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**ADVANCED VISIBLE LIGHT-BASED INDOOR
POSITIONING TECHNIQUES: DEVELOPMENT AND
ANALYSIS**

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2019

ADVANCED VISIBLE LIGHT-BASED INDOOR POSITIONING TECHNIQUES: DEVELOPMENT AND ANALYSIS

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A thesis submitted to the Nanyang Technological
University in partial fulfilment of the requirement for the
degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

07/01/2019

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Date

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Authorship Attribution Statement

This thesis contains material from 4 papers published in the following peer-reviewed journals and conferences where I was the first author.

Chapter 3 is related to journal paper: **R. Zhang**, W. D. Zhong, K. Qian, and D. Wu, "Image sensor based visible light positioning system with improved positioning algorithm," *IEEE Access*, vol. 5, pp. 6087–6094, Apr. 2017.

The contributions of the co-authors are as follows:

- Prof. Zhong Wen-De and Prof. Qian Kemao supervised me on the overall research direction and edited the manuscript drafts.
- I proposed the idea, developed the algorithm, conduct the experiments and prepared the manuscript drafts.
- Dr. Wu Dehao helped me on the experimental evaluation part and gave comments on the manuscript.

Chapter 4 is related to conference paper: **R. Zhang**, W.-D. Zhong, D. Wu, and K. Qian, "A novel sensor fusion based indoor visible light positioning system," in *Globecom Workshops (GC Wkshps)*, IEEE, 2016, pp. 1–6.

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The contributions of the co-authors are as follows:

- Prof. Zhong Wen-De and Prof. Qian Kemao supervised me on the overall research direction and edited the manuscript drafts.
- I proposed the idea, developed the algorithm, conduct the experiments and prepared the manuscript drafts.
- Mr. Zhang Sheng helped me on the experiments and gave comments on the manuscript.

Chapter 6 is related to journal paper: **R. Zhang**, W. D. Zhong, K. Qian, S. Zhang, D. Peng, “A Reversed Visible Light Multitarget Localization System via Sparse Matrix Reconstruction”. *IEEE Internet of Things Journal*, vol. 5, pp. 4223-4230, Aug. 2018.

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- Prof. Zhong Wen-De and Prof. Qian Kemao supervised me on the overall research direction and edited the manuscript drafts.
- I proposed the system, developed the algorithm, conduct the simulations and prepared the manuscript drafts.
- Mr. Zhang Sheng helped me on the algorithm derivation and simulation results analysis and gave comments on the manuscript.
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Abstract

In recent years, light-emitting diodes (LEDs) enabled indoor visible light positioning (VLP) technology has attracted ever-increasing attention, due to its numerous potential applications including pedestrian localization and navigation, robots monitoring, assets tracking and even industrial manufacturing. Compared with traditional radio-frequency (RF)-based systems (e.g., Wi-Fi, Bluetooth), LED-based VLP systems has many inherent advantages, such as energy efficiency, long life-time, better positioning accuracy, etc. Besides, it can be integrated with visible light communication (VLC) system which is a fast-developing next-generation wireless communication system. Nevertheless, the development of accurate, stable and low-cost indoor VLP systems faces many challenging issues: i) limited mobility and robustness due to the line-of-sight constraint; ii) conflict of the complex processing and positioning algorithms with the limited memory, power and calculation ability of mobile devices; iii) high positioning accuracy to centimeter, even to millimeter level; iv) interruption/outage during the localization; v) the integration of VLC and VLP systems. This thesis addresses part of the pressing issues mentioned above to improve the overall performance of a VLP system.

Both photodiodes (PDs) and camera/image sensors (ISs) are commonly used as the receivers of a VLP system. In an IS-based VLP system, a target's location is calculated by solving a camera projection model which describes the one-one correlation between an object point and its projected image point. However, the conventional iterative solvers, e.g., Levenberg-Marquardt (LM), which are used to solve the projection model, are found sensitive to the initial guesses. It means the algorithm may fail to converge or suffer a long response time and a large positioning error when starting from a bad guess. To speed up the positioning process and guarantee the success rate, we derive a closed-form solution to the receiver's position and orientation using singular value decomposition (SVD) technique and propose an SVD based positioning algorithm. Extensive tests verify that the proposed algorithm is 50-80 times faster than the conventional LM algorithm and avoids convergence failures caused by the bad initial guesses.

Meanwhile, we derive the Cramer-Rao lower bound (CRLB) as the theoretical accuracy limit. Simulations are performed to reveal the impact of the various system parameters on the positioning error bound. A robust and fast 3D IS-based VLP system with centimeter-to-decimeter level accuracy is demonstrated experimentally.

On the basis of the above IS-based VLP system, a sensor fusion-based positioning algorithm is proposed to enhance positioning accuracy by effectively fusing data from the image sensor and the motion sensors, both of which are widely accessible in common mobile devices. The issue of sensor fusion is formulated as a multi-objective non-convex optimization problem and mathematically solved using SVD technique, again. Experiment results show that the proposed singular value decomposition-based sensor fusion (SVD-SF) algorithm brings approximate 44% positioning accuracy improvement compared with the algorithm employing a single image sensor under the same experimental settings. Simulations are conducted to further evaluate its positioning error performance under different noises conditions.

The IS-based VLP systems mentioned above requires capturing at least three LEDs in every picture to calculate the targets' positions, otherwise positioning is interrupted or fail. The requirement greatly reduces the system robustness and flexibility. To deal with this problem, we propose a single circular LED positioning (SCLP) system where a common circular LED lamp with a point marker is used as a transmitter. The LED image is no longer treated as a point as in the existing works, but as an image whose geometric features are exploited to determine the receiver's orientation and location relative to the reference LED lamp. The expressions for determining the receiver's orientation and location are derived in terms of the geometric parameters. Our algorithm effectively enhances the system robustness by reducing the minimum number of required LEDs. The SCLP system is experimentally validated and centimeter-level positioning accuracy is achieved in an area of $3\text{m} \times 3\text{m}$.

The IS-based VLP system can be classified as a self-localization system where targets detect, process and calculate the positions of themselves. This system structure may not be appropriate for portable mobile devices having limited power,

memory and computation capability. Besides, the self-localization system lacks the overview of a global view of real-time locations of all targets on the map which is required in some application such as simultaneous navigation and monitoring of multiple targets. Considering these problems, a multi-target localization system is proposed for the first time in terms of visible light positioning (VLP). PDs are mounted on the ceiling to detect light signals from mobile targets (MTs) that carry LEDs, while a central server processes the received signals and calculates the positions of MTs and sends their location information back to them. By exploiting the sparse nature of a localization problem, the multi-target localization is converted into a problem of sparse matrix reconstruction. A 3-step workflow is developed to solve the problem by employing the compressive sensing (CS) theory. The proposed system and supporting algorithms are verified through extensive simulations. The impact of various parameters including the PD number, SNR, grid square size on system positioning performance is investigated.

List of Abbreviations

LBS	Location Based Services
GPS	Global Positioning System
VLC	Visible Light Communication
VLP	Visible Light Positioning
PD	Photo-Diode
IPS	Indoor Positioning Systems
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
RFID	Radio Frequency Identification
UWB	Ultra-Wide Band
RF	Radio Frequencies
DD	Direct Detection
IM	Intensity Modulation
TOA	Time of Arrival
TDOA	Time Difference of Arrival
POA	Phase of Arrival
PDOA	Phase Difference of Arrival
AOA	Angle of Arrival
RSS	Received Signal Strength
TOF	Time of Flight
IS	Image Sensor
LM	Levenberg-Marquardt
SVD	Singular Value Decomposition
CRLB	Cramer-Rao lower bound
RMSE	Root Mean Square Error
LED	Light Emitting Diodes
SCLP	Single Circular LED Positioning
CDF	Cumulative Probability
PDF	Probability Density Function
FOV	Field-of-View
ICI	Inter-Cell Interference
CS	Compressive Sensing

NLLS	Non-Linear Least-Square
LED	Light-Emitting Diode
MT	Mobile Target
LOS	Line-of-Sight
GCS	Global Coordinate System
OOK	On-Off-Keying
TDM	Time Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
RSE	Rolling Shutter Effect
CMOS	Complementary Metal Oxide Semiconductor
OOK	On-Off Keying
FBMC-SCM	Filter Bank Multicarrier-based Subcarrier Multiplexing
AWG	Arbitrary Waveform Generator
SVD-SF	Singular Value Decomposition-based Sensor Fusion
MMSE	Minimum Mean Square Error
LIM	Location Information Matrix
UGV/UAV	Unmanned Ground/Aerial Vehicles
IoT	Internet of Things
WSN	Wireless Sensor Network
ASK	Amplitude Shift Keying
WLAN	Wireless Local Area Networks
SNR	Signal-to-Noise Ratio
$RW_{l_1}M$	Reweighted l_1 -norm Minimization
l_1M	l_1 -norm Minimization
VLC	Visible Light Communication
VLP	Visible Light Positioning
Wi-Fi	Wireless-Fidelity

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Chapter 1 Introduction

1.1 Background and motivations

1.1.1 Background

During the last two decades, we have witnessed the increasing demand of various location-based services (LBSs). Examples of such applications include pedestrian navigation, inventory management, asset tracking, and cargo/robot navigation, automatic driving, tourist route planning, hotel/restaurant recommendation, etc. The global positioning system (GPS), as a mature technology, has been widely adopted to offer ubiquitous and reliable LBSs in outdoor scenarios, which greatly facilitates our life. However, GPS doesn't work well in the indoor environment as the satellite signals are blocked by obstructions or suffer a severe attenuation inside buildings [1]. The issue of indoor positioning/localization remains unresolved and attracts significant attention from both industry and academia.

Researchers have made encouraging progresses in recent years. A number of indoor positioning systems (IPSs) are proposed and experimentally verified using different technologies, e.g., Wi-Fi [2-5], Bluetooth [6-8], radio frequency identification (RFID) [9-12] & fingerprinting [13], Ultra-Wide Band (UWB) [14-15], ultra-sonic and so on, among which Wi-Fi and Bluetooth are mostly investigated. The localization algorithms of radio frequencies (RF)-based IPSs can be roughly divided into two categories, i.e., range-based and range-free algorithms. A common range-based localization process usually consists of three phases. First, detect one or multiple features of RF signals, e.g., time/phase of arrival (TOA/POA), received signal strength (RSS). Second, estimate the object's distances with respect to the multiple reference transmitters based on the RF signal propagation model. Finally, localize the object using the trilateration method. In general, Wi-Fi and Bluetooth-based IPSs have a limited positioning accuracy from a few decimeters to meters. The effect of shadowing and multipath reflection by furniture and walls makes the received RF signals unpredictable (doesn't follow propagation model) which harms the positioning accuracy (PA). A reasonable solution is to do pre-calibration or apply correction algorithms to refine the RF propagation model, which can be labor-intensive and

computational complex. UWB (bandwidth larger than 500 MHz) empowered IPSs are reported to achieve a high PA up to centimeters using time of flight (TOF) by emitting extremely narrow pulse which can be measured precisely and is less vulnerable to the multi-path effect. However, they are costly for implementation and widespread deployment [16]. RFID and fingerprinting-based approaches belong to the range-free approach, which, as the name suggests, doesn't localize objects using geometric relations but by proximity. Specifically, when a target detects signals from transmitters with known locations, it is regarded to locate somewhere close to the transmitters. The proximity approaches simply take the location of the transmitter corresponding to the received strongest signal as the location of the targets [17]. Obviously, it only provides rough location information and may suffer a long response time if the database is large. Various RF-based indoor positioning technologies are compared in terms of scale and positioning accuracy (resolution), as illustrated in Fig. 1.1 [1].

As the light-emitting diodes (LEDs) are gradually adopted to replace the conventional incandescent and fluorescent lighting, a new application of LEDs in communication field is being extensively developed, namely visible light communication (VLC) [18], which uses modulated LED light waves instead of RF signals to transmit information. Compared with RF, white LEDs enabled VLC has many inherent advantages including potential high data rate up to a few Gbit/s [19], license-free bandwidth (400–700 nm), high security, low power consumption and long lifetime. Recent works successfully demonstrate high-speed VLC system and it is considered as a promising candidate of new high-speed wireless communication technology that may collaborate with the conventional RF-based wireless networks especially in short-range indoor applications. Besides its superiority in communication, VLC systems are found potential to provide accurate positioning services, generating a new research topic, i.e., visible light positioning (VLP).

VLP has a number of advantages over the RF-based positioning systems: i) LEDs are energy-efficient and have long lifetime [20]; ii) visible light is less vulnerable to multipath effect leading to a more accurate propagation model and higher PA; iii) VLP system can be built by leveraging the existing lighting infrastructure with small modifications, which can be ubiquitous and low cost; iv) VLP systems can also be

deployed in the RF-restricted environment such as airports, hospitals, etc. Last but not the least, VLP is potential to be naturally integrated with the rapidly-developing VLC system to provide fast communication and positioning services simultaneously [16].

Fig. 1.1. Overview of the performance of RF-based positioning technologies [1].

Positioning accuracy is not the only concern for a positioning system, various aspects have to be considered which are summarized as follows:

- *Positioning accuracy:* Positioning accuracy is no doubt the most important assessment criteria of a positioning system. Usually, we adopt mean Euclidean distance between the true positions and the estimated positions as the positioning error. The inaccuracies of the estimation are caused by many factors: i) the limited precision of devices /quantization error of devices; ii) environment noises (e.g., multi-path reflection); iii) image/signal processing algorithms; iv) approximations in channel model or calculation model, etc. Positioning accuracy can be enhanced by improving one or more problems mentioned above. For example, using multiple sensors, using filters to suppress noise and make prediction, etc. Up to now, centi-to-decimeter level of accuracy can be realized in many VLP work using both PD and camera. Improving accuracy is the most frequently investigated issue in the reported VLP works.

- *System stability and robustness:* Stability and robustness are also important metrics in the evaluation of a VLP system, which, however, receives limited attention and investigation. Stability describes how consistently the system works, i.e., the variation of the positioning performance over many trials. This can be examined by observing the positioning error distribution (cumulative probability (CDF) or probability density function (PDF)) in extensive experiments. When we compare two positioning algorithms/techniques with the similar or close mean error, we prefer the algorithm/technique whose CDF curve is steeper or whose PDF is more concentrated. Because it means the system's performance is stable and has low chance to have extremely large errors. Robustness measures the ability of the system working normally under disturbance of large noises or in the absence of some signals. A robust positioning algorithm/technique should be able to use noisy or incomplete detected data to calculate locations and provide continuous localization services with low chance of outage/interruption.
- *Complexity:* The complexity of a VLP system includes: i) the complexity of software (data processing and positioning algorithms/techniques); ii) the complexity of hardware devices. A computationally complex positioning algorithm may lead to long response time far below the real-time requirement. Also, it can be memory and power-consuming usually requires the support of the powerful hardware which restricts its applications. Meanwhile, in most cases, there is a trade-off between complexity and positioning accuracy.
- *System structure:* VLP system structures can be divided into two categories, i.e., centralized and distributed, depending on where the main calculations are performed. Different system structures are applicable to different applications. For example, mostly reported VLP systems are distributed system, where mobile targets detect and process data to localize themselves. This structure is straightforward and has good security. A typical application of this structure is human localization and navigation. However, for those compact mobile devices and IoT applications which is featuring limited memory, power supply and calculation ability, a centralized positioning system, where a central server calculates the real-time locations of all the devices and send back to them, can be more appropriate. Besides, as it provides overall observations of all access

devices' real-time locations, it is extremely suitable for the robots tracking and control. Therefore, the system structure should be properly designed according to its applications.

- *Scalability*: The scalability of a system ensures that the system functions well when the positioning area gets larger. Existing VLP works usually consider the localization for single user in a cell with a small group of LEDs. The large-scale indoor positioning scenario is simply treated as the duplicate of the localization problem of a single cell. However, in practical large-scale IPS supporting multiple users, new questions/problems may arise. For example, how to efficiently reuse the ID or frequency label? how to mitigate inter-user inferences? can users cooperate to localize each other? An ideal positioning system should be able to scale in terms of both space and techniques.
- *Cost*: The cost of a VLP system includes the cost on money, time, space and energy. The time factor is related to installation and maintenance. For example, a VLP system or a Wi-Fi-based positioning system can be deployed on the existing illumination system/wireless communication system; meanwhile, the camera and Wi-Fi module are widely used in smartphones and other portable devices. In such a case, the cost of the money, time, and labor can be saved. On the contrary, despite the high positioning accuracy of the UWB positioning technology, the large-volume and high expense of UWB devices limits its application. Besides, energy cost/consumption is another important factor. It is predicted that indoor localization technology is potential to be applied in future IoT applications. An ideal IPS should be energy-saving especially for the mobile users. For example, passive RFID tags are completely energy passive and only respond to external fields and, thus, could have an unlimited lifetime.
- *System integration*: The future VLP system are expected to be integrated with VLC systems, i.e., LEDs have should provide the illumination, high-speed communication and reliable positioning services simultaneously for indoor applications. This requires LED signals conveys information for both communication and position. In the integrated system, the modulation and detection scheme, power allocation, pre-processing and post-processing scheme would affect the coverage bandwidth efficiency, bit error rate (BER),

positioning accuracy and robustness. The scheme to integrate VLC and VLP is a critical issue.

1.1.2 Motivations

So far, researchers have explored many possibilities in the design of VLP systems and algorithms. Although many potential solutions have been reported to address the challenges of VLP, there still exist many problems in developing a VLP system which can provide high-accuracy, real-time, stable and uninterrupted localization services for multiple users simultaneously. Therefore, this thesis is motivated to investigate the VLP system in terms of algorithm running speed, positioning accuracy, outage problem, and system structure. Specifically, the motivations are listed below:

- In practical implementation of a VLP system, the response time greatly influences the quality of the localization services. For a typical indoor positioning system for either human beings or robots, real-time or near real-time response is highly desired. However, localization/positioning is usually performed in mobile devices with limited memory, computation complexity and power storage, which can be time- and power-consuming to process sensor data and perform computationally complex algorithms. Besides, success rate determines the system stability, a positioning algorithm should have certain immunity to parameter settings, environmental noise, etc. Therefore, it is of great significance to pay attention to the employed positioning algorithms in terms of the running speed and robustness.
- Nowadays, mobile devices are usually equipped with various sensors. For example, the accelerometer, gyroscope, geomagnetic sensor, camera, etc. Sensor fusion is widely adopted in many fields (e.g., robotics, virtual reality) which combines different sources of sensor data with noise and uncertainties to obtain a more accurate measurement. Similarly, the idea of sensor fusion can be employed in indoor positioning. The scheme to fuse various sensor data effectively to enhance the positioning accuracy is interesting to investigate.
- In conventional IS-based VLP systems, at least three reference LEDs have to be captured in a single picture to determine the receiver's location. From our extensive experiments, it is found that the requirement is not easy to be satisfied in complex indoor environment and in mobile cases for receivers (cameras) with limited field of view (FoV), otherwise localization may be

interrupted or fail. To guarantee a sufficient number of LED lamps are captured by a camera, researchers usually deploy LEDs densely and fix the orientation of the sensors to face towards the LEDs in their experiments, which greatly reduces the system practicability and flexibility. Therefore, the robustness and flexibility of a VLP system can be improved by reducing the minimum number of required reference LEDs.

- Existing VLP works usually consider the localization for single user in a cell with a small group of LEDs. The large-scale indoor positioning scenario is simply treated as the duplicate of the localization problem of a single cell. However, performance may degrade when system coverage is extended and more transmitters and receivers are involved. Meanwhile, most of the VLP systems require mobile targets (MTs) to detect and process signals to localize themselves. These “self-localization” or “active localization” systems are appropriate for people’s localization/navigation in supermarkets, shopping malls, etc. Yet they may not scale well when directly applied to objects tracking and monitoring where the objects have limited power and computational capability, and a global view of real-time locations of all MTs on a map is required. In such case, a reversed “centralized localization” system supporting multiple users is worthy to investigate, where a central server process and perform localization algorithm and then sends location information back to MTs.

1.2 Major contributions

In order to address the above-mentioned problems, novel advanced systems and algorithms are explored and developed in this thesis. The main contributions of this thesis are summarized as follows. (The studied issues and the comparison of related works and our contribution are specially highlighted in Fig. 2.6, Chapter 2.)

- Most of the reported VLP systems focus on achieving high positioning accuracy and pay much less attention to the employed positioning algorithms, which largely determine the system acquisition time and the success rate. Specifically, in an IS-based VLP system, a positioning result is the solution of a non-linear least-square (NLLS) problem obtained from a camera projection

model and an iterative Levenberg-Marquardt (LM) solver is widely adopted to solve the NLLS problem. However, from extensive experiments, the iterative solver is found sensitive to the initial guesses, which means the algorithm may fail to converge or suffer a long response time and a large positioning error when starting from a bad guess. Therefore, in this thesis, we derive a closed-form expression to determine the receiver's position and orientation using the singular value decomposition (SVD) technique and propose an SVD-based positioning algorithm which is proved to be 50-80 times faster and has higher success rate than the LM-based positioning algorithm. (Chapter 3)

- To realize high positioning accuracy, a straightforward and effective way is to use advanced sensors and devices (high-sensitivity PDs, high-resolution camera), however, the high cost will largely hinder their widespread implementation and usage. An economical alternative is to make full use of various accessible technologies and common sensors. With this idea, some hybrid systems involving various technologies are proposed accordingly. However, these hybrid systems usually simply perform the localization using different technologies separately and take average/weighted average of the positioning results. None of them investigated the schemes to acquire extra performance gain by substantially fusing various sensors' readings. In this thesis, a simple and accurate sensor-fusion positioning algorithm is proposed, which effectively fuses the data from image sensor and motion sensors. The issue of sensor fusion is formulated as a multi-objective non-convex optimization problem. We derive a closed-form optimal solution to the optimization problem and propose a low-complexity SVD-SF algorithm accordingly. Both experiments and simulation results verify that our SVD-SF algorithm always perform better (at least the same) accuracy compared with the conventional single IS-based positioning algorithm. (Chapter 4)
- In an IS-based VLP system, at least three LEDs have to be captured in a picture, in order to set up equations required for determining the receiver's location. However, experiments show that this requirement is too stringent to satisfy in practical applications due to many reasons: (i) limited field of view (FoV); (ii) sparse deployment of LEDs; (iii) obstruction of indoor objects such

as walls and furniture. iv) orientation is changing when a receiver is moving. Localization services may be interrupted or fail if less than three LED lamps are within the FoV of the camera, which greatly reduces the system robustness and flexibility. In this thesis, for the first time we propose a single LED-based VLP system without using angular sensors. Unlike the existing IS based VLP systems which treat the LED lamps as point sources with only location information, our approach makes full use of the geometric features of an LED image. Specifically, a circular LED lamp is projected into an ellipse in an image. The geometric features of the captured ellipse image (e.g., centroid, major/minor axis length, major axis orientation), can be exploited to determine the receiver's pose and location relative to the reference LED lamp. In addition, a marginal point of a circular LED lamp is marked to provide the relative azimuthal information. We theoretically derive the receiver's orientation and location in terms of the geometric features and marker's location which can be obtained through proper image processing and propose a circle projection based single LED positioning (CPSL) algorithm. Experimental results demonstrate that a centimeter-to-decimeter level positioning accuracy is achieved in an area of 3m x 3m. (Chapter 5)

- Common VLP systems can be classified as “self-localization” or “distributed localization” systems where mobile targets (MTs) detect and process signals and then localize themselves. This kind of system structure is appropriate for pedestrian localization/navigation in supermarkets, shopping malls, etc., but they may not function well when applied to objects tracking and monitoring. For one thing, the conventional self-localization systems lack a global view of real-time locations of all MTs on the map. Moreover, it is inefficient for multiple MTs to constantly calculate and report their locations to a central controller. The limited power, memory and computation resources of MTs may cause large system latency. In this thesis, we propose a novel VLP system with reversed centralized structure where PDs are mounted on the ceiling to detect light signals from MTs that carry LEDs, while a central server processes the received signals and calculates the positions and sends location information back to them. By exploiting the natural spatial sparsity of an MT, the problem of the simultaneous localization of multiple MTs is converted into

a sparse matrix reconstruction problem and solved by employing compressive sensing theory. The proposed system and supporting algorithms are verified through simulations. The impact of various parameters including PD number, SNR, grid square size on system positioning performance is investigated. (Chapter 6)

1.3 Thesis organization

This thesis consists of seven chapters, which is organized as follows

Chapter 1 briefly introduces the background and the motivations of indoor VLP systems and summarizes the major contributions of the work in this thesis.

Chapter 2 presents a comprehensive literature review. The system model and channel model are introduced, and the positioning algorithms/techniques of VLP are classified and compared. Specially, the recent widely studied issues and related works are summarized, where our contribution is also illustrated clearly. After that, various potential applications of VLC are also introduced.

Chapter 3 investigates the positioning algorithm of an IS-based VLP system. A non-iterative singular value decomposition (SVD)-based positioning algorithm is proposed, which has higher success rate and runs 50-80 times faster the conventional LM-based positioning algorithm. Meanwhile, we derive the expressions of the Cramer-Rao lower bound (CRLB) and root mean square error (RMSE) bound of the estimated position, which sets theoretical limit for PA and reveals the impact of system parameters such as the relative height, image processing noise and the number of LED lamps on the positioning error.

