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COGNITIVE DYNAMIC AIRSPACE MANAGEMENT

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SCHOOL OF COMPUTER SCIENCE AND ENGINEERING
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COGNITIVE DYNAMIC AIRSPACE MANAGEMENT

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A thesis submitted to Nanyang Technological University
in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Authorship Attribution Statement

This thesis contains material from 4 papers published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

Chapter 3 is published as C.S.Y. Wong and S. Suresh. Preference-based 3-dimensional en-route airspace sectorization. Genetic and Evolutionary Computation Conference (GECCO 2018), Kyoto, Japan, July 2018.

The contribution of the co-authors are as follows:

- A/Prof Suresh provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts.
- I designed the study and performed all the experimental work at the School of Computer Science and Engineering.
- The relevant data was cleaned prepared by me.
- The results were analysed by me with inputs from A/Prof Suresh.

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The contributions of the co-authors are as follows:

- A/Prof Suresh provided the initial project direction.
- I prepared the manuscript drafts. The manuscript is revised together with A/Prof Suresh and Dr. Sundararajan.
- The final proposed architecture was developed through discussions between A/Prof Suresh, Dr. Sundararajan and me.
- I conducted all the experimental work at the School of Computer Science and Engineering.

- The results were analysed by me with inputs from A/Prof Suresh and Dr. Sundararajan.

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- A/Prof Suresh provided the initial project direction.
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- The methodology was derived by Mr Venugopalan and A/Prof Suresh.
- I performed the experiments and analysed the results on the 2nd experimental study done on the Singapore Flight Information Region. Mr Venugopalan performed and analysed the results on the 1st experimental study.

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- A/Prof Suresh and Dr. Al-Dujaili provided the initial project direction.
- I prepared the manuscript drafts. The manuscript is revised together with A/Prof Suresh, Dr. Al-Dujaili and Dr. Sundararajan.
- I co-designed the algorithm with Dr. Al-Dujaili, with inputs from A/Prof Suresh.
- I conducted all the experimental work at the School of Computer Science and Engineering.
- The results were analysed by me with inputs from A/Prof Suresh and Dr. Al-Dujaili.

02/03/2019

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Date

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Cheryl Wong Sze Yin

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*Trust in the Lord with all your heart and lean not on your own understanding;
in all your ways submit to him, and he will make your paths straight.*

Proverbs 3:5-6

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Abstract

This thesis focuses on optimisation methods in airspace management. A given airspace is typically divided into sectors where a pair of air traffic controllers are assigned to ensure safety in each sector. The controllers' workload can be generally divided into interactions with (a) pilots in the given sector and (b) other controllers when the aircraft exits their sector and enters into another sector. These interactions are referred to the monitoring and coordination workloads respectively. This research work specifically studies the optimisation methods applied to the optimal design of sector shapes in four aspects.

- Static state: The formulation of finding sector shapes have been mostly proposed in a single objective optimisation framework. However, given the conflicting nature of the considerations, the problem should be formulated as a *multi-objective* problem incorporating the *preferences* of the user.
- Dynamic state: Re-sectorization strategies for reconfiguration or change in sector shapes have been suggested to handle the changing air traffic flow. However, the dynamic airspace sectorization problem (DAS) has not been explored as a *system* that has varying traffic flows over time.
- Weather: Weather plays a significant role in the availability of airspace. However, most work that addresses changing weather conditions focuses on the re-routing of affected airways, which could, in turn, cause an imbalance of controllers workload. The *combination of re-routing and resectorization* should be considered *simultaneously* under the effect of limited available airspace.
- Multi-objective solver: Most multi-objective solvers stem from evolutionary algorithms (EAs) due to its population-based nature. Multi-objective EAs are funda-

mentally stochastic in nature, hence solutions are likely to be irreproducible. For *reproducible and reliable* solutions, *deterministic* algorithms are preferable.

In this thesis, a *preference-based bi-objective* optimisation model that optimises sector shapes for a given set of traffic flows and airspace is first presented. The two objectives are i) minimizing the standard deviation of the monitoring workload (within the sector) among pairs of controllers and ii) minimizing the total coordination workload between sectors. The proposed model aims to obtain traffic flow conforming sectors while equally distributing the monitoring workload among controllers as much as possible. Furthermore, preference-based methods were used to help the solver focus on the particular region of interest on the approximate Pareto front. The proposed preference-based strategy was found to obtain a wider range of feasible solutions when compared to a constraint-based strategy.

Given dynamically changing traffic flows, a single set of sector shapes could not remain optimal over time. Hence, the airspace management problem should be deemed as a system over time, re-sectorizing sectors when needed. The cognitive decision making architecture for dynamic airspace sectorization (CDAS) is first proposed to answer the questions on *when-to-do* and *how-to-do* a resectorization. With a multi-objective framework, CDAS provides the decision maker with the predicted performance objectives (based on flight plans) of available optimal sector shapes (for selection) for the next time period. This could allow the decision maker to avoid a need for resectorization in the next time period. However, there are still some uncertainties present in the feasibility of the sector shapes in future time intervals. Focusing on the benefits of planning, the airspace management problem is fitted into a rolling horizon optimisation framework with a single objective optimisation model. This approach optimizes the sector shapes with the consideration of traffic flows in the next few time intervals. In comparison with the single time interval optimisation, the proposed method is able to provide better feasible solutions over a time horizon.

On top of varying traffic flows, dynamic weather conditions can affect the availability of airspace. The Simultaneous Optimisation of the AirWay and AirSpace (SAWAS) is proposed to address this changing availability of airspace. The model seeks to balance

the ATC monitoring workload, minimize the total coordination workload and maximize the similarity of sector shapes with the initial sectorization. An experimental study was performed to compare this methodology with a sequential design approach. It was found that the concurrent consideration of sector re-design and flow re-routing on the design objectives yields better and more optimal solutions.

The reproducibility and reliability of solutions in real-world problems are not guaranteed with stochastic solvers. Therefore, a deterministic indicator-based multi-objective Multi-scale Search Optimisation (MSO) algorithm, Pareto-Aware Dividing Rectangles (PA-DIRECT) is proposed to tackle this issue. PA-DIRECT is benchmarked against non-dominance-based multi-objective MSO algorithm, MO-DIRECT and popular evolutionary algorithms on a bi-objective test set on the Comparing Continuous Optimisers (COCO) platform. The study results affirm the performance of PA-DIRECT in providing a high-quality approximate set, particularly for multi-modal problems. Further, PA-DIRECT is used to solve the aforementioned preference-based bi-objective optimisation model with general constraint handling techniques.

In summary, the thesis first introduces a preference-based approach for the designing of optimal sector shapes. Next, two different frameworks that consider future traffic flows are proposed for DAS. Following that, the changing availability of airspace is tackled by the simultaneous optimisation of re-routing and resectorization. Last but not least, a deterministic multi-objective solver is developed to find optimal airspace sector shapes.

Future works include: i). a reference-point-based MSO algorithm for optimizing sector shapes; ii). interactive multi-objective optimisation approaches for optimizing sector shapes; iii). extending the rolling horizon optimisation framework with a multi-objective optimisation model; iv). solving the rolling horizon optimisation framework with a reinforcement learning approach; v). implementing SAWAS as a module in the CDAS framework; and vi). simulations for a better measurement of controllers' workload.

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Chapter 1

Introduction

1.1 Motivation

The growth of the global economy has not only increased the popularity of air travel, but also reduced the cost of air travel. As such, the amount of air travel has increased significantly over the past few years. Both International Air Transport Association (IATA) and Federal Aviation Administration (FAA) have projected air traffic to increase in the next 20 years [56, 41]. Thus, there is a concern to optimise the use of airspace for more air vehicles.

One of the fundamental issues in airspace management is the optimal design of sector shapes. Studies on designing optimal sectors such that the workloads among sectors are balanced in the airspace (e.g. [38, 118, 106, 123, 67, 101]) are generally referred to the airspace sectorization problem. Over the years, the problem has been modelled as a graph partitioning problem consisting of air traffic flows [38, 17, 109, 80, 77, 24, 98], a combinatorial problem of elementary building blocks [21, 123, 26, 39, 76, 18, 58, 74, 67] and a continuous black-box problem with the use of computational geometry [106, 119, 107]. The use of Voronoi Diagram as a computational geometry tool in the airspace sectorization problem posed as a continuous black-box problem is shown to be effective in previous studies [106, 107]. The variation of objectives and constraints used in the formulation of the airspace sectorization problem have been built into mostly single

objective [38, 37, 98, 119, 24, 101] and multi-objective [106] optimisation models. For a clearer picture of the considerations addressed in the problem, one can generalize the concerns into three categories: (a) workload within the sector, (b) workload between sectors and (c) safety concerns. In a given airspace, a smaller sector would usually indicate a lower workload within a sector, but a higher workload between sectors and vice-versa. With two conflicting considerations, it would be effective to formulate the airspace sectorization problem as a Multi-Objective Problem (MOP).

The ownership of airspace is mostly based on national sovereignty. In Singapore, the airspace is permanently divided into about 7 given sectors, where a pair of air traffic controllers (ATCs) are in charge of the safety of aircraft in a particular sector. Traffic flows change based on demand and supply of the passengers, varying over time. At times when a sector becomes overloaded, an additional ATC is brought into to help in the management of air traffic. The idea of dynamic airspace sectorization (DAS) is proposed to tackle this problem by changing sector shapes according to traffic flows [76, 18, 105]. Most, if not all, of the previous literature have only proposed resectorization methodologies, quantifying the changes in sectors. No work has looked or analysed the DAS as a problem over time.

Besides changing traffic flows, uncertainties due to extreme weather and the presence of restricted airspace introduce an increase in load in the available airspace. Aircraft are required to fly a different route to avoid these restricted regions. Many works have been done to re-route the aircraft [93, 73, 117, 63, 87]. However, only Yousefi et al. [122] addressed the need for resectorization to provide a well-balanced workload for the ATC after re-routing the airways. However, given the coupled nature of airways and airspace, optimising both components simultaneously could provide a better solution.

In the light of this discussion, three key points arise with regard to airspace management:

- The design of sector shapes has been formulated with different considerations to represent the real-world scenario in varying perspectives. With a growing number of different considerations, there is a need to have a framework to organize and identify the rationale behind these considerations. In other words, a principled methodology is required to select important elements used to design optimal sector shapes such that it reflects the real-world scenario.
- To address the changing traffic flows across time, there is a need to look at the DAS problem in a system architecture over time and evaluate the effectiveness of the concept of resectorization.
- The changes in weather limit the availability of airspace at times, requiring a re-routing of airways and resectorization of airspace to ensure balanced workload among ATCs. In other words, there is a need to investigate and explore new strategies for re-routing and resectorization when a region of airspace becomes unavailable.

In this thesis, the airspace management problem is generally posed as a multi-objective black-box/derivative-free problem. This is because the relationship between the variables (Voronoi sites) and objectives of the airspace sectorization problem is too complex to formulate mathematically.

Continuous black-box optimisation In order to closely model real-world problems, where it would be almost impossible to obtain information about the system such as an analytical expression, its structure or derivatives, the problem is considered to be in a black-box scenario. These scenarios are commonly referred to as black-box or derivative-free optimisation.

Without the loss of generality, the continuous black-box minimization problem with n decision variables has the form:

$$\begin{aligned} & \text{minimize} && f(x) \\ & \text{subject to} && x \in \mathcal{X} \\ & \text{given} && v \text{ function evaluations,} \end{aligned} \tag{Eq. 1.1}$$

where $f : \mathcal{X} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ is the black-box objective function defined on decision space $\mathcal{X}$ subset of $\mathbb{R}^n$. The only source of information about f is point-wise evaluations: one can query f at any point $x \in \mathcal{X}$ and the black box will return the corresponding objective value $f(x)$. In the case of multi-objective optimisation, the corresponding objective values, $f_1(x), \dots, f_m(x)$, where m represents the number of objectives, would be returned. Thus, the black-box optimisation problem can be viewed as a search for the set of optimal solutions within the decision space $\mathcal{X}$ given a finite number of function evaluations, v , as only the zero-order information is available. Due to this feature, black-box optimisation is also commonly referred to as derivative free optimisation.

Multi-objective optimisation (MOO) MOO is the process of solving a problem with conflicting objectives, where there is a number of optimal solutions arising from the tradeoffs of objectives, also known as the Pareto optimal solutions. There are generally two approaches to solve MOPs:

- **Priori:** User preference for the objectives is taken in before the solution process, problem is formulated into a single-objective problem and could be solved using single objective optimisation
- **Posteriori:** A set of Pareto optimal solutions is obtained before the user selects the final solution based on preference

The priori approach requires the user to quantify their preferences on the objectives using weights, which might not be accurate or realistic when resolving real-life applications. On the other hand, the posteriori approach allows the user to see the possible

combinations of solutions before choosing their most preferred solution. For the ease of real-world application, the thesis focuses on posteriori approaches to MOO.

Pareto Optimal Solutions: Concept of Domination Pareto optimal solutions essentially represents the set of best solutions in MOO and they are determined by the concept of domination.

Concept of Domination

A solution x_1 is said to dominate another solution x_2 , if both the following conditions are true:

- (i) The solution x_1 is no worse than x_2 in all objectives.
- (ii) The solution x_1 is strictly better than x_2 in at least one objective

Based on the concept of domination, the properties of a non-dominated set is given below,

Conditions for a non-dominated set, P_1 :

1. Any two solutions of P_1 must be non-dominated with respect to each other
2. Any solution not belonging to P_1 is dominated by at least one member of P_1

Solvers in Multi-Objective Continuous Black-box Optimization Population-based evolutionary algorithms such as genetic algorithm (GA) and particle swarm optimisation (PSO) that are able to store multiple solutions at once are naturally preferred in solving MOPs [35]. Using the concept of non-dominance, GA and PSO are turned into multi-objective solvers Non-dominated Sorting Algorithm II (NSGA-II) [32] and Multi-Objective Particle Swarm Optimisation (MOPSO) [25]. These methods are easy to implement and are commonly used in solving MOO problems [43, 124, 14]. However, they are stochastic and might not return a similar set of optimal solutions. Therefore, to improve the reliability and reproducibility of solutions, deterministic multi-objective solvers must be explored.

1.2 Objectives

This research is primarily concerned with the optimisation methods used in the design of sector shapes in the airspace under different scenarios. The objectives of the thesis can be summarized as follows.

- **Optimal sector shapes.** In literature, various methodologies and varying considerations are suggested independently in the design of optimal sector shapes. There is no specific work that analysed and compared the selection of considerations used in the formulation of the problem. Hence, getting a deeper insight on the objectives and constraints used, if they are a suitable consideration and a reflection of the real-world scenario is a major objective of this research.
- **Dynamic airspace sectorization.** The need for resectorization of sector shapes is motivated by two factors.
 - **Changing traffic flows.** Two fundamental issues that need to be tackled are *when-to-do* and *how-to-do* a resectorization. Most of the previous work address the issue of *how-to-do* a reconfiguration or resectorization of sector shapes when required but did not address the question of *when-to-do* it. Therefore, this research seeks to find solutions that address both questions.
 - **Changing availability of airspace.** Bad weather conditions usually limit the airspace where aircraft can fly. Re-routing strategies for such scenarios have been researched extensively, but the same can not be said of re-sectorization strategies. Thus, this work explores the combination of re-route and re-sectorization under the changing availability of airspace.
- **Multi-objective deterministic solver.** This work focuses on a particular group of deterministic solvers known as the Multi-scale Search Optimisation (MSO)] [8]

algorithms. Given the space partitioning feature of MSO algorithms, they generally tend towards being exploratory and hence taking more function evaluations to find the optimal solution(s). This research focuses on improving the sampling strategies for faster convergence through the indicator-based approach in the multi-objective scenario.

1.3 Thesis Contributions

This thesis provides several contributions to the optimisation methods in the design of optimal sector shapes of the airspace. These new methodologies are tested and analysed on the Singapore regional airspace. These contributions can be categorized into two major components: i). modelling; ii). solver. The contributions are highlighted next, respectively.

1.3.1 Problem Modelling Contributions

This thesis first builds a structure for the common considerations used in the design of optimal sector shapes through studying previous models. The analysis identified conflicting considerations between balancing the monitoring workload within the sector and having less coordination workload between sectors. Thus, an MOO approach is proposed for the purpose of optimising sector shapes.

- *Preference-based bi-objective optimisation model.* The problem of finding optimal sector shapes is modelled as a bi-objective optimisation problem, incorporated with a preference towards the balance of monitoring workload. Further, a dynamic density metric, that accounts for traffic complexities such as changes in altitude, speed and heading, densely populated regions and potential conflicts, on top of traffic density was introduced as a metric for a better reflection of monitoring workload in the real world.

- *CDAS: Cognitive decision-making architecture for Dynamic Airspace Sectorization.* CDAS provides the decision maker with a set of approximate Pareto-optimal solutions with estimated monitoring and coordination workloads (using flight plans) for the next time period. This helps the decision maker in the selection of sectors, allowing one to see which sectorization does not require resectorization in the next time period (if there is any). Hence, the architecture provides an overarching framework for DAS, where a resectorization is suggested only when necessary.
- *Rolling horizon optimisation for dynamic airspace sectorization.* This approach focuses on tackling the DAS problem through planning. By taking into consideration the flight plans for the next few time intervals, the rolling horizon optimisation framework allows us to find sector shapes that best balances the monitoring workload over these time periods while fulfilling the constraints of maximum coordination workload and minimum sector similarity.
- *SAWAS: Simultaneous optimisation of airway and airspace.* SAWAS considers the re-routing of airways and resectorization of airspace simultaneously when addressing changes in the availability of airspace. It generates multiple possibilities of re-routing strategies, where each strategy undergoes the resectorization optimisation model. The combined final solutions are then compared according to the proposed objectives.

1.3.2 Multi-objective Solver Contributions

The airspace sectorization problem is generally modelled as a MOP. To this end, the thesis contributes a new multi-objective deterministic solver.

- *PA-DIRECT: Pareto-aware dividing rectangles.* PA-DIRECT is a deterministic global search based solver extended with the indicator-based approach from DIRECT.

PA-DIRECT was evaluated and compared with other state-of-the-art algorithms on the bi-objective test set on the Comparing Continuous Optimisers (COCO) platform. The results show that PA-DIRECT typically converges more rapidly than the state-of-the-art algorithms.

- *Application of PA-DIRECT to find optimal sector shapes* In addition to testing PA-DIRECT on a benchmark set, PA-DIRECT is used to optimise the airspace sector shapes in the preference-based bi-objective optimisation model with some constraint handling methods.

1.4 Thesis Organisation

The thesis is organised as follows:

- In **Chapter 2**, an overview of the modelling approaches used in finding optimal sector shapes are given in terms of i) generation of sector shapes; and ii). considerations used in defining the problem. This is followed by a literature review on multi-objective derivative-free solvers. Here, the algorithms are categorized based on their characteristics such as its search strategy being *local/global*, *direct/model-based* and *deterministic/local*. Further, they are grouped into how they have been extended to a multi-objective solver, through *non-dominance-based/indicator-based/decomposition-based* methods.
- **Chapter 3** provides a preference-based bi-objective model for optimising sector shapes. The preference of the decision maker was expressed in two different approaches, i). Constraint-based and ii). Reference-point-based. The use of a preference-based bi-objective model allows the decision maker to obtain a set of useful optimal solutions. Moreover, a dynamic density metric is used to capture the

traffic complexities present in the monitoring workload to better represent the real-world scenario. The two methods of expressing preferences are compared against each other using historical flight data in the Singapore regional airspace.

- **Chapter 4** addresses the *when-to-do* and *how-to-do* resectorization in DAS. The Cognitive decision-making architecture for DAS (CDAS) provides the decision maker with the predicted performance objectives of the next time period (based on flight plans) for all the available optimal sector shapes (found by the optimisation solver for selection). This could allow the decision maker to avoid a need for resectorization in the next time period. However, CDAS does not necessarily guarantee feasible solutions in the future time intervals. Thus, a rolling horizon optimisation framework is suggested to address the uncertainties in future time intervals. The rolling horizon optimisation framework considers the flight plans in future time intervals when optimising the sector shapes. The proposed methodologies are evaluated on historical flight data in the Singapore Flight Information Region (FIR).
- In **Chapter 5**, the re-routing of airway and resectorization of airspace is optimised simultaneously during a change in the availability of airspace. The proposed methodology is compared than the sequential case of selecting the best re-routing solution before performing a resectorization in both a simulated and a real-world (Singapore FIR) environment.
- **Chapter 6** introduces a new indicator-based derivative-free solver, Pareto-aware dividing rectangles (PA-DIRECT) for MOO. PA-DIRECT is evaluated and compared against state-of-the-art algorithms on the COCO platform. Further, PA-DIRECT is used to optimise sector shapes with the constraint-based preference-based bi-objective model.
- **Chapter 7** concludes the thesis and provides directions for future research work.

Chapter 2

Literature Review

This chapter is intended to give a general overview of optimisation methods used in tackling airspace sectorization. The problem can generally be explored in two aspects: the modelling of the airspace sectorization problem and use of different optimisation solvers. Therefore, the chapter is divided into two main sections, focusing on previous methodologies in the modelling of the airspace sectorization problem and the various optimisation solvers available, particularly in the area of multi-objective black-box optimisation.

2.1 Modelling the Airspace Sectorization Problem

Airspace sectorization is defined as the virtual division of airspace for the decomposition of air traffic services provided by Air Traffic Controllers (ATCs). Hence, the airspace sectorization problem can be given as automatically finding an optimal sectorization of the airspace given the trajectories of flights and airspace boundaries.

The two main issues in the modelling of airspace sectorization are

- How to generate sectors?
- What defines optimal sectorization?

The section is organised as follows. An analysis of previous methodologies used in generating sector shapes for airspace sectorization is first provided. This is followed by

the literature review of the objectives and constraints used in formulating the airspace sectorization problem.

2.1.1 Sector Generation

Various methods used to generate sectors had been classified into two groups, the *graph-based* model and *region-based* model in a survey on airspace sectorization [45]. In a *graph-based* model, the trajectories of flights are constructed into a graph, where vertices represent the intersection of trajectories. Hence, resolving the sectorization problem is equivalent to solving an NP-complete combinatorial problem of graph partitioning. Additionally, a geometric post-processing step is required to define the sector boundaries. In a *region-based* model, the targeted sectors are formed from elementary blocks of airspace and the problem is formulated as a combinatorial problem. These elementary blocks of airspace are usually represented by grids or ATC functional blocks. However, not all methodologies can fit into the given classification of *graph-based* and *region-based* models. Therefore, we decided to generalize the sector generation approaches into three broad categories. They are as follows:

- Top-down Approach: Division of airspace to form smaller sectors
 - Graph-based [38, 17, 109, 80, 77, 24, 98]: Modelled as a NP-complete combinatorial problem of graph partitioning, usually require post-processing
 - (Computational) Geometry-based [37, 105, 119, 106, 101, 107]: Sectors are formed using geometry methods and are usually paired with evolutionary algorithms
- Bottom-up Approach [21, 123, 26, 48, 39, 76, 18, 58, 74, 67, 101]: Combination of elementary shapes or blocks to form sectors

- Hybrid Approach [101]: Utilizes both top-down and bottom-up approaches to form final sectors

The graph-based model from the top-down approach is one of the oldest methods that have been proposed [38]. Graphs are generally formed using information about the standard trajectories and flows in the given airspace, forming a graph partitioning problem. After the problem have been resolved, the final sectors are to be constructed in geometric post-processing [38]. Following this fundamental framework, works focusing on different methods of computation [109, 24, 17] and computational geometry [77, 80, 98] have been explored.

The bottom-up approach for airspace sectorization is primarily based on the combination of simple polygon mesh and is mostly modelled as a combinatorial problem [123, 26, 21, 58, 74, 67]. Further, the combination of various elementary sectors (from some existing sectorization) is optimised based on varying traffic flows in [101, 48, 39, 18]. Moreover, the combination of sectors at different flight levels was explored [76].

Recently, there is a growing attention on the computational geometry-based methods due to the ease of computation [37, 105, 106, 119, 107, 101]. They are usually paired with evolutionary algorithms and are easy to implement, not requiring a structure built by air traffic flows. In [106], computational geometry models namely KD-tree-based, bisection and Voronoi diagrams were proposed. The models are being able to satisfy convexity constraints required by ATCs to prevent re-entries of flights into the same sector, hence are popular choices in airspace sectorization. Based on various experimentations, Voronoi Diagram seems to obtain the best sectorization when compared to other computational geometry methods [106, 107]. More detail on Voronoi Diagram is provided in Section 3.1.1.

Last but not least, there is a unique hybrid approach in sector generation used in [101]. It starts with the creation of functional airspace block (elementary sectors) with the

use of Voronoi Diagram (computational geometry-based), followed by the combination of these functional airspace blocks into operational airspace sectors, dependent on air traffic requirements at that time.

2.1.2 Formulating the Airspace Sectorization Problem

A single sector in the airspace is generally managed by two ATCs, a radar controller and a planning controller. The radar controller is in charge of transmitting all necessary information for a safe execution of flight within the sector while the planning controller coordinates transition information of inbound and outbound traffic between sectors. Simplifying this idea, we could classify the air traffic controllers workload into monitoring (radar) and coordination (planning) workload.

- Monitoring Workload: Ensure the safety of all aircraft **in given sector**
- Coordination Workload: Safe transition of flights between sectors (**coming into/going out of given sector**)

Before going further, we present a summary of common considerations used in literature. They are broadly classified into three categories: (1) workload in the sector, (2) workload between sectors and (3) safety.

For a given workload in the sector, considerations include the equal distribution of workload among sectors (balanced workload), limiting the maximum workload such that no additional ATC is required to manage the given workload (bounded workload).

For workload between sectors, the number of flights entering/exiting a sector is considered (coordination workload). Further, the more time a flight spends in a particular sector, the higher possibility of a lower coordination workload (minimum dwell time). Moreover, having fewer sectors could also indirectly reduce the coordination workload (no. of sectors).

Last but not least, potential conflicts between flights should not be near sector boundaries such that the ATCs have sufficient time to perform conflict resolution.

Table 2.1 provides a summary of the popular features used in literature when formulating the airspace sectorization problem. Out of all features, it can be observed that all papers have included the measurement of workload in sector. This indicates that the fundamental aim of the airspace sectorization problem is to ensure that the workload in each sector can be managed by the ATCs.

The workload in each sector is more than just the traffic density and can be divided into monitoring and conflict workload [45]. However, only some of the papers in literature consider the conflict workload [109, 17, 58], and the complexity [26, 48, 17] of the airspace. In particular, a metric that measures complexity in airspace known as dynamic density from [72] was used in [17]. However, no specific information about the measurement of human factors such as staffing and communication in the model was described. In [75], the dynamic density metric is divided into three components, namely the traffic density, traffic complexities and ATC intent. However, ATC intent is a human factor and could vary between different ATCs. Thus, the proposed dynamic density in [75] only consists of traffic density and complexities.

Dynamic density [75] The dynamic density, (ρ) for a given sector is defined as:

$$\begin{aligned} \rho = & W_1(\theta_{HC}) + W_2(\theta_{SC}) + W_3(\theta_{AC}) \\ & + W_4(\theta_{MD5}) + W_5(\theta_{MD10}) + W_6(\theta_{CP25}) \\ & + W_7(\theta_{CP40}) + W_8(\theta_{CP70}) + W_9(\theta_{TD}) \end{aligned} \quad (\text{Eq. 2.1})$$

where W_i represents the weights assigned to each factor. θ_{TD} represents the traffic density and the traffic complexity factors are defined as follows:

- (i) Heading Change (θ_{HC}) - The number of aircrafts that made a heading change of more than 15 degrees during a sample interval of two minutes

Table 2.1: Common considerations used in the formulation of airspace sectorization problem in literature. $\circ$ represents the feature being formulated as an objective and $\times$ represents the feature being formulated as a constraint.

