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**COMPUTATIONALLY EFFICIENT METHODS
FOR SOLVING OPTIMAL POWER FLOW
PROBLEMS BY EXPLOITING
SUPPLEMENTARY INFORMATION**

ALI REZA BARZEGAR

School of Electrical and Electronic Engineering

2019

**COMPUTATIONALLY EFFICIENT
METHODS FOR SOLVING OPTIMAL
POWER FLOW PROBLEMS BY
EXPLOITING SUPPLEMENTARY
INFORMATION**

ALI REZA BARZEGAR

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

May 24, 2019

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Date

A handwritten signature in blue ink, reading "Ali Reza Barzegar", is written over a horizontal dotted line.

ALI REZA BARZEGAR

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

May 24, 2019

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Date

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Assoc. Prof. RONG SU

Authorship Attribution Statement

This thesis contains material from 3 papers submitted in the following peer-reviewed journals and from 1 paper accepted at conference in which I am listed as an author.

Chapter 3 is published as A. Barzegar, A. Sadollah, L. Rajabpour, and R. Su, Optimal power flow solution using water cycle algorithm. 14th International Conference on Control, Automation, Robotics and Vision (ICARCV), IEEE, 1-4 (2015) and submitted as A. Barzegar, A. Sadollah, R. Su, S. Srinivasan. A Novel Fully Informed Water Cycle Algorithm for Solving Optimal Power Flow Problem in Energy Grids. Applied Intelligence, Manuscript under review.

The contributions of the co-authors are as follows:

- Dr. Sadollah provided the initial project direction as the creator of traditional Water Cycle Algorithm.
- I proposed the novel fully informed algorithm and prepared the manuscript drafts.
- Prof. Su Supervised the project and revised the manuscript.
- Ms. Rajabpour assisted in collection of the simulation data.
- Dr. Srinivasan revised the manuscripts and proposed some ideas to improve the paper.

Chapter 4 is submitted as A. Barzegar, D. K. Molzahn, R. Su. A Method for Quickly Bounding the Optimal Objective Value of an OPF Problem using a Semidefinite Relaxation and a Local Solution. Electric Power Systems Research, Manuscript under review.

The contributions of the co-authors are as follows:

- Assis. Prof. Molzahn provided the initial idea of this manuscript and the required data and revised the manuscript draft.
- I implemented the idea and performed the required simulation results and wrote the drafts of manuscript.
- Prof. Su supervised the project and revised the manuscript draft.

Chapter 5 is submitted as A. Barzegar, S. Srinivasan, A. Maffei, R. Su, L. Iannelli, L. Glielmo. A Computationally Efficient Approach for Solving Receding-Horizon Optimal Power Flow Problem in Distribution Grids. IEEE/CAA Journal of Automatica Sinica, Manuscript under review.

The contributions of the authors are as follows:

- Prof. Justin Dauwels co-designed the study, provided the initial project direction, guided throughout the project, and helped in revising the manuscript.
- I made the main contribution and wrote the manuscript drafts.

- Dr. Srinivasan and Dr. Maffei provided the required data of real test case “Steinkjer Network”; and revised the manuscript drafts.
- Dr. Prof. Su, Assoc. Prof. Iannelli and Prof. Glielmo supervised the project.

May 24, 2019

.....

Date

ALI REZA BARZEGAR

To
my parents Aziz & Zahra,
my wife Leila,
and my sisters Marziyeh & Raziye,

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List of Abbreviations

ABC Artificial Bee Colony

ACO Ant Colony Optimization

ACOPF Alternating Current Optimal Power Flow

BB-MOPSO Bare-Bones Multi-Objective Particle Swarm Optimization

BBO Biogeography Based Optimization

CI-PSO Constant Inertia Particle Swarm Optimization

CSA-DHS Chaotic Self-Adaptive Differential Harmony Search Algorithm

DA-PSO Dragonfly Algorithm and Particle Swarm Optimization

DSA Differential Search Algorithm

DCOPF Direct Current Optimal Power Flow

DE-HS Differential Evolution and Harmony Search Algorithm

DE-PS Differential Evolution and Pattern Search

EADDE Evolving Ant Direction Differential Evolution

EEA Efficient Evolutionary Algorithm

EGA-DQLF Enhanced Genetic Algorithm for Decoupled Quadratic Load Flow

ESS Energy Storage Systems

FIDEA Forced Initialized Differential Evolution Algorithm

FIWCA Fully-Informed Water Cycle Algorithm

FL Flexible Loads

GA Genetic Algorithm

GSO Glowworm Swarm Optimization

GSP Grid Supply Point

GWO Grey Wolf Optimizer

HSA Harmony Search Algorithm

IPSO Improved Particle Swarm Optimization

KKT Karush-Kuhn-Tucker

LDI-PSO Linearly Decreasing Inertia Weight Particle Swarm Optimization

LP Linear Programming

MDE Modified Differential Evolution Algorithm

MNSGA Modified Non-Dominated Sorting Genetic Algorithm

MOTLBO Multi-Objective Teaching-Learning Based Optimization

MPSO Modified Particle Swarm Optimization

MPC Model Predictive Control

MSCA Modified Sine-Cosine Algorithm

NESTA The NICTA Energy System Test Case Archive

NPSO New Particle Swarm Optimization

OPF Optimal Power Flow

PCC Point of Common Coupling

PSO Particle Swarm Optimization

PSOGSA Particle Swarm Optimization and Gravitational Search Algorithm

QC Quadratic Convex

QCQP Quadratically Constrained Quadratic Programming

RES Renewable Energy Sources

SCADA Supervisory Control And Data Acquisition

SFLA Shuffled Frog-Leaping Algorithm

RHC Receding Horizon Control

SDP Semidefinite Programming

SOCP Second Order Cone Programming

SOS Sum of Squares

TLBO Teaching-Learning Based Optimization

VD Voltage Deviation

WCA Water Cycle Algorithm

List of Symbols

Symbol	Description
$\mathbf{A}_i$	i -th submatrices of matrix $\mathbf{A}$ associated with clique $\mathcal{C}_i$.
$b_k(t)$	State of Charge (SOC) of the storage unit at bus k at time step t
b_k^{loss}	Storage energy loss at bus k .
c_{k0}	Constant cost coefficient of OPF problem.
c_{k0}^{SOX}	Constant SOX emission coefficient of OPF problem.
c_{k1}	Linear cost coefficient of OPF problem.
c_{k1}^{SOX}	Linear SOX emission coefficient of OPF problem.
c_{k2}	Quadratic cost coefficient of OPF problem.
c_{k1}^{SOX}	Quadratic SOX emission coefficient of OPF problem.
$C^{\text{DAM}}(t)$	Day-ahead market cost at time step t .
$C^{\text{MC}}(t)$	Market clearing prices at time step t .
$C^p(t)$	Cost during the peak-periods at time step t .
$C^b(t)$	Contracted storage costs (row vector) at time step t .
$C^g(t)$	Cost for dispatchable generation row vector at time step t .
$C^d(t)$	Cost for flexible loads row vector at time step t .
d_k	Valve point loading effect coefficient of OPF problem.
d_{ik}^{NOX}	Constant NOX emission coefficient of OPF problem.
e_k	Valve point loading effect coefficient of OPF problem.

d_k^{NOX}	Exponential NOX emission coefficient of OPF problem.
$\mathcal{C}_t$	Maximal clique t .
H	Control horizon length.
H_{lm}	Dual 3×3 matrix variables associated with the line flow constraints.
$\mathcal{L}$	Set of all the network lines.
$\mathcal{L}_p$	Set of all lines in the problematic submatrices $\mathcal{S}_p$.
(l, m)	line that connects bus l to bus m .
$(\cdot)^{min}$	Lower limit of intended parameter.
$(\cdot)^{max}$	Upper limit of intended parameter.
$\mathcal{N}$	Set of all the network buses.
$\mathcal{N}_p$	Set of all buses in the problematic submatrices $\mathcal{S}_p$.
N_{pop}	Total number of population in fully informed water cycle algorithm.
NS_n	Number of streams which flow toward a specific river or sea.
N_{sr}	Population of rivers and sea.
N_{Stream}	Population of streams.
$p^{contr}(t)$	Power contracted with the utility the day before.
P_{Dk}	Active load demand at bus k .
$\bar{p}_{Dk}(t)$	Forecasts on active power load demand at bus k at time step t .
P_k	Injected active power of bus k .
P_{lm}	Active power that flows through line (l, m) .
$p_{Rk}(t)$	Forecasts on renewable active power at bus k .
Q_{Ck}	Shunt VAR compensator located at bus k .
Q_{Dk}	Reactive load demand at bus k .
Q_k	Injected reactive power of bus k .

Q_{lm}	Reactive power that flows through line (l, m) .
R_k	Dual 2×2 matrix variables associated with Schur complement formulation of quadratic cost function.
$r_k(t)$	Power exchanged with the storage unit at bus k at time step t .
S_k	Injected apparent power of bus k .
S_{lm}	Apparent power that flows through line (l, m) .
T_{lm}	Transformer tap setting of line (l, m) .
$\mathcal{S}_p$	Set of problematic cliques, i.e. non-PSD submatrices.
$\mathcal{T}_{tu}$	Set of all linking constraints between any two maximal cliques t and u .
$V_{dk} + jV_{qk}$	Rectangular complex representation of voltage at bus k .
$ V_k $	Voltage magnitude at bus k .
$\mathbf{Y}$	Network admittance matrix.
$\vec{X}_{Stream}^i(t)$	Position of stream i at step t .
$\vec{X}_{River}^i(t)$	Position of river i at step t .
$\underline{\beta}_{lm}$	Dual scalar variable associated with lower bound of phase angle difference between bs l and bus m .
$\overline{\beta}_{lm}$	Dual scalar variable associated with upper bound of phase angle difference between bs l and bus m .
$\underline{\gamma}_k$	Dual scalar variable associated with lower bound of injected reactive power at bus k .
$\overline{\gamma}_k$	Dual scalar variable associated with upper bound of injected reactive power at bus k .
$\delta_k(t)$	Maximum percentage of load that can be cut in each period t at bus k .
θ_k	Voltage phase angle at bus k .

θ_{lm}	Phase angle difference between two buses connected by line (l, m) .
$\underline{\lambda}_k$	Dual scalar variable associated with lower bound of injected active power at bus k .
$\bar{\lambda}_k$	Dual scalar variable associated with upper bound of injected active power at bus k .
$\underline{\mu}_k$	Dual scalar variable associated with lower bound of voltage magnitude at bus k .
$\bar{\mu}_k$	Dual scalar variable associated with upper bound of voltage magnitude at bus k .
ρ	Cost function of dual SDP relaxation of OPF problem.
$\varphi_{ij}^{(tu)}$	Lagrange multiplier associated with linking constraint $(i, j) \in \mathcal{T}_{tu}$.
ΔT	Time step.

Summary

Motivated by the important role of electrical energy in the quality of life in cities, the electric power flow management, scheduling, and optimization is a critical task, especially in large-scale power systems. The power flow equations as the heart of power system operation translate the power injections and voltages steady-state relationship. The nonlinearity of the power flow equations makes the optimal power flow (OPF) problem a nonconvex NP-hard optimization problem which usually has multiple optima. The complexity of the OPF problem increases significantly as the size of the power system increases, so obtaining an optimal solution for an OPF problem within a reasonable time is one of the main challenges in the optimization of the power system research area.

Therefore, a variety of deterministic and non-deterministic techniques have been applied to OPF problems over the last 50 years of research on power networks. Although the mature deterministic algorithms are usually able to quickly find a solution, most of them do not guarantee global optimality of the obtained solution and they may get trapped in any local optima and no additional information is provided regarding the solution. Certifiably obtaining a globally optimal solution is important for certain applications of OPF problems.

In this thesis, firstly, a meta-heuristic optimization approach, called ful-

ly informed water cycle algorithm (FIWCA), is proposed with the idea of exchanging global and local information among the individuals in the populations with the goal of achieving a reasonable solution within a smaller number of iterations and shorter time, and also avoiding trapping in any local optima.

Many global optimization techniques, such as semidefinite programming (SDP), compute an optimality gap that compares the achievable objective value corresponding to a feasible point from a local solution algorithm with an objective value bound from a convex relaxation technique. Rather than the traditional practice of completely separating the local solution and convex relaxation computations, this thesis next proposes a method that exploits information from a local solution to speed up the computation of an objective value bound using a SDP relaxation. The improvement in computational tractability comes with the trade-off of reduced tightness for the resulting objective value bound.

Recent development in power industry leads to a more complicated optimization problem to solve. The conventional way of solving static OPF problem is unsuitable for power networks, especially at the distribution level, having renewable energy sources (RES), energy storage systems (ESS) and flexible loads (FL) owing to the time-coupled and stochastic dynamics. On the other hand, convex relaxation approaches are not suitable for distribution grids owing to their high line resistance. This thesis next proposes a computationally efficient approach for solving receding horizon control (RHC) based Alternating Current OPF problems in energy grids having intermittent renewable energy generation, storage devices and flexible loads. The proposed method decomposes the solution into two stages. A Direct

Current OPF (DCOPF) problem is solved using an RHC approach at the first stage by embedding forecasts on renewable generation and demand. A single-period ACOPF to optimize both line-losses and operating cost is solved at the second stage with the storage and flexible loads fixed at the optimal values computed in the first stage. In addition, hot-start is provided to the second stage optimization problem using the generation schedule computed at the first stage.

Chapter 1

Introduction

Quality of life in modern societies is directly and closely related to different types of energy, especially electrical energy. Nowadays, electric power plays an important role in almost every aspect of life in cities, e.g. industry, commercial, household, transportation, information technology and etc., and it is clear that any electric power interruption may cause a large amount of financial damage; as a literature in 2006 [1] stated that the annual cost of electric power failure or interruption to the United States consumers is approximately about 79 billion dollars, Fig. 1.1.

Thus, a stable and reliable electric power supply to the consumers is critical in power network management to prevent any blackout which is achievable by considering engineering and physical constraints while keeping the network performance far enough from any instability which may cause by any uncertainty such as failure in generation units or transmission line, unexpected load demands especially in peak hours and etc.

The other important aspect of concerns in electric power management is the economic cost of power generation, transmission and distribution, and obtaining a suitable and reliable operating point of the network as well as

Figure 1.1: Approximate Annual Power Interruption Cost in the US by Different Sections [1].

minimizing the total generation fuel cost and power loss through the transmission lines. These costs are more significant in large size power network systems. Therefore, decreasing the cost of generated power while supporting all the consumers is one of the main objectives in power network engineering.

In addition to the reliability and economic costs, other goals such as minimizing the emissions, demand side management and distributed renewable resources management are some of the major problems in electric power networks.

In Singapore, electricity plays the main role in energy supply for consumers. The annual Singapore energy statistics, published by Energy Market Authority (EMA) [2] shows that the total generated electric power by the different variety of resources was about 51.6 TWh in 2016 which is 2.6% higher than 2015. Fig. 1.2 demonstrates that the total electric power used

■ **ELECTRICITY FLOW, 2016**

Unit: GWh

Figure 1.2: Electrical Power Generation and Consumption in Singapore, 2016. [2].

by consumers is about 48.6 TWh and 2.4 TWh is utilized within the transformation sectors or converted to power loss.

1.1 Motivation

The power flow management is very critical to enhance the reliability, performance and efficiency of the electric networks. The power flow equations thus play the main role in the engineering and management of the power network. These equations translate the sinusoidal steady state power flow through an electrical network into a nonlinear equilibrium equation model. This mathematical model represents the relationship between the active and reactive power injections and the voltage phasors at each bus of the power network. As the size of the power system increases, the set of these nonlinear power flow equations becomes larger and more complex and thus no analytical approach is available to find the solution to these equations and only iterative tools such as Newton-Raphson method [9] or Gauss-Seidal techniques [10] are able to solve these equations.

The power flow equations are directly correlated with the reliability, stability and economic costs of the electric power systems. Determining a steady-state operating point for an electric power system which satisfies the network physical constraints, i.e. power flow equations, and engineering-based inequality constraints, e.g. voltage magnitude limits, generation limits, line-flow limits, etc., while optimizing an objective function, e.g. minimization of generation cost, total power loss, total emission, etc., requires to define an optimization problem. Thus, optimal power flow (OPF) problem, as an important engineering tool in the operation of electric power networks, has been presented by Carpentier in 1962 [11].

The complexity of the OPF problem increases significantly as the size of electric power network increases. Recent developments in the power in-

dustry, such as renewable resources, distributed power generation, power storage systems, flexible load demands, etc., also lead to a more complicated optimization problem to solve. Therefore, The OPF problem is generally a nonconvex NP-hard problem with the possibility of having multiple local and global optima.

Research conducted for over fifty years has developed many deterministic and non-deterministic optimization methods applicable to the OPF problems.

The deterministic algorithms including Newton-based techniques, sequential linear programming and sequential quadratic programming algorithms, gradient-based methods, interior point methods, etc. [12–15] often quickly find local optima for the large OPF problems. For a more comprehensive review of the deterministic optimization tools for OPF problem see [15,16].

Some of the deterministic methods can provide globally optimal solutions by implementing some techniques such as Lasserre hierarchy [17], branch-and-bound algorithms [18,19], etc. But, in general, the deterministic algorithms may or may not find the globally optimal solution. Local solvers, such as interior point methods, do not guarantee to find a globally optimal solution although in some cases the solution from these mature algorithms may satisfy the sufficient conditions for global optimality [20]. If the solution from a local solver satisfies a sufficient condition for global optimality (such as the KKT conditions for a relaxation [20]), then the local solution is certifiably globally optimal. Otherwise, if the solution may not satisfies any sufficient condition for global optimality, it cannot be concluded that the solution is not globally optimal.

On the other hand, these deterministic approaches may not solve the OPF problems including discrete or integer variables, and in case of having integer variables, the problem can be solved by relaxing or approximating these variables with continuous variables.

The non-deterministic algorithms have been developed and widely applied to many optimization problems for the past three decades due to the ability of these techniques to avoid trapping in local optima. Many of these non-deterministic algorithms such as genetic algorithms (GA) [21], simulated annealing (SA) [22], ant colony optimization (ACO) [23], particle swarm optimization (PSO) [24] and differential evolution (DE) [25], have been applied to the OPF problem. Some of these techniques perform quite well in global searching for the solution but most of them suffer from the computational efficiency, i.e. solver time, especially in large optimization problems. This motivates to develop these meta-heuristic algorithms by improving their exploration strategy using supplementary information so that they may reliably converge to the solution in less number of iterations. For more details on non-deterministic algorithms and their application on OPF problem see [26–28].

The computational difficulty of nonconvex optimization problems, which prevents a user from obtaining the globally optimal solution in a reasonable time period, enforces the researchers to relax the problem in a way that reduces the complexity of the original problem. Convex relaxation as a powerful tool is an approximation of the original nonconvex problem by relaxing some of the constraints and extending the feasibility region of the problem in a way that eventually results in a convex optimization problem. To compute global optimality certificates, recent research has focused on

convex relaxations of OPF problems [29]. Convex relaxations bound the objective value of the original OPF problem, providing lower (upper) bounds for minimization (maximization) problems. The main relaxations applied to the OPF problems are: semidefinite programming (SDP) relaxations, second order cone programming (SOCP) relaxations, quadratic convex (QC) relaxations and linear programming (LP) relaxations.

Since SDP solvers are able to reliably determine a globally optimal solution in polynomial time, the (SDP) relaxation has become a hot topic of interest for the power optimization researchers [30,31]. If the relaxed problem can satisfy the related assumption and constraints, which are considered to relax the original problem, then the relaxation is tight with the original OPF problem, means that there is no gap between the solution to the relaxed problem and the original OPF problem, therefore the global solution for the original classical OPF problem can be obtained.

Solving the SDP relaxation can be computationally challenging due to the presence of a positive semidefinite constraint on a large matrix. A matrix completion technique mitigates this computational challenge by decomposing the single positive semidefinite constraint into constraints on many submatrices [32,33]. However, the resulting formulation is still less tractable than many local solution algorithms. This motivates the development of techniques for exploiting information in a local solution to further speed computations of SDP relaxations.

Conventional way of solving static OPF problem is also not suitable for distribution systems due to the presence of renewable energy sources, energy storage systems and flexible loads because of their time-coupled and stochastic dynamics. The convex relaxation approaches are also not suit-

able for distribution networks due to their high line resistance to reactance ratio [34]. The proposed multi-period OPF approaches to overcome this problem cannot tackle uncertainties owing to the lack of feedback, i.e. measurements. Therefore, the receding-horizon control (RHC) based approach, which solves a multi-period optimization problem during each time-horizon by taking measurements has been proposed recently. But again the main challenge in RHC methods is the computation complexity as the multi-period optimization model needs to be solved during each time-horizon and it is a time consuming process. These electric components characterized by dynamical behaviors, such as storage and flexible loads, cause additional complexity to the OPF problem which already needs to tackle the nonlinear power flow equations. This motivates the development of techniques to decompose the RHC problem into multi stages, for time-coupled and time-decoupled variables, and exploit the required information from the first stage to increase the computation efficiency of solving multi-period OPF problems.

1.2 Contributions

In this thesis, the following contributions to the solution techniques for optimal power flow (OPF) problems, with the goal of increasing the computational efficiency, are provided:

1. An improved non-deterministic optimization algorithm, called fully informed water cycle algorithm (FIWCA) is proposed in Chapter 3 to solve the OPF problem. This algorithm is an improved version of the water cycle algorithm (WCA) by using the idea of sharing more comprehensive information, i.e. using the information of best candidate solutions in each iteration, during the exploitation phase. The

proposed idea changes the structure of the classic water cycle algorithm in updating the new position of each individual by strategically utilizing the information of a set of best candidate solutions which provides a more reliable diversity in the selection of solutions during the searching phase and prevents the possibility of trapping in local optima. The diversity in the exploitation phase is controllable and this diversity decreases as the number of iterations increases while reaching the best solution. Using more comprehensive information to update the position of individuals also helps to move toward the global solution within a smaller number of iterations. Thus, the proposed modification increases the reliability and efficiency of the original algorithm significantly by decreasing the computational time and avoiding getting trapped in any local optima.

2. Existing OPF semidefinite programming (SDP) relaxation methods which consider the voltage phasors as a vector of real and imaginary elements do not consider the phase angle difference limits, so the primal and dual semidefinite programming (SDP) relaxation of OPF problem including the phase angle difference limits is formulated and presented in Chapter 4 which increases the applicability of this relaxation technique to more power systems.
 3. Due to the OPF problem's non-convexity, there may exist multiple local optima. Certifiably obtaining the global solution is important for certain applications of OPF problems. Many global optimization techniques compute an optimality gap that compares the achievable objective value corresponding to the feasible point from a local solution algorithm with the objective value bound from a convex relaxation
-

technique. Rather than the traditional practice of completely separating the local solution and convex relaxation computations, Chapter 4 proposes a method that exploits information from a local solution to speed up the computation of an objective value bound using a semidefinite programming (SDP) relaxation. The improvement in computational tractability comes with the trade-off of reduced tightness for the resulting objective value bound.

4. Using the matrix completion theorem in solving semidefinite programming relaxation of the OPF problem leads to adding some linking constraints to the primal and dual SDP relaxation problem. In chapter 4, the mathematical modification of primal and dual formulation of SDP relaxation of OPF problems by considering the linking constraints is provided.
 5. In order to avoid wasting time in test cases that the proposed method for quickly bounding the objective function may not obtain a suitable solution, some heuristics are proposed that helps to quickly check whether it is reasonable to solve the proposed problem or it is more convenient to directly solve the original SDP relaxation problem.
 6. A computationally efficient approach is proposed in Chapter 5 for solving receding horizon control (RHC) based multi-objective alternating current optimal power flow (ACOPF) problems in distribution networks having intermittent renewable energy generation, storage devices and flexible loads. Comparing to the previous research works, the proposed method decomposes the solution into two stages. A direct current optimal power flow (DCOPF) problem is solved using a
-

RHC approach at the first stage by embedding forecasts on renewable generation and demand. A single-period ACOPF to optimize both line-losses and operating cost is solved at the second stage with the storage and flexible loads fixed at the optimal values information obtained in the first stage. In addition, hot-start is provided to the second stage optimization problem using the generation schedule computed at the first stage. The computation time reduces significantly when compared to a conventional nonlinear RHC solution without significant drop in optimality. While the method preserves the advantages of the RHC, it brings down the computation complexity significantly.

1.3 Thesis Overview

This thesis contains 5 chapters which are organized in the following order:

Chapter 2: This chapter explains the different available representations of power flow equations as the heart of a power system. Then by adding a minimization objective function and engineering-based constraints, the optimal power flow problem is explained. The complexity and non-convexity of the OPF problem are then discussed by demonstrating the feasible space of some small power test cases.

Chapter 3: This chapter proposes a meta-heuristic approach, fully informed water cycle algorithm (FIWCA), for solving the OPF problem. First, a revised OPF problem formulation is presented by adding the transformer tap setting and shunt reactive compensator constraining that in order to have a better comparison of the performance of the proposed algorithm with oth-

er non-deterministic algorithms implemented on an OPF problem. Then, the proposed fully informed water cycle algorithm (FIWCA) is presented along with the standard water cycle algorithm (WCA) and how it originates from nature. Finally, the proposed fully informed water cycle algorithm is implemented on two reported test systems under different scenarios and optimization results, comparisons and discussions are provided.

Chapter 4: This Chapter first describes the modeling issues of an OPF problem and the approximation and especially relaxation approaches to overcome the difficulties of this problem. Then the OPF problem formulation and its semidefinite programming (SDP) relaxation is presented as well as the matrix completion decomposition theorem and its preliminary materials. The sufficient condition for checking the global optimality of a candidate solution from any mature algorithm is also described. Then the method for quickly computing objective value bounds using the SDP relaxation in combination with a local solution is proposed, which is the main contribution of this chapter. Eventually, numerical experiments demonstrate this method using a variety of large power system test cases.

Chapter 5: This chapter first describes the modeling issues of the OPF problem in distribution networks including the mathematical implementation of storage devices and flexible load demands. Then, the proposed multi-stage RHC based approach for solving the ACOPF problem in distribution grids is presented by decomposing the problem into a multi-period DCOPF problem stage and a single-period ACOPF problem stage. Finally, the simulation results of the implementation of the proposed method using data obtained

from a Norwegian microgrid in “Steinkjer” are presented to demonstrate the performance of the proposed method.

Chapter 2

Optimal Power Flow Problem

The optimal power flow (OPF) problem is known as one of the key planning and scheduling tools introduced by Carpentier, 1962, [11] which aims to determine an optimal steady state working point for electric power systems. The OPF problem tries to minimize an objective function, usually the generation cost while satisfying a set of engineering based inequality constraints and the nonlinear power flow equations of the power network.

