Spin-orbit torque effective fields and spin accumulation in TA/CO/PT structure with in-plane magnetic anisotropy

Luo, Feilong

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**NANYANG
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**SPIN-ORBIT TORQUE EFFECTIVE FIELDS AND SPIN
ACCUMULATION IN TA/CO/PT STRUCTURE WITH
IN-PLANE MAGNETIC ANISOTROPY**

FEILONG LUO

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

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SPIN-ORBIT TORQUE EFFECTIVE FIELDS AND SPIN ACCUMULATION IN TA/CO/PT STRUCTURE WITH IN-PLANE MAGNETIC ANISOTROPY

FEILONG LUO

FEILONG LUO

School of Physical and Mathematical Sciences

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ABSTRACT

The magnetic recording technique of magnetoresistive random access and domain-wall memories requires a clear understanding of spin-orbit torque (SOT) which is applied for writing. In the writing unit where the structure of heavy metal (HM)/ferromagnetic metal (FM)/HM or oxide is commonly used, the SOT strength is represented by an effective field comprised of fieldlike and dampinglike terms. Generally, two distinct measurement approaches are used to characterize the effective field magnitude. In this thesis, a self-validating approach is proposed and demonstrated, which enables concurrent quantification of both the fieldlike and dampinglike terms in structures with in-plane magnetic anisotropy. Based on this method, in Ta/Co/Pt structure the dependences of the effective field on the thickness of Ta layer and the magnetization magnitude of Co layer are investigated. The investigation results reveal that the thickness of Ta varies the magnitude of the effective field. The fieldlike term decreases while the dampinglike term increases with the increase of the Co magnetization magnitude, indicating that the SOT effective field is tunable by the magnetization amplitude of FM layer. Angular dependence of the effective field is investigated. The results show that the fieldlike term consists of a component with fixed value and another component with azimuthal angle dependence, which experimentally supports that both Rashba and spin Hall effects contribute to the SOT. The dampinglike term is independent of the angle, indicating that the dampinglike term cannot be tuned by the azimuthal orientation of the magnetization. The SOT effective field, which has been characterized, is caused by spin accumulation in HM/FM interfaces. In this thesis, the spin accumulation is quantified by means of harmonic Hall resistance measurement. Spin accumulation up to 10% of the local magnetization is recorded when the applied electric current density is 10^{11} Am^{-2} .

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List of abbreviations

MRAM	Magnetoresistive random access memory
DW	Domain-wall
STT	Spin-transfer torque
SOT	Spin-orbit torque
HDD	Hard disk drive
SSD	Solid state drive
GMR	Giant magnetoresistance
TMR	Tunnel magnetoresistance
MTJ	Magnetic tunnel junction
IMA	In-plane magnetic anisotropy
PMA	Perpendicular magnetic anisotropy
FM	Ferromagnetic metal
HM	Heavy metal
SHE	Spin Hall effect
LLG	Landau–Lifshitz–Gilbert
FMR	Ferromagnetic resonance
AMR	Anisotropy magnetoresistance
MR	Magnetoresistance
MOKE	Magneto-optical Kerr effect
AHE	Anomalous Hall effect
PHE	Planar Hall effect
AC	Alternating current
DC	Direct current
SOC	Spin-orbit coupling
EBL	Electron beam lithography
RMSE	Root-mean-square-error

List of publications

Journal publications

1, “*Simultaneous determination of effective spin-orbit torque fields in magnetic structures with in-plane anisotropy*”

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2, “*Characterizing the spin orbit torque field-like term in in-plane magnetic system using transverse field*”

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3. “*Spin accumulation magnetoresistance probed by mixed harmonic Hall technique*”

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I. Introduction

The technology of magnetic memory has made significant evolvement over the last decades, transforming from the traditional spinning-disk-flying-head approach to solid state devices. The push for that originates from the research progress of Spintronics. The need for solid state non-volatile magnetic memory is obvious, in view of the rapid advancement of portable and wearable electronics gadgets, which has high conditional demand for on power, dimension and reliability. Among the proposed, or some has been materialized, are magnetic random-access memory (MRAM), domain-wall (DW) memory. Irrespective of the type of magnetic memory proposal, control of the memory recording processes, *i.e.*, writing, reading, or even driving the memory element, is the most critical development. In the early MRAM development, stripline-based magnetic field was used and this was quickly replaced when an electrically-controlled magnetic switching approach was introduced, *i.e.* spin torque transfer (STT). STT method may not be perfect, *e.g.* current issue, but it has made a leapfrog of solid state magnetic memory development. Lately, a new transport magnetism phenomenon, *i.e.*, spin-orbit torque (SOT), which has shown higher power efficiency than the STT approach, has been perceived as a better alternative. Though there are many reports on the materials investigation of SOT phenomenon, the in-depth physics understanding of SOT remains unexplored. In the thesis, the focuses of my research are quantification of the SOT effective fields, dependence of the SOT effective fields on the magnetization vector, and quantification of the spin accumulation causing SOT. The structure of this thesis is described in the last section of this chapter.

1.1. Spin-transfer torque in application

The widely used storage technique is hard disk drive (HDD), where mechanical movement of recording unit, which is magnetic grain [1], is the main factor that reduces its performance. Solid state disk (SSD) is a better alternative of the HDD, where recording unit which is transistor, has no mechanical movement so that its performance is higher. However, the lifespan of SSD is not as long as that of HDD due to a finite number of transistor program/erase cycles [2]. The promising magnetic storage are the MRAM and the DW memory which have not only the high performance of SSD but also the long lifespan. The key point for the two memories is the controlling of magnetic recording units by electric current *via* electron spin. Magnetic tunnel junction (MTJ), which is the mainly investigated structure in Spintronics, is the recording unit in the architecture of MRAM. The junction is comprised of a free layer, a space layer and a fixed layer, as shown in Fig. 1-1. The free and fixed layers are magnetic while the space layer is nonmagnetic [3-9]. The junction exhibits low resistance when the magnetization orientations are parallel for the two magnetic layers, while it exhibits high resistance when they are antiparallel, due to the giant magnetoresistance (GMR) effect firstly introduced in 1988 and later a better alternative tunnel magnetoresistance (TMR) effect. The origins of GMR and TMR have been elaborated by the following references, and interested readers may wish to refer to them for detailed explanations [10-13]. The states of the low and high resistances as 0 and 1, are used to record information. Similar to the case of SSD, access and read time of the information data are short, as the data can be retrieved and read directly from the recording unit location as shown in the MRAM architecture.

Fig. 1-1 Schematic of MRAM recording unit and MRAM architecture.

Writing of the states is achieved *via* switching the magnetization orientation of the free layer by electric current in the junction structure, where the magnetization orientation of the fixed layer is pinned. While there are various write schemes available to perform magnetic reversal of the free layer, such as toggle-field, STT or SOT, which will be explained in detail in subsequent sections, the key concept for electrically-induced switching requires a spin polarized current to transfer angular momentum to the magnetization free layer.

The other type of magnetic memory was proposed by Parkin in IBM as racetrack memory or DW memory, which is based on the DW motion driven by STT, as shown in Fig. 1-2 [14, 15]. In the architecture of the DW memory, arrays of magnetic nanowires are the recording medium. Separated by DWs, magnetic domains in the wires are the recording units, where the opposite magnetization orientations give the recording states of 0 and 1. Writing of the recording units is realized by applying short pulses of electric current orthogonal to the wires. The electric current creates DW to form two domains which give the opposite magnetization orientations. While, reading of them is achieved by measuring

TMR of a MTJ attached on the wires, concurrently moving the DMs by spin-polarized current which is naturally formed in magnetic materials [16-20]. Instead of using the mechanical motion of recording units, which reduces the performance of the widely used HDD, both the writing and reading in this magnetic recording scheme are carried out by using only electric current. The currently used HDD as well as SSD allows to reduce the recording unit size or expand recording areas to increase storing density. Compared with the SDD and HDD, the DW memory allows to expand in perpendicular direction to significantly increase magnetic storage density [14, 15]. The proposed architecture is an efficient alternative of the HDD and SSD, which has not only the low cost of the HDD but also high performance and reliability of SSD [14, 21].

Fig. 1-2 The architecture of domain wall memory. The magnetic domains with different magnetization orientation, which are driven by electric current, are the recording unit.

The DW memory focus on moving the DWs by the spin-polarized current or STT initially. Driving the DWs at high speed by low electric current density has been targeted. The DW motion driven by electric current was realized by Berger in Permalloy wires with in-plane

magnetic anisotropy (IMA) first [22-30]. Later, in the IMA material Permalloy, other researchers also reported the DW motion driven by electric current [31-35]. The critical current densities for the DW motions were in the order of $\sim 10^{12} \text{ Am}^{-2}$ [22-30]. In other IMA materials such as CoFe, DW motion has been driven by the same current density $\sim 10^{12} \text{ Am}^{-2}$ [36]. In IMA semiconductor GaMnAs, DW motion can be driven by the current density as low as $\sim 10^9 \text{ Am}^{-2}$, which is an exception in IMA materials [37]. The low current density is due to the low saturation magnetization of GaMnAs, as saturation magnetization is proportional to the critical current density for driving DW motion [37]. However, in the commonly used metal materials, the high current density in the order of 10^{12} Am^{-2} leads to much energy consumption [38]. The solution for the problem of high current density was found to drive DW motion in the materials with perpendicular magnetic anisotropy (PMA) [38-41]. Relatively lower current density of 10^{11} Am^{-2} has been recorded in the PMA materials such as amorphous TbFeCo, multilayer Co/Ni and CoB/Ni [38]. Additionally, smaller magnetic domains, which were reported in PMA nanowires compared with IMA nanowires, allow higher storing density for DW memory [41]. The high PMA of the nanowires also ensures the stability of recording units [41].

Regardless, the operating principle behind STT in DW memory (and MRAM) is based on the interaction between $4s$ and $3d$ electrons in the magnetic metals such as Fe, Co and Ni. The $4s$ electrons form the spin polarized current as $4s$ electrons are free electrons. Conversely, $3d$ electrons are relatively localized so as to form the local magnetization [42-45]. Due to the spin-selective scattering with $3d$ electrons, the spins of the $4s$ electrons are polarized to be along the orientation of the local magnetization at equilibrium [16-19, 44, 46, 47]. As such, when the current crosses typical magnetic structures, the magnetic domain

which carries a uniform magnetization (and the fixed layer in the MTJ) as shown in Fig. 1-3 [43, 48-50], the spins of the 4s electrons are aligned parallel and antiparallel to the local magnetization [16-19, 44, 46, 47]. Alternatively, the magnetic domain (and the fixed layer) polarize the spins of the incoming electric current. This spin-polarized current enters the neighbor non-uniform magnetization zone which is the magnetic DW for the magnetic wire (and the free layer of the MTJ) [43, 48-50]. In the DW, the spins of the electric current are also aligned with the local magnetization at equilibrium, although the initial orientations of the spin and local magnetization are misaligned instantaneously. Similar alignment acts in the MTJ, when the magnetization of the free layer is not parallel to that of the fixed layer.

Fig. 1-3 Schematic of spin accumulation induced by the spin polarized current in the magnetic wire (up) and the MTJ (down).

In the two architectures of magnetic memories, the spin-polarized current modulates the magnetizations of the free layer and DW with the assistance of the instantaneous misalignment. The modulation mechanism is based on the STT model proposed by Slonczewski, Berger, Heide, S. Zhang and so on [4, 5, 51-53]. In this model, when moving across the free layer and DW, the electrons of the spin-polarized current transfer angular momentum to the local magnetization, by adiabatically re-orientating their spins to be along the local magnetization [4, 51]. This angular momentum transfer gives one of the STT components, adiabatic spin-transfer torque [54, 55], on the local magnetization, as a result, modulates the magnetization orientation. Conversely, due to the instantaneous misalignment, the electrons of the spin-polarized current, which emit from the fixed layer and the magnetic domain, can also be reflected back by the free layer and the DW [56-58]. The reflection of the incoming electrons transfers linear momentum to the local magnetization. The linear momentum transfer provides the other STT component, non-adiabatic spin-transfer torque [54, 59]. S. Zhang proposed an alternative model which is based on spin accumulation that naturally gives the adiabatic and non-adiabatic components, where the spin accumulation as shown in Fig. 1-3 is formed at the DWs and the free layers due to magnetoresistive (MR) effect [53]. Collectively, when the electric current density reaches a critical value, the modulation of magnetization by the STT will be able to achieve magnetization reversal in the case of free layer switching in MTJ, or the DW propagation in DW-related memory applications [60-74]. However, the critical value, which are generally in the order of 10^{12} Am^{-2} , induces Joule heating, consequently, affects the stability of recording units [38, 75-84]. Moreover, performance of MTJ is limited by the pulse width of the applied writing current. While shorter pulse width is expected to

increase the performance of MRAM (write speed), the endurance of the device decreases exponentially due to the increase in the critical current required for a short pulse width. A larger critical current results in a greater stress on the insulating space layer, affecting the Time-Dependent Dielectric Breakdown of the MTJ device [85, 86].

1.2. Spin-orbit torque as an efficient alternative of STT

While STT is able to resolve some technical challenges such as writing speed and scalability, it has reported that SOT which is another spin torque [82, 87-104], is able to induce even faster magnetization switching within MTJ using lower current density [76, 78, 80, 89, 105, 106], as well as its ability to move DW at high speed in FM nanowires sandwiched by two layers of nonmagnetic heavy metals (HM) or sandwiched by HM and oxide [76, 94, 95, 107-109]. The speed of DW motion can be 10 times larger driven by the SOT than the conventional STT [94]. To understand the reason/phenomena behind the enhanced effects/benefits of SOT, I will first briefly describe the generation of SOT. As shown in Fig. 1-4, in the sandwiched structure for SOT, the conduction electrons of FM and HMs exhibit strong spin-orbit coupling (SOC) which leads to two well-known phenomena, *i.e.*, the Rashba effect and the spin Hall effect (SHE) [76, 78, 79, 89, 90, 92, 96, 102, 109-122]. The two effects cause spin accumulation at the FM layer. Due to the exchange interaction of spins, the spin accumulation acts SOT on the local magnetic moment [98]. Analogue to the spin accumulation induced spin torque model which is proposed by S. Zhang, the SOT is reflected in the revised Landau–Lifshitz–Gilbert (LLG)

equation $\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M} \times (\mathbf{H}_{\text{eff}} + J\mathbf{s}) + \alpha\mathbf{M} \times \frac{d\mathbf{M}}{dt}$ by the term $-\gamma\mathbf{M} \times J\mathbf{s}$, where γ is the

gyromagnetic coefficient, $\mathbf{M}$ is the magnetization of the FM layer, $\mathbf{H}_{\text{eff}}$ is the effective magnetic field including the external field, anisotropy field and magnetostatic field, $\mathbf{s}$ is the spin accumulation, and J is a coefficient related to spin diffusion length of accumulated spins in the FM layer [53]. When the initial orientation of spin accumulation, $\mathbf{p}$, is not parallel to $\mathbf{M}$, the spin accumulation $\mathbf{s}$ which is perpendicular to the magnetization $\mathbf{M}$ at the FM layer depends on $\mathbf{p}$, hence, the field $J\mathbf{s}$ can be written as $J\mathbf{s} = H_F \mathbf{p} \times \mathbf{m} + H_D (\mathbf{m} \times \mathbf{p}) \times \mathbf{m}$. Substituting the expression of $J\mathbf{s}$ in the revised LLG equation above gives

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} + H_F \mathbf{p}) - \gamma H_D \mathbf{M} \times (\mathbf{p} \times \mathbf{m}) + \alpha \mathbf{M} \times \frac{d\mathbf{M}}{dt} \quad [53].$$

Therefore, the term $-\gamma \mathbf{M} \times J\mathbf{s}$ can be decomposed into a fieldlike torque $\boldsymbol{\tau}_F = -H_F \mathbf{M} \times \mathbf{p}$ which is similar to the adiabatic STT and a dampinglike torque $\boldsymbol{\tau}_D = H_D \mathbf{M} \times (\mathbf{m} \times \mathbf{p})$ which is similar to the non-adiabatic STT, where $\mathbf{p}$ represents the spin orientation of the electrons diffusing into the FM layer, and $\mathbf{m}$ is the unit vector of $\mathbf{M}$ [5, 53, 111, 123]. The corresponding effective fields arising from the SOT are called as the fieldlike term $\mathbf{H}_F = H_F \mathbf{p}$ and dampinglike term $\mathbf{H}_D = H_D \mathbf{m} \times \mathbf{p}$ [53, 76, 91-93, 111, 112, 120, 124, 125]. Therefore, the SOT is generally represented by the two corresponding effective fields [53, 76, 91-93, 111, 112, 120, 124, 125].

Fig. 1-4 Spin accumulation provided by the heavy metals induces faster DW motion in magnetic wire and faster switching of magnetization in MTJ.

While SOT has already been actively exploited for device applications, accurate quantification has eluded the scientific community due to the limitation of current characterization techniques. Notable examples of characterization techniques listed below, such as current-induced domain wall motion [108, 113, 124, 126, 127], ferromagnetic resonance (FMR) techniques [79, 101, 103, 114, 115, 117-119], and SOT-assisted magnetization switching [76, 78, 79, 92, 128] are based on ferromagnetic heterostructures consisting of ultrathin FM layer with strong PMA, sandwiched between a nonmagnetic HM and/or an oxide layer [78, 82, 91, 93, 95, 99, 100, 102, 124-126]. In the FMR technique, microwave is applied in the stack, where the radio frequency electric component generates an oscillating transverse spin current *via* the HM layer. This spin current exerts an oscillating torque to induce the precession of magnetization. The precession results in an oscillating magnetoresistance. Multipole by the current due to radio frequency electric component, the magnetoresistance leads to a direct voltage which can be detected by voltmeter. The detected voltage therefore gives the information of SOT. However, the characterized amplitudes of SOT effective field are dependent on the approaches applied for quantification [100, 110, 112, 114-118, 124], when the amplitudes are converted to be the material parameter spin Hall angle, the ratio of spin polarized current to charge current [88, 90, 94, 112]. The commonly adopted measurement technique for SOT effective fields characterization is harmonic Hall resistance measurement [82, 93, 113, 122, 125]. In this technique, the respective terms are extracted from the harmonic Hall voltages with respect to externally applied magnetic fields along transverse and longitudinal directions of electric current.

Recent development in magnetic heterostructures with IMA has also attracted academic

interest as large spin Hall angles (larger spin Hall angle indicates stronger SOT) have been obtained in topological insulator structures with IMA. This paves the way for promising three-terminal SOT devices where the electric current applied is in-plane [90]. Furthermore, in topological insulator structures which have IMA, large spin Hall angles have been obtained [89, 104, 129]. Different from PMA systems, the corresponding torques in IMA system are along both in-plane and out-of-plane orientations, making the effective fields characterization difficult. As such, the conventional approach for characterizing the SOT in IMA systems is *via* FMR technique [101, 103]. However, in the FMR technique, the values of characterized SOT effective fields depend on the geometry of the device under test [119]. For quantitative characterization of the SOT effective fields, Hayashi *et al.* proposed a low frequency harmonic Hall voltage measurement method [92]. Using two distinct measurements, the two terms in ferromagnetic materials can be obtained. Sweeping a small externally applied magnetic field along the easy axis of the magnetic wire under test provides the fieldlike term, while the dampinglike term is obtained through measuring anisotropy magnetoresistance (AMR) effect [11, 130-139]. Here, the easy axis indicates that the magnetization prefers to be collinear with this axis. Alternatively, sweeping a magnetic field along the direction normal to the IMA film can provide the two terms. A low-field regime sweep gives the fieldlike term whereas a high-field regime sweep gives the dampinglike term. However, such characterization configurations could lead to different values for the two terms and that could lead to inconsistency during measurement. Additionally, a uniform magnetization configuration must be ensured when using this technique. Any deviation in the magnetization while the external magnetic field is swept will corrupt the signal and result in the subsequent calculations to be erroneous. Moreover,

due to the random arrangement of crystal grains in polycrystalline magnetic films, the value of the magnetization will vary when the external magnetic field is swept [43]. For external fields lower than the saturation field, magnetic moments within crystal grains could be orientated in different directions due to the crystalline magnetic anisotropy [43]. As such, the measurement of effective SOT fields in polycrystalline films can lead to inaccurate results.

The dependence of the two effective fields on the orientation of magnetization $\mathbf{M}$ has been studied, especially in materials with PMA [91, 95, 100, 122, 123, 140-143]. The theoretical work based on free-electron or tight-binding electron model shows that H_F and H_D are dependent on the polar and azimuthal angles of the magnetization [141]. Reported SOT measurements on Ta/CoFeB/MgO structures have shown that $\mathbf{H}_D$ changed direction according to the expression $\mathbf{H}_D = H_D \mathbf{m} \times \mathbf{p}$ when the magnetization orientation is reversed [91, 95, 122, 123, 142, 143]. The fieldlike term has been proposed to arise mainly from the constant Rashba effect [111, 122, 123, 144]. The fieldlike term expression, $\mathbf{H}_F = H_F \mathbf{p}$, is independent on the magnetization vector, hence, H_F has been generally treated as a constant when the magnetization orientation varies within a small angle [91, 95, 100, 122, 141]. However, the dependence of H_F on the polar angle of magnetization was observed experimentally in ultrathin films with PMA [91, 100]. Such dependence indicates that H_F has an additional polar component in addition to a constant transverse component induced by Rashba effect [91, 100].

The SOT effective fields also show a dependence on the thicknesses of the FM and HM layers in the stacks with PMA [125]. In Ta/CoFeB/MgO structure, it has been reported that both fieldlike and dampinglike terms increased with respect to the thickness of Ta, which

is due to larger amount of current in a thicker Ta layer [125]. For the dependence on the thickness of FM, the fieldlike term decreases with increasing CoFeB thickness, while the dampinglike term remains constant [125]. Such dependence was ascribed to GMR effect [125]. The theories of free-electron and tight-binding electron models predict that H_D and H_F should vary as a function of the polar angle of the magnetization, while being impervious to the azimuthal angle of the magnetization [141, 143].

The effective field, $\mathbf{J_s} = H_F \mathbf{p} + H_D \mathbf{m} \times \mathbf{p}$, which is a combination of spin accumulation and a spin-diffusion related coefficient, has been widely characterized by using different approaches. However, quantification of the spin accumulation, which plays a crucial role in the origins of the SOT, has remained elusive.

1.3. Objective of this work

Several approaches have been proposed to characterize the effective fields of SOT, however, the reported amplitudes of the effective fields of SOT depend on the characterization methods. The commonly used harmonic Hall resistance measurement, which could not ensure the uniform magnetization configuration, can lead to inaccuracy characterization of the effective fields. Providing a precise characterization which can avoid the artefacts is one of the objectives of this work.

As elaborated in the previous section, the SOT effective fields may have a direct correlation with the spontaneous magnetization M . To experimentally investigate this correlation, one may choose to either vary the thickness of the magnetic material, or adjust the magnetic content through doping or interlayer diffusion to induce a different saturation magnetization M_s . However, such experimental study will not allow us to accurately

attribute to either the intrinsic saturation magnetization M_s or the thickness of the deposited film. Therefore, the second objective of this work is to validate the M dependence on SOT fields by applying various static external magnetic fields to induce different spontaneous magnetization on a sample instead.

The dependence of the SOT effective fields on the magnetization orientation has been widely investigated in magnetic stacks with PMA. The Rashba effect which contributes to the SOT effective fields, is an interface effect which also supports the configuration of PMA. As such, investigating the dependence on the magnetization orientation in IMA stack is the third objective of this work.

Although the SOT effective fields have been widely characterized, the spin accumulation, which acts SOT on the magnetization, has remained unquantified. Providing a method for quantifying the spin accumulation and experimentally giving the magnitude of the spin accumulation is the third objective of this work.

1.4. This thesis

Chapter 2 introduces the basic knowledge for understanding my work which is about SOT and the corresponding spin accumulation. First, the origin of the ferromagnetism of the widely used metals Fe, Co and Ni is ascribed to the asymmetric band structure of the relatively localized $3d$ electrons with opposite spin orientations. Scattered with the $3d$ electrons, the $4s$ electrons which are free electrons to construct the electric properties, are spin-polarized to generate the spin-polarized current which transfer STT to the magnetization. As the scattering is spin-selective, the Two-current model will serve as the basis to explain the transport properties of the magnetic metals. Due to the magnetic flux

ascribed to the magnetization or $3d$ electrons, the behavior of the $4s$ electrons is modulated to change the light polarization incident on the magnetic films. This change of light polarization, which is so called magneto-optical Kerr effect (MOKE), supports the detection of magnetization orientation by the light, for ultrathin films. However, the MOKE is unable to obtain the magnetization magnitude. This issue is solved by using FMR measurement. At FMR state, with the energy supplied from applied microwave, the magnetization sustains the precession around an applied magnetic field. The precessional frequency, which is also as a function of the magnetization magnitude, depends on the applied field. The magnetization magnitude can be extracted from the relationship between the frequency and the applied field. Without applying the microwave, the magnetization is damped as along the easy axis of the magnetic metals. The damping time is estimated as in the order of nanosecond. Beyond the time scale of nanosecond, the magnetization is at equilibrium state and its orientation is determined by the total magnetic energy. One component of the total magnetic energy is the Zeeman energy due to applied magnetic field. The magnetization follows the orientation of the magnetic field which includes the SOT effective fields induced by the applied electric current, when the period of the electric current is in milliseconds. The electric current induced SOT is observed in magnetic multilayer structures composed of a FM layer sandwiched by two HM layers such as Ta and Pt layers [82, 92, 122]. In this structure, the conduction electrons of FM and HMs exhibit strong SOC which leads to the Rashba effect and the SHE [108, 124]. When $4s$ electrons travel in an asymmetric interface which induces an electric field perpendicular to the interface, a magnetic field acts on the spin of the $4s$ electrons. This is so-called Rashba effect. Due to SOC, skew scattering with an impurity nucleus and side-jump from an energy

band to another energy band of the impurity, lead to anomalous spin flow transverse to applied electric current in metal wires. This is the so-called spin Hall effect. The anomalous Hall effect (AHE) and planar Hall effect (PHE) are from the SHE in magnetic metals. As a result of SHE and Rashba effect, spin accumulation arises in the FM layer that leads to the magnetization switching *via* the SOT represented by two effective fields, *i.e.*, fieldlike and dampinglike terms [82, 92, 108, 109, 111, 122, 124].