Papers	Wkl in sector		Wkl between sectors			Safety
	Balanced wkl	Bounded wkl	Coord. wkl	Min. dwell time	No. of sectors	Min. dist
Delahaye et al. (1998) [38]	$\circ$		$\circ$	$\times$		$\times$
Yousefi and Donohue(2004) [123]	$\circ \times$			$\times$		$\times$
Trandac et al. (2005) [109]	$\times$		$\circ$	$\times$		$\times$
Bichot and Durand(2007)[17]	$\times$		$\circ$			
Conker et al.(2007) [26]	$\times$					
Martinez et al. (2007) [80]		$\times$			$\circ$	
Delahaye and Puechmorel(2008) [37]	$\circ$		$\circ$	$\times$		$\times$
Brinton et al. (2009) [21]		$\times$			$\circ$	
Drew (2009) [39]		$\times$			$\circ$	
Leiden et al. (2009) [76]	$\times$ (v1)				$\circ$ (v2)	
Li et al.(2009) [77]	$\times$	$\times$	$\circ$			$\times$
Kicingner et al. (2009) [67]	$\circ$					
Bloem and Gupta(2010) [18]		$\circ$				
Gianazza(2010) [48]	$\circ$	$\times$			$\circ \times$	
Sabhnani et al. (2010) [98]	$\circ$		$\circ$			$\times$
Xue(2010) [119]	$\circ$			$\circ$		$\circ$
Jägare(2011) [58]	$\times$			$\times$		$\times$
Kulkarni et al. (2011) [74]	$\circ$	$\times$				$\times$
Tang et al.(2012) [106]	$\circ$			$\circ$		$\circ$
Chen at al.(2013) [24]	$\circ$		$\circ$	$\circ$		
Ines (2015) [107]	$\circ$	$\times$		$\times$		
Sergeeva et al. (2015) [101]	$\circ \times$		$\circ$	$\circ$		

- (ii) Speed Change (θ_{SC}) - The number of aircrafts that had a computed airspeed change of greater than 10 knots or 0.02 Mach during a sample interval of two minutes
- (iii) Altitude Change (θ_{AC}) - The number of aircraft that made an altitude change of greater than 750 feet during a sample interval of two minutes.
- (iv) Minimum Distance 0-5 n. mi (θ_{MD5}) - The number of aircrafts that had a Euclidean distance of 0-5 n. mi. to the nearest aircraft at the end of each two minute sample interval, excluding conflicting aircrafts.
- (v) Minimum Distance 5-10 n. mi (θ_{MD10}) - The number of aircrafts that had a Euclidean distance of 5-10 n. mi. to the nearest aircraft at the end of each two minute sample interval, excluding conflicting aircrafts.
- (vi) Conflict Predicted 0-25 n. mi. (θ_{CP25}) - The number of aircrafts predicted to be in conflict with another aircraft whose lateral distance at the end of each two minute sample interval was 0-25 n mi. (with a vertical separation of less than 1000 feet)
- (vii) Conflict Predicted 25-40 n. mi. (θ_{CP40}) - The number of aircrafts predicted to be in conflict with another aircraft whose lateral distance at the end of each two minute sample interval was 25-40 n mi. (with a vertical separation of less than 1000 feet)
- (viii) Conflict Predicted 40-70 n. mi. (θ_{CP70}) - The number of aircrafts predicted to be in conflict with another aircraft whose lateral distance at the end of each two minute sample interval was 40-70 n mi. (with a vertical separation of less than 1000 feet)

The parameter vector, $\mathbf{W}$ is set as [2.40, 2.45, 2.94, 2.45, 1.83, 4.3, 2.11, 1]. These values are taken from an experimental study with 65 qualified ATCs from the Oakland Air Route Traffic Control Center (ARTCC). In the study, the ATCs were asked to give

ratings for each of the eight traffic complexity factors based on a Likert scale of 1-5 (min-max workload). The mean values of the study are reflected in the parameter vector. More details of the experimental study can be found in [75].

Referring back to Table 2.1, it can be seen that workload between sectors is being considered in 18 out of 22 papers. Further, 15 papers have considered it as an objective to be optimised. This is because the reduction of workload between sectors indirectly helps to shape sectors according to the traffic flows. Therefore, the reduction of workload between sectors is commonly used in the airspace sectorization model to obtain flow conforming sector shapes.

Conflicting considerations Both considerations of workload in a sector and between sectors are essential in airspace sectorization. Many previous works have considered both workloads as an objective-constraint pair [123, 109, 17, 80, 21, 39, 77, 107], optimising only one of the workload while bounding the other. Some works considered optimising both workloads using a weighted sum in a single-objective optimisation model [38, 37, 98, 119, 24, 101]. However, the workloads are conflicting in nature and should be modelled as a multi-objective optimisation problem like in [106]. The following example illustrate this:

Figure 2.1: Trade-offs between monitoring and coordination workload [116]

Figure 2.1 illustrates a situation where reducing coordination workload increases monitoring workload. In Figure 2.1(a), the plane re-enters the same sector, causing extra

coordination workload for the planning controller. In order to reduce this coordination workload without changing the trajectory of the plane, the triangular sector in the middle has to be grouped into the same sector as illustrated in Figure 2.1(b). In this situation (Figure 2.1(b)), the reduction of coordination workload leads to an increase in monitoring workload. Depending on how experienced the air traffic controllers are, the balance between monitoring and coordination workload required for each sector could be different. Therefore, it would be ideal to have a bi-objective optimisation model to obtain all the Pareto optimal solutions and display all possible combinations of sector shapes for the user to choose from.

Last but not least, there is a safety consideration on Table 2.1, which is important for the feasibility of airspace sector shapes in the real world. In previous works, papers that considered the minimum distance from areas of potential conflicts to sector boundaries are taken as a constraint [38, 123, 109, 37, 77, 98, 58, 74]. The use of constraint is appropriate because if the solution does not satisfy the safety consideration, the sectors are highly likely to be unsafe for implementation in the real world.

Another consideration on the feasibility of found solutions is the sector shape, whether they can be managed by the ATCs with ease. The following are the four considerations that previous works raised [45]:

- Convexity: convex sector shapes generally prevent flights from re-entering into the same sector
- Connectedness: the sector is fully connected
- Compactness: sector shape is easy to manage e.g. polygon
- Non-jagged boundaries: sector boundaries are smooth

These considerations are mainly raised and addressed in bottom-up approaches as the combination of many polygon meshes allows a plethora of different sector shapes. As for computational geometry, in particular the use of Voronoi diagrams, the above considerations are naturally satisfied.

Dynamic airspace sectorization (DAS) Due to changing weather conditions and consumer preferences, air traffic flows differ across time. To the best of our knowledge, this was first addressed in the work of [39], who proposed merging under-utilized sectors so ATCs can be allocated to help out at busy sectors and in [76], who suggested the use of splitting and merging of sectors using altitude when necessary.

Studies have shown that changing sector boundaries affects the ATCs performance [62]. Following this, measurements for changes in the reconfiguration of sector shapes were proposed in [18, 74, 105, 24]. In particular, the general concept of sector similarity was proposed in [105] to measure how different the sectors are.

Sector Similarity [105] Sector similarity is measured by the overlapping area between 2 sector shapes using Weiler-Atherton Clipping Algorithm [113]. The maximum overlapping area of a sector with all sectors in the previous airspace is used as the similarity of this sector to the previous sectorization, which was calculated based on Eq. 2.2.

$$a_k = \max_{j=1}^K (S_{k,d+1} \cap S_{j,d}) \quad k \in [1, K] \quad (\text{Eq. 2.2})$$

where $S_{j,d}$ is one of the sectors generated from the previous airspace sectorization (d) and $S_{k,d+1}$ is a sector produced in the current airspace sectorization ($d+1$). a_k is the maximum overlapping area of $S_{k,d+1}$ with all sectors from the previous airspace sectorization.

However, using the absolute value of maximum overlapping area is unfair to sectors of smaller size. Hence, the value should be normalized to provide an accurate measure. This normalization is given by Eq. 2.3.

$$r_k = \frac{a_k}{S_{j,d}} \quad (\text{Eq. 2.3})$$

where $S_{j,d}$ is the original sector which has the biggest overlap with the current sector S_k and a_k is an area calculated by Eq. 2.2. Using this metric as an objective to be maximized, the change in sector shape can be reduced, allowing the solutions found to be more applicable in real-world scenarios.

The use of sector similarity addresses the question of *how* to perform a resectorization when required but did not answer the question of *when* a resectorization is required. Hence, we believe that the dynamic airspace sectorization problem should be explored over a period of time.

2.1.3 Under Weather Uncertainty

A common issue faced in air traffic management is the unpredictability of weather. Research has been done to make weather information available to pilots [47] and to interpret how the severity of weather conditions affects the availability of airspace [82].

The presence of bad weather conditions can affect the current capacity of sectors [104] and airports [66]. Furthermore, flights need to be re-routed in order to avoid bad weather conditions [93, 73, 117, 63, 87].

The use of a modified A* path planning algorithm for re-routing proved to find the safest (in terms of weather) and the shortest flight path around the region affected by weather in a short time [19]. Moreover, the A* algorithm was used in [93] to design a Flow-Based Route Planner (FBRP) to obtain optimal routes for multiple flows of aircraft in the transition airspace. Furthermore, the A* algorithm was paired with ant colony optimisation technique to tackle dynamically moving weather in [87]. Other than the A* algorithm, the improved artificial potential field model, a popular technique in robot path planning, was proposed to resolve the re-routing problem [117].

Recent works have been focused on modelling the dynamic weather to improve the feasibility of re-routing strategies in the real world [88, 87, 63, 13]. Various methodologies

including stochastic dynamic programming [88], receding horizon trajectory generation [63] and even modelling the problem as a Partially Observable Markov Decision Processes (POMDPs) were proposed [13]. In a different perspective, classification of aircraft on whether they are able to handle moderate turbulence or icing was performed to optimise the use of airspace in a real-world scenario [120].

A large amount of work has been done on re-routing the flight plans under changing weather. However, these implemented changes in flight trajectories would, in turn, affect the workload of ATCs in the affected sectors. This could require additional ATCs or a resectorization of the airspace. Hence, it can be said that the air traffic flow and airspace sectorization is a coupled problem. Klein et al. [69] identified the demand (air traffic flow) and supply (airspace) relationship and explored the problem using dynamic airspace configuration (DAC) and traffic flow management (TFM). They experimented with a TFM-DAC-TFM loop to check if the use of airspace can be fully optimised. Results indicate that a single loop of TFM-DAC-TFM could lower TFM costs, but more iterations in the loop did not improve the results. In a more specific scenario of weather uncertainty, a change in airspace design is proposed to be made sequentially after obtaining the new weather avoidance traffic flows from the TFM in [122].

With the air traffic flow and airspace being a coupled demand and supply problem, a simultaneous re-routing of airways and resectorization of the airspace could yield a better solution than performing a sequential re-routing and resectorization.

2.2 Multi-objective Derivative-Free Optimisation

The airspace sectorization problem modelled with the use of Voronoi Diagram is typically considered a derivative-free/black-box optimisation problem. In these problems, only the objective function values are available, derivative information is unavailable, unreliable or impractical to obtain [96].

Figure 2.2: Classification of derivative free solvers

2.2.1 Classification of Derivative Free Solvers

Figure 2.2 shows a framework for derivative-free optimisation methods that was introduced in [96] based on various characteristics such as *local/global*, *direct/model-based* and *deterministic/stochastic*. The difference between *local* and *global* search is that algorithms in the latter group generally have the ability to refine the search domain arbitrarily. *Local* search techniques tend to converge to local optima and are unlikely to escape from a local optimum to find the global optimum. *Direct* algorithms determine search directions by computing values of the function f directly, whereas *model – based* algorithms build and use a surrogate model of f to guide the search process. Lastly, *deterministic* algorithms follow a set of rules and have reproducible results while *stochastic* algorithms contain random search steps.

Direct local search methods Direct local search methods broadly consist of two types, (a) the Nelder-Mead simplex algorithm [86] and (b) generalized pattern search

(GPS) [108] or more generally generating set search (GSS) methods [71]. The Nelder-Mead algorithm was analysed and found converging to a non-stationary point, failing to find the local optimal point [83]. Modifications for improvement of the Nelder-Mead algorithm can be found in [27]. On the other hand, GPS was proven to converge to stationary points under mild conditions [108]. Amongst the multiple improvements on GPS, Mesh Adaptive Direct Search (MADS) [9] stood out with an adaptation of poll step to generate an asymptotically dense set of search directions. This change allowed MADS to converge to second-order stationary points under the assumption that f is continuously differentiable with Lipschitz derivatives near the limit point [3] i.e. successfully finding the local optimal point under reasonable assumptions. Other modifications to the pattern search include improving efficiency by giving preference to directions that are closest to the negative of simplex gradient [30]. Simplex gradient is an approximation of the real gradient and is calculated based on previously evaluated points. Recently, the pattern search based method is coupled with multi-start strategies to find the global optimal point in an optimisation problem [28].

Model-based local search methods Model-based methods start by sampling the search space and building a surrogate model of the problem. The algorithm continues to iteratively optimise the surrogate model (picking the optimal point according to the model), evaluate the selected solution points and updating the surrogate model. Generally, surrogate models are built to model the search space (trust region methods) or the gradient (implicit filtering) of the function f . The trust region method uses a surrogate model in a neighbourhood (trust region) around the current iterate. Simple linear [90] and quadratic models [92] are usually used to build the surrogates. Implicit filtering approximates the gradient based on forward or centred differences of neighbouring points and follows the steepest descent method using the obtained gradient approximation [49, 114].

Deterministic global search methods Deterministic global search methods generally consist of space partitioning algorithms as they provide a structure to keep a record of previously evaluated points. Further, this very structure serves as a measure to quantify the amount of sampling performed in the region of any sampled point, which can help in balancing the exploration and exploitation dilemma in global optimisation. A popular algorithm is DIRECT [60], which essentially represents DIviding a hyperRECT-angle. DIRECT follows a unique method of partitioning space with a single dimension at a time, creating varying sizes of hyperrectangles. The different sizes help to indicate the amount of sampling performed around the point and are in turn used to help identify which regions to sample next. Recently, Abdullah et al. [8] proposed a Multi-Scale Optimization (MSO) framework for algorithms like DIRECT that partition the search space over multiple scales. Moreover, finite-time convergence analysis was established and given for Lipschitzian optimisation methods [102], DIRECT [60], Multi-level Coordinate Search [54] and Deterministic Optimistic Optimization (DOO) [85] and Simultaneous Optimistic Optimization (SOO) [85]. One should also note that MCS consists of a local search strategy to help in refining the search. Following the MSO framework, Abdullah and Suresh [6] proposed a current state-of-the-art algorithm called Naive Multi-scale Search Optimization (NMSO) algorithm, together with a theoretical finite-time and asymptotic convergence analysis.

Model-based global search methods In the global search, the surrogate model is developed for the entire search space or subsets that are dynamically refined through partitioning [96]. This approach is broadly referred to response surface methods (RSMs) or Bayesian optimisation. Response surfaces can be non-interpolating (minimize the sum of squared errors from some predetermined functional form) or interpolating (pass through all points). Interpolating surfaces are shown to be more reliable than non-interpolating surfaces [61]. Interpolating methods are usually a linear combination of basis functions

such as thin plate splines, linear, cubic and kriging [81]. The use of a kriging basis function in an interpolating model is commonly referred to as a *Design and Analysis of Computer Experiments* (DACE) model [99]. Radial basis functions are used to build interpolating surfaces in RSM too [91]. Moreover, surrogate models are paired with pattern search method [20] and randomization [55] to solve derivative free optimisation. Another significant contribution in this area is the Bayesian Multi-Scale Optimistic Optimization (BaMSOO) algorithm [112] which combines elements of Bayesian optimisation with SOO.

Stochastic global search methods The literature on algorithms in this category is rather extensive, particularly in applications because their implementation is relatively straightforward compared to deterministic algorithms [96]. They can be broadly classified into hit-and-run algorithms [23], simulated annealing [68], genetic algorithms [53] and particle swarm algorithms [40, 65]. Hit-and-run algorithms sample new points using two random components, direction d and step s with $x + ds$ and only update the current solution when the newly sampled solution is better. Simulated annealing, on the other hand, allows the update of a worse off solution with a probability [96]. Population-based methods like genetic algorithms (GA) and particle swarm optimisation (PSO) algorithms are more popular as they are able to store multiple solutions, allowing one to keep the current best solution and explore other regions in the search space. GA is primarily made for binary variables and was extended to continuous problems in [15]. A significant improvement in the GA approach is the covariance matrix adaption method proposed by Hansen [50]. The resulting search distribution is shaped to the contours of the objective function by updating the covariance matrix deterministically using information from evaluated points. Unlike GA, PSO algorithms are primarily proposed for continuous problems, aimed to find both local and global optima in an optimisation problem. PSO methods are commonly combined with local search methods for faster convergence [111, 42].

Even though there are distinct types of algorithms, it can be observed from above that the state-of-the-art algorithms tend to be hybrid algorithms that combine the methods used in global and local search [28, 54, 20, 55, 111, 42, 112].

2.2.2 Multi-Objective Optimisation

Multi-objective optimisation (MOO) is solving a class of problems where there are two or more conflicting objectives. Solutions in such problems become incomparable when a solution is better than another in one objective but worse off in another. Hence, there is usually a set of optimal solutions for these problems, where the final solution would be eventually selected by the decision maker. Therefore, population-based algorithms such as GA and PSO that are able to store multiple solutions at once are naturally preferred in solving Multi-Objective Problems (MOPs) [35].

Most MOO solvers, if not all, are extended from single objective optimisation solvers. There are broadly three approaches that are used to manage the multiple objectives:

- Non-dominance-based
- Indicator-based
- Decomposition-based

The dominance-based, indicator-based and decomposition-based approaches have been applied to the basic genetic algorithm in Non-dominated Sorting Genetic Algorithm II (NSGA-II) [32], S metric selection Evolutionary Multi-Objective Algorithm (SMS-EMOA) [16] and Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [125] respectively. The different approaches are explained below using the aforementioned algorithms.

Algorithm 1: Pseudo code of GA, adapted from [121]

```
1: Input: Population size  $pop$ ,
2:     Total no. of generations  $max\_gen$ ,
3:     Crossover probability( $p_c$ ),
4:     Mutation probability ( $p_m$ )
5: Initialization: Generate the initial population randomly.
6: Evaluate the initial population.
7: while ( $t < max\_gen$ ) do
8:   Create new population:
9:   while new population size  $< pop$  do
10:    Select two parent chromosomes from current population.
11:    Generate child chromosome(s) by crossover and mutation of selected parent
        chromosomes.
12:    if  $p_c > rand$  then
13:      Perform crossover operation.
14:    end if
15:    if  $p_m > rand$  then
16:      Perform mutation operation.
17:    end if
18:    Add child chromosome(s) to new population.
19:  end while
20:  Evaluate the new population.
21:  Merge the current and new population.
22:  Select the best solutions to keep for the next generation (elitism). (Keep
        population size to  $pop$ )
23: end while
24: Return the population of the last generation.
```

Genetic algorithms GA is a heuristic method that developed from the idea of biological evolution. Solutions are modelled in chromosomes and they undergo change through operations such as crossover, mutation and selection.

Algorithm 1 provides the skeleton of GA. Here, the solutions are made to evolve, keeping a population of the best solutions in each iteration. GA is a stochastic algorithm that uses probability to decide whether a crossover or mutation operation should be performed on the solution. Therefore, different runs of the same algorithm might provide different solutions. However, when a sufficient number of iterations is run, solutions

would converge towards the optimal solution.

The algorithm starts with a random initialization of solutions. A well spread initial population allows greater diversity and has a higher probability of reaching the global optimal point while a closely populated set of initial solutions might quickly converge towards a local optimal point.

Selection operator The selection operator is used to select the parent chromosome for crossover or mutation operations. One of the most common selection operator is the fitness proportionate selection, also known as roulette wheel selection. The selection is based on probability, with better solutions having a higher chance of getting picked. Another popular selection operator is the k tournament selection where k solutions are randomly picked and the best out of all k solutions gets selected. In the case of NSGA-II, binary tournament selection is used, where the *better* of two randomly selected solutions becomes the parent for a crossover or mutation operation. The quality of the solution is determined differently in the three different approaches, elaborated later in the section.

Crossover operator For real-coded chromosomes, simulated binary crossover (SBX) operator is typically used. SBX creates child chromosomes similar to one-point crossover operation in binary-coded GA. The effect of the crossover operation is dependent on the location of the parent chromosomes. The closer the parents are, the smaller the spread of the offsprings and vice-versa. More details on how the SBX operator works can be found in [33, 34].

Mutation operator For real-codes chromosomes, polynomial mutation is usually used. It follows a polynomial probability distribution to modify a single variable of the original solution to form a new solution. This step is usually used to diversify the solutions in the population, especially in the case when the population had started converging towards an optimal point. The detailed formulation can be found in [34].

Therefore, the crossover and mutation probabilities p_c and p_m , which are parameters set by the user can affect the rate of convergence.

NSGA-II: Non-dominated Sorting Genetic Algorithm II [32] NSGA-II follows the non-dominance-based strategy to determine the quality of solutions. The solutions are ranked based on this criteria:

- (i) Ranking based on non-dominated fronts:

A solution x is said to be dominated when there exist any solution y that is not worse off in all objectives and better than solution x in at least one objective. If solution y does not exist in the population, solution x is considered a non-dominated solution. All the non-dominated solutions are ranked first (rank = 1), after that they are removed and the next set of non-dominated solutions are ranked (rank = 2), the procedure continues until all solutions are ranked.

- (ii) Crowding distance based on the spread of solutions:

Given that the solutions are in the same rank, the solutions are first sorted according to an objective function value. The boundary solutions are set to infinity distance value, hence they would always be chosen. The other solutions are assigned a distance value equal to the absolute normalized difference in the function values of two adjacent solutions. This calculation is continued with other objective functions. The overall crowding distance value is calculated as the sum of individual distance values corresponding to each objective.

If two solutions have differing ranks, the solution with lower rank is preferred. When both solutions have the same rank, solution in a less crowded region (longer crowding distance) is preferred. More details on the implementation can be found in [32].

SMS-EMOA: $\mathcal{S}$ Metric Selection Evolutionary Multi-Objective Algorithm [16]

SMS-EMOA follows the indicator-based strategy to measure the quality of solutions. SMS-EMOA is popular due to its simplicity and effectiveness. Instead of generating a new population of solutions, one new solution is generated and a solution from the population is discarded in each iteration. It uses variation operators such as SBX crossover and polynomial mutation operator introduced in Section 2.2.2 to generate a new solution. For the selection operator, it uses the Pareto compliant hypervolume (HV) indicator [128], also known as the $\mathcal{S}$ metric, as the indicator to select which non-dominated individual to discard.

Algorithm 2: SMS-EMOA: Selecting individual to discard [16]

```

1:  $\{\mathbb{R}_1, \dots, \mathbb{R}_v\} \leftarrow \text{nondominated-sort}(Q)$  /* all  $v$  fronts of  $Q^*$  */
2: if  $v > 1$  then
3:    $r \leftarrow \text{argmax}_{s \in \mathbb{R}_v} [d(s, Q)]$  /*  $s \in \mathbb{R}_v$  with highest  $d(s, Q)$  */
4: else
5:    $r \leftarrow \text{argmin}_{s \in \mathbb{R}_1} [\Delta_{\mathcal{S}}(s, \mathbb{R}_1)]$  /*  $s \leftarrow \mathbb{R}_1$  with lowest  $\Delta_{\mathcal{S}}(s, \mathbb{R}_1)$  */
6: end if
7: return  $(Q \setminus \{r\})$  /* eliminate detected element */

```

The selection of a solution to discard is illustrated in Algorithm 2. The population of solutions is first ranked using non-dominated sorting introduced above in NSGA-II. If all the solutions in the population are non-dominated, the individual with the least HV is discarded. Else, the point with the most number of dominating points, $d(s, Q)$, on the worst ranked front would be discarded.

The purpose of using HV as a selection operator is to ensure points that are well-spread and diversified. However, HV can only truly reflect this diversity on the non-dominated front and may not clearly reflect this in dominated fronts. An example of this situation is being illustrated in Figure 2.3. Looking at the points y_8 and y_9 , it can be seen that y_8 has a higher contributing HV than y_9 (green shaded area). On the other hand, y_8 are being dominated by 4 points (y_2, y_3, y_4, y_5) while y_9 is being dominated by 1 point

Figure 2.3: Illustration of dominating points and contributing HV [16]

(y_5). From this, it can be observed that using HV on dominated fronts does not capture any information of the solutions on the non-dominated front while using dominating points favours fronts that are sparsely populated, allowing diversification and in turn the exploration of unexplored areas.

MOEA/D: Multi-objective Evolutionary Algorithm based on Decomposition

[125] MOEA/D follows the decomposition-based approach in managing multiple objectives. The general concept is to decompose the task of approximating the Pareto front into N subproblems. Using the population-based technique, N agents are used to solve N subproblems collaboratively using the relationship between subproblems. These subproblems are usually divided using weight vectors (applied on objectives) with a neighbourhood structure. Two subproblems are considered neighbours if their weight vectors are similar. With the underlying assumption that neighbours have similar optimal solutions,

- the selection operation in a subproblem allows parent chromosome to be selected from its neighbours

Table 2.2: Multi-objective optimisation solvers and their characteristics

Algorithm	Characteristics			
	Local Global Hybrid	Direct Model-based Hybrid	Deterministic Stochastic	Non-dominance Indicator Decomposition
NSGA-II (2002) [32]	G	D	S	N
MOPSO (2002) [25]	G	D	S	N
DEMO (2005) [97]	G	D	S	N
SMS-EMOA(2007) [16]	G	D	S	I
MOEA/D (2007) [125]	G	D	S	D
MO-CMA-ES (2007) [57]	G	D	S	N,I
RM-MEDA (2008) [126]	H	H	S	N
MultiMads (2010) [10]	L	D	D	D
DMS (2011) [31]	L	D	D	N
MO-DIRECT (2016) [4]	G	D	D	N
BMOO (2017) [44]	G	M	S	I
SOCEMO (2017) [84]	G	M	S	N
MO-SOO (2018) [7]	G	D	D	N
MultiGLODS (2018) [29]	H	D	D	N

- the crossover and mutation operations allow a mix of parent chromosomes from its own and neighbouring populations
- the new solution is shared with its neighbours, and it replaces the old solution if it is better based on the given subproblem objective

With the rise of many objectives problems, the concept of MOEA/D is becoming increasingly popular. This is because the decomposed subproblems can be resolved in a parallel manner. The many developments in MOEA/D can be found in [78].

Multi-objective optimisation solvers Table 2.2 provides a list of MOO solvers in chronological order. This is by no means an exhaustive list but hopefully captures the wide range of available MOO solvers with varying characteristics. The list starts with three different evolutionary algorithms based on non-dominated-based strategy, namely NSGA-II[32], MOPSO [25] and DEMO [97]. Soon after, different strategies on managing multiple objectives were implemented in SMS-EMOA [16] and MOEA/D [125]. The

popular evolutionary algorithm, CMA-ES [50] has also been extended to MO-CMA-ES by Igel et al. [57] with non-dominance-based and indicator-based approaches. Further, a combination of evolutionary algorithms and the model-based local search was proposed in RM-MEDA [126] to tackle MOO. In 2010, there seems to be a shift of focus from evolutionary algorithms to deterministic algorithms, which are generally more mathematical-based. The proposed local search based algorithms MultiMads [10] and DMS [31] come with convergence studies. Deterministic global-search based algorithms such as MO-DIRECT [4] and MO-SOO [7] were also proposed, with the latter providing a convergence study. In recent years, global model-based algorithms such as BMOO [44] and SOCEMO [84] were too proposed using the indicator-based and non-dominance-based approach respectively. Last but not least, the hybrid algorithm, GLODS [28] was extended to MultiGLODS [29] using the non-dominated-based approach in 2018.

For the sake of reliability when applied in real-world applications such as the air traffic sectorization, the focus of our research is on the improvement of deterministic algorithms in MOO. With no known extension of deterministic algorithms though the indicator-based approach yet, this work proposes a deterministic algorithm with the use of the Hypervolume indicator [128].

2.3 Summary

In this section, a summary of the research challenges in the airspace sectorization problem and in MOO is discussed based on the literature review in this chapter.

Dynamic airspace sectorization. The airspace sectorization can be divided into (a) the approach taken in generating the sectors and (b) the elements used in the formulation of the airspace sectorization optimisation model. The use of Voronoi diagrams has been proven to be a straightforward and effective method for the generation of sectors

[106, 107]. As for the formulation of the problem, many different objectives and constraints were proposed in various papers. The common considerations were taken and classified into three general groups of (i) workload in the sector, (ii) workload between sectors and (iii) safety to provide an overall picture of previous works. The constant consideration across all papers is the workload in the sector, which we refer to as monitoring workload. The next consideration of workload between sector is found to aid sector shapes in conforming to traffic flows. The last consideration of safety is important for the feasibility of solutions and hence should be kept as a constraint. Further, the relationship between the workload in the sector and workload between sector is analysed and found to be conflicting. Therefore, the airspace sectorization optimisation problem should be posed as an MOO problem, similar to [106, 105]. Consolidating all the important features after analysis, a new preference-based MOO formulation of the airspace sectorization problem is proposed in Chapter 3. Another concern in airspace sectorization is the changing traffic flows over time, which causes unchanged airspace sector shapes to become sub-optimal. This hence requires the change of sector shapes over time for efficient operations. This issue is addressed in Chapter 4 with the help of a sector similarity metric [105] to prevent significant changes in sector shapes.