2.1 The Power Flow Equations

The power flow equations translate the sinusoidal steady state power flow of an electrical network into a mathematical equilibrium equation model. Generally, the power system can be modeled and analyzed by defining the network parameters, voltage and current at each bus which have a linear relationship but in reality the generated, injected or required active and reactive powers are specified instead of the current which needs a power flow model based on the nonlinear voltage-power representation instead of voltage-current. In order to provide a better sense of specified network

quantities at each bus, the power system buses are classified into three main groups:

1. “Load buses” or “**PQ** buses”: The required active power, P , and reactive power, Q , demands are specified for these buses.
2. “Generation buses” or “**PV** buses”: The voltage magnitude, $|V|$, and generated active power, P , are specified on these buses.
3. “Reference bus” or “**Slack** bus”: The voltage magnitude, $|V|$ and phase angle, $\angle V$, are specified for this bus (typically 0 phases angle).

2.2 Power Flow Equations Representation

There are two main classical approaches to represent the power flow equations, the bus injection model and the branch flow model. There are some other newly presented power flow formulations such as the power divide model and the elliptical power flow model. Let $\mathbb{C}^n$ and $\mathbb{R}^n$ donate the set of complex and real numbers and $\overline{(\cdot)}$ represent the complex conjugate.

2.2.1 Bus Injection Model

The bus injection model is mainly based on the injected power on each bus and its relation with the voltage. To formulate the power flow equations, consider a power network with $\mathcal{N} = \{1, 2, \dots, n\}$ as the set of buses and $\mathcal{L}$ as the set of lines where the pair (l, m) represents the line connecting bus l to bus m . To import the network topology and parameters into the formulation, the network admittance matrix $\mathbf{Y}$ is defined [35].

The injected apparent power of energy bus k , S_k , is constructed by active and reactive parts denoted as $S_k = P_k + jQ_k$, where $j = \sqrt{-1}$. The voltage

at every bus k , V_k , is also a complex phasor. The bus injection power flow equation is thus given as

$$P_k + jQ_k = V_k \sum_{i=1}^n (\bar{\mathbf{Y}}_{ik} \bar{V}_i). \quad (2.1a)$$

The voltage magnitude equation also must be satisfied as follows

$$|V_k|^2 = V_k \cdot \bar{V}_k. \quad (2.1b)$$

The complex voltage phasors can be represented in rectangular coordinates or polar coordinates. By considering the rectangular representation of voltage phasors $V_k = V_{dk} + jV_{qk}$ and splitting the equation (2.1a) into the active power and reactive power parts, the bus injection power flow equations are given as

$$P_k = V_{dk} \sum_{i=1}^n (\mathbf{G}_{ik} V_{di} - \mathbf{B}_{ik} V_{qi}) + V_{qk} \sum_{i=1}^n (\mathbf{B}_{ik} V_{di} + \mathbf{G}_{ik} V_{qi}), \quad (2.2a)$$

$$Q_k = V_{di} \sum_{i=1}^n (-\mathbf{B}_{ik} V_{di} - \mathbf{G}_{ik} V_{qi}) + V_{qk} \sum_{i=1}^n (\mathbf{G}_{ik} V_{di} - \mathbf{B}_{ik} V_{qi}), \quad (2.2b)$$

where the admittance matrix is also represented in the rectangular form $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$ and the voltage magnitude equation also needs to be rewritten as

$$V_k^2 = V_{dk}^2 + V_{qk}^2. \quad (2.2c)$$

The power flow equations can also be expressed by considering the polar representation of the complex voltage phasors: $V_k = |V_k|e^{j\theta_k} = |V_k|\angle\theta_k$ where $|V_k|$ and θ_k are the magnitude and phase angle of phasor V_k . The

polar coordinate representation of bus injection power flow equations is thus given as

$$P_k = |V_k| \sum_{i=1}^n |V_i| \left(\mathbf{G}_{ik} (\cos(\theta_i - \theta_k)) + \mathbf{B}_{ik} (\sin(\theta_i - \theta_k)) \right), \quad (2.3a)$$

$$Q_k = |V_k| \sum_{i=1}^n |V_i| \left(\mathbf{G}_{ik} (\sin(\theta_i - \theta_k)) - \mathbf{B}_{ik} (\cos(\theta_i - \theta_k)) \right). \quad (2.3b)$$

The magnitude equation is not required in the polar coordinate representation since the voltage magnitude $|V_k|$ varies in this representation.

2.2.2 Branch Flow Model

For the power systems with tree topology structure, radial power systems, the branch flow model is proposed to present the power flow equations. This model is structured on the quantities flowing through the branches despite the bus injection model that considers bus quantities as the base of the formulation. Consider $\mathcal{L}$ as the set of branches where (i, k) or $i \rightarrow k$ donates the branch flow from bus i to bus k . P_{ik} and Q_{ik} denote the active and reactive power flow through the branch $i \rightarrow k$.

By defining the series impedance of each branch as $Z_{ik} = R_{ik} + jX_{ik}$ and I_{ik} as the current flow from bus i to bus k . The branch flow equations are given as:

$$S_{ik} = Z_{ik} |I_{ik}|^2 - S_k + \sum_{m:k \rightarrow m} S_{km}, \quad (2.4a)$$

$$V_i - V_k = Z_{ik} I_{ik}, \quad (2.4b)$$

$$S_{ik} = V_{ik} \overline{I_{ik}}. \quad (2.4c)$$

By considering the rectangular representation of complex phasors the

branch flow equation is expressed as:

$$P_{ik} = R_{ik}|I_{ik}|^2 - P_k + \sum_{m:k \rightarrow m} P_{km}, \quad (2.5a)$$

$$Q_{ik} = X_{ik}|I_{ik}|^2 - Q_k + \sum_{m:k \rightarrow m} Q_{km}, \quad (2.5b)$$

$$|V_k|^2 = |V_i|^2 - 2(R_{ik}P_{ik} + X_{ik}Q_{ik}) + (R_{ik}^2 + X_{ik}^2)|I_{ik}|^2, \quad (2.5c)$$

$$P_{ik}^2 + Q_{ik}^2 = |V_i|^2 |I_{ik}|^2. \quad (2.5d)$$

It must be noted that the branch flow equations (2.4) or (2.5) are nonlinear equations.

The nonlinear power flow equations (2.1) or (2.4) play an important role in power system control and optimization. Owing to the large size of power systems, the power flow problems include a large set of nonlinear equations to be solved. Thus, no analytical closed-form solution is available for this problem and the iterative solution techniques are the only tool to tackle the power flow equations. The most common iterative techniques used for solving the power flow equations are Newton-Raphson method [9] or Gauss-Seidal technique [10]. The latter technique is a simple approach which is usually used for small power systems while the former as a more powerful method is mostly used nowadays. The convergence of these techniques is extremely dependent on the initial guess of the solution.

The nonlinear power flow equations play an important role in the control and optimization of power systems which are usually large-scale systems and they import a large set of nonlinear equations into the problem formulation. Thus finding an analytical closed-form solution for this nonlinear equation is not possible and the iterative techniques are the only approach to solve these problems. Which are locally convergent techniques and the convergence of

them is highly dependent on the initial solution of the algorithm. Newton-Raphson methods and Gauss-Seidal are the most famous iterative techniques used in the power flow problem.

The Gauss-Seidal technique is a simple algorithm with a slow convergence rate so it is normally used in small systems, while the Newton-Raphson method is a powerful tool which is used in most of the large-scale systems. But the simplicity of Gauss-Seidal algorithm leads to be useful in large-scale systems as a starting algorithm to generate an initial solution for the Newton-Raphson algorithm.

2.3 Optimal Power Flow Problem Formulation

The power flow equations (2.3) can be employed to find the voltage magnitude and the phase angle of every bus while the generated active power and the demand power are determined at each bus. But, in a practical power system, the power flow equations are generally formulated with some inequality constraints on their variables, e.g. active and generated power, voltage magnitude, etc., which leads to a bounded feasibility region for the parameters. Finding a feasible solution as an input for the power flow equations can be expressed as an optimization problem. The OPF problem points to determine a solution that satisfies a set of physical equality constraints, i.e. power flow equations, and technical inequality constraints, e.g. generator capacity limits, bus voltage magnitude limits, line thermal limits, voltage phase angle difference limits, etc., while minimizing an objective function, e.g. generation cost, total power loss, etc.

The OPF problem proposed in 1962 by Carpentier would be presented in this section. Consider a power network with $\mathcal{N} = \{1, 2, \dots, n\}$ as the set of all buses (nodes), $\mathcal{G}$ as the set of all generators and the following nodal network parameters:

- $P_{Gk} + jQ_{Gk}$: active and reactive power of generation unit $k \in \mathcal{N}$.
- $P_{Dk} + jQ_{Dk}$: active and reactive load demand at bus $k \in \mathcal{N}$.
- $V_k = V_{dk} + jV_{qk}$: rectangular complex voltage at bus $k \in \mathcal{N}$.

The set of flow lines can be represented by $\mathcal{L}$ where the pair (l, m) shows the line connecting bus l to bus m . Note that parallel lines between two buses are allowed. The following parameters can be defined between any two connected buses:

- $S_{lm} = P_{lm} + jQ_{lm}$: apparent power flow of line $(l, m) \in \mathcal{L}$ in terms of active and reactive power flows.
- θ_{lm} : phase angle difference between two connected buses by line $(l, m) \in \mathcal{L}$.

Note that the upper and lower limits of each variable are defined with the *max* and *min* superscripts, respectively. By considering the equivalence Π model of each transmission line, Fig. 2.1, the network admittance matrix is defined as $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$. Considering a typical generation cost function for each generator, $\mathcal{F}_k(P_{Gk})$, the optimal power flow problem is thus given as:

Figure 2.1: II-Circuit Model of Transmission line (l, m) .

$$\min \sum_{k \in \mathcal{G}} \mathcal{F}_k(P_{Gk}) \quad (2.6a)$$

subject to

$$P_{Gk}^{min} \leq P_{Gk} \leq P_{Gk}^{max} \quad \forall k \in \mathcal{N}, \quad (2.6b)$$

$$Q_{Gk}^{min} \leq Q_{Gk} \leq Q_{Gk}^{max} \quad \forall k \in \mathcal{N}, \quad (2.6c)$$

$$(V_k^{min})^2 \leq V_{dk}^2 + V_{qk}^2 \leq (V_k^{max})^2 \quad \forall k \in \mathcal{N}, \quad (2.6d)$$

$$|S_{lm}| \leq S_{lm}^{max} \quad \forall (l, m) \in \mathcal{L}, \quad (2.6e)$$

$$(S_{lm})^2 = (P_{lm})^2 + (Q_{lm})^2 \quad \forall (l, m) \in \mathcal{L}, \quad (2.6f)$$

$$\theta_{lm}^{min} \leq \theta_{lm} \leq \theta_{lm}^{max} \quad \forall (l, m) \in \mathcal{L} \quad (2.6g)$$

$$\begin{aligned} P_{Gk} - P_{Dk} = & V_{dk} \sum_{i=1}^n (\mathbf{G}_{ik} V_{di} - \mathbf{B}_{ik} V_{qi}) \\ & + V_{qk} \sum_{i=1}^n (\mathbf{B}_{ik} V_{di} + \mathbf{G}_{ik} V_{qi}) \quad \forall k \in \mathcal{N}, \end{aligned} \quad (2.6h)$$

$$\begin{aligned} Q_{Gk} - Q_{Dk} = & V_{dk} \sum_{i=1}^n (-\mathbf{B}_{ik} V_{di} - \mathbf{G}_{ik} V_{qi}) \\ & + V_{qk} \sum_{h=1}^n (\mathbf{G}_{ik} V_{di} - \mathbf{B}_{ik} V_{qi}) \quad \forall k \in \mathcal{N}. \end{aligned} \quad (2.6i)$$

Note that the OPF problem formulation is originally a nonconvex, NP-hard optimization problem [36,37] which has opened a wide research area on methods, techniques and tools to solve this problem since 1962 and it is still an

ongoing field of research.

2.4 Feasible Space of Optimal Power Flow Problem

The OPF problem is generally a nonconvex NP-hard problem [31,36,38]. However, the literature [31,36,38] proves that the NP-hardness of the problem for radial power networks is weak but generally the OPF problem is strongly NP-hard.

Therefore, the feasible solution space of the OPF problem is a nonconvex space, mainly because of the nonlinear power flow equations (2.6h) and (2.6i). By fixing the power injection and voltage magnitude of each bus, the feasible solution of the power flow equations builds a finite set of discrete points [39]. By generalizing the problem to the OPF problem and considering a bound for the power injection and voltage magnitude, the feasible solutions form a single continuous space or a group of disconnected spaces, because of the nature of power flow equations.

Finally, the OPF problem tries to find a solution within this feasible space which minimizes the (generation) cost.

The OPF problem is a multi-local optima problem. The existence of multi optima solution can be as a result of the following reasons [40]:

- Disconnected feasible solution region: as mentioned the feasible space of optimal power flow problem might be constructed by several disconnected regions and consequently every region might have at least one local optimum solution point and the whole space could be a multi-local optima space [40].

- Existence of loop in the power network: if a group of nodes (buses) forms a loop in a network, that network might have multiple optima due to the possible phase shift difference in the flow equations [40].
- Excess reactive power: the excess amount of reactive power occurs when the loads become capacitive or a load reduction occurs while the lower bound of the generator reactive power is positive. In these situations, the excess reactive power shall be absorbed by lines. On the other hand, the amount of absorbed reactive power in lines is proportional to “ $1 - \cos \Delta\theta$ ”, in which $\Delta\theta$ is the phase angle difference of lines. Thus, the significant amount of excess reactive power can lead to a higher value of “ $1 - \cos \Delta\theta$ ” and consequently, increases the possibility of multi optima solutions. For more details and examples refer to [40].
- Wide generator and line limits: the large bound of generator and line variables may cause the possibility of large voltage angle differences through the lines that can lead to multi optima.

From the mathematical point of view, the wider limits construct a larger feasible space and it increases the possibility of multi-local optima in a non-convex problem. In this section, next, the feasible spaces of a few test cases are presented as examples of nonconvex problems. The feasible spaces for these systems are computed and constructed using the proposed approach in [41]. As the first example, the five-bus test case system “WB5” [40] is considered to compute the feasible space. Fig. 2.2(a) demonstrates the one-line diagram of this test case. The voltage magnitude limits, V^{min} and V^{max} , are set to 0.95 and 1.05, respectively. The line flow limits, S_{lm}^{max} , are set to “*inf*”, i.e. no line flow limits applied to this test system. The generation

Table 2.1: Cost Coefficients for Test Cases “WB5” and “case9mode”.

System Model	Bus	c_{2i} [\$/ (p.u.-hr) ²]	c_{0i} [\$/ (p.u.-hr)]	c_{0i} [\$/hr]
WB5	1	0.0	400.0	0.0
	5	0.0	100.0	0.0
case9mode	1	1100.0	500.0	150.0
	2	85.0	1200.0	600.0
	3	122.5	100.0	335.0

objective function for this case is $\sum_{i=1,5} = c_{i2}P_{Gi}^2 + c_{i1}P_{Gi} + c_{i0}$ and the cost coefficients can be found in Table 2.1.

The feasible space of this system is demonstrated in Fig. 2.2(b). For this system, the lower bound on the reactive power of the generation unit at bus 5 can affect the number of feasible space parts and the number of local optima. For $Q_{G5} \geq -0.30$, i.e. the upper space of the gray surface in Fig. 2.2(b), the feasible space splits into two separated subspaces as shown in Fig. 2.2(b). The colors of feasible space indicate the value of cost function. The globally optimal solution is specified by the green star, “★”, and a local optimum is demonstrated by the blue triangle, “▼”.

The 9-bus system “case9mod” [40] with the one-line diagram presented in Fig. 2.3(a) is considered as the next illustrative example to compute and construct its feasible space. The voltage magnitude is limited within the range [0.90 1.10]. The line flow limits, S_{lm}^{max} , for for lines (5, 6) and (6, 7) are set to 150 MVA and for line (3, 6) it is set to 300 MVA. The line flows are limited to 250 MVA for other lines. For this systems the quadratic cost function is considered as $\sum_{i=1}^3 = c_{i2}P_{Gi}^2 + c_{i1}P_{Gi} + c_{i0}$ with the cost coefficients tabulated in Table 2.3(a).

The black ellipse in Fig. 2.3(b) indicates the border of feasible space which is defined by binding the constraints $Q_{G1} \geq -0.05$, $Q_{G2} \geq -0.05$, $Q_{G3} \geq -0.05$ and $|V_9| \geq 0.9$ (p.u.) at the same time. The resulted feasible

space is constructed by three disconnected parts in which the colors show the value of cost function at each point. The green star, “★” represent the global solution and three local optima are indicated by the blue triangles, “▼”.

2.5 Summary

This chapter describes the power flow equations and their different mathematical representations. Then the importance and main challenge of these equation are discussed. The optimal power flow (OPF) problem formulation is presented. And finally the nonconvex feasible space of the OPF problem and possibility of having multi-optima in this problem are demonstrated in two small test cases.

(a) One-line Diagram.

(b) Feasible Space.

Figure 2.2: The 5-bus System “WB5”: a) One-line Diagram. b) Feasible Space. “★” Indicates the Global Solution and “▼” Represents the Local Optimum.

(a) One-line Diagram.

(b) Feasible Space.

Figure 2.3: The 9-bus System “case9mode”: a) One-line Diagram. b) Feasible Space. “★” Indicates the Global Solution and “▼”s Represent the Local Optima.

Chapter 3

Solving the OPF Problem Using The Proposed Fully Informed Water Cycle Algorithm

Typically, an optimal power flow (OPF) problem determines the values of grid operating or planning decision variables by solving a non-convex and nonlinear optimization problem [42] with underlying physical and operating constraints. The non-convex and nonlinear terms in the OPF optimization model are due to power flow conditions, generator loading effects, voltage cost, piecewise cost function and emission cost. This makes the OPF problem computationally intensive to solve and, usually, determining the globally optimal solution is a time-consuming process.

In literature, the mathematical programming approaches such as linear programming [43], quadratic programming [44], mixed integer programming [45], nonlinear programming [46] and Newton-based approaches and interior

point methods [47] have been developed and employed to solve the OPF problem. However, these approaches usually provide a feasible solution, rather than a global one [48]. To overcome the computational difficulty as well as to obtain a global solution, meta-heuristic approaches such as genetic algorithms (GA) [21], simulated annealing (SA) [22], particle swarm optimization (PSO) [24], ant colony optimization (ACO) [23], and others have been studied in the literature.

Owing to the ability of these meta-heuristic techniques in finding the global or near-global optimum of complex constrained optimization problems, their applications in electrical networks and especially planning and scheduling problems such as power allocation, load management, economic dispatch, and the OPF have been attracted researchers in recent years. For instances, using the evolutionary optimization methods such as SA [49], PSO [6], fuzzy GA [50], biogeography-based optimization (BBO) [51], teaching-learning based optimization (TLBO) [52], efficient evolutionary algorithm (EEA) [53], glowworm swarm optimization (GSO) [54], and hybrid differential evolution and harmony search algorithm (Hybrid DE-HA) [55].

However, since these approaches use meta-heuristics, performance improvements in these algorithms are sought to make them more deterministic and optimal. More recently, the water cycle algorithm (WCA), an optimization algorithm inspired by the water cycle process and observation of how streams and rivers flow into the sea, has been implemented for finding better optimal solutions of power flow optimization problems [4].

Typically the WCA simulates the surface run-off process, i.e., one of the main phases in the water cycle process observed in nature, as updating formulation for generating new individuals during iterative optimization

process [56].

So far, many applications in different fields of research have utilized the efficiency of WCA for solving complex optimization problems in the literature. For instance, some modified versions of WCA have been implemented in different applications such as rough set theory [57], detecting optimum reactive power dispatch problems [58], optimal operation of reservoir systems [59] and antenna array pattern synthesis [60]. While improvements on WCA continue to evolve, its use in solving the OPF problem has also not been explored.

First introduced by Eberhart and Kennedy [24] in 1995, the swarm optimization methods play an important role as the nature-inspired meta-heuristic optimization tools. Each of these algorithms is mainly induced by a phenomenon in nature such as fish schooling or bird flocking. In the canonical PSO algorithm, every individual particle learns to update its velocity using the influence of the particle with the best so far obtained solution as well as the best-obtained solution obtained by itself.

The idea of population organization and structure in swarm algorithm was first presented in [61]. Recent works show the significant impact of population structure and topology in improving the performance of these algorithms [3,62–66]. Figure 3.1 demonstrates the most common population topologies in particle swarms and how each individual communicate with other members during the exploitation phase of swarm algorithms [3].

Implementing the new communication topologies into the classical PSO results in new optimization meta-heuristics which aim to improve the performance of the original algorithm by modifying the transferred information among individuals. Mendes et al. proposed a fully informed particle swar-

Figure 3.1: Population Topologies in Particle Swarms [3].

m optimization (FIPSO) [67] in which every particle's update strategy is influenced by all other members in the swarm. Some other algorithms also have been proposed based on the other communication topologies such as Firefly Algorithm (FA) [68,69] which benefits from ring/cluster topology or glowworm swarm optimization (GSO) [70] in which every individual is influenced only by its better-performing neighbors within a specific radius. Therefore, considering the similarity between these algorithms and FIPSO, all these techniques can be represented as a family of Fully informed swarm (FIS) algorithms [71]. For more details and example of FIS algorithms please see [72–74].

Since communicating with all the swarm members may cause to considering redundant information, some FIS algorithms are proposed to utilize a more efficient of information from the neighbors instead of absorbing com-

plete information of all the swarm members [75–77].

Owing to the population-based properties of water cycle algorithm (WCA), communication topology among the individuals plays an important role in this optimization algorithm. The population individuals classified into three categories of sea, rivers, and streams in which sea is a stream with the best solution and rivers are the next best solutions among the streams. Each stream may flow toward one of the rivers or sea, i.e. influenced by the information from only one of the rivers or sea, and the rivers may only flow towards the sea means that each stream receives the information only from one of the rivers and/or sea and the rivers only communicate with the sea during the exploitation phase.

This investigation proposes an improved WCA that uses the idea of sharing global information among individuals during the exploitation phase, i.e. every stream benefits from the information not only from its allocated river/sea but also from all other rivers/sea during the exploration phase. Consequently, the proposed approach provides diversity in the selection of solution and eliminates the possibility of getting trapped at local optima, thereby enhancing the efficiency and accuracy of the algorithm significantly.

In the original WCA populations are clustered into a few sets, so we utilize this feature in the fully-informed algorithm and instead of using a complete communication topology among all the streams, only consider the information from the leader of each cluster, i.e. rivers or sea which leads to a more efficient algorithm in term of computation complexity while obtaining a higher performance toward finding the global solution.

The main contributions of this investigation are: (i) proposing an improved WCA with a concept of exchanging global and local information

among individuals in the population, i.e. fully informed water cycle algorithm (FIWCA), (ii) solving the OPF problem considering nonlinearities introduced due to generator loading effect, line-flows, piece-wise cost, voltage quality cost and emission cost, and (iii) comparing the performance of the FIWCA with other meta-heuristic techniques including the traditional WCA for IEEE 30-bus and 57-bus test cases.

Looking at obtained optimization results, the proposed FIWCA emerges as a promising technique for solving OPF with nonlinear cost/constraints and has the ability to reach the global optimum within a more reasonable time than existing techniques.

3.1 Problem Formulation

In this chapter, the OPF problem is considered by adding the shunt VAR compensators Q_{Ck} and transformer tap settings T_{lm} constraints to (2.6). The intended OPF problem is

$$\min \quad J \tag{3.1a}$$

subject to

$$(2.6b)-(2.6i),$$

$$Q_{Ck}^{min} \leq Q_{Ck} \leq Q_{Ck}^{max} \quad \forall k \in \mathcal{N}, \tag{3.1b}$$

$$T_{lm}^{min} \leq T_{lm} \leq T_{lm}^{max} \quad \forall (l, m) \in \mathcal{L}. \tag{3.1c}$$

The set of independent variables, i.e. control variables u , contains the generator voltage V_{Gk} , generator active power outputs P_{Gk} , except for the slack bus P_{G1} , transformer tap settings T_{lm} , and shunt VAR compensations Q_{Ck} .

3.2 Water Cycle Algorithm Using the Concept of Global Information

In this section, the standard Water Cycle Algorithm (WCA) is described in detail. Subsequently, Fully informed WCA, denoted as FIWCA, is proposed by equipping the standard WCA with the concepts of moving strategies toward the best solutions. In order to explain the FIWCA, first, the whole processes of WCA are given in the following section.

3.2.1 Standard Water Cycle Algorithm

The water cycle algorithm (WCA), as a meta-heuristic optimization technique, which emulates the natural water flow process first has been presented in [4]. The algorithm initiates with the rain or precipitation phenomena by generation of a random population of design variables, or the streams population between lower and upper bounds. Then, the best stream, i.e. the stream with the minimum objective function value (for minimization problems) is selected as the sea.

Afterward, a set of streams with the closest objective values to the best objective value are selected as the rivers. Note that it is assumed that the remaining streams move toward the rivers and sea. Now let's see the mathematical explanation of the water cycle algorithm. Let us assume the $1 \times D$ dimensional array as a candidate solution (i.e., stream)

$$X = [x_1, x_2, x_3, \dots, x_D], \quad (3.2)$$

where D is the number of design variable. Hence, the initial randomly

generated population can be represented by an $N_{pop} \times D$ matrix as given follows

$$\text{Total Population} = \begin{bmatrix} Sea \\ River_1 \\ River_2 \\ River_3 \\ \vdots \\ Stream_{N_{sr}+1} \\ Stream_{N_{sr}+2} \\ Stream_{N_{sr}+3} \\ \vdots \\ Stream_{N_{pop}} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & \dots & x_D^1 \\ x_1^2 & x_2^2 & x_3^2 & \dots & x_D^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{N_{pop}} & x_2^{N_{pop}} & x_3^{N_{pop}} & \dots & x_D^{N_{pop}} \end{bmatrix}, \quad (3.3)$$

where N_{pop} is the population size. To choose the rivers and sea, first of all the cost value of every generated population, i.e. every stream, need to be determined. The cost of every stream X^i is calculated by evaluating the problem cost function for that stream as given follows

$$C_i = Cost_i = f(x_1^i, x_2^i, \dots, x_D^i) \quad i = 1, 2, \dots, N_{pop}. \quad (3.4)$$

A user parameter which is summation of number of rivers and sea so called N_{sr} is selected. Among this set, the individual stream with the smallest objective value is selected as the sea and the others are chosen as the rivers, given in the following equations

$$N_{sr} = \text{Number of Rivers} + \underbrace{1}_{Sea}, \quad (3.5)$$

$$N_{Stream} = N_{pop} - N_{sr}. \quad (3.6)$$

Therefore, the population of streams that flow towards the rivers and sea is given as follows

$$\text{Population of Streams} = \begin{bmatrix} Stream_1 \\ Stream_2 \\ \vdots \\ Stream_{N_{Stream}} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_D^1 \\ x_1^2 & x_2^2 & \dots & x_D^2 \\ \vdots & \vdots & \vdots & \vdots \\ x_1^{N_{Stream}} & x_2^{N_{Stream}} & \dots & x_D^{N_{Stream}} \end{bmatrix} \quad (3.7)$$

In fact (3.7) which is a part of the total population with the size of N_{Stream} , (3.6). The sea and every river absorb water from the streams. As a matter of fact, water is flowing from streams toward the rivers and the sea by its own nature. The amount of water flow that enters a river or/and sea is different from one stream to another. The rivers also flow towards the sea which is in the deepest location. Therefore, the group of streams designated for every river and sea is determined by the following equation [56]

$$C_n = Cost_n - Cost_{N_{sr}+1} \quad n = 1, 2, \dots, N_{sr}, \quad (3.8)$$

$$NS_n = Round \left\{ \left| \frac{C_n}{\sum_{n=1}^{N_{sr}} C_n} \right| \times N_{Stream} \right\} \quad n = 1, 2, \dots, N_{sr}, \quad (3.9)$$

where NS_n is the number of streams which move toward the specific rivers and sea, i.e. NS_1 is the number of streams move toward the sea and $NS_2, \dots, NS_{N_{sr}}$ are the number of streams that flow toward their corresponding rivers.

