(x, y, z, t) &= \bar{\rho}(z) + \tilde{\rho}(x, y, z, t), \\
p(x, y, z, t) &= \bar{p}(z) + \tilde{p}(x, y, z, t), \\
T(x, y, z, t) &= \bar{T}(z) + \tilde{T}(x, y, z, t).
\end{aligned} \tag{A.10}$$

Boussinesq approximation is restricted to the study of systems whose background variables and motion induced fluctuations do not vary much from the mean values. This is an excellent approximation for the oceans and a reasonable approximation for the atmosphere.

$$\frac{\Delta\rho_0}{\rho_m} \simeq \frac{\Delta\tilde{\rho}}{\rho_m} \equiv \epsilon \ll 1 \tag{A.11}$$

where $\Delta\rho_0$ and $\Delta\tilde{\rho}$ are the variation in ρ_0 along the depth of the fluid and variation in $\tilde{\rho}$ due to the fluid motion respectively.

Now let us have a look at the mass conservation equation. Equation A.4 can be written as

$$\frac{\partial\rho}{\partial t} + \mathbf{U} \cdot \nabla\rho + \rho\nabla \cdot \mathbf{U} = 0, \tag{A.12}$$

expanding $\mathbf{U}$ as $\mathbf{U} = \mathbf{U}_0 + \epsilon\mathbf{U}_1$ and using ρ from equation A.10, equation A.12 can be written as

$$\frac{\partial\tilde{\rho}}{\partial t} + (\mathbf{U}_0 + \epsilon\mathbf{U}_1) \cdot \nabla (\rho_0 + \tilde{\rho}) + (\rho_m + \rho_0 + \tilde{\rho}) \nabla \cdot (\mathbf{U}_0 + \epsilon\mathbf{U}_1) = 0, \tag{A.13}$$

dividing the equation by ρ_m .

$$\frac{1}{\rho_m} \frac{\partial \tilde{\rho}}{\partial t} + (\mathbf{U}_0 + \epsilon \mathbf{U}_1) \cdot \nabla \left(\frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) + \left(1 + \frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) \nabla \cdot (\mathbf{U}_0 + \epsilon \mathbf{U}_1) = 0, \quad (\text{A.14})$$

upon expanding the terms

$$\frac{1}{\rho_m} \frac{\partial \tilde{\rho}}{\partial t} + \mathbf{U}_0 \cdot \nabla \left(\frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) + O(\epsilon^2) + \nabla \cdot \mathbf{U}_0 + \epsilon \nabla \cdot \mathbf{U}_1 + \nabla \cdot \mathbf{U}_0 \left(\frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) + O(\epsilon^2) = 0, \quad (\text{A.15})$$

By using equation A.11, all the terms in equation A.15 are of $O(\epsilon)$ or $O(\epsilon^2)$ except $\nabla \cdot \mathbf{U}_0$, therefore $\nabla \cdot \mathbf{U}_0$ must be order of ϵ too. In the limit of $\epsilon \rightarrow 0$ the mass conservation equation is approximated as $\nabla \cdot \mathbf{U} = 0$ (writing $\mathbf{U}_0$ as $\mathbf{U}$ for convenience). This implies that there is no convergence or divergence of flow, therefore no compression or expansion. Such an approximation will eliminate all possibility of sound waves.

Now let us consider the momentum equation in the horizontal direction. Using ρ from equation A.10 and neglecting the friction terms for convenience, equation A.1 can be written as

$$(\rho_m + \rho_0 + \tilde{\rho}) \frac{Du}{Dt} = (\rho_m + \rho_0 + \tilde{\rho}) f v - \frac{\partial \tilde{p}}{\partial x}, \quad (\text{A.16})$$

dividing the equation by ρ_m

$$\left(1 + \frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) \frac{Du}{Dt} = \left(1 + \frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) f v - \frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial x}, \quad (\text{A.17})$$

making the approximation $\left(1 + \frac{\rho_0}{\rho_m} + \frac{\tilde{\rho}}{\rho_m} \right) \simeq 1$ equation A.17 can be written as

$$\frac{Du}{Dt} = f v - \frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial x}. \quad (\text{A.18})$$

Similar argument can be applied to the momentum equation in meridional direc-

tion.