Air traffic management under weather uncertainty. The re-routing of aircraft during bad weather conditions has been previously addressed by [93, 73, 117, 63, 87]. However, little attention has been given to reshaping sectors, even when the air traffic flow and airspace is identified as a demand and supply coupled problem [69]. To the best of our knowledge, only Yousefi et al. [122] suggested the change of airspace design sequentially after obtaining the re-routed air traffic flows. Chapter 5 provides a novel approach of optimising the re-routed airway(s) and airspace simultaneously.

Multi-objective optimization. Derivative-free optimisation solvers can be broadly characterized as *local/global*, *direct/model-based* and *deterministic/stochastic* [96]. Most

popular multi-objective derivative-free optimisation solvers are based on evolutionary algorithms, which are stochastic global search methods. However, deterministic methods generally have convergence studies [7, 31, 29, 10] and thus tend to be more reliable when applied to real-world applications. Chapter 6 examines the gaps in one of the multi-objective deterministic algorithm, MO-DIRECT and introduces indicator-based strategies to improve the search.

Following the literature review, the next chapter will start off by introducing the basic model used in designing optimal sector shapes for a given airspace in this thesis. The fundamental aspects of the model are used in the next few chapters when addressing other issues in airspace management.

Chapter 3

Preference-based Airspace Sectorization

In this chapter, a preference-based model of the airspace sectorization problem is presented. The main objectives of airspace sectorization are to balance the monitoring workloads among the sectors and reduce the total coordination workload between the sectors. Given the conflicting nature of the two objectives, a bi-objective optimisation formulation is used for the dynamic airspace sectorization problem. However, not all solutions on the Pareto front are operationally feasible. Thus, two different approaches - (a) a constraint-based strategy and (b) a reference-point based strategy are proposed to streamline the process of optimisation and obtain a particular set of ATC preferred solutions.

This chapter is organised as follows. The bi-objective optimisation formulation of airspace sectorization is introduced in Section 2.1.2. The preference-based strategies are explained in Section 3.2. An experimental study on the preference-based strategies is conducted and the results are presented in Section 3.3. Finally, Section 3.4 provides a summary of this chapter.

3.1 A Bi-Objective Optimisation Formulation

This section provides the optimisation formulation for the airspace sectorization problem. The methodology for sector generation is introduced, followed by the problem formulation of airspace sectorization.

3.1.1 Sector Generation: Voronoi Diagrams

In our approach, the sectors are being generated using Voronoi Diagrams. Voronoi diagram is defined as the partitioning of a plane with K points into convex polygons (internal angles are less than 180°) such that each polygon contains exactly one generating point. Every point in a given polygon is closer to its generating point than to any other. These generating points are commonly referred to as Voronoi sites. The generated convex polygons are able to satisfy the operational requirement of convexity and connectivity of airspace sectors, hence suitable for the airspace sectorization problem. The set of Voronoi sites used to generate the sector shape is denoted as $V = \{\mathbf{v}_1, \dots, \mathbf{v}_K\}$ where

$$\mathbf{v}_i = \begin{pmatrix} v_i^\lambda \\ v_i^\phi \end{pmatrix} \text{ for } i = 1 \dots K \quad (\text{Eq. 3.1})$$

Here, K is the number of 2D sectors being designed. Superscript λ and ϕ denote the longitude and latitude of the site's location respectively. The sector boundaries are determined through the Voronoi decomposition of V . For the preservation of convexity, a vertical split of a 2D sector is performed for the creation of 3D sectors. The total number of sectors is referred to N .

3.1.2 Problem Formulation: Objectives and Constraint

Objection Function 1: Minimizing the standard deviation of monitoring workload across all sectors The measurement of monitoring workload in previous literature [118, 106, 101] is usually given as the traffic density i.e. the number of traffic hit

counts. However, the required work for each flight differs. A greater effort from the ATCs is necessary when flights require a change in altitude, speed or heading. For a more accurate reflection of the ATC workload, the dynamic density metric [75] which accounts for traffic complexity is being used in the model.

The dynamic density, (ρ) for a given sector is given in Eq. 2.1.

The monitoring workload of sector j can be given as $M_j = \rho_j$. Hence, the objective of balancing the monitoring workload can be presented as minimizing the normalized standard deviation of M_j and is formally defined as:

$$Obj_M = minimize \left(\frac{\sqrt{\frac{1}{N-1} \sum_{j=1}^N (M_j - \mu_m)^2}}{\mu_m} \right) \quad (\text{Eq. 3.2})$$

where $\mu_m = \frac{1}{N-1} \sum_{j=1}^N M_j$. One should note that the standard deviation is normalized to the average monitoring workload.

Objective Function 2: Minimizing the total coordination workload The workload of a planning controller is typically quantified by the number of aircraft transiting between sectors. There are generally two approaches applied to minimizing the number of aircraft flying through multiple sectors [45].

- Direct: Minimize the total sum of flights leaving a sector
- Indirect: Maximize the average sector flight time

The distinction between the direct and indirect method is that the latter is relative to the speed of aircraft. The use of average sector flight time might not provide an accurate measure of coordination workload if aircraft are flying at different speeds. This is especially so in sectors consisting of both cruising and climbing aircraft. Therefore, given the nature of cruising and climbing aircraft in the Singapore FIR, the metric used

for measuring the coordination workload is the number of flights leaving a particular sector j and is defined as:

$$C_j = \#aircrafts \text{ leaving sector } j \quad (\text{Eq. 3.3})$$

Given that $C = [c_1, c_2, \dots, c_N]$ be coordination workloads for all sectors, the objective can be represented as the minimization of total coordination workload summed up over all sectors and is formally defined as

$$Obj_C = minimize \left(\sum_{i=1}^N C_i \right) \quad (\text{Eq. 3.4})$$

Safety Constraint: Minimum distance of CP to boundaries The crossing point (CP) is established as an area prone to conflicts due to overlapping flows. CP are areas where aircraft are within a lateral distance of 0-10 nautical miles and a vertical separation lower than 1000 feet. For safety operations, ATC requires a minimum distance between the sector boundary and CP. The minimum Euclidean distance (δ_j) between the crossing points (**CP**) and the sector boundary is defined as:

$$\delta_j = \min ||\mathbf{CP} - B_j||, \quad j = 1, 2, \dots, K \quad (\text{Eq. 3.5})$$

where $\mathbf{CP}$ is a vector representing the crossing points, B_j stores information on the sector boundaries of sector j and K is the total number of areas (2D sectors).

In this study, the minimum distance has been set as the operational requirement of 10 nautical miles (NM). Following this, the subsequent safety constraint is set:

$$\delta_j > 10NM, \forall j \quad (\text{Eq. 3.6})$$

The following provides the objectives and constraint of the airspace sectorization problem.

Objectives:

1. Minimize standard deviation of monitoring workload between sectors
2. Minimize total coordination workload across all sectors

Constraints:

1. Distance between crossing points and sector boundaries $> 10NM$

3.2 Preference-based Optimisation Strategies

Before providing the two preference-based optimisation strategies, the evolutionary-based solver used for airspace sectorization is first introduced.

3.2.1 Optimisation Solver

As given in the literature review of multi-objective solvers in Table 2.2, there is a range of efficient multi-objective solvers that can be used to solve the preference-based optimisation model. For the sake of simplicity, the solver used for this implementation is a popular multi-objective evolutionary algorithm known as Non-dominated Sorting Genetic Algorithm II (NSGA-II) [32]. NSGA-II is a heuristic method that developed from the idea of biological evolution. Solutions are modelled in chromosomes and they undergo change through operations such as crossover, recombination, mutation and selection.

3.2.1.1 NSGA-II: Non-dominated Sorting Genetic Algorithm II

In this experiment, the NSGA-II undergoes selection, crossover and mutation operations to form a new population. NSGA-II follows a binary tournament selection process, where two random chromosomes are picked and the one with better objective values will be kept. The ranking of chromosomes is based on the concept of non-domination in multi-objective optimisation. Given that two solutions are of the same rank, they are further sorted based on their crowding distance. The crowding distance is normalized to the objective values of the current approximate front. Non-feasible solutions that do not satisfy the constraints will be ranked worse off than any feasible solution. This

selection process is performed as many times as the size of the population (N_p). Then, the population undergoes an intermediate crossover process described below.

$$\mathbf{s}_1 = \mathbf{q}_1 + \mathbf{k}_c * \beta_c * \mathbf{x}_c * (\mathbf{q}_2 - \mathbf{q}_1) \quad (\text{Eq. 3.7})$$

$$\mathbf{s}_2 = \mathbf{q}_2 - \mathbf{k}_c * \beta_c * \mathbf{x}_c * (\mathbf{q}_2 - \mathbf{q}_1) \quad (\text{Eq. 3.8})$$

$\mathbf{s}_1$ and $\mathbf{s}_2$ are the new child chromosomes produced by the parent chromosomes $\mathbf{q}_1$ and $\mathbf{q}_2$ in the population. The crossover operation is ran $0.5 * N_p$ times, giving a new population of N_p . $\mathbf{k}_c$ is a vector of binary values, that reflects 1 if a randomly generated value exceeds the crossover fraction of $2/nVars$ where $nVars$ represents the number of variables. β_c is a user-defined value, solutions tend to differ more when set above 1 and x_c is a randomly generated vector with values between 0 and 1.

The Gaussian mutation process which follows after the intermediate crossover process is given below. First, the variable that controls the amount of mutation, $scale_m$, is calculated.

$$scale_m = scale_m - shrink_m * \frac{G_i}{G_{max}} \quad (\text{Eq. 3.9})$$

where $scale_m$ and $shrink_m$ are user-defined parameters. $scale_m$ generally controls the amount of mutation and $shrink_m$ allows one to decrease the amount of mutation towards the end of optimisation.

The following shows how each variable in the chromosome is mutated, given that a randomly generated value exceeds the mutation fraction of $2/nVars$.

$$s(i) = q(i) + scale_m * x(N(0, 1)) \quad (\text{Eq. 3.10})$$

where $s(i)$ and $q(i)$ are the i variable of the child and parent chromosome respectively and $x(N(0, 1))$ is a normally distributed random number.

After going through the binary tournament selection, intermediate crossover and Gaussian mutation operations, a new population is formed. This new population is

v_1^λ	v_1^ϕ	v_2^λ	v_2^ϕ	$\dots$	v_{K+1}^λ	v_{K+1}^ϕ	alt_1	$\dots$	v_N^λ	v_N^ϕ	$alt_{(N-K)}$
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Figure 3.1: Input chromosome for N sectors and K areas for a single time period

then merged with the initial population, where the best solutions are kept, forming the final population for the current generation. This cycle then repeats for a user-defined number of generations.

3.2.1.2 Chromosome structure

Since the 2D sectors are created with the use of Voronoi diagrams, the decision variables for the NSGA-II algorithm are the Voronoi site locations and vertical splits for 3D sectors. Figure 3.1 provides a regular chromosome characterising a solution of the airspace sectorization problem in the NSGA-II solver. The total number of 2D sectors is given by K and the total number of vertical splits is specified by $N - K$, giving a total of N sectors. For a total of $N = 7$ sectors with $K = 4$ areas (2D sectors), a total of $2 * N + (N - K) = 17$ input variables are needed. v_i^λ and v_i^ϕ for $i = 1, \dots, K$ represents the Voronoi sites used for generation of 2D sector shapes. v_i^λ and v_i^ϕ for $i = K + 1, \dots, N$ are points used to decide which sector should the vertical split occur in. In other words, the sector with Voronoi site that is closest to $(v_{K+1}^\lambda, v_{K+1}^\phi)$ will experience a vertical split at alt_1 . In the case where $v_{K+1}^\lambda, v_{K+1}^\phi$ and $v_{K+2}^\lambda, v_{K+2}^\phi$ points to the same sector, there would be 2 vertical splits in the selected sector at alt_1 and alt_2 .

3.2.2 Preference-based Strategies

Solving the formulated airspace sectorization problem would allow us to obtain a set of solutions that tend towards the Pareto front. However, only some of the solutions obtained will be of interest to the ATC. For example, a possible extreme solution that could be on the Pareto front with a total coordination workload of 0 is given in Figure

Figure 3.2: Possible solution with 0 total coordination workload

3.2. This suggests that there are no flights transiting between sectors and all the flights are contained in a single sector. Figure 3.2 shows the given airspace sectorization (on the left) along with the monitoring and coordination sector workloads of this solution (on the right). All flights in this solution are contained in Sector 2, hence there are no monitoring workloads reflected in Sector 1 and 3. Obtaining such a solution is equivalent to having no sectorization, which in turn defeats the purpose of airspace sectorization.

On the other extreme, the ideal workload for ATC is to have equal sector monitoring workload as shown in Figure 3.3(a). However, such solutions are often unattainable or with the cost of high total coordination workload. In terms of airspace sectorization, there are two general guidelines that ATC considers: no sectors should be no monitoring workload (empty) and a sector should not hit an upper limit such that another controller is required to manage the traffic. Hence, there is a need to implement strategies so to ensure useful solutions that fall under these guidelines are obtained and allow for faster convergence towards these ATC preferred solutions.

3.2.2.1 Strategy 1: Constraint-based

The constraint-based strategy has been previously used in [116], through ensuring the monitoring workload of each sector is within a user-defined deviation of the average monitoring workload. In this study, we propose a constraint motivated by the ATC general guidelines. Consider an ideal equally divided monitoring workload of 5 sectors to be $\mathbf{M} = [1, 1, 1, 1, 1]$ in Figure 3.3(a). Given that one sector's workload (sector 5) is empty and its workload to be equally divided between the other sectors, $\mathbf{M}$ would be $[1.25, 1.25, 1.25, 1.25, 0]$ as shown in Figure 3.3(b). For the prevention of a possible empty sector explained above, we set a ceiling for the maximum amount of workload for each sector, assuming that we have no information on the upper limit of the monitoring workload. This can be formally given as:

$$M_j \leq (1 + \frac{1}{N-1}) \quad \forall j \quad (\text{Eq. 3.11})$$

The constraint-based strategy is implemented in NSGA-II as an additional constraint to satisfy. Solutions that do not satisfy the upper limit of the workload will be deemed worse off than solutions that do.

3.2.2.2 Strategy 2: Reference-point-based

Another method to obtain preferred solutions is through the use of reference point(s). They indicate the decision maker's desired solution(s). The reference point(s) helps to focus the search towards preferred solution(s). Consider the case of $\mathbf{M} = [1.25, 1.25, 1.25, 1.25, 0]$, the standard deviation is 0.559. Thus, solutions of interest should at least have $Obj_M < 0.559$ when there are 5 sectors. The total coordination workload (Obj_C) on the other hand should be dependent on the total number and length of flights.

The use of reference point(s) is less restrictive as compared to the constraint-based because it allows for a larger range of feasible solutions. Consider a solution with

Figure 3.3: Variations of monitoring workload distribution

$\mathbf{M} = [1.3, 0.9, 0.9, 0.9, 1]$ in Figure 3.3(c). Sector 1, in this case, would have violated the constraint of $M_j \leq 1.25$, but the Obj_m is only 0.1732. However, the given solution would be considered by ATC if the sector workload of $1.3 * \mu_m$ is not above the upper limit and the given sector shapes in the sectorization are feasible.

R-NSGA-II [36] The reference-point-based strategy is implemented through the use of R-NSGA-II. The generation of new solutions such as crossover and mutation operations in R-NSGA-II is similar to that of NSGA-II. However, the crowding distance in NSGA-II is replaced by the preference distance in R-NSGA-II, which measures the L2 distance from the chromosome to the reference point defined by the user. Therefore, R-NSGA-II aims to

find multiple solutions near the region of interest indicated by the user instead of finding the whole Pareto-optimal front through diversity measures. Similarly, the preference distance is normalized to the objective values of the current approximate front. The replacement of crowding distance with preference distance allows one to obtain preferred solutions near the Pareto front.

3.3 Simulation Study

A set of one-hour historical flight data in the Singapore regional airspace is used for the purpose of this study [46]. The region of airspace that is of interest is bounded by latitude $[-1, 7]$, longitude $[99, 107]$ and altitude $[24000, 42000]$ ft. The 2D sectors are being generated using Voronoi diagrams and 3D sectors are formed using a vertical split on a particular sector. We refer to the 2D sector shapes as areas (K) and the total number of sectors as sectors (N).

3.3.1 Simulation Details

The study has been carried out using the software implementation of NSGA-II, NGPM v1.4 (NSGA-II Program for MATLAB) [79]. The constraint-based and reference-point-based strategy are solved using NSGA-II [32] and R-NSGA-II [36] respectively. Table 3.1 provides the parameter settings used in the experiments.

The total number of input variables is $2 * (5) + (5 - 4) = 11$. The constraint is set according to the proposed concept of having no empty sector. The reference point is set as an estimated value that is optimal to the ATCs. As it is a real-world problem, we are uncertain whether there is a set of global optimal solutions that the solver can converge to. Thus, the experiment is repeated 8 times for a fair comparison. With respect to constraint handling in the algorithms, solutions that do not satisfy the constraints are deemed worse off than feasible solutions.

Table 3.1: Parameter settings

Optimisation Settings	Value/Methodology
NGPM Settings	
Population Size	50
Maximum Generations	2000
Selection	Binary Tournament
Crossover	Intermediate Crossover ($\beta_c = 1.2$)
Mutation	Gaussian ($scale_m = 0.1$, $shrink_m = 0.5$)
Model Settings	
Number of sectors, N	5
Number of areas, K	4
Constraint: $M_j \leq$	1.25
Reference point	[0.1, 28]

3.3.2 Results

Figure 3.4: Constraint-based strategy

Figure 3.4 and 3.5 shows results of the constraint-based and reference-point-based strategies respectively. From Figure 3.4, it can be observed in 7 out of the 8 runs (with the exception of Run 4), the values of Obj_M are all below 0.3. Additionally, 4 out of 8 runs (Run 1,3,4,5) managed to find values of Obj_C above 50. It should also be noted that in Run 7 the solutions found seems to dominate those in other runs but the range of solutions in Run 7 is much smaller. Figure 3.5 shows that the solutions found by the

Figure 3.5: Reference-point-based strategy

reference-point-based strategy find Obj_M above 0.3 in all 8 runs. Also, all solutions found using this strategy have a value of Obj_C below 50. Comparing the two strategies, we can conclude that the reference-point-based strategy allows one to find a more diversified set of solution as compared to the constraint-based strategy.

Solvers such as NSGA-II tends towards a region of current optimal solutions when it could not find better solutions in the next few rounds of generations. Hence, solutions from a single run tend to be similar to one another. Furthermore, it is possible for different runs of NSGA-II to provide different sets of solutions with similar objectives, especially with a different set of solutions for initialization. This point is being supported by the results in Run 7 and 8 presented in Figure 3.6. In Figure 3.6, visualization of the 2D sectors are labelled and being shown in different colours. When the given 2D sectors are labelled as 'Sectors 3-4' (in Figure 3.6(a)), this indicates that the blue area, in this case, is being divided into 2 sectors vertically at an altitude. The bottom sector is always the smaller number, in this case - Sector 3 and the top sector is Sector 4. Both solutions in Figure 3.6 have the objectives $Obj_M = 0.19$ and $Obj_C = 30$, but the shapes of sectors are vastly different. Additionally, it is good to note that both solutions, in this case, satisfy the constraint used in the constraint-based strategy where $M_j \leq 1.25$. In

(a) Run 7

(b) Run 8

 Figure 3.6: Solutions of reference-point-based strategy where $Obj_M = 0.19$ and $Obj_C = 30$

Figure 3.6(a), it can be seen that the coordination workload is high for sector 4, while the coordination workload reflected in Figure 3.6(b) is slightly more balanced. Here, the decision maker would make the final say based on his experience, which could be due to geographical location requirements, traffic patterns in other hours or even the coordination workload. Therefore, we believe that it is important to provide options for the decision maker.

3.3.3 Pareto Front

For a deeper analysis between the two strategies, all found solutions are combined to obtain the Pareto front of best solutions.

Figure 3.7: Pareto front of all solutions from constraint-based and reference-point-based strategies

Figure 3.7 shows the Pareto front of all found solutions. It can be observed that the constraint-based strategy dominates solutions where Obj_M is close to 0 while the reference-based strategy provides a more diversified range of solutions from $Obj_M = 0.0035 - 0.67$. Before going deeper into airspace sectorization, the feasibility of solutions has to be verified first. For this, we consider a hard constraint where no sector should be empty and a soft constraint where $M_j \leq 1.25$. Solutions that do not satisfy the soft constraint is reflected in (a) in Figure 3.7 while solutions that do not satisfy both the hard and soft constraint is reflected in (b) in Figure 3.7. This, in turn, explains why a majority of the solutions found using the constraint-based strategy are below 0.3 - many solutions with $Obj_M > 0.3$ are unable to satisfy the constraint where $M_j \leq 1.25$.

However, we believe that solutions in (a) should be provided for the decision maker to make an informed decision.

(a) $Obj_M = 0.115$, $Obj_C = 24$

(b) $Obj_M = 0.011$, $Obj_C = 28$

Figure 3.8: Solutions closest to reference point $[0.1, 28]$

Next, the solutions near reference point $[0.1, 28]$ are analyzed in Figure 3.8. Figure 3.8(a) and 3.8(b) display solutions with the closest value of Obj_M to 0.1 and Obj_C to 28 respectively. The sector shapes in both solutions look similar to each other. This might indicate that this particular airspace sectorization provides the Pareto-optimal solutions for the ATCs given this traffic scenario. It is worthy to note that the solution in Figure 3.8(b) dominates the selected reference point of $[0.1, 28]$. Given that there could be other factors that could contribute to the decision maker's choice of sectorization, we believe

that non-optimal solutions (solutions not on the Pareto front) that are near the reference point should be considered by the decision maker for selection.

3.3.4 Region of Interest

Figure 3.9: Region of interest for airspace sectorization

Figure 3.9 provides all solutions in the region of interest ($0 \leq Obj_M \leq 0.2$) obtained by the constraint-based and reference-point-based strategies in the simulation study. The reference point of $[0.1, 28]$ is indicated by a black cross. Results indicate that the reference-point-based strategy is generally able to find better solutions in terms of Pareto-optimality. These findings are supported by another study [1, 2], where the use of bliss points and interaction are used to improve the search for Pareto-optimal solutions. The study concluded that the use of a desired solution (bliss point) can aid the search towards finding better solutions in terms of Pareto-optimality, overcoming possibilities of poor parameter settings or genetic drift present in evolutionary methods. However, we believe that all solutions, especially vastly different shapes of sectors, should be presented to the decision maker for selection.

3.4 Summary

This chapter proposes two preference-based approaches to airspace sectorization. This direct use of a reference-point-based strategy for preference indication was compared to the indirect use of constraint-based strategy in a simulation study. The key findings in the study are solutions with similar objectives in multiple runs can provide significantly different airspace sectorization. Moreover, results show that the reference-point-based strategy provides both a larger range of solutions on the Pareto front and better solutions in the region of interest. Therefore, we believe that parallel runs of reference-point-based optimisation should be carried out and all solutions within the region of interest should be presented to the decision maker for selection. Instead of presenting the found solutions to the decision maker, it would be interesting to involve the decision maker in the optimisation process in a interactive multi-objective optimisation framework [1, 2].

The next chapter extends the proposed optimisation model for designing airspace sector shapes to a dynamic real-world scenario that addresses varying traffic flows over time.

Chapter 4

Cognitive Methods For Dynamic Airspace Sectorization

This chapter provides cognitive methods for Dynamic Airspace Sectorization (DAS). The main objective in DAS is to partition the airspace into multiple sectors in such a way that the workload of air traffic controller for each sector is minimized and balanced at all times. Hence, this involves the changing of sector shapes - resectorization according to air traffic flows. However, drastic changes in sector shapes could create problems for the air traffic controller during operations. This concern was previously addressed in literature with the use of the sector similarity concept [105] during resectorization. However, there were not many details on the implementation of dynamic resectorization to tackle the DAS problem.

This chapter addresses the present gap in the literature of DAS with two different methodologies. First, the flight data used in the experimental studies of both methodologies is introduced in Section 4.1. In Section 4.2, we provide a Cognitive decision-making architecture for DAS (CDAS) that addresses (1) whether there is a need for resectorization and (2) provides a resectorization with high sector similarity if required. The experimental study of CDAS is provided in Section 4.3. However, CDAS does not necessarily guarantee the operational feasibility of solutions obtained from the optimisation solver. To this end, a rolling horizon optimisation approach for DAS that considers future

traffic flows is presented in Section 4.4. The accompanying experimental study for the rolling horizon optimisation approach is given in Section 4.5. Lastly, a summary of the chapter is given in Section 4.6.

4.1 Flight Data: Singapore Flight Information Region (FIR)

In the experiments, a set of historical flight data on 1st June 2015 obtained from FlightAware [46] is being used. The flight data is made up of ADS-B data with 2-4 positions per minute depending on the coverage of the area and ANSP radar data from governments that are about 1 position per minute. Unfortunately, the tracks given only comprises of location up to 108 in longitude. Thus, the flight data from the longitude of 108 to 116.3 were extrapolated from the provided flight plans to show a better representative illustration of traffic in the Singapore FIR. Moreover, the missing data on the altitude of certain flights is rectified with the use of historical information and flight plans. This study considers flight data that is above flight level FL180.

4.1.1 Traffic Complexities in Singapore FIR

The flight trajectories during the peak and non-peak hours in the Singapore FIR are shown in Figure 4.1. Only trajectories in the upper airspace (above FL 180) have been plotted in these figures. The shape of the Singapore FIR is also shown at the bottom of these figures. Singapore Changi airport is represented by a blue dot at the bottom of the figure. Departing and arriving flights from and to Singapore Changi airport are represented by the green and red trajectories respectively. The blue trajectories in the figures represent the passing flights from other airports in the region.

From Figure 4.1, it can be observed that

Figure 4.1: Flight trajectories in Singapore FIR on 1st June 2015. Green trajectories represents departures from Singapore Changi Airport. Red trajectories represents arrivals into Singapore Changi Airport. Blue trajectories represents flights between other airports.

- Flights originating and terminating at Singapore Changi Airport are climbing and descending respectively. The gradients of the departing flights are generally steeper than those of the arriving flights.
- Traffic densities vary greatly during the peak (Figure 4.1a) and non-peak hours (Figure 4.1b)

Therefore, there are significant amounts of altitude changes in the Singapore FIR which are evident in flights departing from and arriving at the Singapore Changi airport.

Figure 4.2a presents three trajectories in the 2nd hour based on the radar flight data in the vertical plane and Figure 4.2b the corresponding horizontal traces. A heading change is observed from Figure 4.2b, where the aircraft indicated by the blue trajectory shifted from a north-east direction towards an east direction. A speed change is observed in both the views. The aircraft indicated by the green trajectory is illustrated by an inconsistent gap between the scatter points. This indicates that the aircraft's speed is changing given

(a) Vertical Plane

(b) Plan View

Figure 4.2: Illustration of heading, speed and altitude changes (HC,SC,AC) on part of the flight data used

that the duration between each scatter point is constant. Altitude changes are observed in Figure 4.2a. Both aircraft indicated by the red and green trajectories undergo altitude changes. This illustrates the traffic complexity components in the flight data used.

It is vital that the metric employed accurately express the true ATC monitoring workload of the given sector. The traffic complexity components of the dynamic density metric are calculated using Eq. 2.1 in the experiments. The changes in heading, speed and altitude (θ_{HC} , θ_{SC} , θ_{AC}) are stored as locations where these changes occurred. The other factors (θ_{MD5} , θ_{MD10} , θ_{CP25} , θ_{CP40} , θ_{CP70}) are pegged to the flight and point at which they occurred.

4.1.2 Changing Traffic Flows in Singapore FIR

Figure 4.3 shows the 4 hours of air traffic flows being considered in the experiments. The 2D projections and 3D trajectories of the trajectories (on latitude and longitude) are provided on the left and right of Figure 4.3 respectively. It can be observed that the traffic density varies over time and the traffic flows differ spatially in the different sampling time periods. Looking at Figure 4.3, one can see that the traffic density increased rather significantly from Hour 1 to Hour 2. From the 2D projections of trajectories, the spread of traffic flows from Hour 2 to 3 can be seen to increase, with traffic flows from Hour 3 covering over a wider area. The traffic flows again changes from Hour 3 to Hour 4. A greater number of intersecting flights can be observed.