Chapter 3 provides the preparation details of the sample Ta/Co/Pt Hall crosses. The choice of the stack Ta/Co/Pt was because of the strong SHE of Ta and Pt, and the asymmetric interface Ta/Pt for Rashba effect. The magnetron sputtering parameters of film Ta/Co/Pt deposition is given. The IMA property of the deposited films, in which SOT exhibit, are experimentally confirmed by MOKE measurements. The saturation magnetization of the films is obtained by measuring FMR, which is used to calculate the SOT effective fields in the following chapters. Finally, the films were patterned to be Hall crosses with the techniques of electron beam lithography (EBL) and ion-milling. The procedure of EBL is described. The basis about harmonic Hall resistance measurement is provided through the example of measuring AHE resistance of the Hall crosses by a lock-in amplifier. The effective in-plane fields, which support the magnetic configurations of the films as IMA, are obtained from the AHE measurement, which is for calculating the SOT effective fields in the following chapters.

Chapter 4 gives the derivations of the proposed harmonic Hall resistance measurement which can simultaneously characterize the SOT effective fields. The method is validated in the patterned Hall crosses and enables the results to be insulated from measurement artefacts. Details about the characterization of the SOT effective fields are provided. The

dependence of the SOT effective fields on the on the thickness of Ta layer is investigated and explained.

Chapter 5 presents that the SOT effective fields depend on the magnetization magnitude in Ta/Co/Pt wires. The means of varying the magnetization and characterizing the SOT effective fields concurrently in each single wire is described. This dependence is investigated by analogizing the SOT to STT. Electron diffusion constant is then introduced to explain the dependence.

Chapter 6 describes the investigation of the dependence of H_D and H_F on the azimuthal angle of the magnetization. Based on the proposed method in Chapter 4, a revised approach, which is used for the angular dependence investigation, is derived. The experimental details, which are about the SOT effective field characterization at different value of the magnetization azimuthal angle, is provided. The experimental results are discussed *via* considering the Rashba effect and SHE which contribute to the SOT effective fields.

Chapter 7 describes quantification of the spin accumulation in the Ta/Co/Pt wires. The principle of the quantification method is introduced, which predicts that the spin accumulation contributes to the second harmonic Hall resistance. Observations in the Ta/Co/Pt samples validate the quantification method. Spin accumulation up to 10% of the local magnetization is recorded, when the applied current density is in the order of 10^{11} Am^{-2} . The spin accumulation, which shows dependence on the thickness of the Ta layer, coincides with the spin Hall angle respect to the thickness. The coincidence indicates the spin accumulation is from Ta and Pt. Additionally, the magnitude of the spin accumulation is independent on the magnetization orientation.

II. Background theory

In this chapter, the basic knowledges about the origin of magnetism for Fe, Co and Ni are briefly described from the energy band structures of the three metals. The methods we used in our work to characterize the magnetic properties of magnetic materials are MOKE and FMR. The mechanisms of MOKE and FMR are described through deriving corresponding analytical expressions. SOT is from the exchange interaction between local magnetization and current induced spin accumulation. The electric current induces the spin accumulation by SHE and Rashba effect, which are due to SOC. The mechanism of the SOC effect of heavy metals and ferromagnetic metals are introduced. SOT, which is a combination of the magnetic and SOC effects, is introduced. Characterization techniques and phenomenon used for this thesis are also briefly described.

2.1. Magnetism origin of Fe, Co and Ni

In the magnetic metals such as Fe, Co and Ni, the magnetism is due to the unequal amount of electrons with opposite spin orientations [16, 43, 44, 48-50]. The electron circulating around the atomic nucleus forms a closed circuit of electric current which induces magnetic field, as described in the classical image of atom structure [145, 146]. According to electromagnetic theory, the magnetic moment of the circuit, which is proportional to the orbital angular momentum of the electron, is expressed as $\mu_B l$, where μ_B is the Bohr magneton and l is the orbital angular momentum quantum number [43, 44, 49, 50]. We can understand that the orbital angular momentum quantum number gives a magnetic moment of $\mu_B l$. Similarly, the spin angular momentum quantum number, which is due to the spin of electrons, provides a magnetic moment. However, in a system with many atoms, the magnetic moment due to the orbital angular momentum quantum number approximately does not contribute to the system magnetism, as this contribution is partially suppressed [43, 44] (As the contribution of orbital angular momentum quantum number can be considered as negligible for subsequent discussion, interested readers may wish to refer to pages 95 and 526-528 in Ref. [44] for more details). Instead, the magnetic moment due to spin angular momentum of the electron fully contributes to the magnetism, which is expressed as $\pm\mu_B$, where $\pm$ indicates signs of the spin angular momentum quantum number [16, 43, 44, 48-50]. Hence, each electron is allowed to contribute $+\mu_B$ or $-\mu_B$ to the magnetic moment of the system. Stoner-model which is a simplest band-like model explains the contribution amount from the electrons spins, as shown in Fig. 2-1 [44]. We can observe that not all the electrons are in charge of the magnetic moment of the system, such as 4s electron [43, 44]. The schematic of the electron density per unit energy is shown in Fig. 2-

1 [44, 147], where the density of $4s$ electrons with $+\mu_B$ approximately equals to that of $4s$ electrons with $-\mu_B$ (The band width of $4s$ electrons is above 10 eV, which includes not only the $4s$ electrons but also $4p$ electrons. However, in order to relate to the classical atom configuration which gives a classical picture of atom magnetism, we will only consider $4s$ electrons here. Readers who are interested can further take reference of page 523 in Ref. [44] and Chapter 3 in Ref. [43]). The total magnetic moment of the $4s$ electrons with $-$ orientation approximately cancels the total magnetic moment of those with $+$ orientation, consequently, leading to about zero contribution to the magnetic moment of the system (Readers who are interested in the numerical value of the little contribution can further take reference of page 527 in Ref. [44]).

The magnetic moment of the system is nearly entirely contributed by the $3d$ electrons in the magnetic metals due to the asymmetric energy band structure of the $3d$ electrons, the band width is around 3 eV [43, 44]. As shown in Fig. 2-1, the energy band of the $3d$ electrons splits off [147] because of the exchange interactions which is around 1 eV [44]. This split leads to the inequality of the numbers of the $3d$ electrons with $+\mu_B$ and $-\mu_B$. Hence, in the expression of the magnetic moment of the system, $\mu_B n_+ - \mu_B n_- = \mu_B \Delta n$, Δn is nonzero, where $n_{\pm}$ can be the number of the electron with $\pm\mu_B$ per atom. As such, the net amount, Δn , quantifies the magnetic moment of an atom as $m_a = \mu_B \Delta n$ ($n_{\pm}$ also can be the number of $3d$ hole with $\pm\mu_B$ per atom. However, the moment calculated from hole number is smaller than that calculated by $3d$ electrons because of hybridization of d states with s – p states. As this is not the focus of this thesis, interested readers are referred to page 529 of Ref. [44]). In the magnetic metals, Δn is 2.2, 1.7 and 0.7 for Fe, Co and Ni, and gives the magnetic moment of an atom as $2.2\mu_B$, $1.7\mu_B$ and $0.7\mu_B$, respectively [44]. In experiments,

saturation magnetization of the bulk magnetic metals, which are equivalent to the corresponding magnetic moment of the atom at the temperature of 0 K, have been commonly used. They are 2.2 T (1751 in unit of emu/cc), 1.8 T (1432 in unit of emu/cc) and 0.7 T (557 in unit of emu/cc) for bulk Fe, Co and Ni at 4.2 K [44]. Compared to Δn , the number of saturation magnetization in the unit of Tesla at 4.2 K, which is close to Δn , implies the number of 3d electrons contributing the magnetism of metals in an atom [49].

Fig. 2-1 Schematic of electron density per energy unit and Two-current model in Fe, Co and Ni. The spin states marked in red have larger electron number, and the corresponding filled spins are named “majority spins”, while these marked in blue have small electron number are named “minority spins”. Note that the 4s band is filled with *s-p* electrons.

2.2. Magneto-optical Kerr Effect

Vibrating sample magnetometer (VSM) has been widely used to measure the magnetization of relatively thin films [148-155]. However, it is insufficient to measure the magnetization of an ultrathin magnetic films *e.g.* an atomic layer, as the sensitivity of VSM is low, which is $\sim 10^{-7}$ emu [156-159]. Superconductivity quantum interference device (SQUID) is an

alternative of VSM, which has high sensitivity [26, 160-163], though the SQUID works at low temperature which is $\sim 10\text{K}$. In 1887, John Kerr discovered that the polarization of a linear polarized light was altered when the light was reflected from magnetic materials, the polarization alteration is so called magneto-optical Kerr effect which provides the magnetization information of magnetic films [164-167]. As the light can penetrate into the magnetic films typically 10~20 nm and the setup of MOKE is relatively simple, the measurement of MOKE has become an efficient alternative of VSM [168], which works for magnetic ultrathin films at room temperature.

In the classical picture of electrodynamic, MOKE is attributed to an induced transverse displacement of the 4s electrons under the action of the incident light. The electric field of the incident light excites electrons to oscillate along the light polarization. This gives rise to the primary component of the reflected light. The additional transverse component arises from a transverse electron oscillation. The transverse oscillation is due to the spin-orbit coupling which deflects incoming electrons to move along the direction perpendicular to its initial moving direction. As I will discuss in detail in Section 2.5, the magnitude of transverse oscillation or component is dependent of the local magnetization. Generally, the two components of light are not in-phase, a magnetization dependent rotation of the polarization arise from the superposition of these two components. The above description can be analytically understood *via* the following example: polar MOKE, where the local magnetization is perpendicular to film plane and is parallel to the incident plane as shown in Fig. 2.2(a). A linear polarized light is incident upon a magnetic film, the motion equation of the 4s electron e^- is

$$m_0 \frac{d^2 \mathbf{r}}{dt^2} + m_0 \nu \frac{d\mathbf{r}}{dt} = -e\mathbf{E} + e \frac{d\mathbf{r}}{dt} \times \mathbf{B}, \quad (2-1)$$

where $\mathbf{E}$ is the electric field component of the light, $\mathbf{r}$ is the position vector of the 4s electron, m_0 is the mass of the electron, ν is the scattering frequency as $\sim 10^{15}$ Hz due to the empty 3d band [44], and $\mathbf{B}$ is understood as an “effective magnetic field” which gives rise to an “effective Lorentz force” to induce a transverse motion of 4s electrons. Hence, this “effective Lorentz force” is equivalent to the spin-orbit coupling which induces the transverse motion. We define $\mathbf{B}$ to be along the z-axis, which means the local magnetization is along the z-axis. $\mathbf{E}$ propagates in the x-z plane. The polarization is set along the x-axis to oscillate electrons along the x-axis, consequently, an oscillation along the y-axis can be induced by the effective field B . $\mathbf{E}$ in the magnetic film is expressed as $E_0 e^{-i\omega t} \mathbf{x}$, where ω is the angular frequency of the light. The motion equation of the 4s electron is rewritten as

$$m_0 \begin{pmatrix} \frac{d^2 x}{dt^2} \\ \frac{d^2 y}{dt^2} \end{pmatrix} + m_0 \nu \begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = -e \begin{pmatrix} E_0 e^{-i\omega t} \\ 0 \end{pmatrix} + e \begin{pmatrix} \frac{dy}{dt} B \\ -\frac{dx}{dt} B \end{pmatrix}, \quad (2-2)$$

By taking typical solutions $x = x_0 e^{-i\omega t}$ and $y = y_0 e^{-i\omega t}$ into Eq. (2-2), Eq. (2-2) is revised as

$$m_0 \omega^2 \begin{pmatrix} -x_0 e^{-i\omega t} \\ -y_0 e^{-i\omega t} \end{pmatrix} + m_0 \nu \omega \begin{pmatrix} -x_0 i e^{-i\omega t} \\ -y_0 i e^{-i\omega t} \end{pmatrix} = -e \begin{pmatrix} E_0 e^{-i\omega t} \\ 0 \end{pmatrix} + e \omega B \begin{pmatrix} -y_0 i e^{-i\omega t} \\ x_0 i e^{-i\omega t} \end{pmatrix}. \quad (2-3)$$

In our experiment, the light was chosen as ~ 800 nm laser which gives ω as $\sim 10^{15}$ Hz, hence, $m_0 \omega^2 \approx m_0 \omega \nu$ allows to combine the left two terms of Eq. (2-3) to further simplify the equation. By substituting $m_0 \nu$ and e with the approximated values of 10^{-31} kg, 10^{15} Hz and 10^{-19} C in Eq. (2-3), respectively, Eq. (2-3) is rewritten as

$$\begin{pmatrix} (1+i)x_0 - 10^{-3} B i y_0 \\ 10^{-3} B i x_0 + (1+i)y_0 \end{pmatrix} = \begin{pmatrix} 10^{-18} E_0 \\ 0 \end{pmatrix}. \quad (2-4)$$

The solutions of Eq. (2-4) are $x_0 = 10^{-18} E_0 \frac{1+i}{2i-10^{-6} B^2}$ and $y_0 = 10^{-21} E_0 \frac{B i}{2i-10^{-6} B^2}$,

which give the primary and transverse relative dielectric permittivity of the magnetic films as

$$\varepsilon_x = 1 + A \left[(2 - 10^{-6} B^2) - (2 + 10^{-6} B^2) i \right] \text{ and } \varepsilon_y = 1 + 10^{-3} A \left[2B - 10^{-6} B^3 i \right], \quad (2-5)$$

respectively, where $A = \frac{10^{-37} N}{\varepsilon_0 (4 + 10^{-12} B^4)}$ and N is the 4s electron density in the unit of

number per cubic meter, ε_0 is the absolute permittivity in the unit of F/m. Consequently,

the real parts of the permittivity are $\varepsilon_{r,x} = 1 + A(2 - 10^{-6} B^2)$ and $\varepsilon_{r,y} = 1 + 2 \times 10^{-3} AB$, and

the imaginary parts of that are $\varepsilon_{i,x} = A(2 + 10^{-6} B^2)$ and $\varepsilon_{i,y} = 10^{-9} AB^3$. The real parts are

related to refractive index n as $n_{x,y} = \sqrt{\varepsilon_{r,(x,y)}} = \frac{c}{v_{x,y}}$, where c is the light speed in vacuum,

and $v_{x,y}$ are the light speeds in the magnetic films [169]. As $\varepsilon_{r,x} \neq \varepsilon_{r,y}$ for a fixed value of B

according to the expressions of $\varepsilon_{r,(x,y)}$, the primary reflection is not as fast as the transverse

reflection. Therefore, a phase difference is formed between the reflections. Generally, B is

around two orders magnitude of the local magnetization [170]. Alternatively, if the local

magnetization is 1 T, B is around 100 T. However, here B is assumed to equal to Weiss field

(~1000 T) which is the largest magnetic field we can observe in magnetic materials [44],

$\varepsilon_{r,x} = 1 + 2A - 10^{-3} A$ and $\varepsilon_{r,y} = 1 + 2A$. Hence, the difference between $\varepsilon_{r,x}$ and $\varepsilon_{r,y}$ is $10^{-3} A$.

It is quite smaller than $1 + 2A$, leads to $n_x \approx n_y$, according to the expression of $n_{x,y}$. As $\varepsilon_{i,x} \neq \varepsilon_{i,y}$

for a fixed value of B , the extinction coefficient of κ_x is not equal to that of κ_y , where $\kappa_{x,y}$

are expressed as $\kappa_{x,y} = \frac{\varepsilon_{i,(x,y)}}{2n_{x,y}}$ which is related to the light absorption of films [169].

Consequently, the emission intensity of the primary reflection is not equal to that of the

transverse reflection. Due to the phase and emission intensity differences of the two

reflections, which exist only when B is nonzero, the reflection light is a elliptically polarized and the main polarization axis deviates from the polarization axis of the incident linear light. The similar analysis can be applied in the longitudinal MOKE, where the magnetization is in the film plane and is parallel to the incident plane as shown in Fig. 2.2(b). The transverse electron oscillation which gives rise to the transverse component of reflection is still along the y -axis. For the measurement of MOKE, B is related to the local magnetization, the incident linear light is realized by a light polarizer, and the deviation is compared by another polarizer. In conclusion, MOKE can be used to detect the magnetization of magnetic materials.

Fig. 2-2 Schematics of (a) polar and (b) longitudinal MOKE.

2.3. Ferromagnetic resonance

While MOKE can be used to check the preferred orientation of the magnetization, it is unable to provide the absolute magnetization magnitude for ultrathin films. However, FMR in which microwave and magnetic field are used [43, 49, 50], provides a solution for measuring the magnetization magnitude. The FMR can be derived from the LLG equation which describes the dynamics of the saturation magnetization $\mathbf{M}_s$ under the action of the

magnetic field $\mathbf{H}_{\text{eff}}$,

$$\frac{d\mathbf{M}_s}{dt} = -\gamma \mathbf{M}_s \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M} \mathbf{M}_s \times \frac{d\mathbf{M}_s}{dt}, \quad (2-6)$$

where α is the Gilbert damping constant which is typically material dependent [171-175].

The precessional motion of $\mathbf{M}_s$ as a function of time is due to a combination of the field

torque $-\gamma \mathbf{M}_s \times \mathbf{H}_{\text{eff}}$ and the damping torque $\frac{\alpha}{M} \mathbf{M}_s \times \frac{d\mathbf{M}_s}{dt}$ acting upon the magnetization,

with the damping torque aligning the $\mathbf{M}_s$ along $\mathbf{H}_{\text{eff}}$ much like a viscous damping [43, 44,

49, 50]. $\mathbf{H}_{\text{eff}}$ comprises of all possible contributions to the instantaneous axis of

magnetization precession. Notable contributions include demagnetizing anisotropy,

interface anisotropy, magnetic crystalline anisotropy, and Zeeman energies. In FMR

experiments, $\mathbf{H}_{\text{eff}}$ is contributed from the externally applied magnetic field $\mathbf{H}_{\text{ext}}$ and the

magnetic component of the microwave $\mathbf{h} = \mathbf{h}_0 e^{i\omega t}$, where ω is the angular frequency of the

microwave. As the magnitude of $\mathbf{H}_{\text{ext}}$ is far larger than that of $\mathbf{h}$, $\mathbf{h}$ just induces a small

magnetization oscillation around $\mathbf{H}_{\text{ext}}$. Hence, when $\mathbf{H}_{\text{ext}}$ is applied along the x -axis, $\mathbf{M}_s$ is

expressed as $\mathbf{M}_s = (M_0 + m_x e^{i\omega t}) \mathbf{x} + m_y e^{i\omega t} \mathbf{y} + m_z e^{i\omega t} \mathbf{z}$, where $m_{x,y,z}$ are the small oscillation

components of $\mathbf{M}_s$ along the three basis axes and M_0 is the average value of $\mathbf{M}_s$ along the

x -axis. For our experiments as will be shown in the schematic of measurement setup of Fig.

3-4, $\mathbf{H}_{\text{ext}}$ and $\mathbf{h}$ are applied along the x -axis and the y -axis in the film plane, respectively,

which lead to $\mathbf{H}_{\text{ext}} = H_{x\text{-ext}} \mathbf{x}$ and $\mathbf{h} = h_0 e^{i\omega t} \mathbf{y}$. For the wide and thin magnetic film, the

demagnetizing factors along the x and y axes are approximately zero [43, 44, 49, 50]. While,

the demagnetizing factor along z -axis is approximately 1 [43, 44, 49, 50], which results in

demagnetizing field along z -axis as $-m_z e^{i\omega t} \mathbf{z}$. Here, since the magnetization m_z orientating

along the $+z$ direction, induces magnetic free poles which appear on the corresponding two

surfaces, the free poles produce the demagnetizing field directed opposite to the magnetization. As such, $\mathbf{H}_{\text{eff}}$ is comprised of $H_{x-\text{ext}}\mathbf{x}$, $h_0e^{i\omega t}\mathbf{y}$ and $-m_z e^{i\omega t}\mathbf{z}$. Through substituting $\mathbf{H}_{\text{eff}}$ and $\mathbf{M}_s$ by their components into Eq. (2-6), Eq. (2-6) is rewritten as

$$i\omega(m_x\mathbf{x} + m_y\mathbf{y} + m_z\mathbf{z}) = -\gamma \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_0 + m_x & m_y & m_z \\ H_{x-\text{ext}} & h_0 & -m_z \end{vmatrix} + \frac{\alpha}{M_s} \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_0 + m_x & m_y & m_z \\ i\omega m_x & i\omega m_y & i\omega m_z \end{vmatrix}, \quad (2-7)$$

for our experiments. Equation (2-7) can be further rewritten as

$$i\omega \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = -\gamma \begin{pmatrix} -m_y m_z - m_z h_0 \\ m_z H_{x-\text{ext}} + m_z (M_0 + m_x) \\ (M_0 + m_x) h_0 - m_y H_{x-\text{ext}} \end{pmatrix} + \frac{\alpha}{M_s} \begin{pmatrix} m_y i\omega m_z - i\omega m_y m_z \\ m_z i\omega m_x - (M_0 + m_x) i\omega m_z \\ i\omega (M_0 + m_x) m_y - m_x i\omega m_y \end{pmatrix}. \quad (2-8)$$

Neglecting the small quantities of the second order such as $m_{x,y,z}m_{x,y,z}$ and considering $M_0 \approx M_s$, we simplify Eq. (2-8) to be

$$i\omega \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = -\gamma \begin{pmatrix} 0 \\ m_z H_{x-\text{ext}} + m_z M_s \\ M_s h_0 - m_y H_{x-\text{ext}} \end{pmatrix} + \frac{\alpha}{M_s} \begin{pmatrix} 0 \\ -M_s i\omega m_z \\ i\omega M_s m_y \end{pmatrix}. \quad (2-9)$$

Simplifying Eq. (2-9) gives Eq. (2-10) as

$$\begin{cases} m_x = 0 \\ i\omega m_y + (\omega_s + \omega_{\text{ext}} + \alpha i\omega) m_z = 0 \\ -(\omega_{\text{ext}} + i\omega\alpha) m_y + i\omega m_z = -\omega_s h_y \end{cases}, \quad (2-10)$$

where $\omega_s = \gamma M_s$ and $\omega_{\text{ext}} = \gamma H_{x-\text{ext}}$. The solutions of Eq. (2-10) are obtained as

$$\begin{cases} m_y = \frac{(\omega_s + \omega_{\text{ext}} + \alpha i\omega) \omega_s}{\omega_{\text{ext}}^2 - \omega^2 + \omega_s \omega_{\text{ext}} + 2i\alpha \omega_{\text{ext}} \omega + \omega_s \omega i\alpha} h_0 \\ m_z = \frac{i\omega \omega_s}{\omega_{\text{ext}}^2 - \omega^2 + \omega_s \omega_{\text{ext}} + 2i\alpha \omega_{\text{ext}} \omega + \omega_s \omega i\alpha} h_0 \end{cases}. \quad (2-11)$$

From Eq. (2-11), the permeability of the magnetic film is expressed as

$$\left\{ \begin{array}{l} \mu = 1 + \frac{m_{y,z}}{h_0} = \mu_r + \mu_i i \\ \mu_r = 1 + \frac{O_{y,z}(\omega^2)}{(\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2 - \omega^2)^2 + (2\omega_{\text{ext}} + \omega_s) \alpha^2 \omega^2}, \\ \mu_i = \frac{O_{y,z}(\omega^3)}{(\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2 - \omega^2)^2 + (2\omega_{\text{ext}} + \omega_s) \alpha^2 \omega^2} \end{array} \right. , \quad (2-12)$$

where $O_{y,z}(\omega^2)$ $O_{y,z}(\omega^3)$ are as functions of ω^2 and ω^3 . $(\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2 - \omega^2)^2$ is in the denominator while $O_{y,z}(\omega^3)$ is numerator, hence the extremum of μ_i occurs when $(\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2 - \omega^2)^2 = 0$, resulting in $\omega_{\text{res}} = \sqrt{\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2}$. The imaginary part μ_i of μ indicates the absorption of the microwave by the magnetic material [43, 44, 49, 50, 169]. Therefore, the maximum or resonance absorption occurs at

$$\omega_{\text{res}} = \gamma \sqrt{H_{x-\text{ext}}^2 + M_s H_{x-\text{ext}}} \quad (2-13)$$

which is obtained by substituting ω_s by γM_s and ω_{ext} by $\gamma H_{x-\text{ext}}$ in the expression of $\omega_{\text{res}} = \sqrt{\omega_s \omega_{\text{ext}} + \omega_{\text{ext}}^2}$, as shown in Fig. 2-3. In Eq. (2-13), M_s is a parameter of the resonance frequency ω_{res} , hence, we can obtain M_s by fitting ω_{res} as a function of $H_{x-\text{ext}}$.

Fig. 2-3 The intensity of microwave absorption as a function of the microwave frequency ω .