In Chapter 4, a sensor fusion algorithm is proposed to enhance the positioning accuracy of conventional IS-based VLP algorithm, by using sensors which are widely available in a smartphone (image and motion sensors). The issue of data fusion from both image and motion sensors is formulated as a multi-objective non-convex optimization problem. Instead of employing the conventional time-consuming multi-start iterative solver to solve it, we derive a closed-form optimal fused solution and propose an SVD-SF algorithm, which ensure the acquisition rate and support mobility.

In Chapter 5, we propose and demonstrate a single circular LED positioning (SCLP) system which reduces the minimum number of required LEDs in an IS-based positioning system, thus to reduce the probability of failure and interruption during localization. In our system, a common circular LED lamp is projected into an ellipse image whose geometric features are exploited to determine the receiver's orientation and location relative to the reference LED lamp. The expressions for determining the receiver's orientation and location are derived in terms of the geometric parameters, and the SCLP algorithm is developed accordingly. We experimentally evaluate and analyze the performance of the proposed VLP system. Centimeter-level positioning accuracy is achieved in an area of 3m x 3m.

Chapter 6 presents a novel reversed structured VLP system aiming to realize a centralized localization with a global view of all mobile targets (MTs) on a map. By exploiting the natural spatial sparsity of targets' positions, the problem of the simultaneous localization of multiple mobile targets is reformulated into a sparse matrix reconstruction problem and solved. To support this system, a 3-step workflow is developed accordingly by employing compressive sensing (CS) theory. The system and supporting algorithms are verified through simulations. The impact of various parameters including PD number, SNR, grid square size on system positioning performance is investigated and discussed in this chapter.

Chapter 7 gives the conclusions of the work and provides some recommendations on the future work.

Chapter 2 Literature Review

2.1 Introduction of VLP

As GPS cannot provide satisfying positioning performance in indoor environments, indoor positioning technology aims to fill the gap. Unlike traditional RF-based indoor positioning techniques which use RF signals to replace GPS signals, VLP systems use visible light (mainly refer to LED light) in the spectrum from about 400 to 700 nm as the media to carry information. Then, various features of the light signal are extracted, and positioning algorithms are performed to localize an object. In this section, the fundamentals of white LEDs based VLP systems are briefly reviewed. A typical VLP system architecture is introduced, then the optical channel model and a three-phase positioning process are discussed in detail.

2.1.1 System model

Fig.2.1 A typical diagram of a visible light positioning system

Fig. 2.1 depicts a typical VLP system, where multiple LEDs (transmitters) are mounted on the ceiling for both illumination and positioning purposes. At the receiver side, optical sensor, either photodiode or camera, is placed on the object to be localized to detect the light signals from the multiple LED sources (Tx1~Tx4). Usually, reference LEDs in the system are labeled with unique IDs and their locations in the global coordinate system (GCS) are pre-measured and a look-up table of “identification-location” is stored in a database. The LEDs’ light wave is modulated to transmit their IDs, for receivers to distinguish the LED sources from mixed signals.

Once an ID is demodulated, the corresponding reference LED's location can be acquired by searching the look-up table, based on which the receiver device performs the positioning algorithm and shows the positioning results in the users' interface.

2.1.2 Optical channel model

Fig.2.2 (a) Lambertian radiation model, and (b) LED light propagation and reflection model

The modulated LED light signals are transmitted indoor via the free-space channel. Without loss of generality, the LED is often assumed as a Lambertian lighting source which has a Lambertian radiation pattern and the radiation intensity at a receiver surface is given by [21],

$$I(\phi) = I(0) \cos^m(\phi) \quad (2.1)$$

where ϕ is the angle of irradiance with respect to the axis normal to the transmitter surface as shown in Fig.2.2 (a), $I(0)$ is the central luminous intensity of the LED and m is the Lambertian order which is defined by

$$m = \frac{\ln(2)}{\ln(\cos \Phi_{1/2})} \quad (2.2)$$

where $\cos \Phi_{1/2}$ is the semi-angle of half illuminance of the LED.

It has been verified in [22] that the weakest line-of-sight (LOS) component is at least 7-dB higher in electrical power than the strongest diffuse component in typical indoor environments. Therefore, we normally neglect the diffuse components and consider the dominant LOS component in the analysis of an indoor VLP systems. Note that

this assumption has been widely adopted in the literature [22-26]. The LOS optical channel DC gain $H(0)$ is given [22] by

$$H(0) = \begin{cases} \frac{(m+1)A}{2\pi d^2} \cos^m(\phi) T_s(\varphi) g(\varphi) \cos(\varphi), & 0 \leq \varphi \leq \varphi_c \\ 0, & \varphi > \varphi_c \end{cases} \quad (2.3)$$

where m is the Lambertian order; A is the physical area of the PD; d is the distance between a transmitter and a receiver; ϕ is the angle of incidence and φ is the angle of irradiance; φ_c denotes the field of view of the PD (receiver); $T_s(\varphi)$ and $g(\varphi)$ are the gains of an optical filter and an optical concentrator, respectively.

However, when the indoor environment is complex containing objects or walls with high reflection factor, the signals strength can be less predictable and the conventional Lambertian propagation model may cause large positioning error. To deal with this problem, some recent works refine the VLP channel by taking into consideration of non-line-of-sight scenario and multipath effect. As depicted in Fig. 2.2(b), the optical channel DC gain of the first reflection is calculated by Eq. (2.4) [21, 27],

$$dh_{ref} = \frac{(m+1)A_d}{2\pi^2 d_1^2 d_2^2} \rho dA_{wall} \cos^m(\phi_r) \cos(\alpha) \cos(\beta) T_s(\theta_r) g(\theta_r) \cos(\theta_r), \quad 0 \leq \theta_r \leq \Theta_{1/2}, \quad (2.4)$$

where d_1 is the distance between the LED and the reflective point (RP), d_2 is the distance between the RP and the receiver, ρ is the reflectance factor, dA_{wall} is the reflective area of a small region, ϕ_r is the angle of irradiance from the LED, α and β are the angle of incidence to the RP and the angle of irradiance from the RP, respectively, and θ_r is the angle of incidence to the receiving plane.

2.1.3 Three-phase positioning process

Fig.2.3 The workflow of a VLP system

Positioning processes can be totally different when different types of receivers (photodiode or camera) are employed. Fig. 2.3 illustrates the workflow of a VLP system. Generally, the whole positioning process involves three phases: i) modulation and transmission; ii) signal detection and processing; iii) position estimation.

In the phase I, a microcontroller encodes the LED ID data into a binary sequence (frame). The LED light is modulated to transmit the sequence using “on-off-keying” (OOK) scheme, where high voltage represents code “1” and low voltage represents “0”. This modulation scheme belongs to intensity modulation (IM). Normally, a binary frame consists four parts: 1. a preamble indicating the beginning of the frame; 2. LED ID associated with the LED source; 3. location coordinates and transmitted power of the LED; 4. Error correction codes, e.g. CRC code. (Noting that the part 3 and part 4 can be removed in some situation.) DC bias is then added to the to guarantee the signal is non-negative. After that, the resultant electrical signal is used to drive the LED that emits light into the free-space channel. The multiplexing schemes such as frequency division multiplexing (FDM), time division multiplexing (TDM) are employed to allow signals from multiple LEDs transmit in parallel and receivers to distinguish/ extract signals. The binary frame is short and is broadcast repeatedly in the free space.

In the phase II, both PDs and image sensors can serve as the receivers. Specifically, PD is a semiconductor device that converts light into electrical current so that to detect the intensity change of LED light which carries information. Detected signals

are processed to demodulate the ID information to distinguish the signals sources and acquire their location information in global coordinate system.

When an image sensor (camera) is used as a receiver, the principle of signal detection is a little bit different. Rolling shutter effect (RSE) of a complementary metal oxide semiconductor (CMOS) sensor can be exploited to detect the light signal [28].

Fig. 2.4. (a) the principle of rolling shutter effect, and (b) the captured image exploiting rolling shutter effect.

The work in [28] experimentally detected continuous OOK-modulated frames (LED ID information) in a single captured image by leveraging RSE. Specifically, unlike charge-coupled device (CCD) camera which captures a whole picture at once, a CMOS camera capture an image by scanning pixels column by column in a certain line scan rate [29-30]. Through the intensity of light signals changes so fast and is invisible to naked eyes as well as auto-exposure photography, it can be detected in short-exposure time (shorter than $1/6000$ s) and generates dark and bright stripes as illustrated in Fig.2.3(a). The width of the stripes is determined by binary code duration or carrier frequency which should be designed in advance. Each code roughly occupies $T/1/r = r \cdot T$ pixel columns. Take the rear camera of Nokia Lumia 1020 as an example, it has line scan rate of 47540 column/s, then when we use carrier with frequency of 4.754 kHz, then in the image, each binary code occupies 10 columns of pixels.

In the Phase III (position estimation), the features of received signals can be extracted, e.g., received signal strength, time of arrival, angle of arrival, etc., and send into the position estimator, where the positioning algorithms are operated, and the location is calculated various positioning algorithms which is introduced and compared detailly in Section 2.2.

2.2 VLP algorithms and comparisons

Most of the positioning algorithms adopted in RF-based IPSs can be transferred into VLP systems, such as RSS, TOA/TDOA. Besides, due to the special features of visible light, some new techniques, e.g., angle of arrival and image transformation, can also be leveraged for positioning. In this section, VLP algorithms are classified into three categories as shown in Fig.2.4: i) Geometric approach; ii) proximity; iii) vision-based approach; iv) filtering/ probabilistic approach. The works using different positioning techniques will be compared and their respective advantages and deficiencies will be analyzed.

Fig. 2.5. Taxonomy of VLP algorithms

2.2.1 Geometric approaches

Generally, geometric approaches utilize the geometric properties of a triangle to estimate position, and it has two subcategories: lateration and angulation. Lateration estimates the position of an object by measuring its distances from multiple reference points. Instead of measuring the distance and angle directly, we usually use some techniques to assist measurements which include RSS [31-34], TOA/TDOA [35-37], POA/PDOA (phase of arrival/phase difference of arrival) algorithms [38-40].

Angulation locates an object by computing angles relative to multiple reference points and finds the intersection of multiple space line [1]. Angle of arrival (AOA) algorithm belongs to angulation. These approaches are introduced below.

- A. RSS: RSS is a widely adopted algorithm in RF-based indoor positioning systems and VLP systems because RSS is easy to acquire using a cheap and lightweight PD. As discussed in Section 2.1, the visible light signal propagates in free-space channel with light intensity following Lambertian propagation model and based on which the distance from corresponding transmitters can be calculated. The object locates in the intersection of multiple circles (for 2D positioning) and spheres (for 3D positioning) with distances as the radius as shown as in Fig.2.5 (a). Specifically, for a 3D positioning problem, the object's location (x, y, z) satisfies

$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = d_1^2 \\ (x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2 = d_2^2 \\ (x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2 = d_3^2 \end{cases} \quad (2.5)$$

where (x_1, y_1, z_1) , (x_2, y_2, z_2) , (x_3, y_3, z_3) are location of three reference LEDs. Normally, the intersection is a region rather than a precise point, due to the measurement noise and model imperfections. The object's location is the solution with least-square error E [17],[28].

$$E = \sum_{i=1}^N \left\{ (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \right\} - d_i^2 \quad (2.6)$$

- B. TOA/POA: TOA measures the distances d to reference points by $d = t \cdot c$, where t is the propagation time and c is the light speed. POA algorithm assumes the initial phase of carrier is known so that the phase shift with respect to the initial phase can be obtained as $\Delta\varphi$, then the distances to multiple reference LEDs are calculated as $\lambda \cdot \Delta\varphi / 2\pi$, where λ is the carrier wavelength. Then, position is estimated as lateration in RSS. The resolution of TOA should be quite high at the receiver side to guarantee the positioning accuracy, which requires expensive and high-precision equipment. Besides, a significant deficiency of both TOA and POA is the requirement of strict synchronization between transmitters and receivers,

where a small difference cause a large error. Therefore, TOA/POA is conceptually feasible but hard to implement, VLP works use TDOA/PDOA instead to avoid synchronization.

- C. TDOA/PDOA: Instead of using absolute distances to reference points in TOA/POA, TDOA/PDOA algorithm measures distance difference (obtain by time/phase difference) to every two reference points, then the intersection of multiple hyperboloid is the location of receiver, as illustrated in Fig.2.5 (b).
- D. AOA: In AOA, the location of the target can be found by the intersection of space angle direction lines which are emitted from reference transmitters as depicted in Fig.2.5(c) [41]. Normally, AOA cannot be detected directly by using a single camera or a single PD. AOA positioning approach has some advantages. First, no synchronization is required; Second, since it is not based on Lambertian propagation model, the fluctuation of transmit power, background lighting noise, multipath and reflections have not impact on the positioning results. However, a small error of detected angle will be exaggerated along with the light propagation.

2.2.2 Proximity

Both Fingerprinting and ID matching belong to proximity approach [42-48], which usually consists of two stages: i) off-line survey; ii) online positioning. During offline-survey, the LED ID or the signal features such as AOA, RSS, POA are collected as fingerprinting and stored in local data based. In the stage of on-line positioning, the target is located by matching the detected features with the records in the database [43]. Apparently, proximity approach can only offer a rough localization which is suitable for the scenario which has low requirement of accuracy.

RSS is the widely adopted feature as fingerprinting [43]. The testing area is divided into grids and the RSS values from LEDs are collected at each grid. Smaller grid size leads to larger database and more precise localization, however, the offline survey can be labor-intensive accordingly. There is a trade-off between the positioning accuracy and survey workload. During the online-positioning stage. A distance D is defined

as Eq. (2.7) to denote the difference between measured RSS values and fingerprinting database.

$$D(i) = \sum_{i=1}^N \left(r_{ss_i} - r_{ss_i}' \right)^2 \quad (2.7)$$

The target's location is determined as the point with smallest D . In practical implementation, transmitters' power may vary with time and temperature, which affect the positioning accuracy. This problem can be solved by introducing a normalization factor [43].

Fig.2.5 (a) TDOA/ PDOA algorithm diagram (b) RSS/TOA algorithm diagram, (c) AOA algorithm diagram

Meanwhile, some techniques can be used together with fingerprinting for enhancing the PA [43-45]. For example, classifier techniques, e.g., random forest, k-NN, and machine-learning based classifiers [45] and so on, can be combined with fingerprinting algorithm to make better estimations.

ID matching is an even simpler positioning approach. No labor-intensive RSS survey is required, only “ID-location” table is stored in database. LEDs broadcast their IDs in the free space repeatedly where a receiver can receive multiple ID simultaneously with different strength. The receiver can take the location of the LED corresponding to the ID signal with strongest strength as the location of itself or take the weighted summation of multiple LEDs.

2.2.3 Vision-based approaches

Image transformation positioning is a vision-based positioning algorithm [28][49-51], where a camera is used as a receiver, and the captured images are processed to localize target based on pinhole projection model.

Generally, image transformation-based VLP systems have relatively accurate and stable positioning results compared with RSS-based VLP systems. This is because the image transformation approach localizes object based on projection model, where the fluctuation of transmission power, reflections, diffusion, and multipath effect have no impact on it. Meanwhile, pinhole camera projection model is widely adopted in camera-based VLP systems, however, it is a general and simplified imaging model, which doesn't consider the distortions and imperfections caused by optical components. The approximation and assumptions in the model bring errors. The distance to the object, the image resolution determined by camera pixel number and quantization error determined by camera pixel size and image processing algorithm will influence the PA.

2.2.4 Probabilistic approaches

As discussed, all the positioning approach require a LOS condition and stable environment, otherwise many outliers (unsmoothed points) or outage (no positioning results) occur during the localization. To deal with this problem, filters, such as Kalman filter (KF) series (including extended KF, iterated extended KF or unscented KF) and particle filters, are often adopted in collaboration with the algorithms mentioned above to improve the positioning accuracy, smooth results or realize real-time tracking. All the filtering approaches are essentially the probabilistic approach.

KF can achieve the theoretical optimal estimation performance when the system is assumed to be a linear Gaussian system. System can use the kinetic model to lower

the effect of the system noise. The research [52] applied hybrid PF/KF to enhance the performance of mobile devices tracking based on AOA. The research [53] introduced EKF in the TDOA-based positioning. Particle filter is a Monte Carlo method used when the explicit expression of posterior probability is hard to obtained. PF uses weighted particles to approximate the posterior distribution of the problem. System is not necessarily to be linear and has Gaussian noise in the PF process. Hybrid PF/KF scheme is also reported in [54], where a PF is used to estimates orientation and a KF for position. Experiments shows that it obtains more accurate orientation estimation and better positioning accuracy than using EKF.

2.3 Studied issues and related works

This section summarizes the widely studied issues of VLP systems, and the proposed potential solutions/ related works for these issues are classified and discussed. As illustrated in Fig.2.5, the greatest attention has been drawn on improving the positioning accuracy of white LEDs based VLP systems. A lot of techniques have been reported in literature, which can be mainly classified into six categories as explained below.

- (1) Hybrid system [55-56]. Different positioning algorithms/technologies have their own features and are applicable for different scenarios, thus it is a straightforward way for two or more algorithms/technologies collaborate with each other to achieve a better accuracy. Hybrid systems are commonly found in literatures. For example, a hybrid RSS/AOA technique is reported for indoor localization where RSS provides a coarse estimation and AOA is used to further refine the estimation [55]. Another hybrid RSS/AOA system is presented in [56]. In such system, AOA algorithm localizes an object by a least square estimator and RSS information is utilized to further enhance the positioning accuracy using a weighted least squares solution. Besides the combination of two positioning techniques, system of hybrid technologies (RF-based positioning and visible light-based positioning) is also presented in works [57-58].
- (2) Multipath reflection model. Multipath effect is one of the major concerns for complex indoor scenarios which limits the accuracy of RSS based indoor VLP

systems in practical application because most VLP systems work under the assumption of the line-of-sight condition and the signals after reflections are simply neglected. The impact of multipath reflections on the positioning error is investigated in [59-61] in order to obtain a more precise VLP channel model. An iterative algorithm is proposed in [59] and [60] to reduce the influence of multipath reflection by subtracting the estimated reflection signal from the received signal. Besides, work in [61] applies a combined deterministic and modified Monte Carlo (CDMMC) approach to estimate the channel impulse response considering reflections, based on which, the calibration approach is proposed to mitigate the effect of multipath reflections.

- (3) Filtering techniques. Filtering techniques [62-65] are commonly used in VLP systems, including Kalman filter, particle filter, extended Kalman filter and various modified versions of these filters. Filtering techniques can effectively decrease the signals noises, fuse sensor data, make state estimation [62] thus to improve the accuracy, enable the tracking capability and support the mobility of the receiver [64].
- (4) Pre-calibration. Since system parameters may change for different environments and devices, or even along with time, a process of pre-calibration is generally required before an experimental evaluation in order to obtain an accurate position estimation. For example, many VLP systems set Lambertian order $M=1$ for the sake of simplicity, based on the assumption that the half power semi-angle of the receiver is 60° [64][70-74]. Yet, in practical systems, Lambertian order varies when different receivers are employed. A pre-calibration method for obtaining a Lambertian order is summarized in [17], and a better positioning performance is demonstrated after calibration.
- (5) Differential algorithm. The problem of positioning instability caused by LED light intensity fluctuation is addressed in work [69]. A differential detection positioning based algorithm is developed, where a base detectors' reading is used to correct the errors of mobile detectors' positioning results. A differential PDOA algorithm with a similar idea is found in work in [70].

Fig. 2.6. The summary of VLP studied issues, related works and our contributions.

(6) Machine learning approach. Machine learning and neural networks are used in a wide range of applications. In VLP system, machine learning technique is able to model the unpredictable noises and the sophisticated channel model considering multiple factors. Specifically, Zhang et al. in [70] apply it to predict and amend unpredicted noises and drift in localization in a PDOA-based VLP system, which experimentally demonstrates a sub-decimeter positioning accuracy. A machine learning-enhanced RSS-based VLP system is proposed in work [71], which is expected to be adaptive to different environments with proper hypothesis function of the estimated position and reliable training data.

As mentioned in Section 1.1.1, the flexibility and robustness of a VLP system could be significantly reduced by the stringent requirement of capturing multiple LED in every single image. Reducing the minimum number of required LEDs can be an effective way to reduce the possibility of localization outage. Therefore, a few single LED based VLP works are proposed in [72-74]. Specifically, the specially designed receiver in [74] is composed of a horizontal photodetector (PD) and two tilted PDs where the tilting angle is manually set and pre-known. Positions can be calculated from RSS readings and geometric relationships. An average and maximum error of 2.15 and 4.01 cm are achieved respectively in an area of 1 m×1 m with the LED height being 1.5m. SmartLight [74] modifies the transmitter (a single LED lamp) and exploits the light splitting properties of convex lenses to create a one-to-one mapping between a receiver's location and the set of orthogonal light signals received at that location. These works require either multiple PDs as the receiver or complicated modifications for LED lamps, both of which can be expensive and time-consuming. Compared with thesis schemes, our proposed circle projection based VLP system is cheap and simple in terms of the hardware setting and the computational complexity. Yet, it has a limitation that the reference LEDs should be circular.

In the image transformation-based positioning approaches (using image sensor), the equations are set based on the pinhole camera projection model and the location can be determined by solving the equations. Due to the nonlinearity and imperfection in measurement, the equation set has no exact solution but instead has an optimal solution with a minimum square fitting error. Most of the related publications [75]-[79] utilize software solver to address this least-square problem by iterative

algorithms, e.g., the steep gradient descent algorithm. However, these iterative algorithms are found to be sensitive to initial guesses; furthermore, it can be quite complicated and time-consuming. In this thesis, a non-iterative SVD-based algorithm has been developed to obtain the closed-form least-square solution. It is more robust and time-saving than conventional algorithms since no initial guess and iterations are required, and it is easier to be implemented for mobile devices, which cannot be integrated with such iterative solvers.

Theoretical works are also carried out to analyze the positioning error using different positioning techniques, which can be insightful for system design [56][80-82]. A Cramer-Rao lower bound (CRLB) is derived in [80] for a TOA-based synchronous VLP system. The impact of various parameters on the positioning error bound is investigated including LED transition power, center frequency, and the area of PD. In [81], the authors derive the CRLB expression for an RSS-based VLP system. The results reveal how the LED configuration, transmitter's height and signal bandwidth influence can influence the system error performance. Also, [56] and [83] derive the analytical expression of the CRLB for RSS-based localization, wherein [56] considers the different deployment scenarios in 3D geometry, while [83] provides the error bound for VLP using aperture-based receiver (consisting of several PDs with each having an associated aperture) which has both RSS and angle information. However, the theoretical analysis for an IS-based VLP system error performance is rarely found in existing works. To fill this gap, the expressions of the CRLB and root mean square error (RMSE) bound of the estimated position using an image sensor are derived, which sets the theoretical PA limit and indicates the impact of relative height, image processing noise and the number of LED lamps on the positioning error. Moreover, simulation results show that a centimeter-level CRLB can be achieved in a typical setting of IS-based localization parameters.

As the development of VLC technology, an integrated of VLP and VLC system are expected to provide simultaneous high-speed communication and accurate localization services in various indoor scenarios. A combination of orthogonal frequency division multiplexing access (OFDMA) scheme and RSS positioning technique is a potential solution which has been investigated in several works, [84-89]. For example, Lin et al. in [84] experimentally demonstrated an integrated

OFDMA/RSS-based VLCP system, which offers a mean positioning error and an error vector magnitude of 1.68 cm and more than 15 dB, respectively. However, this OFDM-based subcarrier multiplexing scheme usually has high out-of-band interference thus requires large guard band. To address this problem, Yang et al. in [87] proposed a quasi-gapless integrated VLCP system based on filter bank multicarrier-based subcarrier multiplexing (FBMC-SCM) which has lower out-of-band interference compared with conventional OFDM-based subcarrier multiplexing scheme thus saving the guard band, and thereby improving the bandwidth efficiency.

Some issues such as outage problem, centralized/distributed system structures and route planning and optimization have not yet been adequately investigated, and limited works have been reported so far. These problems are the gaps we are aiming to address in our research.

2.4 Applications of VLP system

While most of the RF-based IPSs are still in the stage of testing, some companies have commercialized the VLP system. For example, ByteLight, launched in 2011, is one of the few companies started early in the VLP field [88]. It provides LED integrated indoor positioning services for smartphones helping retailers enhance their customer engagement. In 2015, Carrefour in Lille, France, firstly deployed the Philips' indoor LED positioning system using VLC technology, as shown in Fig. 2.7.

Philips VLP system successfully demonstrate its application in pedestrian indoor navigation with 30-50cm accuracy which could support customers to track discount information and find the products [89]. Moreover, Qualcomm released a VLP system named Lumicast, which reported tens-of-centimeter-level positioning accuracy. To list some potential application scenarios of VLP systems:

- A. Museum, galleries and exhibition centers: People may easily get lost in the huge indoor public places with complex floor plan such as shopping centers, galleries, opera houses, etc. VLP can lead them quickly to find toilet/seats and can navigate people to the artworks they are interested in. Furthermore, the location-aware guide can be delivered base on the real-time position

information. When a tourist stands in front of a painting, VLP system can automatically play the corresponding audio commentary.

- B. Markets and shopping malls: VLP system can offer benefits for both retailers and customers in a shopping center. The retailers can deliver their promotion and sales information to the potential customers accurately. And the customers can find the shop and a product they want quickly.