4.2 Cognitive Decision-making Architecture for Dynamic Airspace Sectorization (CDAS)

Cognitive decision-making architecture for Dynamic Airspace Sectorization (CDAS) intends to manage growing traffic flow and supply an effective decision-making process for operations. The primary objective of CDAS is to find optimal 3-dimensional sector

Figure 4.3: Air traffic flows used for experimental study

shapes such that the conflicting workloads of air traffic controllers are optimised along with minimal changes to sector shapes over a time horizon.

Figure 4.4: Cognitive decision making architecture for dynamic airspace sectorization

The overview of CDAS is depicted in Figure 4.4. CDAS is broadly made up of two major components, viz., a cognitive engine at the base level and a metacognitive decision maker at a higher level. The cognitive engine is made up of a 3D Dynamic Airspace Re-sectorization (3D-DARS) model which can calculate the different workloads for varying sectorizations based on the traffic (using the sector shape and the traffic data as input). The problem of sectorization is built as a multi-objective optimisation problem with constraints. The NSGA-II solver is utilised to get the Pareto optimal solutions (output of the cognitive engine). With the approximate Pareto-optimal solutions achieved for the given traffic data, the metacognitive engine refines the group of optimal solutions to meet any ATC requirements (for example controller preferences). Moreover, the metacognitive decision maker decides on a decision logic to achieve the best solution for strategic

planning goals based on the predicted future traffic patterns. A typical time frame in a pre-tactical flight planning phase in air traffic management of 6 hours is chosen for the analysis of future traffic patterns. Furthermore, the metacognitive decision maker sends a request to the cognitive engine to perform a resectorization when the current solution does not meet the requirements for certain future traffic periods. In the case of resectorization, the chosen sectorization in the current time period t is needed as input for calculation of sector similarity for the time period $t + 1$.

4.2.1 Cognitive Engine

Before presenting the multi-objective optimisation formulation of the 3D-DARS problem, the concept of sector similarity is first defined.

Sector similarity model The change in sector shapes can be measured by the sector similarity metric [105]. The overlapping areas of two sector shapes from two different airspace sectorizations is measured. Details on the calculation of sector similarity metric is previously given in Eq. 2.2 and Eq. 2.3 in Section 2.1.2.

Figure 4.5: Adapted from [105]. Matching of sectors from previous and current sectorization. Dashed boundaries are sectors from previous airspace sectorization. Solid boundaries are sectors from current sectorization

However, one current sector may overlap with multiple earlier sectors and vice-versa, making it difficult to measure sector similarity. This issue is illustrated Figure 4.5. Both

sectors $S1$ and $S2$ have overlapping areas (A and C) and (B and D) respectively with two previous sectors. In this case, only the maximum area (A), highlighted in red, is considered as the overlapping area of $S1$ with previous sectors. As for area $S2$, even though area (B) is bigger than area (D), area (D) is considered as the overlapping area as the previous sector area (B) has been taken by $S1$.

In order to ensure that the same parent (old) sector is not mapped to two children (new) sectors, a one-to-one parent-child relationship between the old and new sectors is established before finding the sector similarity. This relationship is found by treating the mapping as an assignment problem where one needs to maximize the sum of similarity ratios of all sectors. The assignment is found by using integer linear-programming with the constraint that every old sector is mapped to only one new sector. It is formulated as

$$\begin{aligned}
 \text{Max} : & \sum_{i=1}^{\eta} \sum_{j=1}^{\eta} (r_{ij} * x_{ij}) \\
 \text{Subject To} : & \sum_{i=1}^{\eta} (x_{ij}) \leq 1 \quad \forall j, \\
 & \sum_{j=1}^{\eta} (x_{ij}) \leq 1 \quad \forall i, \\
 & \text{and } x_{ij} \in \{0, 1\}
 \end{aligned}$$

Here, i is the index of old sector and j is the index of the new sector. r_{ij} is the similarity ratio between sector- i and sector- j , where $r_{ij} = \frac{S_i \cap S_j}{S_i}$. x is the design variable where $x_{ij} = 1$ implies sector- i is mapped as the parent of sector- j . The values found would reflect the familiarity of the ATCs with the new sectors based on the mapping, providing a more accurate formulation of sector similarity and is denoted as r_j .

4.2.1.1 Multi-objective optimisation formulation of 3D-DARS problem

The multi-objective optimisation formulation of the 3D-DARS problem is extended from the constraint-based strategy model proposed in the preference-based airspace sector-

ization. For the convenience of the reader, the key aspects of the model is highlighted below.

Objective 1: Minimizing the standard deviation of monitoring workload across

all sectors For a given traffic pattern in j sector, the monitoring workload ($M_j = \rho_j$) is computed using the dynamic density measure given in Eq. 2.1. The first objective is to balance the monitoring workload across the ATCs in various sectors. Here, the objective is formulated as a minimization of the standard deviation of monitoring workload and is formally defined as

$$Obj_M = minimize \left(\sqrt{\frac{1}{N-1} \sum_{j=1}^N (M_j - \mu_m)^2} \right) \quad (\text{Eq. 4.1})$$

where $\mu_m = \frac{1}{N} \sum_{j=1}^N M_j$.

Objective 2: Minimizing the total coordination workload summed up over all

sectors The second objective is to minimize the total coordination workload of ATC for a given traffic pattern. Let $C = [c_1, c_2, \dots, c_N]$ be coordination workloads for all sectors. The objective can be defined as minimization of the total coordination workload summed up over all sectors and is formally defined as

$$Obj_C = minimize \left(\sum_{i=1}^N C \right) \quad (\text{Eq. 4.2})$$

Objective 3: Maximizing the total sector similarity

In the case of dynamic resectorization, the shapes of previous sectors seen by ATC should be similar to the current shapes of the sectors. To achieve maximum similarity, the third objective has been formulated as the maximization of the average sector similarity (measure) and is given as

$$Obj_S = maximize \frac{1}{K} \sum_{j=1}^K r_j \quad (\text{Eq. 4.3})$$

Note that the sector similarity is measured in the 2D plane. To satisfy the right prism constraints, the sector similarity of 3D sectors are calculated based on the 2D mapping of the respective 3D sector shape.

Constraint 1: Minimum distance between the crossing points (within a sector) to the boundaries of that sector For the safe operation of aircraft and avoidance of potential conflicts between aircraft in a sector, the ATC requires a minimum distance between the sector boundaries and the crossing traffic flow. The crossing point is defined based on a range of points where aircraft are within a lateral distance of 0-10 nautical miles and a vertical separation lower than 1000 feet.

The minimum Euclidean distance (δ_j) between the crossing points (**CP**) and the sector boundary is defined as

$$\delta_j = \min ||\mathbf{CP} - B_j||, \quad j = 1, 2, \dots, K \quad (\text{Eq. 4.4})$$

where **CP** is a vector representing the crossing points and B_j stores information on the sector boundaries of sector j .

The minimum distance is also set as 10 nautical miles (NM) in this study. Based on this, the following safety constraints are imposed:

$$C1 : \delta_j > 10NM, \forall j \quad (\text{Eq. 4.5})$$

Constraint 2: Maximum monitoring workload in a sector As raised in Chapter 3, there are a set of ATC preferred solutions. With multiple objectives, minimizing the standard deviation of the monitoring workload may not be sufficient to ensure no overloading of monitoring workloads for controllers. To avoid this overloading, a constraint

has been imposed in the optimisation problem based on the percentage deviation from an average monitoring workload (M_a) and is defined as

$$C2 : M_j < (1 + \epsilon) * M_a, \quad j = 1, 2, \dots, N \quad (\text{Eq. 4.6})$$

where M_j is the monitoring workload of sector j , $M_a = \frac{1}{K} \sum M_j$ and ϵ is a user-defined value that is between 0 and 1. The term ϵ determines the limit on maximum monitoring workload. The lower value of ϵ produces more balanced monitoring workload. On the other hand, it provides fewer solution on the Pareto front. The higher value of ϵ provides greater variation and hence provide more solutions on the Pareto front.

Summary of multi-objective optimisation problem The multi-objective optimisation for the 3D-DARS model can be summarized as follows.

Objectives:

- (i) Minimize the standard deviation of monitoring workload between sectors (Obj_M)
- (ii) Minimize the total coordination workload across all sectors (Obj_C)
- (iii) Maximize the average sector similarity between current and previous sectors (Obj_S)

Constraints:

- (i) Distances between the crossing points and the sector boundaries $> 10NM$ ($C1$)
- (ii) Maximum monitoring workloads are kept within user-defined limits ($C2$)

4.2.2 Metacognitive Decision Maker

The metacognitive decision maker obtains Pareto optimal solutions for a given traffic during time period $t + 1$, ATC requirements and the predicted future traffic pattern based on a time horizon ($t + 2$ to $t + 7$). The prediction of future flight track data can be found with flight plans and a simple aircraft model.

The metacognitive decision maker first picks those solutions that satisfy the ATC requirements and determines the decision logic created using the predicted traffic. There are two essential ATC requirements. For better operations, ATCs prefer sectors shapes where traffic flows parallel with the sector boundaries are far away from the sector boundaries. In addition, ATCs prefer continuous airspaces in a sector. The Pareto optimal solutions may have discontinuous airspaces in non-convex airspace like the Singapore Flight Information Region (FIR). Such discontinuous smaller airspaces can be merged with other sectors in cases where there would be minimal adjustments in workloads. Otherwise, that solution would be rejected.

The creation of a decision logic begins with the observation of future traffic pattern and is devised using the workload models given in the cognitive engine block. Using the future workloads in $t+2$ to $t+7$, the metacognitive decision maker derives a decision logic to pick a solution out of many Pareto optimal solutions filtered by ATC requirements and also decides when to call the cognitive engine to perform a resectorization of the airspace. The decision logic consists of rules that are aligned with the objectives of the model, viz., the monitoring and coordination workload of ATCs. They are as follows:

(i) Standard Deviation of Monitoring Workload ($SD(\mathbf{M})$):

$$SD(\mathbf{M}) \leq \gamma_m * \frac{1}{N} \max_{j=t+1}^{t+7} \left(\sum_{i=1}^N \mathbf{M} \right) \quad (\text{Eq. 4.7})$$

(ii) Maximum Coordination Workload of a Sector ($\max(C_j)$):

$$\max(C_j) \leq \gamma_c * \max_{j=t+1}^{t+7} (\#aircrafts), \quad j = 1, 2, \dots, N \quad (\text{Eq. 4.8})$$

where $t + 1$ is the time period of the solution for implementation, γ_m and γ_c are user-defined values ranging from 0 to 1. Higher values of γ_m and γ_c lead to a bigger selection pool of solutions and also decrease the probability of resectorization, while lower values

help the user to identify the solution he is looking for. $\mathbf{M}$ is calculated using the chosen solution in the current time period (t), in the case of $t = 0$, any Pareto-optimal solution in $t = 1$ could be utilised to estimate $\mathbf{M}$.

4.3 CDAS Experimental Study

Detailed experimental studies conducted using CDAS in the Singapore FIR using actual radar flight data is given in this section. The Voronoi sites in the solutions are generated within the given Singapore FIR.

4.3.1 Implementation Details

The software implementation of NSGA-II, NGPM v1.4 (NSGA-II Program for MATLAB) [79] is used for the experimental study of CDAS. The parameter settings used in this experiment is presented in Table 4.1.

Table 4.1: Parameter settings

Optimisation Settings	Value/Methodology
NGPM Settings	
Population Size	50
Maximum Generations	2000
Selection	Binary Tournament
Crossover	Intermediate Crossover ($\beta_c = 1.2$)
Mutation	Gaussian ($scale_m = 0.1$, $shrink_m = 0.5$)
Model Settings	
Number of sectors	7
Number of areas	4
ϵ (Constraint 2 (Eq. 4.6))	0.5

4.3.2 Results and Discussion

The performance of CDAS is evaluated over different time horizons in this section. In particular, the metacognitive decision-making process of CDAS is illustrated with the

results and analysis of the final selected solution. For this purpose, three cases, namely, a) initialization of the optimal sectors for a given traffic and evaluation of the suitability of found solutions for future traffic; b) the resectorization approach and its effectiveness; c) how metacognitive solution picks a selection, that is suitable for a longer time horizon based on the predicted air traffic are considered. The three cases are presented using the Singapore FIR flight data in a) Hour 1, b) Hour 2 and c) Hour 3 and 4 respectively. Therefore, the comparison of solutions in these three cases would allow one to understand the decision-making process in CDAS.

4.3.2.1 Initial optimal 3D sectorization

First, the optimal sectors (shapes) for the first hour (First hour 7 am on 1st June 2015) of traffic flow need to be acquired. The 3D-DARS model is transformed into a bi-objective optimisation (only the objectives related to ATC workloads are taken into account and sector similarity is ignored) problem with the safety and workload constraints for this purpose. The NSGA-II solver discovers solutions that approximate the Pareto-optimal front. Here, a total of 50 unique solutions are found on the Pareto front.

Figure 4.6: Pareto front of Hour 1

The approximate Pareto front on the objective space (the standard deviation of the monitoring workload vs the total coordination workload) is given in Figure 4.6. The

Figure 4.7: Objectives of optimal solutions in Hour 1

plot reflects the optimal performance attained by all these 50 solutions. There are nine different clusters on the Pareto front. Most clusters consist of a few solutions, with the exception of one cluster with 30 solutions. For a better understanding of the two extreme solutions of the Pareto-front (points marked in the figure as A and B), the monitoring and coordination workloads of the individual sectors are given (as points) in Figure 4.7. The extreme point A of the front represents the case of a balanced monitoring workload (the standard deviation is 1.09) and a relatively high total coordination workload ($Obj_c = 17$). Figure 4.7(a) corresponds to the extreme point A with the most balancing monitoring workloads across all the sectors, where the coordination workload varies between 1 to 5 in all these sectors. On the other hand, the extreme point B represents a lower total coordination workload ($Obj_c = 4$) whereas monitoring workload is highly unbalanced (the standard deviation is 9.07). Figure 4.7(b) corresponds to the extreme point B with a minimum total coordination workload. Note that the minimum total coordination workload leads to a large variation in the monitoring workload (7 to 35) and some sectors do not have any coordination workload. Solutions chosen in the neighbourhood of point C in Figure 4.6, have a better balance and suggest a compromise between the two conflicting objectives. Choosing one solution for final operational use is a challenging problem in

practice. In the next sections, we address how the proposed metacognitive decision maker aids in choosing the best solution from a pool of Pareto-optimal solutions.

4.3.2.2 Metacognitive Decision Maker for Hour 1

The metacognitive decision maker processes a pool of Pareto-optimal solutions in two steps. The metacognitive decision maker first narrows down the pool of solutions to those that meet the two ATC requirements. Here, 4 solutions that did not satisfy the ATC requirements were discarded. The second step includes the computation of sector monitoring workloads and coordination workloads based on the predicted future traffic flows.

Figure 4.8: Estimated average monitoring Workload, estimated total coordination workload and total number of flights for Hour 1 to 7

For a better understanding, the average monitoring workload, total coordination workload and the total number of flights in Hour 1 and the next 6 hours of a randomly chosen solution are shown in Figure 4.8. The total number of flights (in blue) provides a baseline for the traffic density in the airspace. The comparison of average monitoring workload and the total number of flights in Hour 2 and 3 allows us to notice that the average monitoring workload is higher for Hour 3 despite having fewer flights than Hour 2. This pattern again can be found in Hour 5 when compared to Hour 6. This

observation accentuates the need to gauge traffic complexity in the airspace. Another insight that can be derived from Figure 4.8 is how the total coordination workload is not always correlated with the total number of flights. The total coordination workload in Hour 3 is higher than Hour 2 despite having fewer flights. This indicates that the sectors used for this estimation are more suitable for the traffic flows in Hour 2 (less total coordination workload) as compared to Hour 3 because the total coordination workload is highly dependent on the sector shapes. Hence, the coordination workload rule is created using the total number of aircraft instead of the total coordination workload. One should also note that using different solutions too produce slightly dissimilar average (or total) monitoring workload. However, experiments carried out concluded that this variation is rather insignificant for our purpose.

With the consideration of how the sector shapes and traffic patterns affect the coordination and monitoring workloads, the metacognitive decision maker utilises two decision rules to choose the most suitable sector shape for operational purpose. Following the ATC specifications, the parameters γ_m and γ_c are set as 0.3 and 0.2 respectively. The decision rules are modified accordingly and shown below:

(i) Monitoring Workload: $SD(\mathbf{M}) \leq 27.93$

(ii) Coordination Workload: $\max(C_j) \leq 12, \quad j = 1, 2, \dots, N$

All 46 solutions can only satisfy the decision rules for Hour 1 because of the significant increase in predicted traffic for the next hour, i.e., if we use the Pareto-optimal solution obtained in Hour 1 for the next hour, the workloads in the next hour will not be balanced. Figure 4.9 illustrates the above scenario through presenting the monitoring and coordination workloads for Hour 2 based on one solution obtained from Hour 1. The total coordination workload is found to be 21, with the maximum coordination workload of a sector being only 6 (in Sector 4). This complies with the coordination workload

Figure 4.9: Objectives of unchanged airspace sectorization in Hour 2 using a selected solution in Hour 1

Figure 4.10: Solution selected by metacognitive decision maker for the first hour

rule generated by the decision logic. However, the monitoring workload varies from as low as 17.99 in Sector 6 to as high as 150.41 in Sector 3. The standard deviation of the monitoring workload among all sectors is 51.31 (indicating a high imbalance), which does not fulfill the decision logic criteria of 27.93 that was given earlier. Therefore, the metacognitive decision maker picks a solution which gives one of the balanced solutions for the first hour. Additionally, it sends a request to the cognitive engine to execute the dynamic resectorization for the second hour.

The plan and elevation view of the final selected sectors along with traffic flow for Hour 1 is presented in Figure 4.10. The Voronoi site is represented as '*' in the plan view

of the solution given in Figure 4.10(a). There are a total of four 2D sectors named as Sector 1-2, Sector 3, Sector 4-5, and Sector 6-7. Sector 1-2, 4-5 and 6-7 have one division in flight level, which are clearly illustrated in Figure 4.10(b). In an airspace with vertical splits, the lower airspace sector is represented by the smaller sector number. The lower and upper sector is Sector 4 and 5 respectively in the blue area. The non-convex shape of Singapore FIR is outlined in black. The blue scatter points provide the flight trajectories formed using the radar flight data. From Figure 4.10(b), one can observe that the flights are either climbing or descending. Figure 4.11 depicts the corresponding workloads of these sectors for Hour 1.

Figure 4.11: Objectives of selected solution in Hour 1

4.3.2.3 Resectorization for Hour 2

The resectorization is carried out with the third objective of sector similarity for the second hour. The cognitive engine discovers the Pareto front for this 3-objective optimisation problem. The found Pareto front consisting of 50 different solutions are then passed to the metacognitive decision maker. Out of the 50 solutions, 11 solutions that do not meet the two ATC requirements or the decision rules are taken out of deliberation.

Figure 4.12 shows the solutions that satisfy the ATC requirements in red and removed in blue. From Figure 4.12(a), it can be observed that solutions that satisfy both ATC

Figure 4.12: Pareto-optimal solutions in Hour 2. Red: Satisfies the ATC requirements. Blue: Does not satisfy ATC requirements.

requirements are closer to lower values of Obj_m and Obj_c . These solutions are also inherently more dissimilar to the previous sectors, as reflected in Figure 4.12(b) and 4.12(c).

Using the same parameters $\gamma_m = 0.3$ and $\gamma_c = 0.2$, the decision rule for Hour 2 are altered as given below:

- (i) Monitoring Workload: $SD(\mathbf{M}) \leq 34.36$
- (ii) Coordination Workload: $\max(C_j) \leq 15, \quad j = 1, 2, \dots, N$

Similar to the occurrence seen in the first hour, none of the remaining 39 solutions meet the current decision logic for the next hour viz. for Hour 3. This leads to a resectorization for the 3rd hour. The metacognitive decision maker then picks a solution which has a better workload balance with maximum sector similarity for Hour 2, marked by (x) in Figure 4.12(b).

The performance objectives of the chosen solution in Hour 2 obtained by resectorization is given in Figure 4.13. The solution has a good balance of monitoring workloads among all the sectors, with the values of all sectors varying between 60 and 80 (refer to Figure 4.13). Further, this observation is supported by the value of standard deviation among monitoring workloads being 3.53. The total coordination workload is 23, with the

Figure 4.13: Objectives of selected solution in Hour 2

maximum coordination workload of a sector being 8. This small increase in coordination workload is exchanged for a better balance of monitoring workloads.

Figure 4.14: Comparison of sectors between Hour 1 and 2

The comparison of sectors chosen in Hour 1 and 2 is given in Figure 4.14. The sectors in Hour 2 is highlighted by the different colours in bold lines and sectors in Hour 1 is outlined by the dotted lines. The blue and green sectors have become slightly smaller while the red and yellow sectors have become a little bigger. In general, the sectors are similar. A sector similarity value of 90.50% affirms this judgement.

One conventional way to manage the workload in the overloaded sector is to divide the sector into two. Figure 4.15 depicts how sector 3 (green area in Figure 4.10a) can

Figure 4.15: Splitting of sector 3 into 2 sectors through bisection in Hour 2

be divided into two sectors, viz. sector 3 and sector 8. Figure 4.15a and 4.15b distinctly displays the split and the final shape of sector 3 and new sector 8 in plan and elevation view respectively.

Figure 4.16: Objectives of bisected sectorization in Hour 2

The obtained performance objectives of sectors after the splitting of sector 3 is shown in Figure 4.16. The monitoring workload remains unaffected for sector 1,2,4,5,6,7 and the monitoring workload for the initial sector 3 is divided into sector 3 and 8. The standard deviation of the monitoring workload is reduced from 51.31 to 35.37. However, this split incremented the total coordination workload from 21 to 24.

Table 4.2: Comparison of objectives for sectors obtained through bisection and resectorization

	No. of Sectors	Standard Deviation of Monitoring Workload	Total Coordination Workload
Bisection	8	35.37	24
Resectorization	7	3.53	23

The comparison between the solutions acquired through the conventional split-and-merge approach[103] and the proposed resectorization approach is given in Table 4.2. The split-and-merge approach tackles the sector with the highest monitoring workload with a division of the sector, keeping the other sector shapes constant. On the other hand, resectorization goes through the cognitive engine again, optimising all the three objectives given in the 3D-DARS model. The results indicate that the conventional approach increases the number of sectors thereby increment the number of controllers in ATC for that hour. From Table 4.2, it can be seen that the proposed CDAS gives a better solution with the slightest changes in sector shapes when compared to the split-and-merge approach. The sole advantage in the conventional approach is that the other sector shapes remain unchanged.

4.3.2.4 Sectors for Hour 3 and 4

The number of solutions on the Pareto front generated for Hour 3 is 50. However, 7 of the solutions could not satisfy the ATC requirement of having a minimum distance between traffic flows parallel to the sector boundaries and the sector boundaries used in Hour 3. Thus, 43 solutions are kept for selection. The rules on monitoring and coordination workloads objectives that form the decision logic for selecting the final solutions in Hour 3 are summarized below:

(i) Monitoring Workload: $SD(\mathbf{M}) \leq 34.54$

(ii) Coordination Workload: $\max(C_j) \leq 15 \quad \forall j$

where $\mathbf{M}$ is the vector consisting of monitoring workloads for all sectors, C_j is the coordination workload of sector j .

Using these rules, the set of Pareto-optimal solutions is reduced from 43 to 38. Next, these 38 solutions are checked to see if any of them satisfy the same rules in the following hours, starting with Hour 4. A total of 37 solutions satisfy these rules in Hour 4. However, no solutions satisfy these rules in Hour 5. Therefore, the final sectorization for Hour 3 and 4 is chosen from the 37 solutions.

Figure 4.17: Pareto-optimal solutions in Hour 3. Yellow: Satisfies decision logic for Hour 3 and 4. Red: Satisfies decision logic for Hour 3. Blue: Does not satisfy decision logic.

Figure 4.17 shows the Pareto-optimal solutions obtained in Hour 3. 37 solutions that satisfy the decision logic are highlighted in yellow. It can be observed these solutions have better performance objectives in Figure 4.17(a). Furthermore, these solutions are less similar to the sectors used in Hour 2 (look at Figure 4.17(b) and 4.17(c)). In this case, a random solution ((x) in Figure 4.17(b)) with a good balance of monitoring workloads and high similarity is selected for illustration.

Figure 4.18a and 4.18b provide the performance objectives for the selected solution that satisfies the rules in Hour 3 and 4 respectively. The monitoring workloads are relatively well balanced among the sectors. This is too indicated by the standard deviation of the monitoring workload, $SD(\mathbf{M})$ being 30.28 and 21.19 for Hour 3 and 4 respectively.

Figure 4.18: Objectives of selected solution by resectorization in Hour 3 and 4

The total coordination workload are 42 and 47 for Hour 3 and 4 respectively. The maximum coordination workload are 12 in Sector 5 and 11 in Sector 3 and 7 in Hour 3 (Figure 4.18(a)) and Hour 4 (Figure 4.18(b)) respectively.

Table 4.3: Operational threshold characteristics for selected solution in Hour 3 and 4

Operation Thresholds	$SD(\mathbf{M})$ ≤ 34.54	$max(C_j)$ ≤ 15
Hour 3	30.28	12
Hour 4	21.19	11

Table 4.3 summarises the characteristics of a chosen solution that meets the operational thresholds set by the decision rules in Hour 3 and 4. The standard deviation of the monitoring workload is below 34.54 and the maximum coordination workload for a sector is below 15. Hence, the sectors remain constant from Hour 3 to 4.

The comparison between sectors in Hour 3 and 4 and in Hour 2 is given in Figure 4.19. A high similarity in sector shapes is observed. This is supported by a sector similarity value of 90.21%.

CDAS is able to aid the decision maker by providing a set of optimal sector shapes for the current time period, with the future predictive workloads for each particular solution. However, it does not guarantee that an operationally feasible solution is available

Figure 4.19: Comparison of sectors between Hour 2 and 3

for future time periods. For this purpose, a rolling horizon optimisation approach for dynamic airspace sectorization is presented in the next section.

4.4 Rolling Horizon Optimisation Approach for Dynamic Airspace Sectorization

Rolling horizon optimisation allows optimisation of the systems at the current time while taking into account the planned traffic flows in future based on a finite horizon, but only implementing the solution for the current (first) interval at the current time. This process is repeated at the next time instant with planned traffic flows in the next time horizon. Hence, rolling horizon optimisation has the ability to take appropriate control actions now keeping the future in mind.

In the Dynamic Airspace Sectorization (DAS) problem, the states are traffic flows in the airspace and the control actions are the determination of proper sector shapes for the airspace. The objective function for the optimisation is to keep the monitoring workloads for the sectors balanced. The ultimate objective of the airspace sectorization problem is to provide optimal sector shapes for each time period such that the monitoring workload between the controller are balanced when the traffic flows change and also meeting the

Figure 4.20: Rolling horizon optimisation framework for dynamic airspace sectorization

required safety and coordination workload constraints with minimum changes in the shapes of the sectors between these time periods. The use of rolling horizon optimisation in DAS allows one to ensure that the shapes of sectors meet a user-defined sector similarity in the given finite-time horizon, which otherwise would not be captured when optimising only for a single time period at a time. Being a non-linear optimisation problem which cannot be fully represented in mathematical models, the actual optimisation tool used in this work is a genetic algorithm.

4.4.1 Overall Framework of Rolling Horizon Optimisation

Fig. 4.20 gives an overall description of the rolling horizon optimisation framework for the DAS problem. The time interval considered for this study is 1 hour. At the current time period t , the sectors used in $t + 1$ to $t + p$ are being considered (where the finite

horizon is p). The sectors in $t + 1$ to $t + p$ periods are being optimised with the objective of balancing the ATC monitoring workload M over all the periods. The optimal sectors in each time period are subjected to the spatial constraints of the Singapore FIR, safety constraint and the maximum coordination workloads of the sectors $\mathbf{C} = [c_1, c_2, \dots, c_N]$ where N is the total number of sectors. The number of sectors is being kept unchanged throughout such that the shapes of sectors can be similar and comparable. Between the current and next time period, the optimisation of sectors is subjected to a lower bound on the sector similarity metric. A normalized standard deviation of monitoring workload $(M^{t+1}, M^{t+2}, \dots)$ is calculated for each period based on the sectors in the solution set. With that, the mean M over $p + 1$ periods is utilized as the performance index for minimization.