As it can be observed in nature, raining and precipitation build the streams and the rivers are created by joining the streams to each other. Some of the streams might even move to the sea, directly. Eventually, all

Figure 3.2: Schematic View of the WCA Optimization Process [4].

the streams and rivers reach the sea which is considered as the current best solution. Fig. 3.2 illustrates the process of WCA in which diamond, stars and circles represent the sea, rivers and streams, respectively. The black circles and stars show the current position of streams and rivers and the white ones denote their new position at the next iteration [4].

3.2.2 Fully Informed Water Cycle Algorithm

In the standard WCA, each stream corresponds to one of the rivers and/or sea without considering the influence of other rivers and the sea. The rivers also just consider the sea as their reference to exchange information for flowing. In this section, a fully informed water cycle algorithm has been proposed which every stream receives the influence of all rivers and/or sea as the information reference for the flowing and updating the

new positions. The information in rivers flows not only to the sea, but these information flows can also be affected by all rivers and the sea to update their new positions and to intensively exploit the searching space. Fig. 3.3 displays the schematic view of the movement process used in FIWCA.

Now, mathematical explanations of updating the streams and rivers positions can be expressed. Let us assume N_{pop} is the total number of streams, of which $N_{sr} - 1$ are selected as the rivers and one individual is chosen as the sea. Let $(\vec{\cdot})$ represent a vector, the new position of streams and rivers are then given as follows

$$\begin{aligned} \vec{X}_{Stream}^i(t+1) &= \vec{X}_{Stream}^i(t) \\ &+ rand \times C \times (\vec{X}_{Sea}(t) - \vec{X}_{Stream}^i(t)) \quad i = 1, 2, \dots, NS_1, \end{aligned} \quad (3.10a)$$

$$\begin{aligned} \vec{X}_{Stream}^i(t+1) &= \vec{X}_{Stream}^i(t) \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^{N_{sr}-1} rand \times C \times (\vec{X}_{River}^j(t) - \vec{X}_{Stream}^i(t)) \quad i = 1, 2, \dots, NS_{2-N_{sr}}, \end{aligned} \quad (3.10b)$$

$$\begin{aligned} \vec{X}_{River}^i(t+1) &= \vec{X}_{River}^i(t) + rand \times C \times (\vec{X}_{Sea}(t) - \vec{X}_{River}^i(t)) \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^{N_{sr}-1} rand \times C \times (\vec{X}_{River}^j(t) - \vec{X}_{River}^i(t)) \quad i = 1, 2, \dots, (N_{sr} - 1), \end{aligned} \quad (3.10c)$$

where $1 < C < 2$ as proposed in [4]. For the best performance C can be equal to 2, “*rand*” is a uniformly distributed random number between 0 and 1, t is the iteration index, and i is the number of individuals. Note that (3.10a) defines the new position for the streams that directly move toward the sea, see Fig. 3.3(a), in the same way as the one used in standard WCA [4]

(a) New Position of the Streams which Directly Flow toward the Sea.

(b) New Position of the Streams which Flow toward the Rivers.

(c) New Position of Every River by Considering the Influence of Sea and all other Rivers.

Figure 3.3: Schematic View of the Fully Informed Water Cycle Algorithm (FIWCA) Optimization Process.

and (3.10b) shows the new position for the streams which flow toward the rivers, not only its corresponding river, see Fig. 3.3(b). In fact, in (3.10b), a stream flowing to the river, has been influenced not only by its specific river, but other rivers are also affecting the stream to provide full information of all the rivers for its new position. (3.10c) represents the new position of every river by considering the influence of the sea and all other rivers, see Fig. 3.3(c).

Note that whenever the cost value of any stream becomes better (smaller) than the cost value of its corresponding river, the status of that stream and river are exchanged, i.e. the stream turns into a river and the river becomes a stream. The same replacement will occur between a river and the sea.

In order to avoid getting trapped in local optima, the evaporation process has been introduced [56]. In nature, the evaporation process occurs as the streams and rivers run toward the sea, which leads to new precipitations. The evaporation condition in the WCA is defined by whether any stream or river gets close enough (using Euclidian distance) to the sea. This idea is implemented in the standard WCA and FIWCA by the following criteria for the rivers flowing into the sea as well as the streams flowing into the sea, respectively, given as follows [56]

```

if  $|\vec{X}_{Sea} - \vec{X}_{River}| < d_{max}$  or  $rand < 0.1$ ,  $i = 1, 2, \dots, N_{sr} - 1$  then
    Perform precipitation process using (3.11).
end if
if  $|\vec{X}_{Sea} - \vec{X}_{Stream}| < d_{max}$ ,  $i = 1, 2, \dots, NS_1$  then
    Perform precipitation process using (3.11).
end if

```

which states that if any river (stream) gets closer than d_{max} to the sea,

Algorithm 3.1 Fully Informed Water Cycle Algorithm

Input: Set user parameters of the WCA: N_{pop} , N_{sr} , d_{max} , and Max Iteration.

Output: Optimized objective value and design variables.

```
1: Initialisation: Create a random initial population of streams between
    $LB$  and  $UB$  (3.3).
2: while “( $t \leq \text{Max Iteration}$ )” or (any assumed stopping condition) do
3:   for “ $i = 1 : N_{pop}$ ” do
4:     Update the position of stream directly toward the sea using (3.10a).
5:     Calculate the objective function of the new stream.
6:     if “ $\text{Objective}(\text{New Stream}) < \text{Objective}(\text{Sea})$ ” then
7:       “ $\text{Sea} = \text{New Stream}$ ”.
8:     end if
9:     Update the position of stream to its corresponding river while con-
       sidering the information flow from the other rivers using (3.10b).
10:    Compute the objective function value of the new stream.
11:    if “ $\text{Objective}(\text{New Stream}) < \text{Objective}(\text{River})$ ” then
12:       $\text{River} = \text{New Stream}$ .
13:      if “ $\text{Objective}(\text{River}) < \text{Objective}(\text{Sea})$ ” then
14:        “ $\text{Sea} = \text{River}$ ”.
15:      end if
16:    end if
17:    Update the position of river to the sea while considering the infor-
       mation flow from other rivers using (3.10c).
18:    Compute the objective function value of the newly generated river.
19:    if “ $\text{Objective}(\text{New River}) < \text{Objective}(\text{Sea})$ ” then
20:      “ $\text{Sea} = \text{New River}$ ”.
21:    end if
22:  end for
23:  %% Evaporation Condition for rivers
24:  for “ $i = 1 : N_{sr} - 1$ ” do
25:    if “ $|\vec{X}_{\text{Sea}} - \vec{X}_{\text{River}}| < d_{max}$  or ( $\text{rand} < 0.1$ )” then
26:      Create new streams and a river (the best one) using (3.11).
27:    end if
28:  end for
29:  %% Evaporation Condition for streams who directly flow to the sea
30:  for “ $i = 1 : NS_1$ ” do
31:    if “ $|\vec{X}_{\text{Sea}} - \vec{X}_{\text{Stream}}| < d_{max}$ ” then
32:      Create New streams using (3.11) and the best one is considered
       as the sea.
33:    end if
34:  end for
35:  Reduce the  $d_{max}$  using (3.12).
36: end while
37: return Post-process the results and visualization.
```

then the evaporation process occurs. After the evaporation condition is satisfied, it is required to randomly form the new streams in a different position between lower and upper bounds. This phenomenon happens by the precipitation process which provides a new sub-population for which the best stream becomes the river (sea) and the other streams moves toward this river (sea). Indeed, the evaporation and raining process help in the exploration phase of the WCA. The location of newly generated sub-population of the corresponding evaporated river (stream) can be specified as follows

$$\vec{X}_{Stream}(t+1) = \vec{LB} + rand \times (\vec{UB} - \vec{LB}), \quad (3.11)$$

in which $\vec{LB}$ and $\vec{UB}$ represent the lower and upper bounds of the optimization problem, respectively. Since large values of d_{max} cause extra exploration and very small values lead to the search gravity around the sea, i.e. more exploitation, thus, this parameter, d_{max} , may directly affect the search intensity near the sea. For the sake of convergence, it is recommended in [56] to decrease the value of d_{max} adaptively as given follows

$$d_{max}(t+1) = d_{max}(t) - \frac{d_{max}(t)}{\text{Max Iteration}} \quad t = 1, 2, \dots, \text{Max Iteration}. \quad (3.12)$$

Furthermore, Algorithm 3.1 shows the pseudocode of FIWCA in details.

Talking about the new evolutionary optimization algorithms usually arises questions regarding the convergence of the proposed algorithm. Some of the literature, presents the condition under which a specific algorithm converges, e.g PSO [78–80], GA [81], etc. Recently, the iterative convergence of WCA has been investigated in [82] and the sufficient condition for this purpose has been derived. It is out of the scope of this thesis but the same

work can be done for the FIWCA algorithm to prove the convergence of the proposed algorithm and determine the sufficed conditions.

3.2.3 Constraint Handling and Feasibility Rules

The OPF problem includes a set of constraints on the independent or design variables, such as generators active power outputs and generators voltages, and a set of constraints on the dependent variables, such as line power flows, generators reactive power outputs, and voltages of non-generators buses. The optimization tools and techniques have to be able to handle all these constraints.

Here, the proposed FIWCA utilizes different procedures to maintain each of these groups of constraints to maintain the feasibility of the candidate solutions at each iteration.

To handle the constraints on the design variables which are directly considered by the FIWCA, the following rules are defined [83]:

- **Rule 1:** Between any two feasible candidate solutions the one with the best objective function value is preferred.
 - **Rule 2:** Any feasible candidate solution is preferred to any infeasible candidate solution.
 - **Rule 3:** Infeasible solutions containing a slight violation of the constraints (from 0.01 in the first iteration to 0.001 in the last iteration) are considered as feasible candidate solutions.
 - **Rule 4:** Between two infeasible solution the one with the lowest sum of constraint violation is preferred.
-

As discussed, the OPF problem also includes a set of constraints on the independent variables which are not directly considered in the proposed algorithm. To overcome these constraints, the penalty function approach is utilized, i.e. a penalty function of each constraint violation is defined and added to the original objective function with a large penalty multiplier.

3.3 Optimization Results

The proposed FIWCA and WCA have been implemented in OPF problems for the IEEE standard 30-bus and 57-bus test systems to obtain better optimal solutions using the proposed improved version and also for the sake of comparison among previous works. The optimization methods considered in this chapter have been coded in “MATLAB” and solved on an “Intel 3.40 GHz” computer with “8 GB of RAM”. For the original WCA and the fully informed WCA, the population size and the maximum iteration number are fixed to 200 and 100, respectively, i.e. the maximum number of function evaluation of 20,000, in order to have a fair comparison with other algorithms. All numerical test systems are performed for 50 independent replications. Note that in the FIWCA and WCA, the total number of sea and rivers, N_{sr} , is considered as 5.

In the literature, a variety of meta-heuristic optimization techniques have been studied for the OPF problems. In this work, the following optimizers have been considered: shuffled frog-leaping algorithm (SFLA) [52], constant inertia particle swarm optimization (CI-PSO) [84], dragonfly algorithm and particle swarm optimization (DA-PSO) [85], teaching-learning based optimization (TLBO) and Levy TLBO (LTLBO) [52], multi-objective TLBO (MOTLBO) [86], particle swarm optimization and gravitational search algo-

rithm (PSOGSA) [87], grey wolf optimizer (GWO) and developed grey wolf optimizer (DGWO) [88], enhanced differential evolution (Enhanced DE) [89], artificial bee colony (ABC) [90], Jaya algorithm [91], evolving ant direction differential evolution (EADDE), efficient evolutionary algorithm (EEA), enhanced GA for decoupled quadratic load flow (EGA-DQLF) [92], forced initialized differential evolution algorithm (FIDEA) [93], glowworm swarm optimization (GSO) [54], hybrid differential evolution and harmony search algorithm (Hybrid DE-HS) [55], modified sine-cosine algorithm (MSCA) [94], bare-bones multi-objective particle swarm optimization (BB-MOPSO) [95], biogeography-based optimization (BBO) [96], chaotic self-adaptive differential harmony search algorithm (CSA-DHS) [97], modified non-dominated sorting genetic algorithm (MNSGA-II) [95], hybrid differential evolution and pattern search (DE-PS) [98], modified particle swarm optimization (MP-SO) [99,100], improved particle swarm optimization (IPSO) [101], differential search algorithm (DSA) [102], hybrid modified PSO-SFLA (HMPSO-SFLA) [103], gravitational search algorithm (GSA) [104], linearly decreasing inertia weight PSO (LDI-PSO) [90], harmony search algorithm (HSA) [105].

3.3.1 IEEE 30-bus Test System

The IEEE 30-bus test system, as shown in Fig. 3.4, has six generators fixed at buses 1, 2, 5, 8, 11, and 13, four transformers with the off-nominal tap ratio at lines 6-9, 6-10, 4-12, and 28-27. Load data and line data for the IEEE 30-bus test system can be found in [6]. According to the literature [6], buses 10, 12, 15, 17, 20, 21, 23, 24, and 29 are considered as shunt VAR compensation buses. The minimum and maximum limits on control variables can be found in Appendix A, Table A.1. The proposed FIWCA is

Figure 3.4: Schematic View of IEEE 30-bus Test System.

implemented on OPF problem with different objective function scenarios to evaluate its effectiveness, as presented in the following subsections.

Case 1: Minimization of Quadratic Fuel Cost

In this case, the objective function of the OPF problem is to minimize the total generation fuel cost by defining a quadratic objective function J_1 as given follows

$$J_1 = \sum_{i=1}^{NG} f_i(P_{Gi}) = \sum_{i=1}^{NG} (c_{i0} + c_{i1}P_{Gi} + c_{i2}P_{Gi}^2), \quad (3.13)$$

where P_{Gi} and f_i are the output power and the generation fuel cost of i -th generator, respectively, and c_{i0} , c_{i1} , and c_{i2} are the cost coefficients of the i -th generator which can be found in Appendix A, Table A.2. The optimal

Table 3.1: Optimal Control Settings for Case 1 on IEEE standard 30-bus Test System Using the WCA and FIWCA.

Design Variables	WCA	FIWCA
P_{G1} (MW)	177.0771	177.0756
P_{G2} (MW)	48.6765	48.6800
P_{G5} (MW)	21.2971	21.2965
P_{G8} (MW)	21.0805	21.0806
P_{G11} (MW)	11.8404	11.8390
P_{G13} (MW)	12.0000	12.0000
V_{G1} (p.u.)	1.1000	1.0999
V_{G2} (p.u.)	1.0877	1.0877
V_{G5} (p.u.)	1.0614	1.0614
V_{G8} (p.u.)	1.0693	1.0693
V_{G11} (p.u.)	1.1000	1.1000
V_{G13} (p.u.)	1.0999	1.0999
T_{6-9} (p.u.)	1.0311	1.0311
T_{6-10} (p.u.)	0.9000	0.9000
T_{4-12} (p.u.)	0.9679	0.9679
T_{28-27} (p.u.)	0.9584	0.9583
Q_{C10} (MVAR)	5.0000	5.0000
Q_{C12} (MVAR)	5.0000	5.0000
Q_{C15} (MVAR)	5.0000	5.0000
Q_{C17} (MVAR)	5.0000	5.0000
Q_{C20} (MVAR)	4.3034	4.3069
Q_{C21} (MVAR)	5.0000	5.0000
Q_{C23} (MVAR)	2.6507	2.6422
Q_{C24} (MVAR)	5.0000	5.0000
Q_{C29} (MVAR)	2.3028	2.3045
Fuel cost (\$/h)	798.8608	798.8608
Emission (ton/h)	0.3661	0.3661
Power loss (MW)	8.5718	8.5719
VD (p.u.)	2.0379	2.0381

values of design variables for Case 1, achieved by the proposed FIWCA, can be found in Table 3.1. The total generation fuel cost achieved by FIWCA is \$798.8608 per hour, while the total emission and power losses are 0.3661 ton per hour and 8.5719 MW, respectively. Comparison of the optimized fuel cost using the WCA and FIWCA along with the other reported optimization methods is tabulated in Table 3.2.

Table 3.2: Comparison of Statistical Optimization Results for Case 1, i.e. Fuel Costs (\$/h), Using Several Optimizers.

Algorithms	Minimum Fuel Cost	Average Fuel Cost	Maximum Fuel Cost
SFLA [52]	802.2100	N/A	N/A
DA-PSO [85]	802.1241	N/A	N/A
MTLBO [106]	801.8925	801.95	N/A
PSO [52]	801.8900	N/A	N/A
PSO-GSA [87]	801.4986	N/A	N/A
GWO [88]	801.2590	802.6630	804.8980
Enhanced DE [89]	801.2300	N/A	N/A
ABC [90]	800.6600	800.8715	801.8674
Jaya [91]	800.4986	N/A	N/A
DGWO [88]	800.4330	800.4674	800.4989
EADDE [8]	800.2041	N/A	N/A
EEA [53]	800.0831	800.1730	800.2123
EGA-DQLF [92]	799.5600	N/A	N/A
LTLBO [52]	799.4369	799.7186	800.2578
FIDEA [93]	799.0827	799.0827	799.0827
GSO [54]	799.0519	799.0642	799.9132
Hybrid DE-HS [55]	799.0514	799.0856	799.1198
WCA (This Study)	798.8614	798.8620	799.1945
FIWCA (This Study)	798.8608	798.8609	798.8612

By observing Table 3.2, the minimum total fuel cost obtained by WCA and FIWCA are same but the comparison of both methods shows better stability in terms of statistical results where the average fuel cost for WCA is \$798.8620 per hour, while the average fuel cost for FIWCA is \$798.8609 per hour. The best attainable results of Case 1 have been highlighted in bold in Table 3.2.

Case 2: Improvement of Voltage Profiles

One of the critical variables of any power flow system, closely correlated with the stability and security of the power system, is bus voltages of the grid. Hence, improving the quality of the bus voltage profiles and keeping

them close to the reference value of 1.0 p.u. can be considered as one of the desired goals of OPF problem. In this case, the improved algorithm has been implemented to minimize the total generation fuel cost, while forcing the bus voltages close to 1.0 p.u.. Thus, the objective function can be defined as given follows

$$J_2 = \sum_{i=1}^{NG} (c_{i0} + c_{i1}P_{Gi} + c_{i2}P_{Gi}^2) + w \sum_{i=1}^{NPQ} |V_i - 1.0|. \quad (3.14)$$

The first term of (3.14) demonstrates the total fuel cost and the second term shows the cost on voltage profile tracking which tries to minimize the total voltage deviation (V.D.) and w is the weighting factor and considered as 100 in this simulation. The FIWCA has been applied to this case to determine the optimal values of design variables by minimizing the fuel cost, as presented in Table 3.3, while keeping the bus voltage profiles close to the desired profile.

The total voltage deviation obtained by the FIWCA is 0.0969 p.u., while using the WCA, the value of 0.0974 p.u. is obtained. Table 3.4 demonstrates a comparison of performances among the previous studies and FIWCA on the cost of voltage profiles. The best attainable voltage deviation cost values have been highlighted in bold in Table 3.4 which shows the minimum total voltage deviation is obtained by FIWCA is smaller than the other algorithms. The obtained statistical results also show better performance of FIWCA with average total voltage deviation of 0.0971 p.u. which is very reasonable comparing to the other algorithms.

Table 3.3: Optimal Control Settings for Case 2 on IEEE Standard 30-bus Test System Using the WCA and FIWCA.

Design Variables	WCA	FIWCA
P_{G1} (MW)	177.1920	176.0293
P_{G2} (MW)	50.0000	50.0007
P_{G5} (MW)	21.7403	21.6417
P_{G8} (MW)	23.2284	22.4090
P_{G11} (MW)	10.0009	12.2134
P_{G13} (MW)	11.2310	11.0339
V_{G1} (p.u.)	1.0347	1.0369
V_{G2} (p.u.)	1.0189	1.0218
V_{G5} (p.u.)	1.0105	1.0141
V_{G8} (p.u.)	1.0056	1.0071
V_{G11} (p.u.)	1.0273	0.9973
V_{G13} (p.u.)	0.9978	0.9977
T_{6-9} (p.u.)	1.0433	1.0111
T_{6-10} (p.u.)	0.9000	0.9000
T_{4-12} (p.u.)	0.9540	0.9541
T_{28-27} (p.u.)	0.9694	0.9688
Q_{C10} (MVAR)	5.0000	4.9877
Q_{C12} (MVAR)	0.0000	0.0000
Q_{C15} (MVAR)	5.0000	5.0000
Q_{C17} (MVAR)	0.0213	0.0000
Q_{C20} (MVAR)	5.0000	5.0000
Q_{C21} (MVAR)	5.0000	5.0000
Q_{C23} (MVAR)	5.0000	5.0000
Q_{C24} (MVAR)	5.0000	5.0000
Q_{C29} (MVAR)	2.6755	2.6105
Fuel cost (\$/h)	803.9741	803.9370
Emission (ton/h)	0.3671	0.3634
Power loss (MW)	9.9928	9.9282
VD (p.u.)	0.0974	0.0969

Case 3: Piecewise Quadratic Fuel Cost Function

In real power generation units, it is possible that thermal generating units can be supplied by various types of fuel sources, e.g., natural gas, coal, and oil. Therefore, the total fuel cost function of this generation units may be composed of piecewise quadratic cost functions specified by the type of

Table 3.4: Comparison of Statistical Optimization Results for Case 2, i.e. Total Voltage Deviation (V.D.) (p.u.), Obtained by Several Optimizers.

Algorithms	Minimum V.D.	Average V.D.	Maximum V.D.
ABC [84]	0.1351	N/A	N/A
CI-PSO [84]	0.1350	N/A	N/A
MABC [84]	0.1292	N/A	N/A
Jaya [91]	0.1243	N/A	N/A
GWO [107]	0.1076	N/A	N/A
MSCA [94]	0.1031	N/A	N/A
BB-MOPSO [95]	0.1021	N/A	N/A
BBO [96]	0.1020	0.1105	0.1207
CSA-DHS [97]	0.1006	N/A	N/A
MNSGA-II [95]	0.0989	N/A	N/A
WCA (This Study)	0.0974	0.0995	0.1018
FIWCA (This Study)	0.0969	0.0972	0.0998

consuming fuel. Here, we suppose that the generation units at buses 1 and 2 may be fed by different fuel types depending on their ranges of working. Hence, the generation fuel cost function of these units can be redefined using the following equation

$$f_i(P_{Gi}) = \begin{cases} c_{i0_1} + c_{i1_1}P_{Gi} + c_{i2_1}P_{Gi}^2 & P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi_1} \\ c_{i0_2} + c_{i1_2}P_{Gi} + c_{i2_2}P_{Gi}^2 & P_{Gi_1} \leq P_{Gi} \leq P_{Gi_2} \\ \vdots & \vdots \\ c_{i0_3} + c_{i1_3}P_{Gi} + c_{i2_3}P_{Gi}^2 & P_{Gi_{k-1}} \leq P_{Gi} \leq P_{Gi}^{max} \end{cases} \quad i = 1, 2, \quad (3.15)$$

where c_{i0_k} , c_{i1_k} , and c_{i2_k} are the cost coefficients of i -th generator with fuel type k which their values can be seen in Appendix A, Table A.3. Thus, the total generation fuel cost function can be defined as follows

$$J_3 = \sum_{i=1}^2 (c_{i0_k} + c_{i1_k}P_{Gi} + c_{i2_k}P_{Gi}^2) + \sum_{i=3}^{NG} (c_{i0} + c_{i1}P_{Gi} + c_{i2}P_{Gi}^2). \quad (3.16)$$

Table 3.5: Optimal Control Settings for Case 3 on IEEE Standard 30-bus Test System Using the WCA and FIWCA. "N/A" Stands for Not Available.

Design Variables	WCA	FIWCA
P_{G1} (MW)	139.9999	139.9995
P_{G2} (MW)	54.9998	54.9988
P_{G5} (MW)	24.1771	25.0704
P_{G8} (MW)	34.9303	34.8609
P_{G11} (MW)	18.8638	16.3878
P_{G13} (MW)	17.4422	18.8228
V_{G1} (p.u.)	1.0500	1.0862
V_{G2} (p.u.)	1.0404	1.0711
V_{G5} (p.u.)	1.0157	1.0377
V_{G8} (p.u.)	1.0260	1.0536
V_{G11} (p.u.)	1.1000	1.0152
V_{G13} (p.u.)	1.0909	1.0506
T_{6-9} (p.u.)	0.9247	0.9949
T_{6-10} (p.u.)	1.0205	1.0065
T_{4-12} (p.u.)	0.9563	1.0343
T_{28-27} (p.u.)	0.9281	0.9997
Q_{C10} (MVAR)	N/A	N/A
Q_{C12} (MVAR)	N/A	N/A
Q_{C15} (MVAR)	N/A	N/A
Q_{C17} (MVAR)	N/A	N/A
Q_{C20} (MVAR)	N/A	N/A
Q_{C21} (MVAR)	N/A	N/A
Q_{C23} (MVAR)	N/A	N/A
Q_{C24} (MVAR)	N/A	N/A
Q_{C29} (MVAR)	N/A	N/A
Fuel cost (\$/h)	647.5310	646.6892
Emission (ton/h)	0.2834	0.2835
Power loss (MW)	7.0133	6.7404
VD (p.u.)	1.1870	0.4675

In order to have a fair comparison with the previous works, the upper limit of bus voltages is set to 1.05 and also no VAR compensation bus is considered. As presented in Table 3.5, the total fuel cost obtained by the FIWCA is \$646.6892 per hour, while it is \$647.5310 per hour for the WCA. Table 3.6 demonstrates the statistical comparison using different optimizers for solving this specific case of OPF problem, which shows better perfor-

Table 3.6: Comparison of Statistical Optimization Results for Case 3, i.e. Fuel Costs (\$/h), Obtained by Several Optimizers.