Fig. 2.7 Philips LED-based indoor positioning in Carrefour in France

- C. Factories, warehouses, logistics: VLP systems can be deployed in factories and warehouses to track and manage the assets and ensure efficient and secure logistics; it can also localize and navigate cargo robots to move and sort products without conflicts.
- D. Healthcare facilities: VLP systems can be used to track or remotely control the hospital assets such as trolleys and wheelchairs. Also, such system can quickly localize emergencies and deliver first aid/rescue on time. Noting that, many medical equipment/devices are sensitive to radio frequency, which makes the RF-based IPS infeasible in hospitals and potentially making VLP the most feasible solution.

Obviously, for different application types, targets sizes, space scales, the positioning requirement can be so different in terms of response time, accuracy, cost, etc. For example, the navigation for a cleaning robot in a room should have higher positioning accuracy than the navigation for customers in a shopping mall. Taking all these

potential benefits of LVP technology and its potential versatility to be employed in a wide range of applications, there is an increasing impetus for greater research and investment in this fast-growing field.

2.5 Summary

In this chapter, a brief review of the VLP system model using camera/PD as the receiver is presented. Meanwhile, positioning algorithms are classified into four categories based on its principle. The principles of the multiple positioning algorithms are introduced and compared in terms of accuracy, robustness, and cost. Finally, some commercialized VLP systems in the world and the related companies are presented. At present, VLP systems are mainly utilized in the retail industry (supermarket/shopping malls) to engage customers and improve their shopping experience. Additionally, we discuss potential applications of VLP in the healthcare sector manufacturing industry and in the logistics field.

Chapter 3 An Image Sensor (IS) based VLP System: Non-iterative SVD Technique and Error Bound Analysis

3.1 Introduction

It has been discussed in Chapter 2 that PDs are widely used as the receivers in VLP systems. A PD is able to estimate the transmission distance by detecting and analyzing the light properties such as RSS [31], TOA/TDOA [36], and POA/PDOA [38]. However, the RSS based techniques rely on the condition that the LED's transmitted power is perfectly known and does not change with time, while both TOA/TDOA and POA/PDOA based techniques require extremely precise devices and measurements [16]. Also, PDs are found susceptible to the direction of a light beam, which may greatly limit the users' mobility [90]. To overcome these problems, image sensor (IS) is considered as a good alternative as the receiver can be easily found in smartphones. Image transformation-based positioning is generally more accurate and stable as it is not sensitive to the fluctuation of transmission power, light diffusions, and reflections. Many camera-based VLP systems have been reported in recent years [28,77-78,90]. The work in [77] utilized a fish-eye lens-equipped camera to capture more reference lamps in order to improve the PA. Furthermore, the work presented in [91] demonstrated a VLP system with centimeter-level accuracy, which employed PD and camera as the receivers simultaneously. However, most of these reported systems focus on acquiring high PA but pay much less attention to the employed positioning algorithms, which largely determine the system acquisition time and the success rate. In the literature, iteration-based solvers, (e.g., Levenberg-Marquardt [92] and Newton solvers), were commonly adopted to solve non-linear least-square (NLLS) positioning problems. However, they are found sensitive to the initial guesses, which means that the algorithm may fail to converge or suffer a long response time and a large positioning error when starting from a bad guess. Recently, error performance of the VLP systems using different techniques is studied in some reported works. Cramer-Rao lower bounds (CRLB) were derived as the theoretical PA limits of the VLP systems based on RSS technique [81] and hybrid TDOA/RSS technique [82],

respectively. However, error analysis is rarely found in the IS-based VLP systems especially for the 3D positioning scenario.

In this Chapter, the VLP system using a smartphone camera as an IS is analyzed and improved. The content of this Chapter is related to the author's work published in [50]. The main contributions of our work are summarized as follows:

- a) A new non-iterative positioning algorithm: we derive a closed-form expression to determine the receiver's position and orientation using the singular value decomposition (SVD) technique and propose a SVD-based positioning algorithm, which is 50-80 times faster and more robust than the LM-based positioning algorithm [92].
- b) Theoretical positioning error analysis: the expressions of the CRLB and root mean square error (RMSE) bound of the estimated position are derived, which sets theoretical limit for PA and reveals the impact of system parameters such as the relative height, image processing noise and the number of LED lamps on the positioning error.

The rest of the chapter is organized as follows. Section 3.2 introduces the system model and the problem formulation. Section 3.3 derives the closed-form solution and proposes a new non-iterative SVD-based positioning algorithm. Section 3.4 evaluates the performance of proposed positioning algorithm and compares it with conventional LM-based algorithm in terms of running speed, accuracy, and robustness. The CRLB/RMSE bound are derived in Sections 3.5 and positioning error bounds under different system configuration is investigated through simulation. Finally, the experimental set-up and positioning results and conclusion are provided in Section 3.6.

3.2 System model and problem formulation

The studied VLP system has similar hardware settings with the work in [28], which includes three LED lamps as transmitters, and a smartphone camera as the image sensor. Each LED lamp is assigned with a unique ID, which is related to its location and stored in an ID-location database. The light intensity of each LED is modulated to broadcast its ID frame repeatedly. The CMOS camera in the smartphone is utilized to detect the modulated signals by making use of the rolling shutter effect (RSE) [78].

When one holds a smartphone, the front camera will periodically capture pictures of LED lamps mounted on the ceiling. These pictures are then processed to acquire the centroids and IDs of the captured LED images. Consequently, both receiver's location and orientation can be obtained.

The VLP system is based on the pinhole camera projection model [28]. As illustrated in Fig. 3.2, LED lamps 1, 2 and 3 are placed at A, B and C, which are projected to the image plane at three separate spots centered at A', B' and C', respectively. The locations of the LED lamps are recorded in the global coordinate system (GCS) as $(x_i, y_i, z_i), i = 1, 2, 3$. Meanwhile, the three LED images' centroids are denoted in the receiver coordinate system (RCS) as $(m_i, n_i, l), i = 1, 2, 3$, respectively, where (m, n) are 2D coordinates on the image plane and l is the distance between the lens center (O) and the image plane. In this system, the LED lamps' locations (x_i, y_i, z_i) are pre-determined, and the corresponding images' centroids coordinates (m_i, n_i) are obtained by image processing. Assuming that all the captured images are well focused on the focal plane, we have $l = f$. The transformation from an object point (LED lamp centroid) to its corresponding image point (lamp image centroid) can be described as a compound procedure of scaling, rotation and translation [29-30].

The scaling factors associated with three LED lamps, i.e., K_1, K_2 and K_3 , can be calculated using the method in [28]. Briefly, let (m_i, n_i, f) be the coordinates of the i -th image's center in the receiver's coordinate system (GCS), and (u_i, v_i, w_i) represent the coordinates of the i -th image's center after multiplying scaling factor K_i , that is,

$$(u_i, v_i, w_i) = K_i(m_i, n_i, f) \quad i = 1, 2, 3. \quad (3.1)$$

Representing the distance between LED1 and LED2 in RCS and GCS respectively, we obtain,

$$\begin{aligned} d_{1,2}^2 &= (u_1 - u_2)^2 + (v_1 - v_2)^2 + (w_1 - w_2)^2 \\ &= K_1^2 |\vec{O_{i1}}|^2 + K_2^2 |\vec{O_{i2}}|^2 - 2K_1K_2 (\vec{O_{i1}} \cdot \vec{O_{i2}}). \\ &= (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \end{aligned} \quad (3.2)$$

Where the only remaining unknowns are the scaling factors K_i . Three transmitters would yield three equations as above, allowing us to find K_1, K_2 and K_3 ,

These two coordinates, (x_n, y_n, z_n) and (u_n, v_n, w_n) then have the same scale after scale transformation. They are related by a rotation matrix $\mathbf{R}$ and a translation $\boldsymbol{\theta}$ [93],

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} = [\mathbf{R} \quad \boldsymbol{\theta}] \times \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \\ 1 & 1 & 1 \end{bmatrix} \quad (3.3)$$

where $\boldsymbol{\theta} = [\theta_x, \theta_y, \theta_z]^T$ is a 3-by-1 vector which is the estimated receiver's location and $\mathbf{R}$ is a 3-by-3 matrix representing the receiver's orientation, which is given by Eq. (3.4),

$$\mathbf{R} = \mathbf{R}_x(\alpha) \cdot \mathbf{R}_y(\beta) \cdot \mathbf{R}_z(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \times \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \times \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.4)$$

Fig.3.1. LED lamps projection model: the global coordinate (x, y, z) is projected to the image plane (m, n) .

where α , β and γ represent the receiver's rotation angles around X-, Y-, and Z-axes, respectively. Noting that, the transmitters coordinate system is set the same as the global coordinate system (GCS) in our system, then the receiver's orientation in the GCS is the same as the rotation from GCS to RCS.

Eq. (3.4) contains six unknowns, i.e., α , β and γ in $\mathbf{R}$ and θ_x , θ_y and θ_z in $\boldsymbol{\theta}$. Since each lamp gives two independent equations, at least three lamps are required to work out the 6 unknowns. The Frobenius norm is used to measure the following squared cumulative error (SCE),

$$E^2 = \sum_{i=1}^3 \left\| (u_i, v_i, w_i)^T - \mathbf{R}(x_i, y_i, z_i)^T - \boldsymbol{\theta} \right\|_F^2 \quad (3.5)$$

which should be zero ideally. However, it is usually greater than zero due to the measurement and image processing errors. The location and orientation of the receiver are then estimated by minimizing the SCE. In the literature, Levenberg-Marquardt (LM) solver is employed widely to solve the above NLLS positioning problem [77]. However, it only finds a local minimum which may not be the global optimum. Meanwhile, it may take a long time to converge when given a bad initial guess. To address the above problems, we propose a fast and robust non-iterative positioning algorithm by deriving a closed-form solution to the receiver's orientation and location using the SVD technique.

3.3 Improved non-iterative SVD-based positioning algorithm

3.3.1 SVD-based positioning algorithm

A non-iterative algorithm for solving the pinhole camera projection model presented in Section 3.2 is derived in this section. Considering the positioning problem as a 3D point fitting problem [94], we define two new vectors $\mathbf{P}_0$ and $\mathbf{P}'_0$ to represent the centroid coordinates of 3 objects and their image points, respectively,

$$\begin{aligned} \mathbf{P}_0 &= \frac{1}{3} \sum_{i=1}^3 (x_i, y_i, z_i)^T \\ \mathbf{P}'_0 &= \frac{1}{3} \sum_{i=1}^3 (u_i, v_i, w_i)^T \end{aligned} \quad (3.6)$$

where $\mathbf{P}_0$ and $\mathbf{P}'_0$ should satisfy $\mathbf{P}'_0 = \mathbf{R}\mathbf{P}_0 + \boldsymbol{\theta}$. Two matrices $\mathbf{Q}_i$ and $\mathbf{Q}'_i$ ($i=1, 2, 3$) are defined based on $\mathbf{P}_0$ and $\mathbf{P}'_0$,

$$\begin{aligned}\mathbf{Q}_i &= (x_i, y_i, z_i)^T - \mathbf{P}_0 \\ \mathbf{Q}'_i &= (u_i, v_i, w_i)^T - \mathbf{P}'_0\end{aligned}\quad (3.7)$$

Recall that the receiver's position and orientation are estimated by minimizing the square cumulative error (SCE) given in Eq. (3.5). By substituting Eq. (3.7) into Eq. (3.5), the SCE is rewritten into Eq. (3.8),

$$\begin{aligned}E &= \sum_{i=1}^3 \left\| (u_i, v_i, w_i)^T - (\mathbf{R}(x_i, y_i, z_i)^T + \mathbf{P}'_0 - \mathbf{R}\mathbf{P}_0) \right\|_F \\ &= \sum_{i=1}^3 \left\| (u_i, v_i, w_i)^T - \mathbf{P}'_0 + (\mathbf{R}(x_i, y_i, z_i)^T - \mathbf{R}\mathbf{P}_0) \right\|_F \\ &= \sum_{i=1}^3 \left\| \mathbf{Q}'_i - \mathbf{R}\mathbf{Q}_i \right\|_F^2\end{aligned}\quad (3.8)$$

where the translation matrix $\boldsymbol{\theta}$ is successfully canceled. The optimization is divided into two steps: (i) find $\mathbf{R}_{opt}$ to minimize Eq. (3.8); (ii) calculate $\boldsymbol{\theta}_{opt}$ as $\boldsymbol{\theta}_{opt} = \mathbf{P}'_0 - \mathbf{R}_{opt}\mathbf{P}_0$. We denote $\text{Tr}(\cdot)$ as the trace of a matrix. Since $\|\mathbf{A}\|_F^2 = \text{Tr}(\mathbf{A}\mathbf{A}^T)$ holds for an arbitrary matrix $\mathbf{A}$, Eq. (3.8) can be further expanded as,

$$E^2 = \sum_{i=1}^3 \text{Tr}(\mathbf{Q}'_i{}^T \mathbf{Q}'_i + \mathbf{Q}_i{}^T \mathbf{Q}_i - \mathbf{Q}'_i{}^T \mathbf{R}^T \mathbf{Q}_i - \mathbf{Q}_i{}^T \mathbf{R} \mathbf{Q}'_i) \quad (3.9)$$

It is noticed that both $\mathbf{Q}'_i{}^T \mathbf{Q}'_i$ and $\mathbf{Q}_i{}^T \mathbf{Q}_i$ are known and fixed. According to the property that $\text{Tr}(\mathbf{A}\mathbf{B}) = \text{Tr}(\mathbf{B}\mathbf{A})$ for $\mathbf{A}$ and $\mathbf{B}$ being square matrices, $\mathbf{Q}_i{}^T \mathbf{R} \mathbf{Q}'_i$ and $\mathbf{Q}'_i{}^T \mathbf{R}^T \mathbf{Q}_i$ share the same trace, and minimizing Eq. (3.9) is equivalent to maximizing Eq. (3.10),

$$\begin{aligned}E^2 &= \sum_{i=1}^3 \text{Tr}(2\mathbf{Q}'_i{}^T \mathbf{R} \mathbf{Q}_i) = \text{Tr}\left(\sum_{i=1}^3 2\mathbf{Q}'_i{}^T \mathbf{R} \mathbf{Q}_i\right) \\ &= \text{Tr}\left(2\mathbf{R}\left(\sum_{i=1}^3 \mathbf{Q}_i \mathbf{Q}'_i{}^T\right)\right) = \text{Tr}(2\mathbf{R}\mathbf{H})\end{aligned}\quad (3.10)$$

where $\mathbf{H} = \sum_{i=1}^3 \mathbf{Q}_i \mathbf{Q}_i'^T$. Let the singular value decomposition (SVD) of $\mathbf{H}$ be

$$\mathbf{H} = \sum_{i=1}^3 \mathbf{Q}_i \mathbf{Q}_i'^T = \mathbf{U} \mathbf{\Omega} \mathbf{V}^T \quad (3.11)$$

where both $\mathbf{U}$ and $\mathbf{V}$ are 3×3 orthonormal matrices, while $\mathbf{\Omega}$ is a diagonal matrix. Substituting Eq. (3.11) into Eq. (3.10) yields

$$\text{Tr}(\mathbf{R}\mathbf{H}) = \text{Tr}(\mathbf{R}\mathbf{U}\mathbf{\Omega}\mathbf{V}^T) = \text{Tr}(\mathbf{V}^T \mathbf{R}\mathbf{U}\mathbf{\Omega}) \quad (3.12)$$

Let $\mathbf{W} = \mathbf{V}^T \mathbf{R}\mathbf{U}$, and $\mathbf{W}$ is found to be an orthogonal matrix. Then the SCE function (E^2) turns to,

$$\text{Tr}(\mathbf{V}^T \mathbf{R}\mathbf{U}\mathbf{\Omega}) = \text{Tr}(\mathbf{W}\mathbf{\Omega}) \quad (3.13)$$

Lemma: Given an arbitrary diagonal matrix $\mathbf{\Omega}$ and an orthogonal matrix $\mathbf{W}$, the value of $\text{Tr}(\mathbf{W}\mathbf{\Omega})$ reaches maximum if and only if $\mathbf{W} = \mathbf{I}$, where $\mathbf{I}$ denotes the identity matrix.

Proof:

Let $\mathbf{\Omega}_i$ be the i -th column of N -dimensional diagonal matrix $\mathbf{\Omega}$, i.e., $\mathbf{\Omega} = (\mathbf{\Omega}_1, \mathbf{\Omega}_2 \cdots \mathbf{\Omega}_N)$, we have,

$$\mathbf{W}\mathbf{\Omega} = (\mathbf{W}\mathbf{\Omega}_1, \mathbf{W}\mathbf{\Omega}_2 \cdots \mathbf{W}\mathbf{\Omega}_N) \quad (3.14)$$

According to the isometry property of an orthogonal matrix $\|\mathbf{\Omega}_i\|_F = \|\mathbf{W}\mathbf{\Omega}_i\|_F, i = 1, 2, \cdots N$, we have

$$|\delta_i| = \sqrt{\sum_j |\theta_{ij}|^2} \quad (3.15)$$

where δ_i is the non-zero element of $\mathbf{\Omega}_i$ and θ_{ij} is the j -th element of $\mathbf{W}\mathbf{\Omega}_i$. Thus,

$$\delta_i \geq \theta_{ij}, \forall j,$$

$$\text{Tr}(\mathbf{\Omega}) = \text{Tr}(\mathbf{I}\mathbf{\Omega}) = \sum_i \delta_i \geq \sum_i \theta_{ii} = \text{Tr}(\mathbf{W}\mathbf{\Omega}), \quad (3.16)$$

where $\mathbf{I}$ is an identity matrix. Hence, the lemma is proved.

Based on this *lemma*, we obtain,

$$\mathbf{W} = \mathbf{U}\mathbf{R}\mathbf{V}^T = \mathbf{I}. \quad (3.17)$$

Therefore, the closed-form least-square solution to $\mathbf{R}$ is given by,

$$\mathbf{R}_{opt} = \mathbf{V}\mathbf{U}^T. \quad (3.18)$$

Before the position is estimated by $\mathbf{\theta}_{opt} = \mathbf{P}'_0 - \mathbf{R}_{opt}\mathbf{P}_0$, we have to check the determinant of the orthogonal matrix $\mathbf{R}_{opt}$. In the case of $\det(\mathbf{R}_{opt}) = 1$, $\mathbf{R}_{opt}$ is the optimal estimation of rotation matrix; while in the case of $\det(\mathbf{R}_{opt}) = -1$, $\mathbf{R}_{opt}$ is called improper rotation, which should be discarded. The proposed low-complexity non-iterative SVD-based positioning algorithm is summarized in Algorithm I. Furthermore, the specific Euler angles can be calculated in the same fashion as in [95].

Algorithm I: SVD-based Positioning Algorithm

- a. Calculate two vectors $\mathbf{P}_0$ and $\mathbf{P}'_0$ according to (3.6) and two matrices $\mathbf{Q}_i$ and $\mathbf{Q}'_i$, $i = 1, 2, 3$ according to (3.7);
 - b. Generate a 3×3 matrix, $\mathbf{H} = \sum_{i=1}^3 \mathbf{Q}_i \mathbf{Q}'_i{}^T$;
 - c. Conduct SVD to $\mathbf{H}$ to obtain $\mathbf{H} = \mathbf{U}\mathbf{\Omega}\mathbf{V}^T$, and then estimate the rotation matrix as $\mathbf{R}_{opt} = \mathbf{V}\mathbf{U}^T$;
 - d. Record $\mathbf{R}_{opt}$ if $\det(\mathbf{R}_{opt}) = 1$ and discard it if $\det(\mathbf{R}_{opt}) = -1$;
 - e. Estimate the translation vector as $\mathbf{\theta}_{opt} = \mathbf{P}'_0 - \mathbf{R}_{opt}\mathbf{P}_0$.
-
-

3.3.2 Performance Comparison: SVD-based vs. LM-based positioning algorithms

In this section, we carry out simulations to compare the LM and SVD-based positioning algorithms. Computer simulations have been carried out using a laptop (Intel Core i5-4210M CPU @2.60GHz) to evaluate and compare the performance of the LM and SVD based positioning algorithms in terms of average running time,

accuracy and robustness. A room with dimensions of $5m \times 5m \times 3m$ is studied. A set of LED lamps are mounted uniformly on the ceiling. In the study, the receiver's location is set at $(1, 1, 1.7)$. To guarantee its front camera generally facing up to capture the ceiling lamps, the rotation angles should satisfy $|\alpha| \leq 40^\circ, |\beta| \leq 40^\circ, |\gamma| \leq 180^\circ$. We set it as $(10^\circ, 35^\circ, 70^\circ)$ without losing generality. Other related parameters are set the same as the experimental specifications given in Table 3.3. The image points' coordinates are generated based on the pinhole projection model. Based on the experimental statistics, Gaussian distributed error with standard deviation of 30 pixels are added on the resulting image points as noise. Then, two algorithms are run 10,000 times respectively to estimate $\mathbf{R}_{opt}$ and $\mathbf{T}_{opt}$ while the initial guesses of the LM-based algorithm, i.e., $(\alpha_0, \beta_0, \gamma_0, x_0, y_0, z_0)$, are randomly generated within the range of $(\leq \pm 40^\circ, \leq \pm 40^\circ, \leq \pm 180^\circ, \leq 5m, \leq 5m, \leq 3m)$. The results are provided in Table 3.1 and Table 3.2.

Table 3.1 gives the average and maximum time per run using various number of LED lamps from 3 to 9. As shown, the SVD-based algorithm is averagely 50-80 times faster than the iterative LM-based algorithm. The reason is that the 'bad initial guesses' involved in the LM-based algorithm tend to result in a large number of iterations. It may be even worse in the case of smartphones with slower processors. Besides, the bad guesses are likely to cause large errors. As observed in Table 3.2, the LM-based algorithm incurs roughly 90 failures (error > 50 cm) out of 10,000 runs, while the SVD-based algorithm works robustly in all trials. The error of the LM-based algorithm is found larger than that of the SVD-based algorithm in the presence of positioning failures. However, it is comparable with the SVD-based algorithm if we exclude those errors of failed runs. That is, the SVD-based algorithm achieves better or at least the same positioning accuracy using much less time.

Table 3.1 Average time consumption per run of LM-based & SVD-based positioning algorithms (in second)

Number of LED lamps	LM-based algorithm				SVD-based algorithm	
	Mean (s)	Maximum (s)	Iterations (mean)	Iterations (max)	Mean (s)	Maximum (s)
3	0.0161	0.8312	9.1520	29	2.2056e-04	0.0182
4	0.0155	0.8249	9.5650	25	2.6297e-04	0.0239
5	0.0193	0.8564	9.8009	24	2.0774e-04	0.0184

6	0.0131	0.8318	9.1939	29	2.2966e-04	0.0194
7	0.0156	0.8194	9.0998	27	2.6381e-04	0.0178
8	0.0149	0.8605	9.2045	25	2.5090e-04	0.0217
9	0.0171	0.8160	9.3107	29	2.9145e-04	0.0182

Table 3.2 Accuracy and robustness of LM-based & SVD-based positioning algorithms

Number of LED lamps	LM-based algorithm				SVD-based algorithm		
	Mean (cm)	Mean Excluding Failures(cm)	Maximum (cm)	NoF /10000 runs	Mean (cm)	Maximum (cm)	NoF /10000 runs
3	12.5225	10.2201	954.1043	81	9.2756	9.2756	0
4	10.4194	7.8117	1.0381e+03	84	7.6698	7.6698	0
5	11.4216	7.1723	978.6117	102	7.1723	7.1723	0
6	10.2551	6.3142	951.5375	94	6.2095	6.2095	0
7	11.0002	4.4705	880.62	97	4.4705	4.4705	0
8	7.8459	2.5618	994.1190	97	2.5618	2.5618	0
9	9.4179	2.0929	1.0075e+03	116	2.0929	2.0929	0

3.4 RMSE bound of an IS-based VLP system: derivation and analysis

In the above mathematical derivation and simulation evaluation, a non-iterative SVD-based positioning algorithm is proposed and verified for an IS-based VLP system to effectively speed up the positioning speed. Then in this section, we investigate the error bound of this algorithm and the impact of the system parameters on the positioning accuracy.

3.4.1 Theoretical error bound of an IS-based VLP system

To analyze the IS-based positioning system, we derive the CRLB and RMSE bound of the estimated 3D position. Here, the CRLB denotes the variance bound of three respective coordinates (x-, y-, z-) errors, while the RMSE bound represents the overall 3D error bound of the estimated position.

