Grouting prediction based on analytical and numerical approaches

Xiao, Fei

2019

Xiao, F. (2019). Grouting prediction based on analytical and numerical approaches.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/102664>

<https://doi.org/10.32657/10220/47808>

**GROUTING PREDICTION BASED ON
ANALYTICAL AND NUMERICAL
APPROACHES**

**XIAO FEI
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING**

2019

GROUTING PREDICTION BASED ON ANALYTICAL AND NUMERICAL APPROACHES

XIAO FEI

School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

8 FEB,2019

.....

Date

XIAO FEI

.....

XIAO FEI

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

14 Feb 2019

.....
Date

.....
ZHAO Zhiye

Authorship Attribution Statement

This thesis contains material from 8 paper(s) published in the following peer-reviewed journal(s) where I was the first and/or corresponding author.

Chapter 3 is published as

- [1] F Xiao, Z Zhao, H Chen. A simplified model for predicting grout flow in fracture channels [J]. Tunnelling and Underground Space Technology, 2017, 70: 11-18.
- [2] F. Xiao, and Z. Y. Zhao. Grout Flow in Fracture Channel Considering Fracture Deformation. 51st US Rock Mechanics/Geomechanics Symposium. American Rock Mechanics Association, 25-28 June 2017.

The contributions of the co-authors are as follows:

- For both papers ([1] and [2]), Associate Professor Z Zhao provided valuable ideas and suggestions during the problem solving process and revised the manuscript.
- I derived all the equations and developed the algorithm for equation solving, then prepared the manuscript.
- Dr H Chen gave me some directions during discussion with her (paper [1]).

Chapter 4 is published as

- [3] Xiao Fei, Zhiye Zhao, Influence of fracture deformation on grout penetrability in fractured rock Masses, CouFrac 2018, 12-14 November 2018, Wuhan.
- [4] Fei Xiao, Zhiye Zhao, Hezhen Yang. Interaction between Grout Penetration and Fracture Deformation, ISOPE 2018 Conference, 11-15 June 2018, Sapporo, Japan.
- [5] Yan-xu Guo; Fei Xiao; Qingsong Zhang. Grouting Rock Fractures with Flowing Water, European Journal of Environmental and Civil Engineering. (Under review)

The contributions of the co-authors are as follows:

- In the first two papers ([3] and [4]), Associate Professor Zhiye Zhao helped me to form the paper structure, the direction for theoretical validation, and revised the paper.
- For all three papers, I was responsible for theoretical derivation and validation, and manuscript preparation.
- Yan-xu Guo and Professor Qingsong Zhang conducted all the laboratory testing and provided data for theoretical validation and explanation (paper [5]).
- Assistant Professor Hezhen Yang revised the manuscript and provided suggestions for algorithm development (paper [4]).

Chapter 5 is published as

- [6] Fei XIAO, Junlong SHANG, Zhiye ZHAO. Linkage between Fracture Aperture Distribution and Grouting Characteristics. Computers and Geotechnics. (Under revision)

The contributions of the co-authors are as follows:

- Associate Professor Zhiye ZHAO provided the initial direction for the problem to be solved and function of the algorithm and revised the manuscript.
- Dr Junlong SHANG provided valuable suggestions for the focus and structure the manuscript and revised the manuscript.
- I was responsible for developing theoretical framework and algorithm, data analysis, and preparing manuscript.

Chapter 6 is published as

- [7] X Wang, F Xiao, Q Zhang. Development of a new grouting test system for rough fracture, Science China Technological Sciences. (Submitted)
- [8] F Xiao, Z Zhao. Evaluation of Equivalent Hydraulic Aperture (EHA) for Rough Rock Fractures, Canadian Geotechnical Journal. (Published on line)

The contributions of the co-authors are as follows:

- X Wang involved in development of experimental equipment, grouting laboratory testing, and manuscript preparation (paper [7]).
- Professor Q Zhang provided guidance and suggestions for the design of laboratory testing and financial support (paper [7]).
- Associate Professor Z Zhao provided precious suggestion for paper structuring and revised the manuscript (paper [8]).
- I designed the experimental equipment and conducted some of the laboratory testing and drafted the original manuscript for paper [7]; developed numerical models for investigating the influence of fracture roughness on fluid transportation in rough rock fractures and prepared the manuscript (paper [8]).

8 FEB,2019

.....

Date

XIAO FEI

.....

XIAO FEI

ACKNOWLEDGEMENT

First, I would like to express my sincere gratitude to my supervisor, A/P Zhao Zhiye, for his confidence in me, allowing me to join his group and conduct research in grouting simulation at NTU; and for various support from him to facilitate my research; for setting an excellent example to me.

Meanwhile, many thanks to Prof. Zhang Qingsong at Shandong University for his academic and financial support. Prof Zhang generously funded me to realize my idea and conduct experiments in his laboratory.

Appreciation should also be given to my seniors Chen Huimei and Mao Dawei, as discussions with them can always help me to figure out solutions to many problems, give me new ideas for my research work, and force me to learn more and go further. Many thanks to another two seniors Liu Xiaoying and Choo Ling Qian. Sometimes, casual conversation with them about some small problems can always inspire me and help me solve some intractable problems immediately.

I should deeply appreciate my parents Xiao Yiping and Lai Chuanxiu, who have not received any high level of education but managed to support me both spiritually and financially all the way.

Finally, I have received a lot of support from Kang Xiaoxi, and learnt quite a lot from her, which will benefit me for my future research and life.

TABLE OF CONTENTS

ACKNOWLEDGEMENT	I
TABLE OF CONTENTS.....	III
SUMMARY	VII
LIST OF TABLES	IX
LIST OF FIGURES	X
LIST OF SYMBOLS	XV
CHAPTER 1 INTRODUCTION	1
1.1 Background	1
1.2 Research objectives and scopes	2
1.3 Thesis organization	3
CHAPTER 2 FRACTURED ROCK MASSES AND GROUTING.....	4
2.1 Introduction.....	4
2.2 Characteristics of fractured rock masses.....	4
2.2.1 Fracture network	5
2.2.2 Fracture surface	6
2.2.3 Fracture aperture	7
2.2.4 Fracture deformation.....	8
2.2.5 Groutability	9
2.3 Characteristics of grouting materials	10
2.3.1 Grout rheology	10
2.3.2 Time dependent properties.....	14
2.3.3 Penetrability	14
2.4 Models for grouting prediction and estimation	14
2.4.1 Empirical model - GIN method.....	15
2.4.2 Semi-analytical models	16
2.4.3 Numerical models	20
2.5 Grouting test and experiments	21
2.6 Grouting technical terms	22
2.6.1 Grouting pressure	22
2.6.2 Injection flow rate	23

2.6.3	Grout volume	23
2.6.4	Grouting targets.....	24
2.6.5	Stop criteria.....	25
2.7	Practical grouting execution.....	26
2.7.1	Pre-grouting principle	27
2.7.2	Grouting determination	28
2.7.3	Grouting design.....	32
2.8	Summary	34
CHAPTER 3 ONE-DIMENSIONAL PENETRATION IN FRACTURE CHANNEL ..		36
3.1	Introduction.....	36
3.2	Constant injection pressure method	36
3.2.1	Governing equation for grout flow in a single fracture.....	37
3.2.2	Algorithm for numerical calculation.....	39
3.2.3	Mathematical model for grouting of a fracture network	40
3.3	Constant injection flow rate method	41
3.3.1	Governing equation for grout flow in a single fracture.....	42
3.3.2	Mathematical model for grouting in a fracture network	43
3.3.3	Algorithm for numerical prediction	44
3.4	Simplified constant injection pressure method	46
3.4.1	Simplified model for grouting prediction.....	46
3.5	Grout penetration considering fracture deformation.....	52
3.5.1	Deformation of fracture channel	54
3.5.2	Grout flow in curved fracture channel	56
3.5.3	Results and discussion.....	59
3.6	Summary	63
CHAPTER 4 TWO-DIMENSIONAL PENETRATION IN ROCK FRACTURE.....		65
4.1	Introduction.....	65
4.2	Derivation of governing equation	65
4.3	Real-time grout penetration and fracture deformation.....	70
4.3.1	Basic Concept and numerical model.....	71
4.3.2	Model for real-time grout penetration.....	72
4.3.3	Model for fracture deformation.....	76
4.3.4	Results and discussion.....	78
4.4	Grouting rock fractures with flowing water	89

4.4.1	Theoretical model.....	90
4.4.2	Experimental results and theoretical explanation.....	95
4.4.3	Determination of grouting strategy	100
4.5	Summary	103
CHAPTER 5 GROUTING OF FRACTURE NETWORK BASED ON DDA.....		104
5.1	Introduction.....	104
5.2	Principle of DDA	106
5.3	Methodology	107
5.3.1	Derivation of governing equations.....	107
5.3.2	Algorithm development	110
5.3.3	Hydromechanical coupling	114
5.4	Realization of Algorithm	117
5.4.1	Special boundary conditions	117
5.4.2	Illustrative example.....	117
5.4.3	Realization in DDA.....	119
5.5	Summary	125
CHAPTER 6 IMPACT OF FRACTURE ROUGHNESS ON GROUTING		126
6.1	Introduction.....	126
6.2	Generation of 3D fracture surfaces	127
6.2.1	Introduction to SynFrac.....	127
6.2.2	MATLAB processing.....	129
6.3	Grout injection system and equipment.....	131
6.3.1	Grouting barrel.....	131
6.3.2	Power system	132
6.3.3	Resetting of grouting barrel	134
6.4	Setup of experimental equipment.....	135
6.4.1	Aperture adjustment and supporting system	136
6.4.2	Installation of 3D printed rough surfaces and waterproofing system	138
6.4.3	Pressure and deformation measurement.....	140
6.5	Testing results and data analysis	142
6.5.1	Two types of grouting test.....	142
6.5.2	Data analysis and conclusions.....	146
6.6	Modelling of rock fractures.....	155
6.7	Simulations of grout flow through rough fractures	159

6.8	Model for predicting equivalent aperture.....	161
6.8.1	Analytical model derivation.....	161
6.8.2	Predicative model development	164
6.8.3	A potential engineering application	170
6.9	Summary	174
CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS.....		175
7.1	Conclusions.....	175
7.2	Recommendations.....	177
LIST OF PUBLICATIONS		179
REFERENCES		180
APPENDICES		195

SUMMARY

Rock masses are discontinuous, due to the existence rock fractures, like faults, cracks, joints, and other similar geological features, which could not only affect the structural strength, but also provide interconnected channels for water inflow. The discontinuities in rock mass pose a great threat to rock excavation, and could lead to economic loss or even casualties, if they are not properly handled.

One way to improve the rock mass quality is grouting, which helps to seal those discontinuities, and strengthen the rock mass simultaneously. However, the grouting cost accounts for a considerable percentage of the total cost for construction of underground facilities, especially for rock masses with complex geological conditions. One reason for the high cost is that the stop criteria cannot be properly determined, which is normally based on either empirical criteria or experience; moreover, the accuracy of existing methods for grouting simulation is poor, due to the lack of effective theoretical models and insufficient consideration of geological conditions.

A lot of practical factors should be considered to better model the grouting process, including the influence of fracture deformation (referring to normal deformation in this thesis) and flowing water, the existence of interconnected fracture network, and fracture roughness and tortuosity. Previous work on grouting investigation is summarized in the thesis, introducing fundamental information on grouting technology, including characteristics of rock masses and cement grout; existing theoretical and empirical models used for grouting prediction; and some basic concepts related to grouting practice.

For grout flow in rock fractures not directly intersected by the injection hole, channel flow can represent the grout flow, a one-dimensional (1D) model with grout flow within two parallel plates. This model is very useful and popular, due to its simplicity and its application in evaluating laboratory testing data. The governing equation for the 1D grouting model is derived analytically, and numerical algorithm is developed for solving the corresponding equations. Two different methods are defined under two boundary conditions: constant injection pressure or constant injection rate, as the

corresponding algorithms for equation solving are significantly different. Thereafter, the characteristics of grout penetration using the two methods are discussed.

For rock fractures directly intersected by the injection hole, the distribution of cement grout around the injection hole is in radial direction, which is two-dimensional (2D). Therefore, a governing equation for radial grout penetration initiating from the injection hole is derived. Real time grout penetration within two neighboring fractures directly intersected by the injection hole is calculated based on numerical algorithm, and the influence of fracture deformation (normal deformation) induced by grout pressure on the penetrability of rock fracture with smaller aperture is analyzed. Then, an analytical model is proposed to predict grouting of single rock fracture with flowing water based on the theory of potential flow, which is used for explaining the data obtained from grouting test in the laboratory considering flowing water. A module for predicting grout penetration in a 2D fracture system is developed based on the Discontinuous Deformation Analysis (DDA), which can present good visualization of real-time grout penetration in intersected rock fractures and real-time pressure variation at various intersection nodes.

Finally, the influence of fracture roughness on grout penetration and fluid transportation is investigated based on laboratory test and numerical simulation. Most existing models take rock fractures as two smooth parallel plates, neglecting the characteristics of fracture surfaces, like fracture roughness, which, however, do exist and can have substantial impact on fluid flow within real rock fractures. Therefore, with assistance of 3D printing, large scale rock fracture with rough surfaces are generated and used for grouting test in laboratory, which are compared with grouting test using traditional fracture channel with smooth parallel surfaces. It is found from the laboratory test results that the impact from fracture roughness is considerable, so the influence is further investigated with numerical simulations.

LIST OF TABLES

Table 2.1 Evaluation of difficulty in terms of sealing effect and conductivity (Stille et al., 2012)	9
Table 2.2 Nonlinear rheological models for grout.....	12
Table 2.3 Sealing targets for grouting of jointed rock fractures (Widmann, 1996). 24	
Table 2.4 Inflow requirements in nordic countries and Hongkong	30
Table 2.5 Inflow requirements in tunnel cases	31
Table 2.6 Trigger value of water inflow from probe holes for grouting corresponding to limit of residual inflow	31
Table 3.1 Relationship between k and $\psi(k, t^*)$	50
Table 3.2 Characteristics of channels and grout slurry (Fujita, 2012).....	60
Table 4.1 Calculation of flow rates at radius r and $r + dr$	66
Table 4.2 Pressure of grout flow	67
Table 4.3 Shear stress	67
Table 4.4 Inertial force.....	67
Table 4.5 Model comparison between 1D channel flow and 2D radial flow	70
Table 4.6 Specifics of seven factors used for DOE	78
Table 4.7 Comparison of grouting data among three grouting types	99
Table 5.1 Characteristics of grouting prediction under different methods	119
Table 6.1 Function and meaning of characteristic parameters (Ogilvie et al., 2006)	128
Table 6.2 Feasibility test of grouting barrel.....	133
Table 6.3 Basic properties and chemical components of Portland cement 42.5....	142
Table 6.4 Information on particle size distribution.....	142
Table 6.5 Details on lab grouting test	145
Table 6.6 Update preset fracture aperture and maximum injection pressure	152
Table 6.7 Equations to be used for calculating equivalent aperture	164
Table 6.8 WPT data and the corresponding fracture characteristics	172
Table 6.9 Determination of the fracture width W	173

LIST OF FIGURES

Figure 2.1 Illustration of fracture network: (a) 2D; (b) 3D (Alghalandis, 2017).....	5
Figure 2.2 Demonstration of real rock fracture with rough surfaces	6
Figure 2.3 Illustration of the formation of filter cake	8
Figure 2.4 Behavior of Newtonian and Bingham fluids (Shuttle et al., 2007)	11
Figure 2.5 Bingham fluid flow within a fracture channel: a plug area is located at the fracture centre (Dalmalm, 2004).....	12
Figure 2.6 1D channel filled with only on Bingham fluid (Hässler et al., 1992a)...	17
Figure 2.7 Inclined channel filled with Bingham fluids and water (Hässler et al., 1992a)	17
Figure 2.8 Part of the preliminary grouting design (Stille, 2012).....	28
Figure 2.9 Flowchart of determination of grouting (Garshol et al., 2012).	28
Figure 2.10 Typical probing and pre-grouting setting (Norwegian Tunneling Society, 2014).....	29
Figure 2.11 Correlation between trigger value of water inflow from probe drilling hole and limit of residual inflow	32
Figure 3.1 Force diagram for fully developed grout flow in channels	37
Figure 3.2 Part of a fracture network with node pressure as unknowns	40
Figure 3.3 Flow chart for simulating grouting of a fracture system	41
Figure 3.4 Schematic of a fracture network.....	43
Figure 3.5 Stress analysis of grout flow (Xiao, 2017)	46
Figure 3.6 Relationship between y_0^* and penetration length L	50
Figure 3.7 Schematic diagram of grout flow pressure on fracture channel	54
Figure 3.8 Deformation of channel upper surface	56
Figure 3.9 Schematic diagram for grout flow in a deformed fracture	56
Figure 3.10 Relative difference of average velocity between the original and simplified formulas	58
Figure 3.11 Comparison of simulation results with experimental data	63
Figure 4.1 Schematic diagram of grouting rock fractures (2D radial penetration)..	66
Figure 4.2 Illustration of rock mass with two parallel fractures under grouting	71
Figure 4.3 Flow chart for investigating normal deformation of rock fractures	72

Figure 4.4 One-dimensional illustration of radial grout spreading in two neighboring rock fractures intersected by a grout hole	73
Figure 4.5 Illustration of real-time grout penetration and variation of injection pressure. Injection flow rate: $Q = 0.8$ (kg/s); fracture aperture: $H_1 = 800$ (μm) and $H_2 = 200$ (μm)	75
Figure 4.6 Illustration of mechanical model for evaluating fracture deformation...	76
Figure 4.7 Deflection of rock slab w during the grouting process within 180s. Injection flow rate: $Q = 1.6$ kg/s; fracture aperture: $H_1 = 500$ (μm) and $H_2 = 200$ (μm); Young's modulus of rock slab: $E = 48$ GPa; Fracture stiffness: $K_1 = K_2 = 10$ (10 GPa); Length and thickness of rock slab: 7.95m and 0.8m.....	77
Figure 4.8 Relationship between injection flow rate Q and grout penetration L_1	80
Figure 4.9 Grout penetration against aperture ratio: (a) $L_1 \sim R_H$; (b) $L_2 \sim R_H$	80
Figure 4.10 Statistics of the injection flow rate Q and pressure P_0	82
Figure 4.11 Distribution of injection pressure P_0 under various aperture ratio R_H	82
Figure 4.12 Statistics of derivative variables related to value and location of maximum deflection	83
Figure 4.13 Distribution of the dimensionless maximum deflection $100w_{\max}/H_2$ under various fracture aperture H_2	84
Figure 4.14 Combined effect of fracture aperture H_2 and fracture aperture ratio R_H on the dimensionless maximum deflection $100w_{\max}/H_2$, which is illustrated with color contour	85
Figure 4.15 The relationship between the dimensionless location where maximum deflection appears $100r(w_{\max})/SL$ and penetration slab length ratio R_L	86
Figure 4.16 Influence of stiffness ratio on the dimensionless location where maximum deflection appears $100r(w_{\max})/SL$	87
Figure 4.17 Grouting rock fractures with flowing water (side elevation view).....	90
Figure 4.18 Grouting rock fractures with flowing water (top plan view).....	91
Figure 4.19 Semi-plane streamline of grout flow in fracture with flowing water ...	94

Figure 4.20 Three types of grout penetration in fractures with flowing water	96
Figure 4.21 The half plane streamline corresponding to three types of grout penetration found from flowing water grouting experiments	98
Figure 4.22 Injection flow rate required to seal certain fractures with flowing water based on simulation: (a) $Q \sim v_0$; (b) $Q/v_0 \sim v_0$	102
Figure 5.1 Schematic diagram of fracture network: (a) 3D; (b) 2D	107
Figure 5.2 Schematic diagram of a node information.....	108
Figure 5.3 Gauss elimination for solving the inverse matrix of the derivative matrix: B and A represent matrix element before and after transformation, respectively ..	114
Figure 5.4 Pressure distribution along fracture $i \leftrightarrow j$	114
Figure 5.5 Illustration of different stages of grout penetration in a simple fracture network: black bold line represents fracture segment, the intersection between bold lines are nodes (one node is marked by light blue dot), and the node marked by red dot is chosen as the injection point	118
Figure 5.6 Setup of the fracture network used for real-time grouting prediction ..	120
Figure 5.7 Illustration of real-time grouting penetration based on DDA	123
Figure 6.1 Interface of SynFrac used for generating rough fractures	128
Figure 6.2 The original data file exported from SynFrac	129
Figure 6.3 Generation of fracture surfaces for grouting experiment	130
Figure 6.4 The rough fracture surface: (a) 3D model; (b) 3D printing	130
Figure 6.5 Grouting equipment designed for grout injection under constant flow rate	132
Figure 6.6 Power system pushing downward the plunger cover	133
Figure 6.7 (a) Tool set for pulling the plunger out: ① and F are screw and lead screw with the same outer screw thread, A is the corresponding screw cap, B is a wooden beam, C and D are a jack and the associated lever, E is a steel plate working together with G, a flange, which are set above the jack; (b) Plunger inside the cylinder; (c) Plunger pulled out using tool set.....	134
Figure 6.8 Overall experimental setup for grouting of rough rock fracture	135
Figure 6.9 Grouting of single fracture with fixed aperture (Fujita et al., 2012)	136
Figure 6.10 Aperture adjustment and supporting system	137

Figure 6.11 Deformation of aluminum alloy plates under 0.4MPa injection pressure	137
Figure 6.12 Exploded view of the aperture setup and waterproofing system (a) in longitudinal direction, (b) in transversal direction.....	139
Figure 6.13 Measurement of injection pressure and fracture deformation	140
Figure 6.14 Flow chart for grouting test	141
Figure 6.15 1D Grout penetration within fracture channel: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at the left end of the experimental platform.....	143
Figure 6.16 2D Grout penetration within fracture channel: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at the middle of the experimental platform.....	143
Figure 6.17 Bleeding during 1D grouting: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at one end of the experimental platform	144
Figure 6.18 Definition of fracture aperture H corresponding to fracture channel with two rough surfaces and smooth surfaces (lateral view)	145
Figure 6.19 Equipment setup and testing result for grouting of fracture with two rough surfaces	146
Figure 6.20 Collection of cement grout for measuring volume concentration distribution after grouting	146
Figure 6.21 Volume concentration of cement grout at 15 different locations for smooth fracture: (a) Aperture=1mm; (b) Aperture=3mm.....	147
Figure 6.22 Volume concentration of cement grout at 15 different locations for rough fracture: (a) Aperture=1mm; (b) Aperture=3mm	148
Figure 6.23 Real-time variation of injection pressure for grouting of smooth fracture	151
Figure 6.24 Real-time variation of injection pressure for grouting of rough fracture	154
Figure 6.25 Four typical synthetic models of rough rock fractures	156
Figure 6.26 Percentage frequency of the aperture of synthetic rock fractures	156

Figure 6.27 Fracture models for simulating grout flow within rough rock fractures	160
Figure 6.28 Velocity distributions presented in five different slices (Case 1).....	160
Figure 6.29 Comparison between predicted and the original equivalent apertures: (a) $Q_m = 0.2\text{kg/s}$, (b) $Q_m = 0.3\text{kg/s}$, (c) $Q_m = 0.4\text{kg/s}$, and (d) $Q_m = 0.5\text{kg/s}$	167
Figure 6.30 Parameters of prediction model	168
Figure 6.31 Linear relationship between parameter b_i and flow rate via fitting.	168
Figure 6.32 Validation of the prediction model for calculating the equivalent aperture based on the characteristics of rock fracture and the corresponding grout injection mass flow rate	170
Figure 6.33 Simplified schematic diagram of WPT	171
Figure 7.1 Dimension transfer for grout penetration in rock fractures directly intersected by an injection hole.....	177
Figure 7.2 Demonstration of filling materials in rough rock fracture.....	178

LIST OF SYMBOLS

<u>Symbol</u>	<u>Description</u>
(x, y)	Point coordinates in DDA
(x_0, y_0)	Centroid coordinates in DDA
(u, v)	Point displacement in DDA
(u_0, v_0)	Centroid displacement in DDA
ε_x	Normal strain (X direction)
ε_y	Normal strain (Y direction)
γ_{xy}	Shear strain
$[K_{ij}]$	Coefficient matrix
$[D_i]$	Displacement matrix
$[T_i]$	Deformation matrix
$[F_i]$	Load matrix
$i \leftrightarrow j$	Fracture segment between nodes i and j
$i \rightarrow j$	Flow direction: from node i to node j
$j \rightarrow i$	Flow direction: from node j to node i
L_{ij}	Length of fracture segment $i \leftrightarrow j$
$L_{ij}(t)$	Grout penetration along direction $i \rightarrow j$
$L_{ji}(t)$	Grout penetration along direction $j \rightarrow i$
h_{ij}	Aperture of fracture segment $i \leftrightarrow j$
(P_x, P_y)	Pressure vector in X-Y plane
Π_p	Potential energy of pressure loading
b and y_0	Half aperture and Bingham plug for fracture with 1D channel flow
H and H_C	Half aperture and Bingham plug for fracture with 2D radial flow

h_e	Equivalent aperture
h_m	Mean mechanical aperture
γ	Shear rate
τ	Shear stress
τ_0	Yield stress
μ	Viscosity
P_0	Injection pressure
P_i	Node pressure
ΔP	Pressure gradient
Q	Flow rate for 3D model
q	Flow rate for 2D model
Δt	Time interval
Δx_i	Segment interval
x_i	Location along rock fracture
$\bar{u}$ or U	Average velocity
R_H	Aperture ratio between two neighboring fractures
R_K	Stiffness ratio between two neighboring fractures
$w_{\max}$	Maximum fracture deformation
W_f and W_g	Complex potential of water and grout flow
$\frac{\partial f_x}{\partial y}$	Partial differentiation of f_x with respect to y
K_i	Fracture stiffness
R_m and T_m	Surface roughness and tortuosity
JRC	Joint Roughness Coefficient
V_{frac}	Fracture volume

Chapter 1 Introduction

1.1 Background

Practical rock masses are not intact but with various faults, joints, fractures, or other discontinuous characteristics, the existence of which can greatly reduce the corresponding comprehensive quality (Lombardi, 2002); and can provide perfect channel for water transportation (Yang et al., 2002; Saeidi et al., 2012). When these discontinuities were encountered during underground construction, water inflow could take place, which would pose great threat to safe and smooth excavation and cause huge potential economic loss if they are not well handled (Brantberger et al., 2000; Lombardi, 2002; Panthi and Nilsen, 2005; Emmelin et al., 2007). Nevertheless, all those potential problems can be avoided or greatly alleviated if grouting is introduced to seal those discontinuities within a certain range. In the meanwhile, grouting can also strengthen the fractured rock masses. Comparatively, pre-grouting cost much lower than other methods, and is more environment-friendly (Eriksson, 2002).

However, the cost of grouting accounts for a considerable portion of the total cost for constructing underground facilities (Hendrickson and Au, 1989). One reason is that the grouting design is often not scientifically derived and lacks theoretical guidance; most of the time, it is based on either empirical criteria or engineers' experiences gained from other similar projects (Kobayashi and Stille, 2007; Rafi, 2014), which is usually conservative and not cost-effective. Therefore, further development of grouting theory is in great need for making better grouting design.

In this research, new methods are used to solve real-time grout penetration in 1D fracture channel. A grouting module is developed to predict real-time grout penetration and pressure variation in a 2D fracture network, based on Discontinuous Deformation Analysis (DDA), which is a very useful tool for analysis of fractured rock masses (Jing et al., 2001; Chen et al., 2013). The algorithm used for developing the grouting module based on DDA choose different variables to form matrix equation, which can greatly simplify the problem. The pressure around the injection hole is the highest, so the potential fracture deformation induced by high pressure is

of great concern (Gothäll and Stille, 2009;Gothäll and Stille, 2010;Rafi and Stille, 2014), a model is proposed to demonstrate this problem. The surfaces of real rock fractures are rough (Tsang and Witherspoon, 1983;Brown, 1987;Hakami, 1995;Brush and Thomson, 2003;Zhao et al., 2014), which could influence grout penetration, so laboratory testing and numerical simulation are introduced to evaluate this problem.

1.2 Research objectives and scopes

The objective of this study is prediction of grout penetration in fractured rock masses, so models for grout penetration in single rock fractures directly and indirectly intersected by an injection hole were developed, respectively; then, DDA based grouting module was developed to predict the characteristics of grout penetration in a 2D fracture network around injection holes, where the model for grout penetration in a single fracture channel discussed in Chapter 3 is adopted; whereas the grouting process, including grout penetration and pressure distribution, can be greatly affected by fracture deformation (referring to normal deformation in this thesis), fracture roughness, and flowing water, which therefore were investigated in succeeding chapters. Therefore, the main objectives for grouting prediction are listed below.

1. Derive the theoretical basis for grout penetration in 1D fracture channel under constant injection pressure/flow rate.
2. Investigate the influence of fracture deformation on grout penetration in simplified 1D fracture channel.
3. Study the characteristics of grout penetration under flowing water based on the theory of potential flow.
4. Investigate the features concerning grouting of rock fractures intersected by the grout hole and the relevant fracture deformation.
5. Derive the algorithm for grout penetration in 2D fracture network and develop the grouting module based on DDA.
6. Conduct laboratory test and numerical simulation on grouting of rock fractures considering practical roughness.

1.3 Thesis organization

There are seven chapters in the thesis, ranging from background introduction in Chapter 1 to future work expectation in Chapter 7.

In Chapter 2, literatures are introduced on the features of fractured rock masses and cement grout, the theoretical model for grouting simulation, and some fundamentals related to grouting operation.

In Chapter 3, grout penetration in fracture channel under constant injection pressure or flow rate is studied. Simplified model for grouting prediction is proposed and the relevant advantages are discussed. The influence of fracture deformation on grout penetration is discussed and compared with experimental data.

In Chapter 4, theoretical model for grout spread in fractures directly intersected by an grout hole is proposed, where the grout pressure is the highest and fast increasing, so the influence of fracture deformation on grouting is investigated.

In Chapter 5, a new model for simulating grout penetration in a 2D fracture network, and the corresponding algorithm is derived and programmed based on Discontinuous Deformation Analysis (DDA) and the time varying grout penetration in a fracture network and node pressure are predicted via the DDA based grouting module.

In Chapter 6, 3D printing is used to generate artificial rock fractures with rough surfaces, which are used for laboratory testing to quantify the influence of fracture roughness on fluid flow in real rock fractures with uneven surfaces. Numerical models are also developed

In Chapter 7, a summary on the current work is presented, including different models for solving specific problems, development of grouting module based on DDA, and results in experimental study. Shortcomings of the present study are pointed out; suggestions and recommendations for future study are also given.

Chapter 2 Fractured Rock Masses and Grouting

2.1 Introduction

Grouting has been used to control water seepage for the construction of underground structures for a long time (Warner, 2004). Effective grouting can significantly decrease the cost of construction and maintenance and enhance engineering safety. The “grouting” in the present work represents pre-excavation grouting for constructing underground facilities in rock masses, which, as its name indicates, mainly involves two aspects: the fractured rock masses and the grout flow inside rock fractures. Therefore, theoretical research for grouting prediction mainly focuses on investigating the characteristics of rock fractures (Tse and Cruden, 1979; Mandelbrot, 1985; Power and Tullis, 1991; Odling, 1994; Belem et al., 2000) or fracture network (Tsang and Tsang, 1987; Watanabe and Takahashi, 1993; Klimczak et al., 2010; Carter et al., 2012), the grout properties and corresponding fluid models (Håkansson et al., 1992; Yahia and Khayat, 2001; Rosquoët et al., 2003; Eriksson et al., 2004; Miltiadou-Fezans and Tassios, 2013; Mohammed et al., 2014; Rahman, 2015), grout flow within rock fractures (Gustafson and Stille, 1996; El Tani, 2012; Singh et al., 2015).

In this chapter, the following contents will be discussed: characteristics of fractured rock mass, including fracture roughness, 2D and 3D fracture system; properties of grouting materials, such as grout rheology, fluid model, time dependent behavior, and groutability; prediction model for grout penetration in one direction or radial direction; concepts related to grouting process, such as grout pressure, stop criteria and mechanism for grouting, and sealing requirement.

2.2 Characteristics of fractured rock masses

Rock fractures in nature appear in the form of fracture networks, where intersected fractures are of different angle, length, width, and aperture; moreover, normally, the fracture surfaces are rough and tortuous, which can greatly affect its hydraulic properties and penetrability of cement grout.

2.2.1 Fracture network

There are two forms of fracture network discussed in academic papers: Figure 2.1(a) shows a 2D fracture network composed of 1D fractures (Watanabe and Takahashi, 1993; Eriksson et al., 2000; Yang et al., 2002; Klimczak et al., 2010), Figure 2.1(b) shows a 3D fracture network with 2D fractures (Munier, 1993; Lee et al., 2000; Carter et al., 2012). Different forms of fracture systems correspond to different models for grouting prediction. As for 2D fracture network, the grouting model involved is only channel flow. As for 3D fracture network, the model is a combination of radial flow (for Type I fracture) and channel flow (for Type II fracture), where Type I fractures are defined as those directly intersected by the injection hole, and Type II fractures are those only intersected by Type I fractures and indirectly intersected by the injection hole. Nevertheless, for a 3D fracture network, the detailed flow type could be very complex, as the connectivity of rock fractures are very complex. The concept of fracture network is proposed for predicting the range of grout penetration in fractured rock masses based on numerical simulation, then for evaluating the potential grouting quality, finally for providing guidance to determine the stopping criteria for grouting (Saeidi et al., 2012).

Figure 2.1 Illustration of fracture network: (a) 2D; (b) 3D (Alghalandis, 2017)

It should be noticed that the fracture connectivity can affect both water and grout flow significantly (Moon and Song, 1997). Under the assumption of constant injection pressure, the flow rate could be decreasing when fractures with large apertures are

sealed during the grouting process; the grout spread under given fracture connectivity could obviously enhance the sealing effect, as grouting can be well controlled (Hernqvist et al., 2008). Additionally, the flow in a fracture network is influenced by the fracture frequency, orientation, length, and other geological characteristics, whereas these factors are of high uncertainty. Thus, it was feasible to apply statistical methods to describe and generate interconnected fractures for grouting simulation.

However, the approach based on fracture network may not be efficient when the rock volume was densely jointed, as the importance of detailed properties of individual joint would be greatly weakened (Andersson et al., 1984), so a highly fractured rock mass could be considered as a porous media, being homogeneous or/and isotropic (Hässler et al., 1992a). Therefore, the conductivity of a rock volume is introduced, which is determined by both the property of individual joint and connectivity of joints.

2.2.2 Fracture surface

The models used for predicting grout penetration in rock fractures trend to assume that their surfaces are smooth, which can be seen in most analytical and numerical models for grouting prediction (Hässler et al., 1992b; El Tani, 2012) and lab testing (Esaki et al., 1999; Mohammed et al., 2015). However, the surfaces of real rock fractures are rough rather than smooth. One rough fracture is taken as an example and demonstrated in Figure 2.2, where there are two rough surfaces.

Figure 2.2 Demonstration of real rock fracture with rough surfaces

Fortunately, the potential influence from practical characteristics of rock fractures on fluid flow has been studied by many researchers in recent years, including many small-scale laboratory tests on the influence of roughness. Some laboratory tests used real rock fractures (Akhavan and Rajabipour, 2012; Singh et al., 2015), and some

employed artificial rock fractures (Baraka-Lokmane and Liedl, 2006; Develi and Babadagli, 2015; Rong et al., 2017), where the rock fractures with rough surfaces are obtained via splitting intact rock samples or making moulds based on existing rough fracture surfaces (Ogilvie et al., 2003).

However, the number of rough fractures generated based on the abovementioned methods is limited, which may not help us derive the most general conclusions concerning the impact of fracture roughness. Therefore, scientists and mathematician started to investigate the characteristics of rock fractures from 2D to 3D step by step (Mandelbrot, 1985; Power and Tullis, 1991; Odling, 1994; Lanaro, 2000; Ogilvie et al., 2003; Ogilvie et al., 2006), they found that fracture surfaces are self-affine fractals, therefore, many theoretical models and algorithms were developed to generate artificial rough fractures, which were used to study the impact of roughness and tortuosity on fluid transportation (Brown, 1987; Ge, 1997; Brush, 2001; Brush and Thomson, 2003; Jeong and Song, 2005; Ogilvie et al., 2006; Noiriél et al., 2007; Crandall et al., 2010; Hanssen, 2013; Briggs et al., 2014; Zhao et al., 2014). However, most of the work done so far were focusing on the investigation of phenomenon or qualitative study, and very few works were conducted on grouting related study, so more work should be carried out to provide theoretical basis or practical guidance to engineering grouting.

2.2.3 Fracture aperture

Fracture aperture is of great significance in various analytical models, lab testing, and onsite field testing, so a lot of work have been done, not only on testing and simulation of water flow, but also on grouting. As the flow rate of fluid within rock fractures is a function of the third power of fracture aperture (Xiao, 2017), a small change in aperture can lead to large variation of flow rate. Therefore, proper grouting design can help prevent excessive grout spread in wide fractures (Kobayashi and Stille, 2007).

In field testing, water pressure test (WPT) is usually conducted to evaluate the equivalent aperture of rock fractures within a certain area, which then can be used for evaluating the grout spread and grout take (Gustafson and Stille, 1996). The aperture

variation of rock fracture (2D curve model) was considered in their work, and the grout take is found to increase with increasing standard deviation of aperture variation. The aperture distribution of nature rock fractures is widely taken as log-normal distribution or Gaussian distribution (Hakami, 1995; Park et al., 1995; Eriksson et al., 2000; Lee and Cho, 2002; Ogilvie et al., 2003).

“Critical aperture” is a very important concept in grouting. As for a certain area where the fracture aperture is far smaller than its surrounding area, namely a constriction area, filtration of cement grout and formation of filter cake would take place, if the average aperture of this area is lower than the “critical aperture” of a certain cement grout. Then, accumulation of filter cake would block the fracture and prevent cement grout from further spreading in surrounding areas (Eriksson et al., 2000). A schematic diagram of grout filtration is presented in Figure 2.3.

Figure 2.3 Illustration of the formation of filter cake

2.2.4 Fracture deformation

The term “fracture deformation” in the present thesis is also called as “fracture jacking” (Vidstrand et al., 2008) or “fracture dilation” (Gothäll and Stille, 2009). Fracture deformation is important to both lab testing and field grouting. As for grouting experiment, failing to control deformation of artificial fracture is very harmful, as the data obtained cannot be used to derive reliable conclusions (Xiao and Zhao, 2017). Nevertheless, compared with on-site rock fractures, the mechanism and controlling of fracture deformation in lab testing is not that hard, so more researches are focusing on fracture deformation during field grouting (Noorishad et al., 1982; Hopkins, 2000; Gutierrez and Lewis, 2002; Cornet et al., 2003; Rutqvist and Stephansson, 2003; Cammarata et al., 2006; Li et al., 2008; Svenson et al., 2008; Vidstrand et al., 2008). It is found that fracture deformation occurs when the

hydraulic pressure within rock fractures is higher than a certain pressure rather than the initial in-situ stress (Rutqvist and Stephansson, 1996).

As for field grouting, the most vulnerable locations are rock fractures close to tunnel surface, where the constraint or support from the neighbouring rock is weak (Fransson et al., 2010), and locations with fractures directly intersected by the injection hole, where the hydraulic pressure is the highest and could increase during the grouting process (Gothäll and Stille, 2010). Then, the major concerns of fracture deformation are the safety and economic efficiency: 1) fast accumulation of injection pressure could pose threat to the stability of rock masses under grouting; 2) fracture deformation can increase aperture of larger fracture, thus larger volume of grout is required for certain grouting range.

2.2.5 Groutability

Houlsby outlined the properties of rock masses that could affect the groutability, including the spacing, dipping, and persistency of joints, filling materials, fracture aperture, rock soundness, strength and in situ stress (Houlsby, 1990). Apart from the properties of rock masses, the cement grout rheology is critical to the groutability of rock masses.

Table 2.1 Evaluation of difficulty in terms of sealing effect and conductivity (Stille et al., 2012)

		Sealing Efficiency		
		<90%	90-99%	>99%
Conductivity	$>10^{-7}$ m/s	Uncomplicated	Fair	Difficult
	10^{-8} to 10^{-7} m/s	Fair	Difficult	Very Difficult
	$<10^{-8}$ m/s	Difficult	Very Difficult	Very Difficult

Lombardi introduced the groutability factor K to account for the geological influence on grout spread (Brantberger et al., 2000). Therefore, K depends on the number of joints per meter of borehole, fracture roughness, and feature variation of fracture geometry (Lombardi, 1996), which are correlated to the conductivity of rock masses.

Moreover, the required conductivity and sealing efficiency determine the difficulty of grouting work (Stille et al., 2012), as shown in Table 2.1.

Saeidi et al. found that the factors affecting the groutability of fractured rock mass can be evaluated by various parameters. The controllable parameters were related to properties of grout material, injection pressure, and configuration of grouting system, whereas the uncontrollable parameters involved geological and hydrogeological conditions (Saeidi et al., 2013). Both the geo-mechanical factors of the rock mass and grout properties are important in controlling the groutability (Saeidi et al., 2014).

2.3 Characteristics of grouting materials

There are various types of grouting materials used in engineering project, such as cement grout, chemical grout, or an admixture of both, but cement grout is the most commonly used (Axelsson et al., 2009). Therefore, the properties of cement grout have been intensively studied, including rheology, time dependent properties, and groutability and bleeding effect.

For sealing fractures with very small apertures, very fine-grained cements are required, due to high level of sealing requirements (Gustafson et al., 2013). To enhance grout spread in finer joints, a pad of lean grout is recommended to initiate grouting, though the borehole maybe very conductive at the very beginning, followed by medium and finally very stiff grout (Gustafson and Stille, 1996).

To reduce water inflow or strengthen the foundation structure, a stable grout with low degree of setting is preferable (Shuttle et al., 2007). For grouting with unstable grout, the fracture will be filled with the grout materials due to particle precipitation (Fu et al., 2013). Thereafter, the sediment gradually accumulates at the bottom of fracture walls, and the effective opening of fractures decreases accordingly.

2.3.1 Grout rheology

The rheology of cement grout is very critical to the prediction of grout spread in fractured rock masses, as the rheological model is the basis of both analytical and numerical models for grouting simulation. Water cement ratio (defined as W/C) is usually used for the classification of cement grout. Simple models, like Newtonian

model (very light cement, $W/C \geq 2$), and Bingham model (relatively dense cement, $0.8 \leq W/C \leq 1$), are popularly used to derive analytical model for grouting prediction, as succinct form of governing equation expressing grout flow rate can be derived (Dai and Byron Bird, 1981; Hässler et al., 1992a; Eriksson, 2002; El Tani, 2012; Saeidi et al., 2013; Xiao and Zhao, 2017). However, it is very hard to get convergent solution if Bingham model is chosen as the fluid model for CFD analysis, since Bingham model is a stepwise function. Fortunately, there are a lot of other models, including modified Bingham model, Power law model, Herschel-Bulkley model, Casson model (Yahia and Khayat, 2001; Banfill, 2006; Mitsoulis, 2007), which are nonlinear but consecutive, thus they can be used in CFD model.

2.3.1.1 Linear model

Newtonian and Bingham fluid models are linear models. Apart from light cement grout, several chemical materials used for grouting present Newtonian behaviour, such as silicates, polyurethanes, and acrylates, which have been used for a long time (Krizek et al., 1992; Nokken, 2003; Xiang et al., 2006). Though chemical grouts can be taken as Bingham fluid, their viscosity is time-increasing, which is called as gelling (Funchag and Gustafson, 2008).

In fact, the theoretical Newtonian fluid should have constant viscosity under various shear rates (Kazemian et al., 2010), and the relationship between shear stress and shear rate is linear, as shown in Figure 2.4. The Newtonian model is applicable to grouts with very high content of water and/or super-plasticizer, ignoring the existence of yield stress (very low value).

Figure 2.4 Behavior of Newtonian and Bingham fluids (Shuttle et al., 2007)

Being a little different from the Newtonian model, the Bingham model is also linear but having intersection with the vertical axis, as shown in Figure 2.4, and it is used for describing the properties of cement grout within a certain range of W/C ratio (0.8 to 1). The Bingham grout will not start to move unless the internal shear stress overcomes the yield strength. The location where the shear stress lower than the yield stress would form a rigid core and move forward together, called as Bingham plug, as given in Figure 2.5. The relationship between shear rate and shear stress of Bingham fluid can be expressed as,

$$\dot{\gamma} = \begin{cases} 0 & (\tau < \tau_0) \\ \tau_0 - \tau / \mu & (\tau \geq \tau_0) \end{cases} \quad (2.1)$$

where τ and τ_0 are the shear stress and yield stress, respectively; μ is the viscosity; $\dot{\gamma}$ is the corresponding shear rate. Bingham model is usually used to derive analytical solution owing to its simplicity.

Figure 2.5 Bingham fluid flow within a fracture channel: a plug area is located at the fracture centre (Dalmalm, 2004)

2.3.1.2 Nonlinear model

Table 2.2 Nonlinear rheological models for grout

Model name	Expression	Description
Modified Bingham	$\tau = \tau_0 + \mu\dot{\gamma} + c\dot{\gamma}^2$	Better describe nonlinear behavior of cement grout when the shear rate is low than Bingham (Yahia and Khayat, 2001)
Herschel-Bulkley	$\tau = \tau_0 + K\dot{\gamma}^n$	Cement grout, W/C ranges from 0.30 to 0.45 (Yahia and Khayat, 2003)
Casson	$\tau = \tau_0 + \mu\dot{\gamma} + 2\sqrt{\tau_0\mu\dot{\gamma}}$	Not applicable to neat cement grout with W/C range between 0.40 and 0.80 (Yahia and Khayat, 2003; Banfill, 2006)
De kee	$\tau = \tau_0 + \mu\dot{\gamma}e^{\alpha\dot{\gamma}}$	Hydrolyzed polyacrylamide solutions with 20g/L NaCl (Yahia and Khayat, 2003)

Yahia -Khayat	$\tau = \tau_0 + 2\sqrt{\tau_0 \mu \gamma e^{a\gamma}}$	Better prediction of rheological profiles of high performance grout (Yahia and Khayat, 2003)
------------------	---	--

Notes: c is an insignificant constant, K is consistency, α is a time dependent parameter, μ is viscosity, τ_0 is yield stress.