Considering the vertical momentum equation A.3, using ρ and p from equation A.10 it can be written as

$$\frac{\partial p_0}{\partial z} + \frac{\partial \tilde{p}}{\partial z} = -\rho_m g - \rho_0 g - \tilde{\rho} g, \quad (\text{A.19})$$

the steady background state is already assumed to be in hydrostatic equilibrium, subtracting which gives us

$$\frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial z} = -\frac{\tilde{\rho}}{\rho_m} g. \quad (\text{A.20})$$

Now let us consider the equation of state for the ideal gas by using the variables from equation A.10.

$$p_m + p_0 + \tilde{p} = (\rho_m + \rho_0 + \tilde{\rho}) R (T_m + T_0 + \tilde{T}), \quad (\text{A.21})$$

rearranging the terms gives us

$$\left(1 + \frac{T_0}{T_m} + \frac{\tilde{T}}{T_m}\right) = \left(1 - \frac{\rho_0}{\rho_m} - \frac{\tilde{\rho}}{\rho_m}\right) \left(1 + \frac{p_0}{p_m} + \frac{\tilde{p}}{p_m}\right), \quad (\text{A.22})$$

neglecting the product of terms (which is of the order of ϵ^2) gives us

$$\left(1 + \frac{T_0}{T_m} + \frac{\tilde{T}}{T_m}\right) = \left(1 + \frac{p_0}{p_m} + \frac{\tilde{p}}{p_m} - \frac{\rho_0}{\rho_m} - \frac{\tilde{\rho}}{\rho_m}\right), \quad (\text{A.23})$$

subtracting the basic state from the equation (basic state can be obtained by doing the same calculation without including the fluctuation variables with tilde, it gives an equation similar to equation A.23 but without the fluctuation variables).

$$\frac{\tilde{T}}{T_m} = -\frac{\tilde{\rho}}{\rho_m} + \frac{\tilde{p}}{p_m}, \quad (\text{A.24})$$

using the formula for speed of sound $c_s = \gamma R T_m$, where γ is the adiabatic index and

equation of state $p_m = \rho_m R T_m$.

$$\frac{\tilde{T}}{T_m} = -\frac{\tilde{\rho}}{\rho_m} + \frac{\gamma \tilde{p}}{\rho_m c_s^2}. \quad (\text{A.25})$$

Since we are considering the flow to be incompressible in this approximation, this implies that the speed of sound should tend towards infinity. Using this approximation equation A.25 can be written as

$$\frac{\tilde{T}}{T_m} \simeq -\frac{\tilde{\rho}}{\rho_m}, \quad (\text{A.26})$$

which implies that the fluctuations in density are related to the fluctuations in temperature only.

Finally let us consider the energy equation A.5 within the scope of this approximation. Equation A.5 is given as

$$\rho \frac{De}{Dt} = -p \nabla \cdot \mathbf{U} + \rho J. \quad (\text{A.27})$$

Although $\nabla \cdot \mathbf{U}$ is equated to zero in the mass conservation equation however the term $p \nabla \cdot \mathbf{U}$ is not negligible as compared to the RHS. This can be shown by a simple argument as follows

$$-p \nabla \cdot \mathbf{U} = \frac{p}{\rho} \frac{D\rho}{Dt} \simeq \frac{p}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \frac{DT}{Dt} = -p \alpha \frac{DT}{Dt}, \quad (\text{A.28})$$

where α is the thermal expansion coefficient, and the approximation is based on considering ρ as a function of temperature only which is in accord with equation A.26. The term $p \nabla \cdot \mathbf{U}$ can be approximated as

$$p \nabla \cdot \mathbf{U} \simeq -\frac{p_m}{\rho_m} \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) (\bar{\rho} + \tilde{\rho}), \quad (\text{A.29})$$

using the background equation of state for eliminating $\bar{\rho}$ and equation A.26 for elim-

inating $\tilde{\rho}$ gives

$$p\nabla \cdot \mathbf{U} \simeq \frac{p_m}{\rho_m} \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \left(\rho_m \frac{\tilde{T}}{T_m} - \frac{p_m + p_0}{R(T_m + T_0)} \right), \quad (\text{A.30})$$

