Surface wave propagation in coastal waters with viscoelastic boundaries

Dharma Sree K K

2019

Dharma Sree K. K. (2019). Surface wave propagation in coastal waters with viscoelastic boundaries. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/106427>

<https://doi.org/10.32657/10220/47962>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**SURFACE WAVE PROPAGATION IN COASTAL
WATERS WITH VISCOELASTIC BOUNDARIES**

**DHARMA SREE K K
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING**

2019

SURFACE WAVE PROPAGATION IN COASTAL WATERS WITH VISCOELASTIC BOUNDARIES

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School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Doctor of Philosophy

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

Mar 29, 2019

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Authorship Attribution Statement

This thesis contains material from 5 papers published in the following peer-reviewed journals / from papers accepted at conferences for which I am listed as an author.

Chapter 3 is published as: Sree, D. K., Law, A. W. K., and Shen, H. H. (2017). An experimental study on the interactions between surface waves and floating viscoelastic covers. *Wave Motion*, 70, 195-208.

The contributions of the co-authors are as follows:

- Assoc/Prof. Law and Prof. Shen provided the project direction and edited the manuscript drafts.
- I co-designed the study with Assoc/Prof. Law and Prof. Shen and performed all the laboratory works at the Environmental Process Modeling Centre (EPMC), Nanyang Environment & Water Research Institute.
- I analyzed the data and prepared the manuscript drafts. The manuscript was revised by Assoc/Prof. Law and Prof. Shen.

Chapter 4 is published as: Sree, D. K., Law, A. W. K., and Shen, H. H. (2018). An experimental study on gravity waves through a floating viscoelastic cover. *Cold Regions Science and Technology*, 155, 289-299. Some results of the chapter are included in: Sree, D. K., Law, A. W. K., and Shen, H. H. (2016). Attenuation of surface wave by viscoelastic floating cover: An experimental study. *Proceedings of 23rd IAHR International Symposium on Ice (IAHR-ICE 2016)*, Ann Arbor, Michigan, USA.

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- I analyzed the data, wrote the MATLAB codes for SA and RP models and prepared the manuscript drafts. The manuscript was revised by Assoc/Prof. Law and Prof. Shen.

Some results of Chapter 7 are published in: a) Sree, D. K., Law, A. W. K., and Shen, H. H. (2017). An experimental study on surface wave interactions with viscoelastic boundaries. *Proceedings of the 24th International Conference on Port and Ocean Engineering under Arctic Conditions (POAC2017)*, Busan, South Korea, and b) Sree, D. K., Law, A. W. K., and Shen, H. H. (2019). An experimental study on surface wave modulation due to viscoelastic bottom. *Proceedings of the 4th International Conference in Ocean Engineering (ICOE2018)*, Vol: 2, Springer Madras, India.

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ACKNOWLEDGEMENT

First and foremost, I want to express my deepest gratitude to my supervisor, Dr. Law Wing Keung, Adrian. His continuous support for my research, the patience to hear the difficulties I faced, motivation provided when I was stressed, and his immense knowledge helped me all the time of research. I am thankful to him for his countless revisions of the manuscript and encouragement provided for me to improve my writings. His guidance has been invaluable for me to grow as a researcher.

I am deeply grateful to Prof. Hayley Shen, Clarkson University who has always been there to listen and give advice. The lengthy discussions with her helped me refine the technical details of my work along with getting updated about the related works done by many others around the globe. I am grateful to her for teaching me how to do research and enriching my ideas with constructive criticisms.

Besides my advisors, I would like to thank my Thesis Advisory Committee members: Assoc. Prof. Edmond Lo Yat-Man, and Assoc. Prof. Xiao Zhongmin for their insightful comments and encouragement to widen my research from various perspectives. I express my sincere thanks to the three thesis examiners for their comments and suggestions to improve the thesis, the precious time and effort they spent giving the advice is invaluable. I gratefully acknowledge the funding provided by the School of Civil and Environmental Engineering, Nanyang Technological University along with the Ministry of Education, Singapore through the AcRF Tier 2 Grant No. MOE2013-T2-1-054 and, the US Office of Naval Research Global Grant #N00014-13-1-0294 and N62909-15-1-2069. These funding allowed me to carry out this research work and provided me the opportunity to attend four international conferences, two international workshops and two international research attachments, and to meet and work in collaboration with many renowned persons in the research area.

The wave-ice experiments discussed in this dissertation would not have been possible without the groups from Ocean Engineering group, University of Melbourne and Naval Research Laboratory, United States. I would like to thank Prof Alexander V. Babanin, University of Melbourne for organizing the SIWWI (Melbourne Sea-Ice-Wave-Wind Interaction Facility) experiments and Prof. Jason Monty for providing the required experimental facilities. I also thank Dr. Jung Lee, Dr. Grant Skidmore, and Dr. Filippo Nelli from the University of Melbourne, Dr. David Wang, Dr. Parra Sabrina, Dr. Eric Rogers, and Dr. Tripp Collins from the US Naval Research Laboratory, and Dr. Lucas Yiew from the Nanyang Technological University for their significant contributions to the experiments.

I express my deepest thanks to the Nanyang Environment & Water Research Institute (NEWRI) administration who provided access to the laboratory and research facilities to conduct this research. I am grateful to the former colleague Dr. Firdaus Prabowo for the necessary advice and guidance provided at the beginning of my research life. I am also thankful to Ms Li Min, Lab Manager EPMC Lab, NEWRI who maintained the laboratory well so that I never had to worry about conducting experiments there. I would also like to thank Ms Ng Soo Ching, Ms Jamillah Bte Sa'adon, other administrative and technical staff members of the School of Civil and Environmental Engineering (CEE), Nanyang Technological University for the help in their respective roles.

I would like to thank Sukun Cheng at Clarkson University for providing the code for the viscoelastic model and Malavarayan Sankarasubramanian at Clarkson University for measuring the polypropylene parameters. I appreciate the work of Ms Teo Wan Lin Jane, Ms Neo Ying Xue, and Mr. Peh Zhisheng all final year undergraduate students who helped me with the experiments, without their assistance it would be tiresome and monotonous. The numerous discussions with them helped me improve my knowledge in the area. I thank my fellow laboratory mates Ms Li Tian and Mr Jiang Mingtao for all the helps they provided in the laboratory and making a peaceful environment to work together.

Last but not the least, I would like to thank my family for their unconditional love, sacrifices and support. My parents, Sheela and Krishnan Kutty, and sister, Dhanusree raised me with love to learn and encouraged me to choose what I desired; so that I can grow as a responsible and independent person. I also feel happy to mention my little nephews Karthik and Krishna whose innocence refreshed me during my difficult times by bringing smiles. I owe special thanks to my better half, Swaroop, for his unparalleled love and understanding during my pursuit of Ph.D. degree.

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SUMMARY

Mathematical models on the wave interactions with viscoelastic boundaries, including ice covers and muddy bottoms, are available in literature. The dispersion relation and attenuation of surface waves are predicted to depend on the rheological properties of these boundaries. In the present study, a novel experimental approach is developed that can enable the verification of existing theoretical predictions in the laboratory. The floating covers are prepared with the blended mixture of white oil and Polydimethylsiloxane (PDMS) material, while the bottom beds are prepared by doping glycerol with PDMS. The targeted viscoelastic properties are achieved by varying the mass percentages of a curing agent. Special curing procedures are also established to create large uniform sheets. The rheological properties are determined using oscillation tests in a rheometer. They show that the mechanical behaviour of the viscoelastic sheets is close to a Voigt material, an assumption commonly used in the existing mathematical models.

Wave flume experiments are conducted to study the wave interactions with unbroken viscoelastic floating covers, viscoelastic floating segments, viscoelastic bottoms, as well as combined viscoelastic covers and bottom layers. It is found that the rheological properties of the viscoelastic covers play a key role in the modification of surface waves with respect to open water case. Both wave lengthening and wave shortening are observed depending on the cover properties, cover thickness as well as wave period. The experimental results match well with the theoretical predictions on wavelength changes, with the unbroken viscoelastic cover behaving primarily as a low-pass filter, damping high frequency surface waves effectively. However, significant discrepancy between the experimental and theoretical results is observed on the wave attenuation coefficient. It is later determined to be the contribution of the boundary layer underneath the floating cover which is not considered in the mathematical models. The introduction of discontinuities to the floating covers alters the wave modification as compared to an unbroken cover. The experimental results on the wavelength change do not agree well

with existing empirical equations available for broken covers. It is found that the discontinuities enhance the attenuation of surface waves with scattering and boundary layer effects, which in turn depend on the rheological properties.

Experiments of wave interactions with fresh water ice are conducted at the University of Melbourne with several types of ice covers. The preparation method for each ice type is distinct, with calm environment for continuous covers and rough conditions for grease and pancake ice covers. Grease ice covers are characterized based on the volume concentration of ice in the slurry, whereas continuous covers are categorized based on their thickness. Three-point bending tests are conducted to determine the Young's modulus of the continuous ice cover which causes a meaningful change in wavelength, and the experimental results on the dispersion relation agree well with the theoretical predictions. Grease and pancake ice covers damp the surface waves very effectively, but no notable change in wavelength is observed.

Finally, wave interactions with flexible viscoelastic bottoms are studied in shallow waters. It is found that the thickness of the viscoelastic bed as well as its rheological properties play a key role in the wavelength modification of surface waves. Notable wave attenuation due to flexible bed is observed compared to rigid bed. The non-linearity of waves enhances the experimental results on attenuation which are generally higher than the theoretical predictions by the linear wave theory. A three-layer system with both viscoelastic top and bottom boundaries is also studied in intermediate waters. The experimental results on wavelength modification for the three-layer system match well with theoretical predictions, but the attenuation is again underpredicted.

LIST OF PUBLICATIONS

Peer-reviewed articles in international journals

1. Sree, D. K., Law, A.W. K., and Shen, H. H. (2017). An experimental study on the interactions between surface waves and floating viscoelastic covers. *Wave Motion*, 70, 195-208.
2. Sree, D. K., Law, A.W. K., and Shen, H. H. (2018). An experimental study on modulation of surface waves due to a flexible viscoelastic floating cover. *Cold Regions Science and Technology*, 155, 289-299.

Peer-reviewed papers in conference proceedings

1. Sree, D., Law, A. W. K., and Shen, H. H. (2019). An experimental study on surface wave modulation due to viscoelastic bottom. In *Proceedings of the Fourth International Conference in Ocean Engineering (ICOE2018)*, Vol.2, 199-206.
2. Sree, D. K., Law, A. W. K., and Shen, H. H. (2017). An experimental study on surface wave interactions with viscoelastic boundaries. *Proceedings of the 24th International Conference on Port and Ocean Engineering under Arctic Conditions*. Busan, Korea.
3. Sree, D. K., Law, A. W. K., and Shen, H. H. (2016). Modification of surface wave by viscoelastic floating cover: an experimental study. *Proceedings of 23rd IAHR International Symposium on Ice (IAHR-ICE)*, Ann Arbor, Michigan, USA.
4. Prabowo, F., Sree, D. K., Law, A. W. K., and Shen, H. H. (2014). A laboratory study of wave-ice interactions in the marginal ice zone using polydimethylsiloxane (PDMS) as a viscoelastic model. *Proceedings of 22nd IAHR International Symposium on Ice (IAHR-ICE)*, Singapore.

Conferences / Workshops presentations:

1. Sree, D. K., Law, A. W. K., and Shen, H. H. (2017). An experimental study on surface wave modulation due to visco-elastic boundaries: a three-layer system. *Ocean Waves and Wave Coupled Processes*, Qingdao, China.

2. Sree, D. K., Law, A. W. K., and Shen, H. H. (2016). Attenuation of surface wave by viscoelastic floating cover: an experimental study. *Ocean Waves and Wave Coupled Processes*, University of Melbourne, Australia.
3. Sree, D. K., Law, A. W. K., and Shen, H. H. (2015). Sea ice model: Wavelength modulation under a finite length viscoelastic cover. *KOZWaves*, The 2nd Australasian Conference on Wave Science, Adelaide, Australia.

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LIST OF ABBREVIATIONS

ADV	Acoustic doppler velocimetry
BL	Boundary layer
BVS	Broadband viscoelastic spectroscopy
DMA	Dynamic mechanical analysis
EP	Elastic plate
HFVS	Hyper-frequency viscoelastic spectroscopy
IC	Inversely calculated
ICESat	Ice, Cloud, and land Elevation Satellite
LAOS	Large amplitude oscillatory shear
LVR	Linear viscoelastic regime
MIZ	Marginal ice zone
ML	Mass loading
NSIDC	National Snow and Ice Data Centre
PDMS	Polydimethylsiloxane
PIV	Particle Image Velocimetry
PMMA	Polymethylmethacrylate
PVC	Polyvinylchloride
RCP	Representative Concentration Pathways
RP	Robinson and Palmer (1990)
RUS	Resonant ultrasound spectroscopy
SA	Squire and Alan (1980)
SAR	synthetic-aperture radar
SAOS	Small amplitude oscillatory shear
TSR	Transpolar sea route
US	Ultrasound
VE	Viscoelastic
WS	Wang and Shen (2010b)

LIST OF SYMBOLS

a	wave amplitude in open water
B	breadth of wave flume
b_b	breadth of ice beam
c_o	open water wave celerity
c_s	wave celerity under the floating cover
c_g	group velocity of wave
D	depth of wave flume
d	water depth
E_w	wave energy
E	Young's modulus of elasticity
F	applied load
g	acceleration due to gravity
G	shear modulus
G'	storage modulus
G''	loss modulus
G_V	complex Voigt shear modulus
h	thickness of upper viscoelastic layer
h_b	thickness of bottom layer
h_s	thickness of PDMS cover
H	depth of lower water layer
k_o	wavenumber in open water
k	complex wavenumber
k_i	imaginary part of k / attenuation coefficient
k_s	real part of k / wavenumber along the floating cover
L	length of wave flume
L_s	length of floating cover
l_s	length of segment

L_r	flexural rigidity
L_b	span for three-point loading
M_B	mass of base
m_{CA}	mass percentage of curing agent
m_s	mass of ice slurry
m_i	mass of ice
M_{CA}	mass of curing agent
M_o	mass of white oil
M_g	mass of glycerol
N_c	number of waves free from reflection
N_d	number of transient waves
T	wave period
t_b	thickness of ice beam
u	horizontal orbital velocity of wave
ω	angular frequency of rheometer
σ	wave frequency
η	vertical surface displacement
ξ	vertical interface displacement
A_x	amplitude along the cover at x
A_o	amplitude along the cover at $x = 0$
α_o	attenuation coefficient for open water condition
α	experimentally determined attenuation coefficient
ϑ	Poisson's ratio
ρ_w	density of water
ρ_m	density of mud
ν_e	complex viscosity of floating cover
ν_{me}	complex viscosity of mud
ν	real part of complex viscosity
ν_w	viscosity of water

κ	normalized wavenumber, k_s/k_o
δ_s	Stokes boundary layer thickness
δ_{ice}	ice deflection
λ	wavelength
Δ	gap between PDMS cover and side wall
γ	viscous term in RP model
α_{IN}	attenuation due to inextensible film
α_{CL}	clean water attenuation
α_{SB}	attenuation due to flume boundaries
α_e	attenuation due to floating cover of finite elasticity and negligible thickness

CHAPTER ONE

Chapter 1

Introduction

1.1 Background

1.1.1 Floating sea ice

The average global temperature in Year 2017 has increased by 0.9°C relative to the 1951-1980 average temperature. Almost all the warmest years occurred after 2001 (Global Climate Change, NASA) in the 136-year record. The rapid ocean surface warming in the Arctic region is also causing a speedy reduction in sea ice extent at a maximum rate of $13.0 \pm 2.4\%$ per decade during summer and $2.7 \pm 0.5\%$ per decade during winter (National Snow and Ice Data Centre, NSIDC), relative to the 1981-2010 average. Arctic amplification, driven by declining sea ice extent, increases the warming rate in the Arctic by roughly twice as that of global average. This sequential cycle of rise in temperature causing sea ice melt and vice versa could bring an early “ice-free” ($< 10^6$ km² of sea ice) summer Arctic sea by 2020-2050 (Wang and Overland, 2009).

The melting of sea ice directly contributes to global sea level rise though in a minor manner compared to land-fast ice, with 2.6% more water to the ocean (4 cm of sea level rise) due to the density difference between saline water and fresh ice (Noerdlinger and Brower, 2007). Furthermore, the sea level rise due to the melting of ice is not occurring uniformly across the globe (Nicholls and Cazenave, 2010), with relatively smaller magnitude (< 3 mm) observed in the Arctic region (National Oceanic and Atmospheric Administration, 2018). The indirect effects of sea ice melting through global climate change is significant, including variation in weather patterns through change in ocean circulation and wind directions, as well as extreme weather conditions with much colder

winters and hotter summers (Meehl et al., 2000). At the same time, the sea ice retreat has also led to the opportunity of new transpolar sea routes (TSR) connecting Atlantic Ocean to Pacific Ocean (Smith and Stephenson, 2013). These shorter sea routes could bring more potential business opportunities through enhanced international trade. Compared to the conventional sea route (via Suez Canal) connecting Europe and Asia, the shorter North Sea route or Northeast Passage helps in the cost reduction of shipping (Schoyen and Brathen, 2011). Currently only icebreakers can make voyages through these sea routes. With the “ice-free” summer sea, the opportunity would be available for even the common open water ships.

Figure 1.1: Proposed summer navigation routes across the Arctic Ocean for two different climate change scenarios (Representative Concentration Pathways, RCP). Red lines indicate TSR for polar class ships and blue lines indicate fastest routes for common open water ships. (Source: Smith and Stephenson, 2013).

For a safe navigation in the Arctic Ocean, it is crucial to have a good understanding of the wave-ice conditions and their forecast, especially in the marginal ice zone (MIZ) which links the open water to the pack ice cover. More open water in the Arctic Ocean also implies longer fetches for the growth of surface waves which can affect the Arctic shipping. The larger sea waves can travel for longer distances into the ice field with limited attenuation and impart fractures to the ice covers. This increases the width of MIZ especially during warm seasons, with thick multiyear ice replaced (in summer) by thin first-year ice (in winter) (Strong and Rigor, 2013).

Many models have been developed in the past to represent the mechanical properties of the ice cover in the MIZ for wave-ice interaction analysis. The thin elastic model by Greenhill (1886) and Wadhams (1973a), and later expanded by others as reviewed by Squire (2007), assumes that the ice cover acts as an elastic layer. The model analysis was validated by the laboratory experiments performed by Sakai and Hanai (2002) using elastic covers made with polyethylene. Comparisons between theory and field data obtained in the MIZ off the east coast of Newfoundland suggested that wave lengthening occurred under the sea ice for longer wave periods (1991), which implied that the elastic plate model can be a good representation of the field conditions with an effective Young's modulus. Keller (1998) proposed a two-layer viscous model to represent the grease ice in the MIZ as a viscous liquid over inviscid water. His theory was also experimentally validated for grease ice by Newyear and Martin (1999), and the predictions agreed qualitatively with remote sensing data from SAR images taken from a frazil-pancake ice field by Wadhams et al. (2002). However, the pancake-grease ice experiments performed by Wang and Shen (2010a) found that a pure viscous layer of constant viscosity was insufficient to explain the resulting change of wave characteristics, indicating that an additional elastic parameter might be necessary. A viscoelastic sea ice model can be considered for broader generalization that can include both the viscous and elastic properties of the ice cover (Squire and Alan, 1980; Robinson and Palmer, 1990; Wang and Shen, 2010b).

1.1.2 Muddy bottom

Muddy coastal zones are usually found near large river mouths with plentiful discharges of fine particles. They are dynamic with land-water interactions and rich in bio-diversity. Based on the physical geography, muddy coasts are classified into: a) tidal flats, b) enclosed sheltered bay deposits, c) estuarine drowned valley deposits, d) inner deposits of barrier-enclosed lagoons, e) supra-tidal (storm surge) deposits, f) swamp marsh and wetland deposits, g) mangrove forest and swamp deposits, h) chenier plains, i) mud deposits veneering eroded shore platforms, j) ice deposited mud veneer, and k) sub-littoral mud deposits (Healy et al; 2002). Muddy coasts have a wide-range of sizes extending from few metres to several kilometres of width, and morphogenetic environment are present in all continents except the Antarctica with fewer exposed rocky coasts (see Figure 1.2). The largest muddy coastal area is the shoreline between the Amazon and Orinoco rivers in South America (1500 km long) (Anthony et al., 2010) with mangrove vegetation. Unlike a rigid or sandy bottom, the nearshore muddy bottom attenuates waves very effectively (Tubman and Suhayda, 1976). The muddy beaches experience erosion, but it is typically less compared to sandy beaches (Gopinath, 2010). The relatively stable muddy bed provides habitat to macrofauna living in burrows and tubes (Bassindale and Clark; 1960).

The Southwest coast of India has a seasonal mudbank. It appears during the South west monsoon period when the offshore wave action is severe (Mathew, 1992; Jiang and Mehta, 1996). The mud banks retreat and move offshore by the end of monsoon leaving a sandy sloping rigid substrate. The waves experience typically a 30% dissipation of wave energy during non-mudbank period, while nearly complete dissipation had been observed within a short distance in the presence of mud bank (Mathew et al., 1995). This helps the small vessels carry on fishing during the harsh monsoon conditions, termed as chakara, thus influencing the financial state of the area.

Figure 1.2: Distribution of muddy coasts across the globe (Source: Encyclopædia Britannica, Inc., 1997).

Since the muddy beaches play a significant role in ecological as well as economic aspects of the region, it is important to understand the wave interaction with non-rigid flexible bottoms. Wave damping induced by muddy bottom depends on the mud properties, and several predictive models have been developed in the past. Similar to the sea-ice models, the two material properties considered for the bottom bed are elasticity and viscosity, along with other aspects such as porosity, plasticity, concentration etc. The elastic model for muddy bottoms, with zero damping, was developed by Mallard and Dalrymple (1977) which was later improved by Dawson (1978) with the addition of soil inertia. Viscous model is the widely adapted one, which was first developed by Gade (1958) assuming the mud as a thick viscous layer. The extension of the work by Dalrymple and Liu (1978) considered the viscosity for both the mud and overlying water layers. The viscoelastic model is a combination of both the viscous and elastic effects of mud. It was first developed independently by Hsiao and Shemdin (1980) and Macpherson (1980), and later expanded by many others (Maa and Mehta, 1988; Zhang and Ng, 2006; Ng et al., 2007).

1.2 Purpose and Scope

The current study contributes to the overall goal of understanding the effect of viscoelastic boundaries on surface wave propagation. The two boundaries are floating viscoelastic covers for the wave-ice interactions and viscoelastic bottom beds for the muddy bottom scenario. They can occur independently or in combination with each other. The first objective of the present study is to develop a novel experimental approach for the verification of the theoretical predictions by the viscoelastic models, for both floating covers and bottom beds. New experimental methods are developed in this study to create large uniform viscoelastic sheets with the required range of properties. These methods include the determination of individual components, their proportion, type of curing and finally testing. The rheological properties of the viscoelastic materials are tested using a rheometer.

The second objective of this study is to understand the effect of material properties on surface waves propagating along a region with viscoelastic boundaries. Experiments are conducted in wave flumes, equipped with wave generators, and the test conditions cover a range of wave periods. The theoretical models assume the ice cover or muddy bottom as homogeneous layers, although they are not in reality. The verification of these theoretical models can be performed with high accuracy under a controlled environment in a laboratory with parameters close to the theoretical assumptions. Laboratory experiments in the present study offer two main advantages over field studies: 1) they are relatively inexpensive and 2) the complexity is reduced. It is noted that the present study is the first experimental investigation on surface wave interactions with homogeneous viscoelastic boundaries of targeted properties. In the literature, almost all the validation of theoretical predictions with laboratory wave experiments, using floating covers or viscoelastic mud, were carried out by matching the experimentally obtained wavenumber and attenuation to theoretical predictions and the viscoelastic properties were inversely calculated and might not be the actual material property. In the present

study, the rheological properties of the floating covers and bottom layers are first determined by methodical testing before the experiments are conducted. This enables the rigorous examination of the model predictions.

The discrepancies between the theoretical predictions and experimental results serve to review possible significant factors missing in the theory. Thus, the laboratory experiments performed in the study could aid in the further improvement of theoretical analysis. From the results, the importance of the boundary layer underneath the floating flexible cover is identified which opens new areas of further research along this direction for field cases. The effect of discontinuities of the floating cover on the wave modification is also considered by using floating segments of different lengths. So far, there is no theoretical analysis on broken viscoelastic floating covers reported in the literature.

Finally, an extensive laboratory study of wave-ice interaction is also carried out at the University of Melbourne. Diverse types of fresh water ice covers are prepared under different environmental conditions in a closed refrigerated room. The study enables the analysis of wave dissipation as well as wavelength modification in the presence of these different ice cover types. The results are compared with the existing theoretical models.

1.3 Outline

This thesis is organized into 8 chapters for the above-mentioned objectives. Chapter 1 describes the background information on the necessity of the research work. The purpose and objectives are also introduced. Chapter 2 covers the literature review. A summary of theoretical works, laboratory experiments and field studies in the areas of both sea-ice and muddy bottom are presented. The basic principles of rheometric testing are also included. In Chapter 3, the newly developed experimental methods to prepare the viscoelastic material for floating cover are introduced. The testing of material properties using small amplitude oscillatory shear tests in a rheometer is also provided. Chapter 4

presents the experiments performed with continuous floating viscoelastic covers. A comparison of the experimental results with the existing theoretical models is also provided. The effect of discontinuities on wave modification along with material property is examined in Chapter 5. The wave-ice experiments in the temperature-controlled wave flume are described in Chapter 6. The preparation procedures for the ice types as well as the wave-ice interaction are discussed here. In Chapter 7, the preparation procedures and testing of viscoelastic bottom layers are presented. The experiments to study the surface wave modification due to viscoelastic bottom layers are described. Wave flume experiments as a three-layer system, with both top and bottom viscoelastic layers, are also carried out. Finally, Chapter 8 summarizes the major findings of the present study along with future recommendations.

CHAPTER TWO

Chapter 2

Literature Review

2.1 Sea Ice

Marginal Ice Zone (MIZ) is a transition zone between open water and confined pack ice, with less dense and low concentration smaller ice floes near open water, that grows its concentration and floe size towards the interior (Collins et al, 2016). MIZ is always dynamic in nature with wave ice interactions and plays a vital role in damping the waves from penetrating long distances into the ice-covered sea, thus preventing wave-induced ice breakup of compact ice cover. The field experiments done by Wadhams (1972, 1975 and 1978) suggest that the wave energy decays exponentially with distance along the MIZ and the decay coefficient, α is frequency dependent.

$$S_{\sigma}(x) = S_{\sigma}(0)e^{-\alpha x} \quad (2.1)$$

where $S_{\sigma}(x)$ is the energy density of a spectral component centred at frequency σ . The attenuation coefficient decreases with increasing wave period, T . Thus, MIZ acts as a low pass filter for waves, unless there is a roll-over effect (Wadhams et al. 1988) defined as a rapid increase of α with increasing T . The possible causes of roll-over in fields could be wind input or nonlinear wave energy transfer (Li et al., 2017).

The possible mechanisms for wave-induced attenuation can be classified into scattering and dissipation (Collins et al, 2016). The scattering mechanism considers the effect of wave energy reflection and transmission during wave-ice interactions for single floe or multiple scatterers (Squire, 2007). This is a conservative energy re-distribution mechanism. The dissipation mechanism on the other hand includes the effects of viscosity, internal friction (Zhao et al., 2015), bottom boundary layer turbulence (Liu and Mollo-Christensen, 1988), elastic hysteresis etc. Based on the dominant mechanism, sea

ice models are broadly classified into continuum models and solitary floe models. In the present study, the focus is mainly on the continuum model. The dispersion as well as the rate of decay of surface waves entering an ice-covered region, based on the properties of ice cover, are the main aspects of consideration.

Continuum models treat the ice cover as a floating layer over water which satisfies certain constitutive law based on the type of ice being represented (see Figure 2.1). The dispersion relation can be derived based on the the principles of conservation of mass, momentum and energy depending on the assumptions made for the floating cover and underlying water.

Figure 2.1: Schematic diagram showing ice floating over water.

2.1.1 Elastic plate (EP) model

The thin elastic plate theorem developed by Greenhill (1886) considered the ice cover as an elastic beam of uniform thickness resting on water of uniform depth. The bending of continuous ice cover as an elastic plate was first considered by Wadhams (1974, 1986). The vibration of the cover due to wave action was based on the Euler-Bernoulli beam theory.

$$L_r \frac{\partial^4 \eta}{\partial x^4} + \rho_i h \frac{\partial^2 \eta}{\partial t^2} = P \quad (2.2)$$

where $L_r = \frac{Eh^3}{12(1-\vartheta^2)}$ denotes the flexural rigidity of ice, E is the Young's modulus of elasticity, ϑ is the Poisson's ratio, ρ_i is the density of ice cover, η is the vertical surface displacement and P is the pressure. The problem of unbroken shore fast sea ice as a thin elastic plate, satisfying the governing Laplace's equation (Equation 2.3), can be solved using the linear wave theory (Fox and Squire, 1990).

$$\nabla^2 \varphi = 0 \quad (2.3)$$

where φ denotes the velocity potential. For the thin elastic plate, the submergence of floating cover was not considered. The linearized kinematic and dynamic boundary conditions were used to solve the governing equation to obtain the dispersion relation.

The continuity of pressure yielded the surface boundary condition.

$$\frac{\partial \varphi_1}{\partial y} - \sigma^2 \varphi_1 = 0 \quad \text{at } y = 0 \text{ and } -\infty < x < 0 \quad (2.4a)$$

$$L_r \frac{d^4}{dx^4} \left(\frac{\partial \varphi_2}{\partial y} \right) + (\rho g - \rho_i h \sigma^2) \frac{\partial \varphi_2}{\partial y} - \rho \sigma^2 \varphi_2 = 0 \quad \text{at } y = 0 \text{ and } 0 < x < \infty \quad (2.4b)$$

where ρ is the density of water and g is acceleration due to gravity. The subscripts 1 and 2 indicate the domain considered, 1 for open water region and 2 for ice covered region as shown in Figure 2.1.

The rigid sea floor, $y = -H$ and $-\infty < x < \infty$, provided the zero-penetration boundary condition:

$$\frac{\partial \varphi}{\partial y} = 0 \quad (2.5)$$

The dispersion relation for the ice-covered region can be obtained by solving the governing equation (Equation 2.3) with the boundary conditions (Equations 2.4b and 2.5) as follows:

$$\sigma^2 - \frac{L_r k^5 + \rho g k}{\rho \coth(kH) + \rho_i h k} = 0 \quad (2.6)$$

where k represents the wavenumber along the ice-covered region. This theory was later extended to include normal as well as oblique reflection and transmission (Fox and Squire, 1990; 1994).