Apart from simple linear models, there are several complex nonlinear models, including Modified Bingham model, Herschel-Bulkley model, Casson model, De kee model, and Yahia model, as presented in Table 2.2.

2.3.1.3 Limitations of Grout Model

Both the linear and nonlinear models have limitations. Linear model is simple, thus the corresponding solution derived is succinct and the correlated computation efficiency is high. Nevertheless, the flow curve of cement grout derived from experimental data is usually of certain nonlinearity, especially at a low shear rate, which cannot be illustrated exactly by the Bingham model (Yahia and Khayat, 2003). Like chemical grouts, due to complex shear stress-shear rate behaviour, cement grout behaves like plastic body under low shear stress (Newman and Choo, 2003). However, grouting tests show that the discrepancy is small even though Bingham model does not give an exact description of the behavior of a cement-based grout (Hakansson, 1995). Hakansson indicated that the approximation and simplification of Bingham model could be acceptable, considering that there are other effects that could have larger impact on the grout flow prediction than the choice of rheological model (Hakansson, 1995). Meanwhile, it could be very time consuming if the nonlinear model was used for grouting simulation, especially in CFD simulation.

Moreover, the existing models for grouting prediction failed to consider the increasing resistance when the fracture aperture is approaching the magnitude of grain size. It is found that the Bingham model is valid for suspension flow when the fracture aperture is around five to ten times the largest particle size (Mezger, 2000). Hansson modified the Bingham model to predict the grout flow in rock fractures with aperture equal to the largest particle diameter (Hansson, 1995).

2.3.2 Time dependent properties

Normally, the properties of cement grout are time varying (Håkansson, 1993; Schwarz, 1998). It is of great significance to consider the time-dependent behaviour of cement grout for predicting practical grout penetration (Hässler et al., 1992a).

Moon and Song derived temporal shear stress and viscosity (Moon and Song, 1997),

$$\begin{cases} \tau(t) = e^{\alpha t} + A \\ \mu(t) = B e^{\beta t} \end{cases} \quad (2.2)$$

where $\tau(t)$ and $\mu(t)$ are shear strength and viscosity, respectively; t is time; α , β , A and B are constants, which can be obtained from grout rheological tests.

2.3.3 Penetrability

Penetrability is defined as the ability of a cement grout to pass through a filter or channel with specific size or aperture, which is normally evaluated by two parameters: the minimum and critical apertures. The penetrability of one grout material in fractured rock mass is mainly affected by three properties: grain size, viscosity, and stability. The grain size is critical to the penetrability of cement grout in dealing with fine joints (Hakami, 1995; Holter and Hognestad, 2012). In engineering grouting, the type of cement grout required by a certain fractured rock masses should be determined considering both the penetrability of cement grout and groutability of rock masses, otherwise filtration might take place and affect the sealing quality.

2.4 Models for grouting prediction and estimation

There are three types of methods for grouting simulation: empirical method, semi-analytical models, and numerical models. Grouting Intensity Number (GIN) (Lombardi, 1996; Lombardi, 1997) is an empirical method, and it is cost and computation efficient. The governing equations derived for the semi-analytical method would be nonlinear, which require numerical algorithm for problem solving. There are two different assumptions concerning the boundary conditions: constant injection flow rate (Nokken, 2003; Dano et al., 2004; Rahmani, 2009), and constant injection pressure (Hässler et al., 1992a; El Tani, 2012; Xiao and Zhao, 2017). Meanwhile, many numerical models were also built to model the process of grout

penetration in fracture networks based on some self-developed programs or commercial software (Yang et al., 2002; Sochi, 2010; Saeidi et al., 2013). Compared with the latter two methods, GIN method is not used so common in academic research, because one cannot directly derive the grout penetration, grout flow rate and grout volume, and so on.

2.4.1 Empirical model - GIN method

The grouting intensity number (GIN) method was introduced by Lombardi and Deere, which can be used for controlling the grout penetration (Lombardi and Deere, 1993). The GIN method is defined via introducing a constant grouting intensity G ,

$$G = PV \quad (2.3)$$

where P is the injection pressure and V is the grout take. Here, the constant G can be used to control the grout penetration regardless of fracture aperture.

Low viscosity and yield strength are required for the GIN method (Lombardi, 2003). The force on the grout caused by injection pressure is (Rombough and Shuttle, 2006),

$$F = P \cdot A = Pwb \quad (2.4)$$

where A is the area of fracture cross-section; w and b are the width and aperture of specific rock fracture, respectively.

The force F is resisted by the shear stress of grout (assumed as Bingham fluid),

$$R = 2wL\tau_0 \quad (2.5)$$

where τ_0 is the yield stress of cement grout, and L is the range of grout spread. As the resistance force is equal to the corresponding driving force caused by injection pressure, L can be calculated easily,

$$L = Pb / 2\tau_0 \quad (2.6)$$

The injected volume of cement grout can be expressed as,

$$V = wbL \quad (2.7)$$

Thus, the aperture b can be derived,

$$b = V / Lw \quad (2.8)$$

Substituting Eq. (2.8) to Eq. (2.6), the penetration length can be obtained,

$$L^2 = PV / 2w\tau_0 \quad (2.9)$$

where PV is GIN defined in Eq. (2.3). According to Eq. (2.9), the length of grout spread L is independent of the fracture aperture, which becomes the basis of the

GIN method. The grout should be injected at a constant and low to medium rate, but Lombardi and Deere did not clearly explain the flow rate in their research work when they initialized the theory. Determination of the impact of injection rate on grout penetration remains a problem (Rombough and Shuttle, 2006).

In fact, the length of grout penetration predicted by GIN method may overestimate the grout spread. The shear stress in Eq. (2.5) for evaluating the resistance force should be the practical shear stress at the wall surfaces, which is far larger than the yield stress of cement grout. Consequently, the penetration length calculated according to Eq. (2.9) could be larger than practical grout spreading range.

2.4.2 Semi-analytical models

Based on the characteristics of fluid properties and the corresponding rheological models, the relationship between grout flow rate and pressure distribution can be derived, which can be used for predicting the range of grout spread. Grout penetration around an injection hole is radial, so two dimensional (2D) models should be developed for grouting prediction; for grout penetration in rock fractures not directly grouted by the injection hole but connected to fractures directly intersected by an injection hole (defined as guide fracture), or grout flow changed from radial flow to channel flow when the width of a guide fracture is far smaller than its length, 1D channel model should be developed. In engineering grouting, grout injection pressure is normally controlled by adjusting the corresponding grout injection flow rate. Hence, there are two different assumptions for the boundary conditions: constant injection pressure and constant injection flow rate, which can lead to different solutions.

2.4.2.1 Hassler, Hakansson and Stille's method

Hassler et al. developed a channel model for simulating grout flow within fractured rock masses, as presented in Figure 2.6, where the channel is horizontal and stiff, and the grout flow in it was assumed to be laminar (Hässler et al., 1992a).

Figure 2.6 1D channel filled with only on Bingham fluid (Hässler et al., 1992a)

As shown in Figure 2.6, sharp boundaries were set between different fluids (water and grout), and the cement grout in a horizontal channel was considered as Bingham fluid. The expression for flow rate is given below,

$$Q = \frac{\rho_w g b^3 W (h_0 - h_L)}{12 \mu_B(t) L} (1 - 3Z + 4Z^3), \quad Z = \min\left(\frac{\tau_0(t)L}{b \rho_w g |h_0 - h_L|}, \frac{1}{2}\right) \quad (2.10)$$

where Q is the flow rate, b is fracture aperture, ρ_w is the water density, g is the gravitational acceleration, μ_w and τ_0 are the plastic viscosity and yield stress of the cement grout, which are time dependent. The effects of different factors on grout spread were investigated, including time-dependent rheology, the inclination of joints, groundwater, and varying aperture.

Figure 2.7 Inclined channel filled with Bingham fluids and water (Hässler et al., 1992a)

Normally, joints to be grouted are inclined and probably filled with water, as shown in Figure 2.7. Additionally, grout and water would mix in various channel intersections. The expression for inclined joints filled with several different fluids,

$$Q = \frac{\rho_w g b^3 W}{12} \left[\frac{(HX_0 - HX_N) + \frac{(n_0 - n_i)}{(X_N - X_0) \rho_w} \sum_{i=1}^N (X_i - X_{i-1}) (\rho_{Bi} - \rho_w)}{\sum \frac{(X_i - X_{i-1}) \mu_{Bi}}{(1 - 3Z_i + 4Z_i^3)}} \right] \quad (2.11)$$

Eq. (2.11) can be applied for the Newtonian fluid when Z_i is zero. Both Eq. (2.10) and Eq. (2.11) can describe the grout flow along a single fracture. Assemble the equations for a whole system, the unknown grout pressures at each intersection of a fracture network can be solved according to the mass conservation, namely, the sum of the inflow and outflow are equal,

$$\sum_{j=1}^n Q_{ij} = 0 \quad (2.12)$$

2.4.2.2 Moon and Song's method

Moon and Song analysed the transportation of groundwater, grout and solute in joints with variable apertures, and investigated the characteristics of channelized flow, residence time and the concentration of nuclide particles in cracks under grouting (Moon and Song, 1997). The time taken for the particles running to the ground surface was explained as well.

Any joint within a fracture network was represented by an element with constant width but variable height. The hydraulic head of individual intersection is the arithmetic mean value of the heads of four neighbouring nodes. A criterion is proposed for determining penetration condition in each fracture,

$$\frac{b}{D_{95}} \begin{cases} > 5, & (\text{penetration}) \\ < 5, & (\text{blockage}) \end{cases} \quad (2.13)$$

where b is aperture of rock fracture, D_{95} is the grain size of cement with a 95% cumulative probability.

According to Eq. (2.10), grout can flow under positive pressure gradient along flow direction. The volumetric flow rate in individual fracture can be obtained via a formula of Hassler et al. (Hässler et al., 1992a). The grout flow would not stop unless the hydraulic gradient is less than $2\tau_0 / b\gamma_w$, with γ_w the unit weight of water.

2.4.2.3 Saeidi, Stille and Torabi's method

Saeidi et al. developed a model to evaluate grout spread in fractured rock masses, where the Buckingham's equation was implemented into the FISH program to model grout spread in a rectangular channel (Saeidi et al., 2013),

$$Q = \frac{1}{\mu} \left[-\frac{\Delta P}{12L} b^3 + \frac{\tau_0^3}{3(\Delta P / L)^2} - \frac{\tau_0 b^2}{4} \right] \quad (2.14)$$

where $\Delta P/L$ is the pressure gradient.

The influence of many factors on the characteristics of grout penetration around an injection hole was discussed, such as the hydraulic aperture, characteristic parameters related to joints, injection pressure, and grout rheological properties. It was discovered that the range of grout penetration would increase with increasing joint aperture, joint spacing, and the angle between two joint sets. The maximum range of grout spread could be obtained with the joint angle ranging between 40° and 60° . The penetration ability would weaken if the angle exceeded 60° (Saeidi et al., 2013).

With increasing yield stress or viscosity, the range of penetration would also be reduced. Thus, the penetration length could be written as a function of different parameters concerning the properties of rock mass and grout (Saeidi et al., 2013),

$$L = f[P, b(JRC, E), \theta, s, l, \tau_0, \mu, t] \quad (2.15)$$

where L is length of grout spread, b is equivalent aperture, JRC is joint roughness coefficient, E is joint mechanical aperture, θ is angle between joint sets, s is joint spacing, l is joint trace length, τ_0 and μ are yield stress and viscosity of cement grout, P is the grout pressure, and t is the grouting time. The value of hydraulic aperture is related to fracture roughness and mechanical aperture.

2.4.2.4 Rahmani's method

In practical grouting, the injection pressure is normally controlled by adjusting the injection flow rate, so some researchers proposed that the injection flow rate was constant during the grouting process; then, in their models, the injection pressure was increasing with time. Therefore, the corresponding stop criteria became the filling of all fractures in a joint system or a preset pressure limit (Rahmani, 2009). Other assumptions are the same as the models discussed above, namely, the grout is taken as Bingham flow, and the flow is laminar and not compressible. The equations required to derive grout distribution in a fracture network were continuity equations and force balance equations, which will be described in the following sections.

Based on force equilibrium between grouting pressure and shear stress, the flow rate per unit width of fracture is (Rahmani, 2009),

$$q = -\frac{2\Delta PH^3}{3\mu_B L} \left(1 + \frac{1}{2}\Psi^3 - \frac{3}{2}\Psi \right) \quad (2.16)$$

where $\Psi = -\tau_0 L / (H\Delta P)$ and $H = b/2$ is fracture half aperture.

Eq. (2.16) can be modified as cubic equation for Ψ ,

$$\Psi^3 + \lambda \cdot \Psi + 2 = 0 \quad (2.17)$$

Thus, the relationship between pressure drop and flow rate can be expressed as,

$$\Psi = 2\sqrt{\frac{-\lambda}{3}} \cdot \cos \frac{\cos^{-1} \left(\sqrt{27/-\lambda^3} \right) + \pi}{3}, \quad \lambda = -3(1 + q\mu_B / \tau_0 H^2) \quad (2.18)$$

Then, Eq. (2.16) to (2.18) were used to calculate the real time grout penetration in a specific rock fracture based on iteration.

2.4.3 Numerical models

Numerical models have been used for grouting simulation for a long time (Rahmani, 2009; Carter et al., 2012; Saeidi et al., 2012). Generally, they are used to solve complex problems like single fractures with rough fracture surfaces (Hanssen, 2013; Lavrov, 2013; Ping and Xueqing, 2015; Wang et al., 2016), where the fractures are of self-affine fractal surfaces; therefore, the characteristics of fluid flow inside rock fractures are very complicated, it is hard to derive simple models to solve this kind of problems, like analytical or semi-analytical models. In addition, to predict the range of grout penetration in fractured rock masses, where rock fractures form an interconnected network, which may be 2D or 3D (Tsang and Tsang, 1987; Munier, 1993; Lee et al., 2000; Yang et al., 2002; Rahmani, 2009; Klimczak et al., 2010; Carter et al., 2012; Saeidi et al., 2012; Zhao et al., 2014). Normally, the geological conditions of practical rock masses are unknown, so the characteristics of a fracture network, like fracture aperture, length, width, density, dip and orientation, etc., should be determined based on some assumptions that they follow certain probability distribution, like lognormal, normal, or gaussian distribution (Tsang and Tsang, 1987; Rahmani, 2009), or the distribution of rock fractures is just obtained through Monte Carlo method (Yang et al., 2002), or based on some statistical data from field mapping (Zhao et al., 2014).

Once the distribution of rock fractures is known, algorithm can be developed to generate the corresponding numerical models for grouting simulation, where many programs can be used, like commercial software UDEC (Saeidi et al., 2012), Discrete Fracture Model (DFM) (Zhao et al., 2014; Alghalandis, 2017), and other self-developed programs (Yang et al., 2002; Rahmani, 2009; Carter et al., 2012), together with some algorithm for fluid model.

2.5 Grouting test and experiments

Laboratory test or experiments are an important part of grouting research, including investigation of the grouting process and grouting materials. Regarding the grouting process, a lot of experiments or tests have been conducted, including 1D channel model, where the grout penetration in rock fracture is taken as fluid flow between two smooth parallel plates (Eriksson, 2002; Fujita et al., 2012); 2D radial model, where grout spread around the injection hole between two parallel plates (Mohammed et al., 2015); and 3D fracture network, where fracture joints are generated by crashing rock samples (Guo et al., 2015), in terms of the dimension of experimental models. Most of the models are simple or/and of small-scale, which can, to some extent, help validate some 1D or 2D theoretical models, but cannot contribute to substantial development of grouting theory or/and practice.

Therefore, many practical factors were included for grouting investigation, like the impact of surface features (fracture roughness) on grout penetration or other fluid transportation, where significant channeling effect was found and the concept of equivalent concept was proposed (Lee and Cho, 2002; Chen and Horne, 2006; Akhavan and Rajabipour, 2012; Develi and Babadagli, 2015; Singh et al., 2015), but most of them were of small scale modeling, and the influence of filling materials was not considered, which is of significant influence, at least increasing the resistance to grout spread; another factor is the existence of flowing water, which can greatly affect the grouting process and sealing effectiveness (Andersson, 1999; Gailing et al., 2011; Sui et al., 2015), as flowing water can greatly change the trace of grout penetration and dilute the cement grout, thus increasing the difficulty and cost of grouting. Apart from laboratory experiments and tests, there are also grouting field test conducted in cooperation with theoretical analysis (Fransson, 2001; Gothäll and Stille, 2009).

2.6 Grouting technical terms

There are several concepts involved in the grouting process, among which grouting pressure, injection flow rate, grout take, stop criteria and mechanism, and sealing requirements are key concepts.

2.6.1 Grouting pressure

In fact, grouting pressure is a by-product of the grouting process, as grout is injected into rock masses through the grouting pump under a certain pumping rate. Grout should be pushed into rock fractures via relatively high pressure to overcome water pressure and shear stress related resistance force, due to the existence of fracture roughness, and certain water pressure; and increase of grouting pressure corresponds to increase of pumping rate. With increasing grouting pressure, jacking of the rock fractures could occur, which might lead to undesirable grout take.

There were some research on the criteria for determining the right range of grouting pressure. Houlsby suggested that the pressure provided must be higher than the weight of rock mass above the region affected by grouting to make sure that the grout penetration reach a required range (Houlsby, 1990). Gustafson and Stille pointed out that the injection pressure should be higher than the groundwater pressure ($P_g > 2P_w$), but lower than the minimum in situ stress ($P_g < \sigma_s$). Between these two limits, the pressure can be determined arbitrarily, but the range of grout penetration is positively correlated to the overpressure (Gustafson and Stille, 1996).

Barton and Quadros found that there was difference between injection pressure measured at the injection lance and the practical grout pressure in rock fractures directly intersected by the injection hole, in the immediate vicinity of which a significant pressure drop would occur (Barton and Quadros, 2003).

It is commonly suggested that high pumping pressure could be used to achieve a good penetration and high productivity (Gothäll and Stille, 2010). However, some scholars indicated that the fracture deformation could strengthen grout penetration in sealable fractures but weaken the penetrability of fine fractures, which implies that high

pressure would not benefit grout diffusion in small fractures when there are fractures with larger apertures (Gothäll and Stille, 2009; Gothäll and Stille, 2010).

2.6.2 Injection flow rate

The injection pressure is directly correlated to the injection flow rate, which is subjected to the pumping rate. Normally, the pumping rate can be easily known from grouting parameter setting, so the real-time pressure variation is recorded. The signal fluctuation recorded can be used for determining the performance of grouting pump. Meanwhile, the pressure would be increasing due to the increase of resistance during the grouting process, thus the injection rate should be properly chosen to ensure that the injection pressure does not go beyond the excessive pressure. Moreover, the injection pressure is also positively proportional to the size of the delivery system: the injection pressure is positively correlated with the diameter of the delivery pipe. Therefore, attention should also be paid to the size of the delivery system, which should be chosen properly to exclude the possibility of exceeding the limit pressure (Widmann, 1996; Warner, 2004).

Moreover, it is found that many grouting materials, especially the cement grout, have the tendency to accumulate or “grow up” on the surface of the delivery pipe, thus leading to continual decrease of pipe diameter (Warner, 2004). As a result, the injection pressure would increase due to worsening conditions of the injection system, rather than the increase of effective grout penetration. Therefore, we should pay attention to the importance of injection pressure, whereas the function of which should not be overestimated, namely, a certain injection pressure reached cannot guarantee the achievement of certain grouting quality. In summary, the variation of injection pressure should be given together with the corresponding injection flow rate, otherwise specifying only the injection pressure is meaningless.

2.6.3 Grout volume

Grout volume is the total volume of grout injected into one injection hole, and it is a very important parameter. First, the grout volume is directly correlated to the cost of grouting; hence, it is very sensitive to all stakeholders, especially to the onsite engineers. As large volume of grout take may indicate that there were bad geological

conditions, like rock faults or large fracture joints, where rock masses are directly or indirectly intersected by the injection hole, especially when these conditions had not been detected by probe holes or site investigation; it may also indicate the grout flow enters some open areas, like the surface of the ground or even somewhere on the wall of a tunnel or rock cavern. Practically, the recordings of injection pressure should be taken as a reference, bad geological conditions may be encountered if large volume of grout has been used, but the corresponding injection pressure does not increase that much (Warner, 2004). Generally, action should be taken if abnormal situation occurs, like checking of leakage from the tunnel wall, or the Lugeon value in different boreholes or probe holes.

On the other hand, the grout volume is a critical value used by the method of Grout Intensity Number (GIN), which equals the product of injection pressure and grout volume. GIN is generally used to help determine the time to stop grouting, when the grouting path curve (relationship between grout take and injection pressure) touches the GIN limitation curve (Lombardi and Deere, 1993; Widmann, 1996; Rombough and Shuttle, 2006). Therefore, as mentioned above, special attention should be paid to the occurrence of unusual or abnormal situations, when grouting should be stopped in advance rather than under predefined criteria, or special measures should be taken, to ensure the grouting quality at the lowest cost.

2.6.4 Grouting targets

According to different project requirements and geological conditions, the corresponding grouting targets can be varying, some may have strict requirement due to special functionality, like storage of oil or nuclear waste, etc. Several sealing targets are listed in Table 2.3.

Table 2.3 Sealing targets for grouting of jointed rock fractures (Widmann, 1996)

Grouting targets	Description
Permeability reduction	Water sensitive structure like dam and tunnel, the seepage or water inflow should be effectively controlled, where the anti-erosion property of grouting material is essential and significant

Strengthening	Increase the strength of fault rock; the grouting material should have adhesive strength, which is affected by the condition of surfaces surrounding grout
Compressibility reduction	Compressibility should be controlled when certain structure is sensitive to settlement under external force

Though three principal grouting targets have been described in Table 2.3, the detailed standards for initiating grouting and stopping grouting still differ from one project to another. Grouting design should be made according to specific project requirement, and onsite engineers should conduct and stop grouting accordingly, referring to both grouting design and onsite monitoring and engineering experience (Emmelin et al., 2004; Gustafson and Stille, 2005; Eklund and Stille, 2008; Fransson, 2008).

2.6.5 Stop criteria

According to grouting practice, there are mainly three forms of stop criteria in terms of grout penetration, grout flow, and grout take, respectively. For grout penetration based stop criterion, the grouting time estimated was uncorrelated to the rock types if the flow rate was not higher than the grouting equipment capacity; and the estimated grout volume and flow rate would increase with increasing rock mass permeability. For the stop criterion based on grout flow rate, it was found that there was positive correlation between grouting parameters (grout spread, grouting time, and grouted take) and the permeability of rock mass. For grouted volume based stop criterion, the rock permeability is negatively correlated to grout spread, grouting time, and final flow rate of cement grout, if the maximum grouted take is more than the required volume.

The most important stop criterion might be the one based on penetration range, even though it is very hard to be determined accurately in grouting practice, other than in theoretical analysis. Gustafson and Stille proposed an analytical solution as a robust stop criterion, to ensure the sealing efficiency and avoid consuming extra grouting materials and time (Gustafson and Stille, 2005). Thereafter, grouting was taken as complete when the grout spread in the fracture with the smallest aperture was larger than a specific target value or the grouting range of fracture with the largest aperture is more than a certain limiting quantity (Kobayashi and Stille, 2007; Kobayashi et al.,

2008). Moreover, Rafi found that the grouting time and the material properties could be optimized by using this stop criterion, which could be more time and cost efficient if the corresponding geological conditions were enough (Rafi, 2013).

Three different stop criteria come into effect simultaneously when there is no effective grout penetration, due to clogging, resistance, and filtration (Axelsson et al., 2009). Clogging occurs when grout grains are unable to pass through rock fracture with certain aperture, when the grains in the cement grout are approximately 1/3 or more than the available aperture. Resistance happens when the maximum grout penetration is reached and there is a force balance between grout pressure and shear stress from fracture walls nearby, when the capacity of grouting pump is reached. The maximum penetration length was usually calculated based on this stop mechanism. Filtration is a phenomenon caused by successive separation of water and cement particles. Filtration could occur once a grout flow encounters a constriction with aperture less than a certain critical value, when the separated grout grains gradually forms a filter cake and blocks the constriction and pathway finally.

It can be found that the fracture aperture is critical to the stop criteria for grouting. Therefore, laboratory tests were conducted to define fracture aperture of two levels. The upper level is critical fracture aperture b_{crit} , and filter cake would start forming for aperture less than b_{crit} ; the lower level is the minimum aperture b_{min} , and no material could pass through the constriction area when the aperture is less than b_{min} (Eriksson, 2002). It was stated that a crack was groutable if the aperture is three times larger than D_{95} of the cement used for grouting (Eklund and Stille, 2008). Brandberger et al. summarized the conclusions from different researchers on apertures of groutable fracture in terms of grain sizes (Brandberger et al., 1998).

2.7 Practical grouting execution

Grouting is always regarded as a handicraft. Typically, uncertainties are always found in actual conditions and geotechnical constructions. Thus, the practical guidance should be easy to follow for on-site the grouting execution. This section mainly focusses on key parts of grouting as disciplines related to rock grouting are tremendous. Before the construction, the necessity of grouting should be verified.

Once grouting is considered necessary, all the aspects related to the grouting should be studied thoroughly to perform the grouting satisfactorily.

2.7.1 Pre-grouting principle

The pre-grouting is an empirical and iterative observational design, where monitoring, recording and verification are indispensable. The design-feedback progress can be described as follows:

- Once the ingress requirements are defined, the project data and all available information on geological and hydrogeological conditions can be analyzed. This often includes indicative calculations for the potential ground water ingress under typical situations. Based on the empirical data from previous projects, a complete pre-grouting method statement can be compiled, which presents how to execute the pre-grouting to achieve the required residual ingress in detail.
- The method statement of the preliminary grouting design is only a prognosis. The actual water ingress can be measured during the excavation. Comparison between the targeted water ingress and real-time measurement can indicate whether the inflow requirement is satisfied or not. If the requirement is achieved, the work can be continued without any changes and the ingress measurements can be carried out continuously for further verification.
- If the volume of measured water ingress is too large, it needs to be figured out how to improve the tightness of the measured section and how to modify the design to achieve the requirement for the similar sections not yet excavated. The improved work may have to be executed step by step via local post-grouting till the ingress values are satisfied.

It is important to transform the general requirements of permissible water ingress/velocity/gradients into grouting specifications for the preliminary grouting design. The correlation between hydraulic conductivity and thickness of the grouted zone may be established. Then, the grout mix with adequate penetrability and suitable rheological properties should be defined. After that, the geometry of grouting sequences with proper grouting pressure limitations and stop criteria are needed for

assessment. If modifications are needed to achieve the target residual inflow limits, the trigger values set for control holes will be reviewed and may be modified based on the ground response and grout fan performance. The overall grouting strategy and all execution parameters should also be evaluated. A preliminary flowchart of the grouting design presented by Stille (2012) is given in Figure 2.7.

Figure 2.8 Part of the preliminary grouting design (Stille, 2012)

2.7.2 Grouting determination

2.7.2.1 Grouting strategy

Pre-grouting is time-critical work for most tunnels, which inevitably leads to the delay of excavation. The value of water inflow is used to determine if grouting is required or not. If the measured water inflow is not larger than the allowable maximum water inflow, grouting can be neglected to save time and reduce cost; otherwise, grouting should be carried to meet the strict water inflow requirement.

Figure 2.9 Flowchart of determination of grouting (Garshol et al., 2012).

Practically, systematic grouting and grouting are two alternatives for grouting strategy. When the allowable limit of water inflow is not that low, the grouting target can be inevitably achieved, so systematic grouting is appropriate. Pre-grouting will be performed all along the tunnel. The grouting materials are chosen according to the water ingress from the probe holes. The trigger value regarding measured water inflow is critical to the initiation of grouting during underground construction, as illustrated in Figure 2.9.

2.7.2.2 Probe holes

The flow during the tunnel/cavern excavation is random and varies with different types of rock masses. The probe holes hitting the water bearing joints can lead to water inflow. Determination of the number of probe holes depends on the characteristics of underground structures, the groundwater regime, the risk involved and the required tightness. The Norwegian Group for Rock Mechanics (2000) recommended that the number of probe holes should not be less than two in most cases and can be four or more in high-risk areas. Practically, six to seven probe holes are common during tunnel excavation (Holmøy, 2008). The length of probe holes can be flexibly adapted according to practical scenarios, affected by the equipment, shift sequences, round length, ground conditions and water head, etc., which is normally less than 30m in length covering 4 - 5 blasting rounds. The diameter of drilling rods is typically larger than 48mm (e.g., 48mm, 51mm, 54mm, etc.).

Figure 2.10 Typical probing and pre-grouting setting (Norwegian Tunneling Society, 2014)

2.7.2.3 Allowable water ingress

The water inflow criteria are affected by various factors such as purpose, location, overburden and structure dimension, consequences of leakages, safety issues,

permanent and temporary functional requirements, and financial aspects (Norwegian Tunnelling Society, 2011). If the tunnel is constructed in the area with lowering ground water level, serious consequences can be caused. Under this condition, the water inflow requirement could be strict. In some rural areas with a surplus of water from heavy rainfall, much higher levels of water ingress can be accepted. Particularly, the function of tunnel can affect the acceptable level of water ingress. Tunnels for conveying water, such as headrace and tailrace tunnels to hydropower plants, can tolerate quite high ingress.

For the final water ingress, it is difficult to achieve the specified limit. It strongly depends on the required water tightness level rather than the level of water ingress before any grout injection. To achieve the absolute water tightness, pre-grouting may be quite costly and time consuming. The experience indicates that an attempt to seal the last 10 % of potential water inflow may cost more than the total cost for sealing off the first 90 % (BASF, 2011). Therefore, the required amount of water reduction must be reasonably considered. A greater reduction in seepage rather than a rationally required value will result in unnecessarily high cost. In many cases, tunnel draining is carried out with water pumping. However, water pumping can be expensive when high volumes of water (or high pumping heads) are encountered (or required). For sub-sea tunnels where there is unlimited sea water supply, the water inflow is often required below 30 L/min/100 m of the tunnel in Norway (Blindheim and Oevstedal, 2002). Ingress higher than 50 to 100 L/min/100m of tunnel will normally lead to practical problems related to pumping. The residual water inflow requirements used in the Nordic countries and Hong Kong are given in Table 2.4. In addition, the requirements in the cases from Norway and Finland are shown in Table 2.5.

Table 2.4 Inflow requirements in nordic countries and Hongkong

Allowable Water Inflow Limit	Notes
< 2 L/min/100 m (Finnish Standard, Sievanen et al, 2012) 2-4 L/min/100m (Norwegian Standard, NTS, 2011) 2-10 L /min/100m (Norwegian Standard, Grov et al, 2012) 2-5 L/min/100m (Swedish Experience)	For Critical areas in Road/Railway Tunnel
< 20 L/min per 100m (General, Kristin and Nilsen, 2014)	

30 L or above /min/100m (without specific requirements) (Grov et al, 2012) 100m (Subsea Tunnel, Blindheim and Oevstedal, 2002) 10-30 L/min/100m (Subsea Tunnel, Grov et al., 2012)	Subsea Tunnels
Typically, 5–50 L/min/100m (Guidance Note on Pre-Excavation Grouting for Underground Construction in Hard Rock, Association of Geotechnical & Geo-environmental Specialists, Hongkong)	

Based on the above discussion, it can be concluded that 2 L/min/100m to 5 L/min/100m could be used as the water inflow criteria for critical areas in road/railway tunnel projects. 30 L/min/100m can be defined as the water inflow target for subsea tunnel. Water inflow higher than 50 L/min/100m can be used after validation with caution.

Table 2.5 Inflow requirements in tunnel cases

Allowable Water Inflow Limit
< 7-14 L/min/100m (T-Baneringen Metro Tunnel, Stage One, Norway, Excavation completed 2002)
< 10-20 L/min/100m (Lunner Road Tunnel, Norway; Excavation completed 2002)
< 4-16 L/min/100m (Skaugum Railway Tunnel, Norway, Excavation completed 2004)
< 10-20 L/min/100m (Storsand Road Tunnel, Norway, ; Excavation completed 2004)
< 1-2L/min/100m (ONKALO Project, Finland)

2.7.2.4 Trigger value of water inflow

Table 2.6 Trigger value of water inflow from probe holes for grouting corresponding to limit of residual inflow

Limit of Residual Inflow (L/min/100m)	Inflow Measured from probe holes (L/min)		Note
	Single Hole	All Holes	
15	> 1	> 3	Norwegian Tunnel Standard (Slides from Kristin) (For Probe hole not less than 20 m long)
30	> 2	> 6	
50	> 4	> 10	
5–50	0.1-5	-	Guidance Note on Pre-Excavation Grouting for Underground Construction in Hard Rock, Association of Geotechnical & Geo-environmental Specialists, Hong Kong
15	1	3	Harbour Area Treatment Scheme Stage 2A (HATS2A), Hong Kong, Garshol et al. 2014
$\approx 27.8^a$	5	10	Qingdao Subsea Tunnel, Wang X. et al. 2012

Notes*: a-Converted from 0.4 m³/day/m; b-any group of 4 holes

For different levels of allowable water inflow, the measured inflow from the probe hole for initiating grouting is different. Records of these values are collected from some available standards/cases (Table 2.6), which can benefit the correlation study between the grouting trigger values and the targeted residual inflow. A preliminary correlation is formed utilizing data from the Norwegian Subsea Tunnel Standards, Hong Kong (China) and Qingdao (China), as shown in Figure 2.11. The available data for establishing such correlation is rather limited, so more data are needed.

Figure 2.11 Correlation between trigger value of water inflow from probe drilling hole and limit of residual inflow

2.7.3 Grouting design

Grout spread is governed by a few complex relationships among properties of grout mix used, the fracture geometry, and grouting parameters.

2.7.3.1 Drill pattern

The drill pattern includes the number of drill holes, spacing, length, overlap, location, inclination angle, hole diameter, and straightness requirement.

Large continuous defects that facilitate the flow of large amounts of water can be readily filled with a minimum number of relatively widely spaced grout holes. Conversely, it is very hard to seal tight apertures and crevices, so, typically, many closely spaced holes are required (Stille, 2015). The length of the pre-grouting holes is affected by the capability and efficiency of drilling equipment used. The length of grout hole, together with the inclination angle, determine the range of the grouted

zone. The overlap distance between subsequent grout fans is typically between 6m to 10m (Lu et al, 2005). The diameter of grouting holes depends on the drilling rods used, which normally has a diameter not less than 48mm.

Based on the values of grouting parameters indicated in available design codes and projects (Holmoy and Nilsen, 2014; Holmoy, PhD thesis), the length of grouting hole is normally not less than 18m. The length generally ranges from 22m to 24m. Zone depth ranging from 3m to 10m can be considered as a start, and 4m to 6m are the typical values. For the overlap of grouting fans, 5m is normally used as a minimum value, and 10m to 11m could be used in extremely poor areas.

2.7.3.2 Grout Recipes

Rheological properties, quantified by parameters like the yield value and viscosity, will have a significant influence on the grout spread rate. These properties depend on many factors and result from a complex chemical reaction after cement, water and other additives are mixed together (Hakansson, 1993). These factors can be categorised as: cement type, grout type, cement conditions, water-cement ratio, type and content of super-plasticizer, mixer type, mixing time and temperature.

Simple common cement-water grouts can be readily injected into the large defects, while the smaller ones require much slower injection rates and more expensive grout constituents, such as ultrafine cement or perhaps a chemical solution grout. Experiences from past cases and theoretical calculations clearly indicate that grout based on ordinary cement can only produce a sealing effect down to 1 Lugeon (1×10^{-8} m/s) (Stille, 2015). With good mixing and controlled execution, much lower results down to 0.2×10^{-8} m/s can be obtained when micro-fine cement is used (Stille, 2015).

Types of cement and super-plasticizers have important effect on the grout properties, such as the viscosity and yield value. The water cement ratio also has a considerable influence on the flow properties, as the penetrability of grout materials would increase with an increasing w/c ratio (water/cement ratio). However, to avoid bleeding and fingering and to ensure the longevity, the water-cement ratio should be kept as low as possible. Using a modern super-plasticizer, the w/c ratio can be reduced to 1 or even lower, but the grout workability and penetrability remain good. A standard field

mixer may produce different properties compared with that obtained via the mixers in laboratory. The performance of mixers is time-dependent, so, in the field/site, the mixers should be examined during grouting to ensure acceptable performance.

Generally, the micro-fine cement is most commonly used. The w/c ratios equal to 1.0 and 0.8 are often chosen. In each grouting stage, normally, the w/c ratios of grout is not fixed, but vary from thin to thick one.

2.8 Summary

Grouting is a complex process and involves rock fracture, grout, and water. For better understanding and prediction of the grouting process, properties of fractured rock masses and grout should be studied first. For a grouting model, it is hard to predict the range of grout penetration in certain rock masses and the final grouting quality, if the distribution of rock fractures or faults is unknown. Therefore, features of discontinuous rock masses are introduced, including the characteristics of fractures (components of a fracture network), the roughness of fracture surfaces which can increase the resistance to grout flow, the significance of fracture aperture and fracture deformation on grouting, finally, groutability evaluation of certain rock fracture.

With known rock mass conditions, the properties of cement grout are critical to a grouting model. Thus, the rheology of cement grout is introduced, including current popular rheological models used for analytical and numerical analysis, and the time dependent behaviors and groutability are also described. All these aspects can help build predictive models of higher accuracy.

Then the rheological models of cement grout and the characteristics of rock fractures can be used to develop theoretical or empirical model for grouting prediction. It is found that grout should be injected into rock masses under certain pressure to overcome the water pressure (due to high water table in neighboring rock masses) and shear stress (due to fracture roughness). Moreover, the injection pressure should be much higher than the water pressure measured in certain rock masses to ensure good grouting efficiency, as grout would not flow unless the shear stress caused by excess pressure is higher than the yield stress.

Generally, there are different sealing requirements determined before grouting according to some grouting codes or engineering experiences, based on which a stop criterion can be chosen: grouting can be stopped when a force balance is reached, or all fractures involved are grouted, or stop when the grouting pressure reach a specific value when filtration occurs under certain cement grout.

Chapter 3 One-dimensional Penetration in Fracture Channel

3.1 Introduction

Rock fractures in rock masses are distributed in 3D space, but 3D models (corresponding to 3D fracture networks) for fluid dynamics analysis should be developed, which are of high computation burden, thus the process could be time consuming. Therefore, 2D fracture network models are usually used for investigating the characteristics of grout penetration in fractured rock masses. Whereas 2D models are only simplifications of real situations, so the conclusions thus obtained, though cannot provide exact prediction, can provide guidance for practical grouting. One-dimensional fracture channel is the basic component of a 2D fracture network, so grout penetration in a 1D fracture channel becomes the emphasis of many researchers. Currently, there is a gap between academic research and practical engineering application regarding some boundary conditions: grout injection pressure is generally taken as constant during the grouting process for the convenience of study, but it is positively correlated with injection rate during the grouting process, and the maximum injection pressure is recorded together with an injection rate (Warner, 2004). Namely, the grout injection rate is the one that can be controlled in practical grouting operation, for both automatic and hand pump. Therefore, grouting under both constant injection pressure (following research practice) and constant injection flow rate (following engineering practice) will be introduced and discussed in this chapter. A simplified model for the fast evaluation of grout penetration in a single fracture channel is proposed, which can be used under certain condition. Moreover, the occurrence of fracture deformation under high grouting pressure can substantially affect grout spread in fractures, especially for grouting test in laboratory, so a model is developed to consider the coupling effect of fracture deformation and grout penetration.

3.2 Constant injection pressure method

Strictly speaking, there is no constant injection pressure method in real grouting practice, but it has been used as a precondition for grouting prediction in many theoretical models, where the grout penetration can be easily derived.

3.2.1 Governing equation for grout flow in a single fracture

As mentioned in Chapter 2, currently, constant injection pressure method is one method for grouting simulation, this chapter will give details by deriving the relevant governing equation. The following assumptions should be made concerning grout flow,

1. It is treated as laminar flow;
2. It is incompressible;
3. It is a kind of Bingham flow, and no variations of fluid properties are considered during short time grouting process in the current work.

Figure 3.1 Force diagram for fully developed grout flow in channels

As shown in Figure 3.1, there is force balance between pressures on two lateral surfaces and shear stress on upper and lower surfaces,

$$2Pw dy - 2 \left(P + \frac{\partial P}{\partial x} dx \right) w dy - 2d\tau dx w = 0 \quad (3.1)$$

Where, P and $\partial P/\partial x$ are grout pressure and pressure gradient along the direction of grout flow, τ is shear stress, and w is channel width.

Simplify Eq. (3.1), it can be found that $d\tau = (-\partial P/\partial x) dy$, and integrate $d\tau$ with respect to dy , the relationship between shear stress and pressure gradient can be derived,

$$\tau = -\frac{\partial P}{\partial x} y \quad (3.2)$$

As cementitious grout used for grouting is usually taken as Bingham flow, the relationship between shear stress τ and velocity gradient $\partial u/\partial y$ along positive

Y can be expressed by Eq. (2.1). τ_0 is the shear stress at y_0 , which equals half width of Bingham plug, as shown in Figure 3.1.

Substitute Eq. (3.2) into (2.1) under non-zero shear rate,

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \left(\tau_0 + \frac{\partial P}{\partial x} y \right) \quad (3.3)$$

Thus, the flow velocity above Bingham plug in the upper half can be calculated through integrating Eq. (3.3) within interval $[y, b]$,

$$\begin{aligned} u_{\tau \geq \tau_0} &= \int_y^b \frac{\partial u}{\partial y} dy = \int_y^b \frac{1}{\mu} \left(\tau_0 + \frac{\partial P}{\partial x} y \right) dy \\ &= \frac{\tau_0}{\mu} (-y + b) + \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) (-y^2 + b^2) \end{aligned} \quad (3.4)$$

The velocity of Bingham plug at $y = y_0$ can be described as,

$$u_{\tau < \tau_0} = \frac{\tau_0}{\mu} (-y_0 + b) + \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) [-y_0^2 + b^2] \quad (3.5)$$

Therefore, integration of velocity over half aperture, average velocity of grout flow can be derived,

$$\bar{u} = \frac{\int_0^{y_0} u_{\tau < \tau_0} dy + \int_{y_0}^b u_{\tau \geq \tau_0} dy}{b} = \frac{1}{b} \left[\frac{\tau_0}{\mu} \left(\frac{y_0^2}{2} - \frac{b^2}{2} \right) + \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} \right) \left(\frac{2y_0^3}{3} - \frac{b^3}{3} \right) \right] \quad (3.6)$$

According to Eq. (3.2), $\tau_0 = (-\partial P / \partial x) y_0$ when $y = y_0$, which can be substituted into Eq. (3.6), thus the final form of the average velocity can be expressed as,

$$\bar{u} = \frac{1}{3\mu b} \left(-\frac{\partial P}{\partial x} \right) [b^3 - 3b^2 y_0 + 4y_0^3] \quad (3.7)$$

After obtaining average velocity in Eq. (3.7), it is not hard to derive the corresponding temporal grout spread and grout take.

Meanwhile, grout flow will stop when $\bar{u} = 0$ and $y_0 = b$. This indicates the maximum spread $L_{\max}$ will be achieved. The detailed value of $L_{\max}$ is to be derived.

As the stop of grout flow also means that the relevant velocity gradient is zero at all locations. Thus, the velocity at any location is equal to the velocity at the channel surface, which is zero. If the maximum absolute velocity gradient $|\partial u / \partial y|_{\max} = 0$,

then at any location $\partial u / \partial y = 0$. Thus, the maximum value is reached at the channel surface,

$$\left| \frac{\partial u}{\partial y} \right|_{\max} = \frac{1}{\mu} \left| \tau_0 + \frac{\partial P}{\partial x} y \right|_{\max} = \frac{1}{\mu} \left| \tau_0 + \frac{\partial P}{\partial x} y_{\max} \right| = \frac{1}{\mu} \left| \tau_0 + \frac{\partial P}{\partial x} b \right| \quad (3.8)$$

Therefore, it can be found that $-\partial P / \partial x = \tau_0 / b$ when Eq. (3.8) equals zero. Meanwhile, when grout flow stops, the relevant pressure gradient would be $\partial P / \partial x = -P_0 / L_{\max}$, which can be substituted into Eq. (3.8) to obtain the maximum grout penetration $L_{\max}$,

$$L_{\max} = P_0 b / \tau_0 \quad (3.9)$$

where P_0 is the relevant grout injection pressure, which is assumed to be constant.