2.4. Magnetization at equilibrium state

With the assistance of $\mathbf{h}=h_0e^{i\omega t}\mathbf{y}$, the magnetization maintains a steady-state precession around $\mathbf{H}_{\text{ext}}=H_{x\text{-ext}}\mathbf{x}$. Hence, $\mathbf{h}=h_0e^{i\omega t}\mathbf{y}$ acts on $\mathbf{M}_s$ as an anti-damping torque to nullify the damping torque $\frac{\alpha}{M}\mathbf{M}_s \times \frac{d\mathbf{M}_s}{dt}$. Without the microwave, $\mathbf{M}_s$ is aligned along the direction of $\mathbf{H}_{\text{eff}}$ by the damping torque. In the following, the time of $\mathbf{M}_s$ being aligned along the direction of $\mathbf{H}_{\text{eff}}$, τ , is estimated.

A. Initially, $\mathbf{M}_s$ is set to be along the $+x$ direction. $\mathbf{H}_{\text{eff}}$ includes $-H_{x\text{-ext}}\mathbf{x}$ and $-M_z\mathbf{z}$, as $\mathbf{H}_{\text{ext}}$ is applied along the $-x$ direction, consequently, the LLG equation becomes to

$$\begin{pmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{pmatrix} = -\gamma \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_x & M_y & M_z \\ -H_{\text{ext}} & 0 & -M_z \end{vmatrix} + \frac{\alpha}{M_s} \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_x & M_y & M_z \\ \frac{dM_x}{dt} & \frac{dM_y}{dt} & \frac{dM_z}{dt} \end{vmatrix}, \quad (2-14)$$

where $M_{x,y,z}$ are the components of $\mathbf{M}_s$ along the three basic axes. Equation (2-14) can be rewritten as

$$\begin{pmatrix} \frac{dM_x}{dt} + \frac{\alpha}{M_s} M_z \frac{dM_y}{dt} - \frac{\alpha}{M_s} M_y \frac{dM_z}{dt} \\ -\frac{\alpha}{M_s} M_z \frac{dM_x}{dt} + \frac{dM_y}{dt} + \frac{\alpha}{M_s} M_x \frac{dM_z}{dt} \\ \frac{\alpha}{M_s} M_y \frac{dM_x}{dt} - \frac{\alpha}{M_s} M_x \frac{dM_y}{dt} + \frac{dM_z}{dt} \end{pmatrix} = \begin{pmatrix} \gamma M_y M_z \\ \gamma M_z H_{x\text{-ext}} - \gamma M_z M_x \\ \gamma M_y H_{x\text{-ext}} \end{pmatrix}. \quad (2-15)$$

Solving Eq. (2-15) gives the solutions as

$$\begin{cases} \frac{dM_x}{dt} = -\frac{1}{1+\alpha^2} \left[\gamma M_y M_z + \alpha \gamma \frac{M_x}{M_s} M_z^2 + \alpha \gamma \frac{H_{x-\text{ext}}}{M_s} (M_y^2 + M_z^2) \right] \\ \frac{dM_y}{dt} = -\frac{1}{1+\alpha^2} \left[-(\gamma H_{x-\text{ext}} M_z + \gamma M_x M_z) + \alpha \gamma \frac{M_y}{M_s} M_z^2 - \alpha \gamma \frac{H_{x-\text{ext}}}{M_s} M_x M_y \right] \\ \frac{dM_z}{dt} = -\frac{1}{1+\alpha^2} \left[\gamma H_{x-\text{ext}} M_y - \alpha \gamma \frac{H_{x-\text{ext}}}{M_s} M_x M_z - \alpha \gamma \frac{M_z}{M_s} (M_y^2 + M_x^2) \right] \end{cases} \quad (2-16)$$

The value of α , which is generally in the order of 10^{-2} [171-175], is far less than 1. Hence,

$\frac{dM_x}{dt}$ in the solution expression is mainly ascribed to the term $\gamma M_y M_z$. As such, in physics,

the switching from $+M_x$ to $-M_x$ is mainly controlled by the precession of M_y around the field M_z , as the huge demagnetizing factor along the z -axis forces $\mathbf{M}_s$ orientating in the x - y plane. Phenomenologically, M_s follows a sine behavior to rotate from $+x$ -axis to $-x$ -axis.

Similar analysis can be applied for $\frac{dM_y}{dt}$ and $\frac{dM_z}{dt}$. As such, neglecting the small quantity

α^2 , equation (2-16) is simplified to

$$\begin{cases} \frac{dM_x}{dt} \approx -\gamma M_y M_z \\ \frac{dM_y}{dt} \approx (\gamma H_{x-\text{ext}} M_z + \gamma M_x M_z) \\ \frac{dM_z}{dt} \approx -\gamma H_{x-\text{ext}} M_y \end{cases} \quad (2-17)$$

According to Eq. (2-17), the initial condition, which limits $\mathbf{M}_s$ to be along the $+x$ -axis,

allows the average solution of $\frac{dM_z}{dt} \approx -\gamma H_{x-\text{ext}} M_y$ as $\overline{M_z} \approx -\gamma H_{x-\text{ext}} M_y t$. Substituting M_z

by the solution, the expression of $\frac{dM_x}{dt}$ is revised to be $\frac{dM_x}{dt} \approx \gamma^2 M_y^2 t$. The average of M_y

is $\frac{1}{2} M_s$ in the switching time τ , as M_s follows a sine behavior in the x - y plane, meanwhile

the variation of M_x is $2M_s$. Therefore, $\frac{2M_s}{\tau} = \gamma^2 H_{x-\text{ext}} \left(\frac{M_s}{2} \right)^2 \tau$, and it gives

$$\tau = \frac{1}{\gamma} \sqrt{\frac{8}{H_{x-\text{ext}} M_s}}. \text{ We estimate the time for a film with } M_s \sim 1.8 \text{ T. } \gamma \text{ is } 2.2 \times 10^5 \text{ mA}^{-1} \text{s}^{-1}$$

and $H_{x-\text{ext}}$ is set to be a relatively small value of 5 Oe, as a result, τ is calculated to be 0.7 ns. The value of 0.7 ns indicates that $\mathbf{M}_s$ is damped to be along the $+x$ direction from $-x$ direction in 0.7 ns.

B. Initially, $\mathbf{M}_s$ is set to be along the $+z$ direction. As $\mathbf{H}_{\text{ext}}$ is not applied, $\mathbf{H}_{\text{eff}}$ is equal to $-M_z \mathbf{z}$. The LLG equation becomes to

$$\left(\frac{dM_x}{dt} \mathbf{x} + \frac{dM_y}{dt} \mathbf{y} + \frac{dM_z}{dt} \mathbf{z} \right) = -\gamma \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_x & M_y & M_z \\ 0 & 0 & -M_z \end{vmatrix} + \frac{\alpha}{M_s} \begin{vmatrix} \mathbf{x} & \mathbf{y} & \mathbf{z} \\ M_x & M_y & M_z \\ \frac{dM_x}{dt} & \frac{dM_y}{dt} & \frac{dM_z}{dt} \end{vmatrix}. \quad (2-18)$$

Equation (2-18) is rewritten as

$$\begin{pmatrix} \frac{dM_x}{dt} + \frac{\alpha}{M_s} M_z \frac{dM_y}{dt} - \frac{\alpha}{M_s} M_y \frac{dM_z}{dt} \\ -\frac{\alpha}{M_s} M_z \frac{dM_x}{dt} + \frac{dM_y}{dt} + \frac{\alpha}{M_s} M_x \frac{dM_z}{dt} \\ \frac{\alpha}{M_s} M_y \frac{dM_x}{dt} - \frac{\alpha}{M_s} M_x \frac{dM_y}{dt} + \frac{dM_z}{dt} \end{pmatrix} = \begin{pmatrix} \gamma M_y M_z \\ -\gamma M_z M_x \\ 0 \end{pmatrix}, \quad (2-19)$$

which gives a solution as

$$\frac{dM_z}{dt} = \gamma \alpha M_s m_z - \frac{\gamma \alpha}{M_s} m_z^3, \quad (2-20)$$

where we take α^2 to be negligible. The final orientation of $\mathbf{M}_s$, which is in the x - y plane, indicates that M_z is damped to be zero in the damping time τ . The average of

$$\gamma\alpha M_s m_z - \frac{\gamma\alpha}{M_s} m_z^3 \text{ from } M_s \text{ to } 0, \text{ which is } \frac{1}{M_s} \int_0^{M_s} \left(\gamma\alpha M_s m_z - \frac{\gamma\alpha}{M_s} m_z^3 \right) dm_z = \gamma\alpha \frac{M_s^2}{4},$$

indicates $\frac{M_s}{\tau} = \gamma\alpha \frac{M_s^2}{4}$. Consequently, the time can be estimated by the expression of

$$\tau = \frac{4}{\gamma\alpha M_s} [176, 177]. \text{ For a film with } M_s \sim 1.8 \text{ T, } \tau \text{ is calculated to be 4.6 ns when } \alpha \text{ is set}$$

as 0.01. The value of 4.6 ns indicates that $\mathbf{M}_s$ needs 4.6 ns to be relaxed in the x - y plane from the z -axis.

As concluded from the above, $\mathbf{M}_s$ relaxes to be at equilibrium state in the time scale of nanoseconds [44]. At equilibrium state, the magnetic energy E is given by

$$E = (N_z M_s^2 - K_{\perp}) \sin^2 \theta - M_s H_z \sin \theta - M_s H_x \cos \theta \cos \varphi - M_s H_y \cos \theta \sin \varphi, \quad (2-21)$$

where N_z is the demagnetizing factor along z direction, $K_{\perp}$ is the interface perpendicular magnetic anisotropy energy density which may be present in ultrathin films [44, 178], for a magnetic wire system comprised of IMA thin films, depicted in Fig. 2-4. $H_{(x,y,z)}$ are the effective magnetic fields along the three basic vector directions. φ and θ are the azimuthal and polar angles of M_s . The three terms with the field components are the Zeeman energy of the wire. The time scale of changing the magnetization state is in the order of nanosecond. Hence, on condition that the time of magnetic fields acting on magnetization is more than nanoseconds, the equilibrium angle between the magnetization and magnetic fields, can be obtained by: carrying out partial derivatives of the total magnetic energy of the wire with respect to the variables φ and θ .

$$\frac{\partial E}{\partial \theta} = 0 \text{ and } \frac{\partial E}{\partial \varphi} = 0. \quad (2-22)$$

For an instance which will be used in Chapter 4, a magnetic field $H = H_0 \sin \omega t$, which is

assumed to be induced by a current $I=I_0\sin\omega t$, is applied on the magnetization. On condition that ω is far lower than GHz (corresponding to nanosecond), the modulation of θ and φ by H can be calculated by Eq. (2-22).

Fig. 2-4 The magnetization orientation under the acts of applied fields.

2.5. Mechanism of spin-orbit torque

The effective field $\mathbf{H}_{\text{eff}}$ as ascribed above can also be induced by SOT, which is attributed to an accumulation of polarized spins due to Rashba and spin Hall effects. As mentioned in Chapter 1, the SOT moves DW at high speed in a FM wire sandwiched by two layers of nonmagnetic HM or HM and oxide [76, 94, 95, 107-109], although STT initially moves DW in simple FM wires [94]. The HMs can be Ta, Pt and W which have larger spin Hall angles (the ratio of spin current to charge current), MgO has been used as the oxide [76, 94, 95, 107-109]. Due to the sandwiched structure, such as Ta/Co/Pt, electric current induces spins to accumulate in the FM layer with the assistance of Rashba and spin Hall effects. The spin accumulation can then transfer torque to the local magnetization by the exchange interaction between spins [98].

2.5.1. Spin accumulation

2.5.1.1. *Rashba effect*

In the two effects leading to the spin accumulation, Rashba effect originates from the $4s$ electrons travelling in the sandwiched structure as shown in the schematic of Fig. 2-5 [44]. When an electric field is applied in the $-x$ direction in the sandwiched wire, the amount of $4s$ electrons travelling in the $-x$ direction is less than that of $4s$ electrons travelling in the $+x$ direction. Hence, in the electron sea of the wire, most of the electrons with opposite travelling directions give zero contribution to the electric current [42, 98]. Only few $4s$ electrons at Fermi level, which are responsible for forming the electric current, are allowed to travel.

The speed v of the $4s$ electrons is in the order of $0.1c$ [42, 107], where c is the light speed in vacuum, hence, the relativistic effect is predominant for the $4s$ electrons. As shown in Fig. 2-5, an electric current is applied along the x -axis. Besides in the electric field E_x which is used to induce the electric current, $4s$ electrons travels in another electric field E_z which is generated due to the asymmetric interfaces [121]. This electric field is E_z in the reference frame of the structure, which changes to $E_z + \mathbf{v} \times \mathbf{H}$ in the reference frame of the $4s$ electrons through using a Lorentz transformation [179, 180], where $\mathbf{H}$ is a magnetic field expressed as $\mathbf{H} = \frac{1}{2c^2} (\mathbf{v} \times E_z \mathbf{z})$. The expression $\mathbf{H} = \frac{1}{2c^2} (\mathbf{v} \times E_z \mathbf{z})$ indicates that $\mathbf{H}$ acts upon the electrons moving in the electric field E_z . We notice that this magnetic field is perpendicular to the travelling direction of the $4s$ electrons and the direction of the electric field. As each electron has a spin which is equivalent to a magnetic moment, the magnetic field $\mathbf{H}$ acts on the spins of $4s$ electrons *via* an interaction of $\mathbf{H}$ and the spins, and

the magnitude of this action is $E_{\text{Rashba}} = \eta_{\text{Rashba}} \boldsymbol{\mu}_B \cdot (\mathbf{v} \times \mathbf{E})$ with the interaction coefficient η_{Rashba} [98]. A spin accumulation $\mathbf{s}$, which is along the y -axis, can be formed by $\mathbf{H}$ through the action. This is the so-called Rashba effect. We notice that the magnetic field due to Rashba effect only acts on the spin of $4s$ electrons directly rather than the magnetization which is formed by $3d$ electrons.

Fig. 2-5 Schematic of the Rashba effect.

2.5.1.2. Spin Hall effect in heavy metals

On the other hand, the spin accumulation generated from SHE arises from the conduction electrons within the HM instead of the $4s$ electrons in FM due to the Rashba effect [78, 79, 110, 112, 114-119]. The accumulation process is similar to the charge accumulation induced by the ordinary Hall effect, as shown in Fig. 2-6(a). For the ordinary Hall effect, electric current carriers are deflected to the top and bottom surfaces of a wire by the Lorentz

force, according to the sign of the carriers. As a result, charges with opposite signs accumulate at the top and bottom surfaces separately. The same analogy can be applied for SHE, where spins with opposite signs, instead of the charges, accumulate at the two surfaces separately. However, the deflection of the electrons with different spin orientations is not due to the Lorentz force but due to the spin-dependent scattering [181-185]. As the amounts of the deflected electrons going to the top and bottom surfaces are the same, there is no charge accumulation at the two surface [16]. The skew and side-jump scatterings of electrons, which are introduced by the impurities of material, are proposed to be the basis of the spin-dependent scattering, although the relative contributions to the SHE are challenging to quantify and distinguish experimentally [16].

As shown in Fig. 2-6(b) for the skew scattering, the nucleus of an impurity gives an additional potential $V_{\text{impurity}}(r)$ which leads to a non-uniform electric field $\nabla V_{\text{impurity}}(r)$. Similarly to the Rashba effect, the electrons which are coming across the additional potential feel a non-uniform magnetic field $\mathbf{H} = \frac{\hbar}{2m_0c^2} [\mathbf{k}(v) \times \nabla V_{\text{impurity}}(r)]$, where $\mathbf{k}(v)$ is the wave vector of the electrons. Alternatively, this magnetic field can also be understood as the magnetic field created by the positive nucleus of the impurity going across the electrons. The non-uniform magnetic field exerts a force $\mathbf{F} = \nabla(\boldsymbol{\mu}_B \cdot \mathbf{H})$ on the electron spin which is equivalent to a magnetic dipole [44]. For the electrons travelling along the x -axis in the x - z plane and above the nucleus, the force is computed as $\mathbf{F} = \mp \mu_B \frac{\partial H}{\partial z} \mathbf{z}$ when the electron spins are aligned along the y -axis, while it is $\mathbf{F} = 0$ when the electron spins aligning along the x and z axes. According to the above expression, the force which orientates in the $-z$ direction leads to a deflection of electrons with $+y$ spin to the bottom

surface of the HM. Conversely, the electrons with $-y$ spin will be deflected upwards due to the force which orientates in the $+z$ direction. Similar forces are felt by the electrons passing through the nucleus of the impurity atom from other positions, leading to spin accumulation with different spin orientations at different surfaces of the wire.

The same spin accumulation can also be generated by the side-jump scattering which is related to the collisions between the electrons in different bands as well [16, 186]. As shown in Fig. 2-6(c), electrons are scattered from one side (or band) to another side (or band). The inter-band scattering gives a momentum transfer $\delta\mathbf{k}$ for electrons. Due to the SOC $\eta_{\text{SOC}}\mu_B \cdot [\delta\mathbf{k} \times \nabla V_{\text{impurity}}(r)]$ with the coefficient η_{SOC} between the magnetic field $\delta\mathbf{k} \times \nabla V_{\text{impurity}}(r)$ and spin μ_B , the scattering is also spin-dependent, which consequently results in spin accumulation at the surfaces of the wire [187].

Besides the potential $\nabla V_{\text{impurity}}(r)$ due to the impurity, the periodic potential $\nabla V_{\text{lattice}}(r)$ in terms of the lattice of materials also gives an effective magnetic field $\mathbf{k} \times \nabla V_{\text{lattice}}(r)$ acting on the conduction electrons, where $\mathbf{k}$ is the crystal wave vector [16]. As the electrons are accelerated in the lattice under the influence of an external E field, the wave vector varies by $\delta\mathbf{k}$. Ascribed to the SOC $\eta_{\text{SOC}}\mu_B \cdot [\delta\mathbf{k} \times \nabla V_{\text{lattice}}(r)]$, the electrons are deflected according to their spin orientations, consequently, spin accumulation arises at the surfaces of the wire. As this spin accumulation arises in the absence of impurities, the deflection by $\eta_{\text{SOC}}\mu_B \cdot [\delta\mathbf{k} \times \nabla V_{\text{lattice}}(r)]$ is referred to as intrinsic effect. On the other hand, the skew and side-jump scatterings are referred to as extrinsic contributions. Therefore, strong SHE observed in HMs such as Ta and Pt is due to large η_{SOC} observed in HMs as compared to light metals [16].

Fig. 2-6 Schematics of (a) the ordinary and spin Hall effects, (b) skew and (c) side-jump scatterings. The positive and negative symbols in (a) indicate the electron charge, while the red and blue arrows in (b) and (c) indicate the electron spin with different orientations.

Another example of the spin accumulation induced by spin-dependent inter-band scattering is known as AHE observed in FMs [188]. Although the $4s$ electrons do not contribute to the magnetization, they are in charge of the transport and scattering properties of the metals [45, 189-191]. In the magnetic metals, $4s$ electrons are freely mobile so as to configure the transport property, whereas $3d$ electrons are localized due to huge effective mass [44]. However, the scattering of $4s$ electron is mainly due to the interaction with the $3d$ electrons rather than with nucleus, where scattering with nucleus is common in other metals such as Al [42]. The mechanism of the scattering is explained as the Two-current model proposed by Mott, which is due to the hybridization between $3d$ and $4s$ band [45, 189-191]. In this model, spin-dependent scattering of $4s$ electrons can only be achieved when the itinerant electrons have the same spin orientation as the empty $3d$ band with available states. Applying this model in Fig. 2-1, only the $4s$ electrons with $-\mu_B$ can be scattered into the $3d$ empty band to configure the electric properties, whereas those with $+\mu_B$ are not scattered. When electric current is applied in the FMs, the minor $4s$ electrons at Fermi level are strongly scattered from the $4s$ band to the minority $3d$ empty band. Hence, the minority $4s$ electrons are deflected to one side of a wire as shown in Fig. 2-7. The majority $4s$ electrons at Fermi level are slightly deflected to the other side of the wire, due to less empty band of the majority $3d$ electrons [16, 44]. The two deflections result in a net electron flow as well as a spin flow transverse to the current and perpendicular to the local magnetization [192]. The magnitude of the local magnetization is approximately equivalent to the amount of positive holes in the empty $3d$ band [44]. As emptier $3d$ band allows more scattering, the electron and spin flows are proportional to the magnitude of the local magnetization [170]. An anomalous Hall voltage, which is generated by the electron flow, can be detected by

voltmeter to characterize the intensity of AHE. Generally, the anomalous Hall voltage is written as

$$V_{\text{AHE}} = R_{\text{A}} I = k M_s \sin \theta = R_{\text{AHE}} \sin \theta , \quad (2-23)$$

where R_{A} is AHE resistance which is proportional the magnetization component $M_s \sin \theta$ along the z -axis with a constant coefficient k [170]. The amount of positive holes in the empty $3d$ band can be adjusted *via* orientating the local magnetization by an externally applied magnetic field, with the assistance of the SOC of $3d$ electrons. Hence, another asymmetric scattering of the minority $4s$ electrons arises when the orientation of magnetization is altered, which gives the PHE resistance or AMR [44]. The PHE resistance is expressed as

$$R_{\text{p}} = \zeta M_x M_y \cos^2 \theta = \zeta M_s^2 \sin 2\varphi \cos^2 \theta = R_{\text{PHE}} \sin 2\varphi \cos^2 \theta , \quad (2-24)$$

where ζ is a coefficient, M_x and M_y are the magnetization components along the x -axis and the y -axis, respectively [125, 130, 193].

Fig. 2-7 Schematic of anomalous Hall effect. The positive and negative symbols indicate the electron charge properties. The red and blue arrows indicate the opposite spin orientations.

2.5.2. Exchange interaction and STT

According to the origin of the spin accumulation, two microscopic pictures of the SOT have been proposed [111]. One picture is related to the Rashba effect, which is described in wave vector space as shown in Fig. 2-8. The top arrows indicate the majority spins while the bottom arrows indicate the minority spins, as discussed in Fig. 2-1. Figure. 2-8(a) shows the space filled with $4s$ electrons. The center of Fermi surface is not at the origin point of its coordinate, which indicates that an electric current is applied. The Rashba effect orientates the spin of $4s$ electrons along the direction orthogonal to the velocities or wave vectors $k_{x,y}$ of the $4s$ electrons, as discussed in Section 2.5.1.1. Figures 2-8(b) and (c) show the spaces filled with $3d$ electrons, where the difference between the majority and minority spins give rise to the local magnetization. The $3d$ electron Fermi spaces are plotted to be at the centers of their coordinates, in order to indicate the zero contribution to the electric current from the $3d$ electrons. Irrespective of the interaction between $4s$ spins and the $3d$ spins, the local magnetization state is only determined by the exchange split (refer to Section 2.1) shown in Fig. 2-8(b). The $3d$ spins are parallel and antiparallel to the local magnetization. However, when considering the interaction, the $3d$ spins are deviated by the Rashba effect as shown in Fig. 2-8(c). As this deviation does not involve the spin angular momentum transfer of the $4s$ electrons to the local magnetization, the Rashba effect acts as an effective field H_F giving a fieldlike component of SOT on the local magnetization [98, 123]. As this effective field is only due to $4s$ electrons, it does not depend on the local magnetization which is formed by the $3d$ electrons.

In the other picture of SOT, the spin accumulation induced by SHE diffuses into the FM layer, hence, forms a spin current. The electrons which carry the spin current can be

reflected back due to spin scattering by the FM layer [56-58]. This reflection transfers linear momentum to the local magnetization, as such, gives the dampinglike term $\tau_D = -H_D \mathbf{M} \times (\mathbf{m} \times \mathbf{p})$ [54, 59]. The spin current going through the FM layer transfers angular momentum to the local magnetization, by adiabatically re-orientating their spins to be along the local magnetization [4, 51], as such, gives a fieldlike torque [54, 55]. In conjunction with the contribution from the Rashba effect, this fieldlike torque constructs the fieldlike component of SOT as $\tau_F = -H_F \mathbf{M} \times \mathbf{p}$.

Fig. 2-8 [98] (a) The spin orientation under the action of the Rashba effect. (b) The magnetization orientation (b) without and with (c) the interaction between 4s and 3d spins. The top arrows indicate the majority spins while the bottom arrows indicate the minority spins.

iii. Sample preparation

Ta and Pt were chosen as the heavy metals for generating spin accumulation, since they have strong SHE. In this chapter, the preparation details of the sample Ta/Co/Pt is provided. The method for films Ta/Co/Pt deposition is given. The processes and results of MOKE measurement as well as the FMR measurement, are described. The procedures of patterning the films into Hall crosses are schematically presented and described. The principle of harmonic Hall resistance measurement is explained through measurement of anomalous Hall resistance in Hall crosses using a lock-in amplifier.

3.1. Film deposition

As Ta and Pt have strong SHE, the stack for the films under investigation was chosen as Ta/Co/Pt which also provides an asymmetric interface for generating Rashba effect. Films in this work were deposited and characterized by Gerard Joseph Lim. The films Ta(t nm)/Co(2 nm)/(5 nm) were deposited using DC magnetron sputtering deposition technique, where $t=2, 4, 6, 8$ and 10 . The film deposition was carried out at room temperature in AJA magnetron sputtering system as shown in Fig. 3-1. The chamber base pressure was typically lower than 2.2×10^{-8} Torr, and the sputtering pressure is 2×10^{-3} Torr. For each of the films, the Ta layer was deposited on a 6-inch Si/SiO₂ wafer, followed by the Co layer and ending off with the Pt layer as a cap. The thickness of the three layers were controlled by deposition time, where the sputtering power was kept at 50 Watts with argon flow rate of 2 sccm, and the sputtering rates of Ta, Co and Pt targets were 2.4, 1.2 and 3.6 nm/min, respectively, which are calibrated by atomic force microscopy and X-ray reflectometry techniques. Atomic force microscopy (AFM) result for a 5 nm thick Ta film is as shown in Fig. 3-2, the typical roughness (arithmetical mean deviation of the assessed profile) of deposited film is ~ 1.3 nm. The roughness is comparable to the thickness of Co layer. This indicates that the interface perpendicular magnetic anisotropy is negligible in our films, since the large roughness reduces the interface anisotropy substantially [194-198]. Consequently, it allows us to measure the saturation magnetization using the FMR method which is proposed in Section 2.3. Additionally, the roughness of the film helps us in the investigation of the dependence between SOT effective fields and the magnetization uniformity with various Ta thicknesses, this will be further discussed in Chapter 5.