The validation of thin elastic plate theorem was done by Sakai and Hanai (2002) using polyethylene plates of two different thicknesses. Along with the validation, they also derived an empirical formula from Equation 2.6 for a broken ice cover. The magnitude of principal/ dominant wavelength obtained by the continuous thin elastic plate analysis depends on the supremacy between inertial effect and the elastic effect (Squire, 1993). For semi-infinite elastic plate with high rigidity, the dominant wavenumber is usually larger than that in open water. Sakai and Hanai observed that the phase velocity of wave decreased (or wavenumber increased) gradually with the decrease in the length of segments, l_s . Thus, for different segment lengths, an equivalent elastic modulus (E_{eq}) could be obtained by substituting their corresponding wavenumber in Equation 2.6. It was found that the non-dimensional equivalent elastic modulus $E_{im} = \frac{E_{eq}}{E}$ showed a linear relation with a parameter IF that considers the effects of both the cover thickness (h) as well as segment length (l_s).

$$IF = \sqrt{\frac{h}{l_c}} \log\left(\frac{l_s}{l_c}\right) \quad \text{where } l_c = \left(\frac{Eh^3}{12\rho g(1-\theta^2)}\right)^{1/4} \quad (2.7)$$

The value of E_{im} for different combinations of h and l_s can be reversely calculated from the linear relation as

$$E_{im} = \begin{cases} 0 & IF \leq 0.145 \\ \frac{IF-0.145}{0.455} & 0.145 < IF \leq 0.6 \\ 1 & 0.6 < IF \end{cases} \quad (2.8)$$

The wave-ice experiments done by Cheng (2018) using ice floes of different lengths, showed a similar trend in the variation of wave celerity with l_s . However, unlike the linear relation obtained by Sakai and Hanai (2002), the reverse calculated E_{im} values for ice floes using Equation 2.6 showed a wider spread not satisfying Equation 2.8.

2.1.2 Mass loading (ML) model

The mass loading model assumes the ice field as a floating mat composed of non-interacting point masses with no cohesion (e.g. Peters, 1950; Weitz and Keller, 1950; Shapiro and Simpson, 1953, Keller and Weitz, 1953). Thus, the ML model is applicable to pancake ice, broken ice and brash ice of low concentration. Weitz and Keller (1950) studied the partial reflection of waves from a floating material made of independent point masses. The surface boundary condition was based on the vertical velocity as:

$$\frac{\partial \phi}{\partial y} = \phi \left(\frac{\rho \sigma^2}{\rho g - \rho_i h \sigma^2} \right) \quad (2.9)$$

Higher frequency waves would experience a total reflection from the ice edge, with a maximum frequency limit of $\sigma^2 = \frac{\rho g}{\rho_i h}$ for the propagation into ice field (Peters, 1950). Shapiro and Simpson (1953) studied the effects of ice field on the angle of wave incidence, angle of reflection and wave period. Squire (1993) made a comparison between the ML model and thin EP model, he showed that the ML model is a limiting case of the EP model with zero rigidity. The dispersion relation, Equation 2.6, thus reduces to

$$\sigma^2 - \frac{\rho g k}{\rho \coth(kH) + \rho_i h k} = 0 \quad (2.10)$$

A wave entering such a medium experiences wave shortening with respect to the open water case (Wadhams, 1986). From the experiments by Sakai and Hanai (2002) using broken polyethylene sheets, it was evident that with the reduction in the length of floating cover, there was a gradual shift in phase velocity towards the mass loading velocity. The value of dimensionless equivalent elastic modulus (E_{im}) decreased with the decrease in the value of IF or l_s and finally became equal to zero. Similar trend was observed for the experiments by Cheng et al. (2018) using real ice floes of different lengths.

The experiments by Newyear and Martin (1997) showed that the ML model is not applicable for grease ice. The interaction among the ice crystals violates the assumption of non-interaction in the ML model. Finally, the ML model always predicts wave

shortening, but the experimental results showed that wave lengthening can occur closer to the viscous model, which will be discussed in the next section.

2.1.3 Viscous model

The hydrodynamic interactions among the ice crystals provide a viscous nature to grease ice (Martin and Kauffman, 1981) that helps it act as an excellent wave absorber. The continuous turbulence provided during grease ice formation in the laboratory prevents the crystals from forming bonds to coalesce. Similarly, in real ice fields, the presence of cold windy conditions favours the grease ice formation. The initial theoretical attempts on the wave propagation through a viscous medium were done independently by Stokes (1851) and Lamb (1932) for an infinite deep layer with constant viscosity. The first two-layer viscous model was developed by Weber (1987) assuming the ice as a thin viscous layer over infinitely deep inviscid water. This was later extended to finite water depth by Keller (1998) by solving the linear Navier-Stokes equation in the upper viscous layer and Euler equation in the lower inviscid layer.

The boundary conditions are:

1. At the free surface, the kinematic boundary condition applies along with the normal stress continuity (= 0 due to inviscid air).
2. At the water ice interface, the continuity of normal and tangential stress components applies along with the kinematic condition (continuity of vertical velocity).
3. Rigid bottom with zero penetration condition.

The corresponding dispersion relation obtained is as follows:

$$\left\{ (2\nu k^2 - i\sigma) - \frac{4\nu k^3 \alpha_{kv}}{(k^2 + \alpha_{kv}^2) \sinh \alpha_{kv} h} (e^{kh} - e^{\alpha_{kv} h}) + \left[\frac{2k^3 (-2\nu \alpha_{kv} \sigma + ig)}{\sigma(k^2 + \alpha_{kv}^2)} + \frac{gk}{i\sigma} \right] \right\} \cdot \left\{ (2\nu k^2 - i\sigma) e^{-kh} + \left[\frac{i\sigma\rho}{\rho_i \tanh kH} + \frac{g(\rho - \rho_i)k}{i\sigma\rho_i} \right] \left[e^{-kh} - \frac{2k^2}{(k^2 + \alpha_{kv}^2)} e^{\alpha_{kv} h} \right] + \right.$$

$$\begin{aligned}
& \frac{4\nu k^3 \alpha_{kv}}{(k^2 + \alpha_{kv}^2)} e^{\alpha_{kv}h} + m(e^{\alpha_{kv}h} - e^{-kh}) \Big\} - \Big\{ 2\nu k^2 - i\sigma - \\
& \frac{4\nu k^3 \alpha_{kv}}{(k^2 + \alpha_{kv}^2) \sinh \alpha_{kv}h} (e^{\alpha_{kv}h} - e^{-kh}) - \left[\frac{2k^3(-2\nu \alpha_{kv} \sigma + ig)}{\sigma(k^2 + \alpha_{kv}^2)} + \frac{gk}{i\sigma} \right] \Big\} \cdot \Big\{ (2\nu k^2 - i\sigma) e^{kh} - \\
& \left[\frac{i\sigma\rho}{\rho_i \tanh kH} + \frac{g(\rho - \rho_i)k}{i\sigma\rho_i} \right] \left[e^{kh} - \frac{2k^2}{k^2 + \alpha_{kv}^2} e^{\alpha_{kv}h} \right] - \frac{4\nu k^3 \alpha_{kv}}{(k^2 + \alpha_{kv}^2)} e^{\alpha_{kv}h} + m_v(e^{\alpha_{kv}h} - \\
& e^{-kh}) \Big\} = 0
\end{aligned} \tag{2.11}$$

where ν is the kinematic viscosity, $\alpha_{kv} = \sqrt{k^2 - i\sigma/\nu}$ and $m_v = \left\{ 2\nu k \alpha_{kv} (e^{\alpha_{kv}h} + e^{-\alpha_{kv}h}) + \left[\frac{i\sigma\rho}{\rho \tanh kH} + \frac{g(\rho - \rho_i)k}{i\sigma\rho} \right] \cdot (-ie^{\alpha_{kv}h} + ie^{-\alpha_{kv}h}) \right\} \frac{ik^2}{(k^2 + \alpha_{kv}^2) \sinh \alpha_{kv}h}$.

Two limiting conditions, i.e. short and long waves with respect to the top layer thickness, were considered in Keller (1998). The short-wave equation reduced to the infinitely deep viscous liquid condition (Lamb, 1945). The model predicted strong attenuation. In terms of wave length, no change was predicted for low wave frequencies while shortening for high wave frequencies. This theoretical model was further extended by De Carolis and Desiderio (2002), with the introduction of viscosity to the underlying water. A theoretical calculation of frequency dependent viscosity for grease ice was derived by De Carolis et al (2005). Both the models of Keller (1998) and De Carolis and Desiderio (2002) were validated using grease ice experiments conducted by Newyear and Martin (1999). They also found that the contribution of water viscosity on attenuation was negligible compared to that of grease ice.

For thicker grease ice, the flocculation of ice crystals along with the vertical compaction caused an increase in the value of viscosity. The grease-pancake wave-ice experiments by Wang and Shen (2010a) showed that the two-layer viscous model cannot be used to model this condition. The experimental results did not match well with the predictions for both constant viscosity as well as frequency dependent viscosity.

2.1.4 Viscoelastic (VE) model

Unlike the above-mentioned models, the viscoelastic (VE) model with the inclusion of both viscous and elastic properties can be considered as a generalised model for any type of ice cover. The EP model, ML model as well as viscous model are all limiting cases of the VE model (Wang and Shen, 2010b). At present, there are three existing VE models found in the literature: Squire and Alan (1980), Robinson and Palmer (1990), and Wang and Shen (2010b). Throughout this thesis, the three models will be referred as SA model, RP model and WS model, respectively. The physical configuration of all the three models is the same as provided in Figure 2.1. The models differ in how the top layer is formulated. The dispersion relations for all the three models show the dependence of complex wavenumber, $k = k_s + ik_i$ on the wave frequency, where k_s is the real wavenumber and k_i is the attenuation / decay coefficient.

The SA model assumes the floating ice cover as a thin viscoelastic plate that satisfies the Maxwell-Voigt condition over deep water ($H \rightarrow \infty$). This system was simplified to a Voigt model and finite water depth by Li et al. (2015). The floating plate of the simplified model has a complex shear modulus, $G_V = G - i\sigma\rho_i\nu$, where G is the shear modulus. The real part of G_V considers the elastic nature while the imaginary part deals with the viscous nature. The dispersion relation for the SA model is as follows:

$$(\sigma^2 - Q_{SA}gk\tanh kH) = 0 \quad (2.12)$$

$$Q_{SA} = \frac{G_V h^3}{6g\rho(1-\vartheta)} k^4 - \frac{\rho_i h \sigma^2}{\rho g} + 1 \quad (2.13)$$

Robinson and Palmer (1990) developed a viscoelastic theory for the wind-induced vibration of floating roofs on oil storage tanks. The analogous fundamental principles enabled the RP model to be applied for sea ice as well. The floating cover was assumed to be a purely elastic Euler Bernoulli beam, but with a viscous term, γ , proportional to the velocity of the vertical displacement under wave action (Mosig et al., 2015). The dispersion relation for the RP model is

$$(\sigma^2 - Q_{RP}gk\tanh kH) = 0 \quad (2.14)$$

$$Q_{RP} = \frac{Gh^3}{6g\rho(1-\vartheta)}k^4 - \frac{\rho_i h \sigma^2}{\rho g} + 1 - \frac{i\sigma}{\rho g}\gamma \quad (2.15)$$

Since γ is not purely a material property but also includes the effects of other damping mechanisms including frictional losses, its value can only be determined by fitting the attenuation with the experimental results.

Unlike the SA and RP models which assumed the ice cover as a Euler Bernoulli beam, Wang and Shen (2010b) extended the two-layer viscous model by Keller (1998) to derive the viscoelastic theory. The floating ice was considered as a Voigt-type viscoelastic fluid with a complex kinematic viscosity, $\nu_e = \nu + iG/\rho_i\sigma$. The dispersion relation for the WS model is

$$(\sigma^2 - Q_{WS}gk \tanh kh)J = 0 \quad (2.16)$$

$$\text{with } Q_{WS} = 1 + \frac{\rho_i g^2 k^2 S_k S_\alpha - (N_{ve}^4 + 16k^6 \alpha_k^2 \nu_e^4) S_k S_\alpha - 8k^3 \alpha_k \nu_e^2 N_{ve}^2 (C_k C_\alpha - 1)}{J} \quad (2.17)$$

$$\text{and } J = gk(4k^3 \alpha_k \nu_e^2 S_k C_\alpha + N_{ve}^2 C_k S_\alpha - gk S_k S_\alpha) \quad (2.18)$$

where $\alpha_k = k^2 - i\sigma/\nu_e$, $S_k = \sinh(kh)$, $S_\alpha = \sinh(\alpha_k h)$, $C_k = \cosh(kh)$, $C_\alpha = \cosh(\alpha_k h)$ and $N_{ve} = \sigma + 2ik^2 \nu_e$. The WS model predicts multiple roots for the dispersion relation, but only the dominant root is considered in the present study. The dominant root, referred to as the flexural-gravity wave mode, has the wave number k_s close to open water wavenumber. Note that the least damped roots for the dispersion relation are the pressure roots and gravity roots. Both roots have equal importance only when $\sigma \approx g\sqrt{\rho_i/4G}$ (Zhao et al., 2017). The wave frequency considered in the present study is away from this value, thus only the flexural-gravity mode dominates the vertical wave motion.

Zhao and Shen (2015) conducted wave experiments with three distinct types of ice covers: frazil/pancake, pancake and fragmented ice. The experimental results matched well with the WS model for equivalent viscosity and shear modulus. The values of inversely determined equivalent viscosity and elasticity were maximum for the fragmented ice and minimum for frazil/pancake ice.

Apart from the laboratory experiments, various wave-ice field experiments were also conducted and compared with existing theoretical models to determine the best fit. The field experiments conducted on land-fast ice at Main Tickle near Twillingate showed good agreement on wavelength with the theoretical predictions by thin elastic plate model (Squire and Alan, 1980). The Mass loading model, also termed as the inertial surface model, was used to characterise pancake ice by Wadhams et al. (2002). However, the inversely derived thickness of pancake ice cover from the east Greenland coast showed unrealistic results compared to in-situ measurements. Wadhams et al. (2004) used the Keller's two-layer viscous model to measure the ice thickness at the Southern Ocean. The inversely calculated thickness showed good agreement with in situ measurements. However, the viscous model provided unsatisfactory results for the data collected from the east Greenland coast (Wadhams et al., 1999; 2004). Field ice experiments conducted at the MIZ of the Weddell Sea by Doble et al. (2015) found that the inversely calculated viscosity value was unrealistically high for pancake ice cover using the viscous model. This motivated the use of viscoelastic model for pancake ice cover instead, and the viscosity was reduced by ~10-15 times for a shear modulus of $G = 2.0 \text{ GPa}$.

2.2 Muddy Bottom

Extraordinary calming effect in the mudbanks of the Mississippi Delta was studied by Tubman and Suhayda (1977). They found that the energy loss by the muddy bottom was one order larger than with a sandy bottom or due to bottom friction effect alone. Similarly, the south west coastal region of India experiences the wave energy decay of around 90% along a 1 km stretch of muddy bottom, which enables a protected coastal zone for fishing especially during monsoon (Mathew et al, 1995).

The first quantitative wave-mud laboratory experiments were performed by Yamamoto and Schuckman (1984) using bentonite clay beds. It was found that the surface wave

attenuation increases non-linearly with the wave height. The bed displacement also increases nonlinearly with the wave height, and damping becomes maximum when the bed exhibits damped resonance motion. The laboratory experiments by Yamamoto et al. (1986) using bentonite clay showed that the wave length along the mud bed region was 14-29% longer than that obtained with a rigid bottom or sandy bottom. Wave flume experiments by Soltanpour et al. (2011) using kaolinite beds showed a similar dependence of wave height on the attenuation coefficient. On the contrary, clay bed experiments by Nagai et al. (1984) showed that attenuation decreases with the increase in wave height. A similar trend was observed in the experiments by Sakakiyama and Bijker (1989). The reason was attributed to the nonlinearity of the shear stress and shear strain relation in the mud layer.

In the present study, the effect of rheological properties of the bed layer on the surface wave damping as well as phase velocity is considered. Different models for the muddy bottom are available in the literature that suit the different rheology of mud. In the following sections, three different models based on the physical properties of mud are introduced. The schematic diagram representing the wave-mud interaction is shown in Figure 2.2.

2.2.1 Viscous model

An infinite two-layer viscous system for muddy bottom was first introduced by Gade (1957) for shallow water conditions. The bottom mud was assumed to be a homogeneous viscous fluid layer of finite thickness resting on a rigid bottom. The top layer of water was assumed to be inviscid, which unlike the bottom layer cannot develop any shear stresses in motion to cause wave dissipation. De Wit and Kranenburg (1996) modified the Gade's model to deep water waves over a shallow mud layer and validated the model with wave flume experiments with two different artificial muds.

Figure 2.2: Schematic diagram of wave propagation over a muddy bottom.

Dalrymple and Liu (1978) extended the Gade's model to form a “complete model” for intermediate water with the incorporation of viscosity to the upper layer. The governing equations were the linearized Navier-Stoke's equation for incompressible fluid under laminar flows and the conservation of mass.

$$\frac{\partial u_i}{\partial t} = -\frac{1}{\rho_i} \frac{\partial P_i}{\partial x} + \nu_i \left(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2} \right) \quad (2.19)$$

$$\frac{\partial v_i}{\partial t} = -\frac{1}{\rho_i} \frac{\partial P_i}{\partial y} + \nu_i \left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} \right) \quad (2.20)$$

where u and v represent the velocity along x - and y - directions and subscripts $i = 1, 2$ represents the upper (water) and lower (mud) layers respectively. The dynamic pressure $P_i = P_i^t + \rho_i g y + P_i^0$, where P_i^t is the total pressure, $P_1^0 = 0$ and $P_2^0 = (\rho_2 - \rho_1) g H_w$.

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0 \quad (2.21)$$

The boundary conditions adopted were:

At the fixed bottom, $y = -(h_m + H_w)$, the “no penetration” condition is satisfied

$$v_2 = 0 \quad (2.22)$$

At the interface, $y = -H_w$, the kinematic condition, continuity of velocities as well as continuity of normal and tangential stresses were satisfied

$$v_2 = v_1 = \frac{\partial \xi}{\partial t} \text{ and } u_2 = u_1 \quad (2.23)$$

$$P_1 - 2\rho_1 v_1 \frac{\partial v_1}{\partial y} - \rho_1 g \xi = P_2 - 2\rho_2 v_2 \frac{\partial v_2}{\partial y} - \rho_2 g \xi \quad (2.24)$$

$$\rho_1 v_1 \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right) = \rho_2 v_2 \left(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x} \right) \quad (2.25)$$

At the free surface, $y = -\eta$, the kinematic conditions along with zero normal and tangential stresses were applied.

$$v_1 = \frac{\partial \eta}{\partial t} \quad (2.26)$$

$$P_1^t - 2\rho_1 v_1 \frac{\partial v_1}{\partial y} = 0 \quad (2.27)$$

$$\rho_1 v_1 \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right) = 0 \quad (2.28)$$

The dispersion relation by Gade (1958) was derived as:

$$k = \pm \sigma \left\{ \frac{\left(1 + \Gamma \frac{h_m}{H_w} \right) \pm \sqrt{\left(1 + \Gamma \frac{h_m}{H_w} \right)^2 - 4\Gamma \gamma \frac{h_m}{H_w}}}{2\gamma g \Gamma h_m} \right\}^{1/2} \quad (2.29)$$

where $\Gamma = 1 - \frac{\tanh(m h_m)}{m h_m}$, $m = (1 - i) \sqrt{\frac{\sigma}{2\nu_m}}$ and $\gamma = \frac{\rho_2 - \rho_1}{\rho_2}$.

The theory was experimentally validated with a two-layer system of kerosene over sugar solution. It was found that the wave decays exponentially with distance along the viscous bottom layer, and the dissipation increases with the increase in bottom layer thickness until a peak value, with the corresponding normalized mud layer thickness $\left(\sqrt{\sigma/2\nu_m} \right) h_m = 1.2$. After the peak value was attained, the dissipation decreased asymptotically to a limiting value. Winterwerp et al. (2007) used the Gade's model in the software SWAN to predict the wave attenuation for the Guyana mud coast.

Dalrymple and Liu (1978) also introduced a boundary layer (BL) model which agreed well with their "complete model". The two models showed reasonable agreement with

the experiments by Gade (1958) in terms of wavenumber and attenuation, except the phase difference between the interface and free surface displacements.

The dispersion relation for the BL model is as follow:

$$\sigma^2 = gk \frac{(-p - (p^2 - 4qr)^{\frac{1}{2}})}{2q} \quad (2.30a)$$

$$p = -\frac{\rho_2}{\rho_1} \tanh kh_m \tanh kH_w \times (1 + \tanh kh_m \tanh kH_w), \quad (2.30b)$$

$$q = \frac{\rho_2}{\rho_1} + \tanh kh_m \tanh kH_w, \quad (2.30c)$$

$$r = \left(\frac{\rho_2}{\rho_1} - 1\right) \tanh kh_m \tanh kH_w. \quad (2.30d)$$

The damping coefficient for the BL model is given by

$$k_i = \frac{(\sigma v_1)^{1/2} \left[C_1^2 + \frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} (C_2^2 + C_3^2) \right]}{2gC_g a^2 \left\{ 1 + \left[\frac{\rho_2}{\rho_1} - 1 \right] \left(\frac{kg}{\sigma^2} \right)^2 \cosh^2 kH_w \left[\frac{\sigma^2}{gk} - \tanh kH_w \right]^2 \right\}} \quad (2.31a)$$

$$C_1 = \frac{gka}{\sigma \sinh kH_w} \frac{\left[\frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} \right]}{\left[1 + \frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} \right]} \times \left\{ \frac{\sigma^2}{gk} - \frac{1}{2} \sinh 2kH_w [\coth kH_w + \coth kh_m] \times \right. \\ \left. \frac{\sigma^2}{gk} - \tanh kH_w \right\}, \quad (2.31b)$$

$$C_2 = -\frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} C_1, \quad (2.31c)$$

$$C_3 = \frac{gka}{\sigma} \frac{\cosh kH_w}{\sinh kh_m} \left(\frac{\sigma^2}{gk} - \tanh kH_w \right). \quad (2.31d)$$

For shallow water cases, Equations 2.30a and 2.31a reduce to Equations 2.32 and 2.33 respectively.

$$\sigma^2 = \frac{gk^2}{2} \left(h_m + H_w + \left\{ [H_w - h_m]^2 + 4 \frac{\rho_1}{\rho_2} h_m H_w \right\}^{1/2} \right) \quad (2.32)$$

$$k_i = \frac{(2d)^{-2} \left(\frac{kg}{\sigma^2} \right) \left(\frac{v_1}{\sigma} \right)^{1/2} \left\{ \left[k(h_m + H_w) - \left(\frac{\sigma^2}{gk} \right) \right]^2 + \left[1 + \frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} \right] \left[\frac{\sigma^2}{gk} - kH_w \right]^2 \right\}}{\left[1 + \frac{\rho_2}{\rho_1} \left(\frac{v_2}{v_1} \right)^{1/2} \right] \left\{ 1 + \left[\frac{\rho_2}{\rho_1} - 1 \right] \left[1 - \left(\frac{kg}{\sigma^2} \right) kH_w \right]^2 \right\}} \quad (2.33)$$

Ng (2002) provided an approximation of the BL model by Dalrymple and Liu (1978) when the thickness of mud layer is comparable to the Stokes boundary layer ($h_m \sim \delta_{BL}$) and much thinner than overlying water layer ($h_m \ll H_w$). He concluded that the maximum dissipation of wave by mud occurs when the viscosity of mud is high, $h_m \approx$

1.5 δ_{BL} , and $\rho_2 \sim \rho_1$. The laboratory experiments by Jiang and Mehta (1992) measured the viscosity of mud from the Lake Okeechobee, Florida, at different bulk densities. The comparison of measured mud particle acceleration with that predicted by the shallow water wave model suggested that the viscous fluid assumption for mud is reasonable.

2.2.2 Elastic model

Mallard and Dalrymple (1977) developed an elastic model for the mud layer, with the two-layer system as water resting over a perfectly elastic soil of infinite depth. They used the elastic displacement equations for the mud layer with stress equilibrium within the soil. The soil deformation due to the wave action can modify the wave speed, but there is no energy dissipation. Dawson (1977) extended this model with the addition of soil inertia. The equations for an elastic solid subjected to initial stresses due to the self-weight and overlying water were considered in his study as:

$$\rho_m \frac{\partial^2 X}{\partial t^2} = (\Lambda_l + N_l) \frac{\partial}{\partial x} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right) + N_l \left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2} \right) \quad (2.34)$$

$$\rho_m \frac{\partial^2 Y}{\partial t^2} = (\Lambda_l + N_l) \frac{\partial}{\partial y} \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right) + N_l \left(\frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} \right) \quad (2.35)$$

where X and Y represent the horizontal and vertical displacements of soil respectively, and Λ_l and N are the Lamè elastic constants. The boundary conditions at the mud-water interface are similar to that for the viscous model: a) zero shear stress, b) normal stress equal to dynamic water pressure, and c) continuity of vertical velocity at the water-mud interface.

The wave-speed relation obtained by Dawson (1978) is given below

$$\left[\frac{\alpha_2^2 + k^2}{\alpha_1 k} \left(\frac{c}{c_2} \right) - \frac{2(\alpha_2^2 + k^2)}{\alpha_1 k} + \frac{4\alpha_2}{k} \right] \left[\sin kH_w - \frac{kc^2}{g} \cosh kH_w \right] = \frac{\rho_1}{\rho_2} \left(\frac{c}{c_2} \right)^4 \left[\cosh kH_w - \frac{kc^2}{g} \sinh kH_w \right] \quad (2.36)$$

where, $\alpha_1 = \left[1 - \left(\frac{c}{c_1}\right)^2\right]^{1/2} k$, $\alpha_2 = \left[1 - \left(\frac{c}{c_2}\right)^2\right]^{1/2} k$, $c_1 = \left[\frac{\Lambda+2N}{\rho_2}\right]^{1/2}$, $c_2 = \left[\frac{N}{\rho_2}\right]^{1/2}$, and $c = \sigma/k$ is the phase velocity of the wave.

Mohapatra and Sahoo (2011) considered the wave interaction with an elastic bottom plate. The two-layer system was modelled as the inviscid water resting on a semi-infinite incompressible Euler-Bernoulli beam. The dispersion relation obtained is given below,

$$\begin{aligned} &\rho_1 \sigma^4 \tanh kH_w - \sigma^2 k [\rho_1 g + E_m I k^4 + (\rho_1 g - \rho_2 h_m \sigma^2)] + \\ &g k^2 \tanh kH_w [E_m I k^4 + (\rho_1 g - \rho_2 h_m \sigma^2)] = 0 \end{aligned} \quad (2.37)$$

where $E_m I$ is the flexural rigidity of the elastic plate.

2.2.3 Viscoelastic models

The first viscoelastic model for muddy bottom was independently developed by Hsiao and Shemdin (1980) and Macpherson (1980). Both models assumed that the mud behaved as a viscoelastic layer that satisfies the Voigt body criteria, $\tau = G_m \epsilon + \nu_m \rho_m \dot{\epsilon}$, where G_m is the shear modulus of mud, τ is shear stress, ϵ is shear strain and, $\dot{\epsilon}$ is strain rate. The behaviour of mud was provided by the following linear 2-D equations of motion

$$\frac{\partial^2 u}{\partial t^2} = J_m \nabla^2 u + \nu_m \nabla^2 \frac{\partial u}{\partial t} - \frac{1}{\rho_m} \frac{\partial^2 p}{\partial x \partial t} \quad (2.38)$$

$$\frac{\partial^2 v}{\partial t^2} = J_m \nabla^2 v + \nu_m \nabla^2 \frac{\partial v}{\partial t} - \frac{1}{\rho_m} \frac{\partial^2 p}{\partial y \partial t} \quad (2.39)$$

where $J_m = G_m / \rho_m$. These equations reduce to the Navier-Stokes equation for the viscous model. The tests conducted on soft mud bed by Maa and Mehta (1988) agreed that the Voigt model fits well for mud with a linear response to strain and resistance to shear force.

The linearized boundary conditions for the Macpherson's model at the mud-water interface are as follows: a) shear stress continuity ($= 0$, due to inviscid water), b) continuity of normal velocity (the continuity of horizontal velocity i.e., no slip condition,

is not considered), and c) the continuity of normal stress balanced with the surface tension. Hsiao and Shemdin's model had the same boundary conditions except the dynamic boundary condition where the surface tension effect was not considered.

The dispersion relation obtained by Hsiao and Shemdin (1980) is given below

$$k = \frac{\sigma^2}{g} \frac{1 + \tanh kH_w \Omega}{\tanh kH_w + \Omega} \quad (2.40)$$

$$\Omega = r \frac{(n^2 - k^2) [\sinh kh_m \cosh nh_m - k/n \cosh kh_m \sinh nh_m]}{(n^2 + k^2) [\cosh kh_m \cosh nh_m - k/n \sinh kh_m \sinh nh_m] - 2k^2} \quad (2.41)$$

where $r = \frac{\rho}{\rho_m}$, and $n = k \left[1 - \frac{\sigma^2}{k^2(J - i\sigma v_m)} \right]$. Similar to the sea ice case, for the viscoelastic mud models, k is a complex number with the real part representing the wave number and imaginary part representing the wave attenuation. The viscoelastic model by Hsiao and Shemdin (1980) generally predicted wave shortening and high wave attenuation.

The dispersion relation obtained by Macpherson (1980) is given below

$$\begin{aligned} & \frac{\rho(\sigma^4 - g^2 k^2) \tanh kH_w}{gk \tanh kH_w - \sigma^2} + \rho_m gk + T_s k^4 + \rho_m (2k^2 v_{me} - \\ & i\sigma)^2 \left\{ \frac{\left((2k^2 - \frac{i\sigma}{v_{me}}) - [l_k C_k C_l - k S_k S_l] - 2k^2 l_k \right)}{\left((2k^2 - \frac{i\sigma}{v_{me}}) [l_k S_k C_l - k C_k S_l] \right)} \right\} - \\ & 4\rho_m k^3 v_{me}^2 l_k \left\{ \frac{\left((2k^2 - \frac{i\sigma}{v_{me}}) - 2k [l_k C_k C_l - k S_k S_l] \right)}{2k [l_k S_k C_l - k C_k S_l]} \right\} = 0 \end{aligned} \quad (2.42)$$

where T_s is surface tension, the complex viscosity of mud $v_{me} = v_m + i G_m / \sigma \rho_m$, $C_k = \cosh kh_m$, $C_l = \cosh l_k h_m$, $S_k = \sinh kh_m$, $S_l = \sinh l_k h_m$, and $l_k = \left(k^2 - \frac{i\sigma}{v_{me}} \right)^{1/2}$.