4.4.2 Details on the Rolling Horizon Optimisation Formulation

4.4.2.1 Objective: Minimizing the standard deviation of monitoring workload across different time periods

For a given traffic pattern in j sector, the normalized monitoring workload ($M_j = \sqrt{\frac{\rho_j^2}{\sum_{j=1}^N \rho_j^2}}$) is computed using the dynamic density measure (ρ) [75] given in Eq. 2.1. This normalization allows for a fair comparison of monitoring workloads across different time periods. The objective for each time period, t , is to balance the monitoring workloads across the controllers in different sectors. This is mathematically expressed as

$$M^t = \min\left(\sqrt{\frac{1}{N-1} \sum_{j=1}^N (M_j - \mu_m)^2}\right), \quad (\text{Eq. 4.9})$$

where $\mu_m = \frac{1}{N} \sum_{j=1}^N M_j$.

The main objective of the optimisation is to minimize M , the mean standard deviation of monitoring workloads over p time periods and is formally defined as

$$M = \frac{\sum_{t=t+1}^{t+p} M^t}{p} \quad (\text{Eq. 4.10})$$

Next, a brief description of the constraints are given.

4.4.2.2 Intra time period constraints 1: spatial constraints

The Voronoi sites $\mathbf{v}_i = v_i^\lambda, v_i^\phi$ of all K 2D sector shapes must exist within the constrained airspace S . More details on sector generation using Voronoi sites can be found in Section 3.1.1. This can be mathematically presented as

$$C1 : \mathbf{v}_i \in S \quad \forall i = 1, \dots, K \quad (\text{Eq. 4.11})$$

4.4.2.3 Intra time period constraint 2 (safety constraint): minimum distance between the crossing points (within a sector) to the boundaries of that sector

In this study, the minimum distance has been set as 10 nautical miles (NM). Based on this, the following safety constraints are imposed:

$$C2 : \sigma_j > 10NM, \quad \forall j \quad (\text{Eq. 4.12})$$

where σ_j is defined in Eq. 4.4.

4.4.2.4 Intra time period constraint 3: maximum sector coordination workload

This constraint assigns a limit on the maximum coordination workload of each sector, ensuring that there is no overload of coordination workloads for controllers. More details on the coordination workload metric can be found in Section 3.1.2.

$$C3 : c_j \leq c_{lim}, \quad \forall j = 1, 2, \dots, N \quad (\text{Eq. 4.13})$$

4.4.2.5 Inter time period constraint: sector similarity

For operational purposes, the shapes of previous sectors seen by the controllers should be similar to the current shapes of the sectors. Details on the sector similarity can be found in Section 4.2.1. The sector similarity SS^t for hour t is the average sector similarity (measure) of all k sector shapes and is given by

$$SS^t = \frac{1}{K} \sum_{i=1}^K r_i \quad (\text{Eq. 4.14})$$

Note that the sector similarity is measured in a 2D plane. To satisfy the right prism constraints, the sector similarity of 3D sectors are calculated based on the 2D mapping of the respective 3D sector shapes. The sector similarity constraint is provided as follows

$$C4 : SS^t \geq SS_{lim}, \quad \forall t = t + 1, \dots, t + p \quad (\text{Eq. 4.15})$$

4.4.2.6 Summary of rolling horizon optimisation model of DAS problem

In a nutshell, the 3-Dimensional Dynamic Airspace Resectorization (3D-DARS) model in the rolling horizon optimisation framework can be summarized.

Input: Voronoi sites and altitude split (continuous-valued variables)

Output: 3D Sector shapes

Objective:

- (i) Minimize the standard deviation of monitoring workload between sectors across time (M)

Constraints:

$C1 - C3$: Implemented within and during each time period

$C4$: Implemented using the sector shapes in the previous time period (if available)

- (i) Voronoi sites must exist within the constrained airspace ($C1$)
- (ii) Distances between the crossing points and the sector boundaries $> 10NM$ ($C2$)
- (iii) Maximum Sector Coordination Workloads are kept within user-defined limits ($C3$)
- (iv) Sector Similarity between different time periods are kept within user-defined limits ($C4$)

4.5 Performance Evaluation of the Rolling Horizon Optimisation Approach

In this section, we provide the details in a experimental study conducted in the Singapore FIR using historical flight data.

4.5.1 Experimental Details

In this section, the details in the experimental study conducted using the Singapore FIR and the historical flight data are presented. The study has been conducted using a GA with a modified version of the software implementation, NGPM v1.4 (NSGA-II Program for MATLAB) [79]. The parameter settings used in this study is shown in Table 4.4.

The population size is 50 and 150 for the single interval and rolling horizon optimisation approach respectively, varied according to the number of input variables. The maximum number of generations is set to ensure that the solutions found by the genetic

Table 4.4: Rolling horizon optimisation parameter settings

Optimisation Settings	Value/Methodology
NGPM Settings	
Population Size (N_p)	50/150(Pred Opt)
Maximum Generations	2000
Selection	Binary Tournament
Crossover	Intermediate Crossover ($\beta_c = 1.2$)
Mutation	Gaussian ($scale_m = 0.1$, $shrink_m = 0.5$)
Model Settings	
Number of sectors (N)	7
Number of areas (K)	4
Number of hours (p) (Pred Opt)	3
Max. coordination workload (c_{lim})	15
Min. sector similarity (SS_{lim})	80%

algorithm converge. The ratio for immediate crossover (β_c) is set to 1.2 (above one) for the diversification of solutions. The scale ($scale_m$) and shrink ($shrink_m$) variables for mutation are set to act as a local search to improve the potentially optimal solutions and allow for convergence towards the end of a run.

In the initialisation, the input variables are set to satisfy the spatial constraint (C1). In the rolling horizon optimisation approach, the input variables for each time period (1,2,3) are initialized with the same values, i.e. the same sector shapes are given for each time period at the start of the optimisation. Sector similarity is applied in all circumstances except in the first time period. In the rolling horizon optimisation approach, the chromosomes previous time period sectorization is used for the measurement of sector similarity in the future time intervals. Further, the selected solution of the future time intervals ($t + 2, t + 3$) will be carried over to the optimisation of the next time interval as part of an individual of the new population.

4.5.2 Results and Discussion

Table 4.5 presents the overall results of the rolling horizon optimisation approach. It can be observed that the mean standard deviation of the monitoring workload over p

Table 4.5: Overall results of rolling horizon optimisation approach

Current interval (t)	M	M^{t+1}	M^{t+2}	M^{t+3}
0	0.016	0.031	0.017	0.001
1	0.008	0.017	0.001	0.005
2	0.007	0.001	0.005	0.014
3	0.013	0.005	0.014	0.022
4	0.014	0.014	0.022	0.006

time periods, M is always below 0.02 from $t = 0$ to $t = 4$. Further, it can be observed that the standard deviation of monitoring workload, $M^{0+2} = M^{1+1}$ and $M^{0+3} = M^{1+2}$. This similar occurrence is also observed in the next time intervals. This means that the solutions found in the earlier time interval are being selected and implemented in the future time intervals. With the sector similarity constraint set at 80%, this observation could indicate that the change in air traffic is gradual.

Next, the implemented solutions are extracted from the rolling horizon optimisation approach and are compared with the results obtained from the single interval optimisation approach in Table 4.6. In period $t = 0, 1, 3$, the M^{t+1} values for the single state and rolling horizon optimisation approach are comparable to each other. However, in the period $t = 2$, the solution obtained for the single interval optimisation approach is worse off than the rolling horizon optimisation approach. Moreover, the single interval optimisation approach is unable to find a solution that meets the sector similarity constraint at period $t = 4$. This shows that the consideration of future time intervals is important in the DAS problem, in order to continuously obtain feasible solutions in the future time horizons. Further, there could be trade-offs in earlier time intervals, to implement lesser optimal solutions (in this case for $t = 1$) so that there are feasible solutions for future time intervals ($t = 4$).

(a) Single interval optimisation approach

(b) Rolling horizon optimisation approach

 Figure 4.21: 2D airspace sectorization (current sectors outlined in bold, previous hour sectors indicated with dotted lines) and monitoring workloads of selected solutions at $t = 2$

Table 4.6: Comparison of objective between the single interval and rolling horizon optimisation approach

Current period (t)	M^{t+1}	
	Single interval	Rolling Horizon Optimisation ($p = 3$)
0	0.037	0.031
1	0.013	0.017
2	0.087	0.001
3	0.006	0.005
4	Unable to meet sector similarity constraint	0.014

4.5.2.1 Detailed results for the time interval $t = 2$

The 2D airspace sectorization and monitoring workloads of solutions obtained by the single interval and rolling horizon optimisation approach in period $t = 2$ is presented in Figure 4.21(a) and 4.21(b) respectively. The variation of monitoring workload in the single interval optimisation approach is greater than the rolling horizon optimisation approach. This is also reflected by the standard deviation of the monitoring workload (M^{2+1}) values in Table 4.6. The 2D shapes of sectors used in $t = 1$ (indicated by the dotted lines) for both approaches are vastly different. Due to the differences in shapes of sectors in $t = 1$, the single interval optimisation method has to trade off the objective value in $t = 2$ in order not to break the sector similarity constraint. Therefore, it can be said that the consideration of future time intervals in the rolling horizon optimisation method helps to obtain better and feasible solutions over a time horizon and that the use of rolling horizon optimisation approach is appropriate for the DAS problem.

4.6 Summary

This chapter presented two different methodologies to tackle the problem of DAS, (1) Cognitive decision-making architecture for Dynamic Airspace Sectorization (CDAS) and (2) A rolling horizon optimisation approach for DAS. CDAS introduces a metacognitive

decision maker to aid the user selecting a solution for the current time period with predictive monitoring and coordination workload for the future time periods. At the same time, CDAS identifies situations where a change in traffic flow is insignificant to reduce the number of resectorization required. On the other hand, the rolling horizon optimisation approach formulates the DAS problem as a single objective optimisation problem, transforming the coordination workload to a constraint. In this way, one can optimise the airspace such that it takes into account the predicted future traffic flows to ensure minimal changes in sector shapes.

In short, CDAS involves the user in the process of decision making by providing a set of approximate Pareto-optimal solutions in a multi-objective setting. On the other hand, the current rolling horizon optimisation approach ensures the feasibility of solutions in the future time intervals in a single-objective setting. However, these two approaches are not mutually exclusive and it would be interesting to extend the rolling horizon optimisation approach into one in multi-objective setting and merge it into CDAS.

In the next chapter, the thesis continues to address the gap in airspace management when there is a change in airspace availability due to weather.

Chapter 5

Simultaneous Optimisation Of Airway And Sector Design For Air Traffic Management

With flight plans made available days in advance, there is adequate time to plan and design optimal sectors. Thus, air traffic flow management (traffic routing) and sectorization are not naturally coupled problems due to the flexibility available in sectorization. The process of air traffic routing and airspace sector design is usually sequentially performed [122, 69]. However, there are frequent scenarios of weather impacted or restricted airspaces, the knowledge of which is available to an ATCC only a few hours in advance. Performing a heuristic re-routing of air traffic to avoid flights through these regions, causes imbalances in the workloads of ATCs. In this scenario, the flexibility in changing the areas of control of ATCs, or increasing the available manpower, is limited. These limitations make the routing and sectorization problems coupled, where it is also highly preferable to perform any re-routing by considering its impact on sectorization. Hence, there is a real need to look at the problem of simultaneous optimisation of both sectorization and routing to minimize and balance the controller's workloads, while also satisfying other sector or flow constraints.

The Dynamic Airspace Routing Tool (DART) is a large scale simulation model for the US National Airspace System (NAS) flow management used in several areas of research

[69]. It is capable of incorporating various re-routing strategies to generate optimal routes that avoid both hazardous weather conditions as well as overloaded sectors. The sector shapes are considered fixed and overloading of sectors is prevented purely through re-routing. Using both sector design and flow management as options to prevent overloading of sectors, [70] investigated a Dynamic Airspace Configuration-Traffic Flow Management (DAC-TFM) strategy. They used the NAS airspace dynamic analysis and partitioning tool for DAC, combined with DART for the total flight management. They propose a sequential strategy - first re-routing to avoid weather restrictions, followed by resectorization for the newly generated routes with sector capacity constraints in mind, and a final round of re-routing with the additional objective to avoid any overloaded sector. Due to the coupled nature of the problem, they also investigated an iterative approach of DAC-TFM loop, but found that it does not improve the solution. One may argue that this is the inherent problem associated with independently designing the sector shapes and routes by keeping one of them constant while designing the other.

In order to overcome the above problem, an approach to simultaneously optimise both the sector design and flow re-routing is proposed and carried out for en-route airspace with restricted airspaces. The major contribution of this work is in the development of a framework called Simultaneous optimisation of both AirWay and AirSpace (SAWAS). This framework incorporates the design objectives of the 3D Dynamic Airspace Resectorization (3D-DARS) such as balancing the ATC traffic monitoring workload, minimizing the ATC coordination workloads, and also maximizing the similarity of sector shapes with a given sectorization. Further, it includes sector design constraints to provide convex sector shapes and sector boundaries that are far from traffic flow crossing points. SAWAS concurrently considers the effects of re-routing and resectorization on the desired design objectives. As a result, it finds both the airspace sectors and re-route strategies that are satisfying the desired objectives. Further, a novel algorithm has been developed for traffic

re-routing-referred to as Conflict Avoidance and Airway Re-Design algorithm (CAARD). This algorithm is used to design airways that follow specified waypoints, avoiding unavailable regions in the airspace. Further, the routing of multiple airways with minimal conflicts is also handled.

This chapter is organised as follows. The basic modelling of airspace and air traffic flow used is first described in Section 5.1. The simultaneous optimisation framework and its sub-components for resectorization and re-routing are presented in Section 5.2. Details on the performance evaluation of SAWAS based on two experimental studies are provided in Section 5.3. The first compares the proposed simultaneous optimisation approach with a sequential optimisation approach, for a typical air traffic management scenario. The second study implements SAWAS in the busy Singapore FIR and evaluates its performance under a more realistic scenario. Finally, Section 5.4 summarizes the main conclusions of the chapter.

5.1 Modelling of Airspace and Traffic Flow

Figure 5.1: Stages of decision making at an air-traffic control centre

Decision-making processes performed in a typical ATCC are categorized based on the frequency at which they are performed, i.e., strategic, pre-tactical, and tactical. Figure

5.1 illustrates the vision for introducing the use of dynamic sector shapes and airway routes into this framework. At the strategic stage, flight plans and knowledge based on past experiences are utilised to find the best sector shapes that balance the ATC workloads across all the sectors. At this stage, the airway route design is independent of the sectorization. At the pre-tactical stage, the sectorization and airways (designed at the strategic level) are modified based on the airspace restrictions and weather forecast data obtained by the ATCC. This modification is applied for a subsequent 2 to 6 hour time window. This is the stage where the combined optimisation of airway design and sectorization is envisioned to be beneficial. At the tactical stage, the individual aircraft pilot and the ATC coordinate to take immediate decisions regarding the flight. At this stage, the sectorization is frozen. Experiences acquired from implementing dynamic sector shapes and routes feed to a knowledge base that helps in future design preferences and decision making.

As an example, a simulated traffic flow pattern in Singapore's southern airspace consisting of three airways is shown in Figure 5.2. The designed sectors with balanced ATC workloads for this traffic flow are also shown. Consider a scenario in which a weather forecast indicates that airway-1 traffic flow would be affected by a weather disturbance (the red dotted circle). This calls for a re-route of the flow. To avoid the weather disturbance, the flow may be diverted either to the north or south of the disturbance. Each of these alternatives will cause a change in the workloads of neighboring sectors and thus an imbalance in the overall workload distribution. One option is to design the re-route strategy first, following which one can handle the imbalance in workloads through sector re-design. However, in this work, the aim is to develop a concurrent strategy involving both airspace and air route changes for results that minimize the sector workload imbalances.

Figure 5.2: Air traffic management scenario in Singapore airspace with airways and sectors indicated (Source: SkyVector, 2015)

5.1.1 Airspace Modelling

Constructed sector shapes are typically presented in 2 (latitude and longitude) or 3 dimensions (including the altitude). In case of 2D representations, the shape is usually considered uniform for all altitude layers. 3D construction, however, allows for more complex sector shapes, especially as required in the case of terminal airspace whose sector shapes resemble an inverted wedding cake. The sectorization component in this work considers designing the 2D sectors for en-route airspace. The sector shapes are generated through the use of Voronoi diagrams which was previously defined in Section 3.1.1.

Weather disturbance and restricted airspace: Unavailable airspaces are abstracted by their locations in a bounding circle. The location is taken as the latitude and longitude of their centers, and their shape as a circular bounding area. Even though

weather regions are previously represented using grid-based and polygon-based shapes [93], the choice of circle is used here in order to maximize the area of available airspace around the periphery. The set of such disturbances is being denoted as D

$$D = \{(\mathbf{d}_1, rad_1), (\mathbf{d}_2, rad_2), \dots, (\mathbf{d}_\delta, rad_\delta)\} \text{ where } \mathbf{d}_k = \begin{pmatrix} d_k^\lambda \\ d_k^\phi \end{pmatrix} \quad (\text{Eq. 5.1})$$

Here, $\mathbf{d}_k$ is the location of the disturbance centre and rad_k is the radius of the circular bounding area. δ is the total number of disturbances. It should also be noted that any arbitrary shape (represented by grid/polygon/circle) can be implemented in CAARD with some modifications as CAARD can simply avoid the placement of waypoints or routes in the hazard areas during the generation of new waypoints.

5.1.2 Traffic Flow Modelling

Airways: The en-route air traffic flow over Singapore is simulated based on the information derived from Jeppesen charts of Singapore FIR. The region considered is 0°N to 2.5°N and 102°E to 106°E . Major routes, also known as airways have been identified as a sequence of waypoints (or fixes). These airways usually allow unidirectional flow and are vertically separated for traffic flowing in the opposite direction. For the simulation, the airways that belong to a particular flight level with a constant uni-directional flow rate are used.

Let $A = \{a_1, a_2, \dots, a_\alpha\}$ be the set of airways and each airway be represented as a sequence of waypoints,

$$a_i = (\mathbf{w}_{ij})_{j=1}^{\beta_i} \text{ where } \mathbf{w}_{ij} = \begin{pmatrix} w_{ij}^\lambda \\ w_{ij}^\phi \end{pmatrix} \quad (\text{Eq. 5.2})$$

Here, the number of airways is denoted as α and the number of waypoints that belong to a_i is denoted as β_i . The flow rate associated with a_i is denoted as f_i .

Figure 5.3: Flow diagram for SAWAS

5.2 SAWAS: Simultaneous Optimisation of AirWay and AirSpace

SAWAS is a framework developed in this work to determine an efficient sector and airway design that balances the workload distribution across sectors. An overview diagram of how SAWAS framework is cast into NSGA-II optimisation is provided in Figure 5.3. It depicts the parallel application of the CAARD for airway design and the Voronoi sector shape generator for airspace design. Using the flight plans and information on the unavailability of airspaces, multiple feasible airway designs are first generated using the CAARD algorithm. With one of the multiple airway designs and Voronoi sector shapes as inputs, the solution is evaluated using a multi-objective optimisation model. This obtained objectives and constraints serve as feedback to the optimiser. The optimiser (NSGA-II) uses this feedback to improve the design through the variations of airway

routes and Voronoi sectors until the termination condition is reached. The best design for the objectives is obtained as the output. The following subsections describe each of these sub-components in more detail.

5.2.1 CAARD Algorithm for Airway Design

A novel path planning algorithm is presented here, referred to as the CAARD. The foundation of CAARD draws its inspiration from A* path planning algorithm [52] that leverages upon classic Dijkstra's shortest path (that guarantees optimal solution) and greedy breadth-first-search (that provides faster solutions using heuristics) algorithms.

The airways are modelled as a sequence of waypoints or fixes in CAARD. It has been developed giving consideration to certain specific requirements for its incorporation into SAWAS. The basic elements of CAARD, viz., search space discretization, airway prioritization, and design of individual airways are elaborated below.

Search space Discretization: The lateral separation criteria for aircraft is 5NM (nautical miles). To have a granularity higher than this, the airspace is divided into regular square grids of side lengths of 2NM or smaller. These grids are used to discretize the search space. A connected graph of finite nodes is constructed from these grids. The connectivity of the grids is based upon the possibility of inter-grid traversal. The four cardinal directions (N, S, E, W) and the four inter-cardinal directions (NE, NW, SE, SW) are taken as the 8 possible directions of traversal from one grid to another.

The airspace is divided into γ unit grids and $N = \{\mathbf{n}_1, \mathbf{n}_2, \dots, \mathbf{n}_\gamma\}$ is the set of all grids. The individual grids are represented by the location of their centers as

$$\mathbf{n}_i = \begin{pmatrix} n_i^\lambda \\ n_i^\phi \end{pmatrix} \text{ for } i = 1 \dots \gamma \quad (\text{Eq. 5.3})$$

Airway Prioritization: The design problem of CAARD involves path planning for multiple airways. In this work, the path planning is done sequentially by designing the

first airway independent of others, followed by the rest. Each subsequent airway attempts to avoid the set of all the previously designed airways. The sequence in which the airways are designed can influence the optimality of the final solution. Hence, CAARD preprocesses the major routes to determine the degree to which each route is affected by the unavailable airspace. Airways that are most affected are assigned higher priorities and are routed first. Contrary to this approach, if unaffected airways were to be designed first, the flexibility in the design of the affected airway will be compromised. However, one may explore alternative approaches of prioritization to suit other requirements.

Individual Airway Design: The design of individual airways presented in Algorithm 3 consists of two steps. The first step is to find the feasible waypoints for the given airway, avoiding all unavailable airspaces (line 9-21 in Algorithm 3). The second step is to generate the route between these waypoints, maintaining sufficient clearances from other airways that have already been designed (line 23-29 in Algorithm 3).

Figure 5.4: Alternate waypoint generation in the presence of unavailable airspaces with multiple disturbances

Generating Feasible Waypoints: To avoid unnecessary path planning, a line-of-sight

Algorithm 3: CAARD

Data: Set of airways $A = \{a_1, a_2, \dots, a_\alpha\}$, airspace restrictions

$D = \{(d_1, rad_1), (d_2, rad_2), \dots, (d_\delta, rad_\delta)\}$ and clearance matrix C

Result: Set of airway routes $R = \{r_1, r_2, \dots, r_\alpha\}$

```

1 Function Main( $A, D, C$ ):
2    $\hat{R} = \phi$ ;                                /* Set of previously designed routes */
3   ;
4   for each airway  $a_i \in A$  do
5      $a'_i = \phi$ ;                             /* Empty sequence of waypoints */
6     put waypoint  $w_{i1}$  into  $a'_i$ ;
7     ;
8     /* Determine feasible waypoints */;
9     ;
10    for each  $w_{ij} \in a_i$  do
11      if LOS between  $w_{ij}$  &  $w_{i(j+1)}$  is obstructed by any  $d \in D$  then
12        Generate alternate waypoint  $w'_{ij}$  to avoid  $d$  based on  $C$ ;
13        while LOS between  $w_{ij}$  &  $w'_{ij}$  is obstructed by any  $d \in D$  do
14          Regenerate alternate waypoint  $w'_{ij}$  to avoid  $d$ 
15          put  $w'_{ij}$  into  $a'_i$ 
16          if  $d_k$  is close to  $w_{i(j+1)}$  then
17            replace  $w_{i(j+1)}$  with  $w'_{ij}$  in  $a_i$ 
18          else
19            put  $w_{i(j+1)}$  into  $a'_i$ 
20        else
21          put  $w_{i(j+1)}$  into  $a'_i$ ;
22    ;
23    /* Determine route of airway */;
24    ;
25     $r_i = \phi$ ;                                /* route (sequence of nodes) of airway  $a_i$  */
26    for each  $w_{ij} \in a'_i$  do
27      if LOS path between  $w_{ij}$  &  $w_{i(j+1)}$  is close to any  $r \in \hat{R}$  then
28        routeNodes = pathPlan( $w_{ij}, w_{i(j+1)}$ )
29      else
30        routeNodes = LOS path between  $w_{ij}$  and  $w_{i(j+1)}$ 
31        put routeNodes in  $r_i$ 
32     $\hat{R} = \hat{R} \cup r_i$ 
33  return  $\hat{R}$ ;

```

check is performed between the waypoint pairs belonging to a particular airway. This helps to pre-determine the presence of disturbances (untraversable airspaces) in the route. Supposing that the straight line path between the waypoints w_{ij} and $w_{i,(j+1)}$ is obstructed, a simple geometric construction is done to stretch the path and avoid the disturbance by introducing a new waypoint w'_{ij} , as illustrated in Figure 5.4(a). The location of this waypoint is determined by the vector from the centre of the disturbance, orthogonal to the line-segment joining the waypoints w_{ij} and $w_{i,(j+1)}$. The length of this vector is determined by the optimisation algorithm. This length could be chosen such that it is just sufficient to clear the obstruction. However, a higher clearance may be desired if it helps to balance the ATC's workload. Further, the two possible directions of this vector is another variable, which is the same as allowing for positive and negative vector lengths. The variable length (and direction) of this vector allows for the much needed ability to generate multiple solutions from the path planning algorithm. In the experimental studies, the length is chosen such that it is just sufficient to clear the obstruction with the direction varied, giving us a total of $2 \times \text{no. of affected routes}$ permutations of re-routing strategies.

The clearance an airway needs to maintain from the unavailable airspace is represented by matrix C, given by

$$C = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1\delta} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\alpha 1} & c_{\alpha 2} & \dots & c_{\alpha \delta} \end{bmatrix} \quad (\text{Eq. 5.4})$$

where, $c_{ik} = |w'_{ij} - d_k|$, computed for $i = 1 \dots \alpha$ and $k = 1 \dots \delta$. The index j is based on the waypoint of a_i that is closest to d_k , along the direction of the traffic flow.

After determining the new waypoint, the routing can be updated from $(w_{ij} \rightarrow w_{i,(j+1)})$ to $(w_{ij} \rightarrow w'_{ij} \rightarrow w_{i,(j+1)})$. However, the disturbance may reside very close to or even encompassing the destination waypoint $w_{i,(j+1)}$. In this case, one needs to skip $w_{i,(j+1)}$ and route to the subsequent waypoint. Following this, a check is performed to ensure that

the newly constructed path is unobstructed. Presence of multiple nearby disturbances might make the new waypoint infeasible, forcing one to iterate the above process for the new disturbance location. This is done until the new route is free of any disturbance. In Figure 5.4(b), this has been illustrated for the presence of two nearby disturbances. The airway a_i being designed is thus modified based on the feasible waypoints obtained and is denoted as a'_i . Once the feasible waypoints for an unobstructed path are obtained, the airway route between the waypoints can be generated.

Routing of Airways: The waypoints obtained earlier only ensure traversable airspace in a straight line path connecting these waypoints. However, it does not ensure a conflict-free path with other airways. Thus, it is necessary to determine if the straight line path between these waypoints maintains sufficient clearances from other airways that were designed earlier. If so, a simple straight line path between the waypoints can be incorporated into the current airway design. Otherwise, the path with least conflicts has to be determined. In a case of originally parallel flows, the newly designed airway needs to maintain a clearance from the previously (re)-designed airways. For originally intersecting flows, orthogonal intersections are preferred in the re-design.

Path planning with these requirements is an optimisation problem that needs to be solved by CAARD. A separation clearance check, similar to the LOS check performed earlier, helps to reduce the scope of this optimisation problem to only a select waypoint pairs. This helps to speed up the execution of the path planning algorithm. Let r_i be the route of a_i that is currently being designed. This is nothing but a sequence of nodes (grids) $n \in N$. The set of all routes is denoted as R and the set of previously designed airways is denoted as $\hat{R} \subset R$.

The path planning between the waypoints is cast as a cost minimization problem. In the context of the graph constructed by discretizing the airspace, this corresponds to finding the lowest cost path from the source to the destination. CAARD finds the

sequence of nodes with the minimum cost of travel from the source to the destination waypoint. This is presented in Algorithm 4. Similar to A* path planning, starting from the source, it preferentially expands the nodes that minimize the estimated cost of the total path to the destination. This is done by maintaining a list of nodes (*FRONT*) that are potential candidates for expansion. The preferential node n_{cur} for expansion at each stage is formulated as

$$n_{cur} = \underset{n \in FRONT}{\operatorname{argmin}} (g(n) + h(n) + f_o(n) + \epsilon_1 f_c(\hat{R}, n) + \epsilon_2 f_{LOS-D}(n)) \quad (\text{Eq. 5.5})$$

Here, $g(n)$ is the distance cost from the node n to the source node. $h(n)$ is the estimated distance (heuristic) from the node n to the destination node. This heuristic distance is computed as the shortest diagonal-distance (distance while traveling along the 8 permissible directions) from the node n to the destination node. This distance is only an estimate since there is no guarantee that this direct path is unobstructed. $f_o(n)$ is the cost due to the presence of an unavailable airspace encompassing node n . Since paths that pass through unavailable airspaces are not permitted, this cost is infinity for such nodes and zero for the rest. $f_c(\hat{R}, n)$ is the cost due to the presence of a previously designed airway within a specified clearance distance threshold from node n . ϵ_1 is a scaling factor associated with this cost. $f_{LOS-D}(n)$ is the perpendicular distance from node n to the line segment connecting the source and destination waypoints. In other words, this is the cost of deviating from the straight line path connecting the waypoints. This is added as a small perturbation cost to prefer straighter paths. ϵ_2 is a scaling factor associated with this cost. During the process of graph exploration, the parent node of every explored node is stored in order to trace back the shortest path from the source. The search stops once the destination node is reached through expansion.