Fig. 3-1 AJA magnetron sputtering system.

Fig. 3-2 AFM image of a deposited Ta film.

3.2. IMA characterization

The IMA property of the films $\text{Ta}(t \text{ nm})/\text{Co}(2 \text{ nm})/(5 \text{ nm})$ were confirmed by measuring the longitudinal MOKE, where the local magnetization is in film plane and is parallel to the incident plane. The principle of longitudinal MOKE is same as the polar MOKE as explained in Section 2.2.

Fig. 3-3 Schematic of the MOKE measurement setup.

In the MOKE measurement as schematically shown in Fig. 3-3, the polarized laser was obtained by passing the laser from the laser source through the first THORLABS PRM1/M Polarizer. After which, the polarized laser was chopped by a NEW FOCUS 3501 Optical Chopper in the frequency of 157 Hz which was as the input reference frequency for the SIGNAL RECOVERY 7265 DSP Lock-in Amplifier. With the assistance of the THORLABS CM1-BS013 Spectroscope and mirror, it was then incident on the magnetic films to interact with the local magnetization. The magnetization orientation, which determines the polarization of the emission laser from the films, was controlled by the externally applied magnetic field which swept from -1000 to $+1000$ Oe along the film plane. The external magnetic field was provided by the electromagnet with ~ 40 Oe remanence field which was measured by HIRST GM08 Gaussmeter. The electromagnet was powered by KIKUSUI Bipolar Power Supply PBZ20-20. The polarization of the emission laser from the films was detected by measuring the intensity of the emission laser by MELLES GRIOT CCD detector after it passed through the second polarizer, which was recorded by the lock-in amplifier. The 40 Oe net magnetic field may possibly orientated

the magnetization before the MOKE measurement, as such, it could have induced a pre-polarization change of the emission laser. For each characterization of the films, this pre-polarization was treated as a reference polarization for the following MOKE measurement. Hence, minimum intensity of the emission laser was as a reference intensity, which was obtained by adjusting the second polarizer before applying electric current into the electromagnets. After applying the electric current into the electromagnets to sweep the magnetic field, I obtained the MOKE signals from the lock-in amplifier as shown in Fig. 3-4. The MOKE loops indicate that the magnetization of all the films is orientated in the film plane, and the magnetization switching occurs when the externally applied in-plane magnetic field is around ± 20 Oe.

Fig. 3-4 The measured MOKE loops for the films of Ta(t nm)/Co(2 nm)/(5 nm).

3.3. Saturation magnetization measurement

The saturation magnetization of the films Ta(t nm)/Co(2 nm)/(5 nm) were obtained by measuring the FMR.

Fig. 3-5 Schematic of the FMR measurement setup.

Fig. 3-6 (left) KEYSIGHT PNA Network Analyzer N5524A and (right) coplanar waveguide

In the FMR measurement as schematically shown in Fig. 3-5, the magnetic field which was provided by LakeShore electromagnet was applied along the plane of the film under test by the electromagnet. The field swept in a 188 Oe increment from 0 Oe to 9000 Oe for the films with $t=2$ and 6, and from 0 Oe to 6000 Oe for the films with $t=4, 8$ and 10. The value of magnetic field was measured by LakeShore 475 DSP Gaussmeter. At each value of the applied magnetic field, broadband microwave frequencies were swept from 2GHz to 43.5GHz for each value of applied magnetic field ramped from the Lakeshore electromagnet. The microwave was supplied by KEYSIGHT PNA Network Analyzer N5524A. For ensuring the penetration of the microwave into the films, the films were firmly attached on the coplanar waveguide, as shown in Fig. 3-6, which cancels the electric

component of the microwave but keeps the magnetic component. As shown in the schematic of the measurement setup, two coaxial cables were each connected to the terminals of the coplanar waveguide through the end launch connectors, terminating at the two VNA ports on the other end. The scattering parameters were recorded by the VNA, of which I analyze the S21 magnitude in order to evaluate the transmission intensity of the microwaves at the frequency range mentioned above. The S12 measured at H_{app} was subtracted from that measured at $H_{\text{app}}+188$ Oe to reduce background noises. As shown in the schematic of Fig. 3-7(a), the subtraction $\Delta S12$ also represents the transmission intensity, and indicates that the resonance transmissions are at H_{app} and $H_{\text{app}}+188$ Oe. The obtained $\Delta S12$ exhibits the resonance transmissions or FMR in each measurement of the films, as shown in Figs. 3-7(b)~3-7(f). The resonance equation $\omega_{\text{res}} = \gamma \sqrt{H_{x-\text{ext}}^2 + M_s H_{x-\text{ext}}}$ was used to fit the angular frequencies at resonance transmission and corresponding magnetic fields, where the gyromagnetic ratio is taken as $1.76 \times 10^7 \frac{\text{rad}}{\text{s} \cdot \text{Oe}}$ [43]. The fittings, which are shown in Figs. 3-7(b) ~ 3-7(f) for each of the film, give the magnetization magnitudes as provided in Table 3-1.

Table 3-1 The magnetization magnitudes for the films Ta(t nm)/Co(2 nm)/Pt(5 nm)

t (nm)	2	4	6	8	10
M_s (T)	0.73	0.59	0.55	0.58	0.74
M_s (Oe)	7300	5860	5480	5750	7440
M_s (emu/cc)	580	466	436	457	592

Fig. 3-7 (a) Schematic for comparing S_{12} at H_{app} , S_{12} at $H_{app}+188$ and corresponding ΔS_{12} . (b)~(f) The ΔS_{12} obtained from the measured transmission intensity (color), and the fitting of Eq. (2-13) to the resonance frequency and corresponding magnetic field (black dot+line), where t indicates the Ta thickness in the measured films.

3.4. Patterning

The IMA films were patterned into wires with the width of 5 μm and the length of 100 μm , by using Raith electron beam lithography system as shown in Fig. 3-8 and AJA Ar⁺-ion milling system as shown in Fig. 3-9. The procedures of patterning are schematically shown in Fig. 3-10.

Fig. 3-8 Raith EBL system

Fig. 3-9 AJA Ar⁺-ion milling system

The samples created for testing and characterization purposes are firstly subjected to 10 minutes acetone and isopropyl alcohol (IPA) cleansing treatment prior to any coating or deposition processes. Following which, a ~100 nm Ma-N negative photoresist was coated on each of the films by the spin coater, after cleaning the films for 10 mins by acetone and following 10 mins by IPA. The coated films were then pre-baked at 100 Celsius for 1 min. On the solid Ma-N, areas of 5 $\mu\text{m} \times 100 \mu\text{m}$ for shaping devices in the form of rectangular magnetic wires, were exposed by the electron beam in Raith EBL system. After which, the development process involves soaking the sample in a beaker containing a magnetic stirrer and Ma-D for 90 seconds followed by another beaker with deionized water for 120 seconds. On the developed films, under the protection of the solid Ma-N, the Ta/Co/Pt stacks were shaped to wires by using Ar⁺ ion milling in the AJA ion milling system. The solid Ma-N on top of the devices was then removed by leaving the patterned film in MR-remover for

6 hours. After this, a ~200 nm PMMA positive photoresist to be used in patterning the two Hall bars, was coated on each of the patterned films by the spin coater. The films coated with PMMA were then cured at 180 degrees for 5 minutes before being sent for second exposure to pattern 20um wide hall bars. The areas were then developed by leaving the films in the stirring liquid of MIBK for 30 seconds and in the stirring IPA for 2 mins. Following the development stage, electrode contacts of Ta(10 nm)/Cu(100 nm)/Ta(10 nm) were deposited using the AJA magnetron sputtering system. The final structure of the Hall cross as shown in Fig. 3-11 was obtained after lifting off the solid PMMA by acetone.

Fig. 3-10 The schematic of patterning procedures.

Fig. 3-11 The scanning electron microscope (SEM) image of the patterned Hall cross patterning.

3.5. Harmonic Hall resistance measurement for AHE

The harmonic Hall resistance measurement was carried out on the patterned Hall cross structure which was attached on a sample holder as shown in Fig. 3-12, for obtaining the AHE resistance. Before going to the experimental procedure of AHE resistance measurement by lockin amplifier, the principle of lockin amplifier technique is explained. The lockin amplifier characterizes an AC voltage inputting into the A terminal with assistant of a reference signal inputting into the Ref-in terminal, as shown in Fig. 3-11. Assuming the AC voltage is $V_{\text{Hall}} = j_0 R_{\text{Hall}} \sin(\omega t + \theta_{\text{Hall}})$ and the reference signal is $V_{\text{ref}} = V_0 \sin(\omega_{\text{ref}} t + \theta_{\text{ref}})$, the cross product $V_{\text{Hall}} V_{\text{ref}}$ which is operated by the lockin amplifier, gives $V_{\text{Hall}} V_{\text{ref}} = j_0 R_{\text{Hall}} V_0 \sin(\omega t + \theta_{\text{Hall}}) \sin(\omega_{\text{ref}} t + \theta_{\text{ref}})$. V_0 is set a 1, this expression can be expanded as

$$V_{\text{Measu}} = j_0 R_{\text{Hall}} \cos[(\omega - \omega_{\text{ref}})t + (\theta_{\text{Hall}} - \theta_{\text{ref}})] - j_0 R_{\text{Hall}} \cos[(\omega + \omega_{\text{ref}})t + (\theta_{\text{Hall}} + \theta_{\text{ref}})]. \quad (3-1)$$

The lockin amplifier actually measures V_{Measu} . If ω_{ref} is not equal to the signal frequency of ω , the two right terms of Eq. (3-1) are all AC signals, which leads to zero output when

passing V_{Measu} through a low pass filter. However, when the reference frequency ω_{ref} is set to be equal to the signal frequency of ω , V_{Measu} is comprised of a DC component $j_0 R_{\text{Hall}} \cos(\theta_{\text{Hall}} - \theta_{\text{ref}})$, and a AC component $j_0 R_{\text{Hall}} \cos[2\omega t + (\theta_{\text{Hall}} + \theta_{\text{ref}})]$ which can be eliminated by passing V_{Measu} through a low pass filter. Consequently, V_{Measu} remains the DC component $j_0 R_{\text{Hall}} \cos(\theta_{\text{Hall}} - \theta_{\text{ref}})$ which is the final output of the lockin amplifier. The final output indicates a vector with X -component $X_{\text{Measu}} = j_0 R_{\text{Hall}} \cos(\theta_{\text{Hall}} - \theta_{\text{ref}})$, Y -component $Y_{\text{Measu}} = j_0 R_{\text{Hall}} \sin(\theta_{\text{Hall}} - \theta_{\text{ref}})$ and the magnitude $R_{\text{Measu}} = j_0 R_{\text{Hall}}$. By adjusting $\theta_{\text{Hall}} = \theta_{\text{ref}}$, the output is maximum, which is the case of our experiments. Similarly, when the inputting signal has the frequency of 2ω , this signal amplitude can be obtained by adjusting reference frequency ω_{ref} to be equal 2ω . Extensively, if a signal includes several components with the frequency of $n\omega$, $V_{\text{Hall}} = \sum_{n=1}^N V_n$ and $V_n = j_n R_{\text{Hall}} \sin(n\omega t + \theta_n)$, where $n=1, 2, 3, \dots, N$, each component can be extracted by adjusting reference frequency ω_{ref} to be equal to the frequency of each component. Namely, $V_1 = j_1 R_{\text{Hall}} \sin(\omega t + \theta_1)$ is called as the first harmonic voltage, and $V_2 = j_2 R_{\text{Hall}} \sin(2\omega t + \theta_2)$ is called as the second harmonic voltage. If $j_{1,2}$ has the unit of current in the above expressions, $\frac{V_{1,2}}{j_{1,2}} = R_{\text{Hall}} \sin([1,2]\omega t + \theta_2)$ have the unit of resistance, hence, correspond to the first and second harmonic resistances which will be used in Chapter 4.

Fig. 3-12 Schematic of harmonic Hall resistance measurement.

In our measurements of the AHE resistance, the voltage is input into the terminal A from the Hall bar. The voltage is calculated as $V_{\text{Hall}} = R_{\text{Hall}} j_0 \sin(\omega t)$, where j_0 is the applied current density in the magnetic wire. Due to $\theta_{\text{Hall}} - \theta_{\text{ref}} = 0$, X_{Measu} was measured with sweeping magnetic field perpendicularly to the magnetic wires.

Fig. 3-13 Sample holder

Fig. 3-14 KEITHLEY 6221 DC and AC Current Source

Fig. 3-15 SIGNAL RECOVERY 7265 DSP Lock-in Amplifier

The alternating electric current (AC), with the current density amplitude $j_0 = 5 \times 10^{10} \text{ Am}^{-2}$ and frequency of $\omega = 307.1 \text{ Hz}$, was applied in the magnetic wires by 6221 DC and AC Current Source as shown in Fig. 3-14. A 7265 DSP Lock-in Amplifier, as shown in Fig. 3-15, was used to measure the harmonic Hall voltages, where the reference signal is from the Ref-out of 6221 DC and AC Current Source. The harmonic Hall resistances were calculated by taking the ratio of the harmonic Hall voltage to the amplitude of the applied current.

Due to the orientation of the sweeping field, the contribution from the AHE resistance dominates the PHE resistance, as we will discuss in Chapter 4. As a result, we are able to determine the AHE resistance with respect to the applied field as shown in Fig. 3-15. The expression of the AHE resistance is $R_A = R_{\text{AHE}} \sin \theta$, where $\sin \theta_0 = \frac{H_{z-\text{ext}}}{H_{\perp}}$ with the IMA effective field $H_{\perp}$ [Chapter 4]. According to the expression, the AHE resistance increases with increasing $H_{z-\text{ext}}$ which was provided by Lakeshore electromagnet and measured by LakeShore 475 DSP Gaussmeter, and comes to saturation at $H_{z-\text{ext}}=H_{\perp}$. Hence, the saturation values of the AHE resistance and corresponding positions give the amplitude of AHE R_{AHE} and the effective fields of IMA $H_{\perp}$. From Fig. 3-16, we obtain R_{AHE} and $H_{\perp}$ for each of the wires, as shown in Table 3-1.

Table 3-1 R_{AHE} and $H_{\perp}$ for each of the wires with Ta(t nm)/Co(2 nm)/Pt(5 nm)

t (nm)	2	4	6	8	10
R_{AHE} (m Ω)	33.2	27.9	24.4	25.9	27.8
$H_{\perp}$ (Oe)	6000	4760	5180	5790	7520

Fig. 3-16 The measured AHE resistances for the samples.

IV. Concurrent quantification of SOT effective fields

In this work, I have developed and experimentally tested a harmonic Hall technique, where a single measurement can concurrently provide quantitative information on both the fieldlike and dampinglike SOT terms in IMA structures. This technique enables the results to be insulated from measurement artefacts, such as magnetization variation which is not expected but will be introduced due to different methods in sweeping of magnetic field (see Chapter 5). Starting from the standard total energy of a magnetic system, the equations to compute the corresponding SOT effective fields are derived. By fitting the measured second harmonic Hall resistance with the derived equation, the fieldlike and dampinglike terms can be extracted. The first harmonic Hall resistance verifies the rotation of magnetization and is used to obtain the parameters in the derived equations. Experimental verification was carried out in the Ta/Co/Pt Hall crosses with IMA.

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4.1. Analytical derivation for measurement

This section describes how the analytical formulas are obtained. The derivation starts from the magnetic energy expression E of a magnetic system with IMA, which is used to obtain the relationship between the magnetization angles φ , θ and magnetic fields $H_{(x,y,z)}$. The modulation of the magnetization angle induced by SOT effective fields is then introduced, and converted into measurable changes of harmonic Hall resistance. The terms for calculating SOT fields are obtained through fitting second harmonic Hall resistance with respect to the cosine of the azimuthal angle of magnetization, instead of the widely used transverse sweeping field.

Fig. 4-1 Orientation of magnetization under magnetic fields.

For a magnetic wire system with IMA, as depicted in Fig. 4-1, the magnetic energy E is given by Eq. (2-21) as:

$$E = (N_z M_s^2 - K_{\perp}) \sin^2 \theta - M_s H_z \sin \theta - M_s H_x \cos \theta \cos \varphi - M_s H_y \cos \theta \sin \varphi. \quad (4-1)$$

where $H_{(x,y,z)}$ are the effective magnetic fields along the three basic vector directions, inclusive of the externally applied magnetic fields ($H_{(x\text{-ext}, y\text{-ext}, z\text{-ext})}$) and the SOT fields (H_F

and H_D). For derivation simplicity, the expression $2(N_z M_s^2 - K_\perp)$ is written as $M_s H_\perp$, where $H_\perp$ represents the effective field that align the magnetization along the in-plane orientation. To obtain the relationship between the magnetization angles and magnetic fields $H_{(x,y,z)}$, partial derivatives of Eq. (4-1) with respect to the variables φ and θ are carried out,

$$\frac{\partial E}{\partial \theta} = M_s H_\perp \sin \theta \cos \theta - M_s H_z \cos \theta + M_s H_x \sin \theta \cos \varphi + M_s H_y \sin \theta \sin \varphi = 0, \quad (4-2)$$

$$\frac{\partial E}{\partial \varphi} = M_s H_x \cos \theta \sin \varphi - M_s H_y \cos \theta \cos \varphi = 0. \quad (4-3)$$

In order to simplify Eqs. (4-2) and (4-3), θ is assumed to be very small which results in $\cos \theta \approx 1$. Equations (4-2) and (4-3) are solved to obtain the equilibrium angles of φ and θ ,

$$\sin \theta = \frac{H_z}{H_\perp + H_x \cos \varphi + H_y \sin \varphi} \text{ and } \tan \varphi = \frac{H_y}{H_x}. \text{ In the regime where } H_\perp \gg H_{(x,y)}, \text{ the}$$

term $H_x \cos \varphi + H_y \sin \varphi$ in the expression of $\sin \theta$ can be ignored. The stable angles, φ_0 and θ_0 , can be determined by the externally applied fields $H_{(x\text{-ext}, y\text{-ext}, z\text{-ext})}$ as;

$$\sin \theta_0 = \frac{H_{z\text{-ext}}}{H_\perp}, \quad (4-4)$$

$$\tan \varphi_0 = \frac{H_{y\text{-ext}}}{H_{x\text{-ext}}}. \quad (4-5)$$

For IMA system, the SOT effective fields are given by: fieldlike term $\mathbf{H}_F = H_F \mathbf{y}$ and dampinglike term $\mathbf{H}_D = H_D \mathbf{m} \times \mathbf{y}$, where $\mathbf{m}$ is along in-plane direction and $\mathbf{y}$ represents $\mathbf{p}$ which is the in-plane unit vector pointing towards transverse to charge current direction

[92]. When an AC current with a low frequency ω , $I=I_0\sin\omega t$, is applied in the wire, the synchronous fieldlike term is $\mathbf{H}_F=(H_F\sin\omega t)\mathbf{y}$. The dampinglike term is $\mathbf{H}_D=(H_D\sin\omega t)\cos\varphi_0\mathbf{z}$, where $\cos\varphi_0\mathbf{z}$ arises from the expression $\mathbf{m}\times\mathbf{y}$. Due to the SOT effective fields, small modulations in the magnetization from the stable angles are induced, $\Delta\theta_0$ and $\Delta\varphi_0$. The modulations can be estimated through partial derivative of Eqs. (4-4) and

$$(4-5) \quad \text{with respect to } \varphi_0 \text{ and } \theta_0, \quad \Delta\theta_0 \approx \Delta \sin \theta_0 \approx \frac{1}{H_{\perp}} \Delta H_{z\text{-ext}},$$

$\Delta \tan \varphi_0 = \frac{\Delta\varphi_0}{\cos^2 \varphi_0} = \frac{1}{H_{x\text{-ext}}} \Delta H_{y\text{-ext}}$. As the effective fields, H_D and H_F act along the z and y axis respectively, $\Delta H_{z\text{-ext}}$ and $\Delta H_{y\text{-ext}}$ can be replaced with H_D and H_F . The modulations of magnetization angle induced by SOT effective fields is then given as,

$$\Delta\theta_0 = \frac{1}{H_{\perp}} H_D \cos \varphi_0 \sin \omega t, \quad (4-6)$$

$$\Delta\varphi_0 = \frac{\cos^2 \varphi_0}{H_{x\text{-ext}}} H_F \sin \omega t. \quad (4-7)$$

From the above discussion, the tilt angle of the magnetization with respect to the easy axis is determined by the externally applied fields and SOT effective fields, $\theta=\theta_0+\Delta\theta_0$ and $\varphi=\varphi_0+\Delta\varphi_0$.

The modulations of the magnetization angle are reflected as measurable changes in the harmonic Hall resistance. The total Hall resistance, R_{Hall} , of the wire consists of two components: AHE resistance, $R_A=R_{\text{AHE}}\sin\theta$, and PHE resistance, $R_P=R_{\text{PHE}}\cos^2\theta\sin 2\varphi$, where R_{AHE} and R_{PHE} are the amplitudes of AHE and PHE resistances [7, 12, 50, 51]. In the presence of an AC current, Hall voltage is given by

$V_{\text{Hall}} = R_{\text{AHE}} I_0 \sin \theta \sin \omega t + R_{\text{PHE}} I_0 \cos^2 \theta \sin 2\varphi \sin \omega t$. Substituting θ and φ with $\theta = \theta_0 + \Delta\theta_0$

and $\varphi = \varphi_0 + \Delta\varphi_0$ respectively into the expression of V_{Hall} gives

$$\begin{aligned} \frac{V_{\text{Hall}}}{I_0} = R_{\text{Hall}} = & R_{\text{AHE}} \sin(\theta_0 + \Delta\theta_0) \sin \omega t \\ & + R_{\text{PHE}} \cos^2(\theta_0 + \Delta\theta_0) \sin[2(\varphi_0 + \Delta\varphi_0)] \sin \omega t \end{aligned} \quad (4-8)$$

From Eq. (4-8), the SOT induced magnetization modulations, $\Delta\theta_0$ and $\Delta\varphi_0$, change the Hall resistance R_{Hall} .

To obtain a clear relationship between the SOT fields and R_{Hall} , Eq. (4-8) can be further simplified. In the absence of an external field acting along z -axis ($H_{z-\text{ext}} = 0$), θ_0 can be set to 0, as seen from Eq. (4-4). From the Pythagorean trigonometric identity and the assumption that $\Delta\theta_0$ and $\Delta\varphi_0$ are small, Eq. (4-8) is simplified to

$$R_{\text{Hall}} = R_{\text{AHE}} \Delta\theta_0 \sin \omega t + R_{\text{PHE}} (\sin 2\varphi_0 + 2\Delta\varphi_0 \cos 2\varphi_0) \sin \omega t. \quad (4-9)$$

According to Eqs. (4-4)-(4-7) and using the identity $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$, Eq. (4-9) can be re-written with the SOT induced effective fields being the variables,

$$\begin{aligned} R_{\text{Hall}} = & R_{\text{PHE}} \sin 2\varphi_0 \sin \omega t \\ & - R_{\text{AHE}} \frac{H_{\text{D}}}{2H_{\perp}} X \cos 2\omega t - R_{\text{PHE}} \frac{H_{\text{F}}}{H_{x-\text{ext}}} (2X^4 - X^2) \cos 2\omega t + C, \end{aligned} \quad (4-10)$$

where $X = \cos \varphi_0$, and C is a constant. The SOT effective fields H_{D} and H_{F} contribute only to the second harmonic Hall resistance amplitude which is given by

$$R_{2\text{ndHall}} = R_{\text{AHE}} \frac{H_{\text{D}}}{2H_{\perp}} X + R_{\text{PHE}} \frac{H_{\text{F}}}{H_{x-\text{ext}}} (2X^4 - X^2). \quad (4-11)$$

Experimentally, H_D and H_F can be extracted from Eq. (4-11) by keeping $H_{x\text{-ext}}$ constant and varying X by sweeping $H_{y\text{-ext}}$. When $H_{x\text{-ext}}$ is fixed, the coefficient $R_{\text{PHE}} \frac{H_F}{H_{x\text{-ext}}}$ of Eq. (4-11) is constant. By sweeping the transverse field $H_{y\text{-ext}}$, X becomes the only variable and can be calculated through Eq. (4-5). The value of X can also be validated by the first harmonic Hall resistance measurement $R_{1\text{stHall}} = R_{\text{PHE}} \sin 2\varphi_0$. Rewriting the expression of $R_{2\text{ndHall}}$, $R_{2\text{ndHall}} = aX + b(2X^4 - X^2)$ and fitting the measured $R_{2\text{ndHall}}$ with respect to the measured X by the above expression, the coefficients, a and b , can be computed. Thus, H_D and H_F can be calculated from the values of a and b as

$$H_D = \frac{2H_{\perp}}{R_{\text{AHE}}} a \text{ and } H_F = \frac{H_{x\text{-ext}}}{R_{\text{PHE}}} b. \quad (4-12)$$

In Eq. (4-12), the amplitude of planar Hall resistance R_{PHE} can be directly extracted from the first harmonic Hall resistance $R_{1\text{stHall}}$, while $\frac{H_{\perp}}{R_{\text{AHE}}}$ can be obtained through measuring

the AHE. From the expression of AHE and Eq. (4-4), when the system is magnetically saturated along the normal direction of the film plane, $\sin \theta$ is equal to 1 which results in

$$\frac{H_{z\text{-ext}}}{H_{\perp}} = 1. \text{ Thus, } \frac{H_{\perp}}{R_{\text{AHE}}} = \frac{H_{z\text{-ext,satur}}}{R_{\text{AHE}}}, \text{ where } H_{z\text{-ext,satur}} \text{ is the out-of-plane saturation field.}$$

4.2. SOT fields in Ta(6 nm)/Co(2 nm)/Pt(5 nm)

To validate the proposed technique to obtain SOT effective fields using a single measurement, the structured samples Ta(t nm)/Co(2 nm)/Pt(5 nm) with IMA were used. The measured AHE resistance, with respect to the out-of-plane field, is shown in Fig. 3-8.