The dispersion relation predicted the strong dependence of attenuation on both viscous and elastic properties of the bottom layer. The peak attenuation was predicted when $\rho_m \sigma v_m / G_m \approx 1$. This model was later extended by Piedra Cueva (1993) considering a thin water boundary layer at the mud-water interface. For $G_m > 1$, a system resonance can occur, that amplifies the interface amplitude on the expense of surface energy

dissipation causing very high damping of surface waves. The occurrence of resonance for certain frequencies was also evident during the wave experiments using the Kaolinite bed by Sakakiyama and Bijker (1989). It was found that the damping rate increases with the viscosity for zero elasticity, while the opposite happens with elasticity (Piedra Cueva, 1993). Zhang and Ng (2006) developed a “boundary layer approximated” Voigt viscoelastic model for wave-mud interactions. The mud layer thickness was comparable to the Stokes boundary layer thickness, $\delta_{BL} \equiv \left(2\nu_m/\sigma\right)^{1/2}$ and both were assumed to be of the same order as the wave amplitude. The model matched well with the physical data considered by Piedra Cueva (1993).

The effects of thin viscoelastic mud on the propagation of small amplitude waves were studied by Liu and Chan (2007). The viscosity of both the top and bottom layers were considered with $\rho \ll \rho_m$. Similar to the previous models, maximum attenuation was observed when $h_m \approx \delta_{BL}$, and a small elastic modulus of mud can cause significant attenuation. Higher wave decay was obtained for larger elastic modulus cases due to resonance effect. A linearized multi-layered water-mud system was developed by Maa and Mehta (1990), with the mud bed composed of several layers of Voigt material. The model showed a reasonably good agreement with laboratory experiments using the Kaolinite and natural mud from the Cedar Key, Florida (Maa and Mehta, 1987).

Tubman and Suhayda (1976) conducted field experiments at East Bay, Louisiana. The measured wave attenuation was well predicted using the viscoelastic model by Hsiao and Shemdin (1980). On the other hand, the predictions by viscous models (Gade, 1958; Dalrymple and Liu, 1978) were one order higher in magnitude. The rheological experiments (oscillatory Frequency Sweep tests) by Soltanpour and Samsami (2011) on the kaolinite and Hendijan Coast mud from the Persian Gulf also showed viscoelastic behaviour. They found that the complex shear modulus, $G_m^* = G_m - i\nu_m\rho_m\sigma$, was frequency dependent. The linearized multi-layered viscoelastic model (Maa and Mehta, 1990) provided a reasonably good estimate of the attenuated wave height. The

experimental verification of viscoelastic models with a real viscoelastic bed has not been done so far. An objective of the present study is to fill this research gap by developing a novel experimental method using the PDMS material.

2.3 Material

2.3.1 Sea ice

The properties of sea ice cover can vary widely depending on various parameters such as temperature, salinity, age, grain size, etc. The mean Arctic sea-ice thickness was 1.89 m as per the ICESat (Ice, Cloud, and land Elevation Satellite) records. A larger variation in sea ice thickness can be observed in the MIZ (National Snow & Ice Data Center, 2018). The Poisson's ratio (ν) for sea ice cover is typically high for young ice and decreases with age (salinity reduction). The average Poisson's ratio for fresh ice is 0.33 and for sea ice is 0.295 (Weeks and Assur, 1967). The static measurements on fresh ice conducted by Hirayama et al. (1974) found that the Poisson's ratio was 0.3 under elastic deformation and can go up to 0.8 under plastic deformation. The variation of Poisson's ratio with the state of sea ice is less compared to variation in elastic modulus, thus the linear relation between the shear modulus (G) and elastic modulus (E) (*i. e.* $G = \frac{E}{2(1+\nu)}$) is typically assumed to be valid for sea ice (Weeks and Assur, 1967).

The in-situ cantilever tests conducted at the Okhotsk Sea Coast found that the Young's modulus for sea ice was 100-1200 MPa ($\sim G = 40 - 460$ Mpa) during the winter (Tabata et al., 1967). Static uniaxial compression tests performed on ice samples at 10°C in the Cold Regions Research and Engineering Laboratory in Hanover, New Hampshire (Richter-Menge, 1984) showed that the Arctic sea ice had a Young's modulus of ranging from 5.0 GPa to 7.2 GPa ($\sim G = 1.9 - 2.8$ GPa). Wadhams (1973b) used an elastic modulus of 6.0-8.0 GPa to determine the attenuation coefficient in fields of sea ice assuming the steady state creep mechanism.

Table 2.1 provides the effective values of material properties obtained by inverse calculation (IC) from theoretical models. The field experiments by Wadham et al. (2004) on 24-cm thick Antarctic pancake ice using synthetic aperture image (SAR), inversely calculated the optimum ice viscosity by assuming the ice cover as a viscous layer. The effective viscosity obtained was in the range of the laboratory experiments performed by Newyear and Martin (1999).

The effective viscosity and shear modulus from the laboratory experiments by Zhao and Shen (2015) using the viscoelastic model showed that both properties increased from frazil to pancake ice to fragmented ice cover. Field ice experiments conducted at the MIZ of the Weddell Sea by Doble et al. (2015) used an effective ice viscosity of $0.01 \text{ m}^2/\text{s}$ and $765 \text{ m}^2/\text{s}$ for two different regimes to compare with the predictions from the two-layer viscous model. The very high viscosity value for the second regime motivated the use of viscoelastic model for pancake ice cover, which the viscosity was reduced to $58 \text{ m}^2/\text{s}$ for a shear modulus of $G = 2.0 \text{ GPa}$. The experimental and theoretical studies available in the literature clearly point out that the rheological characterisation of sea ice cover remains uncertain, especially the viscosity measurements.

2.3.2 Mud

A muddy sea bottom has very complex rheological properties, and its viscoelastic properties can vary widely. Sea mud can be modelled in various manners as discussed earlier in Section 2.2. Mud belongs to the category of granular materials with grain size or particle size less than $62.5\mu\text{m}$. Based on particle size, mud can further be sub-divided into silt, clay and colloid.

Table 2.1: Properties of ice cover from laboratory experiments

Study	Ice type	Ice thickness (cm)	Shear modulus (Pa)	Viscosity (m²/s)	Remarks (IC: Inverse calculation)
Martin and Kauffman (1981)	Grease ice	5.0 – 16.0	-	-	Relative ice volume was measured.
Newyear and Martin (1997)	Grease ice	11.3 - 14.6	-	1.27×10^{-2} - 2.4×10^{-2}	IC: Lamb (1932)
Newyear and Martin (1999)	Grease ice	11.0 – 15.0	-	2.5×10^{-2} - 3.0×10^{-2}	IC: Keller (1998)
De Carolis and Desiderio (2005)	Grease ice	11.3 – 14.6		2.0×10^{-2} – 5.0×10^{-2}	IC: De Carolis and Desiderio (2005)
Wang and Shen (2010a)	Grease-pancake ice	7.8 – 11.5	-	0.18 - 0.01	IC: Keller (1998), ice thickness and pancake diameter were measured
Zhao and Shen (2015)	Frazil/pancake ice mixture	2.5	21	1.40×10^{-2}	IC: Wang and Shen (2010b)
	Pancake ice	4.0	5.1×10^5	61.10	
	Fragmented ice	7.0	1.0×10^6	139.30	

The electrostatic bond among the mud particles, termed as cohesion, provide the shear strength against any sort of shear deformation. With the increase in grain size, the effect of cohesion reduces, and internal friction becomes dominant for sand. Depending on the density, mud can be classified as plastic mud and elastic mud. For plastic mud, the density is less than 1250 kg/m^3 while those with density $> 1250 \text{ kg/m}^3$ are termed as soft mud (the elastic property of mud is usually considered as soft).

The rheological properties measured in the previous laboratory studies on wave-mud interactions are provided in Table 2.2. Unlike the sea ice cover, where definitive procedures for viscosity measurements have not been established yet, the non-Newtonian mud properties can be measured to reasonable accuracy. The shear modulus for marine sediments was determined by Coleman et al. (1980) and Dawson et al. (1981) based on their movement under wave action. The shear modulus was frequency dependent ranging between 3.0 MPa to 6.0 MPa.

2.3.3 Polydimethylsiloxane

Polydimethylsiloxane (PDMS) $[(\text{CH}_3)_2\text{SiO}]_n$ is a widely used polymer in siloxane elastomers due to its unique properties. PDMS networks are formed by the hydrosilation reactions taking place between the hydrosilane (SiH) bond of the cross-linker or curing agent and carbon-carbon double bond in the vinyl group ($-\text{CH}=\text{CH}_2$) of the prepolymer, in the presence of catalyst (Stafie, et al., 2005). The amount of cross linking depends on the ratio of hydrosilane to vinyl group, in other words the pre-polymer/cross-linker ratio (also known as stoichiometric ratio, r_s). Venkataraman et al. (1989) found that the storage modulus of fully cured PDMS samples (at 34°C) increased with the stoichiometric ratio until $r_s = 1$. For $r_s > 1$, the shear modulus decreased.

PDMS is widely used in the field of microfluidics, electronics, textiles, automobiles, construction, medicine, cosmetics and so on. The mold-releasing property with low shrinkage ($\sim 1\%$) and ability to reproduce features to smaller scales make it a good casting

material for microfluidic devices using preformed moulds (Friend and Yeo, 2010). The optical transparency of PDMS along with its gas permeability enables cell culturing in microscopic devices (Halldorsson, et al., 2015).

Table 2.2: Properties of mud for laboratory experiments

Study	Mud type	Mud thickness (cm)	Shear modulus (Pa)	Viscosity (m²/s)	Remarks (IC: Inverse calculation)
Maa and Mehta (1988)	Kaolinite	-	20 - 500	0.001 – 0.4	Measured
Soltanpour and Samsami (2011)	Kaolinite	8.0	200 - 19000	0.005 -2.0	Measured
	Hendijan mud	8.0	800 - 35000	0.009- 5.0	Measured
Hsu et al. (2013)	Hendijan mud	8.0	800 - 35000	0.009- 5.0	Measured
				0.80×10^{-3} – 3.03×10^{-3}	IC: Dalrhymple and Liu (1978)

In the field of electrical engineering, PDMS is commonly used for PV encapsulants for solar applications (Leem et al., 2014) due to its high durability and high transparency. The frequency independent shear modulus and sufficiently good adhesion (increased with the usage of primer) towards oxidized silicon enable its use as the interconnection layer between silicon wafers for microsystem assembly (Lotters et al., 1997). The chemical inertness, high compressibility, high thermal stability, stable properties (less variation with temperature and time) qualifies the usage of PDMS in capacitive accelerometers as springs with linear response and adjustable high sensitivity (Lotters et al., 1996). With the low surface tension, the usage of PDMS as antifoaming agents and protective coatings / polishes find applications in the oil and gas industry as well as automobile industry (Bergeron et al., 1997; Rome and Hueston, 2002). The inertness and high diffusivity of PDMS also make it suitable for the development of gas chromatographic columns with high performance (Seethapathy and Gorecki, 2012), and

the extraction of off-flavour chemicals in beer with sufficient accuracy (Marsili et al. 2007).

The effect of curing temperature on the mechanical properties of PDMS was studied by Liu et al. (2009) with a constant curing agent to base ratio (1:10). The tensile tests conducted based on ASTM specifications showed that the strength of sample prepared was inversely proportional to the heating temperature, and a prolonged heating at elevated temperatures ($>200^{\circ}\text{C}$) caused thermal decomposition. The effects of temperature and the mass of cross linker (curing agent) on the rheological properties of the viscoelastic material were studied by Prabowo et al. (2015). Small amplitude oscillatory shear tests showed that the curing rate was directly proportional to the curing agent mass percentage and curing temperature.

The different methods to determine the mechanical properties of polymers / viscoelastic materials are reviewed below.

2.3.4. Characterisation

2.3.4.1 Dynamic mechanical analysis (DMA)

Dynamic mechanical analysis (DMA) can be described as applying a periodic force to a sample and analysing the resultant response of the material (strain) to the stress experienced. The modulus, viscosity and damping can be calculated by measuring the amplitude and phase difference (δ) of the peak of the sinusoidal strain with respect to stress as shown in Figure 2.3. For pure elastic material, phase difference, $\delta = 0^{\circ}$ and for pure viscous materials, $\delta = 90^{\circ}$. For real world materials that are viscoelastic, the value of δ lies between these two extremes.

Figure 2.3: Periodic force creating oscillatory stress and strain in DMA.

Based on the parameter to be controlled, applied force (stress) or displacement (strain), DMAs can be classified as stress controlled (combined motor transducer system) and strain controlled (separate motor transducer system) respectively. For the small amplitude oscillatory shear (SAOS) tests carried out in the linear viscoelastic regime (LVR), the properties are independent of the strain and hence both measurement types (Bae et al., 2013) provide the same output. For large amplitude oscillatory shear (LAOS) tests, the two measurement types are interchangeable only for reduced inertia torque.

Based on the way which the stress/strain is applied, there are two types of DMA analysers: torsional analyser (twisting motion/shear) and axial analyser (linear force) (Figure 2.4). Torsional analyser is usually employed for liquids, melts and soft solids whereas the axial analyser for solid and semi-solid materials. Torsional analysers can perform creep-recovery, stress-relaxation and stress-strain experiments (Menard, 2008), while the axial analysers are employed for testing flexure, tension and compression. Axial analysers can also be used for shear tests by the proper usage of fixture.

Figure 2.4: Types of DMA analysers, a) Torsional analyser and b) Axial analyser.

To accurately determine the rheological properties, the proper selection of geometry plays a crucial role. The most commonly used geometries with their analyser type are provided below:

- Three-Point and Four-Point Bending (axial)
- Dual and Single Cantilever (axial)
- Tensile test (axial)
- Parallel Plate (axial for compression and torsional for shear)
- Shear Plates and Sandwiches (axial)
- Cone-and-Plate (torsional)
- Bulk (dilatometer-like fixture)
- Couette/ Concentric cylinders (torsional)
- Torsional Beam and Braid (torsional)

The three-point bend (flexural) test can be used for polymers with medium to high elastic modulus. A smooth rectangular test sample is supported at both ends without any clamping, and the force is applied at the middle (Figure 2.5a). In case of four-point bending, instead of a single point loading, the sample is loaded at two points such that

the applied stress spread over a larger area (Figure 2.5b). Single or dual cantilever tests are used for highly damped materials like elastomers. Both ends of the sample are clamped, and the load is applied at the middle for dual cantilever (Figure 2.5c), while one end is flexed for a single cantilever (Figure 2.5d).

The tensile testing is done on samples with high elastic modulus (mostly films and fibres). The test specimen is placed in tension between a fixed and a movable clamp (Figure 2.5e). The sides of the specimen are perpendicular to the bottom plate to prevent any twisting. The buckling of sample is prevented by providing a small static force. Any damage on the edges of the sample results in an inaccurate measurement. Shear plates or sandwich geometry are mainly used for high viscosity resins, gels and adhesives. For the testing, equal sizes of the test specimen are sheared between a fixed outer plate and movable centre plate (Figure 2.5f).

There are two types of parallel plate geometries: a) for compression testing in axial rheometers and b) for shear testing in the torsional rheometers. Compression testing is suitable for low to moderate modulus materials like foams. The test specimen is placed on a fixed flat surface, and the periodic force is applied by the upper plate (Figure 2.5g). For the shear test, the bottom plate is fixed over which the specimen is held, and the oscillating top plate provides the shearing to the specimen (Figure 2.5h), with maximum shear near the edges of the sample and minimum at the centre. Plates of different texture are available that can effectively eliminate slip, such as smooth, sandblasted, crosshatched and serrated. Parallel plate (PP) geometry can be used for soft solids as well as highly viscous liquids.

Cone and plate (CP) geometry is similar to parallel plate geometry except the movable upper plate has an angled cone profile which is in contact with the specimen (Figure 2.5i). The rotation generates a uniform shear across the entire sample. This geometry is mostly used for non-Newtonian fluids. Some other variants of cone plate geometry are double cone plate and cone-cone geometry. The dilatometer-like fixture consists of a

bottom cup instead of fixed plate as in a PP geometry. The restrained cup allows the measurement of bulk modulus of the material (Figure 2.5j).

Figure 2.5: DMA analyser geometries or fixtures.

Torsional beam or braid of material impregnated with resins are clamped with their long axis coaxial with rheometers rotational axis (Figure 2.5k). The testing is done by providing a twisting about their long axis. Couette/ Concentric or coaxial cylinders (CC) geometry is used for low viscosity. The geometry consists of a fixed outer cylinder into which the sample is poured, and an inner cylinder or bob is immersed into it (Figure 2.5l). The oscillating bob shears the test specimen. The settling and evaporation effects are less for this type of geometry. This type of geometry allows high shear rate without edge failure as in cone-plate or plate-plate geometry. Another type of geometry similar to CC geometry is double gap cylinder.

2.3.4.2 Dynamic indentation

The mechanical properties of glassy viscoelastic materials can be tested using instrumented indentation techniques. Traditional static indentation test determines the hardness based on the relative penetration depth (Rockwell hardness scale). The dynamic indentation considers the velocity of the indenter thereby measuring both viscosity and elasticity properties. The dynamic indentation can be represented by a Kelvin-Voigt model (Vriend and Kren, 2004) satisfying the equation,

$$m\ddot{x} + \mu\dot{x} + c_E x = 0 \quad (2.43)$$

The effect of inertia is represented by the first term (m is the mass of the indenter), the viscous effect which is proportional to the indenter's velocity is represented by the second term and the effect of elasticity is represented by the last term (c_E is the rigidity of the spring). Both viscosity and elasticity depend on the geometrical shape of the indenter tip.

Nano indentation employs the usage of small indentation tip of known geometry (Berkovich tip) with areas in the range of square nanometres to micrometres (Li and Bhushan, 2002). Thus, mechanical properties of small volume materials can also be determined with sufficient accuracy. A comparison of dynamic nano-indentation with DMA was done by White et al (2005), which showed that an excellent agreement was

obtained for glassy materials, Polymethylmethacrylate (PMMA) and epoxy, but the results showed larger variations for PDMS samples.

2.3.4.3 Spectroscopy and imaging

Various advanced methods have also been used to determine the viscoelastic properties of materials such as spectroscopy, imaging etc. The different types of spectroscopy included the Broadband viscoelastic spectroscopy or BVS (Xu et al, 2009), Infrared spectroscopy (Kelley et al, 2004), Resonant ultrasound spectroscopy or RUS (Lee et al, 2000), hyper-frequency viscoelastic spectroscopy or HFVS (Henni et al, 2011) etc, while that for imaging techniques include dynamic elastography imaging (Ophir et al, 2002), and the Forced Oscillation Atomic Force Microscopy (Kajiyama, 1994). The imaging techniques are usually used to determine the distribution of elastic properties of soft body tissues.

2.3.5 Rheological tests

In the present experimental study, the rheological properties (viscosity and elasticity) of the oil/glycerol doped PDMS material are determined using the Small Amplitude Oscillatory Shear test (torsional analyser) using a parallel plate geometry. The rotational rheometer enables the rheological measurement of a wide variety of materials by changing the geometry. The PDMS material used in the present study belongs to the category of soft solids, for which the recommended geometry for testing is the Parallel Plate (PP) geometry. The cone and plate geometry on the other hand is recommended for melts, paste-like and gel-like materials. The precured soft solid test samples prevent the usage of cone and plate geometry without the permanent distortion of the material, hence only PP geometry could be used. The rotational rheometer offers better control of temperature and long duration of shearing; approximately 1-hour is needed to complete each test for the required range of rotational frequencies. The strain-controlled

measurements allow the determination of rheological properties in the linear regime without any damage to the test sample.

CHAPTER THREE

Chapter 3

Viscoelastic Floating Cover Preparation and Testing

3.1 Introduction

Our present study of wave propagation through viscoelastic floating covers is driven by an important application, namely wave forecasts in ice covered waters. As discussed in Chapter 1, this problem is motivated by the observed rapid decline of sea ice, and the concomitant increase of shipping activities in the Arctic region (Smith and Stephenson, 2013). The propagation of ocean waves through floating structures has also been an active area of research, e.g. for very large floating structures (VLFSs) (Cheng et al., 2006; Wang et al., 2008; Watanabe 2004a and 2004b). VLFS technology has been developed for floating airports and oil storage bases in Japan, and bridges in the US, Norway, and the United Kingdom. It is in the plan for many more applications including clean energy facilities and space craft launch. There is a large body of literature covering the theories, and existing potential applications. Most of the VLFSs are for structures that must remain nearly rigid. The negligible deformation supports the assumption of a pure linear elastic response of these structures. However, more flexible structures are sometimes desirable due to the light weight requirement (Hermans, 2003; 2004; 2011). For future applications of VLFSs, considerations of viscoelastic and more complicated responses to ocean waves may be necessary.

Based on the records of National Snow and Ice Data Centre, the areal extent of summer ice in the Arctic is declining at a rate of $-12.9 \pm 1.47\%$ per decade (Stroeve et al., 2012).

*This chapter is written based on Sree, D. K., Law, A. W. K., and Shen, H. H. (2017). An experimental study on the interactions between surface waves and floating viscoelastic covers. *Wave Motion*, 70, 195-208.

The decreasing trend of sea ice extent in the Arctic Ocean is increasing the opportunity of Arctic shipping. This opportunity is not just for ice-capable vessels. With the large amount of open water, it might be possible to travel through the summer Arctic even for open water vessels by 2050 (Smith and Stephenson, 2013). Thus, Arctic shipping can become commonplace soon. The study of wave propagation under ice covers is necessary to provide reliable wave forecasts for safer Arctic marine operations.

Current wave forecast models such as WAVEWATCH III® (Tolman et al., 2014) need a formulation to account for the changes of wave speed and amplitude under different ice conditions. There is a long history of theoretical studies of wave propagation under ice covers, dating back to the work of Greenhill (1886). These theoretical studies are based on different idealizations of an ice cover. One type of idealization considers an ice cover as a distribution of individual elastic thin plates (Meylan and Squire, 1996; Meylan et al., 1997; Montiel et al., 2016; Williams et al., 2013a and 2013b) when the size of plate is comparable to the wave length. The response of finite length individual plate is studied by Meylan and Squire (1994) and, Kohout and Meylan (2008). These plates act as wave scatterers (Bennetts and Williams, 2015). As waves propagate through them, the energy is redistributed in different directions. Wave attenuation results from the directional redistribution. The other type of idealization considers an ice cover as a continuum that follows certain constitutional law. The properties of the ice cover determine the wave speed and the rate of attenuation. Three classic theories of this type are the mass loading (Weitz and Keller, 1950), thin elastic plate (Wadhams, 1973a), and viscous fluid (Keller, 1998). There is extensive development in both types of theories. The most recent review of these theories and related studies can be found in Squire (2007).

Ice covers are not uniform homogeneous and isotropic materials. They are conglomerates of a wide range of features depending on the location, time, and history of the ice cover. A partial list of these features includes compression ridges, leads (tensile or shear cracks), rafted level ice, floes (broken pieces of ice), nilas (newly formed thin ice in quiescent water), pancake ice (circular ice disks usually with raised rims), and grease ice (ice slurry

formed in turbulence). The glossary and associated images of different types of ice cover may be found at <http://aspect.antarctica.gov.au/home/conducting-sea-ice-observations>. In the marginal ice zone (MIZ) that borders the open ocean and pack ice, the ice cover may consist of pure pancake/grease ice, or a collection of floes from kilometres down to tens of meters mixed with slushy ice in between. Since most Arctic shipping takes place in the MIZ, where the wave is also the strongest, there is a high interest to find a model that can be applied to the MIZ.

The continuum model is a plausible candidate for different types of ice covers. The three classical types of continuum model mentioned earlier in this chapter each appears to capture some physical properties of ice covers, with mass loading considering the inertia of the ice floes, thin-elastic considering the bending of ice cover, and viscous model considering the energy dissipation in the ice cover. All these processes are present in the MIZ. Recognizing the coexistence of these processes, a viscoelastic ice cover model is developed by Wang and Shen (2010b). This model synthesized the three continuum models: mass loading, thin-elastic, and pure viscous. There are other types of viscoelastic models in the literature (Squire and Allan, 1980). Comparisons of several viscoelastic theories may be found in Mosig et al. (2015). Other types of nonlinear flow rules that can describe energy dissipation have also been proposed (Wadhams, 1986).

In this study, the viscoelastic model developed in Wang and Shen (2010b) is investigated. The WS model is shown to converge to all three types of classical continuum models in proper limits. Mass loading model is obtained by equating complex kinematic viscosity of ice cover to zero whereas elastic plate (with finite thickness) and two-layer viscous model are obtained by substituting shear modulus and viscosity of ice cover to zero respectively. However, results from this pure theoretical work have never been tested experimentally. The main challenge is the difficulty to control and measure the mechanical properties of ice covers. In this chapter, a laboratory study using precisely controlled viscoelastic floating materials to measure the wave celerity will be described. Then comparison of the results with the theoretical predictions is carried out. The same

experimental procedures are also applied to test a commercially available and much stiffer polypropylene (PP) cover. The following sections begin with the material preparation, its rheological properties, experimental design of the wave tests, data analysis and results, followed by comparisons with theory, and finally discussion and summary.

3.2 Viscoelastic Material Preparation

To produce precisely controlled floating viscoelastic materials, a technique using an oil-doped Polydimethylsiloxane (PDMS) material is developed, which is widely used in microfluidics for biomedical applications (Fujii,2002). The density of PDMS material is generally slightly greater than water, hence it needs to be doped with oil to create a floating cover.

In the present experiments, the main components for the preparation of the oil-doped PDMS viscoelastic floating covers are the prepolymer elastomer base, curing agent, and white oil. The prepolymer (base) and cross linker (curing agent) are available as the commercial two-part elastomer kit, Sylgard 184 (Dow Corning, USA). The white oil, also called paraffin oil, is a colourless, transparent oily liquid soluble in organic solvents. The mass percentage of oil and curing agent can be defined respectively as follows:

$$m_o (\%) = \frac{M_o}{M_o + M_{CA} + M_B} \quad (3.1)$$

$$m_{CA} (\%) = \frac{M_{CA}}{M_{CA} + M_B} \quad (3.2)$$

where M_o , M_{CA} and M_B represent the mass of white oil, curing agent, and base respectively. m_o , is maintained at 30% for all the experiments to achieve a target density of 940 kg/m³. The viscoelastic properties are varied by letting m_{CA} be 4%, 6%, 8% and 10%. The material properties of these components are described in Prabowo et al. (2014) who investigated the standing wave behaviour in a small rocking tank with the viscoelastic cover.

The typical preparation of PDMS material involves a curing process whereby the elastomer base is mixed with the curing agent at a ratio, and then the material is heat-treated in the oven at high temperatures. Thus, PDMS applications in the literature so far predominantly involved small sample sizes with a length much less than 30 cm that can be accommodated in a typical oven (2014). To measure the change of wave propagation through a PDMS cover, however, a large sample that can span a few meters in a typical wave flume in the laboratory is needed. Earlier, attempts were made join smaller oven-cured PDMS pieces to form the large cover (Deng et al., 2015). Unfortunately, the joints created strong non-uniformity and localized discontinuity in the mechanical characteristics of the cover. A new curing method is then established in the present study as described below.

The preparation procedures for the PDMS viscoelastic cover are as follows. After carefully measuring all the three components, they are then mixed using an electric mixer (IKA RW 20 digital) for 10 minutes at 500 rpm to ensure a well-mixed solution. The mixing period of 10 minutes at 500 rpm was determined by trial and error. Preliminary tests showed that a shorter duration resulted in oil remaining separate from the PDMS, mix while a longer duration cannot be used due to the pot life of the material. The well-mixing condition was further confirmed based on repeatable rheological tests conducted using samples from different locations. Degassing is applied at room temperature (25°C) inside a vacuum oven for 30 minutes. The mixture is then poured to an aluminium tray with dimensions equal to the size of the cover required for the specific experiment in the wave flume. For the specific PDMS kit used in the present study, the manufacturer has recommended a maximum preparation time of 2 hours at 25°C to ensure the proper mixing of the components. Our overall preparation time for the PDMS cover is limited to less than 1.5 hours, thus fulfilling the recommendation.

The prevalent oven-cured approach for PDMS limits the size of the sample as described earlier. To achieve our objective for larger covers, other approaches were attempted for the curing through trials and errors. These included layer-by-layer preparation over large

areas with surface heating, and sequential block-by-block preparation. Finally, a time-curing approach is settled whereby, the freshly prepared PDMS material is simply allowed to cure for 7 days at room temperature without disturbance. Parallel to the larger size PDMS cover, smaller pieces of the identical material and preparation procedure are prepared in rectangular trays (35 cm x 25 cm x 1 cm) for rheological characterization. Through rheological testing, it is found that sufficient crosslinking can develop under this low-heat long duration curing method. Freshly mixed sample was allowed to cure, with material testing conducted every other day for a week after the mix starts to solidify. The same sample could cure for three weeks and rheological tests were conducted at the end of each week. The rheological properties at the end of each week were compared, to see the variation of properties with curing period. It is found through trials and errors that a long curing period of at least 7 days is essential for the viscoelastic properties to stabilize.

3.3 Rheological Behaviour

Viscoelastic materials can be modelled using combinations of springs and dashpots (Schramm, 1994). The simplest among the combinations is the Voigt model (parallel combination of spring and dashpot) and Maxwell model (series combination of spring and dashpot). In both cases, the coefficient representing the spring, G and dashpot μ are constants, resulting in linear relations between stress and strain/strain rate. The constitutive relation for a Voigt model is given by

$$\tau = G\gamma + \mu \frac{d\gamma}{dt} \quad (3.3)$$

while that for a Maxwell model is

$$\frac{d\gamma}{dt} = \frac{\tau}{\mu} + \frac{1}{G} \frac{d\tau}{dt} . \quad (3.4)$$

In the above, τ is stress, γ is deformation, G is elastic modulus and, μ is dynamic viscosity. To measure the properties of a viscoelastic material in a rheometer directly, a torsional test with oscillatory shearing is commonly used (Barnes et al., 1989). For an oscillatory shear test, the material is subject to an angular frequency ω and a given

maximum strain, whereby the resistance torque is measured to calculate the storage modulus G' and loss modulus G'' . These moduli are related to the elastic modulus and dynamic viscosity. For a Voigt material, the relationships are (Schramm, 1994)

$$G' = G, \text{ a constant}; \quad G'' = \omega\mu \quad (3.5)$$

and for a Maxwell material, the relationships are

$$G' = \frac{G\xi^2\omega^2}{1+\xi^2\omega^2}; \quad G'' = \frac{G\xi\omega}{1+\xi^2\omega^2}. \quad (3.6)$$

In the above $\xi = \mu/G$ is the relaxation time. Thus, during an oscillatory rheometer test, for a Voigt model, G' remains constant when a frequency sweep from low to high ω is applied, while G'' increases linearly with ω resulting in a constant dynamic viscosity μ . The Maxwell model is however very different, with G' increasing with ω and approaching an asymptote when $\omega > 1/\xi$, while G'' increases with ω to a maximum at $\omega = 1/\xi$ and then decreases.

