Fracture assessment of cracked clad pipelines with weld misalignment

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Fracture Assessment of Cracked Clad Pipelines with Weld Misalignment

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SUMMARY

With increasing demand of recoverable corrosive hydrocarbons, metallurgically bonded double walled pipelines, referred to as clad pipelines, are widely used to transport the corrosive natural resources. The clad pipelines consist of a carbon steel outer pipe providing the load capacity and a corrosion resistant alloy liner protecting the carbon steel outer pipe from corrosion. The pipeline is constructed by joining one end of the pipe to the other end by girth welding process, and potential cracks are usually detected to initiate from the welding region. Furthermore, weld misalignment, centerline offset misalignment or angular misalignment, is also invariably introduced into the pipeline during the welding process, which further reduces the integrity of the pipes due to the introduction of local bending stress. Therefore, it's crucial to investigate the fracture responses of cracked clad pipelines with weld misalignment. Although BS7910–Amendment 1 [1] has proposed a fracture assessment equation for misaligned structures, the equation actually uses fracture parameters of aligned configurations to assess the corresponding parameters of misaligned ones based on superposition principle, resulting in an over-conservative prediction. Therefore, the fracture analysis for cracked configurations with weld misalignment needs to be investigated in detail.

In this study, a finite element (FE) mesh generator is developed to serve for the cracked clad pipelines with weld misalignment, and a small-scale experimental test is carried out to verify the validity of the mesh generator. Based on the mesh generator, extensive 3-D FE analysis is performed to determine the stress intensity factors (SIFs) of aligned and misaligned welded joints. The corresponding SIF equations are proposed using multiple regression analysis, which consider the effect of weld misalignment on the SIFs directly, not by the superposition principle. Then, the fracture responses for oval-shaped clad pipelines with a surface crack are analyzed, and an equivalent J -estimation procedure is built up. Finally, engineering critical assessment is conducted to investigate the effect of weld misalignment on the cracked clad pipelines, and a fracture assessment framework covering the commonly used pipe configurations is established to predict the fracture parameters.

LIST OF PUBLICATIONS

1. **H.S. Zhao**, S.T. Lie, Y. Zhang*. Fatigue assessment of cracked pipes with weld misalignment by using stress intensity factors. *International Journal of Fatigue* 116 (2018) 192-209.
2. **H.S. Zhao***, S.T. Lie. Determination of dimensionless stress intensity factor of plate-to-plate butt welds between axially aligned members of different thickness. *Engineering Fracture Mechanics* 172 (2017) 90-105.
3. **H.S. Zhao***, S.T. Lie, Y. Zhang. Stress intensity factors for semi-elliptical surface cracks in plate-to-plate butt welds with parallel misalignment of same thickness. *Marine Structures* 53 (2017) 148-163.
4. **H.S. Zhao***, S.T. Lie, Y. Zhang. Elastic-plastic fracture analyses for misaligned clad pipeline containing a canoe shape surface crack subjected to large plastic deformation. *Ocean Engineering* 146 (2017) 87-100.
5. **H.S. Zhao***, S.T. Lie, Y. Zhang. Fracture assessment of mismatched girth welds in oval-shaped clad pipes subjected to bending moment. *International Journal of Pressure Vessels and Piping* 160 (2018) 1-13.
6. **H.S. Zhao**, S.T. Lie, Y. Zhang*. Strain-based J -estimation scheme for fracture assessment of misaligned clad pipelines with a surface crack in fusion line. *Marine Structures* 61 (2018) 238-255.
7. S.T. Lie, **H.S. Zhao***, S.P. Vipin. New weld toe magnification factors for semi-elliptical cracks in plate-to-plate butt-welded joints. *Fatigue & Fracture of Engineering Materials & Structures* 40 (2017), 207-220.
8. S.T. Lie, **H.S. Zhao***. Fracture analysis of load-carrying cruciform fillet welded joints with multiple cracks. *Engineering Fracture Mechanics* 193 (2018) 90-105.
9. **H.S. Zhao**, S.T. Lie, Y. Zhang*. Prediction of stress intensity magnification factors of a weld toe crack in load-carrying cruciform fillet welded joints under axial loading. *International Journal of Steel Structures* (2018). (Revision Submitted to Journal)
10. **H.S. Zhao***, S.T. Lie, Y. Zhang. Development of finite element mesh generator for misaligned clad and lined pipes with cracks. *International Journal of Pressure Vessels and Piping* (2018). (Under Review)
11. S.T. Lie, Y. Zhang*, **H.S. Zhao**. Fracture response of girth-welded pipeline with canoe shape embedded crack subjected to large plastic deformation. *Journal of Pressure Vessel Technology* 139 (2017) 021406.
12. S.T. Lie, **H.S. Zhao***, Y. Zhang. Weld toe magnification factors for a semi-elliptical surface crack in load-carrying cruciform fillet welded joints under bending moment. *Fatigue & Fracture of Engineering Materials & Structures* (2018). (Submitted to Journal)

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LIST OF SYMBOLS

The notations used in the main text are briefly defined in the following list.

Nomenclature

a	= crack depth
a_c	= characteristic crack dimension
a_f	= final crack depth
a_i	= initial crack depth
b	= un-cracked ligament length
B	= plate width
c	= crack half length in plate-to-plate butt welds
C_O	= degree of ovality
D or D_e	= outer diameter of pipe
$D_e^{\max}$	= maximum measured outer diameter
$D_e^{\min}$	= minimum measured outer diameter
D_i	= inner diameter of pipe
e	= weld misalignment
e_{eff}	= effective weld misalignment
E	= Young's modulus
f_w	= finite width correction factor
h	= weld leg length
J	= J -integral
J_{el}	= elastic component of J -integral
J_{pl}	= plastic component of J -integral

k_m	= stress concentration factor caused by misalignment
k_{mp}	= stress concentration factor resulting from pressure
k_{mp}^{ccja}	= k_{mp} factor for a cylinder, circumferential joint, angular misalignment
k_{mp}^{ccjc}	= k_{mp} factor for a cylinder, circumferential joint, centerline offset
k_{mp}^{clja}	= k_{mp} factor for a cylinder, longitudinal joint, angular misalignment
k_{mp}^{cljc}	= k_{mp} factor for a cylinder, longitudinal joint, centerline offset
k_{ms}	= stress concentration factor resulting from supplemental loads
k_{ms}^{ccja}	= k_{ms} factor for a cylinder, circumferential joint, angular misalignment
k_{ms}^{ccjc}	= k_{ms} factor for a cylinder, circumferential joint, centerline offset
k_{ms}^{clja}	= k_{ms} factor for a cylinder, longitudinal joint, angular misalignment
k_{ms}^{cljc}	= k_{ms} factor for a cylinder, longitudinal joint, centerline offset
k_{ms}^{pa}	= k_{ms} factor for a plate, angular misalignment
k_{ms}^{pc}	= k_{ms} factor for a plate, centerline offset
k_{ta}	= axial stress concentration factor
k_{tb}	= bending stress concentration factor
K_I	= Mode-I stress intensity factor
K_{II}	= Mode-II stress intensity factor
K_{III}	= Mode-III stress intensity factor
K_m	= stress intensity factor of a semi-elliptical surface crack in a misaligned pipe

K_p	= stress intensity factor of a semi-elliptical surface crack in a plain pipe
K_r	= ratio of stress intensity factor to fracture toughness
K_{root}	= stress intensity factor of the crack at the weld root
K_{toe}	= stress intensity factor of the crack at the weld toe
K_λ	= stress intensity measure
	= weld length for butt welds
L	= weld root crack depth in load-carrying cruciform weld joints
	= half length of clad pipe
L_r	= ratio of applied load to plastic collapse load
M	= bulging correction factor
	= bending moment
M_a, M_b, M_{ka}, M_{kb}	= stress intensity magnification factors
M_k	= weld toe magnification factor
M_L	= limiting bending moment
M_y	= weld strength mismatch factor
n	= strain hardening exponent
n_1	= fatigue ductility coefficient
n_2	= fatigue strength exponent
P	= applied load
P_0	= limit load
P_0^b	= plastic limit load of the homogeneous configuration composed of the base material
P_0^{bi}	= plastic limit load of the bi-material pipe
P_0^{clad}	= plastic limit load of the welded clad pipe

P_0^{in}	= plastic limit load of the inner CRA material
P_0^{mis}	= plastic limit load of the mismatched configuration
P_0^{out}	= plastic limit load of the outer carbon steel
P_0^{w}	= plastic limit load of the homogeneous configuration composed of the weld metal
P_a	= primary axial loading
P_b	= primary bending loading
Q_a	= secondary axial loading
Q_b	= secondary bending loading
R_m	= pipe mean radius
t or T	= main plate thickness = weld thickness in clad pipe
T_i	= components of traction vector
u_i	= components of displacement vector
w	= strain energy density
Y	= geometry correction factor
Y_a	= geometry correction factor for the aligned joint under axial loading
Y_b	= geometry correction factor for the aligned joint under bending loading
Y_p	= geometry correction factor of a semi-elliptical surface crack in a plain pipe
α	= dimensionless constant in Ramberg-Osgood model = thickness transition slope
β_c	= critical intersection angle

ΔK_0	= threshold stress intensity factor range
ΔK_{eff}	= effective stress intensity factor range
ΔK_{I}	= Mode-I stress intensity factor range
ΔK_{m}	= stress intensity factor range of a semi-elliptical surface crack in a misaligned pipe
ΔK_{p}	= stress intensity factor range of a semi-elliptical surface crack in a plain pipe
$\Delta \sigma$	= applied stress range
$\Delta \sigma_{\text{a}}$	= axial component of stress range
$\Delta \sigma_{\text{b}}$	= bending component of stress range
ε'_{f}	= fatigue ductility
ε_{nom}	= applied nominal strain
ε_{p}	= plastic strain
ε_{ref}	= reference strain
ε_{u}	= uniaxial strain at the tensile strength σ_{u}
ε_{uc}	= un-cracked body equivalent strain
ε_{y}	= yield strain
θ	= weld angle
θ_{p}	= angle associated with angular misalignment
ν	= Poisson's ratio
$\bar{\sigma}$	= true stress
σ_{a}	= applied axial stress
$\sigma_{\text{b}}(\varepsilon_{\text{p}})$	= stress-strain relationship of the base material
$\sigma_{\text{eq}}(\varepsilon_{\text{p}})$	= equivalent stress-strain relationship

$\sigma_{eq}^{bi}(\epsilon_p)$	= equivalent stress-strain relationship of the bi-material pipe
$\sigma_{eq}^{clad}(\epsilon_p)$	= equivalent stress-strain relationship of the welded clad pipe
σ_f	= flow stress
σ'_f	= fatigue strength
σ_{in}	= stress-strain relationship of the inner CRA material
σ_{ls}	= local notch stress
$\sigma_{in}(\epsilon_p)$	= mean local stress at the weld toe
σ_{nom}	= applied nominal stress
$\sigma_{out}(\epsilon_p)$	= stress-strain relationship of the outer carbon steel
σ_{ref}	= reference stress
σ_s	= local bending stress caused by misalignment
σ_u	= tensile strength
σ_{uc}	= uncracked-body equivalent stress
$\sigma_w(\epsilon_p)$	= stress-strain relationship of the weld metal
σ_y	= yield stress
σ_y^{out}	= yield stress of the outer carbon steel
σ_y^w	= yield stress of the weld metal
σ_Y	= limit of proportionality with $\sigma_Y \approx 0.9\sigma_y$
μ	= shear modulus

CHAPTER 1

INTRODUCTION

1.1 OFFSHORE INSTALLATION

Due to population growth and economic development, the demand for crude oil and natural gases has increased drastically for the past decades, and it is expected to keep on growing in the coming years. As a result, more pipelines are needed to transport these natural resources, and because major oil producing fields are depleting rapidly, the oil and gas industry are forced to explore in towards more difficult reservoirs—deepwater, harsher environment, resulting in pipeline installation facing greater challenges. In this section, the most commonly used installation methods are introduced briefly.

Fig. 1.1 Schematic representation of S-lay installation method [2].

The first installation method is called the S-lay. Fig. 1.1 presents the installation process in which the pipeline is suspended from the vessel to the seabed. These S-lay pipelines are constructed by joining one end of the pipe to another end by girth welding. Then, a series of steps are arranged to inspect and coat the welds. After that the welded pipe joints are held by tensioners and supported by the stringer to leave the vessel from the stern. The purpose of the stringer is to control the overbend radius of the pipelines. After the pipeline leaves the stringer, it is straightened and then bent in the opposite direction in the sagbend. The tension applied by the vessel controls the bending radius. Excessive bending caused by the vessel movement and

external water pressure leads to local buckling or crack growth exist in the pipeline. Therefore, selection of stringer length, movement speed of the vessel, water depth, etc, are all the issues that need to be considered. The characteristic of the S-lay installation method is its high production rate, up to 6.5 km/day. However, due to the restriction of the stringer length, this method is usually used in shallow and intermediate water depth.

Fig. 1.2 Schematic representation of J-lay installation method [2].

J-lay is the second installation method. In this installation process, the pipeline leaves the vessel nearly vertical and is laid down on the seabed as shown in Fig. 1.2. At first the pre-welded pipe joints are raised into the tower and then connected to the free end of the pipeline by girth welding. After the weld is inspected and coated by the pre-determined steps, the pipe joints are lowered as the vessel moves forward, making the segment of pipeline lay on the seabed. As illustrated in Fig. 1.2, the pipeline undergoes tension in the process of suspension and combined loading in the sagbend region. Excessive loading during the installation leads to local buckling and fracture. The production rate for the J-lay installation method is lower than that of S-lay, up to 3.2 km/day. However, this method is more suitable for deeper water because it reduces the lay tension force caused by the shorter suspended pipeline length.

Fig. 1.3 Schematic representation of Reel-lay installation method [2].

Fig. 1.4 Moment-curvature response of pipeline during reeling installation.

The last installation method is called the Reel-lay installation method as shown in Fig. 1.3. Different with the previous two installation methods, the Reel-lay method allows the pre-welded pipe joints welded together onshore, which provides greater convenience for pipeline installation. After inspecting and coating the welds, the pipeline is spooled onto a large diameter reel mounted onto the installation vessel. Then, the pipeline is gradually unspooled from the reel to start installing at the predetermined site. Significant reduction in installation time and costs due to welding onshore makes the Reel-lay method the most efficient installation method with laying rates up to 3.5 km/hour. However, the pipeline is plastically deformed due to the

reeling process, experiencing high strain of nearly up to the order of 2%. The reeling installation process has a significant impact on the integrity of the pipeline, which can be illustrated by the following moment-curvature ($M-\kappa$) diagram of a pipeline under reeling, as depicted in Fig. 1.4.

During the reeling process, three stages are distinct: spooling on, unspooling and straightening. For the first stage, the pipeline end is attached to the reel and the reel rotates slowly. The pipeline is plastically deformed with the rotation of the reel (0-1). For the second stage, as the pipeline is pulled of the reel to the aligner (1-2), it is straightened and bent reversely. On the aligner, the pipeline is bent to a fixed bending curvature (2-3). As the pipeline moves from aligner to the straightener (3-4), the pipeline is straightened and reversely bent in the straightener (4-5), resulting in zero moment and curvature after unloading (5-0). The multiple bending cycles induce plastic deformation in the pipeline, posing tremendous stresses on the pipeline.

1.2 INTRODUCTION OF CLAD PIPES

In recent years, the need for corrosion mitigation grows with increasing volume of recoverable corrosive hydrocarbons. When the hydrocarbons are mildly corrosive, ordinary steel pipelines are still suitable to be used in practice. However, when the corrosivity of the hydrocarbons increases and the safety conditions need to be guaranteed, ordinary steel pipelines cannot suffice. In this case, a corrosion resistant alloy pipeline should be used instead. Due to the high costs in solid corrosion resistant alloy pipelines, double walled pipe is a more cost efficient option. The double walled pipe consists of a carbon steel outer pipe that provides the load capacity and a corrosion resistant alloy liner, protecting the carbon steel outer pipe from corrosion. There are mainly two kinds of double walled pipes: metallurgically bonded double walled pipe, referred as clad pipe, and mechanically-bonded double walled pipe, referred as lined pipe. The focus of the present research works is on clad pipes.

The metallurgically clad pipe is mainly produced from clad plate, consisting of a base material and a corrosion resistant alloy material. The plate is firstly produced on a press brake. Then, the clad plate is formed into an open-seam pipe by pressing in J form first, then, in C form and in O form eventually, called JCO process for short (Fig. 1.5). The open-seam pipe is seamed together by longitudinal welding process and the metallurgically clad pipe (Fig. 1.6) is finished eventually.

Fig. 1.5 Different steps from plate to open-seam pipe by JCO process.

Fig. 1.6 The metallurgically clad pipe.

1.3 FAILURE MODES OF CLAD PIPES

One of the failure modes for clad pipes is local buckling or collapse. The offshore pipeline exhibits significant deformation during reeling installation or in-service operation, which may cause local buckling and collapse of its wall. A buckled part is not acceptable because it prevents hydrocarbon flow and induces stress concentration at the buckled area, resulting in fatigue cracks under cyclic loading. For clad pipes, the local buckling and collapse responses are similar with those of single walled pipes because the cladding layer is metallurgically bonded together with the carbon steel outer pipe. The behaviour of these pipelines has been extensively studied by many researchers [3-10].

Another failure mode is fracture. Pipelines are constructed by joining one end of the pipe to the other end by girth welding process. During welding, some defects such as slag inclusions, porosity or imperfect fusion, may be introduced into the welds. The potential defects pose a great challenge to the reliability of the pipelines during installation or in-service operation.

Therefore, engineering assessment methods are needed to assess the integrity of pipelines. Currently, several important codes and standards such as R6 and British Standard 7910:2013 (BS 7910:2013) [1], have been designed to check the fitness-for-service assessment of cracked or defective pipelines. However, the generic Engineering Critical Assessment (ECA) cannot be directly used for reliability analysis of double walled clad pipes and special guidelines must be developed in practice.

1.4 ASSESSMENT OF WELD MISALIGNMENT ON CLAD PIPES

Weld misalignment invariably exists in many welded plates or pipes. Two common types of misalignment, centreline offset and angular misalignment, are presented in Fig. 1.7. In these two cases, the misalignment causes the local bending stress which may increase the risk of brittle fracture and shorten the fatigue life of the welded joint. Hence, the assessment due to weld misalignment plays an important role in the current applications of fitness-for-service (FFS) procedures.

Fig. 1.7 Examples of weld misalignment: (a) centreline offset and (b) angular misalignment.

Many existing assessment equations for weld misalignment have been obtained for various plate or pipe geometries for different types of misalignment. However, there are very few related investigations on the effect of weld misalignment on clad pipes. Moreover, these assessment equations use fracture parameters of aligned configurations to evaluate the corresponding parameters of misaligned ones by introducing the stress concentration effect induced by misalignment into them, which usually produces over-conservative results, especially for fully plastic analysis. Hence, the effect of weld misalignment on the fracture responses of cracked clad pipes deserves special attention and detailed investigations.

1.5 RESEARCH OBJECTIVES

Two major failure modes, plastic collapse and fracture, of clad pipes are described in the previous sections. For local buckling and collapse of pipes due to large plastic deformation, they are not covered in the current research works. Fracture analysis of welded pipes with cracks is the focus of this research. Most importantly, the influence of weld misalignment on the prediction of fracture and fatigue in clad pipes will be investigated and discussed in the following chapters. In order to accomplish the goal, the research works are sub-divided into the following objectives:

1. To develop a robust finite element (FE) mesh generator for modelling aligned and misaligned welded joints such as plate-to-plate butt joints and load-carrying cruciform fillet-welded joints.
2. To conduct extensive parametric study on the cracked plate welded joints and propose new equations for determining the stress intensity magnification factors (M_k) of these configurations.
3. To conduct extensive parametric study on misaligned pipes with cracks and propose new stress intensity factor (SIF) equations for the pipe joints with various weld misalignment.
4. To develop a FE mesh generator for creating FE models of aligned clad pipelines with surface cracks.
5. To modify the mesh generator mentioned in the above to serve for the misaligned clad pipelines by introducing a coordinate transformation method.
6. To verify the validity of the coordinate transformation method by carrying out a small-scale experimental test on misaligned steel pipes.
7. To conduct fracture analyses of mismatched girth welds in oval-shaped clad pipelines containing a surface crack and develop an equivalent engineering critical assessment methodology.

8. To perform fracture analyses of misaligned clad pipelines with a circumferential surface crack and propose J -estimation formulations for fracture assessment.

1.6 ORIGINAL CONTRIBUTIONS

The original contributions of this study are summarized as follows:

1. Development of a new and robust FE mesh generator to model aligned and misaligned plate or pipe welded joints with cracks. Mesh generation and parametric study of misaligned clad pipes containing a surface crack are carried out for the first time.
2. Proposal of new M_k equations for plate-to-plate butt welds and load-carrying cruciform fillet-welded joints using multiple regression analysis. These equations are applicable for a wide range of crack size and weld geometry.
3. Proposal of new M_k equations for cracked pipes with centerline offset weld misalignment of equal thickness but unequal diameters based on linear elastic FE analysis.
4. Determination of M_k equations for cracked pipes with centerline offset weld misalignment of unequal thickness but aligned inside surface. The proposed M_k equations can be employed in the fatigue strength calculation and fatigue life prediction for the most commonly used pipe configurations encountered in practice.
5. Proposal of a strain-based failure assessment diagram (FAD) for aligned clad pipes containing a circumferential surface crack because the fracture assessments presented in BS 7910:2013 [1] produce an over-conservative prediction on the evolution of crack tip opening deformation (CTOD).
6. Development of notch stress concentration factor (NSCF) equations for misaligned steel pipes. A coordinate transformation method developed in the study is employed in the FE mesh generator to convert an aligned configuration to a misaligned one. Then, a small-scale experimental test on misaligned pipes is carried out to verify the validation of the transformation method, demonstrating the accuracy of the mesh generator aiming for misaligned pipes.

7. Development of an engineering fracture assessment of oval-shaped clad pipes with a circumferential part-through surface crack based upon equivalent stress-strain relationship method (ESSRM) in conjunction with EPRI J -estimation procedure. Plastic limit load solutions are developed specially to determine the equivalent stress-strain relationship of the welded clad pipe.
8. Development of a reference strain J -estimation formulation for the misaligned clad pipes containing a circumferential part-through crack at the interface of weld metal and outer pipe incorporating the effects of various parameters (pipe diameter, crack depth, crack length, centerline offset, strength mismatch level, and strain hardening exponent) on the J -integral.

1.7 THESIS OUTLINE

The thesis is organized as follows:

- Chapter 2 briefly reviews the past and present related research works, including the stress-based fracture assessment procedures, strain-based fracture assessment procedures and the effect of misalignment on the prediction of fracture and fatigue in welded joints.
- Chapter 3 describes the development of the FE mesh generator for aligned and misaligned clad pipes. A coordinate transformation method is employed in the mesh generator to transform an aligned configuration to a misaligned one. Then, a small-scale experimental test on misaligned steel pipes is conducted to verify the accuracy of the transformation method.
- Chapter 4 proposes the M_k equations of aligned plate welded joints. The equations for a semi-elliptical crack in plate-to-plate butt joints and load-carrying cruciform fillet-welded joints, respectively, are established using multiple regression analysis.
- Chapter 5 proposes the M_k equations for misaligned pipe joints based on linear elastic FE analysis, and the fatigue life assessment for the misaligned pipes is carried out by using the M_k equations.

- Chapter 6 presents the fracture assessment of mismatched girth welds in oval-shaped clad pipelines containing a surface crack, and an ESSRM/EPRI estimation framework is provided for the cracked pipeline.
- Chapter 7 proposes the fracture assessment methodologies for misaligned clad pipelines with a circumferential part-through surface crack. A reference-strain J -estimation formulation is developed for the cracked clad pipelines with centerline offset weld misalignment.
- Chapter 8 summarizes the discussions, conclusions and future works on the fracture assessment of misaligned clad pipes configurations.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Over the past decades, with the development of oil and gas industries, large numbers of offshore pipelines are used for transportation purposes. During the reeling installation or in-service operation, the pipelines undergo large plastic deformation, resulting in plastic collapse or fracture failure. Therefore, extensive related research works have been conducted on the reliability of offshore pipelines. This chapter covers the literature review for the current study. The three fracture parameters is presented briefly in this section. Section 2.2 illustrates the stress-based fracture assessment procedures on cracked pipelines used to assess the acceptability of cracks. This section includes two subsections. Subsection 2.2.1 illustrates the elastic-plastic J -integral analysis which incorporates the Electric Power Research Institute (EPRI) J -estimation procedure and reference stress approach. Subsection 2.2.2 describes the mostly widely used assessment procedure for elastic-plastic fracture analysis of structural components—failure assessment diagram. The development on strain-based fracture assessment methodology is outlined in Section 2.3. Section 2.4 reviews the effect of weld misalignment on the fracture and fatigue responses of cracked weld joints. During the welding process, there exists invariably some degree of misalignment. The presence of weld misalignment has a significant influence on the fracture and fatigue analysis of any structural component. The fracture analysis of aligned and misaligned pipes is presented in Section 2.5.

Fracture mechanics is a field of mechanics aiming to investigate the crack propagation in structural members. Three parameters are always used to describe the crack-tip conditions in materials. These are the stress intensity factor (K) proposed by Irwin [11], crack tip opening displacement (CTOD) published by Wells [12] and Cottrell [13], and line contour integral (J -integral) originally developed by Rice [14]. Stress intensity factor (K) is used to describe the intensity of the stress field at the crack tip, which is applicable only to linear elastic materials. A subscript is usually given in the stress intensity factor to represent the mode of loading, viz., K_I , K_{II} or K_{III} (Fig. 2.1). The expression for K_I is given by

$$K_I = Y\sigma\sqrt{\pi a} \quad (2.1)$$

where Y is the geometrical correction factor, σ is the characteristic stress and a is the characteristic crack depth.

Fig. 2.1 The three modes of loading applied to a crack [15].

Linear elastic fracture mechanics (LEFM) is valid only when the nonlinear deformation occurs in a small region around the crack tip. However, for many materials, it is impossible to apply LEFM to describe the fracture behaviors at the crack tip. Therefore, elastic-plastic fracture mechanics (EPFM) is developed to characterize the fracture responses of materials exhibiting large nonlinear deformation. Two elastic-plastic parameters (CTOD and J -integral) are introduced to describe the crack-tip conditions in elastic-plastic materials. There are several methods to define the CTOD. The displacement at the crack tip and the 90° intercept are the two most commonly used definitions as shown in Fig. 2.2, and the latter is frequently used to calculate the CTOD in finite element (FE) simulation. Rice [14] developed a path-independent contour integral called J -integral to analyze the crack propagation, which is expressed as

$$J = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} ds \right) \quad (2.2)$$

where w is the strain energy density, T_i is the components of the traction vector, u_i is the displacement vector components, and ds is the length increment along the contour Γ , as shown in Fig. 2.3. Based on the fracture mechanics theory, a number of fracture mechanics-based

assessment procedures are used to assess the acceptability of flaws in pipeline girth welds, which will be covered in the following subsections.

Fig. 2.2 Definitions of CTOD: (a) displacement at the original crack tip and (b) displacement at the intersection of a 90° vertex [15].

Fig. 2.3 Arbitrary contour around the tip of the crack [15].

2.2 STRESS-BASED FRACTURE ASSESSMENT PROCEDURES

Most of the configurations under external loadings are in load-controlled conditions. Hence, stress-based fracture assessment methodologies are employed to evaluate the reliability of cracked structural components.

2.2.1 Elastic-plastic J -integral analysis

Two J -estimation methods, viz., the Electric Power Research Institute (EPRI) J -estimation procedure [15] and the reference stress approach [16], are presented in this subsection. In order to enable users to determine the fracture driving force of elastic-plastic materials, a series of J handbooks were developed by EPRI to satisfy the need [17-21]. The EPRI procedure proposes

a means to extend the J -estimation procedure over the full range of elastic-plastic regime by assuming that the total J is equal to the sum of the elastic and plastic components:

$$J = J_{el} + J_{pl} \quad (2.3)$$

The first term on the right side of Eq. (2.3) is actually the elastic energy release rate which can be calculated from K_I by the following equation:

$$J_{el} = \frac{K_I^2}{E'} \quad (2.4)$$

where K_I is the Mode-I stress intensity factor, $E' = E/(1-\nu^2)$ with E and ν representing the Young's modulus and Poisson's ratio, respectively. The second term of Eq. (2.3), fully plastic equation for J , can be directly derived from finite element analysis.

For a fully plastic cracked structure, the elastic strain remains small in comparison to the plastic strains. The elastic strain is usually negligible when the pipelines are subjected to large plastic deformation. Hence, it's reasonable to assume that the material follows a pure power law, which is actually the second term of Ramberg-Osgood model [22]:

$$\frac{\varepsilon_p}{\varepsilon_y} = \alpha \left(\frac{\bar{\sigma}}{\sigma_y} \right)^n \quad (2.5)$$

where $\bar{\sigma}$ defines the true stress value, ε_p is the plastic strain, σ_y is the yield stress, $\varepsilon_y = \sigma_y/E$ is the strain corresponding to the yield stress, α is a dimensionless constant, and n is the strain hardening exponent.

Based on the above pure power-law stress-stain relationship and close to the crack tip under J -controlled conditions, the stresses are obtained by the Hutchinson-Rice-Rosengren (HRR) singularity [23,24]:

$$\sigma_{ij} = \sigma_y \left(\frac{J}{\alpha \varepsilon_y \sigma_y I_n r} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(n, \varphi) \quad (2.6)$$

where (r, φ) are the cylindrical coordinate centered at the crack tip, I_n is an integration constant that depends on n and $\tilde{\sigma}_{ij}(n, \varphi)$ is a dimensionless function of n and φ . Then, the above equation is transformed into the following form:

$$J = \frac{\alpha \varepsilon_y \sigma_y I_n r}{[\tilde{\sigma}_{ij}(n, \varphi)]^{n+1}} \left(\frac{\sigma_{ij}}{\sigma_y} \right)^{(n+1)} \quad (2.7)$$

For J -controlled conditions, the local stress must increase in proportion to the remote load P . Therefore, the above expression can be written in form of P :

$$J_{pl} = \alpha \varepsilon_y \sigma_y b h_1 (a/W, l, n) \left(\frac{P}{P_0} \right)^{(n+1)} \quad (2.8)$$

where a is the crack length, W denotes the specimen width, $b = W - a$ is the un-cracked ligament length, l represents the characteristic length for the specimen and P_0 is the corresponding limit load. h_1 is a dimensionless parameter dependent on the crack size, configuration geometry and strain hardening exponent. Eq. (2.8) is the estimate of fully plastic J -integral.

The EPRI J -estimation procedure [15] mentioned in the above assumes that the material analyzed follows a simple power law. However, the flow behavior exists in many materials, which deviates significantly from the power law. In this case, applying Eq. (2.8) for fracture assessment produces significant errors. Therefore, a reference stress J -estimation scheme is developed by Ainsworth [16] to reflect more closely the flow behavior of materials by defining a reference stress σ_{ref} as follows:

$$\frac{\sigma_{ref}}{\sigma_y} = \frac{P}{P_0} \quad (2.9)$$

Substituting Eq. (2.9) into Eq. (2.8) obtains

$$J_{pl} = \sigma_{ref} b h_1 \left(\varepsilon_{ref} - \frac{\sigma_{ref} \varepsilon_y}{\sigma_y} \right) \quad (2.10)$$

Eq. (2.10) can predict the same J -integral value with Eq. (2.8) for a material obeying the power law. However, for other types of stress-strain relationships, Eq. (2.10) is equally applicable, which is more general in comparison with Eq. (2.8).

It is observed that Eq. (2.10) still contains h_1 , which is the dimensionless parameter depending on strain hardening exponent. Ainsworth [16] noticed that h_1 displayed less sensitive to n except at high n values. Hence, the following approximation was proposed,

$$h_1(n) \approx h_1(1) \quad (2.11)$$

where $h_1(1)$ is the corresponding parameter for a linear elastic material. Substituting Eq. (2.11) into Eq. (2.8) obtains the following relationship between the J_{pl} and the K_I ,

$$J_{pl} = \frac{\mu K_I^2}{E} \left(\frac{E \varepsilon_{ref}}{\sigma_{ref}} - 1 \right) \quad (2.12)$$

where $\mu = 0.75$ for plain strain and $\mu = 1.0$ for plain stress. For the EPRI J -estimation procedure [15], the solutions of stress intensity factor and h_1 must be determined to calculate the elastic and plastic term of J -integral, respectively. However, the reference stress approach [16] makes it possible to evaluate the J from the elastic stress intensity factors, which is not only simpler, but also more applicable by comparing with the EPRI J -estimation procedure [15].

2.2.2 Failure assessment diagram

Fig. 2.4 The strip-yield failure assessment diagram [15].

The failure assessment diagram (FAD) is probably the most commonly-used fracture assessment method for the elastic-plastic analysis of structural components. The basis of FAD was originally derived from the strip-yield model [25,26], which is illustrated in Fig. 2.4. Based on the FAD, a structure can fail by two mechanisms: brittle fracture and plastic collapse. The acceptability of the crack is determined by the relative position of the calculated assessment point in FAD. If the point lies within the FAD curve, the crack is acceptable. If the point lies on or outside the FAD curve, the crack is unacceptable.

BS 7910:2013 [1] includes three alternative assessment approaches for general applications, Options 1, 2 and 3. From Options 1 to 3, the complexity and the accuracy of the results are gradually increased based on the required materials and analysis data, and for every approach, the assessment line is determined by the equation of a curve, $K_r = f(L_r)$ and the definitions of K_r and L_r are as follows:

$$K_r = \frac{K}{K_{\text{mat}}} \quad (2.13)$$

$$L_r = \frac{P}{P_0} = \frac{\sigma_{\text{ref}}}{\sigma_y} \quad (2.14)$$

In order to prevent plastic collapse, the cut-off is set at the assessment line at which $L_r = L_{r,\text{max}}$, and the mean tensile properties of materials are used to define the cut-off value, which is shown in the following:

$$L_{r,\text{max}} = \frac{\sigma_y + \sigma_u}{2\sigma_y} \quad (2.15)$$

where σ_u is the tensile strength of the material.

Option 1 is a simplified assessment approach and does not require detailed material or stress analysis data, but provides a conservative result for the integrity assessment of structures. The assessment line is described by the following equations:

$$f(L_r) = \left(1 + \frac{1}{2}L_r^2\right)^{-1/2} \left[0.3 + 0.7 \exp(-\mu L_r^6)\right] \quad \text{for } L_r \leq 1 \quad (2.16)$$

$$f(L_r) = f(1)L_r^{(N-1)/(2N)} \quad \text{for } 1 < L_r \leq L_{r,\text{max}} \quad (2.17)$$

$$f(L_r) = 0 \quad \text{for } L_r \geq L_{r,\max} \quad (2.18)$$

where

$$\mu = \min \left(0.001 \frac{E}{\sigma_y}, 0.6 \right) \quad (2.19)$$

$$N = 0.3 \left(1 - \frac{\sigma_y}{\sigma_u} \right) \quad (2.20)$$

The curve depicted by the above equations is unsuitable for materials exhibiting a yield discontinuity.

Option 2 failure assessment approach can produce more accurate results compared with Option 1 with detailed stress-strain data needed, and it is suitable for all materials regardless of whether yield discontinuity exist. The Options 2 failure assessment curve is described by the following equations:

$$f(L_r) = \left(\frac{E\varepsilon_{\text{ref}}}{L_r\sigma_y} + \frac{L_r^3\sigma_y}{2E\varepsilon_{\text{ref}}} \right)^{-1/2} \quad \text{for } L_r < L_{r,\max} \quad (2.21)$$

$$f(L_r) = 0 \quad \text{for } L_r \geq L_{r,\max} \quad (2.22)$$

Option 3 failure assessment approach is specific to particular material, structure configuration and loading condition which can be determined by elastic and elastic-plastic analyses of the cracked structure. Unlike Option 1 and Option 2, Option 3 is not for general use, just for specific cases. However, the option presents the most accurate result compared with the previous two options, and the corresponding equations defining the curve shape are as follows:

$$f(L_r) = \sqrt{\frac{J_{\text{el}}}{J}} \quad \text{for } L_r < L_{r,\max} \quad (2.23)$$

$$f(L_r) = 0 \quad \text{for } L_r \geq L_{r,\max} \quad (2.24)$$

where J_{el} is the value of J -integral based on the elastic analysis and J is the values of J -integral according to the elastic-plastic analysis.