From $R_A = R_{\text{AHE}} \sin \theta = R_{\text{AHE}} \frac{H_{z\text{-ext}}}{H_{\perp}}$, for the stack Ta(6 nm)/Co(2 nm)/Pt(5 nm), the magnetic moments rotate from in plane at $H_{z\text{-ext}}=0$ Oe to out of plane at $H_{z\text{-ext}}=5180$ Oe. Thus, the effective anisotropy field of the wire is $H_{\perp}=5180$ Oe. The first and second harmonic Hall resistances for extracting the SOT effective fields were measured simultaneously using a single 7265 DSP lock-in amplifier connected across the Hall bars. A constant longitudinal field of $H_{x\text{-ext}}=500$ Oe is applied by a permanent magnet along the wire direction to ensure uniform magnetization along the FM wire long axis, while a transverse field of $H_{y\text{-ext}}$ is swept from -2000 Oe to $+2000$ Oe by a electromagnet powered by KIKUSUI Bipolar Power Supply PBZ20-20. AC current with frequency 307.1 Hz and current density amplitudes ranging from $3 \times 10^{10} \text{Am}^{-2}$ to $1 \times 10^{11} \text{Am}^{-2}$ in steps of $1 \times 10^{10} \text{Am}^{-2}$ were applied across the wire by KEITHLEY 6221 DC and AC Current Source. The value of AC frequency results in an AC periodicity in the order of several milliseconds, which indicates that the varying period of SOT effective fields is in the order of millisecond. This period is much larger than the magnetization relaxation time which is in the order of nanoseconds as discussed in Section 2.4. Therefore, the magnetization oscillation is in-phase with the AC current. This allows us to characterize the SOT effective field by the proposed harmonic Hall resistance measurement. By taking the ratio of the Hall voltages over the amplitudes of the AC current, the first and second harmonic Hall resistances can be obtained.

The dependence of the harmonic Hall resistances on the azimuthal angle φ_0 , cosine X and constant field $H_{x\text{-ext}}$, are studied. The Hall resistances measured at the current density of $1 \times 10^{11} \text{Am}^{-2}$, are shown in Fig. 4-2. The angle φ_0 is calculated simultaneously by Eq. (4-

5), $\tan \varphi_0 = \frac{H_{y-\text{ext}}}{H_{x-\text{ext}}}$, while the transverse field $H_{y-\text{ext}}$ is swept. From Fig. 4-2(a), we note that

the first harmonic Hall resistance $R_{1\text{stHall}}$ measured with a fixed field $H_{x-\text{ext}}=+500$ Oe shows a typical sinusoidal trend with respect to the azimuthal angle of φ_0 . For $H_{x-\text{ext}}=-500$ Oe, where the angle sweeps from $+\varphi_0$ to $-\varphi_0$, a similar trend is observed. By fitting the curves with the expression of $R_{1\text{stHall}}=R_{\text{PHE}}\sin 2\varphi_0$, we obtain $R_{\text{PHE}}=5.1$ m Ω . The corresponding second harmonic Hall resistances are shown in Fig. 4-2(b). For $H_{x-\text{ext}}=+500$ Oe the resistance increases with increasing X . This is consistent with the prediction of Eq. (4-11)

$$R_{2\text{ndHall}} = R_{\text{AHE}} \frac{H_{\text{D}}}{2H_{\perp}} X + R_{\text{PHE}} \frac{H_{\text{F}}}{H_{x-\text{ext}}} (2X^4 - X^2). \text{ The expression for the dampinglike term}$$

is $\mathbf{H}_{\text{D}}=H_{\text{D}}\mathbf{m}\times\mathbf{y}$. By reversing the direction of magnetization $\mathbf{m}$, the sign of X should also change. To verify this, we measured the resistance $R_{2\text{ndHall}}$ at $H_{x-\text{ext}}=-500$ Oe as shown in Fig. 4-2(b). The resistance is symmetric with respect to the zero resistance axis, as compared to the $R_{2\text{ndHall}}$ at $H_{x-\text{ext}}=+500$ Oe. This is consistent with the prediction of the

expression of $R_{2\text{ndHall}}$. The term $R_{\text{PHE}} \frac{H_{\text{F}}}{H_{x-\text{ext}}} (2X^4 - X^2)$ becomes negative with the

negative sign of $H_{x-\text{ext}}$, as the sign of H_{F} is independent of the direction of $\mathbf{m}$. The AHE

component $R_{\text{AHE}} \frac{H_{\text{D}}}{2H_{\perp}} X$ becomes negative as well, since the sign of X changes when the

direction of magnetization is changed. Thus, $R_{2\text{ndHall}}$ at $-H_{x-\text{ext}}$ is symmetric with $R_{2\text{ndHall}}$ at $+H_{x-\text{ext}}$. In summary, the SOT effective fields, H_{F} and H_{D} , guarantee the existence of these dependencies.

Fig. 4-2 The first harmonic Hall resistances (a) and the second harmonic Hall resistances (b) under two constant fields $H_{x\text{-ext}}$ with opposite directions. The applied current density is $1 \times 10^{11} \text{ Am}^{-2}$. For (a), both curves show sine function with respect to the azimuthal angle of magnetization; the inset shows the resistance with the external fields $H_{y\text{-ext}}$, which corresponds to the angle. For (b), the resistance at $+H_{x\text{-ext}}$ is approximately symmetric to that at $-H_{x\text{-ext}}$ about 0 Ohm; the inset represents the second harmonic Hall resistance with respect to the external field $H_{y\text{-ext}}$ which corresponds to the cosine of azimuthal angle. The solid line is a fit to the experimental data.

Fig. 4-3 The second harmonic Hall resistances under two constant fields $H_{x\text{-ext}}$ with opposite directions and different current densities. The current density starts from 3×10^{10} to $1 \times 10^{11} \text{ Am}^{-2}$ with a step size $1 \times 10^{10} \text{ Am}^{-2}$. The solid line is a fit to the experimental data.

Through fitting the second harmonic Hall resistances with the cosine of azimuthal angle X with Eq. (4-11), the SOT effective fields are calculated for each applied current density in the wire. Figure 4-3 shows the second harmonic Hall resistances as the current density is varied from $3 \times 10^{10} \text{ Am}^{-2}$ to $1 \times 10^{11} \text{ Am}^{-2}$. The resistance with respect to X exhibits similar dependencies as in Fig. 4-2(b), where the current density is $1 \times 10^{11} \text{ Am}^{-2}$. Hence, the SOT effective fields contribute to the dependencies for each case of current density, $3 \times 10^{10} \text{ Am}^{-2}$ to $1 \times 10^{11} \text{ Am}^{-2}$. The absolute values of these resistances increase with increasing current density. $R_{2\text{ndHall}}$ increases from $\sim 40 \mu\Omega$ at $3 \times 10^{10} \text{ Am}^{-2}$ to $\sim 140 \mu\Omega$ at $1 \times 10^{11} \text{ Am}^{-2}$, when $H_{x\text{-ext}}$ is aligned along the $-x$ direction and $X=1$. Comparing the experimental result of $R_{2\text{ndHall}}$ with its analytical expression of Eq. (4-11),

$$R_{2\text{ndHall}} = R_{\text{AHE}} \frac{H_{\text{D}}}{2H_{\perp}} X + R_{\text{PHE}} \frac{H_{\text{F}}}{H_{x\text{-ext}}} (2X^4 - X^2),$$

we conclude that this increasing trend represents the increase of H_{D} or H_{F} with respect to current densities. For the sample, Ta(6 nm)/Co(2 nm)/Pt(5 nm), R_{AHE} is $24.4 \text{ m}\Omega$ and $H_{\perp}$ is 5180 Oe (shown in Fig. 3-15), R_{PHE} equals to $5.1 \text{ m}\Omega$, and $H_{x\text{-ext}}$ is $\pm 500 \text{ Oe}$. Taking these parameters into Eq. (4-11) to fit the curves in Fig. 4-2(b) and Fig. 4-3, H_{F} and H_{D} are calculated. Figure 4-4(a) shows the calculated H_{F} . The fieldlike term increases from $\sim 2 \text{ Oe}$ at $3 \times 10^{10} \text{ Am}^{-2}$ to $\sim 7 \text{ Oe}$ at $1 \times 10^{11} \text{ Am}^{-2}$, giving a ratio of H_{F} to current density of $7 \text{ Oe per } 10^{11} \text{ Am}^{-2}$. The sign of H_{F} does not change to negative when the longitudinal $H_{x\text{-ext}}$ is reversed, which is consistent with the expression of $\mathbf{H}_{\text{F}} = H_{\text{F}} \mathbf{y}$. Figure 4-4(b) shows the calculated H_{D} . The absolute value of the dampinglike term increases from $\sim 14 \text{ Oe}$ at $3 \times 10^{10} \text{ Am}^{-2}$ to $\sim 45 \text{ Oe}$ at $1 \times 10^{11} \text{ Am}^{-2}$. The ratio of H_{D} to current density is $44 \text{ Oe per } 10^{11} \text{ Am}^{-2}$. H_{D} becomes negative when the longitudinal field $H_{x\text{-ext}}$ is reversed. This is consistent with the expression of $\mathbf{H}_{\text{D}} = H_{\text{D}} \mathbf{m} \times \mathbf{y}$. Meanwhile, the values of the two ratios (H_{D} and H_{F} to current density) are in the order of

those reported in a similar stack [82], which validates the proposed method.

Fig. 4-4 Amplitudes of fieldlike term (a) and dampinglike term (b) with respect to the applied current density. The dampinglike term changes its sign when $H_{x\text{-ext}}$ is reversed whereas the fieldlike term does not change its direction. The slopes of fieldlike term and dampinglike term over the current density are $\sim 7 \text{ Oe per } 10^{11} \text{ Am}^{-2}$ and $\sim 44 \text{ Oe per } 10^{11} \text{ Am}^{-2}$. The error bars indicate the uncertainty in fitting the Eq. (4-11).

4.3. Dependence of SOT effective fields on Ta thickness

To study the dependence of SOT fields on the thickness of Ta, the measurements were repeated in Ta(t nm)/Co(2 nm)/Pt(5 nm), where $t = 2, 4, 8$ and 10. The IMA property of all the samples are confirmed by measurement of longitudinal MOKE and AHE, as shown in Fig. 3-3 and Fig. 3-15. R_{AHE} and H_{L} are obtained by measuring AHE as shown in the Table 3-1. Similar experimental conditions as for Ta(6 nm)/Co(2 nm)/Pt(5 nm) were used to quantify the SOT effective fields. In Fig. 4-5, the first harmonic Hall resistances are presented for the four samples investigated. For each of the samples, the first harmonic Hall resistances show the typical sinusoidal trend with respect to the azimuthal angle of φ_0 . The R_{PHE} values can be extracted from the Fig. 4-5, as shown in the Table 4-1. In Fig. 4-6, the second harmonic Hall resistances are presented. The second harmonic Hall resistances $R_{2\text{ndHall}}$ increase with increasing X . The first and second harmonic Hall resistances of the three samples present the similar behavior as predicted by the expression of $R_{1\text{stHall}}$ and $R_{2\text{ndHall}}$, hence, the SOT effective fields can be extracted by the proposed method.

Fig. 4-5 The first harmonic resistance with respect to the azimuthal angle (a) and applied transverse field (b) for the samples Ta(t nm)/Co(2 nm)/Pt(5 nm), where t indicates the thickness of Ta.

Table 4-1 The obtained R_{PHE} in the samples Ta(t nm)/Co(2 nm)/Pt(5 nm)

t (nm)	2	4	6	8	10
R_{PHE} (m Ω)	7.5	5.6	5.1	5.7	6.1

Fig. 4-6 The second harmonic Hall resistances under different current densities starting from 3×10^{10} to $1 \times 10^{11} \text{ Am}^{-2}$ with a step size $1 \times 10^{10} \text{ Am}^{-2}$ for the samples Ta(t nm)/Co(2 nm)/Pt(5 nm). The solid line is a fit to the experimental data.

The SOT effective fields are characterized in the four samples with different thicknesses of Ta. Figure 4-7 shows the SOT effective fields for each of the samples. For comparison, the SOT effective fields of Ta(6 nm)/Co(2 nm)/Pt(5 nm) are included. Both H_F (Fig. 4-7(a)) and H_D (Fig. 4-7(b)) show general increasing trends with respect to the thickness for all

current densities. The trends are consistent with that reported in stacks with PMA [111, 115, 125]. It is proposed that the amount of current flowing in the Ta layer increases as t is increased, as such, the SOT effective fields increase with t . This point is also applicable in our case, as the resistance of the patterned wires reduces with increasing the thickness of Ta as shown in Fig. 4-8. However, from the sample with 8 nm to that with 10 nm, H_F keeps approximately constant, while H_D shows significant increases, compared to the H_D values with thinner Ta thickness. H_F at the current density of $1 \times 10^{11} \text{ Am}^{-2}$, for instance, keeps $\sim 6.7 \text{ Oe}$; while the change rate of H_F to t is $\sim 4 \text{ Oe} \cdot \text{nm}^{-1}$ for $t=6$ and $t=8$, increases to $\sim 11 \text{ Oe} \cdot \text{nm}^{-1}$ for $t=8$ and $t=10$. This abnormal behavior is due to the dependence of SOT fields on the saturation magnetization of Co layers, which will be discussed in Chapter 5. As shown in the inset of Fig. 4-7(b), the saturation magnetization exhibits a significant increase from $t=8$ to $t=10$. This implies the dampinglike term increases while the fieldlike term decreases with respect to the magnetization of Co layers. The behaviors can also be observed in samples with $t=2$, $t=4$ and $t=6$. A larger decrease in magnetization is found from $t=2$ to $t=4$ compared to the decrease from $t=4$ to $t=6$. As such, in addition to the influence from the thickness dependence, the increase in H_F for $t=2$ to $t=4$ is more significant than $t=4$ to $t=6$, while H_D shows insignificant increase. The magnetization dependence is consistent with the expectation that the SOT effective fields are likely related to the efficiency of spin diffusion across the Ta/Co interface instead of an intrinsic property [82, 115], as the asymmetric spin scattering can be enhanced by increasing the magnetization of magnetic film [199]. In conclusion, the dependence of the effective SOT fields are ascribed to the increasing of current flowing in the Ta layer and the change of efficiency of spin diffusion across the Ta/Co interface.

Fig. 4-7 Amplitudes of fieldlike term (a) and dampinglike term (b) with respect to the thickness of Ta for different current densities.

Fig. 4-8 The resistance and corresponding resistivity of patterned wires with different Ta film thicknesses.

4.4. Spin Hall angles of Ta/Co/Pt stacks

The effective spin Hall angles are estimated to enable comparison with other methodologies. The spin Hall angle, θ_{SH} , in our film stack is computed by using the following expression as $\theta_{\text{SH}} = \frac{2eM_s t_F}{h} \frac{H_D}{j}$ [88, 90, 94, 112], where e is the electron charge, $\hbar$ is the reduced Planck constant and t_F is the thickness of Co layer. Here, j is the average current density applied in the stack. The calculated θ_{SH} in our film stack, as a function of Ta film thicknesses is plotted in Fig. 4-9. For Ta thickness of $t \leq 6$ nm, θ_{SH} is computed to ~ 0.11 . As the bottom Ta layer in our film stack was used as a seed layer, the interaction with SiO_2 on the substrate leads to the oxidation of the first few atomic layers of Ta at the interface [200, 201]. As such, for small Ta thicknesses, the SHE is mainly due to the Pt layer, and the estimated value is slightly larger than the reported θ_{SH} of Pt. The θ_{SH} of Pt has been reported to be ~ 0.07 as measured by ST-FMR [79], 0.06 ± 0.02 as measured by SOT induced magnetization switching [78], and 0.06 measured by current induced domain-wall motion [124]. For $t > 6$ nm, θ_{SH} increases from 0.15, reaching a maximum of 0.25 for $t = 10$ nm. This is attributed to the contribution of SOT from the Ta film. The percentage of the β -phase Ta, which is another kind of structure compared to the normal body-centered cubic Ta and has been reported to have a large θ_{SH} , increases with the thickness of Ta [79, 82, 115]. Additionally, the growth of Ta on SiO_2 has been reported to promote the Ta β -phase [202, 203]. The resistivity of β -phase Ta is around five times that of body-centered cubic Ta [204]. To ascertain the presence of β -phase Ta in our film stack, the resistance of the film stack was monitored as described by Ref. [82]. As shown in Fig. 4-8, the wire resistance does not reduce significantly with increasing t , which suggests that the fraction

of β -phase Ta increases in the film stacks [82]. The increase of wire resistivity further confirms that the fraction of β -phase increases, as it is known to have a higher resistivity. The largest spin Hall angle in our film stack is 0.25 and was obtained in the stack comprising of Ta(10 nm)/Co(2 nm)/Pt(5 nm). Considering that effective spin Hall angle is equal to individual contribution from both Ta and Pt, the spin Hall angle of Ta is calculated to be $(0.25-0.07=)$ 0.18, where θ_{SH} of Pt is chosen as ~ 0.07 . This value is less than the reported value ~ 0.12 as measured by ST-FMR [79, 115]. For the stack of Ta(8 nm)/Co(2 nm)/Pt(5 nm), the effective spin Hall angle is ~ 0.15 , and the spin Hall angle contribution of Ta is calculated to be 0.08. This value is larger than ~ 0.025 as measured by other harmonic Hall voltage technique [205], ~ 0.02 measured by ST-FMR [110]. The two different values of spin Hall angle for Ta as measured in our film stacks suggest that the percentage of β -phase Ta can be a contributing factor in different spin Hall angles of Ta.

Fig. 4-9 The computed effective spin Hall angles of the Ta/Co/Pt stacks as a function of Ta film thicknesses.

v. Dependence of SOT effective fields on magnetization magnitude

Here, the dependence of the SOT effective fields on the magnitude of magnetization in the Ta/Co/Pt wires with IMA is demonstrated. The impact of HM and FM layers thickness dependence of SOT of the HM and FM [82, 95, 97, 125] is eliminated, as varying the magnetization and characterizing the SOT effective fields were carried out concurrently in each wire. The variation of the magnetization magnitude of the wire was achieved by varying the applied longitudinal fields along the wire long axis, while the amplitude of PHE resistance amplitude was measured simultaneously. The magnetization magnitude increases with respect to the longitudinal field, and the increase is ascribed to the polycrystalline of the wire. Experimental results show that the fieldlike term decreases with respect to the magnetization whereas the dampinglike term increases. It is proposed that the magnetization magnitude increase leads to an increase of electron diffusion constant to decrease and increase the fieldlike and dampinglike terms respectively.

5.1. Characterization of SOT effective fields and PHE resistance

Harmonic Hall resistance measurement technique which was proposed in Chapter 4 is employed. The measurements were carried out in the wires with stacks of Ta(t nm)/Co(2 nm)/Pt(5 nm). The SOT fields were firstly quantified as a function of the longitudinal fields in the sample of Ta(4 nm)/Co(2 nm)/Pt(5 nm). In this quantification, the constant longitudinal field $H_{x\text{-ext}}$ was applied in a range of 250 Oe to 650 Oe in a 50 Oe increment. For each value of $H_{x\text{-ext}}$, the ratio of the maximum value of the sweeping field $H_{y\text{-ext}}$ to $H_{x\text{-ext}}$ was fixed. The AC frequency was set at 307.1 Hz for fulfilling the low frequency requirement of the harmonic Hall resistance measurements. The amplitudes of the AC current were in the range $3 \times 10^{10} \sim 10 \times 10^{11} \text{ Am}^{-2}$ in increment of 10^{10} Am^{-2} , and the Joule heat induced by electric current in this range is expected to be negligible. The first and second harmonic Hall resistances were calculated by the quotient of the first and second harmonic Hall voltages, which were measured by using a 7265 DSP lock-in amplifier, over the amplitudes of AC. The measured harmonic Hall resistance at the current density $1 \times 10^{11} \text{ Am}^{-2}$ and the applied fields $H_{x\text{-ext}}$ with different values are shown in Fig. 5-1. As shown in Fig. 5-1(a), the measured first harmonic Hall resistances $R_{1\text{stHall}}$ exhibit typical $\sin 2\varphi_0$ behaviors as functions of the azimuthal angle φ_0 , and the minimum and maximum values of $R_{1\text{stHall}}$ are at $\varphi_0 = \pm 45$ degrees. Correspondingly, for each value of $H_{x\text{-ext}}$ shown in Fig. 5-1(b), the minimum and maximum values of $R_{1\text{stHall}}$ occur at $H_{x\text{-ext}} = \pm H_{y\text{-ext}}$, which give the values of R_{PHE} and the ratio of $\frac{R_{\text{PHE}}}{H_{x\text{-ext}}}$. R_{PHE} with respect to $H_{x\text{-ext}}$ is plotted in Fig. 5-1(c) where the range of $H_{x\text{-ext}}$ was extended to 100 Oe.

Fig. 5-1 The measured first harmonic resistances $R_{1stHall}$ with respect to (a) the azimuthal angle ϕ_0 of magnetization and (b) the applied transverse field H_{y-ext} . (c) R_{PHE} with respect to the longitudinal field, which are obtained by experiments and fitting, and the calculated magnetization M with respect to the longitudinal field.

The measured second harmonic Hall resistances at the current density $1 \times 10^{11} \text{ Am}^{-2}$ and the applied fields H_{x-ext} with value ranging from 250 Oe to 650 Oe are shown in Fig. 5-2. In Fig. 5-2(a), the measured second harmonic Hall resistances, $R_{2ndHall}$, are shown to increase

with increasing X for each value of $H_{x\text{-ext}}$. Substituting the values of R_{PHE} and $\frac{R_{\text{PHE}}}{H_{x\text{-ext}}}$ from Fig. 5-1(c) into Eq. (4-11) to fit the experimental $R_{2\text{ndHall}}$, we obtain the two SOT effective fields, H_{F} and H_{D} shown in Fig. 5-3. For each value of $H_{x\text{-ext}}$, the fieldlike and the dampinglike terms increase with the strength of applied current density, which follow a linear relationship between SOT effective field and current density. The values of SOT effective fields are in the order of those reported in Chapter 4. However, the measured SOT effective fields vary with the longitudinal field for each value of the applied current densities, *e.g.*, the fieldlike term H_{F} decreases from ~ 6 Oe at $H_{x\text{-ext}} = 250$ Oe to ~ 5 Oe at $H_{x\text{-ext}} = 650$ Oe for $1 \times 10^{11} \text{ Am}^{-2}$; similarly, H_{D} increases from ~ 34 Oe at $H_{x\text{-ext}} = 250$ Oe to ~ 39 Oe at $H_{x\text{-ext}} = 650$ Oe. Such variations are in contrast to the conventional understanding that fieldlike and dampinglike terms are constant with respect to the magnitude of magnetization. Similar characterizations of the SOT effective fields are carried out for samples $\text{Ta}(t \text{ nm})/\text{Co}(2 \text{ nm})/\text{Pt}(5 \text{ nm})$, where $t = 6, 8$ and 10 , as shown in Figs. 5-4~5-6.

Fig. 5-2 The measured second harmonic Hall resistances $R_{2\text{ndHall}}$ with respect to (a) the cosine X of the azimuthal angle and (b) the applied transverse field $H_{y\text{-ext}}$.

Fig. 5-3 The measured fieldlike term (a), and dampinglike term (b), with respect to the longitudinal field for each value of applied current densities.

Fig. 5-4 The measured second harmonic Hall resistances with respect to (a) X and (b) the transverse magnetic field for the sample Ta(6 nm)/Co(2 nm)/Pt(5 nm). The obtained fieldlike term (c) and dampinglike term (d) from (a).

Fig. 5-5 The measured second harmonic Hall resistances with respect to (a) X and (b) the transverse magnetic field for the sample Ta(8 nm)/Co(2 nm)/Pt(5 nm). The obtained fieldlike term (c) and dampinglike term (d) from (a).

Fig. 5-6 The measured second harmonic Hall resistances with respect to (a) X and (b) the transverse magnetic field for the sample Ta(10 nm)/Co(2 nm)/Pt(5 nm). The obtained fieldlike term (c) and dampinglike term (d) from (a).