Assuming that M LED lamps are deployed as transmitters in a VLP system. A smartphone is placed at a certain location as the receiver. After taking a picture, N

($3 \leq N \leq M$) lamps are captured on the image plane of the smartphone camera. The picture is then processed to acquire the captured lamps centers' coordinates as the observation vector $\mathbf{r}$. In the presence of noises (image processing errors), the projection relationship (3.3) can be written as,

$$\mathbf{r}_j = f_j(\boldsymbol{\theta}) + n_j, j = 1, 2 \dots 3N, \quad (3.19)$$

where $\mathbf{r}_j$ is the j -th observation; $n_j (j = 1 \dots 3N)$ are the mutually independent image processing noise and assumed to be Gaussian, i.e., $n_j \sim \mathcal{N}(0, \sigma_j^2)$; $\boldsymbol{\theta} = (\theta_x, \theta_y, \theta_z)^T$ denotes the receiver's location in GCS and f_j is defined as,

$$\begin{aligned} f_{3(i-1)+1}(\boldsymbol{\theta}) &= s_i \times (\mathbf{R}_1 \mathbf{W}_i + \theta_x); \\ f_{3(i-1)+2}(\boldsymbol{\theta}) &= s_i \times (\mathbf{R}_2 \mathbf{W}_i + \theta_y); \\ f_{3(i-1)+3}(\boldsymbol{\theta}) &= s_i \times (\mathbf{R}_3 \mathbf{W}_i + \theta_z); \\ &\text{for } i = 1, 2, \dots, N \end{aligned} \quad (3.20)$$

In (3.20), $\mathbf{W}_i \in \mathcal{R}^{3 \times 1}$ is the 3-dimensional coordinates of the i -th lamp in the GCS, which is pre-known. $\mathbf{R} \in \mathcal{R}^{3 \times 3}$ describes the orientation of the receiver as given in (3.4). $\mathbf{R}_m$ denotes the m -th row of $\mathbf{R}$. $s_i \in \mathcal{R}$ is the i -th scaling factor, which in practice is approximately the ratio of the camera focal length to the lamp's relative height (the lamp's height minus the camera's height), i.e., $s_i \approx f / H_r = s_0$. We further define the following vectors:

$$\begin{aligned} \mathbf{f}(\boldsymbol{\theta}) &= [f_1(\boldsymbol{\theta}), f_2(\boldsymbol{\theta}), \dots, f_{3N}(\boldsymbol{\theta})]^T \in \mathcal{R}^{3N \times 1} \\ \mathbf{r} &= [r_1, r_2, \dots, r_{3N}]^T \in \mathcal{R}^{3N \times 1} \\ \mathbf{n} &= [n_1, n_2, \dots, n_{3N}]^T \in \mathcal{R}^{3N \times 1} \end{aligned} \quad (3.21)$$

Hence, (3.20) can be written in a more compact form as

$$\mathbf{r} = \mathbf{f}(\boldsymbol{\theta}) + \mathbf{n} \quad (3.22)$$

When the image processing errors are mutually independent Gaussian distribution variables, the probability density function (p.d.f) of $\mathbf{r}$ conditioned on $\boldsymbol{\theta}$ is given by,

$$p(\mathbf{r}|\boldsymbol{\theta}) = \frac{\exp\left\{-\sum_{j=1}^{3N} [r_j - f_j(\boldsymbol{\theta})]^2 / 2\sigma_j^2\right\}}{(\sqrt{2\pi})^{3N} \prod_{j=1}^{3N} \sigma_j}. \quad (3.23)$$

We use $\mathbf{r}$ to estimate the receiver's position $\boldsymbol{\theta} = (\theta_x, \theta_y, \theta_z)^T$. Let $\hat{\boldsymbol{\theta}} = (\hat{\theta}_x, \hat{\theta}_y, \hat{\theta}_z)^T$ be the unbiased estimation of true receiver's location. According to the Cramer-Rao theory, the variance of the k -th parameter of $\hat{\boldsymbol{\theta}}$ is lower bounded by

$$\text{CRLB}(\hat{\theta}_k) = [\boldsymbol{\Gamma}^{-1}(\boldsymbol{\theta})]_{k,k}, \quad k = 1, 2, 3, \quad (3.24)$$

and the RMSE of the estimated position $\hat{\boldsymbol{\theta}}$ is lower bounded by,

$$\varepsilon = E[\sqrt{(\theta_x - \hat{\theta}_x)^2 + (\theta_y - \hat{\theta}_y)^2 + (\theta_z - \hat{\theta}_z)^2}] = \sqrt{\text{Tr}(\boldsymbol{\Gamma}^{-1})}, \quad (3.25)$$

where $\boldsymbol{\Gamma}$ is the fisher information matrix (FIM). Here FIM is a 3-by-3 symmetric square matrix with elements defined in terms of $p(\mathbf{r}|\boldsymbol{\theta})$,

$$\Gamma_{k,l} = -E\left[\frac{\partial^2 \ln p(\mathbf{r}|\boldsymbol{\theta})}{\partial \hat{\theta}_k \partial \hat{\theta}_l}\right], \quad k, l = 1, 2, 3. \quad (3.26)$$

Substituting (3.23) into (3.26), after some mathematical manipulations, the FIM elements are calculated,

$$\Gamma_{k,l} = \frac{s_0^2}{\sigma_j^2} \sum_{j=1}^N \left(\frac{\partial \boldsymbol{\theta}}{\partial \theta_k} \cdot \frac{\partial \boldsymbol{\theta}}{\partial \theta_l} \right), \quad k, l = 1, 2, 3. \quad (3.27)$$

Since the camera configuration and the image processing algorithm are fixed in capturing and processing one picture, the error (noise) power on each dimension of $\mathbf{r}$ is considered to be equal, i.e., $\sigma_j^2 = \sigma_n^2$. Therefore, FIM is reduced to a simple diagonal matrix,

$$\boldsymbol{\Gamma} = \frac{Ns_0^2}{\sigma_n^2} \mathbf{I}_{3 \times 3}. \quad (3.28)$$

Then, CRLB of each dimension of the estimated position $(\hat{\theta}_x, \hat{\theta}_y, \hat{\theta}_z)$ is readily calculated with the obtained FIM,

$$\begin{aligned}\hat{\theta}_{x\text{CRLB}} &= E[(\theta_x - \hat{\theta}_x)^2] = \det^{-1}(\mathbf{\Gamma}) \cdot (\Gamma_{2,2}\Gamma_{3,3} - \Gamma_{2,3}^2) \\ \hat{\theta}_{y\text{CRLB}} &= E[(\theta_y - \hat{\theta}_y)^2] = \det^{-1}(\mathbf{\Gamma}) \cdot (\Gamma_{1,1}\Gamma_{3,3} - \Gamma_{1,3}^2) \\ \hat{\theta}_{z\text{CRLB}} &= E[(\theta_z - \hat{\theta}_z)^2] = \det^{-1}(\mathbf{\Gamma}) \cdot (\Gamma_{2,2}\Gamma_{3,3} - \Gamma_{2,3}^2)\end{aligned}\quad (3.29)$$

It is easy to see that,

$$\hat{\theta}_{x\text{CRLB}} = \hat{\theta}_{y\text{CRLB}} = \hat{\theta}_{z\text{CRLB}} = \frac{\sigma_n^2}{Ns_0^2}, \quad (3.30)$$

and the RMSE bound of 3D positioning is

$$\varepsilon = \left(\frac{3\sigma_n^2}{Ns_0^2} \right)^{1/2}. \quad (3.31)$$

Equations (3.30) and (3.31) specify the lower limit of the positioning error in an IS-based VLP system. It is evident that the CRLB and RMSE bound depends on i) the number of LED lamps, ii) the image processing error power which is influenced by CMOS sensor model, camera configuration, image processing algorithm, etc. iii) the scaling factor (s_0). Although this conclusion is applicable to the case with the knowledge of orientation, the theoretical analysis provides useful insight for the future system investigation, e.g., estimating the mean PA in a system, designing the LED lamps deployment based on the PA requirement. The influence of the rotation angle error will be considered in our future work.

3.4.2 Impact of factors on RMSE bound

Based on the derived theoretical expression of the RMSE bound, we depict the curve which visualizes how various parameters impact the positioning error bound under different system settings. Specifically, the related parameters are set the same as in the experiment specifications in Section 3.5. The results show that the mm-cm level of PA is achievable using a smartphone camera in typical indoor environment.

Fig. 3.2. (a) RMSE bound versus number of LED lamps with different heights (H). (b) RMSE bound versus number of LED lamps with different noise power ($H=3\text{m}$). (c) RMSE bound versus noise power with different relative height ($H=3\text{m}$).

As shown in Fig. 3.2(a), the RMSE bound decreases as the number of captured lamps increases. The relative height between LED lamps and smartphone camera, noted as H , is around 3m in indoor when a person holds a smartphone. One-meter drop in the H leads to 0.7 cm decrease in the RMSE bound. RMSE bound is also influenced by the noise power. We process 80 captured images. The minimum, mean and maximum center searching errors are 5, 30 and 84 pixels, which are used as the standard variance of noise in the simulations. The corresponding noise power in dB is then given as -105dB~-80dB as shown in Fig. 3.2(b). The typical (average) case is given

by the black curve with -90 dB noise power. The positioning error is 1.3 cm when 3 lamps are employed, and it gradually decreases to 0.7 cm when 10 lamps are utilized. Fig. 3.2(c) shows the impact of H together with the noise power on the positioning error bound. When both typical values (-90 dB, 300 cm) are taken, it shows an achievable PA of 1.4 cm.

3.5 Experimental evaluation

In this section, we experimentally verify the effectiveness of the proposed imaging processing pipeline and SVD-based positioning algorithm and evaluate the PA of the improved system in a practical scenario. The positioning results are shown and discussed.

3.5.1 Experimental Set-up and Parameters

As illustrated in Fig. 3.3, three LED lamps are mounted on the floor instead of the ceiling for the convenience. The locations of LED lamps (in cm) are (40.7, 83.6, 9.2), (72.0, 62.3, 9.7), (41.2, 46.7, 9.9), respectively, in GCS. Note that, the transmitters' heights are not required to be the same in our system. An arbitrary waveform generator (AWG) is used to generate the waveforms of amplitude-shift keying (ASK) ID frames which are then added with the DC bias of LEDs through bias tees. The rear camera of a smartphone with resolution around 40 Mega pixels is used as the receiver. A laptop is used to process the captured images using the proposed algorithms. The specifications and modulated LED frames are given in Tables 3.3 and 3.4.

3.5.2 Experiment Results

We evaluate the PA in two planes with the heights from the floor setting to be 120 cm (Case I) and 160 cm (Case II), respectively. 35 grid testing points are deployed on each plane. Fig. 3.3 illustrates the X-Y and X-Z views of 3D positioning results in two cases, respectively. Generally, it's found that the paths of estimated positions in both cases match well with that of real positions. Yet, the variance of the localization error for Case II is slightly larger than that for Case I. The errors of the X and Y coordinates are quite small. More specifically, the mean errors on X-axis are about 2 cm for $H=120$ cm and 3 cm for $H=160$ cm. Even smaller errors are observed on Y-axis which are averagely 1 cm and 2.5 cm respectively. However, the Z-axis errors are found to be much larger than that on the X- and Y-axes. According to the statistics,

they are averagely 5 cm for $H=120$ cm and 7 cm for $H=160$ cm. This is probably because the movement on the X- and Y- axes would result in a big difference in the locations of LED images, while the movement along Z-axis bring about relative smaller changes on the images' locations as the Z-axis is vertical to the image plane. Thus, larger positioning errors are observed on the Z-coordinates. In addition, the results suggest that positioning errors on the marginal area of testing planes are generally larger than those of the central area. Due to the limited FoV (51°) of the lamp shades, part of the light is blocked when the camera moves towards the marginal area, where the signal-to-noise ratio (SNR) decreases.

Fig. 3.5 gives cumulative distribution function (CDF) of the positioning errors for both cases. For Case I ($H=120$ cm), the system positioning error ranges from 2.2 cm to 9.7 cm with the median error of 6 cm. For Case II ($H=160$ cm), the positioning error ranges from 2.2 cm to 13.7 cm with the median error of 8 cm. The results demonstrate that, on the one hand, the system accomplishes highly accurate 3D positioning in centimeters in practical environment, on the other hand, the proposed algorithms and system work robustly and reliably as the positioning error varies within 12 cm.

Fig. 3.3. Experimental setup for an IS-based VLP system

Table. 3.3 Modulated LED ID frames

LED #	Preamble + ID (11 digits)
LED1	1101001 1001
LED2	1101001 1011
LED3	1101001 1010

Table 3.4 Experiment specification

Component/Parameter	Model/Value
LED	Cool White (5650K) LUXEON Rebel ES LED, 40mm Round 7-Up Base
Smart Phone	Nokia Lumia 1020
Bias Tee	ZFBT-6GW, Mini-Circuits 15542
Arbitrary Waveform Generator	Tabor Electronics WW2074, 200M/s
Focal Length	7.25mm
Image pixel resolution	7712×4352
Pixel size	1.12μm
FoV of LED optics	$\varphi = 51^\circ$

Fig.3.4. (a) X-Y view of positioning results (H=120 cm), (b) X-Y view of positioning results

(H=160 cm), (c) X-Z view of positioning results (H=120 cm), and (d) X-Y view of positioning results (H=160 cm)

Fig. 3.5 Cumulative distribution function (CDF) of 3D positioning error at two layers (H=120 cm v.s. H=160 cm).

3.6 Conclusion

In this work, we investigate and demonstrate an accurate, fast and robust VLP system using a common smartphone camera as the receiver. We propose a non-iterative SVD-based positioning algorithm to estimate the receiver's location and orientation, which is more robust and 50-80 times faster than the conventional iterative LM-based algorithm. In the positioning error analysis, the CRLB and the RMSE bound of the estimated receiver's position are derived as the theoretical accuracy limits of the IS-based VLP system. Simulation results suggest that the system is able to achieve a positioning accuracy of around 1 cm under typical parameter settings. In practical experiment, the average 3D positioning errors on two testing heights are 6 cm (H= 120 cm) and 8.5 cm (H= 160 cm), with the maximal error around 10 cm and 14 cm, respectively.

Chapter 4 Enhancing Positioning Accuracy of an IS-based VLP System Through Sensor Fusion: SVD-SF Algorithm

4.1 Introduction

Recently reported indoor VLP systems, using either techniques of RSS [32, 33] or AOA [28, 77, 96], usually rely on a single type of sensors, e.g., PD or image sensor. Kim et al. [32] implemented a VLC based positioning system using intensity modulation and direct detection (IM/DD) scheme. In this system, a PD was utilized to recover the modulated optical signals and also measure the RSS. Instead of using a single PD as the receiver, the authors in [33] employed multiple PDs with predetermined tilting angles and performed 3D positioning by making use of the gain difference of these tilted PDs. Besides, cameras (image sensors) based VLP systems are also investigated in several existing studies, where a smartphone camera [4], a fish-eye lens-equipped camera [77], a high-speed camera [97], a dual image sensor [98] have been used as receivers. However, to achieve high positioning accuracy (PA) in these systems, high-sensitivity PDs and high-resolution cameras are demanded, whose high cost will largely hinder their widespread implementation and usage.

A more effective and economical alternative is thus to make full use of various accessible technologies and common sensors. In [99], a hybrid system was proposed to achieve highly accurate localization by fusing RFID systems with ultrasonic sensors. The uncertainties in RFID systems were partially removed by integrating the distance information obtained from ultrasonic sensors. The authors in [31] reported an indoor positioning system that includes both an accelerometer and a PD as receivers. Specifically, the PD was employed to detect the optical signal intensity and an external accelerometer to measure the irradiance and incidence angles of the optical signal. However, the algorithm reported in [31] requires users to collect the RSS information twice at each location by varying the receivers' posture. The operation is not user-friendly and hard to support users' mobility. Meanwhile, in both works of [31] and [99], the data collected by additional sensors are simply substituted into their

respective models directly. None of them investigated the schemes to acquire extra performance gain by fusing different sensor readings.

In this chapter, we propose a simple but smart indoor VLP system to enable highly accurate positioning. The system is simple as we use a common smartphone which is cheap and easily available. It is however different from the existing smartphone based VLP systems [77, 97] which rely on single smartphone cameras since we utilize various types of common smartphone-embedded sensors (camera, accelerometer, and gyroscope) to collect data, which makes the system smarter. The obtained data are then fused to enhance the PA by using the proposed sensor fusion algorithm. The issue of fusing the data from both image and motion sensors is formulated as a multi-objective non-convex optimization problem. Instead of employing the conventional time-consuming multi-start iterative method [100] to solve it, we derive a closed-form optimal fused solution and propose a low-complexity SVD-SF algorithm to ensure the acquisition rate and support mobility. More importantly, the proposed algorithm guarantees a better (or at least the same) PA compared with the original single-camera-based positioning algorithm. Both experiments and simulations are conducted to verify the validity of the proposed white LED-based indoor VLP system and also evaluate error performance of the SVD-SF algorithm under different noise conditions.

The rest of the chapter is organized as follows. The overview of the proposed system and the reformulation of sensor fusion problem are presented in Section 4.2. Then, in Section 4.3, a low-complexity solution to the optimization problem is derived using SVD technique. In Section 4.4, the positioning performance of the proposed algorithm is evaluated by both experiments and simulations. Finally, Section 4.5 concludes this chapter. The content of this chapter is related to the author's work shown in [51].

4.2 System model and sensor fusion problem reformulation

Fig. 4.1. Overview of the proposed sensor fusion-based VLP system

4.2.1 System model

As illustrated in Fig. 4.1, the proposed indoor VLP system includes multiple LED lamps on the ceiling as transmitters and a smartphone as the receiver. Each LED lamp is assigned with a unique ID, and its location is predetermined. An ID-location table is then established and stored. The LED lamps are modulated individually to broadcast their IDs through visible light communication (VLC). At the receiver side, both image sensor (CMOS camera) and motion sensors (accelerometer and gyroscope) embedded on a smartphone are utilized to conduct measurements for position estimation. In particular, the CMOS camera is used to capture images by employing the rolling shutter effect [54], from which both the image centroids' location and the related ID information of each LED can be obtained through image processing. Meanwhile, the receiver's posture information can be detected by the motion sensors in real-time. As depicted in Fig. 4.2, the accelerometer measures the pitch and roll angles while the gyroscope provides the readings about the yaw angle. Once all the sensor data are collected, they are fused to calculate the optimal estimation of the receiver's location and orientation.

Fig. 4.2. Workflow diagram of data fusion of image sensor and motion sensor

4.2.2 Sensor fusion problem reformulation

Fig. 4.3 Pinhole camera projection model

A. Positioning based on pinhole camera projection model

In Chapter 3, we have introduced the principle of estimating the receiver's location based on pinhole camera model as shown in Fig. 4.3 [4]. Briefly, the 3D coordinates of LED lamps (x_i, y_i, z_i) and their corresponding images (m_i, n_i, l_i) ($i=1,2,3$) observe the relationship given by Eq. (4.1) [97],

$$\lambda \begin{bmatrix} m_i & n_i & l_i \end{bmatrix}^T = [\mathbf{R}_{im} \quad \mathbf{T}_{im}] \times \begin{bmatrix} x_i & y_i & z_i & 1 \end{bmatrix}^T + \mathbf{N}_i \quad (4.1)$$

($i=1,2,3$)

Where $\mathbf{R}_{im}$ is a 3-by-3 rotation matrix which indicates the camera's orientation; $\mathbf{T}_{im}$ is a 3-by-1 translation vector which represents the camera's location; $\mathbf{N}_i$ is a 3-by-1 noise vector caused by imperfect measurements of lamps' location as well as the inaccuracies in determining the images' centroids; and λ is a scaling factor.

Because the lens center (O') is set as the origin of the receiver's coordinate system, we have $l_1 = l_2 = l_3 = f$, where f represents the camera focal length. The scale factor λ can be easily calculated by referring to the distance between each pair of the object and image points [28]. After obtaining λ , we use $(u_i, v_i, w_i) = (\lambda m_i, \lambda n_i, \lambda f)$ with $i = 1, 2, 3$ to denote the three image points' coordinates after scale transformation.

For simplicity, $\mathbf{P}_i$ and $\mathbf{P}'_i$ are defined as shown in Eq. (4.2) with each column representing a 3D coordinates,

$$\begin{aligned}\mathbf{P}_i &= [(x_1, y_1, z_1)^T, (x_2, y_2, z_2)^T, (x_3, y_3, z_3)^T] \\ \mathbf{P}'_i &= [(u_1, v_1, w_1)^T, (u_2, v_2, w_2)^T, (u_3, v_3, w_3)^T]\end{aligned}\quad (4.2)$$

with which Eq. (4.1) is rewritten as follows,

$$\mathbf{P}'_i = \mathbf{R}_{im} \mathbf{P}_i + \mathbf{T}_{im} + \mathbf{N}_i (i = 1, 2, 3) \quad (4.3)$$

Meanwhile, $\mathbf{P}_0$ and $\mathbf{P}'_0$ are also defined for later usage,

$$\mathbf{P}_0 = \frac{1}{3} \sum_{i=1}^3 \mathbf{P}_i; \mathbf{P}'_0 = \frac{1}{3} \sum_{i=1}^3 \mathbf{P}'_i \quad (4.4)$$

As can be seen, each index i in Eq. (4.3) provides 3 equations, resulting a total of 9 equations, while there are 6 unknowns (3 for the rotation matrix $\mathbf{R}_{im}$ and 3 for the translation vector $\mathbf{T}_{im}$). Then, the receiver's position and orientation can be solved by employing the non-linear least-square solvers.

In the above algorithm, only an image sensor (IS) is needed, hence, we call it an IS algorithm. The disadvantage is that the accuracy is largely limited by the error of images centroids' coordinates obtained through image processing. To overcome this

problem, a new algorithm fusing data of both IS and motion sensor is proposed as follows.

B. Sensor Fusion Problem Formulation

Common embedded motion sensors include accelerometer and gyroscope, with which one can detect receiver's rotation angles around X-, Y-, and Z- axes, i.e., roll pitch and yaw, which are marked as α , β and γ respectively. Rotation matrix $\mathbf{R}_s$ is then obtained according to Eq. (4.5).

$$\mathbf{R}_s = \mathbf{R}_X(\alpha)\mathbf{R}_Y(\beta)\mathbf{R}_Z(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \times \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \times \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.5)$$

Obviously, $\mathbf{R}_s$ must be an orthonormal matrix. The angles around three axes are independent with measuring errors up to several degree using common embedded motion sensors. According to our experiments, $\mathbf{R}_{im}$ has similar error range with $\mathbf{R}_s$, which makes the idea of fusion feasible.

In order to involve in the motion sensor data, we firstly rewrite Eq. (4.3) as the following optimization problem,

$$\begin{aligned} \min \Sigma^2 &= \sum_i \|\mathbf{P}'_i - (\mathbf{R}_{im}\mathbf{P}_i + \mathbf{T}_{im})\|_F^2 \\ \text{s.t. } \det(\mathbf{R}_{im}) &= 1; \mathbf{R}_{im}^T = \mathbf{R}_{im}^{-1} \end{aligned} \quad (4.6)$$

The optimal estimations thus should be the $\mathbf{R}_{im}$ and $\mathbf{T}_{im}$ which minimize the cumulative fitting error (CFE) measured by the Frobenius norm (F-norm), under the constraints of the rotation matrix property [94].

Then, the issue of sensor fusion is modeled as a multi-objective optimization problem on the basis of Eq. (4.6) by adding another term which evaluates the similarity of the readings of two sensors,

$$\begin{aligned} \min \Sigma^2 &= (1-\varepsilon) \sum_i \left\{ \left\| \mathbf{P}'_i - (\mathbf{R}_{im} \mathbf{P}_i + \mathbf{T}_{im}) \right\|_F^2 / \Gamma_i \right\} + \varepsilon \left\| \mathbf{R}_{im} - \mathbf{R}_s \right\|_F^2 \\ \text{s.t. } \det(\mathbf{R}_{im}) &= 1; \mathbf{R}_{im}^T = \mathbf{R}_{im}^{-1} \end{aligned} \quad (4.7)$$

where the F-norm measures the difference between two sensors' data and Γ_i is the normalization factor which ensures the two terms in the same scale,

$$\Gamma_i = \left\| (\mathbf{P}_i - \mathbf{P}_0)(\mathbf{P}'_i - \mathbf{P}'_0)^T \right\|_F^2 \quad (4.8)$$

Moreover, the parameter $\varepsilon \in [0, 1]$ is used to balance the weight of two terms. The value of ε indicates different emphases of the imaging sensor and the motion sensors, depending on the measurement condition and the sensor accuracy (or reliability). The method of finding the optimal ε will be introduced in section IV. It is worth mentioning that, ε can also be regarded as a characteristic variable of a certain condition. Once the optimal value is calibrated in one trial, it does not change unless the condition is changed.

Moreover, unlike the naive averaging algorithm which may further degrade the PA in the case of bad angle measurements, our fusion algorithm will always outperform the IS algorithm with a proper value of ε . The special case happens only when the readings of motion sensors are extremely inaccurate, then the algorithm with $\varepsilon = 0$ will give the same PA with the IS algorithm. Nevertheless, to realize the idea, an optimization algorithm is necessary, which is proposed and described in detail in the next section.

4.3 SVD-SF algorithm

Detailed mathematical derivation of the optimal solution to the reformulated sensor fusion problem is presented in this section. And the workflow of the proposed SVD-SF algorithm is summarized accordingly.

4.3.1 Optimal solution derivation

As observed, the feasible domain of Eq. (4.6) is non-convex, which means that this problem cannot be solved directly by employing the convex optimization solvers. Generally, two methods can be chosen for a multi-objective non-convex optimization

problem: (i) optimizing the first and second terms alternatively and obtaining a local optimum; (ii) using the multi-start algorithm [100]. Since non-convex problems are usually sensitive to the initial guess and different initial guesses will result in different local optimal solutions. The global optimum can then be chosen among these solutions. Theoretically, if initial guesses are reasonable and sufficient, we are able to find the global optimum, however, at the cost of massive computation. Obviously, the first method is likely to lead to a large positioning error while the second method may take more time and consume more power. Hence, they both are not suitable for an indoor positioning system. To address this issue, we propose a non-iterative closed-form solution for this optimization problem.