2.3 STRAIN-BASED FRACTURE ASSESSMENT PROCEDURES

The fracture assessment approaches mentioned in the above are usually obtained from the load-controlled conditions. However, in several circumstances, the structure is subjected to displacement- or strain-controlled loads deforming into the plastic regime, such as pipelines during the reeling installation stage. In this case, strain-based approach is preferably employed for fracture assessment. Moreover, based on the fact presented by the Electric Power Research Institution (EPRI) J -estimation procedure [15], the plastic component J_{pl} of J -integral varies with P^{n+1} for a power-law material in the fully plastic regime. Hence, a slight error in the load results in a significant error in the estimated J . However, J_{pl} varies with $\Delta_p^{(n+1)/n}$ based on the EPRI methodology where Δ_p is the plastic component of load-line displacement (Fig. 2.5). The strain hardening exponent n is usually much larger than unity for most commonly used pipelines, resulting in nearly a linear dependence of J_{pl} on Δ_p in the fully plastic regime. Therefore, the displacement or strain is a much more accurate indicator of the estimated J for a cracked configuration.

Fig. 2.5 Comparison of J -load and J -displacement curves for a centre-cracked panel: (a) J vs. load and (b) J vs. load-line displacement [15].

In 1990s, Schwalbe [27] proposed a strain-based engineering treatment model on the CTOD and J -integral for plane stress conditions, which was obtained based on a symmetric crack with a total length of $2a$ in a plate of width $2W$. The J for uniform tensile loading is given as

$$J = 0.81\sigma_Y^{1-1/n} (1-a/W)^2 E^{1/n} \pi a F_{\text{eff}}^2 \left[1 + 0.405(1-a/W)^2 F^2 \right] \varepsilon_a^{1+1/n} \quad (2.25)$$

and

$$J = 1.14\pi a \sigma_Y (E/\sigma_Y)^{1/n} \varepsilon_a^{1+1/n} \quad (2.26)$$

where σ_Y is the limit of proportionality with $\sigma_Y \approx 0.9\sigma_y$, F is the dimensionless constant that depends on the structure geometry and the mode of loading, F_{eff} denotes $F(a_{\text{eff}})$ where $a_{\text{eff}} = a + (K/\sigma_Y)^2/2\pi$, ε_a is the applied strain. As an extension of reference stress method, a reference strain J -estimation procedure was developed by Linkins *et al.* [28] to deal with strains directly based on the un-cracked body strain, which was expressed in the following:

$$J = 2F^2 \pi a \sigma_{\text{uc}} \varepsilon_{\text{uc}} \quad (2.27)$$

where σ_{uc} and ε_{uc} are the uncracked-body equivalent stress and strain, respectively. Then, a more accurate approximation [28] was proposed for small cracks in tension,

$$J = F^2 \pi a (\sigma_{\text{uc}} \varepsilon_{\text{uc}} + \frac{\sigma_{\text{uc}}^5}{2E^2 \varepsilon_{\text{uc}} \sigma_y^2}) \quad (2.28)$$

Subsequently, strain-based FAD methods were developed by Budden [29] to perform the fitness-for-service defect assessment of cracked configurations subjected to strain- or displacement-controlled loads in excess of yielding. Different from the stress-based FAD, the abscissa of the strain-based FAD is defined as D_r instead of L_r ,

$$D_r = \varepsilon_{\text{ref}} / \varepsilon_y \quad (2.29)$$

By Substituting Eq. (2.14) into Eq. (2.29), D_r becomes

$$D_r = L_r (E \varepsilon_{\text{ref}} / \sigma_{\text{ref}}) \quad (2.30)$$

The collapse limit of D_r in the strain-based FAD, which is consistent with the stress-based limit of $L_{r, \max}$, is given as

$$D_{r, \max} = \frac{\varepsilon_y + \varepsilon_u}{2\varepsilon_y} \quad (2.31)$$

where ε_u is the uniaxial strain at the tensile strength σ_u based on the true stress-strain curve. The following is to introduce the Option 1, Option 2 and Option 3 failure assessment approaches of strain-based FADs developed based on the corresponding failure assessment options of stress-based FADs.

Option 1 is a general assessment approach and the assessment line is described by the following equations:

$$\begin{aligned} f^*(D_r) &= (1 + 0.5D_r^2)^{-0.5}, \quad D_r \leq 1 \\ f^*(D_r) &= f^*(1)D_r^{-0.5}, \quad D_r > 1 \end{aligned} \quad (2.32)$$

Obviously, Option 1 does not require detailed material or strain analysis data.

The failure assessment equations for Options 2 are given as

$$f^*(D_r) = \left(\frac{\sigma_{uc}^2}{2E\sigma_f\varepsilon_{uc}} \right)^{0.5} = \left(\frac{\sigma_{uc}^2}{2ED_r\sigma_f\varepsilon_y} \right)^{0.5} \quad (2.33)$$

and

$$f^*(D_r) = \left(\frac{E\varepsilon_{uc}}{\sigma_{uc}} + \frac{\sigma_{uc}^3}{2E\sigma_y^2\varepsilon_{uc}} \right)^{-0.5} = \left(\frac{ED_r\varepsilon_y}{\sigma_{uc}} + \frac{\sigma_{uc}^3}{2E^3D_r\varepsilon_y^3} \right)^{-0.5} \quad (2.34)$$

where σ_f is the flow stress with $\sigma_f = (\sigma_y + \sigma_u)/2$.

The function f^* of Option 3 is defined as

$$f^*(D_r) = (J_{el}/J)^{0.5} \quad (2.35)$$

which is obtained based on detailed elastic and elastic-plastic FE analysis.

Budden [29] applied the proposed strained-based FADs to a plate with $a/t = a/c = 0.2$ and $n = 5$, and the results were shown in Fig. 2.6. It is noted that the Option 2 assessment curve of Eq. (2.34) is outside the FE results, which means that Eq. (2.34) underestimates the J values. Hence, Eq. (2.34) should be employed with caution when applying it for the failure assessment, and further consideration is required to validate and refine the strain-based FADs before using it in engineering practice.

Fig. 2.6 Comparison of the Option 1, Option 2 and Option 3 strain-based FADs for the plate with $a/t = a/c = 0.2$ and $n = 5$ under tension [29].

2.4 EFFECT OF MISALIGNMENT ON FRACTURE AND FATIGUE IN WELDED JOINTS

Misalignment, axial or angular, or both, exists frequently in welded configurations. For welded joints subjected to axial loading, local bending stress is introduced due to the presence of weld misalignment. Therefore, the actual stress is the sum of the axial stress and local bending stress. In order to take into account the effect of weld misalignment, the stress concentration factor (SCF), k_m , is defined by the following expression [1]:

$$k_m = \frac{\sigma_a + \sigma_s}{\sigma_a} = 1 + \frac{\sigma_s}{\sigma_a} \quad (2.36)$$

where σ_a is the axial loading and σ_s is the local bending stress due to the presence of misalignment. For combined misalignment cases, the total bending stress is the sum of the

bending stress induced by each case. Using axial and angular misalignments as an example, the expression of k_m is given by [1]:

$$k_m = 1 + (k_m - 1)_{\text{axial}} + (k_m - 1)_{\text{angular}} \quad (2.37)$$

Misalignment has an influence on the stress intensity factor (SIF) for the cracked structure configurations. For the past decades, extensive research works have been conducted by experimental tests or finite element simulations to investigate the effect of misalignment on fracture and fatigue responses of weld joints. Maddox [30] investigated the influence of misalignment on the fatigue strength of transverse butt welds, and he found that the fatigue results for aligned joints subjected to axial loading could be used to predict the results for misaligned joints if the bending stress induced by misalignment was obtained by introducing the stress concentration factor k_m , which was expressed as follows:

$$K_I = Y_a \sigma_a \sqrt{\pi a} + (k_m - 1) Y_b \sigma_a \sqrt{\pi a} \quad (2.38)$$

where Y_a and Y_b are the geometry factors for the aligned joint subjected to axial and bending loading, respectively, and σ_a is the axial stress. In order to determine whether the above analysis results were applicable to other geometry configurations, Andrews [31] considered transverse load-carrying cruciform joint as shown in Fig. 2.7, and obtained the same conclusion.

Fig. 2.7 Cruciform joint geometry: (a) axially misaligned joint and (b) cruciform joints with (i) axial or (ii) angular misalignment [31].

BS 7910:2013 [1] presents the detailed assessment procedures for the SIF of cracked structures with misalignment. The form of SIF has been given in Eq. (2.1). The following equation is used for fracture assessment:

$$Y\sigma = (Y\sigma)_p + (Y\sigma)_s \quad (2.39)$$

where $(Y\sigma)_p$ and $(Y\sigma)_s$ are the contributions of primary stress and secondary stress, respectively. Their expressions are listed in the following:

$$(Y\sigma)_p = Mf_w \left\{ k_{ta} M_{ka} M_a P_a + k_{tb} M_{kb} M_b \left[P_b + (k_m - 1) P_a \right] \right\} \quad (2.40)$$

$$(Y\sigma)_s = M_a Q_a + M_b Q_b \quad (2.41)$$

where M is the bulging correction factor, f_w is the finite width correction factor, k_{ta} and k_{tb} are the axial stress and bending stress concentration factors, respectively, M_a , M_b , M_{ka} and M_{kb} are all the stress intensity magnification factors, P_a and P_b are the primary axial stress and bending stress, respectively, Q_a and Q_b are the secondary primary stress and bending stress, respectively. Clearly, the misalignment is introduced into Eq. (2.40) by k_m .

For fatigue assessment, the corresponding stress intensity factor range is given by

$$\Delta K_I = Y(\Delta\sigma)\sqrt{\pi a} \quad (2.42)$$

where $\Delta\sigma$ is the applied stress range, and the corresponding expression for $Y\Delta\sigma$ is given as follows:

$$(Y\Delta\sigma)_p = Mf_w \left\{ k_{ta} M_{ka} M_a \Delta\sigma_a + k_{tb} M_{kb} M_b \left[\Delta\sigma_b + (k_m - 1) \Delta\sigma_a \right] \right\} \quad (2.43)$$

where $\Delta\sigma_a$ and $\Delta\sigma_b$ are the axial and the bending component of stress range, respectively. Obviously, the determination of SCF caused by misalignment is significant due to the effect of misalignment on the fracture and fatigue analysis of structural components. Therefore, greater emphasis on research has been placed to determine the SCFs for various geometry configurations [32-34].

API 579-1/ASME FFS-1 [35] has provided many SCF equations for various misaligned geometry configurations. In the following equations, the quantity k_{mp} denotes the SCF resulting

from pressure of misaligned welded joints, and the quantity k_{ms} represents the SCF resulting from supplemental loads.

1) Flat plates:

$$k_{mp} = -1 \quad (2.44)$$

$$k_{ms} = k_{ms}^{pc} + k_{ms}^{pa} \quad (2.45)$$

where the superscripts “pc” and “pa” denote a plate with centerline offset and a plate with angular misalignment, respectively.

For centreline offset misalignment (Fig. 2.8):

$$k_{ms}^{pc} = 1 + \left(\frac{6e}{t_1} \right) \left[1 + \left(\frac{t_2}{t_1} \right)^{1.5} \right]^{-1} \quad (2.46)$$

For angular misalignment (Fig. 2.9):

$$k_{ms}^{pa} = \left(\frac{3\delta}{t} \right) C_f \quad (2.47)$$

$$C_f = \frac{\tanh(\beta/2)}{(\beta/2)} \quad \text{for fixed ends} \quad (2.48)$$

$$C_f = \frac{2 \tanh \beta}{\beta} \quad \text{for pinned ends} \quad (2.49)$$

$$\beta = \frac{L}{t} \sqrt{\frac{3\sigma_m}{E}} \quad (2.50)$$

$$\delta = \frac{L\theta_P}{4} \quad (2.51)$$

(a) Same Thickness -- Inside and Outside Surfaces Not Aligned

(b) Different Thickness -- Alignment With One Surface

(c) Different Thickness -- Inside and Outside Surfaces Not Aligned

Fig. 2.8 Centreline offset misalignment in butt-welded joints in flat plates [35].

(a) Angular Weld Misalignment

(b) Angular and Centerline Offset Weld Misalignment

Fig. 2.9 Angular misalignment in butt-welded joints in flat plates [35].

2) Cylinders – Circumferential joints:

$$k_{mp} = k_{mp}^{ccjc} + k_{mp}^{ccja} \quad (2.52)$$

$$k_{ms} = k_{ms}^{ccjc} + k_{ms}^{ccja} \quad (2.53)$$

where the superscript “ccjc” denotes a cylinder with circumferential joints and centerline offset, and “ccja” denotes a cylinder with circumferential joints and angular misalignment.

For centerline offset misalignment (Fig. 2.10):

$$k_{mp}^{ccjc} = \text{abs} \left\{ \frac{12}{R_1 t_1} \left[0.25672 R_2 t_2 \left(\frac{C_1}{C_3} \right) + \frac{(R_1^2 - R_2^2)}{4} \left(\frac{C_2}{C_3} \right) \right] \right\} \quad (2.54)$$

$$k_{ms}^{ccjc} = 1 + \frac{6(R_1 - R_2)}{t_1} [1 + \rho^{1.5}]^{-1} \quad (2.55)$$

$$C_1 = (\rho - 1)(\rho^2 - 1) \quad (2.56)$$

$$C_2 = \rho^2 + 2\rho^{1.5} + 1 \quad (2.57)$$

$$C_3 = (\rho^2 + 1)^2 + 2\rho^{1.5}(\rho + 1) \quad (2.58)$$

$$\rho = \frac{t_2}{t_1} \quad \text{where } t_2 \geq t_1 \quad (2.59)$$

For angular misalignment (Fig. 2.11):

$$k_{mp}^{ccja} = \max \left(\frac{C_1}{C_2}, \frac{C_3}{C_4} \right) \quad (2.60)$$

$$C_1 = 0.023748 - 0.010087 \ln(S_p) + 0.0014571 [\ln(S_p)]^2 + 11.631\theta_p + 10.476\theta_p^2 - 23.792\theta_p^3 \quad (2.61)$$

$$C_2 = 1.0 - 0.36581 \ln(S_p) + 0.062036 [\ln(S_p)]^2 - 0.0044239 [\ln(S_p)]^3 + 0.20821\theta_p \quad (2.62)$$

$$C_3 = -0.037285 - 0.0051687 \ln(S_p) + 0.0072395 [\ln(S_p)]^2 + 14.865\theta_p - 33.1636\theta_p^2 + 91.061\theta_p^3 \quad (2.63)$$

$$C_4 = 1.0 - 0.35912 \ln(S_p) + 0.065885 [\ln(S_p)]^2 - 0.0054959 [\ln(S_p)]^3 + 0.044263\theta_p \quad (2.64)$$

$$S_p = \sqrt{\frac{12(1 - \nu^2)PR^3}{Et^3}} \quad (2.65)$$

$$\theta_p = \arctan \left(\frac{2\delta}{L} \right) \quad (2.66)$$

Note: Equations for R_{ms}^{ccja} are currently under development.

Fig. 2.10 Centreline offset misalignment in circumferential weld joints in cylindrical shell [35].

Fig. 2.11 Angular misalignment in circumferential weld joints in cylindrical shell [35].

3) Cylinders – Longitudinal joints:

$$k_{mp} = k_{mp}^{cljc} + k_{mp}^{clja} \quad (2.67)$$

$$k_{ms} = -1 \quad (2.68)$$

where the superscript “cljc” denotes a cylinder with longitudinal joints and centerline offset, and “clja” denotes a cylinder with longitudinal joints and angular misalignment.

For centerline offset misalignment (Fig. 2.8):

$$k_{mp}^{cljc} = \frac{C_1}{C_2} \quad (2.69)$$

$$C_1 = 0.0038392 + 3.1636\left(\frac{e}{t}\right) + 1.2377\left(\frac{e}{t}\right)^2 - 0.0040582S_p + 0.00034647S_p^2 + 0.0000031205S_p^3 \quad (2.70)$$

$$C_2 = 1.0 + 0.41934\left(\frac{e}{t}\right) + 0.009739S_p \quad (2.71)$$

For angular misalignment, local peaking (Fig. 2.12(a)):

$$k_{mp}^{clja} = \left(\frac{6\delta}{t}\right) C_f \quad (2.72)$$

$$C_f = 1 - \frac{\theta_p}{3\pi} - \frac{4}{\pi\theta_p^2}(\theta_p - \sin\theta_p) - \frac{4S_p^2}{\pi\theta_p^2} \sum_{n=2}^{\infty} \frac{(n\theta_p - \sin n\theta_p)}{n^3(n^2 - 1 + S_p^2)} \quad (2.73)$$

with

$$\theta_p = \arccos\left(\frac{1}{1 + \delta/R}\right) \quad (2.74)$$

For angular misalignment, global peaking (Fig. 2.12(b)):

$$k_{mp}^{clja} = \left(\frac{6\delta}{t}\right) C_f \quad (2.75)$$

$$C_f = 0.5 - \frac{\pi}{2k} \cot k\pi - \frac{k^2 - 1}{2k^2} + \frac{1}{2k^2 - 1} \quad \text{for } S_p^2 < 1 \quad (2.76)$$

$$C_f = 0.5 + \frac{\pi}{2k} \coth k\pi - \frac{k^2 + 1}{2k^2} - \frac{1}{k^2 + 1} \quad \text{for } S_p^2 \geq 1 \quad (2.77)$$

$$k^2 = 1 - S_p^2 \quad \text{for } S_p^2 < 1 \quad (2.78)$$

$$k^2 = S_p^2 - 1 \quad \text{for } S_p^2 \geq 1 \quad (2.79)$$

SCFs for various misaligned geometry configurations are given in the above.

Fig. 2.12 Angular misalignment in longitudinal weld joints in cylindrical shell [35].

2.5 FRACTURE ANALYSES OF ALIGNED AND MISALIGNED PIPES

Fracture is a common failure mode of offshore pipelines. During in-service operation or welding fabrication, crack-like defects are often introduced into the pipes, especially into the welds, which poses tremendous challenges to the reliability of pipelines [36]. Considerable research has been conducted to investigate the fracture responses of cracked pipelines subjected to large plastic strain [37-45]. Jayadevan et al. [37] and Østby et al. [38] investigated the fracture responses of homogeneous pipes with a surface crack based on 3-D FE analysis, and it is noted that the CTOD varies linearly with the global strain. Structural integrity analysis of axially cracked pipelines under high pressure was carried out by using conventional and constraint-modified FADs [39]. Berg et al. [40] considered the effect of biaxial loading on the ultimate fracture capacity based on large scale bending tests and numerical analyses, and the effects of varying wall thickness and yield stress were also studied in detail. Netto et al. [41,42] performed the fatigue analyses of pre-strained pipes with defects by means of full scale bending tests (Fig. 2.13) and FE analysis. The fracture responses and crack tip constraint of cracked pipelines under extreme plastic bending were investigated, and the J -CTOD relationship was examined [44]. Paredes and Ruggieri [45] provided an engineering assessment approach for a circumferential surface crack in girth weld pipes by taking into account a wide range of crack sizes and weld strength mismatch levels.

Recently, there is a large demand of corrosion resistant alloy pipeline with the increased volume of corrosive hydrocarbons. Xiao's group [36,46-48] had carried out research on the fracture capacity of girth welded clad pipes with 3D surface or embedded cracks subjected to axial or bending loading. It was noted that the fracture assessment results obtained from the

assessment equation of BS 7910:2013 [1] were conservative by 30%-40% in comparison with the 3-D FE results as the global strain is loaded up to 3%. Hence, a new assessment equation was proposed to evaluate the evolution trend of CTOD with the global strain. However, the proposed equation is only suited for a specific pipe configuration. Based on a strain-based J -estimation scheme in conjunction with the equivalent stress-strain relationship method, Souza and Ruggieri [49] provided a concise fracture assessment framework for under-matched pipe girth welds by considering a wide range of pipe geometries, crack sizes and material properties.

Fig. 2.13 Full scale bending test [41].

Fig. 2.14 (a) Crack placements in clad welds [50], (b) clad pipes with centreline offset weld misalignment and (c) moment distribution induced by weld misalignment in clad pipes.

During the welding process, weld misalignment such as centerline offset, angular or both is invariably introduced into the pipeline; this induces an additional bending moment along the pipe section (Fig. 2.14). It is noted from Fig. 2.14(c) that the total load in misaligned pipes is

larger than the load in aligned ones due to the additional moment caused by the misalignment, which means that the misaligned configuration will undergo greater challenges in structural integrity when applying a same load to aligned and misaligned ones, respectively. However, the aforementioned research works [36-40,43-49] are all about fracture analyses of aligned pipes with a crack located at the weld center line (WCL), and very limited investigations are involved in the fracture assessment of misaligned clad pipes containing a crack located at the WCL or the weld toe. Even though BS 7910:2013 [1] provides fracture and fatigue assessment equations for cracked pipes with weld misalignment, they actually use the superposition method on the fracture and fatigue results of aligned joints to predict the corresponding results of misaligned joints by including the stress concentration factors into these equations. The assessment equations from BS 7910:2013 frequently produces an over-conservative prediction on the J or CTOD values, especially for fully plastic analysis of misaligned components. Hence, the effect of weld misalignment on the fracture responses of cracked clad pipes deserves special attention and detailed investigations, and more accurate assessment equations incorporating the effect of misalignment are required to be proposed in practice.

CHAPTER 3

DEVELOPMENT OF FINITE ELEMENT MESH GENERATOR FOR MISALIGNED CLAD PIPES

3.1 INTRODUCTION

Finite element (FE) method is an important tool used for the fracture analysis of different types of welded joints containing a crack. Because a large amount of FE models are usually needed to be created in a parametric study, a special mesh generator is developed to simplify the cumbersome and time-consuming task. In this study, the mesh generator can generate input files which are compatible with ABAQUS software [51]. Then, these files are imported into the software to process them based on the ABAQUS Solver [51]. Fig. 3.1 presents the flowchart of the FE models generated and analysed. The purpose of this chapter is to detail the development of the mesh generator for performing elastic and elastic-plastic fracture analysis of cracked clad pipes with weld misalignment.

Fig. 3.1 Flowchart for the FE model generated and analysed.

3.2 TYPES OF CRACK TIP

Fig. 3.2 Node numberings sequence at the crack tip.

The mesh generation around the crack tip is the most pivotal in the FE simulation of flawed structures. According to elastic analysis or elastic-plastic analysis, different mesh strategies are used to mesh the region near the crack tip. In elastic analysis, the nodes at the crack tip are normally tied and the mid-side nodes moved to the $1/4$ points resulting in a $1/r^{1/2}$ strain singularity as presented in Fig. 3.2(a). Fig. 3.3(a) shows the details of the crack tip elements in elastic analysis.

Fig. 3.3 Three different methods used to model the crack tip.

For elastic-plastic analysis, the $1/r^{1/2}$ strain singularity no longer exists at the crack tip because of the formation of the plastic zone. Consequently, elastic singularity elements cannot be used for elastic-plastic analyses. In fact, a $1/r$ strain singularity is produced for the element geometry around the crack tip in elastic-plastic problems. Two methods are usually employed to construct crack tip elements of elastic-plastic problems. For the first method, the crack tip nodes are coincident but untied as shown in Fig. 3.3(b). The untied nodes can move apart with the elements deforming. The above method is appropriate when the deformation around the crack tip is relatively small. However, if the deformation is very large, the method may result in inaccurate J -integral results because of the collapse of crack tip elements. In addition, the convergence of the numerical solution is frequently encountered in the case of large plastic deformation. As a result, Anderson [15] proposed to introduce a key-hole to overcome the convergence problem as illustrated in Fig. 3.3(c), which has a finite radius within the plastic zone of the crack tip. In the present study, the crack tip elements with the key-hole are adopted to solve the elastic-plastic problems. The determination of radius value of key-hole is an important consideration. Using too small finite radius may still result in the collapse of the element around the crack tip. However, too large finite radius may produce inaccurate J -integral results. The only effective way is to observe and check the deformation near the crack tip and

then determine the appropriate finite radius. In the new mesh generator of the present study, the finite radius of the key-hole is set to be a variable. Therefore, the users can continuously adjust to get an optimum value for the elastic-plastic problem.

3.3 MESH GENERATION OF CRACK BLOCK

Two types of cracks are commonly encountered in existing welded joints, viz., semi-elliptical crack (Fig. 3.4) and circumferential part-through crack (Fig. 3.5). The shape of semi-elliptical crack is determined by the elliptical curve. The circumferential part-through crack is assumed to be constant crack depth along the circumferential crack length and the end-radius of the crack is equal to crack depth, which is frequently found in offshore pipelines. In this chapter, the new mesh generators for the two types of cracks are both proposed, but the second one is introduced briefly because of the similar meshing strategy.

Fig. 3.4 Semi-elliptical crack.

Fig. 3.5 Circumferential part-through crack.

Fig. 3.7 The second cross-section for elastic analysis.

Fig. 3.8 The first cross-section for elastic-plastic analysis.

Fig. 3.9 The second cross-section for elastic-plastic analysis.

Fig. 3.10 (a) 20 nodes hexahedral element and (b) 15 nodes collapsed prism element.

A local coordinate UV shown in Fig. 3.6 is defined to generate the crack tube. All the nodes in the cross-section of the crack tube are described by the UV coordinate. In order to number the nodes and determine the geometry of the crack tube, several parameters are defined in the following. $Rtube$ is used to define the radius of the inner circular crack tube. $LSquare$ is used to determine the geometry of the outermost rectangular cross-section of the crack tube. $Square$ denotes the dimension of the middle rectangular cross-section between the outermost rectangular cross-section and the inner circular crack tube. The above three parameters are all variables based on the crack shape and structural geometry. For example, when the crack is very shallow, the value of $LSquare$ should be reduced accordingly so as to still enable the crack to be constructed. $N2R$ is a parameter to control the rings of elements of the inner circular crack

tube. $N2S$ is used to determine the rings of elements of the region between the inner circular crack tube and the middle rectangular cross-section. $N2LS$ is employed to control the rings of elements of the region between the middle rectangular cross-section and the outermost rectangular cross-section. Appropriate setting for the values of the above six parameters can accomplish the transition from a refined mesh near the crack tip to a coarse mesh remote from the crack tip. $N2A$ is the last parameter used to define the segments divided along the crack front, which determines the numbers of cross-section. As shown in Fig. 3.11, the numbers of the cross-section are 46. The value of $N2A$ is 45, meaning that the crack tube is divided into 45 parts. Until now, the general geometry of the crack tube has been determined. The following is to number the nodes in all the cross-sections of the crack tube. As depicted in Fig. 3.6 and Fig. 3.7, the nodes in the first cross-section are numbered firstly and the number of the nodes is $(N2R+N2S+N2LS)*17+1$, and then, the nodes in the second cross-section are numbered and the corresponding number is also $(N2R+N2S+N2LS)*17+1$, and so on. The total number of all the nodes in the crack tube can be calculated, which is $(N2A+1)*[(N2R+N2S+N2LS)*17+1]$. Similarly, the number of all the elements can also be obtained.

Fig. 3.8 and Fig. 3.9 show the first cross-section and the second cross-section of the crack tube in the elastic-plastic analysis, respectively. The parameters used in the analysis are nearly the same with the case of elastic analysis except the additional parameter which is the radius of key-hole ($r2r$). As mentioned in the above, the presence of the key-hole is to avoid the element around the crack tip from collapsing. The calculation around the collapsed elements may result in an inaccurate J -integral. In the present mesh generator, $r2r$ is set as a variable so that its value can be changed based on the crack shape and loading situations. For example, when the structure undergoes a larger plastic deformation, a larger $r2r$ is required to avoid the collapse of the elements around the crack tip and overcome the convergence problem. In addition, because the key-hole exists at the crack tip instead of the point used in the case of the elastic analysis, all the elements in the crack tube are 20 nodes hexahedral elements. The nodes numbering in the elastic-plastic analysis is also the same with the case of the elastic analysis. The total number of all the nodes in the crack tube is $(N2A+1)*[(N2R+N2S+N2LS+1)*17]$, and the number of all the elements is $N2A*[(N2R+N2S+N2LS)*16]$.

Fig. 3.11 A quarter of crack tube.

The crack tube is generated by the “spider-web” design technique, which is shown in Fig. 3.11. In the following, the crack block is constructed by connecting the crack tube with Part A and Part B of the crack block using transition zones. Fig. 3.12 presents Part A of the crack block, and a transition zone is used to transform 3 elements into 1 element between the crack tube and Part A. Then, Part B is generated and connected to the crack tube by another transition zone, whose purpose is the same with the first transition zone. Fig. 3.13 presents the entire mesh of the crack block.

Fig. 3.12 Part A of crack block.

Fig. 3.13 Mesh view of crack block.

3.4 FINITE ELEMENT MESH GENERATION FOR ALIGNED CLAD PIPES

Fig. 3.14 Cracked part of pipe segment.

In Section 3.3, the crack block is constructed by the “spider-web” design technique and transition zones. This section presents the details to connect the crack block with other parts of the pipe, constructing the entire clad pipe mesh. For the crack case at the weld center line (WCL), only a quarter of the pipe is meshed because of symmetry; this greatly reduces the computation time. At first, the crack block mesh configuration is extended along the longitudinal direction of the pipe to obtain the meshed component shown in Fig. 3.14, which is named as the cracked part of pipe segment. Then, the cracked part is rotated around the circumferential direction of the pipe to get the entire pipe mesh as depicted in Fig. 3.15. The number of elements along the longitudinal and circumferential directions of the pipe are both set as variables in the mesh

generator to conveniently adjust the corresponding values based on different requirement in mesh refinement. The construction procedures of the FE model for the aligned clad pipes with a crack is illustrated in Fig. 3.16.

Fig. 3.15 Entire mesh view of pipe segment with a crack at the WCL.

Fig. 3.16 Construction procedures of the FE model for aligned pipes with a crack.

3.5 VARIOUS CRACK LOCATIONS FOR ALIGNED CLAD PIPES

The FE mesh generator is developed to create FE models of structural details with various crack placements (Fig. 2.14). Fig. 3.17 shows the mesh generation of the aligned pipes with a surface crack located at the WCL and interface of the weld metal and outer pipe, respectively. Moreover, the cracks or flaws are located at the surface or embedded into the weld. Hence, the

two cases with a surface crack and an embedded crack are incorporated both in the function list of the mesh generator (Fig. 3.18). Different from single wall pipe, clad pipes consist of two layers having different material properties. Hence, the outer and inner layers should be meshed separately so as to enable assigning the material constants to the corresponding layer. The thickness of the outer and inner layers are both set as variables in the mesh generator to adjust the value depending on the specific situation. Fig. 3.19 present the case of varying inner thickness for the clad pipes.

Fig. 3.17 Mesh generation of aligned pipes with a surface crack at the WCL and interface of the weld metal and outer pipe.

Fig. 3.18 Cases with a surface crack and an embedded crack.

Fig. 3.19 Change of inner thickness implemented by the mesh generator.

3.6 FINITE ELEMENT MESH GENERATION FOR MISALIGNED CLAD PIPES

The FE mesh generator aiming for aligned clad pipes is developed in the previous section. In this section, the mesh generator is further developed to serve for the misaligned pipes with a crack by including a coordinate transformation method into the existing program code. Three types of weld misalignment [35], viz., centreline offset weld misalignment with equal thickness and diameters (Fig. 3.20(a)), centreline offset weld misalignment with equal thickness but unequal diameters (Fig. 3.20(b)), and centreline offset weld misalignment with unequal thickness but aligned inside surface (Fig. 3.20(c)), are investigated herein, which are named as Type-I, Type-II and Type-III for convenience, respectively. The coordinate transformation expressions converting from an aligned configuration (x -, y -, and z - coordinate system) to a misaligned one (X -, Y -, and Z - coordinate system) are derived in the following.

Fig. 3.20 Three types of centreline offset weld misalignment [35].

For the coordinate transformation, the welded pipes are divided into three regions, viz., Region 1 (right pipe), Region 2 (girth weld) and Region 3 (left pipe). At the Region 3, the coordinates in misaligned pipes are the same with that in the aligned pipes. At Regions 1 and 2, the coordinate transformation is required and the adopted method is dependent on the types of weld misalignment. For the Region 2 of cracked pipes with Type-I and Type-II, the centerline offset is a variable as the X - coordinate changes and it linearly increases from zero to e ; hence, an effective centerline offset (e_{eff}) is defined as

$$e_{\text{eff}} = \left(\frac{2x + L_v + L_e}{2L_v} \right) e \quad (3.1)$$

where L_v denotes the varying weld width along the pipe thickness direction; $L_v = L_i$ at the inside surface of girth weld, and $L_v = L_e$ at the outside surface; the expression of L_v can be obtained based on homothetic triangle theory as

$$L_v = L_i + \frac{[2(r + t_2) - D_{e2}](L_e - L_i)}{2t_2} \quad (3.2)$$

r is the radial polar coordinate of a point at the cross-section of pipe as shown in Fig. 3.21, and the other polar coordinate is θ ; the relationship between r , θ and y , z is

$$y = r \sin(\theta) \quad (3.3)$$

$$z = r \cos(\theta) \quad (3.4)$$

Fig. 3.21 A r - θ polar coordinate system at the cross-section of a pipe.

Fig. 3.22 Transformation schemes for Type-I, Type-II and Type-III.

The presence of weld misalignment changes the radial polar coordinate of corresponding point, resulting in the original r replaced by the r plus the centerline offset. Hence, the coordinate transformation method for the cracked pipe with Type-I (Fig. 3.22(b)) is written as

$$\begin{aligned} \text{Region 1: } & X = x, \quad Y = y, \quad Z = z - e \\ \text{Region 2: } & X = x, \quad Y = y, \quad Z = z - e_{\text{eff}} \\ \text{Region 3: } & X = x, \quad Y = y, \quad Z = z \end{aligned} \quad (3.5)$$

For the cracked pipe with Type-II (Fig. 3.22(b)), the corresponding coordinate transformation method is given as

$$\begin{aligned} \text{Region 1: } & X = x, \quad Y = (r - e) \sin(\theta), \quad Z = (r - e) \cos(\theta) \\ \text{Region 2: } & X = x, \quad Y = (r - e_{\text{eff}}) \sin(\theta), \quad Z = (r - e_{\text{eff}}) \cos(\theta) \\ \text{Region 3: } & X = x, \quad Y = y, \quad Z = z \end{aligned} \quad (3.6)$$

For the cracked pipe with Type-III (Fig. 3.22(c)), the thickness transition slope (α) is used to characterize the weld misalignment, and it is defined as

$$\alpha = \frac{t_2 - t_1}{L_e} \quad (3.7)$$

At the Region 2 of misaligned pipes (Fig. 3.23(b)), the transition slope at the outside surface of welded pipes is indicated by α , while no slope is observed at the inside surface. Thus, the transition slope is varying along the thickness direction, and an effective transition slope (α_{eff}) is defined to describe the variation as

$$\alpha_{\text{eff}} = \frac{[2r - (D_{e2} - 2t_2)]}{2t_2} \alpha \quad (3.8)$$

Moreover, the centerline offset is different with the change of X - coordinate. Based on Eq. (3.1), the centerline offset for each point at the Region 2 can be expressed as

$$e_{\text{eff}}^* = \left(\frac{2x + L_v + L_e}{2L_v} \right) \alpha_{\text{eff}} L_e \quad (3.9)$$

At the Region 1 (Fig. 3.22(c)), the centerline offset for each point is $\alpha_{\text{eff}} L_e$ because it is constant with varying X . Based on the above analyses and the expression form in Eq. (3.6), the coordinate transformation method for the cracked pipe with Type-III is proposed as

$$\begin{aligned} \text{Region 1: } & X = x, Y = (r - \alpha_{\text{eff}} L_e) \sin(\theta), Z = (r - \alpha_{\text{eff}} L_e) \cos(\theta) \\ \text{Region 2: } & X = x, Y = (r - e_{\text{eff}}^*) \sin(\theta), Z = (r - e_{\text{eff}}^*) \cos(\theta) \\ \text{Region 3: } & X = x, Y = y, Z = z \end{aligned} \quad (3.10)$$

Fig. 3.23 Schematic view of converting from an aligned girth weld to a misaligned one at Region 2 of cracked pipes with Type-III.