Algorithms	Minimum Fuel Cost	Average Fuel Cost	Maximum Fuel Cost
MDE [100]	650.2800	N/A	N/A
ABC [90]	649.0855	654.0784	659.7708
TLBO [52]	647.8125	647.8335	647.8415
BBO [96]	647.7437	647.7645	647.7928
DE-PS [98]	647.5500	647.6029	647.9253
LTLBO [52]	647.4315	647.4725	647.8638
MPSO [99,100]	646.7300	N/A	N/A
WCA (This Study)	647.5310	648.7809	650.2206
FIWCA (This Study)	646.6892	647.4275	647.5760

mance of the FIWCA comparing to the other algorithms in case of the minimum and average cost values over 50 replications, where the average cost value by FIWCA is \$647.4275 per hour. The best attainable fuel cost values have been highlighted in bold in Table 3.6.

Case 4: Quadratic Cost function with Valve Point Loading Effect

To improve the optimal solution of power flow problems with non-convex cost functions, the valve point loading effect on generators can be considered in the generation fuel cost function of the generators. As reported in the literature [6], the new fuel cost function can be re-written for Generators 1 and 2 and the other generators cost functions would be kept as the same with Case 1.

The new fuel cost function of Generators 1 and 2 can be given as follows

$$f_i(P_{Gi}) = c_{i0} + c_{i1}P_{Gi} + c_{i2}P_{Gi}^2 + \left| d_i \sin(e_i(P_{Gi}^{min} - P_{Gi})) \right| \quad i = 1, 2. \quad (3.17)$$

The cost coefficients of quadratic part of cost function c_{i0} , c_{i1} and, c_{i2} and the coefficients of valve point loading effect term, d_i and e_i , can be found in

Table 3.7: Optimal Control Settings for Case 4 on IEEE Standard 30-bus Test System Using the WCA and FIWCA.

Design Variables	WCA	FIWCA
P_{G1} (MW)	199.5996	199.5996
P_{G2} (MW)	20.0000	0.0000
P_{G5} (MW)	21.9998	22.5907
P_{G8} (MW)	26.1736	24.5433
P_{G11} (MW)	12.7518	13.5448
P_{G13} (MW)	12.0616	12.2930
V_{G1} (p.u.)	1.1000	1.1000
V_{G2} (p.u.)	1.0845	1.0846
V_{G5} (p.u.)	1.0589	1.0592
V_{G8} (p.u.)	1.0684	1.0682
V_{G11} (p.u.)	1.1000	1.1000
V_{G13} (p.u.)	1.0999	1.1000
T_{6-9} (p.u.)	1.0297	1.0297
T_{6-10} (p.u.)	0.9000	0.9000
T_{4-12} (p.u.)	0.9678	0.9666
T_{28-27} (p.u.)	0.9572	0.9570
Q_{C10} (MVAR)	5.0000	5.0000
Q_{C12} (MVAR)	5.0000	5.0000
Q_{C15} (MVAR)	5.0000	4.9450
Q_{C17} (MVAR)	5.0000	5.0000
Q_{C20} (MVAR)	5.0000	4.9677
Q_{C21} (MVAR)	5.0000	5.0000
Q_{C23} (MVAR)	5.0000	2.5560
Q_{C24} (MVAR)	0.0003	5.0000
Q_{C29} (MVAR)	4.9994	2.2920
Fuel cost (\$/h)	917.0858	917.0740
Emission (ton/h)	0.4405	0.4401
Power loss (MW)	9.1866	9.1715
VD (p.u.)	1.9943	2.0398

Appendix A, Table A.4. So, the modified objective function for Case 4 is defined as

$$J_4 = \sum_{i=1}^2 \left(c_{i0} + c_{i1} P_{Gi} + c_{i2} P_{Gi}^2 + \left| d_i \sin(e_i(P_{Gi}^{min} - P_{Gi})) \right| \right) + \sum_{i=3}^{NG} (c_{i0} + c_{i1} P_{Gi} + c_{i2} P_{Gi}^2). \quad (3.18)$$

To find the detailed optimized control parameters by WCA and FIW-

Table 3.8: Comparison of Statistical Optimization Results for Case 4, i.e. Fuel Costs (\$/h), Obtained by Several Optimizers.

Algorithms	Minimum Fuel Cost	Average Fuel Cost	Maximum Fuel Cost
MPSO [99,100]	952.3000	N/A	N/A
ABC [90]	945.4495	960.5647	973.5995
MDE [100]	930.94	N/A	N/A
BBO [96]	919.7647	919.8389	919.8876
PSOGSA [87]	919.65785	N/A	N/A
TLBO [52]	919.3943	919.4710	919.6483
DE-PS [98]	919.0175	919.4750	919.9842
WCA (This Study)	917.1858	917.4408	918.5977
FIWCA (This Study)	917.0740	917.3205	918.1678

CA for this case readers are referred to Table 3.7. Table 3.8 tabulates the obtained statistical optimization results using the different optimization algorithms which demonstrate better performance of the WCA and especially FIWCA comparing to the previous algorithms

The best fuel cost value obtained by the FIWCA is \$917.0740 per hour, while using the WCA, the value of \$917.1858 per hour is attained. The emission and power loss obtained by the FIWCA are 0.4401 ton per hour and 9.1715 MW, respectively, which are lower values comparing to the WCA, i.e. the FIWCA resulted in smaller fuel cost value, emission and power loss. The average cost values obtained by the WCA and FIWCA are \$917.4408 per hour and \$917.3205 per hour, respectively.

The best-obtained fuel cost values have been highlighted in bold in Table 3.8. As can be seen in Table 3.8, the only algorithm that could compete in terms of solution quality with the proposed method is DE-PS with the average of \$919.0175 per hour against the value of \$917.3205 per hour obtained by the FIWCA. The remaining optimization methods show worse optimal solutions.

Table 3.9: Comparison of Statistical Optimization Results for Case 5, i.e. Total Emission (ton/h), Obtained by Several Optimizers.

Algorithms	Minimum Emission	Average Emission	Maximum Emission
TLBO [52]	0.2124	0.2141	0.2156
MDE [100]	0.2093	N/A	N/A
IPSO [101]	0.2058	N/A	N/A
DSA [102]	0.2058	N/A	N/A
DE-PS [98]	0.2052	0.2071	0.2177
HMP SO-SFLA [103]	0.2052	N/A	N/A
DA-PSO [85]	0.2049	N/A	N/A
ABC [90]	0.2048	N/A	N/A
WCA (This Study)	0.2047	0.2051	0.2083
FIWCA (This Study)	0.2047	0.2047	0.2047

Case 5: Minimization of Fuel Cost and Emission

In this case, we minimize the emission function for the OPF problem as well as the fuel cost. For this purpose, the emission function of *SOX* and *NOX*, two impressive emitted gas of electrical power generation processes, are considered. The emission function can be presented as given follows

$$f_{E_i}(P_{Gi}) = c_{i0}^{SOX} + c_{i1}^{SOX} P_{Gi} + c_{i2}^{SOX} P_{Gi}^2 + d_i^{NOX} \exp(e_i^{NOX} P_{Gi}), \quad (3.19)$$

where c_{i0}^{SOX} , c_{i1}^{SOX} , and c_{i2}^{SOX} are the *SOX* emission coefficients, and d_i^{NOX} and e_i^{NOX} are the *NOX* emission coefficient of *i*-th generator. The values of these coefficients can be found in Appendix A, Table A.5. Hence, the objective function of this case can be expressed as given follows

$$J_5 = \sum_{i=1}^{NG} (c_{i0} + c_{i1} P_{Gi} + c_{i2} P_{Gi}^2) + \tau \sum_{i=1}^{NG} (c_{i0}^{SOX} + c_{i1}^{SOX} P_{Gi} + c_{i2}^{SOX} P_{Gi}^2 + d_i^{NOX} \exp(e_i^{NOX} P_{Gi})). \quad (3.20)$$

Note that in this simulation, the value of τ is set to 50000. The optimized

Table 3.10: Optimal Control Settings for Case 5 on IEEE Standard 30-bus Test System Using the WCA and FIWCA.

Design Variables	WCA	FIWCA
P_{G1} (MW)	64.9297	64.9350
P_{G2} (MW)	66.4293	66.4241
P_{G5} (MW)	50.0000	50.0000
P_{G8} (MW)	35.0000	35.0000
P_{G11} (MW)	30.0000	30.0000
P_{G13} (MW)	40.0000	40.0000
V_{G1} (p.u.)	1.1000	1.1000
V_{G2} (p.u.)	1.0956	1.0957
V_{G5} (p.u.)	1.0781	1.0781
V_{G8} (p.u.)	1.0855	1.0856
V_{G11} (p.u.)	1.1000	1.1000
V_{G13} (p.u.)	1.1000	1.1000
T_{6-9} (p.u.)	1.0343	1.0340
T_{6-10} (p.u.)	0.9000	0.9000
T_{4-12} (p.u.)	0.9597	0.9600
T_{28-27} (p.u.)	0.9623	0.9625
Q_{C10} (MVAR)	5.0000	5.0000
Q_{C12} (MVAR)	5.0000	5.0000
Q_{C15} (MVAR)	5.0000	5.0000
Q_{C17} (MVAR)	5.0000	5.0000
Q_{C20} (MVAR)	4.1321	4.1505
Q_{C21} (MVAR)	5.0000	5.0000
Q_{C23} (MVAR)	2.5322	2.5603
Q_{C24} (MVAR)	5.0000	5.0000
Q_{C29} (MVAR)	2.1391	2.1450
Fuel cost (\$/h)	941.8130	941.8048
Emission (ton/h)	0.2047	0.2047
Power loss (MW)	2.9591	2.9591
VD (p.u.)	2.2653	2.2671

value of control variables for this case also can be observed in Table 3.10.

Table 3.9 represents the total amount of emission of generator by optimizing the power flow problem attained by reported methods in the literature.

The total emission obtained by the WCA and FIWCA is 0.2047 ton per hour, while the total fuel cost achieved by the FIWCA is \$941.8048 per hour and lower than the fuel cost obtained by the WCA. Looking at Table 3.9,

the FIWCA shows the best stability in obtained solutions, with the average emission of 0.2047 ton per hour, compared with the other algorithms. The best obtained total emission values have been highlighted in bold in Table 3.9.

3.3.2 IEEE 57-bus Test System

As mentioned in Section 1, one of the main features of the OPF problem, which makes it more complicated, is the size of the problem especially in large-scale power systems. To figure out the performance of the FIWCA in a larger system, the IEEE standard 57-bus test system has been considered. The IEEE standard 57-bus test system, as shown in Fig. 3.5, consists of seven generators, 17 transformers with off-nominal tap ratio, and three shunt VAR compensation buses. The lower and upper limits on control variables are available in Appendix A, Table A.6. The WCA and FIWCA have been applied to this problem for a quadratic cost function given below

$$J_{57-Bus} = \sum_{i=1}^{NG} f_i(P_{Gi}) = \sum_{i=1}^{NG} (c_{i0} + c_{i1}P_{Gi} + c_{i2}P_{Gi}^2), \quad (3.21)$$

where the coefficients of the i -th generator c_{i0} , c_{i1} and c_{i2} are presented in Appendix A, Table A.7. The optimal control variables achieved by implementing the WCA and FIWCA for the IEEE 57-bus test system can be found in Table 3.11.

By evaluating the total optimized costs using the FIWCA, comparing with the previous works, as presented in Table 3.12, the superior performance of the FIWCA against the WCA and other considered optimizers is visible. The minimum value of the total generation fuel cost for this test system is \$41,675.07 per hour, best optimal solution obtained by different

Figure 3.5: Schematic View of IEEE 57-bus Test System.

Table 3.11: The Best Control Design Variables Obtained by the FIWCA and WCA for IEEE 57-bus Test System.

Design Variables	WCA	FIWCA	Design Variables	WCA	FIWCA
P_{G1} (MW)	143.7551	142.9590	T_{24-25} (p.u.)	1.1000	0.9977
P_{G2} (MW)	90.3183	91.4265	T_{24-25} (p.u.)	0.9691	1.0486
P_{G3} (MW)	44.8778	45.0336	T_{24-26} (p.u.)	1.0206	1.0231
P_{G6} (MW)	69.4730	71.0449	T_{7-29} (p.u.)	0.9766	0.9827
P_{G8} (MW)	460.7114	460.3489	T_{34-32} (p.u.)	0.9621	0.9652
P_{G9} (MW)	96.2347	94.5440	T_{11-41} (p.u.)	0.9000	0.9001
P_{G12} (MW)	360.5641	360.5006	T_{15-45} (p.u.)	0.9619	0.9668
V_{G1} (p.u.)	1.0600	1.0597	T_{14-46} (p.u.)	0.9491	0.9540
V_{G2} (p.u.)	1.0558	1.0574	T_{10-51} (p.u.)	0.9563	0.9613
V_{G3} (p.u.)	1.0440	1.0494	T_{13-49} (p.u.)	0.9055	0.9267
V_{G6} (p.u.)	1.0545	1.0566	T_{11-43} (p.u.)	0.9543	0.9610
V_{G8} (p.u.)	1.0589	1.0600	T_{40-56} (p.u.)	0.9866	0.9949
V_{G9} (p.u.)	1.0326	1.0362	T_{39-57} (p.u.)	0.9813	0.9708
V_{G12} (p.u.)	1.0314	1.0393	T_{9-55} (p.u.)	0.9687	0.9745
T_{4-18} (p.u.)	1.0438	0.9000	Q_{C18} (MVAR)	0.0699	0.1289
T_{4-18} (p.u.)	0.9313	1.0792	Q_{C25} (MVAR)	0.1478	0.1445
T_{21-20} (p.u.)	1.0269	1.0116	Q_{C53} (MVAR)	0.1318	0.1312
Fuel cost (\$/h)	41,678.9350	41,675.0794			
Power loss (MW)	15.1347	15.0578			

Table 3.12: Obtained Optimization Results for The IEEE 57-bus Test System, i.e. Fuel Costs (\$/h), Obtained by Several Optimizers.

Algorithms	Fuel Cost
LDI-PSO [90]	41,815.50
GSA [104]	41,695.87
TLBO [52]	41,695.66
ABC [90]	41,693.96
DSA [102]	41,686.82
DE-PS [98]	41,685.29
HSA [105]	41,684.36
FIDE [93]	41,683.00
WCA (This Study) [108]	41,678.93
FIWCA (This Study)	41,675.07

optimization algorithms. As demonstrated in Table 3.11, not only the total fuel cost but also the total power loss obtained by the FIWCA is smaller than the standard WCA.

3.3.3 Multi-period OPF problem with Renewable Resources

Inserting the renewable energy resources into the power grid has many advantages, e.g., reducing the total network emission as the renewable sources are non-pollutant, requiring less maintainable comparing to the traditional power generation units, and decreasing the total power loss in transmission lines as the renewable resources are possible to be distributed within whole the network which decreases the amount of transmitted power through the lines. The total generated power by these renewable resources is dependent on the natural condition, i.e. wind speed, solar radiation, etc., which leads to time and location variant power output. On the other hand, the required power demand is variant during different periods of the day so it is required to solve a multi-period OPF problem.

The performance of FIWCA in solving the multi-period OPF problem including the renewable energy resources is the other challenge which would be investigated in this section. For this aim, the IEEE 30-Bus test system is considered for a period of 24 hours with 1-hour time steps with assumed variable load demands and distributed renewable resources with time and location variant power inserted at the load buses.

To illustrate the performance of FIWCA in solving multi-period OPF problem with integrated renewable resources, Fig. 3.6 shows how to algorithm works in presence of renewable resources. Fig. 3.6(a) represents the power demand at each time step and total generated power by the traditional generators in two different scenarios, with integrated renewable resources and without them. The difference between the generated power of these scenarios is shown in Fig. 3.6(b) which shows a total reduction of about

Figure 3.6: Simulation Results of Solving Multi-period OPF Problem for IEEE 30-Bus Test System with and without Renewable Resources Using FIWCA

Table 3.13: Comparison of Total Fuel Cost, Power Loss and Emission for Multi-period OPF problem with and without Renewable Resources Obtained by FIWCA.

Test System	Fuel Cost (\$)	Power Loss (MW)	Emission (tons)
30-Bus without Renewables	14111.73	130.10	7.2178
30-Bus with Renewables	12402.43	102.99	6.6917

557.64 \$ per day. The total power loss at each time is also demonstrated in Fig. 3.6(c) and total cost saving percentage at each time period is presented in Fig 3.6(d) where the average saving during the day is about 10.4 %.

Table 3.13 presents the comparison results of total fuel cost, total power loss and total emission for the period of 24 hours between the renewable integrated grid and the grid without any renewable resources. The simulation results show that total fuel cost is reduced about 12 %, total power loss is approximately decreased 21 % and total emission from the fossil fuels is reduced around 7 %.

3.3.4 Discussion

The simulation results obtained for the IEEE 30-bus and 57-bus test systems show the better performance of FIWCA in case of finding almost smaller cost function in different conditions, e.g. minimization of fuel cost, emission and voltage deviation compared with the standard WCA and the other reported optimizers. It is worth mentioning that the OPF problems are categorized as well-studied problems, whose solution quality is very tight to the globally optimal solution in some cases.

Therefore, the expected improvement level can be marginal, and no further major improvements can be performed. However, the statistical results validate the better quality of the proposed algorithm in term of stability

of solution over replications which guarantees the reliability of the FIWCA comparing with the WCA and the other compared algorithms.

Furthermore, Fig. 3.7 depicts the fuel cost value (\$/h) vs. iteration numbers for the IEEE 30-bus system, i.e. case 1, and IEEE 57-bus system which show the convergence characteristic of the proposed FIWCA. Looking at Fig. 3.7, the FIWCA has faster and mature convergence rate compared with the standard WCA, and it has converged within a reasonable number of iterations, i.e. around 20 iterations which is faster than the WCA with almost 30 iterations.

Table 3.14 presents the comparison of computational time for solving the OPF problem obtained by different optimization algorithms. The optimal solutions and computational times show that the proposed FIWCA helps to enhance the quality of obtained solutions while being faster than the classical WCA and some other algorithms presented in Table 3.14.

Solving the OPF problem for large scale power networks leads to a more computationally expensive optimization problem which is more difficult to solve owing to the possibility of getting trapped in local optima and/or requiring more powerful processors to take over the convergence of evolutionary optimization methods. Indeed, the complexity level of WCA is discussed in the several references by increasing the dimensionality for each benchmark [82,109]. It is clear that this issue is a common challenge in all the evolutionary optimization algorithms [110] and to obtain the optimal solution it is required to increase the number of iterations which increases the computation time and to overcome this issue it is required to benefit from parallel computing and high-performance computing (HCP) to reduce the CPU time and perform higher number of iterations within faster time.

(a) IEEE 30-bus Test Case System.

(b) IEEE 57-bus Test Case system.

Figure 3.7: Convergence Rate and Cost Reduction History Obtained by WCA and FIWCA.

3.4 Summary

This chapter proposed an improved water cycle algorithm (WCA), called Fully Informed WCA (FIWCA) to solve the optimal power flow problem, a nonlinear and nonconvex optimization problem. The improvement to the WCA was the idea of using global and local information exchange for updat-

Table 3.14: Comparison of Computational Time for The IEEE 30-bus and 57-bus Test Systems, Obtained by Different Optimizers.

Algorithms	Computational Time (sec)	
	30-Bus	57-Bus
DE-PS [98]	27.33	N.A.
TLBO [52]	24.27	N.A.
MDE [111]	23.25	N.A.
LTLBO [52]	22.14	N.A.
Fuzzy-GA [112]	22.07	N.A.
NPSO [7]	20.45	N.A.
GSA [90]	19.71	N.A.
WCA (This Study)	20.38	48.70
FIWCA (This Study)	17.71	43.35

ing the positions of each individual in the population. Using fully informed algorithms helps to increase the diversity in the selection of solution and consequently decrease the possibility of getting trapped at local optima, thereby enhancing the efficiency and accuracy significantly. The OPF problem with a variety of different nonlinear constraints/objectives is studied to evaluate the optimality and efficiency of the FIWCA. The OPF solution to IEEE 30-bus and 57-bus bus systems was used to study the performance of the FIWCA with existing approaches. Obtained optimization results demonstrated the computation benefits and efficiency of the FIWCA in reaching global optimal solutions with reasonable computation time that is better than that of existing techniques such as conventional WCA, genetic algorithm, particle swarm optimization, ant colony and others explored in the literature.

Chapter 4

Quickly Bounding the Optimal Objective Value of an OPF Problem using a Semidefinite Relaxation and a Local Solution

First formulated by Carpentier [11] in 1962, the optimal power flow (OPF) problem determines a steady-state operating point for an electric power system by optimizing an objective function, such as generation cost, while satisfying constraints that model the nonlinear power flow equations as well as limits on line-flows, voltage magnitudes, and generator outputs. The OPF problem is non-convex, NP-Hard [36,37], and may have multiple *local optima* (i.e., feasible points that are superior to all nearby feasible points but possibly inferior to the *global optimum*) [40].

Research conducted for over fifty years has developed many mature local solution algorithms for OPF problems, including Newton-based techniques, sequential quadratic programming algorithms, interior point methods, etc. [12–14]. These algorithms often quickly find local optima for large OPF problems.

Certifiably finding global solutions to OPF problems (or, at least, strong objective value bounds) is important for many applications. To compute global optimality certificates, recent research efforts have focused on convex relaxations of OPF problems [29]. Convex relaxations bound the objective value of the original OPF problem, providing lower (upper) bounds for minimization (maximization) problems. Comparing the bound provided by a relaxation and the objective value obtained from a local solver yields an *optimality gap*. A sufficiently small optimality gap certifies global optimality of the corresponding local solution.

Many OPF problems seek to minimize generation cost. Local optima in generation-cost-minimizing OPF problems may be significantly more expensive than the global solutions [40], and thus provably computing global optima is relevant to improving power system economics. Algorithms that use spatial branch-and-bound techniques to globally solve OPF problems are proposed in, e.g., [113–115]. These algorithms iteratively apply local solvers and convex relaxation techniques in order to shrink the optimality gap until global optimality is certifiably achieved.

Global optimality guarantees are also important to OPF problems that are solved as subroutines in certain bi-level optimization algorithms, e.g., the robust OPF algorithm in [116]. Examples of other applications where rigorous objective value bounds are particularly relevant include proving

insolvability of power flow problems [117], bounding the errors associated with various power flow approximations [118], certifying security in the face of power injection variability for distribution systems with limited measurement and control capabilities [119], and computing the feasible spaces of OPF problems for visualization purposes [41].

Various convex relaxation techniques provide a range of trade-offs between bound tightness and computational tractability. Simple relaxations, such as copper plate and network flow formulations [120], quickly provide crude bounds. More sophisticated approaches, such as those based on second-order cone programming (SOCP) [121–124] and semidefinite programming (SDP) [31,125], provide tighter bounds with associated trade-offs in computational speed. The literature details a variety of approaches for refining trade-offs in computational speed and tightness of the relaxations (e.g., adding valid cuts [123] and exploiting problem structure [125]). A comprehensive survey is provided in [29].

Building on the convex relaxation literature, this chapter proposes a new method for balancing trade-offs between computational tractability and tightness in the calculation of objective value bounds for OPF problems. To speed computation of an objective value bound, our method takes as input an OPF solution obtained from a local solver. Our method therefore does not attempt to compute values for the decision variables; we emphasize that only the objective value bound is of interest in this chapter. The proposed method is applicable as a subroutine in the bounding steps of global solution algorithms, such as those in [113–115], as well as in other algorithms, such as those in [41,116–119], that also require rigorous bounds on certain quantities. Reducing the time required to calculate objective value bounds thus has the

potential to improve computational tractability for a variety of power system algorithms.

We specifically consider a method for obtaining an objective value bound using the SDP relaxation in [31] and the dual variables from a local solution. The SDP relaxation in [31] is derived from an OPF formulation where all non-convexity is contained in a rank condition. Following Shor’s approach [126], an SDP relaxation is formed by neglecting this rank condition. A relaxation whose solution satisfies the rank condition is “exact”, thus providing a tight bound on the objective value and the globally optimal decision variables. While the SDP relaxation in [31] is exact for some OPF test cases, the rank condition is not always satisfied [127]. Nevertheless, the objective value from the SDP relaxation always bounds the objective of the original OPF problem.

Solving the SDP relaxation can be computationally challenging due to the presence of a positive semidefinite constraint on a large matrix. A matrix completion technique mitigates this computational challenge by decomposing the single positive semidefinite constraint into constraints on many submatrices [32,33]. However, the resulting formulation is still less tractable than many local solution algorithms. This motivates the development of techniques for exploiting information in a local solution to further speed computations of SDP relaxations.

Related work [20] proposes a quickly checkable sufficient condition for global optimality of a candidate OPF solution from a local solver. Satisfaction of the Karush-Kuhn-Tucker (KKT) optimality conditions for the SDP relaxation certifies that a local solution is, in fact, globally optimal. Since the KKT conditions for the SDP relaxation can be quickly evaluated for

a candidate OPF solution without explicitly solving an SDP, the condition in [20] provides a tractable method for certifying global optimality.

While the approach in [20] provides significant computational advantages for certain problems, the condition is sufficient but not necessary for global optimality and thus does not provide any information regarding global optimality when the condition is not satisfied. This chapter proposes another method for leveraging information from a local solution to speed the computation of an objective value bound. Specifically, we seek a dual feasible point for the SDP relaxation. The objective value of any dual feasible point bounds the objective value of the primal SDP relaxation [128], which itself bounds the objective value of the original OPF problem. Rather than attempt the computationally difficult task of computing the best achievable bounds, we instead leverage information from local solutions to more tractably obtain (possibly weaker) bounds. In particular, we fix certain dual variables in the SDP relaxation to their values from a local solution and solve the resulting simpler SDP problem.

Application to a variety of large test cases empirically demonstrates that the proposed method is faster than the SDP relaxation of [31], often without excessive degradation in the quality of the objective value bound. Conversely, empirical results indicate that the proposed method is slower than the SOCP relaxation in [121] and the QC relaxation in [124] but can provide significantly tighter bounds. Thus, the proposed method can be viewed as a “middle ground” between the SDP relaxation and the SOCP and QC relaxations in terms of the trade-off between computational speed and tightness.

4.1 Optimal Power Flow Problem Modeling Issues

The computational difficulty of solving the OPF problem, as a nonconvex NP-hard problem with multiple optima, is the main challenge in power flow optimization and the problem is still open for researches and scientists.

Thus, a number of approaches has been proposed to decrease the difficulty of computational issues. The main goal of these approaches is to transform the original nonconvex problem into a new convex formulation which would be able to be solved by the convex optimization tools, such as linear programming, semidefinite programming, second order cone programming, etc.