Algorithm 4: Path Planning

```

1 Function pathPlan(startWaypoint, endWaypoint):
2    $n_{start} = \text{geoToGrid}(\text{startWaypoint})$  ;           /* lat-long to grid ID */
3    $n_{goal} = \text{geoToGrid}(\text{endWaypoint})$ ;
4   ;
5    $FRONT = \phi$  ;           /* candidate nodes with node attributes */
6   insert ( $n_{start}, 0, \phi$ ) into FRONT;
7   ;           /* attributes: (node, g(n), parent(n)) */
8    $n_{cur} = n_{start}$  ;           /* current node for expansion */
9   ;
10  while ( $n_{cur} \neq n_{goal}$ ) AND ( $n_{cur} \neq \phi$ ) do
11    for all  $n' \in \text{neighbor}(n_{cur})$  do
12      if  $n' \in FRONT$  then
13        ;           /* multiple paths found to same node */
14        ;
15        if  $g(n_{cur}) + \text{dist}(n_{cur}, n') < g(n')$  then
16          ;           /* choose shorter distance path */
17          ;
18          replace ( $n', g(n'), \text{parent}(n')$ ) with
19            ( $n', g(n_{cur}) + \text{dist}(n_{cur}, n'), n_{cur}$ ) in FRONT
20        else
21          insert ( $n', g(n_{cur}) + \text{dist}(n_{cur}, n'), n_{cur}$ ) into FRONT
22        remove  $n_{cur}$  from FRONT
23       $n_{cur} = \underset{n \in FRONT}{\text{argmin}} (g(n) + h(n) + f_o(n) + \epsilon_1 f_c(\hat{R}, n) + \epsilon_2 f_{LOS-D}(n))$ 
24  if  $n_{cur} = n_{goal}$  then
25     $PATH = \phi$  ;           /* min-cost path found */
26    while  $\text{parent}(n_{cur}) \neq n_{start}$  do
27      insert  $n_{cur}$  into PATH;
28       $n_{cur} = \text{parent}(n_{cur})$ 
29     $PATH = \text{reverse}(PATH)$ ;
30    return  $\text{gridToGeo}(PATH)$  ;           /* grid ID to lat-long */
31  else
32    return "no path found"

```

5.2.2 Multi-objective Optimisation Model: Objectives and Constraints

The objectives in the design of sectors and airways are directed at balancing the ATC workloads across all sectors, as well as at designing sector shapes that do not change drastically from the original sectorization. Further, constraints are incorporated to obtain realizable sectors.

Objectives: The frequently used metrics to quantify the workload of ATCs include monitoring workload (representative of the number of aircraft monitored within a sector in a given time) and coordination workload (representative of the number of aircraft handed over to an adjacent sector in a given time) [45]. However, these formulations consider individual flight trajectories and not airways. Hence, for the purpose of studying airways, the metrics have been redefined as follows.

A. Monitoring Workload: This metric is used to evaluate the ATC workload in a particular sector owing to the presence of flights within that sector. The time required to monitor a particular airway is proportional to the length of the airway within that sector as well as the rate of flights within that airway. Hence, the monitoring workload $mWkl$ of sector s_i is defined as

$$mWkl(s_i) = \sum_{m=1}^{\alpha} (\text{length of } a_m \text{ within } s_i) * (f_m) \quad (\text{Eq. 5.6})$$

B. Coordination Workload: This metric is used to calculate the number of handshakes between ATCs of adjacent sectors while transferring control of flights transitioning from one sector to the other. The coordination workload of a particular sector is proportional to the number of airways that have their flows exiting that sector, and to the flow rate within each airway. Thus, the coordination workload $cWkl$ is defined as

$$cWkl(s_i) = \sum_{m=1}^{\alpha} (\theta_m) * (f_m) \quad (\text{Eq. 5.7})$$

where $\theta_m = 1$, if a_m is an outgoing flow for s_i . Otherwise, it is zero.

C. Sector Similarity: Dynamic airspace sectorization involves periodic re-configuration of sector shapes in order to deal with changing traffic patterns and unavailable airspaces. Since this is the aim of the current work, the sector shapes should be disallowed to change drastically from the previous, thus allowing ATC to monitor a region close to their area of specialization. Sector similarity is the metric used to calculate how similar a sectorization solution is to the original sectorization. A simple methodology for finding the 2D shape similarity has been proposed in [105]. For every sector, the maximum overlapping area with all previous sectors is first computed. The ratio between this overlapping area and the original sector's area (denoted as $sectSim(s_i)$) is used as the similarity metric for the corresponding sector. Details on the calculation of sector similarity are given in Section 4.2.1. Note that r_j is equivalent to $sectSim(s_i)$ used here.

The objective functions are based on the earlier described ATC workload and sector similarity metrics. The first objective is to balance the monitoring workload of ATCs across all sectors. Thus, the objective function to minimize is formulated as the standard deviation of the workload across sectors

$$Obj_{mWkl} = minimize \left(\sqrt{\frac{1}{N-1} \sum_{i=1}^{\eta} (mWkl_i - \mu_m)^2} \right) \quad (\text{Eq. 5.8})$$

where $\mu_m = \frac{1}{N} \sum_{j=1}^N mWkl_j$

The next objective function is the total number of hand-overs between ATCs, transpiring in the entire airspace, which needs to be minimized. This is formulated based on the coordination workload as

$$Obj_{cWkl} = minimize \left(\sum_{i=1}^{\eta} (cWkl(s_i)) \right) \quad (\text{Eq. 5.9})$$

The third objective function is based on the similarity ratios of the new sectors. Since the aim is to maximize the average sector similarity with the original sectorization, the

objective function is formulated as

$$Obj_{sectSim} = maximize \left(\frac{\sum_{i=1}^{\eta} (sectSim(s_i))}{\eta} \right) \quad (\text{Eq. 5.10})$$

Constraints: The key features that are required for the designed sector shapes are incorporated as constraints of the optimisation and they include,

A. Convexity and Connectivity: The internal angles of the sector shape must be less than 180 degrees to avoid re-entries and short dwell times for aircraft. Also, one sector must not be contained completely within another sector. Voronoi diagrams satisfy both these requirements. Since the sector shapes are found through Voronoi diagrams, no explicit convexity or connectivity constraint have been imposed.

B. Minimum Distance from Crossing Points: Crossing points are points of intersection between major traffic flows, and are critical areas monitored by ATCs. Such regions need to be sufficiently far from sector boundaries to provide higher response times to the ATC for ensuring the safety of operations. The intersection points between the designed airways are determined and denoted as cp_i for i ranging from $(1 \dots \psi)$. ψ is the total number of crossing points found for a given air traffic flow. A constraint is imposed to ensure the crossing points are at least 10 NM from the nearest sector boundary.

The minimum distance metric is formulated as

$$minD = \min_{i=1}^{\psi} \|cp_i, boundaries(s_m)\| \quad (\text{Eq. 5.11})$$

where $cp_i \in s_m$. The constraint can be written as $minD < 10NM$.

The optimisation problem can be expressed as follows.

Find: The Voronoi sites and the Clearance matrix $[V, C]$

Subject To:

Objectives:

- (i) Minimize the standard deviation of monitoring workload between sectors (Obj_mWkl)
- (ii) Minimize the total coordination workload across all sectors (Obj_cWkl)
- (iii) Maximize the average sector similarity between current and previous sectors ($Obj_{sectSim}$)

Constraints:

- (i) Maintain a minimum distance of 10 NM from all crossing points to their nearest sector boundaries

Bounds:

- (i) $minLat \leq v_i^\phi \leq maxLat$ AND $minLon \leq v_i^\lambda \leq maxLon$, $\forall v_i \in V$
- (ii) $-maxClearance \leq c_{ik} \leq -minClearance$ OR $minClearance \leq c_{ik} \leq maxClearance$, $\forall c_{ik} \in C$

5.2.3 NSGA-II Optimisation

This problem is a multi-objective, constrained optimisation, with a decision space of $(2 * \eta + \alpha * \delta)$. Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [32] has been employed to solve this optimisation problem. The design variables for NSGA-II is represented by a hybrid chromosome. As depicted in Figure 5.5, it consists of two segments:

- (i) **Voronoi Segment:** The locations of Voronoi sites $\mathbf{v}_i$ that are needed to generate the sector shapes make up the first part of the chromosome. The length of this segment of a chromosome is determined by the number of sectors desired and is equal to $2 * \eta$.
- (ii) **Clearance Segment:** The matrix C which determines how much clearance an airway must maintain from unavailable airspaces is the second design variable that

is incorporated into the segment-2 of the chromosome. This distance is used to calculate the location of the alternate waypoint during re-routing. This value may be positive or negative allowing for clockwise and anti-clockwise re-routing around the disturbance. The length of this segment is equal to $\alpha * \delta$.

Figure 5.5: Representation of a hybrid chromosome used in NSGA-II

Multi-objective optimisation produces multiple solutions (Pareto optimal solutions) that vary in the trade-offs between specified objectives. This will provide options for ATCC manager to select the best-suited solution for a given scenario. NSGA-II uses non-dominated sorting to reduce the computational complexity, and a crowding distance metric to increase the diversity of solutions. It also defines a constrained-domination principle to handle the constraints specified. The ranking of solutions is first based on feasibility using constraint violation. Feasible solutions are further subdivided into sets of non-domination, which are internally ranked based on the crowding distance metric. The set of infeasible solutions is then ranked based on the value of constraint violation.

5.3 SAWAS Experimental Study

In this section, the performance evaluation of SAWAS is given in two experimental studies. The first study is in a simulated environment is described to highlight its working and also its benefits. This second study is implemented using the Singapore FIR to show its working in a real-world scenario.

5.3.1 Simultaneous VS Sequential Approach

For this study, the three major airways in Singapore's en-route airspace as in Figure 5.2, were chosen for the simulation. The air traffic within these airway routes were simulated, assuming equal flow rates of 10, for a 2-hour time window. First, a strategic sectorization using this flight data was performed, to serve as a basis for comparison. Subsequently, the study was done under a pre-tactical stage where a section of airspace was made unavailable, requiring re-routing. For this scenario, the performance comparison of a sequential solution with the proposed simultaneous solution (by SAWAS) for the airway and sector design is presented.

5.3.1.1 Strategic sectorization

Strategic sector design is a decision making process for synthesizing efficient sectors for a given flight plan. To find the efficient strategic sector design, NSGA-II optimisation was used, using the framework elaborated in Section 5.2. However, since the entire airspace is considered available at the strategic stage, the CAARD algorithm does not play any role in this optimisation. This optimisation was performed with the monitoring and coordination workload objectives as well as minimum distance constraint. The sector similarity objective was not included since at the strategic stage there is flexibility in the sector design and the aim is to find the best static solution for the given air traffic flow. For the software implementation of NSGA-II, NGPM v1.4 (NSGA-II Program for MATLAB) [79] was utilised. The parameter settings are population size = 50, maximum generations = 200, immediate crossover ratio = 1.2, Gaussian mutation scale = 0.1 and shrink = 0.5.

Figure 5.6 shows the plot of few representative solutions on the obtained Pareto front. The variety of solutions presented here depicts the advantage of casting the design problem as a multi-objective optimisation. Multiple solutions, with differing trade-offs

Figure 5.6: Pareto front for strategic sectorization

among objectives, are thus made available for the ATCC to evaluate based on their preferences.

The chosen strategic solution, along with a bar graph of the variation sectorization metrics is shown in Figure 5.7. The bar graph shows an even distribution of monitoring workload across sectors with a standard deviation of 169. The coordination workload ranges from 0 to 20, the sum of which is 60. An alternate way to view this metric is as being proportional to the total number of outgoing airways that are cut by the sector boundaries (which in this case is 6). With reference to Figure 5.6, one can observe that the objectives of the chosen solution can be represented by the 3rd solution from the left on the Pareto front.

5.3.1.2 Pre-tactical airway and sector design

Having obtained the strategic sector shapes, a pre-tactical scenario was simulated in which an airspace along airway-1 was excluded, deeming it unavailable. In reality, this

(a) Visualization of sector shapes

(b) Variation of sectorization metrics across sectors

Figure 5.7: Selected strategic sectorization solution and its objectives

may be invoked by a military aviation authority or due to a weather forecast. The unavailability of airspace can be of any shape. In our experiments, this is modelled by a circle for convenience. One approach to design both airways and sectors for a given air traffic is to design them sequentially; the airway re-routing performed first followed by the sector design for the newly generated airways. This approach has been used in [70] where such a sequential strategy has been proposed to handle this problem. In the following subsection, the results of this approach is first presented. In the subsequent subsection, these results are then compared with those obtained from a concurrent optimisation by

SAWAS.

Sequential solution (re-routing followed by resectorization) A sequential solution requires one to find feasible routes for flights, prior to sectorization. Without changing the strategic sector shapes, CAARD algorithm was used independently to find feasible routes. Two possible re-route strategies generated by CAARD, where airway-1 maintains the minimum allowed clearance on either side of the unavailable airspace is generated.

Figure 5.8: Variation of sectorization objectives across sectors for different re-route strategies

The visualization of these two re-routing strategies which we refer to as southward and northward are shown in Figure 5.9(a) and Figure 5.10(a) respectively. The bar graphs in Figure 5.8(a) and Figure 5.8(b) show the variation of sectorization metrics for the two

re-route strategies (northward and southward respectively) without any change in sector design. Comparing the two strategies in terms of these metrics, the southward re-route strategy has a lower monitoring workload variation (163 as opposed to 172). This is due to the fact that northward re-route strategy involves a longer path and hence higher monitoring workload for sectors 1 and 5. The sum of coordination workloads is the same for both strategies since they both involve an extra entry and exit into an adjacent sector. However, the range is higher for the former (0-30 as opposed to 0-20). The sector shapes being unchanged with respect to the strategic sectorization, the average sector similarity is 100% for both cases. In order to decide which strategy to use for the resectorization step, one can base the choice on the solution that performs better on the sectorization metrics. Based on the three metrics, the southward re-route strategy for airway-1 seems more attractive.

Next, an independent sector optimisation was performed where the airway design was fixed. All three objectives were included in the optimisation, where the sector similarity was evaluated with respect to the strategic sectorization. The minimum distance constraint was also imposed. The solution with a reasonable trade-off between the objectives was chosen, whose sector shapes are visualised in Figure 5.9.

Figure 5.9(a) shows that the new sector design (black unbroken lines) is similar to that of the original strategic sectorization (light dotted lines). The bar graph in Figure 5.9(b) shows the variation of sectorization metrics, for this solution. Since airway-1 was re-routed south of the unavailable airspace, it passes close to the crossing flow of airways-2 and 3. Due to the minimum distance constraint, sectors 2 and 3 are unable to expand to take up their original flow passing through airway-1. This restricts the ability to re-design the sector to reduce the monitoring workload and creates a high standard deviation of 305. The only improvement possible through resectorization was the reduction in coordination workload of sector-1, through the expansion of sector-2 in the west. The total coordination workload is also reduced.

Figure 5.9: Best sequential airway and sector design solution and its performance metrics (Re-routing followed by resectorization)

Concurrent Solution by SAWAS SAWAS overcomes the above limitations of the sequential design approach which involves intermediary decisions that tend to be greedy in nature. To illustrate this claim, the SAWAS architecture was used for optimising both airways and sector design concurrently. All the three objectives were included in the optimisation. The minimum distance constraint was also imposed. The solution with a reasonable trade-off between the objectives was chosen, whose sector shapes are visualised in Figure 5.10(a). The new sector shapes are reflected with black unbroken

lines, and the previous sector design is shown with light dotted lines.

(a) Visualization of sector shapes and northward re-routing (old sectors are indicated as dotted lines)

(b) Variation of sectorization metrics across sectors

Figure 5.10: Best SAWAS airway and sector design and its performance metrics

From Figure 5.10(a), it may be noted that the northward re-route strategy was preferred instead, by the optimiser. The bar graph in Figure 5.10(b) indicates an improvement in both monitoring workload imbalance (which is reduced to 132) and coordination workload sum (which is reduced to 60), post resectorization. In this resectorization,

sector 3 has expanded towards sector 1 in the northern airspace to take up the redirected flow within airway-1. This reduces the extra coordination workload for sector 1. To prevent overloading of sector 3, sector 1 has expanded towards sector 3 in the western region, taking up additional control of airway-1. Consequently, a drop in sector similarity is noted for sectors 1 and 3. Compared to the sequential solution that could only improve the coordination workload, the SAWAS solution was able to achieve an improvement in both monitoring as well as coordination workloads. Both sequential and SAWAS solutions maintain a high sector similarity above 85%.

Though the northward re-direction of airway-1 yields un-optimal fitness function values for the strategic sector shapes, it yields a more optimal solution after resectorization. The sequential approach requires the assumption that an optimal airway design will yield an overall optimal solution. SAWAS, on the other hand, concurrently considers the effect that resectorization and re-routing have on the objectives. It evaluates which combined strategy is best suited for the objectives. Thus, SAWAS manages to outperform the sequential approach.

5.3.2 Performance Evaluation of SAWAS on Singapore FIR

The airways used in this experiment are under the Singapore Air Traffic Control in the upper airspace (above flight levels - FL135) of the Singapore FIR. There is a total of 27 flight routes (airways), which have been extracted from the Civil Aviation Authority of Singapore Aeronautical Information Publication [89] and given in Figure 5.11.

Next, the flow rates of the airways are estimated using the historical flight data recorded on 1st June 2015 [46]. The flight data consists of ADS-B data with 2-4 positions per minute depending on the coverage of area and ANSP radar from governments that are about 1 position per minute. However, the provided tracks only cover up to 108 in longitude, which does not cover the whole Singapore FIR. Therefore, flight data from

Figure 5.11: All the 27 routes in the upper airspace of the Singapore FIR

the longitude of 108 to 116.3 were extrapolated from the provided flight plans. The flow rates are then obtained through matching the combined set of flight tracks to the airways using waypoints. It is also worth noting that by this approach, approximately 70% of flights have been captured, hence these rates may be slightly lower than the actual rates. The total flow rates vary significantly throughout the day. Furthermore, the flow rate for each airway changes almost every hour and not all airways are utilised at all time. Thus, it can be said that air traffic vary throughout the day and resectorization is required to balance the workloads. For this particular experiment, we took an hour worth of flights when the flow rates are heaviest from 4 pm to 5 pm. This covered the use of 20 airways with varying flow rates.

5.3.2.1 Strategic sectorization

This section provides the results for sectorization with the assumption of no weather disturbances. Similar parameter settings for NGPM NSGA-II optimisation as in Section 5.3.1 were selected with an exception for the choice of the maximum generations of 500 due to more number of airways and varying flow rates.

Due to the unique and non-convex shape of the Singapore FIR, the boundaries of the airspace is no longer a box but a polygon shaped like the Singapore FIR. Further, the generated Voronoi sites must lie inside the Singapore FIR. Therefore, the convexity of the sector shape in this study cannot be guaranteed. Out of 50 solutions, a total of 27 solutions lies on the Pareto front. The standard deviation of monitoring workloads among all the sectors for all Pareto-optimal solutions ranges from 334 to 6643. However, huge imbalance of monitoring workloads among sectors is not preferable as it could reflect having empty sector(s) at the expense of overloading other sectors. Therefore, it is in our interest to focus on the solutions with a low standard deviation of monitoring workload on the Pareto front.

Figure 5.12 provides the selected solution A with the monitoring and coordination workloads of the sectors. The blue lines in Figure 5.12(a) represent all considered routes in the Singapore FIR (shape outlined in bold black) and the black lines represent the sector design. Each sector is the overlapping area between the sector design and the Singapore FIR. Figure 5.12(b) shows that the monitoring workload is well balanced among all 5 sectors between 3500 to 5000, with a standard deviation of 343. The coordination workload of sectors ranges from 0 to 28, giving a total of 53 among all sectors. Similar to the previous experiment (in Section 5.3.1), this solution acts as a baseline. Looking at Figure 5.12(a), sector 4 has a small separate airspace that is connected to sector 2. This is the consequence of the non-convexity constraint not being met. In this case,

(a) Visualization of sector shapes

(b) Variation of sectorization metrics across sectors

Figure 5.12: Selected (solution A) sectorization for Singapore FIR

a post-processing of assigning the small area of airspace from sector 4 to sector 2 is performed.

5.3.2.2 Change in Airspace Availability

In this section, a pre-tactical scenario where a particular airspace becomes unavailable is provided. In this given scenario, two airways are being affected by the unavailability of this airspace. A re-routing of the affected airways is first performed using CAARD. A total of 2^n possible re-routing strategies would be generated where n represents the number of affected airways. In this case, a total of 4 ($2^2 = 4$) possible re-route strategies is generated. Figure 5.13 shows the initial affected routes with 4 possible re-route strategies generated by CAARD. Table 5.1 shows the corresponding performance objectives and

Figure 5.13: Affected routes and possible re-route strategies generated by CAARD

characteristics of the re-routed solutions. Overall, the monitoring workload among all sectors becomes more imbalanced, with the standard deviation increasing from 343 to 600-800. The coordination workload increases for solutions A1, A2 and A4 but decreased by one for solution A3. Furthermore, the safety constraint of minimum distance to crossing points has to be checked. The re-routing solution A2 in Figure 5.13(c) and A4 in Figure 5.13(e) display a crossing point very close to the sector boundary between sector 1 and 5, hence not satisfying the safety constraint. Last but not least, the route feasibility of the solutions has to be checked. Visually, we can observe that the re-routed solution A1 in Figure 5.13(b) is infeasible as part of Airway 2 is out of the Singapore

FIR. One should note that this occurrence is because of the non-convex shape of the Singapore FIR. Given a convex airspace, this situation is unlikely to happen. Also, it can be seen that solution A2 in Figure 5.13(c) and A4 in Figure 5.13(e) is similar to each other. This similarity in routes is also reflected in their closely matched performance objectives in Table 5.1. With this, the only feasible solution without any resectorization is solution A3 in Figure 5.13(d).

Table 5.1: Performance characteristics of CAARD solution

Route	stdDev(mWKL)	sum(cWKL)	Constraint	Feasibility
A1	789.5	56	Yes	No
A2	688.3	55	No	Yes
A3	773.0	52	Yes	Yes
A4	663.4	55	No	Yes

Figure 5.14 provides the solution A3 and its sectorization metrics across all sectors. The red lines represent airways that have been re-routed. The monitoring workloads range from 2500 to 5000 with a standard deviation of 773. The coordination workloads range from 0 to 27, giving a sum of 52. The sectors remained unchanged, thus 100% similarity for all sectors.

Concurrent solution by SAWAS To get more balanced monitoring workloads, we perform SAWAS on the current scenario using feasible re-routing strategies A2, A3, and A4 in Figure 5.13 with a maximum generation of 2000. The number of generations is being increased here again to capture the possible variation of new re-routing strategies, which were not present in strategic sectorization.

Figure 5.15 displays the performance objectives of 50 Pareto-optimal solutions obtained using SAWAS using 'o' and four solutions with only re-routing A1, A2, A3 and A4 using '+'. The x-axis and y-axis represent the standard deviation of the monitoring workload and the total coordination workload among all sectors respectively. This range of solutions obtained by SAWAS presents the user with more permutations of re-routing

(a) Visualization of sector shapes

(b) Variation of sectorization metrics across sectors

Figure 5.14: Solution A3 (Re-routing solution without resectorization)

and resectorization compared to performing a re-routing followed by resectorization after choosing a set of particular routes. Figure 5.15 shows how solution P dominates solutions A1, A2 and A4 but is incomparable to solution A3 in terms of the performance objectives. This allows us to conclude that the solution A3 is a good choice of solution should the user decide that sectors are to remain unchanged.

Figure 5.16 shows the sectors in solution P and its sectorization metrics across all sectors. As seen from Figure 5.16(a), the selection of routes is the same as solution A4 in Figure 5.13. Comparing solution P to solution A4, we are able to show how a change in sector shapes can help to optimise ATC workloads and satisfy the safety constraint. The standard deviation of monitoring workloads among all sectors is 593. The coordination

Figure 5.15: Pareto front obtained using NSGA-II with restricted airspace

workload ranges from 0 to 31, giving a sum of 54. The sector similarity of sectors to solution A is 81%. This change in sector shapes is visualised in Figure 5.16(a).

Next, we present a solution with more balanced monitoring workload among sectors (lower standard deviation of monitoring workload) - solution Q. In comparison with solution P, it can be said that solution Q has gained a larger coordination workload as a trade-off for a better balance of monitoring workload between sectors (refer to Figure 5.15).

Figure 5.17 displays the sectors in solution Q, a solution that has 318 as the standard deviation of monitoring workloads among sectors, similar to that of the original sectorization. This is reflected in Figure 5.17(b) where the sector monitoring workload is much more balanced compared to solution A3 in Figure 5.14(b). Again, solution Q is using the same set of routes as solution A4. The affected routes for re-routing are represented with red lines in Fig 5.17(a) (same as re-route strategy A4 in Figure 5.13(e)) and the blue lines represent the other unaffected routes. In this case, the coordination workload

(a) Visualization of sector shapes

(b) Variation of sectorization metrics across sectors

Figure 5.16: Solution P (Lower coordination workload)

ranges from 0 to 31 and the total incurred is 60. The sector similarity to solution A is 74%. This sector shape change is being visualised in Figure 5.17(a), where the black unbroken lines represent the new sector design and the light dotted lines show the initial sector design.

Ultimately, SAWAS provides the user more choices in the selection of solutions. Further, just rerouting did not guarantee constraint satisfaction (eg: safety constraint) while SAWAS gives solutions that guarantee this through consideration of the interactions between rerouting and sector geometry. If the user has a preference for balanced monitoring workloads among sectors, solution Q would be preferred. However, if the user prefers unchanged sectors, solution A3 would be chosen. The choice of sectors here would have

(a) Visualization of sector shapes

(b) Variation of sectorization metrics across sectors

Figure 5.17: Solution Q with better balancing of monitoring workloads

an impact on the air traffic operations in the future. Therefore, the final choice of sectors should be determined with the use of information on predicted future air traffic.

5.4 Summary

In this chapter, a methodology to simultaneously optimise sector design and traffic routing during the changing availability of airspace is explored. The proposed methodology consists of three key components: airspace sector design, traffic flow design, and optimisation. Voronoi diagrams was employed for the sector design. The traffic flow design had unique requirements to allow its usage as a design variable, such as the need to generate multiple solutions, and to resolve conflicts between multiple airways. Thus, a novel

algorithm was developed to satisfy these requirements. Multi-objective constrained optimisation using evolutionary algorithm was employed to find the optimal sector re-design and traffic re-routes. To test the efficacy of the algorithm, a simulation environment was set up with synthetic air traffic and airspace disturbances, in the context of Singapore air traffic control. A sequential strategy where the traffic flow design precedes the sector design was compared with the simultaneous optimisation. In the sequential strategy, the traffic flow design lacks prior knowledge on future sector shapes. Hence, it requires the assumption that optimal traffic flow design would lead to an overall optimal solution for the design objectives. SAWAS overcomes this shortcoming by concurrently considering the effect of sector and flow re-design on the objectives.

The Cognitive decision-making architecture for Dynamic Airspace Sectorization (CDAS) tackles the changing traffic flows in the airspace over time by providing a resectorization when needed. On the other hand, SAWAS finds optimal solutions when the availability of airspace changes through simultaneously optimising re-routing and resectorization strategies. Therefore, the incorporation of SAWAS into the Cognitive decision-making architecture framework would be a fascinating direction to explore because it would allow one to obtain a complete model tackling the main issues present in dynamic airspace management.

Till now, all the methodologies proposed utilised the stochastic solver, NSGA-II. However, solutions obtained from NSGA-II are generally irreproducible due to the random nature of the algorithm. In the next chapter, the thesis would explore deterministic solvers, particularly algorithms following the multi-scale search optimisation framework [8].