3.2.2 Algorithm for numerical calculation

In this section, the grout injection pressure is set as constant, this means that all fractures originating from the injection node have the same pressure drop at a certain time. However, the pressure drop per unit length is not known in advance, so an initial value of grout penetration in each fracture linked to the same node should be assumed, thus the final penetration length can be obtained based on iteration. Here, Newton's method is used for iterative calculation, and the detailed formula for calculation is modified from Eq. (3.7), where the average velocity is $\bar{u} = I / \Delta t$.

$$\frac{I}{\Delta t} = -\frac{1}{3\mu b} \left(\frac{\partial P}{\partial x} \right) [b^3 - 3b^2 y_0 + 4y_0^3] \quad (3.10)$$

Substitute $\partial P / \partial x = -P_0 / I_{n+1}$, $y_0 = \tau_0 I_{n+1} / P_0$, $I = I_{n+1} - I_n$, and $\Delta t = t_{n+1} - t_n$ into the right side of Eq. (3.10). The following expression can be derived after some simplification,

$$v(I_{n+1}) = \frac{P_0 b^2}{3\mu I_{n+1}} \left[1 - \frac{3\tau_0 I_{n+1}}{P_0 b} + 4 \left(\frac{\tau_0 I_{n+1}}{P_0 b} \right)^3 \right] \quad (3.11)$$

As the initial value of $I_0 = 0$ correlated to the initial time $t_0 = 0$, the penetration length at time can be obtained from the following iteration accordingly,

$$I_n = I_{n+1} - v(I_{n+1}) \cdot (t_{n+1} - t_n) \quad (3.12)$$

3.2.3 Mathematical model for grouting of a fracture network

For grout flow in a fracture network, the grout penetration in each fracture can be calculated if the relevant pressure head at each node is known. Figure 3.2 demonstrates the concept of a simple fracture network, with four fractures of different fracture apertures, which is represented by red lines of different width.

Figure 3.2 Part of a fracture network with node pressure as unknowns

In Figure 3.2, P_0 is the injection pressure (taken as constant), and the other pressures from P_1 to P_4 require to be calculated. This process is solved based on mass conservation at each node, namely the mass of grout flow into a node is equal to that out of the node,

$$\sum_{i=1}^n Q_i = 0 \quad (3.13)$$

where the flow rate in each fracture can be derived based on Eq. (3.8) as $Q = 2\bar{u}wb$.

The flow chart for predicting grout penetration in a fracture network is presented in Figure 3.3, where detailed guidance from iteration within each time step to final criteria for stopping grouting is given.

Figure 3.3 Flow chart for simulating grouting of a fracture system

3.3 Constant injection flow rate method

It is called constant injection flow rate method mainly in comparison with constant injection pressure method, as presented in Section 3.2. An approximately constant injection rate is achieved under certain pumping rate in practical grouting operation, so it is of great significance to see, more practically, the characteristics of grout penetration within a 1D fracture channel, and in a complex fracture system.

3.3.1 Governing equation for grout flow in a single fracture

For constant injection flow rate method, the derivation of velocity and average velocity is the same as Eq. (3.6) in Section 3.2.1, so the injection flow rate can be expressed as,

$$q = \bar{u}b = -\frac{b^3}{3\mu} \left(\frac{\partial P}{\partial x} \right) \left[1 - 3 \left(\frac{y_0}{b} \right) + 4 \left(\frac{y_0}{b} \right)^3 \right] \quad (3.14)$$

Here, the pressure gradient is $\partial P / \partial x = \Delta P / L$, and let $\Psi = y_0 / b$ with $y_0 = -\tau_0 L / \Delta P$, then substitute Ψ into Eq. (3.14),

$$q = \frac{b^2}{3\mu} \left(\frac{\tau_0}{\Psi} \right) [1 - 3\Psi + 4\Psi^3] \quad (3.15)$$

Rewrite Eq. (3.15) into cubic equation of Ψ , after some simplification we can derive,

$$\Psi^3 + \left(-\frac{3}{4} - \frac{3\mu q}{4b^2\tau_0} \right) \Psi + \frac{1}{4} = 0 \quad (3.16)$$

Assuming $\lambda_1 = (-3/4 - 3\mu q / 4b^2\tau_0)$ and $\lambda_2 = 1/4$, then Eq. (3.16) becomes

$\Psi^3 + \lambda_1 \Psi + \lambda_2 = 0$. According to the principle for solving cubic polynomial equation, here the relevant discriminant $\Delta = [(-\lambda_2/2)^2 + (\lambda_1/3)^3] < 0$. Therefore, the root of Eq. (3.16) is,

$$\begin{cases} \Psi_1 = 2\sqrt{-\lambda_1/3} \cos\{\alpha/3\} \\ \Psi_2 = 2\sqrt{-\lambda_1/3} \cos[(\alpha + 2\pi)/3] \\ \Psi_3 = 2\sqrt{-\lambda_1/3} \cos[(\alpha - 2\pi)/3] \end{cases} \quad (3.17)$$

Where, $\alpha = \cos^{-1} \left[-\sqrt{-27/64\lambda_1^3} \right]$ and $\alpha \in [\pi/2, \pi]$.

As $\alpha \in [\pi/2, \pi]$, the range of angle Ψ in Eq. (3.17) can be estimated as,

$$\begin{cases} \alpha/3 \in [\pi/6, \pi/3] \\ (\alpha + 2\pi)/3 \in [5\pi/3, \pi] \\ (\alpha - 2\pi)/3 \in [-\pi/2, -\pi/3] \end{cases} \Rightarrow \begin{cases} \Psi_2 < 0 \\ \Psi_1 > \Psi_3 > 0 \end{cases}$$

The negative solution Ψ_2 should not be accepted as a useful solution, as Ψ should be positive to ensure positive pressure drop per unit length $-\Delta P / L = \tau_0 / \Psi b > 0$. Meanwhile, considering the pressure drop $-\Delta P / L = \tau_0 / \Psi b$,

then $\Psi_1 > \Psi_3 > 0$ means that the relevant pressure drop corresponding to Ψ_3 is larger than Ψ_1 for the same penetration length L . If a lower pressure can lead to a penetration length of L , a higher pressure can make it. Therefore, Ψ_1 is taken as the only feasible root of Eq. (3.16),

$$\begin{cases} \Psi = 2\sqrt{-\lambda_1/3} \cos(\alpha/3) \\ \alpha = \cos^{-1} \left[-\sqrt{-27/64\lambda_1^3} \right] \\ \lambda_1 = -3/4 - 3\mu q / (4b^2\tau_0) \end{cases} \quad (3.18)$$

3.3.2 Mathematical model for grouting in a fracture network

For grouting of a fracture network, as shown in Figure 3.4, grout penetration in each fracture at any time can be calculated if the relevant grout flow rate in individual channel can be determined. This is an iterative process, since the penetration length and pressure drop are interconnected. To solve these two series of unknowns, two categories of equations are involved: continuity equation and pressure drop equation.

Figure 3.4 Schematic of a fracture network

Considering $(n+1)$ fractures connected to a node, among them one fracture is for inflow and other n fractures are for outflow. Three kinds of relationship between flow rate and penetration length must be satisfied,

1. Continuity equation at network node

$$Q_{ij} = \sum_{N=k}^{k+n-1} Q_{jN} \quad (3.19)$$

where Q_{ij} and Q_{jN} are flow rates in fracture $i-j$ and $j-N$, n is the relevant fractures connected to node j .

2. Continuity equation in fracture network

$$L_{ij}(t) = \begin{cases} L_{ij}(t-\Delta t) + \frac{Q_{ij}}{A_{ij}} \Delta t & L_{ij}(t) < L_{ij} \\ L_{ij} & L_{ij}(t) \geq L_{ij} \end{cases} \quad (3.20)$$

where $L_{ij}(t)$ and L_{ij} are grout penetration length in fracture $i-j$ at time t and the length of fracture $i-j$, respectively.

3. Pressure drop in fractures connected to one node

$$\Delta P = f(Q_{jm})L_{jm}(t) = f(Q_{jm1})L_{jm1}(t) = \dots = f(Q_{jm+n-1})L_{jm+n-1}(t) \quad (3.21)$$

If $L_{jk}(t) > L_{jk}$, the pressure drop is $\Delta P = f(Q_{jk})L_{jk} + f(Q_{kN})L_{kN}(t)$, where $f(Q) = \Delta P/L = -\tau_0/b\Psi$, and Ψ can be calculated based on Eq. (3.18).

3.3.3 Algorithm for numerical prediction

It can be found from Section 3.3.2 that there are $2n$ equations corresponding to n network fractures, which also introduce $2n$ unknowns, including the penetration length and the flow rates in network fractures. As relationships involved in the above equations are nonlinear, iterative methods should be used for equation solving. In this section, Newton's method is adopted to solve the above $2n$ equations concerning the range of grout penetration and grout flow rate, and determination of initial values is very important, which help greatly reduce the computation time. Here, initial values of new time step are kept as the same with values of previous time step.

The mechanism of Newton's method is as follows,

$$[x_{N+1}] = [x_N] - [f'(x_N)]^{-1} [f(x_N)] \quad (3.22)$$

where,

$[x_{N+1}]$ is the solution vector after N times of iteration;

$[x_N]$ is the solution vector after $(N - 1)$ times of iteration;

$[f'(x_N)]^{-1}$ is Jacobean matrix of all $2n$ equations corresponding to solution vector after N times of iteration. When N is zero, it is Jacobean matrix related to initial assumption;

$[f(x_N)]$ is the equation vector of all $2n$ equations correlated to solution vector after N times of iteration; When N is zero, it is an equation vector related to initial assumption.

In the present work, Eq. (3.22) is a matrix equation for a node shared by n fractures. The detailed matrix form can be expressed as follows,

$$\begin{bmatrix} x_{1N+1} \\ \vdots \\ x_{nN+1} \\ \vdots \\ x_{2nN+1} \end{bmatrix} = \begin{bmatrix} x_{1N} \\ \vdots \\ x_{nN} \\ \vdots \\ x_{2nN} \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial x_{1N}} & \dots & \frac{\partial f_1}{\partial x_{nN}} & \dots & \frac{\partial f_1}{\partial x_{2nN}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_{1N}} & \dots & \frac{\partial f_n}{\partial x_{nN}} & \dots & \vdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial f_{2n}}{\partial x_{1N}} & \dots & \dots & \dots & \frac{\partial f_{2n}}{\partial x_{2nN}} \end{bmatrix}^{-1} \begin{bmatrix} f(x_{1N}) \\ \vdots \\ f(x_{nN}) \\ \vdots \\ f(x_{2nN}) \end{bmatrix} \quad (3.23)$$

The symbols f_1 to f_{2n} involved in Eq. (3.23) are related to equations defined in Section 4.3.2, and the detailed description is as follows,

1. Continuity equation of flow rate in all fractures connected to one common node

$$f_1 = \sum Q \quad (3.24)$$

2. Grout penetration in fractures

$$f_N = \begin{cases} L(t) - L(t - \Delta t), & L(t) < L_N \\ L(t) - L_N, & L(t) \geq L_N \end{cases} \quad (2 \leq N \leq n+1) \quad (3.25)$$

3. Pressure drop in fractures connected one common node

$$f_N \Big|_{(n+2 \leq N \leq 2n)} = \begin{cases} f(Q_{ij})L_{ij}(t) - f(Q_{ik})L_{ik}(t), & L_{ik}(t) < L_{ik} \\ f(Q_{ij})L_{ij}(t) - f(Q_{ik})L_{ik}(t) - f(Q_{ik})L_{ik}(t) - f(Q_{kl})L_{kl}(t), & L_{ik}(t) \geq L_{ik} \end{cases} \quad (3.26)$$

Regarding variables in the above equations, they are defined as follows,

$$\begin{cases} x_N = Q_{iN} \\ x_{N+n} = L_{iN}(t) \end{cases} \quad (1 \leq N \leq n)$$

Since the relationship between pressure drop and grout flow rate is $f(Q) = -\tau_0/b\Psi$, whose partial derivative with respect to the flow rate Q (flow rate within a channel with unit width) can be expressed,

$$\begin{aligned} \frac{\partial f(Q)}{\partial Q} &= \frac{\partial f(Q)}{\partial \Psi} \left(\frac{\partial \Psi}{\partial Q} \right) = \left(\frac{\tau_0}{b\Psi^2} \right) \left(\frac{\partial \Psi}{\partial \lambda_1} \right) \left(\frac{\partial \lambda_1}{\partial Q} \right) \\ &= \left(\frac{\tau_0}{b\Psi^2} \right) \left[\sqrt{\frac{-1}{3\lambda_1}} \cos(\alpha/3) - \frac{2}{3} \sqrt{\frac{-\lambda_1}{3}} \sin\left(\frac{\alpha}{3}\right) \left(\frac{-64\lambda_1^3}{64\lambda_1^3 + 27} \right) \left(\frac{-3}{16} \sqrt{\frac{27}{-\lambda_1^5}} \right) \right] \left(\frac{-3\mu}{b^2\tau_0} \right) \end{aligned} \quad (3.27)$$

3.4 Simplified constant injection pressure method

The computation time taken for simulating grout penetration in a single fracture is very short, but it takes quite long for predicting the grouting process of a fracture network consisting of hundreds of fractures. It is of interest to find out whether there are any simplified models with acceptable degree of accuracy. Therefore, a new model considering the influence of the inertia term is derived and solved.

3.4.1 Simplified model for grouting prediction

Figure 3.5 Stress analysis of grout flow (Xiao, 2017)

The coordinate system is shown in Figure 3.5, where the horizontal axis is parallel with the bottom plate. The fluid velocity ranges from zero to the maximum within $0 \leq y \leq y_1$, which makes the current problem similar with heat transfer in a one-dimensional rod with time-varying length (Petrov, 2000). In addition, y_1 here is not

a constant, but a time varying parameter $y_1(t)$, it is written as y_1 in the following sections for simplification.

Two parallel plates with distance of $2b$ are fixed. The velocity of grout flow is taken as zero when $t=0$. According to the Newton's Second Law $F=ma$, the relationship among pressure, shear stress and velocity can be expressed as,

$$Pw dy - \left(P + \frac{\partial P}{\partial x} dx \right) w dy + \left(d\tau + \frac{\partial \tau}{\partial y} dy \right) w dx - w d\tau dx = \rho w dx dy \frac{\partial u}{\partial t} \quad (3.28)$$

The expression on the left-hand side of Eq. (3.28) represents external force applied on the control volume of grout flow, while the expression on the right-hand side represents the inertia force due to acceleration. After some simplifications we have,

$$\left(\rho \frac{\partial u}{\partial t} + \frac{\partial P}{\partial x} \right) = \frac{\partial \tau}{\partial y} \quad (3.29)$$

The flow field is symmetric about the middle surface located at $y=b$ between two boundary plates; meanwhile, the velocity within $y_1 \leq y \leq 2b - y_1$ at any time is the same and equal to $u(t, y_1)$ (velocity at location $y = y_1$), thus we only need to study the characteristics of the velocity variation within $0 \leq y \leq y_1$.

Regarding Eq. (2.1), the relationship between velocity gradient in vertical direction and the shear stress is $\tau = \tau_0 + \mu(\partial u / \partial y)$ when $0 \leq y \leq y_1$; substitute it into Eq. (3.29) and simplify the consequent expression,

$$\rho \frac{\partial u}{\partial t} - \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial P}{\partial x} = 0, (0 \leq y \leq y_1) \quad (3.30)$$

The relevant boundary conditions and initial conditions are as follows,

$$u(0, y_1) = 0, \quad u(t, 0) = 0, \quad u_{y=y_1} = 0, \quad \left. \frac{\partial^2 u}{\partial y^2} \right|_{y=y_1} = -\frac{\tau_0}{b - y_1} \quad (3.31)$$

The last boundary condition is derived as follows: differentiate $\tau = \tau_0 + \mu(\partial u / \partial y)$ with respect to y when $y \leq y_1$, we have $\partial^2 u / \partial y^2 = \partial \tau / \partial y$, which is actually the shear stress gradient along vertical direction, and it is linearly varying within the plug area $y_1 \leq y \leq b$ (Dalmalm, 2004). Thus, the relevant shear gradient is

$\partial\tau/\partial y = \tau_0/(y_1 - b)$, as shear stress at the location $y = y_1$ is $\tau = \tau_0$ and the half width of the plug is $(b - y_1)$.

After all boundary conditions are determined, the problem of solving grout velocity is now a boundary-value problem. To make the problem solving easier, the following dimensionless parameters are introduced (Petrov, 2000),

$$(u^*, P^*, y^*, y_0^*, t^*) = \left(u / \left[-\frac{\partial P}{\partial x} \left(\frac{b^2}{\mu} \right) \right], -\frac{\partial P}{\partial x} \left(\frac{b}{\tau_0} \right), \frac{y}{b}, \frac{y_0}{b}, \frac{\mu}{\rho b^2} t \right) \quad (3.32)$$

After the nondimensionalization, motion equation and boundary condition in Eqs. (3.31) and (3.32) can be expressed as follows,

$$u_t^* - u_{y^* y^*}^* - 1 = 0, (t^* \geq 0, 0 \leq y^* \leq y_1^*) \quad (3.33)$$

$$u^*(t^*, 0) = 0, u_{y^*}^*(t^*, y_1^*) = 0 \quad (3.34)$$

$$u^*(0, y^*) = 0, y_1^*(0) = 0 \quad (3.35)$$

$$u_{y^* y^*}^*(t^*, y_0^*) = -1/P^*(1 - y_1^*) \quad (3.36)$$

To solve Eq. (3.33) with boundary conditions in Eqs. (3.34) to (3.36), separation of variables is adopted here, by assuming $u^*(t^*, y^*) = U(y^*)T(t^*)$ and $y_1^*(t) = 1$. First consider the solution of U , the following boundary-value equation can be obtained,

$$\begin{cases} U'' + \lambda U = 0 \\ U(0) = 0, U'(1) = 0 \end{cases} \quad (3.37)$$

There would be no solution for Eq. (3.37) when $\lambda \leq 0$; whereas its solution can be expressed as follows if $\lambda > 0$,

$$U = C_1 \cos \sqrt{\lambda} y^* + C_2 \sin \sqrt{\lambda} y^* \quad (3.38)$$

According to the boundary condition given in Eq. (3.34), it can be found that,

$$\begin{cases} C_1 = 0 \\ C_2 \sqrt{\lambda} \cos \sqrt{\lambda} = 0 \end{cases} \quad (3.39)$$

There would be no solution if $C_2 = 0$ and $\lambda = 0$, so $\cos \sqrt{\lambda} \equiv 0$ must be satisfied, then $\sqrt{\lambda} = (k - 1/2)\pi$. The solution of U is,

$$U = C_2 \sin(k - 1/2)\pi y^* \quad (3.40)$$

Then the general solution u^* can be expressed as,

$$u^* = \sum_{k=1}^{\infty} T_k(t^*) \sin(\sqrt{\lambda_k} y^*), (t^* \geq 0, 0 \leq y^* \leq 1) \quad (3.41)$$

$$\lambda_k = [(k-1/2)\pi]^2, (k=1, 2, 3, \dots)$$

Though the solution in Eq. (3.41) satisfies boundary condition in Eq. (3.34), it should be further validated to determine the final solution. Substitute Eq. (3.41) into Eq. (3.33),

$$\sum_{k=1}^{\infty} \frac{dT_k(t^*)}{dt^*} \sin(\sqrt{\lambda_k} y^*) + \sum_{k=1}^{\infty} \lambda_k T_k(t^*) \sin(\sqrt{\lambda_k} y^*) - 1 = 0 \quad (3.42)$$

To derive the expression of $T_k(t^*)$, multiply Eq. (3.42) by $\sin(\sqrt{\lambda_n} y^*)$, then integrate the equation with respect to y^* within interval $[0, 1]$,

$$dT_n(t^*)/dt^* + \lambda_n T_n(t^*) - 2/\sqrt{\lambda_n} = 0 \quad (3.43)$$

Considering the initial condition $T_k(0) = 0$, the solution of Eq. (3.44) can be obtained,

$$T_n(t^*) = 2(1 - e^{-\lambda_n t^*}) \lambda_n^{-3/2} \quad (3.45)$$

Therefore, the general solution of Eq. (3.34) is given as follows,

$$u^*(t^*, y^*) = 2 \sum_{k=1}^{\infty} (1 - e^{-\lambda_k t^*}) \lambda_k^{-3/2} \sin(\sqrt{\lambda_k} y^*), (t^* \geq 0, 0 \leq y^* \leq 1), \quad (3.46)$$

$$\lambda_k = [(k-1/2)\pi]^2, (k=1, 2, 3, \dots)$$

Since $\sum_{k=1}^{\infty} [\sin(\sqrt{\lambda_k} y^*) / \lambda_k^{3/2}]$ can be substituted by $(2y^* - y^{*2})/4$, thus we have,

$$u^*(t^*, y^*) = y^* - \frac{y^{*2}}{2} - \frac{2}{\pi^3} \sum_{k=1}^{\infty} \frac{\sin(k-1/2)\pi y^*}{(k-1/2)^3} e^{-[(k-1/2)\pi]^2 t^*}, (t^* \geq 0, 0 \leq y^* \leq 1) \quad (3.47)$$

Then the average velocity $\bar{u}^*(t^*)$ is an integration of $u^*(t^*, y^*)$ with respect to y^* ,

$$\bar{u}^*(t^*) = \int_0^1 u^* dy = \int_0^{y_1^*} u^*_{\tau \geq \tau_0} dy + \int_{y_1^*}^1 u^*_{\tau \leq \tau_0} dy \quad (3.48)$$

For simplification, integration can be conducted within interval $[0, y_1^*]$, when $(-\partial P / \partial x) \rightarrow \infty$, namely the dimensionless height of Bingham plug $y_1^* = 1$. To

validate the feasibility of this simplification, the relationship between penetration length and the dimensionless height $y_1^* = \tau_0 b / (-\partial P / \partial x)$ is presented in Figure 3.6. Generally, the order of magnitude of τ_0 , P and b are 1Pa, 10^6 Pa and 10^{-4} m, based on statistical data from research work and projects (Houlsby, 1990; Stille et al., 2009; Holmberg et al., 2012; Rafi and Stille, 2014). It can be seen from Figure 3.6 that the dimensionless height $y_1^* > 0.9$ within 10 m of grout penetration, indicating that the errors caused by the simplification is not that large.

Figure 3.6 Relationship between y_0^* and penetration length L

Therefore, the dimensionless average velocity can be expressed as follows,

$$\bar{u}^*(t^*) = \begin{cases} 1/3 - 2\pi^{-4} \sum_{k=1}^{\infty} (k-1/2)^{-4} e^{-[(k-1/2)\pi]^2 t^*} & (y_1^* \neq 0) \\ 0 & (y_1^* = 0) \end{cases} \quad (3.49)$$

Table 3.1 Relationship between k and $\psi(k, t^*)$

k	$\Psi(k, t^*)$
1	1.356879559537821
2	4.481535297488440e-11
3	4.156931835119260e-29
4	2.071917127768009e-55
5	3.883977122804150e-90

Note*: Set $t=10^{-3}$ to reduce the impact of t^* on $\psi(k, t^*)$ as much as possible, since the grouting time before nondimensionalization satisfies $t \gg 1s$; therefore, the order of magnitude of dimensionless $t^* = \mu t / \rho b^2$ is 1, as the order of magnitude of μ , ρ and b are generally 10^{-2} Pa·s, 10^3 kg/m³ and 10^{-4} m, based on incomplete statistics from research work and projects (Houlsby, 1990; Stille et al., 2009; Holmberg et al., 2012; Rafi and Stille, 2014).

Further simplify Eq. (3.49), define $\psi(k, t^*) = (k-1/2)^{-4} e^{-[(k-1/2)\pi]^2 t^*}$, ($k=1, 2, 3, \dots$). To determine the influence of k on the dimensionless average velocity $\bar{u}^*(t^*)$, the value of $\psi(k, t^*)$ regarding different k is calculated, and shown in Table 3.1.

According to Table 3.1, the value of $\psi(k, t^*)$ ($k > 1$) can be neglected compared with $\psi(1, t^*)$, so only $k=1$ matters regarding the influence of k on the dimensionless average velocity $\bar{u}^*(t^*)$. Therefore, Eq. (3.48) can be further simplified as,

$$\bar{u}^*(t^*) = \begin{cases} \frac{1}{3} - \frac{32e^{-\pi^2 t^*/4}}{\pi^4} & (y_0^* \neq 0) \\ 0 & (y_0^* = 0) \end{cases} \quad (3.50)$$

Then the average velocity can be calculated based on $\bar{u} = [-(\partial P / \partial x)(b^2 / \mu)] \bar{u}^*$.

Moreover, it can be seen from Eq. (3.49) that the second term $32e^{-\pi^2 t^*/4} / \pi^4 \approx 0$, since $t^* = \mu t / \rho b^2 > 1000$ when $t > 1s$. Therefore, the average velocity can be expressed as $\bar{u} = -(\partial P / \partial x)(b^2 / 3\mu)$, which resembles the cubic law for the Newtonian flow, indicating that when the injection pressure or the relevant injection flow rate is very high, the influence of yield stress can be neglected. Referred to $\bar{u} = -(\partial P / \partial x)(b^2 / 3\mu)$, the range of grout spread can be quickly determined, according to real time measurement of grout flow rate and injection pressure during the grouting process.

Thus, the penetration length can be calculated through the following expression,

$$\begin{aligned} x_n &= x_{n+1} - \bar{u}_{n+1} \cdot (t_{n+1} - t_n) \\ \bar{u}_{n+1} &= P_0 b^2 / 3\mu x_{n+1} \end{aligned} \quad (3.51)$$

Where t_{n+1} and t_n are the (n+1)th and nth time steps, respectively; $\bar{u}_{n+1}$ is the average velocity at time step t_{n+1} ; x_{n+1} and x_n are penetration lengths at time steps t_{n+1} and t_n , respectively.

It can be concluded from the discussion in this section that the influence of inertia term on grout penetration can be neglected, since the velocity of grout flow is low. The final form of the simplified model, in fact, treats grout flow as Newtonian fluid, under the assumption that the thickness of Bingham plug is so small that it can be ignored, and the resulting governing equation for predicting grout penetration can be accepted with high degree of accuracy, when the relevant fracture aperture is relatively large, or the corresponding injection pressure is relatively high.

3.5 Grout penetration considering fracture deformation

Groundwater is very important to our ecosystem, but too much seepage of water into underground tunnels during rock excavation can lead to large pumping cost or even greatly influence the project progress. To predict and control water inflow, the fracture geometry in rock masses under investigation or excavation should be known as a prerequisite. In order to determine the distribution and characteristics of rock fractures, many research works have been done (Witherspoon et al., 1980; Smith and Schwartz, 1984; La Pointe, 1988; Hirata, 1989; Tsang, 1992; Piggott and Elsworth, 1993; Hakami, 1995; Renshaw, 1995; Zimmerman and Bodvarsson, 1996; Odling, 2001; Baghbanan and Jing, 2007; Zhu et al., 2013), and the equivalent aperture was proposed and commonly accepted to describe the transmissivity of rock masses (Witherspoon et al., 1980; Tsang, 1992; Zimmerman and Bodvarsson, 1996). The equivalent aperture is usually obtained from hydraulic test in the field or laboratory (Zimmerman and Bodvarsson, 1996; Di Federico, 1998; Odling, 2001; Baraka-Lokmane, 2002; Cappa et al., 2008; Zhu et al., 2013), and is often called as hydraulic aperture (Tsang, 1992). The introduction of equivalent aperture can be beneficial to grouting design, as the grout spread, grout take, grouting time, and the relevant stop criteria can be determined accordingly, once the fracture geometry and aperture distribution are known (Hässler et al., 1992a; Gustafson and Stille, 1996; Hori and

Koyama, 1999; Eriksson et al., 2000; Fransson, 2008; Hernqvist et al., 2008; Fujita et al., 2012; Stille et al., 2012).

Hence, real fractures, which are probably nonparallel and rough, are simplified into parallel and smooth fractures, with the introduction of equivalent aperture (Tsang, 1992; Renshaw, 1995; Zimmerman and Bodvarsson, 1996; Eriksson et al., 2000). However, the shapes, non-uniform deformation, and fracture roughness can influence the relevant grouting process, as grout flow highly relies on the geometry it flows through (Di Federico, 1998; Frigaard and Ryan, 2004; Sharifzadeh et al., 2009; Sochi, 2011; Fusi et al., 2015). Moreover, one equivalent aperture probably correlates to different kinds of fracture profiles, which can lead to grout flow of different characteristics. Additionally, grout flow is generally treated as Bingham flow, which is a kind of viscoplastic flow and different from water flow, so new characteristics may emerge due to variation of fracture geometry (Di Federico, 1998; Denn, 2001; Clemeur et al., 2004; Binding et al., 2006; Morris et al., 2015).

Presently, some research works were done concerning grout flow in fractures taking into consideration of fracture roughness or aperture variation. In Hanssen's PhD thesis, an algorithm was developed to generate 3D fracture surfaces with rough walls, and the influence of shear displacement, yield stress, and plastic viscosity on grout flow was investigated (Hanssen, 2013). In another similar work, multi-fractional Brownian motion was utilized to study grout flow in single fracture with random aperture distribution (Luo et al., 2012). The work from Hanssen and Luo were useful, but they were time consuming with a lot of calculations; moreover, it's hard to conduct large scale grouting simulation to guide grouting practice. In comparison, Eriksson conducted relatively more practical grouting simulation for grout flow in a fracture channel with variable apertures, but the channel was considered as a combination of parallel channels with different apertures (Eriksson et al., 2000; Eriksson, 2002). Saeidi investigated the characteristics of grout and rock joint on grouting process based on both analytical and numerical (UDEC) analysis, whereas fracture roughness was represented by the joint roughness coefficient and effective aperture (Saeidi et al., 2013). Empirical methods were also proposed to investigate the impact of fracture aperture and roughness on grouting (Kim et al., 1997;

Brantberger et al., 2000; Dalmalm, 2004), which could provide some guidance to grouting simulation and design.

The present study will investigate the hydromechanical coupling process, namely both normal deformation of simple fracture under fluid pressure and grout penetration in curved fracture channel are considered, so the governing equations are derived for grout flow in fractures with variable aperture along the flow direction. The predictive models derived considering hydromechanical coupling were compared with both experimental data and traditional model without considering normal deformation.

3.5.1 Deformation of fracture channel

It is known that the normal deformation of fracture channel can occur due to the application of fluid pressure during the grouting process. The fracture channel is simplified into a 1D beam with two ends fixed to investigate the impact of grout flow on fracture deformation. The fluid pressure is taken as a linear loading, where it is the maximum at the injection end and zero at the front of grout flow. The schematic diagram of the application of fluid pressure on the fracture channel is shown in Figure 3.7. Only the normal deformation of channel upper surface is considered here, since it is mainly composed of acrylic board, while the bottom surface is made of steel, and it is fixed at the test-bed.

Figure 3.7 Schematic diagram of grout flow pressure on fracture channel

In Figure 3.7, L is the length of the fracture channel. $L(t)$ is the length of grout penetration and the influence area of grout flow pressure at time t , and it is updated during the grouting process, resulting in a fracture with curved shape. The velocity of grout flow depends highly on the shape of the fracture channel, and the fracture shape is also affected by the grout penetration. Therefore, grouting of a fracture channel is an interactive process between grout flow and fracture channel.

The shape of channel upper surface under linear loading can be calculated based on the solid mechanics. The relevant governing equation can be expressed as follows,

$$EIh_e^{IV} = q(x) = q_0[1 - x/L(t)], \quad (q_0 = P_0 w) \quad (3.52)$$

where $h_e = h_e(x, t)$ is the change of the channel aperture with $h_e(x, 0) = 0$, E and I are the elastic modulus and moment of inertia of channel surface, respectively. $q(x)$ is the linear loading with q_0 the peak value, which equals to the product of the injection pressure P_0 and the channel width w .

Integrate Eq. (3.51) with respect to x , and apply the boundary conditions $h_e(0, t) = 0$ and $h_e'(0, t) = 0$ at the injection end $x = 0$, then the deformation is,

$$h_e = \frac{Q}{EI} \left(-\frac{x^5}{60L(t)^2} + \frac{x^4}{12L(t)} \right) + \frac{Ax^3}{6EI} + \frac{Bx^2}{2EI} - \parallel_{L(t)} \frac{Q}{EI} \cdot \frac{(a-x)^5}{60L(t)^2} \quad (3.53)$$

where $Q = q(x) \cdot L(t)/2$ is the resultant of the linear loading within $L(t)$, A and B are unknown coefficients, which can be calculated based on boundary conditions $h_e(L, t) = 0$ and $h_e'(L, t) = 0$ at the outlet end $x = L$. $\parallel_{L(t)}$ indicates the following term is included only when $x \geq L(t)$.

Based on these two boundary conditions, two equations concerning A and B can be derived and solved,

$$\begin{aligned} A &= \frac{Q}{10L^3L(t)^2} \left[-2L(t)^5 + 5L(t)^4 L - 10L(t)^2 L^3 \right] \\ B &= \frac{Q}{30L^2L(t)^2} \left[3L(t)^5 - 10L(t)^4 L + 10L(t)^3 L^2 \right] \end{aligned} \quad (3.54)$$

Therefore, the fracture aperture can be updated accordingly and expressed as,

$$h(x, t) = h_0(x) + h_e(x, t) \quad (3.55)$$

where $h_0(x)$ representing the original fracture aperture at $t = 0$.

A series of deformed shapes of the fracture channel corresponding to different grout penetration length $L(t)$ are shown in Figure 3.8, where the order of magnitude of the deformation is 10^{-5} m. The corresponding grout injection pressure is 0.15 MPa, and the elastic modulus of the upper surface is taken as 200GPa. Therefore, it can be

concluded from Figure 3.8 that the aperture distribution of the rock fracture at different locations is varying during the whole grouting process.

Figure 3.8 Deformation of channel upper surface

3.5.2 Grout flow in curved fracture channel

The fracture aperture corresponding to different grout penetration can be obtained from Eqs. (3.52) to (3.55), but grout penetration also depends on the specific aperture distribution of a fracture channel. Therefore, grout penetration in a fracture channel with temporal and spatial variation of aperture should be obtained. The schematic diagram of a fracture channel with variable aperture is shown in Figure 3.9, where the original point of the coordinate system is set at the injection end.

Figure 3.9 Schematic diagram for grout flow in a deformed fracture

A schematic diagram for fully developed grout flow in irregular channel is shown in Figure 3.9. A coordinate system is built taking the symmetric axis of two plates as the horizontal axis, and the direction perpendicular to it as the vertical axis,

$$\rho \frac{\partial u_i}{\partial t} = \mu \frac{\partial^2 u_i}{\partial y^2} - \frac{\partial p}{\partial x} \quad (3.56)$$

where u_i is velocity of grout flow in the x direction; ρ and μ are the density and viscosity of cement grout, respectively, and cement grout is taken as Bingham flow; p is pressure in grout flow, and $\partial p/\partial x$ is pressure gradient in x direction.

As the velocity variation of grout flow is so small within an infinitesimal time interval, thus $\partial u_i/\partial t=0$ is used for further simplification,

$$\mu \frac{\partial^2 u_i}{\partial y^2} = \frac{\partial p}{\partial x} \quad (3.57)$$

According to Eq. (3.7), the average velocity $\bar{u} = \bar{u}(x, t)$ at location x and time t can be obtained with respect to y ,

$$\bar{u} = \frac{h^2}{12\mu} \left(-\frac{\partial p}{\partial x} \right) \left[1 - 3 \left(\frac{y_0}{h} \right) + 4 \left(\frac{y_0}{h} \right)^3 \right] \quad (3.58)$$

Before further deriving the governing equation for grout flow in irregular channel, Eq. (3.57) is further simplified as,

$$\bar{u} = \frac{h}{12\mu} \left[\left(-\frac{\partial p}{\partial x} \right) h - 3\tau_0 \right] \quad (3.59)$$

This simplification is not only simple, but also considers both viscosity and yield stress of cement grout. According to data from literatures and reports (Houlsby, 1990; Houlsby, 1990; Stille, 2009; Holmberg, 2012; Rafi, 2014; Rafi, 2015), generally, the order of magnitude of h , p , μ and τ_0 are 10^{-4} m, 10^6 Pa, 10^{-2} Pa·s, and 1Pa, respectively, and the grout penetration length of x is around 10 m. Based on those estimation, the relative error $|\bar{u}_{10} - \bar{u}_9|/\bar{u}_9$ between average velocity derived from Eq.

(3.57) ($\bar{u}_9$) and Eq.(3.58) ($\bar{u}_{10}$) is shown in Figure 3.10.

Though the relative error is increasing with increasing penetration length, the maximum relative error is less than 0.6%, indicating that the impact due to simplification can be ignored. In the subsequent discussions, the simplified Eq. (3.58) will be used instead of Eq. (3.57).

Figure 3.10 Relative difference of average velocity between the original and simplified formulas

According to Eq. (3.58), the average velocity is a function of x , so the time taken for grout flow to reach location x should be obtained through stepwise calculation. In the present work, the length of channel L is divided into n different infinitesimal intervals $\Delta x_1, \Delta x_2, \dots, \Delta x_n$, and they are defined as follows,

$$\Delta x_i = x_i - x_{i-1}, (1 \leq i \leq n, x_0 = 0) \quad (3.60)$$

Then the corresponding half aperture $h(i)$ at location x_i is given as,

$$h(i) = h(x_i, t)/2, (0 \leq i \leq n) \quad (3.61)$$

The pressure gradient $p_x(i)$ corresponding to infinitesimal interval Δx_i can be expressed as,

$$p_x(i) = \Delta p_i / \Delta x_i, (1 \leq i \leq n) \quad (3.62)$$

The grout flow rate at location x_i can be represented by $Q(t_i) = \bar{u}(i) \cdot 2h(i)$. For the conservation of mass, the grout flow rates in all cross sections of the fracture channel at any time should be the same, so the following relationship should be satisfied,

$$Q(t_i) = \bar{u}(i) \cdot 2h(i) = \bar{u}(n) \cdot 2h(n) \quad (3.63)$$

Substitute Eqs. (58) and (3.61) into Eq. (3.62), the following relationship can be obtained,

$$\left[p_x(i) \cdot h(i) - \frac{3\tau_0}{2} \right] h(i)^2 = \left[p_x(n) \cdot h(n) - \frac{3\tau_0}{2} \right] h(n)^2 \quad (3.64)$$

Through simple transformation, $p_x(i)$ can be derived from Eq. (3.61) as,

$$p_x(i) = p_x(n) \cdot \frac{h(n)^3}{h(i)^3} - \frac{3\tau_0}{2} \cdot \frac{h(n)^2}{h(i)^3} + \frac{3\tau_0}{2} \cdot \frac{1}{h(i)}, (1 \leq i \leq n) \quad (3.65)$$

Then pressure drop Δp_i within interval Δx_i can be expressed as,

$$\Delta p_i = p_x(i) \cdot \Delta x_i, (1 \leq i \leq n) \quad (3.66)$$

The summation of all Δp_i is equal to the injection pressure P_0 ,

$$P_0 = \sum_{i=1}^n \Delta p_i = p_x(n) \cdot h(n)^3 \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)^3} - \frac{3\tau_0}{2} \cdot h(n)^2 \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)^3} + \frac{3\tau_0}{2} \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)} \quad (3.67)$$

Hence, pressure gradient $p_x(n)$ against interval Δx_n can be expressed as,

$$p_x(n) = P_0 + \frac{3\tau_0}{2} \cdot h(n)^2 \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)^3} - \frac{3\tau_0}{2} \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)} \bigg/ h(n)^3 \cdot \sum_{i=1}^n \frac{\Delta x_i}{h(i)^3} \quad (3.68)$$

Therefore, we can obtain the average velocity at location x_n ,

$$\bar{u}(n) = \left[p_x(n) \cdot h(n)^2 / 3\mu \right] \cdot \left[1 - 3y_0(n) / 2h(n) \right] \quad (3.69)$$

Then the time taken for grout moving from location x_{i-1} to x_i can be expressed as,

$$\Delta t_i = \Delta x_i / \bar{u}(i) \quad (3.70)$$

Hence the total time for grout penetration length of L is obtained,

$$t = \sum_{i=1}^n \frac{\Delta x_i}{\bar{u}(i)}, \left(\sum_{i=1}^n \Delta x_i = L \right) \quad (3.71)$$

Therefore, grout penetration in deformed channel and the corresponding grouting time can be calculated. In the following section, the characteristics of grout flow in deformed fracture channel will be compared with that in the undeformed fracture.

3.5.3 Results and discussion

Based on Eqs. (3.52) to (3.54) and Eqs. (3.67) to (3.70), grout penetration considering the coupling of fracture normal deformation and grout penetration can be calculated.

To validate the coupling model proposed, four cases of grout penetration predicted by the current model are compared with that predicted by traditional model (not considering fracture normal deformation) and the corresponding experimental data. The characteristics of the fracture channels in laboratory testing and Ca-type bentonite grout are listed in Table 3.2.

The properties of grout used are measured by rheometer before grouting tests. The elastic modulus of the fracture upper surface is set as 100 GPa, considering the existence of observation windows in steel cover board and the usage of acrylic board, and the effective thickness of the board is taken as 4 mm. Moreover, deformation of the bottom surface is ignored, as it is made of steel board and fixed at the test-bed.

Table 3.2 Characteristics of channels and grout slurry (Fujita, 2012)

Case	Channel		Grout	
	Aperture (mm)	Injection pressure (MPa)	Yield stress (Pa)	Viscosity (mPa·s)
1	0.206	0.15	0.63	43.5
2	0.104	0.5	2.2	84.2
3	0.197	0.15	2.2	86.6
4	0.094	0.15	0.5	38.9

Comparisons of simulation results considering (new model) and not considering (traditional model) fracture normal deformation with the experimental data are presented in Figure 3.11, where four cases are conducted based on grouting parameters given in Table 3.2.

It can be found from Figure 3.11 that, compared with the experimental data, the simulation results of all four cases predicted by the proposed new model considering fracture deformation are much better than the traditional model taking the fracture channel as un-deformed. The time taken for grouting the whole channel (1m) predicted by the traditional model is much longer than the new model and experimental data, and all of which are two times that predicted by the new model; moreover, the shapes of the curve predicted by the new model can better describe the trend of experimental data, the curve slope of which is not decreasing throughout the grouting process as predicted by the traditional model.

It can be noticed that there are still some discrepancies between the results predicted by the new model and the experimental data. One of the main reasons may be simplification of the channel model, where a 2D channel is simplified as a 1D beam, which ignores the influence from two lateral walls, thus introducing some errors and affecting the simulation results. Wall slip may also take place as the fracture surface of the steel board is probably not rough enough.

It is known that the computation efficiency of 1D model is much higher than the relevant 2D model, therefore the simulation results of 1D model are acceptable if the computation efficiency is of priority and the trend of grout penetration is good enough for practical judgement.

From the discussion made above on the coupling effect, it can be found that the shape of fracture surfaces can be time-varying during the grouting process, as the pressure distribution varies with time varying grout penetration; the characteristics of grout penetration predicted by models with and without considering fracture normal deformation are greatly different, indicating that the influence of fracture deformation is considerable. The proposed new model considering fracture deformation can better predict the experimental data, especially the trend of grout penetration, which also shows that fracture deformation occurred during the grouting process.

Figure 3.11 Comparison of simulation results with experimental data

In future study, 3D full scale fluid models should be developed to consider the impact from both lateral boundaries and wall slip effect, to fully consider the coupling effect between grout penetration and fracture deformation.

3.6 Summary

In this chapter, models for grouting prediction are developed under two different assumptions: constant injection pressure and constant injection flow rate, including derivation of governing equations. Though the governing equation for the two methods is the same, the algorithms used for calculating grout penetration are significantly different. The algorithm for grouting of a 2D fracture network based on two methods are also introduced. The process is more complex for predicting grout penetration under constant injection flow rate, since the grout flow rate is a nonlinear function of the pressure gradient.

A simplified model under constant injection pressure is proposed, the accuracy would be high when the fracture under investigation has relatively large aperture, or the injection pressure is relatively high. The simplified model is applicable to prediction of grout penetration in a 2D fracture network, when qualitative analysis is enough for

practical engineering usage. Compared with the traditional model, the new simplified model can achieve much higher computation efficiency.

The coupling effect between grout flow and fracture normal deformation is also investigated. The expression for calculating fracture deformation under linear grout pressure is derived, which can be used for calculating shapes of a fracture channel under time varying grout penetration. The algorithm for calculating grout penetration within deformed shape of a specific fracture channel is also derived. Prediction of grout penetration considering the coupling effect between grout flow and fracture deformation is conducted, which are compared with experimental data (deformation of experimental equipment occurred during the grouting process,) and the theoretical prediction not considering deformation. It is found that the trend of experimental data can be well revealed by the coupling model proposed.

It can be concluded that the characteristics of grout flow predicted by different theoretical models can be different, even in the same fracture channel. If different boundary conditions are considered: for constant injection pressure method, the grout flow rate will be decreasing with time, so the maximum length of grout penetration would be reached when there is a force balance; for constant injection flow rate method, grout flow will not stop unless reaching a preset injection pressure. The grouting process is accompanied by hydro-mechanical coupling, which can greatly affect the corresponding grouting effect.

Chapter 4 Two-dimensional Penetration in Rock Fracture

4.1 Introduction

As for rock fractures directly intersected by an injection hole, the grout flow around it is in radial direction (Barker, 1988; Eriksson, 2002; Mohammed et al., 2015), if there is no flowing water involved or it is not considered. Whereas grout flow in rock fractures not directly intersected by an injection hole is usually taken as 1D channel flow. The flow type is important: firstly, the governing equations of two types of flow can be greatly different; secondly, pressure concentration is at the injection hole for radial flow and the value is the largest, so pressure induced fracture deformation could be a tough problem that we cannot ignore (Gothäll and Stille, 2009; Gothäll and Stille, 2010; Runslätt and Thörn, 2010; Rafi and Stille, 2014); finally, a rock fracture would not be sealed unless the region close to the injection hole could be sealed, so it would be a big problem if there is flowing water and its effect is so considerable as to flush out grout material injected (Zhang et al., 2010; Gailing et al., 2011; Liu et al., 2011; Zhang et al., 2012; Sui et al., 2015).