These approaches are classified into the OPF approximations and relaxations.

4.1.1 Optimal Power Flow Problem Approximation

The approximation of the OPF problem would be achieved by considering a set of assumptions on power flow equations and variable derivations to simplify the problem into the intended convex approximation.

Different studies have been done on approximation of OPF problem. The linear approximation of OPF problem has been implemented by different ideas such as considering DC load flow [118,129–133], linearizing the power flow equations near an operating point [134–137], decoupling the power flow equations [138,139], etc. Some other studies use semidefinite programming approximation [140,141] and second order cone programming [142,143] to simplify the OPF problem.

The solution of an approximated OPF problem may be infeasible for the original OPF problem and even the infeasibility of the approximated problem does not result in the infeasibility of the original problem. Therefore, some studies have been done to quantify in which condition it is safe to utilize the approximation solutions [118]. Also, it should be considered that generally the solution of an approximation approach does not guarantee a lower bound on the original OPF problem.

4.1.2 Optimal Power Flow Problem Relaxation

The other method of simplifying a difficult optimization problem is relaxation which can be defined as an approximation whose solution provides some information about the main problem.

Definition (*Relaxation*): Consider the minimization problem

$$\min_x f_c(x) \quad \text{subject to} \quad (4.1a)$$

$$x \in \Omega \subseteq \mathcal{R}^n. \quad (4.1b)$$

A relaxation of (4.1) is defined as

$$\min_x f_{c_R}(x) \quad \text{subject to} \quad (4.2a)$$

$$x \in \Omega_R \subseteq \mathcal{R}^n, \quad (4.2b)$$

with the following properties:

1. The feasible region of the original problem is a subset of the feasible region of the relaxed problem, i.e. $\Omega \subseteq \Omega_R$.
 2. For every $x^* \in \Omega_R$, the cost value of the relaxed problem is less
-

than/equal to the cost value of the original problem, i.e. $f_{c_R}(x^*) \leq f_c(x^*)$.

It can be derived from this definition that:

1. If the relaxation of an optimization problem is infeasible, then the original problem is infeasible too.
2. The optimal solution of the relaxed problem provides a lower/upper bound for the original minimization/maximization convex problem.

The computational difficulty of nonconvex optimization problem, which prevents to achieve the optimal solution in a reasonable time period, enforces the researchers to relax the problem in a way that it lessens the complexity of the original problem. Convex relaxation as a powerful tool is an approximation of the original nonconvex problem obtained by relaxing some of the constraints and extending the feasibility region of the problem in a way that eventually results in a convex optimization problem.

Recently implementing the relaxation techniques on the OPF problems has become a hot topic of interest for the power optimization researchers. Most often, the objective functions of the OPF problem (4.10) are considered as quadratic or linear functions which are naturally convex and then the main source of nonconvexity in the OPF problem is the power flow equations. Therefore the main focus of relaxation techniques in the OPF problem is to relax the power flow equations.

The main relaxations applied on the OPF problems are semidefinite programming (SDP) relaxations, second order cone programming (SOCP) relaxations, quadratic convex (QC) relaxations and linear programming (LP) relaxations.

The SDP relaxation of OPF problem is mainly based on the Shor relaxation [144] which was proposed in 1987 for quadratically constrained quadratic programming (QCQP) problems. Using the close feature of the OPF problem formulation to the QCQP problem, Bai et al. proposed the first SDP relaxation of OPF problem in 2008 and applied the proposed relaxation on a few numbers of test cases with up to 300 buses [30]. Lavaei and Low presented the general convex SDP relaxation of the OPF problem in [31,145]. They also derived the dual problem of the relaxed problem and provided the efficient conditions that guarantee the zero duality gap. They also demonstrated how to recover the original OPF solution from the dual solution.

If the solution from a relaxed problem can satisfy the related assumptions and constraints, which are considered to relax the original problem, then it can be claimed that the relaxation is exact. The proposed SDP relaxation in [31] is exact for most of the IEEE standard OPF problem test cases. But there are still some test cases that their SDP relaxations are not exact [40,41,146–148]. For more details on exactness of the Shor relaxation in the OPF problem, refer to [149,150] in which the authors provide some of the sufficient conditions under which the relaxation is exact for the OPF problem.

The second order cone programming (SOCP) [151] relaxation is the other convex relaxation which has been applied to the bus injection model OPF problem by Jabr in 2006 [121]. A strong SOCP relaxation of the OPF problem has been proposed in [152] which provides an SOCP relaxation which can guarantee a lower bound very close to (the lower bound) from SDP relaxation. Some other works on tighter (stronger) SOCP relaxation of the

OPF problem can be found in [124,153–155]. The SOCP relaxation also has been applied to the branch flow modeled OPF problem, recently [156,157]. The sufficient conditions which result in an exact SOCP relaxation also have been investigated in some research works [158–161].

The linear programming (LP) relaxation is the other technique to achieve a convex formulation of OPF problem. The LP relaxation usually leads to a weaker convex problem comparing to the SDP and SOCP relaxations. And it may not be able to obtain a tight lower bound but because of the computational benefits of this approach, it is still interesting for researchers. Different ideas and techniques have been used to yield an LP relaxation in recent works such as network flow relaxation [162], the copper plate relaxation which totally omits the power flow equations [162], the Tylor-Hoover relaxations [163], McCormick relaxations [164,165], Bienstock Munoz LP relaxations [166], etc. [167,168].

The other convex relaxation that can be defined as an extension of the semidefinite relaxation is the moment/sum of squares (SOS) relaxation. The moment-based relaxation utilizes the polynomial optimization theory to generalize the Shor relaxation and provides a convex relaxation.

As discussed, though solving the SDP relaxation of the OPF problem may result in the global solution for many test cases but still, there are some cases for which the global solution may not be guaranteed by SDP solution. Thus using the Lasserre hierarchy [169] and Shor relaxation, the moment-based relaxation of OPF problem has been proposed by Molzahn [170]. The moment-based relaxation attempts to define the OPF problem as a polynomial in term of the voltage phasors.

The degree of polynomial determines the order of the moment relaxation.

Note that the semidefinite relaxation considers all degree-two components of the voltage-phasors higher degree of the polynomial or, briefly higher order moment relaxation improves the ability to achieve the global optimal solution.

The only limit of moment-based relaxation is the increasing size of the matrices and curse of dimension, i.e. for an n -bus network the γ -order semidefinite matrix of size $k = (2n + \gamma)! / ((2n)! \gamma!)$ [17]. The computational issue of solving higher order relaxations is the main difficulty in this approach.

Semidefinite Programming (SDP) Relaxation

In semidefinite programming (SDP) the problem would be modeled as the minimization of a linear cost function subject to a set of positive semidefinite (PSD) matrices forming an intersection of a cone which is convex while it may be nonlinear or nonsmooth. Comparing to the linear programming modeling of problems, the semidefinite programming is a more general and stronger tool in finding an exact solution especially in relaxation and optimization techniques.

The SDP is a useful tool for modeling different problems such as linear matrix inequalities, eigenvalue optimization, singular value optimization [171], convex relaxation and etc. and their applications in power system optimizations and stability analysis and control systems.

The primal SDP can be defined as

$$\min_X \quad \text{tr}\{C \cdot X\} \quad \text{subject to} \quad (4.3a)$$

$$\text{tr}\{A_i \cdot X\} = b_i \quad i = 1, \dots, r, \quad (4.3b)$$

$$X \succeq 0, \quad (4.3c)$$

where X is an asymmetric positive definite matrix of variables and C and A_i are symmetric square matrices of coefficients. Note that $\text{tr}\{\cdot\}$ shows the trace operator and

$$\text{tr}\{AB\} = \sum_i \sum_j a_{ij} b_{ij}.$$

By defining the Lagrange function as

$$L(X, \lambda) = \text{tr}\{C \cdot X\} - \sum_{i=1}^r \lambda_i \left(\text{tr}\{A_i \cdot X\} - b_i \right), \quad (4.4)$$

the dual SDP problem is then as follows

$$\max_{\lambda} \quad b^T \lambda \quad \text{subject to} \quad (4.5a)$$

$$C - \sum_{i=1}^r A_i \lambda_i \succeq 0, \quad (4.5b)$$

where λ is the vector of dual variables.

There are fewer numbers of algorithms for solving semidefinite programming problems comparing to the linear or the second order cone programming problems. Some of these codes, mostly based on the interior point method, are SeDumi [172], SDPT3 [173], SDPA [174], MOSEK [171] and some others are such as ConicBundle [175] and SCS [176] solvers.

Some of the properties of the positive semidefinite (PSD) matrices which

are helpful in continue are as follows:

- All of the diagonal entries of a PSD matrix are nonnegative.
- In a block diagonal PSD matrix, all the diagonal submatrices are PSD.
- Any submatrix of a PSD matrix is PSD.
- Any quadratic transformation of a PSD matrix is PSD, i.e. if $A \succeq 0$ then any $B^T A B \succeq 0$.
- For any two PSD matrices, $A, B \succeq 0$, we have $\text{tr}\{A.B\} \geq 0$
- If matrix A is PSD, the inverse of A is also PSD.

4.2 The OPF Problem and an SDP Relaxation

This section first formulates the OPF problem as well as its SDP relaxation [31] and then discusses the matrix completion decomposition used to exploit network sparsity [32].

4.2.1 Optimal Power Flow Formulation

Consider an n -bus system with the set of buses $\mathcal{N} = \{1, \dots, n\}$. Each bus $k \in \mathcal{N}$ has an associated complex voltage phasor $V_k = V_{dk} + jV_{qk}$, power generation $P_{Gk} + jQ_{Gk}$, and specified power demand $P_{Dk} + jQ_{Dk}$, where $j = \sqrt{-1}$. The bus which sets the reference angle is denoted with the subscript “ref”. The set of lines is denoted as $\mathcal{L}$, where $(l, m) \in \mathcal{L}$ indicates the line connecting buses l and m . Each line $(l, m) \in \mathcal{L}$ is modeled as a Π circuit with mutual admittance y_{lm} and shunt susceptance $b_{sh,lm}$.

Extending our method to more general line models and multiple generators per bus is straightforward as presented in [33]. The numerical results in Section 4.6 use MATPOWER's line model [14]. The complex power flow and angle difference on each line $(l, m) \in \mathcal{L}$ are denoted by $S_{lm} = P_{lm} + jQ_{lm}$ and $\theta_{lm} = \theta_l - \theta_m$, respectively. The network admittance matrix is denoted as $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$. Superscripts “max” and “min” indicate specified upper and lower limits. Buses without any generation units have the corresponding generation limits set to zero.

Denote e_k as the k^{th} standard basis vector in $\mathbb{R}^n$. Define the matrix $Y_k = e_k e_k^\top \mathbf{Y}$, where $(\cdot)^\top$ denotes the matrix transpose. Following the notation in [31], define the matrices $\mathbf{Y}_k$, $\overline{\mathbf{Y}}_k$, $\mathbf{M}_k$, and $\mathbf{N}_k$ used to construct expressions for the active and reactive power injections, the squared voltage magnitudes, and the angle reference, respectively, at bus $k \in \mathcal{N}$:

$$\mathbf{Y}_k = \frac{1}{2} \begin{bmatrix} \text{Re}(Y_k + Y_k^\top) & \text{Im}(Y_k^\top - Y_k) \\ \text{Im}(Y_k - Y_k^\top) & \text{Re}(Y_k + Y_k^\top) \end{bmatrix}, \quad (4.6a)$$

$$\overline{\mathbf{Y}}_k = -\frac{1}{2} \begin{bmatrix} \text{Im}(Y_k + Y_k^\top) & \text{Re}(Y_k - Y_k^\top) \\ \text{Re}(Y_k^\top - Y_k) & \text{Im}(Y_k + Y_k^\top) \end{bmatrix}, \quad (4.6b)$$

$$\mathbf{M}_k = \begin{bmatrix} e_k e_k^\top & 0 \\ 0 & e_k e_k^\top \end{bmatrix}, \quad (4.6c)$$

$$\mathbf{N}_k = \begin{bmatrix} 0 & 0 \\ 0 & e_k e_k^\top \end{bmatrix}. \quad (4.6d)$$

For each line $(l, m) \in \mathcal{L}$, let $Y_{lm} = \left(\frac{b_{sh,lm}}{2} + y_{lm} \right) e_l e_l^\top - y_{lm} e_l e_m^\top$. To formulate the expressions for active and reactive power flows and phase

angle differences associated with the line $(l, m) \in \mathcal{L}$, define the matrices

$$\mathbf{Y}_{lm} = \frac{1}{2} \begin{bmatrix} \operatorname{Re}(Y_{lm} + Y_{lm}^\top) & \operatorname{Im}(Y_{lm} - Y_{lm}^\top) \\ \operatorname{Im}(Y_{lm}^\top - Y_{lm}) & \operatorname{Re}(Y_{lm} + Y_{lm}^\top) \end{bmatrix}, \quad (4.7a)$$

$$\overline{\mathbf{Y}}_{lm} = -\frac{1}{2} \begin{bmatrix} \operatorname{Im}(Y_{lm} + Y_{lm}^\top) & \operatorname{Re}(Y_{lm} - Y_{lm}^\top) \\ \operatorname{Re}(Y_{lm}^\top - Y_{lm}) & \operatorname{Im}(Y_{lm} + Y_{lm}^\top) \end{bmatrix}, \quad (4.7b)$$

$$\mathbf{M}_{lm} = \frac{1}{2} \begin{bmatrix} e_l e_m^\top + e_m e_l^\top & 0 \\ 0 & e_l e_m^\top + e_m e_l^\top \end{bmatrix}, \quad (4.7c)$$

$$\overline{\mathbf{M}}_{lm} = \frac{1}{2} \begin{bmatrix} 0 & -e_l e_m^\top + e_m e_l^\top \\ e_l e_m^\top - e_m e_l^\top & 0 \end{bmatrix}. \quad (4.7d)$$

By denoting the vector of voltage coordinates x as

$$x = \begin{bmatrix} V_{d1} & V_{d2} & \dots & V_{dn} & V_{q1} & V_{q2} & \dots & V_{qn} \end{bmatrix} \quad (4.8)$$

and defining the rank-one matrix

$$\mathbf{W} = x x^\top, \quad (4.9)$$

all quantities of interest can be written in terms of $\mathbf{W}$. Let $\operatorname{tr}(\cdot)$ denote the trace of a matrix. The active and reactive power injections at each bus k are $\operatorname{tr}(\mathbf{Y}_k \mathbf{W})$ and $\operatorname{tr}(\overline{\mathbf{Y}}_k \mathbf{W})$, respectively. The squared voltage magnitude at bus k is $\operatorname{tr}(\mathbf{M}_k \mathbf{W})$. The active and reactive power flows into terminal l of line $(l, m) \in \mathcal{L}$ are $\operatorname{tr}(\mathbf{Y}_{lm} \mathbf{W})$ and $\operatorname{tr}(\overline{\mathbf{Y}}_{lm} \mathbf{W})$, and the phase angle difference is $\arctan(\operatorname{tr}(\overline{\mathbf{M}}_{lm} \mathbf{W}) / \operatorname{tr}(\mathbf{M}_{lm} \mathbf{W}))$.

We consider a convex quadratic cost function of active power generation with specified quadratic, linear, and constant coefficients $c_{k2} \geq 0$, c_{k1} , and

c_{k0} for each generator $k \in \mathcal{N}$. Buses without generators have $c_{k2} = c_{k1} = c_{k0} = 0$.

Using these definitions, the OPF problem (2.6) can be represented as

$$\min \sum_{k \in \mathcal{N}} \alpha_k \quad (4.10a)$$

$$\text{subject to } (\forall k \in \mathcal{N}, (l, m) \in \mathcal{L})$$

$$P_{Gk}^{min} - P_{Dk} \leq \text{tr}(\mathbf{Y}_k \mathbf{W}) \leq P_{Gk}^{max} - P_{Dk}, \quad (4.10b)$$

$$Q_{Gk}^{min} - Q_{Dk} \leq \text{tr}(\bar{\mathbf{Y}}_k \mathbf{W}) \leq Q_{Gk}^{max} - Q_{Dk}, \quad (4.10c)$$

$$(V_k^{min})^2 \leq \text{tr}(\mathbf{M}_k \mathbf{W}) \leq (V_k^{max})^2, \quad (4.10d)$$

$$\text{tr}(\mathbf{N}_{ref} \mathbf{W}) = 0, \quad (4.10e)$$

$$\begin{bmatrix} -(S_{lm}^{max})^2 & \text{tr}(\mathbf{Y}_{lm} \mathbf{W}) & \text{tr}(\bar{\mathbf{Y}}_{lm} \mathbf{W}) \\ \text{tr}(\mathbf{Y}_{lm} \mathbf{W}) & -1 & 0 \\ \text{tr}(\bar{\mathbf{Y}}_{lm} \mathbf{W}) & 0 & -1 \end{bmatrix} \preceq 0, \quad (4.10f)$$

$$\begin{aligned} \tan(\theta_{lm}^{min}) \cdot \text{tr}(\mathbf{M}_{lm} \mathbf{W}) &\leq \text{tr}(\bar{\mathbf{M}}_{lm} \mathbf{W}) \\ &\leq \tan(\theta_{lm}^{max}) \cdot \text{tr}(\mathbf{M}_{lm} \mathbf{W}), \end{aligned} \quad (4.10g)$$

$$\begin{bmatrix} c_{k1} \text{tr}(\mathbf{Y}_k \mathbf{W}) + a_k - \alpha_k & \sqrt{c_{k2}} \text{tr}(\mathbf{Y}_k \mathbf{W}) + b_k \\ \sqrt{c_{k2}} \text{tr}(\mathbf{Y}_k \mathbf{W}) + b_k & -1 \end{bmatrix} \preceq 0, \quad (4.10h)$$

$$\mathbf{W} = xx^\top, \quad (4.10i)$$

where $a_k = c_{k1}P_{Dk} + c_{k0}$, $b_k = \sqrt{c_{k2}}P_{Dk}$, and $(\cdot) \succeq 0$ indicates positive semidefiniteness. Constraints (4.10b)–(4.10d) limit the power injections and squared voltage magnitudes, (4.10e) sets the reference angle, and (4.10f) and (4.10g) limit the apparent power flows and phase angle differences. The objective (4.10a) and constraint (4.10h) represent the generation cost us-

ing a Schur complement formulation with auxiliary variables α_k . All non-convexity is contained in the rank-one condition (4.10i).

4.2.2 Semidefinite Programming Relaxation of the OPF

Problem: Primal and Dual Formulation

The approach in [31] forms a Shor relaxation [126] of (4.10) by replacing the rank-one condition (4.10i) with a less stringent positive semidefinite matrix constraint,

$$\mathbf{W} \succeq 0. \quad (4.11)$$

This yields the primal formulation of the SDP relaxation:

$$\min \sum_{k \in \mathcal{N}} \alpha_k \quad \text{subject to} \quad (4.10b)-(4.10h), (4.11). \quad (4.12)$$

If the solution to the SDP relaxation, $\mathbf{W}^*$, satisfies the rank condition $\text{rank}(\mathbf{W}^*) = 1$, then the relaxation is *exact* and thus provides both the globally optimal objective value and decision variables $V^* = \sqrt{\lambda} (\eta_{1:n} + j\eta_{n+1:2n})$, where λ denotes the non-zero eigenvalue of $\mathbf{W}^*$ with corresponding unit-length eigenvector η . While solutions for which $\text{rank}(\mathbf{W}^*) > 1$ do not provide globally optimal decision variables, the optimal objective value from the solution to (4.12) still lower bounds the objective value of the original OPF problem (4.10).

Our method for speeding computations is based on the dual form of the SDP relaxation. To formulate the dual of (4.12), we define (for each bus $k \in \mathcal{N}$ and each line $(l, m) \in \mathcal{L}$) the dual scalar variables $\underline{\lambda}_k$, $\underline{\gamma}_k$, $\underline{\mu}_k$, and $\underline{\beta}_{lm}$ associated with the lower bounds of (4.10b), (4.10c), (4.10d) and (4.10g); the dual scalar variables $\bar{\lambda}_k$, $\bar{\gamma}_k$, $\bar{\mu}_k$, and $\bar{\beta}_{lm}$ associated with the

upper bounds of (4.10b), (4.10c), (4.10d) and (4.10g); the dual 3×3 matrix variables $\mathbf{H}_{lm}$ associated with line-flow constraints (4.10f); and the dual 2×2 matrix variables $\mathbf{R}_k$ associated with the Schur complement formulation of the quadratic cost function (4.10h).

For notational convenience, denote

$$\begin{aligned}\lambda_k &= \bar{\lambda}_k - \underline{\lambda}_k + c_{k1} + 2\sqrt{c_{k2}}R_k^{12}, & \gamma_k &= \bar{\gamma}_k - \underline{\gamma}_k, \\ \mu_k &= \bar{\mu}_k - \underline{\mu}_k, & \beta_{lm} &= \bar{\beta}_{lm} - \underline{\beta}_{lm}.\end{aligned}$$

The dual formulation of the SDP relaxation (4.12) is

$$\max \quad \rho \tag{4.13a}$$

$$\text{subject to} \quad (\forall k \in \mathcal{N}, (l, m) \in \mathcal{L})$$

$$\mathbf{A} \succeq 0, \tag{4.13b}$$

$$\mathbf{H}_{lm} \succeq 0, \tag{4.13c}$$

$$\mathbf{R}_k \succeq 0, \quad \mathbf{R}_k^{11} = 1, \tag{4.13d}$$

$$\underline{\lambda}_k \geq 0, \bar{\lambda}_k \geq 0, \underline{\gamma}_k \geq 0, \bar{\gamma}_k \geq 0, \underline{\mu}_k \geq 0, \bar{\mu}_k \geq 0, \tag{4.13e}$$

$$\underline{\beta}_{lm} \geq 0, \bar{\beta}_{lm} \geq 0, \tag{4.13f}$$

where the scalar- and matrix-valued functions ρ and $\mathbf{A}$ are

$$\begin{aligned} \rho = \sum_{k \in \mathcal{N}} \left\{ \lambda_k P_{Dk} + \underline{\lambda}_k P_{Gk}^{min} - \bar{\lambda}_k P_{Gk}^{max} + \gamma_k Q_{Dk} + \underline{\gamma}_k Q_{Gk}^{min} \right. \\ \left. - \bar{\gamma}_k Q_{Gk}^{max} + \underline{\mu}_k (V_k^{min})^2 - \bar{\mu}_k (V_k^{max})^2 \right\} + \sum_{k \in \mathcal{N}} (c_{k0} - \mathbf{R}_k^{22}) \\ - \sum_{(l,m) \in \mathcal{L}} \left\{ (S_{lm}^{max})^2 \mathbf{H}_{lm}^{11} + \mathbf{H}_{lm}^{22} + \mathbf{H}_{lm}^{33} \right\}, \end{aligned} \quad (4.14)$$

$$\begin{aligned} \mathbf{A} = \sum_{k \in \mathcal{N}} \left\{ \lambda_k \mathbf{Y}_k + \gamma_k \bar{\mathbf{Y}}_k + \mu_k \mathbf{M}_k \right\} \\ + \sum_{(l,m) \in \mathcal{L}} \left\{ 2\mathbf{H}_{lm}^{12} \mathbf{Y}_{lm} + 2\mathbf{H}_{lm}^{13} \bar{\mathbf{Y}}_{lm}^{13} - \bar{\mathbf{M}}_{lm} \beta_{lm} \right. \\ \left. + \tan(\theta_{lm}^{max}) \mathbf{M}_{lm} \bar{\beta}_{lm} - \tan(\theta_{lm}^{min}) \mathbf{M}_{lm} \underline{\beta}_{lm} \right\}. \end{aligned} \quad (4.15)$$

Note that $\mathbf{R}_k^{cd}$ and $\mathbf{H}_{lm}^{cd}$ represent the (c, d) elements of matrices $\mathbf{R}_k$ and $\mathbf{H}_{lm}$, respectively.

Any feasible point for the dual SDP relaxation (4.13) provides a lower bound for the objective value of the primal SDP relaxation (4.12) [128], and thus a lower bound for the original OPF problem (4.10). The solution to (4.13) provides the best achievable lower bound among all dual feasible points.

4.3 Checking the Global Optimality of a Candidate Solution

Many mature OPF algorithms provide locally optimal solutions which are actually the globally optimal solution in a wide range of power systems test cases but these algorithms are not able to provide any information regarding

the quality of their solution. Thus, finding a method that may quickly check the quality of a local solution from mature OPF algorithms would be really efficient comparing with solving a semidefinite programming relaxation OPF problem. Checking the sufficient condition for global optimality of a candidate local solution has been presented in [20]. If a semidefinite programming relaxation becomes feasible then there exists a global solution that satisfies the KKT conditions, as described in [177]. To check the global optimality of a candidate solution it is required to form the matrices $\mathbf{W}$ and $\mathbf{A}$ from (4.9) and (4.15) using the local candidate solution.

A solution of the classical OPF formulation (2.6) obtained by any mature OPF algorithm provides the vector of rectangular voltage phasors, $V_k = V_{dk} + jV_{qk}$, power injections, $P_k + jQ_k$, and Lagrange multipliers associated with the problem formulation (2.6). To form the matrix $\mathbf{A}$, it is required to utilize the Lagrange multipliers associated with the semidefinite relaxation formulation of OPF problem (4.12). Thus, some transformation is needed to be applied to the candidate local solution to be suitable for checking its global optimality.

For this aim, consider ξ_k , λ_k , γ_k and ζ_{lm} as the Lagrange multipliers of the voltage magnitude constraint, active power constraint, reactive power constraint and apparent power flow constraint associated with equations (2.6d), (2.6b), (2.6c) and (2.6e). The dual semidefinite OPF problem is constructed by the Lagrange multipliers μ_k , λ_k , γ_k , α_{lm} and η_{lm} associated with the constraints on square of voltage magnitudes, active powers, reactive powers, active power flows and reactive power flows, respectively. Note that $\alpha_{lm} = 2\mathbf{H}_{lm}^{12}$ and $\eta_{lm} = 2\mathbf{H}_{lm}^{13}$. Therefore, the following transformations are

needed before checking the global optimality of a candidate local solution

$$\mu_k = \xi_k \left(\frac{1}{V_k^*} \right), \quad (4.16)$$

where V_k^* is the voltage solution at bus k .

$$\alpha_{lm} = \zeta_{lm} \left(\frac{\partial S_{lm}}{\partial P_{lm}} \right) = \zeta_{lm} \left(\frac{\partial P_{lm}^*}{\partial S_{lm}^*} \right), \quad (4.17a)$$

$$\alpha_{lm} = \zeta_{lm} \left(\frac{\partial S_{lm}}{\partial Q_{lm}} \right) = \zeta_{lm} \left(\frac{\partial Q_{lm}^*}{\partial S_{lm}^*} \right), \quad (4.17b)$$

where P_{lm}^* and Q_{lm}^* are the the active and reactive power flows through line (l, m) from the local solution. Be noted that $S_{lm} = \sqrt{P_{lm}^2 + Q_{lm}^2}$. For more details see [20].