First, we define another two matrices, i.e., $\mathbf{Q}$ and $\mathbf{Q}'$, with the column vectors obtained according to Eq. (4.9),

$$\mathbf{Q}_i \triangleq \mathbf{P}_i - \mathbf{P}_0; \mathbf{Q}'_i \triangleq \mathbf{P}'_i - \mathbf{P}'_0 (i = 1, 2, 3) \quad (4.9)$$

$\mathbf{P}_0$ and $\mathbf{P}'_0$ also satisfy the imaging relationship,

$$\mathbf{P}'_0 = \mathbf{R}_{im} \mathbf{P}_0 + \mathbf{T}_{im} \quad (4.10)$$

Substituting Eq. (4.9) and Eq. (4.10) into Eq. (4.7) yields,

$$\Sigma^2 = (1 - \varepsilon) \sum_i \left\{ \left\| \mathbf{Q}'_i - \mathbf{R}_{im} \mathbf{Q}_i \right\|_F^2 / \Gamma_i \right\} + \varepsilon \left\| \mathbf{R}_{im} - \mathbf{R}_s \right\|_F^2 \quad (4.11)$$

where the translation matrix $\mathbf{T}_{im}$ is successfully canceled and Γ_i is simplified as

$\left\| \mathbf{Q}_i \mathbf{Q}_i^T \right\|_F^2$. For $\mathbf{A}$ as either a vector or a matrix, we have $\|\mathbf{A}\|_F^2 = \text{trace}(\mathbf{A}\mathbf{A}^T)$, with

which Eq. (4.11) is expanded as,

$$\begin{aligned} \Sigma^2 = & (1 - \varepsilon) \sum_i \text{trace}(\mathbf{Q}'_i \mathbf{Q}'_i^T + \mathbf{Q}_i \mathbf{Q}_i^T - 2\mathbf{R}_{im} \mathbf{Q}_i \mathbf{Q}'_i^T) / \Gamma_i \\ & + \varepsilon \text{trace}(2\mathbf{I} - 2\mathbf{R}_{im} \mathbf{R}_s^T) \end{aligned} \quad (4.12)$$

Since $\text{trace}(\mathbf{Q}'_i \mathbf{Q}'_i^T + \mathbf{Q}_i \mathbf{Q}_i^T)$ and $\text{trace}(\mathbf{I})$ are known constants, they can be dropped from the CFE function without changing the unknown estimation. Then, Σ^2 can be simplified as,

$$\begin{aligned}\Sigma^2 &= -\text{trace} \left\{ \mathbf{R}_{im} \left[(1-\varepsilon) \sum_i (\mathbf{Q}_i \mathbf{Q}_i^T / \|\mathbf{Q}_i \mathbf{Q}_i^T\|_F^2) + \varepsilon \mathbf{R}_s^T \right] \right\} \\ &= -\text{trace}(\mathbf{R}_{im} \mathbf{W})\end{aligned}\quad (4.13)$$

where we define $\mathbf{W} = (1-\varepsilon) \sum_i (\mathbf{Q}_i \mathbf{Q}_i^T / \|\mathbf{Q}_i \mathbf{Q}_i^T\|_F^2) + \varepsilon \mathbf{R}_s^T$, which is also known. The singular value decomposition (SVD) of $\mathbf{W}$ is given by $\mathbf{W} = \mathbf{U} \mathbf{\Omega} \mathbf{V}^T$, where both $\mathbf{U}$ and $\mathbf{V}$ are 3-by-3 orthonormal matrices, and $\mathbf{\Omega}$ is a diagonal matrix. Next, we substitute the SVD of $\mathbf{W}$ into the CFE function. Due to the matrix trace property $\text{trace}(\mathbf{A} \mathbf{A}^T) = \text{trace}(\mathbf{A}^T \mathbf{A})$, we have,

$$\begin{aligned}\text{trace}(\mathbf{R}_{im} \mathbf{W}) &= \text{trace}(\mathbf{R}_{im} \mathbf{U} \mathbf{\Omega} \mathbf{V}^T) \\ &= \text{trace}(\mathbf{V}^T \mathbf{R}_{im} \mathbf{U} \mathbf{\Omega}) = \text{trace}(\mathbf{Z} \mathbf{\Omega})\end{aligned}\quad (4.14)$$

where $\mathbf{Z} = \mathbf{V}^T \mathbf{R}_{im} \mathbf{U}$ is an orthogonal matrix. Given an arbitrary diagonal matrix $\mathbf{\Omega}$, $\text{trace}(\mathbf{Z} \mathbf{\Omega})$ reaches its maximum if and only if $\mathbf{Z} = \mathbf{I}$, where $\mathbf{I}$ denotes an identity matrix, i.e.,

$$\mathbf{Z}_{opt} = \mathbf{V}^T \mathbf{R}_{opt} \mathbf{U} = \mathbf{I} \quad (4.15)$$

Therefore, the closed-form optimal rotation matrix ($\mathbf{R}_{opt}$) is obtained by using the SVD technique,

$$\mathbf{R}_{opt} = \mathbf{R}_{fusion} = \mathbf{V} \mathbf{U}^T \quad (4.16)$$

Finally, we should verify if the obtained $\mathbf{R}_{opt}$ satisfies the rotation matrix requirement, i.e., $\det(\mathbf{R}_{opt}) = 1$, since $\mathbf{R}_{opt}^T = \mathbf{R}_{opt}^{-1}$ is naturally satisfied according to Eq. (4.16). If $\det(\mathbf{R}_{opt}) = 1$, our positioning is successful and the optimal solution to the position vector is finally given by

$$\mathbf{T}_{opt} = \mathbf{T}_{fusion} = \mathbf{P}' - \mathbf{R}_{fusion} \mathbf{P} \quad (4.17)$$

However, if $\det(\mathbf{R}_{opt}) = -1$, $\mathbf{R}_{opt}$ is called improper rotation or reflection [101]. In such case, the system discards the data and starts the next round of calculation. According to our experiments, this case almost doesn't occur.

4.3.2 SVD-SF algorithm and complexity

Based on the above derivation, a non-iterative SVD-SF algorithm is summarized in Algorithm II. According to [102], the time complexity of a $m \times n$ matrix SVD is $O(\min\{m^2n, mn^2\})$. Thus, the proposed SVD-SF algorithm which requires the SVD of a 3-by-3 matrix $\mathbf{W}$ has a fixed low time complexity of $O(27)$. At the same time, the SVD-SF algorithm, which provides the closed-form solution, has more robust performance compared with the conventional iterative-based algorithm and the multi-start algorithm [100] for solving combinatorial non-convex problems whose accuracy and complexity largely depend on the initial guesses.

Algorithm II: SVD-based Sensor Fusion Algorithm

1. Calculate matrices $\mathbf{P}$ and $\mathbf{P}'$, and then $\mathbf{Q}$ and $\mathbf{Q}'$.
 2. Generate a 3×3 matrix, $\mathbf{W} = (1 - \varepsilon) \sum_i (\mathbf{Q}_i \mathbf{Q}_i'^T / \|\mathbf{Q}_i \mathbf{Q}_i'^T\|_F^2) + \varepsilon \mathbf{R}_s^T$
based on $\mathbf{Q}$, $\mathbf{Q}'$ and $\mathbf{R}_s$.
 3. Conduct SVD on $\mathbf{W}$, $\mathbf{W} = \mathbf{U} \mathbf{\Omega} \mathbf{V}^T$, and calculate
 $\mathbf{R}_{opt} = \mathbf{R}_{fusion} = \mathbf{V} \mathbf{U}^T$.
 4. Check if $\det(\mathbf{R}) = 1$, otherwise, discard the current data and start next round of calculation.
 5. Calculate $\mathbf{T}_{opt} = \mathbf{T}_{fusion} = \mathbf{P}' - \mathbf{R}_{fusion} \mathbf{P}$
-
-

4.4 Performance evaluation and complexity analysis

Both experiments and simulations are carried out to evaluate the performance of the proposed system and the SVD-SF algorithm. The performance gain brought by our algorithm depends on the accuracy of both the motion sensor and the image processing. Among them, the motion sensor accuracy is determined by the sensor model and can be obtained by checking its datasheet directly. Yet, the error of the obtained images centroids coordinates is affected not only by the characteristics of the

image sensor (e.g., pixels numbers and size, focal length) but also by the experimental settings (e.g., LED lamp power, LED lamps field of view (FoV), distance to LED lamps plane (d)). In our experiment, we regard the error of the obtained centroids' coordinates related to all the aforementioned factors as images processing noise without tracing the explicit sources of the error.

Our evaluation is divided into two parts. The first part presented in subsection 4.4.1 studies the method of obtaining the optimal balancing parameter and the PA improvement under our experimental settings. The second part 4.4.2 investigates the positioning performance of the proposed system in more general cases through simulations, where image processing noise and angle error deviation vary under different conditions.

4.4.1 Experimental set-up and results

To evaluate the SVD-SF algorithm in practical environments, we carry out intensive experiments by using the image sensor (rear camera) and motion sensors embedded in Nokia Lumia 1020 smartphone. Both the smartphone and the LEDs are chosen from the typical civil used models. The experimental specifications are given in Table I.

We consider the large amount of measurements as a random process. The image processing errors can then be modeled as additive noises that follow Gaussian distribution. The noise variance is easily estimated using massive samples, with which the image processing noise (N_{ip}) power can be calculated. The average power of N_{ip} is 12 dBW under the settings of Table I. Similarly, the detected angle errors of a motion sensor (e_{agl}) are modeled as Gaussian white noise since the angle errors around three axes are independent. According to the datasheet, the embedded motion sensor in the Nokia Lumia 1020 has angle errors within 2° , 2° and 3° for pitch, roll and yaw respectively. Hence, its angle noise deviation is set to be 0.7° , 0.7° and 1° around three axes to guarantee the generated errors within the given ranges with the possibility of 99.7%.

Fig. 4.4 compares the 3D positioning accuracy of the IS algorithm and the proposed SVD-SF algorithm with the balancing parameter ε taking values from 0 to 1. As can be seen, the error performance of IS algorithm labeled in red, which relies on a single

image sensor, doesn't change with the varying balancing parameter ε . It achieves 3D positioning accuracy of roughly 9.7 cm. The blue curve records the positioning errors of the SVD-SF algorithm with different values of ε . It starts from the same PA as the IS algorithm when $\varepsilon = 0$, which implies motion sensor's reading is neglected. Then, it gradually decreases and reaches minimum when $\varepsilon \approx 0.68$ before increases slightly till $\varepsilon = 1$. From this figure, it is found that the optimal value of ε associated with our settings is approximate 0.68. As illustrated, the SVD-SF algorithm improves the PA by 4.3 cm, which is roughly 44% performance improvement compared with the IS algorithm.

Table 4.1. Experimental specifications for SVD-SF algorithm

Component/Parameter	Model/Value
LED	Cool White (5650K) LUXEON Rebel ES LED, 40mm Round 7-Up Base
Power (per LED lamp)	10 W
Height of LED plan h (cm)	180 cm
FoV of LED optics	$\varphi = 51^\circ$
Smart Phone	Nokia Lumia 1020
Focal Length	7.25 mm
Motion sensor angle error	$\leq 2^\circ; 2^\circ; 3^\circ$
Image pixel resolution	7712×4352
Pixel size	$1.12 \mu m$
Shutter Speed	1/16000 s

Fig. 4.4. 3D positioning error under different ε

4.4.2 Simulation results

In the above subsection, PA gain under our experimental settings is evaluated. However, the measured error (noise) power varies when different models of sensors and experimental settings are employed. Without losing generality, the performance of the IS and SVD-SF algorithms are compared in more general cases where both the N_{ip} power and e_{agl} deviation take larger range of values. As depicted in Fig. 5, each testing point corresponds to a pair of (N_{ip}, e_{agl}) value. The positioning error of the SVD-SF algorithm at each point in Fig. 4.5 (b) represents the performance under the optimal setting of ε which can be found the same way as introduced in subsection A. By comparing subgraph Fig. 4.5 (a) and Fig. 4.5 (b), it is found that the proposed SVD-SF algorithm has overall higher PA than the IS algorithm under the same condition.

Additionally, the PA improvement percentage is calculated according to the simulation results, and some sample values are recorded in Table II. The recorded data proves that the proposed SVD-SF algorithm never degrades the PA in any condition. Also, the SVD-SF algorithm shows its superiority in most situations especially for the cases with large image processing noise. A practical example of this case is that LED lamps are mounted on the ceiling and low-resolution front cameras with high noise (error) power are naturally utilized as receivers. One step further, the

SVD-SF algorithm shows great potential to lower the hardware requirement of the indoor positioning system. One can acquire the equivalent or even higher PA by using two or more ordinary sensors rather than purchasing expensive sophisticated devices. Meanwhile, it is found that our algorithm shows limited improvement when image processing noise power is extremely small and/or angle error deviation is large. The reason is that these settings represent the special cases that the images' centroids coordinates are precisely obtained while the data of motion sensors is quite unreliable. In that scenarios, the PA of a single camera based IS algorithm is already as high as millimeter. It is hard to achieve high PA gain through fusing inaccurate angle information.

(a)

(b)

Fig 4.5. (a) 3D positioning error using IS (original) algorithm. (b) 3D positioning error using SVD-SF algorithm

Table 4.2 Positioning Accuracy Improvement Percentage (%)

Angle Error Deviation	Image processing noise power (dBW)								
	20	16	12	8	4	0	-4	-8	-12
1°	62.35	61.51	61.29	58.68	53.5	44.69	32.39	21.43	10.55
2°	61.53	59.98	56.61	49.09	39.59	26.55	16.78	7.1	3.354
3°	60.77	57.47	50.44	41.2	29.7	15.42	7.51	3.15	2.1
4°	58.07	54.62	43.78	33.91	20.11	10.79	4.70	1.75	0.27
5°	56.46	50.36	38.56	25.47	14.74	8.01	4.01	1.76	0.35
6°	55.08	46.89	34.3	21.28	11.39	5.73	2.56	0.87	0
7°	52.47	42.46	31.21	18.82	9.89	3.45	1.63	0.31	0

4.5 Conclusion

In this chapter, we propose a novel image sensor-based VLP algorithm which achieves higher PA by fusing the data of common embedded sensors on a smartphone: the image sensor and the motion sensor. The issue of sensor fusion is formulated as a multi-objective non-convex optimization problem. The closed-form optimal solution to the sensor fusion problem is derived using the SVD technique, and the low-complexity non-iterative SVD-SF algorithm is proposed accordingly. Experiment results have shown that the proposed SVD-SF reduces the positioning error from 9.7 cm to 4.4 cm which is equivalent to 44% performance improvement over the IS algorithm based on a single camera. Furthermore, simulations are conducted to analyze the performance of the proposed SVD-SF algorithm under different noise conditions.

Importantly, the proposed system is not limited to the cases with image sensors and motion sensors. It is easy to be extended to the future hybrid positioning systems which involve more other types of common sensors, such as barometers and magnetometers. Also, the SVD-SF algorithm has the potential to fuse data for even

wider purpose besides indoor positioning, e.g., monitoring in the large-scale internet of things (IoT) system.

Chapter 5 An Anti-outrage IS-based VLP System: Circle Projection based Single LED (CPSL)

Positioning

5.1 Introduction

In Chapters 3, we investigated the approach to speed up the IS-based positioning algorithm and the impact of various factors on the positioning accuracy. Then in Chapter 4, based on the same VLP system architecture, we proposed an advanced positioning algorithm which effectively improves the positioning accuracy by fusing the data from both image sensor and motion sensors. However, during the extensive experiments, we find that it is hard for a camera to capture more than three reference LED lamps every time, due to limited FoV or arbitrary orientation of the camera or the shade from indoor objects. In such situation, IS-based positioning services may fail or be interrupted. To deal with this issue, in this chapter, we propose a single LED positioning algorithm based on circle projection. By reducing the minimum number of required LEDs, we can reduce the possibility of positioning failure thus to improve the robustness of the VLP system.

Decimeter/centimeter-level accuracy has been reported in some image sensor (IS) based VLP systems [28, 77]. Yet, all these IS based VLP systems treat LED lamps as point sources without geometric information. Thus, at least three LEDs have to be captured in a picture, in order to set up equations required for determining the receiver's location. However, this requirement is too stringent to satisfy in practical applications due to many reasons: (i) the majority of smartphone cameras have very limited field of view (FoV); (ii) LED lamps are usually deployed sparsely inside buildings; (iii) LED lamps are likely to be blocked by some objects such as walls and furniture in indoor environment. Positioning may be interrupted or fail if less than three LED lamps are within the FoV of the camera, which greatly reduces the system robustness and flexibility.

To reduce the number of required LEDs, a straightforward way is to employ angular sensors to measure the receiver's orientation information, thus to “compensate” the

missing information due to the reduced number of LED lamps. Two recent works on single LED based localization [103-104] provide the potential solution by using this method. However, the use of angular sensors brings about another problem. Previous researches have shown that a main source of the positioning and navigation errors is the inaccuracy of azimuthal angles, either measured by magnetometers or gyroscopes [105-106]. Magnetometers are sensitive to the magnetic disturbance which is particularly severe inside buildings; while gyroscopes' output will drift over time, especially in mobile cases. That is, both types of angular sensors have problem providing stable and reliable orientation measurements for positioning in practical mobile indoor environment.

Motivated by the pressing problems above, we propose a single LED based VLP system without using angular sensors for the first time. Unlike the existing IS based VLP systems [28, 51] which treat the LED lamps as point sources with only location information, our approach makes full use of the geometric features of an LED image. Specifically, a common circular LED lamp is used as the reference LED in our system, which is projected into an ellipse in the image plane [107]. The geometric features of the captured ellipse image (e.g., centroid, major/minor axis length, major axis orientation), are exploited to determine the receiver's pose and location relative to the reference LED lamp. In addition, a marginal point of a circular LED lamp is marked to provide the relative azimuthal information. Then, after proper image processing, one can mathematically determine the receiver's orientation and location using a single reference lamp.

The proposed single LED based positioning algorithm is conceptually simple, easy to implement, and computationally efficient because no iterations are involved. It avoids two practical difficulties at the same time: multiple LEDs requirement and angular sensor dependence. It allows to provide robust and accurate estimations for receiver's location and orientation in practical mobile indoor environment with sparse LED lamps deployment.

The rest of the chapter is organized as follows. In Section 5.2, the VLP system model based on circle projection is introduced. The theory of a circle projection is presented and the supporting positioning algorithm CPSL is derived in Section 5.3. Then we experimentally evaluate the positioning accuracy in an area of $3\text{m} \times 3\text{m}$ and the

performance is given and analyzed in Section 5.4. Finally, Section 5.5 concludes this chapter. The content of this chapter is related to the author’s work reported in [49].

5.2 Circle projection model

Fig. 5.1. The schematic diagram of the single LED based indoor VLP system

Fig. 5.2. Image processing workflow for ellipse feature extraction

Fig. 5.1 shows an overview of the proposed indoor VLP system using a single LED lamp. In this system, common circular LED lamps, with each having a marginal marker, are installed on the ceiling. Each LED lamp is assigned with a unique ID,

which is associated with its location and stored in an ID-location database. The LED light intensity is modulated to broadcast its ID frame repeatedly. When one holds a smartphone, the front camera will periodically capture pictures of LED lamps and meanwhile detect the modulated signals by making use of the rolling shutter effect (RSE) [29]. Since the smartphone is naturally held by a human, it can be arbitrarily tilted as long as the reference LED is in the FoV of the camera.

The captured pictures are processed at the receiver according to the image processing workflow given in Fig.5.2. Firstly, the red marker (pixels) is recognised from the original RGB color image. The center of these red pixels is taken as the marker's coordinates in the receiver's coordinate system (RCS). Then, we turn the RGB image into grayscale and demodulate the ID of the reference LED lamp in the same way as that in [28, 50]. By looking up the ID-location database, the pre-stored reference LED's and its marker's locations in the global coordinate system (GCS) are obtained. Meanwhile, the LED image is segmented from the grayscale picture and the ellipse fitting is conducted based on the minimum mean square error (MMSE) criteria [108] to find the ellipse function that is best matched with the LED image. Consequently, the geometric features of the ellipse image such as eccentricity, the length, and orientation of major/minor axis, and the projected marker's coordinates can be obtained. The details of the image processing procedures described above can be found in [28, 50]. In this chapter, we focus on the theory of the receiver's 3D location estimation using the geometric parameters extracted from the ellipse image of the circular LED, which is highlighted in red in Fig. 5.2.

5.3 Circle projection based single LED (CPSL) positioning: theory and algorithm

In the proposed system, a circular LED lamp is placed on the x-y plane with its centroid being coincided with the origin of the GCS, i.e., Oxyz coordinate system. It has a marginal marker point m on the +x-axis, as shown in Fig. 5.3(a). The circular LED lamp is then projected as an ellipse image on the i-j plane in the RCS, i.e., O'ijk coordinate system, with the origin at the center of the image plane. The marker m is mapped into M accordingly as shown in Fig. 5.3(b).

Let $\mathbf{P}=(x,y,0)$ be an arbitrary point on the circle and $\mathbf{P}'=(i,j,0)$ be the corresponding projected point on the ellipse on the i - j plane. Since the object (LED lamp) size is usually much smaller than the distance from the camera to the object, the projection can be treated as weak perspective projection [109], according to which, the projection point pair ($\mathbf{P}$ and $\mathbf{P}'$) are related as,

$$\begin{bmatrix} i & j & 0 \end{bmatrix}^T = \text{diag}(s, s, 0) \left\{ \mathbf{R}_{3 \times 3} \begin{bmatrix} x & y & 0 \end{bmatrix}^T + \mathbf{T}_{3 \times 1} \right\} \quad (5.1)$$

where $(\cdot)^T$ denotes transpose of a vector or matrix, which is used throughout this paper; $\mathbf{R}$ denotes the 3×3 rotation transformation from the GCS to the RCS; $\mathbf{T}$ denotes the translation, which is equivalent to the receiver's position in the GCS [28]; s is the ratio between the image scale and the object scale.

Fig. 5.3. (a) The circular LED lamp with a marker in the GCS, (b) the ellipse image of the circular LED lamp in the RCS.

To solve Eq. (5.1), a common method is to substitute multiple circle-to-ellipse point pairs as in the literatures [104], which is however inefficient as one has to mark multiple points on the object and distinguish them at the receiver side. In this section, we propose an alternative way to mathematically and sequentially determine the scaling factor s , the rotation matrix $\mathbf{R}$, and finally the receiver location $\mathbf{T}$.

A. Scaling factor calculation

To calculate the scaling factor, one has to figure out the correspondence between the circle and the ellipse image. Under the assumption of the weak perspective projection, the circular LED center O is approximately projected into the ellipse image center E [107]. Linking two centers O and E , there always exists a diameter which is perpendicular to OE in the circle. Meanwhile, the major axis of the ellipse image is always perpendicular to OE regardless how the camera is placed [110]. Thus, the major axis of the ellipse image coincides with the diameter of the circular LED lamp which is perpendicular to OE , from which the scaling factor s can be calculated as,

$$s = a/r \quad (5.2)$$

where r and a are the radius of the circular LED lamp and the length of the semi-major axis of the ellipse, respectively.

B. Rotation matrix determination

The rotation from GCS to RCS is firstly decomposed as a series of consecutive rotations in the order of x-y-x (can also be decomposed in other orders such as x-y-z, z-y-z, etc. x-y-x is chosen here for computational simplicity). Let α , β and γ denote the three consecutive rotation angles around x-, y- and x-axis, respectively. Then the x-y-x rotation transformation can be represented by matrix multiplications:

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & -\sin \gamma \\ 0 & \sin \gamma & \cos \gamma \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad (5.3)$$

where the rotation angles obey the right-hand rule in Cartesian coordinate and are restricted within the ranges of $-\pi \leq \alpha, \gamma \leq \pi$, $0 \leq \beta \leq \pi$, which cover all the possible rotations. Our task is to solve the three rotation angles so that $\mathbf{R}$ can be determined.