using binomial expansion and neglecting the product of terms (which will be of the order of ϵ^2) we get

$$p\nabla \cdot \mathbf{U} \simeq \left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right) \left(p_m \frac{T_0 + \tilde{T}}{T_m} - p_0 \right), \quad (\text{A.31})$$

combining this with the RHS of energy equation, using hydrostatic equation to replace $\frac{dp_0}{dz}$ and approximating ρ with ρ_m

$$\rho_m c_p \frac{D}{Dt} (T_0 + \tilde{T}) + w g \rho_m = \rho_m J, \quad (\text{A.32})$$

where c_p is the specific heat at constant pressure given by $c_p = c_v + R = c_v + \frac{p_m}{\rho_m T_m}$.

Dividing by ρ_m the energy equation can then be written as

$$\frac{D\tilde{T}}{Dt} + w \left(\frac{dT_0}{dz} + \frac{g}{c_p} \right) = \frac{J}{c_p}. \quad (\text{A.33})$$

This complete set of approximations in the mass, momentum and energy conservation equations is known as *Boussinesq approximation*. The complete set of equations under Boussinesq approximation is written as follows.

$$\frac{Du}{Dt} = fv - \frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial x}, \quad (\text{A.34})$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial y}, \quad (\text{A.35})$$

$$\frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial z} = \frac{\tilde{T}}{T_m} g, \quad (\text{A.36})$$

where equation A.26 have been used to replace $\frac{\tilde{\rho}}{\rho_m}$ with $\frac{\tilde{T}}{T_m}$

$$\nabla \cdot \mathbf{U} = 0, \tag{A.37}$$

$$\frac{D\tilde{T}}{Dt} + w \left(\frac{dT_0}{dz} + \frac{g}{c_p} \right) = \frac{J}{c_p}. \tag{A.38}$$

Appendix B

Correlation between r_0 and TRMM precipitation

This appendix shows the results of correlation between r_0 calculated from the $Q1$ heating field in chapter 3 and TRMM precipitation filtered for MJO for the eight phase of MJO. The TRMM data correlates well with r_0 over Indian Ocean and Maritime continents. Figure B.1 shows the results for Kelvin waves over Indian Ocean and Maritime continents to Pacific Ocean for DJFM and over Maritime continents for JJAS with regression slope and coefficient of determination (r^2) given in the figure.

Correlation results for WIG n=2 waves over Indian Ocean are given in figure B.2, while for WIG waves over Maritime continents for DJFM and JJAS are given in figure B.3.

Correlations over Pacific Ocean are also found to be good for Kelvin and WIG waves and are given in figure B.4.

Correlations near Africa and South America are lower than before. This is because the influence on r_0 due to MJO's diabatic heating is weak in these regions and therefore the signal due to MJO is overwhelmed by the noise.

Figure B.1: Correlation results for Kelvin waves (a) over Indian Ocean (b) from Maritime continents to Pacific Ocean for DJFM and (c) over Maritime continents for JJAS.

Figure B.2: Correlation results for WIG n=2 waves over the two range of Indian Ocean.

Figure B.3: Correlation results for WIG n=1 waves over Maritime continents (a) for DJFM, (b) for JJAS and (c) for WIG n=2 for JJAS.

(a)

(b)

(c)

(d)

Figure B.4: Correlation results over Pacific Ocean for (a) Kelvin waves in JJAS, (b) WIG $n=1$ waves (c) WIG $n=2$ waves in DJFM and (d) for WIG $n=2$ waves in JJAS.

Figure B.5: Correlation results for JJAS (a) WIG $n=1$ near South America, (b) WIG $n=1$ near Western Africa and (c) WIG $n=2$ near central Africa.