Then the KKT conditions need to be checked for global optimality of the solution. First, the complementarity condition needs to be satisfied as follows

$$\text{tr} \{ \mathbf{A} \mathbf{W} \} = 0. \quad (4.18)$$

The second condition is the feasibility of $\mathbf{W} = xx^\top$ and $\mathbf{A}$. The positive semidefiniteness of $\mathbf{W} = xx^\top$ is naturally satisfied since $\mathbf{W} = xx^\top \succeq 0$ so only the feasibility of matrix $\mathbf{A}$ is required as

$$\mathbf{A} \succeq 0. \quad (4.19)$$

If a candidate solution satisfies the conditions (4.18) and (4.19) then the candidate solution is a globally optimal solution. But if the KKT conditions are not satisfied, it is unknown whether the solution is global or not and no more information on quality of the candidate solution can be provided.

4.4 Matrix Completion Decomposition and its Application in OPF Problem

The positive semidefinite constraint on a $2n \times 2n$ matrix ($\mathbf{W} \succeq 0$ in the primal form, $\mathbf{A} \succeq 0$ in the dual form) challenges the computational tractability of the SDP relaxation. To improve computational speed, a matrix completion theorem [178] enables the decomposition of this single positive semidefinite constraint into constraints on many smaller submatrices [32,179]. Our method leverages this decomposition. This section summarizes the matrix completion decomposition.

4.4.1 Relevant Graph Theory Definitions

In the beginning several definitions from graph theory are described. A “graph” $G = (V, E)$ is composed of a set of “vertices” or “nodes”, V , and a set of “edges” or “arcs”, E . The number of vertices, $n_O = |V|$, represents the “order” of graph and the number of edges, $n_S = |E|$, shows the “size” of graph.

The vertices v and w are said to be adjacent if an edge, $e = (v, w)$, connects them together. A graph $G = (V, E)$ is called “undirected” if the edge (v, w) is identical to the edge (w, v) .

A “walk” is a sequence of vertices and edges in a graph in which $e_k = (v_{k-1}, v_k)$ for $k = i + 1, \dots, j$. A walk with no repeated edges is called a “trail” and a trail with no repeated vertices, except the initial and final vertices, is called a “path”. The number of edges in a walk, trail or path is the “length” of them. A “cycle” is a closed path with the minimum length of one.

Figure 4.1: Example of the One-line Diagram of a Four-bus Power Network (Black Lines) and a Chordal Extension (Black Lines plus the Dashed Red Line).

A “clique” is a set of nodes which are adjacent to all other nodes in the clique. A clique that is not contained in a larger clique is a “maximal clique”. An edge that connects two non-adjacent vertices in a cycle is called a “chord”. A graph is “chordal” if every cycle with the length of four or more nodes has a chord. We use $\mathcal{S}$ to denote the set of all maximal cliques and m to indicate the number of maximal cliques. For more details on graph theory refer to [180].

As a simple example, consider the one-line diagram of a four-bus system shown in Fig. 4.1. The network graph is not chordal since the cycle $\{1, 2, 3, 4\}$ does not have a chord. The graph formed by adding the dashed red line is chordal with two maximal cliques $\mathcal{C}_1 = \{1, 2, 3\}$ and $\mathcal{C}_2 = \{1, 3, 4\}$. Thus, the graph including the dashed red line is a “chordal extension”. See [32] and [180] for further details on these definitions.

4.4.2 Matrix Completion Decomposition

The matrix completion theorem in [178] considers a partially specified symmetric matrix with an associated undirected chordal graph. For our

purposes, this graph is (a chordal extension of) the power system network and the unspecified matrix entries correspond to non-adjacent pairs of buses. The theorem states that the partially specified matrix can be “completed” to be positive semidefinite (i.e., there exist values for the unspecified entries such that the matrix is positive semidefinite) if and only if all the submatrices corresponding to the graph’s maximal cliques are positive semidefinite.

For non-chordal power system networks, the matrix completion decomposition is applied by forming a chordal extension of the network graph. An appropriate chordal extension is constructed using the sparsity pattern of a Cholesky factorization of the network’s adjacency matrix [32,179]. A minimum degree ordering reduces the number of additional edges required in the chordal extension [181]. Further computational improvements are possible via heuristics for constructing alternative chordal extensions [33].

Using the matrix completion theorem, the positive semidefinite constraint is equivalently replaced by constraints on smaller submatrices associated with the maximal cliques of the chordal extension of the network graph. For the system in Fig. 4.1, this implies that the constraint $\mathbf{W} \succeq 0$ in (4.11) can be replaced by the constraints on the two submatrices whose rows and columns are associated with buses in the maximal cliques $\mathcal{C}_1$ and $\mathcal{C}_2$.

4.4.3 Primal and Dual Linking Constraints

Due to the possibility of non-empty intersections among maximal cliques, note that different submatrices may contain elements which refer to the same element in the original $2n \times 2n$ matrix $\mathbf{W}$. “Linking constraints” ensure consistency among variables contained in the submatrices associated with

multiple cliques [32,179].

As an illustrative example, again consider the chordal extension of the four-bus system in Fig. 4.1. The matrix completion theorem enables the decomposition of the 8×8 positive semidefinite matrix constraint (4.11) into two constraints on the 6×6 submatrices, denoted $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$, that are associated with the cliques $\mathcal{C}_1$ and $\mathcal{C}_2$. For notational simplicity, we next show the upper-left 3×3 diagonal blocks and corresponding linking constraints for these submatrices. The remaining entries are treated analogously.

$$\mathbf{W}^{(1)} = \begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} & \cdots \\ w_{12}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} & \cdots \\ w_{13}^{(1)} & w_{23}^{(1)} & w_{33}^{(1)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \succeq 0, \quad \mathbf{W}^{(2)} = \begin{bmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} & \cdots \\ w_{12}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} & \cdots \\ w_{13}^{(2)} & w_{23}^{(2)} & w_{33}^{(2)} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \succeq 0$$

Since buses 1 and 3 are the first and third elements in $\mathcal{C}_1$ and the first and second elements in $\mathcal{C}_2$, the linking constraints are

$$w_{11}^{(1)} = w_{11}^{(2)}, \quad w_{13}^{(1)} = w_{12}^{(2)}, \quad w_{33}^{(1)} = w_{22}^{(2)}, \quad \dots$$

Applying this decomposition to the matrix $\mathbf{W}$ in (4.11) significantly improves the computational tractability of the SDP relaxation [32]. Taking the dual of the resulting SDP yields a chordal-sparsity-exploiting version of (4.13), which is the formulation used by our proposed method. We hereafter denote the submatrices in the sparsity-exploiting version of (4.13) as $\mathbf{A}_i$, $i = 1, \dots, m$, each corresponding to a maximal clique contained in the set $\mathcal{S} = \{\mathcal{C}_1, \dots, \mathcal{C}_m\}$. Note that the sparsity-exploiting version of (4.13) includes dual variables in $\mathbf{A}_i$ that correspond to the linking constraints in

the primal version.

It is also required to implement the effect of primal linking constraints (4.4.3) on the dual problem. Let $\mathcal{T}_{tu}$ as the set of all linking constraints between any two maximal cliques $\mathcal{C}_t$ and $\mathcal{C}_u$, and $\varphi_{ij}^{(tu)}$ as the Lagrange multipliers for the linking constraint $(i, j) \in \mathcal{T}_{tu}$. The dual modified submatrices $\tilde{\mathbf{A}}_t$ and $\tilde{\mathbf{A}}_u$ are then

$$\tilde{\mathbf{A}}_t = \mathbf{A}_t + \sum_{(i,j) \in \mathcal{T}_{tu}} \left\{ \varphi_{ij}^{(tu)} \mathbf{Z}_{ij}^{(t)} \right\}, \quad (4.21a)$$

$$\tilde{\mathbf{A}}_u = \mathbf{A}_u - \sum_{(i,j) \in \mathcal{T}_{tu}} \left\{ \varphi_{ij}^{(tu)} \mathbf{Z}_{ij}^{(u)} \right\}, \quad (4.21b)$$

where $\mathbf{Z}_{ij}^{(t)}$, with the same size of $\mathbf{A}_t$, is corresponding submatrix of $\mathbf{Z}_{ij}$ for maximal clique $\mathcal{C}_t$, where $\mathbf{Z}_{ij}$ is a $2n \times 2n$ symmetric matrix

$$\mathbf{Z}_{ij} = \frac{1}{2} \left(\hat{e}_i \hat{e}_j^\top + \hat{e}_j \hat{e}_i^\top \right), \quad (4.22)$$

where $\hat{e}_k$ is the k^{th} standard basis vector in $\mathbb{R}^{2n}$. Note that there is no change to the dual objective function ρ (4.14). For further details, see [29,32,33].

4.5 Proposed Method

Despite the computational improvements from the matrix completion decomposition, sparsity-exploiting SDP relaxations are slower than many local solution algorithms for typical OPF problems. This section describes the proposed method for leveraging the speed of local solution algorithms to more quickly compute an objective value bound using the SDP relaxation. We present a high-level summary, detail each step, and discuss several features of the proposed method.

4.5.1 Description of the Proposed Method

The proposed method is based on the fact that the objective value associated with any dual feasible point for the SDP relaxation (4.13) bounds the objective of the primal SDP relaxation (4.12) and therefore also bounds the objective of the original OPF problem (4.10). Rather than perform the computationally expensive calculation of the best achievable bound via application of an SDP solver to (4.13), we instead use certain information from a local solution to form a simplified SDP problem. Applying an SDP solver to this simplified problem can quickly provide a dual feasible point whose corresponding objective value is a bound on the objective of the original OPF problem (4.10). The trade-off inherent to this computational improvement is potential suboptimality in the obtained bound.

To form this simplified problem, we fix certain dual variables in the SDP relaxation (4.13) to their corresponding values from a specified local solution, leaving only a subset of the dual variables to vary. The fixed dual variables are identified using a heuristic that computes “problematic” submatrices of the matrix $\mathbf{A}$ in (4.13) evaluated using the dual variable values from the local solution. Consider the submatrices $\mathbf{A}_i, i = 1, \dots, m$, resulting from the matrix completion decomposition. A submatrix is deemed “problematic” if it has any eigenvalues that are either negative or small positive numbers. Dual variables associated with “problematic” submatrices are allowed to vary, while the other dual variables in (4.13) are fixed to their values from the local solution.

Algorithm 4.1 summarizes the proposed method. Each step is described in further detail below.

Algorithm 4.1 Quickly Bounding the Objective Value

Input: Dual variables from a local OPF solution.

Output: A bound for the OPF's optimal objective value.

- 1: Convert the local solution's dual variables to be consistent with the OPF formulation (4.10) using [20].
 - 2: Obtain the matrix $\hat{\mathbf{A}}$ matrix by substituting the dual variable values from the local solution into (4.15).
 - 3: Compute the matrix completion decomposition, i.e., the submatrices $\mathbf{A}_i$ for all $i \in \mathcal{S}$ using the approach in [32,33]. Substitute in the values of the dual variables from the local solution to form $\hat{\mathbf{A}}_i$.
 - 4: Identify the set of problematic submatrices, $\mathcal{S}_p$.
 - 5: Create the sets of all buses, $\mathcal{N}_p$, and lines, $\mathcal{L}_p$, included in $\mathcal{S}_p$.
 - 6: Identify the dual variables that are fixed and those that are allowed to vary based on $\mathcal{N}_p$ and $\mathcal{L}_p$.
 - 7: Formulate and solve the simplified SDP problem (4.23). Return the objective value as a bound on the original OPF problem (4.10).
-

Input: Our method takes as input the values of the dual variables from a local solution to the OPF problem (4.10). These values can be obtained using any primal/dual local OPF solver.

Note that solutions to equivalent but differently formulated OPF problems can have different values for the dual variables. For instance, the OPF formulation in MATPOWER [14] limits the voltage magnitudes rather than the squared voltage magnitudes as in (4.10d). If the local solution is computed for a different OPF formulation than (4.10), use conversions such as those in [20] to obtain dual variables that are consistent with (4.10).

1) Evaluate the dual matrix: Substitute the values of the dual variables into the $\mathbf{A}$ matrix defined in (4.15). Denote the resulting matrix as $\hat{\mathbf{A}}$.

2) Construct the matrix completion decomposition: As summarized in Section 4.4, use the approach in [32,33] to compute the maximal cliques of a chordal extension of the network graph and form the corre-

sponding submatrices of $\hat{\mathbf{A}}$, i.e., $\hat{\mathbf{A}}_1, \dots, \hat{\mathbf{A}}_m$. When constructing $\hat{\mathbf{A}}_i$, the dual variables corresponding to the linking constraints are set to zero.

3) Identify problematic submatrices: Use the following heuristic to identify a set of “problematic” cliques, which are denoted $\mathcal{S}_p$. Let $|\cdot|$ indicate the cardinality of a set. Let $\text{ceil}(\cdot)$ denote the ceiling function which returns the smallest integer greater than or equal to the argument. Let $\text{eig}(\cdot)$ return the eigenvalues of a matrix. Let σ be a specified scalar parameter that dictates the minimum percentage of submatrices that are identified as “problematic”.

First, identify as problematic all submatrices that are not positive semidefinite, i.e., if $\min(\text{eig}(\hat{\mathbf{A}}_i)) < 0$, include i in $\mathcal{S}_p$. Then, if $|\mathcal{S}_p| < \sigma \cdot |\mathcal{S}|$, list all the submatrices $\mathbf{A}_i$ in increasing order of their smallest eigenvalues. Select the first $\text{ceil}(\sigma \cdot |\mathcal{S}|)$ submatrices in this ordering and add them to $\mathcal{S}_p$. Thus, at least σ percent of the submatrices are identified as problematic. Note that $\sigma = 0$ does not necessarily imply that $\mathcal{S}_p$ is empty. Rather, $\sigma = 0$ indicates that we only identify the submatrices that are not positive semidefinite as problematic.

4) Identify the fixed and non-fixed dual variables: Identify the set $\mathcal{N}_p$ that contains all buses which are in problematic cliques $\mathcal{S}_p$. Similarly, let $\mathcal{L}_p$ denote the set of all lines for which either terminal bus is in a problematic clique $\mathcal{S}_p$. The dual variables associated with buses in $\mathcal{N}_p$ and lines in $\mathcal{L}_p$ are decision variables in the simplified SDP problem. Dual variables associated with the remaining buses and lines, i.e., $\mathcal{N} \setminus \mathcal{N}_p$ and $\mathcal{L} \setminus \mathcal{L}_p$, are fixed to their corresponding values from the local solution.

5) Formulate and solve the simplified SDP relaxation: Fixing

the dual variables for buses in $\mathcal{N} \setminus \mathcal{N}_p$ and lines in $\mathcal{L} \setminus \mathcal{L}_p$ yields the following simplified version of (4.13)

$$\max \quad \rho \tag{4.23a}$$

$$\text{subject to} \quad (\forall k \in \mathcal{N}_p, \forall (l, m) \in \mathcal{L}_p)$$

$$\tilde{\mathbf{A}}_i \succeq 0, \quad \forall i = 1, \dots, m \tag{4.23b}$$

$$\mathbf{H}_{lm} \succeq 0, \tag{4.23c}$$

$$\mathbf{R}_k \succeq 0, \quad \mathbf{R}_k^{11} = 1, \tag{4.23d}$$

$$\underline{\lambda}_k \geq 0, \bar{\lambda}_k \geq 0, \underline{\gamma}_k \geq 0, \bar{\gamma}_k \geq 0, \underline{\mu}_k \geq 0, \bar{\mu}_k \geq 0, \tag{4.23e}$$

$$\underline{\beta}_{lm} \geq 0, \bar{\beta}_{lm} \geq 0, \tag{4.23f}$$

where the submatrices in the sparsity-exploiting dual formulation are denoted as $\tilde{\mathbf{A}}_i$. These submatrices include constant terms which correspond to the values of the dual variables associated with the buses in $\mathcal{N} \setminus \mathcal{N}_p$ and lines in $\mathcal{L} \setminus \mathcal{L}_p$, i.e., the fixed dual variable values from the local solution. Furthermore, all matrices $\tilde{\mathbf{A}}_i$, $i = 1, \dots, m$, are functions of the dual variables corresponding to the linking constraints, as discussed in Section 4.4 and [29,32,33]. The dual variables corresponding to the linking constraints are decision variables in (4.23) whose values are not provided by the local solution. Thus, (4.23b) considers the submatrices $\tilde{\mathbf{A}}_i$ corresponding to *all* maximal cliques, $\mathcal{S}$, rather than just those corresponding to the problematic cliques $\mathcal{S}_p$.

A solution to (4.23) is a dual feasible point for the (sparsity-exploiting) SDP relaxation (4.13). Thus, the objective value for the solution to (4.23) is a lower bound on the objective value of the original OPF problem (4.10).

4.5.2 Discussion

With many variables fixed to their corresponding values from the local solution, the SDP problem (4.23) is easier to solve than the original SDP relaxation (4.13), which results in a computational speed advantage for the proposed method. This advantage comes at the cost of reduced tightness since the dual problem has fewer degrees of freedom with which to improve the objective ρ . The value of σ controls this trade-off between the solution speed and bound tightness. The tightest lower bound occurs for $\sigma = 100\%$, which yields the (sparsity-exploiting) complete dual SDP relaxation (4.13), i.e., all dual variables are allowed to vary. Conversely, $\sigma = 0\%$ fixes the largest number of dual variables in (4.23) and thus yields the fastest computational speed at the cost of a weaker bound.

The proposed method performs well with small values of σ when the local solution is close to the SDP solution. To numerically evaluate this claim, we compute the two-norm $\|\cdot\|_2$ of the ratio between the dual variable values for the local solution and the SDP solution

$$\text{Dual Correspondence Ratio (\%)} = \left\| \frac{X_{local}^{Dual} - X_{SDP}^{Dual}}{X_{SDP}^{Dual}} \right\|_2 \times 100, \quad (4.24)$$

where X denotes the vector of the dual variables defined in Section 4.2.2. When the local solution is far from the SDP solution, fixing variables to the values from the local solution overly constrains (4.23), resulting in an inferior bound and potentially even infeasibility. Such cases can be addressed by increasing the value of σ .

The dual correspondence ratio in (4.24) provides an a-posteriori explanation of our method's performance. We also consider a heuristic for estimat-

ing the quality of our method’s bound without prior knowledge of the dual SDP solution. Specifically, we empirically find that our method performs well when more than 95% of the submatrices $\hat{\mathbf{A}}_i$ obtained using the local solution’s dual variables are positive semidefinite.

4.6 Numerical Experiments

A key advantage of the proposed method is its ability to control the trade-off between computational speed and tightness of the objective value bound. This trade-off is determined by how well the dual variables from the local solution match the dual variables for the SDP relaxation’s solution. The closeness of these solutions is problem dependent. Thus, evaluating the effectiveness of the proposed method requires numerical experimentation. After providing our implementation details and evaluation methodology, this section applies the proposed method to a variety of large test cases. The results indicate that the proposed method is a valuable “middle ground” in that it is often tighter but slower than the SOCP and QC relaxations while weaker but faster than the SDP relaxation.

4.6.1 Implementation Details and Evaluation Methodology

Algorithm 4.1 is implemented using MATLAB R2016a and YALMIP [182] and solved using MOSEK 8 on a computer with a “64-bit Intel Quad Core 3.40 GHz CPU” with “8 GB of RAM”. The local solutions are obtained using the interior point method in MATPOWER [14]. The numerical results include comparisons to the SOCP relaxation [122], the QC relaxation [124]

augmented with the “Lifted Nonlinear Cuts” from [153], and the sparsity-exploiting SDP relaxation [32,33]. For our experiments, we consider the large test cases (≥ 118 buses) in the NESTA 0.6.0 archive [183].

We evaluate bound tightness via the *optimality gap*, which compares the objective value from the local solution to the bound from a dual feasible point obtained using the proposed method or the bound from a relaxation

$$\text{Optimality Gap (\%)} = \frac{\text{Local Sol. Obj.} - \text{Obj. Value Bound}}{\text{Local Sol. Obj.}} \times 100.$$

4.6.2 Test Case Results

To illustrate the trade-off in computational speed and bound tightness, Fig. 4.2 shows how the solver times and optimality gaps vary with σ for selected test cases. For some test cases, such as “case3012wp_mp_api”, “case2736sp_mp_api”, and “case3375wp_mp”, the solver times decrease by approximately a factor of nine to ten as σ varies from 100% (equivalent to the sparse version of the SDP relaxation (4.13)) to 0% (only submatrices that are not positive semidefinite are identified as problematic). Varying σ has a much smaller impact on the solver times for other cases, such as “case2746wop_mp”, “case2383wp_mp”, and “case3120sp_mp_api”.

The improvement in speed with smaller σ has a trade-off with respect to the tightness of the objective value bound. As with the solver times, the impact of σ on the optimality gap is case dependent. For “case2746wop_mp”, “case3012wp_mp_api”, and “case2736sp_mp_api”, the optimality gaps decrease by approximately a factor of two to three as σ varies from 0% to 100%, while for “case3375wp_mp”, “case2383wp_mp”, and “case3120sp_mp_api”, the optimality gaps decrease by factors of 8, 20 and 200, respectively, as σ

Table 4.1: Optimality Gaps and Solver Times for $\sigma = 20\%$.

System Model	MATPOWER Obj. Value (\$/h)	Optimality Gap (%)				Solver Time (sec)			
		Our Method	SDP	QC	SOCP	Our Method	SDP	QC	SOCP
Typical Operating Condition (TOC)									
case118_ieee	3718.64	0.862	0.064	1.568	2.069	0.33	0.65	0.19	0.11
case300_ieee	16891.28	0.148	0.078	1.175	1.179	0.86	2.80	0.56	0.18
case1354_pegase	74069.35	0.107	0.010	0.077	0.081	4.46	13.58	2.31	0.52
case2383wp_mmp	1868511.83	6.643	0.375	1.039	1.049	58.48	181.81	6.60	2.27
case2736sp_mmp	1307883.13	0.000	0.000	0.288	0.297	50.00	277.30	6.29	1.29
case2737sop_mmp	777629.30	0.001	0.000	0.246	0.249	53.10	230.97	3.49	1.43
case2746wop_mmp	1208279.81	0.001	0.001	0.356	0.366	55.45	318.05	6.94	1.31
case2746wp_mmp	1631775.10	0.000	0.000	0.318	0.324	51.55	239.08	6.21	1.34
case2869_pegase	133999.29	0.085	0.008	0.088	0.091	13.02	41.13	4.52	1.82
case3012wp_mmp	2600842.77	0.850	0.143	1.002	1.021	74.01	362.15	6.97	1.27
case3120sp_mmp	2145739.43	0.153	0.088	0.536	0.545	95.78	435.99	6.52	1.34
case3375wp_mmp	7435697.54	0.372	0.073	14.907	14.916	103.92	449.06	8.76	1.74
Congested Operating Conditions (API)									
case118_ieee_api	10325.27	37.888	31.502	43.650	44.082	0.62	0.84	0.35	0.14
case300_ieee_api	22866.01	0.004	0.003	0.818	0.837	0.68	1.97	0.66	0.23
case2383wp_mmp_api	23499.48	0.547	0.101	1.118	1.120	97.02	194.25	3.80	1.15
case2736sp_mmp_api	25437.70	0.175	0.069	1.320	1.328	55.25	191.73	3.96	1.07
case2737sop_mmp_api	21192.40	0.318	0.007	1.049	1.056	41.77	196.74	3.94	1.13
case2746wop_mmp_api	22814.86	0.000	0.000	0.488	0.491	65.85	282.61	4.34	1.40
case2746wp_mmp_api	27291.58	0.000	0.000	0.569	0.491	53.99	199.81	4.02	1.15
case3012wp_mmp_api	27917.36	0.061	0.025	0.891	0.894	77.84	266.51	4.29	1.25
case3120sp_mmp_api	22874.98	15.416	0.542	3.023	3.029	104.09	458.98	5.76	1.75
case3375wp_mmp_api	48898.95	0.035	0.014	45.765	45.766	89.74	284.44	5.04	1.63
Small Angle Difference Conditions (SAD)									
case118_ieee_sad	4324.17	10.687	7.652	8.286	15.783	0.68	0.93	0.43	0.17
case2383wp_mmp_sad	1935308.26	22.467	1.259	2.967	4.464	82.19	224.38	6.66	1.73
case2736sp_mmp_sad	1337042.83	38.915	0.635	2.009	2.472	66.92	314.64	4.39	1.18
case2746wop_mmp_sad	1241955.37	20.126	0.999	2.482	3.067	78.31	458.02	5.25	1.13
case3012wp_mmp_sad	2635451.43	15.537	0.464	1.916	2.320	103.00	439.56	5.19	1.34

Figure 4.2: Solver Times (seconds) (●) and Optimality Gaps (%) (▲) vs. $\sigma(\%)$.

varies from 0% to 100%. These variations are explained by the quality of the local solution. When the local solution is close to the SDP solution, increasing σ only has a small impact on the bound.

In contrast to the solver times, which vary gradually with σ , the optimality gaps exhibit more significant variations. As an example, the optimality gap for “case2736sp_mp_api” does not change significantly from $\sigma = 0\%$ until a steep decrease at $\sigma = 60\%$. This is explained by a group of buses whose dual variable values from the local solution are far from their values in the SDP solution. Choosing $\sigma < 60\%$ fixes these dual variables to the values from the local solution, restricting our method’s ability to achieve a tight bound. Jumps in the optimality gaps for other test cases have similar explanations.

Table 4.1 shows results for representative test cases, specifically comparing the proposed method using $\sigma = 20\%$ with the SDP, QC, and SOCP relaxations. Three categories of NESTA test cases are considered: typical operating conditions (TOC), congested operation (API), and small angle difference (SAD).

Table 4.2 presents the average optimality gaps and solver times for each category of test cases. Our method performs well for most of the TOC and API test cases, as indicated by smaller optimality gaps than the QC and SOCP relaxations with a significant (approximately factor of five) improvement in computation speed relative to the SDP relaxation. The average results show that the proposed method yields better optimality gaps than QC and SOCP relaxations, while being significantly faster than the SDP relaxation, suggesting the method’s utility as a middle ground between the SDP relaxation and the SOCP and QC relaxations.

However, despite promising performance on many problems, there are test cases which challenge our method. For instance, our method yields large gaps relative to the SOCP, QC, and SDP relaxations for the SAD test cases,

Table 4.2: Average Optimality Gaps and Solver Times.

Approach	Average Optimality Gaps (%)			Average Solver Times (sec)		
	TOC	API	SAD	TOC	API	SAD
Our Method	0.77	1.84	21.55	46.9	65.6	66.2
SDP	0.07	0.08	2.20	212.7	230.8	287.5
QC	1.80	6.12	3.53	5.0	4.0	4.4
SOCP	1.85	6.11	5.62	1.2	1.2	1.1

Table 4.3: Dual Correspondence Ratios and Percentages of Positive Semidefinite Submatrices for Selected Test Cases.