The viscoelastic properties of oil doped PDMS are measured using a commercial rheometer (Anton Paar, MCR 302, Germany, with a parallel plate geometry of 20 mm diameter). The test samples are cut out of the smaller PDMS pieces using circular punch of diameter 20 mm. Our measurements are done with 1.6 mm thickness based on the required specifications (Schramm, 1994).

Amplitude sweep is first performed in the rheometer tests to determine the linear viscoelastic range, where the material property is independent of strain amplitude. The viscoelastic properties of the PDMS material are then analysed using a frequency sweep analysis from 0.01 to 100 rad/s with a constant strain amplitude of 1.0%, which is within the linear range. Three samples are tested for each m_{CA} to ensure the repeatability of the test results which are shown in Figure 3.1. From the figure, the rheological behaviour of the PDMS material closely resembled the Voigt model with G' almost independent of rotational frequency and G'' varying with rotational frequency nearly linearly. Note that both G' and G'' increase with m_{CA} . Figure 3.1 also shows that G'' is slightly nonlinear with a gradual increase over four orders of magnitude in the shear frequency. The WS

model used the Voight model to define the viscoelastic cover, hence our PDMS material is suitable for testing this viscoelastic theory. The evaluation of dynamic viscosity from the loss modulus will be discussed in Section 3.6.

Figure 3.1. Viscoelastic properties against angular frequency for floating covers. Curing Period: circles = 1 week; asterisk = 2 week; diamond = 3 week. m_{CA} = (a) 4%, (b) 6%, (c) 8% and, (d) 10%.

For the wave periods considered in the present experiments, the viscoelastic floating covers have larger values of G' compared to G'' (see Table 3.1), and the ratio of G''/G' decrease with increasing m_{CA} . From $m_{CA} = 4\%$ to 10%, G''/G' drops by a factor of 2, i.e. the material becomes relatively more elastic than viscous. Finally, the rheological behaviour of the PDMS material over a three-week duration after curing is measured and

found that it remained stable. Thus, the wave experiments could be performed assuming that the viscoelastic properties remained unchanged within this duration.

Table 3.1: Ratio of loss modulus to storage modulus for different m_{CA} .

T (s)	G''/G'			
	4%	6%	8%	10%
0.5	0.1161	0.0544	0.0428	0.0331
0.6	0.1096	0.0515	0.0407	0.0313
0.7	0.1039	0.0489	0.0387	0.0297
0.8	0.0992	0.0467	0.0372	0.0285
0.9	0.0955	0.0450	0.0359	0.0275
1.0	0.0924	0.0436	0.0349	0.0267

Experiments with the PDMS as well as commercially available polypropylene (PP) covers are performed for comparison. Both types of covers have the same thickness of 0.01 m. The properties of the PP cover are determined using dynamic mechanical analysis (DMA Q800 V20.9 Build 27) to be $G' = 870$ MPa and $G'' = 24$ MPa, and the density is 910 kg/m^3 . As a viscoelastic material, PP is thus much stiffer than the PDMS with $m_{CA} = 10\%$. The storage modulus of the PP cover is nearly constant and so is the loss modulus over the entire range of frequency test from 0.1 to 100 Hz. Hence, the PP cover is strictly speaking not a Voigt type or a Maxwell type material.

3.4 Experimental Setup

The experiments are conducted in a wave flume made of tempered transparent glass panels supported with a stainless-steel framework. The wave flume is 8 m long, 0.3 m wide and 1.0 m deep. It is equipped with a piston-type wave generator (see Figure 3.2)

that can generate regular waves with a wide range of wave periods. Mesh type wave absorbers are installed at both ends of the flume. The vertical upstream absorber filtered energy of the standing wave formed behind the wave generator. The downstream absorber has a slope of 1:1.25 and it acted as the dissipation beach to minimize the reflected wave energy.

Figure 3.2: Schematic diagram of the wave flume, with stopper rod.

The surge as well as the drift motion of the floating cover is prevented by a vertical stopper rod at the trailing edge of the cover. The stopper rod is kept in position by tightening to the steel framework of tank and with its square base resting on the bottom of tank. It should be noted that in the literature, various methods had been employed to constrain surge. The experimental study on floating plates by Sendil and Graf (1974) used mooring lines attached to the floor to keep the plate in position. In the laboratory experiments conducted by Bennetts et al. (2015), loose moorings attached to the side walls were used. The wave basin experiments conducted by Montiel et al. (2013a) for the hydroelastic analysis of floating disks used two vertical rods (one passed through a hole at the centre of the disk and the other was close to the edge) to restrict horizontal

motion (surge and sway) as well as rotations (yaw). Both rods acted like vertical cantilevers with their top end fixed to a platform above. The influence of the centre rod on flexure modes was found to be insignificant (Montiel et al., 2013b). Despite the different approaches, the stopper rod used in the present experiments does not have any physical attachment with the cover, hence its effects on the vertical displacement and deformation of the floating cover is assumed to be negligible.

In real sea ice fields, the normal wave periods vary over a larger range from 5 s to 25 s with wave amplitudes from 0.5 m to 2.0 m (Liu et al., 1992; Wadhams et al., 2004). The present experimental conditions included a range of monochromatic waves with periods from $T = 0.5$ to 1.0 s, while the water depth is held constant at 0.30 m. The stroke of the wave generator is controlled by a capacitance wave gauge to produce an incident wave of constant amplitude. The repeatability of the wave generation is checked to be excellent. Wave conditions generated by the same setting always produced nearly identical time series (see Figure 3.3). The wave period range is selected such that the reflection from the absorbing beach is small and the waves are not too steep. The optimum wave amplitude $a = 0.0085$ m is chosen (with ~2% error due to sensor resolution) because smaller amplitudes would cause relatively larger errors in the analysis, while larger amplitudes would lead to significant nonlinear effects and wave breaking. Table 3.2 provides the values of wave steepness used, in which the open water wave number k_0 and the wave steepness $k_0 a$ are provided.

Ultrasound sensors are installed above the floating cover to monitor the surface displacement during the passage of surface waves. The ultrasound sensors are synchronized by a common data acquisition system (NI 9215, National Instruments), and the time series data are recorded using LABVIEW. A total of three ultrasound sensors (US325, General Acoustics) are used with resolution of 0.18 mm and sampling frequency of 50 Hz (Ultra lab 40D, General Acoustics). Vertically, they are held at 5 cm above the cover as the blind area for the sensor is 3 cm. With the 1.0 m PDMS and PP covers, the position of the first ultrasound sensor (US1) is fixed at 10 cm from the leading edge. The

two other ultrasound sensors are placed at 10 cm and 50 cm from US1 respectively, in the first set of experiments conducted for different wave periods. Each experiment is performed at least three times to check for repeatability. Subsequently, the two sensors are moved to 20/60 cm, 30/70 cm and 40/80 cm from US1 in the second, third and fourth sets of experiments, respectively. In each of these subsequent sets of experiments, the wave conditions are identical. In this manner, the time series of surface displacement data at 9 locations of the cover are obtained with a total of 72 experiments for each test condition (i.e. 6 wave periods $\times$ 4 position changes $\times$ 3 repetitions). With the longer 2.45 m PP cover, US1 is fixed at 75 cm from the leading edge instead, while US2 and US3 are shifted by 20 cm after each set of experiment. Thus, the time series of surface displacement data at 7 locations are obtained (6 wave periods $\times$ 3 position changes $\times$ 3 repetitions).

Figure 3.3: Time series of the wave record at US1 ($T = 0.7$ s) for three tests with identical wave generator setting. The boxed portion is used for analysis. Repeatability of tests is checked since the three-time series overlapped exactly.

Overwash is always observed at the edges of PP covers in initial trial experiments conducted. Both G' and G'' of the PP cover are very large compared to the PDMS covers,

which restricted its deformation with the wave. Skene et al. (2015) developed a theoretical model for overwash with a thin elastic plate model. The model predictions were validated by laboratory experiments using PP and PVC sheets. To prevent the overwash, a 1 cm thick Styrofoam edge barrier is attached to the leading and trailing edges of the PP cover for the subsequent experiments. All results reported here for the 1.0 m and 2.45 m long PP covers had the barriers. A similar approach is used before for the experiments conducted by Montiel et al, (2013a) using neoprene foam edge barrier. Because of this modification, the wave characteristics near the edges of the cover might be influenced.

Table 3.2: Input wave characteristics.

T (s)	λ (m)	k_o (rad/m)	$k_o a$
0.5	0.390	16.08	0.137
0.6	0.560	11.19	0.095
0.7	0.755	8.32	0.071
0.8	0.960	6.53	0.056
0.9	1.168	5.37	0.046
1.0	1.373	4.57	0.039

3.5 Experimental Results

3.5.1 Wave celerity along viscoelastic covers

The data rate is first increased from 50 Hz to 200 Hz by numerical interpolation using the *PCHIP* function (Piecewise Cubic Hermite Interpolating Polynomial) in MATLAB to enable more precise determination of the wave crests. Since the trailing edge of the 1 m long cover in the experiments is positioned at 2.90 m from the end wall of the wave

flume, the minimum time required for the longest wave ($T = 1.0$ s) to reach the cover after reflection from the end of the flume is approximately 4 seconds. Hence, the first three waves which are judged to be fully developed in the wave record free from reflection are considered in the analysis as shown in Figure 3.3.

The wave celerity is calculated as

$$c_s = \frac{\Delta x_i}{\Delta t} \quad (3.7)$$

where Δx_i is the distance of the i^{th} ultrasound sensor from US1, and Δt is the corresponding time lapse for the crest to travel. The average celerity for the three waves considered is calculated for each position of ultrasound sensor. The error plot for the celerity is drawn using *errorbar*(Y_m, E_s) function of MATLAB where Y_m is the mean of the 3 repetitions and E_s is the corresponding standard deviation. Figure 3.4 shows a comparison of the experimentally determined open water celerity with the linear wave theory.

Figure 3.4: Open water wave celerity. Asterisk represents experimental data and solid line represents linear wave theory.

Figure 3.5: Wave celerity under the PDMS cover. The solid line represents the open water case. $T =$ (a) 1.0 s, (b) 0.7 s, and (c) 0.5 s.

Figure 3.5 shows the measured c_s along the viscoelastic cover region which is clearly different from the open water celerity, c . The difference is also highly dependent on both the material and incident wave characteristics. For longer wave periods, $T = 0.7$ and 1.0

s, c_s is smaller than c as shown in Figures 3.5a and 3.5b which implied wavelength shortening in the dispersion relationship. However, for the shortest wave period tested, $T = 0.5$ s, c_s became larger than c for the more elastic covers (with larger m_{CA}) as shown in Figure 3.5c. For all the wave periods, c_s generally increases with shear modulus.

The determination of wave celerity involved the examination of the wave profile at different locations. Attempts are made to determine the reduction of wave magnitude, or wave attenuation, along the cover at the same time. However, it is found that the cover length is too short for a reliable attenuation measurement.

3.5.2 Spectral analysis

Figure 3.6 shows the spectral analysis of the surface displacement at different locations along the covers based on the ultrasound sensor data for the PDMS viscoelastic covers as well as the 1.0 m long PP cover. The discrete Fourier transform ($|Y(\sigma)|$) of the time series data is calculated using *fft()* function of MATLAB. As discussed previously, only the first three fully developed waves are considered.

With the PDMS covers (see Figure 3.6a), the wave spectrum at different locations varied slightly. Comparatively, the wave spectrum for the 1.0 m PP cover shown in Figure 3.6b had obvious larger peaks towards the two free edges of the cover and smaller peaks in the middle. Using a thin-elastic plate theory, Fox and Squire (1990)) and Andrianov (2005) as well as many other studies found that due to the existence of evanescent and two damped travelling waves at the edges of an elastic plate, the vertical displacement near the edges is larger than the middle section of the plate. In addition, for waves that were longer than the floating plate, significant pitching might occur (Bennetts and Squire, 2012; Meylan and Squire, 1994) along with oscillations due to trapped wave energy under the cover. Both phenomena are possible sources of the observed non-uniform spectral energy along the cover. Theoretical study of deflecting and pitching of a finite size viscoelastic cover over water waves, to the authors' knowledge, has not been done.

Figure 3.7 provides the spectral analysis results for the 2.45 m PP cover. As mentioned earlier, to prevent overwash, Styrofoam barriers are installed on the leading the trailing edges. Two cases are shown, one with $T = 0.7$ s and the other with $T = 0.5$ s. Comparing

Figure 3.6: Spectral analysis of the surface elevation at different locations along the cover ($T = 0.7$ s): (a) PDMS ($m_{CA} = 10\%$) and (b) 1.0 m PP cover. The locations are marked with A-E from the leading edge, with distance in meters. The data are vertically displaced by increments of 0.5 and 0.25 for (a) and (b) respectively to aid visualization.

Figure 3.6 with Figure 3.7a, both for $T = 0.7$ s, it is seen that the variation of peaks of the spectra is reduced significantly with a longer cover size. Comparing Figure 3.7a with 3.7b, it is observed that the finite size effects are slightly diminished when the wavelength reduces. The effect of finite length of cover on the celerity and wave number measurement is discussed in Section 3.6

3.5.3 Wave celerity along 2.45 m polypropylene cover

The results of the average wave celerity along the 2.45 m long PP cover are shown in Figure 3.8. The recorded wave celerity is higher than the open water celerity, implying that wave lengthening occurred for all wave period cases. The average celerity for the first two locations closer to US1 is typically faster. With increasing distance from the leading edge, the average celerity approximated a constant. It is noted that longer separation distances between two locations would yield more accurate wave celerity estimates due to the longer traveling time.

3.6 Comparison with Theoretical Model

As mentioned in the Introduction, the key objective of this study is to provide the necessary experimental data to verify the existing theory. A comparison of the above experimental results with the model predictions developed in Wang and Shen (2010b) is performed. However, this theory is developed based on an infinite cover without edge effects, whereas in the present experimental study the floating cover is of a finite extent. The potential influence of the edge effects as mentioned earlier is present in the data.

Figure 3.7: Spectral analysis of the surface elevation at different locations for 2.45 m long PP cover. $T =$ (a) 0.7 s and, (b) $T = 0.5$ s. The data are vertically displaced from A to G by increments of 0.1 to aid visualization.

The transcendental dispersion relationship, Equation 2.12, is solved numerically using the Muller method (Wang and Shen, 2010b) with the values of shear modulus and viscosity of the PDMS material used in the experiments. This dispersion relationship has multiple modes, which is common in wave propagation through layered materials, e.g. pure viscous fluid over inviscid water (Keller, 1998). The mode with its real part closest to the open water case is selected and compared to the wave celerity measured in the experiments. As discussed in Wang and Shen (2010b) this mode is the dominant one.

The value of G' and G'' obtained from the frequency sweep analysis is independent of strain applied within the linear viscoelastic regime. Hence the wave frequency is matched with the angular frequency of oscillatory test to get the required G' and G'' from the obtained data. Table 3.3 summarizes the viscoelastic parameters of the PDMS covers for the range of wave periods in the experiments. In the present experiments with the PDMS covers, $\rho_1 = 940 \text{ kg/m}^3$ is the density of the viscoelastic cover and $\rho_2 = 1000 \text{ kg/m}^3$ is the density of water.

Table 3.3: Variation of the viscoelastic properties of PDMS material for different m_{CA} .

T (s)	G (Pa)				$\nu = \mu / \rho_1$ (m^2/s)			
	4%	6%	8%	10%	4%	6%	8%	10%
0.5	22465	55854	89542	146442	0.22	0.26	0.32	0.41
0.6	22248	55565	89181	146081	0.25	0.29	0.37	0.46
0.7	22033	55333	88878	145722	0.27	0.32	0.41	0.51
0.8	21851	55151	88635	145418	0.29	0.35	0.45	0.56
0.9	21709	55009	88446	145182	0.32	0.38	0.48	0.61
1.0	21595	54893	88292	144989	0.34	0.41	0.52	0.65

Figure 3.8: Wave celerity along the length of the long PP cover. Solid line = open water wave celerity; dashed line = WS model predictions and circles = experimental data. $T =$ (a) 0.5 s, (b) 0.6 s, (c) 0.7 s, (d) 0.8 s, (e) 0.9 s and, (f) 1.0 s.

3.6.1 Viscoelastic cover

A comparison of the normalized wave number between the experimental data and the WS model predictions for the PDMS covers is summarized in Table 3.4, in which k_{si} , $i = 1, 2, 3$ denotes the real part of the three most significant roots with the least attenuation among the infinite roots obtained as described in Wang and Shen (2010b). Many wave

modes are generated in the wave-ice interaction processes, which include evanescent, gravity, shear, pressure, interfacial, and Rayleigh-Lamb (Zhao et al., 2017). All modes other than pressure, shear and gravity modes ($i=1,2,3$) have very large damping coefficients. Hence, they do not have significant effects on the vertical surface displacement of the cover except near the edges. Shear mode contains no energy; hence it can be discarded. In addition, the wave period in the experiments for the current study was away from the mode swap region ($\omega = g \sqrt{\frac{\nu}{4G}}$) between gravity mode and pressure mode, hence only the gravity mode dominated the vertical motion. By choosing the one closest to the open water wave number from the three roots, the comparison of theoretical results of $\kappa = k_s/k_o$ with experimental data is performed (Figure 3.9), where k_s is real part of the wave number under the cover and k_o is that for open water. Note that $\kappa = 1$ for open water, and for wave lengthening $\kappa < 1$ and wave shortening $\kappa > 1$. In general, within the present range of tested PDMS, the model predicted mostly $\kappa > 1$ along the cover which agrees with the experimental data. The two cases with $\kappa < 1$ are again consistent with the experimental measurements.

Individual plots in Figure 3.9 correspond to PDMS covers with different rheological properties. Figure 3.9a shows the cover with $m_{CA} = 4\%$ thus the lowest shear modulus and viscosity. The model closely predicted the normalized wave number with the shortest wave period of $T = 0.5$ s. However, noticeable differences can be observed with $T = 0.7$ s and beyond, and in general the model predicted a lower κ compared with the experimental measurements. Similar trends can also be identified in the other wave period cases of Figure 3.9b to 3.9d. The differences are less for higher shear modulus. Excellent agreement between the model predictions and experiment data can in fact be observed for the two covers with $m_{CA} = 8\%$ and 10% for $T = 0.6$ to 0.8 s. The differences for the longer wave periods of 0.9 s and 1.0 s remained however generally unchanged throughout the tested range.

Figure 3.9: Normalized wave number under the PDMS cover against wave period. Dashed line = open water case, circles = experimental data, and asterisk = WS model. m_{CA} = (a) 4%, (b) 6%, (c) 8% and, (d) 10%. The symbol enclosed in a box corresponds to the case when overwash is observed.

Two important reasons might have contributed towards the differences between the theoretical results and experimental data. The first reason is due to the overwash at the leading and trailing edges of the cover, which occurred for the experiments with the shorter wave periods and for covers with larger value of shear modulus, $m_{CA} = 10\%$ (Figure 3.9d and Figure 3.10). The study by Skene et al. (2015) showed that the overwash increases with wave steepness. In the present case, the importance of overwash is only noted after the experiments with viscoelastic covers are completed, and the experiments with additional edge protection to eliminate the overwash was not done. Since overwash is not observed for most cases with PDMS, the discrepancies are most likely due to a second reason, which was the finite size of the cover. The finite extent is not included in the theoretical model as discussed earlier. To study the wave propagation through a finite floe would require, the inclusion of damped travelling modes from the two free edges

and possible pitching motion of the floating cover. From the spectral analysis shown in Section 3.5.2, the insignificant variation of spectral energy suggested that the edge effects and pitching motion are minimum. Visual observation also confirmed the absence of pitching. Another source of the discrepancies could involve the deviation of the PDMS covers from that of a Voigt model and nonlinear effects for the higher frequency cases.

Figure 3.10: Over wash at the leading edge of the PDMS cover for the higher frequency case, $T = 0.5$ s and $m_{CA}=10\%$.

3.6.2 Polypropylene cover

As discussed before, the 1.0 m PP cover is subjected to relatively strong edge effects and pitching. Thus, the measurements with the 2.45 m PP cover only performed. For this case, wave periods from 0.5 to 1.0 s are tested with increments of 0.05 s. Figure 3.11 summarizes the comparison between theoretical and experimental results of the dimensionless wave number, in which the results from thin-elastic plate theory from Fox and Squire (1990) are also plotted for comparison. The predictions for the two theories are very close, as expected from the asymptotic behaviour examined in WS model. Ignoring the viscosity, when the elastic modulus increases the dispersion from WS model would converge to that of the thin-elastic plate in (Fox and Squire, 1990).

As shown in Figure 3.11, κ is always less than 1 which implied wave lengthening. A close agreement is observed between the measurements and theoretical predictions for the shorter wave periods from 0.5 to 0.6 s. In this range, with the length of the cover being much longer than the wavelength, with small pitching observed visually. Nevertheless, for periods greater than 0.6 s, differences between the measurements and

predictions began to be noticeable. This trend is consistent with the observations along the cover discussed earlier in Figure 3.9.

The relative discrepancies between theoretical results and experimental data are greater for the PP case than the PDMS case. With a shear modulus more than three orders of magnitude greater than that of PDMS, the finite size effects of the PP cover, even with a longer cover size, are likely be much greater than the PDMS cover. For the PP cover, to avoid overwash, the installed Styrofoam barrier also changed the boundary condition. In addition, the rheological property of the PP cover does not strictly follow the ideal Voigt model either. These are all potential sources for the observed differences between the theoretical and experimental results.

Figure 3.11: Normalized wave number under the 2.45 m long PP cover. Solid line = open water, asterisk = WS model, diamonds = Fox and Squire (1990) and circles = experimental data.

Table 3.4: Comparison of wave numbers obtained from wave experiments and theoretical model (WS model).

T (s)	Real part of wave number (rad/m)							
	$m_{CA} = 4\%$				$m_{CA} = 6\%$			
	Experiment	Wang and Shen (2010b)			Experiment	Wang and Shen (2010b)		
	k_s	k_{s1}	k_{s2}	k_{s3}	k_s	k_{s1}	k_{s2}	k_{s3}
0.5	17.220	1.278	17.541	2.558	16.455	0.814	1.628	16.386
0.6	12.617	12.301	1.071	2.143	11.959	0.680	12.008	1.361
0.7	9.581	8.956	0.923	1.847	9.063	8.888	0.584	1.169
0.8	7.315	6.902	0.811	1.623	6.980	6.882	0.512	1.025
0.9	5.871	5.595	0.724	1.448	5.884	5.587	0.456	0.912
1.0	4.907	4.716	0.653	1.307	4.866	0.411	4.711	0.822
T (s)	$m_{CA} = 8\%$				$m_{CA} = 10\%$			
	Experiment	Wang and Shen (2010b)			Experiment	Wang and Shen (2010b)		
	k_s	k_{s1}	k_{s2}	k_{s3}	k_s	k_{s1}	k_{s2}	k_{s3}
0.5	15.640	0.643	1.287	15.654	13.694	0.503	1.006	14.825
0.6	11.653	0.537	1.075	11.771	11.308	0.420	0.840	11.448
0.7	8.962	0.461	8.828	0.923	8.751	0.360	8.735	0.721
0.8	6.848	0.404	6.865	0.808	6.934	0.316	0.631	6.838
0.9	5.789	0.360	5.582	0.719	5.789	0.281	0.561	5.573
1.0	4.838	0.324	4.709	0.648	4.829	0.253	0.506	4.705

3.7 Summary

In this study, an experimental approach using Polydimethylsiloxane (PDMS) material mixed with oil is developed to produce a viscoelastic float material. The rheological characteristics of this material approximated the Voigt model, thus could be used to verify the viscoelastic model for sea ice by Wang and Shen (2010b) which model ice cover as a Voigt material. A range of targeted viscoelastic properties are created by using different amount of curing agent.

Due to the large coverage required for the wave flume experiments, special curing procedures are also established for the PDMS preparation beyond the prevalent procedures for small PDMS applications in the literature. Polypropylene (PP) cover with more than three orders of magnitude higher shear modulus than the PDMS covers is also used in the experiments, to contrast the very flexible PDMS covers.

Wave flume experiments showed that wave shortening is observed under the PDMS cover with lower shear modulus for the range of wave periods tested. When the shear modulus increases, wave lengthening is observed for lower wave periods. The model predictions by the WS model are generally consistent with the experimental data, both in terms of the switching of normalized wavelength from less than to greater than 1, as well as the magnitude of the change. Wave lengthening is observed for the PP cover for all cases studied and approached the open water wave celerity with increase in the wave period. These results are also consistent with theoretical predictions.

For both PDMS and PP covers, comparisons between theoretical and experimental results are better for shorter wave periods, despite the higher wave steepness and thus some nonlinear effects. Discrepancies for longer wave periods can be attributed to the finite extent of the cover in the experiments and edge barriers for PP cover. The rheometer measurements for PDMS and PP cover showed that both are not strictly linear Voigt material. The deviations of experimental results from the theoretical model might

be due to this discrepancy which is more for the PP cover than PDMS. Squire (1993) provided a relationship to determine the nature of wavelength modification for thin

elastic plates. The normalised wavenumber can be simplified as $\kappa = \frac{1 + \frac{\rho_i h_s k_o}{\rho}}{1 + \frac{L_r k_o^4}{\rho g}}$, the value

of which depends largely on the flexural rigidity $\left(L_r = \frac{E h_s^3}{12(1-\nu^2)}\right)$ of the cover. The

condition for wave lengthening is $\frac{L_r k_o^3}{g} > \rho_i h_s \left(\text{for deep water } \frac{L_r (\omega^2/g)^3}{\rho_i h_s g} > 1 \right)$,

otherwise wave shortening would occur. This clearly shows that flexural rigidity and wave period are the decisive factors.

Overall, the present study provides a general confirmation of the viscoelastic theory by WS model in terms of the dispersion relationship and illustrates the level of predictive accuracy of the theory within a range of viscoelastic properties and wave characteristics. The next step would be to examine theoretically the influence due to finite extent of the cover, and experimentally determine the wave attenuation under the viscoelastic cover. Despite these efforts, there is still a major gap between the current viscoelastic theory, even verified experimentally, and the ice cover in the MIZ. To determine whether real ice covers that consist a variety of different ice features can or cannot be modeled as a Voigt or other types of viscoelastic continuum, different approaches may be taken: either from using first principles to derive such a theory, or from a large amount of direct measurements in the laboratory or from field experiments. For both cases, considerable future work needs to be done to fill this gap.

CHAPTER FOUR

Chapter 4

Wave Modification Due to Viscoelastic Floating Cover

4.1 Introduction

Arctic ice reduction has created longer fetches, which aid in the generation of larger gravity waves that could penetrate kilometres of distance through ice covers. With increased interests in Arctic shipping (Smith and Stephenson, 2013), better knowledge of the wave climate in the Marginal Ice Zone (MIZ) is required to aid navigation near the ice edge. Thus, there is an appreciable demand to understand wave propagation through diverse types of ice covers in the MIZ (the glossary and associated ice cover images can be found at <http://aspect.antarctica.gov.au/home/conducting-sea-ice-observations>).

Field studies based on observations from satellites (Wadhams et al., 2002) and in-situ measurements (Wadhams et al., 1986, 1988; Meylan et al., 2014) showed that ice covers behaved primarily as a low-pass filter, which scattered and attenuated the high frequency wave components. Satellite observations also suggested changes of wavelength and thus the speed of energy propagation (Schulz-Stellenfleth and Lehner, 2002). To determine the wave propagation under diverse types of ice covers from first principles is a challenging task. Because of these difficulties, various theories have been proposed based on different assumptions.

Three classical theories that describe wave dispersion through an idealized floating cover are of particulate relevance here. The first assumed the large unbroken ice cover as a thin

*This chapter is written based on Sree, D. K., Law, A. W. K. and Shen, H. H. (2018). An experimental study on gravity waves through a floating viscoelastic cover. Cold Regions Science and Technology (accepted), DOI: 10.1016/j.coldregions.2018.08.013.

elastic plate (Greenhill, 1886). This theory demonstrated the changes of wavelength, but it did not allow wave attenuation. The second theory, the mass loading model, assumed that the ice cover as a surface composed of non-interacting mass points (Peters, 1950; Weitz and Keller, 1950; Keller and Weitz, 1953). This theory was later pointed out to be a special case of the thin elastic plate theory when the elasticity is set to zero (Squire, 1993). The mass loading theory predicts only wave shortening and no attenuation. In the third theory, the two-layer viscous model, the momentum equation of the floating high viscous layer is coupled with that of the inviscid layer underneath to solve for the dispersion relation (Keller, 1998). Of the three theories, this is the only one that produced attenuation.

The wave energy exponentially decreases with distance along MIZ (Wadhams, 1972). The hydrodynamic interactions between ice crystals provide a viscous nature to grease ice (Martin and Kauffman, 1981) that help it to act as an excellent wave absorber. The initial theoretical attempts on wave propagation through a viscous medium and resulting wave attenuation are done independently by Stokes (1851) and Lamb (1932) for an infinite deep single layer with constant viscosity. The first two-layer viscous model was developed by Weber (1987) assuming ice as a thin viscous layer over infinitely deep inviscid water. This was later extended to inviscid finite deep water by Keller (1998). Newyear and Martin (1999) used this theory to inversely determine the viscosity of grease ice. De Carolis and Desiderio (2002) and De Carolis et al. (2005) later extended Keller's model to include the viscosity of water under ice covers.

Each of the three classical theories partially captures the mechanical processes associated with an ice cover that moves and deforms in a wave field. The thin elastic theory focused on the elastic strain as well as the inertia of the ice cover, the mass loading theory considered only the inertia, and the viscous theory focused on the strain-rate-dependent dissipation. Recognizing all these processes may co-exist, several viscoelastic theories have been proposed to model sea ice in a wave field. Squire and Allan (1980) extended the thin elastic plate theory by introducing a complex elasticity. Wang and Shen (2010b)

extended Keller (1998) model to consider the ice cover as a Voigt-type viscoelastic continuum. This continuum model is based on the concept that in any representative region of an ice cover, part of it would experience elastic deformation while the rest would undergo some rate dependent dissipation. The resulting dispersion relation is shown to converge under proper limiting conditions to all three classical theories mentioned above. In addition to these two viscoelastic theories, Mosig et al. (2015) investigated another relevant theory developed for wind-induced vibration of floating roofs on oil storage tanks (Robinson and Palmer, 1990). The fundamental principles of this theory are analogous to the existing models on sea ice.