Thus, the FE mesh generator used to generate the numerical models of aligned and misaligned clad pipes with a crack is developed, and a function list is depicted in Fig. 3.24 to better describe the functional modules of the mesh generator.

Fracture analysis	Pipe type	Pipe connection	Crack location
☐	☐	☐	☐
☐ Elastic analysis	☐ Single wall pipe	☐ Aligned	☐ Weld centre line
☐ Elastic-plastic analysis	☐ Clad pipe	☐ Centreline offset weld misalignment	☐ Surface crack
		☐ Equal thickness and diameters	☐ Embedded crack
		☐ Equal thickness but unequal diameters	☐ Interface of weld and outer pipe
		☐ Unequal thickness but aligned inside surface	☐ Surface crack
			☐ Embedded crack

Fig. 3.24 Function list of the FE mesh generator.

3.7 PRELIMINARY VALIDATION OF THE COORDINATE TRANSFORMATION METHOD

3.7.1 Hot spot stress concentration factors

Considerable effort has been made to investigate the fracture responses of aligned pipes with a crack [36-38,45-49,52]. Therefore, it is convenient to verify the accuracy of the FE models of aligned pipes generated by the FE mesh generator in comparison with the J -integral or CTOD values tabulated in these references [36-38,45-49,52]. In Section 3.6, the coordinate transformation method is appended in the mesh generator to convert the aligned pipes to the misaligned ones. However, there are limited published works on the fracture assessment of misaligned pipes with a crack subjected to large plastic deformation, especially for clad pipes. Because the only difference between the mesh generator aiming for aligned and misaligned pipes is the adoption of the coordinate transformation, the availability of FE models of misaligned pipes can be confirmed if the coordinate transformation method is validated. The presence of weld misalignment raises a stress concentration at the weld toe, and various stress concentration factor (SCF) equations have been proposed to show the effect [33,53,54], which is an alternative method to ascertain the transformation method. Therefore, the mesh generator is employed to create FE models of misaligned pipes without cracks, and the SCFs obtained are used to compare with the results given by the available equations [33,53,54].

For a welded structure, the hot spot is usually located at the weld toe, and the hot spot stress is the value of the structural stress at the hot spot, which can be determined according to the stress evaluation points at distances $0.5t$ and $1.5t$ away from the hot spot, where t is the plate thickness at the weld toe [55]. Based on the shell theory and FE analysis, Lotsberg [33] established a hot spot SCF equation for misaligned butt welds in shell structures, and it is expressed as

$$SCF = 1 + \frac{6(e + e_t)}{t_1} \frac{1}{1 + (t_2/t_1)^\beta} \exp(-\alpha) \quad (3.11)$$

where

$$\alpha = \frac{1.82L_e}{\sqrt{D_{el}t_1}} \frac{1}{1 + (t_2/t_1)^\beta} \quad (3.12)$$

$$\beta = 1.5 - \frac{1.0}{\text{Log}(D_{e1}/t_1)} + \frac{3.0}{[\text{Log}(D_{e1}/t_1)]^2} \quad (3.13)$$

$$e_t = \frac{t_2 - t_1}{2} \quad (3.14)$$

It is noted that the equation is suitable for misaligned pipes with Type-I, Type-II and Type-III, and the stress concentration effects induced by Type-I and Type-II are identical, which is also observed in the FE analysis conducted by the authors. For Type-I and Type-II, the hot spot SCFs obtained from the Lotsberg's equation [33] and FE analysis agree very well with each other for varying pipe diameter, weld geometry and misalignment level (Fig. 3.25), and so does Type-III, which is not shown here in the interest of conserving spaces. Hence, the accuracy of the coordinate transformation method is assured to some extent.

Fig. 3.25 Comparison of hot spot SCFs obtained from Lotsberg's equation [33] and FE analysis for Type-I and Type-II.

3.7.2 Small-scale experimental test on stress concentration curve

The coordinate transformation method is confirmed earlier by means of the comparison of hot spot SCFs. However, the hot spot SCF only characterizes the spot stress concentration effect at the weld toe. In this section, a small-scale experimental test on the stress concentration curve along the longitudinal direction of misaligned pipes is carried out to further verify the validity of the transformation method.

Table 3.1 Geometric parameters of specimens

Group	Material	Specimen	Diameter (mm)	Thickness (mm)	Centreline offset (mm)
1	API 5L X60	PType-I	89.3	5.3	1.0
2	API 5L X60	PType-I	134.2	10.1	1.0
		PType-III	133.9	10.1	1.0

The experimental test is composed of two groups totaling three specimens, and Table 3.1 tabulates the geometric parameters of the three specimens. For the Group 1 test, two pipe segments with a diameter of 89.3mm and thickness of 5.3mm are welded together with weld misalignment of Type-I. The material properties are: $E = 206\text{GPa}$, $\nu = 0.3$ and $\sigma_y = 483\text{MPa}$ with E , ν and σ_y being the Young's modulus, Poisson's ratio and yield stress, respectively. After welding, the weld cap is removed to produce a smooth transition from the weld to the pipe surface [42]. Prior to each experiment, electrical strain-gages of FLA-3-11-5LT type with gage length of 3mm are mounted on the external surface of PType-I to monitor the local strain history, and the strain gage layout for the specimen is shown in Fig. 3.26(a). The strain gages at location "180°" are used to measure the total strains induced by the tension and weld misalignment. The strain gages at location "90°" and "270°" are used to determine whether location "180°" is the maximum offset position. Then, the specimen is gripped in the hydraulic testing machine (Fig. 3.27), and a tension of 100kN is applied to the ends of the specimen under quasi-static loading. The SCFs at different X are determined based on the strains measured and stress-strain relationship.

The FE mesh generator is employed to create FE models of misaligned pipes with Type-I, and the models are analysed by ABAQUS software [51]. A Python script is written to extract and calculate the SCFs along the longitudinal direction of pipes. The comparison of stress concentration curves obtained from FE analysis and experimental test is presented in Fig. 3.28. It is noted that the experimental results are higher than the numerical values with the percentage difference in value being 9.7%, and only one point extracted from the test is available with the SCF of the second point being 1.0, indicating negligible stress concentration effect. In Fig. 3.28, the SCF obtained from FE analysis is nearly 1.0 when X/t_1 is close to 2.5. Hence, a thicker pipe and smaller strain gage should be used in the experimental test so as to obtain as many data points as possible.

Fig. 3.26 Strain gage layouts for PType-I and PType-III.

Fig. 3.27 Experimental setup of misaligned pipes.

Fig. 3.28 Comparison of SCFs obtained from FE analysis and experimental test for Group 1 test.

In the Group 2 test, a thicker pipe with a nominal diameter of 134mm and thickness of 10mm is employed. Two specimens (PType-I and PType-III) are fabricated with weld misalignment of Type-I and Type-III, respectively, due to the similarity for the pipes with Type-I and Type-II. Eq. (3.14) is used to calculate the centreline offset value of PType-III. In the experimental test, the pipe surface is treated more carefully, especially near the weld region because the pipe and weld roughness can affect the local stress values. Electrical strain-gages of FLA-1-11-3LJC type with gage length of 1mm are used to measure the local strain history. The strain gage layout for PType-I is depicted in Fig. 3.26(a). Due to the axial symmetry of PType-III (Fig. 3.26(b)), the strain gages are uniformly arranged around the circumferential direction to measure four strains at one X . Then, the average strain can be computed and the error is obtained by comparing the average strain with each value. Fig. 3.29 presents the comparison of SCFs obtained from FE analysis and experimental test. The experimental results are slightly higher than the numerical values with the largest percentage difference in value being 5.3%, and the difference between one another tends to decrease with increasing X . In general, Fig. 3.29 shows a good correlation between the stress concentration curves obtained by FE analysis and experimental test. Therefore, the accuracy of the coordinate transformation method is further ascertained. However, the validations mentioned in the above may be not a necessary and sufficient condition because the crack driving force is not the same with the SCF value. Hence, a comparative study of the fatigue life obtained from present FE analysis,

theoretical predictions of Ferreira and Branco [56] and experimental results [57] is carried out to well prove the accuracy of the coordinate transformation method, which is given in Sub-section 5.3.2. Even though the accuracy of the coordinate transformation method is verified based on linear elastic analysis, the proposed method is still valid for the pipelines subjected to large plastic deformation. This is because the same pipeline configuration is created by the mesh generator no matter for elastic case or fully plastic case.

Fig. 3.29 Comparison of SCFs obtained from FE analysis and experimental test for Group 2 test.

3.8 NOTCH STRESS CONCENTRATION FACTOR EQUATIONS

As mentioned previously, a large amount of hot spot SCF equations have been proposed to assess the structural reliability [33,53,54], but there are almost no relevant equations on notch stress concentration factor (NSCF) [55]. In fact, the NSCF is an alternative method for fatigue design by linking with a notch stress S-N curve and it can reveal the actual stress distribution at the weld toe [55], which is defined as

$$\text{NSCF} = \frac{\sigma_{ls}}{\sigma_a} \quad (3.15)$$

where σ_a is the applied axial stress and σ_{ls} is the local notch stress evaluated by extrapolating the in-plane stress to the weld toe (Fig. 3.30) [58]. In Sub-section 3.7.2, the stress concentration curve along the longitudinal direction of misaligned pipes is obtained by FE analysis and

verified by comparing with the experimental results. In the section, the FE analysis is conducted to propose the NSCF equations for the pipes with Type-I, Type-II and Type-III.

Fig. 3.30 Stress evaluation of hot spot SCF and NSCF at the weld toe.

For the pipe cases with Type-I and Type-II, the diameter ratios (D_{e1}/t_1) analysed are ranging from 10 to 100, which covers the most commonly used pipe configurations in practice. Width ratios (L_e/t_1) of weld beads of 0.2, 0.8, 1.4 and 2.0 are used according to AWS D1.1:2000 [59] recommendations. A typical value for centreline offset employed in many fabrication standards is $e = 0.15t_1$ or maximum 3-4 mm [33]. Thus, centreline offset ratios (e/t_1) of 0.00, 0.05, 0.10 and 0.15 are adopted in this study. The mesh design of the FE models is based on the recommendations for the notch stress methodology given in Appendix D of DNV-RP-C203 [55]. The notch at the weld toe is modelled using a radius of 1.0 mm to avoid infinite stress. The element type used in these models is 20-node brick element with reduced integration (C3D20R). An axial loading is applied to the end of the pipe model, and the NSCF is determined based on Eq. (3.15). Fig. 3.31 presents the effect of e/t_1 on the NSCF, and it is noted that the linear relationship between the NSCF and e/t_1 clearly holds true for varying L_e/t_1 and D_{e1}/t_1 . The increase of D_{e1}/t_1 enlarges the NSCF, and the enlargement gradually slows down with the further increase of D_{e1}/t_1 . However, a larger L_e/t_1 results in a smaller NSCF (Fig. 3.32). This is because a larger L_e/t_1 for constant e/t_1 reduces the steepness of the girth weld, raising a lower stress concentration effect. Based on the above variation trends, the NSCF equation for the pipes with Type-I and Type-II is proposed as

$$\text{NSCF} = 1 + \frac{e}{t_1} \left[0.9424 \left(\frac{L_e}{t_1} \right)^{2.0776} + 6.6295 \left(\frac{L_e}{t_1} \right)^{-0.4424} \right] \exp \left\{ \left[-0.9734 \left(\frac{L_e}{t_1} \right)^{0.7446} \right] \frac{L_e}{\sqrt{D_{e1} t_1}} \right\} \quad (3.16)$$

Fig. 3.33 shows the comparison of NSCFs obtained from Eq. (3.16) and FE analysis for varying e/t_1 , L_e/t_1 and D_{e1}/t_1 , and a good agreement between them is always observed, which confirms the accuracy of Eq. (3.16).

Fig. 3.31 Effect of e/t_1 on NSCFs for varying L_e/t_1 and D_{e1}/t_1 for the pipes with Type-I and Type-II.

Fig. 3.32 Effect of D_{e1}/t_1 and L_e/t_1 on NSCFs for $e/t_1 = 0.05$ for the pipes with Type-I and Type-II.

Fig. 3.33 Comparison of NSCFs obtained from Eq. (3.16) and FE analysis for the pipes with Type-I and Type-II.

For the pipe case with Type-III, the ranges for D_{e1}/t_1 and L_e/t_1 are the same with that of Type-I and Type-II. As for the centreline offset, the thickness transition slope (δ_t) is employed to describe the variation of misalignment caused by pipe thickness transition (e_t), and the relationship between δ_t and e_t is expressed as

$$\delta_t = \frac{t_2 - t_1}{L_e} = \frac{2e_t}{L_e} \quad (3.17)$$

Based on AWS D1.1:2000 [59] recommendations, the maximum permitted transition slope is 0.4. Hence, δ_t values ranging from 0.0 to 0.4 are used in this study. It is noted from Fig. 3.34 that the NSCF varies nonlinearly with the increase of δ_t for the entire ranges of L_e/t_1 and D_{e1}/t_1 . Fig. 3.35 presents the effects of D_{e1}/t_1 and L_e/t_1 on the NSCF. In comparison with that shown in Fig. 3.32, the variation trend between the NSCF and D_{e1}/t_1 is similar, but the NSCF changes with L_e/t_1 in an opposite trend. It is because a larger L_e/t_1 for same δ_t induces a larger e_t/t_1 according to Eq. (3.17), resulting in a larger stress concentration effect at the weld toe. The NSCF equation for the pipe with Type-III is proposed in the following,

$$\text{NSCF} = 1 - 2.3911\delta_t^2 + \delta_t \left[5.3461 \left(\frac{L_e}{t_1} \right)^{0.3526} \right] \exp \left\{ \left[0.0409 \left(\frac{L_e}{t_1} \right)^{-2.6831} - 0.4636 \right] \frac{L_e}{\sqrt{D_{e1}t_1}} \right\} \quad (3.18)$$

The accuracy of Eq. (3.18) is assured by observing the very close correlation of NSCFs obtained from the equation and FE analysis as depicted in Fig. 3.36.

Fig. 3.34 Effect of δ_t on NSCFs for varying L_e/t_1 and D_{e1}/t_1 for the pipes with Type-III.

Fig. 3.35 Effect of D_{e1}/t_1 and L_e/t_1 on NSCFs for $\delta_t = 0.133$ for the pipes with Type-III.

Fig. 3.36 Comparison of NSCFs obtained from Eq. (3.18) and FE analysis for the pipes with Type-III.

3.9 SUMMARY OF THE CHAPTER

In this chapter, a FE mesh generator is first developed for cracked clad pipes with various crack locations, viz., a surface or embedded crack at the WCL and a surface or embedded crack at the interface of the weld metal and outer pipe. A coordinate transformation method is incorporated in the mesh generator to convert the FE models of aligned configurations to that of misaligned ones. In order to verify the accuracy of the transformation method, a comparison study on hot spot SCFs obtained by Lotsberg's SCF equations [33] and FE analysis is carried out, and a good agreement between one another is observed. Then, a small-scale experimental test on the stress concentration curve along the longitudinal direction of misaligned steel pipes is conducted to further ascertain the validity of the transformation method. Hence, the mesh generator with the coordinate transformation is considered to be well suited to serve for misaligned clad pipes. Moreover, the NSCF equations for misaligned pipes are proposed based on the developed mesh generator. In the following chapters, the mesh generator will be used for the fracture analysis of aligned and misaligned configurations containing a crack.

CHAPTER 4

STRESS INTENSITY FACTORS OF ALIGNED WELDED JOINTS

4.1 INTRODUCTION

The new FE mesh generator is first used to perform the extensive parametric study of aligned plate welded joints containing a surface crack before it is used for the fracture analysis of offshore pipelines. The weld toe magnification factor (M_k) equations for a semi-elliptical surface crack in aligned plate-to-plate butt-welded joints and load-carrying cruciform fillet-welded joints are proposed.

4.2 WELD TOE MAGNIFICATION FACTORS FOR A CRACK IN PLATE-TO-PLATE BUTT-WELDED JOINTS

BS 7910:2013 [1] proposes a single set of equations for estimating the M_k of different types of welded joints with a semi-elliptical surface crack. In this section, extensive FE analyses are performed to determine the M_k of plate-to-plate butt-welded joints subjected to axial and bending loading. The M_k values at the crack deepest point and the crack ends are determined. It is because that the maximum stress intensity factor (SIF) value is obtained either at the crack deepest point or at the crack ends; for a wider crack (smaller crack aspect ratio), the maximum SIF is located at the crack deepest point; for a penny shape crack (larger crack aspect ratio), the SIF at the crack ends is maximum. The M_k results of butt welds are different with those of single T-butt joints with percentage difference in values being as high as 63.8% and 97.4% for axial and bending loading cases, respectively. Percentage differences of 60.4% and 358.2% for axial and bending loading cases are observed between butt welds and X-joints. Hence, new M_k equations for butt welds are proposed using multiple regression analysis.

4.2.1 Background of plate-to-plate butt-welded joints

The failure of structural components is usually caused by the propagation of semi-elliptical surface cracks existing at the weld toe of the welded joints. Therefore, it is of practical importance to assess the structural integrity of cracked welded joints. Stress intensity factor (SIF), which controls the intensity of the stress field at the crack tip, is usually used to assess

the fracture and fatigue resistance of such flawed structures. Newman and Raju [60] presented an empirical equation for the SIFs for a surface crack in the plain plate subjected to tension and bending loads. However, for welded toe cracks, the SIF solutions are frequently described in terms of weld toe magnification factor (M_k) [61], which is expressed as the product of the ratio of the SIF with stress concentration to the SIF for the same crack in component without stress concentration, as shown in the following:

$$M_k = \frac{K_{(\text{in plate with stress concentration})}}{K_{(\text{in same plate but without stress concentration})}} \quad (4.1)$$

2-D finite element (FE) analysis has been used to calculate M_k at the weld toe of the welded joint geometry. However, Lie et al. [62] found that the 2-D M_k overestimated and underestimated the 3-D M_k at the crack deepest point and the crack ends by approximately 15% and 65%, respectively. Fu et al. [63] performed 3-D linear elastic fracture analysis of surface cracks in single T-butt joints using FE methods, but their solution was restricted to the weld geometry with 45° weld angle and sharp notch at the weld toe. An extensive parametric study on M_k of single T-butt joints was conducted by Bowness and Lee [64], which covered the most commonly used weld profiles found in practice and has been incorporated in BS7910–Amendment 1 [1]. On the basis of the work done by Bowness and Lee [64], Lie et al. [65] subsequently established the M_k equations for a semi-elliptical crack in double-sided T-butt joints and cruciform X-joints. Until now, M_k equations for T-butt geometry with one-side weld [64], T-butt geometry with double-sided weld and non-load carrying cruciform X-joints [65] have been established. However, the corresponding equations for plate-to-plate butt-welded joints do not exist. Therefore, the main purpose of this section is to propose a new set of M_k equations for butt welds under axial and bending loading.

4.2.2 Parametric study of cracked plate-to-plate butt welds

In order to perform the extensive numerical analysis of cracked butt welds, four basic parameters crack depth ratio (a/t), crack aspect ratio (a/c), weld length ratios (L/t) and weld angle (θ) are considered in this study (see Fig. 4.1 for notation). The parameters used are based on works done by Bowness and Lee [64] and AWS D1.1:2000 [59] recommendations, taking into account a wide range of crack and weld dimensions for butt welds.

4.2.2.1 Details of crack parameters

The crack depth ratios studied are 0.05, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9, varying from a very shallow crack depth ratio of 0.05 to a deep crack depth ratio of 0.9. In this study, the plate thickness (t) is kept constant and the crack depth (a) is gradually changed to get the required crack depth ratio. The crack aspect ratios (a/c) used in this study are 0.1, 0.2, 0.4 and 1.0, which makes allowance for most crack sizes encountered in practice.

Fig. 4.1 Nomenclature of parameters used in butt weld.

Table 4.1 Table of analyses

Loading	L/t	θ	a/t	a/c
m = axial, b = bending	0.2	$0^\circ, 15^\circ,$ $30^\circ, 45^\circ$	0.05,0.1,0.15,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m = axial, b = bending	0.8	$0^\circ, 15^\circ,$ $30^\circ, 45^\circ$	0.05,0.1,0.15,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m = axial, b = bending	1.4	$0^\circ, 15^\circ,$ $30^\circ, 45^\circ$	0.05,0.1,0.15,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m = axial, b = bending	2.0	$0^\circ, 15^\circ,$ $30^\circ, 45^\circ$	0.05,0.1,0.15,0.2,0.3,0.4, 0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0

4.2.2.2 Details of weld parameters

For T-butt joints and cruciform X-joints as shown in Fig. 4.2, the attachment footprint width ratios (L/t) investigated are 0.5, 1.25, 2.0 and 2.75. However, for butt welds without attachments considered in this study, the weld length ratios of 0.2, 0.8, 1.4 and 2.0 are employed based on

AWS D1.1:2000 [59] recommendations, which covers the most commonly-used butt-welded plate configurations. This parameter includes the main plate thickness (t) and weld length (L). As referred in the above, the plate thickness (t) is kept constant. Hence, the variation of weld length ratio is obtained by altering the values of weld length. The last parameter is the weld angle. Considering that butt welds should be made with no change in thickness unless otherwise specified, 0° is proposed as the minimum weld angle [59]. The maximum proposed weld angle is 45° . Therefore, weld angles of 0° , 15° , 30° and 45° are used, covering a wide range of butt weld profiles found in practice.

Fig. 4.2 Schematics of double-sided T-butt joint and X-joint.

4.2.2.3 Modelling details

In the present study, about 1400 cases of different geometries of butt welds are analyzed. Therefore, a new FE mesh generator is developed to create these models, which is based on some on-going research works in Nanyang Technological University, Singapore [66-70]. The new mesh generator can generate input files compatible with ABAQUS software [51]. In this study, the difference for J -integral values of contours 2, 3, 4 and 5 is nearly negligible (less than 1%) satisfying path independence. Therefore, the SIF is determined from the average J value of these contours, ignoring the first contour value due to numerical errors. The plane strain condition is employed along the crack front except at the crack ends [64,71], and the SIF is calculated by the following expression:

$$K_{eq} = \sqrt{K_I^2 + K_{II}^2} = \sqrt{\frac{JE}{(1-\nu^2)}} \quad (4.2)$$

where K_I and K_{II} are the Mode-I and Mode-II SIF, respectively. Since the J values obtained are the total ones, an equivalent mixed-mode SIF (K_{eq}) defined in Eq. (4.2) is used to characterize the crack-tip singularity [72]. The Mode-III SIF (K_{III}) is not included in this expression due to its insignificant contribution to the total J . For the SIF values at the crack ends, the plane stress condition is assumed and $E/(1-\nu^2)$ in Eq. (4.2) is replaced by E to determine the corresponding SIF values [64,71]. Then the calculated SIFs are used to calculate the M_k values of butt welds. Eventually using multiple regression analyses, the relationship between the M_k and the independent variables a/T , a/c , L/T and θ are established.

In the analyses, the element type used is a 20-node brick element with reduced integration (C3D20R), accounting for compatibility and convergent issues. Based on symmetry, only half of the plate model is considered. A typical butt weld mesh generated by the mesh generator is shown in Fig. 4.3. The butt welds are subjected to both axial and bending loadings. For axial loading cases, one end of the main plate parallel to crack face is restrained and a uniform stress is applied at the other end of the main plate [65,73]. Bending is achieved through pure bending [65,73]. The Young's modulus and Poisson's ratio used in the analyses are 210 kN/mm² and 0.3, respectively.

Fig. 4.3 Typical FE mesh of butt weld.

4.2.2.4 Validity of the FE mesh generator

The accuracy of the FE mesh generator must be verified before it can be used for extensive parametric study of cracked plate-to-plate butt welds. At first, a mesh generator for a plain plate containing a semi-elliptical surface crack [71,74] is created to produce the corresponding FE models. The SIF values obtained are compared with the values obtained from Newman and

Raju equation [60] at the crack deepest point and at the crack ends, and a good agreement is observed (Fig. 4.4), indicating that the present mesh generator is well suited for further analyses of more complicated configurations where sufficient elements are used to guarantee good convergence. Therefore, FE models of plate-to-plate butt welds are created by inserting a subroutine to the previous main program to generate the butt weld, and convergent tests covering the range of parameters outlined in Table 4.1 are performed to determine that the number of elements ranging from 3,000 to 8,000 depending on the crack size and plate geometry are adequate. The extensive validation works provide confidence in the present FE analyses to attain accurate SIF values.

Fig. 4.4 Comparison of SIF values obtained from Newman and Raju equation [60] and FE analyses for a plain plate.

4.2.3 Results and Discussion

The section is divided into two subsections. The first subsection gives a comparison of M_k values between butt welds with single T-butt welds and X-joints, and the second one covers the influence of the four parameters (a/t , a/c , L/t and θ) on M_k values.

4.2.3.1 Comparison of M_k results for butt welds with single T-butt joints and X-joints

The weld angles (θ) and weld length ratios (L/t) used for butt welds in this study are 0° , 15° , 30° , 45° and 0.2, 0.8, 1.4, 2.0, respectively. However, weld angles (θ) of 30° , 45° , 60° , 75° and attachment footprint width ratios (L/t) of 0.5, 1.25, 2.0, 2.75 are employed for single T-butt joints and X-joints. It is found that butt welds, single T-butt joints and X-joints have the same parameters for $\theta = 30^\circ$, 45° and $L/T = 2$. Therefore, M_k vs a/t plots comparison for the same

parameters are carried out to investigate the influence of different weld profiles on M_k values. Figs. 4.5-4.6 outline the plots comparison between butt welds with single T-butt joints and X-joints subjected to axial and bending loading. The representative plots cover different crack depth ratios for $a/c = 0.4$, $L/t = 2$, $\theta = 45^\circ$. For the deepest point under axial loading cases (Fig. 4.5), the M_k values for shallower cracks are lower for the butt welds when compared to the X-joints, and the reverse trend is observed for deeper cracks. X-joint has two additional attachments in comparison with butt weld. The stress intensity factor for shallower cracks in X-joints is magnified by the presence of the notch stress. As the cracks opens, the upward and opening deformation of the plain plate take place. However, for deeper cracks with a/t ratios greater than 0.2, the presence of the two attachments in X-joints restrains the upward and opening deformation of the crack. Therefore, the M_k values tend to fall below that of the butt welds for deeper cracks. A similar trend is observed at the crack ends under axial loading (Fig. 4.6). For the bending loading cases, the M_k values of butt welds are always lower than that of the X-joints through the whole crack depth ratios. As mentioned above, the parameter ranges used in the analyses for butt welds are different from single T-butt joints and X-joints. Even for the same parameter values ($L/t=2$, $\theta=30^\circ$ and 45°), all the plots shown in Fig. 4.5-4.6 distinctly indicate the difference between M_k values of butt welds with single T-butt joints and X-joints. The above mentioned points establish the need for having separate M_k equations for butt welds.

Fig. 4.5 M_k vs a/t plot comparison between butt weld with single T-butt joint and X-joint – for axial loading ($a/c = 0.4$, $L/t = 2$, $\theta = 45^\circ$).

Fig. 4.6 M_k vs a/t plot comparison between butt weld with single T-butt joint and X-joint – for bending loading ($a/c = 0.4$, $L/t = 2$, $\theta = 45^\circ$).

In order to further validate the need for proposing new M_k equations for butt welds, the largest percentage difference is calculated in this study to quantitatively describe the difference between M_k values of butt welds with single T-butt joints and X-joints, which is given by the following expression:

$$\% \text{ Difference} = \max \left(\frac{|M_{k(\text{butt weld})} - M_{k(\text{single T-butt joint or X-joint})}|}{M_{k(\text{butt weld})}} \right) \times 100\% \quad (4.3)$$

Table 4.2 Comparison of M_k values for butt welds with single T-butt joints and X-joints

	Axial loading		Bending loading	
	Deepest point	Crack ends	Deepest point	Crack ends
$M_{k(\text{butt weld})}$	0.9237	1.9027	0.8459	1.6513
$M_{k(\text{single T-butt joint})}$	0.8239	3.1162	0.6801	3.2596
$a/t, a/c, L/t, \theta$	0.7,0.1,2,30°	0.05,0.2,2,45°	0.5,1.0,2,30°	0.05,0.2,2,45°
Largest percentage difference (%)	10.8	63.8	19.6	97.4
$M_{k(\text{butt weld})}$	1.2924	1.9027	0.9042	1.6513
$M_{k(\text{X-joint})}$	2.0732	2.4551	4.1430	5.2478
$a/t, a/c, L/t, \theta$	0.05,0.1,2,45°	0.05,0.2,2,45°	0.2,0.1,2,45°	0.05,0.2,2,45°
Largest percentage difference (%)	60.4	29.0	358.2	217.8

The Comparison of M_k values for butt welds with single T-butt joints and X-joints are outlined in Table 4.2. It is found that the largest percentage differences between butt welds and single T-butt joints are 63.8% and 97.4% for axial loading and bending loading cases, respectively. Percentage differences of 60.4% and 358.2% for axial and bending loading cases

are observed between butt welds and X-joints. All the above comparison results reconfirm the need for having separate M_k equations for butt welds.

4.2.3.2 Influence of different parameters on M_k results

Figs. 4.7-4.10 show the representative plots depicting the variations of M_k values over crack depth ratio (a/t) on the influence of different values of crack aspect ratio (a/c), weld angle (θ) and weld length ratio (L/t). Due to the similar trend for axial loading cases with bending loading cases, only M_k versus a/t plots under axial loading are presented. In the following subsections, the effect of the above mentioned parameters are discussed individually.

Fig. 4.7 illustrates the effect of crack aspect ratio on M_k values at the deepest point of the crack under axial loading. It is observed that M_k has higher values for lower a/c values (wider cracks) and lower values for higher a/c values (penny shape cracks). As the axial loading is applied, the crack gradually opens. The crack opening deformation is greater for wider cracks. Therefore, M_k values are higher for lower a/c values. It is also observed that M_k values gradually decrease with increasing a/t ratio and fall below unity for deep cracks. The plain plate deforms upward at the crack region under axial loading (Fig. 4.8(a)). For the butt welds, the stress intensity factors for shallow cracks are magnified by notch stress caused by the presence of butt welds. With crack depth increasing, the presence of butt welds restrains the upward and opening deformation of crack (Fig. 4.8(b)), resulting in M_k values falling below unity. However, the gradually decreasing trend of M_k values with increasing a/t values is not observed at the deepest point of the crack for a/t values of 0.8 and 0.9. This is due to negative stress intensity factors caused by crack face penetrating each other. For the case at the deepest point of the crack under bending, the trend is similar to the axial loading cases except that decreasing tendency of M_k versus increasing a/t is not observed when a/t is greater than 0.5. Therefore, in the regression analyses, very deep crack a/t values of 0.8 and 0.9 are omitted for the deepest point of the crack under axial loading. The numerical data with a/t ratio above 0.5 are not used to fit the M_k equations for the bending cases.

For the crack ends under axial loading cases, the variation tendency of M_k values with a/c at the crack ends is more noticeable than that at the deepest point of the crack. It is also noted that the decreasing trend of M_k values with increasing a/t is applicable through the whole a/t range. A similar trend is also observed for bending loading cases. Therefore, all the numerical

data obtained can be employed in the regression analyses for the crack ends under axial and bending loading.

Fig. 4.7 Effect of crack aspect ratio on M_k values – under axial loading ($L/t = 0.8$, $\theta = 15^\circ$).

Fig. 4.8 Deformed models under axial loading using the same scale.

At the deepest point of the crack under axial loading (Fig. 4.9), the M_k values increase with the increasing weld angle (θ) for shallow cracks ($a/t < 0.2$). This is because the steeper weld angle induces a larger stress concentration effect at the crack position. However, with the crack depth ratio increasing ($a/t > 0.2$), increasing weld angle results in a reduction of M_k values, presenting an opposite trend with the case of shallow cracks. The stiffening effect of the butt welds prevents the further opening deformation of the crack, bringing about lower M_k values for steeper weld angle for deep cracks. At the crack ends under axial loading, the M_k values

increase with the increasing weld angle (θ) through the whole crack depth range, and this increase becomes larger as the crack depth becomes shallower.

The effect of varying weld length ratio (L/t) on M_k values is shown in Fig. 4.10. The variable trend of M_k versus a/t for different weld length ratio is very similar to the tendency affected by variation of weld angle at the deepest point and the crack ends. However, it is noteworthy that the effect of weld length ratio becomes marginal beyond $L/t = 0.8$, especially for the case at the crack ends. This is due to the butt welds stiffening the top surface of the main plate and raising the weld toe notch stress. The above tendency also exists at the deepest point and the crack ends under bending loading.

Fig. 4.9 Effect of weld angle on M_k values – under axial loading ($a/c = 0.2$, $L/t = 0.8$).

Fig. 4.10 Effect of weld length ratio on M_k values – under axial loading ($a/c = 0.2$, $\theta = 15^\circ$).

4.2.4 Multiple Regression Analyses

In this section, the new M_k equations for butt welds are obtained by multiple regression analyses. However, not all the numerical data for the parametric study are used in the multiple regression analyses. The exception to the data that is not used for the study is listed in the following:

- Deepest point of the crack for axial loading: The data for $a/t > 0.7$ are not used in the multiple regression analyses.
- Deepest point of the crack for bending loading: The data when a/t is greater than 0.5 are excluded in the regression analyses.
- Crack ends for axial and bending loading: The regression analyses include all the M_k values with a/t ranging from the shallowest case 0.05 to the deepest one 0.9.

4.2.4.1 Regression analyses

Various parameters (a/t , a/c , L/t and θ) are included in the multiple regression analyses. Therefore, the new M_k equations are developed in stages allowing for the effects of these parameters progressively. In the first stage of the equation development, the M_k data for a constant $\theta=15^\circ$ and $L/t=0.8$ are involved in the regression analyses. The second development stage takes account of the effect of varying weld angle (θ) for a constant L/t ($=0.8$). In the final stage of the equation development, all the M_k data are included in the regression analysis, making allowance for varying θ and varying L/t . The new parametric M_k equations are built up, in stages, from the following four basic functions:

- The crack depth to a power $C_1 \left(\frac{a}{t} \right)^{C_2}$
- One minus the crack depth to a power $C_1 \left[1 - \left(\frac{a}{t} \right) \right]^{C_2}$
- Exponent of the crack depth to a power $C_1 \exp \left[\left(\frac{a}{t} \right)^{C_2} \right]$
- Polynomial functions $C_1 + C_2 \left(\frac{a}{t} \right) + C_3 \left(\frac{a}{t} \right)^2 + \dots$

where C_1 , C_2 etc. are coefficients. Based on the basic functions, a trial equation with an initial guess to the coefficients values are given first [64,71,74,75]. Then, least square fitting method is used to minimise the differences between the FE data and the equation predictions by changing the coefficients in the equation. Finally, the required equations are proposed in three stages mentioned in the above. The development procedures are implemented by a MATLAB codes. The newly proposed M_k equations are established between the dependent variable M_k and the independent variables a/t , a/c , θ and L/t as shown in Appendix A, which are similar in form to the M_k equations outlined in BS 7910:2013 [1].

4.2.4.2 Goodness of fit of proposed new equations

Due to the complex form for the newly proposed M_k equations, percentage error frequency histograms are used to demonstrate the goodness of fit of the equations. In the histogram, the abscissa is represented by the percentage error which is described by the following expression:

$$\% \text{ Error} = \frac{[M_{k(\text{equation})} - M_{k(\text{FEA})}]}{M_{k(\text{FEA})}} \times 100\% \quad (4.4)$$

and the ordinate is defined by the percentage of data points inside the range of the particular percentage error.

Figs. 4.11 are the histograms describing the relative difference between the proposed M_k equations and the data used in the regression analyses. Overall, these equations are a good fit to the used data due to the histograms exhibiting good normal distribution. In detail, for the deepest point of the crack under axial loading, the maximum percentage of over-prediction is 2.3% and the maximum percentage of under-prediction is 3.9%. For the crack ends under axial loading, the maximum percentage of over-prediction is 7.7% and the maximum percentage of under-prediction is 7.8%. For the deepest point of the crack under bending loading, the maximum percentage of over-prediction is 2.7% and the maximum percentage of under-prediction is 3.7%. For the crack ends under bending loading, the maximum percentage of over-prediction is 7.9% and the maximum percentage of under-prediction is 5.1%.