System Model	Dual Correspondence Ratio (%)	Percentage of PSD Submatrices	Optimality Gaps (%)
case2736sp_mp	0.16	99.3	0.000
case2746wop_mp	0.45	98.4	0.001
case3375wp_mp__api	0.93	97.5	0.035
case3012wp_mp	1.32	97.3	0.850
case2869_pegase	4.32	96.7	0.085
case3375wp_mp	7.00	95.7	0.372
case2383wp_mp	28.91	78.0	6.643
case3120sp_mp__api	462.70	92.0	15.416
case2383wp_mp__sad	105.10	58.4	22.467
case2736sp_mp__sad	61.30	89.8	38.915
case3012wp_mp__sad	104.31	86.2	15.537

“case2383wp_mp”, and “case3120sp_mp__api”. For these cases, the local solution is not close to the SDP relaxation’s solution such that large values of σ (with correspondingly slow solution times) are necessary to achieve small optimality gaps. Explaining our method’s poor performance for these test cases, the dual correspondence ratios (4.24) presented in Table 4.3 have smaller values for the test cases where our method performs poorly relative to the test cases with tighter optimality gaps.

Finally, we demonstrate the a-priori heuristic proposed in Section 4.5.2

via selected test cases in Table 4.3. Our method performs well for the test cases where the percentages of positive semidefinite submatrices are greater than 95%. For these cases, our method yields optimality gaps between those from the SDP relaxation and the SOCP and QC relaxations with significant speed improvements compared to the SDP relaxation. Conversely, our method often performs poorly for test cases that do not satisfy this heuristic, such as “case2383wp_mp”, “case3120sp_mp_api”, and the SAD cases. This shows the appropriateness of our heuristic recommendation for applying our method to problems where the percentage of submatrices $\hat{\mathbf{A}}_i$ that are positive semidefinite is greater than 95%.

To quantify this observation, we compute the dual correspondence ratio (4.24) defined in section 4.5.2. For the test cases “case2746wop_mp”, “case3012wp_mp_api”, “case2736sp_mp_api”, and “case3375wp_mp”, this ratio is 0.45%, 1.32%, 2.46% and 7.00%, respectively, indicating that the local solutions’ dual variables are reasonably close to the dual variables from the SDP relaxations’ solutions, thus explaining the good performance for these cases (optimality gaps of 0.001%, 0.061%, 0.175%, and 0.372%, respectively, for $\sigma = 20\%$). Conversely, our method exhibits poor performance for test cases with large values of this ratio, e.g., 28.91% for “case2383wp_mp” and 462.70% for “case3120sp_mp_api” with optimality gaps of 6.643% and 15.416%, respectively, for $\sigma = 20\%$. This illustrates our method’s dependence on the closeness of the dual variables between the local solution and the SDP relaxation’s solution.

The dual correspondence ratio in (4.24) provides an a-posteriori explanation of our method’s performance. We next present an a-priori heuristic that is useful for estimating the quality of our method’s bound. Specifically, we

empirically find that our method performs well when more than 95% of the submatrices $\hat{\mathbf{A}}_i$ obtained using the local solution's dual variables are positive semidefinite. For instance, the percentages of positive semidefinite submatrices in test cases “case2736sp_mp”, “case2869_pegase”, “case3012wp_mp”, “case2736sp_mp_api”, and “case3375wp_mp_api” are 99.3%, 96.7%, 97.3%, 98.9% and 97.5%, respectively. Our method performs well for these test cases (for $\sigma = 20\%$, optimality gaps between those from the SDP relaxation and the SOCP and QC relaxations with significant speed improvements compared to the SDP relaxation). Conversely, our method performs poorly for “case2383wp_mp” and “case3120sp_mp_api”, for which only 78% and 92% of the matrices $\hat{\mathbf{A}}_i$ are positive semidefinite. We, therefore, recommend applying our method to problems where the percentage of submatrices $\hat{\mathbf{A}}_i$ that are positive semidefinite is greater than 95%.

4.7 Summary

This chapter has proposed an SDP-based method that leverages knowledge of a local solution to quickly compute objective value bounds for optimal power flow problems. Numerical experiments indicate that the proposed method is approximately four to five times faster than the SDP relaxation for a variety of test cases without excessive degradation in the quality of the objective bound. These experiments also show that the proposed method is slower than the SOCP and QC relaxations but can obtain tighter bounds. Thus, the proposed method provides a middle ground between the SDP relaxation and the SOCP and QC relaxations with a trade-off between computational speed and tightness.

Chapter 5

A Computationally Efficient Approach for Solving Receding-Horizon OPF Problem in Distribution Networks

5.1 Introduction

The optimal power flow (OPF) problem has emerged as a key tool for planning and operating energy networks. The conventional way of solving static OPF problem is unsuitable for power networks having renewable energy sources (RES), energy storage systems (ESS) and flexible loads (FL) due to the time-coupled and stochastic dynamics [184]. The multi-period OPF proposed to overcome this problem cannot tackle uncertainties owing

to the lack of feedback, i.e. measurements. The MPC optimization technique with its closed-loop nature provides significant benefits comparing to open-loop optimization problems in the sense of robustness to model uncertainties, network failures, etc. [185,186]. The receding horizon approach (or *Model Predictive Control*, *MPC*) that solves a multi-period OPF during each time-horizon taking measurements is fast emerging as a promising solution in power networks [187]. A major challenge in MPC is the computational complexity because a multi-period optimization model is solved during each time-horizon to make the decisions. Approaches such as convex relaxation [188], DC approximation [184], sub-optimal solutions using brute force optimization [48] or simplified assumptions are used to overcome this problem [189,190]. While the convex relaxation approaches are not suitable for distribution networks due to their high line resistance [191], the DCOPF approximation cannot model the line-losses. Consequently, computationally efficient approaches for solving alternating current OPF (ACOPF) problem without using convex relaxation or DC approximations, yet performing RHC based optimization are required for distribution networks.

In recent years, a significant number of research works has focused on implementation of MPC in power and energy grids. These efforts have mainly targeted two major scopes in power grids including:

1. Application of MPC for energy management and power allocation in power networks which focuses on the optimal performance of the network [48,192–194].
2. Application of MPC for voltage and frequency stability and control goals which aims a reliable and secure performance of the power grid [195–199].

From the viewpoint of MPC, the main difference between these two class of problem is the time-scale or horizon-time, where the time-scale of energy management problem is about 15 minutes to 1 hour while the time-scale of the stabilization problem is less than few seconds [200]. Hierarchical MPC approaches are also presented to handle both time-scales and solve a comprehensive control and management system [201], e.g. in [202] a hierarchical scheme is proposed to concurrently handle the electric vehicles (EVs) charging and frequency control for smart grids, or a three-level MPC scheme is presented in [203] for different time-scales covering power flow management at the transmission grid, power management at the distribution grid, and frequency control. A two-level MPC based voltage regulation scheme is also proposed in [204] to keep the voltages within the safe limits.

As seen in the previous chapter, solving the single-period OPF problem in large-scale power grids is still an ongoing challenge, so adding dynamic variables which leads to a multi-period OPF problem is more complicated and challenging to solve.

The computational complexity of multi-period OPF problem enforces the researchers to approximate, relax or simplify the multi-period OPF problem in a way that the obtained problem or proposed algorithm may still satisfy the required functional objectives of the power grid, e.g. the research efforts in [205,206] are focused on convex relaxation of dynamic OPF problem where the authors propose a two-step convex relaxation to solve an hourly day-ahead power distribution in radial networks. Linearization of the multi-period OPF problem and solving a multi-period DCOPF problem was one of the first implemented ideas to overcome this complexity, e.g. in [207,208] the authors solved the linearized problem using dynamic programming and

in [209] Lagrangian relaxation is implemented to solve the problem. A two-stage approach is also presented in [210] to reduce the computational effort of stochastic multi-period DCOPF problem in which the stages are linked together using Bender Cuts.

The lower computational complexity of the linearized multi-period DCOPF is at the cost of losing the decision variables of reactive power and voltage for the real network. Thus, the request of this information enforces the engineers to solve the multi-period ACOPF which is highly nonlinear [211–213]. While most of the literature on multi-period OPF problem benefits from convex approximations or relaxations, the authors in [214] implement the MPC for solving the multi-period ACOPF problem considering renewable resources. Going beyond [214], the authors in [200] proposes an economic nonlinear MPC scheme to solve the OPF problem including storage desires besides the renewable resources.

Some recent works have focused on the application of multi-period OPF via MPC technique. The implementation of multi-period OPF formulation in power dispatch problem has been reviewed in [215]. The application of MPC approach in multi-energy systems, including electrical and thermal energy, is presented in [216,217]. One recent research [218] investigates the application of receding horizon control for storage integration management in high voltage networks. In [219] a receding horizon control based technique is utilized to solve a multi-period OPF, aiming to schedule the voltage controllers and recover the unstable transmission voltages. Authors in [220] proposed implementing MPC scheme to prevent violating ramp rate constraints between discrete time periods of the scheduling horizon. A novel MPC technique is proposed in [189,190] to lessen the severe line-overload

fluctuation consequences in an electric power grid. Energy storage dispatch problem in low voltage networks has been solved using deterministic and stochastic MPC in [221] considering the demand forecasts.

In [222] a hierarchical distributed MPC-based algorithm is proposed for energy management of microgrids with energy storage integration and in real-time pricing signal condition. A hierarchical distributed alternating direction method of multipliers (ADMM) for MPC with the application in power grids also is presented in [223]. The authors in [224] proposed a distributed receding horizon control (D-RHC) technique and an adaptive receding horizon control (A-RHC) scheme to handle the power flow within the distribution grid in presence of storage devices and solar photovoltaic (PV) panels, aiming to reduce the peak demands and reverse power flow while increasing the operational savings for customers.

Recently some other research works have been done in this field. An RHC-based approach has been applied to solve the multi-period ACOPF problem where a network state estimator is designed to close the RHC loop [225]. Owing to the stochastic behavior of renewable resources, researchers have been focused on the implementation of probabilistic MPC approaches in multi-period OPF problems [218,226,227]. Compared to grid-connected networks, the isolated grids are more sensitive to any load demand or renewable generation fluctuations so the authors in [228] proposed using an MPC based energy management for these distribution grids. By clustering the generation units of a wind farm and implementing MPC approach, a power distribution optimization scheme with the possibility of power tracking within a wind farm is proposed in [229].

In contrast to the existing literature, this chapter aims to optimize both

the operating cost and line-losses by embedding forecast within RHC based ACOPF using a two-level optimization process. The first stage contracts the storage and flexible loads in a receding horizon manner without considering the line-flow constraints; the second stage solves a single-period ACOPF problem to reduce both line-losses and operating cost by fixing the storage and flexible loads to the values computed in the first stage. Additionally, the first stage solution provides a hot-start to the second stage optimization. This decomposition introduces significant computation benefits on one hand and retains the desirable features of predictive control methodology, i.e. embedding forecast information within optimization driven decisions and making it more robust to fluctuations.

The proposed controlling solution is applied to a simulator of the Steinkjer distribution network, Norway. Real measurements obtained from the network using supervisory control and data acquisition (SCADA) systems are used as input to the controller. The ACOPF combined with SE is modeled in Julia [230], a high-performance and open source programming language for technical computing. The language is designed to exploit the advantages of the “Just In Time” compiler to generate software and algorithms fundamental to the field of operations research, with a focus on mathematical optimization.

5.2 OPF Problem Modeling Issues in Distribution Networks

A physical electrical network model is a building block of the RHC approach. This section proposes a mathematical model for different compo-

nents of the network.

5.2.1 Power network model

The power network could be modeled by an undirected graph $\mathbf{G} = (\mathcal{N}, \mathcal{L})$. The set of nodes describes the set of network substations (or nodes) indexed by $i \in \{1, 2, \dots, N\}$ with $N = |\mathcal{N}|$, while the set of edges describes the directed transmission lines $(i, j) \in \mathcal{L}$, with $E = |\mathcal{L}|$. $\mathcal{N}_i$ denotes the set of the neighbors of node i , i.e, connected to the node i through a connection line. Further, the index $i = 1$ refers the reference bus (slack bus). Note that in this chapter, owing to the time variable network parameters, the powers, voltages and etc. are represented in small letters.

The network transmission lines are modeled with a common branch model [231], consisting of a standard Π equivalent circuit, Fig. 2.1, with series admittance $y_{ij} = \frac{1}{r_{ij} + jx_{ij}} = g_{ij} + jb_{ij}$.

By considering the polar representation of the complex voltage phasors: $v_i = |v_i|e^{j\theta_i} = |v_i|\angle\theta_i$, where $|v_i|$ and θ_i are the magnitude and phase angle of phasor v_i , the active and reactive power flow equations due to the AC power flow model for the line $(i, j) \in \mathcal{L}$ are,

$$p_{ij} = v_i^2 g_{ij} - |v_i||v_j| \left(g_{ij} \cos(\theta_{ij}) + b_{ij} \sin(\theta_{ij}) \right), \quad (5.1a)$$

$$q_{ij} = -v_i^2 b_{ij} - |v_i||v_j| \left(g_{ij} \sin(\theta_{ij}) - b_{ij} \cos(\theta_{ij}) \right), \quad (5.1b)$$

where $\theta_{ij} = \theta_i - \theta_j$ denotes the phase angle difference between bus i and bus

j . The complex power loss on the transmission line $(i, j) \in \mathcal{L}$ is then

$$p_{(i,j)}^{\text{loss}} = (v_i^2 + v_j^2) g_{ij} - 2|v_i||v_j|g_{ij} \cos(\theta_{ij}), \quad (5.2a)$$

$$q_{(i,j)}^{\text{loss}} = -(v_i^2 + v_j^2) b_{ij} + 2|v_i||v_j|b_{ij} \cos(\theta_{ij}). \quad (5.2b)$$

The constraint ensuring secure line-flows (constraints due to thermal capacity etc.) is

$$p_{ij}^2 + q_{ij}^2 \leq (s_{ij}^{\max})^2 \quad \forall (i, j) \in \mathcal{L}. \quad (5.3)$$

The active and reactive power balance considering line-flows is given by

$$p_{Gi} - \sum_{j \in \mathcal{N}_i} p_{ij} = p_{Di} \quad \forall i \in \mathcal{N}, \quad (5.4a)$$

$$q_{Gi} - \sum_{j \in \mathcal{N}_i} q_{ij} = q_{Di} \quad \forall i \in \mathcal{N}, \quad (5.4b)$$

where p_{Di} and q_{Di} are the measured (or predicted) active and reactive power load after the power cut as can be seen in Section 5.2.4.

5.2.2 Buses

Generally, buses are classified as PV, PQ, and slack based on the connection of generators, loads, and utility grid, respectively. Independent from the type of bus, it is necessary to verify that none of the voltage magnitude limits are violated

$$|v_i|^{\min} \leq |v_i| \leq |v_i|^{\max} \quad \forall i \in \mathcal{N}. \quad (5.5)$$

The voltage at the secondary substation can be controlled by using the so called on-load tap-changing (OLTC) mechanism, that consists in manually

or automatically selecting the tap at the power transformer. However, for the PV and slack bus it is required to keep its voltage to a known target value

$$|v_i|^{min} = |v_i| = |v_i|^{max} = \text{constant}. \quad (5.6)$$

The voltage phase angle limits for each bus also need to be considered as follows

$$\theta_i^{min} \leq \theta_i \leq \theta_i^{max}. \quad (5.7)$$

For convenience the slack bus plays the role of a phase angle reference for the remaining sinusoidal waveforms associated with each bus, therefore it is seen as a controlled source given by

$$\theta_1^{min} = \theta_1^{max} = 0. \quad (5.8)$$

The slack bus is usually chosen among those generating buses with the largest capacity and performs frequency regulation.

5.2.3 Generators

In a distribution network a certain number of generation sources must be present in order to compensate the total amount of power required by the loads, and possibly sell extra energy to the external grids through the “point of common coupling” (PCC) bus. Traditionally, the generation sources are non-renewable generators that are controllable sources and can be switched on or off and modulated in order to balance the loads demand.

The generator unit is modeled as a manipulable complex power injection

at a specific PV or slack bus, i.e.,

$$s_{Gi} = p_{Gi} + jq_{Gi}, \quad (5.9)$$

subject to the following lower and upper bounds:

$$\begin{aligned} p_{Gi}^{min} &\leq p_{Gi} \leq p_{Gi}^{max}, \\ q_{Gi}^{min} &\leq q_{Gi} \leq q_{Gi}^{max}. \end{aligned} \quad (5.10)$$

Note that for PQ buses $s_{Gi} = 0$.

In modern power networks, electrical power injection can be supplied also by RES; generators and RES can work in parallel to meet the load power demand. Unlike the non-renewable generators, RESs are uncontrollable and their own power injections will not be considered as optimization variables. In other words, a renewable energy source can be described as a non-dispatchable distributed generator efficiently predictable using several mathematical forecasting techniques [232]. Consequently, they are modeled by a given active and reactive power values over a time horizon delivered at a specific bus i , and they give a positive contribution to the power balance equations

$$s_{Ri} = p_{Ri} + jq_{Ri}. \quad (5.11)$$

Note that if there is not RES at bus i , then the forecasted energy is $s_{Ri} = 0$.

5.2.4 Load Demands

The complex power consumption at the PQ bus is modeled to be a negative power source. Therefore, for each PQ bus we have:

$$s_{Di}(t) = p_{Di}(t) + jq_{Di}(t). \quad (5.12)$$

Considering that the time axis is discretized uniformly, usually with ΔT , loads whose power consumption over the time can be cut are called *flexible*. Their control can help the power network during stressful and abnormal situations. For example, some of the loads could be able to be cut to match the power generation in the most economic way, namely to alleviate the power generation peaks during intervals of high power demand. The parameter $\delta_i \in [0, 1]$, negotiated with the power consumers in a preliminary phase, is introduced to model the maximum percentage of load that can be cut at the bus i for a time interval. Therefore, at all time t

$$\bar{p}_{Di}(t)(1 - \delta_i(t)) \leq p_{Di}(t) \leq \bar{p}_{Di}(t), \quad (5.13)$$

where $p_{Di}(t)$ is the actual dispatched load while $\bar{p}_{Di}(t)$ refers to the predicted maximum amount of load. Note that for the non-flexible bus i , $\delta_i(t) = 0$.

The flexibility cost associated with each load is assumed to be a linear function of the cut, i.e.,

$$C_i^d(t) = c_i^d(\bar{p}_{Di}(t) - p_{Di}(t)). \quad (5.14)$$

5.2.5 Storage Dynamics

In a distribution power network, energy storage systems are used essentially for *i*) mitigating the intermittent renewable energy resource; *ii*) avoiding peaks of generation when the cost is high or during congestions.

The ESS at the generic substation $i \in \mathcal{N}$ is modeled by the equations

$$\begin{aligned} b_i(t+1) &= b_i(t) - \Delta T r_i(t) - b_i^{\text{loss}}, \\ |r_i(t)| &\leq r_i^{\text{max}}, \\ 0 < B_i^{\text{min}} &\leq b_i(t) \leq B_i^{\text{max}}. \end{aligned} \tag{5.15}$$

Here, $b(t)$ denotes the stored energy level, the State of Charge (SoC) at time t , with $b(0)$ being the initial storage capacity. The power exchanged with the network at time t , $r(t)$ can either be negative (charging phase) or positive (discharging phase) and the b^{loss} term denotes a constant SoC decrease during the sampling interval. ESS is considered to be a load during charging phases and a generator during discharging phases (5.4). The lower bound B_i^{min} can be strictly greater than zero for technological reasons or to overcome unexpected network faults.

The cost for the storage units is assumed linearly dependent on the SoC [233], i.e.

$$C_i^b(t) = c_i^b b_i(t). \tag{5.16}$$

Coefficient costs of different ESS technologies are provided in [234].

5.2.6 Interaction with the Utility Grid

The distribution network has external connections at the Grid Supply Point (GSP) substation also called PCC. It can import/export power

from/to the utility depending on the grid conditions. The prices of this power are provided by the grid operator a day-ahead. The grid operator may add an additional cost for meeting the grid infrastructure. This feature of the distribution network is modeled using a piecewise linear cost function

$$C^{\text{DAM}}(t) = C^{\text{MC}}(t) \min(p^{\text{contr}(t)}, p_{G1}(t)) + C^p(t) \max(0, p_{G1}(t) - p^{\text{contr}(t)}), \quad (5.17)$$

where $C^{\text{DAM}}(t)$, $C^{\text{MC}}(t)$, and $C^p(t)$ denote the total day-ahead market cost, unitary market clearing prices and cost during peak-periods, while $p^{\text{contr}}(t)$ and $p_{G1}(t)$ are the power contracted with the utility and actual power purchased/sold from/to the utility through the PCC. To account for the bi-directional power flows (power bought/sold from/to utility grid), a negative lower bound $p_{G1}^{\min} \leq 0$ is employed in (5.10).

5.3 Proposed Method

The RHC based ACOPF problem aims at reducing the operating cost and line-losses considering the underlying physical and operating conditions. To account for the fluctuations in demand and renewable generation, it uses the forecast tools (a detailed review of the forecast tools used in this chapter are presented in [232]).

In general, the OPF problem is difficult to solve, since the power flow constraints are both non-linear and non-convex. The presence of multiple objectives of operating costs and line-losses complicates the problem further. There are in general three ways to deal with this non-convexity: relaxing the non-convex power flow constraints, linearizing the power flow constraints or

solving the DCOPF problem and looking for a local optimum of the OPF problem.

As stated earlier, relaxation methods [184,187,205] or DC network model [189,190,235] have been studied by most authors in the literature. Convex relaxation of the ACOPF problem relies on certain assumptions on network parameters such as low x/r , the ratio of line resistance to reactance as in distribution networks. Similarly, the DC network model cannot capture the line-losses. Therefore, when it is desired to optimize both line-losses and operating cost in distribution networks, both the relaxations and approximations fail. Therefore, new techniques to solve the OPF problem is required.

The main objective of this chapter is to propose a computationally efficient solution technique that finds a local solution efficiently for the RHC based multi-objective ACOPF problem without using relaxation or DCOPF problem assumptions. Furthermore, the method should preserve the advantages of the RHC method while reducing the computation complexity.

The main disadvantage of the heuristic/local search methods for finding local optimum is their long computation time, the most time-consuming part being the satisfaction of the nonlinear line-flow constraints. Considering that the RHC based ACOPF problem is solved using commercial solvers, the nonlinear power flow constraints are strict equality constraints and satisfying them is usually a hard task. Thus, the convergence to the optimum slows down. Another cause of slowing down is the multi-period nature of the RHC-based ACOPF problem. The time-coupling of storage, renewable and flexible loads make the problem overwhelming. To remedy this problem, a new two-stage solution technique is here proposed.

Figure 5.1: Two level optimization problem.

The main idea is to decouple electric components characterized by dynamical behaviors, such as storage and flexible loads, from nonlinear line-flow constraints. This means deciding the commands to dynamical components without solving the line-flow constraints. As a consequence, only the operating costs are optimized and line-losses cannot be accounted for. Then, a static optimization problem or single-stage ACOPF problem—now involv-

ing both line constraints and operating costs—can suffice to provide a local optimal value of the RHC-based ACOPF. The idea behind the proposed approach is illustrated in Fig. 5.1.

5.3.1 First Stage

The DCOPF problem employs forecasts on renewable generation and demand. The DC model assumes that:

- the difference between adjacent bus voltage phase angles are small, such that $\sin \theta_{ij} \approx \theta_{ij}$ and $\cos \theta_{ij} \approx 1$;
- the bus voltage profile is considered flat and all voltages are put to $|v_i| = 1\text{p.u.}$;
- all branch resistances are approximately zero, that is the power system is assumed to be lossless;
- the reactive power flow is neglected.

One can verify that the above assumptions are not always valid for distribution networks.

The DC assumptions lead to the simplified power balance equation

$$p_{Gi}(t) - \sum_{j \in \mathcal{N}_i} b_{ij} \theta_{ij}(t) = p_{Di}(t) \quad \forall i \in \mathcal{N}, \quad (5.18)$$

where b_{ij} is the susceptance of the distribution node (i, j) .

According to the proposed model, the power dispatch, with the integration of the energy storage dynamics, is formulated as a finite-horizon optimal control problem of dynamic power allocation over a prediction horizon and is implemented using an RHC framework.

Therefore, the RHC DCOPF output is an optimal operational schedule over a time horizon T consisting of decisions on how the flexibilities of generators, storage and loads can be exploited to reach the power balance while minimizing costs. The control variables for the generic horizon $[\hat{t}, \hat{t} + T]$ are denoted as

$$z_t^{\hat{t}} = \left(p_G(\hat{t} + t), p_D(\hat{t} + t), r(\hat{t} + t) \right) \quad \forall t \in [0, T - 1], \quad (5.19)$$

where p_D is the vector of active power load demands for the flexible buses and r denotes the vector of the power exchanged with the ESS. Given the equation (5.19), all of other quantities at time t , such as power flows, currents and SoC can be determined, formulating the power flow equations as constraints.

The cost function to be optimized over the horizon $[\hat{t}, \hat{t} + T]$ is the economic objective:

$$J_{DC}^{\hat{t}}(z) := \sum_{t=\hat{t}}^{\hat{t}+T} f^E(t). \quad (5.20)$$

In this case, the cost function is time varying and the economic terms take into account also the contribution of the flexible loads and the power stored in the ESS at each time, i.e.,

$$f^E(t) = s(t) + \sum_{i=2}^N (C_i^g(t) + C_i^b(t) + C_i^d(t)), \quad (5.21)$$

where $s(t)$ is an epigraph variable introduced to obtain a standard LP problem, (see Fig. 5.2).

Figure 5.2: Piecewise Linear Cost Function, C^{DAM} .

Therefore, the RHC based DCOPF can be written as

$$\min_{z_t^{\hat{t}}} J_{DC}^{\hat{t}}(z) \quad (5.22a)$$

s. t.

$$p_{Gi}(t) - \sum_{j \in \mathcal{N}_i} b_{ij} \theta_{ij}(t) = p_{Di}(t), \quad (5.22b)$$

$$b_i(t+1) = b_i(t) - \Delta T r_i(t) - b_i^{\text{loss}}, \quad (5.22c)$$

$$|r_i(t)| \leq r_i^{\text{max}}, \quad (5.22d)$$

$$0 \leq B_i^{\text{min}} \leq b_i(t) \leq B_i^{\text{max}}, \quad (5.22e)$$

$$p_{Gi}^{\text{min}} \leq p_{Gi}(t) \leq p_{Gi}^{\text{max}}, \quad (5.22f)$$

$$\theta_i^{\text{min}} \leq \theta_i(t) \leq \theta_i^{\text{max}}, \quad (5.22g)$$

$$s(t) \geq C^{\text{MC}}(t) p_{G1}(t), \quad (5.22h)$$

$$s(t) \geq C^p(t) p_{G1}(t) + (C^{\text{MC}}(t) - C^p(t)) p^{\text{contr}}(t), \quad (5.22i)$$

$$b_i(\hat{t}) = \hat{b}_i. \quad (5.22j)$$

The inputs to the RHC based DCOPF problem are forecast information on renewable generation and demand, current network measurements from

the SCADA system, and operational constraints of the network components. The outputs of the first stage are storage rate and flexible load values, and generator schedules. The solution of (5.22) computed in a fast and easy way can be seen as an initial guess for the second stage ACOPF problem.