A viscoelastic dispersion relation is appealing for wave modelling in MIZ, because it can predict different wavelength changes and wave energy attenuation by choosing different combinations of elastic and viscous parameters. By fitting the observational data obtained from different ice cover types, it is envisioned that one may connect different ice types to their distinct viscoelastic properties, as demonstrated in the laboratory studies (Newyear and Martin, 1999; Zhao and Shen, 2015) and a field study (Cheng et al., 2017). However, the validity of the dispersion relation from these viscoelastic theories needs to be established to confidently apply it to the field. Such model validation can only be done in the laboratory under controlled conditions with true viscoelastic materials and clean wave forcing. Laboratory experiments, closer to theoretical conditions with lesser complexities, enable the comparison of the experimental results with different viscoelastic theories to validate the existing theories and point out missing mechanisms to be considered in future. In real ice field, other than the rheological properties of sea ice, bottom roughness of floating ice floes (Kohout et al, 2011), ice pack compression (Liu and Mollo Christensen, 1988) etc. could also contribute to wave damping in though they are not considered in the above-mentioned continuum models. In addition to enhancing our understanding of wave propagation through ice covers, the finding is also relevant in wave interaction with very large floating structures (Hermans, 2004).

The first attempt of experimental validation of viscoelastic theories was performed by Sree et al. (2016a). A set of laboratory experiments are conducted using viscoelastic covers with different properties, which are measured in-situ in the laboratory using a rotational rheometer. The covers are made of oil-doped polydimethylsiloxane (PDMS). Different properties are achieved by varying the percentage of a curing agent added during preparation. The model predictions by Wang and Shen (2010b) are found to be generally consistent with the experimental data from wave flume experiments, both in terms of the switching of normalized wavelength from less than to greater than 1, as well as the magnitude of the change. Thus, the study provided a general confirmation of the viscoelastic theory by Wang and Shen (2010b) in terms of the dispersion relation with respect to the viscoelastic properties and wave characteristics. However, there are uncertainties introduced by the free-body motion of the floating cover due to its finite extent. In addition, it is not possible to examine the wave attenuation through the viscoelastic cover because the magnitude of wave decay through the cover is relatively insignificant compared to the motion uncertainties.

The wave ice experiments by Zhao and Shen (2015) utilized the Wang and Shen model by fitting the model with measured wavenumber and attenuation coefficient, to inversely determine the effective ice properties. In the current study, the laboratory validation effort using the PDMS approach with a much longer viscoelastic cover for attenuation as well as different cover thicknesses is continued. The additional combinations of viscoelastic cover properties and wave characteristics performed in the new experiments provided more data for further confirmation of the dispersion relation predicted from viscoelastic theories. In addition, experimental measurements on wave attenuation through a true viscoelastic cover are also reported for the first time, which enables the close examination of another aspect of predictions of critical importance from viscoelastic theories.

The following sections will begin by describing the material preparation along with its mechanical properties and the experimental setup for wave experiments. Data analysis

for wave celerity and attenuation, and comparison of the experimental results with theoretical models are presented, followed by the discussion and conclusions.

4.2 Experiments

4.2.1 Material preparation

In the present study, the viscoelastic covers are 3 m long, which span from the center of the flume to the beach. Wave experiments using 1 m long covers are reported in Chapter 3 earlier, which assessed the dispersion relation under the viscoelastic cover and the results are found to agree well with the predictions by WS model. However, the 1 m cover length do not allow the determination of wave attenuation since the damping is too slight to be accurately measured.

The greatest challenge in the current experiments is the natural curing of the 3 m long covers. The process of transferring a long and soft PDMS cover prepared outside the wave flume with a curing tray, as used in the previous study (Sree et al. 2016a), often damages the cover. Another new set of in-situ procedures is developed to make the cover inside the wave flume as described in Sree et al. (2016b).

Covers with various levels of viscoelasticity are produced by varying the percentage of curing agent used, $m_{CA} = 4, 6, 8$ and 10%. The freshly prepared PDMS material is left to cure for 7 days at room temperature. Figure 4.1 shows the cover inside the flume under curing. The inside walls of the 3 m long confined region for the PDMS cover to cure are lined with transparent plastic films to prevent the adhesion of the PDMS cover to the flume. The layer thickness is $h_s = 0.01$ m for covers with $m_{CA} = 6, 8$, and 10%. For the 4% mass of curing agent, two thicknesses, $h_s = 0.01$ m and 0.02 m, are used.

Figure 4.1: Curing of PDMS inside wave flume (view from above).

The rheological properties of oil-doped PDMS covers are analysed once the covers became stable after curing. The test results are shown in Table 4.1. When m_{CA} is increased from 4 to 10% the material become stiffer with larger G' but smaller G''/G' . The values obtained for G' and G'' shows that the PDMS material properties can be approximated by the Voigt model with G' nearly constant and G'' monotonically increasing with frequency (Sree et al. 2016a). The viscoelastic parameters used in the SA and WS models are related to the storage and loss modulus as $G = G'$, $\nu = G''/\rho_1\sigma$. Table 1 shows that viscosity increases with m_{CA} . The Poisson ratio is not measured, the value $\vartheta = 0.5$ for PDMS neglecting the influence of oil in the property (Folch, 2012) is used. The irrelevance of finite mechanical properties for PVC film in this study will be discussed in Section 4.4.2. Instead the PVC film experiments are performed to estimate the influence of factors other than the rheological properties of a cover on surface waves, i.e. the boundary layer dissipation under a floating cover.

For the present experiments, two types of floating covers are used: PVC film and PDMS covers. The thickness of the viscoelastic PDMS layer h is at either 0.01 m or 0.02 m, $\rho_1 = 940 \text{ kg/m}^3$, water depth $H = 0.3 \text{ m}$ and $\rho_2 = 1000 \text{ kg/m}^3$. The viscosity and shear modulus for the PDMS covers are provided in Table 4.1. These values of material properties will be used to obtain the theoretical wave speed and attenuation from SA and WS models. The thickness of the PVC film is on the order of 10^{-4} m , its mechanical properties are not directly measured. As will be discussed later, for such thin film its mechanical properties become irrelevant for the SA and WS models. The RP model requires a phenomenological parameter which can only be calibrated.

4.2.2 Wave flume experimental setup

The laboratory experiments are conducted in a wave flume (see Figure 4.2) at the Environmental Process Modelling Centre (EPMC) Laboratory of Nanyang Environment and Water Resources Institute, Singapore. Monochromatic waves with periods from $T = 0.5$ to 1.0 s at 0.05 s intervals are generated for the experiments. A static calibrated capacitance wave gauge is installed near the wave generator to control the stroke of the piston to produce incident waves of the required amplitude. The values of wave steepness ($k_o a$) for the experiments are provided in Table 4.2, where k_o is the wavenumber and a is the wave amplitude in open water. Smaller amplitudes are used for high frequency cases, to prevent the water running over the cover, referred to as overwash at the leading edge. The repeatability of the experiments is checked by performing each experiment at least three times (see Figure 4.3).

It is noted that for the range of waves tested, the effect of surface tension of water, T_s , on the dispersion relations can be neglected as $\frac{T_s k^2}{\rho g} \sim 0.002 \ll 1$.

Table 4.1: Mechanical properties of PDMS cover. Viscosity and shear modulus obtained for Voigt model.

T (s)	ω (rad/s)	G'' (kPa)				$G' = G$ (kPa)				$\nu = G''/\rho\omega$ (m ² /s)			
		4%	6%	8%	10%	4%	6%	8%	10%	4%	6%	8%	10%
0.50	12.56	2.70	3.12	3.92	4.92	23.18	55.64	89.79	145.11	0.23	0.26	0.33	0.42
0.55	11.42	2.61	3.02	3.81	4.77	23.06	55.47	89.60	144.91	0.24	0.28	0.35	0.44
0.60	10.47	2.53	2.9	3.71	4.64	22.95	55.33	89.45	144.75	0.26	0.30	0.38	0.47
0.65	9.67	2.45	2.863	3.62	4.53	22.85	55.21	89.29	144.58	0.27	0.32	0.40	0.50
0.70	8.98	2.37	2.79	3.53	4.40	22.73	55.10	89.15	144.39	0.28	0.33	0.42	0.52
0.75	8.38	2.31	2.72	3.45	4.30	22.64	55.00	89.02	144.23	0.29	0.35	0.44	0.55
0.80	7.85	2.25	2.66	3.37	4.21	22.55	54.92	88.90	144.09	0.30	0.36	0.46	0.57
0.85	7.39	2.19	2.61	3.31	4.13	22.48	54.84	88.80	143.96	0.32	0.38	0.48	0.59
0.90	6.98	2.15	2.56	3.26	4.05	22.41	54.78	88.71	143.85	0.33	0.39	0.50	0.62
0.95	6.61	2.11	2.52	3.21	3.99	22.35	54.72	88.63	143.75	0.34	0.41	0.52	0.64
1.00	6.28	2.07	2.48	3.16	3.93	22.30	54.66	88.56	143.66	0.35	0.42	0.54	0.67

Figure 4.2: Schematic diagram of wave flume.

In the present study, the 3-m long PDMS covers extend up to the beach. The beach and the inertia for the long covers minimize possible surge or drift as well as free-body rotation without the aid of additional restrictions. Seven ultrasound sensors (US325, Ultra lab 40D, General Acoustics, Germany) with resolution of 0.18 mm and sampling frequency of 50 Hz are installed above the floating covers to record its surface deflection upon wave action. The synchronized data from all ultrasound sensors are then collected from LABVIEW, using a common data acquisition system (NI 9215, National Instruments, Texas).

The first ultrasound sensor is held 20 cm away from the leading edge to avoid possible edge effects like flapping due to damped travelling waves. The positioning of ultrasound sensors over the 0.01 m thick covers is as follows:

- (i) The first ultrasound sensor, US1, is fixed at 20 cm from the leading edge.

- (ii) The second ultrasound sensor, US2, is placed at 50 cm from the leading edge and the other five sensors are kept at 40 cm distance apart from each other following US2 towards the beach.
- (iii) The first set of experiments are conducted with the position of the ultrasound sensors as mentioned above for the required wave periods.
- (iv) The second set of experiments are conducted after moving the ultrasound sensors other than US1 by 20 cm towards the beach from their previous position. Each experiment from the second set is also repeated three times. Because of the repeatability of the experiments, the two sets under the same wave condition are then combined to effectively increase the monitoring density along the floating covers. In all, each wave test is monitored at 13 locations with respective distance from the leading edge as: 20, 50, 70, 90, until 270 cm.

For the 0.02 m thick PDMS cover, the distance between the sensors is maintained at 20 cm for the first set of experiments. In the second set of experiments, all the sensors are shifted by 10 cm from their previous position (except US1), resulting in the 13 locations as: 20, 50, 60, 70, until 160 cm. This positioning is done so that the surface elevation under stronger attenuation is still within the measurable range of the ultrasound sensors.

For each PDMS cover, the time series of surface displacement data at 13 locations along the length of the cover are obtained with a total of 66 experiments (i.e. 11 wave periods $\times$ 2 position changes $\times$ 3 repetitions). Open water experiments and experiments with floating PVC film are also conducted with the same input wave conditions to compare with the viscoelastic floating cover cases. For each PVC cover, the time series of surface displacement data at only 7 locations along the length of the cover are obtained with no shift provided for the sensors. Thus, a total of 33 experiments (i.e. 11 wave periods $\times$ 1 position $\times$ 3 repetitions) are conducted. Open water experiments and experiments with floating PVC film are also conducted with the same input wave conditions to compare with the viscoelastic floating cover cases.

To increase the time resolution of the ultrasound sensor records, prior to data analysis, numerical interpolation with the *PCHIP* function (Piecewise Cubic Hermite Interpolating Polynomial) is used to increase the data rate from 50 Hz to 200 Hz. Only the first three fully developed waves are considered in the analysis (see Figure 4.3) to avoid the effect of reflection from the beach.

Figure 4.3: The surface profile data obtained using US1 for wave period, $T = 0.5$ s, $m_{CA} = 10\%$ and cover thickness, $h_s = 0.01$ m. The three fully developed waves considered are enclosed in the rectangle box. The repeatability of test is confirmed using three experiments (cross symbol, asterisk and circle).

4.3 Data Analysis

4.3.1 Wave celerity

Wavenumbers are calculated in the same way as mentioned in Sree et al. (2016a) for the 1 m long PDMS covers. The use of only three waves limits the usage of spectral method for the determination of wave celerity or phase speed. In the present study, the celerity is determined by measuring the times corresponding to the peaks of the three waves with the *findpeaks* function at each sensor and is calculated using Equation 3.7.

Table 4.2: Input wave characteristics.

T (s)	λ (m)	k_o (rad/m)	$a \times 10^{-2}$ (m)	$k_o a$	h_s (m)
0.50	0.39	16.20	0.43	0.07	0.02
			0.51	0.08	0.01, 0.02
0.55	0.47	13.32	0.43	0.06	0.02
			0.51	0.07	0.02
			0.60	0.08	0.01, 0.02
0.60	0.56	11.24	0.51	0.06	0.02
			0.60	0.07	0.02
			0.68	0.08	0.01, 0.02
0.65	0.65	9.72	0.72	0.07	0.02
			0.85	0.08	0.01, 0.02
0.70	0.76	8.29	0.85	0.07	0.01, 0.02
0.75	0.86	7.31	0.85	0.06	0.01, 0.02
0.80	0.96	6.56	0.85	0.06	0.01, 0.02
0.85	1.05	5.99	0.85	0.05	0.01, 0.02
0.90	1.14	5.50	0.85	0.05	0.01, 0.02
0.95	1.26	4.99	0.85	0.04	0.01, 0.02
1.00	1.37	5.57	0.85	0.04	0.01, 0.02

For the PVC film cover, $c_s/c_o \approx 0.99 - 1.01$ for all wave periods tested. Hence the resulting dispersion is unchanged from the open water condition. For the PDMS covers, the measured celerity behaves almost the same as with the 1 m case (Sree et al., 2016a). As shown in Figure 4.4, c_s along covers is different from the open water celerity, c_o . For $T = 0.5$ s, $c_s < c_o$, for $m_{CA} = 4\%$. It increases with increasing m_{CA} and finally become greater than c_o . The increase of celerity with increasing m_{CA} is less dramatic with increasing wave period. The celerity obtained using the sensors near the free edges shows deviation from the values obtained away from the edges, due to the edge effect from the

damped travelling waves (Fox and Squire, 1990). For the 1 m cover studied in Sree et al. (2016a), the proportion of the data affected by the edge is greater.

Figure 4.4: Wave celerity along the length of the PDMS cover, $h_s = 0.01$ m. To aid visualization, figure for each wave period is separated by dotted lines into 4 plots for different values of m_{CA} : Circles = 4%, asterisk = 6%, diamonds = 8%, and triangle = 10%. Solid line = open water wave velocity.

4.3.2 Wave attenuation

The vertical locations of crests and troughs from the interpolated data (200 Hz) are determined using the *findpeaks* function. The wave height is then obtained as the distance between the adjacent crest and trough. The wave amplitude, A_x , calculated as half of the wave height, is plotted against the distance along the cover as shown in Figure 4.5. For wave amplitude measurements, especially at small amplitudes in our study, sensor resolution is an important issue. With the sensor resolution at ~ 0.18 mm, less than 10% of the minimum amplitude shown in Figure 4.5, the data are adequate for our attenuation analysis.

To compare with the three viscoelastic theories, the measured amplitude data is fitted with an exponential curve, included in Figure 4.5. The attenuation coefficient for each case is defined as the parameter, α , of the fitting curve

$$A_x = A_o e^{-\alpha x} \quad (4.1)$$

where, A_x is the wave amplitude at the distance x from the leading edge and A_o is the amplitude at $x = 0$. From Figure 4.5, the wave amplitude clearly decreases with distance from the leading edge for $T = 0.5$ s. The dependence of attenuation coefficient on wave period and thickness of the cover is clearly visible. The root mean square error for the fitted curve is confirmed to be less than 10^{-4} for all the cases tested.

4.4 Results and Discussion

4.4.1 Wavenumber

The variation of normalized wavenumber, $\kappa = k_s/k_o$ with wave period is provided in Figure 4.6. In this figure and all subsequent figures, the theoretical curves are from Equations 2.12, 2.14 and 2.16 for the three models considered. The required elastic parameters for all the three models and the viscous parameters for the SA and WS models are from Table 4.1. But for the RP model, the viscous parameter is inversely calculated

by fitting the attenuation data for one wave period ($T = 0.55$ s for $m_{CA} = 4\%$) with the theoretical model. This value is applied for all other cases to calculate the real and imaginary parts of the complex wavenumber, k , for other wave periods.

Figure 4.5: Normalized wave amplitude along the length of the floating covers. PDMS cover with $m_{CA} = 4\%$, $h_s = 0.01$ m and $T =$ a) 0.5 s and b) 0.7s. Wave attenuation along: c) PDMS cover with $m_{CA} = 4\%$, $h_s = 0.02$ m and $T = 0.5$ s, and d) PVC film for $T = 0.5$ s. Overlapping circles confirm the repeatability of tests. The solid line indicates the exponential fit curve and dashed line indicates the 95% simultaneous functional prediction bounds of the fitted curve.

The experimental results with the 1 m PDMS cover reported in Sree et al. (2016a) are also superimposed in Figure 4.6. It is seen that for the longer PDMS covers, with much

less edge effects, the measured wavelength compares even better with the theoretical predictions than Sree et al. (2016a). There is practically no difference among the three predictions within the tested parameter range.

Figure 4.6: Variation in normalized wavenumber with wave period for PDMS cover ($h_s = 0.01\text{ m}$). The three model predictions are practically identical.

For open water, $\kappa = 1$; wave lengthening compared to open water case is represented by $\kappa < 1$ and wave shortening by $\kappa > 1$. The normalized wavenumber depends on both the material property and wave period. For $h_s = 0.01$ m, all three viscoelastic models predict the experimental results well. Figure 4.6 shows that for $m_{CA} = 4\%$ and thickness, $h_s = 0.01$ m, wave shortening occurs for all wave periods. However, the behaviour of κ is not monotonic. The dimensionless wavenumber increases from $T = 0.5$ s to 0.6 s to reach the maximum then drops with further increase of T . For $m_{CA} = 6\%$, the wavenumber is similar to the open water case for $T = 0.5$ s. With increasing wave period, an increase in κ followed by a decrease is again seen. For $m_{CA} = 8$ and 10%, wave lengthening is observed for $T = 0.5$ s, while all other cases show wave shortening. This non-monotonic change of wavenumber with wave period is shown in all three viscoelastic models and the experimental data. Hence, the modification of wavenumber in a covered region, with respect to open water case, depends largely on the combination of wave period and the material property of the covers. This non-monotonic behaviour is mentioned in the theoretical studies by Squire (1993) and Collins et al. (2017). However, for the thicker PDMS cover ($h_s = 0.02$ m), unlike the thinner covers with the same $m_{CA} = 4\%$, differences between all three theoretical predictions and experimental data are noted. Additional discussion on this wave number discrepancy is provided in the following sections.

4.4.2 Wave attenuation

A comparison of the attenuation coefficient, α , obtained from the experiments (circles) and the imaginary part of the complex wavenumber, k_i , obtained from the theories (lines) are shown in Figure 4.7. The error bars show the 95% confidence bound for the coefficient obtained when fitting the experimental data for average amplitude of the three fully developed waves with an exponential curve. Qualitatively, the attenuation coefficient decreases with increasing wave period, except for $m_{CA} = 10\%$ and the thicker sheet with $m_{CA} = 4\%$, where a rollover is seen. This rollover phenomenon is not

predicted by two of the three theoretical models, only the RP model demonstrates a mild rollover for larger m_{CA} or a thicker cover. Quantitatively, except with RP model, a significant difference between the experimental results and theoretical predictions is observed. As explained earlier, unlike the other two models where the viscous parameter is from the actual rheometer data, the empirical viscous parameter, γ for the RP model is calibrated from data fitting. A good comparison between the RP model and the experimental data is thus expected.

A common challenge in a wave flume facility to analyse the wave parameters is to avoid contamination from the beach reflection. Reflection can produce partial standing waves that can influence both celerity and attenuation measurements. The only way to avoid the influence of reflection is to truncate the data at the time before the arrival of the reflected waves, which may be estimated using the wave celerity. Hence, the number of “clean waves” is first determined in the data analysis. The time needed for the wave energy to travel from each sensor with the group velocity, reflect from the vertical wall near the beach and return to the sensor is calculated. The time taken for each wave to make a round trip from a sensor to the wall is denoted by t_r . This time span is calculated as twice the distance between the sensor location and wall (x_b) divided by c_g , $t_r = 2x_b/c_g$. The number of waves considered free from reflection is calculated as $N_c = \frac{t_r}{T} - N_d$, where N_d is the number of waves generated before the fully developed waves, which is estimated from the ultrasound signal. Using this method, it is found that for $T = 0.5$ s, N_c is 33 for US1 and 8 for US13. For $T = 1.0$ s, N_c is 4 for the US1 and 3 for the US4. Since only three fully developed waves are considered for the analysis of wave attenuation, the portion of the PDMS cover where $N_c < 3$ are excluded from further analysis. Thus, the effect of reflection can be ruled out as the reason for discrepancy between the experimental data and theoretical results.

Figure 4.7: Attenuation coefficient against wave period ($h_s = 0.01$ m). The circles are the mean of each test and the error bars represent the 95% confidence interval limit for fitted curve.

Some of the plausible reasons for the observed discrepancies in wave attenuation are provided as follows:

4.4.2.1 Linear constant viscoelasticity

The three theories considered are all based on linear viscoelastic assumptions. The dispersion relation in the WS model is based on treating the ice cover as a linear Voigt material, with G' independent of rotational frequency and G'' linearly varying with rotational frequency. In the SA model, the Euler-Bernoulli assumption is used with a linear complex modulus. When the ice thickness becomes very thin and the elastic modulus is sufficiently high, these two models essentially converge to each other (Mosig et al., 2015). The RP model (1990), though quite different from the above two, also assumes a linear combination of the elastic and viscous stresses.

For a strict linear Voigt model, the storage modulus G' is constant with respect to varying frequency ω , while G'' is linearly proportional to ω , implying a constant viscosity with respect to ω . Table 4.1 shows that our PDMS material is not strictly a linear Voigt material. The SA and WS models are re-examined and found that for a small amplitude monochromatic wave field, it is inconsequential whether the material is a linear Voigt material or frequency-dependent material. The same conclusion is also made in Mei et al. (2010) for wave-mud interaction. Because each test has a single frequency with the associated viscosity, there is no change in the form of the dispersion relation. The results shown in Figures. 4.6 and 4.7 already adopt these frequency dependent viscosities.

To check whether a small amplitude wave assumption does apply, several different wave amplitudes are analysed, the results are shown in Table 4.3. Under different wave amplitudes, the variation of the attenuation is much lower than the discrepancies observed in Figure 4.7. Thus, the observed discrepancy cannot be attributed to the material properties of the PDMS covers or the linearity of the experimental regime.

The data analysis and theoretical assumptions are thus both ruled out to be the main cause of the possible discrepancies observed between the experimental data and theoretical predictions. Extra mechanisms that could be present in the laboratory but not in the theories are analysed next.

4.4.2.2 Damping due to water and wave flume interaction

It is known that in wave flumes, even under open water, there are several mechanisms that could produce damping (Miles, 1967). These mechanisms include: viscous dissipation by the BL with rigid boundaries (walls and bottom) and at the top surface, and the meniscus effect at the wall-air-water interface. For the top surface, minute impurities (surfactant or other films) could increase the wave damping from the clean water condition. Theoretical models were developed to account for these energy losses (Henderson et al., 2015). Using $\nu_w = 1.1 \times 10^{-6} \text{ m}^2/\text{s}$ as the kinematic viscosity of clean water, the attenuation from the wave surface is $\alpha_{CL} = \frac{2\nu_w k^2}{c_g} = \sim 0.001 \text{ m}^{-1}$ and the

boundaries of the flume $\alpha_{SB} = \sqrt{\frac{\omega \nu_w}{2}} \frac{k}{c_g} \left(\frac{1}{\sinh(2kd)} + \frac{1}{k_B} \right) = \sim 0.002 \text{ m}^{-1}$ for $T = 0.5 \text{ s}$, where

c_g is the group velocity (Sutherland et al., 2017). Both are two orders of magnitude lower than the discrepancies in Figure 4.7. For the possible impurity effect at the surface, using the maximum dissipation due to the presence of massless floating viscoelastic film

$\sqrt{2} \left[\frac{\nu_w^2 |k|^7}{g} \right]^{1/4} \sim 0.08 \text{ m}^{-1}$ for $T = 0.5 \text{ s}$ (Henderson et al. 2015). For the present study,

the attenuation under open water conditions is measured directly using the exact same procedure as with floating covers. The maximum attenuation, α_o , obtained is for $T = 0.5 \text{ s}$, which yield $\alpha_o = 0.05 \text{ m}^{-1}$ (see Figure 4.8). All these values due to the various mechanisms are much too low to explain the discrepancies observed in Figure 4.7.

4.4.2.3 Boundary effects due to the moving cover between walls

The effect of the floating covers moving close to the side walls is not considered for the model predictions. Between the PDMS cover and glass walls there is a clearance, $\Delta \cong 0.005$ m, on both sides. The attenuation coefficient due to the side wall may be obtained by considering the viscous dissipation per unit time per unit length caused by the fixed side wall on either side of the cover, $\frac{1}{T} \int_0^T 2\nu_w \left(\frac{\partial u}{\partial y} \right)^2 dt \sim \frac{2\nu_w L_s t_s}{\Delta} \left(2 \frac{\pi a}{T} \right)^2$. The corresponding energy dissipation per unit time per unit length is then $L_s \frac{d}{dx} (E_w c_g) \sim L_s \frac{a^2 \alpha e^{-2\alpha L_s} \rho_w g}{T} \frac{\lambda}{2T}$, where E_w is the wave energy density and L_s is length of the covers. The coefficient α calculated for $T = 0.5$ s is 10^{-6} m^{-1} , which is three orders of magnitude lower than all other mechanisms and thus insignificant.

4.4.2.4 Boundary layer under the covers

For wave attenuation under ice covers, Weber (1987) and Liu and Mollo-Christensen (1988) considered damping mechanisms in the water body below the ice cover. They used an eddy-viscosity to describe the boundary layer dynamics and the associated attenuation rate. Note that the effect of a boundary layer beneath the viscoelastic covers is not considered in the three viscoelastic models. Here, the laminar Stokes boundary layer theory is first considered which is only applicable to an oscillating rigid plate in a fluid. The viscous boundary layer for this situation has the thickness $\delta_s = 2\pi \sqrt{\frac{2\nu_w}{\sigma}}$. Using the kinematic viscosity and wave frequencies in our study, this thickness is in the range of 0.0025 m to 0.0035 m for $T = 0.5$ s to 1.0 s. Hence the attenuation coefficient from this boundary effect, $\frac{\pi \nu_w k}{\delta_s c_g} < O(10^{-2})$, is at least one order below the measured attenuation. However, even though the dissipation due to laminar viscosity next to a flat and horizontally oscillating surface is very low, the PVC cover tests show significantly higher dissipation as shown in Figure 4.5(d) and Figure 4.8 (circles). Without direct measurements of the flow structure under the viscoelastic covers, it can only be

speculated that for the PDMS covers, the dissipation due to this boundary layer effect is also significant.

Sutherland et al. (2017) provided another estimate of the wave attenuation from the laminar boundary layer effect. For a floating but inextensible thin film (with modulus $E \rightarrow \infty$), the theoretical attenuation coefficient is given by $\alpha_{IN} = \frac{1}{2} \nu_w \gamma_a k / c_g$ where $\gamma_a = \sqrt{\frac{\sigma}{2\nu_w}}$. If a finite elasticity is considered, Christensen (2005) showed $\alpha_e = \alpha_{IN} \left[\frac{2\varepsilon^2 + 4 \frac{k}{\gamma_a (1-\varepsilon)}}{(1-2\varepsilon+2\varepsilon^2)} \right]$ where $\varepsilon = \frac{Ek^2 \gamma_a}{\rho_w \sigma^2}$. For the present experiments with PVC film ($E \approx 1$ GPa), both formulas yield approximately the same value for the range of waves considered here. This boundary layer attenuation, α_{IN} , is shown as the solid line in Figure 4.8.

The estimated boundary layer attenuation in the present study, α_{IN} , is much larger than the sum of the theoretical predictions for clean water attenuation and side boundaries, $\alpha_{CL} + \alpha_{SB}$. Hence, the latter can be ignored from further analysis. Figure 4.8 shows the measured PVC attenuation, calculated PVC attenuation using the inextensible assumption discussed above, and measured open water attenuation (from different wave amplitudes). The measured attenuation from the water body under the film is on the same order as the measured attenuation under the PDMS, which is not considered in the SA and WS models but could be parameterized by the RP model.

Notable discrepancy between α_{IN} and attenuation caused by PVC cover is evident in Fig 8. Quantitative discrepancies from 20% to 90% for higher frequencies were also observed in the results by Sutherland et al (2017) when they tested very thin (0.25 to 0.5 mm) polypropylene and latex covers. A comparable discrepancy of almost twice the value of α_{IN} was observed in experiments conducted by Henderson et al. (2015) using floating cling wrap. Increased attenuation may be due to the resonance effect with energy

transfer from transverse to longitudinal waves when $\sigma = \left(\frac{2\nu\rho^2g^4}{E^2}\right)^{\frac{1}{5}}$ (Sutherland et al., 2017). The resonant frequency is approximately 0.002 rad/s, which is 3 - 4 orders smaller than the experimental frequency range considered, hence the resonant effect can be ignored. At present, a clear explanation is not available for the discrepancy.

Figure 4.8: Wave attenuation due to all additional damping mechanisms. Circles = experimental data for PVC cover, Black solid line = theoretically obtained attenuation due to inextensible sheet (α_{IN}), Black asterisk = experimental data for open water experiment for amplitude, $a = 0.01$ m, and Grey asterisk = experimental data for open water experiment with reduced wave amplitude as mentioned in Table 4.2 for $h_s = 0.01$ m. Each test is repeated at least three times. The circles are the mean of the tests repeated and the error bars represent the 95% confidence interval limit for the fitted curve.