Fig. 4.11 Error histograms for butt weld.

4.2.4.3 Validity limits of proposed equations

Appendix A includes the proposed M_k equations at the deepest point of the crack and at the crack ends under axial and bending loadings. For the axial loading case, the validity range for crack depth ratio (a/t) is from 0.05 to 0.7 at the deepest point and from 0.05 to 0.9 at the crack ends. For the bending case, the validity range for crack depth ratio (a/t) is from 0.05 to 0.5 at the deepest point and from 0.05 to 0.9 at the crack ends.

4.2.5 Summary of the section

The present study indicates that the M_k results for butt welds differ from those of single T-butt joints with differences in values being as high as 63.8% for axial loading cases and 97.4% for bending loading cases, and percentage differences of 60.4% and 358.2% for axial and bending loading cases are observed between butt welds and X-joints. Moreover, due to the differences of various parameters used in the study, the new set of M_k equations are specifically developed at the deepest point and at the crack ends of butt welds subjected to axial and bending loading by multiple regression analyses. The parameters of crack aspect ratio (a/c), weld angle (θ) and weld length ratio (L/t) have significant influence on M_k results. Finally, the goodness of

fit of the proposed M_k equations is demonstrated by percentage error frequency histogram, showing a good correlation between FE results and the proposed equations.

4.3 WELD TOE CRACK MAGNIFICATION FACTORS IN LOAD-CARRYING CRUCIFORM JOINTS

Load-carrying cruciform fillet welded joint is commonly encountered in on-shore and offshore structures. In this section, the M_k of such configuration containing a 3-D semi-elliptical surface crack at the weld toe are analysed using 3-D FE analyses. Due to the non-penetrating fillet welds at the intersection of the main plate and transverse stiffener, the M_k values are found to be much larger than the corresponding results of non-load-carrying cruciform welded joints. Therefore, a new set M_k equations for load-carrying joints are built up using multiple regression analyses, and statistical analyses are carried out to validate the goodness of fit of these equations. It is noted that the existing BS 7910:2013 [1] assessment equation produces an over-conservative prediction on M_k values as compared with the newly proposed M_k equations, and the largest over-estimation percentage difference can be as high as 257.8%.

4.3.1 Background of loading-carrying cruciform joints

Load-carrying cruciform fillet welded joint is formed when two longitudinal main plates are welded to a transverse stiffener in a girder web of many steel structures. The design of this welded joint under fatigue loading conditions is complicated as there are two stress concentration regions from which a fatigue crack can initiate. These two regions are the severe crack-like internal notch formed by the plates end at the root of the fillet welds and at the weld toe. The crack initiation point depends greatly upon the relative magnitude of the nominal stresses in the plate and in the weld. The value of the stress at the root related to that at the toe depends on the size of the weld leg length, and when it is large, failure will always occur at the weld toe. When the weld leg length is small, the stress at the tip of the root may be high enough to propagate a crack from this defect so as to cause failure there before failure at the weld toe. Therefore, considerable research works had been conducted to evaluate the fatigue strength in weld root cracks [76-79]. Frank and Fisher [76] presented an empirical stress intensity factor (SIF) solution of fatigue cracks initiating from the weld root based on 2-D FE analyses, and a range of SIF calculation formula was also proposed for fatigue assessments. However, the previous solutions are only applicable for joints with equal main and transverse plate

thicknesses and isosceles triangle shaped fillet welds. The influence of different main and transverse plate thicknesses on SIF values was examined by Motarjemi et al. [72], and a set of coefficients for SIF equations were tabulated to determine SIF values of specific configurations. Kainuma and Mori [79] considered the effect of weld shape, weld size, non-penetrating region and plate thickness on fatigue strength of the cruciform welded joints with weld root failures, and a corresponding formula was derived to quantitatively assess the fatigue strength.

In addition to the weld root failure, fatigue cracks can also occur from the weld toe. Many studies had been carried out to evaluate the fatigue strength based on SIF calculations of weld toe cracks [31,80-82], but the SIF equations obtained are confined to 2-D configuration cases only. In practice, the crack at the weld toe is found to resemble a semi-elliptical in shape [64,83,84], and the SIF equations for this crack profile have not been established for load-carrying cruciform fillet welded joints. Therefore, the main objective of this study is to propose a new set SIF equations for load-carrying cruciform fillet welded joints containing a semi-elliptical crack at the weld toe based on 3-D linear elastic FE analyses.

4.3.2 Parametric study of cracked load-carrying cruciform fillet-welded joints

The various weld and crack parameters are chosen in accordance with the recommendations given in Annex M of BS 7910:2013 [1] (see Fig. 4.11 for notation), and these parameters take into account a wide range of weld and crack dimensions associated with the cracked load-carrying cruciform fillet-welded joints. The following two subsections show the details of weld and crack parameters and the numerical modelling procedure.

4.3.2.1 Details of weld and crack parameters

Fig. 4.12 Load-carrying cruciform fillet welded joints with a semi-elliptical surface crack at weld toe.

The present study focuses solely on the joints cases with equal main and transverse plate thicknesses (T), and the sharp weld toe is used in the numerical models. For fillet welds, the leg length (h) on the main and transverse plates is equal and the leg length ratio (h/T) is introduced to represent the relative dimension. The non-penetrating region is characterized using the incomplete penetration ratio (L/W), and W is defined as follows:

$$W = T + 2h \quad (4.5)$$

The weld root gaps measured by Kainuma and Kim [78] were less than 0.2 mm in welded structures. It is found in this study that the gap width value varying from 0.0 mm to 0.2 mm has a negligible effect on the SIF results of the weld toe crack. Hence, the root gap is set to be 0.1 mm in all the following FE models. The semi-elliptical surface crack located at the weld toe is perpendicular to the main plate face as indicated in Fig. 4.12. 9 different crack depth ratios (a/T) are analysed, ranging from a very shallow crack depth ratio of 0.05 to a deep crack depth ratio of 0.7, and the crack aspect ratios (a/c) used are varying from 0.1 to 1.0. These wide ranges of parameters cover most of the crack sizes encountered in practice. Therefore, four basic parameters, namely, crack depth ratio (a/T), crack aspect ratio (a/c), incomplete penetration ratio (L/W) and weld leg length ratio (h/T) are used in the parametric study (please refer to Table 4.3 for details).

Table 4.3 Table of analyses

Loading	h/T	a/T	a/c	L/W
m = axial	0.45	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0	0.1,0.3,0.5,0.7
m = axial	0.70	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0	0.1,0.3,0.5,0.7
m = axial	0.95	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0	0.1,0.3,0.5,0.7
m = axial	1.20	0.05,0.1,0.15,0.2,0.3,0.4,0.5,0.6,0.7	0.1,0.2,0.4,1.0	0.1,0.3,0.5,0.7

4.3.2.2 Numerical modelling procedure

Fig. 4.13 Boundary conditions of cracked load-carrying cruciform fillet welded joints subjected to axial loading.

Fig. 4.13 presents the boundary conditions for the cruciform welded joints. Due to symmetry of the model analyzed, a symmetric boundary condition about Y-axis is applied to the plate face at $Y = 0$. One end of the main plate is restrained in the X-axis and Z-axis directions, and the other end is only constrained in the Z-axis direction and a uniform axial stress is applied in the X-axis direction to provide the loading condition. Very fine mesh density around the crack front is necessary to guarantee the convergence of the numerical results. The full mesh model of a typical load-carrying cruciform fillet-welded joint is depicted in Fig. 4.14. The determination of SIFs is based on the descriptions in Sub-section 4.2.2.3. The M_k for the load-carrying cruciform fillet-welded joint is expressed as [64,74,85]

$$M_k = \frac{K_{eq}^{(\text{load-carrying cruciform fillet-welded joints})}}{K_{eq}^{(\text{plain plate})}} \quad (4.6)$$

The relationship between M_k and other independent variables a/t , a/c , L/W and h/t are established by multiple regression analyses.

Fig. 4.14 Typical FE mesh for load-carrying cruciform fillet welded joints.

4.3.2.3 Verification of the FE mesh generator

In this sub-section, the accuracy of the FE mesh generator is verified before it can be used for extensive parametric study of cracked load-carrying cruciform fillet-welded joints. This mesh generator is developed step by step based on the earlier works conducted by the authors [65,71,74]. At first, a mesh generator for a plain plate with a semi-elliptical surface crack [71,74] is developed to create the FE models, and the comparison of SIF values obtained from Newman and Raju equation [60] and present FE analyses has been given in Fig. 4.4. Then, the mesh generator is upgraded to serve for non-load-carrying cruciform fillet-welded joints [65] by adding two attachments to the main plate, and the validation is confirmed by comparing with the results from FEACrackTM software [86]. The previous works indicate that the present mesh generator is found to be well suited for further analyses of more complicated configurations where sufficient elements are used to guarantee good convergence. Therefore, FE models of load-carrying cruciform fillet-welded joints are created by inserting a subroutine to the previous main program to generate the non-penetrating region, and convergent tests covering the range of parameters outlined in Table 4.3 are performed to determine the optimal number of elements ranging from 13,672 to 61,432 depending on the crack size and plate geometry. The extensive validation works provide confidence in the present FE analyses to attain accurate SIF values.

4.3.3 Results and discussion

In this section, the influence of a/c on M_k values is firstly investigated to determine the proper interval for a/c varying from 0.1 to 1.0. Then, the comparison for load-carrying and non-load-carrying cruciform welded joints is performed to demonstrate the need to have separate M_k equations for the former one.

4.3.3.1 Effect of a/c on M_k values

Fig. 4.15 Effect of a/c on M_k values for $h/T = 0.7$ and $L/W = 0.3$.

Fig. 4.15 illustrates the effect of a/c on M_k values at the crack deepest point and at the crack ends. M_k values for $a/c = 0.7$ and $a/c = 1.0$ are nearly identical. Hence, a/c values of 0.1, 0.2, 0.4 and 1.0 are used in the present parametric study. For the crack deepest point cases, the variation of a/c has a marginal influence on M_k values. It is also noted that M_k values gradually decrease with the increase of a/t and fall below unity for intermediate and deep cracks [74]. The SIFs for shallow cracks are magnified by notch stress induced by the presence of fillet welds. However, as crack depth further increases, the presence of fillet welds restrains the upward and opening deformation of crack, leading to M_k falling below unity. For the crack ends, the variation tendency of M_k versus a/t is similar to the crack deepest point cases, while the difference of M_k values for varying a/c is more noticeable with lower a/c values (wider cracks) causing higher M_k results.

4.3.3.2 Comparison results for load-carrying and non-load-carrying cruciform fillet-welded joints

Fig. 4.16 outlines the comparison of K_{II} values from load-carrying and non-load-carrying cruciform welded joints. In the latter, the K_{II} values for the weld toe crack are vanishingly small, approximately $0.2 \text{ N/mm}^{3/2}$ at the crack deepest point, while the corresponding values in the former gradually increase by increasing L/W , up to nearly $2.0 \text{ N/mm}^{3/2}$ for $L/W = 0.7$. It is due to the presence of the non-penetrating region in load-carrying joints significantly magnifying in-plane shear loading applied at the crack region. Therefore, the K_{eq} calculated in Eq. (4.2) is employed to assess the fatigue and fracture resistance of such cracked structures by including K_I and K_{II} , and the K_{III} component is ignored because of its negligible influence. In order to quantitatively clarify the effect of K_{II} and K_{III} values on the total J , J_1 , J_2 and J_3 are defined using the following expressions for plane strain cases:

$$J_1 = \frac{(1-\nu^2) K_I^2}{E} \quad (4.7)$$

$$J_2 = \frac{(1-\nu^2)(K_I^2 + K_{II}^2)}{E} \quad (4.8)$$

$$J_3 = \frac{(1-\nu^2)(K_I^2 + K_{II}^2)}{E} + \frac{K_{III}^2}{2\mu} \quad (4.9)$$

where μ is the shear modulus with $\mu = E/[2(1+\nu)]$, and $(1-\nu^2)/E$ in Eqs. (4.7)-(4.9) is replaced by $1/E$ for plane stress condition. Tables 4.4 and 4.5 show the detailed results and percentage differences of J_1 , J_2 and J_3 at the crack deepest point and the crack ends. For the crack deepest point cases (Table 4.4), the percentage differences between J_2 and J_1 reflecting the effect of K_{II} are noticeable with the largest value being as high as 18.35%, while nearly no difference is observed for J_3 and J_2 , which means that K_{III} has an insignificant influence on the total J . It is also noted that the total J values at the crack ends (Table 4.5) are marginally affected by K_{II} and K_{III} . Therefore, the definition of K_{eq} in Eq. (4.2) is reasonable. Similar to the case shown in Fig. 4.16, a larger L/W induces an increase of the M_k value especially for shallow cracks. Hence, the M_k values for load-carrying joints are always larger than the corresponding values for non-load-carrying joints, and the difference is growing with the increase of L/W as predicted in Fig. 4.17. Therefore, even though the M_k equations for non-load-carrying cruciform fillet-welded joints

have been proposed by the authors, the above comparison confirms the need to have separate M_k equations for load-carrying joints.

Table 4.4 Effect of K_{II} and K_{III} on J values at the crack deepest point

h/t	L/W	a/t	a/c	J_1 ($\times 10^{-4}$ N/mm)	J_2 ($\times 10^{-4}$ N/mm)	J_3 ($\times 10^{-4}$ N/mm)	$\left(\frac{J_2-J_1}{J_1}\right) \times 100\%$	$\left(\frac{J_3-J_2}{J_2}\right) \times 100\%$
0.45	0.1	0.05	0.1	0.2434	0.2472	0.2472	1.56	0.00
			1.0	0.0873	0.0900	0.0900	3.09	0.00
		0.7	0.1	5.9613	5.9614	5.9614	0.00	0.00
			1.0	0.5572	0.5586	0.5586	0.25	0.00
	0.7	0.05	0.1	1.1677	1.2786	1.2786	9.50	0.00
			1.0	0.3864	0.4573	0.4573	18.35	0.00
		0.7	0.1	5.2672	5.4328	5.4328	3.14	0.00
			1.0	0.2361	0.2488	0.2488	5.38	0.00
1.20	0.1	0.05	0.1	0.2536	0.2580	0.2580	1.74	0.00
			1.0	0.0900	0.0932	0.0932	3.56	0.00
		0.7	0.1	5.2529	5.2530	5.2530	0.00	0.00
			1.0	0.5433	0.5442	0.5442	0.17	0.00
	0.7	0.05	0.1	0.3905	0.4114	0.4114	5.35	0.00
			1.0	0.1359	0.1497	0.1497	10.15	0.00
		0.7	0.1	5.7838	5.7924	5.7924	0.15	0.00
			1.0	0.4811	0.4811	0.4811	0.00	0.00

Table 4.5 Effect of K_{II} and K_{III} on J values at the crack ends

h/t	L/W	a/t	a/c	J_1 ($\times 10^{-4}$ N/mm)	J_2 ($\times 10^{-4}$ N/mm)	J_3 ($\times 10^{-4}$ N/mm)	$\left(\frac{J_2-J_1}{J_1}\right) \times 100\%$	$\left(\frac{J_3-J_2}{J_2}\right) \times 100\%$
0.45	0.1	0.05	0.1	0.0459	0.0459	0.0460	0.00	0.22
			1.0	0.1674	0.1678	0.1683	0.24	0.30
		0.7	0.1	1.8187	1.8268	1.8386	0.45	0.65
			1.0	1.3809	1.3856	1.3929	0.34	0.53
	0.7	0.05	0.1	0.2596	0.2597	0.2605	0.04	0.31
			1.0	0.8887	0.8942	0.9052	0.62	1.23
		0.7	0.1	5.3323	5.3540	5.4716	0.41	2.20

			1.0	4.0656	4.0950	4.2187	0.72	3.02
		0.05	0.1	0.0489	0.0489	0.0489	0.00	0.00
			1.0	0.1699	0.1704	0.1710	0.29	0.35
	0.1	0.7	0.1	1.6723	1.6869	1.7015	0.87	0.87
			1.0	1.3371	1.3429	1.3522	0.43	0.69
1.20		0.05	0.1	0.0778	0.0778	0.0780	0.00	0.26
			1.0	0.2623	0.2636	0.2659	0.50	0.87
	0.7	0.7	0.1	1.9056	1.9190	1.9461	0.70	1.41
			1.0	1.6978	1.7138	1.7544	0.94	2.37

Fig. 4.16 Comparison of K_{II} values from load-carrying and non-load-carrying cruciform welded joints for $h/t = 0.7$, $a/t = 0.3$ and $a/c = 0.2$.

Fig. 4.17 Comparison of M_k values from load-carrying and non-load-carrying cruciform welded joints for $h/t = 0.7$ and $a/c = 0.2$.

4.3.4 M_k equations developed using multiple regression analyses

Multiple regression analyses are used to propose the new M_k equations for load-carrying cruciform fillet-welded joints. The M_k equations are developed in stages to consider the effects of four parameters (a/t , a/c , L/W and h/t) progressively. The first development stage allows for the effect of varying a/c for a constant $L/W=0.1$ and $h/t=0.45$. The effect of varying L/W for a constant $h/t=0.45$ is taken into account in the second development stage. In the final stage of the equation development, all the M_k data are included in the regression analysis, making allowance for varying L/W and h/t . The new parametric M_k equations are built up, in stages, according to the four basic functions listed in Sub-section 4.2.4.1. The followings are the M_k equations for the crack deepest point and the crack ends established based on the four basic functions.

At the crack deepest point:

$$M_{k,toe}^{deep} = \left[f_1\left(\frac{a}{t}, \frac{a}{c}\right) + f_2\left(\frac{a}{t}, \frac{L}{W}\right) \right] f_3\left(\frac{a}{t}, \frac{L}{W}, \frac{h}{t}\right) \quad (4.10)$$

where

$$f_1\left(\frac{a}{t}, \frac{a}{c}\right) = -4.187443\left(\frac{a}{t}\right)^{A_1} + A_2 \left[1 - \left(\frac{a}{t}\right) \right]^{A_3} \quad (4.11)$$

$$A_1 = -0.006699\left(\frac{a}{c}\right)^2 + 0.060199\left(\frac{a}{c}\right) + 0.132981$$

$$A_2 = 0.243472\left(\frac{a}{c}\right)^2 - 0.631431\left(\frac{a}{c}\right) + 4.502722 \quad (4.12)$$

$$A_3 = 0.021592\left(\frac{a}{c}\right)^2 - 0.082626\left(\frac{a}{c}\right) - 0.047647$$

$$f_2\left(\frac{a}{t}, \frac{L}{W}\right) = A_4 \left[1 - \left(\frac{a}{t}\right) \right]^{A_5} + 0.971259\left(\frac{a}{t}\right)^{A_6} \quad (4.13)$$

$$\begin{aligned}
 A_4 &= -2.275102 \left(\frac{L}{W} \right)^3 + 5.099321 \left(\frac{L}{W} \right)^2 - 3.768011 \left(\frac{L}{W} \right) - 0.236142 \\
 A_5 &= -0.006393 \left(\frac{L}{W} \right)^{-1.709022} + 0.048061
 \end{aligned} \tag{4.14}$$

$$\begin{aligned}
 A_6 &= -3.907304 \left(\frac{L}{W} \right)^3 + 6.919132 \left(\frac{L}{W} \right)^2 - 4.633721 \left(\frac{L}{W} \right) + 0.873854 \\
 f_3 \left(\frac{a}{t}, \frac{L}{W}, \frac{h}{t} \right) &= \left[A_7 \left(\frac{h}{t} \right)^2 + A_8 \left(\frac{h}{t} \right) + A_9 \right] \left(\frac{a}{t} \right)^{-0.005681} \\
 &\quad - 0.633002 \left[1 - \left(\frac{a}{t} \right) \right]^{A_{10} \left(\frac{h}{t} \right)^2 + A_{11} \left(\frac{h}{t} \right) + A_{12}}
 \end{aligned} \tag{4.15}$$

$$\begin{aligned}
 A_7 &= -3.935803 \left(\frac{L}{W} \right)^3 + 4.905402 \left(\frac{L}{W} \right)^2 - 0.692841 \left(\frac{L}{W} \right) + 0.024166 \\
 A_8 &= 10.565033 \left(\frac{L}{W} \right)^3 - 12.984031 \left(\frac{L}{W} \right)^2 + 1.674406 \left(\frac{L}{W} \right) - 0.048156 \\
 A_9 &= -3.912511 \left(\frac{L}{W} \right)^3 + 4.803722 \left(\frac{L}{W} \right)^2 - 0.603372 \left(\frac{L}{W} \right) + 1.633214 \\
 A_{10} &= 3.927331 \left(\frac{L}{W} \right)^2 - 4.092911 \left(\frac{L}{W} \right) + 0.401612 \\
 A_{11} &= -4.910425 \left(\frac{L}{W} \right)^2 + 9.111931 \left(\frac{L}{W} \right) - 0.973012 \\
 A_{12} &= 1.327904 \left(\frac{L}{W} \right)^2 - 3.213625 \left(\frac{L}{W} \right) + 0.352334
 \end{aligned} \tag{4.16}$$

At the crack ends:

$$M_{k, \text{toe}}^{\text{ends}} = g_1 \left(\frac{a}{t}, \frac{a}{c} \right) g_2 \left(\frac{a}{t}, \frac{L}{W} \right) g_3 \left(\frac{a}{t}, \frac{L}{W}, \frac{h}{t} \right) \tag{4.17}$$

where

$$g_1 \left(\frac{a}{t}, \frac{a}{c} \right) = 0.006645 \left(\frac{a}{t} \right)^{B_1} + B_2 \left[1 - \left(\frac{a}{t} \right) \right]^{B_3} \tag{4.18}$$

$$\begin{aligned}
 B_1 &= 0.025504 \left(\frac{a}{c} \right)^{-0.427822} - 1.413056 \\
 B_2 &= 0.092443 \left(\frac{a}{c} \right)^{-0.533921} + 1.105046
 \end{aligned} \tag{4.19}$$

$$\begin{aligned}
 B_3 &= 0.259243 \exp \left[-0.267633 \left(\frac{a}{c} \right) \right] - 0.431201 \exp \left[-8.547022 \left(\frac{a}{c} \right) \right] \\
 g_2 \left(\frac{a}{t}, \frac{L}{W} \right) &= B_4 \left(\frac{a}{t} \right)^{0.037085} + B_5 \left[1 - \left(\frac{a}{t} \right) \right]^{2.193111}
 \end{aligned} \tag{4.20}$$

$$\begin{aligned}
 B_4 &= -2.929901 \left(\frac{L}{W} \right)^3 + 3.708066 \left(\frac{L}{W} \right)^2 - 0.146582 \left(\frac{L}{W} \right) + 0.981334 \\
 B_5 &= -3.042231 \left(\frac{L}{W} \right)^3 + 4.955614 \left(\frac{L}{W} \right)^2 - 1.001302 \left(\frac{L}{W} \right) + 0.159388
 \end{aligned} \tag{4.21}$$

$$\begin{aligned}
 g_3 \left(\frac{a}{t}, \frac{L}{W}, \frac{h}{t} \right) &= B_6 \left(\frac{a}{t} \right)^{B_7} + B_8 \exp \left[\left(\frac{a}{t} \right)^{B_9} \right] + \left[B_{10} \left(\frac{L}{W} \right) + B_{11} \right] \left(\frac{a}{t} \right) \\
 &\quad + \left[B_{12} \left(\frac{L}{W} \right)^3 + B_{13} \left(\frac{L}{W} \right)^2 + B_{14} \left(\frac{L}{W} \right) + B_{15} \right]
 \end{aligned} \tag{4.22}$$

$$\begin{aligned}
 B_6 &= 0.108712 \left(\frac{h}{t} \right)^3 - 0.325564 \left(\frac{h}{t} \right)^2 + 0.330611 \left(\frac{h}{t} \right) + 0.023104 \\
 B_7 &= 2.427003 \exp \left[-5.451022 \left(\frac{h}{t} \right) \right] + 1.832011 \exp \left[0.023192 \left(\frac{h}{t} \right) \right] \\
 B_8 &= 0.596244 \left(\frac{h}{t} \right)^2 - 1.505243 \left(\frac{h}{t} \right) + 0.461443 \\
 B_9 &= 0.002399 \left(\frac{h}{t} \right)^{-4.387023} + 0.015433 \\
 B_{10} &= -0.782511 \left(\frac{h}{t} \right)^3 + 1.669321 \left(\frac{h}{t} \right)^2 - 0.731284 \left(\frac{h}{t} \right) + 0.062355
 \end{aligned} \tag{4.23}$$

$$\begin{aligned}
 B_{11} &= -0.327223 \left(\frac{h}{t} \right)^3 + 1.070706 \left(\frac{h}{t} \right)^2 - 1.183403 \left(\frac{h}{t} \right) + 0.336634 \\
 B_{12} &= -4.583221 \left(\frac{h}{t} \right)^2 + 10.604032 \left(\frac{h}{t} \right) - 3.838501 \\
 B_{13} &= -3.001621 \left(\frac{h}{t} \right)^3 + 13.304051 \left(\frac{h}{t} \right)^2 - 18.651014 \left(\frac{h}{t} \right) + 5.972304 \\
 B_{14} &= 0.522173 \left(\frac{h}{t} \right)^3 - 2.046621 \left(\frac{h}{t} \right)^2 + 2.073301 \left(\frac{h}{t} \right) - 0.566133 \\
 B_{15} &= -1.654421 \left(\frac{h}{t} \right)^2 + 4.203302 \left(\frac{h}{t} \right) - 0.340781
 \end{aligned}$$

4.3.5 Goodness of fit of proposed M_k equations

The aim of this subsection is to demonstrate that the M_k equations proposed in Sub-section 4.3.4 are a good fit to the numerical data obtained using 3-D linear elastic FE analyses, and percentage error frequency histograms are employed to validate the goodness of fit of these equations. The abscissa in the histogram is denoted by the percentage error, and it is calculated using the following expression:

$$\% \text{ Error} = \frac{[M_{k(\text{equation})} - M_{k(\text{FEA})}]}{M_{k(\text{FEA})}} \times 100\% \quad (4.24)$$

The ordinate is defined as the percentage of data points inside the range of the particular percentage error.

Fig. 4.18 Error histograms for load-carrying cruciform fillet welded joints.

It is observed from Fig. 4.18 that the error histograms show a good normal distribution of the percentage error, indicating that the proposed M_k equations fit very well with these regression data. Statistical parameters relating to the goodness of fit are included in Table 4.6 to further validate the accuracy of these equations. For the crack deepest point, the maximum percentage of over-prediction is 7.95% and the maximum percentage of under-prediction is 7.74%, and the corresponding percentage values for the crack ends are 5.43% and 5.84%, respectively. The mean percentage errors for all the cases are very close to zero, and the percentage error standard deviations are all less than 3%. The above statistical analyses further demonstrate that the proposed M_k equations are a very good fit to these FE data.

Table 4.6 Statistical evaluation of the proposed M_k equations

Statistical parameters	The crack deepest point	Crack ends
Number of FE data used in the analyses	576	576
Maximum percentage over-prediction (%)	7.95	5.43
Maximum percentage under-prediction (%)	-7.74	-5.84
$\sum(\text{Residuals})^2$	0.9093	0.8471
Mean Percentage Error (%)	-0.51	0.04
Percentage Error Standard Deviation (%)	2.96	1.98

4.3.6 Comparison of newly proposed M_k equations with assessment equation from BS 7910:2013

BS 7910:2013 [1] provides a fatigue and fracture assessment equation to apply for a surface crack at the weld toe of load-carrying cruciform fillet-welded joints subjected to axial loading based on 2-D plain strain FE analyses, and the equation is expressed as follows:

$$M_k = v_1 \left(a/t \right)^{v_2} \quad (4.25)$$

where

$$\begin{aligned} v_1 &= 0.615 \left(t/t_w \right)^{0.5}, \quad v_2 = -0.31 \quad \text{for } a/t \leq 0.073 \\ v_1 &= 0.830 \left(t/t_w \right)^{0.5}, \quad v_2 = -0.20 \quad \text{for } a/t > 0.073 \end{aligned} \quad (4.26)$$

t_w is the weld throat thickness. According to the Fatigue Design Recommendations for Steel Structures issued by the Japanese Society of Steel Construction [87], t_w is calculated using the following form:

$$t_w = \frac{W - L}{2\sqrt{2}} \quad (4.27)$$

In Sub-section 4.3.4, the M_k equations for the weld toe crack are built up in terms of 3-D FE analyses, and the goodness of fit of these equations are validated in Sub-section 4.3.5. This subsection is to compare the M_k values obtained from the newly proposed M_k equation (Eq. (4.10)) and the assessment equation given by BS 7910:2013 (Eq. (4.25)). It can be observed clearly from Fig. 4.19 that Eq. (4.25) significantly overestimates the M_k results for the entire range of crack depth. The over-estimation percentage difference is introduced to describe quantitatively the difference of M_k values given by these two equations, and it is determined by the following expression:

$$\% \text{ Difference} = \frac{[M_{k(\text{Eq.}(4.25))} - M_{k(\text{Eq.}(4.10))}]}{M_{k(\text{Eq.}(4.10))}} \times 100\% \quad (4.28)$$

Fig. 4.19 Comparison of Eq. (4.10) with Eq. (4.25) for $h/t = 0.7$ and $L/W = 0.3$ at the crack deepest point.

Figs. 4.20 and 4.21 depict the variation of percentage differences over a/t on the influence of different values of L/W and h/t , and all the points are located in the region above the red line, indicating the conservative prediction of Eq. (4.25) to the M_k values analysed. The differences

between Eq. (4.10) and Eq. (4.25) increase with the increase of L/W , and a reverse trend is observed for h/t . Moreover, a larger a/c value restricts the crack opening deformation in the 3-D configuration case, further enlarging the difference of these two equations. Therefore, the over-estimation percentage difference for $a/t = 0.7$, $a/c = 1.0$, $L/W = 0.7$, $h/t = 0.45$ is the largest one and its value is 257.8%.

Fig. 4.20 Percentage differences between Eq. (4.10) and Eq. (4.25) for $h/t = 0.7$ and $a/c = 0.4$.

Fig. 4.21 Percentage differences between Eq. (4.10) and Eq. (4.25) for $L/W = 0.5$ and $a/c = 0.4$.

4.3.7 Summary of the section

In this study, the FE mesh generator is developed and used for extensive parametric study of cracked load-carrying cruciform fillet-welded joints. The M_k values at the crack deepest point

and the crack ends are determined. The presence of non-penetrating region in the welded joints induces larger M_k values in comparison with the corresponding results of non-load-carrying cruciform welded joints, which illustrates the need to have separate M_k equations for load-carrying joints. Therefore, a new set of M_k equations are specially developed at the crack deepest point and at the crack ends of this cracked configuration using multiple regression analyses. The goodness of fit of the proposed M_k equations is demonstrated by statistical analyses and percentage error frequency histograms, indicating a good correlation between FE results and the proposed equations. Finally, by comparing the proposed M_k equations with the assessment equation from BS 7910:2013 [1], it is found that the latter one produces an over-conservative prediction of M_k values, and the largest over-estimation percentage difference is as high as 257.8%.

4.4 FRACTURE ASSESSMENT OF LOAD-CARRYING CRUCIFORM FILLET WELDED JOINTS WITH MULTIPLE CRACKS

Different from non-load-carrying cruciform fillet welded joints, the crack in load-carrying joints can propagate from the weld root or from the weld toe. Hence, the crack propagation location is critical for a given weld size, transverse and main plates thickness. In this section, the critical crack and weld sizes at which the crack propagates from the weld root to the weld toe will be determined. A new set of SIF equations of the weld toe crack have been reported by the authors in Section 4.3. In this section, the SIF values of the weld root crack are determined from the FE analysis. Then, by comparing the SIFs obtained at the weld root and the weld toe, it enables the authors to predict the critical crack and weld sizes whereby the crack will propagate from the weld root of the load-carrying filled welded joint.

4.4.1 Details of numerical analyses

4.4.1.1 Dimensions of weld and crack parameters

The load-carrying cruciform fillet welded joints containing multiple cracks are investigated in this study (Fig. 4.22). The thickness (T) and leg length (h) of the main and transverse plates are equal, and the leg length ratio (h/T) is introduced to represent the relative dimension. The width (B) of the main plate selected for the analyses are as follows: $B = 10c$ but $\geq 10T$, which minimizes finite geometry effect [73]. The cruciform joint contains a total of three cracks, viz., one weld toe crack and two weld root cracks. The weld toe crack is modelled as a semi-elliptical

shape and it is perpendicular to the main plate face as indicated in Fig. 4.22, which has also been clarified in Section 4.3. The non-penetrating regions at the weld root are assumed to act as two initial cracks [72,88], and the incomplete penetration ratio (L/W) used in the previous work [77,89] is renamed as the weld root crack depth ratio in this study to characterize the crack profile with $W = T + 2h$. Therefore, four basic parameters, namely, weld root crack depth ratio (L/W), weld leg length ratio (h/T), weld toe crack depth ratio (a/T) and weld toe crack aspect ratio (a/c) are used in the parametric study and reported in Tables 4.7.

Fig. 4.22 Load-carrying cruciform fillet welded joints with a semi-elliptical surface crack at the weld toe and two initial cracks at the weld root.

Table 4.7 Table of analyses

Loading	h/T	L/W	a/T	a/c
m = axial	0.45	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	0.70	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	0.95	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0
m = axial	1.20	0.1,0.2,0.3,0.4,0.5,0.6,0.7	0.0,0.1,0.2,0.3,0.5,0.7	0.1,0.2,0.4,1.0

4.4.1.2 Finite element modelling

In this study, a total of different geometries of the welded joints are analyzed, and a FE mesh generator developed earlier [65,71,74] enables the authors to generate all the 3-D models rapidly. The analyses are performed using ABAQUS [51], and the element type used is a 20-node brick element with reduced integration (C3D20R). The boundary conditions for the model

are same with that shown in Fig. 4.12. Fig. 4.23 shows a typical FE model of load-carrying cruciform fillet welded joints for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$. A fine mesh along the crack front is required to ensure convergence, especially at the weld toe crack front ends [64,73]. The mesh design of the crack section is depicted in Fig. 4.24 by using the same crack and weld sizes with that shown in Fig. 4.23. The material employed in the analyses is SM490B [85] with Young's modulus (E) and Poisson's ratio (ν) being 210 kN/mm² and 0.3, respectively.

Fig. 4.23 Typical FE mesh of load-carrying cruciform fillet welded joints with multiple cracks for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$.

Fig. 4.24 Mesh design of weld root and weld toe crack cross-sections.

The SIFs of the weld root crack are computed from the J -integral [90] implemented in ABAQUS [51]. Since the crack front penetrates uniformly through the main plate along the width direction, a plane strain condition is employed at the crack front except the region near the plate boundary. The SIF is calculated along the crack front using the following expression:

$$Y_{\text{root}} = \frac{K_{\text{root}}}{\sigma_a \sqrt{\pi L/2}} \quad (4.29)$$

where σ_a is the applied axial stress.

4.4.1.3 Validity of the FE mesh generator

Fig. 4.25 Comparison of weld root crack SIF values obtained from Frank and Fisher equation [76] and FE analysis for load-carrying cruciform welded joints.

The mesh generator used in Section 4.3 is developed to generate FE models of load-carrying cruciform fillet-welded joints with a weld toe crack and two weld root cracks by including the non-penetrating region. The mesh details at the weld toe crack have been confirmed in Subsection 4.3.2.3. Herein, the mesh strategy at the weld root crack is verified because the weld root crack is absent in the FE models of Section 4.3. It is noted from Fig. 4.25 that the SIF values obtained from FE analysis correlate very well with the values produced by Frank and Fisher equation [76], confirming the validity of the mesh generator for modelling the weld root crack. Therefore, the present FE mesh generator is considered to be well suited for parametric study of load-carrying fillet welded joints with multiple cracks where sufficient elements are used to guarantee good convergence. Convergent tests covering the range of parameters listed in Table 4.7 are performed to determine the optimal number of elements ranging from 20,000

to 68,000 depending on the crack size and plate geometry. The extensive validation works mentioned in the above provide confidence in the present FE analysis to attain accurate SIF values of the weld root and weld toe cracks.