Since the circular LED center $O(0, 0, 0)$ is approximately projected into the ellipse image center $E(i_e, j_e, 0)$ in **solid line**, the following equation can be obtained by substituting the projection pair points into (5.1),

$$\begin{bmatrix} i_e & j_e & 0 \end{bmatrix}^T = \text{diag}(s, s, 0) \left\{ \mathbf{R}_{3 \times 3} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T + \mathbf{T}_{3 \times 1} \right\} = \text{diag}(s, s, 0) \mathbf{T}_{3 \times 1} \quad (5.4)$$

Then, subtracting (5.4) from (5.1), the translation $\mathbf{T}_{3 \times 1}$ is canceled and (5.1) is simplified as,

$$\begin{bmatrix} i - i_e & j - j_e & 0 \end{bmatrix}^T = \text{diag}(s, s, 0) \mathbf{R} \begin{bmatrix} x & y & 0 \end{bmatrix}^T. \quad (5.5)$$

Let $i' = i - i_e$, $j' = j - j_e$, the left-hand side of (5.5) is geometrically equivalent to shift the captured ellipse image $E(i, j)$ to a new ellipse $E'(i', j')$ in ***dash line*** centered at RCS origin (image plane center O') as illustrated in Fig. 5.3(b). Substituting (5.2) and (5.3) into (5.5) and dividing a on both sides of (5.5), we have,

$$\begin{aligned} i' / a &= (x / r) \cos \beta + (y / r) \sin \alpha \sin \beta \\ j' / a &= (x / r) \sin \beta \sin \gamma + (y / r) (\cos \alpha \cos \gamma - \sin \alpha \cos \beta \sin \gamma) \end{aligned} \quad (5.6)$$

Specifying the particular marker $m(r, 0)$ on the $+x$ -axis of the circle in (5.6), its projected marker's location after shifting, i.e., $M'(i'_m, j'_m)$, can be expressed in terms of rotation angles,

$$i'_m / a = \cos \beta; \quad j'_m / a = \sin \beta \sin \gamma \quad (5.7)$$

where the subscript m denotes the marker, and $i'_m = i_m - i_e$, $j'_m = j_m - j_e$. The projected marker's location $M(i_m, j_m)$ and the ellipse center location $E(i_e, j_e)$ can both be acquired through image processing, therefore, two rotation angles β and γ are immediately obtained as,

$$\beta = \arccos\left(\frac{i'_m}{a}\right); \quad \gamma = \arcsin\left(\frac{j'_m}{a \cdot \sin \beta}\right) \quad (5.8)$$

The next step is to find the remaining unknown angle α , for which we rely on the particular shape and orientation of the ellipse image. The length of the semi-major axis has been denoted as a . We further denote the length of the semi-minor axis as b and their ratio as $k = b / a < 1$. Then, a standard ellipse centered at the origin of the i - j plan with a vertical major axis can be expressed as,

$$i_{st} = b \sin t; \quad j_{st} = a \cos t \quad (5.9)$$

Rotating the vertical major axis clockwise by ω around its origin, the general ellipse function representing the captured image is obtained in terms of ω ,

$$\begin{aligned} i &= i_{st} \cos \omega + j_{st} \cdot \sin \omega = b \sin t \cdot \cos \omega + a \cos t \cdot \sin \omega \\ j &= -i_{st} \sin \omega + j_{st} \cdot \cos \omega = -b \sin t \cdot \sin \omega + a \cos t \cdot \cos \omega \end{aligned} \quad (5.10)$$

Dividing a on both sides of the (5.10), the ellipse is scaled into the same size as (5.6),

$$\begin{aligned} i/a &= k \sin t \cdot \cos \omega + \cos t \cdot \sin \omega \\ j/a &= -k \sin t \cdot \sin \omega + \cos t \cdot \cos \omega \end{aligned} \quad (5.11)$$

As shown in Fig. 2(b), the i -coordinate of *rightmost* ellipse point of the shifted ellipse E' can be expressed either from (5.6) as $\sqrt{\cos^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta}$ or from (5.11) as $\sqrt{k^2 \cos^2 \omega + \sin^2 \omega}$ (refer to Appendix A). Obviously, they should be equal as both functions describe the same ellipse, which gives,

$$k^2 \cos^2 \omega + \sin^2 \omega = \sin^2 \alpha \cdot \sin^2 \beta + \cos^2 \beta \quad (5.12)$$

In the same manner, the j -coordinate of the *upmost* ellipse point of the shifted ellipse E' can be expressed from (5.6) as $\sqrt{\sin^2 \beta \cdot \sin^2 \gamma + (\cos \alpha \cdot \cos \gamma - \sin \gamma \cdot \cos \beta \cdot \sin \alpha)^2}$ or from (5.11) as $\sqrt{k^2 \sin^2 \omega + \cos^2 \omega}$. Thus, we have,

$$k^2 \sin^2 \omega + \cos^2 \omega = \sin^2 \beta \cdot \sin^2 \gamma + (\cos \alpha \cdot \cos \gamma - \sin \alpha \cdot \cos \beta \cdot \sin \gamma)^2 \quad (5.13)$$

Substituting the already obtained β and γ into both (5.12) and (5.13), α is solved (refer to Appendix B) and the rotation matrix is subsequently determined.

C. Receiver localization

With s and $\mathbf{R}$ determined above, the receiver's location $\mathbf{T}_{3 \times 1} = [x_r \ y_r \ z_r]^T$ can be calculated by substituting them back into the projection model (1),

$$[i_m \ j_m \ 0]^T = \text{diag}(s, s, 0) \{ \mathbf{R} [x_m \ y_m \ 0]^T + [x_r \ y_r \ z_r]^T \} \quad (5.14)$$

where $M(x_m, y_m)$ is the marker's location in the GCS which is pre-known by measurement and stored in the LED database; $M(i_m, j_m)$ is the projected marker's location in the RCS before shifting.

D. Discussions

The proposed algorithm is not only applicable to the case of a single LED lamp, but also able to localize the receiver using multiple available LED lamps. When two or more LED lamps are captured simultaneously, the receiver's position can be estimated by leveraging the redundant reference LED lamps beyond the minimum requirement, so that a better positioning accuracy can be achieved. Specifically, the position can be estimated by seeking the following minimum mean square error (MMSE),

$$\sum_{n=1}^N \left\| [i_m^n \ j_m^n \ 0]^T - \text{diag}(s^n, s^n, 0) \{ \mathbf{R} [x_m^n \ y_m^n \ z_m^n]^T + [x_r \ y_r \ z_r]^T \} \right\|_2^2, n=1, 2 \dots N \quad (5.15)$$

where $\|\cdot\|_2$ denotes l_2 -norm of a vector; (x_m^n, y_m^n, z_m^n) and $(i_m^n, j_m^n, 0)$ are the locations of the n -th circular LED in the GCS and the n -th ellipse image center in the RCS, respectively; s^n is the scaling factor associated with n -th circle-ellipse pair; N is the number of the captured reference LEDs in a photo.

Appendix A

Since x and y in Eq. (5.6) satisfy a circle of $(x/r)^2 + (y/r)^2 = 1$, let $x/r = \cos \theta$ and $y/r = \sin \theta$, we have:

$$\begin{aligned} i/a &= (x/r) \cos \beta + (y/r) \sin \alpha \sin \beta = \cos \theta \cos \beta + \sin \theta \sin \alpha \sin \beta \\ &= \sqrt{\cos^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta} \sin(\theta + \delta) \end{aligned} \quad (A1)$$

where θ is the variable and δ is a certain angle. The maximum value of i/a , which represents the i -coordinate of the *rightmost* ellipse point, is obtained when $\sin(\theta + \delta) = 1$, that is,

$$(i/a)_{\max} = \sqrt{\cos^2 \beta + \sin^2 \alpha \cdot \sin^2 \beta} \quad (\text{A2})$$

Appendix B

Due to the ambiguity of the trigonometric function, Eq. (5.7) results in two sets of possible solutions (β, γ_1) and (β, γ_2) within the range of $0 \leq \beta \leq \pi, -\pi \leq \gamma_1, \gamma_2 \leq \pi$. To solve α , $\cos^2 \alpha$ is firstly represented from Eq. (5.12) as,

$$\cos^2 \alpha = (1 - k^2) \cos^2 \omega / \sin^2 \beta \quad (\text{B1})$$

Where k , ω and β are already known. It gives four solutions to $\alpha \in [-\pi, \pi]$, i.e., $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, which brings two sets of possible combinations of rotation solutions, $(\alpha_i, \beta, \gamma_1)$ and $(\alpha_i, \beta, \gamma_2)$, $i = 1, 2, 3, 4$, respectively. Among the above eight solutions, half of them do not satisfy Eq. (5.13) and are discarded.

We further consider a practical constraint that the camera should generally face up, i.e., mathematically, the normal vector of the front camera after the x-y-x rotation should have positive z-component. Let $[0 \ 0 \ 1]^T$ be the original vertical up normal vector, it becomes $[n_1 \ n_2 \ n_3]^T = \mathbf{R} [0 \ 0 \ 1]^T$ after the rotation, which can be elaborated as

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha \sin \beta \\ -\sin \alpha \cos \gamma - \cos \alpha \cos \beta \sin \gamma \\ -\sin \alpha \sin \gamma + \cos \alpha \cos \beta \cos \gamma \end{bmatrix} \quad (\text{B2})$$

Requiring the z-component to be positive gives

$$n_3 = -\sin \alpha \sin \gamma + \cos \alpha \cos \beta \cos \gamma > 0 \quad (\text{B3})$$

Meanwhile, people naturally hold mobile phones with phone screens facing themselves, which gives another constraint,

$$n_2 = -\sin \alpha \cos \gamma - \cos \alpha \cos \beta \sin \gamma > 0 \quad (\text{B4})$$

By examining conditions (B3) and (B4), three more solutions will be discarded, with only one possible solution eventually obtained.

5.4 CPSL system: experimental evaluation and analysis

5.4.1 CPSL System setup

We conduct experiments to evaluate the performance of the proposed system. Specifically, we use a common circular LED lamp and slightly modify it by adding a red marker at the circle margin as in Fig. 5.4(b). The experimental setup is shown in Fig. 5.4(a) and the specifications of devices are given in Table I. We test the positioning accuracy in an area of $3\text{m} \times 3\text{m}$ with a single LED lamp being placed at the height of 1.5m and 2m, respectively. Fig. 5.4(c) gives an LED image with modulated signals which is captured in a setting of shutter speed up to $1/8000\text{s}$ $\sim 1/16000\text{s}$ [28]. Such a high shutter speed guarantees clear images with modulated information.

Fig. 5.4. (a) CPSL experimental setup, (b) circular LED lamp with a red marker, (c) captured LED image with modulated signal and a red marker.

Table 5.1. Devices Specifications

Component/Parameter	Model/Value
LED model	Cool White (5650K) LUXEON

	Rebel ES LED, 40mm Round 7-Up Base
Diameter of LED lamp	$d_1 = 11\text{cm}$, $d_2 = 8\text{cm}$
Modulated Code	1101001 1001
Arbitrary Waveform Generator	Tabor Electronics WW2074, 200M/s
Camera Model	Nokia Lumia 1020 rear camera
Focal Length	7.25 mm
Image pixel resolution	7712×4352
Pixel size	$1.12 \mu\text{m}$
FoV of camera	$\varphi \approx 71.1^\circ$

5.4.2 Experimental performance and discussions

In this section, the error performance of the proposed system and supporting CPSL algorithm is evaluated. To avoid random errors, we take six pictures by varying camera's postures at each testing point, and then take the average of the six estimations. Fig. 5.5 gives the mean absolute positioning error at each point. Fig. 5.6 shows the cumulative density function of the estimated errors. Fig. 5.7 gives the X-Y view of the positioning results in the experimental area of $3\text{m} \times 3\text{m}$, where the Z-axis errors are neglected.

(b)

Fig. 5.5. 3D positioning error with an arbitrarily tilted smartphone camera: (a) Height = 2m, (b) Height = 1.5m

Fig. 5.6. The CDF plot of the 3D positioning error (in meter).

When the height is set as 2m (Fig. 5.5(a)), the measured results show that our simple system achieves high 3D positioning accuracy with positioning errors varying from 8.94 cm to 25.12 cm with the mean and standard deviation of 17.52 cm and 3.7 cm, respectively, in the area of $3\text{m} \times 3\text{m}$. The largest errors usually occur at margin points of the coverage area. It is partially because the camera at margin points generally has large tilting angles to capture the reference LED lamp, which brings inaccuracy in the approximation that the circle center projects into the ellipse center. Besides, the ellipse images captured at the margin area are relatively smaller and occupy less

pixels compared with the cases at the central part, which leads to larger fitting errors in the ellipse fitting and features extraction during the image processing. This reason is proved when the height is set to be 1.5 m as given in Fig. 5.5(b). The captured ellipse images occupy a larger area of pixels in the image plane compared with the case with height = 2m, resulting in more accurate image processing and better positioning accuracy. Specifically, a mean error of 12.79 cm is achieved at height = 1.5m, and the minimum and maximum errors are 5.05cm and 27.25cm, respectively. Moreover, some large errors are also found at a few central points. This is probably because the captured ellipses tend to become circles at these points, which lowers the accuracy in determining the orientation of the major axis. For the interest in the 2D error performance of the proposed system, the X-Y view of the positioning results which neglects the Z-axis errors is also provided as shown in Fig. 5.7. Intuitively, the estimated positions match well with the true positions in both cases. The positioning accuracy at central points is slightly higher than that at margin points, which agree with 3D positioning results.

Fig. 5.7. X-Y view of the 3D positioning results: (a) Height = 2m and (b) Height = 1.5m.

5.5 Conclusion

We have proposed a single LED based VLP system that exploits the property of circle projection. The proposed system solves two pressing problems simultaneously: multiple LEDs requirement and angular sensor dependence in the conventional image sensor based VLP systems. In our design, the geometric features of the LED image are fully exploited to determine the receiver's position and orientation relative to the reference LED lamp. A marginal point of the LED lamp is marked which efficiently remove the information ambiguity during the calculation. A comprehensive algorithm is proposed accordingly. The performance of the proposed system has been evaluated by experiments. An average positioning error of 17.52 cm is achieved in an area of $3\text{m} \times 3\text{m}$ when the LED lamp is deployed at the height of 2m.

Chapter 6 A Reversed Multi-target VLP System via Sparse Matrix Reconstruction (SMR)

6.1 Introduction

In previous chapters, we mainly investigate the IS-based VLP system in terms of the problem of running speed and success rate, sensor fusion for improving the positioning accuracy, minimum number of required reference LEDs which affects the positioning outage probability. The proposed algorithms are performed at the receiver side to calculate the position of their own. Similarly, in the related works, the existing VLP systems using either RSS, AOA or TOA/TDOA, usually require mobile targets (MTs) to detect signals from multiple LEDs with prior known positions and then localize themselves. They all can be classified as “self-localization” or “active localization” systems, which can be properly used for people’s localization/navigation in supermarkets, shopping malls, etc. However, they cannot scale well when they are directly extended to objects tracking/monitoring. For one thing, the conventional self-localization systems lack a global view of real-time locations of all MTs on the map. Moreover, it is inefficient for multiple MTs to constantly calculate and report their locations to a central controller, and the limited power, storage and computation resources of equipped processors may cause large system latency [111].

To address this issue, we propose and explore a novel VLP system which works reversely. Specifically, photodiodes (PDs) are mounted on the ceiling to detect light signals from MTs that carry LEDs, while a central processor processes the received signals and calculates the positions of all MTs, and then sends location information back to them. We will show that, interestingly, due to the natural spatial sparsity of an MT (each MT can only occupy one location at a specific time), its real-time location on a map can be represented by a sparse vector, and multiple MTs’ locations can form a location information matrix (LIM). The PD array mounted on the ceiling can serve as a “sensing matrix” to observe the light signals from multiple MTs. Then, the problem of the simultaneous localization of multiple MTs is converted into a sparse LIM reconstruction problem and solved by employing compressive sensing (CS) theory [112]. Our system has potential to be applied to tracking/monitoring cargo

robots in warehouses and unmanned ground vehicles (UGVs) in workshops and factories. Also, it can be easily integrated with the future internet of things (IoT) devices featuring limited storage and processing/computation capability [113]. The content of this chapter is related to the author's work published in [126].

To the best of our knowledge, this is the first work on the reversed VLP powered by CS theory. Similar approaches have been reported in wireless sensor networks (WSNs) [114] and in wireless local area networks (WLAN) [115-117]. However, the work in [10] only gives the spatial distribution of multiple sensors without identifying them, which does not achieve the purpose of tracking and monitoring. Besides, their model cannot handle the scenario when multiple MTs fall into one grid. Simulations presented in [116] and [117] are based on an assumption that each MT locates on an exact grid point in the sensing area, which simplifies the reconstruction problem but is unrealistic in practical applications. Moreover, CS theory is found more desirable for VLP systems compared with the reported RF based systems. This is because the received RF signals can be severely influenced by the multipath effect in an indoor environment [118], while such a problem can be neglected in detecting light signals since the line-of-sight (LoS) signal is dominant [119]. Further, a limited sensing area of a PD naturally filters out the small signals outside the sensing radius which usually have low signal-to-noise ratios (SNRs). Both properties guarantee the accuracy of the RSS measurement and result in a good positioning performance. The main contributions of this work are outlined as follows:

1. A novel reversed multi-target VLP system is proposed, which is modeled as a sparse matrix reconstruction problem. A 3-step workflow is developed accordingly to support this system.
2. A sensing matrix redesign method via QR decomposition is proposed to meet the restricted isometry property (RIP) requirement [120]. It offers comparable performance with the reported “*orth*” approach [115-116], but has lower computational complexity and higher parallelism.
3. An RWL_1M algorithm for LIM reconstruction is proposed by adopting reweighted l_1 -minimization method [121], which significantly outperforms the conventional l_1M algorithm, particularly in the low SNR condition.

4. A “unit 4-sparse” property of a sub-grid location representation vector is discovered, and a sub-grid localization algorithm is proposed accordingly to overcome a common unpractical assumption of on-grid locations and to tackle a false peak problem in a reconstructed matrix.
5. The proposed system and supporting algorithms are verified through simulations. The impact of various parameters including PD number, SNR, grid square size on system positioning performance is investigated.

6.2 A reversed multi-target VLP system based on RSS

In this Section, a novel reversed and centralized RSS-based VLP system is introduced. The system architecture is first presented and then the localization problem reformulated.

6.2.1 System architecture

Fig. 6.1. Schematic diagram of a reversed centralized VLP system for multi-target localization

Considering a scenario illustrated in Fig. 6.1, K MTs (e.g. robots) randomly distribute in a large sensing area which is evenly divided into grid squares with N grid points. Each MT has a LED on its top which is modulated with a sinusoidal signal of a unique frequency as a label. M PDs are mounted on the ceiling to detect light signals from K MTs. The detected mixed sinusoidal signals can be separated by using a series

of digital bandpass filters and the real-time RSS readings corresponding to each MT will be sent to a central processor. Normally, we have $K \ll N$ and $M \ll N$. The proposed system aims to localize all the K MTs simultaneously and accurately on the map using noisy RSS observations.

6.2.2 Localization problem reformulation

As an object can only appear on one location at a specific time, this location can be presented as a sparse vector [115], where the non-zero elements indicate its location on the map. For example, when the k -th target locates exactly at the n -th grid point in the map, its location representation vector $\boldsymbol{\theta}_k$ is an $N \times 1$ sparse vector with all elements equal to zero except for the n -th element $\boldsymbol{\theta}_k(n) = 1$. In this way, we can form a location information matrix (LIM) for all K MTs as $\boldsymbol{\theta}_{N \times K} = [\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_k, \dots, \boldsymbol{\theta}_K]$. The RSS observations $\mathbf{Y}_{M \times K}$ from K MTs detected by M PDs with prior known positions can now be described as,

$$\mathbf{Y}_{M \times K} = \boldsymbol{\Psi}_{M \times N} \boldsymbol{\theta}_{N \times K} + \boldsymbol{\varepsilon}_{M \times K}, \quad (6.1)$$

where $\boldsymbol{\Psi}_{M \times N}$ is the sensing matrix whose element $\psi_{i,j} = \text{RSS}_{i,j}$ is the RSS reading detected by PD i from an MT at grid point j which follows the Lambertian model [119]; $\boldsymbol{\varepsilon}_{M \times K}$ represents channel noises and measurement noises. CS theory has proved that the sparse LIM $\boldsymbol{\theta}$ can be reconstructed from the noisy measurement $\mathbf{Y}$ with high probability via l_0 -norm minimization, in condition that the sensing matrix $\boldsymbol{\Psi}$ obeys the RIP criteria [120]. MTs' positions can be determined consequently.

It is worth noting that, in practical implementation, the modulated frequency corresponding to a certain MT may not be fixed. The frequencies can be dynamically assigned to MTs by the central controller (similar to a cellular mobile network). Also, the modulated signal frequencies should be properly designed by considering both the modulation bandwidth of the employed LEDs and the digital filter design at the receiver side. Specifically, if the interval between two adjacent frequencies is large, the digital filters can easily separate the mixed sinusoidal signals, but the bandwidth efficiency and system capacity could be low. If the interval between two adjacent frequencies decreases, our system is able to support more MTs but has higher requirement for the design of digital bandpass filters.

6.3 Multi-target localization vis sparse matrix reconstruction

A 3-step workflow is developed for solving the multi-target localization problem described in Section II via sparse signal reconstruction. As illustrated in Fig. 2(a), it includes pre-processing of sensing matrix, LIM reconstruction and sub-grid localization.

Fig. 6.2. The workflow for multi-target localization via sparse signal reconstruction

6.3.1 Sensing matrix design via QR decomposition

The sensing matrix Ψ contains coherent elements thus disobeys the RIP criteria [116]. Therefore, a pre-processing operation is required to induce RIP before we apply CS theory. An “*orth*” algorithm was proposed in previous works by multiplying an operator $\mathbf{T} = \text{orth}(\Psi^T)^T \Psi^\dagger$ on both sides of Eq. (6.1) [114-115],

$$\begin{aligned} \hat{\mathbf{Y}} &= \text{orth}(\Psi^T)^T \Psi^\dagger \mathbf{Y} = \text{orth}(\Psi^T)^T \Psi^\dagger (\Psi \boldsymbol{\theta} + \boldsymbol{\varepsilon}), \\ &= \text{orth}(\Psi^T)^T \boldsymbol{\theta} + \hat{\boldsymbol{\varepsilon}} \end{aligned} \quad (6.2)$$

where $\text{orth}(\cdot)$ finds an orthogonal basis of a matrix based on singular value decomposition (SVD); $(\cdot)^T$ returns the transpose of a matrix, $(\cdot)^\dagger$ denotes a pseudo-inverse of a matrix. The obtained equivalent sensing matrix $\text{orth}(\Psi^T)^T$ is an orthogonal matrix which meets the RIP condition.

Here, we offer an alternative sensing matrix redesign method via QR decomposition [122]. First, we conduct QR decomposition to the transpose of the sensing matrix $\Psi_{M \times N}$ as,

$$\Psi^T = \mathbf{Q}\mathbf{R}, \quad (6.3)$$

where $\mathbf{Q}$ is an $N \times N$ unitary matrix and $\mathbf{R}$ is an $N \times M$ upper triangular matrix.

Then, taking $\mathbf{G}_{M \times N} = [\mathbf{I}_{M \times M} \quad \mathbf{0}]_{M \times N} \mathbf{Q}$ as the first M rows of $\mathbf{Q}$, and multiplying $\mathbf{G}\Psi^\dagger$ on both sides of Eq. (6.1), we have ,

$$\hat{\mathbf{Y}} = \mathbf{G}\Psi^\dagger \mathbf{Y} = \mathbf{G}\Psi^\dagger (\Psi \boldsymbol{\theta} + \boldsymbol{\varepsilon}) = \mathbf{G}\boldsymbol{\theta} + \hat{\boldsymbol{\varepsilon}}, \quad (6.4)$$

where $\Psi^\dagger$ can also be solved by QR decomposition. As $\mathbf{Q}$ is an orthonormal unitary matrix, $\mathbf{G}_{M \times N}$ should also be a partial orthonormal matrix which is one of the most common measurement matrices [123], $\boldsymbol{\theta}$ can be well reconstructed from $\hat{\mathbf{Y}}$ with high probability. From extensive simulations, our method shows comparable performance with the conventional *orth* algorithm, while QR decomposition has relatively lower computational complexity than SVD [122] and can easily be parallelized in practical implementation [124].

6.3.2 LIM reconstruction via reweighted l1-minimization

Due to the non-convexity of l_0 -norm, a common approach is to use l_1 -norm as its relaxation. The problem of solving sparse LIM $\boldsymbol{\theta}$ in Eq. (6.1) is converted into the following convex optimization problem P_1 ,

$$P_1: \min_{\boldsymbol{\theta}_k} \|\boldsymbol{\theta}_k\|_{l_1}, \text{ s.t. } \|\mathbf{Y}_k - \Psi \boldsymbol{\theta}_k\| \leq \boldsymbol{\varepsilon}, \quad k = 1, 2 \cdots K, \quad (6.5)$$

where $\mathbf{Y}_k$ is the k -th column of the observations $\mathbf{Y}_{M \times K}$; $\boldsymbol{\theta}_k$ is the representation vector corresponding to the k -th MT. As shown, MTs are treated as individuals in this phase. Each $\boldsymbol{\theta}_k$ is solved based on only the $\mathbf{Y}_k$ without using the other $k-1$ observations. For convenience, we call the conventional approach based on Eq. (6.5) as an l_1 -minimization ($l_1\text{M}$) algorithm [125], which, however, is found vulnerable to noises. The reason is that l_1 -minimization is not a perfect approximation of l_0 -minimization due to uneven penalties on coefficients [121]. Specifically, unlike even penalization of l_0 -minimization, larger elements of $\boldsymbol{\theta}_k$ are penalized more heavily than smaller elements in l_1 -minimization. This fact can severely limit $l_1\text{M}$'s positioning accuracy as, in a positioning problem, large elements usually correspond to MTs' locations (penalty should be small) while small values are noises (penalty should be heavy). It is believed that the positioning performance can be enhanced by rectifying the uneven

penalties in l_1 -minimization. Motivated by this, we adopt the reweighted l_1 -minimization method with $\mathbf{W}_{N \times N}^k = \text{diag}\{w_1^k, w_2^k, \dots, w_N^k\}$ as follows,

$$\mathcal{P}_2: \min_{\boldsymbol{\theta}_k} \|\mathbf{W}^k \boldsymbol{\theta}_k\|_{l_1}, \text{ s.t. } \|\mathbf{Y}_k - \boldsymbol{\Psi} \boldsymbol{\theta}_k\| \leq \epsilon, \quad k = 1, 2, \dots, K, \quad (6.6)$$

where the weight w_i^k is designed to be inversely related to the i -th signal value of $\boldsymbol{\theta}_k$, i.e., $\theta_{k,i}$, thus to encourage the large coefficients and discourage the small coefficients [121]. In particular, we set

$$w_i^k = 1/(|\theta_{k,i}| + \sigma), \quad (6.7)$$

where $\sigma > 0$ is a very small parameter introduced to ensure a non-zero denominator. An iterative reweighted l_1 -norm minimization (RW l_1 M) algorithm based on Eq. (6.6) is then proposed as shown in Algorithm I, which can refine the weights w_i^k based on the estimated $\boldsymbol{\theta}_k$ at each iteration. Empirically, the refined weights usually converge within five iterations. After $\boldsymbol{\theta}_k$ is solved for all K MTs ($k = 1, 2, \dots, K$), the sparse LIM $\boldsymbol{\Theta}$ can be fully reconstructed. This algorithm is more sparsity-encouraging than the l_1 M algorithm, which benefits accurate localization in low-SNR condition.