In this chapter, the governing equation for grout flow around an injection hole (not considering the existence of flowing water) will be derived, which will be used for building the relationship between grout flow rate and grouting parameters, including grout pressure, fracture aperture, and range of grout penetration (radius). Then, this relationship will be used in a theoretical model, investigating the influence of fracture normal deformation on grouting of two neighboring rock fractures intersected by the same injection hole. Finally, a theoretical model for grouting of rock fractures considering the existence of flowing water will be developed, based on the theory of potential flow, and the model will be used to explain some experimental phenomena.

4.2 Derivation of governing equation

Fractures in real rock mass are in three-dimensional (3D) fracture network, where grout penetration can be of one dimensional (1D) channel flow and/or two dimensional (2D) radial flow. There have been many studies on prediction of grout penetration in 1D fracture channel, but very limited research work for 2D radial penetration, partly due to its higher complexity.

The current problem starts from the derivation of two basic equations: mass conservation and momentum conservation. The derivation and calculation are based on the schematic diagram shown in Figure 4.1.

Figure 4.1 Schematic diagram of grouting rock fractures (2D radial penetration)

The equation related to “mass conservation” is derived as follows. Suppose that the velocities at radii r and $r + dr$ are v and $v + \frac{\partial v}{\partial r} dr$, respectively. The items involved in calculating the relevant flow rate at radius r and $r + dr$ are listed in Table 4.1.

Table 4.1 Calculation of flow rates at radius r and $r + dr$

Location	Area	Velocity	Flow rate
r	$2\pi r dz$	v	$2\pi r dz \times v$
$r + dr$	$2\pi(r + dr) dz$	$v + \frac{\partial v}{\partial r} dr$	$2\pi(r + dr) dz \times (v + \frac{\partial v}{\partial r} dr)$

The flow rates are equal at location r and $r + dr$, due to the conservation of mass, thus the following equations is obtained,

$$2\pi r dz \times v = 2\pi(r + dr) dz \times \left(v + \frac{\partial v}{\partial r} dr \right) \quad (4.1)$$

Eq. (4.1) can be further simplified as follows,

$$\left(v + \frac{\partial v}{\partial r} \right) dr = \frac{\partial(rv)}{\partial r} dr = 0 \quad (4.2)$$

Therefore, the first equation for solving 2D radial flow is obtained,

$$\frac{1}{r} \frac{\partial(rv)}{\partial r} = 0, \quad (r \geq r_0) \quad (4.3)$$

It can be concluded from Eq. (4.3) that rv is a function of z , thus

$$rv = f(z); \quad v = \frac{f(z)}{r} \quad (4.4)$$

Considering the conservation of momentum, there are three forces involved: pressure of grout flow, shear stress, and inertia force, which are listed in Table 4.2 to Table 4.4. The conservation of momentum is the force balance achieved among the three forces.

Table 4.2 Pressure of grout flow

Location	Pressure	Loading area
r	P	$S_p = 2\pi r dz$
$r + dr$	$P + \frac{\partial P}{\partial r} dr$	$S_p = 2\pi r dz$

Table 4.3 Shear stress

Location	Pressure	Impact area
Fracture lower surface	τ	$S_\tau = 2\pi r dr$
Fracture upper surface	$\tau + \frac{\partial \tau}{\partial z} dz$	$S_\tau = 2\pi r dr$

Table 4.4 Inertial force

Location	Force	Impact volume
r to $(r + dr)$	$\rho \frac{\partial v}{\partial t}$	$V = 2\pi r dr \times dz$

The relationships among these three forces are expressed as,

$$(P \cdot S_p + \tau \cdot S_\tau) - \left\{ \left(P + \frac{\partial P}{\partial r} dr \right) \cdot S_p + \left(\tau + \frac{\partial \tau}{\partial z} dz \right) \cdot S_\tau \right\} = \rho \cdot (2\pi r dr) dz \cdot \frac{\partial v}{\partial t} \quad (4.5)$$

where ρ and g are the density of cement grout and gravitational acceleration.

Considering that the time variation of velocity is very small, it is reasonable to set $\partial v / \partial t = 0$. Thus, the following equation can be obtained via simplifying Eq. (4.5),

$$\frac{\partial \tau}{\partial z} = -\frac{\partial P}{\partial r}, \quad (\tau \geq c) \quad (4.6)$$

The two equations describing “conservation of mass” and “conservation of momentum” can be put together as follows,

$$\begin{cases} \frac{1}{r} \frac{\partial(rv)}{\partial r} = 0, & (r \geq r_0) \\ \frac{\partial \tau}{\partial z} = -\frac{\partial P}{\partial r}, & (\tau \geq c) \end{cases} \quad (4.7)$$

For comparison, it is necessary to present the corresponding equations for 1D channel flow (Lipscomb and Denn 1984; Papanastasiou 1987; Eriksson 2002; Huilgol 2015),

$$\begin{cases} \frac{\partial v}{\partial r} = 0 \\ \frac{\partial \tau}{\partial z} = -\frac{\partial P}{\partial r}, & (\tau \geq c) \end{cases} \quad (4.8)$$

The cement grout is taken as Bingham flow, so it satisfies

$$\tau = \tau_0 - \mu \frac{\partial v}{\partial z}, \quad (\tau \geq c) \quad (4.9)$$

where τ_0 and μ are the yield stress and viscosity of cement grout, respectively.

Substitute Eq. (4.4) into Eq. (4.9),

$$\frac{\partial \tau}{\partial z} = -\mu \frac{\partial^2 v}{\partial z^2} = -\frac{\mu}{r} \frac{\partial^2 f(z)}{\partial z^2} \quad (4.10)$$

Combine Eqs. (4.7) and (4.10), the following relationship can be derived,

$$\mu \frac{\partial^2 f(z)}{\partial z^2} = r \frac{\partial P}{\partial r} = C \quad (4.11)$$

where C is bound to be a constant to make sure that both sides are equal, since both sides of Eq. (4.11) are function of z and r , respectively.

Therefore, simplify Eq. (4.11), and consider the relevant boundary conditions of pressure distribution,

$$\frac{\partial P}{\partial r} = \frac{C}{r}, \quad \left(P \Big|_{(r=r_0)} = P_0; P \Big|_{(r=L_p)} = P_w \right) \quad (4.12)$$

where r_0 and L are the radius of injection hole and the relevant grout penetration at time t , respectively. P_0 and P_w are the corresponding grout injection pressure and water pressure at the front of grout flow.

Then the pressure distribution P can be expressed as,

$$P = P_0 - \left[\frac{P_0 - P_w}{\log(L/r_0)} \right] \log\left(\frac{r}{r_0}\right) \quad (4.13)$$

It should be noted that Eq. (4.13) is obtained based on both “mass conservation” and “momentum conservation”, and the basic fluid model assuming cement grout as Bingham flow.

Based on Eqs. (4.10), (4.11) and (4.13), the following relationship can be obtained,

$$\frac{\partial^2 v}{\partial z^2} = \frac{\partial P}{\partial r} \quad (4.14)$$

There are two boundary conditions for Eq. (4.14),

$$v(r, H) = 0; \quad \frac{\partial v}{\partial z}(r, H_c) = 0 \quad (4.15)$$

where H_c is the half-height of Bingham plug (Dai and Byron Bird 1981; El Tani 2012; Huilgol 2015).

Integrate Eq. (4.14) twice with respect to z and consider the relevant boundary conditions, the velocity of grout flow can be derived,

$$v = \begin{cases} -\frac{1}{2\mu} \frac{\partial P}{\partial r} (H - z)(H + z - 2H_c) & (|z| \geq H_c) \\ -\frac{1}{2\mu} \frac{\partial P}{\partial r} (H - H_c)^2 & (|z| < H_c) \end{cases} \quad (4.16)$$

Therefore, the injection flow rate can be derived,

$$Q = \int_{-H}^H v dz = \frac{2\pi(P_0 - P_w)}{3\mu \log(L/r_0)} (1 - \lambda)^2 (2 + \lambda), \quad \left\{ \lambda = \frac{H_c}{H} = \frac{(L - r_0)\tau_0}{(P_0 - P_w)H} \right\} \quad (4.17)$$

Regarding theoretical models, a comparison is made between 1D channel flow and 2D radial flow, including pressure distribution along flow direction, velocity of grout flow, and injection flow rate, as listed in Table 4.5.

As can be seen from Table 4.5, the pressure distribution for 1D flow is a linear function, but it is a logarithmic function for 2D flow, which make the expressions of velocity distribution and injection flow rate different, though part of the expressions is similar.

Table 4.5 Model comparison between 1D channel flow and 2D radial flow

Variable	Flow Type	Expression
Pressure distribution	1D channel flow	$P = P_0 - (P_0 - P_w) \frac{x}{L}$
	2D radial flow	$P = P_0 - \frac{P_0 - P_w}{\log(L/r_0)} \log\left(\frac{r}{r_0}\right)$
Velocity distribution	1D channel flow	$u = \frac{1}{2\mu} \left(-\frac{\partial P}{\partial x} \right) (b - y)(b + y - 2y_0)$
	2D radial flow	$v = \frac{1}{2\mu} \left(-\frac{\partial P}{\partial r} \right) (H - z)(H + z - 2H_c)$
Injection flow rate	1D channel flow	$Q = \frac{2Wb^3}{3\mu} \left(\frac{P_0 - P_w}{L} \right) (\alpha - 1)^2 (\alpha + 2)$
	2D radial flow	$Q = \frac{2\pi H^3 (P_0 - P_w)}{3\mu \log(L/r_0)} (\lambda - 1)^2 (\lambda + 2)$

Notes*: Velocity distribution in Table 4.5 is velocity of grout flow above the Bingham plug, $\alpha = y_0/b$, $\lambda = H_c/H$, and b and H are half aperture of rock fracture for 1D and 2D, respectively. Here, the water pressure $P_w = 0$, if there is no flowing water involved or it is not considered.

4.3 Real-time grout penetration and fracture deformation

Grouting is an effective way to seal rock fractures, but it is still difficult to make a reliable and cost efficient grouting design, because many scientific problems remain unsolved. Among them is determination of the right grout injection flow rate, as there are contradictory conclusions existing between the academic research and engineering practices. Some researchers showed that increase of grout injection pressure (or flow rate) can increase the range of grout penetration within certain time, since the higher flow velocity can be obtained (Cornet et al., 2003; Xiao et al., 2017); but other researchers indicated that it would not result in longer distance of grout spreading by increasing injection pressure (or flow rate), which, however, could cause fracture deformation thus weakening the sealing effect of fractures with small apertures, and could also increase the corresponding grout take (Gothäll and Stille, 2010; Rafi and Stille, 2015). However, it is taken as a common practice by site

engineers to raise injection flow rate to accelerate the grouting process and achieve better grouting effect, which seems to work for practical grouting project. Therefore, it is of great interest to estimate the impact of fracture normal deformation on the grouting process.

4.3.1 Basic Concept and numerical model

Fracture deformation would be triggered when the fluid pressure within a rock fracture surpass the original in-situ stress, and the magnitude of deformation depends on many factors. There are two situations where fracture deformation could significantly affect the grouting process: one is the deformation of rock fracture that is close to the excavation surface of rock cavern or tunnel, where large amplitude of fracture deformation could be easily triggered due to low level of geometric constraint or in-situ stress (Fransson et al., 2010); another one is deflection of rock slab between two neighboring fractures, as rock fractures generally appear in series, which in turn can change the apertures of rock fractures involved (Gothäll and Stille, 2010). The schematic diagram of the second situation is shown in Figure 4.2.

Figure 4.2 Illustration of rock mass with two parallel fractures under grouting

For the convenience of discussion, the surfaces of rock fractures are taken as parallel smooth, and the fracture aperture is magnified to a certain scale, as the practical aperture is very small, about hundreds or even tens of micrometers. Cement grout is pumped into the fractures through the injection hole. If the profile of grout penetration in individual rock fracture forms a circle (two blue circles in Figure 4.2), the rock slab between two fractures is subjected to fluid pressure within two circles of different

radii. Additionally, there are other external forces applying on the rock slab from the rock masses above and below the rock slab, which are balanced by each other.

It is known that the ranges of grout penetration in rock fractures with different aperture are different, which can greatly influence the magnitude of fracture normal deformation (Xiao and Zhao, 2017). Therefore, a theoretical model is proposed to determine the grout penetration in two neighboring fractures, which, combined with other grouting parameters obtained, are used as input parameters of a finite element model for investigating the characteristics of fracture deformation; finally, all parameters obtained are used for data analysis to find the corresponding intercorrelations. The detailed process is presented in Figure 4.3.

Figure 4.3 Flow chart for investigating normal deformation of rock fractures

4.3.2 Model for real-time grout penetration

As can be seen from Figure 4.2, grout penetration in individual rock fracture is radial and symmetric with respect to the injection hole, so it can be represented by a 1D model, which is shown in Figure 4.4. The injection flow rate from the injection hole is Q ; the length and thickness of the rock slab are taken as SL and ST , respectively. The flow rates and grout penetrations in the lower and upper fractures are Q_1 and Q_2 , and L_1 and L_2 , respectively; the injection pressure is taken as P_0 . As there are

five unknown parameters, five equations are required to solve all these variables. The magnitudes of fracture apertures in Figure 4.4 are magnified for better illustration.

Figure 4.4 One-dimensional illustration of radial grout spreading in two neighboring rock fractures intersected by a grout hole

The first equation is based on the mass conservation, that is the flow rate from the injection hole equals the sum of flow rates in upper and lower fractures,

$$Q = Q_1 + Q_2 \quad (4.18)$$

The flow rate in a fracture of radial grout spread can be expressed as (El Tani, 2012),

$$Q = f(P, L) = \frac{2\pi PH^3}{3\mu \log(L/r_0)} (1-\lambda)^2 (2+\lambda), \lambda = \frac{(L-r_0)\tau_0}{PH} \quad (4.19)$$

where μ and τ_0 are the viscosity and yield stress of cement grout, which is taken as Bingham flow; L and r_0 are the range of grout penetration in a fracture and the radius of grout hole, respectively; $P = P_0 - P_w$ is the pressure difference between injection pressure P_0 and water pressure P_w , and the latter is taken as zero in the present work, so $P = P_0$. Therefore, Eq. (4.19) can be expressed as $Q = f(P_0, L)$.

Based on Eq. (4.19), two more equations can be found regarding the injection flow rates in two neighboring fractures,

$$Q_1 = f[P_0, L_1(t)] \quad (4.20)$$

$$Q_2 = f[P_0, L_2(t)] \quad (4.21)$$

where $L_1(t)$ and $L_2(t)$ are real-time penetration in lower and upper fractures.

The last two equations required are real-time grout penetration between two adjacent time steps,

$$L_1(t_{n+1}) = L_1(t_n) + \frac{Q_1}{4\pi L_1(t_n)H_1} \Delta t \quad (4.22)$$

$$L_2(t_{n+1}) = L_2(t_n) + \frac{Q_2}{4\pi L_2(t_n)H_2} \Delta t \quad (4.23)$$

where t_{n+1} and t_n are the n th and $(n+1)$ th time steps, respectively, and $\Delta t = t_{n+1} - t_n$.

Note from Eqs. (4.20) to (4.23) that the injection flow rate, injection pressure, and grout penetration in individual fracture are intercorrelated, so they cannot be solved directly from these equations. Therefore, the Newton's method is introduced for solving these variables via iteration:

$$X_{N+1} = X_N - F(X_N)/F'(X_N) \quad (4.24)$$

where $N + 1$ and N represent the number of iterations.

In the present work, X and $F(X)$ have the following expressions:

$$X = \begin{bmatrix} Q_1 \\ Q_2 \\ P_0 \\ L_1(t_{n+1}) \\ L_2(t_{n+1}) \end{bmatrix} \Rightarrow F(X) = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} Q - Q_1 - Q_2 \\ Q_1 - f(P_0, L_1(t_{n+1})) \\ Q_2 - f(P_0, L_2(t_{n+1})) \\ L_1(t_{n+1}) - L_1(t_n) - Q_1 \cdot \Delta t / 4\pi L_1(t_{n+1})H_1 \\ L_2(t_{n+1}) - L_2(t_n) - Q_2 \cdot \Delta t / 4\pi L_2(t_{n+1})H_2 \end{bmatrix} \quad (4.25)$$

Therefore, the derivative matrix $F'(X)$ can be expressed as,

$$F'(X) = \begin{bmatrix} \frac{\partial F_1}{\partial Q_1} & \frac{\partial F_1}{\partial Q_2} & \frac{\partial F_1}{\partial P_0} & \frac{\partial F_1}{\partial L_1} & \frac{\partial F_1}{\partial L_2} \\ \frac{\partial F_2}{\partial Q_1} & \frac{\partial F_2}{\partial Q_2} & \frac{\partial F_2}{\partial P_0} & \frac{\partial F_2}{\partial L_1} & \frac{\partial F_2}{\partial L_2} \\ \frac{\partial F_3}{\partial Q_1} & \frac{\partial F_3}{\partial Q_2} & \frac{\partial F_3}{\partial P_0} & \frac{\partial F_3}{\partial L_1} & \frac{\partial F_3}{\partial L_2} \\ \frac{\partial F_4}{\partial Q_1} & \frac{\partial F_4}{\partial Q_2} & \frac{\partial F_4}{\partial P_0} & \frac{\partial F_4}{\partial L_1} & \frac{\partial F_4}{\partial L_2} \\ \frac{\partial F_5}{\partial Q_1} & \frac{\partial F_5}{\partial Q_2} & \frac{\partial F_5}{\partial P_0} & \frac{\partial F_5}{\partial L_1} & \frac{\partial F_5}{\partial L_2} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & Pf[L_1(t_{n+1}), H_1] & 0 & 0 \\ 0 & 1 & Pf[L_2(t_{n+1}), H_2] & 0 & 0 \\ -\frac{1}{4\pi L_1(t_{n+1})H_1} & 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{4\pi L_2(t_{n+1})H_2} & 0 & 0 & 1 \end{bmatrix} \quad (4.26)$$

$$Pf(L, H) = -\frac{4\pi H^3}{3\mu \log(L/r_0)} - \frac{4\pi(r_0^3 - 3r_0^2 L + 3r_0 L - L^3)\tau_0^3}{3\mu P_0^3 \log(L/r_0)} \quad (4.27)$$

Based on Eqs. (4.24) to (4.27), the injection pressure and grout penetration in two rock fractures with different apertures can be obtained. One example is presented in Figure 4.5 for the real-time grout penetration and pressure variation.

In Figure 4.5, the rates of grout penetration in two fractures are slowing down with time, especially, for the fracture with smaller aperture, as the corresponding average velocity of grout flow is decreasing with increasing penetration radius, which can be easily understood according to Eqs. (4.22) and (4.23), under constant injection flow rate. As a result, the injection pressure is also increasing with time but with a reduced rate; in the meanwhile, the ratio between the two penetration lengths $R_L = L_1/L_2$ is becoming larger, thus R_L should not be taken as a constant value when calculating fracture normal deformation.

Figure 4.5 Illustration of real-time grout penetration and variation of injection pressure. Injection flow rate: $Q = 0.8$ (kg/s); fracture aperture: $H_1 = 800$ (μm) and $H_2 = 200$ (μm)

4.3.3 Model for fracture deformation

The deflection of the rock slab between two fractures, as shown in Figure 4.2 and Figure 4.4, can be used to evaluate fracture deformation, as deformation of rock slab can lead to increase or decrease of aperture of two fractures involved. Therefore, in the present work, we only pay attention to the deformation features of rock slab, which is affected by not only the injection pressure and penetration lengths in two neighboring fractures, but also other factors, including the stiffness of rock fractures, the length and thickness, and Young's modulus of rock slab. The surfaces of real rock fractures are rough with asperities resisting slab deformation and functioning as stiffness of rock fractures (Belem et al., 2000; Brush, 2001; Crandall et al., 2010; Wang et al., 2016; Hopkins, 2000; Cammarata et al., 2006; Gothäll and Stille, 2009).

Figure 4.6 Illustration of mechanical model for evaluating fracture deformation

Forces applied to the rock slab is symmetric, so the force diagram of the rock slab is also simplified as a 1D model, as shown in Figure 4.6, where the elastic supports can be expressed as $-K_2w$ and $-K_1w$, respectively, which serve as elastic base to resist deformation of the rock slab (Gothäll and Stille, 2010). Namely, the lower rock mass would generate an extra force (apart from the original force existed for balancing the in-situ stress) to resist the downwards deformation of the rock slab; likewise, the upper rock mass would generate an extra force to resist the upwards deflection of the

rock slab. The negative sign indicate that the direction of force generated is opposite to the corresponding direction of deflection. K_1 and K_2 are fracture stiffness of the upper and lower fractures, respectively, and w is deflection of rock slab.

Figure 4.7 Deflection of rock slab w during the grouting process within 180s. Injection flow rate: $Q = 1.6\text{kg/s}$; fracture aperture: $H_1 = 500\text{ }(\mu\text{m})$ and $H_2 = 200\text{ }(\mu\text{m})$; Young's modulus of rock slab: $E = 48\text{GPa}$; Fracture stiffness: $K_1 = K_2 = 10\text{ } (10\text{GPa})$; Length and thickness of rock slab: 7.95m and 0.8m

The deformation of rock slab can correlate easily to the change of individual fracture aperture, through adding or subtracting the magnitude of slab deflection correspondingly. Rock slab deformation is calculated based on a finite element software according to mechanical model presented in Figure 4.5, where the fluid pressure is obtained from Section 4.3.2 and the upper and lower boundary are set as elastic support with stiffness K_1 and K_2 , respectively. Figure 4.7 shows one case study on slab deformation under certain grouting conditions, where the wider fracture is filled by grout at around 200s. It can be found that the maximum slab deformation w_{max} is increasing during the grouting process, and the location where the maximum deflection appears $r(w_{max})$ is moving forward along the direction of grout flow. Meanwhile, the rock slab is deforming towards the narrower fracture during the grouting process, indicating that the aperture of narrow fracture could become even

smaller, and the aperture of wider fracture would become larger. Therefore, in some areas, the aperture of narrower fracture at the location r_{wmax} could be less than the critical aperture at certain time, thus causing filtration and no effective sealing (Eklund and Stille, 2008; Draganovic, 2009).

4.3.4 Results and discussion

As mentioned in the previous sections, deformation of rock slab depends on many factors, including the injection flow rate, the fracture apertures of two neighboring fractures, the stiffness of rock fractures, the dimension of rock slab (length and thickness), and Young's modulus of rock slab. Nevertheless, most of the above information is unknown to us regarding practical rock fractures. Therefore, geological statistics and Design of Experiment (DOE) are used for discussion, and an orthogonal array with 7 factors on 9 levels is generated for case study. Compared with full scale study, only 81 cases will be investigated, otherwise $9^7 = 4782969$ cases are required. Seven factors are listed in Table 4.6, including their symbols, descriptions, and parameter ranges.

Table 4.6 Specifics of seven factors used for DOE

Factor	Description	Parameter range
Q	Grout injection flow rate	0.2~1.8 (kg/s)
H_2	Aperture of upper fracture	200~600 (μm)
R_H	$R_H = H_1/H_2$, H_1 : aperture of lower fracture	1.5~5.5
R_K	$R_K = K_2/K_1$, $K_1 = 10\text{GPa}$; K_1 and K_2 : stiffness of upper and lower fractures	0.2~1.8
R_L	$R_L = SL/L_1$; SL : Length of rock slab; L_1 : penetration length in the lower fracture	0.2~1
ST	Thickness of rock slab	0.2~1.8 (m)
R_E	Variation of Young's modulus of rock slab; Benchmark of Young's modulus $E=60\text{ GPa}$	0.6~1

Table 4.6 includes all parameters that will be used both for fluid and mechanical analysis, where Q , H_2 , and R_H are used in the fluid model, and the rest is used for the mechanical model. The stiffness ratio is introduced, since the stiffness of fractures with different apertures might be different, due to the size of bumps on surfaces of fractures with different apertures are different and the existence of potential filling

materials. The aperture of the upper fracture is set to be smaller than the lower fracture in the following discussion, for the convenience of discussion; though, the distribution of fracture apertures is random. The length of rock slab is not included, as it can be calculated according to R_L and grout penetration L_1 from fluid model.

After going through the whole process following the flowchart given in Figure 4.3, all the unknown variables can be obtained, including penetration length, grouting pressure, slab deformation, and the value and location of the maximum deflection, etc., which are presented in Table A1 in Appendix A (the calculation of Case 20 regarding fluid model is not convergent, so it is removed from this table), the meaning of most the symbols are defined in the above sections, and the units, following international unit, are not included, except for $100w_{max}/H_2$ and $100r(w_{max})/SL$, which are the ratio (transferred to percentage) between the maximum deflection and the aperture of upper fracture, and the ratio (transferred to percentage) between the location where the maximum deformation occurs and the length of rock slab. These two ratios are defined to evaluate the degree of impact of slab deformation on grouting, where $100w_{max}/H_2$ indicates whether the slab deformation could lead to serious reduction of penetrability for the smaller fracture; $100r(w_{max})/SL$ indicates the relative location where the maximum deflection occurs, and large value indicates that the smaller fracture can be relatively well grouted before filtration happens.

Among 14 variables listed in Table A1 of Appendix B, some are generated from the fluid model, and some are derived from the mechanical model; and they are intercorrelated. Therefore, correlation analysis is conducted, and the results are presented in Fig. B1 (a) and (b) in Appendix B, where there are 14 rows and 14 columns, which are in one-to-one correspondence with 14 variables in Table A1. Sub-diagrams in the n th row (or column) represent the correlation of the n th variable with other 13 variables, and 14 sub-diagrams, along the diagonal line from lower-right corner to the upper-left corner, are statistic distribution of individual variable. In Figure B1 and Figure B2, the intercorrelations among 14 variables are presented, where the variation patterns of one variable with another can be easily found; the degree of correlation between any two variables is characterized by various colors in

Figure B2, which shade from bright yellow (correlation coefficient is 1) to dark blue (correlation coefficient is -1), ranging from significant positive correlation to significant negative correlation, and the color in the middle (correlation coefficient is 0) indicates no correlation.

Figure 4.8 Relationship between injection flow rate Q and grout penetration L_1

Figure 4.9 Grout penetration against aperture ratio: (a) $L_1 \sim R_H$; (b) $L_2 \sim R_H$

For variables involved in fluid model, some are independent variables, like flow rate Q , fracture aperture H_2 , aperture ratio between two fractures $R_H = H_2/H_1$; some are dependent variables, like grout penetration L_1 and L_2 , and injection pressure P_0 . The correlations between dependent and independent variables are discussed firstly, as they are more useful to grouting design.

As can be found from Figure 4.8 that there is a strong positive correlation between injection flow rate Q and grout penetration L_1 , where there is a quantile-quantile diagram obtained through drawing a line between the first quantile ($QT_1 = 3.8m$) and the third quantile ($QT_3 = 6.71m$). All data points are distributed around the line passing through the two points QT_1 and QT_3 , indicating that the range of grout penetration increases with increasing inject flow rate, which can well demonstrate the practical situation. The grouting time for all cases is at the same level.

Then, Figure B2 also shows strong a negative correlation between aperture ratio R_H and grout penetration L_1 and L_2 , which is shown in Figure 4.9. It can be found that, within a specific time, grout penetration in wider fracture L_1 is decreasing linearly with the increase of R_H , but grout penetration in narrower fracture L_2 is decreasing exponentially with the increase of R_H . The reason is that most of the grout would choose to flow through wider fracture, where it is easier for grout to move forward; the aperture of wider fracture is larger when R_H is higher. That is why “split spacing principle” is used, where there are primary holes, secondary holes, and tertiary holes, and so on. It is known that the primary holes are used to fill fractures with large or very large apertures, since the range of grout penetration in fine fractures would be very small (Warner, 2004), which can be well explained by the conclusions obtained here. Moreover, both grout penetrations L_1 and L_2 are becoming smaller with increasing R_H , as larger volume of cement grout is required when the fracture becomes wider, but the grout injection flow rate is within a certain range.

Apart from the relationship between injection flow rate and grout penetration, on site engineers care more about variation patterns of injection pressure with injection flow rate, which shows weak positive correlation, so we only analyze its statistical characteristics, as shown in Figure 4.10, where the color ranging from bright yellow to dark blue indicates the occurrence frequency of various variable combination. It can be found that there are no color areas regarding variable combination of high injection pressure and low injection flow rate; most color areas fall into lower triangle of the variable domain, among which 90% of all cases fall into the variable combinations with injection pressure less than 0.5MPa. Therefore, it can be concluded that, regardless of the magnitude of fracture aperture and grouting time, it

is highly unlikely that high injection pressure would take place when the injection flow rate is low.

Figure 4.10 Statistics of the injection flow rate Q and pressure P_0

Figure 4.11 Distribution of injection pressure P_0 under various aperture ratio R_H

In the meanwhile, the distribution characteristics of injection pressure under different aperture ratios are presented in Figure 4.11, which shows that the percentage of higher injection pressure zones are gradually shrinking and vanishing, with aperture ratio becoming larger; and all the injection pressures fall into the low-pressure zone when the aperture ratio is higher than 4. These characteristics, together with the conclusions obtained from Figure 4.9 and Figure 4.10, can be used for guiding engineering

grouting: it can be inferred that wide or very wide fractures may be encountered, if the injection pressure remains low (not increasing to a certain pressure level, say 100 MPa in the current model) within a certain time; then longer time or higher injection flow rate is required for grouting the primary holes to ensure that a sufficient range of large fractures are filled and a sufficient rock mass strength can be obtained.

Regarding fracture deformation, apart from dependent variables like the deflection aperture ratio $100w_{max}/H_2$ and location slab length ratio $100r(w_{max})/SL$, other 12 parameters are all independent variables. Before investigating the correlations between independent and dependent variables, first analyze the statistics of maximum deformation w_{max} and the location where it appears $r(w_{max})$ regarding their derivative variables.

Figure 4.12 Statistics of derivative variables related to value and location of maximum deflection

Figure 4.12 shows the distribution of $100w_{max}/H_2$ and $100r(w_{max})/SL$ for all cases. The upper and lower boundaries of the box represent the 25th and 75th percentiles, named as Prc_1 and Prc_3 , respectively; the line in the middle indicates the 50th percentile, named as Prc_2 . The points beyond the range of two whiskers are called outliers and are presented individually with “+” symbol. Extending to the most

extreme data points not counted as outliers, two whiskers correspond to vertical coordinates in Figure 4.12 equaling $W_1 = Prc_1 - 1.5(Prc_3 - Prc_1)$ and $W_2 = Prc_1 + 1.5(Prc_3 - Prc_1)$, respectively. For $100w_{max}/H_2$, the percentiles equal 0 and 87.5th for W_1 and W_2 , respectively; likewise, the percentiles of $100w_{max}/H_2$ equal 0 and 97.5th for W_1 and W_2 , respectively. Therefore, there are no points located outside the lower whiskers for both cases; nevertheless, 10 ($= 80 \times (1 - 87.5\%)$) and 2 ($= 80 \times (1 - 97.5\%)$) points are located outside the upper whiskers of two cases, respectively.

Figure 4.13 Distribution of the dimensionless maximum deflection $100w_{max}/H_2$ under various fracture aperture H_2

In summary, regarding the maximum slab deflection, almost 90% percent of all cases studied are less than 0.4% of the half aperture of narrower fracture, and all of them are less than 1%, which indicates that the influence of fracture deformation induced by grouting is not significant; Concerning the location where the maximum deflection occurs, 75% of all cases are less than 25% of rock slab length, indicating that most of the narrower fracture might not be sealed, if the value of $2H_2 - w_{max}$ is less than the critical aperture for cement to pass through (Eklund and Stille, 2008; Draganovic, 2009). Nevertheless, within the parameter ranges chosen according to Table 4.6,

filtration would not happen for cases studied here, and fracture deformation will not reduce the penetrability of rock fractures with smaller apertures.

Slab deformation (causing fracture normal deformation) is a complex phenomenon resulted from combined effect of all 12 independent variables, so it can be seen from Figure B2 that there are very few independent variables having strong positive or negative correlations with $100w_{max}/H_2$ and $100r(w_{max})/SL$, except fracture aperture H_2 , aperture ratio R_H , stiffness ratio R_L , which are presented in Figure 4.13 to Figure 4.15.

Figure 4.14 Combined effect of fracture aperture H_2 and fracture aperture ratio R_H on the dimensionless maximum deflection $100w_{max}/H_2$, which is illustrated with color contour

In Figure 4.13, the distribution of $100w_{max}/H_2$ is grouped into five categories, which are represented by different color bars; it is obvious that the length of color bar representing smaller magnitude of deflection is gradually extending with increasing fracture aperture, and the length of color bar representing larger magnitude of deflection is gradually shrinking and vanishing. Figure 4.14 further presents the combined effect of fracture aperture H_2 and aperture ratio R_H on the distribution of maximum slab deformation, the magnitude scales of which are represented by color contours, with dark red color and dark blue color representing large and small deformation, respectively. All large deflection cases fall into zones with $H_2 < 400\mu m$ and $R_H < 2$. The impact of R_H is much more significant, as it ensures that

both apertures of the neighboring fractures are sufficiently small, especially the wide fracture. It is known that wide fractures are mainly grouted during grouting of the primary holes, thus narrow fractures are to be grouted during grouting of the secondary holes and afterwards, indicating that large deformation is more likely to occur during the later stages of grouting, when attention to fracture deformation should be paid.

Figure 4.15 The relationship between the dimensionless location where maximum deflection appears $100r(w_{max})/SL$ and penetration slab length ratio R_L

The location where the maximum deformation takes place is also of great importance, so the influence of aperture ratio R_H and penetration slab length ratio on $100r(w_{max})/SL$ are presented in Figure 4.15 and Figure 4.16.

From Figure 4.15, it can be found that the $100r(w_{max})/SL$ is descending with increasing R_H , indicating that the location where the maximum slab deflection occurs is moving closer to the injection hole during this process. It is easy to explain this phenomenon: larger R_H correlates to lower velocity of grout flow in two neighboring fractures, leading to smaller range of grout penetration (see Fig. 8), thus the range subjected to fluid force is becoming smaller, finally leading lower value of $100r(w_{max})/SL$. The conclusion obtained can be further validated by the trend shown in Figure 4.15, where the location of maximum deformation takes place is

moving forward along the direction of grout flow with increasing penetration slab length ratio, namely increasing grout penetration.

In summary, within a certain time, the range of grout penetration L_1 and L_2 and injection pressure P_0 increase with increasing injection flow rate Q , and they become larger and increase faster when the aperture difference between two neighboring fractures is smaller, which could lead to larger fracture deformation. Therefore, larger fracture deformation probably happens in the later stages of grouting, when wide fractures have already been filled and narrow or fine fractures are not well grouted; then in the latter stage, onsite engineers may increase the injection flow rate to accelerate the grouting process, which would lead to higher injection pressure and larger fracture deformation, finally reducing penetrability of fine or very fine fractures.

Figure 4.16 Influence of stiffness ratio on the dimensionless location where maximum deflection appears $100r(w_{\max})/SL$

Finally, the influence of another variable: the stiffness ratio R_k , which has lower negative correlation to fracture deformation, but it is very important variable, as shown in Figure 4.16. It is interesting to find that, apart from few outlier points, the overall trend of maximum deformation is becoming smaller with increasing R_k , namely with increasing fracture stiffness of narrower fracture. This phenomenon

again reduces the occurrence probability of very large fracture deformation that can significantly weaken the penetrability of narrower fractures.

For grouting of various combinations of two rock fractures within a certain time, the difference between grout penetrations in two neighboring fractures would become larger with increasing aperture difference between them; for grouting of two specific neighboring rock fractures, the penetration difference between wider and narrower fractures is enlarging with time; thus, fractures with smaller apertures would be grouted mainly during the later stage of grouting.

Within a certain time, the precondition for the occurrence of high injection pressure is high injection flow rate, but the injection pressure could increase faster when both fractures under grouting are of small apertures, which can be used to evaluate the magnitude of fracture aperture and the real-time grout penetration.

The maximum deflection of rock slab between two neighboring fractures and the locations are important factors to evaluate the influence of fracture deformation on grouting; the former is affected mainly by the aperture magnitude of both fractures under grouting, and the latter is influenced by both aperture difference and range of grout penetration.

Reduction of penetrability of narrower fractures due to fracture deformation would mainly happen during the later stages of grouting, when it is more likely that the injection pressure would become larger and increase faster, but it still depends on the magnitude of fracture stiffness of narrower fracture.

The impact of fracture deformation on grouting is not significant under the conditions assumed in the present thesis, partly because the injection flow rates used are not excessively high.

In the meanwhile, the model presented in this chapter remains in the level of theoretical study, and it represents a very special scenario, where two neighboring fractures are parallel with each other. Rock fracture in nature, however, are very complex and one injection hole could intersect several fractures, many of which are probably not parallel. Moreover, the distribution of complex in-situ stress could also

affect the effect of fracture deformation, which are not taken into consideration in this chapter. As the hydromechanical coupling effect are not only related to the working safety of onsite engineers but also to the relevant sealing quality, complete understanding of the coupling effect is of great significance to engineering practice, which, however, require more comprehensive and systematic investigations in the future work.

4.4 Grouting rock fractures with flowing water

Water may appear and flow into rock tunnel continuously under pressure (flowing water), when fractures connecting with water bearing areas or underground rivers are encountered during rock excavation. Seepage of water is unacceptable for rock tunnels with high standard of watertight requirements, such as subway. Flowing water is a challenging problem faced by field engineers and researchers for a long time, since it can increase the difficulty of rock excavation and the cost of a project, due to the extra effort spent on the treatment of inflow water. Grouting is commonly used to prevent the seepage of water during rock excavation, but it highly relies on personal experiences, no enough and reliable theory is available to guide engineers (Vik et al., 2000; Wang et al., 2007).

A lot of experiments were conducted to investigate the characteristics of grouting process accompanied by flowing water so far. Zhang and Liu studied the grouting of rock fractures with flowing water and proposed the theory of U-shape diffusion of grout material and proposed a formula to determine the diffusion condition (Liu et al., 2011); through further experimental study, Liu found that grout penetration could basically form three typical shapes but did not reveal the mechanism behind (Zhang et al., 2015). Another research team compared the influence of static and flowing water on the diffusion of grout and concluded that flowing water can reduce the penetration length of grout in the upstream but increase the relevant penetration in the downstream (Zhang et al., 2011). As for theoretical study on this subject, Ren proposed the first model, where he used the principle of superposition according to the theory of potential flow, then grouting under flowing water was considered as a superposition of a uniform flow (representing flowing water) and a point source (representing injection of grout) (Ren, 1982); this model is reasonable to some extent,

but the influence of boundary condition was not considered. Another model was proposed by Zhan, where the injection of grout and flowing water were treated as two pressures: grout injection pressure and static water pressure as well as water induced force at the interface between grout and water, then he derived the relationship between the radius of grout penetration and grouting time (Zhan et al., 2011); however, the hypothesis that the diffusion shapes of grout in the upstream and downstream were both semicircles with different radii may not be applicable in many cases. Therefore, in the present work, a new model considering the existence of rock boundaries is proposed, where the grout flow injected into a fracture channel with limited width is represented by a point source confined by two parallel plates, and the flowing water is taken as a uniform flow. Simulations are conducted to explain three phenomena found some laboratory experiments. The flow rates of grout required to seal rock fractures with flowing water within a certain time are investigated.

4.4.1 Theoretical model

Individual rock fracture in practical rock masses is three dimensional (3D), with certain length, width, and height, where the height is normally called as aperture, which is usually of a very small value. Therefore, the space inside a fracture resembles a very thin cuboid, of which the side elevation view and top plan view are presented in Figure 4.17 and Figure 4.18, respectively.

Figure 4.17 Grouting rock fractures with flowing water (side elevation view)

Figure 4.18 Grouting rock fractures with flowing water (top plan view)

The length, width and aperture of the fracture are L , $2h$ and $2b$, respectively. The radius of the injection pipe is r_0 . The upper and bottom boundaries represent the upper and bottom surfaces of a fracture, and the front and back boundaries are the boundaries limiting the width of the fracture channel, which is in the direction of X axis. Grout is injected into the fracture through injection pipe, where grout flow can flow outside radially.

Since the aperture of fracture is very small, being 10^{-4} m in the order of magnitude (Hakami, 1995; Eriksson, 2002; Eklund, 2005; Kobayashi and Stille, 2007), so the current problem can be studied based on the theory of 2D plane flow, where the flowing water can be represented by a uniform flow, and the relevant grout flow can be treated as a point source placed between two parallel walls, according to the model shown in Figure 4.18.

Suppose the velocity of the water comes from the left side is a constant v_0 , so the relevant complex potential of water W_f can be expressed as follows based on the theory of potential flow,

$$W_f = v_0 z, \quad (z = x + iy) \quad (4.28)$$

Assume that the grout injection flow rate Q is a constant, thus the relevant strength is $Q/2\pi b$ (Ren, 1982). The complex potential of injected grout flow W_g can be described as follows before considering the influence of boundary condition,

$$W_g = \frac{Q}{2\pi b} \ln z \quad (4.29)$$

The two lateral boundaries along the flow direction, where the point source is placed, can be taken as two mirrors based on the theory of potential flow. Therefore, infinite number of images of the original point source will be generated, like a man standing between two parallel mirrors, where it will see millions of its images inside both mirrors. Based on the Method of Images, the complex potential W_{gk} below can represent these images,

$$W_{gk} = \frac{Q}{2\pi b} \ln(z - z_k), \quad (z_k = i \cdot 2kh, k \in (-\infty + \infty), k \neq 0) \quad (4.30)$$

where z_k is the location of the point source and injection hole.

Therefore, based on the potential theory, the final form of the complex potential of a point source between two parallel walls is a superposition of the original point source and its images,

$$W_G = \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \ln(z - z_k), \quad (z_k = i \cdot 2kh) \quad (4.31)$$

Considering the influence of flowing water, the complex potential W of the whole flow system can be expressed as follows,

$$W = W_f + W_G = v_0 z + \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \ln(z - z_k), \quad (z_k = i \cdot 2k) \quad (4.32)$$

Differentiate both sides of Eq. (4.32), the velocity along X and Y direction of the whole flow system can be obtained,

$$\frac{dW}{dz} = v_x - iv_y = v_0 + \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \frac{1}{z - z_k}, \quad (z_k = i \cdot 2k) \quad (4.33)$$

It is known that $z - z_k = x + i(y - 2kh)$, so Eq. (4.33) can be further transformed,

$$v_x - iv_y = v_0 + \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \frac{x - i(y - 2kh)}{x^2 + (y - 2kh)^2} \quad (4.34)$$

Then, the expressions of v_x and v_y are as follows,

$$\begin{aligned} v_x &= v_0 + \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \frac{\cos \theta_k}{r_k}; v_y = \frac{Q}{2\pi b} \sum_{k=-\infty}^{+\infty} \frac{\sin \theta_k}{r_k}, \{k \in (-\infty, +\infty)\} \\ r_k &= \sqrt{x^2 + (y - 2kh)^2}, \sin \theta_k = x/r_k, \cos \theta_k = (y - 2kh)/r_k \end{aligned} \quad (4.35)$$

If there are no boundaries in the front and end of the fracture channel as shown in Figure 4.17 and Figure 4.18, namely the width of the channel is infinite or is so large that the influence of boundaries is negligible, then the velocities v_x and v_y can be expressed in simple forms,

$$\begin{aligned} v_x &= v_0 + \frac{Q}{2\pi b} \frac{\cos \theta}{r}, v_y = \frac{Q}{2\pi b} \frac{\sin \theta}{r} \\ r &= \sqrt{x^2 + y^2}, \sin \theta = x/r, \cos \theta = y/r \end{aligned} \quad (4.36)$$

As $v_x = dx/dt$ and $v_y = dy/dt$, and let $\bar{Q} = Q/2\pi b$, thus Eq. (4.36) can be written as,

$$\begin{aligned} \frac{dx}{dt} &= v_0 + \frac{\bar{Q} \cos^2 \theta}{x} \\ \frac{dy}{dt} &= \frac{\bar{Q} \sin^2 \theta}{y} \end{aligned} \quad (4.37)$$

Let $M = \bar{Q} \cos^2 \theta / v_0$, Eq. (4.37) can be written in the form that both sides of equation can be integrated with respect to x (or y) and t respectively,

$$\begin{aligned} \frac{x}{M+x} dx &= v_0 dt \\ y dy &= (\bar{Q} \sin^2 \theta) dt \end{aligned} \quad (4.38)$$

Integrate both sides of Eq. (4.38) with respect to x , y , and t within the interval of $[x_0, x]$ (or $[y_0, y]$) and $[0, t]$, then the following relationships can be derived,

$$\begin{aligned} M \ln \left| \frac{x+M}{x_0+M} \right| &= x - x_0 - v_0 t \\ y^2 - y_0^2 &= 2\bar{Q}t \sin^2 \theta \end{aligned} \quad (4.39)$$

where $x_0^2 + y_0^2 = r_0^2$ with x_0 and y_0 representing the coordinates of the nodes along the injection hole. Eq. (4.39) can describe the path line of grout flow.

The farthest distance the grout can run in the upstream (along the negative direction of X axis) is the location where its velocity becomes zeros, which can be obtained from Eq. (4.36) by letting the left-hand side be zero,

$$x = -Q/2\pi b v_0 \quad (4.40)$$

The minus sign indicates that it is in the negative axis, and this point is also called stagnation point. Nevertheless, for the case when the width is neither very large nor

infinite, which was discussed previously, the location of the stagnation point should be derived based on Eq. (4.34) when v_x is close to zero; and it is taken as zero when $|v_x| \leq 10^{-5} \text{ m/s}$ in the present work.