Appendix C

Convective quasi equilibrium

The concept of Convective quasi equilibrium (CQE) has been developed in order to understand the role of convection in the atmosphere. It has also been used for developing convective parameterisation for various theoretical models. The idea behind CQE is that forcing of the atmosphere towards convective instability by large scale non-convective processes such as large scale dynamics and radiation is statistically balanced by convection. It is assumed that convection occurs on a much faster time scale as compared to the non-convective processes and can adjust almost instantaneously or with a short delay (few hours). Therefore, the convection is almost in equilibrium with the large scale non-convective processes. The transient adjustment of a cloud ensemble in this approach is ignored. Although the time scale for convection is short, CQE is a good approximation when considering time scale of a few days or greater. A consequence of CQE is that the temperature profile of the atmosphere is constrained to be close to neutrally stable to moist convection.

A number of theoretical models have formulated CQE in terms of CAPE (convective available potential energy). CQE implies that CAPE is consumed by convection at nearly the same rate as it is generated by the large scale non-convective processes.

Convective quasi equilibrium is based on invariance of convective available potential energy (CAPE)

$$\frac{\partial CAPE}{\partial t} = 0, \tag{C.1}$$

where CAPE is defined as

$$\int_{z_b}^{z_a} B dz \quad (\text{C.2})$$

where B is the buoyancy of a rising air parcel given as

$$B = g \frac{\rho_a - \rho_p}{\rho_p} = g \frac{T_p - T_a}{T_a}, \quad (\text{C.3})$$

where ρ_a, T_a and ρ_p, T_p are the density and temperature of the atmosphere and air parcel respectively.

Moist static energy of the sub-cloud layer is defined as

$$h_b = C_p T_b + L_v q_{sb} + g z_b, \quad (\text{C.4})$$

where C_p is the specific heat capacity of air at constant pressure, T_b is the absolute temperature of sub-cloud layer, L_v is the latent heat of vaporization, q_{sb} is the saturation mixing ratio, g is the gravitational constant and z_b is the height above sea level of the sub-cloud layer.

As the air parcel rises from the sub-cloud layer to a level z_a , its moist static energy remains constant $\Delta h_b = 0$, see chapter 2 of Andrews (2010).

$$0 = C_p(T_p - T_b) + L_v(q_{sp} - q_{sb}) + g(z_a - z_b) \quad (\text{C.5})$$

where T_p and q_{sp} is the temperature and saturation mixing ratio of the air parcel at level z_a .

Let T_a be the average temperature of a layer above the sub-cloud layer. Calculating T_p using eq. C.5 and using eq. C.1 and C.3

$$\frac{\partial}{\partial t} [C_p T_a + L_v q_{sa} + g z_a] = \frac{\partial}{\partial t} [C_p T_b + L_v q_{sb} + g z_b] \quad (\text{C.6})$$

by defining

$$C_p T_a + L_v q_{sa} + g z_a = \langle h^* \rangle \quad (\text{C.7})$$

as the average saturation moist static energy of a layer above the sub-cloud layer

$$\frac{\partial h_b}{\partial t} = \left\langle \frac{\partial h^*}{\partial t} \right\rangle \quad (\text{C.8})$$

Appendix D

Spectra for DJFM and JJAS

Figure D.1 is similar to figure 2.5, however the spectra are computed separately for DJFM and JJAS. The results for figure D.1 do not appear to be significantly different from the results of figure 2.5, therefore the significant peaks in the spectra are obtained by comparing them with the background spectra given in figure 2.6 (d).

Significant spectral peaks for DJFM and JJAS in the TRMM record considered in this thesis is shown in figure D.2. One obvious difference which can be seen between the two seasons is that there is greater power in the westward moving waves in JJAS for wavenumbers ranging from 5 to 25 and frequency ranging from 0.1 to 0.4 cpd. This is due to the increase in the strength of TD-type disturbances during this time of the year, previously noticed by Dunkerton and Baldwin (1995). WIG waves and westward moving MRG waves are stronger in JJAS, while ER waves and MJO appear to be stronger in DJFM. All these features are consistent with similar spectra produced using OLR given in figure 5 of M. Wheeler and Kiladis (1999). It is interesting to note that the background state during JJAS which favours TD-type disturbances, WIG waves and westward moving MRG waves does not favour ER waves, indicating that the mechanism for ER waves could be very different from the other wave modes.