The PVC attenuation strongly suggests that the damping in the boundary layer is much more significant than what is anticipated from an oscillatory laminar boundary layer. From physical arguments, the no-slip condition generates flows that are determined by the boundary geometry and its dynamics. The cover changes the horizontal velocity from being max (free surface) to zero (if the cover is held in place with only one degree of

freedom: vertical). No slip condition would apply here, which creates strong shear and damping. For the infinitely deep viscous model, the Reynolds number $= \frac{g^2}{4\nu\sigma^2} \sim O(10^5)$ for the wave parameters considered in the present study (Newyear and Martin, 1997). Adopting a Reynolds number in the form of $Re = u_{\max} * \delta/\nu$ instead, where δ is the boundary layer thickness, previous research (e.g. Song and Law, 2015) had shown that the laminar boundary layer will transit to partial turbulence when $Re > \sim 150$ and fully developed turbulence when $Re > \sim 500$ for a fixed boundary. The corresponding energy dissipation in the oscillatory turbulent boundary layer would be much higher than that of the laminar boundary layer. However, how the undulating water surface interacts with this boundary layer such that this critical Re may be reduced to the range in our experiments, is not currently well understood and needs to be further studied. This is a difficult hydrodynamic problem, since even in open water, how waves interact with the flow structure under the free surface is not completely known. Hence, the attenuation due to this effect would be unique for the viscoelastic properties of each cover. Without a complete knowledge of this boundary layer structure, its effect on wave attenuation can only be estimated as follow.

In Figure 4.9, a rough estimate of reduced experimental attenuation due to PDMS covers is obtained by subtracting the attenuation due to the boundary layer effect using the PVC cover from the actual attenuation obtained for the PDMS covers. The underlying assumption is that for the same wave period, the boundary attenuation under the PDMS cover is the same as the PVC cover. An improved correlation between the reduced attenuation and theoretical predictions is obtained, compared to Figure 4.7.

Finally, as discussed above, although the comparison for wavenumbers is good for thin PDMS covers as shown in Figure 4.7, it deteriorates for the thicker PDMS cover. It is unlikely due to the edge effect since the extra wave mode existing near the leading edge is two orders larger than the surface gravity waves with an attenuation rate $= 25 \text{ m}^{-1}$. One possibility for the discrepancy is that for a thicker cover, the boundary layer effect

may begin to not only influence the attenuation, but also the wave speed. Thus, the current findings highlight that a detailed study on the presence of boundary layer underneath an oscillating cover, with its influence on the decay of wave amplitude as well as wavenumber modification, need to be performed in the future.

Figure 4.9: Reduced attenuation coefficient against wave period ($h_s = 0.01$ m). Triangle = reduced experimental data, model predictions: a) Dark grey dotted line = SA model, and b) Light grey solid line = WS model.

Table 4.3: Variation of wavelength and attenuation coefficient with wave amplitude. $m_{CA}=4\%$, $h_s=0.02$ m.

		κ (mean $\pm$ standard deviation)						α (1/m) mean (lower upper, limit of 95% confidence interval)					
$a \times 10^{-2}(\text{m})$		0.43	0.51	0.60	0.68	0.73	0.85	0.43	0.51	0.60	0.68	0.73	0.85
$T(\text{s})$	0.50	0.89 ± 0.02	0.89 ± 0.02	-	-	-	-	0.28 (0.22,0.36)	0.27 (0.19,0.35)	-	-	-	-
	0.55	0.96 ± 0.01	0.96 ± 0.01	0.97 ± 0.02	-	-	-	0.30 (0.25,0.36)	0.24 (0.18,0.30)	0.22 (0.15,0.28)	-	-	-
	0.60	-	1.03 ± 0.01	1.03 ± 0.01	1.04 ± 0.02	-	-	-	0.22 (0.16,0.27)	0.26 (0.21,0.31)	0.26 (0.21,0.30)	-	-
	0.65	-	-	-	-	1.05 ± 0.03	1.07 ± 0.02	-	-	-	-	0.25 (0.21,0.29)	0.25 (0.22,0.28)

4.5 Summary

Floating viscoelastic covers with properties close to a linear Voigt material are prepared from oil-doped PDMS material. Four different PDMS covers are made covering a range of shear modulus between 22 to 145 kPa, and viscosity between 0.23 to 0.67 m²/s. Two different cover thicknesses, 0.01 and 0.02 m, are used in this study. The wavenumber and attenuation along the covers are measured and compared with three different linear viscoelastic theories (Squire and Allan, 1980; Robinson and Palmer, 1990; and Wang and Shen, 2010b).

The theoretical predictions match reasonably well with the experimental results for longer wave periods and the thinner cover. For the thicker cover, some discrepancy between the theoretical predictions and experimental data is observed. As explained in Chapter 4, the decisive factors for wavelength modification are flexural rigidity (fundamentally thickness and elastic modulus), this is confirmed legitimate in the present study with wave lengthening $\left(\frac{L_r(\omega^2/g)^3}{\rho_i h_s g} > 1 \right)$ observed for thicker ($m_{CA} = 4\%$, $h_s = 0.02$ m) as well as stiff floating covers ($m_{CA} = 10\%$, $h_s = 0.01$ m).

The wave attenuation has large discrepancies between the theoretical predictions and experimental data, more so for shorter wave periods and the thicker cover. Several missing mechanisms that could be estimated based on fundamental knowledge in fluid mechanics are examined. The strongest effect appears to be the boundary layer below the flexing floating covers. The experiments with the PVC film is performed to confirm this conjecture. Future study is necessary to directly measure the boundary layer structure under the floating sheet. It is noted that similar effects are also expected in the field under real ice covers. As shown in recent studies (Smith et al., 2016; Marchenko et al., 2017), waves may create complicated flow structures under an ice cover.

To conclude, for given input wave parameters, the modification of wavelength of surface wave in a covered region depends on its flexural rigidity or the easiness to bend. The wave amplitude decay on the other hand is a measure of the viscous dissipation by the floating cover. Nonetheless, to avoid complicated forcing and natural variabilities of material properties in the field, the type of controlled laboratory studies reported in this study is necessary for checking the theories for floating viscoelastic covers over water.

CHAPTER FIVE

Chapter 5

Wave Modification Due to Viscoelastic Floating Segments

The objective of this chapter is to analyse the effect of discontinuities on wave modification along a region covered with a viscoelastic floating layer. Viscoelastic covers of four uniform segment lengths (l_s) are used in the present experiments. The four different segment lengths are $l_s = 1, 0.5$ and 0.25 m as well as the continuous cover 3.0 m long cover mentioned in Chapter 4. In other words, the number of segments is: 3, 9, 12, and 1 with the total length kept at 3 m in all cases. The cover length $l_s = 3$ m represents the unbroken cover for the whole area under consideration. Experiments are performed with the input wave parameters as shown in Table 4.1. Once the tests for a particular segment length are completed, the cover is then cut successively into the next size for the experiments. The same wave parameters and ultra sound sensor positions are used in all tests, and each run is repeated two more times to ensure the repeatability of test results.

This chapter is organised as follows: the subsequent sections will begin by describing the experimental setup for wave experiments along with data analysis for wave celerity, attenuation and leading-edge reflection. A comparison of the experimental results on wavenumber with an existing empirical relation is presented in the discussion, along with attenuation and reflection, followed by summary.

5.1 Experimental Setup and Data Analysis

As mentioned in Chapter 4, the vertical surface displacement of viscoelastic floating covers is monitored with the aid of seven ultrasound sensors held above them along the length of covers. The first ultrasound (US1) sensor, hollow circle in Figure 5.1, is kept at 20 cm from the leading edge and the second sensor is at 30 cm from US1. The other

sensors, represented as solid grey circles, are kept at 40 cm interval. After the completion of the experiments with the required wave parameters, the position of ultrasound sensors, except US1, is shifted by 20cm (solid black circles in Figure 5.1). Thus, each set of tests for a specific floating cover consists of 66 runs ($3 \text{ repetitions} \times 11 \text{ wave periods} \times 2 \text{ positions of ultrasound sensors}$), and data at 13 locations are collected. Four wave gauges are installed in the open water region near the wave generator. Four wave gauges are installed in the open water region near the wave generator.

Figure 5.1: Top figure is the schematic diagram of the wave flume (profile view, not in scale). Bottom figure shows the location of discontinuities for the PDMS cover, with A representing the leading edge (top view). l_s (m): a) Unbroken cover length, $OA = 3.0 \text{ m}$, b) Thick black solid line = $OB = 1.0 \text{ m}$, c) Thin light grey solid line = $OC = 0.5 \text{ m}$ and d) Dashed black line = $OD = 0.25 \text{ m}$. The initial positions of ultra sound sensors are represented using grey circles and the shifted positions by black circles, with hollow circle representing US1.

5.1.1 Celerity

The wave celerity is calculated in the same manner as mentioned in Chapter 3 using Equation 3.7. Here, the first three fully developed waves are also considered for the analysis. Figure 5.2 shows the variation of celerity for $T = 0.5$ s, for two different cover properties and four different cover lengths. For $m_{CA} = 4\%$ and $h_s = 0.01$ m, the unbroken cover shows a slight zig-zag profile for velocity, which vanishes when the cover is cut into smaller pieces. The value of celerity decreases with decrease in l_s , though the magnitude of change is small. For $m_{CA} = 10\%$, a notable change in celerity with segment length is observed. The celerity for longer segment lengths is larger than the open water celerity, c_o . With the reduction in segment length, the celerity reduces and eventually drops below c_o , but still higher than mass loading velocity.

For $m_{CA} = 4\%$, unlike the cover with $h_s = 0.01$ m, the thicker cover ($h_s = 0.02$ m) has a phase velocity larger than the open water case for longer segment lengths. The phase velocity reduces with the reduction in cover length, and becomes equal to the open water wave celerity for $l_s = 0.5$ m. With further reduction in the cover length to $l_s = 0.25$ m, the phase velocity becomes less than the open water case but larger than mass loading velocity.

5.1.2 Attenuation

For the calculation of attenuation coefficient, the interpolated time series data without smoothing are used. The crest (η_{max}) and trough (η_{min}) are obtained as the local maxima and local minima for each wavelength considered. For the unbroken cover, a smooth gradual reduction in wave amplitude with distance is obtained (see Figure 5.3). However, in the presence of discontinuities for the broken cover cases, the variation of wave amplitude along the length of the 3.0 m stretch is uneven.

Larger values for the vertical displacement are obtained near to the discontinuities. This could be due to the pitching motion of segments as well as the presence of any damped travelling wave. For the same l_s , the irregularity directly depends on m_{CA} . For $m_{CA} = 4\%$, both the 3.0 m and 1.0 m long covers show almost the same profile for the amplitude decay, where $m_{CA} = 10\%$ shows larger difference between them.

Figure 5.2: Wave celerity along the length of PDMS cover for $T = 0.5$ s, $h_s = 0.01$ m, $m_{CA} =$ a) 4%, b) 10%, and c) $h_s = 0.02$ m, $m_{CA} = 4\%$.

For the determination of attenuation coefficient, the larger amplitudes obtained from locations closer to discontinuities are discarded as shown in Figure 5.3. The attenuation coefficient is obtained by fitting the amplitude of surface displacement data at various locations of the sensors with an exponential curve (Figure 5.3) using Equation 4.1.

Figure 5.3: Surface displacement along the length of cover. Plot showing all data points along length of cover: a) $m_{CA} = 4\%$, and b) $m_{CA} = 10\%$. Exponentially fitted curves only with the data points away from discontinuities: c) $m_{CA} = 4\%$, and d) $m_{CA} = 10\%$. Solid lines represent the respective exponentially fitted curve, l_s (m): Black = 3.0, Red = 1.0, Blue = 0.5, and Green = 0.25.

5.2. Results and Discussion

5.2.1 Wavenumber

The celerity measurement by Sakai and Hanai (2002) and Cheng et al. (2018) could not consider the data from consecutive sensors due to large error. The notable error was obtained due to the superposition of significant pitching of segments and the vertical surface wave displacement, for rigid floating segments. For the present study, the floating PDMS segments were not rigid enough to tamper the surface wave profile with pitching motion, hence a relatively constant velocity is obtained along the length of cover as shown in Figure 5.2. In this section the variation of normalised wavenumber with segment length and their rheological properties are discussed. The modification in wavelength along the covered region is dependent on the wave period is also discussed.

5.2.1.1 Smaller wave period

Firstly, the effect of material property on the wave length modification for the thinner covers is analysed (See Figure 5.4). For smaller waves, as m_{CA} increases from 4% to 10%, the value of κ decreases. For $T = 0.5$ s, $m_{CA} = 4\%$ κ is larger than one, decreases to ~ 1 for $m_{CA} = 6\%$ and reduces further for $m_{CA} = 8\%$ and 10% . Wave lengthening is observed for the unbroken stiff covers, whereas wave shortening for the flexible ones. For unbroken cover, all the cases, the value of κ increases with T , with the slope increasing along with value of m_{CA} .

For $m_{CA} = 4\%$, only a slight difference in κ is observed with segment lengths. With increase in m_{CA} , notable change in celerity with l_s is visible. For $m_{CA} = 8\%$ and 10% , the unbroken cover shows wave lengthening, while those with $l_s = 0.25$ m experience wave shortening. With increase in m_{CA} and reduction in l_s , κ moves towards the predicted mass loading wavenumber. The reason can be attributed to the fact that the stiffer covers bend less with waves with the decrease in cover length compared to flexible

covers. Thus, they act closer to point masses with celerity moving towards the mass loading velocity. The separation between the wavenumber predicted using thin elastic theory and mass loading theory increases with m_{CA} for the higher frequency cases. For covers with $l_s = 0.25$ m, the slope of increase in κ with T decrease with m_{CA} .

For the thicker cover, $m_{CA} = 4\%$ and $h_s = 0.02$ m, a clear distinction on the variation of normalised wavenumber with segment length is visible, unlike thinner cover. For $T = 0.5$ s, wave lengthening is observed for $l_s = 3.0$ m and 1.0 m, while there is no modification in wavelength for $l_s = 0.5$ m. Wave shortening is observed for $l_s = 0.25$ m. The variation of κ with l_s decreases with the increase in wave period until $T = 0.75$ s.

5.2.1.2 Longer wave period

For longer wave periods, a downward trend of decreasing κ with T is evident. For $m_{CA} = 4\%$, as compared to shorter waves, an exact opposite behaviour is visible. For longer wave periods, the value of κ decreases with the reduction in segment length. For $m_{CA} = 4\%$, the maximum value of κ is obtained for $l_s = 0.25$ m, κ gradually decreases with increase in l_s and a minimum is reached for the unbroken cover. This deviation for longer waves becomes less significant with increase in m_{CA} . For the thicker cover, $m_{CA} = 4\%$ and $h_s = 0.02$ m, a similar trend reversal is observed, the normalised wavenumber increases with segment length. For $l_s = 0.25$ m, the wavenumber moves closer to open water wavelength.

For the present experiments using PDMS, because of the presence of viscous effect in the cover, the applicability of the empirical formula by Sakai and Hanai (2002) needs to be tested. The plots showing the linear relation between E_{im} and IF (discussed in Chapter 2) are provided in Figure 5.5. The Y-axis ranges from -1 to 4, for better visibility of the empirical relation for the present case. It can be seen that for lower values of m_{CA} (4 and 6%), the range of E_{im} goes beyond the Y axis range.

Figure 5.4: Variation of normalised wavenumber with wave period for different floating segments.

Figure 5.5: Relation between E_{im} and IF for different PDMS covers. Segment length, l_s (m): a) Circle = 3, b) Asterisk = 1, c) Diamond = 0.5, and d) Triangle = 0.25. The solid line indicates the empirical relation obtained by Sakai and Hanai (2002).

The experimental study by Cheng et al. (2018) on effective elasticity of ice floes with the same empirical relation shows that the larger range of E_{im} depends on the separation between elastic wavenumber and mass loading wave number. The normalised elastic wavenumber (κ_e) for the current study is represented by the grey solid line and normalised mass loading wave number (κ_m) by the black dotted line in Figure 5.4. For the low wave period cases, the wavenumber separation is significant, and E_{im} values close to the empirical relation are obtained. For longer wave periods, the two wavenumbers overlap and E_{im} values go beyond the Y axis range.

For $m_{CA} = 4\%$, only $T \leq 0.6$ s show a clear distinction between κ_e and κ_m in Figure 5.4. Correspondingly only five points are visible in Figure 5.5. For $l_s = 3.0$ m and 1.0 m, $E_{im} = 1$ shows that the equivalent elasticity is the same as the Young's modulus of the cover for the longer sheets, but the value of E_{im} reduces with further reduction in length. For $m_{CA} = 6\%$, $E_{im} = 1$ is obtained only for the unbroken ice cover. The value of E_{im} thus depends m_{CA} and l_s . A comparison of the two different thicknesses for $m_{CA} = 4\%$ shows that the dependence of effective elasticity on segment length increases with the cover thickness.

The deviation of κ_e and κ_m is found to be dependent on m_{CA} for constant wave parameters. This in turn decreases the deviation of E_{im} for constant IF , thus bringing the points closer to the empirical relation for all wave periods. The value of IF is found to decrease in m_{CA} , which indicates that the dependence of change in equivalent elasticity with Young's modulus increase with m_{CA} .

5.2.2 Wave attenuation

The attenuation coefficient for the wave decay along the length of PDMS covers with discontinuous segments is obtained in the manner as discussed in Section 5.1.2. The fitted plot for the different length of sections for $m_{CA} = 10\%$ is provided in Figure 5.3. Figure 5.6 shows the variation of attenuation coefficient with the wave period and

segment length. It can be seen that $m_{CA} = 4\%$ shows the maximum attenuation for the unbroken cover case, and the attenuation coefficient decreases with increase in m_{CA} . The dependence of attenuation coefficient on l_s is not significant for $m_{CA} = 4\%$, except for $l_s = 0.25$ m. Unlike other cases, a small drop in attenuation coefficient is observed when l_s is reduced from 1 m to 0.5 m, but it increases with further reduction in l_s to 0.25 m. For all the other three values of m_{CA} (6, 8 and 10%), the attenuation coefficient increases with the number of discontinuities (N_s) of cover, though notable increase is observed only for the smaller segments. For the continuous unbroken sheet, α is largest for $m_{CA} = 4\%$ and decreases with increase in m_{CA} . However, when the cover is cut into smaller pieces a reverse trend is observed. The largest value of α is obtained for $m_{CA} = 10\%$, and the value decreases with m_{CA} . For $m_{CA} = 10\%$, roll-over is evident for the unbroken continuous cover, which vanishes for the broken covers. The plots clearly indicate that the attenuation coefficient depends largely on the wave period, number of segments and rheological property of the cover.

Figure 5.7 shows the free edge reflection and transmission coefficients for a surface wave entering a viscoelastic region from open water. The experimentally obtained transmission coefficient is calculated as $T_r = A_o/a$, where A_o is the wave amplitude at $x = 0$ for the unbroken cover using Equation 4.1 and a is the input open water wave amplitude in Table 4.1. It is found that the experimental results for the transmission coefficient match well with the theoretical predictions by Zhao and Shen (2013) except for the stiffest cover $m_{CA} = 10\%$. For all the cases, T_r increases while reflection coefficient (R) decreases with the wave period. It is also evident that both the reflection and transmission coefficients decrease with m_{CA} for same wave period. The rate of decrease in T_r with m_{CA} is more significant compared to R . With the discontinuities, the transmission at each free edge is dependent on the value of m_{CA} for the cover. Since T_r decreases with m_{CA} , one would expect the largest decay for $m_{CA} = 10\%$ and $l_s = 0.25$ m. For the same m_{CA} , the thicker cover reflects waves more significantly back to open water

with lesser transmission. For $m_{CA} = 4\%$ and $h_s = 0.02$ m, the reflection coefficient becomes larger than transmission coefficient for smaller wave periods.

Figure 5.6: Variation of attenuation coefficient with wave period for different viscoelastic floating segments.

Figure 5.7: Reflection and transmission coefficient for viscoelastic floating cover, $h_s = 0.01$ m. Theoretical predictions (Zhao and Shen, 2013): Black circles = reflection (R) and Black asterisk = transmission (T_r), and Grey asterisk represents experimentally obtained transmission coefficient.

Bennets and Squire (2012) provided an expression for the non-dimensional wave attenuation due to scattering in terms of the reflection coefficient.

$$\frac{\alpha_d}{l_s} = -\log(1 - R^2) \quad (5.1)$$

With the values of reflection coefficient obtained using the viscoelastic model, Figure 5.7 shows that flexible covers reflect more than stiffer ones. With an increase in thickness from 0.01 m to 0.02 m, the wave attenuation $\frac{\alpha_d}{l_s}$ increases by an order of magnitude. This is a significant finding as it demonstrates the strong dependence of wave attenuation on the cover thickness with discontinuities.

With discontinuities, the attenuation coefficient α can be split down into two parts, α_m due to the material property of cover and α_d due to the discontinuities. With N_s number of discontinuities, Equation 4.1 can be modified as

$$\frac{A_x}{A_o} = e^{-(\alpha_m x + \alpha_d N_s)} \quad (5.2)$$

$$A_x = A_o e^{-(\alpha_m + \alpha_d \frac{c_{vs}}{l_s})x} \quad (5.3)$$

where, c_{vs} is the concentration (area covered by viscoelastic segments to total area). For the unbroken sheet case, $\alpha = \alpha_m$ since $N_s = 0$. For the broken cases, $\alpha_d = (\alpha - \alpha_m)l_s / c_{vs}$. When the 3.0 m region is fully covered, $c_{vs} = 1$, the value is reasonable for the present case.

Table 5.1 shows a comparison on the value obtained for α_d for different segment lengths. It shows that α_d is dependent on h_s , l_s as well as m_{CA} for a given wave period. The value of α_d increases when l_s decreases from 1.0 m to 0.25 m, but $l_s = 0.5$ m deviates from the trend for some cases. The relative contribution of attenuation due to material property and discontinuities can be analysed using the value of $A_r = \alpha_d / l_s \alpha_m$. For $A_r \ll 1.0$, material property has more influence on the attenuation, while $A_r \gg 1.0$ indicates the larger influence of scattering effect due to discontinuities. $A_r \approx 1.0$ implies that both are significant.

Table 5.1: α_d values obtained for various cases considered.

m_{CA}	4%			6%		
T (s)	l_s (m)					
	1.0	0.5	0.25	1.0	0.5	0.25
0.50	0.01	0	0.01	0.01	0.02	0.05
0.55	0.01	0	0.04	0.05	0	0.07
0.60	0	0	0.01	0	0	0.07
0.65	0.03	0.03	0.01	0	0	0.05
0.70	0	0	0.03	0	0.03	0.06
m_{CA}	8%			10%		
T (s)	l_s (m)					
	1.0	0.5	0.25	1.0	0.5	0.25
0.50	0.05	0.05	0.09	0.09	0.08	0.19
0.55	0.04	0.01	0.10	0.08	0.04	0.14
0.60	0.01	0	0.04	0	0.03	0.09
0.65	0	0.01	0.05	0	0.01	0.09
0.70	0	0.01	0.05	0	0.02	0.05
$m_{CA} = 4\%, h_s = 0.02$ m						
T (s)	l_s (m)					
	1.0		0.5		0.25	
0.50	0.22		0.02		0.45	
0.55	0.10		0		0.28	
0.60	0.09		0		0.28	
0.65	0.07		0.02		0.15	
0.70	0.01		0		0.08	

Figure 5.8 shows that with $m_{CA} = 4\%$, the effect of discontinuities is negligible except for $l_s = 0.25$ m. The material property plays a significant role in wave attenuation along the covered region. When the value of m_{CA} increases to 6%, the effect of scattering becomes more evident, with $A_r > 1$ for $l_s = 0.25$ m. A similar trend in the increase in A_r with m_{CA} is observed for stiffer covers. Though these results are more evident for shorter wave periods, the relation between A_r and wave period is not monotonic. With the thicker cover and shorter wave periods, the attenuation due to scattering is 4 times than due to material property α_m . The relative contribution of scattering to material property decreases with increase in wave period.

5.2.3 Leading edge reflection

The reflection and transmission of waves due to a thin semi-infinite elastic plate was theoretically studied by Fox and Squire (1990). The analysis was later extended to solitary ice floes by Meylan and Squire (1994), who found that both the reflection and transmission are dependent on cover thickness and length. In the present study, the transmission coefficient is found to increase with the wave period as shown in Figure 5.7. Meylan and Squire (1994) showed that the maximum bending of elastic raft occurs when the wavelength tunes perfectly to the floe's length with zero reflection. This is termed as resonance, where $T_r = 1$. For the present experiments, sufficient data points are not available to verify the resonance phenomenon for the viscoelastic floating cover.

5.3. Summary

A novel laboratory experimental study on surface wave attenuation by discontinuous viscoelastic covers is performed. For a wave period, the wave celerity along the viscoelastic covered region depends on the rheological properties of the cover as well as segment length. Both wave lengthening and wave shortening occur depending on the segment length, wave period and cover properties. For smaller wave periods, the normalised wavenumber increases with reduction in segment length. The dependence of

wavenumber on the segment length is significant for stiff covers. For longer waves, the wavenumber clearly decreases with segment length and this reduction diminishes for stiff covers.

Figure 5.8: Relative effect of material property and discontinuity on attenuation of wave along viscoelastic cover.

A comparison between the experimental results and the empirical study by Sakai and Hanai (2002) is performed. It shows that the reverse calculated equivalent elasticity for the present experiments spans over a very large range unlike Sakai and Hanai (2002). The range depends on the value of m_{CA} , which in turn depends on the divergence between the wavelengths predicted by the ML model and the EP model. For stiffer covers, the notable difference between the two wavenumbers is visible, while the span of E_{im} is relatively less. Thus, it can be concluded that the empirical relation can only be used for stiff covers and in a specific range of wave periods.

The wave attenuation due to discontinuous viscoelastic covers is calculated by curve fitting of the experimental results. Only the data obtained from the sensors away from the segment gaps are considered. For flexible covers, the effect of discontinuities on attenuation is not significant. For stiff covers, however, there is an obvious increase in the value of attenuation coefficient with the reduction in segment length. The contribution of scattering and material property on the total attenuation is determined by subtracting the unbroken cover attenuation from the total attenuation. The contribution of scattering for flexible covers becomes notable only for very small segment lengths, even then the material property contributes more towards the wave attenuation. For stiff covers, the effect of scattering increases with the reduction in segment length, up to 4 times larger than that by material property. The total attenuation shows the monotonous relation of low pass filtering effect.

CHAPTER SIX

Chapter 6

Wave-Ice Experiments

MIZ is the most dynamic region in an ice-covered sea. The MIZ morphology and extent is related to wave-ice interactions stemming from surface gravity waves that penetrate, stress, and fracture the ice field. The floes scatter the incoming wave energy and change the frequency content of the wave energy distribution as waves penetrate farther into the ice (Dixon and Squire, 2001). Thus, the wave parameters including wavelengths (and hence the wave celerity) and wave amplitudes, change once they enter the ice-covered sea. Wave-forecast models urgently need the knowledge and understanding of these changes. Additional complication for the wave dynamics in the Arctic is their interactions with ice, whether is it solid ice sheets or a spectrum of ice floes in MIZ. This is a material problem and requires understanding the physical properties of ice. Such problem does not exist in any other Metocean applications. While field experiments are the only conclusive way of understanding the physics of oceanic phenomena and processes and of validating the respective models, laboratory experiments are a useful and perhaps necessary bridge.

Physical modelling in laboratory can aid on a) separating the different features observed in the field, and to produce estimates of the expected effects, b) determine the most essential properties and parameters to be measured and scrutinised in the field to distinguish, verify and quantify various theories available.

*This chapter is written based on Lucas Y.; Parra, S. M., Wang, D., Sree, D.K.K., Babanin, A. V., and Law, A. W. K. Wave attenuation and dispersion due to floating ice cover. Submitted to Applied Ocean Research (2019).

In the laboratory, different mechanisms and conditions, both for waves and ice, can be detached and even singled out and studied separately. Despite the challenges of applying these laboratory results to field conditions (Broström and Christensen, 2008; Zhao and Shen, 2015), such studies would provide a more certain guidance for the future field experiments. This chapter discusses on the wave ice interaction experiments conducted at the Sea-Ice-Wind-Wave-Interaction (SIWWI) facility, in Mitchel Laboratory, University of Melbourne during February 2018.

The experiment set-up consists of a wave flume ($L=15.0$ m, $B = 0.75$ m and $D = 0.6$ m) made up of transparent glass panels supported with wooden frameworks. The tank is housed in a refrigerated room equipped with a piston type wavemaker on one side and an inclined damping beach on the other side. The beach is made of PVC coil matting backed by stainless steel mesh. The room temperature could be varied between -15°C to 30°C to enable the growth and melting of ice. The wooden frameworks allow the usage of saline water in this wave flume set up. However, only freshwater is used for the present experimental study. A water depth of 45 cm is maintained for the current experimental study.

Various stages of ice, from grease to solid, can be produced in the flume itself. In addition, an external freezer facility is also available to produce ice sheets of specified dimensions that can later be transferred to the flume. In the present study, five types of ice conditions are considered: a) Unbroken ice sheet, b) Grease ice, c) Pancake ice, and d) Broken ice sheet. The preparation procedures for each type of ice cover is distinct.

6.1 Ice Cover Preparation

6.1.1 Continuous solid ice

Among all the other ice types considered, the large solid unbroken ice cover is the easiest to prepare. The preparation procedure for the unbroken solid ice is straightforward by

dropping the room temperature to a desired temperature level based on the ice thickness required and allowing the ice to form without any disturbance (see Figure 6.1). Two different thicknesses are considered for the present study, $h_s = 0.01\text{m}$ and 0.02 m . For both the cases, the initial room temperature at the start of preparation is $-1\pm 1^\circ\text{C}$ and which is gradually dropped to $-5\pm 1^\circ\text{C}$ for $h_s = 0.01\pm 0.05\text{ m}$ and to $-10\pm 1^\circ\text{C}$ for $h_s = 0.02\pm 0.05\text{ m}$. For thin ice, a time duration of 13.5 hours and for thick ice a duration of 10.5 hours is taken to achieve the required thickness. Intermittent stoppage of the cooling fans is automatically controlled to prevent the air-conditioning system from breakdown and is referred as DEFROST mode.

Figure 6.1: Unbroken solid ice cover, top view (left) and side view (right).

The ice growth started from the side walls towards the interior, hence the final thickness near the walls is slightly larger than interior. After the desired thickness of ice is obtained all along the length of flume, the ice attached to the wave generator is removed to enable wave generation. About 2.5 m long open water is allocated in front of the wave generator for the growth of fully developed waves.