6.3.3 Sub-grid localization via weighted interpolation

Fig. 6.3. A mobile target localizes inside a unit grid square

Once the LIM $\boldsymbol{\Theta}$ has been obtained, MTs' physical locations can be determined. If the k -th MT locates at a grid point, its corresponding representation vector (k -th column of LIM) $\boldsymbol{\theta}_k$ is exactly 1-sparse with only one peak and the rest elements being near-zero. The location can then be found precisely from the peak index. This

straightforward method has been used in [115-116]. However, in a real situation, an MT is more likely to appear at any point PT inside a unit grid square, as shown in Fig. 2(b). In such a case, the representation vector has more than one peak whose values and indexes jointly determine the locations on the map.

From extensive tests, an interesting “unit 4-sparse” property is observed in a sub-grid localization. Specifically, the representation vector of a sub-grid point PT usually has four peaks on P1-, P2-, P3- and P4-th elements with other elements being near-zero, and these four grid points should just form a unit grid square enclosing PT, as shown in Fig. 2(b). Meanwhile, the corresponding peak’s values (V1, V2, V3, and V4) are inversely related to the distance between the MT and four vertexes. When PT approaches one of the four vertexes, the corresponding peak value increases while others drop. As two special cases, when PT moves to a grid point, its representation vector becomes 1-sparse as stated earlier; when PT appears in the square centroid, four peak values are generally the same. A sub-grid location is thus estimated by a weighted interpolation,

$$P_t = \sum_{i=1}^q V_i P_i / \sum_{i=1}^q V_i, \quad (6.8)$$

where $q=4$ and the sum in the denominator is used for normalization. Yet in practical execution, the presence of noises may introduce false peaks (the largest four peaks’ indexes may not form a unit square), which has to be considered. Thus, we examine the indexes and values to exclude the false peaks before we perform a weighted interpolation. In other words, we allow a partial unit grid square for interpolation and allow $q \leq 4$ in Eq. (6.8). The overall sub-grid localization algorithm is then summarized as Algorithm III, which further enhances the positioning accuracy and anti-noise property of our system.

Algorithm III: Sub-grid Localization

- a. Pick the largest 4 elements’ indexes P1, P2, P3 and P4 and record their values V1, V2, V3 and V4;
- b. Check if P1, P2, P3 and P4 form a grid square:
 - If yes, go to step c;
 - If no, exclude false peaks’ indexes P_i and the corresponding

values V_i ($i=1, 2, 3, 4$).

c. Calculate PT's location by taking weighted average of the remaining vertexes:

$$P_t = \sum_{i=1}^q V_i P_i / \sum_{i=1}^q V_i, \quad q \leq 4;$$

6.4 System verification and performance evaluation

6.4.1 Simulation setup

In our simulations, the sensing matrix $\Psi_{M \times N}$ with elements $\Psi_{i,j} = \text{RSS}_{i,j}$ is generated following the Lambertian model; the simulated system noise is composed of shot noises and thermal noises on a PD as well as the measurement noises.

A. Visible light RSS model

Since the multipath effect of a visible light signal is much smaller compared with WiFi signals, our VLP system mainly considers the LoS condition without the multipath effect. The radiation of a light source follows common Lambertian radiation pattern given by [21, 119],

$$H(\theta) = \begin{cases} \frac{(m+1)A}{2\pi d^2} \cos^m(\phi) T_s(\theta) g(\theta) \cos(\theta), & 0 \leq \theta \leq \theta_c, \\ 0, & \theta > \theta_c \end{cases} \quad (6.9)$$

where m is the Lambertian order; A is the physical area of the PD; d is the distance between a transmitter and a receiver; ϕ is the angle of incidence and θ is the angle of irradiance; θ_c denotes the field of view of the PD (receiver); $T_s(\theta)$ and $g(\theta)$ are the gains of an optical filter and an optical concentrator, respectively. We usually let $C = T_s(\theta)g(\theta)$ as a constant. As shown in Fig.1, when the transmitter LEDs face up to the ceiling, the normal axes of a transmitter and a receiver are parallel, leading to $\theta = \phi$; $\cos(\theta) = \cos(\phi) = h/d$. Thus, Eq. (6.9) is simplified as:

$$H = \frac{(m+1)ACh^{m+1}}{2\pi d^{m+3}}, \quad 0 \leq \theta \leq \theta_c. \quad (6.10)$$

Further, the received light current is calculated as $I = \gamma P_t H$, where γ is the responsivity of the photodiodes; P_t denotes the transmit power of the LEDs.

B. Noise model

Both optical channel noise and measurement noise are considered in this study. The power of received optical signal noises P_N is the sum of thermal noise σ_{ther}^2 power, shot noise power σ_{shot}^2 and measurement noise power $\sigma_{measure}^2$ [21, 35],

$$\begin{aligned} P_N &= \sigma_{ther}^2 + \sigma_{shot}^2 + \sigma_{measure}^2 \\ &= (8\pi kT\eta A I_2 \beta^2 / G + 16\pi^2 kT\Gamma \eta^2 A^2 I_3 \beta^3 / g_m)^{1/2}, \\ &\quad + (2q\gamma P_{BS} A \lambda_{BW} \beta)^{1/2} + \sigma_{measure}^2 \end{aligned} \quad (6.11)$$

where σ_{ther}^2 and σ_{shot}^2 are also calculated according to [22], and the measurement noise is modeled as a Gaussian noise.

C. SNR tests in a typical indoor environment

MTs' LEDs are modulated with sinusoidal signals of different frequencies. At the receivers' side, each PD receives mixed sinusoidal signals from multiple MTs, which can be distinguished using multiple bandpass filters (note that digital filters can be applied to separate signals of different frequencies). The SNR corresponding to a certain MT can then be calculated. To determine the simulated SNR range, we experimentally test the SNR values of a received sinusoidal signal at various distances to an LED light source in a typical indoor environment, when the angle of incidence $\phi \approx 0$. The experiment settings are specified in Table I and the results are summarized in Table II. As can be seen from Table II, the SNR of the received sinusoidal signals varies from 10 dB to 30 dB using an avalanche photodiode (APD) as a receiver. Therefore, in our simulation, the SNR range of the received light signal is set to be 5-40 dB which covers the possible SNR conditions in a typical indoor environment.

Table 6.1. Experiment Specifications

Component/Parameter	Model/Value
LED	Cool White (5650K) LUXEON Rebel ES LED, 40mm Round 7-Up Base

Modulation Depth	0.4
Sinusoidal signal frequency	1 MHz
Avalanche Photo Diode (APD)	Hamamatsu C12702
Incidence angle	$\phi \approx 0^\circ$
APD sensing area	20 mm^2
FoV of LED optics	$\varphi \approx 120^\circ$

Table 6.2. SNR of received LED signal in a typical indoor environment

Distance Receiver	1.0 m	2.0m	3.0m	4.0 m	5.0 m
APD	29.1 dB	26.7 dB	22.1 dB	19.3 dB	12.7 dB

6.4.2 System verification and comparison of the l_1M and the RWl_1M algorithms

In this section, the effectiveness of the proposed system and its supporting algorithms are verified and analyzed through simulations. Besides, the impact of various parameters on positioning performance, e.g., SNR, number of employed PDs, size of grid squares, is also investigated.

(a)

Fig. 6.4. Positioning results using the l_1M and the RWl_1M algorithms: (a) SNR=30 dB, and (b) SNR=15 dB.

A. System verification

Simulations are performed in an area of 6m×6m, with a grid size of 0.2m×0.2m. The number of MTs is $K=20$. MTs' locations are randomly generated and are usually not on exact grid points. The positions of all MTs in the sensing area are estimated using both the l_1M and the RWl_1M algorithms. The positioning results are depicted in Fig. 6.4. As can be seen, the proposed RWl_1M algorithm can find all the MTs accurately regardless the MTs are on exact grid points or sub-grid locations in both settings of SNR=30 dB and SNR=15 dB, which demonstrates the effectiveness of the proposed reversed visible light multi-target localization system and supporting algorithms.

B. Comparisons of two algorithms under various SNR settings

By comparing Fig. 6.4(a) and (b), the proposed RWl_1M algorithm is found more robust to noises than the conventional l_1M algorithm. Specifically, when SNR=30 dB, both l_1M and RWl_1M algorithms localize all MTs accurately. However, when SNR drops to 15 dB, the l_1M algorithm misses many MTs, while the RWl_1M algorithm still achieves a good positioning accuracy, as shown in Fig. 6.4(b).

For quantitative comparison of two algorithms under varying SNR conditions, two parameters are further defined: i) average error (AE) is defined as the average Euclidean distance between the true positions of MTs and their corresponding estimated positions,

$$AE = \frac{1}{K} \sum_{k=1}^K \sqrt{(\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2} \quad ; \quad (6.12)$$

and ii) success rate (SR) is defined as the percentage of the number of MTs with positioning error $Pe \leq 0.2m$,

$$SR = \frac{N(Pe \leq 0.2)}{N_{trial}} \quad . \quad (6.13)$$

These two measurements are given in Fig. 6.5, where each result point takes an average of 500 runs to eliminate accidental errors.

As illustrated in Fig. 6.5, the proposed RWI₁M algorithm achieves a small AE of 7.4 cm and a high SR of 94.5 % by using 25 PDs when SNR= 20 dB. As expected, the AE drops and the SR increases for both algorithms as SNR increases from 5 dB to 40 dB. Specifically, in Fig. 6.5(a), the RWI₁M algorithm shows much better accuracy than the I₁M algorithm in a low-SNR region (5~15 dB) and the gap gets smaller as SNR increases and finally becomes almost the same when SNR $\geq$ 25 dB. Besides, the RWI₁M algorithm achieves a higher SR than the I₁M algorithm when SNR is relatively low and their SRs become comparable when SNR is over 30 dB, as observed in Fig. 6.5(b).

Further, when we have 20 PDs, the RWI₁M algorithm requires an SNR of 15 dB to achieve an SR $\geq$ 90%, while the I₁M algorithm requires a higher SNR of 25~30dB. Both figures demonstrate the superiority of the proposed RWI₁M algorithm over the conventional I₁M algorithm in low-SNR condition (the reason has been analyzed in section 3.2). This improvement in the low-SNR region (5~20 dB) is significant in practical use as the SNR of the received sinusoidal signals in a typical indoor environment generally falls into this region.

(a)

(b)

Fig. 6.5 Positioning performance of the I_M and the $RW I_M$ algorithms: (a) AE vs. SNR (b) SR vs. the SNR

6.4.3 Impact of system parameters

A. The impact of PD numbers

The impact of PD number on the positioning performance using RWLM algorithm is also examined. Since both number and deployment scheme of PDs are found to influence positioning performance, here, we randomly deploy PDs in each trail and take an average of 200 runs to eliminate the influence of PDs' deployment.

As illustrated in Fig. 6.6, when we have a small number of PDs ($N \leq 10$), the achievable accuracy and SR are not acceptable even in high SNR scenarios. Increasing the number of PDs will greatly boost the positioning performance but not infinitely. As depicted, when a PD number increases to around 20, the further increase of the PD number brings limited improvement on positioning accuracy and success rate. SNR also influences the required PD number to meet a certain performance standard. For example, when SNR is lower than 10 dB, the achievable AE and SR are poor even 30 PDs are utilized, but when SNR=30 dB, an AE of 0.13m and an SR of 91% can be achieved using only 15 PDs.

B. The impact of grid square size

An interesting trade-off is found in designing the size of the grid square. On one hand, a small grid size naturally limits the largest positioning error once the unit square where an MT locates is determined correctly; on the other hand, the dimension of LIM increases as a square size goes smaller, thus more measurements (PDs) are required to precisely reconstruct signals under the same SNR condition. This tradeoff is also verified by simulations as given in Fig. 6.7, where every point is an average of 200 runs with newly generated MTs' locations in each run.

As observed, at the very beginning, the positioning accuracy improves as the grid square size decreases. However, the dimension of a LIM will increase exponentially at the same time, therefore, the performance may downgrade severely once the dimension of LIM goes beyond the limit where a LIM can be successfully reconstructed. This tradeoff gives optimal square size for each setting of PD numbers and SNR. Fig. 5 and Fig. 6 not only reveal the relationship between PD number, SNR and grid square size with positioning performance, but also offer an insightful reference for a practical system design, based on which we can allocate resources reasonably according to the requirements and physical conditions.

Fig. 6.6. The impact of PD numbers and SNR on the RWI_M algorithm: (a) AE vs. PD numbers under different SNR settings, and (b) SR vs. PD numbers under different SNR settings.

Fig. 6.7. AE v.s. grid square size using different numbers of PDs when SNR=15 dB.

6.5 Conclusion

We develop a reversed VLP system whose structure is particularly suitable for multi-target tracking and monitoring. The multi-target localization problem is formulated as a sparse matrix reconstruction problem, and a 3-step workflow is proposed accordingly to solve it. The QR decomposition-based sensing matrix redesign simply and effectively enables CS theory. The RWl_1M algorithm enhances the LIM reconstruction accuracy and the robustness against noises of the conventional l_1M algorithm. The sub-grid localization algorithm breaks through a limitation of on-grid locations in the related works. These supporting algorithms work seamlessly to empower the practicability of the proposed system and facilitate its potential applications in robots/objects tracking and monitoring. We set up simulation parameters based on actual measurement data and the effectiveness of the proposed algorithms are validated through extensive tests. Our system demonstrates to localize all MTs accurately with a small AE of 7.4 cm by using 25 PDs when SNR= 20 dB. Further, the impact of various parameters including SNR, PD numbers and grid square size on the positioning performance is investigated. The obtained results offer an insightful reference for a practical system design. The impact of the PD

deployment on the system performance and the potential problems brought by hardware implementations will be studied and addressed in our future works.

Chapter 7 Conclusions and Future Work

7.1 Conclusions

This thesis theoretically and experimentally investigates four aspects of the advanced positioning techniques/algorithms for VLP systems, which are potential to provide highly accurate, fast and robust localization for indoor applications.

In Chapter 3, an SVD-based positioning algorithm is proposed to replace the conventional iterative LM-based algorithm in the IS-based VLP system. Extensive tests verify that the proposed positioning algorithm is 50-80 times faster than the conventional LM-based positioning algorithm. Also, it avoids the convergence failures caused by improper initial guesses in the iterative LM algorithm, leading to a better success rate. Besides, CRLB is derived as the theoretical lower bound of positioning error of an IS-based VLP system, based on which, simulations are performed to depict the impact of the various system parameters on the positioning accuracy bound. A 3D positioning errors of 6 cm ($H=120$ cm) and 8.5 cm ($H=160$ cm) with the maximal error around 10 cm and 14 cm, respectively, are experimentally demonstrated.

In Chapter 4, we investigate the approach to fuse the data from the image sensor and the motion sensors which are commonly embedded in mobile devices, to enhance the positioning accuracy of a conventional IS-based algorithm. The issue of sensor fusion is formulated as a multi-objective non-convex optimization problem. The optimal solution to the sensor fusion problem is derived using the SVD technique, and the SVD-SF algorithm is proposed accordingly. Before localization, a calibration process should be carried out to find the optimal balancing parameter to adapt to the current environment and devices. Experiment results show that, under the optimal balancing parameter value, the proposed SVD-SF algorithm reduces the positioning error from 9.7 cm to 4.4 cm which is equivalent to 44% accuracy improvement over the conventional IS-based algorithm using a single camera. Furthermore, simulations results reveal that the performance the SVD-SF algorithm varies under different noise conditions, but always outperforms the conventional positioning algorithm.

In Chapter 5, we propose a single LED based VLP system that exploits the property of circle projection. The proposed system solves two pressing problems simultaneously: multiple LEDs requirement and angular sensor dependence in the conventional image sensor based VLP systems. In our design, the geometric features of the LED image are fully exploited to determine the receiver's position and orientation relative to the reference LED lamp. A marginal point of the LED lamp is marked, which efficiently removes the information ambiguity during the calculation. A comprehensive algorithm is proposed accordingly. The performance of the proposed system is evaluated by experiments. An average positioning error of 17.52 cm is achieved in an area of $3\text{m} \times 3\text{m}$ when the LED lamp is deployed at the height of 2m.

In Chapter 6, we develop a reversed VLP system whose structure is particularly suitable for multi-target tracking and monitoring. The multi-target localization problem is formulated as a sparse matrix reconstruction problem, and a 3-step workflow is proposed accordingly to solve it. The QR decomposition-based sensing matrix redesign simply and effectively enables CS theory. The $\text{RW}/l_1\text{M}$ algorithm enhances the LIM reconstruction accuracy and the robustness against noises of the conventional $l_1\text{M}$ algorithm. The sub-grid localization algorithm breaks through a limitation of on-grid locations in the related works. These supporting algorithms work seamlessly to empower the practicability of the proposed system and facilitate its potential applications in robots/objects tracking and monitoring. We set up simulation parameters based on actual measurement data and the effectiveness of the proposed algorithms are validated through extensive tests. It is shown that our system can localize all MTs accurately with a small AE of 7.4 cm by using 25 PDs when $\text{SNR} = 20$ dB. Further, the impact of various parameters including SNR, PD numbers and grid square size on the positioning performance is investigated. The obtained results offer an insightful reference for a practical system design.

7.2 Future work

In this thesis, four advanced algorithms/techniques have been proposed and investigated for improving the performance of indoor VLP systems in terms of response speed, robustness, accuracy, stability, and scalability to large-scale scenario

with multiple indoor users. Besides the issues investigated in this thesis, the following topics are worth to be studied and explored in the future.

Either using IS or PDs, either using conventional IS-based algorithms or single LED based algorithm, a LoS condition is a common assumption during localization and navigation process. However, it is unlikely to be guaranteed for all users at every moment in practical indoor environment, especially in mobile state. The localization and navigation services interrupt immediately when VLP signals outage, which may severely influence the user's experience. Although, some approaches to reduce the outage possibility, e.g., single LED-based positioning, are proposed in some works, method to solve the outage problem has been provided so far. Therefore, it would be interesting and necessary to investigate the approach of predicting the trajectory or bridging the missing trajectory data using historical data and real-time measurements of embedded sensors during the VLP signal outage period.

An ideal VLP system should be scalable in terms of space and techniques. In Chapter 3, 4 and 5, the investigated IS-based VLP systems are only examined in a cell with a single user and a small group of LEDs. The large-scale indoor positioning scenario is simply regarded as the duplicates of the localization problem in a cell. However, considering the practical implementation of a large-scale VLP system supporting multiple users, many potential problems/questions are worth studying. For example, i) how to efficiently reuse the IDs or frequency labels, ii) how to deal with the degradation of positioning accuracy caused by the extended distance, iii) how to deal with inter-user or inter-cell interference and how to mitigate it, iv) how users in the near area cooperate to localize each other. These issues should be figured out before a VLP system can be applied practically in a large scenario. Besides, with the indoor map information and real-time location of objects/human, the route planning scheme considering different service features can also be explored.

Since the VLC and VLP systems share the same hardware infrastructure, the integration of these two technologies can be extremely interesting. However, due to the different features and requirements of VLC and VLP signals, we are facing some critical problems. For example, when camera is used as the receiver for VLCP, while the communication and positioning can be achieved by exploiting rolling shutter effect and projection model respectively, the communication speed is limited by the

line scan rate of the employed image sensor, which is far lower than the requirement of VLC. The way to improve the speed of camera-based VLC becomes an issue. Instead, when PDs are used as the receivers for VLCP, the problems are: i) how to allocate the power and frequency band to transmit VLC and VLP signals simultaneously, ii) what is the most proper modulation and demodulation technique to support high-speed VLC and accurate VLP, iii) how to mitigate the interference between VLC and VLP signals. These are the potential topics worthy exploring in the future.

Author's Publications

Journal

- [1] **R. Zhang**, W. D. Zhong, K. Qian, and D. Wu, "Image sensor based visible light positioning system with improved positioning algorithm," *IEEE Access*, vol. 5, pp. 6087–6094, Apr. 2017. (related to Chapter 3)
- [2] **R. Zhang**, W. D. Zhong, K. Qian, and S. Zhang, "A Single LED Positioning System Based on Circle Projection." *IEEE Photonics Journal*, vol. 9, pp. 1-9, Aug. 2017. (related to Chapter 5)
- [3] **R. Zhang**, W. D. Zhong, K. Qian, S. Zhang, D. Peng, "A Reversed Visible Light Multitarget Localization System via Sparse Matrix Reconstruction". *IEEE Internet of Things Journal*, vol. 5, pp. 4223-4230, Aug. 2018. (related to Chapter 6)

Conference

- [4] **R. Zhang**, W.-D. Zhong, D. Wu, and K. Qian, "A novel sensor fusion based indoor visible light positioning system," in *Globecom Workshops (GC Wkshps)*, IEEE, 2016, pp. 1–6. (related to Chapter 4)
- [5] **R. Zhang**, W.-D. Zhong, "Single LED based Indoor Visible Light Positioning System Using Smartphone Camera and Motion Sensors" *Proceedings of the 2017 International Conference on Wireless Communications, Networking and Applications*. ACM, 2017, pp. 220-224. (related to Chapter 5)

References

- [1] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Trans. Systems, Man, and Cybernetics, Part C: Applications and Reviews* vol. 37, pp. 1067-1080, 2007.
- [2] C. Yang and H.-R. Shao, "WiFi-based indoor positioning," *IEEE Commun. Mag.*, vol. 53, no. 3, pp. 150-157, 2015.
- [3] Y. Zhuang and N. El-Sheimy, "Tightly-coupled integration of WiFi and MEMS sensors on handheld devices for indoor pedestrian navigation," *IEEE Sensors J.*, vol. 16, no. 1, pp. 224-234, 2016.
- [4] Y. Zhuang, Z. Syed, Y. Li, and N. El-Sheimy, "Evaluation of two WiFi positioning systems based on autonomous crowdsourcing of handheld devices for indoor navigation," *IEEE Trans. Mobile Comput.*, vol. 15, no. 8, pp. 1982-1995, Aug. 2016.
- [5] P. Bahl and V. N. Padmanabhan, "RADAR: An in-building RF-based user location and tracking system," *Proc. 19th Annu. Joint Conf. IEEE Comput. (INFOCOM)*, Tel Aviv, Israel, vol. 2., pp. 775-784, 2000.
- [6] J. Nieminen, C. Gomez, M. Isomaki, T. Savolainen, B. Patil, Z. Shelby, "Networking solutions for connecting bluetooth low energy enabled machines to the internet of things," *IEEE Netw.*, vol. 28, pp. 83-90, 2014.
- [7] A. K. M. M. Hossain and S. Wee-Seng, "A comprehensive study of Bluetooth signal parameters for localization," in *Proc. IEEE 18th Int. Symp. Pers. Indoor Mobile Radio Commun. (PIMRC)*, Athens, Greece, pp. 1-5, 2007.
- [8] Y. Zhuang, J. Yang, Y. Li, L. Qi, and N. El-Sheimy, "Smartphonebased indoor localization with Bluetooth low energy beacons," *Sensors*, vol. 16, no. 5, 2016.
- [9] H. Zou, L. Xie, Q.-S. Jia, and H. Wang, "Platform and algorithm development for a rfid-based indoor positioning system," *Unmanned Systems*, vol. 2, pp. 279-291, 2014.
- [10] A. R. J. Ruiz, F. S. Granja, J. C. P. Honorato, and J. I. G. Rosas, "Accurate pedestrian indoor navigation by tightly coupling footmounted IMU and RFID measurements," *IEEE Trans. Instrum. Meas.*, vol. 61, no. 1, pp. 178-189, Jan. 2012.

- [11] Y. Po and W. Wu, "Efficient particle filter localization algorithm in dense passive RFID tag environment," *IEEE Trans. Ind. Electron.*, vol. 61, no. 10, pp. 5641–5651, 2014.
- [12] Guan R, Harle R., "Towards a crowdsourced radio map for indoor positioning system" *IEEE International Conference on Pervasive Computing and Communications Workshops (PerCom Workshops)*, pp. 207-212, 2017.
- [13] K. Kaemarungsi, "Efficient design of indoor positioning systems based on location fingerprinting," in *2005 International Conference on Wireless Networks, Communications and Mobile Computing*, pp. 181-186, 2005.
- [14] C. Zhang, M. J. Kuhn, B. C. Merkl, A. E. Fathy, and M. R. Mahfouz, "Real-time noncoherent UWB positioning radar with millimeter range accuracy: Theory and experiment," *IEEE Trans. Microw. Theory Techn.*, vol. 58, pp. 9-20, 2010.
- [15] P. Cheong, A. Rabbachin, J.-P. Montillet, K. Yu, and I. Oppermann, "Synchronization, TOA and position estimation for low-complexity LDR UWB devices," in *IEEE International Conference on Ultra-Wideband (ICU)*, pp. 480–484, 2005.
- [16] J. Armstrong, Y. Sekercioglu, and A. Neild, "Visible light positioning: A roadmap for international standardization," *IEEE Commun. Mag.*, vol. 51, pp. 68-73, 2013.
- [17] Y. Zhuang, L. Hua, L. Qi, J. Yang, P. Cao, Cao Y., H. Haas, "A survey of positioning systems using visible led lights" *IEEE Communications Surveys & Tutorials*, vol. 20, no. 3, pp. 1963-1988, 2018.
- [18] H. Haas, L. Yin, Y. Wang, and C. Chen, "What is LiFi?" *J. Lightw. Technol.*, vol. 34, no. 6, pp. 1533–1544, Mar. 15, 2016.
- [19] G. Cossu, A. M. Khalid, P. Choudhury, R. Corsini, and E. Ciaramella, "3.4 Gbit/s visible optical wireless transmission based on RGB LED," *Opt. Exp.*, vol. 20, no. 26, pp. B501–B506, 2012.
- [20] A. Sevincer, A. Bhattarai, M. Bilgi, M. Yuksel, and N. Pala, "LIGHTNETs: Smart lighting and mobile optical wireless networks— A survey," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 4, pp. 1620–1641, 4th Quart., 2013.
- [21] T. Komine and M. Nakagawa, "Fundamental analysis for visible-light communication system using LED lights," *IEEE Trans. Consum. Electron.*, vol. 50, no. 1, pp. 100–107, 2004.