Figure D.1: Composite symmetric and antisymmetric spectra with wavenumber on x-axis and frequency on y-axis in cycles per day(cpd) for DJFM in sub-figure (a) and (b) and for JJAS in sub-figure (c) and (d).

Figure D.2: Sub-figure (a) and (b) show the contour plot of significant peaks in symmetric and antisymmetric spectra for DJFM and sub-figure (c) and (d) for JJAS. The lowest contour at .1072. Superimposed on these figures are the dispersion curves of equatorial wave modes for different equivalent depths so as to contain the significant peaks between them. The filters for the different wave modes are also highlighted with thick dashed boxes.

Appendix E

Composite wave power anomalies as a function of MJO phase

Composite wave power anomalies as a function of MJO phase superimposed with MJO's convective anomalies is given in figure E.1, for Kelvin waves, figure E.2 for WIG waves and figure E.3 for MRG waves. Higher wave power for CCEW moves eastwards with increasing phase and seems to lie more or less in the MJO's convective envelop.

Figure E.1: Composite Kelvin wave power anomalies as function of MJO phase for (a) DJFM and (b) JJAS. The positive precipitation anomalies for MJO ($> 0.02 \text{ mm/hr}$) is given by the solid black contour and the negative anomalies for MJO ($< 0.02 \text{ mm/hr}$) is given by dashed black contour.

Figure E.2: Composite WIG wave power anomalies as function of MJO phase for (a) DJFM WIG n=1, (b) JJAS WIG n=1, (c) DJFM WIG n=2 and (d) JJAS WIG n=2. The positive precipitation anomalies for MJO ($> 0.02\text{mm/hr}$) is given by the solid black contour and the negative anomalies for MJO ($< 0.02\text{ mm/hr}$) is given by dashed black contour.

Figure E.3: Composite MRG wave power anomalies as function of MJO phase for (a) DJFM MRG westward moving, (b) JJAS MRG westward moving, (c) DJFM MRG eastward moving and (d) JJAS MRG eastward moving. The positive precipitation anomalies for MJO ($> 0.02\text{mm/hr}$) is given by the solid black contour and the negative anomalies for MJO ($< 0.02\text{ mm/hr}$) is given by dashed black contour.

Appendix F

Derivation of governing equations for the theoretical model

Set of PDEs expressing the law of conservation of mass, momentum and energy is given as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial x} \quad (\text{F.1})$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (\text{F.2})$$

$$\frac{\partial \tilde{T}}{\partial t} + u \frac{\partial \tilde{T}}{\partial x} + w \frac{\partial \tilde{T}}{\partial z} + wS = J \quad (\text{F.3})$$

$$\frac{1}{\rho_m} \frac{\partial \tilde{p}}{\partial z} = \frac{\tilde{T}}{T_m} g, \quad (\text{F.4})$$

where u and w are the velocity components in the zonal and vertical directions respectively, J is the diabatic heating, S is $\left(\frac{dT_0}{dz} + \frac{g}{c_p}\right)$ and eq. A.26 from appendix A is used to write the hydrostatic equation as eq. F.4. A set of basis functions for the above set of equations can be obtained by linearising the equations, then using the separation of variables for the vertical (z) dependence and then solving the resulting Sturm-Liouville problem (as done in chapter 1). These basis functions have the forms

of sinusoids.

Taking upto only the first two baroclinic modes, the dependent variables of the governing equations can be written as

$$u = u_1 \cos(\pi z/H) + u_2 \cos(2\pi z/H) + U_0 + U_1 \cos(\pi z/H) \quad (\text{F.5a})$$

$$\tilde{p} = p_1 \cos(\pi z/H) + p_2 \cos(2\pi z/H) \quad (\text{F.5b})$$

$$w = w_1 \sin(\pi z/H) + w_2 \sin(2\pi z/H) \quad (\text{F.5c})$$

$$\tilde{T} = T_1 \sin(\pi z/H) + T_2 \sin(2\pi z/H) \quad (\text{F.5d})$$

$$J = J_1 \sin(\pi z/H) + J_2 \sin(2\pi z/H), \quad (\text{F.5e})$$

where H is the height of the troposphere. Substituting in the zonal momentum equation and linearizing it