4.4.2 Extraction of SIF values and effect of various parameters

As indicated in the previous section, the non-penetrating regions at the weld root are modelled as two initial cracks. Hence, there are four weld root crack fronts existed in the analyzed model (Fig. 4.26). The most critical crack front depends upon the largest SIF value as it is usually used to assess the integrity of the cracked structures [88,91]. In this section, the maximum SIF along the four weld root crack fronts and the SIFs at the weld toe crack front deepest point and the crack front ends are extracted first. Then, the interaction effect of the weld root and weld toe cracks is investigated in detail.

Fig. 4.26 Crack fronts in weld root cracks of load-carrying cruciform fillet welded joint for $h/T = 0.7$, $L/W = 0.3$ and $a/T = 0.4$.

4.4.2.1 Extraction of SIF values along crack fronts of weld root cracks

Fig. 4.27 illustrates a typical variation of SIF values along the Y-axis for the four weld root cracks fronts. Due to the presence of weld toe crack, the weld roots SIF values are influenced significantly when $Y/B \leq 0.23$. With the increase of Y/B ratio, the effect of the weld toe crack gradually weakens and the difference of SIF values for the four crack fronts becomes smaller and smaller. The rapid increase of SIF values at the end of the plate width ($Y/B \geq 0.43$) is induced by the boundary effect, and the SIF values in this region are neglected in the extraction procedure. It can be seen clearly from Fig. 4.27 that the maximum SIF is obtained from the

crack front-1 which is closest to the weld toe crack and the most affected by it. Extensive test cases covering the range of parameters listed in Table 4.7, demonstrate that the presence of weld toe crack always has the most significant influence on the crack front-1. Therefore, the maximum required SIF value is extracted from the variation curve of SIF versus Y/B at the crack front-1. The Y_{root} can be calculated from Eq. (4.29) based on the SIF value obtained.

Fig. 4.27 Variation of SIF values of four weld root cracks fronts for $h/T = 0.7$, $L/W = 0.3$, $a/T = 0.4$ and $a/c = 0.4$.

4.4.2.2 Influence of different parameters on Y_{root} values

Fig. 4.28 Effect of h/T on Y_{root} values for $a/T = 0.3$ and $a/c = 0.2$.

The present numerical study is composed of various parameters, and the aim of this subsection is to illustrate the influence of these parameters on Y_{root} values. Fig. 4.28 shows a representative plot depicting the variation of Y_{root} with L/W for different h/T values. Y_{root} values increase with the decrease of h/T , and the increase becomes larger as h/T tends to be smaller, which means that a smaller weld size induces a larger Y_{root} value for a constant L/W . The effect of the presence of a weld toe crack on the SIF of the weld root crack is presented in Fig. 4.29, and it is noted that a larger a/T results in a larger Y_{root} value. Fig. 4.17 shows the dependence of $M_{k,\text{toe}}$ on L/W by observing that increasing L/W enlarges the $M_{k,\text{toe}}$. In fact, the two figures indicate the interaction effect of the weld root and weld toe cracks, conforming the necessity of proposing the SIF equations for the two types of cracks.

Fig. 4.29 Effect of a/T on Y_{root} values for $h/T = 0.7$ and $a/c = 0.2$.

4.4.3 Multiple regression analysis of Y_{root} equation

The Y_{root} equations are progressively developed in stages by considering the effects of four parameters (L/W , h/T , a/T and a/c). The primary development stage of the equations allows for the effect of varying L/W for a zero weld toe crack depth. Therefore, the Frank and Fisher equation [76] is used as the first function of the Y_{root} equations as shown in Appendix B. The secondary development stage involves the effect of varying a/T for a constant $a/c = 0.1$. The final stage of the equation development takes into account the effect of varying a/c , thereby covering all the Y_{root} data in the regression analysis. The new parametric Y_{root} equations for

the weld root crack in load-carrying cruciform fillet welded joints with multiple cracks are built up as follows:

$$Y_{\text{root}} = f_{\text{wm}} M_{\text{km}} h_1 \left(\frac{a}{T}, \frac{h}{T} \right) h_2 \left(\frac{a}{T}, \frac{h}{T}, \frac{a}{c} \right) \quad (4.30)$$

where

$$h_1 \left(\frac{a}{T}, \frac{h}{T} \right) = D_1 \left[1 - \left(\frac{a}{T} \right)^{-0.123596} \right] + D_2 \left(\frac{a}{T} \right)^{D_3} \quad (4.31)$$

$$D_1 = 0.087604 \left(\frac{h}{T} \right)^2 - 0.146992 \left(\frac{h}{T} \right) + 1.068704$$

$$D_2 = -1.110122 \left(\frac{h}{T} \right)^3 + 3.848106 \left(\frac{h}{T} \right)^2 - 4.392061 \left(\frac{h}{T} \right) + 1.941125 \quad (4.32)$$

$$D_3 = 0.641577 \left(\frac{h}{T} \right)^2 + 1.133506 \left(\frac{h}{T} \right) + 0.487384$$

$$h_1 \left(\frac{a}{T}, \frac{h}{T}, \frac{a}{c} \right) = \left[D_4 \left(\frac{h}{T} \right) + D_5 \right] \left(\frac{a}{T} \right)^{D_6} + \left[1 - \left(\frac{a}{T} \right) \right] \left[D_7 \left(\frac{h}{T} \right)^2 + D_8 \left(\frac{h}{T} \right) + D_9 \right] \quad (4.33)$$

$$D_4 = 1.111012 \left(\frac{a}{c} \right)^{-0.188802} - 1.713016$$

$$D_5 = -0.221411 \left(\frac{a}{c} \right)^2 - 0.034145 \left(\frac{a}{c} \right) + 1.011042$$

$$D_6 = -0.210891 \left(\frac{a}{c} \right)^2 + 0.541566 \left(\frac{a}{c} \right) + 0.950673 \quad (4.34)$$

$$D_7 = -0.188517 \left(\frac{a}{c} \right)^{-0.703801} + 0.951512$$

$$D_8 = 0.510211 \left(\frac{a}{c} \right)^{-0.695732} - 2.525144$$

$$D_9 = -0.009782 \left(\frac{a}{c} \right)^{-1.728042} + 1.519031$$

It can be observed clearly that the proposed equations are an extension of the Frank and Fisher equation [76] by considering the effect of the weld toe crack size on the SIF values of weld root crack.

4.4.4 Goodness of fit of proposed Y_{root} equations

The newly proposed Y_{root} equations are complex in nature due to various influencing parameters as well as the large amount of FE data. Therefore, histograms describing error frequency distributions [64,71,74,75] are used to verify the goodness of fit of the equations, and the percentage errors are calculated using the following expression, respectively,

$$\% \text{ Error} = \frac{[Y_{\text{root}(\text{equation})} - Y_{\text{root}(\text{FEA})}]}{Y_{\text{root}(\text{FEA})}} \times 100\% \quad (4.35)$$

In the histogram, the abscissa is denoted by the percentage error and the ordinate is defined as the percentage of values inside a particular error range. Fig. 4.30 shows that the vast majority of regression data for the weld root crack are predicted to within the percentage errors of $\pm 2\%$, illustrating that the proposed equations are a good fit to the FE data obtained. Statistical parameters associated with the goodness of fit are presented in Table 4.8 to further demonstrate the accuracy of the equations [64].

Fig. 4.30 Error histogram for weld root stress intensity factors of load-carrying cruciform fillet welded joints.

Table 4.8 Statistical evaluation of the proposed Y_{root} equations

Statistical parameters	Weld root crack
Number of FE data used in the analyses	672
Maximum percentage over-prediction (%)	7.93
Maximum percentage under-prediction (%)	-3.96
$\sum(\text{Residuals})^2$	0.1854
Mean Percentage Error (%)	-0.05
Percentage Error Standard Deviation (%)	1.71

4.4.5 Determination of critical crack and weld sizes

For load-carrying cruciform fillet welded joints with multiple cracks, the crack can initiate and propagate from the weld root or the weld toe [92], which depends on the comparison of the SIFs at these two locations [88]. In Sub-sections 4.3.4 and 4.4.4, the Y_{root} and $M_{k,\text{toe}}$ equations for the weld root and weld toe cracks are proposed by considering the effect of various parameters (L/W , h/T , a/T and a/c), and the corresponding SIF values can easily be calculated according to Eqs. (4.6) and (4.29). Based on the above SIF equations attained for the weld root and weld toe cracks, it is easy to determine whether the SIF in the former is higher than that in the latter for given crack and weld parameters (L/W , h/T , a/T and a/c). Fig. 4.31 shows the changes of the crack propagating from the weld root to the weld toe for constant $h/T = 0.7$ and $a/c = 0.4$. The red and the black lines are depicted using the following equality relationships:

$$\begin{aligned} K_{\text{root}} &= K_{\text{toe}}^{\text{deep}} && \text{for red line} \\ K_{\text{root}} &= K_{\text{toe}}^{\text{ends}} && \text{for black line} \end{aligned} \quad (4.36)$$

where $K_{\text{toe}}^{\text{deep}}$ and $K_{\text{toe}}^{\text{ends}}$ are the SIFs for the weld toe crack front deepest point and weld toe crack front ends, respectively. Therefore, the blank area denotes that the crack propagates from the weld root, and the shadow regions on the left and the right of the blue line indicate the crack propagating from the weld toe crack deepest point and weld toe crack ends, respectively. The data points in the solid line represent the critical combination of L/W and a/T values at which the crack propagates from the weld root to the weld toe. The SIF equations proposed for the weld root and weld toe cracks provide a convenient way to construct similar figures as Fig. 4.31,

and to determine the critical combination of L/W and a/T values for each group of h/T and a/c . However, for longer cracks ($a/c = 0.1$), the crack propagates from the weld root or the weld toe crack front deepest point as illustrated in Fig. 4.32 due to the SIF at the crack deepest point always being higher than the value at the crack ends. Fig. 4.32 also indicates that the critical combination values of L/W and a/T are remarkably influenced by the variation of h/T , and a smaller weld size induces the crack more easily propagating from the weld root, which is consistent with previous experiment tests showing that cracks propagate from the weld root for smaller weld size [93,94].

Fig. 4.31 Determination of critical crack and weld sizes for $h/T = 0.7$ and $a/c = 0.4$.

Fig. 4.32 Effect of h/T on determination of critical crack and weld sizes for $a/c = 0.1$.

4.4.6 Summary of the section

In this study, a FE mesh generator is developed to create the 3-D models of load-carrying cruciform fillet welded joint with multiple cracks. Extensive parametric study, covering a wide range of parameters presented in Table 4.7, is carried out to calculate the SIF values of the weld root crack. The dimensionless SIFs (Y_{root}) are obtained according to Eq. (4.29), and the effects of h/T and a/T on Y_{root} values are investigated in details. It is observed that Y_{root} values decrease with increasing h/T while a larger a/T produces larger Y_{root} results. Then, a new set of Y_{root} equations for the weld root crack are proposed by considering the influence of the weld toe crack, and the goodness of fit of the equations is verified using percentage error frequency histograms and statistical analyses. Finally, based on the SIF equations established for the weld root and weld toe cracks [95], the critical combination values of L/W and a/T at which the crack propagates from the weld root to the weld toe for each group of h/T and a/c are determined.

CHAPTER 5

STRESS INTENSITY FACTORS OF MISALIGNED WELDED JOINTS

5.1 INTRODUCTION

In the chapter, fatigue assessment of cracked pipes with weld misalignment is carried out by using fracture mechanics approach. Two types of weld misalignment, viz., centreline offset weld misalignment with equal thickness but unequal diameters, and centreline offset weld misalignment with unequal thickness but aligned inside surface, are considered. A finite element (FE) mesh generator for cracked plain pipes is developed first, and then it is used for the misaligned pipes by incorporating a coordinate transformation method. A small-scale experimental test on the stress concentration curve along the longitudinal direction of misaligned pipes is conducted to ascertain the accuracy of the transformation method. Then, extensive FE analyses are performed to determine the stress intensity magnification factor (M_k) at the crack deepest point and the crack ends of a surface crack in misaligned pipes, which is because that the maximum stress intensity factor (SIF) value is obtained either at the crack deepest point or at the crack ends. For a wider crack (smaller crack aspect ratio), the maximum SIF is located at the crack deepest point; for a penny shape crack (larger crack aspect ratio), the SIF at the crack ends is maximum. A new set of M_k equations for the misaligned pipes are built up using multiple regression analysis, and statistical analyses are carried out to validate the goodness of fit of these equations. The proposed equations can be employed in the fatigue strength calculation and fatigue life prediction based on the fatigue assessment procedure provided by BS 7910:2013.

Over the past decades, with the rapid development of oil and gas industries, large numbers of offshore pipelines are used for transportation purposes. Hence, the demand for welded pipes and risers has also increased dramatically. These pipelines are important structural components which are connected to the offshore platforms. During in-service operation, the pipes and risers are subjected to cyclic loadings induced by the dynamic wave or current, posing tremendous challenge to the integrity and reliability of these components. Fatigue failure is a major failure

mode in the welded pipes and risers, hence, the fatigue assessment of these structural components deserves special attention and detailed investigations.

Linear elastic fracture mechanics (LEFM) is a common method to calculate the fatigue strength and predict the fatigue life based on the Paris' law [15], establishing the relationship between the crack growth rate and the applied stress intensity factor (SIF) range. The key of LEFM approach in fatigue assessment is the determination of SIF solutions for specific geometries [60,64,71,75,84,96,97]. Newman and Raju [60] proposed empirical equations for the SIFs of a surface crack in a plain plate under axial and bending loadings, respectively. The SIF equations of a weld toe crack in single T-butt joints were established by Bowness and Lee [64] based on 3-D finite element (FE) analysis. These equations mentioned in the above have been incorporated in the BS 7910:2013 [1], and the fatigue analysis for corresponding configurations can be carried out by integrating the proposed SIF equations into the fatigue assessment procedure provided by the standard book [1]. Subsequently, Lie et al. [74] developed the SIF equations for the crack deepest point and the crack ends of a surface crack in plate-to-plate butt-welded joints. It is noted that these equations all serve for cracked plate configurations. The SIF solutions for a circumferential surface crack in a plain pipe were derived by Bergman [98] according to FE analysis. Hoh et al. [99] extended Bergman's research work by considering a girth-welded pipe with a weld toe crack, and fatigue crack propagation analysis was conducted by following the fatigue assessment procedure [1].

Fig. 5.1 Effect of weld misalignment on the stress distribution along the longitudinal direction of the pipe.

During the welding process, weld misalignment such as centreline offset, angular or both is invariably introduced into the pipes [31,58]. This weld misalignment induces an additional bending moment along the pipe section (Fig. 5.1), and the maximum value is observed at the weld toe [100]. Hence, the weld toe in the misaligned pipe is a more critical location of crack initiation and propagation in comparison with that in a plain pipe. It is worth noting that the aforementioned research works [60,64,74,84,96-99] are all about fatigue and fracture analyses of aligned configurations with a surface crack at the weld toe. However, the SIF equations of a weld toe crack in misaligned pipes are not available. It can be confirmed in this study that the presence of weld misalignment significantly shortens the fatigue life, which proves the necessity of developing the SIF equations for the misaligned pipes. Moreover, the proposed equations can be applicable for a wide range of pipe diameters, weld sizes, crack sizes and misalignment levels, which covers the most commonly used pipe configurations.

5.2 MESH GENERATION FOR MISALIGNED PIPES WITH A SURFACE CRACK

Finite element method (FEM) is an important tool used for the fatigue and fracture analysis of different types of welded joints containing a crack. Since large numbers of complicated FE models are required to be created, a FE mesh generator is developed specially to simplify the cumbersome and time-consuming task. The mesh generator for creating FE models of cracked pipes with weld misalignment is introduced in this section.

5.2.1 Mesh generation details of a cracked plain pipe

Fig. 5.2 Plain pipe geometry with a semi-elliptical surface crack.

Fig. 5.2 shows the details of a plain pipe with internal diameter (D_i), pipe thickness (t) and weld width (L and L_i). A semi-elliptical surface crack is located at the weld toe with crack depth

of a and crack length of $2c$. The cracked geometry is transformed from a cracked plain plate and the transformation equations are derived in the following.

The crack front of the surface crack in a plain plate is described by the following elliptic equation as

$$\frac{(z' - t)^2}{a^2} + \frac{(y')^2}{c^2} = 1 \quad (5.1)$$

which can be treated as the parameterized expression,

$$\begin{aligned} t - z' &= a \sin \phi \\ y' &= c \cos \phi \end{aligned} \quad (5.2)$$

where ϕ is the parametric angle along the crack front in a plain plate. For the surface crack in a plain pipe, its crack front points are located on a transformed ellipse with the equation as [101]

$$\frac{(D_i/2 + t - r_c)^2}{a^2} + \frac{[(D_i/2 + t)\theta_c]^2}{c^2} = 1 \quad (5.3)$$

where r_c and θ_c are the polar coordinates of a point at the crack front as shown in Fig. 5.3(b).

Fig. 5.3 Schematic views of a cracked plain plate and a plain pipe.

Substituting Eq. (5.2) into Eq. (5.1) and then equating Eq. (5.1) to Eq. (5.3) obtain the following expressions as

$$\begin{aligned} r_c &= D_i/2 + t - a \sin \phi \\ \theta_c &= \frac{c \cos \phi}{D_i/2 + t} \end{aligned} \quad (5.4)$$

Since the relationships between r_c , θ_c and y , z are

$$\begin{aligned} y &= r_c \sin \theta_c \\ z &= r_c \cos \theta_c \end{aligned} \quad (5.5)$$

the transformation equations from a cracked plain plate to a plain pipe are obtained as [98]

$$\begin{aligned} y &= (D_i/2 + t - a \sin \phi) \sin \frac{c \cos \phi}{D_i/2 + t} \\ z &= (D_i/2 + t - a \sin \phi) \cos \frac{c \cos \phi}{D_i/2 + t} \end{aligned} \quad (5.6)$$

Based on Eq. (5.6), a plain plate with a semi-elliptical surface crack is transformed to a plain pipe with the same crack profile as shown in Fig. 5.4.

Fig. 5.4 Transformation procedure from a cracked plain plate to a cracked pipe.

5.2.2 Coordinate transformation method

The mesh generation strategies for a cracked plain pipe have been introduced in the previous section. Then, a coordinate transformation method is incorporated into the mesh generator to convert the aligned pipe to misaligned one. Two types of weld misalignment, viz., centreline offset weld misalignment with equal thickness but unequal diameters (Fig. 5.5(a)) [35], and centreline offset weld misalignment with unequal thickness but aligned inside surface (Fig. 5.5(b)) [35], are considered, which are actually the Type-II and Type-III as introduced in Section

3.6. Hence, herein, the transformation procedure is not described, and the details can be found in Section 3.6.

Fig. 5.5 Two types of misaligned pipes [35].

5.2.3 Finite element modelling

According to AWS D1.1:2000 [59] recommendations, butt welds or girth welds should be made with minimum face reinforcement and the weld surface needs to be flushed for some cases. Hence, the welds with flushed surface are only taken into account, which means that the weld reinforcement is zero. For the cracked pipe with weld misalignment, five basic parameters, namely, diameter ratio (D_i/t), weld width ratio (L/t), crack depth ratio (a/t), crack aspect ratio (a/c), and weld misalignment ratio (e/t for Type-II and α for Type-III), are incorporated in the parametric study and reported in Table 5.1. The D_i/t and L/t values analysed are ranging from 10 to 20 and from 0.72 to 2.0, respectively, which covers the most commonly used pipe configurations in practice [45]. For the pipe with Type-II, the e/t values of 0.0, 0.05, 0.1 and

0.15 are adopted because a typical value for centreline offset employed in many fabrication standards is $e = 0.15t$ or maximum 3-4 mm [33]. For the pipe with Type-III, the maximum permitted transition slope is 0.4 [59,71]. Therefore, the transition slopes used are 0.0, 0.133, 0.267 and 0.4.

Table 5.1 Table of analyses for the cracked pipe with Type-II and Type-III

Misalignment Type	D_i/t	L/t	a/t	a/c	e/t or α
Type-I	10,15,20	0.72,1.36,2.0	0.05,0.1,0.2, 0.3,0.5	0.25,0.5,1.0	0.0,0.05,0.1,0.15
Type-II	10,15,20	0.72,1.36,2.0	0.05,0.1,0.2, 0.3,0.5	0.25,0.5,1.0	0.0,0.133,0.267, 0.4

As indicated in Table 5.1, 1080 different geometries of misaligned pipes are analysed, and the FE mesh generator developed in the previous subsection enables the authors to generate all these 3-D models rapidly. The element type employed in these models is a 20-node brick element with reduced integration (C3D20R), and the Young's modulus (E) and Poisson's ratio (ν) used are 210 kN/mm² and 0.3, respectively. A fine mesh along the crack front is required to ensure convergence, especially at the weld toe crack ends as shown in Fig. 5.4. Based on symmetry, only half of the pipe model is considered. The loading conditions are achieved by fixing one end of the pipe and applying a uniform axial stress to the other end. The SIF (K_m) of a weld toe crack in the misaligned pipe is computed from the J -integral implemented in ABAQUS [51]. For each crack front node, the K_m is determined from the average J -integral value of the second to fifth contours, and the J -integral for the first contour is ignored due to numerical errors. The plane strain condition is used along the crack front except at the crack ends [64,71,74,75,102], and the K_m is defined as

$$K_m = K_I = \sqrt{\frac{JE}{(1 - \nu^2)}} \quad (5.7)$$

where K_I is the Mode-I SIF. At the crack ends, the plane stress condition is assumed [64,71,74,75,102] and $E/(1 - \nu^2)$ in Eq. (5.7) is replaced by E to determine the corresponding K_m . It is noted that only Mode-I SIF is considered due to the negligible effect of Mode-II and Mode-III SIFs (K_{II} and K_{III}) on the total J , which is classified in Sub-section 5.3.3 in detail.

5.3 VERIFICATION OF THE FE MESH GENERATOR

5.3.1 Comparative study with existing SIF solutions

The validity of the FE mesh generator needs to be verified before it is used for the fatigue and fracture analysis of cracked pipes with weld misalignment. At first, the mesh generator is used to generate FE models of cracked plain pipes by setting e/t or α being zero, and the SIFs obtained are compared with Bergman's solutions [98] at the crack deepest point and the crack ends by covering varying diameter ratios and crack sizes. An excellent agreement between one another is observed as shown in Table 5.2 with the largest percentage difference being only 1.08%, confirming the validity of the mesh generator for a cracked plain pipe. Then, the mesh generator is upgraded to create FE models of welded pipes with a weld toe crack by inserting a subroutine of creating the girth weld in Fig. 5.6(b) to the original main program. The details of the girth weld can be found in Ref. [99]. Fig. 5.7 shows the comparative study of the SIFs obtained from present FE analysis and the research work conducted by Hoh et al. [99]. These two sets of results are consistent with one another by considering the largest percentage difference being only 2.2%, which demonstrates the robustness of the mesh generator in incorporating subroutines to serve for more complicated configurations. Subsequently, the mesh generator is developed for the fatigue and fracture analysis of cracked pipes with weld misalignment by introducing the subroutine of describing the coordinate transformation method as indicated in Section 3.6. The validation works mentioned in the above provide confidence in the present FE analysis to attain accurate SIFs if the coordinate transformation method is verified, because the only difference between the mesh generator for aligned and misaligned pipes is the adoption of the coordinate transformation. The validity of the coordinate transformation method has been preliminarily confirmed in Section 3.7.

Fig. 5.6 Schematic view of a welded pipe.

Table 5.2 Comparison of dimensionless SIFs obtained from Bergman [98] and FE analysis

D_i/t	a/t	a/c	Deepest point			Crack ends		
			FE	Bergman [98]	Diff. (%)	FE	Bergman [98]	Diff. (%)
10	0.2	0.25	1.0795	1.0693	0.95	0.6115	0.6097	0.3
		0.5	0.9045	0.9052	-0.08	0.7252	0.7237	0.21
		1.0	0.6557	0.6618	-0.92	0.743	0.7484	-0.72
	0.4	0.25	1.2502	1.2514	-0.1	0.6928	0.6933	-0.07
		0.5	0.9699	0.9788	-0.91	0.7942	0.7943	-0.01
		1.0	0.6679	0.6747	-1.01	0.7726	0.781	-1.08
20	0.2	0.25	1.0746	1.0666	0.75	0.6121	0.6099	0.36
		0.5	0.9022	0.9017	0.06	0.7249	0.7231	0.25
		1.0	0.6571	0.6636	-0.98	0.7433	0.7483	-0.67
	0.4	0.25	1.2447	1.237	0.62	0.703	0.7023	0.1
		0.5	0.9672	0.9716	-0.45	0.7976	0.7972	0.05
		1.0	0.6714	0.6677	0.55	0.7742	0.7825	-1.06

Note: Dimensionless SIF (Y_p) is defined as $Y_p = K_p / \sigma \sqrt{\pi a}$, where σ is the applied stress.

Fig. 5.7 Comparison of dimensionless SIFs obtained from Hoh et al. [99] and present FE analysis for $a/c = 0.5$.

5.3.2 Comparative study of fatigue life predictions

Ferreira and Branco [56] carried out a comparative study of the fatigue life obtained from theoretical predictions and experimental results [57] for a cracked plate with parallel misalignment of equal thickness. It is noted from Fig. 5.4 that the mesh details in plates and pipes are almost the same irrespective of whether it is in the crack region or in the un-cracked parts. When the FE mesh generator is employed to generate FE models of a cracked plain plate, the SIFs obtained are consistent with the values given by the widely accepted Newman and Raju equation [60,71,74,75,102]; when the mesh generator is employed for a cracked plain pipe by adopting the transformation equations listed in Eq. (5.6) and keeping the same mesh details with the plain plate both in the crack region and in the un-cracked parts, the calculated SIFs are in accord with the Bergman's solutions [98]. In other words, the strategies in mesh refinement and number of elements for the pipes can be equally applicable for the plates, and vice versa. Hence, the availability of the mesh generator can be further verified based on the comparative study of fatigue life predictions in the cracked plates. The corresponding FE models can be created by adopting the inverse transformation of Eq. (5.6). Fig. 5.8 presents the comparison of crack propagation life obtained from FE analysis, Ferreira and Branco's predictions [56] and experimental results [57]. The ordinate of this figure is the resulting nominal stress range $\Delta\sigma_N$ [56], and σ_N is the sum of the applied stress (σ) and the local bending stress induced by weld misalignment. The abscissa is the total fatigue life N_t , which is the crack initiation life N_i plus the crack propagation life N_p ; N_p can be determined based on the Paris' law [15]; N_i is calculated using the Coffin-Manson equation modified by Wetzel [103],

$$\frac{\Delta\varepsilon}{2} = \varepsilon'_f(2N_i)^{n_1} + \frac{\sigma'_f - \sigma_m}{2}(2N_i)^{n_2} \quad (5.8)$$

where $\Delta\varepsilon$ and σ_m are the local strain and the mean local stress at the weld toe, ε'_f and σ'_f are the fatigue ductility and fatigue strength, respectively, n_1 is the fatigue ductility coefficient and n_2 is the fatigue strength exponent. The detailed information for the plate sizes, fatigue parameters and assessment procedures can be found in Ref. [56]. It is observed from Fig. 5.8 that the present FE analysis produces a reasonable prediction in fatigue life in comparison to experimental results [57], and more importantly, an excellent agreement is obtained between the FE results and the predictions given by Ferreira and Branco [56]. Finally, the robustness and validity of

the FE mesh generator are proven and it can be accurately used for the fatigue and fracture analysis of misaligned pipes.

Fig. 5.8 Comparison of fatigue life predictions of a weld toe crack in the misaligned plate.

5.3.3 Definition of stress intensity magnification factor

In this study, the SIF (K_m) of a weld toe crack in the misaligned pipe is calculated by only considering the K_I and neglecting the K_{II} and K_{III} as presented in Eq. (5.7). In order to quantitatively illustrate the effect of K_{II} and K_{III} on the J -integral, J_1 and J_3 are defined in the following form for plain strain cases:

$$J_1 = \frac{(1 - \nu^2)K_I^2}{E} \quad (5.9)$$

$$J_3 = \frac{(1 - \nu^2)(K_I^2 + K_{II}^2)}{E} + \frac{K_{III}^2}{2\mu} \quad (5.10)$$

where μ is the shear modulus with $\mu = E/[2(1 + \nu)]$, and $(1 - \nu^2)/E$ in Eqs. (5.9) and (5.10) is replaced by $1/E$ for plain stress conditions. Eq. (5.9) is actually the same as Eq. (5.7). Table 5.3 shows the comparison of J_1 and J_3 generated using the FE mesh generator for the pipe configuration with $D_i/t = 15$, $L/t = 1.36$ and $a/c = 0.5$. It is observed that the values of J_1 and J_3 are almost the same with each other for varying crack depth and misalignment level, indicating negligible influence of K_{II} and K_{III} on the J . More comparative studies covering the range of parameters listed in Table 1 confirm this point by noting the largest percentage

difference of the crack deepest point and the crack ends between incorporating the K_{II} and K_{III} into the J or not being only 2.88%. It is because the presence of weld misalignment induces an additional bending stress (Fig. 5.1), which in turn mainly produces a Mode-I crack due to the bending stress being normal to the crack plane. Besides, Ferreira and Branco [56] investigated the influence of misalignment on the fatigue strength of butt welds by considering a Mode-I dominated failure mode; a good agreement between the analytical and experimental results [57] was obtained, which further demonstrates the reasonability of ignoring the K_{II} and K_{III} . Hence, it is sufficiently accurate to take into account only the effect of K_I on the K_m .

Table 5.3 Effect of K_{II} and K_{III} for the misaligned pipe with $D_i/t = 15$, $L/t = 1.36$ and $a/c = 0.5$

Misalignment Type	a/t	e/t or α	Crack deepest point			Crack ends		
			$J_1 (\times 10^{-4} \text{N/mm})$	$J_3 (\times 10^{-4} \text{N/mm})$	$\left(\frac{J_3 - J_1}{J_1}\right) \times 100\%$	$J_1 (\times 10^{-4} \text{N/mm})$	$J_3 (\times 10^{-4} \text{N/mm})$	$\left(\frac{J_3 - J_1}{J_1}\right) \times 100\%$
Type-II	0.05	0.0	0.1068	0.1068	0.0	0.0747	0.0747	0.0
		0.15	0.1926	0.1928	0.12	0.1841	0.1846	0.29
	0.5	0.0	1.4304	1.4304	0.0	1.1222	1.1222	0.0
		0.15	1.6530	1.6532	0.01	1.9911	1.9962	0.25
Type-III	0.05	0.0	0.1068	0.1068	0.0	0.0747	0.0747	0.0
		0.4	0.2756	0.2792	1.27	0.4598	0.4715	2.57
	0.5	0.0	1.4304	1.4304	0.0	1.1222	1.1222	0.0
		0.4	1.4454	1.4455	0.01	2.7526	2.8120	2.16

Due to changing SIF values with specific crack and weld sizes, stress intensity magnification factor (M_k) [63,74,85] is frequently introduced to characterize the fatigue and fracture strengths of welded joints, which is expressed as the product of the ratio of the SIF with stress concentration to the SIF for the same crack in a pipe without stress concentration as follows:

$$M_k = \frac{K_m}{K_p} \quad (5.11)$$

where K_p is the SIF for a plain pipe with a semi-elliptical surface crack at the weld toe [98,99].

5.4 STRESS INTENSITY MAGNIFICATION FACTOR EQUATIONS

In this section, multiple regression analysis [64,71,74,75] is employed to develop M_k equations of a weld toe crack in the pipe with Type-II and Type-III, respectively, whilst the goodness of fit of the proposed equations is demonstrated.

5.4.1 Multiple regression analysis

Three basic functions, viz., power, exponential and polynomial functions, are employed to construct the M_k equations. Based on the basic functions, a trial equation with an initial guess to the coefficients values are given at first. Then, least square fitting method is used to minimize the differences between the FE data and the equation predictions by changing the coefficients in the equation. Finally, the required equations are proposed progressively in stages by considering the effects of five parameters (D_i/t , L/t , a/t , a/c , e/t or α). The development procedures are implemented by a MATLAB code. The new parametric M_k equations of a weld toe crack in the pipe with Type-II and Type-III are built up and listed in Tables 5.4-5.7, respectively.