The biggest concern for the experiment is to detach the ice sheets from either side of the wall. The cooling fans are turned off for one hour for the ice sheet to detach from the side walls, still it could rub against walls during wave transmission. The ice sheet is then cut from either side using steel strips dipped in hot water. Though the process is time-consuming, free-floating unbroken ice sheet with straight edges is successfully prepared.

6.1.2 Grease ice

Unlike unbroken ice sheets, grease ice is created in rough turbulent conditions. The air temperature in the room is maintained at 1°C overnight with some broken ice pieces. In the morning the air temperature is set to -15°C and higher amplitude waves are generated continuously with intermittent stoppage for few seconds at every 30 minutes to avoid break down. Grease ice of different concentrations are made by changing the amplitude and frequency of waves generated. To get a concentration of 30%, the wave frequency is set at 2 Hz with amplitude 5 V, for a duration of 4 hours. Similarly, for a concentration of 40% the frequency is reduced to 1 Hz with wave amplitude 5 V, for a duration of 6 hours. After grease ice is formed (see Figure 6.2), the cooling fan is turned off for 30 minutes to remove the rough ice crystals stuck to the walls.

Figure 6.2: Grease ice, top view (left) and side view (right).

6.1.3 Pancake ice

Two different procedures are used for the preparation of pancake ice for the present study (see Figure 6.3). The first method involves creating pancake ice from grease ice and the second method is preparing pancake ice from fresh water itself. In the first method, after the grease ice is formed at -15°C under wave conditions of 2 Hz and 5 V, the wave frequency is then reduced to 1.1 Hz with an amplitude of 5.99 V. The pancake ice of length range 250-300 mm (along the direction of wave propagation) and width twice of

the length is formed under the long wave conditions for 2 hours. This type of pancake ice formation is referred as greasy pancake ice.

In the second method, the room temperature is kept at -1.5°C overnight and in the next day morning the temperature is set to -10°C for ice production. Longer waves with frequency 0.7 Hz and amplitude 5.99V are generated. Thin pancake ice started to form in an hour, and after 5 hours solid circular pancakes are fully grown with a diameter of 40-60 cm. The thickness of the pancake ice towards the beach end is 5-7cm with an overlapping layered structure. The pancakes produced using the second method are more “solid” in nature as compared to greasy pancake ice.

Figure 6.3: Pancake ice cover, solid pancakes on the left side and greasy pancake on the right side.

6.1.4 Broken ice cover

The broken ice cover is made from the unbroken continuous ice by breaking the ice cover with a wooden hammer. Care is taken to ensure almost uniform floe size distribution. The photographs of the broken ice taken along with a ruler is used to obtain the floe sizes. The average area for the floes of thickness 1.0 cm and 2.0 cm are found to be $97.0 \pm 15.6 \text{ cm}^2$ and $77 \pm 14.2 \text{ cm}^2$ respectively. After each wave experiment, the ice floes that drifted to the open water are moved back to the ice-covered region to maintain a similar ice edge for all experiments.

6.2 Material Testing

Unlike the rheological testing for PDMS, proper material testing facilities for ice sheets are not available at test site. Three-point bending is performed in a crude way to find the approximate elastic modulus of ice sheets, whereas to identify different concentrations of grease ice, volume fraction of ice in grease ice is used.

6.2.1 Ice sheet

The three-point bending tests are performed at room temperature (27°C) to determine the Young's modulus of unbroken ice sheets. Horizontal strips of ice beam are cut out of the ice sheets for the testing. All the ice beams are kept inside the refrigerator at -8°C and are taken out at the time of experiment. Each experiment took an average time of 3-5 minutes to finish. The ice beams are held horizontal with the help of bottom supports on either side and is loaded on top at the center with weight applied at an increment of 50 g each until failure. The support span is $L_b = 0.503$ m.

A panel marked with precision grid is held behind the experimental setup to determine the deflection as shown in Figure 6.4. The distance between the adjacent markings in the panel is 5 mm. The whole experiment is recorded using a high-resolution video camera. The videos recorded are converted to images and the pixel distances between the known markings are used to obtain the deflection. Six beams are tested to check the repeatability of the test. The dimensions of the ice beams are provided in Table 6.1.

Figure 6.5a shows the plot of applied load F , at the center of the ice beam against the deflection δ_{ice} . Load F varies linearly with δ_{ice} in the elastic region. The ice being a rigid material that breaks easily once reaches the yield stress with less necking effect. The gradient of plot is given by $\frac{dF}{d\delta_{ice}} = \frac{48EI}{L_b^3}$ where the E is the Young's modulus. The average value of Young's modulus obtained from the six ice beams is found to be 2.8

GPa (Figure 6.5b). The measurements done on fresh water ice by Gold (1977) show that Young's modulus varied from 3.0 GPa to 10.0 GPa depending on the grain type. The measurements on salt water done by Langleben (1962) show that the value decreases with brine content.

Figure 6.4: Ice beams subjected to three-point bending, left) without and right) with loading. The beige-coloured rectangle is a wooden block used to separate/insulate ice from weights. The distance between two adjacent dots is 5.0 mm.

Table 6.1: Dimensions and corresponding moment of inertia, I , for the Tested ice beams.

Ice number	b_b (mm)		t_b (mm)		Span (mm)	$I = \frac{t_b^3 b_b}{12} \times 10^4 \text{ mm}^4$
	(min)	(max)	(min)	(max)		
1	54.4	57.3	12.0	15.3	503	1.19
2	50/0	50.2	13.5	16.2	503	1.37
3	46.0	48.4	16.9	16.9	503	1.90
4	44.5	53.0	14.0	15.0	503	1.24
5	31.0	31.0	17.0	18.7	503	1.46
6	17.2	18.6	15.3	19.9	503	0.81

6.2.2 Grease ice concentration

The volume concentration of grease ice is measured before each test run. It is estimated by collecting the ice slurry in a container up to its brim and measuring the whole mass and the ice mass without water in it. The volume concentration is calculated as $m_i/m_s \times 100$ where m_i is the mass of ice alone and m_s is the mass of the ice slurry in the container. 40% and 30% concentrations are then employed in actual tests. As mentioned before, the pancake and broken ice covers are classified based on their dimensions.

Figure 6.5: Determination of Young's modulus. a) Deflection of beam with force applied, for beam of width 47.2 ± 1.2 mm and thickness 16.9 mm (Ice number 4), b) the elasticity value for the six beams considered.

6.3 Wave Experimental Setup

Prior to the wave-ice experiments, the open water calibration of the tank is done. The static calibration of the ultrasound sensors is done to obtain the voltage to displacement conversion factor for the sensors. The dependence of the conversion factor on temperature or humidity is tested. The relative difference in the values at 15°C and -5°C is less than 3% for all the seven sensors used.

The feasible range of wave frequency for the wave generator is between 0.5 Hz and 2.0 Hz. Based on the previous experiences, it is decided to choose the wave frequency range from 0.8 Hz to 1.5 Hz (Table 6.2). Longer waves are susceptible to wave reflection and high frequency waves decay quickly along ice cover, hence the extreme frequencies are not considered. The open water experiment is conducted with the first sensor kept at 2.5 m from the wave generator. Unlike the wave-ice experiments only six sensors are used due to technical reason, and they are kept at 0.75 m apart along the length of the tank.

The second task is the calibration of wave generator, the input for the wave generator is in voltage which need to be calibrated with the required wave heights for the necessary frequencies. The wave parameters used for the present study is provided in Table 6.2, where a and k_o are the wave amplitude and wavenumber for open water case. The voltage corresponding to the required wave height is obtained through linear interpolation from the voltage against amplitude relation. Rather than continuous wave generation, a packet of N_w waves are generated to minimise the effect of beach reflection.

The wave celerity and attenuation for ice covers are done in the same manner as for the ultrasound sensors. The broken ice cover experiments are done during the initial stages of the study; hence the positions of the ultrasound sensors are different for broken ice cover. For the broken ice cover, the sensors are kept at 0.5 m apart over a stretch of 5.0 m. If the same setup is used for pancake or grease ice cover the sensors near to the beach cannot sense any surface displacement for higher frequency cases due to high damping. The positioning of sensors along the length of cover are shown in Figure 6.6.

Figure 6.6: Schematic diagram of wave flume

Table 6.2: Wave parameters for SIWWI experiments.

f (Hz)	N_w	T (s)	$a \times 10^{-3}$ m	k_o (rad/m)	$k_o a \times 10^{-2}$
0.800	7	1.25	7.5, 10, 12.5	6.10	0.22, 0.30, 0.37
0.905	10	1.11	7.5, 10, 12.5, 15	4.52	0.27, 0.36, 0.45, 0.54
1.042	14	0.96	7.5, 10, 12.5, 15	9.05	0.34, 0.45, 0.56, 0.68
1.227	17	0.82	7.5, 10, 12.5, 15	3.57	0.46, 0.61, 0.76, 0.92
1.500	20	0.67	7.5, 10, 12.5, 15	2.96	0.68, 0.91, 0.11, 0.14

6.4 Results and Discussion

6.4.1 Open water

The two forms of wave velocity shall be analysed: phase velocity and group velocity. As discussed in previous chapters, wave celerity or phase velocity ($c_p = \lambda/T$) refers to the speed at which each wave propagates. Group velocity (c_g), on the other hand refers to the speed with which the wave group propagates. Surface waves are dispersive in nature for deep and intermediate waters with a non-linear relation between the wavenumber and wave frequency, $c_g \neq c_p$. To estimate c_g for the present experiments, waves are generated in packets of N_w numbers to have the group wave profile as provided in Table 6.2.

6.4.1.1 Phase velocity

The wave celerity is measured in the same way as mentioned in Equation (3.7), with only the first five fully developed waves considered. The experimentally determined open water celerity matches well with the theoretical predictions by the linear wave theory, as shown in Figure 6.7. The overlapping markers confirm that the wave celerity is independent of the wave amplitudes considered.

Figure 6.7: Open water wave celerity for the SIWWI facility. $a \times 10^{-3}$ (m): Circle = 7.5, Asterisk = 10.0, Triangle = 12.5, and Diamond = 15.0. Dotted line represents linear wave theory.

6.4.1.2 Group velocity

To measure the group velocity, the surface wave displacement (η) recorded by US1 and US2 are considered. The areas under the absolute value of surface displacement plots corresponding to the two sensors ($\sum A(|\eta|_{US1})$ and $\sum A(|\eta|_{US2})$) are calculated using trapezoidal rule. The time taken for the group wave profile to travel from US1 to US2 is calculated as

$$\Delta t_g = \frac{\sum A(|\eta|_{US2})t}{\sum A(|\eta|_{US2})} - \frac{\sum A(|\eta|_{US1})t}{\sum A(|\eta|_{US1})} \quad (6.1)$$

Group velocity, $c_g = \frac{\Delta x}{\Delta t_g}$, where Δx is the distance between US1 and US2. This method is referred as displacement method.

Group velocity is also defined as the speed of energy moving, so a slightly different method referred to as energy method is also used for its calculation. Since the wave energy is proportional to the square of wave amplitude, Δt_g can be obtained as

$$\Delta t_g = \frac{\sum A(|\eta|_{US2}^2)t}{\sum A(|\eta|_{US2}^2)} - \frac{\sum A(|\eta|_{US1}^2)t}{\sum A(|\eta|_{US1}^2)} \quad (6.2)$$

The wave velocity obtained by the two methods of calculations are provided in Figure 6.8. It is shown that the results obtained using the wave energy method fit relatively well the theoretical predictions using the equation $c_g = \frac{c_p}{2} \left(1 + \frac{2kd}{\sinh(2kd)}\right)$. The group velocity is also found to be independent of wave amplitude.

Figure 6.8: Group velocity for open water condition: a) Displacement method and b) Energy method. $a \times 10^{-3}$ (m): Circle = 7.5, Asterisk = 10.0, Triangle = 12.5, and Diamond = 15.0. Dotted line represents linear wave theory.

6.4.2 Continuous solid ice

The wave experiments with continuous ice cover is done after detaching the ice from the side walls. The main problem faced during the experiments is the severe overwash for high amplitude waves, hence in this section only the experiments with the smallest wave height $a = 7.5$ mm are considered. Even with the slight overwash, the tests done to check the repeatability do not agree well as shown in Figure 6.9. The reflection of waves from the leading edge of ice cover back into the open water is significant. The maximum amplitude of superimposed waves recorded by US1 (see Figure 6.9) and US2 is 1.5 times the input wave amplitude of 7.5 mm. For the thin cover, $h_s = 1.0$ cm, both the overwash and leading-edge reflection are relatively small, when the cover thickness becomes 1.5 cm, a simultaneous increase in the two values are observed. With further increase in cover thickness to 2.0 cm, the overwash becomes severe and a reduced leading-edge

reflection is noticed for certain wave periods. Wave parameters, such as wave period and wave amplitude along the thickness of ice cover, determine the non-linear processes happening at the boundary of cover region.

Figure 6.9: Time series data of surface wave recorded by US1 (open water with leading edge reflection) and US4 (transmitted wave) with continuous unbroken cover, $h_s = 2.0$ cm. $a = 7.5$ mm and $T = 1.25$ s, the three fully developed waves considered are enclosed in the rectangular box. Grey and black markers indicate the repeated tests.

The variation of wave celerity with respect to the open water celerity for the same experimental input is shown in Figure 6.10. For all the ice thicknesses and wave periods considered, the celerity of wave along the unbroken ice-covered region, c_s is larger than the open water celerity c_o . This implies that the celerity of waves increases as it enters the ice-covered region. The change in speed is determined by the wavelength modification. The variation of wave number will be discussed in Section 6.4, whereas only the data obtained from the last three ultrasound sensors are considered to reduce the effect of overwash.

Figure 6.10: Wave celerity along the length of continuous ice cover. Cover thickness, h_s (mm): a) 7.5, b) 1.0, and c) 2.0. The dashed line represents the location of leading edge with respect to US ($x = 0$) and dash-dotted line represents open water celerity. T (s): Circle = 1.25, Asterisk = 0.96, and Diamond = 0.67.

For 2.0 cm thick ice cover, for $T = 1.25$ s, the wave amplitude variation with distance appeared as a linear reduction (see Figure 6.11). The attenuation of wave along the continuous unbroken ice cover is obtained by Equation (4.1). The exponentially fitted decay curve, thus provided an attenuation coefficient $\alpha = 0.36 \text{ m}^{-1}$. With reduction in wave number to $T = 0.67$ s, an abrupt exponential reduction in amplitude is evident with $\alpha = 1.74 \text{ m}^{-1}$. In field studies also, a similar wave period/ frequency dependent damping of waves is observed. Ice act as a low pass filter, efficiently damping the high frequency

waves. Thus, waves entering a continuous ice cover experiences change in both wavelength and amplitude.

Figure 6.11: Decay of surface wave along continuous cover, $h_s = 2.0$ cm for $a = 7.5$ mm. T (s): Circle = 1.25 and Diamond = 0.67. The solid lines represent the exponentially fitted curve and the dashed line represents the position of leading edge with respect to US1 at $x = 0$.

6.4.3 Grease ice

The leading-edge reflection for grease ice is less compared to continuous ice cover. The amplitudes of superimposed waves obtained from ultra sound sensors held in the open water region near the leading edge is a maximum of 1.2 times larger than the input wave amplitude of 15.0 mm. With reduced non-linear effects due to overwash at the leading edge, the time series data obtained using sensors held over open water region shows good repeatability (see Figure A6.1). The repeatability in the interior of ice covered region is not good due to the independent movement of frazil ice crystals along with the waves. For the same wave period, the transmission of waves along grease ice significant as compared to continuous cover with leading edge overwash.

Figure 6.12 shows that the celerity of wave along the grease ice cover ($c_v = 30\%$) is almost equal to open water celerity for $T = 1.25$ s. With decrease in wave period to 0.96s, the celerity becomes smaller than c_o . With further reduction in wave period, a reversal of wave period is visible, and c_s becomes greater than c_o . A similar trend is observed in the case of grease ice with $c_v = 40\%$. The decay of surface wave along the grease cover is shown in Figure 6.14. Even for the high amplitude waves $a = 12.5$ mm, the amplitude reduces to the range of sensor resolution when it reached US6. Thus, the data obtained with amplitudes $a = 12.5$ and 15.0 mm is more reliable than $a = 7.5$ mm and 10.0 mm. The dependence of attenuation on wave period is clearly visible with the prompt attenuation of the shorter waves (see Figure 6.13). For $c_v = 40\%$, $a = 12.5$ mm and $T = 0.67$ s, the waves decayed exponentially at a rate of $\alpha = 6.95 \text{ m}^{-1}$, with increase in wave period to $T = 1.25$ s, the decay rate reduced to $\alpha = 0.34 \text{ m}^{-1}$.

6.4.4 Pancake ice

The wave tests with solid pancake ice show reasonably good repeatability. The two possible reasons could be the absence of overwash as in continuous cover and absence of independent movement of frazil ice crystals along with the waves as in grease ice. The repeatability of greasy pancake ice is not as good as solid pancake ice. Like grease ice cover, for shorter waves the wave experiments with both types of pancake ice covers at higher amplitude waves are more reliable due to heavy damping. The absence of overwash led to increase in wave transmission through the cover. With minimal reflection, the amplitude of superimposed waves in open water is almost the same as that of the incident wave amplitude.

Figure 6.12: Wave celerity along the length of grease ice cover. Volume concentration, c_v (%): a) 30, and b) 40. The dashed line represents the location of leading edge with respect to US ($x = 0$) and dash-dotted line represents the open water wave celerity. T (s): Circle = 1.25, Asterisk = 0.96, and Diamond = 0.67. Wave amplitudes (mm): = Black 7.5, Red = 10.0, Green = 12.5, and Blue = 15.0.

Similar to grease ice cover, the celerity of wave along the pancake ice covers $c_s = c_o$ for long waves with wave period, $T = 1.25$ and 0.96 s. For shorter waves, the celerity becomes more than open water celerity. The attenuation by pancake ice follows the low pass filtering trend like continuous ice cover and grease ice (see Figure 6.14). For all the wave periods considered, the wave damping by pancake ice is less than that of grease ice.

Figure 6.13: Decay of surface wave along grease ice cover, $c_v = 40\%$ for $a = 12.5$ mm. Diamond indicates $T = 0.67$ s and circle indicate $T = 1.25$ s. The solid lines represent the exponentially fitted curve and the dashed line represents the position of leading edge with respect to US1 at $x = 0$.

Figure 6.14: Decay of surface wave along solid pancake cover. Diamond indicate $T = 0.67$ s and $a = 15.0$ mm and, circle indicates $T = 1.25$ s and $a = 12.5$ mm. The solid lines represent the exponentially fitted curve and the dashed line represents the position of leading edge with respect to US1 at $x = 0$.

6.4.5 Broken ice

The leading-edge reflection for broken ice cover is negligible, with superimposed wave amplitude in open water nearly the same as incident wave amplitude. With the absence of overwash issues, the tests show a reasonably good repeatability. Except for thicker cover at low wave periods, the celerity obtained along the cover remains unaffected with respect to open water celerity. As shown in Figure 6.15, c_s is slightly smaller than c_o for $h_s = 0.02$ and $T = 0.67$ s. A similar trend in velocity reduction is observed during the polyethylene experiments conducted by Sakai and Hanai (2002) with smaller sections.

Figure 6.15: Wave celerity along broken ice cover, h_s (cm): a) 1.0 and b) 2.0. T (s): Circle = 1.25, Asterisk = 1.0, and Diamond = 0.67. a (mm): Black = 10, Red = 15, and Green = 20. Dashed line indicates the position of leading edge with respect to US1 ($x = 0$) and dash-dotted line represents open water celerity.

The attenuation of wave due to broken ice cover is shown in Figure 6.16, it follows the low pass filtering trend like the other types of ice covers mentioned earlier. When the thickness is increased from $h_s = 1.0$ to 2.0 cm, the attenuation coefficient increases from $\alpha = 0.40 \text{ m}^{-1}$ to $\alpha = 1.26 \text{ m}^{-1}$ for $T = 0.67 \text{ s}$.

Figure 6.16: Decay of surface wave along broken ice cover, $a = 15.0$ mm. h_s (cm): a) 1.0, and b) 2.0. T (s): Diamond = 0.67 and, Circle = 1.25. The solid lines represent the exponentially fitted curve and the dashed line represents the position of leading edge with respect to US1 at $x = 0$.

6.4.6 Wavenumber and attenuation

In the present study, continuous solid ice cover experiments only show significant variation in the wavenumber along covered region with respect to open water wavenumber for the range of wave parameters used. Figure 6.17 shows that, wavenumber decreases along a continuous ice cover i.e., wave length of surface wave increases when it enters an ice-covered region with unbroken continuous ice. The dependence of wavelength to wave period (dispersion relation) decreases for waves propagating along a region covered with solid continuous ice. The variation of wavenumber with wave period decreases with increase in cover thickness. The

theoretical prediction for wavenumber is obtained using the viscoelastic model by Robinson and Palmer (1980). The elasticity of ice measured using the three-point loading mentioned in Section 6.2.1 is used, while the viscous parameter is inversely calculated by fitting the wave attenuation with experimentally obtained results for $T = 0.96$ s. It is found that the viscous parameter increases with cover thickness.

Figure 6.17: Variation of wavenumber with wave period for continuous ice cover of different thicknesses. Circles represent experimental data. Dash-dotted line represents open water wavenumber and dotted line theoretical predictions by Robinson and Palmer (1990) model.

For continuous cover, severe overwash is evident from the leading edge for large input wave amplitudes, $a = 12.5$ mm and 15 mm. A layer of water of notable thickness piles up over the cover along a wave bore. The frequency of wave bore depends on incident wave frequency which makes the data less reliable for determining attenuation coefficient. Figure 6.18 shows the amplitude neutral behaviour of attenuation coefficient for small amplitude waves. The attenuation coefficient decreases with increase in wave period for thicker ice cover, $h_s = 2.0$ cm. The theoretical predictions by Robinson and Palmer (1980) with viscosity determined for $T = 0.96$ s, do not match well with the experimental results.

Figure 6.18: Variation attenuation coefficient with wave period for continuous ice cover. Left figure shows the attenuation coefficient for the thicker cover, $h_s = 2.0$ cm. $a(\text{mm})$: a) Circle = 7.5, and b) Asterisk = 10.0. Right figure shows the dependence of decay on cover thicknesses. Experimental results for h_s (cm): square = 0.5, diamond = 1.0 and triangle = 2.0; theoretical predictions for h_s (cm): dotted line = 0.5, solid line = 1.0, and dash-dotted line = 2.0.

Unlike the continuous ice cover, both grease ice and pancake ice show a downward trend in attenuation coefficient with increasing wave period, see Figure 6.19. The shorter waves are damped very quickly, the waves with smaller amplitudes completely damped before reaching sensors closer to the beach. For grease ice cover attenuation is directly

proportional to the ice concentration. The viscoelastic WS model is fitted for grease ice concentration $c_v = 60\%$, assuming ice as a dense viscous fluid with zero shear modulus and an effective kinematic viscosity of $\nu = 0.05 \text{ m}^2/\text{s}$. The ice-experiments by Zhao and Shen (2015) as well as Newyear and Martin (1999) had also inversely calculated a viscosity of $\nu \sim 0.05 \text{ m}^2/\text{s}$ for grease ice cover, similar range of viscosity was obtained for field experiments conducted by Wadham et al. (2004).

For the grease ice with lower concentration $c_v = 30\%$, sufficient data points are not available for the inverse calculation of material property. In the case of pancake ice cover, greasy pancake ice cover shows larger attenuation than solid pancake for smaller wave periods, while the reverse trend is observed for longer wave periods. The determination of rheological properties of various ice sheets are not done due to limitation in facilities, hence no comparison is done between experimental data and theoretical predictions by viscoelastic models.

Figure 6.19: Variation attenuation coefficient with wave period for a) grease ice and b) pancake ice, $a \times 10^{-3} = 15.0 \text{ m}$. For grease ice cover, the dotted line = WS model for $c_v = 60\%$ with $\nu = 0.05 \text{ m}^2/\text{s}$.

The variation of wavenumber and attenuation coefficient with wave period for broken ice cover are provided in Figure 6.20. In the case of dispersion relation tested for the range of wave parameters, there is no notable change in wavenumber along the ice-covered region with respect to open water. The variation between attenuation coefficient for the two cover thicknesses considered is significant for smaller wave periods. With increase in wave period the magnitude of attenuation coefficient as well as its dependence on cover thickness reduces.

Figure 6.20: Variation wavenumber and attenuation coefficient with wave period for broken ice. a (mm): Circle = 10, Asterisk = 15, and Diamond = 20. Theoretical predictions; Dash-dotted line = open water, Solid line = thin elastic plate, and Dotted line = mass loading. Cover thickness, h_s (m): Black = 0.01 and Grey = 0.02.

6.5 Summary

Floating fresh water ice covers with a wide range of properties are prepared in a refrigerated wave flume. Four different ice covers are made a) Continuous solid ice cover, b) Broken ice, c) Grease ice and d) Pancake ice. The diverse types of ice are made under different environmental conditions. Young's modulus for the continuous ice cover is determined using three-point bending. Characterization of grease ice covers are done using volume fraction of ice while, floe size is considered for broken and pancake ice.

The wave experiments using ice covers disclosed that only continuous covers cause significant wavelength modification for the range of parameters considered. The theoretical predictions using the thin elastic plate model with the elastic modulus obtained from the bending test match well with the experimental results. All types of ice covers show frequency dependent damping of surface waves, low pass filtering. Damping of waves depend on concentration of ice for grease ice cover and cover thickness for continuous and broken ice.

In the context of wave modelling, viscoelastic dissipation and change of the dispersion relationship is an important topic. Part of the dissipation will happen due to production of turbulence in the boundary layer below the ice. This part will have to be evaluated through careful laboratory experiments before measuring and applying to the field data where sources of turbulence are many. The sheer variety of mechanisms relevant and active simultaneously in the ocean dictates further use of laboratory experiments. The mechanisms must be singled out and scrutinised in the laboratory before field observations where everything is mixed together. This should include laboratory tests of ice as a material which is viscoelastic, brittle, porous, whose properties depend on temperature and salinity, and would differ between solid, pancake, frazil and other kinds of ice.

CHAPTER SEVEN

Chapter 7

Viscoelastic Bottom Boundary and Three-layer System

7.1 Introduction

Increasing surface temperature and sea level rise associated with global warming can reshape the shorelines around the globe by altering the climate-driven wave parameters including the wave height, wave period and wave direction (Hemer et al., 2013). A large portion of the muddy Indian coastline which is devoid of vegetation (Baba and Nayak, 2002) can be susceptible to the changes in the direct wave action. This demands the need for a better understanding about the interaction of surface waves with erodible seabed, including muddy bottom.

Several theories have been developed with various approximations made for the soft mud bottom. A two-layer viscous model with inviscid water above viscid bottom mud layer was developed by Gade (1958) for shallow water conditions. This two-layer viscous model was subsequently modified for shallow and intermediate waters by Dalrymple and Liu (1978), with the verification effort using kerosene over sugar solution (Gade, 1958). The elastic model for mud by Mallard and Dalrymple (1977) assumed mud as an infinitely deep elastic bottom layer underlying water of finite depth. This model was later improved by Dawson (1978) with the inclusion of soil inertia for bottom layer.

Recent work on the wave interaction with a bottom elastic plate was performed by Mohapatra and Sahoo (2011), assuming the bottom layer as thin Euler Bernoulli beam.

*This chapter is written based on Sree, D.K., Law, A.W.K. and Shen, H.H. (2018). An experimental study on surface wave interactions with viscoelastic bottom. Proceedings of the International Conference on Ocean Engineering (ICOE) 2018, Springer, Vol:2, Madras, India.

Muds with high sediment concentrations exhibit both viscous and elastic properties (Maa, 1986), and the energy dissipation thus need to be included in coastal wave modelling. A generalized viscoelastic model (Hsiao and Shemdin, 1980; Macpherson, 1980; Piedra Cueva I. 1993) was subsequently proposed, which considered both properties.

The linear viscoelastic model includes Voigt model and Maxwell model. The material testing of Kaolinite and mud in the laboratory by Maa (1986) showed that Voigt model was suitable for soft mud. However, a substantial error was evident between the inversely calculated rheological properties from the experimental results and actual rheometric measurements by Soltanpour and Samsami (2011) for multi-layered viscoelastic models. The discrepancy drives the need for more detailed experimental studies for the verification of theoretical models and to identify possible missing factors. In other words, laboratory experiments need to be performed under controlled environment with similar viscoelastic conditions as in the theory.

To the authors' knowledge, wave interaction experiments with a real homogeneous viscoelastic bottom have not been done so far. In this chapter, a novel laboratory study using a precisely controlled real viscoelastic bottom is described and its influence on the modification of surface waves is quantified. The objective is to obtain measurement to verify the related theories in the future. The scaling of the laboratory parameters with respect to specific field conditions is not considered.

7.2 Material preparation

The 1.0 and 2.0 m long viscoelastic bottom boundaries above the raised platform is made of glycerol-doped Polydimethylsiloxane (PDMS). Glycerol is added to increase the specific gravity of the bottom layer. The preparation procedures include the mixing of the three components (base, curing agent and glycerol) in required proportions, followed by degassing and curing. The fraction of glycerol added is calculated as $m_g(\%) =$

$\frac{M_g}{M_g + M_b + M_{CA}} \times 100$, where M_g is the mass of glycerol. For the present study, $m_g = 10\%$ while two fractions of curing agent are used ($m_{CA} = 1.5\%$ and 2.0%) to alter the properties of the bottom layer. The curing period for $m_{CA} = 1.5\%$ is 32 days and that for $m_{CA} = 2.0$ is 18 days, which is determined using rheological tests.

Figure 7.1: Viscoelastic properties against angular frequency for PDMS bottom bed after the curing period.

The rheological properties of the viscoelastic bottom are determined using small amplitude oscillatory shear tests in a rheometer. Figure 7.1 shows the variation of G' and G'' with rotational frequency of rheometer (ω). Three samples are tested for each m_{CA} to check the repeatability. The stability of the material property is ensured by conducting the tests within two weeks after the curing period. The G' and G'' values are obtained by equating the wave frequency (σ) to the rotational frequency of rheometer (Table 7.1). Within the required range of frequency, G' is always greater than G'' , with the dependence of G' on ω lesser compared to G'' . Both G' and G'' increase with m_{CA} ; and the rate of increase of G' is relatively larger than G'' . The ratio G''/G' decreases with increase in m_{CA} . The G''/G' value for the bottom covers is more than thrice the value for the top cover. The physical appearance of bottom covers is more like gel material than the solid like top cover. The viscosity of bottom covers is one order smaller than the top cover, same is the case with shear modulus.

Table 7.1: Variation of the viscoelastic properties of bottom layer for different m_{CA} .