$$\begin{aligned} \frac{\partial u_1}{\partial t} \cos(\pi z/H) + \frac{\partial u_2}{\partial t} \cos(2\pi z/H) + U_0 \frac{\partial u_2}{\partial x} \cos(2\pi z/H) + U_1 \frac{\partial u_1}{\partial x} \cos(\pi z/H) \cos(\pi z/H) \\ + U_1 \frac{\partial u_2}{\partial x} \cos(\pi z/H) \cos(2\pi z/H) - \frac{\pi}{H} U_1 w_1 \sin(\pi z/H) \sin(\pi z/H) \\ - \frac{\pi}{H} U_1 w_2 \sin(\pi z/H) \sin(2\pi z/H) = -\frac{1}{\rho_m} \frac{\partial p_1}{\partial x} \cos(\pi z/H) - \frac{1}{\rho_m} \frac{\partial p_2}{\partial x} \cos(2\pi z/H). \end{aligned} \quad (\text{F.6})$$

Substituting the dependent variables in equation F.2

$$\frac{\partial u_1}{\partial x} \cos(\pi z/H) + \frac{\partial u_2}{\partial x} \cos(2\pi z/H) + \frac{\pi}{H} w_1 \cos(\pi z/H) + \frac{2\pi}{H} w_2 \cos(2\pi z/H) = 0, \quad (\text{F.7})$$

now substituting the dependent variables in equation F.3 and linearizing it

Figure F.1: Vertical profiles for the modes. Sub-figure (a) and (b) show the vertical profiles for the zonal wind ($u_{1,2}$) and pressure while sub-figure (c) and (d) show the vertical profiles for the temperature ($T_{1,2}$) and vertical heating ($j_{1,2}$) for the first and second baroclinic modes.

$$\begin{aligned}
& \frac{\partial T_1}{\partial t} \sin(\pi z/H) + \frac{\partial T_2}{\partial t} \sin(2\pi z/H) + U_0 \frac{\partial T_1}{\partial x} \sin(\pi z/H) + U_0 \frac{\partial T_2}{\partial x} \sin(2\pi z/H) \\
& + U_1 \frac{\partial T_1}{\partial x} \cos(\pi z/H) \sin(\pi z/H) + U_1 \frac{\partial T_2}{\partial x} \cos(\pi z/H) \sin(2\pi z/H) + w_1 \sin(\pi z/H) S \\
& + w_2 \sin(2\pi z/H) S = J_1 \sin(\pi z/H) + J_2 \sin(2\pi z/H),
\end{aligned} \tag{F.8}$$

and substituting the dependent variables in equation F.4

$$-\frac{\pi}{H} \frac{1}{\rho_m} p_1 \sin(\pi z/H) - \frac{2\pi}{H} \frac{1}{\rho_m} p_2 \sin(2\pi z/H) = \frac{1}{T_m} T_1 \sin(\pi z/H) g + \frac{1}{T_m} T_2 \sin(2\pi z/H) g. \quad (\text{F.9})$$

Now we have a linearised system of equations containing only the first two baroclinic modes and the barotropic and first baroclinic component of the background wind.

The equations can be separated for the first and second baroclinic modes by taking the inner product defined as

$$\langle f(z)g(z) \rangle = \frac{1}{H} \int_0^H f(z)g(z)dz, \quad (\text{F.10})$$

with the vertical functions.

Extracting the first mode from the zonal momentum equation by taking inner product with $\cos(\pi z/H)$

$$\frac{\partial u_1}{\partial t} + U_0 \frac{\partial u_1}{\partial x} + \frac{U_1}{2} \frac{\partial u_2}{\partial x} - \frac{\pi}{2H} U_1 w_2 = -\frac{1}{\rho_m} \frac{\partial p_1}{\partial x}, \quad (\text{F.11})$$

extracting the first mode from the continuity equation

$$\frac{\partial u_1}{\partial x} + \frac{\pi}{H} w_1 = 0, \quad (\text{F.12})$$

extracting from the energy equation

$$\frac{\partial T_1}{\partial t} + U_0 \frac{\partial T_1}{\partial x} + \frac{U_1}{2} \frac{\partial T_2}{\partial x} + w_1 S = J_1, \quad (\text{F.13})$$

and from the hydrostatic equation

$$-\frac{\pi}{H} \frac{1}{\rho_m} p_1 = \frac{1}{T_m} T_1 g. \quad (\text{F.14})$$