Chapter 6

Multi-Objective Multi-Scale Optimisation Algorithm

Stochastic multi-objective evolutionary algorithms are commonly used to solve real-world problems [64, 94, 95] due to the ease of application. On the other hand, deterministic algorithm requires more effort in problem modelling. However, they are able to obtain reproducible results. Hence, they are more reliable when solving real-world problems.

One of the first deterministic multi-objective algorithm proposed for black-box optimisation is the multi-objective mesh adaptive direct search, also known as MultiMads [10]. In MultiMads, the objectives are aggregated with weights to become single objective problems, which was then solved using mesh adaptive direct search (MADS). This method to manage multiple objectives is also known as the decomposition-based approach. Soon after, Direct Multisearch (DMS) [31] was extended with the non-dominance-based approach from a similar search and poll framework used in MADS. Being a local-search based algorithm, DMS could face difficulties in finding the whole set of Pareto-optimal solutions when these solutions are located in different areas of the search space. This characteristic that is common in real-world problems is known as multi-modality. This shortcoming of DMS has recently been addressed with a clever multi-start strategy [29].

Another group of deterministic algorithms that had successfully managed multi-modal problems are the global-search based algorithms based on MSO framework [8]. The MSO

framework follows a similar concept behind the multi-level scheme that was previously used in the proposal of new algorithms for multi-objective optimisation problems with differentiable objectives [100]. The MSO framework allows one to approximate the amount of exploration performed in a region and allows one to derive a sampling mechanism such that one can exploit the areas that have given good solutions previously and explore areas that have yet been sampled. Multi-objective MSO algorithms have been extended using the non-dominance-based approach to MO-DIRECT [4] and in a more general form, MO-SOO [7]. Further, a finite time convergence analysis of MO-SOO was given in [7].

Till date, multi-objective deterministic algorithms have been extended from their single-objective counterparts with the non-dominance-based and decomposition-based approaches. Given the success of indicator-based approach in stochastic evolutionary algorithm in SMS-EMOA [16], an indicator-based multi-objective deterministic algorithm is proposed.

In this chapter, a new multi-objective deterministic algorithm called Pareto-Aware Dividing RECTangles (**PA-DIRECT**) is presented. **PA-DIRECT** is based the Multi-Scale Optimisation (MSO) framework and follows the partitioning method of the DIRECT [60] and MO-DIRECT [4] algorithm. Further, the fundamental issues that affect the convergence rate in MO-DIRECT is identified and improved in the proposed new algorithm **PA-DIRECT**.

The chapter is organised as follows. The MSO framework and the partitioning procedure is first provided in Section 6.1. The **PA-DIRECT** algorithm is then presented with illustrative examples in Section 6.2.3. Details on the performance of **PA-DIRECT** with other state-of-the-art algorithms on the *bbob-biobj* benchmark [110] are given in Section 6.3. **PA-DIRECT** is also applied to the problem of optimizing sector shapes in Section 6.4. Finally, the chapter is summarised in Section 6.5.

6.1 Multi-Scale Optimisation (MSO) Framework

A MSO algorithm is defined as *an algorithm that constructs a hierarchy of partitions on the search space $\mathcal{X}$ whilst looking for f 's global optimiser* [8]. Due to the systematic approach of partitioning, the information obtained is commonly stored as a search tree. Hence, these algorithms are also commonly known as space partitioning tree algorithms. PA-DIRECT follows the partitioning procedure of DIRECT (and MO-DIRECT), therefore DIRECT is first used to illustrate the MSO framework.

6.1.1 Working Principles of DIviding RECTangles (DIRECT)

DIRECT is a single objective algorithm, known for having a balance between exploitation and exploration in its search. This prevents the algorithm from assuming a local optima for a global optimum. The idea of exploitation is to further search a certain area that had previously shown promising results. On the other hand, exploration is the search of areas that have previously not been searched before.

The framework of DIRECT can be divided into two parts: the division of current hyperrectangles into smaller hyperrectangles using the partitioning procedure and the process of selecting Potentially Optimal Hyperrectangles (POHs) for a division in the next iteration. They are elaborated in Section 6.1.1.1 and 6.1.1.2 respectively.

6.1.1.1 Partitioning methodology

The partitioning procedure of DIRECT is explained in Figure 6.1 with a single objective, 2 decision variables problem. The whole decision space is viewed as a huge single hyperrectangle. DIRECT starts by sampling the point in the centre, obtaining a function value of 1. Then, DIRECT samples 2 points in each dimension of the problem such that the 3 points (including the initial centre point) in each dimension is equally spread out. In the 2 decision variable example shown, a total of 4 points are sampled. After all the

Figure 6.1: DIRECT partitioning: 2D example

points are sampled, DIRECT follows a unique partitioning procedure to split the decision space into smaller hyperrectangles. The algorithm divides the decision space equally into one-thirds one dimension at a time, based on a criterion.

Order of dimensions for the partitioning of POHs The order of dimension in which the hyperrectangle is partitioned would decide the final size of the newly sampled hyperrectangles. In DIRECT, the dimension with the best(lowest) function value will be partitioned first. The partitioning then goes on with the next dimension until the worst function value. The smallest function value of newly sampled points in each dimension j , f_j , is used to decide which dimension j would be partitioned first. It is defined as the following:

$$f_j = \min_{k \in \{1, -1\}} f(\mathbf{c}_i + k \cdot \delta \cdot \mathbf{e}_j) , \quad (\text{Eq. 6.1})$$

In other words, a hyperrectangle is divided such that the biggest produced hyperrectangles contain the distant solutions from that of the initial hyperrectangle, increasing the likelihood of visiting unexplored regions of the function space. This is reflected in Figure 6.1 where the hyperrectangle with a function value of 2 is bigger than the hyperrectangle with a function value of 7.

Figure 6.2: DIRECT selection strategy

6.1.1.2 Selection strategy

Consequently, the hyperrectangle(s) for the next iteration of sampling has to be selected. In the DIRECT framework, the local and global score of each sampled point can be measured by the function value and the size of hyperrectangle respectively. Hence, the order in which the POHs are partitioned, which decides the size of newly sampled hyperrectangles can be referred to as the assignment of global score. In order to balance between exploitation and exploration, DIRECT uses the non-dominated front of the lowest function value(s) and biggest size of the hyperrectangles as a criteria to select POHs. Hyperrectangle(s) of every size with the lowest function value is selected given that there exists no bigger hyperrectangle with a lower function value.

Following the previous problem given in Section 6.1.1.1, hyperrectangles with the function values of 1 and 2 (highlighted in red and green in Figure 6.2(a) respectively) are selected. Again, one should note that the size of hyperrectangle with a function value of 2 is bigger. The size of the hyperrectangle allows one to measure the amount

of sampling performed around the given area. Therefore, it can be said that the choice of a bigger hyperrectangle (green hyperrectangle) is an exploration of the search space and the choice of hyperrectangle with the lowest function value (red hyperrectangle) is an exploitation of the search space. This shows that DIRECT both explores and exploits the search space.

6.1.2 Towards Multi-Objective Optimisation

The DIRECT framework is extended into MO-DIRECT to tackle MOPs. The rules for global score assignment (order of dimension in the partitioning of POHs) and the selection of POHs in the multi-objective scenario is addressed below.

6.1.2.1 Global score assignment

The following provides MO-DIRECT's strategy in the assignment of global score, i.e. choosing the order of dimensions in partitioning POHs. This is defined as the following.

The dimension j that is first partitioned in the partitioning procedure is chosen using the lowest value of spread factor w_j . The partitioning then advances with the next dimension until the highest w_j . w_j is the inverse of spread/diversity between the newly sampled solution and the original solution.

$$w_j = \frac{1}{\min_{k \in \{1, -1\}} \|\mathbf{f}(\mathbf{c}_i + k \cdot \delta \cdot \mathbf{e}_j) - \mathbf{f}(\mathbf{c}_i)\|} , \quad (\text{Eq. 6.2})$$

The measure of diversity is measured locally using the minimum distance between the objective values of a newly sampled solution $\mathbf{f}(\mathbf{c}_i + \delta \cdot \mathbf{e}_j)$ and the original solution $\mathbf{f}(\mathbf{c}_i)$.

Using the aforementioned method, a hyperrectangle is divided such that the biggest produced hyperrectangles contains the distant solutions from that of the initial hyperrectangle, increasing the likelihood of visiting unexplored regions of the objective space. Hence, the concept of diversification, to favour search areas having dissimilar objectives,

is being applied. The strategy sees that the most dissimilar points in the objective space have a better global score.

However, this methodology neglects the other equally important goal in MOP - convergence. This observation is further elaborated with a synthetic problem in Section 6.2.3.

6.1.2.2 Selection of POHs

The selection of POHs in MO-DIRECT is similar to that of DIRECT, using the non-dominated front of the lowest objective values and biggest size of the hyperrectangles as a criteria to select POHs. In this case, the non-dominated front is a multidimensional surface instead of a 2D curve. The selection strategy is illustrated with a synthetic problem in Section 6.2.3.

6.2 Pareto-Aware Strategies for Faster Convergence

In this section, we first investigate MO-DIRECT's slow convergence to the set of Pareto-optimal solutions. A easy artificial bi-objective 2-variable problem is introduced. The given problem has two conflicting objectives represented by f_1 and f_2 , the search space is bounded and can be represented by a 2D square (see Figure 6.3(a)). The global score assignment (aka order of partitioning) and sampling (aka selection) strategies in MO-DIRECT are being illustrated in this synthetic problem. Through this, the two main causes of slow convergence, namely Pareto-independent global score assignment and generous sampling are identified and explained.

6.2.1 Issue 1: Pareto-Independent Global Score Assignment

As mentioned earlier, the measure of the global score is the size of hyperrectangle in DIRECT. The size of hyperrectangle is solely determined by the order of dimensions

Figure 6.3: Original partitioning strategy in MO-DIRECT

in the partitioning of POHs. New hyperrectangles that are in the dimensions that are chosen to be partitioned first are larger and hence have a better global score than the other hyperrectangles.

Figure 6.3 illustrates the original strategy of MO-DIRECT (given in Section 6.1.2.1) on the synthetic problem. Given, that point 1 is chosen again due to its non-dominance, it samples points 6, 7, 8, 9 (highlighted in red in Figure 6.3).

With Eq. 6.2, points 8 and 9 would obtain a larger value of w_j as they are further away from point 1 in the objective space. This in turn leads to the partitioning reflected in Figure 6.3a where the hyperrectangles with points 8 and 9 are bigger than those with points 6 and 7. However, when we refer to Figure 6.3b, we observe that points 8 and 9 are dominated by both points 6 and 7. Additionally, the points 4 and 5 are close to points 8 and 9. Thus, if the criteria of non-dominance and greater dissimilarity in objectives (diversification) when compared to the whole set of solutions is considered, points 6 and

7 are regarded as more promising solutions (as compared to points 8 and 9).

Hence, to boost the probability of points 6 and 7 of being picked as POH, the size of hyperrectangles containing points 6 and 7 should be larger than points 8 and 9. Based on this observation, one can conclude that **MO-DIRECT**'s strategy does not account for the non-dominance of newly sampled solutions and overlooks diversity that is relative to the whole objective space. This Pareto-independent strategy could inaccurately assign the global score and hence slow down the search for the set of Pareto optimal solutions.

6.2.2 Issue 2: Generous Sampling

One of the hardest types of problems to resolve in multi-objective optimisation is multi-modal problems, where Pareto-optimal solutions are found in different areas of the search space. While many state-of-the-art algorithms struggle to resolve these multi-modal problems, **MO-DIRECT**'s selection strategy for sampling based on the balance of global and local search ensures that these Pareto-optimal solutions are found eventually. However, this exact strategy samples too generously such that **MO-DIRECT** would eventually reduce to an exhaustive grid search. This observation is being explained in detail using the synthetic problem.

Figure 6.4 shows **MO-DIRECT**'s selection strategy on the same artificial problem following the result of **MO-DIRECT** partitioning strategy. POHs are chosen based on their function values of f_1 , f_2 and size of the hyperrectangle they are in. Figure 6.4(a) depicts the objective space of the problem, where points 1,3,6,7,10,11 are non-dominated solutions. Figure 6.4(b) visualises the function values and the size of hyperrectangles, which we refer to as the ND Space. The chosen points are represented by squares. A total of 10 points (all points except 8) is picked out of 11. All non-dominated solutions are naturally picked based on function values regardless of the size of hyperrectangles [black squares]. Out of 5 points on the dominated fronts, a total of 4 points are selected as

Figure 6.4: MO-DIRECT selection strategy example. Black squares: selected POH on approximate Pareto front. Red Squares: selected POH on dominated fronts

POHs [red squares]. The POHs on the non-dominated front, $\mathcal{I}_{local}$ (points 1,3,6,7,10,11) and dominated fronts, $\mathcal{I}_{global}$ (points 2,4,5,9) indicate MO-DIRECT's local and global search respectively.

Local search MO-DIRECT's selection strategy ensures that all (100%) non-dominated points are selected in the every iteration regardless of its global score (size of hyper-rectangle). Further, it does not consider the concept of diversification in its selection. In this case, points 3 and 10 are close together in the objective space (shown in Figure 6.4(a)), hence it might not be effective to select both points as POHs. Additionally, as the number of non-dominated points becomes larger especially in a near-optimal state, the selection of all non-dominated points would be a huge waste of sampling.

Global search 4 out of 5 points (80%) are selected in the global search of the given problem above. **Consider point 9 (a dominated point), one can observe that it has a worse global score when compared to nondominated points 3,10,11.**

Point 9 is being selected as a POH because it is only being dominated by points 1,6,7 which are smaller than itself. However, selecting a point (point 9) that has a worse global score to points in a local search (point 3,10,11) does not exactly meet the criteria of searching regions that are yet explored as defined by global search. Another characteristic of MO-DIRECT's selection strategy is that the point with the best global score (point 2 in this example) would always be selected as a POH. However, the point with the best global score might not always be a region of interest to explore, especially in the later part of the search. Thus, the selection of POH in a global search can be refined and improved.

6.2.3 The PA-DIRECT Algorithm

PA-DIRECT integrates diversification into the inherent characteristic of balance between local and global search present in MO-DIRECT for a faster convergence to the whole Pareto front. PA-DIRECT addresses the use of diversification in the local and global search separately. Therefore, the selection strategy of PA-DIRECT is divided into two - points on the approximate Pareto front (local search) and those that are not (global search). The PA-DIRECT algorithm is presented below.

Before presenting the pseudo-code of PA-DIRECT, the following terms have to be defined:

Definition 6.1 (Hypervolume Indicator for Diversity measure: $hv_d(\mathbf{y})$) *The hypervolume indicator for diversity measure requires projection of the Pareto-optimal front approximation onto a line (in the bi-objective case) or a hyperplane (in greater number of objectives). This can be done by translating the normalized objectives,*

$$\hat{y}_j(s) = y_j(s) + \frac{1 - \sum_{i=1}^M y_i(s)}{M}, \quad 1 \leq s \leq S, \quad 1 \leq j \leq M \quad (\text{Eq. 6.3})$$

where $\hat{y}_j$ is the projection of an approximate solution in objective j , y_j is the normalised objective j of an approximate solution ($\mathbf{y} \in \mathcal{Y}_*$), S and M are the total number of approximate solutions and objectives respectively.

For any objective vector $\mathbf{y} \in \mathcal{Y}_* \subset \mathbf{R}^m$ and a set of objective vectors $\mathcal{Y}_* \subset \mathbf{R}^m$, $hv_d(\mathbf{y}) = hv_d(\hat{\mathcal{Y}}_*) - hv_d(\hat{\mathcal{Y}}_* \setminus \hat{\mathbf{y}})$. The nadir point required for the calculation of HV is defined as 110% of the maximum value of each objective in the approximate set.

Definition 6.2 (Number of points dominating $\mathbf{c}$: $ndp(\mathbf{c}, \mathcal{Y})$) For every sampled point $\mathbf{c}$, there is an solution objective vector $\mathbf{y} \in \mathcal{Y} \subset \mathbf{R}^m$. Let $B = (\mathcal{Y} \setminus \mathbf{y})$.

$$ndp(\mathbf{c}, \mathcal{Y}) = \sum_{\mathbf{q} \in B} \begin{cases} (\mathbf{q} \prec \mathbf{y}) (\mathbf{q} \text{ dominates } \mathbf{y}) & 1 \\ \text{else} & 0 \end{cases}$$

This order of dimensions which the partitioning strategy will follow starts with the dimension with the lowest value of

$$\text{num_dom_pts}_j = \sum_{k \in \{1, -1\}} \text{num_dom_pts}(\mathbf{c}_i + k \cdot \delta \cdot \mathbf{e}_j), \quad (\text{Eq. 6.4})$$

where num_dom_pts_j represents the sum of the number of dominating points of the newly sampled points in dimension j . The partitioning then advances with the next dimension until the highest num_dom_pts_j .

6.2.3.1 Pseudo-code for PA-DIRECT

Algorithm 5 provides the pseudo-code of our proposed algorithm, PA-DIRECT. The eventual aim of PA-DIRECT is to find solutions on the whole Pareto front. The use of DIRECT framework allows the exploitation of promising regions and the exploration of less explored regions in the search space to be carried out simultaneously. Moreover, the use of proposed indicators, $hv_d(\mathbf{y})$ and $ndp(\mathbf{c})$ can enhance the process of local and global search through the reduction of redundant sampling with the concept of diversification.

Algorithm 5: PA-DIRECT : Pareto-Aware Dividing Rectangles

Input: vectorial function to be minimized $\mathbf{f}$,

search space $\mathcal{X}$,

evaluation budget v ,

% of dominating points threshold k_{dom}

Output: approximation set of $\min_{\mathbf{x} \in \mathcal{X}} \mathbf{f}(\mathbf{x})$, $\mathcal{Y}_*^v$

Initialisation : $\mathcal{H}_1 = \{\mathcal{X}\}$

- 1: **while** evaluation budget v is not exhausted **do**
 - 2: Evaluate all the new hyperrectangles: $\mathbf{y}_i = \mathbf{f}(\mathbf{c}_i), \forall i \in \mathcal{H}_t$.
 - 3: Update the number of dominating points, $\mathbf{ndp}(\mathbf{c}_i), \forall i \in \mathcal{H}_t$.
 - 4: Identify approximate set of Pareto-optimal solutions:
 $\mathcal{Y}_*^t \leftarrow \{\mathbf{ndp}(\mathbf{c}_i) = 0, i \in \mathcal{H}_t\}$.
 - 5: Calculate hypervolume for approximate Pareto-optimal solutions: $\mathbf{hv}_d(\mathbf{y}_i), \forall \mathbf{y}_i \in \mathcal{Y}_*^t$.
 - 6: Identify small contributing hypervolume solutions as terminal nodes:
 $\mathcal{H}_{terminal} \leftarrow \{\mathcal{H}_i : i \in \mathcal{H}_t, \mathbf{hv}_d(\mathbf{y}_i) < \epsilon, \mathbf{y}_i \in \mathcal{Y}_*^t\}$.
 - 7: $\mathcal{H}_t \leftarrow \mathcal{H}_t \setminus \mathcal{H}_{terminal}$.
 - 8: Identify the size of the largest hyperrectangle on the approximate Pareto front:
 $\sigma_{nd} \leftarrow \min(\{\sigma_i : i \in \mathcal{H}_t, \mathbf{y}_i \in \mathcal{Y}_*^t\})$.
 - 9: Local Search: $\mathcal{I}_{local} \leftarrow \text{ND}(\{(\mathbf{hv}_d(\mathbf{y}_i), \sigma_i) : i \in \mathcal{H}_t, \mathbf{y}_i \in \mathcal{Y}_*^t\})$.
 - 10: Global Search: $\mathcal{I}_{global} \leftarrow \text{ND}(\{(\mathbf{ndp}(\mathbf{c}_i), \sigma_i) : i \in \mathcal{H}_t, \mathbf{y}_i \in (\mathcal{Y} \setminus \mathcal{Y}_*^t), \sigma_i \leq \sigma_{nd}, \mathbf{ndp}(\mathbf{c}_i) \leq k_{dom} \times \text{evaluation}_t\})$.
 - 11: $\mathcal{I}_t \leftarrow \mathcal{I}_{local} + \mathcal{I}_{global}$.
 - 12: Partition the hyperrectangles in $\mathcal{I}_t$ according to the DIRECT procedure using Eq. Eq. 6.4.
 - 13: $\mathcal{H}_{t+1} \leftarrow \mathcal{H}_t \setminus \mathcal{I}_t \cup \{\mathcal{I}_t\text{'s newly generated hyperrectangles}\}$
 - 14: $t \leftarrow t + 1$
 - 15: **end while**
 - 16: **return** $\text{ND}(\{\mathbf{f}(\mathbf{c}_i)\}_{i \in \mathcal{H}_t})$
-

6.2.4 PA-DIRECT's Faster Convergence

In this section, the details of PA-DIRECT, particularly the selection and global score assignment strategies, are explained with justifications. Further, the illustration of how PA-DIRECT overcomes the issues of MO-DIRECT viz., the Pareto-independent global score assignment followed by generous sampling is provided using the same synthetic bi-objective 2-variable problem.

Sampled point (c) no.	ndp(c)
6	0
7	0
8	5
9	3

Table 6.1: List of dominating points for PA-DIRECT partitioning strategy

6.2.4.1 Pareto-Aware Global Score Assignment

PA-DIRECT's strategy of order of dimensions in the partitioning of POH(s), which determines the assignment of global score, is examined here. The goal behind the order of dimensions when partitioning POH is to achieve diversification in the search of Pareto-optimal solutions. For this purpose, the use of **ndp(c)** is proposed as a suitable indicator in the order of division in PA-DIRECT. The concept of dominance is inherent in **ndp(c)**. This hence resolves the issue identified in the diversity approach used in MO-DIRECT (raised in Section 6.2.1).

ndp(c) captures the dominance and diversity of the newly sampled point in global context by comparing it against the whole population. Therefore, allowing a fair comparison between the newly sampled points. In addition, the information of **ndp(c)** is utilised for the choice of POH(s) in global search, thus does not contribute to additional computation.

The global score assignment strategy of PA-DIRECT is being illustrated in the same synthetic problem (refer to Figure 6.3) here. Table 6.1 furnishes **ndp(c)** for the newly sampled points (6,7,8,9). Figure 6.5a shows the result of new methodology in the order of partitioning, where the more diverse and less dominated points are in bigger hyper-rectangles. Hence, this proposed strategy for the order of partitioning aligns with the fundamental concept of diversification towards Pareto-optimal solutions and boosts the search in PA-DIRECT.

Figure 6.5: PA-DIRECT strategy for order of partitioning

6.2.4.2 Conservative sampling

An ideal set of Pareto-optimal solutions is not only optimal but also diverse. Therefore, it is wise to look at both convergence and diversity when sampling new points. In PA-DIRECT, the idea of diversity is analyzed on two fronts - the approximate Pareto-optimal solutions and the dominated solutions. This allows sampling in a conservative manner that can lead to a faster convergence to the whole set of Pareto-optimal solutions. The details and reasons behind the two selection strategies applied on the approximate Pareto-optimal and dominated solutions in PA-DIRECT are first elaborated. Next, the use of PA-DIRECT's selection strategies is performed on the same artificial bi-objective problem that was used by MO-DIRECT to show the relatively conservative sampling of PA-DIRECT.

Selection of $\mathcal{I}_{local}$: POH(s) in Local Search In [115], hypervolume (HV), epsilon (EPS) and R2 indicators were embedded into the DIRECT framework and were evaluated in BMOBench test functions [5]. The results were similar, with HV narrowly emerging with the best results. Hence, HV is selected as a metric for diversity on the approximate

Pareto front in PA-DIRECT .

However, it was theoretically established in [11] that the value of HV is reliant on the shape of the Pareto front. The point is favoured if its tangent has a gradient of -45° . This shortcoming of HV was also highlighted in [59], particularly when being used to measure the diversity of solutions. A modified HV measure, $hv_d(\mathbf{y})$, that could overcome this shortcoming was introduced in [59]. Therefore, to fulfill the diversity purpose, the proposed measure $hv_d(\mathbf{y})$ is jointly utilised with the size of hyperrectangles for picking $\mathcal{I}_{local}$ in PA-DIRECT.

Moreover, it should be noted that $HV/hv_d(\mathbf{y})$ returns a value of 0 for solutions on the dominated fronts. Therefore, it is only useful in choosing diversified solutions on the approximate non-dominated front, i.e. in carrying out a local search. The selection strategy for global search is elaborated in the next section.

Selection of $\mathcal{I}_{global}$: POH(s) in Global Search There are many strategies that can be adopted when seeking diversification on the dominated fronts. One such measure is the crowding distance, which is essentially the gap between a particular solution to its nearest neighbours. However, the crowding distance alone is unable to capture the relative dominance of points to one another. Recently, a measure that counts the number of solutions (in a population) that a particular solution is being dominated by serves as an indicator for elimination in SMS-EMOA [16]. This measure is referred to as the number of dominating points, $ndp(\mathbf{c})$. The member with the most number of dominating points on the worse front is eliminated from the population in SMS-EMOA. As the use of dominating points is able to provide a picture of diversity in a global context, $\mathcal{I}_{global}$ is chosen with the non-dominated front of $ndp(\mathbf{c})$ and size of hyperrectangles in PA-DIRECT.

The ideal outcome of this strategy is to pick diversified solutions in the less explored areas of the search space. This, in turn, raises two possible limitations of the aforementioned strategy in the choice of $\mathcal{I}_{global}$:

- (i) All points that have a single dominating point ($\mathbf{ndp}(\mathbf{c})=1$) will always be chosen despite the size of hyperrectangle

This means that how crowded or sparse the region where the point lies in the search space is not taken into consideration when the given point has a single dominating point.

- (ii) One point with the biggest hyperrectangle (indicating the least explored region(s)) will be chosen despite the number of dominating points it has

This signifies that one point in the least explored region(s) will always be chosen despite how unfavourable the solution might be.

Elitism Consequently, to streamline sampling and concentrate on the selection of points that encourages diversification in promising and less explored areas of the search space, two independent filters focusing on the two respective limitations are suggested.

- (i) The chosen POHs in a global search should be bigger than those in the local search
Since the number of dominating points is only being applied in dominated fronts, it will be coherent to only choose POHs that are bigger in size than those on the non-dominated front. This will lessen redundant sampling on well-explored regions of the search space.

- (ii) $\mathbf{ndp}(\mathbf{c})$ for chosen POHs must be less than $k\%$ of the total sampled points

As mentioned earlier, one of the biggest hyperrectangles would always be picked, despite being dominated by a considerable number of points most of the time. Here, a measure is proposed to stop or delay this effect. Only dominated points that are being dominated by less than a certain $k\%$ (e.g. 50%) of the total number of evaluated points in that iteration are contemplated for selection. This filter cut down redundant sampling in unfavourable regions and would, in turn, allow the

solutions to converge at a faster rate.

However, if the value k is set too low, particularly at the initial stages of experiments, the solver could be stuck in a local optimum instead. For example, if $k = 50\%$, with only 10 function evaluations, a point with 5 points dominating it will be discarded. On the other hand, when there is a total of 1000 function evaluations, a point would require 500 points to dominate it before being discarded. Hence, the use of a decreasing exponential function dependent on the number of function evaluations is suggested to decide the number of dominating points required.

$$k = \exp^{-\frac{nfe}{tfe}} \quad (\text{Eq. 6.5})$$

where nfe and tfe represent the current and total number of function evaluations respectively. Eq. 6.5 supplies the formulation of $k\%$ that is dependent on the number of evaluations in the experiment.

Now, the selection strategies of PA-DIRECT are applied to the same bi-objective problem in Figure 6.4 of Section 6.2.2. The selection strategy for the approximate Pareto front using $hv_d(\mathbf{y})$ is first provided in Figure 6.6.

Figure 6.6 displays the values of indicator $hv_d(\mathbf{y})$ against the size of hyperrectangles. The size of hyperrectangle is reflected as the outcome of the new global score assignment of PA-DIRECT shown in Figure 6.5. The characteristic of a diverse solution (on the approximate Pareto front) is a large value of $hv_d(\mathbf{y})$. A point in a sparse region is indicated by a big hyperrectangle size. In this example, the non-dominated front of $hv_d(\mathbf{y})$ and size of hyperrectangle, points 6 and 11, are in the set of $\mathcal{I}_{local}$ in PA-DIRECT. The number of points in $\mathcal{I}_{local}$ drastically reduced from 6 in MO-DIRECT to 2 in PA-DIRECT.