5.3.2 Second Stage

The second stage is given a hot-start by pivoting the storage and flexible loads at values computed in the first stage and then passing the other decision variables as an initial guess to the second stage. The partial pivoting and hot-start provides significant computation benefits as no dynamical equations are to be solved with non-linear power flow constraints. The absence of dynamical components also means that a time-coupled problem need not be solved, just a single stage optimization works. The second stage aims to find the optimal value of the variables

$$w(\hat{t}) = \left(p_G(\hat{t}), q_G(\hat{t}), v(\hat{t}), \theta(\hat{t}) \right)$$

that minimizes the multi-objective cost function:

$$J_{AC}^{\hat{t}}(w) := \alpha f^E(\hat{t}) + \beta f^L(\hat{t}) \quad (5.23)$$

where

$$f^L(t) = \sum_{(i,j) \in \mathcal{E}} p_{(i,j)}^{\text{loss}}(t). \quad (5.24)$$

The solution to the second-stage problem optimizes a combination of the line-losses and the operating cost. Therefore, the ACOPF can be written as

$$\min_w J_{AC}^t \quad (5.25a)$$

$$\text{s. t.} \quad (5.25b)$$

$$p_{ij} = v_i^2(t)g_{ij} - |v_i(t)||v_j(t)|\left(g_{ij} \cos(\theta_{ij}(t)) + b_{ij} \sin(\theta_{ij}(t))\right), \quad (5.25c)$$

$$q_{ij} = -v_i^2(t)b_{ij} - |v_i(t)||v_j(t)|\left(g_{ij} \sin(\theta_{ij}(t)) - b_{ij} \cos(\theta_{ij}(t))\right), \quad (5.25d)$$

$$p_{Gi}(t) + p_{Ri}(t) - \sum_{j \in \mathcal{N}_i} p_{ij}(t) + r_i(t) = p_{Di}(t), \quad (5.25e)$$

$$q_{Gi}(t) + q_{Ri}(t) - \sum_{j \in \mathcal{N}_i} q_{ij}(t) = q_{Di}(t), \quad (5.25f)$$

$$p_{ij}^2(t) + q_{ij}^2(t) \leq \bar{s}_{ij}^2, \quad (5.25g)$$

$$p_{Gi}^{min} \leq p_{Gi} \leq p_{Gi}^{max}, \quad (5.25h)$$

$$q_{Gi}^{min} \leq q_{Gi} \leq q_{Gi}^{max}, \quad (5.25i)$$

$$|v_i|^{min} \leq |v_i| \leq |v_i|^{max}, \quad (5.25j)$$

$$s(t) \geq C^{MC}(t)p_{G1}(t), \quad (5.25k)$$

$$s(t) \geq C^p(t)p_{G1}(t) + (C^{MC}(t) - C^p(t))p^{contr}(t). \quad (5.25l)$$

However, the method is computationally efficient as a multi-objective, temporally coupled, nonlinear and non-convex problem. The method, while retaining the benefits of MPC, reduces the computing time making it a suitable tool, implementable in energy management systems for distribution networks.

5.4 Numerical Experiments

This section presents the implementation results of the proposed method on a distribution radial network. The following scenarios are then compared

to evaluate the performance of the proposed method:

- Traditional RHC based ACOPF solution, i.e. solution of the nonlinear receding horizon problem using off-the shelf optimizer;
- Solution of the RHC based ACOPF using proposed approach.

Both scenarios were implemented in Julia [230], a high-performance and open source programming language for technical computing, working on a “64-bit Intel Quad Core 3.40 GHz CPU” with “8 GB of RAM”. The idea is to exploit the “Just In Time” compiler feature, numerical accuracy, and an user friendly syntax.

5.4.1 Description of the “Steinkjer” Distribution Test Case

The proposed contracting tool is demonstrated on a Norwegian distribution in Steinkjer that has a radial structure. The “Steinkjer” network data obtained on 25th, 2015 with accurate forecast values, i.e. with a mean absolute error less than 5%. It consists of 85 nodes, 3 hydro-generators, 700 consumers, 7 industries acting as flexible loads, photo-voltaic panels, and 1 point of connection with the utility grid (slack bus). In addition, there are consumers having contracts to provide load flexibility to the network. The base MVA of the network is 100 MVA. The Steinkjer network is shown in Fig.5.3.

The maximum value of storage in the network is limited to $B_i^{max} = 0.15$ MWh, $r_i^{max} = 0.05$ MW and $b_i^{loss} = 7.5$ kWh. We have considered 7 flexible loads whose maximum curtailment is $\delta_i(t) = 20\% \forall t$. The step-size is fixed at $\Delta T = 1$ h. The length of the MPC horizon has been chosen

Figure 5.3: Steinkjer Network.

Figure 5.4: Time Varying Power Cost Function.

as $T = 8$ h. Indeed, the choice of T is crucial because reducing it implies a decreasing of the computational burden, but shorter T could make the scheduling shortsightedness [236].

5.4.2 Test Description

The normalized energy costs used in the experiments is shown in Fig. 5.4. These costs are published based on market clearing prices for 24 hours at 12:00 pm each day [237]. The cost is used for simulations. The other cost parameters are $C_i^b(t) = 0.2\text{€/p.u}$ and $C_i^d(t) = 17\text{€/p.u}$, $\forall i, t$.

5.4.3 Direct Solution to the RHC based MO-ACOPF

This section presents the simulation results obtained by directly solving the RHC based ACOPF using Julia with accurate forecasts. The power drawn from the PCC, renewable generation in the MG, demand, and power supplied/consumed by the storage devices storage contracted during differ-

ent hours of the day in p.u. values are shown in Fig. 5.5(a). The hatched area shows the reduction in actual demand owing to the use of flexible loads. The flexible loads scheduled during each hour of the day are shown in Fig. 5.5(b). One can observe that the flexible loads and storage are scheduled during periods of increased prices and the storage is charged during periods where energy prices are low.

The hydro generation (non-dispatchable), power consumed from PCC, and power generated from other renewable sources is shown in Fig. 5.5(c) and a comparison with Fig. 5.5(a) reveals that the flexible loads are used for handling the fluctuations introduced by the renewable generators. The savings (operating cost plus line-losses) obtained during different hours of the day is shown in Fig. 5.5(d), one can observe that the average savings is about 35% over a period of 24 hours. This is a significant saving considering the operating cost of the network. The negative savings in the initial part of the graph is due to charging of the storage devices sensing an increase in energy cost later in the day.

5.4.4 Solution to the RHC based ACOPF with the Proposed Approach

The results of the two-level approach for ACOPF showing the power drawn from the PCC, renewable generation, demand, and storage power discharged/charged during different hours of the day in p.u. values is shown in Fig. 5.6(a). The hatched area shows the reduction in actual demand due to flexible loads used in different hours of the day Fig. 5.6(b). One can see that the flexible loads and storage are judiciously scheduled by the proposed method to reduce the operating cost, i.e. the flexible loads and

Figure 5.5: Simulation Results with RHC Based MO-ACOPF Contracting Tool.

storage energy are used during peak-periods. The hydro generation (non-dispatchable), power consumed from PCC, and power generated from other renewable sources is shown in Fig. 5.6(c), this result illustrates that the flexible loads are scheduled to compensate for the fluctuations in renewable generation as well. The total savings (on operating cost plus line-losses) with the proposed approach for different hours of the day is shown in Fig. 5.6(d), one can observe that the average savings is about 33% over a period of 24 hours. This is not only a significant saving but is very close to the one computed by directly solving the RHC based ACOPF problem. This result illustrates that the behavior and savings of the proposed approach are almost similar to that of the direct solution of the multi-period RHC based ACOPF.

5.4.5 RHC based ACOPF Solution Versus that of the Proposed Method

To study the performance of the proposed approach, its performance was compared with the direct solution method. The difference in the power drawn from PCC, power demand on the MG, and storage power supplied/consumed are shown in Fig. 5.7(a). It can be seen that the difference in these values is in the order of 0.1 p.u., a small deviation from the optimal. The difference between p.u. values of flexible loads during different hours of the day using the two algorithms is shown in Fig. 5.7(b). The amount of flexible load used in the RHC based ACOPF solution is slightly lower than the proposed method. The difference in power drawn from the PCC and hydrogenerator is shown in Fig. 5.7(a), the difference shows that the consumption from PCC is almost the same.

The comparison between the proposed solution technique and conven-

Figure 5.6: Simulation Results With Two-level RHC DC/ACOPF.

Table 5.1: Average difference between ACOPF and DC/ACOPF.

	Contr. Storage(p.u.)	Contr. Flexibility(%)	Savings (%)	Solver Time (sec)
RHC-ACOPF	0.50	20	35	91.54
2-level RHC DC/ACOPF	0.45	20	33	4.7

tional method is shown in Table. 5.1. One can infer that, the total drop in savings over a 24 hour horizon with the proposed approach as compared to RHC based ACOPF is only 2%. However, there is significant reduction in computing time by about 18 times. This is a significant reduction and simplification that may be required for energy management systems of large distribution networks. The proposed method solves the multi-objective OPF problem without involving significant computation complexity and finds a local solution efficiently, in a time shorter than an off-the shelf solver, e.g. Julia. While the proposed approach retains the benefits of the RHC based ACOPF, for the case in hand its computation time is reduced by 18 times without significant drop in performance.

5.5 Summary

This investigation presented a computationally efficient method for solving the RHC based ACOPF problem with multiple objectives of optimizing operating cost and line-losses. The main idea behind the solution technique is to separate out the dynamic components having time-coupled behaviors from the nonlinear and nonconvex line-flow constraints. In addition, the method also separates the objectives: the first stage optimizes only for the operating cost; the second stage takes into account both line-losses and op-

(a) Power Balance in MG

(b) Flexibility Contracts

(c) Power Supplied by Hydro, RES and PCC

Figure 5.7: Difference Between RHC based MO-ACOPF and Two-level RHC DC/ACOPF.

erating cost. In the first stage, the DCOPF problem is solved in a RHC manner employing forecasts to schedule the storage devices, flexible loads and the generators to reduce the operating cost. The storage and flexible loads are fixed at the values computed in the first stage, and the generation schedule is used to hot-start the second stage. A single-stage ACOPF problem is solved in the second stage. The experimental results on a distribution network demonstrate that the proposed approach provides significant reduction in computation by 18 times, without much loss of optimality. This method is clearly applicable to networks requiring fast OPF solutions and having high line-resistance wherein convex relaxation or DCOPF cannot be applied.

Chapter 6

Conclusions and Future Works

This chapter briefly summarizes the obtained contribution of this thesis and then provides an outline for potential future works according to the proposed methods and algorithms in this dissertation.

6.1 Conclusions

A stable and reliable electric power network which operates economically while guaranteeing to satisfy consumer demands is the main purpose of power network engineering. As discussed in Chapter 1, the power flow equations plays the main role in power electric network operation by explaining the physical relationship between network powers and voltages. These nonlinear equations as the main heart of the optimal power flow (OPF) problem lead to a nonconvex NP-hard optimization problem which may have multiple optima.

To date, many deterministic and non-deterministic algorithms have been proposed and applied to the OPF problem. Owing to the nonconvexity of this problem, these techniques do not guarantee to find the globally optimal

solution. In recent years a variety of convex relaxations methods, especially semidefinite programming (SDP) relaxation, have been applied to the OPF problem to compute the global optimality certificates. The solution of a convex relaxed problem provides a lower(upper) bound for the original minimization(maximization) OPF problem, and if the solution can satisfy the relaxation constraints then the relaxation is tight and the obtained solution is a globally optimal solution of the original problem as well. The SDP relaxation has been successfully applied to many electric power systems with a tight relaxation gap. Although using the matrix completion theorem helps to improve the computational speed of the SDP relaxed problem, but still, the solver time is high for large-scale power networks compared to the mature OPF problem algorithms. Recent developments in the power industry, such as renewable resources, storage units and etc., also increases the complexity and difficulty of solving the OPF problem, especially at the distribution networks.

In this thesis, the mathematical representations of power flow equations and optimal power flow problem are described in Chapter 2. The nonconvexity and complexity of the OPF problem are demonstrated by providing the feasible space of some small test cases.

Since the computational effort is one of the main challenges in solving the OPF problem using any techniques, regardless of their classification, e.g. deterministic, non-deterministic, convex relaxation or receding-horizon control based, enhancing the computational speed of any of these techniques is a goal.

Therefore, this thesis proposes three different methods, a meta-heuristic algorithm, a semidefinite relaxation based method and a receding-horizon

control based method, in which the main goal is to improve the computation efficiency comparing to the original methods by exploiting supplementary information for them.

The deterministic algorithms can quickly find a local solution for OPF problem but they do not provide any additional information on the global optimality of obtained solution. On the other hand, these deterministic algorithms are not able to solve the optimization problems including discrete or integer variables. Therefore, non-deterministic or meta-heuristic optimization algorithms are widely proposed and applied on nonconvex optimization problems including the OPF problem due to their global exploration features which can avoid trapping in the local optima. The main weaknesses of these algorithms are their convergence characteristic, computational time and their reliability in term of the stability of the solution over the replications.

Chapter 3 proposes a fully informed water cycle algorithm (FIWCA) which is an enhanced version of the water cycle algorithm (WCA). The statistical results obtained by the implementation of the original WCA show that the algorithm may not perform reliably over replications and the number of iterations for converging to a reasonable solution is not suitable. The original algorithm considers exchanging the local information to update streams and rivers position, e.g. the position of a stream is updated only by considering the information from its assigned river and sea. The FIWCA proposes to utilize the concept of exchanging global and local information among individuals in the population, e.g. the position of each stream can be updated by receiving the information from its own assigned river and other rivers and sea. Using fully informed algorithm helps to in-

crease the diversity in the selection of solution and consequently decrease the possibility of getting trapped at local optima, thereby enhancing the efficiency and accuracy significantly. The numerical optimization results on OPF problem demonstrated the computation benefits and efficiency of the FIWCA in reaching toward global optimal solutions with a more reasonable convergence rate while obtaining slightly better objective values comparing to the other techniques such as conventional WCA, genetic algorithm, particle swarm optimization, ant colony and others explored in the literature.

Certifiably obtaining the global solution is important for certain applications of OPF problems. Many global optimization techniques compute an optimality gap that compares the achievable objective value corresponding to the feasible point from a local solution algorithm with the objective value bound from a convex relaxation technique. Rather than the traditional practice of completely separating the local solution and convex relaxation computations, Chapter 4 proposes a method that exploits information from a candidate local solution, obtained very quickly from a mature OPF algorithm, to speed the computation of an objective value bound using a semidefinite programming (SDP) relaxation.

Using the matrix completion theorem and with many dual variables of a SDP relaxation problem fixed to their corresponding values from the local solution, the new SDP problem is easier to solve than the original SDP relaxation, which results in a computational speed advantage for the proposed method. And since the dual objective value is a lower bound for primary SDP relaxation, and the solution of any convex relaxed problem is a lower bound for the original problem, the obtained objective value from the proposed algorithm is a lower bound for the globally optimal solution of original

OPF problem. Numerical experiments indicate that the proposed method is approximately four to five times faster than the SDP relaxation for a variety of test cases without excessive degradation in the quality of the objective bound. These experiments also show that the proposed method is slower than the SOCP and QC relaxations but can obtain tighter bounds. Thus, the proposed method provides a middle ground between the SDP relaxation and the SOCP and QC relaxations with a trade-off between computational speed and tightness.

The conventional way of solving static OPF problem is unsuitable for distribution systems having renewable energy sources, energy storage systems and flexible loads due to the time-coupled and stochastic dynamics. The convex relaxation approaches are also not suitable for distribution networks due to their high line resistance to reactance ratio, (R/x) . The multi-period OPF approaches proposed to overcome this problem cannot tackle uncertainties as due to lack of feedback, i.e. measurements. Recently, the receding-horizon control (RHC) based approach, which solves a multi-period optimization problem during each time-horizon taking measurements is proposed. A major challenge in RHC approaches is the computation complexity as the multi-period optimization model need to be solved during each time-horizon to make the decisions.

Thus, Chapter 5 proposes a computationally efficient approach for solving receding horizon control (RHC) based multi-objective alternating current optimal power flow (ACOPF) problem in distribution networks having intermittent renewable energy resources, storage devices and flexible loads. The method decomposes the solution into two stages. A direct current optimal power flow (DCOPF) problem is quickly solved using an RHC approach at

the first stage by embedding forecasts on renewable generation and demand without considering the nonlinear power flow equations. Then, a single-period ACOPF to optimize both line-losses and operating cost is solved at the second stage with the storage and flexible loads fixed at the optimal values information obtained from the first stage. In addition, hot-start is provided to the second stage optimization problem using the generation schedule computed at the first stage which also helps the computational effort at the second stage. The numerical results on a distribution network test case demonstrate that the proposed method provides a significant reduction in computation by 18 times, without much loss of optimality. This method is more suitable for networks requiring fast OPF solutions and having high line-resistance, e.g. distribution networks, wherein convex relaxation cannot be applied.

6.2 Future Works

This thesis with the idea of exploiting supplementary information for different OPF problem optimization techniques to improve their computational efficiency can open several scopes of research for future works.

- This thesis proposes a fully informed water cycle algorithm (FIWCA), in which the exploitation phase is improved by leveraging the global information from other rivers in addition to the local information received by each stream from its assigned river, and the same idea for updating the rivers. This process helps to decrease the number of convergence iterations while obtaining more reliable results over replications. Currently, the algorithm uses the same weighting factor for exploiting the local and global information. One open question is that

whether there are optimal values for these weighting factors which enhance the performance of the proposed algorithm to its best.

The proposed version of FIWCA is suitable for the optimization problems consists of continuous variables. So extending the proposed FIWCA such that it may tackle the mixed-integer optimization problems, e.g. voltage stability constrained OPF problem, can be another scope of future work based on the current algorithm.

It is always interesting to solve an optimization problem with multi-objectives, especially in the power grids that there can be different goals for solving the OPF problem, thus one of the potential future works would be extending the proposed FIWCA to solve multi-objective problems and specially multi-objective OPF problem.

- Then, a method to quickly compute a lower bound for the optimal objective value of OPF problem is proposed by leveraging the information from a candidate local solution and solving the dual semidefinite programming (SDP) relaxation problem. Since any feasible solution of the SDP relaxation problem, regardless of the primal or dual problem, is a lower bound for the original problem, there is a question whether it is possible to implement the same idea on the primal SDP problem.

As shown in the simulation results, the solver time for the quadratic convex (QC) relaxation is much smaller than that of SDP relaxation, while the optimality gap is not that bad. So, it can be a good experience to apply the same idea to the dual QC relaxation of OPF problem, the results might be interesting if the solution time will be still that small without excessive degradation in the quality of the objective bound.

- The next chapter proposes a two-stage receding-horizon control (RHC) based method for solving the ACOPF problem in distribution networks in an efficient time since the presence of time-coupled units and the high resistance to reactance ratio of transmission lines in these networks increase the computational complexity of the OPF problem. The proposed method shows a great success in decreasing the solver time without much loss of optimality. As the size of the power network increases the computational difficulty and solver time of the OPF problem rise significantly. Distributed optimization techniques are very useful for large-scale power systems, so extending the proposed RHC-based method in this thesis to a distributed implementation is another future course of the investigation.
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Author's Publications

1. Alireza Barzegar, Seshadhri Srinivasan, Alessio Maffei, Rong Su, Luigi Iannelli, Luigi Glielmo, "A Computationally Efficient Approach for Solving Receding-Horizon Optimal Power Flow Problem in Distribution Grids," IEEE/ CAA Journal of Automatica Sinica, *Manuscript under review*.
2. Alireza Barzegar, Daniel K. Molzahn, Rong Su, "A Method for Quickly Bounding the Optimal Objective Value of an OPF Problem using a Semidefinite Relaxation and a Local Solution," Electric Power Systems Research, *Manuscript under review*.
3. Alireza Barzegar, Ali Sadollah, Rong Su, Seshadhri Srinivasan, "Fully Informed Water Cycle Algorithm for Solving Optimal Power Flow Problem in Energy Grids," Applied Intelligence, *Manuscript under review*.
4. Alireza Barzegar, Ali Sadollah, Leila Rajabpour, and Rong Su, "Optimal power flow solution using water cycle algorithm," 14th International Conference on Control, Automation, Robotics and Vision (ICARCV), IEEE, pp. 1-4, 2016.
5. Alireza Barzegar, Rong Su, Changyun Wen, Leila Rajabpour, Yicheng

Zhang, Amit Gupta, Chandana Gajanayake, and Meng Yeong Lee, “Intelligent power allocation and load management of more electric aircraft,” in Power Electronics and Drive Systems (PEDS), 2015 IEEE 11th International Conference on Power Electronics and Drive Systems (PEDS), IEEE, pp. 533-538, 2015.

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Appendices

Appendix A

Cost Coefficients and Design Variable Limits Used in Chapter 3

The design variable limits and cost coefficients of different test cases for IEEE 30-bus and IEEE 57-bus systems are presented as following:

Table A.1: The Control Variables Limits for IEEE standard 30-bus [5].

Control variables	Min	Max	Control variables	Min	Max
PG1 (MW)	50.00	200.00	T6-10 (p.u.)	0.90	1.10
PG2 (MW)	20.00	80.00	T4-12 (p.u.)	0.90	1.10
PG5 (MW)	15.00	50.00	T28-27 (p.u.)	0.90	1.10
PG8 (MW)	10.00	35.00	QC10 (MVAR)	0.00	5.00
PG11 (MW)	10.00	30.00	QC12 (MVAR)	0.00	5.00
PG13 (MW)	12.00	40.00	QC15 (MVAR)	0.00	5.00
VG1 (p.u.)	0.95	1.10	QC17 (MVAR)	0.00	5.00
VG2 (p.u.)	0.95	1.10	QC20 (MVAR)	0.00	5.00
VG5 (p.u.)	0.95	1.10	QC21 (MVAR)	0.00	5.00
VG8 (p.u.)	0.95	1.10	QC23 (MVAR)	0.00	5.00
VG11 (p.u.)	0.95	1.10	QC24 (MVAR)	0.00	5.00
VG13 (p.u.)	0.95	1.10	QC29 (MVAR)	0.00	5.00
T6-9 (p.u.)	0.90	1.10			

Table A.2: Generation Cost Coefficients for Case 1 (IEEE 30-bus) [5].

Bus No.	Cost Coefficients		
	c_{i0}	c_{i1}	c_{i2}
1	0.00	2.00	0.00375
2	0.00	1.75	0.01750
5	0.00	1.00	0.06250
8	0.00	3.25	0.00834
11	0.00	3.00	0.02500
13	0.00	3.00	0.02500

Table A.3: Generation Cost Coefficients for Case 3 (IEEE 30-bus) [6].

Bus No.	From (MW)	To (MW)	Cost Coefficients		
			c_{i0_k}	c_{i1_k}	c_{i2_k}
1	50	140	55.00	0.70	0.0050
	140	200	82.50	1.05	0.0075
2	20	55	40.00	0.30	0.0100
	55	80	80.00	0.60	0.2000

Table A.4: Generation Cost Coefficients for Case 4 (IEEE 30-bus) [7].

Bus No.	P_{Gi}^{min} (MW)	P_{Gi}^{max} (MW)	Cost Coefficients				
			c_{i0}	c_{i1}	c_{i2}	d_i	e_i
1	50	200	150	2.00	0.0016	50	0.0630
2	20	80	25	2.50	0.0100	40	0.0980

Table A.5: Generation Emission Coefficients used in Case 5 (IEEE 30-bus) [5].

Bus No.	Cost Coefficients				
	c_{i0}^{SOX}	c_{i1}^{SOX}	c_{i2}^{SOX}	d_i^{NOX}	e_i^{NOX}
1	0.04091	-0.05554	0.06490	0.000200	2.857
2	0.02543	-0.06047	0.05638	0.000500	3.333
5	0.04258	-0.05094	0.04586	0.000001	8.000
8	0.05326	-0.03550	0.03380	0.002000	2.000
11	0.04258	-0.05094	0.04586	0.000001	8.000
13	0.06131	-0.05555	0.05151	0.000010	6.667

Table A.6: The Control Variables Limits for the IEEE 57-bus Test System [8].

Control variables	Min	Max	Control variables	Min	Max
PG1 (MW)	0.00	575.88	T24-25 (p.u.)	0.90	1.10
PG2 (MW)	0.00	100.00	T24-25 (p.u.)	0.90	1.10
PG3 (MW)	0.00	140.00	T24-26 (p.u.)	0.90	1.10
PG6 (MW)	0.00	100.00	T7-29 (p.u.)	0.90	1.10
PG8 (MW)	0.00	550.00	T34-32 (p.u.)	0.90	1.10
PG9 (MW)	0.00	100.00	T11-41 (p.u.)	0.90	1.10
PG12 (MW)	0.00	410.00	T15-45 (p.u.)	0.90	1.10
VG1 (p.u.)	0.94	1.06	T14-46 (p.u.)	0.90	1.10
VG2 (p.u.)	0.94	1.06	T10-51 (p.u.)	0.90	1.10
VG3 (p.u.)	0.94	1.06	T13-49 (p.u.)	0.90	1.10
VG6 (p.u.)	0.94	1.06	T11-43 (p.u.)	0.90	1.10
VG8 (p.u.)	0.94	1.06	T40-56 (p.u.)	0.90	1.10
VG9 (p.u.)	0.94	1.06	T39-57 (p.u.)	0.90	1.10
VG12 (p.u.)	0.94	1.06	T9-55 (p.u.)	0.90	1.10
T4-18 (p.u.)	0.90	1.10	QC18 (MVAR)	0.00	30.00
T4-18 (p.u.)	0.90	1.10	QC25 (MVAR)	0.00	30.00
T21-20 (p.u.)	0.90	1.10	QC53 (MVAR)	0.00	30.00

Table A.7: Generation Cost Coefficients for IEEE 57-bus [8].

Bus No.	Cost Coefficients		
	c_{i0}	c_{i1}	c_{i2}
1	0.00	20.00	0.0775
2	0.00	40.00	0.0100
3	0.00	20.00	0.2500
6	0.00	40.00	0.0100
8	0.00	20.00	0.0222
9	0.00	40.00	0.0100