The above equations are for the first mode. Similarly the equations for the second

mode can be extracted. Extracting the second mode from the zonal momentum equation by taking inner product with $\cos(2\pi z/H)$

$$\frac{\partial u_2}{\partial t} + U_0 \frac{\partial u_2}{\partial x} + \frac{U_1}{2} \frac{\partial u_1}{\partial x} - \frac{\pi}{2H} U_1 w_1 = -\frac{1}{\rho_m} \frac{\partial p_2}{\partial x}, \quad (\text{F.15})$$

extracting from the continuity equation

$$\frac{\partial u_2}{\partial x} + \frac{2\pi}{H} w_2 = 0, \quad (\text{F.16})$$

extracting from the energy equation

$$\frac{\partial T_2}{\partial t} + U_0 \frac{\partial T_2}{\partial x} + \frac{U_1}{2} \frac{\partial T_1}{\partial x} + w_2 S = J_2, \quad (\text{F.17})$$

and finally extracting from the hydrostatic equation

$$-\frac{2\pi}{H} \frac{1}{\rho_m} p_2 = \frac{1}{T_m} T_2 g. \quad (\text{F.18})$$

Equations F.15 to F.18 describe the second baroclinic mode. Eliminating w_1, w_2, p_1 and p_2 from the first and second baroclinic modes using eq. F.12, F.14, F.16 and F.18, equations for the first baroclinic mode are written as

$$\frac{\partial u_1}{\partial t} + U_0 \frac{\partial u_1}{\partial x} + \frac{3}{4} U_1 \frac{\partial u_2}{\partial x} = \frac{H}{\pi T_m} \frac{\partial T_1}{\partial x} g, \quad (\text{F.19})$$

$$\frac{\partial T_1}{\partial t} + U_0 \frac{\partial T_1}{\partial x} + \frac{U_1}{2} \frac{\partial T_2}{\partial x} - \frac{H}{\pi} \frac{\partial u_1}{\partial x} S = J_1, \quad (\text{F.20})$$

while the equations for the second baroclinic mode are written as

$$\frac{\partial u_2}{\partial t} + U_0 \frac{\partial u_2}{\partial x} = \frac{H}{2\pi T_m} \frac{\partial T_2}{\partial x} g, \quad (\text{F.21})$$

$$\frac{\partial T_2}{\partial t} + U_0 \frac{\partial T_2}{\partial x} + \frac{U_1}{2} \frac{\partial T_1}{\partial x} - \frac{H}{2\pi} \frac{\partial u_2}{\partial x} S = J_2. \quad (\text{F.22})$$

Scaling T_1, J_1 with $\frac{H}{\pi T_m} g$, T_2, J_2 with $\frac{H}{2\pi T_m} g$ and writing $\frac{H^2 g S}{n^2 \pi^2 T_m}$ as c_n^2 where $n = 1, 2$ the system of equations can be written as

$$\frac{\partial u_1}{\partial t} + U_0 \frac{\partial u_1}{\partial x} + \frac{3}{4} U_1 \frac{\partial u_2}{\partial x} = \frac{\partial T_1}{\partial x} \quad (\text{F.23a})$$

$$\frac{\partial T_1}{\partial t} + U_0 \frac{\partial T_1}{\partial x} + U_1 \frac{\partial T_2}{\partial x} - c_1^2 \frac{\partial u_1}{\partial x} = j_1 \quad (\text{F.23b})$$

$$\frac{\partial u_2}{\partial t} + U_0 \frac{\partial u_2}{\partial x} = \frac{\partial T_2}{\partial x} \quad (\text{F.23c})$$

$$\frac{\partial T_2}{\partial t} + U_0 \frac{\partial T_2}{\partial x} + \frac{1}{4} U_1 \frac{\partial T_1}{\partial x} - c_2^2 \frac{\partial u_2}{\partial x} = j_2. \quad (\text{F.23d})$$