(a) With two boundaries

(b) Without boundary

Figure 4.19 Semi-plane streamline of grout flow in fracture with flowing water

Figure 4.19 shows the comparison of the streamlines of grout flow in rock fractures with and without boundaries, under the influence of flowing water, where the water

is flowing along the horizontal direction. Half plane streamline is drawn here since it is symmetric about the horizontal axis, and the streamline is calculated within 10s. The values of the injection rate and the velocity of flowing water are the same for both cases. The fracture aperture and the radius of injection hole used for calculation are 5mm and 2cm, respectively. As for the boundary in Figure 4.19(a), the relevant fracture width is 2m.

It can be seen from Figure 4.19 that the influence of fracture boundaries is significant, and the patterns of streamline are greatly different between Figure 4.19 (a) and (b), where the direction of streamline in Figure 4.19(a) is becoming horizontal due to the influence of boundary nearby, whereas, in Figure 4.19(b), it is more in radial direction. Moreover, the grout run much farther in Figure 4.19(a) in the horizontal direction, where the maximum penetration in the upstream and downstream are 0.7182m and 2.3194m, respectively; while, in Figure 4.19(b), they are 0.584m and 1.875 in the upstream and downstream, respectively. However, in the perpendicular direction, the maximum penetration in Figure 4.19 (b) is 1.0344m, larger than that in Figure 4.19 (a), being 0.8089m.

Therefore, it can be concluded that the existence of boundaries can increase the penetration along the direction of flowing water both in the downstream and upstream, but, in the meanwhile, it weakens the penetrability of grout in the direction perpendicular to the flowing water.

4.4.2 Experimental results and theoretical explanation

Experiments were conducted to find new characteristics that may appear during grouting of rock fractures with flowing water. A special experimental platform was developed for grouting simulation. The fracture has a length, width, and aperture of 4m, 2m, and 5mm, respectively, of which the bottom surface is fixed, and the upper surface is a removable tempering glass. The radius of the grout injection hole is 2cm. The pneumatic grouting pump used can provide a grout injection flow rate ranging from 0 to 30L/min, and a grouting pressure up to 3MPa, and two pumps are used. The radius of injection hole used for calculation is 2cm.

During the grouting experiment, the injection flow rate is controlled to vary within 20L/min and 50L/min, and the initial velocity of the flowing water ranges from 0.16 m/s to 0.4m/s, and the stopping pressure is set as 0.5MPa.

Due to the existence of flowing water, the location of the injection hole is not set at the geometric centroid, but a position close to the entrance of the flowing water; normally, the distance between the injection hole and the water entrance should be less than half the channel length.

Figure 4.20 Three types of grout penetration in fractures with flowing water

From the experiments conducted, the following three types of grout penetration would appear, as shown in Figure 4.20. For grout penetration type I, the outline of the grout penetration is like an ellipse from the start to the end, the fracture channel is not sealed before the grout flow reaches the downstream end.

For the outline of type II, an elliptical shape forms and develops till the grout flow reaches the two boundaries along the X direction, after which the outline of grout flow becomes a U-shape, and the fracture in type II is sealed finally. As for the type

III grouting, the outline of the grout penetration gradually becomes a U-shape till it reaches the downstream end; the fracture is not sealed.

These three types of grout penetration as shown in Figure 4.20 can be described by the model derived in Section 4.4.1. The fracture aperture and the radius of the injection hole used for calculation are the same with that in the experiment, and the grout injection flow rate and the velocity of flowing water are listed in Table 4.7. Due to the existence of flowing water, the location of the injection hole is not set at the geometric centroid, but a position close to the entrance of the flowing water; normally, the distance between the injection hole and the water entrance should be less than half the channel length.

From the experiments conducted, the following three types of grout penetration would appear, as shown in Figure 4.20. For grout penetration type I, the outline of the grout penetration is like an ellipse from the start to the end, the fracture channel is not sealed before the grout flow reaches the downstream end.

Table 4.7. The grouting time used for calculation is 10s. Therefore, the related half plane streamlines are presented in Figure 4.21 as Type I, II and III correspondingly.

Type I

Type II

Type III

Figure 4.21 The half plane streamline corresponding to three types of grout penetration found from flowing water grouting experiments

Due to the existence of flowing water, the location of the injection hole is not set at the geometric centroid, but a position close to the entrance of the flowing water; normally, the distance between the injection hole and the water entrance should be less than half the channel length.

From the experiments conducted, the following three types of grout penetration would appear, as shown in Figure 4.20. For grout penetration type I, the outline of the grout penetration is like an ellipse from the start to the end, the fracture channel is not sealed before the grout flow reaches the downstream end.

Table 4.7 Comparison of grouting data among three grouting types

Type	Grouting condition	Injection flow rate	Water velocity
I	$Q = 1L/s, v_0 = 0.05m/s$	Low	Low
II	$Q = 10L/s, v_0 = 0.2m/s$	Very high	Relatively high
III	$Q = 4L/s, v_0 = 0.4m/s$	Relatively high	Very high

For the outline of type II, an elliptical shape forms and develops till the grout flow reaches the two boundaries along the X direction, after which the outline of grout flow becomes a U-shape, and the fracture in type II is sealed finally. As for the type III grouting, the outline of the grout penetration gradually becomes a U-shape till it reaches the downstream end; the fracture is not sealed. and Figure 4.21 that: Type I shape of grout penetration appears when both the injection rate and the velocity of flowing water are low, indicating that neither side has overwhelming influence on the other side; therefore, it is probably that grout cannot seal the fracture, or it takes quite long time to seal the fracture, both situations are not acceptable in practical grouting project. Grouting situation of Types II shape happens when the injection of grout flow has overwhelming impact on the flowing water, since the former is very high compared with the latter; hence, the fracture can be sealed in a short time. Contrary to Type II, when the flowing water has overwhelming influence on the injected grout flow will lead to the occurrence of Type III grout penetration; thus, the fracture cannot be sealed however long time it takes, since the streamline is moving forward in the same direction as the flowing water.

Additionally, it can be found from the three types of grouting that, the grout penetration in the upstream is far less than that in the downstream, indicating that the grout hole should also be properly located to ensure efficient grouting; because the total effective length grouted may not be long enough to hold the relevant water pressure, where the effective length of grout penetration is the penetration length in the upstream plus that in the downstream, and the latter is generally very small, thus

requiring sufficient length in the downstream. In the meanwhile, it can be noticed that the total penetration lengths in Types II and III within 10s are larger than the length of the platform in the experiment, since no boundaries are set at the two ends in the theoretical model; moreover, it can be found that the whole fracture channel is sealed in the direction of length if the effective penetration length predicted is larger than the practical fracture length.

Therefore, it can be concluded that the injection flow rate of grout should be properly chosen before grouting operation via the velocity of flowing water measured; theoretical calculation can be conducted to ensure that the grout injected has overwhelming influence on the relevant flowing water, otherwise, there would be no effective grouting.

4.4.3 Determination of grouting strategy

The principle for determining the value of grout injection flow rate according to various flowing water is a key factor in grouting strategy. In this section, three kinds of fractures, with fracture apertures of 0.5mm, 1mm and 5mm, respectively, are to be grouted under flowing water with various velocities. In this section, each fracture with the same velocity of flowing water are required to be sealed within 60s, namely the penetration length in the Y direction should reach 1m, thus proper grout injection flow rate should be determined. The velocities of flowing water in fractures to be sealed are 0.02, 0.05, 0.75, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55 and 0.6m/s.

The injection flow rates of grout required to seal rock fractures with various flowing water within 60s are presented in Figure 4.22(a), and the relationships between injection flow rate divided by the relevant velocity of flowing water are shown in Figure 4.22(b). It can be found from Figure 4.22(a) that, overall, the higher the velocity of flowing water, the higher injection rate of grout should be provided to seal the fractures of interest. More useful patterns can be seen from Figure 4.22(b) that the injection flow rate divided by the relevant water velocity is decreasing with increasing velocity of flowing water; the slope of each curve is approaching zero gradually in each case, where the ratio becomes approximately constant, which are 2,

4 and 20 L/m corresponding to apertures of 0.5, 1 and 5mm, which forms a geometric progression.

Additionally, it can be observed from Figure 4.22 (a) that, for grouting of rock fracture with 0.5mm, part of the curve has negative slope, indicating that grout injection flow rates in these situations are decreasing with increasing water velocities. This phenomenon shows that the influence between grout flow and flowing water is a complex process, but it still follows the basic law in fluid mechanics. In the meanwhile, the ratio between the relevant injection flow rate and water velocity is decreasing without exception.

Therefore, the above analysis can lead to the following grouting strategies: rock fractures with high velocity of flowing water require high injection flow rate. However, it is not the higher the better for determining the relevant grout injection flow rate, since, with faster water inflow, the ratio between injection flow rate (required for effective sealing within a certain time) and water velocity is decreasing. Moreover, high injection flow rate requires the allocation of grouting pumps with high capacity, which cost quite a lot or even are non-existent at the working site.

Under different combinations of flow rates between grout flow and flowing water, three types of grout penetration can be observed during grouting of rock fracture with flowing water. It is found that the injection flow rate of grout should be chosen properly to ensure overwhelming effect on the flowing water, avoiding ineffective grouting. For rock fractures required to be sealed within a certain time, generally, the magnitude of grout injection rate should increase with increasing water velocity, but a decreasing ratio between these two parameters can be chosen with increasing water velocity, which can help reduce overall grouting cost.

In fact, the properties of both grout flow and flowing water can also influence grout spreading, which are ignored in the present model to qualitatively (to a certain extent) explore the basic characteristics of grout penetration in fractures with flowing water. In the meanwhile, the interaction between fluid flow and rock fractures are also not considered, which might influence the grout penetration if the pressure of fluid flow

on rock fractures is very high, or if the elastic modulus of the relevant rock masses is very low, which could lead to considerable rock deformation.

(a)

(b)

Figure 4.22 Injection flow rate required to seal certain fractures with flowing water based on simulation: (a) $Q \sim v_0$; (b) $Q/v_0 \sim v_0$

4.5 Summary

In this chapter, simple process for deriving the governing equation for radial grout flow around an injection hole (not considering flowing water) is presented, where the conservation of mass and momentum is used. Then, the flow rate expression derived is utilized to calculate time dependent grout spread in two neighboring fractures directly intersected by the injection hole. As excessive fluid pressure (higher than the in-situ stress) could lead to jacking (normal deformation) of rock fractures with larger aperture, which might weaken the penetrability of its neighboring fracture with smaller aperture. Thus, a numerical model is further proposed to evaluate the degree of this effect. It is found that grout penetration in rock fracture with larger aperture is much faster than that with smaller aperture, and the difference between grout penetrations in two fractures is increasing with time. The fracture deformation induced penetrability reduction of rock fracture with smaller aperture is limited, since the ratio between the maximum fracture deformation and the aperture of narrower fracture is not high enough to be significant. Here, the maximum jacking is taken into consideration, since the location where it appears is critical to the penetrability of the whole fracture, and it is the point where filtration would most probably take place.

Sealing of rock fractures with flowing water is also investigated through developing a simple model based on the theory of potential flow. The analysis is qualitative to some extent, but it can be used to explain three typical grouting phenomena observed from some experimental data. It is found that, to seal a rock fracture with flowing water within a certain time, the rate of grout injection should be raised with higher water flow rate, but it does not mean that the ratio between injection rate and water velocity should be kept as a constant, which, however, is found to be decreasing with increasing velocity of flowing water. Therefore, the injection rate of grout should be properly chosen to seal a specific rock fracture with flowing water, and it is better that certain measurement data could be used as a reference during the grouting process, like variation of injection pressure and rate of water inflow.

Chapter 5 Grouting of Fracture Network Based on DDA

5.1 Introduction

Since the day when the Discontinuous Deformation Analysis (DDA) was created, it has been used and modified for analysis in many areas (Shi, 1992; Lin et al., 1996; Cheng, 1998; Kim et al., 1999; Jing et al., 2001; Chen et al., 2013), due to its advantages in solving problems in rock engineering, where rock masses are discrete rock blocks separated by all kinds of discontinuities. The relevant 2D and 3D DDA have also been developed to handle more realistic problems and get better results and visual effects (Shi and Goodman, 1989; Yeung et al., 2007; Beyabanaki et al., 2008). In the past two decades, DDA has been applied in the field of hydromechanics (Kim et al., 1999; Jing et al., 2001; Koyama et al., 2011; Ben et al., 2012; Chen et al., 2013), such as water flow in fractures (Jing et al., 2001; Chen et al., 2013) and fracturing process due to hydraulic forces (Koyama et al., 2011; Ben et al., 2012). Grout is also a kind of fluid, generally modelled as Bingham flow (Hässler et al., 1992a; Moon and Song, 1997; Hori and Koyama, 1999; Lombardi, 2002; Yang et al., 2002; Hernqvist et al., 2008; El Tani, 2012; Mohammed et al., 2014), so DDA is used for simulating the grouting process in this chapter, based on constant injection rate method and constant injection pressure method. Finally, some examples and comparisons will be given to validate the capability of DDA grouting prediction.

Grouting is an aged technology, which can be dated back to the nineteenth century (Warner, 2004), but experiences of engineers in grouting application accumulate much faster than that concerning theoretical knowledge on this field. The reason behind partly comes from the fact that it is too hard to know clearly and exactly the detailed geological conditions, such as the fracture distribution in rock masses, which is very important for grouting simulation and providing guidance for grouting design. In the meanwhile, many works on prediction of grout penetration either remain at the level of single fracture channel or on steady state analysis (Dai and Byron Bird, 1981; Barker, 1988; El Tani, 2012; Singh et al., 2015; Sui et al., 2015). Though steady state prediction is important for us to understand the characteristics of grouting process, such as pressure and velocity distribution, it is more critical to know the dynamic grout penetration with varying time under certain grouting setup, such as specific type

of cement grout and grout injection flow rate, so that better grout plan can be made, and onsite engineers can determine the right time to stop grouting. However, research on real-time grouting prediction is far from enough. Simulation on real-time grout penetration in a 2D fracture network first appeared in Hässler's work (Hässler et al., 1992b), but without detail introduction on the algorithm used. Then, Eriksson conducted experimental validation in comparison with simulation results, but theoretical background on the simulation process was not provided in detail (Eriksson, 2002). In 2002, Yang (Yang et al., 2002) also proposed a method to simulate grout penetration in 2D fracture network, but it could only predict the overall sealing conditions of a specific fracture, which can be used to check whether a fracture is filled at a specific time step but cannot show real-time penetration in individual fracture. In recent years, UDEC was used to simulate grouting of 2D fracture network by Saeidi (Saeidi et al., 2012; Saeidi et al., 2013), and the influence of different factors on grout penetration were also discussed, and fracture distributions in their case study were all simulated situations without any practical background; meanwhile, there were no introduction to the mechanism on how grout spreads from node to node.

As one type of Discrete Element Method (DEM) family, Discontinuous Deformation Analysis (DDA) is different from UDEC, since it solves the relationship between displacement and stress more like the Finite Element Method (FEM). More importantly, it is not hard to know how DDA works, and implement whatever modules of different functions into it. Therefore, DDA is chosen as a platform to develop a grouting module. In the present work, a node based model is proposed for solving the real-time grout penetration in fracture segments connected to one node, where the node pressure and grout penetration in individual fracture segment starting from this node are taken as variables. The governing equations are formed using the conservation of mass at each node, where the inflow of grout equals that outflow from it; and based on the relationship between length of grout penetration and velocity of grout flow in a fracture. Real-time grout penetration in a 2D fracture network is taken as a case study to illustrate how the algorithm works. The precondition for the current algorithm is that the connection between nodes in a fracture network should be known in advance, which is easy to know through developing a matrix specially for storing the connection information for individual nodes based on DDA.

5.2 Principle of DDA

Designed for discontinuous block system, DDA derives its system equations like a finite element method but with analytical and discrete properties, as block deformation matrix of individual block in DDA is also derived from combination of its shape function and block deformation parameters. Meanwhile, the motion equation of a block system using DDA method is also obtained from minimization of the system's total energy. The key difference between DDA and FEM is that block deformation parameters in DDA are three initial displacements and initial strains of individual block, rather than two displacements of all nodes in an element as that in FEM.

The displacement matrix of a block is expressed as follows (Shi, 1992),

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 & -(y-y_0) & (x-x_0) & 0 & (y-y_0)/2 \\ 0 & 1 & (x-x_0) & 0 & (y-y_0) & (x-x_0)/2 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ r_0 \\ \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix} \quad (5.1)$$

where u and v are horizontal and vertical displacements of any point with coordinate x and y , x_0 and y_0 are coordinates of a specific position, u_0 , v_0 , and r_0 are the horizontal, vertical and rotational displacements at that position x_0 and y_0 , and ε_x , ε_y , and ε_{xy} are relevant normal and shear strains.

In terms of a block system consisting of n individual blocks with contacts between each other, the system equilibrium equation can be written as (Shi, 1992),

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\ K_{31} & K_{32} & K_{33} & \cdots & K_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{Bmatrix} \Rightarrow [K]\{D\} = \{F\} \quad (5.2)$$

where $[K]$, $\{D\}$ and $\{F\}$ are coefficient matrix, displacement matrix and force matrix of a block system, respectively. D_i and F_i represent deformation of and

loading on block i , which are 6×1 submatrices. K_{ij} is determined by material properties of block i when $i = j$, or by contract between block i and j when $i \neq j$. These individual matrices are obtained from minimization of total potential energy resulted from forces and stresses.

5.3 Methodology

Figure 5.1 (a) shows the concept of 3D fracture network, and Figure 5.1 (b) is a 2D fracture network, resulting from the intersection between the 3D fracture network and a xy plane with a specific z . The model in Figure 5.1(b) can be imported into DDA to develop the grouting module for evaluating real-time grout penetration.

Figure 5.1 Schematic diagram of fracture network: (a) 3D; (b) 2D

5.3.1 Derivation of governing equations

Figure 5.2 shows the most basic element involved for grouting of 2D fracture network, which is a node and fracture segments linked to it. The pressure at each node is set as $P_i (i \in [1, N])$, where N is the total number of nodes in a fracture network; the grout penetration in a segment is taken as $L_{ij}(t), (i \neq j; i, j \in [1, N])$; the length of segment $i \leftrightarrow j$ is L_{ij} , and it is a constant parameter. It is true that $L_{ij} = L_{ji}$, but it holds true for the following expressions if and only if segment $i \leftrightarrow j$ is filled by grout,

$$L_{ij}(t) = L_{ji}(t) = L_{ij}$$

The reason is that grout can not only run from i to j , but also run from j to i , during the grouting process; thus, there are two grout flow velocities $v_{ij}(t)$ and $v_{ji}(t)$ when segment $i \leftrightarrow j$ is under grouting and not filled.

Figure 5.2 Schematic diagram of a node information

The relationship between grout spread $L_{ij}(t)$ in segment $i \rightarrow j$ ($\rightarrow$ represents the flow direction) and the corresponding velocity $v_{ij}(t)$ is as follows

$$\begin{cases} L_{ij}(t) = L_{ij}(t - \Delta t) + v_{ij}(t - \Delta t) \cdot \Delta t & \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \\ L_{ij}(t) = L_{ij} & \text{if } [L_{ij}(t) + L_{ji}(t) = L_{ij}] \end{cases} \quad (5.3)$$

As the time interval Δt is very small, the velocity at time $(t - \Delta t)$ can be used for calculating the grout penetration at time t without reducing the accuracy. Moreover, the segment $i \leftrightarrow j$ is taken as “filled” once $L_{ij}(t) + L_{ji}(t) = L_{ij}$, namely, grout flow in direction $i \rightarrow j$ and $i \leftarrow j$ meets each other.

It is known that the relationship between the injection rate and the grouting parameters and fracture characteristics can be expressed as (Xiao et al., 2017),

$$Q = \frac{Wh^3}{12\mu} \left(-\frac{\partial P}{\partial x} \right) \left[1 - 3 \left(\frac{y_0}{h} \right) + 4 \left(\frac{y_0}{h} \right)^3 \right], y_0 = \frac{\tau_0}{(-\partial P / \partial x)} \quad (5.4)$$

In practical grouting, the magnitude of $(y_0/h)^3$ is very small, compared with other two terms, that it can be ignored without introducing too much errors (Stille et al., 2009; Holmberg et al., 2012; Rafi and Stille, 2014; Rafi and Stille, 2015; Xiao et al., 2017). Thus Eq. (5.4) can be further simplified as

$$Q = \frac{Wh^3}{12\mu} \left[\left(-\frac{\partial P}{\partial x} \right) - 3 \frac{\tau_0}{(-\partial P / \partial x)h} \right] \quad (5.5)$$

The pressure gradient in segment $i \rightarrow j$ is $(-\partial P / \partial x) = (P_i - P_j) / L_{ij}(t)$, thus the grout flow rate $Q_{ij}(t)$ can be expressed as

$$Q_{ij}(t) = \begin{cases} \frac{Wh_{ij}^3}{12\mu} \left[\frac{P_i}{L_{ij}(t)} - 3 \frac{\tau_0 L_{ij}(t)}{P_i h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \\ \frac{Wh_{ij}^3}{12\mu} \left[\frac{P_i - P_j}{L_{ij}} - 3 \frac{\tau_0 L_{ij}}{(P_i - P_j)h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) = L_{ij}] \end{cases} \quad (5.6)$$

where h_{ij} is the fracture aperture of segment $i \leftrightarrow j$, which is taken as a constant.

Thus, the corresponding velocity $v_{ij}(t) = Q_{ij}(t) / Wh_{ij}$ can also be obtained

$$v_{ij}(t) = \begin{cases} \frac{h_{ij}^2}{12\mu} \left[\frac{P_i}{L_{ij}(t)} - 3 \frac{\tau_0 L_{ij}(t)}{P_i h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \\ \frac{h_{ij}^2}{12\mu} \left[\frac{P_i - P_j}{L_{ij}} - 3 \frac{\tau_0 L_{ij}}{(P_i - P_j)h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) = L_{ij}] \end{cases} \quad (5.7)$$

We can find from Eqs. (5.3) and (5.7) that, grout penetration $L_{ij}(t)$ for segment $i \rightarrow j$, is a function of the corresponding flow velocity $v_{ij}(t)$ and P_i , and $v_{ij}(t)$ is also a function of $L_{ij}(t)$ and P_i , which indicates that there are only two dependent variables. In the present work, $L_{ij}(t)$ and P_i are treated as dependent variables. In the meanwhile, it also indicates that the variables cannot be solved directly, since the relationships involved are nonlinear; thus, iterative algorithm should be presented, which will be introduced in detail in the following sections.

For a node connected to a series of fracture segments with a total number of n_i , as shown in Figure 5.2, there are $(n_i + 1)$ variables that should be solved, including n_i grout penetration length $L_{ix}(t)$ and one node pressure P_i .

Therefore, $(n_i + 1)$ equations should be found to solve all the variables. The first equation is the continuity equation at node i : all grout flow into the node will flow out the node, indicating that the sum of flow rate at node i is equal to zero,

$$f(Q_i) = \sum_k^{n_i} Q_{ik} = 0 \quad (5.8)$$

where Q_{ik} represents the flow rate in segment $i \rightarrow k$.

The remaining N equations are the corresponding real-time grout penetration in n_i different fracture segments,

$$f(L_{ik}) = \begin{cases} L_{ik}(t) - L_{ik}(t - \Delta t) - v_{ik}(t) \cdot \Delta t = 0 & \text{if } [L_{ik}(t) + L_{ki}(t) < L_{ik}] \\ L_{ik}(t) - L_{ik} = 0 & \text{if } [L_{ik}(t) + L_{ki}(t) = L_{ik}] \end{cases} \quad (5.9)$$

where the number $k = j_1, j_2, \dots, j_{n_i}$, and each j_x represents the numbering of nodes connected to node i .

5.3.2 Algorithm development

The variables related to node i , namely node pressure and segment penetration can be derived via solving Eqs. (5.8) and (5.9), but the relationships involved are nonlinear, so specific iteration algorithm should be designed to solve this problem. Newton's method is chosen because of its high efficiency in solving nonlinear problems. The original expression for Newton's method is as follows,

$$x^{Ni+1} = x^{Ni} - [f'(x^{Ni})]^{-1} \cdot f(x^{Ni}) \quad (5.10)$$

where x^{Ni+1} and x^{Ni} are the solutions to $f(x)=0$ after $(Ni+1)$ and Ni steps of iteration, respectively; $f'(x)$ is the first-order derivative of $f(x)$.

Two special issues need to be discussed for the nonlinear problem. First, there are more than one variables, so the variables and functions in Eq. (5.10) become array and matrix; so x^{Ni+1} , x^{Ni} , and $f(x^{Ni+1})$ become X^{Ni+1} , X^{Ni} , and $F(X^{Ni})$ with length of Na , and $f'(X^{Ni})$ become matrix $F'(X^{Ni})$ with size of $Na*Na$.

Second, the variables are all time dependent, so the convergent solutions derived from Newton's method are grouting states at specific time step, but the terms representing time step will not be shown in equation derivation in the following sections.

Therefore, Eq. (5.10) can be further expressed in matrix form as,

$$X^{Ni+1} = X^{Ni} - [F'(X^{Ni})]^{-1} \cdot F(X^{Ni}) \quad (5.11)$$

where the derivative matrix $F'(X^{Ni})$ for node i has the following form (matrix inside the red brackets),

$$\left[\begin{array}{ccccccc} x_i & x_{j1} & x_{j2} & \cdots & x_{j3} & \cdots & \\ \frac{\partial f_Q(x_i^{Ni})}{\partial x_i^{Ni}} & \frac{\partial f_Q(x_i^{Ni})}{\partial x_{j1}^{Ni}} & \frac{\partial f_Q(x_i^{Ni})}{\partial x_{j2}^{Ni}} & \cdots & \frac{\partial f_Q(x_i^{Ni})}{\partial x_{j3}^{Ni}} & \cdots & P_i \\ \frac{\partial f_L(x_{j1}^{Ni})}{\partial x_i^{Ni}} & \frac{\partial f_L(x_{j1}^{Ni})}{\partial x_{j1}^{Ni}} & & \cdots & & & L_{ij1}(t) \\ \frac{\partial f_L(x_{j2}^{Ni})}{\partial x_i^{Ni}} & & \frac{\partial f_L(x_{j2}^{Ni})}{\partial x_{j2}^{Ni}} & & & & L_{ij2}(t) \\ \vdots & & & \ddots & & & \vdots \\ \frac{\partial f_Q(x_{j3}^{Ni})}{\partial x_i^{Ni}} & & & & \frac{\partial f_Q(x_{j3}^{Ni})}{\partial x_{j3}^{Ni}} & & P_{j3} \\ \vdots & & & & & \ddots & \vdots \end{array} \right] \quad (5.12)$$

where node i has direct connection with nodes $j1, j2, j3, \dots$; x_i, x_{j1}, x_{j2} , and x_{j3} etc., representing $P_i, L_{ij1}(t), L_{ij2}(t)$, and P_{j3} etc., respectively. The derivative term $f'(x)$ in Eq. (5.10) becomes partial derivative $\partial f_Q / \partial x_i$ or $\partial f_L / \partial x_i$ in Eq. (5.12), since $f(x)$ is a function of more than one variables. f_Q and f_L are corresponding to Eqs. (5.8) and (5.9), respectively, and they are of different forms depending on the real-time grouting situation, since $f(x)$ is time dependent. Here, pressure P_{j3} in the sub-matrix of node i indicates that segment $i \rightarrow j_3$ is filled by grout, where segment $i \rightarrow j_3$ is either a fracture channel providing grout for node i from node j_3 or a channel providing grout from i for node j_3 .

There are two forms of $\partial f_Q / \partial x_i$, if x_i is node pressure P_i of segment $i \rightarrow j$ or P_j of segment $j \rightarrow i$,

$$\frac{\partial f_Q}{\partial P_i} = \begin{cases} \frac{Wh_{ij}^3}{12\mu} \left[\frac{1}{L_{ij}(t)} + \frac{3\tau_0 L_{ij}(t)}{P_i^2 h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \\ \frac{Wh_{ij}^3}{12\mu} \left[\frac{1}{L_{ij}} + \frac{3\tau_0 L_{ij}}{(P_i - P_j)^2 h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) = L_{ij}] \end{cases} \quad (5.13)$$

$$\frac{\partial f_Q}{\partial P_j} = \begin{cases} 0 & \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \\ \frac{Wh_{ij}^3}{12\mu} \left[-\frac{1}{L_{ij}} - \frac{3\tau_0 L_{ij}}{(P_i - P_j)^2 h_{ij}} \right] & \text{if } [L_{ij}(t) + L_{ji}(t) = L_{ij}] \end{cases} \quad (5.14)$$

In Eqs. (5.13) and (5.14), the second equation is used for updating two node pressures information for fracture segment $i \rightarrow j$ in the global information, after it is filled.

If x_i is the real-time penetration $L_{ij}(t)$ in segment $i \rightarrow j$, $\partial f_Q / \partial x_i$ has only one form,

$$\frac{\partial f_Q}{\partial L_{ij}(t)} = \frac{Wh_{ij}^3}{12\mu} \left[-\frac{P_i}{L_{ij}^2(t)} - 3\frac{\tau_0}{P_i h_{ij}} \right] \quad \text{if } [L_{ij}(t) + L_{ji}(t) < L_{ij}] \quad (5.15)$$

As $L_{ij}(t)$ becomes constant when this segment is filled, then penetration length corresponding to segment $i \leftrightarrow j$ will be removed from Eqs. (5.11) and (5.12) and the final global solution matrix for the fracture system. Therefore, the size of the global matrix will not increase significantly but would keep running in a relatively stable level. The procedure is the most important part of the current algorithm, as the computation efficiency could be improved significantly.

When x_i is node pressure P_i of segment $i \rightarrow j$ in terms of $\partial f_L / \partial x_i$, we have,

$$\frac{\partial f_L}{\partial P_i} = -\frac{\partial v_{ik}(t)}{\partial P_i} \Delta t = -\frac{h_{ij}^2}{12\mu} \left[\frac{1}{L_{ij}(t)} + \frac{3\tau_0 L_{ij}(t)}{P_i^2 h_{ij}} \right] \Delta t, [L_{ik}(t) + L_{ki}(t) < L_{ik}] \quad (5.16)$$

Here only derivative with respect to P_i is calculated without considering node pressure P_j at another node, since f_L is only a function of P_i before segment $i \rightarrow j$ is filled, and $f_L \equiv 0$ when the segment is filled, thus P_j will not appear in Eq. (5.16) regarding segment $i \rightarrow j$.

For $\partial f_L / \partial x_i$ regarding node pressure, when x_i is real-time penetration length $L_{ij}(t)$ of segment $i \rightarrow j$,

$$\frac{\partial f_L}{\partial L_{ij}(t)} = 1 - \frac{\partial v_{ik}(t)}{L_{ij}(t)} \cdot \Delta t = 1 + \frac{h_{ij}^2}{12\mu} \left[\frac{P_i}{L_{ij}^2(t)} + \frac{3\tau_0}{P_i h_{ij}} \right] \Delta t, [L_{ik}(t) + L_{ki}(t) < L_{ik}] \quad (5.17)$$

In summary, the partial derivative $\partial f / \partial x_i$ has only one expression when the relevant x_i represents $L_{ij}(t)$, which is no longer a variable when the segment $i \rightarrow j$ is filled, and it will be removed from the global matrix. As a result, the number of variables will increase during the grouting process but accompanied by removing some “known” variables. Therefore, the total number of variables will increase first and then decrease till all the fractures are grouted.

Substitute Eqs. (5.13) to (5.17) into Eq. (5.12), we can derive the matrix for node i ; assemble all matrices of individual node that are involved in grouting at time step t_n can form the global derivative matrix $F'(X)$. Moreover, the size of the global matrix at the initial time depends on how many nodes were taken as the injection point, which indicates that the size of the final global matrix could be $\sum_{k=1}^M n_i(j_k)$, if there are M injection nodes, where $n_i(j_k)$ is the number of nodes connected to node j_k . Nevertheless, the inverse matrix $[F'(X)]^{-1}$ is the target matrix required for calculating real-time grout penetration. It can be inferred from Eq. (5.12) that $F'(X)$ is a highly symmetric matrix, so Gauss elimination is introduced to derive the corresponding inverse matrix $[F'(X)]^{-1}$, and this method is of high efficiency since the original matrix is a sparse matrix. The detailed process and mechanism behind for conducting the Gauss elimination is presented in Figure 5.3. Finally, the real-time grout penetration can be obtained based on Eq. (5.11).

$$\left[\begin{array}{ccc|ccc}
 & & F'(X) & & & I \\
 B & B & B & \dots & 1 & \\
 B & B & & & 1 & \\
 B & & B & & 1 & \\
 \vdots & & \ddots & & & 1 \\
 & B & B & B & B & \dots \\
 & B & B & & & 1 \\
 & B & & B & & 1 \\
 & B & & & B & 1 \\
 & \vdots & & \ddots & & 1
 \end{array} \right] \downarrow \left[\begin{array}{ccc|ccc}
 1 & & & & A & A & A & \dots \\
 & 1 & & & A & A & & \\
 & & 1 & & A & & A & \\
 & & & 1 & \vdots & & \ddots & \\
 & & & & & A & A & A & A & \dots \\
 & & & & & A & A & & \\
 & & & & & A & & A & \\
 & & & & & A & & & A \\
 & & & & & \vdots & & \ddots & \\
 & & & & I & & [F'(X)]^{-1} &
 \end{array} \right]$$

Figure 5.3 Gauss elimination for solving the inverse matrix of the derivative matrix:
 B and A represent matrix element before and after transformation, respectively

5.3.3 Hydromechanical coupling

Figure 5.4 Pressure distribution along fracture $i \leftrightarrow j$

Grout flow within rock fractures would generate fluid pressure on fracture surfaces, which could cause considerable fracture deformation if the pressure is large enough.

Consider the characteristics of rock blocks in DDA, the fluid pressure on fracture surface between two nodes can be taken as line loading, as shown in Figure 5.4, where temporary nodes m and n would disappear when $L_{ij}(t) + L_{ji}(t) = L_{ij}$.

Suppose that the coordinates of the two nodes are (x_i, y_i) and (x_j, y_j) , respectively, so the equation of this fracture is,

$$\begin{cases} x = (x_j - x_i)t + x_i \\ y = (y_j - y_i)t + y_i \end{cases}, \quad (0 \leq t \leq 1) \quad (5.18)$$

The coordinates of temporary nodes m and n are,

$$m: \begin{cases} x_m = x|_{t=L_{ij}(t)/L_{ij}} \\ y_m = y|_{t=L_{ij}(t)/L_{ij}} \end{cases} / n: \begin{cases} x_n = x|_{t=[L_{ij}-L_{ji}(t)]/L_{ij}} \\ y_n = y|_{t=[L_{ij}-L_{ji}(t)]/L_{ij}} \end{cases} \quad (5.19)$$

If $L_{ij}(t) + L_{ji}(t) = L_{ij}$, the pressure vector (P_x, P_y) along this fracture segment can be expressed as follows,

$$\begin{cases} P_x = (P_{xj} - P_{xi})t + P_{xi} \\ P_y = (P_{yj} - P_{yi})t + P_{yi} \end{cases} \quad \left(t = \frac{x - x_i}{x_j - x_i} = \frac{y - y_i}{y_j - y_i} \right) \quad (5.20)$$

If $L_{ij}(t) + L_{ji}(t) < L_{ij}$, the pressure (P_x, P_y) along this fracture segment can be expressed as follows,

$$\text{if } (0 \leq t \leq t_1) \quad \begin{cases} P_x = (1 - C_1 t) P_{xi} \\ P_y = (1 - C_1 t) P_{yi} \end{cases} \quad \left(\begin{aligned} t_1 &= \frac{x_m - x_i}{x_j - x_i} \\ C_1 &= \frac{x_j - x_i}{x_m - x_i} = \frac{y_j - y_i}{y_m - y_i} \end{aligned} \right) \quad (5.21)$$

$$\text{if } (t_1 \leq t \leq t_2) \quad \begin{cases} P_x = 0 \\ P_y = 0 \end{cases} \quad \left(t_2 = \frac{x_n - x_i}{x_j - x_i} \right) \quad (5.22)$$

$$\text{if } (t_2 \leq t \leq 1) \quad \begin{cases} P_x = (C_2 t + C_3) P_{xj} \\ P_y = (C_2 t + C_3) P_{yj} \end{cases} \quad \left(\begin{aligned} C_2 &= \frac{x_j - x_i}{x_j - x_n} = \frac{y_j - y_i}{y_j - y_n} \\ C_3 &= \frac{x_i - x_n}{x_j - x_n} = \frac{y_i - y_n}{y_j - y_n} \end{aligned} \right) \quad (5.23)$$

where $P_{xi} = P_i \sin \theta$, $P_{yi} = P_i \cos \theta$, $P_{xj} = P_j \sin \theta$, and $P_{yj} = P_j \cos \theta$, with $\sin \theta = |y_j - y_i|/L_{ij}$ and $\cos \theta = |x_j - x_i|/L_{ij}$.

The potential energy Π_p of pressure loading is,

$$\Pi_p = \int_0^1 (u \quad v) \begin{pmatrix} P_x \\ P_y \end{pmatrix} L_{ij} dt \quad (5.24)$$

To include the influence of pressure loading into the submatrix of $[F_i]$ presented in Eq. (5.2), the derivatives of Eq. (5.24) are calculated to minimize the potential energy,

$$\begin{aligned} f_r &= -\frac{\partial \Pi_p}{\partial d_{ri}} = \frac{\partial}{\partial d_{ri}} \left([D_i]^T \int_0^1 [T_i]^T \begin{pmatrix} P_x \\ P_y \end{pmatrix} L_{ij} dt \right), \quad (r=1, \dots, 6) \\ &\Downarrow \\ &\int_0^1 [T_i]^T \begin{pmatrix} P_x \\ P_y \end{pmatrix} L_{ij} dt \rightarrow [F_i] \end{aligned} \quad (5.25)$$

If $L_{ij}(t) + L_{ji}(t) = L_{ij}$, the pressure distribution in Eq. (5.20) can be substituted into Eq. (5.25) to derive the submatrix (Chen et al., 2013),

$$\int_0^1 [T_i]^T \begin{pmatrix} P_x \\ P_y \end{pmatrix} L_{ij} dt = L_{ij} \begin{pmatrix} (P_{xi} + P_{xj})/2 \\ (P_{yi} + P_{yj})/2 \\ -\left[\frac{P_{xi} + 2P_{xj}}{6}(y_j - y_i) + \frac{P_{xi} + P_{xj}}{2}(y_i - y_0) \right] + \left[\frac{P_{yi} + 2P_{yj}}{6}(x_j - x_i) + \frac{P_{yi} + P_{yj}}{2}(x_i - x_0) \right] \\ \frac{P_{xi} + 2P_{xj}}{6}(x_j - x_i) + \frac{P_{xi} + P_{xj}}{2}(x_i - x_0) \\ \frac{P_{yi} + 2P_{yj}}{6}(y_j - y_i) + \frac{P_{yi} + P_{yj}}{2}(y_i - y_0) \\ \frac{1}{2} \left[\frac{P_{xi} + 2P_{xj}}{6}(y_j - y_i) + \frac{P_{xi} + P_{xj}}{2}(y_i - y_0) \right] + \frac{1}{2} \left[\frac{P_{yi} + 2P_{yj}}{6}(x_j - x_i) + \frac{P_{yi} + P_{yj}}{2}(x_i - x_0) \right] \end{pmatrix} \quad (5.26)$$

If $L_{ij}(t) + L_{ji}(t) < L_{ij}$, substitute the piecewise pressure distribution, as presented in Eqs. (5.21) to (5.23), into Eq. (5.25) to derive the submatrix,

$$\begin{aligned} \int_0^1 [T_i]^T \begin{pmatrix} P_x \\ P_y \end{pmatrix} L_{ij} dt &\sim \int_0^{t_1} + \int_{t_1}^{t_2} + \int_{t_2}^1 \\ &= L_{ij} \begin{pmatrix} \frac{t_1}{2} P_{xi} + \frac{(1-t_2)}{2} P_{xj} \\ \frac{t_1}{2} P_{yi} + \frac{(1-t_2)}{2} P_{yj} \\ \left[\frac{(y_j - y_i)t_1^2 + 3(y_i - y_0)t_1}{6} P_{xi} + \frac{(x_j - x_i)t_1^2 + 3(x_i - x_0)t_1}{6} P_{xj} \right] + \\ \left[\frac{(y_j - y_i)(1+C_2+C_3t_2) + 3(y_i - y_0)(1+2C_3)t_1}{6} (1-t_2) P_{xj} + \frac{(x_j - x_i)(1+C_2+C_3t_2) + 3(x_i - x_0)(1+2C_3)}{6} (1-t_2) P_{yj} \right] \\ \frac{(x_j - x_i)t_1^2 + 3(x_i - x_0)t_1}{6} P_{xi} + \frac{(x_j - x_i)(1+C_2+C_3t_2) + 3(x_i - x_0)(1+2C_3)}{6} (1-t_2) P_{xj} \\ \frac{(y_j - y_i)t_1^2 + 3(y_i - y_0)t_1}{6} P_{xi} + \frac{(y_j - y_i)(1+C_2+C_3t_2) + 3(y_i - y_0)(1+2C_3)t_1}{6} (1-t_2) P_{yj} \\ \left[-\frac{(y_j - y_i)t_1^2 + 3(y_i - y_0)t_1}{12} P_{xi} + \frac{(x_j - x_i)t_1^2 + 3(x_i - x_0)t_1}{12} P_{xj} \right] + \\ \left[-\frac{(y_j - y_i)(1+C_2+C_3t_2) + 3(y_i - y_0)(1+2C_3)t_1}{12} (1-t_2) P_{xj} + \frac{(x_j - x_i)(1+C_2+C_3t_2) + 3(x_i - x_0)(1+2C_3)}{12} (1-t_2) P_{yj} \right] \end{pmatrix} \end{aligned} \quad (5.27)$$

Submatrix derived in Eqs. (5.26) and (5.27) can be added into the submatrix $[F_i]$ in the global Eq. (5.2) for computing the hydromechanical coupling.

5.4 Realization of Algorithm

5.4.1 Special boundary conditions

Thus far, the framework for calculating the real-time grout penetration has been built, some practical work remains to be done, such as the boundary conditions at the injection point and the model boundary nodes.

As for the boundary conditions of the nodes at the model boundary, the pressure at these nodes were set as zero in the present work. Nevertheless, the boundary condition at the injection point differs with different injection method: constant injection pressure method or constant injection flow rate method.

When the injection flow rate Q_0 is kept as constant, Eq. (5.8) at the injection node should be modified,

$$f(Q_i) = \sum_k^{n_i} Q_{ik} - Q_0 = 0 \quad (5.28)$$

Eq. (5.28) can then be substituted into Eq. (5.11) to form the global matrix; thus, the injection pressure at the injection node can be updated as easily as other normal nodes. The precondition is that the injection nodes should be specially labelled, so that it can be found and Eq. (5.28) can be applied to it.

However, the process would be a little bit different if the injection pressure is kept as constant, as the only variable would be the real-time penetration length in a segment connected to the injection node, and it can be calculated based on Eq. (5.9) only. The global matrix will not be introduced until one segment connected to the injection node is filled, indicating that a new node is involved, where the fluid pressure at this node is unknown. Detailed examples will be used for illustrated in the following sections.

5.4.2 Illustrative example

Grouting of a simple 2D fracture network, with 21 nodes and 24 fracture segments is used for illustration purpose, and different stages of grout penetration are shown in

Figure 5.5. In Figure 5.5, the thin black lines represent model boundaries, whereas the thick black line are rock fractures, the apertures of them are assumed to be the same; the intersection of any two lines is called node (one node is marked by a blue dot in Figure 5.5 (a)), and node pressure is of interest; the water head at the boundary node (intersection of rock fracture with model boundary) is taken as zero; there is only one injection hole and it is marked by a red dot; thick red lines are used to represent the traces of grout penetration. The real time grouting process is demonstrated by four diagrams (a), (b), (c), and (d) shown in Figure 5.5.

The differences between two grouting methods, constant injection flow rate and pressure, are summarized in

Table 5.1, where the number of variables involved in each grouting stage are listed. From the comparison between the two methods in

Table 5.1, it can be found that the only difference is that the global matrix size is smaller for constant injection pressure method, where pressure at the injection node is known; and calculation of grouting initiation at the injection node is different, after that everything is the same.

Figure 5.5 Illustration of different stages of grout penetration in a simple fracture network: black bold line represents fracture segment, the intersection between bold

lines are nodes (one node is marked by light blue dot), and the node marked by red dot is chosen as the injection point

Table 5.1 Characteristics of grouting prediction under different methods

Grouting stages		Constant injection pressure	Constant injection flow rate
(a)	NO. of variables	$P_x : 0; L_{xy}(t) : 4$	$P_x : 1; L_{xy}(t) : 4$
	Characteristics	Global matrix is not introduced	Size of global matrix: 5*5
(b)	NO. of variables	$P_x : 2; L_{xy}(t) : 8$	$P_x : 3; L_{xy}(t) : 8$
	Characteristics	Size of global matrix: 10*10; Removed $L_{xy} : 2$	Size of global matrix: 11*11; Removed $L_{xy} : 2$
(c)	NO. of variables	$P_x : 4; L_{xy}(t) : 10$	$P_x : 5; L_{xy}(t) : 10$
	Characteristics	Size of global matrix: 14*14 Grouted boundary node: 2 Removed $L_{xy} : 4$	Size of global matrix: 14*14 Grouted boundary node: 2 Removed $L_{xy} : 4$
(d)	NO. of variable	$P_x : 8; L_{xy}(t) : 10$	$P_x : 9; L_{xy}(t) : 10$
	Characteristics	Size of global matrix: 18*18 Removed $L_{xy} : 8$	Size of global matrix: 19*19 Removed $L_{xy} : 8$

*notes: P_x and $L_{xy}(t)$ represent unknown node pressure at node x and grout penetration at fracture segment $x \rightarrow y$, respectively; “Grouted boundary node” means that grout front reaches a node located at the model boundary, where the node pressure is set as a constant value, so it is not a variable; “Removed L_{xy} ” means that segment $x \rightarrow y$ is filled by grout, $L_{xy}(t) = L_{xy}$ is no longer a variable, thus it is removed from the global matrix.