Table 5.4 M_k equation at the crack deepest point for the pipe with Type-II

Application range	$M_{k,deep}^{II}$ equation
$10 \leq D_i/t \leq 20$	$M_{k,deep}^{II} = 1 + f_1 \left(\frac{a}{t}, \frac{a}{c}, \frac{e}{t} \right) f_2 \left(\frac{a}{t}, \frac{L}{t} \right) f_3 \left(\frac{D_i}{t}, \frac{L}{t} \right)$
$0.72 \leq L/t \leq 2.0$	where
$0.05 \leq a/t \leq 0.5$	$f_1 \left(\frac{a}{t}, \frac{a}{c}, \frac{e}{t} \right) = \frac{e}{t} \left[36.2626 \left(\frac{a}{t} \right)^{-0.0148} - 35.8174 \left(1 - \frac{a}{t} \right)^{A_1} \right]$
$0.25 \leq a/c \leq 1.0$	$A_1 = 0.011 \left(\frac{a}{c} \right)^{-0.4639} - 0.0303$
$0.0 \leq e/t \leq 0.15$	$f_2 \left(\frac{a}{t}, \frac{L}{t} \right) = A_2 \exp \left[A_3 \left(\frac{a}{t} \right) \right] + A_4 \exp \left[1.4652 \left(\frac{a}{t} \right) \right]$
	$A_2 = 0.2196 \left(\frac{L}{t} \right)^2 - 0.6892 \left(\frac{L}{t} \right) + 1.393$
	$A_3 = -1.9421 \left(\frac{L}{t} \right)^2 + 7.838 \left(\frac{L}{t} \right) - 7.542$
	$A_4 = -0.3983 \left(\frac{L}{t} \right) + 0.6904$
	$f_3 \left(\frac{D_i}{t}, \frac{L}{t} \right) = -0.0006 \left(\frac{D_i}{t} \right)^2 + A_5 \left(\frac{D_i}{t} \right) + A_6$
	$A_5 = -0.0014 \left(\frac{L}{t} \right)^2 + 0.0149 \left(\frac{L}{t} \right) + 0.0156$
	$A_6 = 0.0105 \left(\frac{L}{t} \right)^2 - 0.1368 \left(\frac{L}{t} \right) + 0.8935$

Table 5.5 M_k equation at the crack ends for the pipe with Type-II

Application range	$M_{k,ends}^{II}$ equation
$10 \leq D_i/t \leq 20$	$M_{k,ends}^{II} = 1 + g_1 \left(\frac{a}{t}, \frac{c}{a}, \frac{e}{t} \right) g_2 \left(\frac{L}{t} \right) g_3 \left(\frac{D_i}{t}, \frac{L}{t} \right)$
$0.72 \leq L/t \leq 2.0$	where
$0.05 \leq a/t \leq 0.5$	$g_1 \left(\frac{a}{t}, \frac{c}{a}, \frac{e}{t} \right) = \left[B_1 \left(\frac{a}{t} \right)^{-0.1132} - 0.0865 \left(1 - \frac{a}{t} \right)^{B_2} \right] \left[9.474 \left(\frac{e}{t} \right)^2 + \right.$
$0.25 \leq a/c \leq 1.0$	$\left. 19.8102 \left(\frac{e}{t} \right) \right]$
$0.0 \leq e/t \leq 0.15$	$B_1 = -0.0002 \left(\frac{c}{a} \right)^2 + 0.004 \left(\frac{c}{a} \right) + 0.174$
	$B_2 = 0.0018 \left(\frac{c}{a} \right)^{3.563} - 0.1556$
	$g_2 \left(\frac{L}{t} \right) = 0.2558 \left(\frac{L}{t} \right)^2 - 1.231 \left(\frac{L}{t} \right) + 2.187$
	$g_3 \left(\frac{D_i}{t}, \frac{L}{t} \right) = B_3 \left(\frac{D_i}{t} \right)^2 + B_4 \left(\frac{D_i}{t} \right) + B_5$
	$B_3 = -0.0003 \left(\frac{L}{t} \right) + 0.00004$
	$B_4 = 0.0193 \left(\frac{L}{t} \right) - 0.0032$
	$B_5 = -0.1591 \left(\frac{L}{t} \right) + 1.029$

Table 5.6 M_k equation at the crack deepest point for the pipe with Type-III

Application range	$M_{k,deep}^{III}$ equation
$10 \leq D_i/t \leq 20$	$M_{k,deep}^{III} = 1 + p_1 \left(\frac{a}{t}, \alpha \right) p_2 \left(\frac{L}{t} \right) p_3 \left(\frac{D_i}{t}, \frac{L}{t} \right)$
$0.72 \leq L/t \leq 2.0$	where
$0.05 \leq a/t \leq 0.5$	$p_1 \left(\frac{a}{t}, \alpha \right) = C_1 \left(\frac{a}{t} \right)^{5.424} + C_2 \left(1 - \frac{a}{t} \right)^{5.424}$
$0.25 \leq a/c \leq 1.0$	$C_1 = 13.5402\alpha^3 - 12.5301\alpha^2 + 0.5035\alpha$
$0.0 \leq \alpha \leq 0.4$	$C_2 = 0.2482\alpha^3 - 1.533\alpha^2 + 2.395\alpha$
	$p_2 \left(\frac{L}{t} \right) = -0.1965 \left(\frac{L}{t} \right)^2 + 0.8541 \left(\frac{L}{t} \right) + 0.2019$
	$p_3 \left(\frac{D_i}{t}, \frac{L}{t} \right) = -0.0001 \left(\frac{D_i}{t} \right)^2 + C_3 \left(\frac{D_i}{t} \right) + C_4$
	$C_3 = 0.0018 \left(\frac{L}{t} \right) + 0.0018$
	$C_4 = -0.0174 \left(\frac{L}{t} \right) + 0.9889$

Table 5.7 M_k equation at the crack ends for the pipe with Type-III

Application range	$M_{k,ends}^{III}$ equation
$10 \leq D_i/t \leq 20$	$M_{k,ends}^{III} = 1 + q_1 \left(\frac{a}{t}, \frac{c}{a}, \alpha \right) q_2 \left(\frac{L}{t} \right) q_3 \left(\frac{D_i}{t}, \frac{L}{t} \right)$
$0.72 \leq L/t \leq 2.0$	where
$0.05 \leq a/t \leq 0.5$	$q_1 \left(\frac{a}{t}, \frac{c}{a}, \alpha \right) = \left[H_1 \left(\frac{a}{t} \right)^{-0.3572} + 0.2722 \left(1 - \frac{a}{t} \right)^{H_2} \right] \left[H_3 \left(\frac{a}{t} \right) + H_4 \right]$
$0.25 \leq a/c \leq 1.0$	$H_1 = -0.0008 \left(\frac{c}{a} \right)^2 + 0.0177 \left(\frac{c}{a} \right) + 0.2117$
$0.0 \leq \alpha \leq 0.4$	$H_2 = -0.0018 \left(\frac{c}{a} \right)^{4.636} + 1.771$
	$H_3 = 9.332\alpha^3 - 10.3002\alpha^2 + 2.086\alpha$
	$H_4 = -4.305\alpha^3 + 2.7002\alpha^2 + 3.3311\alpha$
	$q_2 \left(\frac{L}{t} \right) = -0.0952 \left(\frac{L}{t} \right)^2 + 0.4335 \left(\frac{L}{t} \right) + 0.5864$
	$q_3 \left(\frac{D_i}{t}, \frac{L}{t} \right) = H_5 \left(\frac{D_i}{t} \right)^2 + H_6 \left(\frac{D_i}{t} \right) + H_7$
	$H_5 = -0.0002 \left(\frac{L}{t} \right)^2 + 0.0003 \left(\frac{L}{t} \right) - 0.0002$
	$H_6 = 0.005 \left(\frac{L}{t} \right)^2 + 0.0019 \left(\frac{L}{t} \right) + 0.0029$
	$H_7 = -0.0281 \left(\frac{L}{t} \right)^2 - 0.0424 \left(\frac{L}{t} \right) + 0.9904$

5.4.2 Goodness of fit of proposed equations

The newly proposed M_k equations are complex in nature due to various influencing parameters as well as the large amount of FE data. Therefore, the purpose of this sub-section is to demonstrate the goodness of fit of these equations compared with the FE data. Statistical parameters associated with the goodness of fit are presented in Table 5.8 to verify the accuracy of these equations. For the $M_{k,deep}^{II}$ equation, the maximum percentages of over-prediction and under-prediction are 1.05% and 0.86%, respectively. For the $M_{k,ends}^{II}$ equation, the maximum percentages of over-prediction is 3.37% whilst the worst under-prediction is 5.59%. The corresponding percentage values for the cracked pipe with Type-III are 2.87% and 5.28%, respectively. The correlation coefficients for all the cases are very close to unity, and the percentage error standard deviations are all less than 1.5%. The statistical analyses illustrate that the proposed equations fit very well with the FE data. Moreover, percentage error frequency histograms [64,71,74,75] are employed to clearly describe the frequency distribution in different error ranges. The percentage error is determined using the following expression:

$$\% \text{ Error} = \frac{[M_{k(\text{equation})} - M_{k(\text{FEA})}]}{M_{k(\text{FEA})}} \times 100\% \quad (5.12)$$

Table 5.8 Statistical evaluation of M_k equations for the cracked pipe with Type-II and Type-III

Statistical parameters	Type-II		Type-III	
	Deepest point	Crack ends	Deepest point	Crack ends
Number of FE data used in this study	540	540	540	540
Maximum percentage over-prediction (%)	1.05	3.37	2.87	2.22
Maximum percentage under-prediction (%)	-0.86	-5.59	5.28	-2.72
$\sum(\text{Residuals})^2$	0.0040	0.1152	0.2417	0.0734
Mean Percentage Error (%)	-0.02	-0.08	-0.77	0.02
Percentage Error Standard Deviation (%)	0.23	0.9	1.43	0.64
Correlation Coefficient	0.9996	0.9977	0.9947	0.9996

Fig. 5.9 shows the error histograms for the M_k equations relative to the FE data with the abscissa and ordinate being denoted by the percentage error and the percentage of values inside a particular error range, respectively. It is noted that 85% of all the data obtained are located within the percentage errors of $\pm 1\%$, and the percentage value within $\pm 2\%$ is 95%, further confirming that the proposed equations are a very good fit to the FE data.

Fig. 5.9 Error histograms for M_k equations of a weld toe crack in the pipe with Type-II and Type-III, respectively.

5.5 FATIGUE ASSESSMENT OF CRACKED PIPES WITH WELD MISALIGNMENT

Based on the BS 7910:2013 [1] procedures, the Paris' law [15] is frequently employed to predict the crack propagation or the fatigue life:

$$\frac{da}{dN} = C(\Delta K_{\text{eff}})^m \quad (5.13)$$

where C and m are the constants depending on the applied conditions and materials and they are set to $C = 5.21 \times 10^{-13}$ and $m = 3$, respectively [1,99]. ΔK_{eff} is the effective SIF range and it is defined as

$$\Delta K_{\text{eff}} = \frac{\Delta K_m - \Delta K_0}{1 - R} \quad (5.14)$$

where ΔK_0 is the threshold SIF range with $\Delta K_0 = 63 \text{ N/mm}^{3/2}$ [1,99], R is the stress ratio and the case of $R = 0$ is considered. In this study, ΔK_m is the SIF range of the weld toe crack in the misaligned pipe and the expressions for the crack deepest point and the crack ends are given as

$$\Delta K_{m,\text{deep}} = M_{k,\text{deep}} \Delta K_{p,\text{deep}} \quad (5.15)$$

$$\Delta K_{m,\text{ends}} = M_{k,\text{ends}} \Delta K_{p,\text{ends}} \quad (5.16)$$

where $M_{k,\text{deep}}$ and $M_{k,\text{ends}}$ are the stress intensity magnification factors listed in Tables 5-8 for the cracked pipe with Type-II and Type-III, respectively, $\Delta K_{p,\text{deep}}$ and $\Delta K_{p,\text{ends}}$ are the SIF ranges for the cracked plain pipe at the crack deepest point and at the crack ends, respectively [99], and they are expressed as

$$\Delta K_{p,\text{deep}} = Y_{\text{deep}} \Delta \sigma \sqrt{\pi a} \quad (5.17)$$

$$\Delta K_{p,\text{ends}} = Y_{\text{ends}} \Delta \sigma \sqrt{\pi a} \quad (5.18)$$

where $\Delta \sigma$ is the applied stress range, Y_{deep} and Y_{ends} are the corresponding dimensionless SIFs.

The crack propagation life of the misaligned pipe can be determined by integrating the Paris' law [15] shown in Eq. (5.13):

$$N_p = \int_{a_i}^{a_f} \frac{da}{C(\Delta K_{\text{eff}})^m} \quad (5.19)$$

where a_i and a_f are the initial and final crack depth with $a_i = 1.03$ mm ($a/t = 0.05$ and $t = 20.6$ mm) and $a_f = 10.3$ mm ($a/t = 0.5$), respectively. In practice, the Paris' law [15] is usually replaced by a numerical summation as

$$a_f - a_i = \sum_{j=1}^{N_p} c [\Delta K_{\text{eff},j}]^m \Delta N_j \quad (5.20)$$

The fatigue analysis of the misaligned pipe can be carried out by substituting Eq. (5.14) into Eq. (5.20) and following the fatigue assessment procedure given in BS 7910:2013 [1].

The changes of crack aspect ratio a/c for the pipe with Type-II and Type-III are shown in Figs. 5.10 and 5.11, respectively. As the crack propagates, the crack tends towards a same a/c for different initial a/c . Figs. 5.10(b) and 5.11(b) present the effect of L/t and e/t (or α) on the final a/c , which is the crack aspect ratio as the crack reaches a final crack depth a_f of 10.3 mm. For the cracked pipe with Type-II (Fig. 5.10(b)), the final a/c decreases with the increase of e/t and a smaller L/t further reduces the a/c . This is because a larger e/t together with a smaller L/t significantly raises the stress concentration effect at the weld toe, resulting in a wider crack (smaller a/c) in comparison with the case of a plain pipe, which is also observed by Hoh et al [99]. For the cracked pipe with Type-III (Fig. 5.11(b)), a larger α reduces the final a/c , but the a/c decreases with the increase of L/t . As is well known, a larger L/t for same α induces a larger weld misalignment, thereby increasing the stress concentration effect. In conclusion, a wider crack is expected at the region of stress concentration by comparing with the case of a plain pipe.

Figs. 5.12 and 5.13 show the fatigue assessment for the cracked pipe with weld misalignment. It can be confirmed that the presence of weld misalignment significantly reduces the fatigue life, and a larger reduction is observed in low misalignment level for the same misalignment increment, which further proves the necessity of developing the M_k equations for the misaligned pipe. Moreover, the proposed equations in combination with the fatigue assessment procedure [1] can be used to determine the fatigue strength at a required fatigue life as indicated in Figs. 5.12(b) and 5.13(b). Taking the case in Fig. 5.12(b) as an example, the theoretical fatigue strength for $e/t = 0.1$ is 98.1 MPa calculated at 1.0×10^6 cycles. Fig. 5.14 illustrates that the fatigue life is also noticeably affected by the weld size L/t . All these

parameters mentioned in the above are incorporated into the M_k equations with a wide application range. Therefore, the M_k equations can be employed in the fatigue strength calculation and fatigue life prediction for the most commonly used pipe configurations encountered in practice.

Fig. 5.10 Evolution of a/c for the cracked pipe with Type-II: (a) change of a/c with crack growth for different initial a/c values, and (b) variation tendency of final a/c with e/t and L/t as the crack reaches a final crack depth a_f of 10.3 mm.

Fig. 5.11 Evolution of a/c for the cracked pipe with Type-III: (a) change of a/c with crack growth for different initial a/c values, and (b) variation tendency of final a/c with α and L/t as the crack reaches a final crack depth a_f of 10.3 mm.

Fig. 5.12 Fatigue assessment for the cracked pipe with Type-II: (a) fatigue crack growth and (b) fatigue life prediction of a weld toe crack in the misaligned pipe.

Fig. 5.13 Fatigue assessment for the cracked pipe with Type-III: (a) fatigue crack growth and (b) fatigue life prediction of a weld toe crack in the misaligned pipe.

Fig. 5.14 Effect of L/t on the fatigue life for the cracked pipe with (a) Type-II and (b) Type-III.

5.6 IMPLICATION OF CORNER POINT SINGULARITIES ON FATIGUE LIFE PREDICTION

In Section 5, the SIFs obtained are employed in the fatigue life assessment of misaligned pipes with a surface crack, which includes the implicit assumption that the power of singularity of the crack tip stress field is -0.5. In other words, the elastic stress components in the vicinity of the crack tip are proportional to $K/\sqrt{r}$, where K is the SIF with subscripts I, II and III denoting the loading modes, r is the distance from the crack tip. However, the nature of the singularity changes in the vicinity of a corner point where a crack front intersects a free surface. The concept of a corner point singularity was introduced first by Bažant and Estenssoro [104] and later by Pook [105] to describe this point, and the corresponding elastic stress field is defined as

$$\sigma_{ij} = \frac{K_\lambda}{r^\lambda} f_{ij}(\lambda, \varphi, \phi) \quad (5.21)$$

where σ_{ij} is the elastic stress components, r , φ and ϕ are the spherical coordinates with origin at the corner point, K_λ is the stress intensity measure, and λ is the exponent denoting the strength of the singularity. As indicated in Ref. [105], λ is a function of the Poisson's ratio (ν) and the crack front intersection angle (β) (Fig. 5.15). For the special case of $\lambda = 0.5$, K_λ becomes K_I or a combination of K_{II} and K_{III} depending on the loading modes. In this case, the crack front intersection angle is called the critical intersection angle (β_c), and it is clearly noted that β_c is a function of ν and loading modes. Based on the crack surface displacement in the neighbourhood of the crack tip, Pook [105] derived the appropriate solution of β_c for the Mode-II/III case, which is given as

$$\beta_c = \tan^{-1}[(1 - \nu)/\nu] \quad (5.22)$$

An empirical expression of β_c for the Mode-I case was also obtained [105],

$$\beta_c = \tan^{-1}[(\nu - 2)/\nu] \quad (5.23)$$

Fig. 5.16 shows the variation trend of β_c with ν , and it can be seen that β_c increases with ν for the Mode-I case and decreases for the Mode-II/III case. Based on the corner point singularity, a crack front tends to intersect the free surface at β_c [104,105]. Therefore, the widely used crack front of semi-elliptical form (Fig. 5.15(a)) cannot be adequately realistic, and a symmetrical part-elliptical surface crack has been proposed for modelling the part-through crack, especially

for the Mode-II/III case [105]. For this coupled mode loading case, the fatigue crack usually occurs from the corner point region, which means that the intersection angle has a significant implication on the fatigue crack growth. Thus, the corner point singularity cannot be neglected and a symmetrical part-elliptical crack is more appropriate. Considerable research works have been conducted by Pook et al. [106-109] to investigate the coupled fracture mode of a cracked plate and disc under anti-plane loading. However, the corner point singularity does not need to be considered in the fatigue crack analysis subjected to Mode-I loading case. This is because the crack deepest point dominates the fatigue behaviour; thus, the precise crack front profile in the vicinity of a corner point is of little importance [105]. In order to verify whether these conclusions [105] can be applicable for the present case, a comparative study on the fatigue crack analysis is carried out by considering different crack front intersection angles. As indicated in Section 2, the loading mode used in this study is Mode-I and the crack front is modelled as a semi-elliptical shape, viz., $\beta = 90^\circ$. Since $\nu = 0.3$, $\beta_c = 100.4^\circ$ based on Eq. (5.23). Therefore, the surface cracks with three different intersection angles, i.e., $\beta = 80^\circ$, $\beta = 90^\circ$ and $\beta = 100.4^\circ$, are analysed, and Fig. 5.17 presents the FE mesh details of the three crack fronts. It is noted that the SIFs obtained at the crack deepest point are almost identical with the largest percentage difference between $\beta = 90^\circ$ and the other two angles being only 4.5%. However, the discrepancies for the SIFs at the crack ends are relatively large due to the implication of different intersection angles, and even exceed 80% for some results between $\beta = 90^\circ$ and $\beta = 100.4^\circ$. Hence, the changes of a/c shown in Figs. 5.10 and 5.11 provide a qualitative indication of the well-known tendency for fatigue cracks to propagate to a particular stable shape, and the final a/c value depends on the crack front intersection angle. It is worth noting that the qualitative descriptions of fatigue crack propagation for a semi-elliptical surface crack in Section 5 are equally applicable for a symmetrical part-elliptical surface crack. Fig. 5.18 shows the effect of different β on the fatigue life prediction, which is calculated in accordance with the fatigue assessment procedures of Section 5. The predicted fatigue life decreases with the increasing β , but the differences are reasonably small by considering the largest percentage difference being 16.5% in comparison with the differences of SIFs at the crack ends. This is because the fatigue behaviour is dominated by the crack deepest point and the corresponding SIFs obtained are almost the same. More comparative studies confirm the relatively little practical importance of the precise crack front profile for the Mode-I fatigue crack analysis [105]. In conclusion, the predicted fatigue life is not significantly dependent on the crack front intersection angle in the

vicinity of the critical angle as would be indicated by the noticeable change in the SIFs of the corner points. The present SIF equations proposed for a semi-elliptical crack are still available in the fatigue analysis of cracked pipes with weld misalignment.

(a) Semi-elliptical surface crack (b) Symmetrical part-elliptical surface crack

Fig. 5.15 Schematic views of a semi-elliptical and symmetrical part-elliptical surface cracks.

Fig. 5.16 Effect of Poisson's ratio on the critical intersection angle for $\lambda = 0.5$.

(a) Part-elliptical surface crack (b) Semi-elliptical surface crack (c) Part-elliptical surface crack

Fig. 5.17 FE mesh details of the crack region for different intersection angles.

Fig. 5.18 Effect of intersection angle on the fatigue life prediction.

5.7 SUMMARY OF THE CHAPTER

In this study, a FE mesh generator is first developed for a plain pipe with a semi-elliptical surface crack at the weld toe, and the accuracy of the mesh generator is verified by comparing the SIFs obtained from Bergman's solutions [98] and FE analysis. Then, a coordinate transformation method is included in the mesh generator to convert the FE models of aligned configurations to that of misaligned ones. The validity of the coordinate transformation is confirmed by a small-scale experimental test on the stress concentration curve along the longitudinal direction of misaligned steel pipes. Subsequently, a comparative study of fatigue life predicted by the present FE analysis and Ferreira and Branco [56] is conducted, and a good agreement is observed. Hence, the newly developed mesh generator is considered to be well suited for fatigue and fracture analysis of misaligned pipes. Subsequently, extensive FE analyses are carried out to calculate the M_k values at the crack deepest point and at the crack ends of a surface crack in the misaligned pipe. Based on multiple regression analysis, a new set of M_k equations are proposed by considering the effects of pipe diameters, weld sizes, crack sizes and misalignment levels. The goodness of fit of the proposed equations is ascertained by statistical analysis and percentage error frequency histograms. Finally, the fatigue assessment of cracked pipes with weld misalignment is carried out by integrating the M_k equations into the fatigue assessment procedure provided by BS 7910:2013 [1]. It is noted that a wider crack (smaller a/c) is expected at the region of stress concentration induced by the weld misalignment. Moreover, the presence of weld misalignment significantly reduces the fatigue

life, which illustrates the importance of developing the M_k equations for the misaligned pipe. Finally, the implication of corner point singularity on the fatigue life prediction is investigated in detail, and it is concluded that the corner point singularity has relatively little practical importance for the Mode-I fatigue crack analysis. Therefore, the M_k equations proposed for a semi-elliptical surface crack can be available in the fitness-for-service assessment for a wide range of pipe configurations encountered in practice.

CHAPTER 6

FRACTURE ASSESSMENT OF MISMATCHED GIRTH WELDS IN OVAL-SHAPED CLAD PIPELINES

This study provides an engineering assessment of an oval-shaped clad pipes with a circumferential part-through surface crack subjected to bending moment based upon equivalent stress-strain relationship method (ESSRM) in conjunction with EPRI J -estimation procedure. Plastic limit load equations are developed specially to determine the equivalent stress-strain relationship of the welded clad pipe. Then, the conventional EPRI J -estimation procedure is extended to calculate J values of the equivalent configuration with oval cross-section converted from the welded clad pipe using the ESSRM. The validation of the ESSRM/EPRI estimation framework mentioned in the above is carried out by comparing with the results obtained from 3-D elastic-plastic finite element (FE) analysis.

6.1 BACKGROUND OF PIPE OVALIZATION

With increasing demand of recoverable corrosive hydrocarbons, the metallurgically bonded clad C-Mn steel pipes with an inner corrosion resistant alloy (CRA) layer are widely used to transport these natural resources. During the installation process or in-service operation, the pipes undergo complicated loading conditions as shown in Fig. 6.1 [110]. They have to withstand internal pressure induced by the transported oil or gas products and external pressure exerted by the sea water. Moreover, longitudinal and transversal forces generated by seismic waves or other types of ground displacements are applied to the pipes, eventually resulting in the loading combinations of pressure, axial/shear forces and bending moment. However, it is cumbersome and almost impossible to consider all the loading conditions in one study. Hence, the structural response of pipes subjected to bending moment is the focus of the current research due to its importance and universality [38,44,52]. In reality, the cross-section of a pipe under bending moment inevitably exists with some degree of ovality, which results in the reductions of the bending capacity of the pipe. Brazier [111] first investigated the effect of ovalization on the bending capacity of elastic cylindrical shells, and further research was carried out by Ades [112] on the mechanical responses of elastic-plastic tubes with uniform ovalization.

Fig. 6.1 The complicated loading conditions withstood by the pipe [110].

Crack-like defects are frequently discovered in pipes exposed to aggressive environment or during welding fabrication [37,113]. Therefore, the severity of these flaws or defects needs to be evaluated in order to guarantee the structural integrity of offshore pipelines. Considerable research works have been carried out to develop structural integrity assessment procedures according to fracture mechanics [36,39,43,45,46,48,114,115]. Kim et al. [116] proposed a fully plastic J -estimation expression of circumferentially cracked pipes based on EPRI J -estimation procedure [117] and reference stress approach. A modified reference stress solution was established by Tkaczyk et al. [43] to conduct the fracture assessment of elastic-plastic pipes containing external surface defects. NourPanah and Taheri [115] developed a reference strain approach for cracked pipes subjected to plastic bending based on previous works carried out by Linkens et al. [28]. However, the procedures indicated in the above are applicable only for assessing critical homogeneous structures. Zhao et al. [100] studied the evolution tendency of crack tip opening displacement (CTOD) versus global strain for welded clad pipes with a circumferential part-through surface crack, while it is only suited to a specific pipe configuration. An equivalent stress-strain relationship method (ESSRM) is proposed by Lei and Ainsworth [118] to estimate J values of the cracked mismatched structure using J determined from the homogenous structure composed of the 'equivalent' material. Based on the ESSRM, Souza et al. [119] provided a concise fracture assessment framework for dissimilar girth welds by incorporating the influence of weld strength mismatch.

It is worth noting that the previous works are all limited to circular pipes subjected to plastic loading. When the same load is applied to an oval-shaped pipe, a larger plastic deformation is induced due to reduction of the bending capacity caused by ovalization [120,121], significantly

increasing the risk of crack initiation and propagation. Therefore, in the present study, the fracture responses of oval-shaped clad pipes with a circumferential part-through surface crack are investigated based on the ESSRM coupled with the EPRI J -estimation procedure [117]. Extensive 3-D finite element (FE) analyses are carried out first to propose the plastic limit load solutions of the welded clad pipe. Then, the ESSRM is employed to characterize the mechanical properties of the clad pipe in terms of corresponding properties of the homogeneous pipe made of an equivalent material. Subsequently, the J values for a wide range of pipe geometries and crack sizes are calculated by extending the conventional EPRI J -estimation procedure [117] for an oval-shaped homogeneous pipe. Finally, the accuracy of the ESSRM/EPRI estimation framework is validated by comparing it with the FE results.

6.2 FRACTURE ESTIMATION METHODOLOGY OF WELDED CLAD PIPES

This section presents the development of fracture assessment methodology for mismatched clad pipes containing a circumferential part-through surface crack as shown in Fig. 6.2.

Fig. 6.2 Details of welded clad pipes with a circumferential part-through surface crack.

6.2.1 The equivalent stress-plastic strain relationship

The ESSRM is proposed to evaluate the J -integral values of the mismatched configuration using homogeneous one made of an equivalent material, and it is expressed in the following form:

$$\sigma_{eq}(\epsilon_p) = \left[\frac{P_0^w - P_0^{mis}}{P_0^w - P_0^b} \right] \sigma_w(\epsilon_p) + \left[\frac{P_0^{mis} - P_0^b}{P_0^w - P_0^b} \right] \sigma_b(\epsilon_p) \quad (6.1)$$

where ε_p is the plastic strain, $\sigma_{eq}(\varepsilon_p)$ represents the equivalent stress-strain relationship, $\sigma_w(\varepsilon_p)$ and $\sigma_b(\varepsilon_p)$ are the stress-strain relationship of the weld metal and the base material, while P_0^{mis} is the plastic limit load of the mismatched configuration, P_0^w and P_0^b denote the corresponding plastic limit load of the homogeneous configurations composed of the weld metal and the base material, respectively. The welded clad pipe consists of three distinct materials (Fig. 6.2): outer carbon steel, inner CRA material and weld metal. Therefore, in this study, the ESSRM is employed twice to transfer the welded clad pipe into the homogeneous one made of only one material. In the first step, the mechanical response of the bi-material pipe composed of the outer carbon steel and the inner CRA material is treated in terms of the ESSRM using the following expression:

$$\sigma_{eq}^{bi}(\varepsilon_p) = \left[\frac{P_0^{in} - P_0^{bi}}{P_0^{in} - P_0^{out}} \right] \sigma_{in}(\varepsilon_p) + \left[\frac{P_0^{bi} - P_0^{out}}{P_0^{in} - P_0^{out}} \right] \sigma_{out}(\varepsilon_p) \quad (6.2)$$

where $\sigma_{eq}^{bi}(\varepsilon_p)$ is the equivalent stress-strain relationship of the bi-material pipe, $\sigma_{in}(\varepsilon_p)$ and $\sigma_{out}(\varepsilon_p)$ are the stress-strain relationship of the inner CRA material and the outer carbon steel, while P_0^{bi} is the plastic limit load of the bi-material pipe, P_0^{in} and P_0^{out} denote the plastic limit load of the inner CRA material and the outer carbon steel. Then, the ESSRM is employed again to determine the mechanical property of the equivalent material for the welded clad pipe,

$$\sigma_{eq}^{clad}(\varepsilon_p) = \left[\frac{P_0^{bi} - P_0^{clad}}{P_0^{bi} - P_0^w} \right] \sigma_{eq}^{bi}(\varepsilon_p) + \left[\frac{P_0^{clad} - P_0^w}{P_0^{bi} - P_0^w} \right] \sigma_w(\varepsilon_p) \quad (6.3)$$

where $\sigma_{eq}^{clad}(\varepsilon_p)$ and P_0^{clad} are the equivalent stress-strain relationship and the plastic limit load of the welded clad pipe, respectively.

The equivalent stress-strain curve described by $\sigma_{eq}^{clad}(\varepsilon_p)$ can characterize the mechanical response of the welded clad pipe as long as the plastic limit loads (P_0^{bi} , P_0^{in} , P_0^{out} and P_0^{clad}) can be determined, which are proposed in Section 6.3.

6.2.2 The EPRI *J*-estimation procedure of homogeneous pipes with ovalization

As indicated previously, the welded clad pipe is transformed into the homogeneous one made of an equivalent material using the ESSRM. Hence, the *J*-integral estimation values for the homogeneous pipe can be used to predict the fracture responses of the welded clad pipe.

The fracture assessment strategy for the homogeneous pipe with oval cross-section is developed in this sub-section by incorporating the effect of ovality into the conventional EPRI J -estimation procedure [117].

The EPRI J -estimation procedure [117] provides an approach in estimating the J -integral over the full range of elastic-plastic regime by assuming the total J equal to the sum of the elastic and plastic components,

$$J = J_{el} + J_{pl} \quad (6.4)$$

The first term on the right side of Eq. (6.4) is in fact the elastic release rate which can be calculated from the stress intensity factor (K), and it is expressed in the following,

$$J_{el} = \frac{K^2}{E'} \quad (6.5)$$

where $E' = E$ and $E' = E/(1 - \nu^2)$ are for plane stress and plane strain conditions with E and ν representing the elastic modulus and the Poisson's ratio, respectively.

In the FE analyses, a Ramberg-Osgood model [22] is usually employed to describe the uniaxial stress-strain curve of an elastic-plastic material, and it is given as follows:

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha \left(\frac{\sigma}{\sigma_y} \right)^n \quad (6.6)$$

where α is a dimensionless constant, n is the strain hardening exponent, σ_y and ε_y denote the yield stress and the yield strain, respectively. Based on the above power-hardening model and the HRR singularity derived by Hutchinson, Rice and Rosengren [122,123], the plastic component, J_{pl} , of Eq. (6.4) is given in the following to evaluate the J -integral of a circumferential cracked pipe under bending moment,

$$J_{pl} = \alpha \varepsilon_y \sigma_y b h_1 (a/t, D_e/t, \theta, n) \left(\frac{M}{M_L} \right)^{n+1} \quad (6.7)$$

where a is the crack depth, θ represents the surface crack length, D_e and t are the pipe outer diameter and the wall thickness, respectively, $b = t - a$ denotes the un-cracked ligament, M and M_L are the applied bending moment and the limiting bending moment, respectively, h_1 is the dimensionless parameter that depends on pipe geometry, crack size and strain hardening

exponent. The h_1 -factor characterizes the linear relationship between J_{pl} and M^{n+1} quantitatively and can be determined by numerical analysis of the configuration of interest.

For M_L , the following expression is used in the present study [124],

$$M_L = 2\sigma_y R_m^2 t \left(2\sin\beta - \frac{a}{t} \sin\theta \right) \quad (6.8)$$

where R_m is the pipe mean radius with $R_m = (D_e - t)/2$, and β is denoted as

$$\beta = \frac{\pi}{2} \left[1 - \left(\frac{\theta}{\pi} \right) \left(\frac{a}{t} \right) \right] \quad (6.9)$$

It is worth noting that the limit bending moment given in Eq. (6.8) is applicable for $(\theta + \beta) \leq \pi$.

In this study, the effect of ovality on fracture responses in the cracked pipe subjected to bending moment is investigated in detail. The degree of ovality [110] is determined by the difference between the maximum and the minimum measured outer diameters ($D_e^{\max}$ and $D_e^{\min}$) divided by the nominal outer diameter as depicted in Fig. 6.3, and it is expressed as follows:

$$C_O = \frac{(D_e^{\max} - D_e^{\min})}{D_e} \times 100\% \quad (6.10)$$

Since the only difference between the circular pipe and the oval-shaped one is characterized by C_O , it is assumed that the above J -estimation methodology for a circular pipe can be extended directly to serve for the corresponding J calculation of an oval-shaped pipe by inserting C_O into the h_1 -factor in Eq. (6.7), and the new expression is given in the following,

$$J_{pl} = \alpha \varepsilon_y \sigma_y b h_1(a/t, D_e/t, \theta, n, C_O) \left(\frac{M}{M_L} \right)^{n+1} \quad (6.11)$$

The verification of Eq. (6.11) and the determination of h_1 -factor are presented in Sub-section 6.4.3. Extensive 3-D elastic and elastic-plastic FE analyses are conducted to determine the J_{el} and J values of the oval-shaped homogeneous pipe with a circumferential surface part-through crack. Then, the fully plastic part, J_{pl} , of J is calculated using Eq. (6.4), and the values of h_1 -factor in Eq. (6.11) is determined according to the estimation procedure developed by Yun-Jae Kim et al. [116], Chiodo and Ruggieri [52].

Fig. 6.3 Transformation of a circular clad pipe to an oval-shaped one.

6.3 DETERMINATION OF PLASTIC LIMIT LOAD OF WELDED CLAD PIPE

The determination of ESSRM depends strongly on the plastic limit load solutions of the structure. In this section, the limit load solutions for the welded clad pipe with a circumferential part-through external surface crack subjected to bending moment are built up through 3-D elastic-perfectly plastic FE analyses.

6.3.1 Analysis matrix and FE modelling

Fig. 6.2 presents the clad pipe configuration with a V-groove butt weld containing a circumferential part-through external surface crack, and the weld geometry is defined by the root opening width ($2h$) and the groove angle (ϕ). The analysis matrix [119] considers the cracked pipe with a constant wall thickness $t = 20.6$ mm, and the CRA layer thickness of $t_{\text{CRA}} = 3$ mm used in this study. The CRA thickness is representative of the typical value for the commonly-used layer thickness encountered in practice. Three different values for D_e/t are adopted: $D_e/t = 10, 15$ and 20 . The crack depth ratio (a/t) and the crack length ratio (θ/π) are ranging from 0.1 to 0.4 and from 0.04 to 0.20 , respectively. For the weld root opening width, the values of h/t employed in this analysis are 0.1 and 0.2 , and the groove angle varying from 20° to 60° is considered, covering a wide range of butt weld profiles found in real application. The weld strength mismatch factor (M_y) [125] is usually defined as the ratio of the yield stress between the weld metal (σ_y^w) and the outer carbon steel (σ_y^{out}), and it is expressed as

$$M_y = \frac{\sigma_y^w}{\sigma_y^{\text{out}}} \quad (6.12)$$

Five different M_y values, $M_y = 0.5, 0.75, 1.0, 1.25, 1.5$, are included in present analyses. The clad pipe segments are usually welded together using a filler material, such as nickel-chromium alloy 625 (UNS N06625), which eventually has the same mechanical properties and metallurgical composition as the inner CRA layer [119]. Hence, identical material properties are employed for the CRA layer and the weld metal in this study.

Fig. 6.4 Typical FE model of welded clad pipes containing a circumferential part-through surface crack.

A FE mesh generator is developed specially to generate models of the welded clad pipe with a circumferential part-through surface crack, and a typical mesh for this configuration is depicted in Fig. 6.4. 3-D elastic-perfectly plastic FE analyses are performed using ABAQUS, a general purpose FE software [51], and a 20-node brick element with reduced integration (C3D20R) is used throughout the modelling. In this analysis, materials are assumed to be elastic-perfectly plastic with non-hardening J_2 flow theory in small geometry change setting. Symmetric conditions are fully utilized in the FE models to reduce the number of elements and computing time. Multi-point constraint (MPC) is imposed at the end surface of the pipe where a single node located at the center of the surface is connected to the nodes, and a rotation displacement is applied to this node to provide the bending loading condition. The required loads can be obtained directly from the nodal force, and the twice elastic slope method as illustrated in Fig. 6.5 is used to determine the final plastic limit load.

Fig. 6.5 Determination of plastic limit load using the twice elastic slope method for a pipe with $D_e/t = 15$, $a/t = 0.2$, $\theta/\pi = 0.12$, $h/t = 0.15$, $\phi = 20^\circ$ and $M_y = 1.25$.

6.3.2 Accuracy of present FE limit load analyses

The reliability of FE models employed in the present study is verified first before it is used extensively to generate results of plastic limit loads. The mesh generator mentioned previously is modified to create FE models for the homogeneous pipe with a circumferential through-wall crack under bending moment, and the FE results obtained are compared with the analytical limit load (M_0^{Miller}) proposed by Miller [126], which is expressed in the following,

$$\frac{M_0^{\text{Miller}}}{4\sigma_y R_m^2 t} = \cos\left(\frac{\theta}{2}\right) - \frac{\sin\theta}{2} \quad (6.13)$$

Fig. 6.6 illustrates the comparison results and a good agreement is observed. Hence, the mesh generator is considered to be well suited for further analyses of more complicated configurations where sufficient elements are used to guarantee good convergence. FE models for the welded clad pipe with a circumferential part-through external surface crack are generated by the mesh generator, and convergent tests covering the range of parameters are carried out to determine the optimum number of elements which vary from 8,043 to 16,011 based on the crack size and pipe geometry. The case shown in Fig. 6.7 is one of these convergent tests. The above operations provide confidence of the present FE analyses to obtain accurate plastic limit loads.