T (s)	ω (rad/s)	$G''/G' \times 100$		$G' = G$ (kPa)		$\nu = G''/\rho\omega \times 10$	
		1.5%	2%	1.5%	2%	1.5%	2%
0.50	12.56	72.62	33.33	2.96	11.02	0.18	0.31
0.55	11.42	71.33	32.47	2.87	10.85	0.19	0.33
0.60	10.47	70.17	31.70	2.78	10.70	0.20	0.34
0.65	9.67	69.11	30.94	2.71	10.57	0.21	0.36
0.70	8.98	68.14	30.20	2.65	10.46	0.21	0.37
0.75	8.38	67.26	29.51	2.59	10.35	0.22	0.39
0.80	7.85	66.43	28.91	2.54	10.26	0.23	0.40
0.85	7.39	65.66	28.38	2.49	10.17	0.24	0.42
0.90	6.98	64.94	27.89	2.45	10.08	0.24	0.43
0.95	6.61	64.25	27.45	2.41	10.00	0.25	0.44
1.00	6.28	63.59	27.04	2.37	9.93	0.26	0.45

7.3 Experimental setup

The same wave flume as mentioned in Chapter 4 is used for the viscoelastic bottom layer experiment. The water depth inside the tank is 26.5 cm, for the wave parameters considered the depth corresponds to deep and intermediate water depths where the influence of flexible bottom on surface wave is insignificant. Thus, a shallower water depth is set by allowing the viscoelastic bottom to rest on a raised platform (Figure 7.2). The platform is provided with slopes on either side (vertical: horizontal = 1:2 at leading

edge and 1:1 at the trailing edge) for the gradual change of wavelength between two water depths.

Figure 7.2: Schematic diagram of wave flume with viscoelastic boundaries.

In the present study, viscoelastic bottom with two different thicknesses are considered, $h_b = 0.02$ m and 0.03 m, the former one will be referred to as thin bottom while the later one as thick bottom layer. For the thin bottom layer, the water depth above the viscoelastic bottom is 6.0 cm. Seven ultra sound sensors are installed along the length of the flume (Figure 7.2) to monitor the vertical surface displacement due to the progressive waves. The ultrasound sensor, US1, is kept at 20 cm from the leading edge of the platform. All other ultrasound sensors, named in sequential order, are at 20 cm interval from each other.

For the thick viscoelastic bottom, the length of the viscoelastic bed is restricted to 1.0 m. The depth of water above the thick viscoelastic bottom is 0.05 m. The ultrasound sensor, US1, is kept at 20 cm from the leading edge of the platform. All other ultrasound sensors, named in sequential order, are at 20 cm interval from each other, length of bed limited the number of sensors used to 5 for this case. Along with the viscoelastic bottom cases,

a 1.0 m long viscous bottom is also considered using glycerol of viscosity $1.1 \times 10^{-3} \text{ m}^2/\text{s}$. As mentioned before, the viscosity of glycerol was determined using rheometer with concentric cylinder (CC) geometry.

The same set of input wave parameters (T and a) as in Chapter 4 are used for the thin viscoelastic bottom, with the change in water depth due to the raised platform the corresponding open water wavenumber with rigid boundary are provided in Table 7.2. As expected, for a comparison between Table 7.2 and 4.2 with the same input parameters, shows that shortening of wave occurs along the raised bed.

Along with the viscoelastic bottom layer experiments, a three-layer system is also conducted. The three-layer system corresponds to the 6-cm water sandwiched between 2 cm thick viscoelastic bottom layer made of glycerol-doped PDMS and 1 cm thick viscoelastic floating cover made of oil-doped PDMS. A vertical rod of diameter 0.5 cm is held to prevent the surge motion of the floating under wave action, as shown in Figure 7.2.

7.4 Data Analysis

Though the input wave parameters are the same as in Chapter 4, wave shoaling due to reduction in water depth with the incorporation of raised platform, the profile of the time series data depended time is not always sinusoidal. For thin viscoelastic bed at smaller frequency waves, the profile is comparatively linear, but with increase wave period to $T = 0.7\text{s}$, the wave profiles have sharp crest and shallow trough as shown in Figure 7.3.

The surface waves with $0.5 \leq T \leq 0.75$ have λ/H_w and H_o/H_w in the regime corresponding to Stokes waves (Fenton, 1990), where H_o is the wave height at the leading edge of the platform. The nonlinearity for the surface waves could be measured using a non-dimensional parameter, Ursell number $\left(U = \frac{H_o \lambda^2}{H_w^3}\right)$. For the present

experiments, waves with $T > 0.75$ s have $U > 25$, hence fall under the regime of cnoidal waves (Le Méhauté, 1976). With increase in nonlinearity, the small humps are visible at the trough region for $T = 1.0$ s.

Table 7.2: Wave parameters for rigid bottom experiments

$h_b = 0.02$ m					
T (s)	$a \times 10^{-2}$ (m)	k (rad/s)	T (s)	$a \times 10^{-2}$ (m)	k (rad/s)
0.5	0.51	19.21	0.7	0.77	12.48
	0.85	18.95		0.85	12.49
0.55	0.60	16.61	0.75	0.85	11.49
	0.85	16.65	0.8	0.85	10.67
0.6	0.68	14.78	0.85	0.85	10.07
	0.85	14.88	0.9	0.85	9.64
0.65	0.77	13.59	0.95	0.85	9.16
	0.85	13.65	1.0	0.85	8.64
$h_b = 0.03$ m					
T (s)	$a \times 10^{-2}$ (m)	k (rad/s)	T (s)	$a \times 10^{-2}$ (m)	k (rad/s)
0.5	0.64	20.71	0.7	0.64	13.75
0.55	0.64	18.36	0.75	0.64	12.71
0.6	0.64	16.49	0.8	0.64	11.83
0.65	0.64	14.99	0.85	0.64	11.07

Figure 7.3: Time series profile of waves along the region with viscoelastic bottom ($h_b = 0.02$ m, $m_{CA} = 1.5\%$) for different wave periods. Position of ultrasound sensor, x (m): Blue solid line = 0.2 and Red dotted line = 1.0.

With the introduction of the floating cover in the three-layer system, an entirely different wave profile is obtained, see Figure 7.4. For $T = 0.5$ s, the wave profile is linear. When the period is increased to $T = 0.7$ s the troughs become sharper than the crest. With further increase in wave period for $T = 1.0$ s, notable hump is evident at the trough.

Comparing Figure 7.3 and 7.4 a clear distinction in wave profiles with and without the floating covers is visible.

Figure 7.4: Time series profile of waves for the three layer system ($m_{CA} = 1.5\%$ for bottom and $m_{CA} = 4\%$ for top) for different wave periods. Position of ultrasound sensor, x (m): Blue solid line = 0.2 and Red dotted line = 1.0.

7.4.1 Wave celerity

The celerity of the waves that enter the region with viscoelastic bottom is measured in the same manner as in Equation 3.7. The variation of celerity with wave period for the thin viscoelastic bed is provided in Figure 7.5. Here also, only the first three fully developed waves are considered. It is found in the present study that the material property of the bottom cover does not cause any notable variation on the wave celerity with respect

to rigid bottom. The error bar represents the standard deviation of celerity for the three waves considered at various locations along the viscoelastic bottom bounded region.

Figure 7.5: Variation of celerity with wave period for thin viscoelastic bottom ($h_b = 0.02$ m). $m_{CA} =$ a) 1.5%, and b) 2.0%. Wave amplitude, $a \times 10^{-2}$ (m): circle = 0.43, grey asterisk = 0.51, grey diamond = 0.60, grey triangle = 0.68, diamond = 0.68, square = 0.77, and triangle = 0.85. Open water celerity with rigid bottom: grey solid line = linear wave theory and black asterisk = experiments with $a = 0.85 \times 10^{-2}$ m.

The KdV equation could be solved for cnoidal waves; the phase velocity of cnoidal wave (c_n) can thus be determined using the following equation,

$$c_n = \sqrt{gh} \left[1 + \frac{2a}{m_e h} \left(1 - \frac{1}{2} m_e - \frac{3}{2} \frac{E(m_e)}{K(m_e)} \right) \right] \quad 7.1$$

where m_e is the elliptic parameter, $K(m_e)$ is the complete elliptic integral of the first kind and $E(m_e)$ is the complete elliptic integral of the second kind. For infinitesimal waveheight, the phase speed reduce to

$$c_c = \sqrt{gh} \frac{1}{1 - \frac{1}{6}(kh)^2} \quad 7.2$$

The experimentally obtained phase celerity of cnoidal waves ($T > 0.75$ s) along a rigid bottom was compared with phase celerity for linear waves. It was found that the ratio of

both theroretically obtained and experimentally determined cnoidal wave celerity to linear wave celerity was always less than 1.03 for the wave parameters considered.

The variation of celerity due to thick bottom bed is provided in Figure 7.6. With increase in bed thickness and decrease in water depth, notable change in celerity is observed. For the viscous glycerol bottom, the celerity obtained is larger than open water celerity. For the viscoelastic bottom, the flexible bottom with $m_{CA} = 1.5\%$ had celerity larger than open water while the relatively stiffer bottom with $m_{CA} = 2.0\%$ had celerity smller than open water. For the present experiments, the celerity of wave along a flexible bottom increase with decrease in G' and increase in relative modulus G''/G' (G' for glycerol $\rightarrow 0$ for viscous model).

Figure 7.6: Variation of celerity with wave period for thick bottom bed, $h_b = 0.03$ m, $a = 6.4 \times 10^{-2}$ m. Circle = glycerol, Asterisk = $m_{CA} 1.5\%$, Triangle = $m_{CA} 2.0\%$, and Solid line = open water.

Unlike the cases with thin viscoelastic bottom, the three-layer system shows significant changes in wave celerity with respect to open water case with rigid bottom (Figure 7.7). The influence of floating cover on the wave celerity is significant compared to bottom boundary. For the 1.5% bottom layer and 4% top cover case, celerity for the shortest

wave $T = 0.5$ s as well as longest waves $T = 0.95$ and 1.0 s are almost equal to open water celerity with rigid cover (c_o). For all other cases, the celerity of the three-layer system is smaller than c_o .

For the case with 1.5% bottom and 10% top, an entirely different behaviour is observed. For shorter periods, the celerity with the viscoelastic boundaries is larger than c_o . The difference between c_s and c_o decreases with increase in wave period and they overlap for $T \geq 0.7$ s. For the longer cnoidal waves with $T > 0.9$ s, an increase in c_s with respect to c_o is observed.

Figure 7.7: Variation of celerity with wave period for the three-layer system. Bottom $m_{CA} = 1.5\%$ and a) 4% top and, b) 10% top. Wave amplitude, $a \times 10^{-2}$ (m): circle = 0.43, grey asterisk = 0.51, grey diamond = 0.60, triangle = 0.68, diamond = 0.68, square = 0.77, and triangle = 0.85. Open water celerity with rigid bottom: solid line = linear wave theory and black asterisk = experiments with $a = 0.85 \times 10^{-2}$ m.

7.4.2 Wave attenuation

Since the waves are no longer regular sinusoidal waves, the attenuation is not calculated using the Equation 4.1 considering the wave amplitude as half of wave height. Instead,

the attenuation coefficient is calculated by fitting the wave height along the length of viscoelastic bounded region with an exponential curve.

$$H_x = H_0 e^{-\alpha x} \quad (7.3)$$

where, H_x is the wave height at the distance x from the leading edge. The wave height is obtained as the vertical distance between the crest and the lowest point of the trough.

Figure 7.8: Wave decay along the length of bounded region. Viscoelastic thin viscoelastic bottom, $m_{CA} = 1.5\%$: a) $T = 0.5$ s, $a = 0.51 \times 10^{-2}$ m and b) $T = 0.7$ s, $a = 0.85 \times 10^{-2}$ m. Three-layer system with 1.5% bottom and 4% top : c) $T = 0.5$ s, $a = 0.51 \times 10^{-2}$ m and d) $T = 0.7$ s, $a = 0.85 \times 10^{-2}$ m. The solid represents the exponentially fitted curve.

From Figure 7.8, the wave amplitude clearly decreases with distance from the leading edge with thin viscoelastic bed. For $m_{CA} = 1.5\%$, $T = 0.7$ s experienced larger damping than $T = 0.5$ s. With the addition of viscoelastic floating cover ($m_{CA} = 4\%$), the value of

attenuation coefficient increase from $\alpha = 0.30 \text{ m}^{-1}$ to 0.55 m^{-1} for $T = 0.7 \text{ s}$. But for $T = 0.5 \text{ s}$, the addition of viscoelastic floating cover reduces the value of attenuation coefficient from $\alpha = 0.24 \text{ m}^{-1}$ to 0.17 m^{-1} . Attenuation of surface wave depends on both wave period and boundary condition along with material property of the viscoelastic boundaries. For the present experimental conditions, attenuation coefficient is independent of wave amplitude.

7.5 Results and Discussion

7.5.1 Wavenumber

Figure 7.9 shows that the material property of the thin bottom bed does not significantly change the wavenumber of surface wave for the present test conditions. The viscoelastic model for muddy bottom by Hsiao and Shemdin (1980) as well as Liu and Chan (2007) over-predicts the normalised wavenumber. The elastic bottom model by Mohapatra and Sahoo (2011) shows an unusual discrepancy compared to the experimental results. For the present study, the elastic model by Mohapatra and Sahoo is modified with a complex elastic modulus, such that the imaginary part considers the viscous term of the PDMS bottom. It is found that the predicted wavenumber decreases with increase in m_{CA} , but the experimental results remain unaffected with value close to open water.

The wavelength modulation of surface wave due to thick bottom bed is shown in Figure 7.10. It is found that for the viscous glycerol bed, surface waves experience wave lengthening. Though the theoretical predictions by Gade (1958) also agrees with wave lengthening, notable discrepancy with the experimental results is observed.

For the thick viscoelastic bed with $m_{CA} = 1.5\%$, the normalised wavenumber is slightly less than open water case, implying wave lengthening. Mohapatra and Sahoo (2011), and Hsiao and Shemdin (1980) predicted wave shortening, which is not in agreement with the present experimental results. For the stiffer viscoelastic bed with $m_{CA} = 2\%$, the

experimental results show reasonably good agreement with the theoretical predictions for elastic bottom by Mohapatra and Sahoo (2011). Comparing Figures 7.9 and 7.10, thickness of the bottom bed as well as overlying water depth greatly influence the wavelength modification of surface waves for the given experimental parameters.

Figure 7.9: Variation of normalized wavenumber with wave period, thin viscoelastic bed, m_{CA} (%): a) 1.5 and b) 2.0. Theoretical predictions Black solid line: Mohapatra and Sahoo (2011), Grey solid line: Hsiao and Shemdin (1980), and Dotted line: Liu and Chan (2007).

The variation of normalised wavenumber in the three-layer system with viscoelastic top and bottom are provided in Figure 7.11. As discussed before, for the present experimental study the material properties of the thin viscoelastic bottom are insignificant for wavenumber modification, thus the changes visible in Figure 7.11 are contributed by viscoelastic floating cover. A constant $m_{CA} = 1.5\%$ is provided for bottom layer, while two types of floating covers are used, $m_{CA} = 4\%$ and 10% . For the flexible cover with $m_{CA} = 4\%$, the normalised wave number is close to open water wavenumber for $T = 0.5$ s. With increase in wave period, the wavenumber increases till $T = 0.65$ s and then show a downward trend with wavenumber equal to k_o for the longer wave periods considered.

Figure 7.10: Variation of normalized wavenumber with wave period, thick flexible bed, a) glycerol, m_{CA} (%): b) 1.5 and c) 2.0. Theoretical predictions: Grey dotted line: Gade (1958), Black solid line = Mohapatra and Sahoo (2011), Dotted line = Liu and Chan (2007), and Grey solid line = Hsiao and Shemdin (1980).

For the stiffer floating cover with $m_{CA} = 10\%$, wave lengthening is observed for the higher frequency cases till $T = 0.7$ s. For $T = 0.75$ s, wave shortening is observed, with further increase in wave period, the normalised wavenumber reduces, and wave lengthening is observed again. The theoretical predictions for the three-layer system by Das and Sahoo (2016) show good agreement with the experimental results. The model is modified by providing a complex elastic modulus for the floating cover.

Figure 7.11: Variation of normalized wavenumber with wave period for the three-layer system. Bottom layer $m_{CA} = 1.5\%$, floating cover m_{CA} : a) 4%, and b) 10%. Grey solid line represents theoretical predictions by Das and Sahoo (2016) and dashed line represents open water.

7.5.2 Attenuation

As mentioned earlier, attenuation for the thin viscoelastic bed and three-layer system are considered for the present study. With the rigid bottom and linear waves considered, the finite dimensions of the flume can contribute to a maximum attenuation of $\alpha_{sb} = 0.03 \text{ m}^{-1}$ for $T = 0.5 \text{ s}$. The surface waves along the flexible beds for the present study are not small amplitude linear waves (see Figure 7.3) but gets transformed to non-linear Stokes wave and cnoidal waves. The non-linear effects could dissipate the wave energy, thus the attenuation of waves along a rigid bottom need to be considered. In Figure 7.12, the non-linear effects play a key role in wave attenuation.

For viscoelastic bed with $m_{CA} = 1.5\%$, the attenuation coefficient is larger than that of rigid bed, while for $m_{CA} = 2.0\%$ a predominant difference is not evident as compared to rigid bed. The elastic bottom model by Mohapatra and Sahoo (2011) with the complex

elastic modulus over predicts the attenuation for $m_{CA} = 1.5\%$ especially at higher frequencies, while it shows reasonably good agreement for $m_{CA} = 2.0\%$.

Figure 7.12: Variation of attenuation coefficient with wave period, for surface waves along region with viscoelastic bottom, m_{CA} (%): a) 1.5 and b) 2.0. Circles represents experimental results along viscoelastic bed while blue asterisk represents experimental data along rigid bed. Theoretical predictions Grey solid line: Mohapatra and Sahoo (2011), Dotted line: Hsiao and Shemdin (1980), and Dashed line: Liu and Chan (2007). c) Relative difference in attenuation coefficient with m_{CA} .

The viscoelastic model by Hsiao and Shemdin (1980) overpredicts the attenuation coefficient for smaller wave periods, reduced to zero attenuation and overlapped with the

predictions by Liu and Chan (2007) for longer wave periods. For the present wave experiments, the ratio of attenuation coefficient for $m_{CA} = 1.5\%$ to attenuation coefficient for $m_{CA} = 2.0\%$ ($\alpha_{1.5}/\alpha_{2.0}$) is always greater than 1. It indicates that when the attenuation coefficient is dependent on material property and increases with decrease in m_{CA} . The flexible cover with larger value for $\frac{G''}{G'}$ attenuates wave more efficiently.

The variation of attenuation coefficient with wave period for the three-layer system is provided in Figure 7.13. Unlike the wave dissipation caused by floating cover alone mentioned in chapter 4, the three-layer system do not follow any trend. In Figure 7.13, for the floating cover with $m_{CA} = 4\%$, the value of attenuation coefficient gradually increases with wave period while a reverse trend is observed for floating cover with $m_{CA} = 10\%$, both have the same flexible viscoelastic bed with $m_{CA} = 1.5\%$. The larger value of attenuation coefficient for longer periods could be a contribution of the non-linear effects (as shown in Figure 7.4) in addition to material properties of both boundaries, boundary layer effects and other factors.

The relative attenuation coefficient of the floating covers (ratio of α due to floating cover with $m_{CA} = 4\%$ to $m_{CA} = 10\%$) shows that the flexible cover with $m_{CA} = 4\%$ attenuates wave more effectively in the three-layer system. The theoretical model for the three-layer system by Das and Sahoo (2016) underestimate the attenuation for all the cases considered. The larger discrepancy between the theoretical predictions and experiments is expected as the model do not consider the non-linear effects, boundary layer dissipation and so on.

7.6 Summary

Laboratory experiments are conducted to analyse wave interactions with viscoelastic flexible bottom layer; two different material properties and two thicknesses are considered. The results show that thick viscoelastic bed can significantly influence the

wavelength modification, while no notable difference is observed with the thin bed. The waves undergo wave lengthening for viscous bottom bed using glycerol as well as for the thick flexible bed with $m_{CA} = 1.5\%$. While, for the thick bed with $m_{CA} = 2\%$, wave shortening is observed.

Figure 7.13: Variation of attenuation with wave period coefficient for the three-layer system. Bottom layer $m_{CA} = 1.5\%$, floating cover m_{CA} : a) 4%, b) 10%, and c) relative difference in attenuation coefficient. Grey solid line represents theoretical predictions by Das and Sahoo (2016) and dashed line represents open water.

In the case of wave decay, the thin bed with $m_{CA} = 1.5\%$ attenuates more effectively than the one with $m_{CA} = 2\%$. Comparison of the experiments with existing three theories are

also performed which show discrepancy for most cases. Though the goal of the study is to examine the influence of viscoelastic boundaries on linear waves, the presence of nonlinear effect cannot be avoided due to the required wave amplitude to enable reliable measurement. This non-linear nature of waves along with other missing parameters could be the reason for the above-mentioned notable discrepancy, thus these factors also need to be considered for a future study.

Along with the viscoelastic bottom bed, experiments are also carried out for a three-layer system of viscoelastic boundaries. The water layer is sandwiched between viscoelastic bottom bed and viscoelastic floating cover. Floating covers of two different material properties are used along with the two different bottom beds. Notable changes in wave length as compared to open water is observed which match well with the theoretical predictions. Significant wave decay is also observed for the three-layer system, but it shows large discrepancy with the theoretical predictions.

CHAPTER EIGHT

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

The surface wave interactions with viscoelastic boundaries, including floating covers and bottom beds, are investigated in the present study. The advantage of using a real viscoelastic material lies in solving the problem of field to laboratory scaling issue, for experimental studies with real ice or mud. A proper scaling method for sea ice cover is not established due to the unavailability of an accepted method for the determination of material property, especially the viscosity which plays a key role in wave attenuation. Usage of viscoelastic materials with measurable rheological properties enable the verification of viscoelastic models in a controlled environment close to theory, with less complexities. In the current study, the dispersion relation and attenuation along the viscoelastic regions are quantified. The results are compared with existing three theoretical models, and the discrepancies are analysed to gain foresights into necessary future improvement. The key results are summarized as follows:

8.1.1 Viscoelastic material

- A novel method based on PDMS material is developed to prepare large viscoelastic sheets. For floating cover, the density of PDMS was reduced from 1020 kg/m^3 to 940 kg/m^3 by doping with lighter white oil.
- The rheological properties, i.e. the storage modulus and loss modulus, of the floating cover are determined using small amplitude oscillatory shear tests in a rheometer. The

properties are varied by changing the proportion of curing agent. Covers of four different properties are considered for the current study, with $m_{CA} = 4, 6, 8$ and 10% .

- The rheological test results show that the viscoelastic material properties behave close to the Voigt model with relatively large increase in loss modulus with rotational frequency as compared to storage modulus. The shear modulus and viscosity are calculated based on this approximation. The results show that G' increases, and G''/G' decreases with increase in m_{CA} . For the current experiments, the shear modulus varies between 22 to 145 kPa, and viscosity between 0.23 to 0.67 m²/s. In the case of real ice the shear modulus varies from $O(10)$ to $O(10^9)$ Pa for grease ice to unbroken ice cover, whereas viscosity varies from $O(10^{-2})$ to $O(100)$ m²/s.

8.1.2 Wave modification due to viscoelastic floating cover

Wave interaction with an unbroken homogeneous floating viscoelastic cover is studied. The changes happening on a surface wave when it enters a covered region in terms of its wavelength and wave amplitude are determined and compared with theoretical models.

- Experimental results show that modification of wave characteristics along the covered region is found to be dependent on the wave period, rheological properties of the cover as well as cover thickness. Both wave lengthening and wave shortening are observed along viscoelastic floating covers with respect to open water case, with wave lengthening for stiff covers and shorter wave periods.
- Reasonably good agreement is obtained between the experimental results and theoretical predictions by existing viscoelastic models (Squire and Alan, 1980; Robinson and Palmer, 1990 and Wang and Shen, 2010b) for thinner covers. Some discrepancies are evident for the thicker cover. This agrees with the limiting case for

wave length modification $\left(\frac{L_r(\omega^2/g)^3}{\rho_i h_s g} > 1 \right)$ provided by Squire (1993) for thin elastic plate model in terms of flexural rigidity.

- The viscoelastic covers act mainly by low pass filtering of waves, effectively damping the high frequency waves. The wave attenuation is found to be dependent on both the cover properties and cover thickness. The maximum attenuation obtained for the flexible cover goes up to 0.35 rad/s for $m_{CA} = 4\%$, as compared to zero wave energy loss for thin elastic plate model. Larger discrepancies in wave attenuation, up to 50%, are observed between the experimental results and theoretical predictions.
- The possible reasons for the discrepancy are investigated, and the dominant effect is attributed to the effect due to the oscillating boundary layer underneath the floating cover, which is not considered in the theories. This is verified experimentally using a thin polyvinylchloride cover. The study points out the necessity of improvement of theoretical models with the incorporation of boundary layer effect.

8.1.3 Wave modification due to viscoelastic floating segments

The effect of addition of discontinuities in the viscoelastic floating cover on wavenumber modification and wave attenuation are studied. Previous experimental studies have only considered the elasticity (Sakai and Hanai, 2002; Kohout et al, 2007), while the present experiment is the first in the field that included the material viscosity as well. In real field case, this setup closely resembles the broken ice cover during summer and pancake ice cover during winter.

- For shorter waves, the wavenumber along the covered region increases with reduction in segment length. The dependence of wavenumber on segment length increases with shear modulus. The wavenumber moves closer to mass loading velocity with the decrease in segment length and increase in shear modulus.

- For longer waves, the mass loading wave number overlaps with the thin elastic wavenumber. The wavenumber decreases with the reduction in segment length and this effect diminishes for stiff covers.
- The experimental results show a large range for the effective elasticity which does not fit well with the empirical relation by Sakai and Hanai (2002). The range depends on the material properties and wave periods considered. The empirical relation is only applicable for stiff covers and in the range of wave period with notable difference between the wavenumber predicted using ML and EP models.
- For flexible covers, the effect of discontinuities on attenuation is not significant. For stiff covers, there is an abrupt increase in the value of attenuation coefficient with reduction in segment length. For stiff covers, the attenuation due to scattering effect increases with the reduction in segment length, reaching almost four times larger than the continuous cover.

8.1.4 Wave-Ice experiments

Along with real viscoelastic floating cover experiments, wave interaction with fresh water ice covers are also studied. Diverse types of ice covers are prepared in a refrigerated wave flume under different wave conditions and characterised based on their properties.

- The continuous solid ice is prepared in a calm environment, and the thickness of the ice cover is changed by varying the room temperature and the duration of preparation. The Young's modulus of elasticity for continuous ice is determined as 2.8 GPa using three-point loading and the value agrees well with that obtained by Gold (1977) for fresh water ice.

- The grease ice and pancake ice are prepared under a turbulent environment with waves generated continuously under low temperature. Grease ice types are characterised based on the volume concentration while pancake and broken ice are characterised based on their size. Different concentrations of grease ice are obtained by varying the room temperature and amplitude of the high frequency waves. Pancake ice of different dimensions are created by varying the room temperature as well as the frequency of waves.
- All ice covers show frequency-dependent low pass filtering of surface waves, with the most damping by grease ice, and the least by the continuous ice covers. The continuous ice covers also cause the lengthening of surface waves for the range of wave periods considered, and the experimental results match reasonably well with the theoretical predictions. No significant wavelength change is observed for the other ice types considered. The inversely calculated viscosity of $0.05\text{m}^2/\text{s}$ for grease ice agrees well with that predicted by viscous model for Newyear and Martin (1999).

8.1.5 Viscoelastic bottom boundary and three-layer system

The effect of viscoelastic bottom cover on surface wave modification was studied, along with a three-layer system that has water sandwiched between viscoelastic layers of different properties. The three-layer system closely resembles floating ice cover / flexible floating platforms in coastal waters with muddy bottom (Das and Sahoo, 2016).

- A novel method based on the same PDMS material is developed to prepare viscoelastic bottom layer. The density is increased by doping with denser glycerol. Viscoelastic bottom beds of two different properties are prepared ($m_{CA} = 1.5$ and 2%) and tested using the rheometer.
- The wavelength modification is found to be dependent on both the thickness of the bed layer and its rheological properties. Thicker bed layers as well as viscous glycerol

bottom layer cause significant wavelength modification, while with thinner covers no notable change in wavelength is observed.

- The attenuation of surface waves is found to be dependent on the rheological properties of bottom bed, with the flexible bed damping the waves more effectively. The theoretical predictions based on the linear wave theory show significant discrepancies with the experimental results for both the wavelength and attenuation. The non-linear nature of shallow water waves is identified to be the primary reason.
- A three-layer system with water sandwiched between the viscoelastic floating cover and viscoelastic bottom bed is also investigated. Both wave lengthening and wave shortening of surface waves are observed depending on the rheological properties of the boundaries as well as the wave periods considered. The experimental results for wavenumber match well with the theoretical predictions. However, the theoretical models under-predict the attenuation measured.

8.2 Recommendations and Future Works

- The present study is restricted to the experimental determination the surface wave modification along the viscoelastic floating cover. Currently the RP model shows a good fit with the experimental data because the viscous parameter was inversely calculated and not measured directly. On the other hand, the WS and SA models need further improvement with the incorporation of boundary layer. Thus, a detailed study on the boundary layer structure underneath an oscillating floating cover needs to be explored. The current experimental results have already led to the initiation of wave-ice experiments in the University of Melbourne to quantify the turbulence created underneath a continuous ice cover using the acoustic doppler velocimetry (ADV). Preliminary measurements show that significant turbulence is generated underneath the ice cover (results yet to be published).

- The predictions from the existing empirical model for discontinuous cover do not agree well with the experimental results in this study. The only existing continuum model for discrete ice cover in literature is the thin elastic plate model by Kohout et al. (2007), which only considers the elastic property. Thus, a new theoretical model on the wave propagation along a discontinuous viscoelastic floating cover need to be developed, which considers the wave dissipation due to both material property and scattering effect.
- In the case of fresh water ice experiments, at present a suitable testing method for the determination of material properties of the ice cover is not available. A method to quantify the viscosity and elasticity of different ice covers that depends on the volume concentration, thickness, areal coverage etc. without altering the ice cover structure needs to be developed.
- An analytical model for non-linear cnoidal waves in shallow waters with viscoelastic bottom needs to be developed. The effect of bottom bed thickness on both the wavelength and attenuation need to be experimentally determined for a wider range of material properties. A theoretical model of the three-layer system with water sandwiched between top and bottom viscoelastic boundaries is also needed.

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