5.4.3 Realization in DDA

The algorithm developed in Section 5.3 is programmed with C language as one module of DDA for grouting prediction, and one case study is used for illustrating the grouting process. As shown in

Figure 5.6, the fracture network is simple, which make it easier to see how effectively the algorithm works; this fracture network resembles the setup in Jing’s work (Jing et al., 2001), where static analysis was conducted. The fracture network contains 28 square blocks with size of 0.2*0.2m, indicating that the length of all fracture segments is 0.2m; there are in total 45 fracture segments, and all fracture aperture is set as 1mm.

In

Figure 5.6, “Injection node” is the point where grout flows into the fracture network, and the injection pressures at two injection nodes are set as 0.5 MPa; “Boundary node” is located at the boundaries of the fracture network, and the pressure is normally a constant value, which should be determined according to practical hydro-geological conditions, but in the present work they are set as zero; “Pressure node” is the point where the node pressure is unknown, and the pressure will be updated with the grouting progressing.

Figure 5.6 Setup of the fracture network used for real-time grouting prediction

The simulation results are shown in Figure B3 in Appendix B, and grout penetration at eight critical time steps are chosen. The time step interval is 0.1s. The squares in different colors are rock blocks, which are separated by fracture segments. The trace of grout penetration different time t is shown in red color, and it is time varying. The grouting process corresponding to seven time-steps are presented in

Figure 5.7, where interesting and critical situations appear.

At 28th time step, grout penetration at two injection nodes are the same, and the grout penetration in fracture segments connected to them are symmetric, since both the fracture length and the relevant fracture aperture are the same for all fracture segments.

Figure 5.7 Illustration of real-time grouting penetration based on DDA

At 50th time step (5s), eight fracture segments were filled, and the corresponding penetration length variables were removed from the global matrix; grout starts to spread in fracture segments connected to these eight new pressure nodes, and the penetration traces around these eight nodes are the same; the fracture segment inside the white ellipse becomes a segment with both ends under grout inflow, so there are in total four variables involved for this fracture segment, including two node pressures and two penetration lengths. At 160th time step (16s), the fracture segment inside the white ellipse is filled and the corresponding variable concerning penetration length is removed from global matrix.

At 240th time step (24s), four more fracture segments were filled as shown inside the white ellipse, where grout run faster than that in the fracture segments inside that dark ellipse, as nodes inside the red ellipses only have two fracture segments to fill, but nodes inside the dark ellipses have three fracture segments to fill; at 256th time step (25.6s), six fracture segments connected to four nodes inside the white circles become segments with four variables, which also indicates that the current algorithm is reliable and of high degree of accuracy. At 300th time step (30s), four more fracture segments were filled, the corresponding fracture segment variables were removed, the size of the global matrix decreases greatly. At 380th time step (38s), two more fracture segments were filled. At 500th time step (50s), all fracture segments were filled, grouting work should be stopped if the current fracture boundaries are the required range for water sealing.

The real-time pressure variation at each node is presented in Figure B3. The pressures at boundary nodes are set as zero, so their pressures are all horizontal lines; the pressures at the two injection nodes are 0.5MPa, thus they are also horizontal lines; for pressure curves at other nodes, their node pressures do not start to vary till grout front reaches their location, before which their pressure remains zero. The fracture network is asymmetric, and two injection holes are symmetric about its vertical symmetry axis, thus the time dependent variation of node pressure should be the same for the points which are also symmetric. Put symmetric nodes into a same group, several groups can be formed, like nodes 1, 6, 29, and 32; nodes 9,12, 24, and 27; nodes 14, 17, 19, and 22; nodes 4 and 31; the pressure variations are the same for all nodes in the same group. Meanwhile, all boundary nodes are put into the same group, since their pressures are all zero. Additionally, node 16 has no symmetric node, so its node pressure is different from any other nodes in the fracture network.

More importantly, pressure jumps can be seen from Figure B3 (Nodes 1, 4, 6, 9,12, 14, 16, 17, 19, 21, 22, 24,27, 29, 31, and 32), which happens when new fracture segments are filled with grout, namely cement grout start to spread in new fracture segments. Meanwhile, the pressure variation is smooth at individual node between pressure jumps (which can also show the stability of the current algorithm). This phenomenon can be taken as an important signal, based on which the on-site

engineers can know that there are new rock fractures involved, therefore grouting should not be stopped at that moment, which can be used for determining the stopping criteria.

5.5 Summary

In the present work, a grouting module is developed to predict real-time grout penetration in a two-dimensional fracture network for Discontinuous Deformation Analysis (DDA), and the model can be applied to both constant injection pressure and constant injection flow rate situations.

The pressure at a node and the grout penetration in fracture segments connected to it are taken as variables; the relationship between them is nonlinear, so the Newton's method is used for equation solving.

The present model starts from individual node information, which can form individual sub-matrix; then assemble all sub-matrixes to form the global matrix, which can be used for iterative calculation based on the Newton's method. The global matrix includes new nodes when grout flow front reaches them, rather than including all nodes and segments into the global matrix at the initial state of computation. Variables related to grout penetration in a segment will be removed from the global matrix to improve computation efficiency.

The pressure jump during grouting can be used to determine the stopping criteria for grouting. The present model can also consider the hardening effect of cement grout on grout penetration, as the grout viscosity can be expressed as a temporal function and be easily included into the governing equations controlling grout penetration.

Chapter 6 Impact of Fracture Roughness on Grouting

6.1 Introduction

Normally, the surfaces of rock fractures are rough. The roughness of 2D fracture surfaces can be quantified by Joint Roughness Coefficient (JRC), or by referring to some standard profiles (Tse and Cruden, 1979). The concept of surface roughness for rock fracture has been generalized to 3D cases, and the algorithm for evaluating it has been developed (Belem et al., 2000). The fracture roughness was initially investigated in rock engineering to determine its influence on water flow, and usually equivalent aperture is calculated to evaluate its hydraulic properties (Brown, 1987; Lanaro, 2000; Baraka-Lokmane, 2002). For grouting, the existence of fracture roughness may have even greater impact, as significant channeling effect was found for fluid flow within rough surfaces (Brown, 1987; Thompson, 1991; Amadei and Illangasekare, 1994). Additionally, due to the existence of fracture roughness, the mechanical apertures in some area could be very small, where filtration might occur. All these factors might lead to nonuniform distribution of grout material, thus affecting the quality of grouting. What's more, due to the existence of uneven surfaces, grouting of real rock fractures would have faster increase of injection pressure, compared with pressure measured during grouting test for ideal fracture model with two smooth parallel plates; if a certain pressure is taken as stop criterion, the corresponding range of grout penetration in ideal and real rock fractures (with the same equivalent aperture) would be different, and the range would be smaller in real rock fracture. Therefore, for grouting of real rock fracture, a relative higher stop pressure should be set to reach required range of sealing, but excessive pressure would cause harmful or even uncontrollable fracture deformation, which pose great potential threat to project safety.

Therefore, it is of great importance to evaluate the degree of influence of fracture roughness on grouting. There is a lack of comprehensive theoretical study in the field, even though some research works were already done either based on highly idealized or small scale model (Lipscomb and Denn, 1984; Ping and Xueqing, 2015) or even empirical method (Barton and de Quadros, 1997). Hanssen built a theoretical model to investigate the characteristics of Bingham flow within rough walled fractures, with

focus more on relationship between shear displacement and channelization (Hanssen, 2013). So far, large-scale experiments were hardly used for investigating the influence of fracture roughness on grouting. Hence, in this chapter, many lab tests are used to explore the impact of fracture roughness on grout penetration and the corresponding grouting quality, where 3D printing is used to generate rough surfaces of rock fracture, which will work with a precise aperture adjustment system.

As grouting laboratory tests are time consuming and of high cost, so it would take quite a long time and large financial input to collect enough and useful data. Numerical simulation can be used to estimate the influence of fracture roughness on fluid transportation, based on conclusions obtained from laboratory tests. Hence, systematic numerical simulations are designed as supplementary experiments to quantify the impact of surface roughness, namely, expressing the equivalent aperture as a function of surface properties and grouting parameters.

6.2 Generation of 3D fracture surfaces

Rough rock fractures are created by 3D printing, the original data of which is generated and exported from SynFrac (Isakov et al., 2001; Ogilvie et al., 2003; Ogilvie et al., 2006), which is then reproduced by MATLAB programming, converting grid points into coordinate points in micrometer.

6.2.1 Introduction to SynFrac

The software interface of SynFrac is shown in Figure 6.1: there are three methods can be chosen from the **Method** panel, among which AUPG method is more reasonable and reliable (Ogilvie et al., 2006). The resolution of top and bottom surfaces of the numerical rock fracture can be chosen from **Resolution** panel, where there are four choices. The higher the resolution chosen the better the quality of the fracture surfaces generated would be, since more nodes will be used for describing characteristics of a specific zones with the same area; but the relevant point data could increase exponentially if higher resolution is chosen. **Radom number generator** is for generating two seeds, thus generating fracture surfaces through controlling the phase of Fourier components; these three methods listed here can generate random numbers

with reliable quality. The function and meaning of the characteristic parameters in the **Parameter** panel are presented in Table 6.1.

Figure 6.1 Interface of SynFrac used for generating rough fractures

Table 6.1 Function and meaning of characteristic parameters (Ogilvie et al., 2006)

Parameter	Function and meaning
Physical Size	Control the size of numerical fracture
Mismatch Length	A critical scale value determining the matching property of fracture surfaces
Transitional Length	Scale range between low and high matching property
Standard Deviation	Standard deviation of fracture surface height from its mean plane
Anisotropy Factor	Create fractures with anisotropy
Fractal Dimension	Control fracture roughness
Maximum Mismatch Fraction	Quantifying matching at large scale
Minimum Mismatch Fraction	Quantifying matching at small scale

Notes: for more information concerning this software, refer to Ogilvie, Isakov et al. 2006, or go directly to their official website: <http://t.cn/RDbovRI>.

From the above introduction, it can be found that synthetic rock fractures with different sizes and roughness can be generated. The relevant fracture data can be exported and converted to .TXT files easily, and three different files can be generated,

including information of the relevant top and bottom surfaces. Then they can be further processed by MATLAB programming to obtain the original data for preparing “.TSL” files, which can be directly used for 3D printing.

6.2.2 MATLAB processing

The data exported from SynFrac is dimensionless resolution data, and the maximum value of the relevant coordinates is the resolution, regardless of what physical size is chosen. The following data file is taken as an example, where the physical size is 1000mm and the relevant resolution chosen is 1024*1024.

Figure 6.2 The original data file exported from SynFrac

As shown in Figure 6.2, the first two columns show the resolution points related to X and Y coordinates, while the third column represents the practical Z coordinate. In addition, the data should be further processed to obtain two fracture surfaces with size of 500mmx2000mm, which is the size of the experimental equipment. Thus, the following conversion has been made, as shown in Figure 6.3. Namely, data describing one quarter of the Square fracture surface generated by SynFrac is extracted and used for generating a rectangular fracture surface.

The algorithm used by MATLAB programming during the data process is as follows,

$$Width = \frac{Data_{width}}{256} \times 500mm; Length = \frac{Data_{Length}}{1023} \times 2000mm \quad (6.1)$$

where $Data_{width}$ and $Data_{length}$ represent the original data obtained from SynFrac, describing a rectangular with resolution of 256x1024. Both the length and width have been magnified and the corresponding magnification is the same in both directions, which are two times the original data.

Figure 6.3 Generation of fracture surfaces for grouting experiment

Figure 6.4 The rough fracture surface: (a) 3D model; (b) 3D printing

Based on the data generated by MATLAB programming, 3D models for rough fracture surfaces are built for 3D printing, and the 3D model and the corresponding fracture surfaces printed out are shown in Figure 6.4.

6.3 Grout injection system and equipment

In the academic field, especially for analytical analysis, scholars tend to take the injection pressure as constant value (Hässler et al., 1992a; Eriksson et al., 2000; Yang et al., 2002; Kobayashi and Stille, 2007; El Tani, 2012; Rafi, 2014), since it makes the process much easier to derive the governing equation for grout penetration in rock fractures. Whereas it is hard to keep the injection pressure as constant in the field grouting, since grout is injected through grouting pump, of which the speed of revolution can vary within a certain range; thus, a constant injection flow rate can be obtained if a specific speed of revolution of grouting pump is chosen. A grouting power system providing constant injection flow rate is developed, to make sure that our experiment can better describe the characteristics of practical grouting.

6.3.1 Grouting barrel

For the grouting laboratory test, to find the relationship between the grout injection pressure and the corresponding flow rate, a reliable grouting injection equipment must be provided, to guarantee stable grout injection flow rate. For constant injection flow rate, the first choice may be the grouting pump used for field grouting, which is large and not able to provide precise control of grout flow rate. Therefore, a special grout injection equipment is designed for constant injection rate, as shown in Figure 6.5.

The idea for designing this grouting equipment comes from inoculator used in medical field. It is, to some extent, a huge inoculator, but with uniform diameter from top to bottom. The inner diameter D_g is 260mm, and the effective distance of travel L_e is 250mm, indicating that the volume of this grouting barrel is 13.273L. A still pipe is specially designed for pouring grout or water inside, thereafter a cap nut will be used to seal the pipe. Then cement grout with a constant flow rate Q can be pushed out when a constant speed v is applied to the plunger cover, and the relationship between Q and v can be expressed as follows,

$$Q = \pi D_g^2 v / 4 \quad (6.2)$$

Figure 6.5 Grouting equipment designed for grout injection under constant flow rate

6.3.2 Power system

The power system is the source for grout supply, which can provide energy to overcome resistance correlated to fluid pressure and shear stress. In the section, the components involved will be introduced, and the system feasibility will be investigated based on a series of laboratory test.

6.3.2.1 Basic components and function

The power system is adapted from a hydraulic shearing machine, which is used for shearing strength testing, as shown in Figure 6.6. With the grouting barrel placed above the platform within the frame, the vertical part, originally designed for applying normal stress, is now used to push the plunger cover and force it to move downward in a constant speed. The reason to use this machine is that it can provide stable large force, which can indirectly provide large injection pressure; moreover, the advancing speed can vary within a large range.

6.3.2.2 Flow rate testing

A series of water pumping test were done to verify the relationship between injection flow rate and the advancing speed as given in Eq. (6.2). The advancing speed varies from 5mm/min to 20mm/min. The testing results and the relevant theoretical results obtained from Eq. (6.2) is given in Table 6.1.

Figure 6.6 Power system pushing downward the plunger cover

Table 6.2 Feasibility test of grouting barrel

Speed (mm/min)	Test Q_1 (mL/min)	Theory Q_2 (mL/min)	$ Q_1 - Q_2 /Q_2$
5	280/262/250	265.465	0.55
6	300/300/310	318.557	4.78
7	310/345/330	371.650	11.66
8	376/381/389	424.743	10.06
9	405/384/412	477.836	16.22
10	440/423/440	530.929	18.18
11	440/480/460	584.022	21.24

The difference between the testing data and the theoretical value increases with increasing advancing speed, as deformation of experimental equipment becomes significant and gradually not negligible during this process. Part of the water injected into the fracture system would be occupied by increased space due to fracture deformation, therefore water of volume lower than theoretically predicted will come

out from the outlet end of the system. The phenomenon reveals the importance of deformation controlling during the test, which requires special attention during laboratory test design.

6.3.3 Resetting of grouting barrel

Just like an inoculator, you must pull it back when the liquid medicine is pushed all out, and the plunger is on the bottom; likewise, the plunger cover of grouting barrel should be pulled out from the bottom of the barrel after one round of grouting test is finished. A set of tools were specially designed for this purpose, since it cannot be pulled out by bare-handed man. The tools are shown in Figure 6.7.

Figure 6.7 (a) Tool set for pulling the plunger out: ① and F are screw and lead screw with the same outer screw thread, A is the corresponding screw cap, B is a wooden beam, C and D are a jack and the associated lever, E is a steel plate working together with G, a flange, which are set above the jack; (b) Plunger inside the cylinder; (c) Plunger pulled out using tool set

As can be seen from Figure 6.7, the plunger cover can be pulled out relying on the counter-force from the wooden beam, which is supported by the cylinder wall. The jack is placed on the top of the beam, so the upward moving of its head part can force the steel plate E move upward together with the flange G, which can be fixed at any height of the two screw leads F. Two screw leads F are combined with two screws

① fixed in the plunger cover by two screw caps A, thus establishing connection with the jack. Therefore, a strong structure is formed, where the jack can provide large push force, which is transferred to a pull force capable of overcoming the friction between the plunger cover and the cylinder inner wall, like the plunger moving from Figure 6.7 (b) to Figure 6.7 (c).

In summary, this is a detachable king-size inoculator, the plunger of which moves forward and backward relying on two independent system: the hydraulic shearing machine and the resetting tool set.

6.4 Setup of experimental equipment

In the past, most of laboratory testing on fluid flow between two fracture surfaces has only one specific aperture or within a limited and discontinuous range (Esaki et al., 1999; Fransson, 2001; Eriksson, 2002; Fujita et al., 2012; Sui et al., 2015) for one specific equipment, thus their study was lack of representativeness, and some phenomena concerning grouting of narrow or ultranarrow fractures might not be discovered. Therefore, we have designed an experimental platform which has an unlimited and continuous aperture variation within reasonable aperture range (these fracture apertures appear highly frequently in nature rock mass).

Figure 6.8 Overall experimental setup for grouting of rough rock fracture

A specific aperture adjustment and supporting system is designed, where the bottom plate of the system is movable. Waterproofing becomes a critical problem when the

bottom surface is not fixed, which cost us quite a lot of time. After a specific fracture aperture is determined and the corresponding waterproofing is ensured, 3D printed rough fracture surfaces should be fixed onto the bottom and cover plate of the experimental platform. The experiment can be conducted after the above mentioned three procedures are finished successfully. The overall look of the platform after everything is setup is presented in Figure 6.8.

6.4.1 Aperture adjustment and supporting system

As discussed, a system for changing fracture aperture should be designed, since we want to have a continuous and unlimited variation of fracture aperture. Regarding traditional experimental system for grouting single fracture channel, taken Fujita's work as an example (Fujita et al., 2012), as shown in Figure 6.9, which is a fracture channel model with smooth parallel surfaces, where the bottom surface is fixed to a platform bed and not movable.

Figure 6.9 Grouting of single fracture with fixed aperture (Fujita et al., 2012)

In contrast, in Figure 6.10, our bottom plate is movable, so a system that can raise and lower the bottom plate when adjusting aperture, and provide support when the platform is in operation state, and stable vertical elevation is designed, and composed of eight hydraulic jacks (maximum bearing load: 5 tons, and maximum stroke length: 90mm), and four precise elevators (maximum bearing load: 21kg, travel range: 110mm, and minimum adjust distance: 50μm), and two 996*496*20(mm) aluminum alloy plates (elastic modulus: 72GPa) are used for deformation controlling.

Both the hydraulic jack and precise elevator can change the height of the aluminum alloy plates, where the waterproofing system and the 3D printed fracture bottom surface will be placed on. Based on empirical and analytical estimation, the maximum grout injection pressure during the grouting process would be around 0.2MPa. For conservative design, an injection pressure of 0.4MPa is chosen for 3D modeling to find out the relevant deformation of the supporting aluminum alloy plate, where the pressure is taken as linearly distributed from inlet to outlet.

Figure 6.10 Aperture adjustment and supporting system

Figure 6.11 Deformation of aluminum alloy plates under 0.4MPa injection pressure

As shown in Figure 6.11, the eight small circles and four small squares represent the supporting of the hydraulic jack and the precise elevator, and the simulation result

shows that the maximum deformation is only 0.14mm within vary small areas. Thus, it can be concluded that the deformation of the supporting system is small and negligible, since the maximum injection pressure is far less than 0.4 MPa in practical grouting test, and the fracture aperture to be tested ranges from 0.5mm to 3mm.

6.4.2 Installation of 3D printed rough surfaces and waterproofing system

The next step before the final grouting test is to fix the top and bottom fracture surfaces to the experiment platform surface, under the precondition of reliable waterproofing. In the meanwhile, the specific aperture for grouting test is preset in this stage based on the experimental plan, since both the mechanical aperture and hydraulic aperture are known in advance based on pre-testing. Normally, the 3D printed rough surfaces should be placed directly on the aluminum alloy plates, but there would be water or grout leakage due to the existence of gaps (design margin) between the alloy plate and the platform wall. Therefore, an extra layer has been designed for waterproofing, where a silicone rubber bar work together with a specially designed acrylic sheet. Another silicone rubber bar is also used for waterproofing between the platform cover and the platform body. The detailed distribution of the supporting system, waterproofing system, and two rough fractures are presented in Figure 6.12.

Figure 6.12 is a schematic diagram and exploded view of the detailed distribution of platform top cover and bottom plate, the 3D printed fracture surfaces, the waterproofing and supporting system. Actually, the true distance between the lower surface of platform cover ① and the upper surface of the fracture top ③ is zero, since fracture top ③ is pasted onto platform cover through transparent acrylic adhesive tape; the fracture bottom surface ④ is placed on the acrylic sheet ⑤, and the latter is placed on top surface of the aluminum alloy plate ⑦. Each of the 2000mm*500mm rough fracture surfaces (including top and bottom surfaces), is composed of five pieces of 3D printed surfaces of equal size limited by the capacity of the 3D printer utilized, where it is not cost effective to print a sample with length longer than 500mm.

Figure 6.12 Exploded view of the aperture setup and waterproofing system (a) in longitudinal direction, (b) in transversal direction

The height of the fracture top is fixed, since it is placed on the top of the platform, which has a height of 180mm, a fixed value. Therefore, a specific fracture aperture can be obtained through adjusting the height of the aluminum alloy plate relying on the jack and precise elevators. The critical problem involved is the waterproofing between each layer and the platform wall. The gap between the silicone rubber bar ⑥ and the platform vertical wall is sealed by glass glue, after a specific fracture aperture is chosen and aperture adjustment is completed. For the waterproofing between the platform top cover and the platform body, a silicone rubber bars (with a shape of rectangular frame) is placed between them.

In Figure 6.12: ① and ⑤ (in white color) are the platform top cover and bottom sealing plate, respectively; ② and ⑥ (in green color) are the silicone rubber bars for preventing liquid leakage from the top and bottom, respectively; ③ and ④ (in blue color) are top and bottom rough fracture surfaces via 3D printing, respectively; ⑤ is a thick acrylic sheet working together with ⑥ for waterproofing; ⑦ is

aluminum alloy plate for controlling bottom deformation; ⑧ is the jack (simplified representation); ⑨ is the precise elevator (simplified representation).

6.4.3 Pressure and deformation measurement

For grouting under constant flow rate, the relevant injection pressure is of great importance, since it can be used for calculating the corresponding equivalent aperture. Both the injection pressure and the corresponding equivalent aperture derived can be compared with grouting of smooth fracture with identical mean mechanical aperture. Thus, grout injection pressure is measured using pressure transducer together with grout flow pressures at other locations. The deformation of fracture top surface should be controlled to ensure that the injection pressure measured during the grouting test is under a fixed fracture aperture. Hence, dial gauges are placed above the platform cover, on the bottom of which the top fracture surface is attached to, as the fluid pressure applied on the fracture surface can transfer to the platform cover. The detailed setup of the pressure transducers and the dial gauges is shown in Figure 6.13.

Figure 6.13 Measurement of injection pressure and fracture deformation

There are three pressure transducers located at the injection hole, with one located directly over the injection hole, the other two located 100mm apart from the injection hole. These three readings obtained can be used for comparison and determining the steady state of grouting. Regarding the dial gauges, they are distributed along the symmetric axis of the platform, where the deformation is the largest. Reinforcement would be required if the deformation measured during the pre-testing stage is large compared with the predetermined fracture aperture, and the maximum allowable deformation is set as 10% of the corresponding fracture aperture. Steel U-channel is chosen for controlling fracture deformation if it goes beyond the limit, as presented in Figure 6.13 around the injection hole, where the deformation could be unacceptably large if it is not controlled.

Figure 6.14 Flow chart for grouting test

6.5 Testing results and data analysis

For laboratory grouting test, a lot should be done before initiating grouting: platform alignment, determination and adjustment of fracture aperture, waterproofing setup, and determination of water cement ratio. Thereafter, grouting test can be initiated, and the flow chart for grouting of rough rock fractures is presented in Figure 6.14.

The cement used for all the grouting testing in the present work is Portland cement 42.5, and the corresponding properties and particle size distribution are listed in Table 6.3 and Table 6.4, respectively.

Table 6.3 Basic properties and chemical components of Portland cement 42.5

Setting time (h)		Major component and its content/%					
Initial set	Final set	CaO	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃	MgO	SO ₃
≥1.5	≤6	63.5	18.8	5.8	4.4	4.35	3.15

Table 6.4 Information on particle size distribution

Type	Average particle size (μm)	Maximum particle size (μm)	Specific surface area (m ² ·kg ⁻¹)
Portland cement	15~20	44~100	260~400

6.5.1 Two types of grouting test

6.5.1.1 Fracture channel with rough bottom surface

Presently, only one set of rough fracture surfaces are printed and used for grouting test since 3D printing is too expensive, and more representative rough fractures will be generated after enough experience and knowledge of grouting test are gained. However, compared with traditional test of grout flow a fracture channel with two smooth parallel surfaces, a lot of new phenomena were found during the grouting test: firstly, the grout flow front is moving unevenly forward, as shown in Figure 6.15 (1D penetration), and Figure 6.16 (2D penetration); secondly, there is severe bleeding appearing at the grout flow front (shown in Figure 6.17).

Figure 6.15 1D Grout penetration within fracture channel: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at the left end of the experimental platform

Figure 6.16 2D Grout penetration within fracture channel: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at the middle of the experimental platform

Figure 6.17 Bleeding during 1D grouting: top surface is transparent and smooth, and the bottom surface is the 3D printed rough surface; grout injection hole is located at one end of the experimental platform

The fracture surfaces are ivory-white but not transparent, since it cost two or three times more to print with transparent materials. Hence, the upper rough surface (③ as shown in Figure 6.12) is removed or not attached to the platform cover, to see directly the grout penetration with the influence of one rough surface. However, grouting under this condition is only for preliminarily determining the potential influence of rough fracture surfaces on grout penetration.

A reservoir with size of 200mm*500mm is set at the left end, in Figure 6.15 and Figure 6.16, to ensure stable and constant injection flow rate. Both the top and bottom surfaces are acrylic plates, indicating that the first piece of the bottom rough surface is removed, so the grout flow will not enter the fracture channel before it fills up the reservoir.

6.5.1.2 Fracture channel with two rough surfaces

Several grouting tests for rock fractures with two rough surfaces were conducted and compared with rock fractures with two smooth surfaces. The testing details are listed in Table 6.5, where the type of testing is represented by various codes for easier data recording. For example, H1C1T1 indicates that this is the first grouting test (T1) for

fracture with aperture H1 (1mm) under water cement ratio C1 (W/C=0.8). The downward speed of the grouting barrel is 10mm/min, so the corresponding injection flow rate can be calculated based Eq. (6.2).

Table 6.5 Details on lab grouting test

	Rough surface			Smooth surface		
Type	H1C1T1	H1C2T1	H1C3T1	H1C1T1	H1C2T1	H1C3T1
Time	5.20.94	5.56.20	4.55.63	4.23.31	6.51.35	4.24.44
Type	H1C1T2	H1C2T2	H1C3T2	H1C1T2	H1C2T2	H1C3T2
Time	Fail	Fail	5.20.08	5.09.14	7.15.26	4.45.95
Type	H2C1T1	H2C2T1	H2C3T1	H2C1T1	H2C2T1	H2C3T1
Time	7.23.90	6.35.08	6.39.60	7.53.38	6.23.10	6.01.63
Type	H2C1T2	H2C2T2	H2C3T2	H2C1T2	H2C2T2	H2C3T2
Time	Fail	6.56.36	7.19.60	8.25.46	6.48.80	Fail

*Notes: H represents aperture: H1=1mm, H2=3mm; C represents water cement ratio: C1=0.8, C2=1, and C3=1.2; T represents the number of tests: T1=first test, T2=second test. “Fail” represents failed to get useful results or some sensors failed to work. The format of time is “Minute. Second. Centisecond”.

Regarding fracture aperture given in Table 6.5, the definition for smooth fracture is obvious, whereas, for rough fracture, it is specially defined as shown in Figure 6.18, where H is the lowest location of the top rough surface and the highest location of the bottom rough surface. Therefore, the average mechanical aperture of rough fracture is larger than that of the smooth fracture, and it equals 0.95mm when H=0.

Figure 6.18 Definition of fracture aperture H corresponding to fracture channel with two rough surfaces and smooth surfaces (lateral view)

The testing setup and results for grouting of rough fracture are shown in Figure 6.19. As for the control group (two smooth surfaces), the only difference (from that shown in Figure 6.19) is that the two rough surfaces are removed.

The variation of injection pressure and the volume concentration of cement grout at various locations within the rock fracture are of great interest, as they can directly demonstrate the influence of fracture roughness. The real-time injection pressure can

be just obtained from the pressure sensor, which will be discussed in the following section. The volume concentration is defined as the ratio of bleeding water and cement grout, as shown in Figure 6.20. Around 20 ml of cement grout at 15 locations (5 rows and 3 columns) were collected along the direction from the inlet to the outlet, then they were poured into 15 graduated cylinders for quantifying the corresponding volume concentration; the distance between two neighboring rows is 400mm, and the distance between two neighboring columns is 150mm.

Figure 6.19 Equipment setup and testing result for grouting of fracture with two rough surfaces

Figure 6.20 Collection of cement grout for measuring volume concentration distribution after grouting

6.5.2 Data analysis and conclusions

The volumes of bleeding water (V_w) and the cement grout below (V_c) are measured and recorded, then the volume concentration $C_v = V_w / V_c$ is calculated and illustrated in Figure 6.21 for smooth fracture and Figure 6.22 for rough fracture,

where the magnitude of C_V is illustrated by transitional color. The locations of these points are in one-to-one correspondence with that shown in Figure 6.20.

Figure 6.21 Volume concentration of cement grout at 15 different locations for smooth fracture: (a) Aperture=1mm; (b) Aperture=3mm

It can be found from Figure 6.21 and Figure 6.22 that, regardless of the water cement ratio and the fracture aperture, the volume concentration C_V in the flow front area (Coordinate: 1400mm to 1800mm) is larger than other areas, indicating that the concentration of cement grout is lowering, defined as “thinning” phenomenon; moreover, the degree of thinning (magnitude of volume concentration and range of thinning) increases with decreasing fracture aperture and increasing fracture

roughness. This thinning phenomenon might influence the sealing quality or finally the structure strength, as the content of cement in the grout is relatively low.

Figure 6.22 Volume concentration of cement grout at 15 different locations for rough fracture: (a) Aperture=1mm; (b) Aperture=3mm

The real-time variation of injection pressure corresponding to case study (defined in Table 6.5) is shown in Figure 6.23 and Figure 6.24 for smooth and rough fractures, respectively, though no data were obtained in some cases due to human error or equipment failure. For each type of grouting test under the same W/C, only one case study is presented below.

Situation like Case H1C1T1 shown in Figure 6.23 is taken as abnormal and invalid, since there are no normal signal fluctuation and the pressure changes from a large negative value to a large positive value. Normally, there are two stages for pressure variation: ascending and descending stages. Taking H1C2T1 as an example, in the first stage, the grout pressure increases to the peak value, then grouting stops; in the second stage, grout flow pressure drops rapidly due to loss of pressure source (from grout injection). In a normal grouting test, the original signal data collected from pressure transducer are accompanied with noise, so smooth algorithm is introduced to derive more unambiguous signal data: firstly, find outliers (defined as data points located outside three standard deviations from the mean) and replace them with the nearest data which are not outliers; then smooth the noisy data with Gaussian-weighted moving filter, of which the window length used should be large enough, as the data were collected by pressure transducer at very high frequency and there are hundreds of data points even in a small segment of the pressure curve in Figure 6.23.

Figure 6.23 Real-time variation of injection pressure for grouting of smooth fracture

The effective signal data is collected from the smooth data under its ascending stage, which corresponds to real process of grouting test. Both the original and smoothed signal data are presented in Figure 6.23 and Figure 6.24.

As noted from Chapter 3 on grouting under constant injection flow rate, the corresponding injection flow rate is increasing linearly with time, and almost all the injection pressure measured is increasing approximately linearly with time, except Case H2C2T1 for rough fracture where grouting was stopped and restarted during the grouting process due to grout leakage.

After laboratory testing, the most important procedure is to evaluate the quality or even the reliability of the experimental data obtained. Normally, the maximum injection pressure is higher if the corresponding W/C ratio is larger, and/or the fracture aperture is larger, which can be taken as a criterion for determining the quality of experimental data. As can be seen from Figure 6.23, the maximum injection pressure in Case H1C2T1 is far lower than that in Case H1C3T1, but the W/C ratio of the latter is smaller, indicating that something is wrong; the reason might be that the fracture aperture was not exactly equal to 1mm. The preset mechanical aperture of the experiment fracture can be evaluated using the following expression:

$$h_m = V/WL \quad (6.3)$$

where V is the overall grout take: for smooth fracture $V=Qt$, and Q is the injection flow rate, t is the grouting time, $W=0.5\text{m}$, and $L=2\text{m}$; for rough fracture, $V = Qt - 0.32Wh_f$, h_f is the sum of the average thickness of two fracture surfaces within the 320mm range of the inlet zone, $W=0.5\text{m}$ and $L=1.92\text{m}$.

Table 6.6 Update preset fracture aperture and maximum injection pressure

Item	Rough surface			Smooth surface		
Case	H1C1T1	H1C2T1	H1C3T1	H1C1T1	H1C2T1	H1C3T1
Time	5.20.94	5.56.20	4.55.63	4.23.31	6.51.35	4.24.44
Aperture (mm)	3.55	3.94	3.27	2.33	3.64	2.34
Theory (Pa)	3045	1019	995	13435	1598	3321
Test (Pa)	Non	1473	1520	Non	1736	3688
Type	H2C1T1	H2C2T1	H2C3T1	H2C1T1	H2C2T1	H2C3T1
Time	7.23.90	6.35.08	6.39.60	7.53.38	6.23.10	6.01.63
Aperture (mm)	4.91	4.37	4.42	4.18	3.39	3.2
Theory (Pa)	1162	756	433	2339	1980	1308
Test (Pa)	1354	886	562	2595	2196	1454

*Note: "Non" indicates no valid pressure data is obtained, due to human error and equipment fault.

Based on Eq. (6.3), the mechanical aperture of Cases H1C2T1 and H1C3T1 for smooth fracture are 3.8mm and 2.23mm, respectively, which are not equal to 1mm. According to Eq. (6.3), for cases presented in Figure 6.23 and Figure 6.24, all the preset apertures are updated, and the maximum values of the corresponding injection pressure were calculated based on theoretical formula, all the results are shown in Table 6.6Table 6.6.

As can be seen from Table 6.6, most of the average mechanical apertures calculated is larger than what they should be, so the maximum injection pressures measured are all relatively low. What's more, the difference between the maximum pressures measured during the test and that calculated using the mechanical aperture for rough fracture are larger than that for smooth fracture, indicating that the existence of fracture roughness can lead to the increase of resistance force. On the other hand, it also indicates that the effective aperture of rough fracture is much less than its average mechanical aperture. The magnitude of effective aperture (equivalent aperture) and the relevant influence will be discussed in the following sections.

Figure 6.24 Real-time variation of injection pressure for grouting of rough fracture

6.6 Modelling of rock fractures

The natural rock fractures have rough surfaces and are often tortuous at the same time. A lot of work has been done to characterize and parameterize these properties by using the fractal and self-affine features (Tse and Cruden, 1979; Mandelbrot, 1985; Power and Tullis, 1991; Hsiung et al., 1993; Odling, 1994). Researchers have proposed different methods for generating synthetic rock fractures based on these studies (Moon and Song, 1997; Brush, 2001; Brush and Thomson, 2003; Ogilvie et al., 2003; Chen and Horne, 2006; Ogilvie et al., 2006; Crandall et al., 2010; Hanssen, 2013; Lavrov, 2013; Xiao et al., 2013; Wang et al., 2016).

In the present work, Ogilvie's method is adopted for generating synthetic rock fractures (Ogilvie et al., 2006), since their model considers all key factors, including fractal dimension, anisotropy property, and the degree of matching between upper and lower surfaces of a rock fracture, which makes it easy for quantitative study and sensitivity analysis of different impact factors. Moreover, their model is developed based on statistics from natural rock fractures. The procedure for generating the synthetic rock fractures is as follows: 1) The fracture size and the relevant resolution are determined first, which are set as 100x100mm with 512x512 resolution, which is a square model; 2) Then five key parameters are selected, including fractal dimension, mismatch wavelength, transition length, standard deviation of fracture aperture, and anisotropy factor (Ogilvie et al., 2006); 3) A series of rough rock fractures are generated based on orthogonal design table, which is presented in Appendix A. The ranges of all parameters are determined based on statistical analysis from parameterization of natural rock fractures. In total, 58 different fracture models are generated based on parameter combination listed in Table A2 in Appendix A, and the original data are further processed with MATLAB programming to generate a rectangular shape fracture channel, with size of 50x100mm. The direction of grout flow is along the long side, which provides longer channel for fluid transportation and for better observation and analysis.

Four typical models of rough fracture channels, among all 58 models, are presented in Figure 6.25 to illustrate the characteristics of rough rock fracture models. As it can be seen from Figure 6.25, compared with Case 4, Cases 1, 2 and 3 have one feature

in common: they have large aperture and high roughness; and Case 4 is of very small aperture and relatively small roughness. Cases 1 and 2 have similar level of aperture and roughness, but the degree of tortuosity in Case 2 is higher than that in Case 1. In the present work, the second order surface roughness is taken as tortuosity for short.

Figure 6.25 Four typical synthetic models of rough rock fractures

Figure 6.26 Percentage frequency of the aperture of synthetic rock fractures

Another criterion to evaluate the suitability of the fracture model generated is the frequency of its fracture aperture distribution, and Figure 6.26 presents the percentage frequency of the fracture aperture and the relevant probability density distribution. The characteristics of aperture distributions for the four cases are quite different, with different mean aperture and distribution characteristics. Nevertheless, they all satisfy Gaussian distribution, which is taken as reasonable aperture distribution for natural rock fractures (Hakami, 1995; Wang et al., 2016).

To better describe the roughness and tortuosity, a quantitative method is needed. The traditional geological definition for roughness is based on a concept within 2D space consisting fracture aperture and length (Tse and Cruden, 1979), and it is defined as,

$$JRC = 32.2 + 32.47 \log(Z2)$$

$$Z2 = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_{i+1} - y_i)^2} / \Delta x \quad (6.4)$$

where JRC is the well-known Joint Roughness Coefficient; $Z2$ is a dimensionless parameter corresponding to a fracture channel of length $(N-1)\Delta x$, and y_i is the aperture height at location $(i-1)\Delta x$.

It can be seen from Eq. (6.4) that the value of $Z2$ can be used directly to evaluate fracture roughness. However, the natural rock fractures are in 3D domain, thus $Z2$ should be extended to 3D form (Belem et al., 2000),

$$Z2_m = \left\{ \frac{1}{(m-1)(n-1)} \left[\frac{1}{\Delta x^2} \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \frac{(z_{i+1,j+1} - z_{i,j+1})^2 + (z_{i+1,j} - z_{i,j})^2}{2} + \frac{1}{\Delta y^2} \sum_{j=1}^{n-1} \sum_{i=1}^{m-1} \frac{(z_{i+1,j+1} - z_{i+1,j})^2 + (z_{i,j+1} - z_{i,j})^2}{2} \right] \right\}^{1/2} \quad (6.5)$$

where $l_x = (m-1)\Delta x$ and $l_y = (n-1)\Delta y$ are the fracture length in the x and y directions, respectively; $z_{i,j}$ is the fracture aperture at location $(i-1)\Delta x$ and $(j-1)\Delta y$.

Belem also proposed a direct method to evaluate the surface roughness and tortuosity coefficient R_m and T_m , which are modified from the 2D model (Belem et al., 2000),

$$A_r = (\Delta x \cdot \Delta y) \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \left[1 + \left(\frac{z_{i+1,j} - z_{i,j}}{\Delta x} \right)^2 + \left(\frac{z_{i,j+1} - z_{i,j}}{\Delta y} \right)^2 \right]^{1/2} \quad (6.6)$$

$$R_m = \frac{(A_r^t + A_r^b)}{2A_n} = \frac{(R_m^t + R_m^b)}{2} \quad (6.7)$$

where A_r is the area of fracture rough surface, with A_r^t and A_r^b representing the top and bottom surface area, respectively; R_m is the overall fracture roughness coefficient, with R_m^t and R_m^b corresponding to the top and bottom surfaces, respectively.

Likewise, the surface tortuosity coefficient T_m is also defined as the ratio of two areas (Belem et al., 2000),

$$T_m = \frac{A_r}{A_\pi} = \frac{A_r}{A_n} \cos \theta = R_m \cos \theta, (0 < \cos \theta \leq 1) \quad (6.8)$$

where θ will be defined in the following part, and A_π is the area of the best fitting surface that can pass through the four extreme points of the rough surface, called “ π -surface”, which is defined as,

$$\alpha x + \beta y - z + \gamma = 0 \quad (6.9)$$

The coordinates of the four points can be substituted into Eq. (6.9), then the value of α and β can be calculated based on the least square regression method. Then, T_m can be derived, as $\cos \theta$ can be expressed as follows,

$$\cos \theta = 1 / \sqrt{\alpha^2 + \beta^2 + 1} \quad (6.10)$$

If the “ π -surfaces” of the top and bottom surfaces are different, the overall surface tortuosity coefficient of a rough fracture is calculated as,

$$T_m = \frac{A_r^t + A_r^b}{A_\pi^t + A_\pi^b} = \frac{A_r^t + A_r^b}{(A_n / \cos \theta^t) + (A_n / \cos \theta^b)} = 2R_m \frac{\cos \theta^t \cdot \cos \theta^b}{\cos \theta^t + \cos \theta^b} \quad (6.11)$$

As for the mean mechanical aperture of a 3D fracture, it can be defined as follows,

$$h_m = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (z_{i,j}^t - z_{i,j}^b) \quad (6.12)$$

where $z_{i,j}^t$ and $z_{i,j}^b$ are the points located at the top and bottom surfaces with coordinates of $(i-1)\Delta x$ and $(j-1)\Delta y$, respectively.

Therefore, the characteristics of rough rock fractures can be described based on Eqs. (7.5) to (7.12). The results for all cases are given in Table A3 in Appendix A, including the mean mechanical aperture h_m , dimensionless parameter $Z2_m$, the fracture roughness and tortuosity R_m and T_m .

As shown in Table A3, the variation trends of $Z2_m$, R_m and T_m are the same, namely they all increase or decrease simultaneously, and larger values of these parameters indicate larger mean mechanical aperture, namely a wide fracture.

6.7 Simulations of grout flow through rough fractures

To study the characteristics of grout flow through rough rock fractures, including the relationship between injection rate and pressure, the pressure or velocity distribution, and the equivalent fracture aperture, simulations of grout flow through rough rock fracture are conducted based on the Navier-Stokes Equations (the flow is assumed to be isothermal, so conservation of energy is not mentioned here),

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (6.13)$$

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \left[-p\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \right] + \mathbf{F} \quad (6.14)$$

where Eq. (6.13), representing mass conservation, is the continuity equation, where the density ρ is taken as constant so $\partial \rho / \partial t = 0$; Eq. (6.14) is conservation of momentum and a vector equation; $\mathbf{u} = [u_x, u_y, u_z]$ is the velocity vector (m/s); p is the pressure (Pa); $\mathbf{F}$ is the volume force vector (N/m³); $\mathbf{I}$ is a unit vector.

The model for simulation is shown in Figure 6.27, where all four models have the same boundary conditions. Assume that there is no outflow from two lateral boundaries along the flow direction. The top and bottom rough surfaces are assumed to be impermeable; and coupling effect between fluid flow and rock fracture is not considered here. The grout flow is taken as steady state laminar flow.

Following the engineering grouting practice, a constant injection flow rate is assumed as the boundary condition at the inlet end, which is easier to be controlled than keeping the injection pressure as constant (Warner, 2004); the mass flow rate applied is 0.2 to 0.5 kg/s, roughly corresponding to 10 to 30 L/min; the pressure is taken as

zero at the outlet end. Two cuboids are attached to the left and right ends of the fracture model, which are used to reduce the boundary influence, and to make it easier to get convergent solution. The size of the two cuboids is designed in such a way so that the cross section of rock fracture at both ends can be included inside the corresponding cuboids surfaces, as can be seen from Figure 6.27 and Figure 6.28. Normally, their sizes are the same within one fracture model, and they vary among different models.