5.2. Polycrystallinity and magnetization

The variation of the magnetization with respect to the applied longitudinal field was characterized by measuring the amplitude of the first harmonic or planar Hall resistance, since the resistance R_{PHE} is a linear function of M^2 , *i.e.*, $R_{\text{PHE}}=kM^2$, where k is a material related coefficient [206, 207]. The measured R_{PHE} increases with respect to $H_{x\text{-ext}}$, as shown in Fig. 5-1(c). The origin of this observation can be ascribed to the polycrystalline structure of the sputtered Ta/Co/Pt film. Without applying $H_{x\text{-ext}}$, the magnetic moments of crystalline grains orientate randomly due to the effective field H_{cry} of crystalline magnetic anisotropy in the film. Hence, the magnitude of the magnetization equals to the value of remanence magnetization which is due to the demagnetizing field transverse to the wire and the longitudinal component $H_{L,\text{cry}}$ of H_{cry} . However, when $H_{x\text{-ext}}$ is applied, magnetic moments of the grains re-orientate towards the x -axis, consequently, M increases, as schematically shown in Fig. 5-7. As such, due to $R_{\text{PHE}}=kM^2$, R_{PHE} is increased by $H_{x\text{-ext}}$. We

suggest that M is related to $H_{x\text{-ext}}$ through the equation of $M = M_r + \frac{H_{x\text{-ext}}}{\sqrt{H_{x\text{-ext}}^2 + H_{T,\text{crys}}^2}} M_H$,

where M_r is the remanence magnetization, and $\frac{H_{x\text{-ext}}}{\sqrt{H_{x\text{-ext}}^2 + H_{T,\text{crys}}^2}} M_H$ is the magnetization

which can be manipulated by $H_{x\text{-ext}}$. In the equation of M , $H_{T,\text{crys}}$ is the transverse component of H_{cry} , and M_r+M_H is equal to the saturation magnetization M_s of the wires.

Substituting M expression into the equation of R_{PHE} , we obtain

$$R_{\text{PHE}} = k \left(M_r + \frac{H_{x\text{-ext}}}{\sqrt{H_{x\text{-ext}}^2 + H_{T,\text{crys}}^2}} M_H \right)^2, \text{ which is used to fit the measured } R_{\text{PHE}} \text{ with}$$

considering $M_r+M_H=466$ emu/cc, as shown in Fig. 5-1(c). The derived function fits the

measurement data well suggesting a good match of our explanation. From the fitting, the

relationship between M and H_{x-ext} is obtained as $M = 405 + \frac{H_{x-ext}}{\sqrt{H_{x-ext}^2 + 165^2}} 61$ (emu/cc).

Similar characterizations of $R_{1stHall}$, R_{PHE} , $R_{2ndHall}$ and M are carried out for samples Ta(t nm)/Co(2 nm)/Pt(5 nm), as shown in Figs. 5-8, 5-9 and 5-10.

Fig. 5-7 Schematic of a polycrystalline magnetic structure and the orientation of magnetic moment for each of crystalline grain under H_{x-ext} and transverse demagnetizing field.

Fig. 5-8 The measured first harmonic Hall resistances with respect to (a) the azimuthal angle of magnetization, and (b) the transverse magnetic field for the sample Ta(6 nm)/Co(2 nm)/Pt(5 nm).

(c) Plots of the amplitudes of PHE obtained from (a) and corresponding magnetization with respect to the longitudinal magnetic field.

Fig. 5-9 The measured first harmonic Hall resistances with respect to (a) the azimuthal angle of magnetization, and (b) the transverse magnetic field for the sample Ta(8 nm)/Co(2 nm)/Pt(5 nm).

(c) Plots of the amplitudes of PHE obtained from (a) and corresponding magnetization with respect to the longitudinal magnetic field.

Fig. 5-10 The measured first harmonic Hall resistances with respect to (a) the azimuthal angle of magnetization, and (b) the transverse magnetic field for the sample Ta(10 nm)/Co(2 nm)/Pt(5 nm). (c) Plots of the amplitudes of PHE obtained from (a) and corresponding magnetization with respect to the longitudinal magnetic field.

The SOT effective fields per 10^{11} Am^{-2} at each value of $H_{x\text{-ext}}$ were obtained from Fig. 5-3 ~ 5-6, for comparison. Replacing $H_{x\text{-ext}}$ with the corresponding value of M in Fig. 5-1(c), Fig. 5-8(c), Fig. 5-9(c) as well as Fig. 5-10(c), the SOT effective fields per 10^{11} Am^{-2} with respect to M are plotted for sample Ta(t nm)/Co(2 nm)/Pt(5 nm). As shown in Fig. 5-11, the fieldlike term decreases with respect to the magnitude of magnetization while the dampinglike term increases.

Fig. 5-11 The measured fieldlike term (a), and dampinglike term (b), with respect to the calculated magnetization M for Ta(t nm)/Co(2 nm)/Pt(5 nm), where $t = 4, 6, 8$ and 10 .

5.3. Dependence attributed to SHE

The second picture of SOT, which is discussed in Chapter 2, is applied to explain the dependence of the SOT on magnetization magnitude. SOT is analogized to the STT. In the STT model proposed by S. Zhang [53], due to GMR effect, the spin current, which is from a reference layer, leads to a transverse spin accumulation in a free layer. Consequently, the transverse spin accumulation induces two effective fields: $b\mathbf{m}_r$ and $a\mathbf{m}_f \times \mathbf{m}_r$, where $\mathbf{m}_r$ and $\mathbf{m}_f$ are unit vectors of the local magnetization of the reference layer and the free layer, respectively. When $\mathbf{m}_r$ and $\mathbf{m}_f$ are in the plane of the magnetic layers, b and a are expressed as $b = \frac{hj_e}{eM_s t_F} (\sin \xi e^{-\xi})$ and $a = \frac{hj_e}{eM_s t_F} (1 - \cos \xi e^{-\xi})$, respectively, where h is the Planck constant, j_e is the electric current density perpendicular to the plane of magnetic layers, t_F is the thickness of the free layer, and e is the electron charge. In the expressions of a and b , ξ equals to $\frac{t_F}{\sqrt{2}\lambda_J}$ with a spin diffusion length of $\lambda_J = \sqrt{\frac{2hD_0}{J}}$, where J is a coefficient of the contact interaction between the spin accumulation and the local magnetization of the free layer, and D_0 is the electron diffusion constant. Analogously in the Ta/Co/Pt structure, the Ta or Pt layer is used to generate spin current normal to the magnetic Co layer. Hence, the Ta or Pt layer is similar to the reference layer, as such, $\mathbf{y}$ can be considered as $\mathbf{m}_r$. The spins generated by the Ta and Pt layers are accumulated at the Co layer, and that allows us to take the Co layer as analogous to the free layer, similarly, $\mathbf{m}$ is to $\mathbf{m}_f$. Consequently, the fieldlike term, $\mathbf{H}_F = H_F \mathbf{y}$ is equivalent to $b\mathbf{m}_r$, and the dampinglike term, $\mathbf{H}_D = H_D \mathbf{m} \times \mathbf{y}$ is equivalent to $a\mathbf{m}_f \times \mathbf{m}_r$. Thus, we obtain $H_F = \frac{hj_e}{eM_s t_F} (\sin \xi e^{-\xi})$ and $H_D = \frac{hj_e}{eM_s t_F} (1 - \cos \xi e^{-\xi})$ for the Ta/Co/Pt samples, where j_e is the charge current.

We propose the magnetization magnitude of Co layer changes the coefficient of ξ to manipulate the SOT effective fields, considering the above expressions of H_F and H_D . The dampinglike term is related to spin Hall angle θ_{SH} via the expression $H_D = \frac{hj_e}{eM_s t_F} \theta_{SH}$, where θ_{SH} is defined as the ratio of spin current j_s to charge current j_e [88, 90, 94, 112]. Comparing the two expressions of H_D , we obtain $\theta_{SH} = 1 - \cos \xi e^{-\xi}$. As such, ξ is ≤ 1.6 , since the sum of spin Hall angle for Pt and Ta is not larger than 1 [82, 208]. As $\xi = \frac{t_F}{\sqrt{2}\lambda_J}$ and λ_J is about 1.2~2.4 nm for Co [53], we obtain $\xi \geq 0.6$, using the Co layer thickness as $t_F = 2\text{nm}$. Therefore, our samples have values of $0.6 \leq \xi \leq 1.6$. ξ can be rewritten as $\xi = \frac{t_F}{2} \sqrt{\frac{J}{hD_0}}$, where D_0 is related to the magnetization of the wire [53, 199]. Ustinov creates a superlattice model to explain the correlation of D_0 -related MR and magnetization [199]. In this model, the superlattice comprises of several magnetic layers, for any of two neighboring layers, magnetizations are initially antiparallel to each other. A transverse magnetic field, which is perpendicular to the initial magnetization in the plane of the magnetic layers, is applied to change the magnetization amplitude of the superlattice. The model concludes that the MR increases with respect to the magnetization for the superlattice. Hence, D_0 decreases with increasing magnetization in our samples, as it is inverse proportional to MR. Therefore, ξ increases with respect to the magnetization magnitude, as $\xi = \frac{t_F}{2} \sqrt{\frac{J}{hD_0}}$. In the range of 0.6~1.6 for our samples, the increasing of ξ lead to the decreasing of the term $\sin \xi e^{-\xi}$ and increasing of the term $1 - \cos \xi e^{-\xi}$, as shown

in Fig. 5-12. Consequently, in our SOT samples, $H_F = \frac{hj_d}{eM_s t_F} (\sin \xi e^{-\xi})$ and

$H_D = \frac{hj_d}{eM_s t_F} (1 - \cos \xi e^{-\xi})$ decreases and increases with respect to the magnetization,

respectively, as the term $\frac{hj_e}{eM_s t_F}$ is a constant for each sample.

Fig. 5-12 Plots of $\sin \xi e^{-\xi}$ and $1 - \cos \xi e^{-\xi}$ with respect to ξ , where $\sin \xi e^{-\xi}$, $1 - \cos \xi e^{-\xi}$ and ξ correspond to the fieldlike term H_F , the dampinglike term H_D and the magnetization M , respectively.

vi. Dependence of SOT effective fields on magnetization azimuthal angle

This work presents a systematic investigation of the dependence of H_D and H_F on the azimuthal angle of magnetization in the Ta/Co/Pt stack with IMA. By leveraging on our concurrent measurement scheme proposed in Chapter 4, whereby H_D and H_F can be simultaneously extracted, a harmonic Hall resistance measurement which enables the quantization of the effective fields as a function of the azimuthal angle is proposed. For each value of the angle, a magnetic field which orientates the magnetization is swept, and the measured second harmonic Hall resistance as a function of the magnetic field gives the values of H_D and H_F . Experimental results show that the dampinglike term is independent on the azimuthal angle. However, the fieldlike term comprises of a constant component and an azimuthal-angle-dependent component. The two components quantify the contributions of the Rashba effect and SHE to the fieldlike term.

6.1. Revised method for characterizing SOT effective fields

I have previously shown that the fieldlike term and dampinglike term can be extracted simultaneously for samples with IMA in Chapter 4. Starting from the total magnetic energy of a magnetic system with IMA, the modulations of the magnetization angles induced by SOT fields are introduced. Figure 6-1 shows a schematic the structure being measured with information of the magnetization and applied field orientation. An in-plane magnetic field, H_{app} , is applied with an azimuthal angle φ_H with respect to the x -axis. With the saturation magnetization of the wire being M_s , the magnetic energy E is given by

$$E = \frac{1}{2} M_s H_{\perp} \sin^2 \theta - M_s H_z \sin \theta - M_s H_{\text{app}} \cos \theta \cos(\varphi_H - \varphi), \quad (6-1)$$

where $H_{\perp}$ represents the effective field that aligns the magnetization in film plane. In Eq. (6-1), $M_s H_{\perp} = 2(N_z M_s^2 - K_{\perp})$ is positive for IMA (Chapter 4). Partial derivatives of Eq. (6-1) with respect to the variables φ and θ leads to the relationship between the stable angles of the magnetization and the magnetic field H_{app} ,

$$\frac{\partial E}{\partial \varphi} = -M_s H_{\text{app}} \sin(\varphi_H - \varphi) = 0, \quad (6-2)$$

$$\frac{\partial E}{\partial \theta} = H_{\perp} \sin \theta - H_z + H_{\text{app}} \sin \theta \cos(\varphi_H - \varphi) = 0. \quad (6-3)$$

Solution of Eq. (6-2) indicates

$$\varphi_H = \varphi_0. \quad (6-4)$$

As such, M_s is along the direction of H_{app} in the film plane. Consequently, solving Eq. (6-

3) gives $\sin \theta_0 = \frac{H_z}{H_{\perp} + H_{\text{app}}}$. When the magnetic field H_z is far less than the perpendicular

effective field $H_{\perp}$, the stable polar angle of the magnetization can be approximated as

$$\theta_0 \approx \frac{H_z}{H_{\perp} + H_{\text{app}}} . \quad (6-5)$$

Fig. 6-1 Image of scanning electron microscope of patterned Hall cross (dark grey: Ta/Co/Pt wire; light grey: Hall bars) and schematic of the orientation of magnetization under the interactions of several magnetic fields.

The modulations of angles induced by SOT can be derived from Eqs. (6-4) and (6-5). The fieldlike term and the dampinglike term are $\mathbf{H}_F = (H_F \sin \omega t) \mathbf{y}$ and $\mathbf{H}_D = (H_D \sin \omega t) \cos \phi_0 \mathbf{z}$, respectively, when an AC current with a low frequency ω , $I = I_0 \sin \omega t$, is applied in the wire. In the expression of $\mathbf{H}_D$, again $\cos \phi_0 \mathbf{z}$ arises from the expression $\mathbf{m} \times \mathbf{y}$. I estimate the modulation induced by H_D through partial derivative of Eq. (6-4) with respect to θ_0 . From

Eq. (6-4), we obtain $\Delta \theta_0 = \frac{1}{H_{\perp} + H_{\text{app}}} \Delta H_{z-\text{ext}}$. Replacing $\Delta H_{z-\text{ext}}$ with $H_D \sin \omega t$, the

modulation of θ_0 is obtained as

$$\Delta\theta_0 \approx \frac{1}{H_{\perp} + H_{\text{app}}} H_D \sin \omega t. \quad (6-6)$$

We estimate the modulation induced by H_F through partial derivative of Eq. (6-4) with respect to φ_0 . Rewriting Eq. (6-4) in terms of the applied field H_{app} , we obtain

$$\sin \varphi_0 = \frac{H_{y\text{-ext}}}{\sqrt{H_{y\text{-ext}}^2 + H_{x\text{-ext}}^2}}, \text{ where } H_{y\text{-ext}} \text{ and } H_{x\text{-ext}} \text{ are the } y \text{ and } x \text{ components of } H_{\text{app}}. H_F \text{ is}$$

directed along the y -axis, thus, the modulation of φ_0 can be obtained through partial

$$\text{derivative of } \sin \varphi_0 = \frac{H_{y\text{-ext}}}{\sqrt{H_{y\text{-ext}}^2 + H_{x\text{-ext}}^2}} \text{ with respect to } H_{y\text{-ext}} \text{ as } \Delta\varphi_0 = \frac{\cos \varphi_0}{H_{\text{app}}} \Delta H_{y\text{-ext}}.$$

Replacing $\Delta H_{y\text{-ext}}$ with $H_F \sin \omega t$, the modulation of φ_0 changes to

$$\Delta\varphi_0 = \frac{\cos \varphi_0}{H_{\text{app}}} H_F \sin \omega t. \quad (6-7)$$

With the expression of Hall resistance, the modulations are converted into measurable harmonic Hall resistance. The Hall resistance R_{Hall} results from AHE resistance R_{AHE} and PHE resistance R_{PHE} where $R_A = R_{\text{AHE}} \sin \theta$ and $R_P = R_{\text{PHE}} \cos^2 \theta \sin 2\varphi$ [92]. The angles θ and φ of M_s are determined by the external field H_{app} and SOT fields H_D and H_F . As such, we obtain $\theta = \theta_0 + \Delta\theta_0$ and $\varphi = \varphi_0 + \Delta\varphi_0$. In the absence of external field along the z -axis, $H_{z\text{-ext}} = 0$ leads to $\theta = \Delta\theta_0$ according to Eqs. (6-4) and (6-6). Substituting θ and φ with $\Delta\theta_0$ and $\varphi_0 + \Delta\varphi_0$, the Hall resistance is obtained as

$$\frac{V_{\text{Hall}}}{I_0} = R_{\text{Hall}} = R_{\text{AHE}} \sin(\Delta\theta_0) \sin \omega t + R_{\text{PHE}} \cos^2(\Delta\theta_0) \sin[2(\varphi_0 + \Delta\varphi_0)] \sin \omega t. \quad (6-8)$$

Using the Pythagorean trigonometric identity, Eq. (6-8) simplifies to

$$R_{\text{Hall}} = R_{\text{AHE}} \Delta\theta_0 \sin \omega t + R_{\text{PHE}} (\sin 2\varphi_0 + 2\Delta\varphi_0 \cos 2\varphi_0) \sin \omega t. \quad (6-9)$$

Substituting $\Delta\theta_0$ and $\Delta\varphi_0$ with Eqs. (6-6) and (6-7), respectively, the two SOT effective fields are reflected in the amplitude of Hall resistance,

$$R_{\text{Hall}} = R_{\text{1stHall}} + R_{\text{2ndHall}}, \quad (6-10)$$

where $R_{\text{1stHall}} = R_{\text{PHE}} \sin 2\varphi_0 \sin \omega t$ and

$$R_{\text{2ndHall}} = \left[\frac{R_{\text{AHE}} H_{\text{D}} \cos \varphi_0}{2} \frac{1}{H_{\perp} + H_{\text{app}}} + R_{\text{PHE}} H_{\text{F}} (2\cos^3 \varphi_0 - \cos \varphi_0) \frac{1}{H_{\text{app}}} \right] \cos 2\omega t. \quad (6-11)$$

H_{D} and H_{F} can be calculated through fitting second harmonic Hall resistance R_{2ndHall} with respect to the applied magnetic field H_{app} , where the azimuthal angle of H_{app} or M_{s} can be arbitrary in the film plane.

To enable investigation of the angular dependence of the SOT effective fields, the two coefficients D and F are introduced to reflect the azimuthal angle dependence. Defining

$D = \frac{R_{\text{AHE}} H_{\text{D}}}{2} \cos \varphi_0$ and $F = R_{\text{PHE}} H_{\text{F}} (2\cos^3 \varphi_0 - \cos \varphi_0)$, we simplify Eq. (6-11) as

$$R_{\text{2ndHall}}(\varphi_0, H_{\text{app}}) = \left[D \frac{1}{H_{\perp} + H_{\text{app}}} + F \frac{1}{H_{\text{app}}} \right] \cos 2\omega t. \quad (6-12)$$

D and F can be experimentally obtained through fitting the experimental second harmonic resistance with respect to H_{app} by Eq. (6-12). On condition that D with respect to φ_0 can be fitted by the expressions of D , we can conclude that H_{D} is constant with φ_0 , otherwise, it is

dependent on φ_0 . Similar analysis is applicable for H_F .

6.2. SOT effective fields at $\varphi_0 = 0$

The dampinglike term H_D and fieldlike term H_F effective fields were characterized in the sample with Ta(6 nm)/Co(2 nm)/Pt(5nm) with $\varphi_0 = 0^\circ$. Amplitude of the applied AC current density ranges from $3 \times 10^{10} \text{ Am}^{-2}$ to $1 \times 10^{11} \text{ Am}^{-2}$ with an increment of 10^{10} Am^{-2} . The AC frequency was 307.1 Hz, which was provided by the 6221 DC and AC Current Source. Firstly, the applied field H_{app} sweeps from $\sim +500 \text{ Oe}$ to $\sim +3000 \text{ Oe}$, in which range, uniformity of magnetization is ensured. The sign dependency of the dampinglike term on the orientation of M_s can be verified by changing direction of the field H_{app} to orientate M_s along the $-x$ -axis, which sweeps from $\sim -500 \text{ Oe}$ to $\sim -3000 \text{ Oe}$. The SIGNAL RECOVERY 7265 DSP Lock-in Amplifier was used to measure the second harmonic Hall voltages at two Hall bars. The second harmonic Hall resistances are calculated by the measured second harmonic Hall voltage divided by the amplitude of applied current. The second harmonic Hall resistances with respect to the field H_{app} is shown in Fig. 6-2(a).

Each set of $R_{2\text{ndHall}}$ measured at $\pm H_{\text{app}}$ has been adjusted to be symmetric around $R_{2\text{ndHall}} = 0 \Omega$, to eliminate a constant offset of $R_{2\text{ndHall}}$ induced by other sources, such as Nernst–Ettingshausen effect [91]. With constant current density, the magnitudes of the $R_{2\text{ndHall}}$ decrease with respect to H_{app} . For instance, $R_{2\text{ndHall}}$ increases from $\sim 85 \mu\Omega$ to $\sim 150 \mu\Omega$ when the current density is $1 \times 10^{11} \text{ Am}^{-2}$. The trends coincide with the prediction of Eq. (6-11), which indicates that the SOT fields modulate $R_{2\text{ndHall}}$. For different current densities, the variation of the magnitudes of $R_{2\text{ndHall}}$ increases with respect the current

density. At the current density of $1 \times 10^{11} \text{ Am}^{-2}$, the change in the magnitude of $R_{2\text{ndHall}}$, as the external field is increased from $\pm 500 \text{ Oe}$ to $\pm 3000 \text{ Oe}$, is $\sim 65 \mu\Omega$. For a similar field range, the corresponding difference in $R_{2\text{ndHall}}$ at a current density of $3 \times 10^{10} \text{ Am}^{-2}$ is $\sim 20 \mu\Omega$. In line with Eq. (6-11), we conclude that the SOT fields increase with the current density. Substituting R_{AHE} , R_{PHE} and $H_{\perp}$ with the numerical values from Chapter 3 and Chapter 4 in Eq. (6-11), and fitting $R_{2\text{ndHall}}$ by Eq. (6-11), we obtain the SOT fields with respect to the current densities, as shown in Fig. 6-2(b). Both SOT fields increase linearly with respect to the current density in approximation. For fixed current density, the absolute amplitudes of the fieldlike term, as well as the dampinglike term, measured through $+H_{\text{app}}$ approximately equal to those measured through $-H_{\text{app}}$. At the current density of $1 \times 10^{11} \text{ Am}^{-2}$, both of the absolute amplitudes of fieldlike term at $\pm H$ are $\sim 4 \text{ Oe}$, and those of dampinglike term are $\sim 50 \text{ Oe}$. The values of the H_{D} and H_{F} are in the same order as the values presented in Chapter 4 as well as in similar stack with PMA [82]. The fieldlike term does not change sign while the dampinglike does. This result confirms that the sign of dampinglike term depends on the orientation of M_{s} while that of the fieldlike term is independent on the orientation of M_{s} . These results show that the derived model for quantifying the SOT is valid.

Fig. 6-2 The second harmonic Hall resistance with respect to applied magnetic field with different current densities. (b) Fieldlike term (blue for $+H_{app}$, black for $-H_{app}$) and dampinglike term (red for $+H_{app}$, magenta for $-H_{app}$) with respect to current densities.

6.3. Dependence of SOT effective fields in Ta(6 nm)/Co(2 nm)/Pt(5 nm)

The measurements of second harmonic Hall resistances were carried out as a function of the azimuthal angle φ_0 from $5^\circ \sim 80^\circ$ in increments of 5° . Shown in Fig. 6-3(a) are the measured second harmonic Hall resistances as functions of the applied field H_{app} , for a fixed current amplitude of $1 \times 10^{11} \text{ Am}^{-2}$. To mitigate the offset in $R_{2\text{ndHall}}$, which may be induced by sources such as Nernst–Ettingshausen effect, the curves of $R_{2\text{ndHall}}$ measured at $\pm H_{\text{app}}$ respectively, were shifted to be symmetric with respect to $R_{2\text{ndHall}}=0$. For a fixed value of the applied field such as $H_{\text{app}} \sim \pm 1800 \text{ Oe}$, the absolute magnitude of $R_{2\text{ndHall}}$ decreases when φ_0 increases. From Eq. (6-11), the change in $R_{2\text{ndHall}}$ is attributed to the term $\frac{R_{\text{AHE}} H_{\text{D}} \cos \varphi_0}{2}$. For varying H_{app} , absolute magnitude of $R_{2\text{ndHall}}$ increases with respect to H_{app} within the azimuthal angle range of $5^\circ \sim 45^\circ$, while they decrease in the range of $45^\circ \sim 80^\circ$. This trend can be explained by the contribution of the term $R_{\text{PHE}} H_{\text{F}} (2 \cos^3 \varphi_0 - \cos \varphi_0)$, where $2 \cos^3 \varphi_0 - \cos \varphi_0$ reverses sign when angle is greater than 45° . The changes in $R_{2\text{ndHall}}$ with respect to φ_0 and H_{app} imply that H_{D} and H_{F} are reflected at each value of φ_0 .

Substituting R_{AHE} , R_{PHE} and $H_{\perp}$ with the numerical values from Chapters 3 and 4 into Eq. (6-12), and fitting $R_{2\text{ndHall}}$ measured at each value of φ_0 , we obtain D and F , as shown in Figs. 6-3(b) and 6-3(c). The absolute magnitude of D decreases with respect to φ_0 ; while the value of F increases with respect to φ_0 within the range of $0^\circ \sim 60^\circ$. Thereafter, F decreases as φ_0 is increased. For eliminating azimuthal angle offset errors $\varphi_{\text{error}}^{\text{D,F}}$ which can be experimentally introduced by misalignment of wires to ideal x -axis, $D = \frac{R_{\text{AHE}} H_{\text{D}}}{2} \cos \varphi_0$

and $F = R_{\text{PHE}} H_F (2 \cos^3 \varphi_0 - \cos \varphi_0)$ are revised to be $\frac{D}{R_{\text{AHE}}/2} = H_D \cos(\varphi_0 + \varphi_{\text{error}}^D)$ and

$$\frac{F}{R_{\text{PHE}}} = H_F \left[2 \cos^3 (\varphi_0 + \varphi_{\text{error}}^F) - \cos (\varphi_0 + \varphi_{\text{error}}^F) \right], \text{ respectively.}$$

Fig. 6-3 (a) The second harmonic Hall resistance with respect to applied magnetic field with different current densities at different azimuthal angles; Comparisons between the experimental (dots) and the fitted (lines) values of the coefficients D and F for the cases of $-H_{\text{app}}$ (b) and $+H_{\text{app}}$ (c).