Fig. 6.6 Comparison of plastic limit loads obtained from Eq. (6.13) [126] and FE analyses for homogeneous pipes with a circumferential through-wall crack under bending moment.

Fig. 6.7 The convergent test case results.

6.3.3 Plastic limit load solutions of homogeneous and bi-material pipes

Based on present FE analyses, the approximation expression of plastic limit loads for the homogeneous pipe with a circumferential part-through external surface crack is proposed in the following,

$$\frac{M_0^{\text{homo}}}{4\sigma_y R_m^2 t} = 1 + A_1 \left(\frac{a}{t}\right) + A_2 \left(\frac{a}{t}\right)^2 \quad (6.14)$$

where

$$A_1 = 1.338708 \left(\frac{\theta}{\pi} \right)^2 - 0.321014 \left(\frac{\theta}{\pi} \right) - 0.002893 \quad (6.15)$$

$$A_2 = -12.607045 \left(\frac{\theta}{\pi} \right)^2 + 1.335322 \left(\frac{\theta}{\pi} \right) - 0.067332 \quad (6.16)$$

The plastic limit loads given by the developed equation agree very well with the FE results as depicted in Fig. 6.8. It is worth noting that the plastic limit loads for the homogeneous pipe made of outer carbon steel, inner CRA material or weld metal, can all be represented using Eq. (6.14) by substituting the corresponding yield stress into it.

Fig. 6.8 Plastic limit loads of homogeneous pipes with a circumferential part-through surface crack.

For bi-material pipes, the mismatch between the outer pipe and the CRA layer can be denoted by M_y since identical material properties are employed for the CRA layer and the weld metal in this study. Hence, the additional parameter, M_y , is introduced to describe the material properties of bi-material pipe in comparison with the homogeneous pipe. It is assumed that the plastic limit load solution for the bi-material pipe can be developed based on Eq. (6.14) and they are relative to each other by M_y . Extensive FE analyses are conducted to determine the linear relationship between M_0^{bi}/M_0^{homo} and M_y , where M_0^{bi} is the plastic limit load of bi-material pipes. An initial guess to the coefficients values in the linear expression is given at first; then, least square fitting method is employed to minimize the differences between the expression predictions and the FE data by changing the coefficients in the expression, and it is obtained as

$$\frac{M_0^{\text{bi}}}{M_0^{\text{homo}}} = 0.132426M_y + 0.867041 \quad (6.17)$$

Figs. 6.9-6.10 show comparison results of plastic limit loads obtained from the FE results and Eq. (6.17) for varying a/t , θ/π , D_e/t and M_y , and the agreement between them is excellent, indicating that the proposed expression is sufficiently valid to predict the plastic limit loads of the cracked bi-material pipe.

Fig. 6.9 Plastic limit loads of bi-material pipes with a circumferential part-through surface crack for varying a/t and θ/π but constant D_e/t .

Fig. 6.10 Plastic limit loads of bi-material pipes with a circumferential part-through surface crack for varying D_e/t .

6.3.4 Plastic limit load solutions of welded clad pipe

Based on research works performed by Sang-Hyun Kim et al. [127], the plastic limit load solutions for the V-groove welded clad pipe can be characterized by three parameters: the groove weld angle (ϕ), the slenderness of the weld (ψ) and the mismatch factor (M_y). As depicted in Fig. 6.11, the V-groove weld configuration is simplified as an equivalent strip one using the effective weld width (h_{eff}) which is defined as average of the un-cracked weld ligament,

$$h_{\text{eff}} = h + \frac{(t - a)}{2} \tan(\phi/2) \quad (6.18)$$

Therefore, in this case, the limit loads for the simplified strip weld can be represented by ψ and M_y , and ψ is denoted using the following expression,

$$\psi = \frac{(t - a)}{h_{\text{eff}}} + 5 \left[\cos\left(\frac{\theta}{2}\right) - \frac{\sin\theta}{2} \right] \quad (6.19)$$

Eventually, the limit load equation for the welded clad pipe is built up in terms of ψ and M_y , and it is given as

$$\frac{M_0^{\text{clad}}}{M_0^{\text{bi}}} = \frac{24(M_y - 1)}{25} \left(\frac{\psi_1}{\psi} \right) + \frac{(M_y + 24)}{25} \quad (6.20)$$

where

$$\psi_1 = \exp \left[-\frac{2(M_y - 1)}{5} \right] \quad (6.21)$$

It is noted that Eq. (6.20) has a similar expression in form as that established by Sang-Hyun Kim et al. [127] except that the denominator on the left side of this equation is denoted by the limit load solution (M_0^{bi}) of the cracked bi-material pipe. Fig. 6.12 depicts the comparison results of the plastic limit loads obtained from the FE analyses, $M_0^{\text{clad}}/M_0^{\text{bi}}$ (FEA) and Eq. (6.20), $M_0^{\text{clad}}/M_0^{\text{bi}}$ (Eq. (6.20)), and two dotted lines represent a deviation of $\pm 5\%$ illustrating the difference between these two sets of results. The analysis matrix covers the analytical parameters of the pipe with $D_e/t = 10, 20$, $a/t = 0.1, 0.4$, $\theta/\pi = 0.04, 0.20$, $h/t = 0.1, 0.2$ and $M_y = 0.5, 0.8, 1.5$. It can be seen that the limit load solutions given by Eq. (6.20) deviate

significantly from the results obtained by the FE analyses for the weld mismatch case of $M_y = 0.5$. However, for M_y values ranging from 0.8 to 1.5, the limit load values of Eq. (6.20) agree very well with the FE results in view of all the data points located within the deviation of $\pm 5\%$. Therefore, the validity range of the proposed limit load equation for M_y is varied from 0.8 to 1.5.

Fig. 6.11 Converting V-groove butt weld into idealized one having a rectangular cross-section using effective weld width method.

Fig. 6.12 Comparison results of limit loads obtained from FE analyses and Eq. (6.20).

6.4 J-ESTIMATION PROCEDURE BASED ON THE H_I -FACTOR

In this section, extensive 3-D elastic-plastic FE analyses are performed on oval-shaped homogeneous pipes with a circumferential part-through surface crack under bending moment.

6.4.1 Analysis matrix and material models for elastic-plastic analyses

The analysis matrix presented in Table 6.1 includes four crack depths ($a/t = 0.1, 0.2, 0.3$ and 0.4) and three crack lengths ($\theta/\pi = 0.04$ to 0.20 with an increment of 0.08). The American Lifelines Alliance (ALA) [128] considers the maximum allowable degree of ovality to be 15%. Hence, C_O values of 0, 5%, 10% and 15% are employed in the present study. The mechanical properties of pipes are varied based on the typical range of high strength low alloy (HSLA) steels as: $n = 5$ and $E/\sigma_y = 800$ (high hardening material), $n = 10$ and $E/\sigma_y = 500$ (moderate hardening material), $n = 20$ and $E/\sigma_y = 300$ (low hardening material) with $E = 206$ GPa and $\nu = 0.3$, which reflect the upward trend in yield stress with the increase of strain hardening exponent. The above materials in the FE analyses are assumed to follow the Ramberg-Osgood model [22] with $\alpha = 1$ as illustrated in Eq. (6.6), and the FE analyses are carried out using deformation plasticity and small geometry change setting.

Table 6.1 Table of analysis matrix

D_o/t	n	a/t	θ/π	C_O (%)
10	5	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15
	10	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15
	20	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15
20	5	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15
	10	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15
	20	0.1, 0.2, 0.3, 0.4	0.04, 0.12, 0.20	0, 5, 10, 15

6.4.2 FE analysis and validity of FE models

In the present study, 288 cases of different geometries of pipe configurations are analysed, and the FE models are built up using the mesh generator mentioned in Sub-section 6.3.1. The element type, boundary and loading conditions used for the J -integral estimation in the elastic-

plastic analyses are identical with that described in Sub-section 6.3.1 aiming at determining the plastic limit load solutions.

The transformation procedure from the circular pipe to the oval-shaped one is depicted in Fig. 6.13, and the detailed methodology description is listed in the following.

Fig. 6.13 Transformation procedure from the circular cross-section to the oval shape.

The following assumptions are made for the transformation procedure:

- The cross section of the pipe is assumed to be perfectly oval [120];
- The change in the outer perimeter of the pipe is not considered during the ovalization of pipe cross-section [110];
- The variation of the pipe thickness induced by the cross-sectional ovalization is negligible, and this assumption is reasonable especially for thin-walled pipes employed in this analysis.

According to the first assumption, the oval outer contour of the pipe can be expressed using the standard equation of ellipse,

$$\frac{\eta^2}{(D_e^{\max}/2)^2} + \frac{\zeta^2}{(D_e^{\min}/2)^2} = 1 \quad (6.22)$$

and the equation describing the circular profile is written as

$$\frac{Y^2}{(D_e/2)^2} + \frac{Z^2}{(D_e/2)^2} = 1 \quad (6.23)$$

The second assumption indicating equal perimeters between the circular and the oval outer contours of the pipe produces

$$\pi D_e = \pi D_e^{\min} + 2(D_e^{\max} - D_e^{\min}) \quad (6.24)$$

Based on Eq. (6.10) and Eq. (6.24), the expressions for $D_e^{\max}$ and $D_e^{\min}$ are obtained,

$$\begin{aligned} D_e^{\max} &= \left(\frac{\pi + \pi C_0 - 2C_0}{\pi} \right) D_e \\ D_e^{\min} &= \left(\frac{\pi - 2C_0}{\pi} \right) D_e \end{aligned} \quad (6.25)$$

Substituting Eq. (6.25) into Eq. (6.22) and making the corresponding terms in Eq. (6.22) and Eq. (6.23) equal, the conversion relationship for the two coordinate systems shown in Fig. 6.13(a) is given as follows:

$$\begin{aligned} \eta &= \left(\frac{\pi + \pi C_0 - 2C_0}{\pi} \right) Y \\ \zeta &= \left(\frac{\pi - 2C_0}{\pi} \right) Z \end{aligned} \quad (6.26)$$

Eq. (6.26) provides the means for transforming the circular outer contour of the pipe into the oval shape, then in the following, the circular cross-section of the pipe in Y-Z system is projected to the oval cross-section in η - ζ system as depicted in Fig. 6.13(b). The third assumption illustrates that the thickness of the pipe keeps constant during the ovalization process, hence, the following expression is obtained,

$$D_e^{\max} - D_e^{\min} = D^{\max} - D^{\min} \quad (6.27)$$

where $D^{\max}$ and $D^{\min}$ are the maximum and the minimum diameters of an arbitrary ellipse in the pipe cross-section as outlined in Fig. 6.13(b), and the degree of ovality of this ellipse is denoted as

$$C_o^{\text{temp}} = \frac{(D^{\text{max}} - D^{\text{min}})}{D} \times 100\% = \frac{(D_e^{\text{max}} - D_e^{\text{min}})}{D} \times 100\% \quad (6.28)$$

where D is the nominal diameter of the circle relative to this ellipse. The relationship between C_o and C_o^{temp} is built up by substituting Eq. (6.25) into Eq. (6.28),

$$C_o^{\text{temp}} = \left(\frac{D_e}{D}\right) C_o \quad (6.29)$$

Substituting Eq. (6.29) into Eq. (6.26) produces

$$\begin{aligned} \eta &= \left(\frac{\pi D + \pi D_e C_o - 2 D_e C_o}{\pi D}\right) Y \\ \zeta &= \left(\frac{\pi D - 2 D_e C_o}{\pi D}\right) Z \end{aligned} \quad (6.30)$$

It is noted that the circular cross-section of the pipe can be converted into the oval shape by using Eq. (6.30). The FE model for the oval-shaped clad pipes can be created by including the above transformation methodology into the previous mesh generator codes.

Fig. 6.14 Path independence of numerical values of the J -integral.

In order to evaluate the J -integral values accurately, a focused spider web mesh shown in Fig. 6.4 is employed at the crack tip zone, and the crack tip is surrounded by 12 concentric rings of elements. The J -integral at the crack deepest point is determined from the average value of 2nd-12th contours, and the corresponding J -integral for the first contour is ignored due to the numerical errors. Confidence in the FE analyses is gained from the path independence of the J -integral considering that the difference of the J values for these 11 contours is marginal as shown in Fig. 6.14. Moreover, it is worth noting that the J values are sensitive to the mesh

design, especially on the number of elements. Therefore, the convergent tests covering the range of parameters listed in Table 6.1 are carried out to determine the optimal number of elements used in the analyses, and Fig. 6.15 presents one of these cases. It is found that the number of elements ranging from 15,901 to 24,317 is adequate based on the crack configuration and pipe geometry. Further confidence in the validity of FE models is obtained through the above mentioned sensitivity analysis.

Fig. 6.15 The convergent test case results.

6.4.3 Verification of Eq. (6.21) and determination of h_1 -factor

In Sub-section 6.3.2, the conventional EPRI J -estimation expression is developed to serve for the oval-shaped pipe with a circumferential part-through external surface crack by considering the effect of ovality. In this subsection, the validity of the new J -estimation expression given in Eq. (6.11) is demonstrated. Then, the h_1 -factor is determined according to the procedure proposed by Yun-Jae Kim et al. [116], Chiodo and Ruggieri [52].

For the conventional J -estimation expression (Eq. (6.7)), J_{pl} is in a proportion to $(M/M_L)^{n+1}$ based on the power-hardening model and the HRR singularity [122,123]. Hence, the validity of Eq. (6.11) can be ascertained if the linear relationship between J_{pl} and $(M/M_L)^{n+1}$ still holds true. Eq. (6.11) is rewritten into the following form to conveniently facilitate the verification procedure,

$$\frac{J_{pl}}{\alpha \varepsilon_y \sigma_y b} = h_1(a/t, D_e/t, \theta, n, C_0) \left(\frac{M}{M_L} \right)^{n+1} \quad (6.31)$$

Fig. 6.16 shows the evolution trend of $J_{pl}/\alpha \varepsilon_y \sigma_y b$ with $(M/M_L)^{n+1}$ for an oval-shaped pipe with $D_e/t = 10$, $a/t = 0.4$, $n = 10$ and $C_0 = 15\%$, and a linear relationship between them is clearly observed. More verification tests for various parameter combinations are conducted to further confirm the linear dependence, verifying the validity of Eq. (6.11). It is noted that the factor h_1 can be determined directly by measuring the slope of the evolution curve of $J_{pl}/\alpha \varepsilon_y \sigma_y b$ with $(M/M_L)^{n+1}$. The detailed evaluation procedure of the h_1 -factor is also shown in Fig. 6.12, and the corresponding expression is listed in the following,

$$h_1 = [A_1(C_0)^2 + A_2 C_0 + A_3] \left(\frac{a}{t} \right)^{A_4} \quad (6.32)$$

where the values of A_i ($i = 1, 2, 3, 4$) are tabulated in Table 6.2.

Table 6.2 Values of A_i ($i = 1, 2, 3, 4$) in Eq. (6.32)

D_e/t	n	θ/π	A_1	A_2	A_3	A_4
10	5	0.04	0.0218	0.4894	12.8501	1.1570
		0.12	0.0564	1.9240	45.6702	1.8471
		0.20	0.0811	1.7951	47.3512	1.9583
	10	0.04	0.1143	0.8991	17.1204	1.2492
		0.12	0.2529	2.1714	38.8214	1.6480
		0.20	0.2692	2.3821	41.3122	1.8643
	20	0.04	0.7757	-1.3471	19.8322	1.1851
		0.12	0.9674	-1.2662	25.1102	1.3340
		0.20	0.7421	-1.4630	17.4414	1.3872
20	5	0.04	0.0643	1.8171	46.4423	1.7804
		0.12	0.2445	3.9513	133.8022	2.4391
		0.20	0.1759	4.9822	129.2033	2.5732
	10	0.04	0.3736	3.0112	59.6404	1.7553
		0.12	0.9564	9.0573	160.1033	2.3884
		0.20	0.7689	6.7363	129.9022	2.4034
	20	0.04	2.5174	-2.2422	74.8331	1.6573
		0.12	3.4770	-3.9312	102.4055	1.8512
		0.20	2.3974	-2.4453	68.9804	1.9394

The influence of the degree of ovality, C_O , on the h_1 -factor for the cracked pipes with varying a/t and D_e/t is investigated in detail. It is noted from Fig. 6.17 that the h_1 value for a given a/t increases as the C_O increases, and the h_1 -factor displays a relatively strong sensitivity to C_O , especially for deep cracks. It is due to ovalization of the pipe cross-section resulting in reduction of the bending capacity. When the same bending moment is applied to the configurations with circular and oval cross-sections, the oval-shaped pipe undergoes a larger plastic deformation, inducing a larger J -integral value in the crack tip zone. The dependence of h_1 on θ/π for the pipe configuration is not discussed here due to the similar trend observed in the present study with that in the paper published by Chiodo and Ruggieri [52].

Fig. 6.16 Evolution trend of $J_{pl}/\alpha\varepsilon_y\sigma_yb$ with $(M/M_L)^{n+1}$.

Fig. 6.17 Effect of C_O on h_1 -factor with increasing a/t .

6.5 VALIDATION STUDY

In the previous sections, the mechanical properties of the welded clad pipe are characterized by the properties of a homogeneous pipe using the ESSRM. Then, the EPRI J -estimation procedure [117] is employed to assess the J -integral values of the homogeneous pipe configuration, and the h_1 -factor values covering the range of parameters outlined in Table 6.1 are calculated. Finally, the J -integral predictions of the welded clad pipe can be implemented through the ESSRM in combination with the EPRI framework [117]. The aim of this section is to demonstrate the accuracy of the method presented in the above by comparing the J -integral values obtained from the ESSRM/EPRI approach and the FE analyses.

A typical clad pipe geometry with $t = 20.6$ mm , $t_{\text{CRA}} = 3$ mm and $D_e/t = 15$ is considered in the present study, and two weld groove angles ($\phi = 20^\circ$ and 60°) with a fixed root opening width ($h/t = 0.15$) are used, representing the narrow and wide weld configurations. The degree of ovality is varied from 0% to 15% in this analysis, covering the range of the parameter, C_0 , outlined in Table 6.1. A representative pipe steel, API 5L Grade X60, is employed, and its yield stress and strain hardening exponent are 483MPa and 12, respectively. The mismatch level ranging from 0.5 to 1.5 is incorporated in this study to reflect the effects of the weld metal and CRA material mismatch strength. Figs. 6.18-6.20 present the comparison results. It is observed that the J -integral values obtained by the ESSRM/EPRI method are in good agreement with the FE results except for the case of $M_y = 0.5$. It is because the application range of the proposed limit load expression given by Eq. (6.16) is from $M_y = 0.8$ to $M_y = 1.5$. Therefore, the ESSRM coupled with the EPRI J -estimation procedure can predict well the J -integral values for the oval-shaped clad pipe with the mismatch ratio varying from 0.8 to 1.5.

Fig. 6.18 Comparison of J -integral values obtained from ESSRM/EPRI method (red line) and FE analyses (black line) for $M_y = 0.8$.

Fig. 6.19 Comparison of J -integral values obtained from ESSRM/EPRI method (red line) and FE analyses (black line) for $M_y = 1.5$.

Fig. 6.20 Comparison of J -integral values obtained from ESSRM/EPRI method (red line) and FE analyses (black line) for $M_y = 0.5$.

6.6 SUMMARY OF THE CHAPTER

The present study provides an engineering integrity assessment of mismatched girth welds in oval-shaped clad pipes subjected to bending moment based on the ESSRM/EPRI estimation framework. Extensive 3-D FE analyses are performed to determine the plastic limit loads applicable to the welded clad pipes with a circumferential part-through surface crack by considering the effects of weld strength mismatch and weld bevel geometry. The proposed limit load equation agrees very well with the FE results for the mismatch factor varying from 0.8 to 1.5. Based on the limit load equation, the ESSRM is used to convert a complex tri-material structure into a homogeneous configuration made of only one equivalent material, reducing significantly the computational complexity and saving the calculation time. Then, the conventional EPRI J -estimation procedure [117] is extended to serve for the homogeneous pipes with oval cross-section by appending the degree of ovality to the h_1 -factor function. The h_1 -factor for a wide range of pipe geometries and crack sizes are calculated and formulated in Eq. (6.32). Moreover, the influence of ovality on the h_1 -factor is investigated in detail, and the factor displays a relatively strong sensitivity to the ovality of the pipe cross-section. Finally, validation studies are carried out by comparing the results obtained from the ESSRM/EPRI

estimation framework and the elastic-plastic FE analyses, and a good agreement between them is observed for the mismatch factor ranging from 0.8 to 1.5, indicating the validity of the ESSRM/EPRI estimation framework.

CHAPTER 7

FRACTURE ASSESSMENT OF MISALIGNED CLAD PIPELINES CONTAINING AN INTERFACE CRACK

7.1 BACKGROUND OF CLAD PIPELINES

A reference strain J -estimation formulation is developed to perform the fracture assessment of misaligned clad pipes containing a circumferential part-through crack at the interface of weld metal and outer pipe. Extensive parametric study totaling of 896 finite element (FE) models is carried out to investigate the effects of various parameters (pipe diameter, crack depth, crack length, centreline offset, strength mismatch level and strain hardening exponent) on the J -integral. Then, the estimation formulation characterizing the linear relationship between the J -integral and the applied displacement loading is proposed using the standard least square fitting, and a goodness of fit for the J -integral values obtained from the formulation and the FE analyses is endorsed. Finally, the application range of the estimation formulation is then extended by conducting further comparison study.

With dramatic increase demand for recoverable corrosive hydrocarbons, metallurgically bonded clad C-Mn steel pipe with an inner corrosion resistant alloy (CRA) layer has become a major transportation tool for oil and natural gas. These pipes are connected together by joining one end of the pipe to the other end by girth welding procedure, and during the welding process, crack-like defects are often introduced into the pipes, especially into the welds [36]. Moreover, these pipes are subjected to complicated loading conditions during the installation or in-service operation [110]; and this poses tremendous challenge to the reliability of the pipelines. Therefore, it is of great importance to find a suitable fracture assessment methodology for offshore pipelines subjected under large plastic deformation. Existing codes and standards for fracture assessment such as BS7910 [1] and R6 [129] are usually obtained from the load-controlled conditions [43,52]. However, in some situations, the pipes is subjected to displacement- or strain-controlled loading deforming into the plastic regime [130], such as in the reeling installation [110]. In this case, strain-based approach is preferably employed for the fracture assessment [131-133].

In the 1990s, Schwalbe [27] proposed a strain-based engineering treatment model on crack tip opening displacement (CTOD) and J -integral for fully plastic conditions. However, the model can only be used for small strain levels and plane stress problems. As an extension of reference stress method, a reference strain J -estimation procedure was developed by Linkins et al. [28] to deal with strains directly based on the un-cracked body strain. With the advances in computing power, 3-D finite element (FE) analysis has become an alternative method to conduct fracture assessment of cracked pipes subjected to large plastic deformation. Jayadevan et al. [37] and Østby et al. [38] investigated the fracture responses of homogeneous pipes with a surface crack based on 3-D FE analysis, and it is noted that the CTOD varies linearly with the global strain. Jia et al. [131-133] proposed a modified engineering critical assessment method for the metallic pipelines containing a circumferential surface crack [133] and a deeply-embedded crack [132], respectively. The variation tendency of CTOD with the global strain for the welded clad pipes with an external surface crack was studied by Zhang et al. [134], where the proposed assessment equation is only suited to a specific pipe configuration. Based on a strain-based J -estimation scheme in conjunction with the equivalent stress-strain relationship method, Souza and Ruggieri [49] provided a concise fracture assessment framework for under-matched pipe girth welds by considering a wide range of pipe geometries, crack sizes and material properties.

During the welding process, weld misalignment such as centreline offset, angular or both is invariably introduced into the pipeline [33,53,54], and this induces an additional bending moment along the pipe section (Fig. 7.1). Due to the stress concentration effect caused by the misalignment, the maximum bending moment is obtained at the weld toe, while no bending moment is observed at the weld centre line (WCL) [100]. Hence, the weld toe is a more critical location of crack initiation and propagation. It is worth noting that the aforementioned research works [36-38,49,52,134] are all about fracture analyses of aligned pipes with a surface crack located at the WCL, and very limited investigations are involved in the fracture assessment of misaligned clad pipes containing a crack located at the weld toe. As shown in Chapter 5, the fatigue analysis of misaligned pipes with a weld toe crack is carried out by using linear elastic fracture mechanics (LEFM), and the crack is assumed to be normal to the pipe surface; it is a commonly used crack profile at the weld toe due to the loading case and the same elastic properties of weld metal and pipe steel. Herein, the fracture responses of misaligned clad pipes are investigated by elastic-plastic fracture mechanics (EPFM) due to the large plastic

deformation induced by the reeling installation or in-service operation; a surface crack located at the interface of weld metal and outer pipe is investigated by considering the heterogeneity of weld metal and pipe steel in plastic properties and the recommendations in Ref. [50]. Moreover, the equivalent stress-strain method used in Chapter 6 no longer works well for the present interface crack case. Hence, a strain-based J -estimation methodology is proposed in this study based on 3-D elastic-plastic FE analysis [135]. The present study is organized as follows: a reference strain J -estimation formulation is derived in Section 7.2 according to the conventional EPRI J -estimation procedure [117,136]; in Section 7.3, the geometrical configurations, material properties of the pipe segment and FE modelling procedure are described in detail; section 7.4 illustrates the effects of crack sizes and material properties on the J -integral, and the proposed estimation formulation in Section 7.2 is determined based on a standard least square fitting; section 7.5 presents further discussion on the application range of the estimation formulation, and conclusions are given in the Section 7.6.

Fig. 7.1 (a) Crack locations in welds [50]; (b) clad pipes with centreline offset weld misalignment; (c) moment distribution induced by weld misalignment in clad pipes.

7.2 THE REFERENCE STRAIN J -ESTIMATION METHODOLOGY

The path-independent J -integral can satisfactorily characterize the crack-tip stress fields for materials that exhibit time-independent, nonlinear behavior, and it can be used as a fracture criterion. Hence, numerous J -estimation schemes have been developed. The Electric Power Research Institute (EPRI) J -estimation procedure [117,136] provides an approach to estimate the J -integral over the full range of elastic-plastic regime for a material obeying the Ramberg-Osgood model [22,136],

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha \left(\frac{\sigma}{\sigma_y} \right)^n \quad (7.1)$$

where α is a dimensionless constant, n is the strain hardening exponent, σ_y and ε_y denote the yield stress and yield strain, respectively. The J -estimation procedure divides the total J into the sum of the elastic and plastic components,

$$J = J_{el} + J_{pl} \quad (7.2)$$

where J_{el} is actually the elastic release rate which can be calculated from the stress intensity factor (K) using the following equation:

$$J_{el} = \frac{K^2}{E'} \quad (7.3)$$

where $E' = E$ and $E' = E/(1-\nu^2)$ for plane stress and plane strain conditions with E and ν representing the elastic modulus and the Poisson's ratio, respectively. The plastic term, J_{pl} , represents a situation where elastic strains are negligible, thus Eq. (7.1) can be simplified into the following form:

$$\frac{\varepsilon_{pl}}{\varepsilon_y} = \alpha \left(\frac{\sigma}{\sigma_y} \right)^n \quad (7.4)$$

where ε_{pl} is the plastic strain. Based on the above pure power law and the “HRR” singularity proposed by Hutchinson, Rice and Rosengren [122,123,136], J_{pl} can be written in terms of the applied load (P),

$$J_{pl} = \alpha \varepsilon_y \sigma_y L h_1 \left(\frac{P}{P_0} \right)^{n+1} \quad (7.5)$$

where L is the characteristic length dimension for the structure studied, P_0 denotes the reference load, h_1 is the dimensionless parameter depending on crack size, structure geometry and hardening exponent, and this has been tabulated for some simple configurations. However, the uniaxial stress-strain curve for many materials deviates considerably from the power law due to the presence of flow behavior. In this case, applying the EPRI J -estimation procedure [117,136] for fracture assessment results in significant errors. Therefore, a reference stress J -

estimation scheme is developed by Ainsworth [16] to reflect more closely the flow behavior of materials by defining a reference stress (σ_{ref}) as follows:

$$\frac{\sigma_{\text{ref}}}{\sigma_y} = \frac{P}{P_0} \quad (7.6)$$

Substituting Eq. (7.6) into Eq. (7.2), making reasonable approximations and ignoring the minor plasticity correction term, which has a marginal influence in the elastic region and in the fully plastic region, one can obtain

$$J = J_{\text{el}} \left(\frac{E \varepsilon_{\text{ref}}}{\sigma_{\text{ref}}} \right) \quad (7.7)$$

where ε_{ref} is the reference strain corresponding to the reference stress (σ_{ref}) on the stress-strain curve. However, the stress-based assessment strategy mentioned in the above [16,117,136] produces conservatism for configurations under displacement- or strain-controlled loading deforming into the plastic domain. Hence, the interest in strain-based methods has been growing for particular applications, and the reference strain J -estimation methodology [29] is attained according to Eq. (7.7),

$$J = F^2 \pi a \sigma_{\text{nom}} \varepsilon_{\text{nom}} \left(\frac{\varepsilon_{\text{ref}} / \sigma_{\text{ref}}}{\varepsilon_{\text{nom}} / \sigma_{\text{nom}}} \right) \quad (7.8)$$

where F is the dimensionless constant that depends on the structure geometry and the mode of loading, σ_{nom} and ε_{nom} are the applied nominal stress and strain, respectively, a is the characteristic crack dimension. Linkens et al. [28] replace σ_{nom} and ε_{nom} in Eq. (7.8) using the uncracked-body equivalent stress (σ_{uc}) and strain (ε_{uc}), and a factor of 2 is introduced to compensate for some simplifying assumptions. Then, a very straightforward reference strain expression is obtained,

$$J = 2F^2 \pi a \sigma_{\text{uc}} \varepsilon_{\text{uc}} \quad (7.9)$$

The reference strain formulation shown in Eq. (7.9) is a very attractive alternative for reeled pipelines subjected to strain-controlled loading, and it can qualitatively capture the evolution trend of fracture response parameter (J) with the variation of uncracked strain (ε_{uc}). However, the quantitative characterization is not sufficiently accurate. In this study, a more accurate

reference strain J -estimation formulation is developed based on Eq. (7.9) and the research work conducted by Nourpanah and Taheri [115] to predict the fracture responses of misaligned clad pipes containing a circumferential part-through interface crack as depicted in Fig. 7.2 is given by

$$J = \sigma_y b (f_1 \varepsilon_{uc} + f_2) \quad (7.10)$$

where b is the uncracked ligament length in the pipe thickness direction; it is defined as $b = t - a$ with t and a being the pipe thickness and crack depth, respectively. In this formulation, $\sigma_y b$ is used instead of $\sigma_{uc} a$ as a characteristic parameter to nondimensionalize the fracture response parameter J , and the validity of the characteristic parameter is verified in Sub-section 7.2.5.1. f_1 and f_2 are expressions associated with pipe geometry, weld misalignment, crack size and material properties, and a series of parametric FE analyses are carried out to determine the final values of f_1 and f_2 . Various parameters affecting f_1 and f_2 are listed in Sub-section 7.2.3.1, and a more detailed expression form for Eq. (7.10) is also illustrated in this subsection.

Fig. 7.2 Details of the welded clad pipe with centreline offset weld misalignment – equal diameters ($D_{e1} = D_{e2} = D_e$) and a circumferential part-through crack is contained at the interface of weld metal and outer pipe.

7.3 FINITE ELEMENT ANALYSES OF CLAD PIPELINES

7.3.1 Details of pipe parameters and material properties

In this study, a welded clad pipe with centerline offset weld misalignment and containing a circumferential part-through interface crack subjected to axial loading is investigated, and the geometric configuration and parameter definitions are shown in Fig. 7.2. The analysis matrix considers the cracked pipe with constant wall thickness $t = 20.6$ mm, and two different values

for D_e/t are adopted: $D_e/t = 10$ and 20 . The CRA layer thickness of $t_{CRA} = 3$ mm is applied, which is representative typical value for a commonly-used layer thickness encountered in practice. The crack is modelled as a “canoe” shape with its fillet radius equal to the crack depth (a), and the crack depth (a) used herein is defined as the vertical projection depth of the inclined crack along the pipe thickness direction. The non-dimensional parameters, crack depth ratio (a/t) and crack length ratio (θ/π), are employed to describe the crack geometry with values varying from 0.05 to 0.20. The weld geometry is represented by the weld root opening width (h) and weld groove angle (ϕ). For the weld root opening width, the value of h/t used in this analysis is 0.3, and the weld groove angle of 45° is considered, signifying the typical weld profile value found in real application. The centreline offset of the pipe section at the welded joint is denoted by e . Due to a fabrication tolerance $e/t = 0.15$ being frequently used in fabrication standards for offshore structures [33], e/t values of 0.00, 0.05, 0.10 and 0.15 are employed in the present study. Table 7.1 presents the detailed analysis matrix for various parameters.

Table 7.1 Table of analysis matrix

D_e/t	h/t	ϕ	e/t	a/t	θ/π
10	0.3	45°	0.00	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.05	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.10	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.15	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
20	0.3	45°	0.00	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.05	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.10	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20
			0.15	0.05,0.10,0.15,0.20	0.05,0.10,0.15,0.20

The welded clad pipe consists of three distinct materials (Fig. 7.2): outer carbon steel, inner CRA material and weld metal. The narrow heated affected zone (HAZ) is not considered since the HAZ width is much smaller than the uncracked ligament length (b) and weld width, inducing a minimal effect on the fracture responses. Furthermore, as the HAZ is typically over-matched in comparison with surrounding constituents, viz., outer carbon steel and weld metal [137], ignoring the HAZ produces a conservative result for critical defect prediction, which is beneficial for fracture assessment of offshore pipelines. The clad pipe segments are usually welded together using a filler material such as nickel-chromium alloy 625 (UNS N06625)

which eventually has the same mechanical properties and metallurgical composition as the inner CRA layer [119]. Hence, the identical material properties are employed for the CRA layer and the weld metal. The above materials are all assumed to follow the Ramberg-Osgood model [22,136] with $\alpha = 1$ as illustrated in Eq. (1). The FE analyses are carried out using deformation plasticity and small geometry change (SGC) setting. The weld strength mismatch factor (M_y) is usually defined as the ratio of the yield stress between the weld metal (σ_y^w) and the outer carbon steel (σ_y^{out}), and it is expressed as

$$M_y = \frac{\sigma_y^w}{\sigma_y^{\text{out}}} \quad (7.11)$$

The current FE analyses consider three mismatch levels: 20% under-match ($M_y = 0.8$), even-match ($M_y = 1.0$) and 25% over-match ($M_y = 1.25$), covering a typical range of weld mismatch levels found in practice. The yield stress (σ_y^{out}) and strain hardening exponent (n_{out}) of the outer carbon steel are 483 MPa and 12, respectively, which is the material properties of an API X60 pipeline grade steel. Therefore, the corresponding properties of the weld metal for different mismatch levels can be calculated in terms of Eq. (7.11) and they are listed in Table 7.2 with $E = 206$ GPa and $\nu = 0.3$ for all the FE simulation cases. All the non-dimensional parameters describing pipe geometry, crack profile and material property are summarized as D_e/t , e/t , a/t , θ/π , M_y and n_w . Then, the proposed reference strain J -estimation formulation in Eq. (7.10) is detailed as

$$\frac{J}{\sigma_y^{\text{out}} b} = f_1(D_e/t, a/t, \theta/\pi, e/t, M_y, n_w) \varepsilon_{\text{uc}} + f_2(D_e/t, a/t, \theta/\pi, e/t, M_y, n_w) \quad (7.12)$$

Table 7.2 Material properties for the welded clad pipes used in current FE analyses

M_y	Outer carbon steel			Weld metal	
	σ_y^{out} (MPa)	n_{out}		σ_y^w (MPa)	n_w
0.8	483	12		386.4	6
					9
					12
1.0	483	12		483	12
1.25	483	12		603.8	12
					15
					18

7.3.2 Numerical procedures and mesh refinement issues

As stated in the above, various parameters are included in the current FE analyses to determine expressions of f_1 and f_2 , and then to propose the reference strain J -estimation formulation presented by Eq. (7.12). A total of 896 3-D FE models are analyzed using the ABAQUS Version 6.11 software [51]. Therefore, a FE mesh generator is specially developed to generate the FE models of misaligned clad pipe containing a circumferential part-through interface crack. Fig. 7.3 depicts a typical FE mesh for this configuration with $D_e/t = 10$, $a/t = 0.2$, $\theta/\pi = 0.1$ and $e/t = 0.1$. The boundary conditions for the misaligned clad pipe are shown in Fig. 7.4. Due to symmetry of the model analysed, a symmetric boundary condition about Y-axis is applied to the surface at $Y = 0$. The displacements in X-axis and Z-axis directions on the left end of the pipe are constrained to prevent rigid body movements along the corresponding directions. The right end is constrained in the Z-axis direction, and a tensile displacement loading (u) is applied to this end surface providing the loading condition. The relationship between u and ε_{uc} is expressed as

$$\varepsilon_{uc} = \frac{u}{L} \quad (7.13)$$

and ε_{uc} is set up to 3% [36,134] in this study. The pipe length used is equal to six times of the outer diameter ($L = 6D_e$), and exploratory numerical analyses have verified that the adopted pipe length is long enough to have a marginal effect on the J values extracted from the FE simulation.