Figure 6.7 shows $ndp(\mathbf{c})$ against the size of hyperrectangles for solutions on the dominated fronts for the same example problem. Similarly, the size of hyperrectangle is

Figure 6.6: Illustration of the use of $hv_d(\mathbf{y})$ on approximate Pareto front

Figure 6.7: Illustration of use of dominating points on dominated fronts

reflected as the result of the global score assignment of PA-DIRECT in Figure 6.5. The non-dominated front (shown in Figure 6.7) is point 2 (square). Referring back to the decision space in Figure 6.5, one can observe that point 2 denotes the unexplored area in the decision space. The proposed filters are represented by the red lines in Figure 6.7, where the red vertical line represents the first filter of the minimum size of hyperrectangles and the red horizontal line represents the second filter of $\mathbf{ndp}(\mathbf{c})$ where k is set to 50%. The red shaded box represents the filtered region where $\mathcal{I}_{global}$ should be chosen from. In this case, point 2 manages to fall in the red shaded box and is being chosen. In this synthetic example, the effect of these filters is not being shown. However, as the number of sampled points increases, these filters can aid in choosing POHs more conservatively through the exclusion of a solution in a well-explored region in the decision space or a relatively unpromising solution. The number of points in $\mathcal{I}_{global}$ significantly reduced from 4 in MO-DIRECT to 1 in PA-DIRECT.

The use of PA-DIRECT's selection strategies for both local and global search on the same bi-objective example problem reduced the total number of points selected for sampling ($\mathcal{I}_t$) from 10 (in MO-DIRECT) to 3. Thus, it can be gathered that PA-DIRECT has a conservative sampling strategy. For the validation of PA-DIRECT's faster convergence through Pareto-aware global score assignment and conservative sampling, numerical experiments are ran. The results are given in Section 6.3.2.

6.2.5 Efficient Way of Computing Diversity Measures

The whole archive of evaluated solutions is used for the selection of POHs in PA-DIRECT. Hence, the computational complexity grows exponentially when the number of evaluations is raised. For the reduction of computation time required by PA-DIRECT, the following methods are suggested:

- Hypervolume Computation in a Large Archive

Non-dominated solutions that have previously been given a hypervolume of lower than ϵ will be treated as terminal nodes in the tree. This is done to reduce the total number of hypervolume computations. The value of 0.0001 is suggested for ϵ , given that the objective values are normalised. The small value of hypervolume (0.0001) implies that the solution has close neighbours. Therefore, the point should not be chosen when the consideration of diversification. This is reflected in line 6 and 7 of the Algorithm 5.

- Using Global Variables

With an increasing population size in PA-DIRECT after every iteration, the amount of computations for $\mathbf{ndp}(\mathbf{c})$ increases as the number of evaluated solutions increases. However, the value of $\mathbf{ndp}(\mathbf{c})$ could only increase over time due to the emergence of newly sampled points. Thus, $\mathbf{ndp}(\mathbf{c})$ can be kept as a global variable and gets updated when new solutions are evaluated. This step is given in line 3 of the Algorithm 5.

6.3 Numerical Experiments of PA-DIRECT on the *bbob-biobj* Test Suite Benchmark

The details and outcome of the numerical experiments experimental results in evaluating PA-DIRECT are given in this section. PA-DIRECT is compared with the following state-of-the-art algorithms.

- SMS-EMOA[16] : $\mathcal{S}$ Metric Selection Evolutionary Multi-objective Optimisation Algorithm (Indicator-based EA)
- MOEA/D[127] : Multi-objective Evolutionary Algorithm based on decomposition (Decomposition-based EA)

- DMS[31] : Direct MultiSearch (Dominance and diversity DSM)
- MO-DIRECT[4] : Multi-objective Dividing Rectangles (Dominance and diversity MSO)

The function evaluation budget is fixed at 10^5 for all algorithms. The parameter settings for SMS-EMOA and MOEA/D are kept the same as in [12] for a fair comparison. The EA algorithms follow the simulated binary crossover and polynomial mutation operators. The parameters of these algorithms were fixed as: population size of 100; crossover parameter is given as $\eta_c = 15$; mutation parameter is given as $\eta_m = 20$; crossover probability at a value of 0.9; swap probability at a value of 0.5; mutation probability is given as $1/n$ where n is the number of decision variables. In MOEA/D, the neighbourhood index is given a value of 10 and the weighted sum approach is assigned as the decomposition method used here. DMS is ran without a search step and was initialised by sampling along the line between upper and lower bounds. The list in DMS contained all the current feasible non-dominated points. When a point on the list is picked, it became the poll centre. The newly sampled solutions were added to the end of the list with the poll centre being the last on the list. The step size parameter was initialised at 1 and halved during unsuccessful iterations. The hyper-rectangle size threshold, σ_t , in MO-DIRECT (which denotes the minimum size a hyper-rectangle needs to be in order to be considered for selection as a POH) was not defined to encourage exploitation. In PA-DIRECT, the size of POHs considered for selection in a global search was fixed to be bigger than the largest selected POH in the local search.

6.3.1 *bbob-biobj* Test Suite

There are some distinctive features in a multi-objective black-box optimisation problem that can help to define the difficulty behind solving a given problem. A problem with independent variables is also known as a *separable* function. This, in turn, means that the

problem can be resolved by employing successive one-dimensional optimisations along the coordinate axes while keeping the other variables unchanged. Contrarily, a *non-separable* function, which has dependent variables, is much harder to solve. Another characteristic relates to the number of optima in the problem. A *unimodal* function has a single optimum which is the global optimum. On the other hand, *multi-modal* function with multiple optima, is much harder to solve. However, the *multi-modal* feature is prevalent in real-world optimisation problems. *Ill-conditioning* is another challenging feature that exists when a tiny change in variables can generate a function value improvement that varies by orders of magnitudes.

The aforementioned problem features are included in the bi-objective problems from the *bbob-biobj* test suite. In this experiment, the 2D, 5D and 10D problems are evaluated. The test suite consists of a total of 55 test functions, with permutations of ten single objective functions with two from each of the following groups:

- separable functions
- functions with low or moderate conditioning
- functions with high conditioning and unimodal
- multi-modal functions with adequate global structure
- multi-modal functions with weak global structure

There are a total of ten instances for each test function. The test functions are parametrised differently in each instance, allowing the objective values of different instances to differ greatly. This makes it tough to design an algorithm or tune parameters to fit the entire suite of problems. A total of five instances for each test function were run in this experiment.

Figure 6.8: Overall results on all 55 functions in 2D, 5D and 10D using 5 instances

COCO platform allows the aggregation of results based on the features of problems of interest (such as the groups above) over which a uniform distribution is assumed [51]. With this visualisation of results, deeper insights about the various algorithms can be recognised and analysed. The evaluation metric of this benchmark set is dependent on the hypervolume of all non-dominated solutions in the archive of candidate solutions. The runtime in terms of function evaluations until hypervolume value exceeds prescribed target values is recorded [22].

6.3.2 Performance Evaluation

An overview on the performance of algorithms on 2D, 5D and 10D problems in the *bbob-biobj* test suite is presented in Figure 6.8. The x-axis denotes the number of function evaluations executed and the y-axis represents the percentage of all the solutions found

in comparison with the reference Pareto front. This percentage was computed using the hypervolume indicator. A higher value in y at a lower value of x signifies a better performance.

Figure 6.8 shows that PA-DIRECT has the best overall performance given a budget of 10^5 function evaluations in all 2D, 5D and 10D problems. The rank order of the best performance in 2D begins with PA-DIRECT, followed by SMS-EMOA, MO-DIRECT, DMS and MOEAD. Moreover, PA-DIRECT reflects a better performance at any given function evaluation in 2D. This implies that the convergence rate of PA-DIRECT is the best among all the algorithms in 2D. In 5D, the order of rank beginning from the best performance is PA-DIRECT, MO-DIRECT, DMS, SMS-EMOA and MOEAD. In 10D, the order of rank beginning from the best performance is PA-DIRECT, DMS, MO-DIRECT, MOEAD and SMS-EMOA. The performance ranks of the algorithms vary when the number of dimensions increases. Overall, PA-DIRECT has better convergence than other mathematical programming based algorithms.

Figure 6.9 depicts results in the various groups of problems in the test suite in 5D. Given the 5 classes of functions, a total of 15 permutations can be derived assuming that it can be paired with itself. One straightforward method is to look at the performance in problems with both functions in the same category:

- separable-separable

The performances of all the algorithms in this category vary slightly (ranging between 0.5 and 0.7 of total HV). DMS is the best performing algorithm, followed by PA-DIRECT, MO-DIRECT, MOEAD and SMS-EMOA.

- moderate-moderate

The performances of the algorithms in this category range from 0.4 to 0.8 of the total HV. DMS outperforms all the algorithms. This is followed by PA-DIRECT and MO-DIRECT with similar performance, then MOEA and SMS-EMOA.

Figure 6.9: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 58 targets with target precision in $\{-10^{-4}, -10^{-4.2}, -10^{-4.4}, -10^{-4.6}, -10^{-4.8}, -10^{-5}, 0, 10^{-5}, 10^{-4.9}, 10^{-4.8}, \dots, 10^{-0.1}, 10^0\}$ for all function subgroups in 5-D.

- ill-conditioned-ill-conditioned

PA-DIRECT manages to top all the algorithms in this category. It is closely followed by DMS and MO-DIRECT. MOEAD and SMS-EMOA perform rather poorly in this group. DMS is noticed to have good performance in the first 10^3 function evaluations.

- multimodal-multimodal

The performances of all the algorithms are unsatisfactory in this category. The best algorithm, PA-DIRECT, is only able to obtain 0.5 HV of all the given solutions. However, PA-DIRECT clearly performs better than MO-DIRECT, SMS-EMOA, MOEAD and DMS (in that order). DMS is noticed to stagnate after a steady growth in performance in the first 10^2 function evaluations.

- weakstructure-weakstructure

PA-DIRECT distinctly emerged as the best performer followed by MO-DIRECT in this category. SMS-EMOA, DMS AND MOEAD following behind. Similar to multimodal-multimodal problems, DMS starts to stagnate after the first 10^2 function evaluations.

6.3.3 Overall Comparison

This section discusses the results featured in the previous section.

Overall analysis

- Indicator-based methodology:

In 2D problems, indicator-based methods, PA-DIRECT and SMS-EMOA, are ranked 1 and 2. There lies a considerable difference between PA-DIRECT and its dominance-based counterpart, MO-DIRECT. Likewise, a significant difference lies between SMS-EMOA and its decomposition-based counterpart, MOEAD. However, in 5D and

10D problems, the difference between PA-DIRECT and MO-DIRECT diminishes. The results of SMS-EMOA become comparable to MOEAD. This depicts the effectiveness of indicator-based methods in problems with smaller dimensions. This is because, in smaller dimensions, there is a higher probability for the Pareto-optimal solutions to be discovered given the same number of function evaluations. Thus, the use of indicator-based techniques for diversification would have a larger impact in incrementing the HV in 2D as compared to 5D and 10D problems.

- **MP vs EA:**

The results in Figure 6.8 allow us to conclude that MP methods generally performs better than EA. This pattern becomes explicit in higher dimensions (5D, 10D). It is not shocking that MP performs better than EA because MP utilises more knowledge when picking the subsequent point to evaluate. This is also why the computation time of MP methods is much longer compared to EA.

- **Curse of Dimensionality:** Both MP and EA algorithms suffer from the curse of dimensionality. Performances of all the algorithms deteriorate as the problem dimension increases. The search space where the Pareto-optimal solutions exist expands as the number of dimensions increases. This, in turn, makes it harder for both types of algorithms to find the Pareto-optimal solutions.

Detailed analysis

- **Strength of MSO framework:**

Figure 6.9 depicts that MSO algorithms PA-DIRECT and MO-DIRECT achieve better results than the rest of the algorithms in multi-modal problems (multimodal-multimodal and weakstructure-weakstructure categories). Both PA-DIRECT and MO-DIRECT follow the DIRECT framework [60] under the class of MSO algorithms.

The DIRECT framework is popular for a good balance between exploration and exploitation. Therefore, both PA-DIRECT and MO-DIRECT are able to explore unexplored areas in the search space and, in turn, find more Pareto-optimal solutions. Hence, they are able to find a bigger number of Pareto-optimal solutions and are obtaining better results. This is specifically accentuated by the performance in multi-modal functions with weak global structure. The solutions without adequate global structure can be discovered with the DIRECT framework because the partitioning method in DIRECT gives a structure to the search space. Furthermore, the results shown on the graph for multi-modal functions with weak global structure exemplifies the impact of selection strategy in DIRECT. A selection strategy concentrated on diversification (PA-DIRECT) could substantially improve the performance (compared to MO-DIRECT).

- Occasions where MO-DIRECT outperforms PA-DIRECT:

PA-DIRECT performs better than MO-DIRECT on the whole. However, there are some occasions where the performance of PA-DIRECT is poorer than MO-DIRECT. Here, this likelihood is being investigated in the case of functions f12 and f23.

Figure 6.10 shows the results of f12, f23 on the left and the unscaled objective space of the first instance of the relative function on the right (taken from the online documentation of bbob-biobj Test Suite [110]). On the left, the results of PA-DIRECT on both f12 and f23 are depicted to be worse than MO-DIRECT. On the right, the reference set for the Pareto front (indicated in black) is given to be like an L-shape, with many other local optima (indicated by the other colours). The hypervolume indicator for diversity is utilised for the choice of POHs for local search in PA-DIRECT. In f12 and f23, the value of hypervolume indicator for diversity between the points on the black line and the red line is negligible i.e. $hv_d(\text{black}) \approx$

Figure 6.10: Results of f12,f23 in 5D in bbob-biobj suite on COCO Platform

hv_d (red). Thus, the convergence to the real Pareto front is hindered in functions similar to f12 and f23.

- Tight bound on the size of POHs in global search:

In this study, the bound on the size of POHs in global search is the size of the biggest POH selected in the local search. However, this could delay the choice of POHs in optimal areas in exchange for more exploration, particularly in problems of larger dimensions. If given a restricted or small number of function evaluations for a problem with large dimensions, one might want to ease the bound to the smallest POH picked in the local search or even abolish this filter altogether.

6.4 PA-DIRECT for the Optimal Design of Sector Shapes

The constraint-based approach for the optimal design of sector shapes proposed in Chapter 3 is solved by PA-DIRECT with constraint handling methods here. There are two con-

straints, namely the safety (Eq. 3.6) and workload constraint (Eq. 3.11) to be handled.

In NSGA-II, infeasible solutions (that did not satisfy the constraints) are ranked worse than feasible solutions. Given the stochastic nature of NSGA-II, this does not hamper the search of the algorithm. However, the space partitioning nature of PA-DIRECT could potentially lead to finding only sub-optimal solutions if infeasible solutions are neglected. Following this, approaches to handling the two constraints are provided.

- **Safety Constraint:** Solutions that violate the safety constraint is deemed as infeasible. However, if the total hypervolume of the solutions are unchanged (meaning no new optimal solutions are discovered), PA-DIRECT is triggered to sample from this pool of infeasible solutions considering the non-dominated front of the following criteria:
 - (i) Standard deviation of monitoring workload
 - (ii) Total coordination workload
 - (iii) Size of hyperrectangle containing the solution
 - (iv) Minimum distance from crossing point to sector boundaries
- **Workload Constraint:** Instead of considering these solutions as infeasible, they are being penalised in both objective functions. The objective values are re-calculated using the following formula:

$$\zeta = \begin{cases} \sum_{j=1}^N (M_j - (1 + \frac{1}{N-1})), & \text{constraint violated} \\ 0, & \text{otherwise} \end{cases} \quad (\text{Eq. 6.6})$$

$$obj = obj * X^\zeta \quad (\text{Eq. 6.7})$$

where X is a multiplier of the objective function, penalising solutions that have more unbalanced monitoring workloads at an exponential rate. The value of X should be a integer ranging from 2.

Initialization. In PA-DIRECT, the centre of the decision variable space is usually sampled as the first point. However, using the same Voronoi sites for the 4 areas would not allow sectors to be generated. Hence, the Voronoi sites in the decision variable space (variable 0 to 7) are slightly shifted by 0.01% (relative to the longitude and latitude bounds) in equal diagonal directions such that sectors can be generated in the first iteration.

6.4.1 Simulation Study

This study evaluates the effectiveness of PA-DIRECT in the problem of optimizing sector shapes. Similar to the study conducted in Chapter 3, a set of one-hour historical flight data in the Singapore regional airspace is used [46]. The region of airspace that is bounded by latitude $[-1, 7]$, longitude $[99, 107]$ and altitude $[24000, 42000]$ ft. The 2D sectors are being generated using Voronoi diagrams and 3D sectors are formed using a vertical split on a particular sector.

A study is carried out using the software implementation of PA-DIRECT and NSGA-II, NGPM v1.4 (NSGA-II Program for MATLAB) [79] for NSGA-II [32]. A total of 20 000 evaluations is given to each optimisation solver. The proposed deterministic algorithm PA-DIRECT is ran a single time and NSGA2 is ran 30 times to ensure a fair comparison.

Table 6.2 provides the parameter settings used in the experiments.

The population size is set to 100 such that the population is diverse and thus does not converge to a local minimum quickly.

6.4.1.1 Results

The value of X , multiplier was experimented with values of 2,3,5 to penalize unbalanced workload. Figure 6.11 shows that using $X = 3$ enables PA-DIRECT to find better solutions and hence is a suitable value to use in the optimisation model. The following results are obtained using $X = 3$ in the optimisation model when solved with PA-DIRECT.

Table 6.2: Parameter settings

Optimisation Settings	Value/Methodology
NSGA-II: NGPM Settings	
Population Size	100
Maximum Generations	200
Selection	Binary Tournament
Crossover	Intermediate Crossover ($\beta_c = 1.2$)
Mutation	Gaussian ($scale_m = 0.1, shrink_m = 0.5$)
Model Settings	
Number of sectors, N	5
Number of areas, K	4
Constraint: $M_j \leq$	1.25

Figure 6.11: Approximate Pareto front of solutions obtained with $X = 2, 3, 5$ in PA-DIRECT with 20 000 evaluations

The approximate Pareto-optimal solutions found in PA-DIRECT and in 30 different runs of NSGA-II is given in Figure 6.12. The filled $\bullet$ represent solutions found using PA-DIRECT and the unfilled $\circ$ are the those found using NSGA-II over 30 runs. Varying colours are used to represent solutions obtained in a different run. It can be observed that the solutions found using NSGA-II in different runs vary quite widely. For a more quantitative analysis, the hypervolume of each run is calculated with reference to the maximum values obtained over all the results.

Figure 6.12: Approximate Pareto front of solutions obtained in PA-DIRECT (●) and NSGA-II (○) in 30 runs

Figure 6.13 provides the box plot of hypervolumes obtained over the 30 runs in NSGA-II and in PA-DIRECT. The box plot reveals that the spread of hypervolume is quite wide (between 0.7 and 0.95) with a median of 0.78. The short distance between the 25th percentile and the median indicates that the hypervolume is commonly found between 0.76 and 0.78. Further, there is an outlier at 0.48, which shows that NSGA-II can behave poorly occasionally. In comparison, the deterministic PA-DIRECT gives a hypervolume of 0.87, higher than the median of 0.78 in NSGA-II. Hence, it can be said that the performance of the proposed PA-DIRECT is more reliable than NSGA-II.

Analysis For a deeper insight into the problem of optimising sector shapes, the approximate Pareto front of all solutions was filtered from PA-DIRECT and 30 runs of NSGA-II.

Figure 6.14 provides the objective values of all the approximate Pareto front solutions and identifies which solver and run were they obtained from. They are made up of 6 different runs of NSGA-II and from PA-DIRECT. It is worthy to note that all the solutions obtained from PA-DIRECT is on the combined approximate Pareto front. Further, it

Figure 6.13: Box plot of Hypervolume obtained in PA-DIRECT and NSGA-II over 30 runs

is interesting to see that the solutions from various runs/solver are in clusters on the approximate Pareto front with the exception of NSGA 2- Run 12, where solutions are found near the two extremes.

The approximate Pareto-optimal solutions are also illustrated in the search space with the use of parallel coordinates in Fig. 6.15. The plot shows that the solutions in different runs of NSGA-II and PA-DIRECT are found in separate regions of the search space. This allows us to conclude that the problem of optimizing sector shapes has multi-modal characteristics.

6.5 Summary

This chapter has presented a new multi-objective deterministic solver based on the MSO framework, PA-DIRECT. PA-DIRECT uses improved sampling and global score assignment strategies to converge towards the Pareto-optimal front at a faster rate. PA-DIRECT's performance is found to be competitive with the state-of-the-art algorithms on the bi-objective test suite on the COCO platform. Moreover, PA-DIRECT is able to find better

Figure 6.14: Overall approximate Pareto front obtained from PA-DIRECT and 30 runs of NSGA-II

Figure 6.15: Parallel coordinates plot of approximate Pareto-optimal solutions in the search space

solutions than the other algorithms in multi-modal functions with weak global structure. Further, PA-DIRECT has been implemented to find optimal sector shapes given a set of traffic flows and airspace with general constraint handling techniques. When compared to NSGA-II, PA-DIRECT provides a more consistent performance in the airspace sectorization problem.

Chapter 7

Conclusions And Future Directions

The thesis is focused on the development of a cognitive decision-making architecture for dynamic airspace management. The work has raised and addressed various issues present in the implementation of Dynamic Airspace Sectorization (DAS) in the real world. In this chapter, a summary of these research findings and possible future directions is presented.

7.1 Summary of Contributions

With an increasing demand in air traffic, there is a need to optimize the utilisation of airspace. The change in sector shapes over time was proposed to manage varying traffic flows such that the air traffic controllers are able to manage the safety of aircraft. This idea is referred to as DAS. Three key aspects of DAS have been particularly addressed, namely i). the optimisation model in DAS; ii). the possible real-world scenarios in DAS; and iii). the solver used to solve DAS. In essence, the major contributions of this thesis are:

- Development and analysis of preference-based bi-objective optimisation models for airspace sector design. The constraint-based and reference-point-based methodologies are applied to identify optimal solutions preferred by the air traffic controllers.

- Development and analysis of a cognitive-based approach for the DAS problem, which we refer to Cognitive decision-making architecture for Dynamic Airspace Sectorization (CDAS). First time in literature, the question of *when-to-do* a resectorization is addressed.
- Development and analysis of a rolling horizon optimisation approach for DAS. It addresses the uncertainty present in the feasibility of sector shapes (in satisfying the defined sector similarity constraint) in future time intervals.
- Development and analysis of a methodology that simultaneously optimises the affected airways and airspace when there is a change in availability of airspace, which we refer to Simultaneous Optimisation of AirWay and AirSpace (SAWAS).
- Development and benchmarking a new multi-objective deterministic solver, Pareto-Aware Dividing Rectangles (PA-DIRECT).
- Implementation and analysis of the newly developed deterministic PA-DIRECT on the constraint-based bi-objective optimisation model with common constraint handling strategies.

Based on the above contributions, the major conclusions of this thesis are summarised as follows.

Preference-based bi-objective optimisation models for airspace sector design.

In Chapter 3, a new optimisation model for the design of sector shapes is presented. The model is built on the two objectives focusing on the air traffic controllers' workloads, i) balancing the interactions between controllers and pilots among sectors (monitoring workload) and ii) reducing total interactions between controllers (coordination workload). A dynamic density-based metric is proposed to better capture the interactions between controllers and pilots. Further, the model incorporates two different strategies, namely

the constraint-based strategy and the reference-point-based strategy, to focus the search on a particular region of interest on the approximate Pareto front. The reference-point-based strategy is generally able to provide better solutions in the region of interest when compared to the constraint-based strategy.

Cognitive decision-making architecture for DAS. The varying traffic flows might require a change in sector shapes at times, to allow the controllers better manage the safety of aircraft. CDAS addresses the questions of *when-to-do* and *how-to-do* a resectorization of the given airspace. CDAS introduces a metacognitive decision maker to aid the user in selecting a solution from a pool of solutions optimised for the current time period, with predictive monitoring and coordination workload for the future time periods. With this metacognitive feature, CDAS identifies situations where change in traffic flow is insignificant to reduce the number of resectorization required. The conventional split-and-merge approach was compared against the resectorization method used in CDAS and is found to be worse in terms of the model objectives.

Rolling horizon optimisation framework for DAS. Even though CDAS provided a framework to address when and how to perform a resectorization, it does not necessary guarantee the feasibility of solutions found in future time intervals. A rolling horizon optimisation framework is proposed to overcome this uncertainty of feasible solutions. In this framework, DAS is modelled as a single objective optimisation problem, minimising the average standard deviation of monitoring workloads found over a finite time horizon. In this way, one can optimise the airspace such that it takes into account the predicted future traffic flows to ensure feasibility in found sector shapes. The rolling horizon optimisation approach is compared to the single time interval optimisation. The study performed shows that the feasibility of solutions in future time intervals was obtained by taking up slightly worse solutions (compared to single time interval optimisation) in earlier time intervals.

Simultaneous optimisation of airway and airspace for changing availability of airspace. Weather conditions can affect the availability of airspace. This, in turn, affects airways, requiring aircraft to be re-routed. When this happens, the controllers' workloads also changes, which sometimes calls for a resectorization. SAWAS is proposed to find optimal solutions during this situation. Compared to the sequential approach of re-routing and resectorization, SAWAS is able to find better and a greater range of solutions.

Deterministic multi-objective multi-scale search algorithm for airspace sector design. A new deterministic multi-objective multi-scale search algorithm, PA-DIRECT is proposed for deriving reproducible and reliable solution in airspace sector design. PA-DIRECT follows a unique space partitioning procedure found in the DIRECT [60] and MO-DIRECT [4] framework. PA-DIRECT has found new sampling and global score assignment strategies for faster convergence towards the Pareto front. PA-DIRECT is validated on the bi-objective test suite [110] on the Comparing Continuous Optimisers (COCO) [51] platform. PA-DIRECT is found to generally perform better than the state-of-the-art algorithms, particularly in multi-modal functions with weak global structure. Moreover, PA-DIRECT is used to solve the earlier proposed constraint-based bi-objective optimisation model with some general constraint handling techniques. The performance of deterministic PA-DIRECT is competitive with the stochastic NSGA-II, with PA-DIRECT finding better solutions than NSGA-II most of the time.

7.2 Future Directions

Based on the current research, we raise a few promising directions for future work with regard to cognitive dynamic airspace management. One should note that some of these suggestions were previously discussed in previous chapters.

Towards reference-point-based multi-scale optimisation solver. In the preference-based bi-objective model study, the Reference-based Non-dominated Sorting Genetic Algorithm II (R-NSGA-II) is generally found to discover better solutions than the Non-dominated Sorting Genetic Algorithm II (NSGA-II) with constraint. Hence, it would be interesting to develop a deterministic reference-based solver and investigate its performance on the DAS problem.

Towards interactive multi-objective optimisation methods. The optimisation approaches in this study are mainly posterior methods where the decision maker selects the final solution from a set of approximate Pareto-optimal solutions. However, it would be ideal if the decision maker with domain knowledge can participate in the optimisation process. With specific expertise, the decision maker might be able to efficiently guide the search to suitable optimal solutions for the problem.

Towards a rolling horizon optimisation framework with a multi-objective optimisation model. The current work proposed a single objective rolling horizon optimisation framework to handle the uncertainty of obtaining feasible solutions in the future time intervals. However, this current framework restricts the choice of user based on the limits set in the coordination workload constraint. This does not allow the user to select their preferred solutions on the approximate Pareto front. Therefore, it would be fascinating to explore building a multi-objective rolling horizon optimisation framework, which in turn can be fitted into CDAS to address the uncertainty of solutions' feasibility.

Towards a reinforcement learning method for rolling horizon optimisation framework. The rolling horizon optimisation framework in the thesis is resolved by the black-box optimisation approach. Another popular method for solving problems in rolling horizon optimisation framework is the use of reinforcement learning. With the increasingly rapid development of reinforcement learning methodologies, it would

be exciting to explore resolving the dynamic airspace sectorization problem in a rolling horizon optimisation framework through a reinforcement learning approach.

Towards implementation of SAWAS as a module in CDAS framework. CDAS primarily addresses changes in traffic flows over time. On the other hand, SAWAS focuses on finding solutions when there is a change in airspace availability. The implementation of SAWAS as a module in the CDAS framework would allow one to build a complete model that manages the two main issues present in the airspace management.

Towards a better measurement of Air Traffic Controller (ATC) workload The ATC workload used in the thesis is taken from a study previously done in the US [75]. They take into consideration the traffic densities and weighted traffic complexities in the airspace. However, the human factor still plays an important role in the measurement of ATC workload. For the future deployment of proposed technologies into the real-world, simulations need to be conducted for a better measurement of ATC workload.

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