Figure 6.27 Fracture models for simulating grout flow within rough rock fractures

Figure 6.28 Velocity distributions presented in five different slices (Case 1)

Both Bingham model and Newtonian model are used for simulation, and simulation results between these two models are compared, including the corresponding

injection pressure generated, and the velocity fields. Comparisons are presented in detail in Figure B4 in Appendix B. It is observed that the differences between two models are very small under the current constant injection flow rates. Therefore, the fluid model for cement grout is taken as Newtonian flow, since the relevant computational efficiency is much higher.

Take Case 1 as an example, the velocity distributions within fractures are shown in Figure 6.28, where velocity distribution in five longitudinal cross-sections (five slices) are presented, and the red and blue colors refer to high and low velocities, respectively. As can be seen from Figure 6.28, the inlet velocity at the cuboid is stable and uniform, indicating its function for stabilizing the inlet flow into the rough rock fracture.

One important observation is that bright red color concentrates mainly at the fracture middle; and the color close to the fracture wall is dark blue (extremely low velocity), especially at the location with large roughness, indicating that these areas do not effectively contribute to grout transportation. The same phenomena can be found for all other cases. Therefore, we can conclude only part of the rough rock fracture provides effective channel for grout flow, which is called “equivalent aperture”.

6.8 Model for predicting equivalent aperture

6.8.1 Analytical model derivation

It is straightforward to calculate the equivalent aperture if the fluid can be taken as Newtonian flow, since the relationship between the grout injection pressure and the flow rate follows the well-known Cubic law. Thus, the equivalent aperture can be calculated based on the transformed equation

$$h_e = \left(\frac{12\mu Q_m L}{\rho W P_0} \right)^{1/3} \quad (6.15)$$

where h_e is the equivalent aperture of the fracture channel; μ and ρ are the viscosity and density of cement grout, respectively; W and L are the width and length of the fracture channel, respectively; Q_m and P_0 are the grout injection flow rate and the corresponding injection pressure, respectively. P_0 can be derived as the

difference of average pressures on two cross sections (xz plane) located at $y=0$ and 100, respectively, as shown in Figure 6.27 and Figure 6.28.

For cement grout taken as Bingham flow, the grout injection flow rate is expressed as

$$Q_m = \frac{\rho W h_e^3}{12\mu} \left(-\frac{\partial P}{\partial x} \right) \left[1 - 3 \left(\frac{y_0}{h_e} \right) + 4 \left(\frac{y_0}{h_e} \right)^3 \right], y_0 = \tau_0 / (-\partial P / \partial x) \quad (6.16)$$

where τ_0 is the yield stress of cement grout; $-\partial P / \partial x$ is the pressure gradient, and it is taken as P_0 / L , thus the half width of the Bingham plug y_0 is $\tau_0 L / P_0$. Eq. (6.16) is derived by assuming that grout flows within parallel plates with a distance of h_e .

The relationship between Q_m and h_e is nonlinear, so we cannot express h_e as a function of Q_m directly. Thus, some transformations should be used to determine h_e .

We introduce a dimensionless variable $\phi = y_0 / h_e$. Then, substitute ϕ into Eq. (6.16)

$$Q_m = \frac{\rho W \tau_0 y_0^2}{12\mu} \left(\frac{1 - 3\phi + 4\phi^3}{\phi^3} \right) \quad (6.17)$$

Rewrite Eq. (6.17) into cubic equation of ϕ , the following equation can be obtained,

$$(C_0 - 4)\phi^3 + 3\phi - 1 = 0 \quad (6.18)$$

where $C_0 = 12\mu Q_m / \rho W \tau_0 y_0^2$ is a dimensionless constant under specific injection rate.

For a special situation with $\phi = 1/3$ ($C_0 = 4$), the equivalent aperture can be expressed as a simple function of the injection pressure,

$$h_e = 3y_0 = 3\tau_0 L / P_0 \quad (6.19)$$

Here, Eq. (6.19) can be used to calculate the equivalent aperture when the relationship between the injection mass flow rate and the corresponding injection pressure is $Q_m = \rho W L^2 \tau_0^2 / 3\mu P_0^2$, which could be very useful for conducting laboratory testing using Bingham fluid, when the test setup satisfies the condition here.

Let $\lambda_1 = 3/(C_0 - 4)$, and $\lambda_2 = -1/(C_0 - 4)$ when $C_0 \neq 4$, then Eq. (6.18) becomes

$$\phi^3 + \lambda_1 \phi + \lambda_2 = 0 \quad (6.20)$$

To solve the cubic equation, we should determine the discriminant Δ of Eq. (6.20),

$$\Delta = \left(-\frac{\lambda_2}{2}\right)^2 + \left(\frac{\lambda_1}{3}\right)^3 = \frac{C_0}{4(C_0 - 4)^3} \begin{cases} > 0, & \text{if } (C_0 > 4) \\ < 0, & \text{if } (C_0 < 4) \end{cases}$$

There is only one real root from Eq. (6.20) that satisfies $C_0 > 4$, as the other two roots are imaginary numbers, thus the unique solution of Eq. (6.20) is

$$\phi = \sqrt[3]{-\lambda_2/2 + \sqrt{\Delta}} + \sqrt[3]{-\lambda_2/2 - \sqrt{\Delta}} \quad (6.21)$$

For typical grouting projects, C_0 is normally a very large number, so Eq. (6.18) can be simplified as $C_0 \phi^3 + 3\phi - 1 = 0$, then $\lambda_1 = 3/C_0$, $\lambda_2 = -1/C_0$, and $\Delta = 1/4C_0^2$. Finally, we have $\phi = C_0^{-1/3}$, so the equivalent aperture is $h_e = C_0^{1/3} \tau_0 L / P_0$.

In some laboratory tests, the injection flow rate or pressure might be very low, so the condition $C_0 < 4$ may occur. In that case, there would be three real roots for Eq. (6.20)

$$\begin{cases} \phi_1 = 2\sqrt{-\lambda_1/3} \cos\{\alpha/3\} \\ \phi_2 = 2\sqrt{-\lambda_1/3} \cos[(\alpha + 2\pi)/3] \\ \phi_3 = 2\sqrt{-\lambda_1/3} \cos[(\alpha - 2\pi)/3] \end{cases} \quad (6.22)$$

where $\alpha = \arccos[-1/2(4 - C_0)^{5/2}]$, thus $\alpha \in [\pi/2, \pi]$.

As $\alpha \in [\pi/2, \pi]$, the range of angle ϕ in Eq. (6.22) can be estimated as,

$$\begin{cases} \alpha/3 \in [\pi/6, \pi/3] \\ \alpha + 2\pi/3 \in [5\pi/3, \pi] \\ \alpha - 2\pi/3 \in [-\pi/2, -\pi/3] \end{cases} \Rightarrow \begin{cases} \phi_2 < 0 \\ \phi_1 > \phi_3 > 0 \end{cases}$$

As for the negative solution, ϕ_2 should not be accepted as a useful solution, as ϕ should be positive to ensure that the relevant pressure drop per unit length $P_0/L = \tau_0/(\phi h_e) > 0$. Meanwhile, consider the expression of pressure drop $P_0/L = \tau_0/(\phi h_e)$, the conditions $\phi_1 > \phi_3 > 0$ indicate that the injection pressure corresponding to ϕ_3 is higher than ϕ_1 for the same fracture. If a lower pressure can

make the cement grout move with a flow rate of Q_m , a higher pressure is bound to generate higher flow rate. Therefore, ϕ_1 is chosen as the root of Eq. (6.20),

$$\begin{cases} \phi_1 = 2\sqrt{1/4 - C_0} \cos(\alpha/3) \\ \alpha = \arccos\left[-1/2(4 - C_0)^{5/2}\right] \end{cases} \Leftarrow \begin{cases} C_0 = 12\mu Q_m / \rho W \tau_0 y_0^2 \\ y_0 = \tau_0 L / P_0 \end{cases} \quad (6.23)$$

Then, the equivalent aperture can be calculated as $h_e = y_0 / \phi_1$, according to Eq. (6.23).

A summary is presented in Table 6.7 to calculate the relevant equivalent aperture.

Table 6.7 Equations to be used for calculating equivalent aperture

Fluid type of cement grout	Equation Number
Newtonian flow	Eq. (7.15) or Eq. (19)
	Eq. (19) ($C_0 = 4$)
Bingham flow	Eq. (21) ($C_0 > 4$)
	Eq. (23) ($C_0 < 4$)

Notes*: When $C_0 = 4$, the Newtonian model becomes a special case of Bingham model.

6.8.2 Predicative model development

In the present work, the flow can be treated as Newtonian flow based on the numerical simulation conducted in Section 6.7, so the equivalent aperture h_e can be calculated based on Eq. (6.15), where the grout injection pressure P_0 is obtained from the simulation models under four different grout injection flow rates. The simulation results and the equivalent apertures calculated are presented in Table A4 in Appendix A. It can be found that the equivalent aperture decreases with increasing mass flow rate, and it seems that the effective aperture for grout transportation is shrinking with an increasing flow rate, as the corresponding energy lost increase faster than the rate of increase of injection flow rate.

In practical grouting operation, it would be of great significance if we can know the relationship between grout injection flow rate and the corresponding injection pressure, as the injection pressure can be used to evaluate the potential fracture (or rock mass) deformation, which is critical to the stability and safety of tunnel or rock cavern under excavation. However, the effective aperture is affected by both the grout

injection flow rate, and the characteristics of the fracture channel, including its roughness, tortuosity, mean mechanical aperture, the fractal dimension, standard deviation, and so on. Therefore, it is essential to evaluate the importance of each factor on the equivalent aperture.

From stepwise regression analysis based on MATLAB programming, it is found that fracture roughness R_m , standard deviation of fracture aperture σ_m , and the mean mechanical aperture h_m are the three most important impact factors to the equivalent aperture. The fractal dimension and anisotropy factor are not considered separately, since their influences have already been reflected by the three factors Z_2 , R_m and T_m , as defined in Section 6.6. In the meanwhile, among Z_2 , R_m and T_m , only R_m is taken as the most critical factor, since the magnitude and variation trends of these three parameters are more or less the same, which can be seen from Table A3 in Appendix A, so it is reasonable to assume that R_m alone can represent their common influence. The mean mechanical aperture h_m and its standard deviation σ_m describe the magnitude and the degree of concentration of fracture aperture, respectively, so they are bound to be among the most critical factors.

In building a prediction model, correlating the equivalent aperture h_e to fracture roughness R_m , standard deviation of fracture aperture σ_m , and the mean mechanical aperture h_m , we prefer to use an explicit expression, especially a polynomial. In the present work, we use a polynomial to obtain an expression $h_e = \sum_{n=0}^N (b_n R_m^i h_m^j \sigma_m^k)$, where $i \in [0, m_1]$, $j \in [0, m_2]$, and $k \in [0, m_3]$, N is the number of polynomial terms; m_1 , m_2 , and m_3 are the maximum exponents corresponding to R_m , σ_m , and h_m ; a constant term exists when $i = j = k = 0$. The equivalent aperture h_e obtained thus far is under a specific injection flow rate, and it changes with varying injection flow rates, so we need to establish the relationship between injection flow rate Q_m and the coefficients b_n , expressed as $b_n = f_n(Q_m)$, ($n \in [0, N]$). Then, the

equivalent aperture can be expressed as $h_e(Q_m) = \sum_{n=0}^N [b_n(Q_m) R_m^i h_m^j \sigma_m^k]$. Finally, the final expression should be validated with extra cases. The detailed process for deriving the prediction model is described in the following sections.

In the present work, linear regression is tried first

$$h_e = b_0 + \sum_i^N (b_i x_i) \quad (6.24)$$

where $N=3$ is the number of impact factors, x_i and b_i represent the impact factor and regression parameters, x_1 represents standard deviation of aperture σ_m , x_2 represents the fracture roughness R_m , x_3 represents mean mechanical aperture h_m .

The parameter array $B = [b_0, b_1, \dots, b_N]$ is obtained when the minimum value of the sum of squared residual error (SSE or RSS) between the predicted value $\hat{h}_e$ and the practical value h_e is reached,

$$B = \arg \min_B \left(\sum_{i=1}^N (h_e - \hat{h}_e)^2 \right) \quad (6.25)$$

Then, the predicted equivalent aperture can be obtained based on Eq. (6.24). Comparisons between the predicted results and the original data are presented in Figure 6.29. For most cases, there are little differences between the predicted and original values, indicating that the predictive model derived is of high accuracy.

Figure 6.29 Comparison between predicted and the original equivalent apertures:
 (a) $Q_m = 0.2\text{kg/s}$, (b) $Q_m = 0.3\text{kg/s}$, (c) $Q_m = 0.4\text{kg/s}$, and (d) $Q_m = 0.5\text{kg/s}$

Figure 6.30 Parameters of prediction model

To develop a final predictive model, further explore the potential influence of injection flow rate Q_m on the model parameters of the prediction model for four cases, the parameter arrays of B are plotted in Figure 6.30, so as to see the relevant variation patterns. As it can be seen, the value of b_1 and b_2 is negative, indicating that the existence of aperture deviation and fracture roughness can reduce the relevant equivalent aperture, which is reasonable and acceptable.

Figure 6.31 Linear relationship between parameter b_i and flow rate via fitting

Moreover, parameters $b_i (i=0,1,2,3)$ can increase (b_2 and b_3) or decrease (b_1 and b_0) with increasing Q_m . Therefore, it is reasonable to express the relevant relationship with linear equation, which is obtained through finding the best fitting line passing through all the points. The linear regression results are presented in Figure 6.31, which shows very good linear fitting.

The final prediction model, which includes the standard deviation of fracture aperture σ_m , fracture roughness R_m , mean mechanical aperture h_m , and flow rate Q_m , can be expressed as,

$$\begin{aligned} h_e &= b_0(Q_m) + b_1(Q_m) \cdot \sigma_m + b_2(Q_m) \cdot R_m + b_3(Q_m) \cdot h_m \\ \begin{cases} b_0(Q_m) = -0.327Q_m + 0.5 \\ b_1(Q_m) = 0.311Q_m - 0.5 \\ b_2(Q_m) = 0.144Q_m - 0.232 \\ b_3(Q_m) = -0.546Q_m + 0.761 \end{cases} \end{aligned} \quad (6.26)$$

Eq. (6.26) has been used to calculate the equivalent apertures of all cases under grout injection flow rate of 0.1, 0.6, and 0.7 kg/s, respectively. The prediction results are compared with that from numerical simulations, which are presented in Figure 6.32. The parameters taken as input of the predicative model are only the characteristic parameters of rock fractures (R_m , h_m , and σ_m), and the specific grout injection flow rate Q_m . The equivalent aperture obtained from numerical simulations are based on Navier-Stokes equation and Eq. (6.15). It can be found from Figure 6.32 that there are small differences between the equivalent aperture predicted by the derived model and that from numerical simulations for most cases, which verifies that the derived predictive model has a high degree of accuracy, and it can predict the equivalent aperture of rock fracture under a specific injection flow rate.

The accuracy of the prediction model for the cases under injection flow rate $Q_m = 0.7\text{kg/s}$ is slightly lower than that under $Q_m = 0.1\text{kg/s}$ and $Q_m = 0.6\text{kg/s}$, since the latter two injection flow rates are closer to the flow rate interval $[0.2, 0.5]$ (kg/s) used for training. This indicates that a wider range of data used for training can lead to a prediction model of higher accuracy.

Figure 6.32 Validation of the prediction model for calculating the equivalent aperture based on the characteristics of rock fracture and the corresponding grout injection mass flow rate

6.8.3 A potential engineering application

Under the condition that Eq. (6.26) is obtained through many training cases, coefficients a_{i0} and a_{i1} should be reliable to be used in $b_i(Q_m) = a_{i1} \cdot Q_m + a_{i0}$ ($i=0,1,2,3$), which can be used to predict the fracture equivalent apertures based on Water Pressure Test (WPT), then other characteristics of rock fractures investigated can be calculated based on the equivalent aperture obtained, finally we can use all information derived to estimate the volume of grout in need.

In engineering practice, WPT is generally conducted to determine the relevant geological condition, especially the fracture distribution or characteristics. A simplified schematic diagram of the testing holes of WPT is shown Figure 6.33, where the rock fracture is assumed to be a rectangular with limited width and length. The pressure measured at holes $H1$ and $H2$ can be used for calculating the pressure difference between two holes, which is taken as the nominal injection pressure; and the flow rate of water outflow from hole $H2$ can be recorded simultaneously. The top and bottom surfaces are rough but are not illustrated here.

Figure 6.33 Simplified schematic diagram of WPT

The basic information concerning the characteristics of rock fractures, including the mean mechanical aperture h_m , the fracture roughness R_m , and its standard deviation σ_m , can be estimated according to data analysis of the rock samples from early site investigation. The distance between the two holes is $L = 5\text{m}$, which can be taken as the nominal fracture length. The fracture width W is still unknown, but it is a critical parameter for grouting design, as the void volume of the rock fracture V_{frac} can be calculated as $V_{\text{frac}} = W \cdot L \cdot h_m$, which is the volume of grout required for filling the fracture. However, the width W can be calculated through the following equation transformed from Eq. (6.15),

$$W = 12\mu Q_m L / \rho P_0 h_e^3 \quad (6.27)$$

In Eq. (6.27), the equivalent aperture can be calculated based on Eq. (6.26) utilizing the WPT site data. Three sets of WPT data, as shown in Table 6.8, are required for three pairs of P_0^i and Q_m^i ($i=1,2,3$) for better estimation, of which the average is taken as the final value of width W .

Substitute the water flow rate Q_m^i ($i=1,2,3$), and the characteristic information of rock fractures into Eq. (6.26), we have three equivalent apertures h_e^i ($i=1,2,3$)

$$\begin{cases} b_0(Q_m^1) + b_1(Q_m^1) \cdot \sigma_m + b_2(Q_m^1) \cdot R_m + b_3(Q_m^1) \cdot h_m = h_e^1 \\ b_0(Q_m^2) + b_1(Q_m^2) \cdot \sigma_m + b_2(Q_m^2) \cdot R_m + b_3(Q_m^2) \cdot h_m = h_e^2 \\ b_0(Q_m^3) + b_1(Q_m^3) \cdot \sigma_m + b_2(Q_m^3) \cdot R_m + b_3(Q_m^3) \cdot h_m = h_e^3 \end{cases} \quad (6.28)$$

where the coefficients a_{i0} and a_{i1} for expression $b_j(Q_m^i) = a_{j1} \cdot Q_m^i + a_{j0}$ ($i=1,2,3; j=0,1,2,3$) is the same as that presented in Eq. (6.26).

By using the equivalent aperture h_e obtained from Eq. (6.28), the fracture width W via Eq. (6.27) can be calculated, which are presented in Table 6.8. As can be seen, three fracture widths calculated are close to each other, indicating good prediction accuracy.

Table 6.8 WPT data and the corresponding fracture characteristics

Data sets	Testing pressure (Pa)	Water volume (L)	Time taken (min)	Fracture characteristics	
1	212370	120	10	σ_m	0.75
2	480353	126	7	R_m	1.64
3	864671	216	9	h_m	1.13

Thereafter, the volume of cement grout required for sealing this fracture is taken as

$$V_{frac} = W \cdot L \cdot h_m = 2.145e^{-3} (m^3)$$

Then, the total volume of cement grout required for sealing the whole area under investigation can be expressed as,

$$V_{totl} = \sum_{i=1}^N V_{frac}^i = \sum_{i=1}^N (W^i \cdot L^i \cdot h_m^i) \quad (6.29)$$

where V_{frac}^i , W^i , L^i , and h_m^i are the volume, width, length, and mean mechanical aperture of the i -th rock fracture investigated, and N is the total number of fractures discovered in a specific area.

The grout flow under the assumed flow rate is considered as a Newtonian flow, which should be further validated through laboratory testing in the future. If the grout injection flow rate is lower than what is used in this chapter, the flow type probably

cannot be taken as Newtonian flow; however, the corresponding equivalent aperture can still be calculated using Eqs. (6.19), (6.21), and (6.23) for Bingham flow.

Table 6.9 Determination of the fracture width W

Data NO.	h_e (mm)	W (m)	Final W (m)
1	0.53	0.3795	0.3797
2	0.46	0.3850	
3	0.42	0.3746	

It can be found that, for rough rock fracture, larger mean mechanical aperture correlates to higher fracture roughness and tortuosity. With the increase of injection flow rate, the injection pressure is increasing, but the corresponding equivalent aperture is decreasing, the probable reason of which is that the corresponding energy lost increases faster than the rate of energy supply from grout injection. The equivalent aperture can be calculated based on a predictive model consisting of grout injection flow rate, standard deviation and mean mechanical aperture of rock fracture, and fracture roughness, and it is negatively related to the standard deviation and roughness of a rock fracture.

The “engineering application” mentioned here is a generalized concept, not representing any specific engineering project. Namely, one potential technology for engineering application is proposed here, which correlates the data from a special onsite test (resembling Water Pressure Test) to equivalent aperture of rock fractures directly intersected by the hole, and, finally, estimates the width of the fracture under investigation. To some extent, it is more like a thought experiment, since the distribution of rock fractures in nature are very complex, and it is not that possible that one injection hole only intersects one rock fracture. Therefore, the concept proposed here is limited to rock masses with relatively good quality, where there exists a small number of rock fractures, but the corresponding engineering project is of high sealing requirement, like rock cavern designed for storing nuclear waste.

6.9 Summary

In this chapter, an experimental system is developed for grouting test in laboratory, including pumping system, aperture adjustable fracture system, and data collection and processing system. The pumping system can simulate practical pumping system, exporting constant injection flow rate. Top and bottom rough surfaces of rock fracture for grouting test is generated based on 3D printing, and the mean mechanical aperture between the two surfaces can be adjusted by a precision lifting system. It is found that the existence of roughness can change not only the path of grout flow but also the concentration distribution of cement grout; simultaneously, roughness can increase the resistance to grout flow remarkably, as the injection pressure is increasing faster compared with grout flow in a channel with two smooth parallel plates.

A numerical model is developed to supplement the experimental test and to further quantify the influence of fracture roughness and tortuosity. An algorithm is utilized to synthesize various rough surfaces for rock fractures, and the characteristics of the fractures are analyzed based on MATLAB program. A series of synthetic rock fractures are generated based on the design of experiments (DOE) for grouting simulation, then the numerical results are used for calculating the corresponding equivalent apertures of rock fractures. Through analyzing the simulation data, it is found that the roughness and tortuosity of rock fracture increase with increasing mean mechanical aperture. What's more, for rough rock fracture, only part of the fracture space provides effective channel for grout transportation, the aperture of this effective space is taken as "equivalent aperture", which can be written as a function of grouting parameters (grout injection flow rate) and parameters characterizing rough fracture surface (like mean mechanical aperture and its deviation, and fracture roughness). The equivalent aperture is directly proportional to the mean mechanical aperture, but it is negatively correlated to the fracture roughness and standard deviation of fracture aperture, indicating that the resistance from rough surface to grout transportation would increase with increasing degree of fracture roughness.

Chapter 7 Conclusions and Recommendations

7.1 Conclusions

Grouting of rock fractures is a very complex problem, as it involves geological conditions, characteristics of rock fractures, rheological properties of grout, and specific grouting strategies. However, theories that have been developed so far contribute little to grouting practice, which still heavily relies on the experience of on-site engineers. Therefore, the present work investigates more practical problems by considering more practical impact factors involved in grouting practice.

Firstly, two fundamental models for evaluating grout penetration in 1D fracture channel are discussed based on two different boundary conditions: constant injection pressure (CIP) and constant injection flow rate (CIFR), where the former appears more in academic paper, and the latter more correlates to engineering practice. As for the CIP method, the flow rate of grout is decreasing with time, so the grout flow stops when it reaches a force balance between injection pressure and resistance force due to shear stress and water head, and the maximum length of grout penetration is achieved; but for the CIFR method, the flow will not stop but the corresponding injection pressure is increasing with time, so the stop criterion for grout injection will be a preset pressure limit in engineering practice or a certain volume of grout materials consumed. Thereafter, a highly simplified formula is derived for a common determination of grout penetration in engineering application under some specific conditions, which can also reduce computation burden compared with the non-simplified model. Analytical model for considering the coupling effect between grout flow and fracture deformation is also developed, which can help to explain some experimental data where fracture deformation occurred.

One-dimensional channel flow model is only applicable to grout flow in rock fractures not directly intersected by some injection hole, so a 2D radial flow model is proposed for investigating grout spread in rock fractures directly intersected by a grouting hole. Based on the model proposed, the range or radius of real-time grout penetration around a grout hole and real-time injection pressure can be predicted. A model is developed for considering deformation of two neighboring fractures, and for evaluating its potential impact on the penetrability of grout through rock fracture with

a smaller aperture. It is found that grout flow in the fracture with a larger aperture runs much faster than that with a smaller aperture; the ratio between the maximum amplitude of fracture deformation and the smaller aperture is key to the weakening impact on the penetrability of the smaller fracture, but it is highly influenced by aperture and the stiffness of the smaller fracture. As rock fractures are normally accompanied with flowing water, an analytical model is developed to predict grout penetration in rock fractures considering the existence of flowing water based on the theory of potential flow, and the simulation results can explain some phenomena appeared in a grouting test in laboratory.

Prediction of grout spread in 3D fracture network is very complicated, especially when nonlinear and/or nonconsecutive fluid model characterizes the fluid type. Therefore, a model for simulating grout penetration in a 2D fracture network is developed and implemented in Discontinuous Deformation Analysis (DDA) as a grouting module. A new algorithm is developed to predict grout penetration in 2D fracture network under two different assumptions (constant injection pressure and constant injection flow rate). Based on the grouting module developed, real-time grout penetration in interconnected rock fractures and real-time pressure variation at various intersection nodes can be visualized, which can provide important signal or reference data for grouting stop.

Finally, the influence of another practical factor on grout spread within rock fractures is considered: fracture roughness, so a large-scale grouting testing is designed and conducted in the laboratory, where 3D printing is used to generate rough surfaces of real rock fractures. Test data shows that fracture roughness can alter the distribution of grout concentration, especially at the grout flow front; it also introduces more resistance to grout flow, as the injection pressure increases faster compared with smooth surfaces. Further study is conducted via numerical simulation to make up the shortage of laboratory testing. The rough surfaces of rock fractures can be characterized by four parameters: fracture roughness and tortuosity, mean mechanical aperture and its standard deviation, and the influence of rough surface on grout penetration can be quantified by the equivalent aperture, which is a function of the four parameters.

7.2 Recommendations

Even though grouting models of different dimensions have been proposed, where many practical factors have been considered. Many other problems remain to be solved for better grouting prediction.

The first one is the problem of dimension transfer during grouting, as shown in Figure 7.1. If the size in one direction is bigger than that in another direction, the dimension of grout flow would change from 2D radial flow to 1D channel flow after the fracture in the shorter direction is sealed. During this process of dimension transfer, the characteristics of pressure variation would change a lot, which can be taken as a signal that whether a fracture (directly intersected by the injection hole) has been sealed or not.

Figure 7.1 Dimension transfer for grout penetration in rock fractures directly intersected by an injection hole

Another concern is the influence of filling materials in rock fractures, a schematic diagram is shown in Figure 7.2. If the existence of filling material could increase the resistance to grout flow, it is of great significance to define the potential categories of influence, and then it is better to quantify it. In the meanwhile, would filtration occur more easily and frequently for grouting of rough rock fractures with filling materials? Because some of the filling materials might be forced to migrate from their

original location to somewhere else, which might trigger the formation of filter cake, and then accelerate the process of filtration, when grout flow reaches locations where the regional fracture aperture is very small or less than the critical aperture.

Figure 7.2 Demonstration of filling materials in rough rock fracture

Grouting theory nowadays are generally based on assumed geological conditions or under the precondition that the geological condition is known. Nevertheless, practically, it is hard to know the exact geological condition in advance, such as the distribution and connection of rock fractures, not to mention the apertures of specific rock fractures. Therefore, some work should be done to overcome this obstacle. One suggestion is to use more engineering data collected on-site, like probe hole test, grout volume and pressure, to help reconstruct the geological condition or at least the overall distribution of rock mass conductivity, which can facilitate empirical, analytical, or numerical prediction. Another recommendation is through introducing data mining, where data from previous engineering projects can be used to build a data base, which can be further processed through classification and outlier filtration. Then the data after treatment can be used to develop general prediction model, which can predict the range of grout penetration or the volume of grout take within a certain time.

LIST OF PUBLICATIONS

Yan-xu Guo; Fei Xiao; Qingsong Zhang. Grouting Rock Fractures with Flowing Water, *European Journal of Environmental and Civil Engineering*. (Under review)

X Wang, F Xiao, Q Zhang. Development of a new grouting test system for rough fracture, *Science China Technological Sciences*. (Submitted)

Fei XIAO, Junlong SHANG, Zhiye ZHAO. Linkage between Fracture Aperture Distribution and Grouting Characteristics. *Computers and Geotechnics*. (Under reversion)

F Xiao, Z Zhao. Evaluation of Equivalent Hydraulic Aperture (EHA) for Rough Rock Fractures, *Canadian Geotechnical Journal*. (Published on line)

F Xiao, Z Zhao, H Chen. A simplified model for predicting grout flow in fracture channels [J]. *Tunnelling and Underground Space Technology*, 2017, 70: 11-18.

Xiao Fei, Zhiye Zhao, Influence of fracture deformation on grout penetrability in fractured rock Masses, *CouFrac 2018*, 12-14 November, Wuhan.

Zhiye Zhao, Wen Nie, Yasuhiro Yokota , Fei Xiao, Laddu Bhagya Jayasinghe, *Rock Mechanics and Rock Engineering in Singapore, ARMS10 - the ISRM 10th Asian Rock Mechanics Symposium*, Singapore.

Fei Xiao, Zhiye Zhao, Hezhen Yang. Interaction between Grout Penetration and Fracture Deformation, *ISOPE 2018 Conference*, 10-15 June 2018, Sapporo, Japan.

F. Xiao, and Z. Y. Zhao. Grout Flow in Fracture Channel Considering Fracture Deformation. *51st US Rock Mechanics/Geomechanics Symposium*. American Rock Mechanics Association, 25-28 June 2017.

REFERENCES

- Akhavan, A. and Rajabipour, F. (2012). "Quantifying the effects of crack width, tortuosity, and roughness on water permeability of cracked mortars." Cement and Concrete Research **42**(2): 313-320.
- Alghalandis, Y. F. (2017). "ADFNE: Open source software for discrete fracture network engineering, two and three dimensional applications." Computers & Geosciences **102**: 1-11.
- Amadei, B. and Illangasekare, T. (1994). "A mathematical model for flow and solute transport in non-homogeneous rock fractures." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts **31**(6): 719-731.
- Andersson, H. (1999). Chemical Rock Grouting-An Experimental Study On Polyurethane Foams. 9th ISRM Congress, International Society for Rock Mechanics and Rock Engineering.
- Andersson, J., Shapiro, A. M. and Bear, J. (1984). "A stochastic model of a fractured rock conditioned by measured information." Water Resources Research **20**(1): 79-88.
- Axelsson, M., Gustafson, G. and Fransson, Å. (2009). "Stop mechanism for cementitious grouts at different water-to-cement ratios." Tunnelling and Underground Space Technology **24**(4): 390-397.
- Baghbanan, A. and Jing, L. (2007). "Hydraulic properties of fractured rock masses with correlated fracture length and aperture." International Journal of Rock Mechanics and Mining Sciences **44**(5): 704-719.
- Banfill, P. (2006). "Rheology of fresh cement and concrete." Rheology reviews **2006**: 61.
- Baraka-Lokmane, S. (2002). "A new resin impregnation technique for characterising fracture geometry in sandstone cores." International Journal of Rock Mechanics and Mining Sciences **39**(6): 815-823.
- Baraka-Lokmane, S. and Liedl, R. (2006). "From Micro to Meso: an exercise in determining hydraulic conductivity of fractured sandstone cores from detailed characterization of the fractures." Geophysical Journal International **166**(3): 1440-1453.
- Barker, J. (1988). "A generalized radial flow model for hydraulic tests in fractured rock." Water Resources Research **24**(10): 1796-1804.
- Barton, N. and de Quadros, E. F. (1997). "Joint aperture and roughness in the prediction of flow and groutability of rock masses." International Journal of Rock Mechanics and Mining Sciences **34**(3-4): 252.e251-252.e214.
- Barton, N. and Quadros, E. (2003). Improved understanding of high pressure pre-grouting effects for tunnels in jointed rock. 10th ISRM Congress, International Society for Rock Mechanics.

- Belem, T., Homand-Etienne, F. and Souley, M. (2000). "Quantitative parameters for rock joint surface roughness." Rock mechanics and rock engineering **33**(4): 217-242.
- Ben, Y., Xue, J., Miao, Q., Wang, Y. and Shi, G.-H. (2012). Simulating hydraulic fracturing with discontinuous deformation analysis. 46th US Rock Mechanics/Geomechanics Symposium, American Rock Mechanics Association.
- Beyabanaki, S. A. R., Mikola, R. G. and Hatami, K. (2008). "Three-dimensional discontinuous deformation analysis (3-D DDA) using a new contact resolution algorithm." Computers and Geotechnics **35**(3): 346-356.
- Binding, D. M., Phillips, P. and Phillips, T. N. (2006). "Contraction/expansion flows: The pressure drop and related issues." Journal of non-newtonian fluid mechanics **137**(1): 31-38.
- Brantberger, M., Dalmalm, T., Eriksson, M. and Stille, H. (1998). "Styrande faktorer för tätheten kring en förinjekterad tunnel." Report Trita-AMI. Report: 1400-1306.
- Brantberger, M., Stille, H. and Eriksson, M. (2000). "Controlling grout spreading in tunnel grouting—analyses and developments of the GIN-method." Tunnelling and Underground Space Technology **15**(4): 343-352.
- Brantberger, M., Stille, H. and Eriksson, M. (2000). "Controlling grout spreading in tunnel grouting — analyses and developments of the GIN-method." Tunnelling and Underground Space Technology **15**(4): 343-352.
- Briggs, S., Karney, B. W. and Sleep, B. E. (2014). "Numerical modelling of flow and transport in rough fractures." Journal of Rock Mechanics and Geotechnical Engineering **6**(6): 535-545.
- Brown, S. R. (1987). "Fluid flow through rock joints: the effect of surface roughness." Journal of Geophysical Research: Solid Earth **92**(B2): 1337-1347.
- Brush, D. J. (2001). Three-dimensional fluid flow and solute transport in rough-walled fractures. PhD thesis PhD thesis, University of Waterloo.
- Brush, D. J. and Thomson, N. R. (2003). "Fluid flow in synthetic rough-walled fractures: Navier-Stokes, Stokes, and local cubic law simulations." Water Resources Research **39**(4).
- Cammarata, G., Fidelibus, C., Cravero, M. and Barla, G. (2006). "The Hydro-Mechanically Coupled Response of Rock Fractures." Rock Mechanics and Rock Engineering **40**(1): 41-61.
- Cappa, F., Guglielmi, Y., Rutqvist, J., Tsang, C. F. and Thoraval, A. (2008). "Estimation of fracture flow parameters through numerical analysis of hydromechanical pressure pulses." Water resources research **44**(11).
- Carter, T. G., Dershowitz, W., Shuttle, D. and Jefferies, M. (2012). Improved methods of design for grouting fractured rock. Proc. 4th Int. Conf. Grouting and Deep Mixing.

- Chen, C. Y. and Horne, R. N. (2006). "Two-phase flow in rough-walled fractures: Experiments and a flow structure model." Water resources research **42**(3).
- Chen, H., Zhao, Z. and Sun, J. (2013). "Coupled hydro-mechanical model for fractured rock masses using the discontinuous deformation analysis." Tunnelling and Underground Space Technology **38**: 506-516.
- Cheng, Y. (1998). "Advancements and improvement in discontinuous deformation analysis." Computers and Geotechnics **22**(2): 153-163.
- Clemeur, N., Rutgers, R. and Debbaut, B. (2004). "Numerical simulation of abrupt contraction flows using the Double Convected Pom-Pom model." Journal of non-newtonian fluid mechanics **117**(2): 193-209.
- Cornet, F. H., Li, L., Hulin, J. P., Ippolito, I. and Kurowski, P. (2003). "The hydromechanical behaviour of a fracture: an in situ experimental case study." International Journal of Rock Mechanics and Mining Sciences **40**(7-8): 1257-1270.
- Crandall, D., Bromhal, G. and Karpyn, Z. T. (2010). "Numerical simulations examining the relationship between wall-roughness and fluid flow in rock fractures." International Journal of Rock Mechanics and Mining Sciences **47**(5): 784-796.
- Dai, G. and Byron Bird, R. (1981). "Radial flow of a Bingham fluid between two fixed circular disks." Journal of Non-Newtonian Fluid Mechanics **8**(3-4): 349-355.
- Dalmalm, T. (2004). "Choice of grouting method for jointed hard rock based on sealing time predictions."
- Dano, C., Hicher, P.-Y. and Tailliez, S. (2004). "Engineering Properties of Grouted Sands." Journal of Geotechnical and Geoenvironmental Engineering **130**(3): 328-338.
- Denn, M. M. (2001). "Extrusion instabilities and wall slip." Annual Review of Fluid Mechanics **33**(1): 265-287.
- Develi, K. and Babadagli, T. (2015). "Experimental and visual analysis of single-phase flow through rough fracture replicas." International Journal of Rock Mechanics and Mining Sciences **73**: 139-155.
- Di Federico, V. (1998). "Non-Newtonian flow in a variable aperture fracture." Transport in porous media **30**(1): 75-86.
- Draganovic, A. (2009). "Bleeding and filtration of cement-based grout."
- Draganovic, A. (2009). Bleeding and Filtration of Cement-Based Grout. Doctoral thesis, Royal Institute of Technology (KTH).
- Eklund, D. (2005). Penetrability due to filtration tendency of cement based grouts. Stockholm, KTH.

- Eklund, D. and Stille, H. (2008). "Penetrability due to filtration tendency of cement-based grouts." Tunnelling and Underground Space Technology **23**(4): 389-398.
- El Tani, M. (2012). "Grouting Rock Fractures with Cement Grout." Rock Mechanics and Rock Engineering **45**(4): 547-561.
- Emmelin, A., Brantberger, M., Eriksson, M. and Gustafson, G. (2007). Rock grouting. Current competence and development for the final repository, Swedish Nuclear Fuel and Waste Management Co., Stockholm (Sweden).
- Emmelin, A., Eriksson, M., Fransson, Å. and AB., S. k. (2004). Characterisation, Design and Execution of Two Grouting Fans at 450 M Level, Äspö HRL, SKB.
- Eriksson, M. (2002). Prediction of grout spread and sealing effect. 1001, Bygghälsan.
- Eriksson, M., Friedrich, M. and Vorschulze, C. (2004). "Variations in the rheology and penetrability of cement-based grouts—an experimental study." Cement and Concrete Research **34**(7): 1111-1119.
- Eriksson, M., Stille, H. and Andersson, J. (2000). "Numerical calculations for prediction of grout spread with account for filtration and varying aperture." Tunnelling and Underground Space Technology **15**(4): 353-364.
- Esaki, T., Du, S., Mitani, Y., Ikusada, K. and Jing, L. (1999). "Development of a shear-flow test apparatus and determination of coupled properties for a single rock joint." International Journal of Rock Mechanics and Mining Sciences **36**(5): 641-650.
- Fransson, Å. (2001). "Characterisation of a fractured rock mass for a grouting field test." Tunnelling and Underground Space Technology **16**(4): 331-339.
- Fransson, Å. (2008). Grouting design based on characterization of the fractured rock. Presentation and demonstration of a methodology, Swedish Nuclear Fuel and Waste Management Co., Stockholm (Sweden).
- Fransson, Å., Tsang, C. F., Rutqvist, J. and Gustafson, G. (2010). "Estimation of deformation and stiffness of fractures close to tunnels using data from single-hole hydraulic testing and grouting." International Journal of Rock Mechanics and Mining Sciences **47**(6): 887-893.
- Frigaard, I. and Ryan, D. (2004). "Flow of a visco-plastic fluid in a channel of slowly varying width." Journal of non-newtonian fluid mechanics **123**(1): 67-83.
- Fu, P., Zhang, J., Xing, Z. and Yang, X. (2013). "Numerical simulation and optimization of hole spacing for cement grouting in rocks." Journal of Applied Mathematics **2013**.
- Fujita, T., Shinkai, F. and Nobuto, J. (2012). "Fundamental Study on a Grout Penetration Model for a HLW Repository." Journal of Energy and Power Engineering **6**(8): 1191-1203.

- Funehag, J. and Gustafson, G. (2008). "Design of grouting with silica sol in hard rock–New design criteria tested in the field, Part II." Tunnelling and underground space technology **23**(1): 9-17.
- Fusi, L., Farina, A., Rosso, F. and Roscani, S. (2015). "Pressure driven lubrication flow of a Bingham fluid in a channel: A novel approach." Journal of Non-Newtonian Fluid Mechanics **221**: 66-75.
- Gailing, Z., Kaiyu, Z., Yue, G. and Wenxue, W. (2011). "Comparative experimental investigation of chemical grouting into a fracture with flowing and static water." Mining Science and Technology (China) **21**(2): 201-205.
- Ge, S. (1997). "A governing equation for fluid flow in rough fractures." Water Resources Research **33**(1): 53-61.
- Gothäll, R. and Stille, H. (2009). "Fracture dilation during grouting." Tunnelling and underground space technology **24**(2): 126-135.
- Gothäll, R. and Stille, H. (2010). "Fracture–fracture interaction during grouting." Tunnelling and Underground Space Technology **25**(3): 199-204.
- Guo, T., Zhang, S., Ge, H., Wang, X., Lei, X. and Xiao, B. (2015). "A new method for evaluation of fracture network formation capacity of rock." Fuel **140**: 778-787.
- Gustafson, G., Claesson, J. and Fransson, Å. (2013). "Steering parameters for rock grouting." Journal of Applied Mathematics **2013**.
- Gustafson, G. and Stille, H. (1996). "Prediction of groutability from grout properties and hydrogeological data." Tunnelling and underground space technology **11**(3): 325-332.
- Gustafson, G. and Stille, H. (2005). "Stop criteria for cement grouting." Felsbau: Zeitschrift für Geomechanik und Ingenieurgeologie im Bauwesen und Bergbau **25**(3): 62-68.
- Gutierrez, M. S. and Lewis, R. W. (2002). "Coupling of fluid flow and deformation in underground formations." Journal of Engineering Mechanics **128**(7): 779-787.
- Hakami, E. (1995). "Aperture distribution of rock fractures."
- Hakami, E. (1995). Aperture distribution of rock fractures, Royal Institute of Technology Stockholm, Sweden.
- Håkansson, U. (1995). "Rheology of fresh cement-based grouts."
- Håkansson, U. (1993). Rheology of fresh cement-based grouts. PhD thesis, Royal Institute of Technology Stockholm, Sweden.
- Håkansson, U., Hässler, L. and Stille, H. (1992). "Rheological properties of microfine cement grouts." Tunnelling and Underground Space Technology **7**(4): 453-458.
- Hanssen, A. R. (2013). Numerical modelling of Bingham fluid flow and particle transport in a rough-walled fracture. Master thesis, Norwegian University of Science and Technology.

- Hansson, P. (1995). "Filtration stability of cement grout for injection of concrete structures." IABSE Symposium, San Francisco: 1199-1204.
- Hässler, L., Håkansson, U. and Stille, H. (1992a). "Computer-simulated flow of grouts in jointed rock." Tunnelling and Underground Space Technology **7**(4): 441-446.
- Hässler, L., Håkansson, U. and Stille, H. (1992b). "Classification of jointed rock with emphasis on grouting." Tunnelling and Underground Space Technology **7**(4): 447-452.
- Hendrickson, C. and Au, T. (1989). Project management for construction: Fundamental concepts for owners, engineers, architects, and builders, Chris Hendrickson.
- Hernqvist, L., Fransson, Å. and Vidstrand, P. (2008). Numerical modelling of grout spread and leakage into a tunnel in hard rock—A case study. World tunnel Congress.
- Hirata, T. (1989). Fractal dimension of fault systems in Japan: fractal structure in rock fracture geometry at various scales. Fractals in Geophysics, Springer: 157-170.
- Holmberg, M., Tsuji, M., Stille, B. and Stille, H. (2012). Evaluation of pre-grouting for the City Line project using the RTGC method. ISRM International Symposium-EUROCK 2012, International Society for Rock Mechanics.
- Holter, K.-G. and Hognestad, H.-O. (2012). "Modern pre-injection in underground construction with rapid-setting microcements and colloidal silica – applications in conventional and TBM-tunnelling." Geomechanics and Tunnelling **5**(1): 49-56.
- Hopkins, D. (2000). "The implications of joint deformation in analyzing the properties and behavior of fractured rock masses, underground excavations, and faults." International Journal of Rock Mechanics and Mining Sciences **37**(1): 175-202.
- Hori, M. and Koyama, T. (1999). "Numerical simulation of grouting by two grout analysis methods." Doboku Gakkai Ronbunshu **1999**(624): 21-30.
- Houlsby, A. C. (1990). Construction and design of cement grouting: a guide to grouting in rock foundations, John Wiley & Sons.
- Hsiung, S. M., Ghosh, A., Ahola, M. P. and Chowdhury, A. H. (1993). "Assessment of conventional methodologies for joint roughness coefficient determination." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts **30**(7): 825-829.
- Huilgol, R. R. (2015). Fluid mechanics of viscoplasticity, Springer.
- Isakov, E., Glover, P. and Ogilvie, S. (2001). Use of synthetic fractures in the analysis of natural fracture apertures. 8th European Congress for Stereology and Image Analysis, 8th ECS & IA, Leeds.
- Jeong, W. and Song, J. (2005). "Numerical investigations for flow and transport in a rough fracture with a hydromechanical effect." Energy Sources **27**(11): 997-1011.