First, we assume that H_D and H_F are constant with respect to φ_0 in the expressions of

$$\frac{D}{R_{AHE}/2} \text{ and } \frac{F}{R_{PHE}} .$$

Under this assumption, the fitted D and F by the two revised

expressions of D and F , are shown in Figs. 6-3(b) and (c), where R_{AHE} and R_{PHE} are substituted with 24 mΩ and 5 mΩ respectively. D can be well fitted by the function of

$$\frac{D}{0.012} = 52 \cos(\varphi_0 - 4^\circ)$$

with the root-mean-square-error (RMSE) of the fitting reaching

to minimum. This implies that H_D is equal to 52 Oe and is constant with respect to the angle φ_0 . The computed value of H_D is coincident with that quantified by the method in Chapter 4 where H_D has been considered to be constant with respect to φ_0 . The coincidence validates the fitting. However, for the fitting of F by the expression of

$$\frac{F}{0.005} = -4.5 \left[2 \cos^3(\varphi_0 - 7^\circ) - \cos(\varphi_0 - 7^\circ) \right]$$

which reaches minimum of the fitting RMSE,

the fitted values of F deviate from the experimental points. The large deviation suggests

that the magnitude of H_F changes as a function of φ_0 in the expression $\frac{F}{R_{PHE}}$. Hence,

secondly, to investigate the dependence of H_F on φ_0 , we assume that $\frac{F}{R_{PHE}}$ has a constant

component H_{F1} as the first order approximation to F , alternatively

$$\frac{F}{R_{PHE}} = H_{F0} (2 \cos^3 \varphi_0 - \cos \varphi_0) + H_{F1} .$$

The experimental points can be well fitted by the

$$\text{approximated expression of } \frac{F}{0.005} = -5.3 \left[2 \cos^3(\varphi_0 + 2^\circ) - \cos(\varphi_0 + 2^\circ) \right] + 1.9$$

with

RMSE reaching to minimum. Compared with the initial expression of

$$\frac{F}{R_{PHE}} = H_F (2 \cos^3 \varphi_0 - \cos \varphi_0) ,$$

the fitting expression of $\frac{F}{0.005}$ indicates

$$H_F = -5.3 + \frac{1.9}{2\cos^3(\varphi_0 + 2^\circ) - \cos(\varphi_0 + 2^\circ)} \cdot H_F \text{ comprises of a constant component}$$

$$H_{F0} = -5.3 \text{ Oe and a } \varphi_0\text{-dependent component } H_{F1} = \frac{1.9}{2\cos^3(\varphi_0 + 2^\circ) - \cos(\varphi_0 + 2^\circ)} \cdot \text{It is}$$

expected that H_{F0} is due to the constant Rashba effect. H_{F1} is expected to be due to SHE which induces spin accumulation at the interfaces of Ta/Co and Co/Pt. The accumulated spins diffuse into Co layer to generate fieldlike torque. The diffusion is related to the electron diffusion constant [53]. The orientation of magnetization can change the electron diffusion constant *via* MR effect. Hence, the value of H_{F1} varies with respect to φ_0 of the magnetization, consequently, the value of the fieldlike term H_F is changed. H_F are calculated through fitting the experimental F by the initial expression and first order

approximated expression of $\frac{F}{R_{\text{PHE}}}$ where R_{PHE} is a constant, as shown in Fig. 6-4. The

values from the approximated expression follow the trend presented by the values calculated using initial expression. Rewriting the above approximated expression for

universal usage, we obtain $H_F = H_{\text{Rashba}} + \frac{H_{\text{SHE}}}{2\cos^3\varphi_0 - \cos\varphi_0}$ where H_{Rashba} is the

contribution from Rashba effect and $\frac{H_{\text{SHE}}}{2\cos^3\varphi_0 - \cos\varphi_0}$ is from SHE. As such, $\frac{F}{R_{\text{PHE}}}$ to the

first order approximation is rewritten to be $\frac{F}{R_{\text{PHE}}} = H_{\text{Rashba}} (2\cos^3\varphi_0 - \cos\varphi_0) + H_{\text{SHE}} \cdot$

Fig. 6-4 Comparisons between the values calculated by the initial expression and the approximated expression for the cases of $-H_{\text{app}}$ (a) and $+H_{\text{app}}$ (b).

6.4. Dependence of SOT effective fields in Ta(*t* nm)/Co(2 nm)/Pt(5 nm)

Same measurements of $R_{2\text{ndHall}}$ in the sample of Ta(6 nm)/Co(2 nm)/Pt(5 nm) were carried out in samples of Ta(*t* nm)/Co(2 nm)/Pt(5 nm), where $t=2, 4, 8$ and 10 . As shown in the Figs. A6-1~A6-4 in the Appendix of this Chapter, the quantified coefficients of F and D of the samples, behave similarly to the F and D of Ta(6 nm)/Co(2 nm)/Pt(5 nm), respectively. The similarity validates that H_F and H_D contribute to F and D for each of the samples. The experimental D with respect to φ_0 is fitted by using the expression of

$$\frac{D}{R_{\text{AHE}}/2} = H_D \cos(\varphi_0 + \varphi_{\text{error}}^D) \text{ to compute } H_D, \text{ where } R_{\text{AHE}} \text{ is substituted by its numerical}$$

value for each of the samples obtained from Chapter 3 for each sample. The experimentally obtained F with respect to φ_0 is fitted by using the expression

$$\frac{F}{R_{\text{PHE}}} = H_{\text{Rashba}} \left[2 \cos^3(\varphi_0 + \varphi_{\text{error}}^F) - \cos(\varphi_0 + \varphi_{\text{error}}^F) \right] + H_{\text{SHE}} \text{ to compute } H_{\text{Rashba}} \text{ and } H_{\text{SHE}}.$$

The fitting plots match the experimental F and D for each of the samples. The match implies that H_D is independent on φ_0 whereas H_F is dependent on φ_0 in the four samples, similar to the case of the 6 nm Ta sample. The obtained H_D and H_F with respect to the thickness of Ta layer are plotted in Fig. 6-5, where H_D and H_F from Chapter 4 are also plotted for comparison. In Chapter 4, H_D and H_F , which were assumed to be constant with respect to φ_0 , have been characterized by sweeping φ_0 from -76° to $\sim +76^\circ$. As shown in Fig. 6-5, the values of the reported H_D in Chapter 4 are close to the values obtained in this work. Furthermore, the increasing trend of the obtained H_D as a function of the thickness of Ta in this work is consistent with that of the reported H_D . To compare with the reported H_F , the average value of H_F in the φ_0 range from 0° to 76° are obtained for each sample

according to the approximated expression of $H_F = \frac{F}{R_{\text{PHE}}}$ presented in Figs. 6-3(b), 6-3(c)

and A6-1~A6-4. The average values of H_F are close to the reported values as shown in Fig. 6-5. Moreover, H_F obtained in this work, which increases with respect to the thickness of Ta, follows the trend of the reported H_F . Hence, the above coincidences between the obtained and reported SOT effective fields validate the expression

$$H_F = H_{\text{Rashba}} + \frac{H_{\text{SHE}}}{2\cos^3\varphi_0 - \cos\varphi_0}.$$

Fig. 6-5 The obtained values of H_F and H_D from our measurements with respect to the thickness of Ta, and comparisons with those reported in Chapter 4.

The dependence of H_{SHE} on the amplitude of the local magnetization is investigated. The magnetization amplitude influences the electron diffusion constant *via* the MR effect [199],

which affects the diffusion process of the accumulated spins into the FM layer from HM layer [111]. As such, the magnetization amplitude can modulate the spin accumulation which is induced by SHE *via* the electron diffusion constant, which has been verified in Chapter 5. The SHE induced fieldlike term is a function of the diffusion constant. Hence, the value of H_{SHE} should vary with respect to the amplitude of the magnetization. Figure 6-6 shows H_{SHE} and the amplitude of the magnetization with respect to the thickness of Ta. H_{SHE} increases from 1.3 Oe in the sample with 2 nm Ta to the maximum of 2.0 Oe in the sample with 6 nm Ta, and then decreases to 1.3 Oe in the sample with 10 nm Ta. In contrast, the amplitude of the magnetization decreases to minimum for the samples from 2 nm Ta sample to 6 nm Ta sample, and then increases from 6 nm Ta sample to 10 nm Ta sample [24]. H_{SHE} exhibits a reverse trend of the magnetization amplitude with respect to the Ta thickness. Hence, the increasing of the magnetization amplitude leads to the decreasing of H_{SHE} . The relationship between H_{SHE} and the magnetization amplitude supports that the φ_0 -dependent H_{SHE} is attributed to SHE of Ta and Pt.

Fig. 6-6 H_{SHE} and the amplitude of the magnetization with respect to the thickness of Ta.

Appendix

Fig. A6-1 For the sample Ta(2 nm)/Co(2 nm)/Pt(5 nm), (a) The second harmonic Hall resistance with respect to applied magnetic field with different current densities at different azimuthal angles; Comparisons between the experimental (dots) and the fitted (lines) values of the coefficients D and F for the cases of $-H_{app}$ (b) and $+H_{app}$ (c); Comparisons between the values calculated by the initial expression and the approximated expression for the cases of $-H_{app}$ (d) and $+H_{app}$ (e).

Fig. A6-2 For the sample Ta(4 nm)/Co(2 nm)/Pt(5 nm), (a) The second harmonic Hall resistance with respect to applied magnetic field with different current densities at different azimuthal angles; Comparisons between the experimental (dots) and the fitted (lines) values of the coefficients D and F for the cases of $-H_{app}$ (b) and $+H_{app}$ (c); Comparisons between the values calculated by the initial expression and the approximated expression for the cases of $-H_{app}$ (d) and $+H_{app}$ (e).

Fig. A6-3 For the sample Ta(8 nm)/Co(2 nm)/Pt(5 nm), (a) The second harmonic Hall resistance with respect to applied magnetic field with different current densities at different azimuthal angles; Comparisons between the experimental (dots) and the fitted (lines) values of the coefficients D and F for the cases of $-H_{app}$ (b) and $+H_{app}$ (c); Comparisons between the values calculated by the initial expression and the approximated expression for the cases of $-H_{app}$ (d) and $+H_{app}$ (e).

Fig. A6-4 For the sample Ta(10 nm)/Co(2 nm)/Pt(5 nm), (a) The second harmonic Hall resistance with respect to applied magnetic field with different current densities at different azimuthal angles; Comparisons between the experimental (dots) and the fitted (lines) values of the coefficients D and F for the cases of $-H_{app}$ (b) and $+H_{app}$ (c); Comparisons between the values calculated by the initial expression and the approximated expression for the cases of $-H_{app}$ (d) and $+H_{app}$ (e).

vii. Quantification of spin accumulation

In this chapter, a concise solution to quantify the spin accumulation in the Ta/Co/Pt wire with IMA is provided. The spin accumulation s contributes to second harmonic Hall resistance in the harmonic Hall voltage measurement, besides that from the SOT effective field J_s as expected. Based on the proposed method for quantifying J_s , applying a biasing direct current (DC) enables the extraction of the contribution of the spin accumulation from the second harmonic Hall resistances. Analogized to the first harmonic Hall resistance, which is induced by the magnetization, the contribution can be used to compute the spin accumulation. Results of the computation show that the spin accumulation is $\sim 10\%$ of the local magnetization when the applied current density is in the order of 10^{11} Am^{-2} and is dependent on the thickness of the HM layers. This quantification allows us to understand the anatomy of J_s , with a clearer distinction of the roles between J and s .

7.1. PHE resistance induced by spin accumulation

Following the transfer of momentum to the Co magnetization, the accumulated spins s adopt similar polarization as the magnetization orientation in the Ta/Co/Pt structure, as shown in Fig. 7-1. In this structure, the initial polarization of s is induced by Rashba effect due to the asymmetric HM/FM interface, and SHE within the Ta and Pt layers [44, 76, 78, 82, 91, 93, 95, 99, 100, 102, 121, 124-126, 182]. The Rashba effect re-orientates the spin of the conduction electrons of Co layer to provide a net resultant spin in the FM layer [44]. Additionally, the SHE induces a spin-selective separation of electrons in the HM layer; the spin polarized electrons then diffuses into the FM layer to transfer spin torque on the magnetization of Co [111]. Due to the magnetoresistive effect of FM layer, spins are accumulated at the interface of HM/FM. Therefore, in another picture of STT, S. Zhang proposed that the spin accumulation transfers spin torque to the Co magnetization [53]. The transfer occurs on sub-nanosecond time scale [44]. At the end of the transfer, the diffusing electron spins are in relaxation state, hence they adopt the same orientation as the local magnetic moment $M\mathbf{m}$. In experiment, within the low frequency regime of hundreds of Hertz, corresponding to the period of AC current in millisecond scale, it is reasonable to consider that the diffusing spins follow the orientation of $M\mathbf{m}$. Similarly, extending to DC bias regime, an identical approximation can be made, and the diffusing spins similarly align along $\mathbf{m}$ direction. Therefore, after the transfer of the momentum to the local magnetization, the resultant polarization direction of the diffusing spin is aligned along the magnetization orientation of the FM layer. Thus, magnitude of the diffusing spin per unit volume, which is equivalent to the magnitude of SHE induced spin accumulation, can be written as $s\mathbf{m}$ in the experiments with millisecond scale. Therefore, the total magnetization of the stack

becomes $M\mathbf{m}+s\mathbf{m}$.

The magnitude of planar Hall resistance, R_{PHE} due to the local magnetic moment $M\mathbf{m}$ is proportional to the square of the magnitude of the local magnetization M , i.e., $R_{\text{PHE}}=kM^2$ [207, 209]. The diffusing spins, $s\mathbf{m}$, results in additional planar Hall resistance, as $s\mathbf{m}$ is an additional magnetization. Analogized to the local magnetization $M\mathbf{m}$, the magnitude of the planar Hall resistance r_{PHE} induced by $s\mathbf{m}$ should show the same behavior as that induced by $M\mathbf{m}$, hence, $r_{\text{PHE}}=ks^2$. The planar Hall resistance due to $M\mathbf{m}$ is expressed as $R_p=R_{\text{PHE}}\sin 2\varphi_0$, where φ_0 is the azimuthal angle of magnetization $M\mathbf{m}$ [4, 51, 130, 193, 210]. Analogically, the planar Hall resistance, r_p due to the extra magnetization, $s\mathbf{m}$, also follows a similar behavior, $r_p=r_{\text{PHE}}\sin 2\varphi_0$. Taking the k term from the expression R_{PHE} , we obtain $r_{\text{PHE}}=\frac{R_{\text{PHE}}}{M^2}s^2$ which can be used to calculate the magnitude of the spin accumulation.

Fig. 7-1 (a) Schematic of the spin torque transfer from the spin accumulation s to the local magnetization M . (b) s is along the orientation of M after the spin torque transfer.

Applying a DC bias enables the measurement of the magnitude of r_{PHE} . When AC current and the DC bias are applied in the wire concurrently, the harmonic Hall voltage induced by $s\mathbf{m}$ can be written as $v_{s,\text{Hall}} = \left[\left(\frac{R_{\text{PHE}}}{M^2} s^2 \right) \sin 2\varphi_0 \right] (j_{\text{AC}} \sin \omega t + j_{\text{DC}})$, where j_{AC} and ω are

the amplitude and frequency of AC current density, respectively, and j_{DC} is the magnitude of DC current density. At steady state condition, where the rate of spin decay equals to that of spin generation, s is proportional to the current density and can be written as $s = \zeta (j_{\text{AC}} \sin \omega t + j_{\text{DC}})$, where ζ is the coefficient constant. Substituting s by $\zeta (j_{\text{AC}} \sin \omega t + j_{\text{DC}})$ in the harmonic Hall voltage expression gives

$$v_{\text{s,Hall}} = R_{\text{PHE}} \frac{\zeta^2}{M^2} \sin 2\varphi_0 \left[(j_{\text{DC}}^3 + 3j_{\text{DC}}^2 j_{\text{AC}} \sin \omega t + 3j_{\text{AC}}^2 j_{\text{DC}} \sin^2 \omega t + j_{\text{AC}}^3 \sin^3 \omega t) \right]. \quad (7-1)$$

In Eq. (7-1), $\sin^2 \omega t$ can be substituted with $\frac{1}{2} - \frac{\cos 2\omega t}{2}$ to eliminate the constant $\frac{1}{2}$ to

obtain a second harmonic Hall voltage $v_{\text{s,2ndHall}} = \frac{3}{2} R_{\text{PHE}} \sin 2\varphi_0 \frac{\zeta^2}{M^2} j_{\text{AC}}^2 j_{\text{DC}} \cos 2\omega t$.

Consequently, $\mathbf{s}\mathbf{m}$ induces a second harmonic Hall resistance as

$$r_{\alpha}^{\pm\beta} = \frac{v_{\text{s,2ndHall}}}{j_{\text{AC}}} = Z_{\alpha}^{\pm\beta} \sin 2\varphi_0 \text{ and } Z_{\alpha}^{\pm\beta} = \frac{3}{2} R_{\text{PHE}} \frac{\zeta^2}{M^2} j_{\text{DC}} j_{\text{AC}}, \quad (7-2)$$

where α and β correspond to the factors $\alpha \times 10^{10} \text{ Am}^{-2}$ for j_{AC} and $\beta \times 10^{10} \text{ Am}^{-2}$ for j_{DC} , and

$\pm$ indicates the DC sign. Compared with the expression $r_{\text{PHE}} = \frac{R_{\text{PHE}}}{M^2} s^2$, the expression

$Z_{\alpha}^{\pm\beta} = \frac{3}{2} R_{\text{PHE}} \frac{\zeta^2}{M^2} j_{\text{DC}} j_{\text{AC}}$ is another form of r_{PHE} which includes electric current, and $r_{\alpha}^{\pm\beta}$

is equivalent to r_{P} . For a fixed amplitude of j_{AC} , the resistance $Z_{\alpha}^{\pm\beta}$ is proportional to the amplitude of the applied DC current. As such, j_{DC} provides a way to obtain the second harmonic Hall resistance $r_{\alpha}^{\pm\beta}$.

In experiment, the s -induced second harmonic Hall resistance $r_{\alpha}^{\pm\beta}$ can be obtained by

subtracting the measured second harmonic Hall resistance $\Re_{\alpha}^{\pm\beta}$, which is measured by AC and DC concurrently, from $\Re_{\alpha}^0$, which is measured by AC only. The measured second harmonic Hall resistance $\Re_{\alpha}^{\pm\beta}$ is induced by both $s\mathbf{m}$ and $M\mathbf{m}$ concurrently. As such, $\Re_{\alpha}^{\pm\beta}$ is the total second harmonic Hall resistance, which consists of $r_{\alpha}^{\pm\beta}$ due to $s\mathbf{m}$, and $\Re_{\alpha}^{\pm\beta}$ due to $M\mathbf{m}$. As discussed in Chapter 4, the second harmonic Hall resistance $\Re_{\alpha}^{\pm\beta}$ is obtained from the Hall resistance

$$R_{\text{Hall}} \approx R_{\text{AHE}} \Delta\theta_0 + R_{\text{PHE}} \sin 2\varphi_0 + 2\Delta\varphi_0 R_{\text{PHE}} \cos 2\varphi_0, \quad (7-3)$$

where $\Delta\theta_0 = \Delta\theta_{0\text{AC}} \sin \omega t + \Delta\theta_{0\text{DC}}$ and $\Delta\varphi_0 = \Delta\varphi_{0\text{AC}} \sin \omega t + \Delta\varphi_{0\text{DC}}$. In the expressions of $\Delta\theta_0$ and $\Delta\varphi_0$, the terms including $\sin \omega t$ are the orientation modulations induced by AC current, while the others are the orientation modulations induced by DC current. Taking the expressions in to Eq. (7-3), we obtain

$$R_{\text{Hall}} \approx R_{\text{AHE}} (\Delta\theta_{0\text{AC}} \sin \omega t + \Delta\theta_{0\text{DC}}) + R_{\text{PHE}} [\sin 2\varphi_0 + 2(\Delta\varphi_{0\text{AC}} \sin \omega t + \Delta\varphi_{0\text{DC}}) \cos 2\varphi_0]. \quad (7-4)$$

From Eq. (7-4), the Hall voltage is obtained as

$$V_{\text{Hall}} \approx \left[\begin{aligned} &(R_{\text{AHE}} \Delta\theta_{0\text{DC}} + R_{\text{PHE}} \sin 2\varphi_0 + 2\Delta\varphi_{0\text{DC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{AC}} \\ &+ (R_{\text{AHE}} \Delta\theta_{0\text{AC}} + 2\Delta\varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{DC}} \sin \omega t \end{aligned} \right] + (R_{\text{AHE}} \Delta\theta_{0\text{AC}} + 2\Delta\varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{AC}} \sin^2 \omega t. \quad (7-5)$$

Hence, the first and second harmonic Hall voltages are

$$\begin{cases} V_{1\text{stHall}} \approx \left[\begin{aligned} &(R_{\text{AHE}} \Delta\theta_{0\text{DC}} + R_{\text{PHE}} \sin 2\varphi_0 + 2\Delta\varphi_{0\text{DC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{AC}} \\ &+ (R_{\text{AHE}} \Delta\theta_{0\text{AC}} + 2\Delta\varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{DC}} \end{aligned} \right] \sin \omega t \\ V_{2\text{ndHall}} \approx (R_{\text{AHE}} \Delta\theta_{0\text{AC}} + 2\Delta\varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) j_{\text{AC}} \sin^2 \omega t \end{cases}. \quad (7-6)$$

Consequently, the corresponding harmonic Hall resistances are

$$\begin{cases} R_{1\text{stHall}} \approx \left[\begin{aligned} & (R_{\text{AHE}} \Delta \theta_{0\text{DC}} + R_{\text{PHE}} \sin 2\varphi_0 + 2\Delta \varphi_{0\text{DC}} R_{\text{PHE}} \cos 2\varphi_0) \\ & + (R_{\text{AHE}} \Delta \theta_{0\text{AC}} + 2\Delta \varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) \frac{j_{\text{DC}}}{j_{\text{AC}}} \end{aligned} \right] \sin \omega t \\ R_{2\text{ndHall}} \approx (R_{\text{AHE}} \Delta \theta_{0\text{AC}} + 2\Delta \varphi_{0\text{AC}} R_{\text{PHE}} \cos 2\varphi_0) \sin^2 \omega t \end{cases} \quad (7-7)$$

The second harmonic Hall resistance excludes the orientation modulations induced by DC current, and is same as the second harmonic Hall resistance induce by AC only. Hence, we obtain

$$R_{\alpha}^{\pm\beta} = R_{2\text{ndHall}} = R_{\text{AHE}} \frac{H_{\text{D,AC}}}{2H_{\perp}} \cos \varphi_0 + R_{\text{PHE}} \frac{H_{\text{F,AC}}}{H_{x-\text{ext}}} \left[2 \cos^4 \varphi_0 - \cos^2 \varphi_0 \right] \quad (7-8)$$

from Chapter 4. For the IMA wires, $H_{\text{D,AC}}$ is in the z -axis while $H_{\text{F,AC}}$ is in the y -axis. The DC-induced J_s has no effect on $R_{\alpha}^{\pm\beta}$, or $R_{\alpha}^{\pm\beta} = R_{\alpha}^0$. Both $H_{\text{D,AC}}$ and $H_{\text{F,AC}}$ are only determined by the AC component of the applied electric current. Hence, according to Eq. (7-2), the second harmonic Hall resistance measured using AC only, is $\mathfrak{R}_{\alpha}^{\pm\beta} = R_{\alpha}^0 + 0$, while that measured by using both DC and AC concurrently is $\mathfrak{R}_{\alpha}^{\pm\beta} = R_{\alpha}^0 + r_{\alpha}^{\pm\beta}$, where β is nonzero. Therefore, substituting R_{α}^0 from $\mathfrak{R}_{\alpha}^{\pm\beta}$, *i.e.*, $\Delta \mathfrak{R}_{\alpha}^{\pm\beta} = \mathfrak{R}_{\alpha}^{\pm\beta} - R_{\alpha}^0$, gives $r_{\alpha}^{\pm\beta}$. Based on Eq. (7-2), we can conclude that

$$\Delta \mathfrak{R}_{\alpha}^{\pm\beta} = r_{\alpha}^{\pm\beta} = Z_{\alpha}^{\pm\beta} \sin 2\varphi_0 = \frac{3}{2} R_{\text{PHE}} \frac{\zeta^2}{M^2} j_{\text{DC}} j_{\text{AC}} \sin 2\varphi_0. \quad (7-4)$$

As such, theoretically, the coefficient ζ can be quantified by $\Delta \mathfrak{R}_{\alpha}^{\pm\beta}$, which indicates the magnitude of spin accumulation.

7.2. Experimental verification

Measurements of the second harmonic Hall resistances $\Re_{\alpha}^{\pm\beta}$ with respect to the azimuthal angle of magnetization were carried out in Ta(8 nm)/Co(2 nm)/Pt(5 nm) magnetic wires. Figure 7-2 show a schematic of the measurement setup, where the lock-in amplifier was used to measure the harmonic Hall voltage signals. The second harmonic Hall resistance $\Re_{\alpha}^{\pm\beta}$ is calculated by dividing the measured second harmonic Hall voltage with the magnitude of the AC current. Only the Hall resistance modulation has been considered by removing the offset resistance for each measurement. Each measured $\Re_{\alpha}^{\pm\beta}$, as well as the corresponding $\Delta\Re_{\alpha}^{\pm\beta}$ has been adjusted to be around 0 Ω by eliminating a constant offset for ease of comparison. The azimuthal angle of the wire magnetization is field-dependent and is given as $\varphi_0 = \arctan \frac{H_{y-\text{ext}}}{H_{x-\text{ext}}}$ in Chapter 4, where the transverse field $H_{y-\text{ext}}$ sweeps from -1800 Oe to $+1800$ Oe along the y -axis, while $\pm H_{x-\text{ext}}$ maintained at ± 560 Oe to orientate M along the $\pm x$ -axis. In our subsequent discussion, $H_{y-\text{ext}}$ is equivalent to φ_0 for a constant $H_{x-\text{ext}}$.