- [22] L. Zeng, D. O'Brien, H. L. Minh, G. E. Faulkner, K. Lee, D. Jung, Y. Oh, and E. Won, "High data rate multiple input multiple output (MIMO) optical wireless communications using white LED lighting," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 9, pp. 1654–1662, 2009.
- [23] A. Vavoulas, H. Sandalidis, T. Tsiftsis, and N. Vaiopoulos, "Coverage aspects of indoor VLC networks," *J. Lightw. Technol.*, vol. 33, no. 23, pp. 4915–4921, 2015.
- [24] C. Tsai and Z.-Fan Xu, "Line-of-sight visible light communications with InGaN-based resonant cavity LEDs," *IEEE Photon. Technol. Lett.*, vol. 25, no. 18, pp. 1793–1796, 2013.
- [25] I. Stefan and H. Haas, "Analysis of optimal placement of LED arrays for visible light communication," in *IEEE Veh. Technol. Conf. (VTC)*, pp. 1–5, 2013.
- [26] Z. Wang, C. Yu, W.-D Zhong, J. Chen, and W. Chen, "Performance of a novel LED lamp arrangement to reduce SNR fluctuation for multi-user visible light communication systems," *Opt. Exp.*, vol. 20, no. 4, pp. 4564–4573, 2012.
- [27] F. R. Gfeller and U. Bapst, "Wireless in-house data communication via diffuse infrared radiation," *Proc. IEEE*, vol. 67, no. 11, pp. 1474–1486, 1979.
- [28] Y.-S. Kuo, P. Pannuto, K.-J. Hsiao, and P. Dutta, "Luxapose: Indoor positioning with mobile phones and visible light," in *Proceedings of the 20th annual international conference on Mobile computing and networking*, pp. 447–458, 2014.
- [29] C. Danakis, M. Afgani, G. Povey, I. Underwood, and H. Haas, "Using a CMOS camera sensor for visible light communication," in *Globecom Workshops (GC Wkshps), 2012 IEEE*, pp. 1244–1248, 2012.
- [30] M. Liu, K. Qiu, F. Che, S. Li, B. Hussain, L. Wu, "Towards indoor localization using Visible Light Communication for consumer electronic devices," in *Intelligent Robots and Systems (IROS 2014), 2014 IEEE/RSJ International Conference on*, pp. 143–148, 2014.
- [31] M. Yasir, S.-W. Ho, and B. N. Vellambi, "Indoor positioning system using visible light and accelerometer," *J. Lightw. Technol.*, vol. 32, pp. 3306–3316, 2014.

- [32] H.-S. Kim, D.-R. Kim, S.-H. Yang, Y.-H. Son, and S.-K. Han, "An indoor visible light communication positioning system using a RF carrier allocation technique," *J. Lightw. Technol.*, vol. 31, pp. 134-144, 2013.
- [33] S.-H. Yang, H.-S. Kim, Y.-H. Son, and S.-K. Han, "Three-Dimensional Visible Light Indoor Localization Using AOA and RSS With Multiple Optical Receivers," *J. Lightw. Technol.*, vol. 32, pp. 2480-2485, 2014.
- [34] Lv H, Feng L, Yang A, "High accuracy VLC indoor positioning system with differential detection". *IEEE Photon. J.*, vol. 9, pp. 1-13, 2018
- [35] S.-Y. Jung, S. Hann, and C.-S. Park, "TDOA-based optical wireless indoor localization using LED ceiling lamps," *IEEE Trans. Consum. Electron.*, vol. 57, no. 4, pp. 1592–1597, Nov. 2011.
- [36] T.-H. Do and M. Yoo, "TDOA-based indoor positioning using visible light," *Photonic Network Communications*, vol. 27, pp. 80-88, 2014.
- [37] P. Cheong, A. Rabbachin, J.-P. Montillet, K. Yu, and I. Oppermann, "Synchronization, TOA and position estimation for low-complexity LDR UWB devices," in *IEEE International Conference on Ultra-Wideband (ICU)*, 2005, pp. 480–484.
- [38] K. Panta and J. Armstrong, "Indoor localisation using white LEDs," *Electronics letters*, vol. 48, pp. 228-230, 2012.
- [39] P. Du, C. Chen, and W. Zhong, "Demonstration of a low-complexity indoor visible light positioning system using an enhanced TDOA scheme," *IEEE Photon. J.*, vol. 10, no. 4, 2018.
- [40] S. Zhang, W. Zhong, P. Du, C. Chen, "Experimental Demonstration of Indoor Sub-Decimeter Accuracy VLP System Using Differential PDOA", *IEEE Photon. Technol. Lett.*, vol. 30, no. 19, pp. 1703-1706, 2018.
- [41] Hassan, N. U., Naeem, A., Pasha, M. A., Jadoon, T., & Yuen, "Indoor Positioning Using Visible LED Lights: A Survey". *ACM Computing Surveys*, vol. 48, no. 2, pp.1-32, 2018.
- [42] C. Xie, .W. Guan, Y. Wu, L. Fang, Y. Cai, "The LED-ID Detection and Recognition Method Based on Visible Light Positioning Using Proximity Method," *IEEE Photon. J.*, vol. 10, no. 2, pp.1-16, 2018.

- [43] J. Vongkulbhisal, B. Chantaramolee, Y. Zhao, and W. S. Mohammed, "A fingerprinting-based indoor localization system using intensity modulation of light emitting diodes," *Microw. Opt. Technol. Lett.*, vol. 54, no. 5, pp. 1218–1227, 2012.
- [44] A. M. Vegni and M. Biagi, "An indoor localization algorithm in a smallcell LED-based lighting system," in *Proc. Int. Conf. Indoor Position. Indoor Navig. (IPIN)*, Sydney, NSW, Australia, 2012, pp. 1–7.
- [45] X. Guo, S. Shao, N. Ansari, and A. Khreishah, "Indoor localization using visible light via fusion of multiple classifiers," *IEEE Photon. J.* vol. 9, no. 6, pp. 1-16, 2017.
- [46] Z. Yuxin et al., "Particle filtering for positioning based on proximity reports," in *Proc. 18th Int. Conf. Inf. Fusion (Fusion)*, 2015, pp. 1046–1052.
- [47] Y. Zhao, Yuxin Zhao, and Fredrik Gunnarsson, "Proximity report triggering threshold optimization for network-based indoor positioning," in *18th International Conference on Information Fusion*. pp. 1061-1069, 2015.
- [48] Paek J, Kim K H, and Singh J P, "Energy-efficient positioning for smartphones using cell-id sequence matching" *Proceedings of the 9th international conference on Mobile systems, applications, and services. ACM*, pp. 293-306. 2011
- [49] R. Zhang, W. D. Zhong, K. Qian, and S. Zhang, "A Single LED Positioning System Based on Circle Projection." *IEEE Photon. J.*, vol. 9, pp. 1-9, Aug. 2017.
- [50] R. Zhang, W. D. Zhong, K. Qian, and D. Wu, "Image sensor based visible light positioning system with improved positioning algorithm," *IEEE Access*, vol. 5, pp. 6087–6094, Apr. 2017.
- [51] R. Zhang, W.-D. Zhong, D. Wu, and K. Qian, "A novel sensor fusion based indoor visible light positioning system," in *Globecom Workshops (GC Wkshps) IEEE*, pp. 1–6, 2016.
- [52] D. Ganti, W. Zhang, and M. Kavehrad, "VLC-based indoor positioning system with tracking capability using Kalman and particle filters," in *Proc. IEEE Int. Conf. Consum. Electron. (ICCE)*, Las Vegas, NV, USA, pp. 476–477, 2014.
- [53] A. Taparugssanagorn, S. Siwamogsatham, and C. Pomalaza-Raez, "A hexagonal coverage LED-ID indoor positioning based on TDOA with extended Kalman filter," in

Proc. IEEE 37th Annu. Comput. Softw. Appl. Conf. (COMPSAC), pp. 742–747, Kyoto, Japan, 2013.

- [54] Won S P, Melek W W, Golnaraghi F. “A Kalman/particle filter-based position and orientation estimation method using a position sensor/inertial measurement unit hybrid system”. *IEEE Trans. Ind. Electron.*, vol.57. pp. 1787-1798, 2010.
- [55] Prince, Gregory B., and Thomas DC Little. "A two phase hybrid RSS/AoA algorithm for indoor device localization using visible light." *Globecom*. pp. 3347-3352, 2012.
- [56] Şahin, Alphan, et al. "Hybrid 3-D localization for visible light communication systems." *J. Lightw. Technol.*, vol. 33. pp. 4589-4599, 2015.
- [57] Lee, Yong Up, and Mohsen Kavehrad. "Two hybrid positioning system design techniques with lighting LEDs and ad-hoc wireless network." *IEEE Trans. Consum. Electron.*, vol. 58. 2012.
- [58] Lee, Y. U., Baang, S., Park, J., Zhou, Z., Kavehrad, M, "Hybrid positioning with lighting LEDs and Zigbee multihop wireless network." *Broadband Access Communication Technologies VI*. Vol. 8282, *International Society for Optics and Photonics*, 2012.
- [59] Gu, W., Mohammadreza A. Kashani, and Kavehrad.M, "Multipath reflections analysis on indoor visible light positioning system." *arXiv preprint arXiv:1504.01192*, 2015.
- [60] Gu, W., Aminikashani, M., Deng, P., Kavehrad, M., "Impact of multipath reflections on the performance of indoor visible light positioning systems." *J. Lightw. Technol.*, vol. 34. pp. 2578-2587, 2016.
- [61] Huang, Heqing, Feng, L., Guo, P., Yang, A., & Ni, G, "Iterative positioning algorithm to reduce the impact of diffuse reflection on an indoor visible light positioning system." *Opt. Eng.*, vol. 55. pp.066117. 2016
- [62] Li, Z., Yang, A., Lv, H., Feng, L., & Song, W, “Fusion of visible light indoor positioning and inertial navigation based on particle filter,” *IEEE Photon. J.*, vol. 9. no.5. pp.1-13, 2017.
- [63] Z. Vatansever and M. Brandt-Pearce, “Visible light positioning with diffusing lamps using an extended Kalman filter,” in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2017, pp. 1–6.

- [64] Gu W, Zhang W, Kavehrad M, “Three-dimensional light positioning algorithm with filtering techniques for indoor environments,” *Opt. Eng.*, vol. 53, no.10, 2014.
- [65] Gu W, Zhang W, Wang J, “Three-dimensional indoor positioning based on visible light with Gaussian mixture sigma-point particle filter technique”. *Broadband Access Communication Technologies IX. International Society for Optics and Photonics*, vol. 9387, p. 93870O, 2015.
- [66] Z. Zhou, M. Kavehrad, and P. Deng, “Indoor positioning algorithm using light-emitting diode visible light communications,” *Opt. Eng.*, vol. 51, no. 8, pp. 1–6, 2012.
- [67] L. Li, P. Hu, C. Peng, G. Shen, and F. Zhao, “Epsilon: A visible light-based positioning system,” in *Proc. 11th USENIX Symp. Netw. Syst. Design Implement. (NSDI)*, Seattle, WA, USA, 2014, pp. 331–343.
- [68] Zhang R, Zhong W D, Qian K, Zhang S. “A Reversed Visible Light Multitarget Localization System via Sparse Matrix Reconstruction,” *IEEE Internet Things J.*, vol. 5, no. 5, pp. 4223-4230, 2018.
- [69] Lv H, Feng L, Yang A, et al. “High accuracy VLC indoor positioning system with differential detection,” *IEEE Photon. J.*, vol.9, no. 3, pp.1-13, 2017.
- [70] Zhang, Sheng, et al. "Experimental demonstration of indoor sub-decimeter accuracy VLP system using differential PDOA," *IEEE Photon. Technol. Lett.*, vol. 30, no.19 pp. 1703-1706, 2018.
- [71] Hsu, C. W., Liu, S., Lu, F., Chow, C. W., Yeh, C. H., & Chang, G. K, “Accurate Indoor Visible Light Positioning System utilizing Machine Learning Technique with Height Tolerance,” *IEEE Optical Fiber Communications Conference and Exposition (OFC)*, pp. 1-3, 2018.
- [72] Yu X, Wang J, Lu H., “Single LED-Based Indoor Positioning System Using Multiple Photodetectors,” *IEEE Photon. J.*, vol.10, no.6., pp.1-8, 2018.
- [73] Naz A, Hassan N U, Pasha M A, “Single LED ceiling lamp based indoor positioning system” *Internet of Things (WF-IoT), IEEE 4th World Forum on. IEEE*, pp. 682-687, 2018.

- [74] Liu S, He T., "SmartLight: Light-weight 3D Indoor Localization Using a Single LED Lamp", *Proceedings of the 15th ACM Conference on Embedded Network Sensor Systems*, ACM, vol. 11, 2017
- [75] Y.-S. Kuo, P. Pannuto, K.-J. Hsiao, and P. Dutta, "Luxapose: Indoor positioning with mobile phones and visible light," in *Proceedings of the 20th annual international conference on Mobile computing and networking*, pp. 447-458, 2014.
- [76] J. Lim, "Ubiquitous 3D positioning systems by led-based visible light communications," *IEEE Trans. Wireless Commun.*, vol. 22, pp. 80-85, 2015.
- [77] Y. Nakazawa, H. Makino, K. Nishimori, D. Wakatsuki, and H. Komagata, "Indoor positioning using a high-speed, fish-eye lens-equipped camera in Visible Light Communication," in *Indoor Positioning and Indoor Navigation (IPIN), 2013 International Conference on*, 2013, pp. 1-8.
- [78] Y. Nakazawa, H. Makino, K. Nishimori, D. Wakatsuki, and H. Komagata, "LED-Tracking and ID-Estimation for Indoor Positioning using Visible Light Communication," in *International Conference on Indoor Positioning and Indoor Navigation*, 2014, p. 30th.
- [79] M. Yoshino, S. Haruyama, and M. Nakagawa, "High-accuracy positioning system using visible LED lights and image sensor," in *IEEE Radio and Wireless Symposium*, pp. 439-442, 2008.
- [80] T.Wang,Y. Sekercioglu, A. Neild, and J. Armstrong, "Position accuracy of time-of-arrival based ranging using visible light with application in indoor localization systems," *J. Lightw. Technol.*, vol. 31, no. 20, pp. 3302–3308, 2013.
- [81] X. Zhang, J. Duan, Y. Fu, and A. Shi, "Theoretical Accuracy Analysis of Indoor Visible Light Communication Positioning System Based on Received Signal Strength Indicator," *J. Lightw. Technol.*, vol. 32, pp. 3578-3584, 2014.
- [82] M. F. Keskin and S. Gezici, "Comparative Theoretical Analysis of Distance Estimation in Visible Light Positioning Systems," *J. Lightw. Technol.*, vol. 34, pp. 854-865, 2016.
- [83] H. Steendam, T. Q. Wang, and J. Armstrong, "Theoretical lower bound for indoor visible light positioning using received signal strength measurements and an aperture-based receiver," *J. Lightw. Technol.*, vol. 35, no. 2, pp.309–319, 2017.

- [84] Lin B, Tang X, Ghassemlooy Z, “Experimental demonstration of an indoor VLC positioning system based on OFDMA”. *IEEE Photonics J.*, vol.9, no.2, pp. 1-9, 2017.
- [85] M. Aminikashani, W. Gu, and M. Kavehrad, “Indoor positioning with OFDM visible light communications,” *Proc. IEEE Consum. Commun. Netw. Conf. (CCNC)*, pp. 505–510, 2016.
- [86] Y. Xu, Wang, Z., Liu, P., Chen, J., Han, S., Yu, C., & Yu, “Accuracy analysis and improvement of visible light positioning based on VLC system using orthogonal frequency division multiple access,” *Opt. Express*, vol. 25, no. 26, pp. 32618–32630, 2017.
- [87] Helin Yang, Chen Chen, Wen-De Zhong, Arokiaswami Alphones, Sheng Zhang, Pengfei Du, "Demonstration of a Quasi-Gapless Integrated Visible Light Communication and Positioning System", *IEEE Photon. Technol. Lett.*, vol. 30, no. 23, pp. 2001-2004, 2018.
- [88] Acuity Brands. (2015). *ByteLight™ Services: Indoor Positioning System*. [Online] Available: <http://www.acuitybrands.com/solutions/services/bytelight-services-indoor-positioning>.
- [89] Philips. 2015, Carrefour in France Installs Philips LEDBased Indoor Positioning System. [Online]. Available: [http:// www.lighting.philips.com/main/cases/cases/food-and-large-retailers/ carrefour-lille.html](http://www.lighting.philips.com/main/cases/cases/food-and-large-retailers/carrefour-lille.html)
- [90] X. Zhao and J. Lin, "Theoretical Limits Analysis of Indoor Positioning System Using Visible Light and Image Sensor," *ETRI Journal*, vol. 38, pp. 560-567, 2016.
- [91] Wei L, Zhang H, Yu B, “High-accuracy indoor positioning system based on visible light communication”. *Opt. Engineering*, vol. 54, no. 11, pp.110501, 2015
- [92] K. Levenberg, “A method for the solution of certain non-linear problems in least squares,” *Quarterly of applied mathematics*, vol. 2, no. 2, pp.164–168, 1944.
- [93] Z. Zhang, "A flexible new technique for camera calibration," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 22, pp. 1330-1334, 2000.
- [94] K. S. Arun, T. S. Huang, and S. D. Blostein, “Least-squares fitting of two 3-d point sets,” *IEEE Trans. Pattern Anal. Mach. Intell.*, no. 5, pp. 698–700, 1987.
- [95] G. G. Slabaugh, “Computing euler angles from a rotation matrix,” *Retrieved on August*, vol. 6, no. 2000, pp. 39–63, 1999.

- [96] Y. Hou, M. Bi, and Y. Xue, "A Novel Angle-of-Arrival Based Position Estimation Scheme Using Visible Light in Indoor Positioning Systems," in *Asia Communications and Photonics Conference*, poster session, 184, 2014.
- [97] T. Yamazato, M. Kinoshita, S. Arai, E. Souke, T. Yendo, T. Fujii, *et al.*, "Vehicle Motion and Pixel Illumination Modeling for Image Sensor Based Visible Light Communication," *IEEE J. Sel. Areas Commun.*, vol. 33, pp. 1793-1805, 2015.
- [98] M.-g. Moon, S.-i. Choi, J. Park, and J. Y. Kim, "Indoor Positioning System using LED Lights and a Dual Image Sensor," *Journal of the Optical Society of Korea*, vol. 19, pp. 586-591, 2016.
- [99] B.-S. Choi, J.-W. Lee, J.-J. Lee, and K.-T. Park, "A hierarchical algorithm for indoor mobile robot localization using RFID sensor fusion," *IEEE Trans. Ind. Electron.*, vol. 58, pp. 2226-2235, 2011.
- [100] R. Martí, M. G. Resende, and C. C. Ribeiro, "Multi-start methods for combinatorial optimization," *European Journal of Operational Research*, vol. 226, pp. 1-8, 2013.
- [101] L. C. Kinsey and T. E. Moore, "Symmetry, Shape, and Surfaces: An Introduction to Mathematics Through Geometry", *Springer Science & Business Media*, 2002.
- [102] M. Holmes, A. Gray, and C. Isbell, "Fast SVD for large-scale matrices," in *Workshop on Efficient Machine Learning at NIPS*, pp. 249-252, 2007.
- [103] Y. Hou, S. Xiao, M. Bi, Y. Xue, W. Pan, and W. Hu, "Single LED Beacon-based Three-Dimension Indoor Positioning Using Off-the-shelf Devices," *IEEE Photonics J.*, vol. 8, no. 6, pp. 181-194, 2016.
- [104] H. Huang, L. Feng, G. Ni, and A. Yang, "Indoor imaging visible light positioning with sampled sparse light source and mobile device," *Chinese Optics Letters*, vol. 14, no. 9, p. 090602, 2016.
- [105] U. Shala and A. Rodriguez, "Indoor Positioning using Sensor-fusion in Android Devices," Thesis, Kristianstad University, Sweden, 2011.
- [106] F. Li, C. Zhao, G. Ding, J. Gong, C. Liu, and F. Zhao, "A reliable and accurate indoor localization method using phone inertial sensors," in *Proceedings of the 2012 ACM Conference on Ubiquitous Computing*. ACM, pp. 421-430, 2012.

- [107] R. Safaee-Rad, I. Tchoukanov, K. C. Smith, and B. Benhabib, "Three dimensional location estimation of circular features for machine vision," *IEEE Trans. Robot. Autom.*, vol. 8, no. 5, pp. 624–640, 1992.
- [108] A. Fitzgibbon, M. Pilu, and R. B. Fisher, "Direct least square fitting of ellipses," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 21, no. 5, pp. 476–480, 1999.
- [109] Z. Zhang, "Weak perspective projection," in *Computer Vision: A Reference Guide*. New York, NY, USA: Springer, pp. 877–883, 2014.
- [110] L.-F. Li, Z.-R. Feng, and Q.-K. Peng, "Detection and model analysis of circular feature for robot vision," in *Proc. 2004 Int. Conf. Mach. Learn. Cybern.*, vol. 6, pp. 3943–3948, 2004.
- [111] Shang Ma, Qiong Liu, and Phillip C.-Y. Sheu, "Foglight: Visible Light-enabled Indoor Localization System for Low-power IoT Devices," *IEEE Internet of Things J.*, vol.5, no. 1, pp. 175-185, 2017.
- [112] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Sep. 2006.
- [113] Gubbi, J., Buyya, R., Marusic, S., & Palaniswami, M. (2013). "Internet of Things (IoT): A vision, architectural elements, and future directions," *Future generation computer systems*, vol. 29, pp. 1645- 1660, 2013.
- [114] B. Zhang, X. Cheng, N. Zhang, Y. Cui, Y. Li, and Q. Liang, "Sparse Target Counting and Localization in Sensor Networks Based on Compressive Sensing," *Proc. IEEE INFOCOM*, pp. 2255-2263, Apr. 2011.
- [115] C. Feng, S. Valaee, and Z.H. Tan, "Multiple Target Localization Using Compressive Sensing," *Proc. IEEE GlobeCom*, pp. 1-6, Dec. 2009.
- [116] C. Feng, W.S.A. Au, S. Valaee, and Z. Tan, "Compressive Sensing Based Positioning Using RSS of WLAN Access Points," *Proc. IEEE INFOCOM*, pp. 1-9, Mar. 2010.
- [117] C. Feng, W. S. A. Au, S. Valaee, and Z. Tan, "Received-signal-strength based indoor positioning using compressive sensing," *IEEE Trans. Mobile Computing*, vol. 11, no. 12, pp. 1983–1993, Dec. 2012
- [118] A. Goldsmith, "Wireless Communications," first ed. Cambridge Univ., 2005.

- [119] J. R. Barry, “Wireless Infrared Communications,” *Kluwer Academic Press*, Boston, MA, 1994.
- [120] E. Cands and M. Wakin, “An introduction to compressive sampling,” *IEEE Signal Process. Mag.*, vol. 25, no. 2, pp. 21–30, 2008.
- [121] E. Candes, M. Wakin, and S. Boyd, “Enhancing sparsity by reweighted ℓ_1 -minimization,” *J. Fourier Anal. Appl.*, vol. 14, no. 5–6, pp. 877–905, 2008.
- [122] Bernstein and Dennis S., “Matrix mathematics: Theory, facts, and formulas with application to linear systems theory,” Princeton: Princeton University Press, vol. 41, 2005.
- [123] Candès E and Tao T., “Near-optimal signal recovery from random projections: universal encoding strategies,” *IEEE Trans. Information Theory*, vol. 52, no. 12, pp. 5406-5425, 2006.
- [124] Cosnard, Michel, J-M. Muller, and Yves Robert. "Parallel QR decomposition of a rectangular matrix." *Numerische Mathematik*, vol. 48, pp. 239 -249, Mar. 1996.
- [125] S.S. Chen and D.L. Donoho, “Atomic Decomposition by Basis Pursuit,” *SIAM J. Scientific Computing*, vol. 20, no. 1, pp. 33-61, Aug. 1998.
- [126] R. Zhang, W. D. Zhong, K. Qian, S. Zhang, D. Peng, “A Reversed Visible Light Multitarget Localization System via Sparse Matrix Reconstruction”. *IEEE Internet of Things J.*, vol. 5, pp. 4223-4230, Aug. 2018.