- Jing, L., Ma, Y. and Fang, Z. (2001). "Modeling of fluid flow and solid deformation for fractured rocks with discontinuous deformation analysis (DDA) method." International Journal of Rock Mechanics and Mining Sciences **38**(3): 343-355.
- Kazemian, S., Prasad, A. and Huat, B. B. (2010). "Rheological behavior of grout in context of newtonian and non-newtonian fluids." Electronic Journal of Geotechnical Engineering **15**.
- Kim, K., Barton, N. and de Quadros, E. F. (1997). "Joint aperture and roughness in the prediction of flow and groutability of rock masses." International Journal of Rock Mechanics and Mining Sciences **34**(3): 252.e251-252.e214.
- Kim, Y.-I., Amadei, B. and Pan, E. (1999). "Modeling the effect of water, excavation sequence and rock reinforcement with discontinuous deformation analysis." International Journal of Rock Mechanics and Mining Sciences **36**(7): 949-970.
- Klimczak, C., Schultz, R. A., Parashar, R. and Reeves, D. M. (2010). "Cubic law with aperture-length correlation: implications for network scale fluid flow." Hydrogeology Journal **18**(4): 851-862.
- Kobayashi, S. and Stille, H. (2007). Design for rock grouting based on analysis of grout penetration. Verification using Äspö HRL data and parameter analysis, Swedish Nuclear Fuel and Waste Management Co.
- Kobayashi, S., Stille, H., Gustafson, G. and Stille, B. (2008). Real time grouting control method. Development and application using Äspö HRL data, Swedish Nuclear Fuel and Waste Management Co., Stockholm (Sweden).
- Koyama, T., Nishiyama, S., Yang, M. and Ohnishi, Y. (2011). "Modeling the interaction between fluid flow and particle movement with discontinuous deformation analysis (DDA) method." International Journal for Numerical and Analytical Methods in Geomechanics **35**(1): 1-20.
- Krizek, R. J., Michel, D. F., Hetal, M. and Borden, R. H. (1992). "Engineering properties of acrylate polymer grout." Geotechnical Special Publication **1**(30): 712-724.
- La Pointe, P. (1988). A method to characterize fracture density and connectivity through fractal geometry. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, Elsevier.
- Lanaro, F. (2000). "A random field model for surface roughness and aperture of rock fractures." International Journal of Rock Mechanics and Mining Sciences **37**(8): 1195-1210.
- Lavrov, A. (2013). "Redirection and channelization of power-law fluid flow in a rough-walled fracture." Chemical Engineering Science **99**: 81-88.
- Lee, H. and Cho, T. (2002). "Hydraulic characteristics of rough fractures in linear flow under normal and shear load." Rock Mechanics and Rock Engineering **35**(4): 299-318.

- Lee, J., Bang, C., Mok, Y. and Joh, S. (2000). "Numerical and experimental analysis of penetration grouting in jointed rock masses." International Journal of Rock Mechanics and Mining Sciences **37**(7): 1027-1037.
- Li, B., Jiang, Y., Koyama, T., Jing, L. and Tanabashi, Y. (2008). "Experimental study of the hydro-mechanical behavior of rock joints using a parallel-plate model containing contact areas and artificial fractures." International Journal of Rock Mechanics and Mining Sciences **45**(3): 362-375.
- Lin, C. T., Amadei, B., Jung, J. and Dwyer, J. (1996). Extensions of discontinuous deformation analysis for jointed rock masses. International journal of rock mechanics and mining sciences & geomechanics abstracts, Elsevier.
- Lipscomb, G. G. and Denn, M. M. (1984). "Flow of bingham fluids in complex geometries." Journal of Non-Newtonian Fluid Mechanics **14**(0): 337-346.
- Liu, R., Li, S., Zhang, Q., Yuan, X. and Han, W. (2011). "Experiment and application research on a new type of dynamic water grouting material." Chinese Journal of Rock Mechanics and Engineering **30**(7): 1454-1459.
- Liu, R., Li, S., Zhang, Q., Yuan, X. and Han, W. (2011). "Experiment and application research on a new type of dynamic water grouting material [J]." Chinese Journal of Rock Mechanics and Engineering **30**(7): 1454-1459.
- Lombardi, G. (1996). "Selecting the grouting intensity." The International Journal on Hydropower & Dams, Volume Three, Issue Four: 62-66.
- Lombardi, G. (1997). "GIN principle revisited." International water power & dam construction **49**(10): 33-36.
- Lombardi, G. (2002). Grouting of rock masses. Grouting and ground treatment, ASCE.
- Lombardi, G. (2003). Grouting of rock masses. Grouting and ground treatment: 164-197.
- Lombardi, G. and Deere, D. (1993). "Geomechanics abstract Rock and soil reinforcement and support Grouting: Grouting design and control using the GIN principle . Int Water Power Dam Constr " International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts **30**: 372.
- Lombardi, G. and Deere, D. (1993). "Grouting design and control using the GIN principle." International Water Power and Dam Construction IWPCDM **45**(6).
- Luo, P., Wang, L., Fan, B. and Zhang, F. (2012). "Numerical simulation of infiltration laws of grouts in random aperture based on multi-fractional Brownian motion." Chinese J. Geot. Eng **34**(2): 309-306.
- Mandelbrot, B. B. (1985). "Self-affine fractals and fractal dimension." Physica scripta **32**(4): 257.

- Mezger, T. (2000). *Das Rheologie-Handbuch: Für Anwender von Rotations-und Oszillations-Rheometern*, Curt R, Vincentz Verlag, Hannover.
- Miltiadou-Fezans, A. and Tassios, T. P. (2013). "Penetrability of hydraulic grouts." *Materials and Structures* **46**(10): 1653-1671.
- Mitsoulis, E. (2007). "Flows of viscoplastic materials: models and computations." *Rheology reviews* **2007**: 135-178.
- Mohammed, M. H., Pusch, R. and Knutsson, S. (2015). "Study of cement-grout penetration into fractures under static and oscillatory conditions." *Tunnelling and Underground Space Technology* **45**: 10-19.
- Mohammed, M. H., Pusch, R., Knutsson, S. and Hellström, G. (2014). "Rheological Properties of Cement-Based Grouts Determined by Different Techniques." *Engineering* **6**(05): 217.
- Moon, H. K. and Song, M. K. (1997). "Numerical studies of groundwater flow, grouting and solute transport in jointed rock mass." *International Journal of Rock Mechanics and Mining Sciences* **34**(3-4): 206.e201-206.e213.
- Morris, J. P., Chochua, G. G. and Bogdan, A. V. (2015). "An efficient non-newtonian fluid-flow simulator for variable aperture fractures." *The Canadian Journal of Chemical Engineering* **93**(11): 1902-1915.
- Munier, R. (1993). "Four-dimensional analysis of fracture arrays at the Äspöhard rock laboratory, SE Sweden." *Engineering Geology* **33**(3): 159-175.
- Newman, J. and Choo, B. S. (2003). *Advanced concrete technology 3: processes*, Elsevier.
- Noiriel, C., Gouze, P. and Madé, B. (2007). "Time-resolved 3D characterisation of flow and dissolution patterns in a single rough-walled fracture." *J. Krasny and JM Sharp Eds*: 629-642.
- Nokken, M. (2003). "Water transport in brick, stone and concrete." *Cement, Concrete and Aggregates* **25**(1): 1-1.
- Nokken, M. (2003). "Water Transport in Brick, Stone and Concrete."
- Noorishad, J., Ayatollahi, M. S. and Witherspoon, P. A. (1982). "A finite-element method for coupled stress and fluid flow analysis in fractured rock masses." *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts* **19**(4): 185-193.
- Odling, N. (1994). "Natural fracture profiles, fractal dimension and joint roughness coefficients." *Rock mechanics and rock engineering* **27**(3): 135-153.
- Odling, N. E. (2001). "The scaling of hydraulic conductivity in rock fracture zones." *Geophysical research letters* **28**(15): 3019-3022.

- Ogilvie, S., Isakov, E., Taylor, C. and Glover, P. (2003). "Characterization of rough-walled fractures in crystalline rocks." Geological Society, London, Special Publications **214**(1): 125-141.
- Ogilvie, S. R., Isakov, E. and Glover, P. W. J. (2006). "Fluid flow through rough fractures in rocks. II: A new matching model for rough rock fractures." Earth and Planetary Science Letters **241**(3-4): 454-465.
- Panthi, K. and Nilsen, B. (2005). Significance of grouting for controlling leakage in water tunnels—a case from Nepal. Proceedings of ITA-AITES 2005 world tunnelling congress and 31st ITA General Assembly, Istanbul, Turkey.
- Park, C.-K., Keum, D.-K. and Hahn, P.-S. (1995). "A stochastic analysis of contaminant transport through a rough-surfaced fracture." Korean Journal of Chemical Engineering **12**(4): 428.
- Petrov, A. (2000). "The development of the flow of viscous and viscoplastic media between two parallel plates." Journal of Applied Mathematics and Mechanics **64**(1): 123-132.
- Piggott, A. R. and Elsworth, D. (1993). "Laboratory assessment of the equivalent apertures of a rock fracture." Geophysical research letters **20**(13): 1387-1390.
- Ping, Y. and Xueqing, S. (2015). "Single Fracture Grouting Numerical Simulation Based on Fracture Roughness in Hydrodynamic Environment." Electronic Journal of Geotechnical Engineering **20**: 59-66.
- Power, W. L. and Tullis, T. E. (1991). "Euclidean and fractal models for the description of rock surface roughness." Journal of Geophysical Research: Solid Earth **96**(B1): 415-424.
- Rafi, J. Y. (2013). Design approaches for grouting of rock fractures; Theory and practice. 2021, KTH Royal Institute of Technology.
- Rafi, J. Y. (2014). Study of Pumping Pressure and Stop Criteria in Grouting of Rock Fractures. 1020, KTH Royal Institute of Technology.
- Rafi, J. Y. and Stille, H. (2014). "Control of rock jacking considering spread of grout and grouting pressure." Tunnelling and Underground Space Technology **40**(0): 1-15.
- Rafi, J. Y. and Stille, H. (2014). "Control of rock jacking considering spread of grout and grouting pressure." Tunnelling and Underground Space Technology **40**: 1-15.
- Rafi, J. Y. and Stille, H. (2015). "Basic mechanism of elastic jacking and impact of fracture aperture change on grout spread, transmissivity and penetrability." Tunnelling and Underground Space Technology **49**(0): 174-187.
- Rahman, M. (2015). "Rheology of cement grout : Ultrasound based in-line measurement technique and grouting design parameters."

- Rahmani, H. (2009). Estimation of grout distribution in a fractured rock by numerical modeling, University of British Columbia.
- Ren, K. (1982). "Diffusing and grouting method of chemical grout with flowing water." Water Resources and Hydropower Engineering **7**: 57-62.
- Renshaw, C. E. (1995). "On the relationship between mechanical and hydraulic apertures in rough-walled fractures." Journal of Geophysical Research: Solid Earth **100**(B12): 24629-24636.
- Rombough, V. B., Grant and Shuttle, D. (2006). "Penetrability Control of GIN Mixes during Fractured Rock Grouting." Proceeding of Sea to Sky Geotechnique, 59th Canadian Geotechnical Society Conference October 2006: 528-535.
- Rong, G., Hou, D., Yang, J., Cheng, L. and Zhou, C. (2017). "Experimental study of flow characteristics in non-mated rock fractures considering 3D definition of fracture surfaces." Engineering Geology **220**: 152-163.
- Rosquoët, F., Alexis, A., Khelidj, A. and Phelipot, A. (2003). "Experimental study of cement grout: Rheological behavior and sedimentation." Cement and Concrete Research **33**(5): 713-722.
- Runslätt, E. and Thörn, J. (2010). Fracture deformation when grouting in hard rock: In situ measurements in tunnels under Gothenburg and Hallandsås. Master Thesis, Chalmers University of Technology.
- Rutqvist, J. and Stephansson, O. (1996). A cyclic hydraulic jacking test to determine the in situ stress normal to a fracture. International journal of rock mechanics and mining sciences & geomechanics abstracts, Elsevier.
- Rutqvist, J. and Stephansson, O. (2003). "The role of hydromechanical coupling in fractured rock engineering." Hydrogeology Journal **11**(1): 7-40.
- Saeidi, O., Azadmehr, A. and Torabi, S. R. (2014). "Development of a rock groutability index based on the Rock Engineering Systems (res): a case study." Indian Geotechnical Journal **44**(1): 49-58.
- Saeidi, O., Stille, H. and Torabi, S. R. (2013). "Numerical and analytical analyses of the effects of different joint and grout properties on the rock mass groutability." Tunnelling and Underground Space Technology **38**(0): 11-25.
- Saeidi, O., Torabi, R., Ataei, M. and Stille, H. (2012). "Prediction of grout penetration length into the jointed rock mass using regression analyses." Scientific Research and Essays **7**(45): 3931-3943.
- Schwarz, L. G. (1998). "Roles of rheology and chemical filtration on injectability of microfine cement grouts."

- Sharifzadeh, M., Khani, J., JANATI, A. Z. and AHMADLUY, D. M. (2009). "Finite volume numerical analysis of joint geometry effect on grout velocity changes with variable viscosity under unsteady turbulent flow regime."
- Shi, G.-H. (1992). "Discontinuous deformation analysis: a new numerical model for the statics and dynamics of deformable block structures." Engineering computations **9**(2): 157-168.
- Shi, G. h. and Goodman, R. E. (1989). "Generalization of two-dimensional discontinuous deformation analysis for forward modelling." International Journal for Numerical and Analytical Methods in Geomechanics **13**(4): 359-380.
- Shuttle, D., Rombough, V. and Bonin, G. (2007). Impact of grout rheology on GIN. Grouting for Ground Improvement: Innovative Concepts and Applications: 1-10.
- Singh, K., Singh, D. and Ranjith, P. (2015). "Laboratory simulation of flow through single fractured granite." Rock Mechanics and Rock Engineering **48**(3): 987-1000.
- Smith, L. and Schwartz, F. W. (1984). "An analysis of the influence of fracture geometry on mass transport in fractured media." Water Resources Research **20**(9): 1241-1252.
- Sochi, T. (2010). "Pore-Scale modeling of non-Newtonian flow in porous media." arXiv preprint arXiv:1011.0760.
- Sochi, T. (2011). "The flow of power-law fluids in axisymmetric corrugated tubes." Journal of Petroleum Science and Engineering **78**(3): 582-585.
- Stille, B., Stille, H., Gustafson, G. and Kobayashi, S. (2009). "Experience with the real time grouting control method. Erfahrungen mit einer Echtzeit Kontrollmethode für das Injizieren." Geomechanics and Tunnelling **2**(5): 447-459.
- Stille, H., Gustafson, G. and Hassler, L. (2012). "Application of New Theories and Technology for Grouting of Dams and Foundations on Rock." Geotechnical and Geological Engineering **30**(3): 603-624.
- Sui, W., Liu, J., Hu, W., Qi, J. and Zhan, K. (2015). "Experimental investigation on sealing efficiency of chemical grouting in rock fracture with flowing water." Tunnelling and Underground Space Technology **50**: 239-249.
- Svenson, E., Schweisinger, T. and Murdoch, L. C. (2008). "Field evaluation of the hydromechanical behavior of flat-lying fractures during slug tests." Journal of Hydrology **359**(1): 30-45.
- Thompson, M. E. (1991). "Numerical simulation of solute transport in rough fractures." Journal of Geophysical Research: Solid Earth **96**(B3): 4157-4166.
- Tsang, Y. (1992). "Usage of "equivalent apertures" for rock fractures as derived from hydraulic and tracer tests." Water Resources Research **28**(5): 1451-1455.

- Tsang, Y. and Witherspoon, P. A. (1983). "The dependence of fracture mechanical and fluid flow properties on fracture roughness and sample size." Journal of Geophysical Research: Solid Earth **88**(B3): 2359-2366.
- Tsang, Y. W. and Tsang, C. (1987). "Channel model of flow through fractured media." Water Resources Research **23**(3): 467-479.
- Tse, R. and Cruden, D. (1979). Estimating joint roughness coefficients. Int J Rock Mech Min Sci & Geomech Abstr, Elsevier.
- Tse, R. and Cruden, D. M. (1979). "Estimating joint roughness coefficients." Int J Rock Mech Min Sci & Geomech Abstr **16**(5): 303-307.
- Vidstrand, P., Wallroth, T. and Ericsson, L. O. (2008). "Coupled HM effects in a crystalline rock mass due to glaciation: Indicative results from groundwater flow regimes and stresses from an FEM study." Bulletin of Engineering Geology and the Environment **67**(2): 187-197.
- Vik, E., Sverdrup, L., Kelley, A., Storhaug, R., Beitnes, A., Boge, K., Grepstad, G. and Tveiten, V. (2000). "Experiences from environmental risk management of chemical grouting agents used during construction of the Romeriksporten tunnel." Tunnelling and Underground Space Technology **15**(4): 369-378.
- Wang, M., Chen, Y.-F., Ma, G.-W., Zhou, J.-Q. and Zhou, C.-B. (2016). "Influence of surface roughness on nonlinear flow behaviors in 3D self-affine rough fractures: Lattice Boltzmann simulations." Advances in Water Resources **96**: 373-388.
- Wang, Y., Ji, H. and Yang, Y. (2007). "Practices on dynamic water grouting and sealing in coal mine [J]." Coal Science and Technology **2**: 008.
- Warner, J. (2004). Practical handbook of grouting: soil, rock, and structures, John Wiley & Sons.
- Watanabe, K. and Takahashi, H. (1993). Fractal characterization of subsurface fracture network for geothermal energy extraction system. Proceedings, eighteenth workshop on geothermal reservoir engineering, Stanford University, Stanford, CA. Report No. SGP-TR-145-17.
- Widmann, R. (1996). International society for rock mechanics commission on rock grouting. International journal of rock mechanics and mining sciences & geomechanics abstracts, Elsevier.
- Witherspoon, P. A., Wang, J. S., Iwai, K. and Gale, J. E. (1980). "Validity of cubic law for fluid flow in a deformable rock fracture." Water resources research **16**(6): 1016-1024.
- Xiang, X., Qian, J., Yang, W., Fang, M. and Qian, X. (2006). "Synthesis and properties of nanosilica-reinforced polyurethane for grouting." Journal of applied polymer science **100**(6): 4333-4337.

- Xiao, F. and Zhao, Z. (2017). Grout Flow in Fracture Channel Considering Fracture Deformation. 51st US Rock Mechanics/Geomechanics Symposium, American Rock Mechanics Association.
- Xiao, F., Zhao, Z. and Chen, H. (2017). "A simplified model for predicting grout flow in fracture channels." Tunnelling and Underground Space Technology **70**(Supplement C): 11-18.
- Xiao, W., Xia, C., Wei, W. and Bian, Y. (2013). "Combined effect of tortuosity and surface roughness on estimation of flow rate through a single rough joint." Journal of Geophysics and Engineering **10**(4): 045015.
- Yahia, A. and Khayat, K. (2001). "Analytical models for estimating yield stress of high-performance pseudoplastic grout." Cement and Concrete Research **31**(5): 731-738.
- Yahia, A. and Khayat, K. (2003). "Applicability of rheological models to high-performance grouts containing supplementary cementitious materials and viscosity enhancing admixture." Materials and Structures **36**(6): 402-412.
- Yang, M. J., Yue, Z. Q., Lee, P. K., Su, B. and Tham, L. G. (2002). "Prediction of grout penetration in fractured rocks by numerical simulation." Canadian Geotechnical Journal **39**(6): 1384-1394.
- Yeung, M., Jiang, Q. and Sun, N. (2007). "A model of edge-to-edge contact for three-dimensional discontinuous deformation analysis." Computers and Geotechnics **34**(3): 175-186.
- Zhan, K., Sui, W. and Gao, Y. (2011). "A model for grouting into single fracture with flowing water." Rock Soil Mech **32**(6): 1659-1663.
- Zhang, G., Zhan, K., Gao, Y. and Wang, W. (2011). "Comparative experimental investigation of chemical grouting into a fracture with flowing and static water." Mining Science and Technology (China) **21**(2): 201-205.
- Zhang, Q.-s., Han, W., Li, S., Yuan, Y., LIU, R., LI, J. and Sun, H. (2012). "Comprehensive grouting treatment for water gushing analysis in limestone breccias fracture zone." Chinese Journal of Rock Mechanics and Engineering **31**(12): 2412-2419.
- Zhang, Q., Zhang, L., Liu, R., Han, W., Zhu, M., Li, X., Zheng, D. and Xu, X. (2015). "Laboratory experimental study of cement-silicate slurry diffusion law of crack grouting with dynamic water." Rock and Soil Mechanics **36**(8): 2159-2168.
- Zhang, X., Li, S.-c., Zhang, Q.-s., Li, H.-y., Wu, W.-d., Liu, R.-t. and Lin, M.-y. (2010). "Filed test of comprehensive treatment method for high pressure dynamic grouting." Journal of China Coal Society **8**: 019.

- Zhao, Z., Li, B. and Jiang, Y. (2014). "Effects of fracture surface roughness on macroscopic fluid flow and solute transport in fracture networks." Rock mechanics and rock engineering **47**(6): 2279.
- Zhu, H., Xie, H., Yi, C., Jiang, Y., LIU, J., LAI, S. and DONG, X. (2013). "Analysis of properties of fluid flow in rock fractures." Chin J Rock Mech Eng **32**: 657-663.
- Zimmerman, R. W. and Bodvarsson, G. S. (1996). "Hydraulic conductivity of rock fractures." Transport in porous media **23**(1): 1-30.
- Guidance Note on Pre-excavation Grouting for Underground Construction in Hard Rock-Principles & Design Elements. AGS (HK).
- Weaver, Kenneth D. Dam foundation grouting. New York: American Society of Civil Engineers, 1991.
- Garshol, K. F., et al. Deep subsea rock tunnels in Hong Kong. World Tunnel Congress. 2013.
- Eurocode 7: Geotechnical design - Part 1 &2, 2004
- Tattersall, J. W., et al. "Engineering Geological Approach for Assessment of Quantities and Programme for Deep Tunnels in Hong Kong." The HKIE Geotechnical Division 32 Annual Seminar–Geotechnical Aspects of Tunnelling for Infrastructure. 2012.
- Engineering Geological Practice in Hong Kong, GEO Publication No. 1/2007.
- Pre-excavation Grouting of Underground Construction in Hard Rock-Site Practices, Association of Geotechnical & Geo-environmental Specialist (Hong Kong)
- Subsea Tunnels, Publication No. 8 &18, Norwegian Tunnelling Society 2009
- Garshol, K. F., et al. Grouting Techniques for Deep Subsea Sewage Tunnels in Hong Kong. World Tunnel Congress. 2012.
- Norwegian Tunnelling Technology, Publication No. 23, Norwegian Tunnelling Society 2014.
- Rock Mass Grouting in Norwegian Tunnelling, Publication No. 20, Norwegian Tunnelling Society 2011.
- Code for construction on tunnel of railway, China 2002
- Code for Design of Road Tunnel, China 2004
- Code for Design on Tunnel of Railway, China 2005
- Technical code for waterproofing of underground works, China 2008
- Technical specifications for construction of highway tunnel, China 2009
- Code for acceptance of construction quality of underground waterproof, China 2011

APPENDICES

Appendix A

Table A1 Parameters regarding grouting, properties of rock fractures, and fracture deformation

Q	H_2	R_H	R_E	R_K	R_L	ST	SL	P_0	L_1	L_2	t_g	$\frac{w_{max}}{H_2} \%$	$\frac{r(w_{max})}{SL} \%$
0.8	200	4	0.7	0.6	0.3	1.0	15.206	21050.075	4.562	1.287	200.00	0.344	5.156
1	450	2	0.9	0.6	1.0	1.6	7.200	22384.454	7.200	3.772	500.00	0.086	30.242
1	400	3.5	0.6	1.0	0.6	1.0	9.054	8453.079	5.433	1.679	400.00	0.041	13.240
0.4	400	2.5	0.65	1.2	0.5	0.2	7.960	8555.858	3.980	1.688	400.00	0.029	18.257
1.4	550	3	0.9	1.0	0.3	0.2	21.910	8156.143	6.573	2.331	500.00	0.027	9.789
1	250	3	0.7	0.2	0.4	1.2	15.939	32506.830	6.376	2.321	300.00	1.010	5.099
1.2	450	4	0.6	1.2	0.7	1.2	8.420	6215.591	5.894	1.588	500.00	0.024	14.255
1.6	350	1.5	0.85	1.8	0.3	1.2	33.057	108257.649	9.917	6.820	400.00	0.105	17.063
1.2	200	2	1	1.8	0.6	1.4	12.450	177241.201	7.470	3.967	200.00	0.516	23.154
1.8	450	5	0.7	1.0	0.5	1.4	12.969	5278.828	6.484	1.409	500.00	0.026	8.177
1.6	500	2	0.75	0.2	0.5	1.0	17.280	25584.530	8.640	4.525	500.00	0.262	17.120
1.2	600	3	0.65	1.4	0.9	1.0	7.094	6544.251	6.384	2.249	600.00	0.013	24.267
1.8	500	4	0.9	0.8	0.2	0.6	34.241	6658.944	6.848	1.845	500.00	0.035	4.070
0.2	200	1.5	0.6	0.2	0.2	0.2	16.373	59864.249	3.275	2.267	200.00	1.081	10.137
1.8	300	4.5	0.65	0.2	0.3	1.6	21.570	13263.108	6.471	1.604	300.00	0.385	4.111
0.2	300	3	0.75	1.2	1.0	1.4	2.605	5946.177	2.605	0.936	300.00	0.021	32.647
1.4	500	4.5	0.6	1.4	0.8	1.4	7.137	4441.740	5.709	1.366	500.00	0.014	16.293
0.6	300	2.5	0.6	0.6	0.4	0.6	12.175	20549.444	4.870	2.092	300.00	0.167	12.180
1.6	600	3.5	0.9	1.2	0.4	0.4	17.187	5915.131	6.875	2.088	600.00	0.016	10.131
1.2	550	4.5	0.8	0.6	0.5	0.4	8.581	2938.992	4.290	1.027	362.32	0.022	7.270
1.2	250	5	0.85	0.8	1.0	0.2	5.500	11502.458	5.500	1.238	300.00	0.149	20.362
0.2	400	5	0.95	0.6	0.8	1.2	1.352	850.325	1.082	0.239	111.18	0.003	25.380
0.8	350	3	0.6	0.8	0.5	0.8	11.136	13237.388	5.568	2.000	400.00	0.081	13.066
1.2	500	1.5	0.95	1.0	0.4	1.6	20.098	35535.478	8.039	5.513	500.00	0.044	18.102
0.4	200	3.5	0.75	0.8	0.8	1.6	4.291	16015.647	3.433	1.095	200.00	0.138	21.458
0.2	250	4.5	0.9	1.6	0.6	0.8	3.939	3841.535	2.364	0.578	300.00	0.019	11.561
0.8	500	3.5	0.65	1.6	1.0	1.2	4.861	4949.873	4.861	1.481	500.00	0.012	24.389
1.8	600	5.5	0.6	1.8	1.0	1.8	3.633	1850.728	3.633	0.721	229.63	0.003	21.540
0.2	600	4	0.85	0.4	0.5	1.6	1.483	474.676	0.742	0.203	63.17	0.001	31.266
0.6	200	5.5	0.9	1.4	0.5	1.2	6.778	7882.457	3.389	0.707	200.00	0.056	9.334
1.4	250	3.5	0.95	1.8	0.5	0.6	14.075	30033.308	7.037	2.221	300.00	0.128	13.154
0.6	400	3	0.8	1.8	0.2	1.6	22.560	7980.528	4.512	1.611	400.00	0.017	6.104
1.2	350	2.5	0.9	0.2	0.8	1.8	9.206	27356.473	7.365	3.149	400.00	0.479	13.236

1.8	350	3.5	1	0.6	0.9	0.2	8.656	17849.794	7.790	2.427	400.00	0.168	25.216
1	200	4.5	0.85	1.2	0.9	0.6	5.358	19161.678	4.822	1.220	200.00	0.184	16.390
0.6	450	1.5	0.65	0.8	0.6	0.4	9.987	25776.160	5.992	4.109	500.00	0.048	35.162
1.4	600	1.5	0.75	0.6	0.7	0.8	12.411	27547.057	8.688	5.945	600.00	0.048	38.125
1	550	4	0.65	1.8	0.8	0.8	5.632	3480.488	4.505	1.205	428.34	0.008	17.367
1.8	400	2	0.85	1.4	0.4	0.8	22.897	45889.190	9.159	4.819	400.00	0.086	18.089
1.6	400	4.5	0.7	0.8	0.7	1.8	8.718	7065.751	6.102	1.484	400.00	0.042	13.249
0.8	400	1.5	0.9	0.4	0.9	1.4	7.289	41043.151	6.560	4.509	400.00	0.160	33.229
0.6	250	4	0.75	1.0	0.9	1.8	4.810	10991.272	4.329	1.200	300.00	0.072	22.403
0.4	300	5	0.9	1.8	0.7	1.0	4.142	3491.966	2.900	0.639	300.00	0.013	13.522
0.8	550	2	0.95	1.2	0.2	1.8	29.139	12034.553	5.828	3.040	500.00	0.019	7.080
1.4	350	5.5	0.7	1.2	0.6	1.6	9.226	5611.018	5.536	1.110	400.00	0.029	11.241
0.4	550	1.5	0.7	1.4	1.0	0.6	4.429	12001.937	4.429	3.028	500.00	0.011	52.269
0.8	600	5	0.8	0.2	0.6	0.6	3.227	1049.179	1.936	0.424	133.68	0.013	5.730
1.6	550	5	0.6	1.6	0.9	1.6	5.169	2879.873	4.652	1.008	353.83	0.007	18.395
1.6	250	5.5	0.65	0.6	0.2	1.4	30.315	11637.511	6.063	1.248	300.00	0.171	3.080
0.2	500	2.5	0.7	1.8	0.9	0.4	2.937	3496.306	2.644	1.108	440.43	0.006	28.608
0.6	350	4.5	0.95	0.4	1.0	1.0	3.995	4653.032	3.995	0.966	400.00	0.064	11.553
0.2	350	2	0.65	1.0	0.7	0.6	4.665	10525.064	3.265	1.712	400.00	0.036	23.410
0.4	600	4.5	1	1.0	0.2	1.2	5.922	693.488	1.184	0.288	90.32	0.001	6.395
1.6	200	2.5	0.8	1.0	1.0	0.8	7.948	133068.094	7.948	3.442	200.00	0.916	35.204
0.4	350	4	0.8	1.6	0.4	1.4	8.628	4441.529	3.451	0.933	400.00	0.014	10.260
1.8	550	2.5	0.75	0.4	0.6	1.2	13.417	14234.884	8.050	3.407	500.00	0.088	16.156
0.6	550	3.5	0.85	0.2	0.7	1.4	4.990	3044.344	3.493	1.063	378.65	0.031	11.444
0.8	250	2.5	1	1.4	0.7	1.6	8.798	41276.738	6.159	2.655	300.00	0.158	20.227
1	300	1.5	1	1.6	0.5	1.8	14.661	99741.706	7.330	5.051	300.00	0.129	22.133
0.6	500	5	1	1.2	0.3	0.8	6.976	1366.766	2.093	0.458	173.56	0.004	5.339
1.2	400	5.5	0.75	1.6	0.3	0.6	15.982	3857.540	4.794	0.953	400.00	0.016	5.148
1	350	5	0.75	1.4	0.2	0.4	24.507	5374.021	4.901	1.075	400.00	0.028	4.098
1	600	2.5	0.95	0.8	0.3	1.4	20.989	8183.673	6.297	2.644	600.00	0.023	7.111
0.8	300	5.5	0.85	1.0	0.8	0.4	4.893	4675.348	3.914	0.792	300.00	0.043	10.458
1.6	450	3	1	0.4	0.8	0.6	9.708	13836.583	7.766	2.772	500.00	0.127	22.200
1.4	300	2	0.8	0.8	0.9	1.2	8.970	72341.610	8.073	4.267	300.00	0.313	35.181
1.6	300	4	0.95	1.4	0.6	0.2	10.756	15790.777	6.453	1.788	300.00	0.084	15.197
1	500	5.5	0.8	0.4	0.7	0.2	4.142	1751.720	2.899	0.577	219.37	0.026	7.558
1.4	450	2.5	0.85	1.6	0.2	1.0	39.237	18365.684	7.847	3.332	500.00	0.036	7.059
1.8	200	3	0.95	1.6	0.7	0.4	11.152	92375.895	7.806	2.863	200.00	0.488	22.174
0.4	450	5.5	0.95	0.2	0.9	0.8	1.578	916.756	1.420	0.286	118.39	0.011	17.310
0.6	600	2	0.7	1.6	0.8	0.2	6.621	8661.404	5.296	2.750	600.00	0.011	37.237
1.2	300	3.5	0.7	0.4	0.2	0.8	29.743	16971.681	5.949	1.868	300.00	0.266	3.081
1.8	250	1.5	0.8	1.2	0.8	1.0	13.468	293763.316	10.775	7.420	300.00	0.592	46.100

0.8	450	4.5	0.75	1.8	0.4	0.2	10.681	3576.424	4.272	1.023	440.98	0.012	9.212
1.4	200	5	0.65	0.4	0.4	1.8	13.556	19797.699	5.422	1.245	200.00	0.419	7.171
0.4	250	2	0.6	0.4	0.3	0.4	15.753	37344.965	4.726	2.503	300.00	0.427	12.139
0.4	500	3	0.85	0.6	0.6	1.8	6.144	4163.750	3.686	1.298	500.00	0.015	16.340
0.2	450	3.5	0.8	1.4	0.3	1.8	5.752	1769.846	1.726	0.528	226.81	0.002	11.385
1.4	400	4	1	0.2	1.0	0.4	6.038	8193.395	6.038	1.644	400.00	0.217	17.342

Table A2 Orthogonal design table on key parameters for generating fracture model

Case number	Transition length	Mismatch wave length	Standard deviation	Fractal dimension	Anisotropy factor
1	20	7	0.89	2.6	0.96
2	30	7	0.96	2.65	0.92
3	80	8	0.75	2.75	0.96
4	50	4	0.89	2.85	0.98
5	70	5	0.47	2.8	0.96
6	50	7	0.47	2.75	1.02
7	50	3	0.61	2.95	0.94
8	50	2	0.75	2.65	1.06
9	30	6	0.68	2.75	1.06
10	40	7	0.4	2.85	1
11	50	9	0.4	2.9	0.96
12	80	4	0.47	3	1.04
13	70	3	0.96	2.6	0.98
14	80	6	0.4	2.7	0.98
15	80	9	0.61	2.6	1.02
16	40	5	0.89	2.65	1.08
17	70	6	0.54	2.65	1.02
18	30	5	0.61	2.9	1
19	80	5	0.54	2.85	0.92
20	40	2	0.68	2.6	1.04
21	20	2	0.54	2.95	1
22	50	1	0.68	2.8	1
23	60	5	0.82	2.6	1.06
24	20	3	0.4	2.8	1.06
25	10	7	0.82	2.7	0.94
26	30	2	0.4	3	1.02
27	50	5	0.96	2.7	1.04
28	40	4	0.82	2.8	1.02
29	20	6	0.61	2.85	1.04
30	10	2	0.47	2.9	0.98
31	50	8	0.54	2.6	1.08
32	90	8	0.61	2.8	0.92
33	20	9	0.82	2.75	1.08
34	60	7	0.54	2.8	0.98
35	70	4	0.4	2.95	1.08
36	60	9	0.47	2.95	0.92

37	20	1	0.47	2.65	0.94
38	10	4	0.61	2.65	0.96
39	70	9	0.75	2.7	1
40	90	4	0.54	2.9	1.06
41	10	3	0.54	2.75	1.04
42	70	8	0.68	2.85	0.94
43	10	1	0.4	2.6	0.92
44	20	4	0.68	2.7	0.92
45	90	6	0.47	2.6	1
46	90	5	0.4	2.75	0.94
47	30	4	0.75	2.6	0.94
48	30	1	0.54	2.7	0.96
49	90	9	0.68	2.65	0.98
50	60	2	0.61	2.7	1.08
51	40	8	0.47	2.7	1.06
52	90	3	0.89	2.7	1.02
53	40	9	0.54	3	0.94
54	80	3	0.82	2.65	1
55	30	3	0.47	2.85	1.08
56	70	1	0.4	2.9	0.96
57	20	6	0.96	2.7	0.98
58	10	7	0.47	2.6	0.94

Table A3 Characteristics of fracture models generated via data from Table A2

Case number	Mean aperture h_m	Roughness factor $Z2_m$	Roughness R_m	Tortuosity T_m
1	2.03	1.419	1.656	1.656
2	2.68	1.798	1.930	1.930
3	3.33	1.907	2.013	2.013
4	3.41	2.995	2.872	2.872
5	2.18	1.382	1.628	1.628
6	2.13	1.195	1.502	1.502
7	2.14	2.613	2.561	2.561
8	1.13	1.403	1.644	1.644
9	2.53	1.730	1.879	1.879
10	1.88	1.346	1.602	1.602
11	2.29	1.528	1.730	1.730
12	2.24	2.239	2.265	2.265
13	1.92	1.529	1.732	1.732
14	1.6	0.876	1.303	1.303
15	2.71	0.973	1.360	1.360
16	2.19	1.669	1.834	1.834
17	2.13	1.011	1.384	1.384
18	2.41	2.327	2.335	2.335
19	2.79	1.818	1.942	1.942
20	0.91	1.081	1.428	1.428
21	1.58	2.312	2.323	2.323
22	1.04	1.997	2.082	2.082
23	2.36	1.311	1.580	1.580
24	1.01	1.175	1.488	1.488
25	2.24	1.794	1.927	1.927
26	1.28	1.905	2.008	2.008
27	2.87	2.101	2.161	2.161
28	2.46	2.411	2.400	2.400
29	2.19	2.052	2.121	2.121
30	1.25	1.793	1.925	1.925
31	2.49	0.863	1.296	1.296
32	3	1.792	1.925	1.925
33	2.72	2.087	2.150	2.150
34	2.6	1.586	1.773	1.773
35	1.71	1.715	1.865	1.865
36	2.64	2.015	2.092	2.092
37	0.48	0.880	1.305	1.305
38	1.21	1.143	1.467	1.467
39	3.41	1.641	1.813	1.813
40	2.72	2.060	2.127	2.127
41	1.15	1.374	1.623	1.623

42	3.66	2.286	2.303	2.303
43	0.36	0.638	1.176	1.176
44	1.61	1.488	1.703	1.703
45	1.96	0.751	1.233	1.233
46	1.66	1.019	1.387	1.387
47	1.31	1.196	1.501	1.501
48	0.63	1.180	1.493	1.493
49	3.05	1.271	1.553	1.552
50	1.07	1.333	1.596	1.596
51	2.06	1.028	1.394	1.394
52	2.28	1.950	2.043	2.043
53	3.18	2.574	2.529	2.529
54	1.78	1.534	1.736	1.736
55	1.29	1.582	1.769	1.769
56	0.86	1.525	1.730	1.730
57	2.56	2.101	2.162	2.162
58	1.03	0.750	1.232	1.232

Table A4 Grout injection pressure and the corresponding equivalent aperture

Case number	$Q_m = 0.2$		$Q_m = 0.3$		$Q_m = 0.4$		$Q_m = 0.5$	
	P_0	h_e	P_0	h_e	P_0	h_e	P_0	h_e
1	28963	1.034	64879	0.904	116440	0.819	184180	0.757
2	13254	1.342	29518	1.176	52817	1.066	83341	0.986
3	5795	1.768	12704	1.558	22588	1.415	35534	1.311
4	6644	1.689	14444	1.492	25250	1.363	39074	1.270
5	14857	1.291	32139	1.143	56874	1.040	89354	0.964
6	15616	1.270	33679	1.125	59462	1.025	93246	0.950
7	23661	1.106	51201	0.979	89268	0.895	137910	0.834
8	212370	0.532	480353	0.464	864672	0.420	1369121	0.388
9	13443	1.335	29998	1.170	53763	1.060	84993	0.980
10	24545	1.092	54155	0.961	96686	0.872	152699	0.806
11	13776	1.324	30058	1.169	53314	1.063	83816	0.985
12	19707	1.175	44015	1.029	78897	0.933	124666	0.863
13	37317	0.950	84024	0.830	151083	0.751	239187	0.694
14	38937	0.937	84289	0.829	148954	0.755	233838	0.699
15	6807	1.675	14513	1.490	25448	1.360	39750	1.263
16	25777	1.075	58033	0.939	104315	0.850	165102	0.785
17	17836	1.215	38744	1.074	68507	0.978	107484	0.906
18	16785	1.240	37374	1.087	66898	0.985	105585	0.912
19	8561	1.552	18739	1.368	33281	1.244	52344	1.152
20	301014	0.474	671593	0.415	1200117	0.376	1892054	0.348
21	84300	0.724	188300	0.634	335380	0.576	526400	0.534
22	154737	0.591	343362	0.519	608515	0.472	951223	0.438
23	16433	1.249	36538	1.095	65364	0.993	103222	0.919
24	179132	0.563	399964	0.493	716630	0.447	1132287	0.413
25	22405	1.126	50264	0.985	90230	0.892	142700	0.825
26	57338	0.823	129477	0.718	232720	0.650	367815	0.601
27	12203	1.379	27317	1.207	48910	1.094	77148	1.012
28	21145	1.148	47542	1.003	85275	0.909	134639	0.841
29	11538	1.405	25796	1.230	46264	1.114	73111	1.030
30	137010	0.616	309806	0.537	557552	0.486	882623	0.449
31	4153	1.975	8765	1.763	15279	1.612	23770	1.499
32	6871	1.670	15077	1.471	26834	1.336	42264	1.237
33	6703	1.684	15007	1.473	26906	1.335	42493	1.235
34	5305	1.820	11632	1.604	20691	1.457	32583	1.349
35	19630	1.177	44115	1.029	79449	0.930	126023	0.859
36	4997	1.857	10966	1.636	19522	1.486	30754	1.375
37	834949	0.337	1855639	0.296	3307897	0.268	5206459	0.249
38	54772	0.836	122504	0.732	220125	0.662	348565	0.612
39	2319	2.399	5060	2.117	8989	1.924	14150	1.781

40	10113	1.468	22345	1.290	39854	1.171	62820	1.084
41	157749	0.588	354237	0.514	635138	0.465	1002985	0.430
42	4227	1.964	9283	1.729	16504	1.571	25960	1.455
43	2502868	0.234	5442952	0.207	9604089	0.188	15015033	0.175
44	55325	0.833	124456	0.728	223906	0.659	354696	0.609
45	19618	1.177	41671	1.048	72856	0.958	113666	0.890
46	36856	0.954	80339	0.842	142417	0.766	223883	0.710
47	94483	0.697	211312	0.610	378730	0.553	598547	0.511
48	1026360	0.315	2275008	0.276	4038238	0.251	6328947	0.233
49	5652	1.782	12249	1.577	21665	1.435	34004	1.330
50	208066	0.536	466318	0.469	834726	0.425	1316857	0.393
51	16597	1.245	35942	1.101	63522	1.003	99695	0.929
52	26067	1.071	58711	0.935	105489	0.847	166832	0.783
53	6277	1.721	13714	1.518	24321	1.381	38191	1.280
54	48040	0.873	108103	0.763	194155	0.691	307113	0.639
55	111194	0.660	250221	0.577	449330	0.522	710375	0.483
56	417068	0.425	931502	0.372	1661962	0.338	2614934	0.313
57	18196	1.207	40903	1.055	73399	0.955	115978	0.884
58	114158	0.654	248014	0.578	440022	0.526	692705	0.487

Notes: The unit of Q_m , P_0 , and h_e are kg/s, Pa, and mm, respectively.

Appendix B

Figure B1 Intercorrelation among 14 variables listed in Table A1

Figure B2 Strength of intercorrelation among 14 variables listed in Table A1

Figure B3 Real-time pressure variation at different nodes: horizontal axis is time step; vertical axis is real time pressure (Pa)

Figure B4 Comparison of grout flow within two rough surfaces based on Bingham and Newtonian model

Considering numerical simulation for grouting of rock fractures, the most critical parameters of a specific cement grout are its dynamic viscosity μ and yield stress τ_0 . For cement grout taken as Bingham flow, it is known that the theoretical model is a stepwise function, thus it is hard to reach convergent solution. Therefore, Bingham-Papanastasiou model is adopted (Mitsoulis, 2007) in Chapter 6. The relevant apparent viscosity η is expressed as $\eta = \mu + (\tau_0/|\dot{\gamma}|) \left[1 - e^{(-m|\dot{\gamma}|)} \right]$, where $\dot{\gamma}$ is shear rate, and m is a constant number controlling fitting accuracy, and it is normally taken as 3000. The corresponding dynamic viscosity μ and yield stress τ_0 are taken as 0.01 Pa·s and 0.1 Pa. The injection flow rate is 0.1 kg/s. The simulation results based on two fluid models are shown in Figure B4. The relevant velocity distributions look the same. The injection pressure corresponding to Bingham model and Newtonian model are 214.52 Pa and 194.62Pa. The relevant equivalent aperture calculated based on Eq. (6.15) is 1.55mm, which is close to that presented in Figure 6.23, for Case 23 which is 1.50mm.

However, consideration of yield stress in Bingham model will increase the injection pressure, but the influence on the equivalent aperture calculated is negligible using Newtonian model for grouting simulation, if the relevant yield stress is low, which corresponds to relatively large W/C ratio.