Fig. 7-2 Scanning electron microscope image of our sample structure, and a schematic of harmonic Hall resistance measurement setup is superimposed on the image.

The measured second harmonic Hall resistances, $\mathfrak{R}_4^{0,\pm 6}$, with respect to the azimuthal angle of the magnetization at applied field $H_{x\text{-ext}} = -560$ Oe, are shown in Fig. 7-3. For $\mathfrak{R}_4^0$, which was measured by using AC only, has an expression as given in Eq. (7-3).

Fig. 7-3 The measured second harmonic Hall resistance $\mathfrak{R}_4^{\pm 6}$ and the obtained subtraction $\Delta \mathfrak{R}_4^{\pm 6}$ with respect to $H_{y\text{-ext}}$ when $H_{x\text{-ext}} = -560$ Oe, the first harmonic Hall resistances $R_{1stHall}$ with respect to $H_{y\text{-ext}}$ are measured at $H_{x\text{-ext}} = \pm 560$ Oe. Both $R_{1stHall}$ and $\Delta \mathfrak{R}_4^{\pm 6}$ show typical behavior of PHE resistance with respect to $H_{y\text{-ext}}$.

By substituting the experimental values of $H_L = 5790$ Oe, $R_{AHE} = 26$ m Ω and $R_{PHE} = 6$ m Ω into Eq. (7-3), the calculated $\mathfrak{R}_4^0$ is in good agreement with the measured $\mathfrak{R}_4^0$ or Eq. (7-3), which gives $H_{D,AC} = 29$ Oe and $H_{F,AC} = 4$ Oe, as shown in Fig. 7-3. This good agreement suggests that the AC-induced $\mathbf{Js}$ (or $H_{F,AC}\mathbf{y} + H_{D,AC}\mathbf{m} \times \mathbf{y}$) contributes to the symmetric behavior of $\mathfrak{R}_4^0$ with respect to $H_{y\text{-ext}}$. For the second harmonic Hall resistance $\mathfrak{R}_4^{\pm 6}$ measured by using both

DC and AC concurrently, an asymmetric behavior centered at $H_{y\text{-ext}} = 0$ Oe is observed. The measured resistances, $\mathfrak{R}_4^{+6}$ and $\mathfrak{R}_4^{-6}$ are symmetric to each other at $H_{y\text{-ext}} = 0$ Oe. These variations indicate that both magnitude and sign of the DC bias in the wire contribute to the corresponding signals, $\mathfrak{R}_4^{\pm 6}$. According to the values of $H_{D,AC}=29$ Oe and $H_{F,AC}=4$ Oe, $H_{D,DC}$ and $H_{F,DC}$ can be calculated as 44 Oe and 6 Oe at $j_{DC}=6\times 10^{10}$ Am⁻², respectively, as the SOT effective fields are proportional to current density. The magnitudes of $H_{D,DC}$ and $H_{F,DC}$ are far smaller than the values of $H_{\perp}$ and $H_{y\text{-ext}}$, consequently, the possibility of a DC-induced $H_{D,DC}$ and $H_{F,DC}$ in Eq. (7-3) as offsets to $H_{\perp}$ and $H_{y\text{-ext}}$ to result in the behavior of $\mathfrak{R}_4^{\pm 6}$ is excluded.

The subtractions of the measured $\mathfrak{R}_4^0$ from the measured $\mathfrak{R}_4^{\pm 6}$, $\Delta\mathfrak{R}_4^{\pm 6}$, are obtained to verify Eq. (7-4) experimentally. As shown in Fig. 7-4, the obtained $\Delta\mathfrak{R}_4^{\pm 6}$ follow the $\sin 2\varphi_0$ behavior with respect to $H_{y\text{-ext}}$, which is predicted by Eq. (7-4) and is similar to the first harmonic Hall resistance. Furthermore, $\Delta\mathfrak{R}_\alpha^{\pm\beta}$ follow the behaviors of $r_\alpha^{\pm\beta}$ (or $R_{1stHall}$) with respect to the orientation and magnitude of $H_{x\text{-ext}}$. According to the expression

$$\varphi_0 = \arctan \frac{H_{y\text{-ext}}}{H_{x\text{-ext}}}, \text{ where } H_{x\text{-ext}} \text{ is as a parameter, } \sin 2\varphi_0 \text{ should change sign when } H_{x\text{-ext}} \text{ is}$$

reversed, which leads to the sign reverse of $\Delta\mathfrak{R}_\alpha^{\pm\beta}$. In experiment, as shown in Fig. 7-3, $\Delta\mathfrak{R}_4^{+6}$ were measured at $H_{x\text{-ext}} = -560$ Oe, $\Delta\mathfrak{R}_4^{+6}$ changes from $-16 \mu\Omega$ to $+16 \mu\Omega$ when $H_{y\text{-ext}}$ varies from -1800 Oe to $+1800$. However, as shown in Fig. 7-4, $\Delta\mathfrak{R}_4^{+6}$ changes from $+16 \mu\Omega$ to $-16 \mu\Omega$ when $\Delta\mathfrak{R}_4^{+6}$ were measured at $H_{x\text{-ext}} = +560$ Oe. For the dependence of magnitude, the term $\sin 2\varphi_0$ in the expression of $\Delta\mathfrak{R}_\alpha^{\pm\beta}$ indicates that the extremums of $\Delta\mathfrak{R}_\alpha^{\pm\beta}$ should present at $H_{y\text{-ext}} = \pm H_{x\text{-ext}}$. As shown in Fig. 7-5, the extremum values of $\Delta\mathfrak{R}_4^{+6}$

are at $H_{y\text{-ext}} = \pm 360$ and ± 1000 Oe when $\Delta\mathfrak{R}_4^{\pm 6}$ were measured at $H_{y\text{-ext}} = +360$ and $+1000$ Oe,

respectively. The term $Z_a^{\pm\beta} = \frac{3}{2} R_{\text{PHE}} \frac{\zeta^2}{M^2} j_{\text{DC}} j_{\text{AC}}$ in Eq. (7-4) is verified experimentally.

Fig. 7-4 The measured second harmonic Hall resistance $\mathfrak{R}_4^{\pm 6}$ and the corresponding subtraction $\Delta\mathfrak{R}_4^{\pm 6}$ with respect to $H_{y\text{-ext}}$ when $H_{x\text{-ext}} = +560$ Oe.

Fig. 7-5 The measured $\mathfrak{R}_4^{\pm 6}$ and $R_{1stHall}$, and the corresponding $\Delta\mathfrak{R}_4^{\pm 6}$ with respect to $H_{y\text{-ext}}$ at $H_{x\text{-ext}} = +360$ Oe and $+1000$ Oe.

For different DC biases, the obtained Z_4 shows a linear relationship with j_{DC} , as shown in Fig. 7-6(a). Inset shows the fittings of the corresponding $\Delta\mathfrak{R}_4^{\pm[1 \text{ to } 5]}$ which were measured at $H_{x\text{-ext}} = -560$ Oe (The measured $\mathfrak{R}_4^{\pm[1 \text{ to } 5]}$ and $\Delta\mathfrak{R}_4^{\pm[1 \text{ to } 5]}$ are shown in Fig. A7-1 in the Appendix of this Chapter). For different AC density, figure 7-6(b) shows the linear relationship of $Z_4^{\pm 4}$ with respect to j_{AC} . Inset shows the corresponding $\Delta\mathfrak{R}_{[1-6]}^{\pm 4}$ (The measured $\mathfrak{R}_{[1-6]}^{\pm 4}$ and $\Delta\mathfrak{R}_{[1-6]}^{\pm 4}$ are shown in Fig. A7-2 in the Appendix of this Chapter), which were measured $H_{x\text{-ext}} = -560$ Oe.

Fig. 7-6 Plot of $Z_4^{\pm\beta}$ versus the applied DC current density when the AC current density was fixed at $4 \times 10^{10} \text{ Am}^{-2}$. Inset shows the fittings of the experimental data $\Delta\mathfrak{R}_4^{\pm\beta}$. (b) Plot of $Z_{\alpha}^{\pm 4}$ versus the applied AC current density when the DC density was fixed at $\pm 4 \times 10^{10} \text{ Am}^{-2}$. Inset shows the fittings of the experimental data $\Delta\mathfrak{R}_{\alpha}^{\pm 4}$.

7.3. Magnitude of spin accumulation and spin Hall angle

The coefficient, ζ , which indicates the ratio of spin accumulation to electric current, is

extracted from $Z_{\alpha}^{\pm\beta}$. By taking partial derivatives of $Z_{\alpha}^{\pm\beta}$ over j_{DC} and j_{AC} , $\frac{\partial}{\partial j_{AC}} \left(\frac{\partial Z_{\alpha}^{\pm\beta}}{\partial j_{DC}} \right)$

which equals to $\frac{3}{2} R_{PHE} \frac{\zeta^2}{M^2}$ according to Eq. (7-4). Based on Fig. 7-7, $\frac{\partial}{\partial j_{AC}} \left(\frac{\partial Z_{\alpha}^{\pm\beta}}{\partial j_{DC}} \right)$ is

calculated as $130 \left[\frac{\mu\Omega}{(10^{11} \text{ Am}^{-2})^2} \right]$. Substituting $R_{PHE}=5.7 \text{ m}\Omega$ and $M=458 \text{ emu/cc}$ in

$\frac{3}{2} R_{PHE} \frac{\zeta^2}{M^2}$, we obtain ζ as $56 \text{ emu/cc per } 10^{11} \text{ Am}^{-2}$. The value of ζ suggests that 56

emu/cc of magnetization is generated by the electric current with the current density of 10^{11}

Am^{-2} in the sample Ta(8 nm)/Co(2 nm)/Pt(5 nm). Similarly, ζ is quantified for the samples

Ta(t nm)/Co(2 nm)/Pt(5 nm). In the quantification, $\frac{\partial Z_{\alpha}^{\pm\beta}}{\partial j_{DC}}$ which equals to $\frac{3}{2} R_{PHE} \frac{\zeta^2}{M^2} j_{AC}$

according to Eq. (7-4), are obtained at $j_{AC} = 4 \times 10^{10} \text{ Am}^{-2}$, as shown in the inset of Fig. 7-

7, the measured $\Re_4^{\pm\beta}$ and $\Delta \Re_4^{\pm\beta}$ are shown in from Fig. A7-2 to Fig. A7-6 in the Appendix

of this Chapter. Substituting j_{AC} , R_{PHE} and M with the experimental values shown in the

inset table of Fig. 7-7 in $\frac{3}{2} R_{PHE} \frac{\zeta^2}{M^2} j_{AC}$, the obtained values of ζ at 10^{11} Am^{-2} are plotted

in Fig. 7-8, where ζ for $t = 8$ is included for comparison. ζ is $\sim 50 \text{ emu/cc per } 10^{11} \text{ Am}^{-2}$ for

samples with $t \leq 6$, while ζ increases from $56 \text{ emu/cc per } 10^{11} \text{ Am}^{-2}$ for samples with $t > 6$,

and reaches a maximum of $107 \text{ emu/cc per } 10^{11} \text{ Am}^{-2}$ at $t = 10$. This observation indicates

that ζ is dependent on the thickness of Ta.

The ratio $\frac{S}{M}$ is calculated to investigate the influence of the spin accumulation on the local magnetization. As shown in the inset of Fig. 7-8, when the applied current density in the wires is $1 \times 10^{11} \text{ Am}^{-2}$, the ratio is $\sim 10\%$ for the films with $t \leq 8$, and 18% for $t=10$. The critical current density for SOT to switch magnetization and drive domain wall motion is in the order of $1 \times 10^{11} \text{ Am}^{-2}$ [94]. As the initial orientation of spin accumulation is in the y-axis, the spin accumulation can vary the local magnetization by as much as 13% when the local magnetization orientates along the y-axis, in the wire with $t = 10$. Hence, the spin accumulation assisting the SOT in switching magnetization by means of changing the magnitude of the local magnetization is expected. If the current density increases to $1 \times 10^{12} \text{ Am}^{-2}$, which is the critical value for STT to switch magnetization and drive domain wall motion [94, 211], the ratio is 100% for the films with $t \leq 8$, and 180% for $t=10$. Hence, the magnitudes of spin accumulation are close to the magnitudes of the corresponding local magnetization. In this case, the magnetic moments can be reorganized in the wires, hence, the orientation of magnetization could be determined by the spin accumulation.

ζ follows the spin Hall angles of the samples with respect to the thickness of the Ta layer, irrespective of their magnitudes, as shown in Fig. 7-8. ζ represents the spin accumulation generated, while the spin Hall angle of the Ta/Pt indicates the percentage of spin current converted by the Ta/Pt. As such, ζ and spin Hall angle with respect to the thickness of Ta is expected to have similar behavior. Therefore, the observation confirms that the spin accumulation in the Co layer is originated from the Ta and Pt layers.

Fig. 7-7 Plot of $Z_{\alpha}^{\pm\beta}$ versus the applied DC current density under different j_{AC} ($\times 10^{10} \text{ Am}^{-2}$) for sample Ta(8 nm)/Co(2 nm)/Pt(5 nm). Bottom right inset shows plot of $Z_{\alpha}^{\pm\beta}$ for samples Ta(t nm)/Co(2 nm)/Pt(5 nm), with $t=2, 4, 6$ and 10 . Top left inset shows tabulated slopes $\frac{Z_{\alpha}^{\pm\beta}}{j_{\text{DC}}}$ for all the measured samples at applied AC current density of $j_{\text{AC}}=4$ ($\times 10^{10} \text{ Am}^{-2}$).

Fig. 7-8 The obtained magnitude of the spin accumulation and the reported spin Hall angle with respect to the thickness of Ta.

7.4. Dependence on the magnetization azimuthal angle

Figure 7-3 shows the measured second harmonic Hall resistances $\mathfrak{R}_4^{0,\pm 6}$ with respect to the orientation of magnetization, where the orientation varied in the film plane. $\mathfrak{R}_4^0$ was measured by applying DC $4 \times 10^{10} \text{Am}^{-2}$ in the patterned wire, while $\mathfrak{R}_4^{\pm 6}$ was measured by applying AC $4 \times 10^{10} \text{Am}^{-2}$ and DC $\pm 6 \times 10^{10} \text{Am}^{-2}$ concurrently. The subtraction, $\mathfrak{R}_4^{\pm 6} - \mathfrak{R}_4^0$, equals to the second harmonic Hall resistances r_p which is induced by $\mathbf{sm}$. As shown in Fig. 7-3, r_p exhibits the same behavior to that induced by $\mathbf{M} = M\mathbf{m}$, where r_p is expressed as $r_p = r_{\text{PHE}} \sin 2\varphi_0$ and $r_{\text{PHE}} = \frac{R_{\text{PHE}}}{M^2} s^2$. φ_0 is the azimuthal angle of the magnetization, which is determined by the in-plane longitudinal constant field $H_{x\text{-ext}}$ and the in-plane transverse sweeping field $H_{y\text{-ext}}$ as $\varphi_0 = \arctan \frac{H_{y\text{-ext}}}{H_{x\text{-ext}}}$. R_{PHE} is the constant amplitude of PHE resistance induce by $M\mathbf{m}$. As r_p with respect to φ_0 strictly follow the function of $\sin 2\varphi_0$, hence, r_{PHE} does not vary with φ_0 . Base on the expression of r_{PHE} , consequently, s does not vary with the φ_0 . We conclude that the spin accumulation is independent of the azimuthal angle of magnetization.

Appendix

Fig. A7-1 The measured $\mathcal{R}_4^{\pm[1 \text{ to } 5]}$ and $\Delta\mathcal{R}_4^{\pm[1 \text{ to } 5]}$ for the sample Ta(8 nm)/Co(2 nm)/(5 nm).

Fig. A7-2 The measured $R_{[1-6]}^{\pm 4}$ and $\Delta R_{[1-6]}^{\pm 4}$ for the sample Ta(8 nm)/Co(2 nm)/(5 nm).

Fig. A7-3 The measured $\mathfrak{R}_4^{\pm[0.8-6]}$ and $\Delta\mathfrak{R}_4^{\pm[0.8-6]}$ for the sample Ta(2 nm)/Co(2 nm)/(5 nm).

Fig. A7-4 The measured $\mathfrak{R}_4^{\pm[1-6]}$ and $\Delta\mathfrak{R}_4^{\pm[1-6]}$ for the sample Ta(4 nm)/Co(2 nm)/(5 nm).

Fig. A7-5 The measured $\Delta R_4^{\pm[0.8-6.2]}$ and R_4^0 for the sample Ta(6 nm)/Co(2 nm)/(5 nm).

Fig. A7-6 The measured $\mathfrak{H}_4^{\pm[0.8-6]}$ and $\Delta\mathfrak{H}_4^{\pm[0.8-6]}$ for the sample Ta(10 nm)/Co(2 nm)/(5 nm).

VIII. Conclusion and future work

8.1. Conclusion

The analytical derivation has been used to conduct the measurements of SOT fields in materials with in-plane magnetic anisotropy. As the experimental verification showing, both the fieldlike and dampinglike terms can be quantified by using a single harmonic Hall measurement, and the single measurement avoids the artefacts introduced due to using a two-measurement scheme. Additionally, the proposed approach does not need correction of the effective fields by the PHE contribution which was used in conventional scheme of harmonic Hall resistance measurement. The proposed approach realizes the consistent quantifications of the SOT effective fields in IMA materials. As characterization results demonstrate for the first time, in the stack with in-plane magnetic anisotropy, the fieldlike term and dampinglike term can be increased by increasing the thickness of Ta.

The proposed approach has been used to investigate the dependence of the SOT effective fields on the magnetization amplitude. The experimental results show that the SOT effective fields depend on the magnetization uniformity in Ta/Co/Pt structure. The dependence indicates that the SOT effective fields can be manipulated by varying the magnetization uniformity. The change of magnetization uniformity was achieved in each sample by applying magnetic fields along the wire long axis. As the SOT effective fields are concurrently characterized, the characterization method eliminates influences from other SOT dependence effects. In analogy to the STT, the SOT dependence on the magnetization uniformity is attributed to the electron diffusion properties. This dependence

suggests that SHE plays a significant role in the dependence of SOT effective fields on magnetization orientation. It also indicates that the SOT effective fields cannot be considered as constant parameters when analyzing domain wall dynamics *via* SOT.

A revised method for investigating the angular dependence of the SOT effective fields has proposed and validated. Based on the method, the SOT effective fields have been characterized at different azimuthal angles of the magnetization without introducing experimental artefacts. Results of the experiments validate this method for characterizing the angular dependence of SOT in the stack with IMA. The fieldlike term of SOT has been found to consist of not only a constant component but also an angular dependent component, which quantifies the contributions from the constant Rashba effect and the SHE. An analytical expression in terms of the magnetization orientation for the fieldlike term has been demonstrated. The analytical expression reveals that the fieldlike term can be adjusted to extreme by orientating magnetization at $\sim 45^\circ$. This indicates that fast precession magnetization switching could be induced by fieldlike term, and the shortest duration could be realized when magnetization initially derives from charge current direction at $\sim 45^\circ$. The dampinglike term is independent of azimuthal angle. Such independence property helps us realize that the dampinglike term remains constant when SOT induced magnetization switching process occurs in IMA stack, where the dampinglike term is generally considered dominant.

Applying a biasing direct current in the structure induces an anomalous second harmonic Hall resistance which is not from the contribution of the widely investigated spin-orbit torque. Analytical derivation results demonstrate for the first time, besides the SOT, spin accumulation can contribute to second harmonic Hall resistance. The spin accumulation

magnetoresistance indicates that the magnetization of a ferromagnetic material can be altered *via* spin injection from the spin accumulation of nonmagnetic materials. As such, besides the widely investigated SOT-induced magnetization switching, the spin accumulation causing SOT plays a role in the switching process. We have experimentally quantified the spin accumulation induced by electric current in stacks of Ta/Co/Pt. The quantification results show that the spin accumulation is around 10% of the local magnetization when the applied current density is in the order of 10^{11} Am^{-2} . The ratio of the spin accumulation over the applied electric current is consistent with spin Hall angle. Hence, the ratio can be used to evaluate the efficiency of a heavy metal in converting electric current to spin current. As harmonic Hall resistance measurement is a commonly used approach, it provides a simple solution to quantify spin accumulation.

8.2. Future work

On the SOT effective fields:

As shown in Chapters 4~6, both the fieldlike and dampinglike terms can be quantified by using a single harmonic Hall measurement, and the single measurement avoids the artefacts introduced due to using a two-measurement scheme. In the current work, the proposed measurement scheme is only applicable for changing the magnetization orientation in plane. In future, a single harmonic Hall measurement for rotating the magnetization from in plane to out of plane can be further investigated for more accurately characterize SOT dependence on the polar angle of magnetization. As observed in Chapter 5, amplitude of SOT depends on the magnetization uniformity, hence, in the future investigation of polar

angle dependence, the magnetization uniformity must be ensured.

On the spin accumulation:

Applying a DC bias in the Ta/Co/Pt wire with IMA enables the measurement of the PHE resistance induced by spin accumulation, as discussed in Chapter 7. The PHE resistance, which has been used to quantify the spin accumulation, implies the independence of the spin accumulation on the magnetization azimuthal angle. To investigate on the dependence of spin accumulation on the magnetization polar angle, a DC bias was applied in a Ta/Co/Pt wire with PMA to measure $\Re_{\alpha}^{\pm\beta}$. During the measurement, a constant magnetic field was applied along the easy axis of the wire to ensure the uniformity of magnetization, and a magnetic field $H_{x\text{-ext}}$ ($H_{y\text{-ext}}$) was swept along the long (short) axis of the wire to change the polar angle of magnetization. The measured $\Re_{\alpha}^{\pm\beta}$ with respect to the applied sweeping fields which are equivalent to the polar angles are shown in Fig. 8-1. $\Re_4^0$ is linear to the applied fields, when $H_{(x,y)\text{-ext}}$ is close to 0. The linear behavior, which has been used to characterize SOT effective fields, indicate spin accumulation in the test sample. The Hall resistance $\Delta\Re_4^{\pm6}$ induced by the spin accumulation, which are obtained as shown in Fig. 8-1, are symmetric around $H_{(x,y)\text{-ext}}=0$ Oe. The first harmonic Hall resistance $R_{1\text{stHall}}$ induced by the magnetization, as shown in Fig. 8-2, are symmetric around $H_{(x,y)\text{-ext}}=0$ Oe as well. The coincidence of the symmetric behaviors indicates that the spin accumulation follows the magnetization to vary orientation with respect to the applied field. However, irrespective of the magnitudes of $\Delta\Re_4^{\pm6}$ and $R_{1\text{stHall}}$, plots of $\Delta\Re_4^{\pm6}$ is different from $R_{1\text{stHall}}$. As the magnetization magnitude is constant with respect to the polar angle, this difference

indicates that the spin accumulation magnitude varies as a function of the angle.

However, in theory, the spin accumulation cannot induce $R_{\alpha}^{\pm\beta}$. The spin accumulation $s\mathbf{m}$ follows the magnetization $M\mathbf{m}$ to induce AHE. The amplitude of AHE resistance induced by $M\mathbf{m}$, R_{AHE} , is proportional to the magnetization magnitude *via* a coefficient κ as $R_{\text{AHE}}=\kappa M$. Analytically, the AHE resistance, r_{A} , induced by the spin accumulation $s\mathbf{m}$ is

written as $r_{\text{A}} = \left(\frac{R_{\text{AHE}}}{M} s \right) \sin \theta_0$, where θ_0 is the magnetization polar angle. Substituting s

with $\zeta(j_{\text{AC}} \sin \omega t + j_{\text{DC}})$ in the expression of r_{A} , we obtain r_{A} as

$r_{\text{A}} = \frac{R_{\text{AHE}}}{M^2} \sin \theta_0 \zeta(j_{\text{AC}} \sin \omega t + j_{\text{DC}})$. As such, the measurable Hall voltage becomes to

$v_{\text{Hall}} = \frac{R_{\text{AHE}}}{M^2} \sin \theta_0 \zeta(j_{\text{AC}} \sin \omega t + j_{\text{DC}})^2$ which gives the second harmonic Hall voltage as

$v_{2\text{ndHall}} = \frac{R_{\text{AHE}}}{M^2} \zeta^2 \sin \theta_0 j_{\text{AC}}^2 \sin^2 \omega t$. From the expression of $v_{2\text{ndHall}}$, the second harmonic

Hall resistance induced by $s\mathbf{m}$, which should be equivalent to $\Re_{\alpha}^{\pm\beta}$, is obtained as

$r_{2\text{ndHall}} = \frac{R_{\text{AHE}}}{M} \zeta^2 \sin \theta_0 j_{\text{AC}} \sin^2 \omega t$. $r_{2\text{ndHall}}$ shows independence on the DC current density

j_{DC} . However, the measured $\Re_{\alpha}^{\pm\beta}$ shows dependence on j_{DC} in our experiments. Hence, to

obtain the correct expression of AHE induced by spin accumulation is the future work.

Fig. 8-1 The measured $\mathfrak{R}_4^{\pm 6}$ when sweeping (a) $H_{x\text{-ext}}$ (b) $H_{y\text{-ext}}$, and the corresponding $\Delta\mathfrak{R}_4^{\pm 6}$.

Fig. 8-2 The obtained first harmonic Hall resistance $R_{1\text{stHall}}$ when measuring $\mathfrak{R}_4^{\pm 6}$.

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