Fig. 7.3 Typical FE model for the misaligned clad pipe with $D_e/t = 10$, $a/t = 0.2$, $\theta/\pi = 0.1$ and $e/t = 0.1$.

Fig. 7.4 Boundary conditions of the misaligned clad pipe.

In these FE models, the element type used is 20-node brick element with reduced integration (C3D20R), accounting for compatibility and convergent issues. In order to accommodate the large plastic deformation, a focused spider web mesh along the crack front is adopted with a small key-hole geometry (blunt notch) at the crack tip, and the small initial root radius (r_0) is set to 0.001. The crack tip is surrounded by 12 circumferential layers to provide distances sufficiently far from the crack tip so as to obtain path-independent J values (Fig. 7.5). The J value at the crack deepest point, where the maximum J occurs, is determined from the average value of 2nd-12th contours, and the J for the first contour is ignored due to the numerical errors. Confidence in the FE mesh strategies is attained from the path independence of the J by considering that the difference of the J values for these 11 contours is negligible as shown in Fig. 7.6. Then, the evolution of the J -integral with ϵ_{uc} for varying blunt notch radius (r_0) is investigated to further verify the adequacy of the near-tip mesh refinement employed in the parametric study. Fig. 7.7 illustrates the effect of varying blunt notch radius, $r_0 = 0.001$ mm, $r_0 = 0.005$ mm and $r_0 = 0.01$ mm, on J values with $a/t = 0.10$, $\theta/\pi = 0.10$, $e/t = 0.15$ and different strength mismatch levels. It is noted that there are essentially no differences between these curves for the entire range of displacement loading, thus, it demonstrates convincingly the accuracy of the mesh refinement and FE modelling. Moreover, mesh convergent tests covering the range of parameters listed in Table 7.1 and Table 7.2 are performed to determine the adequacy of number of elements by varying it from 20,412 to 31,752 according to the crack size and pipe geometry; they are not listed here. The above sensitivity analyses provide confidence in the present FE study in obtaining accurate J -integral values.

Fig. 7.5 Crack profile details at the interface of weld metal and outer pipe.

Fig. 7.6 Path independence of the numerical values of the J -integral for $\epsilon_{uc} = 3\%$.

Fig. 7.7 Influence of varying blunt notch radius on J -integral values for $a/t = 0.10$, $\theta/\pi = 0.10$ and $e/t = 0.15$.

7.4 RESULTS AND DISCUSSION

7.4.1 Effect of various parameters on $J/(\sigma_y^{\text{out}} b)$ values

The numerical procedure on how to determine the factors f_1 and f_2 is introduced before proposing the reference strain J -estimation formulation presented in Eq. (7.12). Fig. 7.8 shows the evolution curve of $J/(\sigma_y^{\text{out}} b)$ with ϵ_{uc} for $D_g/t = 10$, $a/t = 0.15$, $\theta/\pi = 0.20$, $e/t = 0.15$, $M_y = 0.8$ and $n_w = 9$. The factor f_1 can be determined directly by measuring the slope of this curve, and the intercept at ordinate is used to define the factor f_2 . The same estimation procedure mentioned in the above is adopted to evaluate the f_1 and f_2 factors for various parameter combinations illustrated in Table 7.1 and Table 7.2, and eventually, a series of f_1 and f_2 values changing with D_g/t , a/t , θ/π , e/t , M_y and n_w are obtained. It is noted from Fig. 7.8 that $J/(\sigma_y^{\text{out}} b)$ varies nonlinearly with ϵ_{uc} at the initial range of the applied displacement loading, which is called a short transient region prior to the fully plastic stress state. This phenomenon is also observed for all the other parameter combinations. However, a linear relationship between $J/(\sigma_y^{\text{out}} b)$ and ϵ_{uc} clearly holds

true in the range of 0.5% to 3% for all the cases. Hence, the f_1 and f_2 values proposed in this study are available in this range of ϵ_{uc} .

Fig. 7.8 Determination of factors f_1 and f_2 using the slope of the fitted line to the evolution of $J/(\sigma_y^{out} b)$ with ϵ_{uc} .

Figs. 7.9-7.11 present the effect of various parameters (a/t , θ/π , e/t , M_y and n_w) on J -integral for the misaligned clad pipe with a circumferential interface crack. As shown in Fig. 7.9, $J/(\sigma_y^{out} b)$ displays a strong sensitivity to a/t , and a larger a/t induces a larger $J/(\sigma_y^{out} b)$ value. It is because the crack opening deformation is greater for a deeper crack as the same displacement loading is applied. However, the increase tendency of $J/(\sigma_y^{out} b)$ with θ/π is less noticeable in comparison with the case of a/t , and the effect of θ/π becomes marginal beyond $\theta/\pi = 0.15$, which is due to the crack depth restraining further crack deformation for a wider crack. The $J/(\sigma_y^{out} b)$ values increase with increasing e/t through the whole range of ϵ_{uc} as depicted in Fig. 7.10(a) since the additional bending moment induced by weld misalignment is proportional to e/t [33,54]. Fig. 7.10(b) illustrates the evolution tendency of $J/(\sigma_y^{out} b)$ with e/t for varying a/t and constant ϵ_{uc} ($= 3\%$) to clarify directly the effect of e/t on J -integral values. It is noted that a nearly linear dependence relationship between $J/(\sigma_y^{out} b)$ and e/t is observed for shallow cracks ($a/t = 0.05$ and 0.10), while $J/(\sigma_y^{out} b)$ varies with e/t in a parabolic relationship as the crack gradually deepens. The comparison results depicted in Fig. 7.11 reveal that the mismatch level (M_y) has an appreciable influence on $J/(\sigma_y^{out} b)$ values, and under-match cases ($M_y = 0.8$) cause larger $J/(\sigma_y^{out} b)$ values in comparison with over-match cases ($M_y = 1.25$). For the under-match results ($M_y = 0.8$), $J/(\sigma_y^{out} b)$ presents a strong dependence on the strain hardening exponent (n_w) of the weld metal with larger n_w resulting in larger $J/(\sigma_y^{out} b)$. For the over-match cases ($M_y = 1.25$), even

though the overall trend between $J/(\sigma_y^{\text{out}} b)$ and n_w remains similar with the under-match cases, n_w displays insignificant effect on $J/(\sigma_y^{\text{out}} b)$ by considering the negligible difference of $J/(\sigma_y^{\text{out}} b) - \epsilon_{uc}$ curves for varying n_w . It is worth noting that the above evolution trends exist for all sets of analyses included in the parametric study.

Fig. 7.9 Evolution of $J/(\sigma_y^{\text{out}} b)$ with ϵ_{uc} : (a) for varying a/t and (b) varying θ/π for the pipe model with $D_e/t = 10$, $e/t = 0.10$, $M_y = 0.8$ and $n_w = 9$.

Fig. 7.10 (a). Effect of e/t on J -integral values for the pipe model with $D_e/t = 10$, $a/t = 0.10$, $\theta/\pi = 0.10$, $M_y = 0.8$ and $n_w = 9$; (b) Evolution of $J/(\sigma_y^{\text{out}} b)$ with e/t for the pipe model with $D_e/t = 10$, $\theta/\pi = 0.10$, $M_y = 0.8$ and $n_w = 9$ at $\epsilon_{uc} = 3\%$.

Fig. 7.11 Distribution of $J/(\sigma_y^{\text{out}} b)$ with different M_y and n_w values for the pipe model with $D_e/t = 10$, $a/t = 0.10$, $\theta/\pi = 0.10$ and $e/t = 0.10$.

7.4.2 Determination of reference strain J -estimation formulation in Eq. (7.12)

In Sub-section 7.4.1, the dependence of $J/(\sigma_y^{\text{out}} b)$ on various parameters (a/t , θ/π , e/t , M_y and n_w) for the misaligned clad pipe with a circumferential interface crack is studied in detail. The aim of this subsection is to determine the reference strain J -estimation formulation shown in Eq. (7.12) to provide a simpler result manipulation by proposing the expression relationships of the factors f_1 and f_2 with these parameters analysed. The expression for f_1 and f_2 are constructed as

$$f_1 = A_1 \left(\frac{a}{t} \right)^{A_2 \left(\frac{e}{t} \right) + A_3} + A_4 \left(\frac{e}{t} \right) + A_5 \quad (7.14)$$

$$f_2 = \left[B_1 \left(\frac{e}{t} \right) + B_2 \right] \left(\frac{a}{t} \right)^2 + \left[B_3 \left(\frac{e}{t} \right) + B_4 \right] \left(\frac{a}{t} \right) + B_5 \quad (7.15)$$

where A_1 to A_5 and B_1 to B_5 are dependent on D_e/t , θ/π , M_F and n_w , and these coefficients obtained by a standard least square fitting are tabulated in Tables B1 and B2 of Appendix B. Substituting Eqs. (7.14) and (7.15) into Eq. (7.12) obtains the required reference strain J -estimation formulation. As a total of 896 FE calculation cases are included in the parametric study, the goodness of fit of the estimation formulation needs to be verified due to its complexity. Fig. 7.12 shows the comparison of $J/(\sigma_y^{\text{out}} b)$ values obtained from the estimation

formulation and FE analyses for varying mismatch levels and pipe diameters, and a very good agreement between the two sets of results is observed. More comparison tests covering the range of parameters listed in Tables B1 and B2 have been conducted to validate the availability of the estimation formulation.

Fig. 7.12 Comparison of $J/(\sigma_y^{\text{out}} b)$ values obtained from the proposed estimation formulation and FE analysis for $a/t = 0.10$, $\theta/\pi = 0.10$, $e/t = 0.10$.

7.5 FURTHER DISCUSSION FOR APPLICATION RANGE OF THE ESTIMATION FORMULATION

7.5.1 Effect of yield strength of outer carbon steel on $J/(\sigma_y^{\text{out}} b)$

The reference strain J -estimation formulation shown in Eq. (7.12) for the misaligned clad pipe containing a circumferential interface crack is established. The J -integral in the formulation is nondimensionalized using $\sigma_y^{\text{out}} b$, which is the yield stress of API X60 pipeline grade steel and specified to be 483 MPa in the parametric study. However, it is skeptical whether $\sigma_y^{\text{out}} b$ can be regarded as a characteristic quantity to nondimensionalize the J -integral results. Fig. 7.13 illustrates the effect of varying σ_y^{out} on $J/(\sigma_y^{\text{out}} b)$ values by considering 20% lower σ_y^{out} (386.4 MPa) and 20% higher σ_y^{out} (579.6 MPa) than the original σ_y^{out} (483 MPa), respectively. The J -integral results calculated for the three different σ_y^{out} values are nondimensionalized by the respective $\sigma_y^{\text{out}} b$. It is noted that nearly the same dimensionless quantities $J/(\sigma_y^{\text{out}} b)$ are obtained, indicating that varying σ_y^{out} has a negligible influence on $J/(\sigma_y^{\text{out}} b)$ values through the ε_{uc} range of 0.5% to 3.0%. Hence, $\sigma_y^{\text{out}} b$ is considered to be a suitable as a characteristic quantity of the dimensionless operation. The above analysis is available for all sets of analyses included in this study even though Fig. 7.13 just outlines the comparison results

for $D_e/t = 10$, $a/t = 0.10$, $\theta/\pi = 0.10$ and $e/t = 0.10$ with different values of M_y and n_w . Therefore, the proposed reference strain J -estimation formulation shown in Eq. (7.12) can be applicable for a wide range of σ_y^{out} values, not just limited to $\sigma_y^{\text{out}} = 483$ MPa.

Fig. 7.13 Effect of varying σ_y^{out} on $J/(\sigma_y^{\text{out}} b)$ results for the pipe model with $D_e/t = 10$, $a/t = 0.10$, $\theta/\pi = 0.10$ and $e/t = 0.10$.

7.5.2 Effect of weld groove angle on $J/(\sigma_y^{\text{out}} b)$ values

As stated previously, the reference strain J -estimation formulation in Eq. (7.12) is built up for the misaligned clad pipe with weld groove angle, $\phi = 45^\circ$. The objective of this subsection is to investigate the effect of ϕ on $J/(\sigma_y^{\text{out}} b)$ values so as to extend the application range of the proposed estimation formulation (Fig. 7.14). The comparison study considers the crack profile from a smallest one ($a/t = 0.05$ and $\theta/\pi = 0.05$) to a largest one ($a/t = 0.20$ and $\theta/\pi = 0.20$) with two different D_e/t values ($D_e/t = 10$ and 20), and the mismatch level (M_y) and strain hardening exponent (n_w) used cover the range listed in Table 7.2. The ϕ values varying from 30° to 60° are analysed in this study, signifying the commonly-used groove angles encountered in welded pipelines. It is observed that the $J/(\sigma_y^{\text{out}} b)$ values for $\phi = 45^\circ$ are always larger than the values calculated for $\phi = 60^\circ$, but smaller than most of the values for $\phi = 30^\circ$. It is because a larger ϕ causes a smaller angle between the crack face and the applied tensile stress as indicated in Fig. 7.15, thus resulting in a smaller effective stress in opening the crack when a same tensile loading is applied. Hence, the reference strain J -estimation formulation proposed for $\phi = 45^\circ$ can make a reasonable and reliable prediction of $J/(\sigma_y^{\text{out}} b)$ for the ϕ ranging from 45° to 60° with the largest over-estimation percentage difference being only 7.8%. However, the estimation

formulation is inapplicable in the range of $\phi = 30^\circ$ to $\phi = 45^\circ$ since it underestimates the corresponding $J/(\sigma_y^{\text{out}} b)$ values.

Fig. 7.14 Effect of ϕ on $J/(\sigma_y^{\text{out}} b)$ results for $\epsilon_{uc} = 3\%$ with the variation of various parameters included in the study.

Fig. 7.15 Schematic view of the angle between the crack face and the applied tensile stress.

7.5.3 Effect of weld misalignment type and loading condition on $J/(\sigma_y^{\text{out}} b)$

Two types of centreline offset weld misalignment, misalignment with equal diameters (Fig. 7.16(a)) and misalignment with unequal diameters (Fig. 7.16(b)), are investigated in this subsection. In order to illustrate the reference strain J -estimation formulation proposed for the misaligned clad pipelines with equal diameters equally applicable for the pipelines with unequal diameters, Fig. 7.17 provides the comparison of $J/(\sigma_y^{\text{out}} b)$ values for these two types of weld

misalignment with varying mismatch levels. These analyses consider the pipe configuration with $D_e/t = 10$, $a/t = 0.10$, $\theta/\pi = 0.10$, $e/t = 0.10$ and different mismatch levels ($M_F = 0.8$, 1.0 and 1.25) by including varying strain hardening exponents, n_w . The $J/(\sigma_y^{\text{out}}b)$ values obtained for these two types of weld misalignment are consistent with one another for the entire range of ϵ_{uc} with the largest percentage difference being only 1.1%, and more comparative studies covering widely different crack and pipe dimensions further confirm the consistency. Hence, the proposed J -estimation formulation can be well suited for the misaligned pipelines with unequal diameters. It is worth noting that the fracture analyses mentioned in the above are only for the misaligned pipelines subjected to tensile loading. However, bending moment is a more general loading in pipelines due to the reeling installation. Hence, the misaligned pipelines under bending moment are considered, and the corresponding global strain is defined as

$$\epsilon_{uc} = \frac{D_e \beta}{2L} \quad (7.16)$$

where β is the rotation angle of the pipeline. For the pipeline case subjected to bending moment, a linear relationship between $J/(\sigma_y^{\text{out}}b)$ and ϵ_{uc} is observed, and the same $J/(\sigma_y^{\text{out}}b)$ values are obtained for these two types of weld misalignment as shown in Fig. 7.16, which are similar with the case under tensile loading. Fig. 7.18 presents the typical comparisons of $J/(\sigma_y^{\text{out}}b)$ for the pipelines subjected to tensile loading and bending moment by taking into account varying weld misalignment and mismatch levels. For all the cases, the $J/(\sigma_y^{\text{out}}b)$ values for tension are slightly larger than that for bending with the largest over-estimation percentage difference being 13.7%. This comparative study demonstrates that the J -estimation formulation developed for tension predicts reasonably conservative results for the bending case. In conclusion, the reference strain J -estimation formulation for the misaligned clad pipelines subjected to tensile loading is considered suitable for the pipelines subjected to bending moment.

Fig. 7.16 Geometry of misaligned clad pipes with centreline offset weld misalignment – (a) equal diameters and – (b) unequal diameters.

Fig. 7.17 Comparison of $J/(\sigma_y^{\text{out}} b)$ results for two different centreline offset weld misalignment.

Fig. 7.18 Comparison of $J/(\sigma_y^{\text{out}} b)$ for the pipe model subjected to tensile loading and bending moment at $\epsilon_{uc} = 3\%$.

7.5.4 A realistic stress-strain curve with a Lüders plateau

A realistic stress-strain curve with a Lüders plateau is considered since the material of outer pipe in clad pipelines is low carbon steels and most carbon steels exhibit a plateau in the curve immediately after yielding. In this case study, the Lüders plateau is assumed to extend up to a strain of 1.5% based on the research work conducted by Nourpanah and Taheri [115]. Then, a power-law model is employed to represent the strain hardening portion of the curve. Three Ramberg-Osgood fits are used to approximate the realistic stress-strain curve as shown in Fig. 7.19(a), viz., a best fit to the elastic portion (red line), a best fit to the post-yield strain hardening portion (dark cyan line) and an average fit (blue line). Fig. 7.19(b) shows the evolutions of the J with ϵ_{uc} obtained from the realistic curve and three fitting curves. It is noted that the elastic fit agrees well with the realistic curve for lower strain levels; the post-yield fit presents a good correlation for higher strain levels; an intermediate result is predicted by the average fit. These three fitting methods can reasonably characterize the fracture behaviours within a certain strain range. However, for a material with an apparent Lüders plateau in engineering practice, the elastic fit is recommended for use because it produces a conservative prediction for most of the strain range in comparison with the other two fitting methods. In addition, the CRA material Inconel 625 does not follow Ramberg-Osgood model very precisely, and a bi-linear model is more suitable. In this case, the above three fitting methods used for the outer pipe are employed

for the CRA layer, and a similar result is observed. Therefore, the elastic fit is also recommended for use in the CRA layer due to its slightly conservative prediction on the J -integral. The detailed procedure is not listed here because of the similarity and interest in saving space.

Fig. 7.19 (a) A realistic stress-strain curve with a Lüders plateau and three Ramberg-Osgood fits and (b) corresponding J - ϵ_{uc} curves.

7.5.5 Effect of ratio of pipe outer diameter to thickness on $J/(\sigma_y^{\text{out}} b)$

Fig. 7.20 Effect of D_e/t on $J/(\sigma_y^{\text{out}} b)$ for the pipe model with $a/t = 0.1$, $\theta/\pi = 0.1$, $e/t = 0.1$, $b = 18.54$ mm, $M_y = 0.8$ and $n_w = 9$.

In this study, the D_e/t analysed is in the range of 10 to 20, which is the thick pipe case and commonly employed in offshore pipelines [36-38,49,52,119]. In this section, the effect of

various D_e/t on the $J/(\sigma_y^{\text{out}}b)$ is investigated by considering it varying from 10 to 100, covering thick pipe and thin pipe cases. It is observed from Fig. 7.20 that the linear relationship between $J/(\sigma_y^{\text{out}}b)$ and ε_{uc} still holds true even for a very thin pipe. It is also noted that the $J/(\sigma_y^{\text{out}}b)$ increases with the increasing D_e/t ; the $J/(\sigma_y^{\text{out}}b)$ displays a very sharp rise for D_e/t ranging from 10 to 20; then, the rise slows down as the D_e/t becomes larger and saturates when $D_e/t = 70$. Based on the fact of the increase, the proposed J -estimation formulation in Eq. (7.12) cannot be employed for the pipe model with D_e/t larger than 20. In the follow-up research works, an estimation formulation with a wider application will be developed by incorporating the thick pipe and thin pipe cases.

7.6 SUMMARY OF THE CHAPTER

A reference strain J -estimation formulation is derived for the misaligned clad pipe containing a circumferential part-through interface crack under axial loading. A FE mesh generator is developed specially to generate the required pipe models. The validation of the mesh generator is verified using the path independence of the J -integral and large amounts of convergent tests covering the range of various parameters listed in Tables 7.1 and 7.2. Then, extensive elastic-plastic FE analyses totaling 896 FE models are conducted to investigate the influence of crack size, weld misalignment and material property on $J/(\sigma_y^{\text{out}}b)$ values. The $J/(\sigma_y^{\text{out}}b)$ values increase with the increase of crack depth ratio (a/t), crack length ratio (θ/π) and centreline offset ratio (e/t), and a stronger sensitivity on a/t and e/t is observed. The mismatch level (M_y) has a considerable influence on $J/(\sigma_y^{\text{out}}b)$ values, and under-match cases ($M_y = 0.8$) cause larger $J/(\sigma_y^{\text{out}}b)$ values as compared with over-match cases ($M_y = 1.25$). For the strain hardening exponent (n_w) of the weld metal, a larger n_w results in a larger $J/(\sigma_y^{\text{out}}b)$ for each mismatch level, and the $J/(\sigma_y^{\text{out}}b)$ presents a much stronger dependence on the n_w of under-match cases in comparison with that of over-match cases. It is worth noting that $J/(\sigma_y^{\text{out}}b)$ varies linearly with ε_{uc} in the range of 0.5% to 3% for all sets of analyses. Therefore, the reference strain J -estimation formulation is proposed in the range of $\varepsilon_{\text{uc}} = 0.5\%$ to $\varepsilon_{\text{uc}} = 3\%$, and the polynomial coefficients obtained by a standard least square fitting are tabulated in Tables B1 and B2. Finally, several comparative studies are carried out to extend the application range of the proposed formulation.

CHAPTER 8

CONCLUSIONS AND FUTURE WORKS

8.1 SUMMARY AND CONCLUSIONS

The main objective of this study is to develop fracture assessment methodologies for cracked clad pipelines with weld misalignment. Based on the research works conducted in Chapters 3-7, the main conclusions are summarized as follows:

1. A FE mesh generator is developed for aligned cracked clad pipelines with various crack locations, viz., a surface or embedded crack at the WCL and interface of the weld metal and outer pipe. Then, a coordinate transformation method is incorporated in the mesh generator to convert the FE models of aligned configurations to that of misaligned ones. A small-scale experimental test on the stress concentration curve along the longitudinal direction of misaligned steel pipes is carried out to verify the validity of the transformation method. It is concluded that the mesh generator with the coordinate transformation is found to be well suited to serve for misaligned clad pipelines.
2. BS 7910:2013 [1] proposes a single set of equations to estimate the SIFs of different types of welded joints with a semi-elliptical surface crack. In this study, extensive FE analyses are performed to determine the SIFs for a surface crack in plate-to-plate butt-welded joints and load-carrying cruciform fillet-welded joints. By comparing the SIFs obtained from FE analysis and BS 7910:2013 [1] assessment equation, it is found that the latter produces an over-conservative prediction. Therefore, a new set of SIF equations are proposed using multiple regression analysis, and the goodness of fit of these equations is demonstrated by statistical analyses and percentage error frequency histograms.
3. The fracture and fatigue behaviors of misaligned pipes with a surface crack is investigated in detail. A series of SIF equations are developed using multiple regression analysis, and the proposed equations can be employed in the fitness-for-service assessment for the commonly used pipe configurations encountered in practice.

4. An engineering integrity assessment of mismatched girth welds in oval-shaped clad pipelines subjected to bending moment is produced based on the ESSRM/EPRI estimation framework. The validation of the ESSRM/EPRI estimation framework is ascertained by comparing results obtained from 3-D elastic-plastic FE analysis.
5. A reference strain J -estimation formulation is derived for the misaligned clad pipelines containing a circumferential part-through interface crack under axial loading, and the detailed expressions are determined by extensive parametric study. The effects of the yield stress of the outer carbon steel and weld groove angle on the J -integral are investigated in detail to extend the application range of the estimation formulation. The comparison study for the misaligned clad pipelines with equal diameters and unequal diameters is carried out to confirm that the estimation formulation aiming for the former one can be directly applicable to the latter one.

8.2 FUTURE WORKS

This study has accomplished the main objectives. However, several future works should be extended and preliminary as follows:

1. In this study, the effect of weld misalignment on the fracture responses of clad pipelines with a surface crack is investigated, and the fracture assessment methodology is developed. However, embedded cracks may also be introduced into the welds during construction [46,48]. Hence, the fracture assessment for the misaligned clad pipelines with an embedded crack needs to be carried out in the future works.
2. Interaction between coplanar cracks induces a magnification of SIF [15]. Hence, the research works conducted in this study can be extended for the fracture responses of misaligned clad pipelines with multiple cracks.
3. For lined pipelines, two failure modes need to be considered, viz., fracture and buckling, which is more complicated than clad pipelines. The related investigations for lined pipelines can be conducted in the future works. More importantly, the determination of the critical locus between the two failure modes will be a meaningful piece of research work.

4. Apart from fracture and fatigue analysis, the dynamic behavior of pipelines is of considerable importance in many engineering fields [138-140]. The effect of weld misalignment on the dynamic responses of pipelines should also be investigated.

REFERENCES

- [1] BS7910-Amendment 1, (2013), "Guide to methods for assessing the acceptability of flaws in metallic structures," British Standards Institution, UK.
 - [2] Kyriakides, S., Corona, E., (2007), "Mechanics of offshore pipelines. buckling and collapse," Elsevier, Oxford, UK.
 - [3] Tvergaard, V., (1983), "Plastic buckling of axially compressed circular cylindrical shells," *Thin-Walled Structures*, Vol. 1, No. 2, pp. 139-163.
 - [4] Corona, E., Kyriakides, S., (1988), "On the collapse of inelastic tubes under combined bending and pressure," *International Journal of Solids and Structures*, Vol. 24, No. 5, pp. 505-535.
 - [5] Ju, G.T., Kyriakides, S., (1992), "Bifurcation and localization instabilities in cylindrical shells under bending-II. predictions," *International Journal of Solids and Structures*, Vol. 29, No. 9, pp. 1143-1171.
 - [6] Kyriakides, S., Ju, G.T., (1992), "Bifurcation and localization instabilities in cylindrical shells under bending-I. experiments," *International Journal of Solids and Structures*, Vol. 29, No. 9, pp. 1117-1142.
 - [7] Bardi, F.C., Kyriakides, S., (2006), "Plastic buckling of circular tubes under axial compression---part I: experiments," *International Journal of Mechanical Sciences*, Vol. 48, No. 8, pp. 830-841.
 - [8] Bardi, F.C., Kyriakides, S. and Yun, H.D., (2006), "Plastic buckling of circular tubes under axial compression---part II: analysis," *International Journal of Mechanical Sciences*, Vol. 48, No. 8, pp. 842-854.
 - [9] Paquette, J.A., Kyriakides, S., (2006), "Plastic buckling of tubes under axial compression and internal pressure," *International Journal of Mechanical Sciences*, Vol. 48, No. 8, pp. 855-867.
 - [10] Limam, A., Lee, L.H., Corona, E. and Kyriakides, S., (2010), "Inelastic wrinkling and collapse of tubes under combined bending and internal pressure," *International Journal of Mechanical Sciences*, Vol. 52, No. 5, pp. 637-647.
 - [11] Irwin, G.R., (1957), "Analysis of stresses and strains near the end of a crack traversing a plate," *Journal of Applied Mechanics*, Vol. 24, No. 3, pp. 361-364.
 - [12] Wells, A.A., (1961), "Unstable crack propagation in metals: cleavage and fast fracture," *Proceedings of Crack Propagation Symposium*, Cranfield, UK, pp. 210-230.
 - [13] Cottrell, A.H., (1961), "Theoretical aspects of radiation damage and brittle fracture in steel pressure vessels," ISI Special Report No. 69, Iron and Steel Institute, UK.
 - [14] Rice, J.R., (1968), "A path independent integral and the approximate analysis of strain concentration by notches and cracks," *Journal of Applied Mechanics*, Vol. 35, No. 2, pp. 379-386.
 - [15] Anderson, T.L., (2005), "Fracture mechanics: fundamentals and applications," CRC Press.
-

-
- [16] Ainsworth, R.A., (1984), "The assessment of defects in structures of strain hardening materials," *Engineering Fracture Mechanics*, Vol. 19, No. pp. 633-642.
- [17] Shih, C.F., (1981), "Relationship between the J -integral and the crack opening displacement for stationary and extending cracks," *Journal of the Mechanics and Physics of Solids*, Vol. 29, No. 4, pp. 305-326.
- [18] Kumar, V., German, M.D. and Shih, C.F., (1981), "An engineering approach for elastic-plastic fracture analysis," EPRI Report NP-1931, Electric Power Research Institute, Palo Alto, CA.
- [19] Kumar, V., German, M.D., Wilkening, W.W., Andrews, W.R., deLorenzi, H.G. and Mowbray, D.F., (1984), "Advances in elastic-plastic fracture analysis," EPRI Report NP-3607, Electric Power Research Institute, Palo Alto, CA.
- [20] Kumar, V., German, M.D., (1988), "Elastic-plastic fracture analysis of through-wall and surface flaws in cylinders," EPRI Report NP-5596, Electric Power Research Institute, Palo Alto, CA.
- [21] Zahoor, A., (1989), "Ductile fracture handbook, volume 1: circumferential throughwall cracks," EPRI Report NP-6301-D, Electric Power Research Institute, Palo Alto, CA.
- [22] Ramberg, W., Osgood, W.R., (1943), "Description of stress-strain curves by three parameters," Technical Note No. 902. National Advisory Committee for Aeronautics, Washington DC.
- [23] Rice, J.R., Rosengren, G.F., (1968), "Plane strain deformation near a crack tip in a power-law hardening material," *Journal of the Mechanics and Physics of Solids*, Vol. 16, No. 1, pp. 1-12.
- [24] Hutchinson, J.W., (1968), "Singular behavior at the end of a tensile crack tip in a hardening material," *Journal of the Mechanics and Physics of Solids*, Vol. 16, No. 1, pp. 13-31.
- [25] Barenblatt, G.I., (1962), "The mathematical theory of equilibrium cracks in brittle fracture," *Advances in Applied Mechanics*, Vol. 7, No. pp. 55-129.
- [26] Dugdale, D.S., (1960), "Yielding in steel sheets containing slits," *Journal of the Mechanics and Physics of Solids*, Vol. 8, No. 2, pp. 100-104.
- [27] Schwalbe, K.H., (1994), "The crack tip opening displacement and J integral under strain control and fully plastic conditions estimated by the engineering treatment model for plane stress tension," ASTM STP, 1207. Philadelphia: ASTM.
- [28] Linkens, D., Formby, C.L. and Ainsworth, R.A., (2000), "A strain-based approach to fracture assessment-example applications," In: Proceedings of the 5th international conference on engineering structural integrity assessment, Churchill College, Cambridge, UK.
- [29] Budden, P.J., (2006), "Failure assessment diagram methods for strain-based fracture," *Engineering Fracture Mechanics*, Vol. 73, No. 5, pp. 537-552.
- [30] Maddox, S.J., (1985), "Fitness-for-purpose assessment of misalignment in transverse butt welds subject to fatigue loading," Welding Institute Research Report 279.
-

-
- [31] Andrews, R.M., (1996), "The effect of misalignment on the fatigue strength of welded cruciform joints," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 19, No. 6, pp. 755-768.
- [32] Cui, W.C., Wan, Z.Q. and Mansour, A.E., (1999), "Stress concentration factor in plates with transverse butt-weld misalignment," *Journal of Constructional Steel Research*, Vol. 52, No. 2, pp. 159-170.
- [33] Lotsberg, I., (2009), "Stress concentrations due to misalignment at butt welds in plated structures and at girth welds in tubulars," *International Journal of Fatigue*, Vol. 31, No. 8, pp. 1337-1345.
- [34] Brabin, T.A., Christopher, T. and Rao, B.N., (2010), "Finite element analysis of cylindrical pressure vessels having a misalignment in a circumferential joint," *International Journal of Pressure Vessels and Piping*, Vol. 87, No. 4, pp. 197-201.
- [35] API 579-1/ASME FFS-1, (2007), "Fitness for service," American Petroleum Institute (API), USA.
- [36] Yi, D., Idapalapati, S., Xiao, Z.M. and Kumar, S.B., (2012), "Fracture capacity of girth welded pipelines with 3D surface cracks subjected to biaxial loading conditions," *International Journal of Pressure Vessels and Piping*, Vol. 92, No. pp. 115-126.
- [37] Jayadevan, K.R., Østby, E. and Thaulow, C., (2004), "Fracture response of pipelines subjected to large plastic deformation under tension," *International Journal of Pressure Vessels and Piping*, Vol. 81, No. 9, pp. 771-783.
- [38] Østby, E., Jayadevan, K.R. and Thaulow, C., (2005), "Fracture response of pipelines subject to large plastic deformation under bending," *International Journal of Pressure Vessels and Piping*, Vol. 82, No. 3, pp. 201-215.
- [39] Cravero, S., Ruggieri, C., (2006), "Structural integrity analysis of axially cracked pipelines using conventional and constraint-modified failure assessment diagrams," *International Journal of Pressure Vessels and Piping*, Vol. 83, No. 8, pp. 607-617.
- [40] Berg, E., Østby, E., Thaulow, C. and Skallerud, B., (2008), "Ultimate fracture capacity of pressurised pipes with defects – comparisons of large scale testing and numerical simulations," *Engineering Fracture Mechanics*, Vol. 75, No. 8, pp. 2352-2366.
- [41] Netto, T., Lourenco, M. and Botto, A., (2008), "Fatigue performance of pre-strained pipes with girth weld defects: full-scale experiments and analyses," *International Journal of Fatigue*, Vol. 30, No. 5, pp. 767-778.
- [42] Netto, T.A., Botto, A. and Lourenco, M.I., (2008), "Fatigue performance of pre-strained pipes with girth weld defects: local deformation mechanisms under bending," *International Journal of Fatigue*, Vol. 30, No. 6, pp. 1080-1091.
- [43] Tkaczyk, T., O'Dowd, N.P. and Nikbin, K., (2009), "Fracture assessment procedures for steel pipelines using a modified reference stress solution," *Journal of Pressure Vessel Technology*, Vol. 131, No. 3, pp. 031409.
- [44] Nourpanah, N., Taheri, F., (2011), "A numerical study on the crack tip constraint of pipelines subject to extreme plastic bending," *Engineering Fracture Mechanics*, Vol. 78, No. 6, pp. 1201-1217.
-

-
- [45] Paredes, M., Ruggieri, C., (2015), "Engineering approach for circumferential flaws in girth weld pipes subjected to bending load," *International Journal of Pressure Vessels and Piping*, Vol. 12, No. pp. 49-65.
- [46] Yi, D., Xiao, Z.M., Idapalapati, S. and Kumar, S.B., (2012), "Fracture analysis of girth welded pipelines with 3D embedded cracks subjected to biaxial loading conditions," *Engineering Fracture Mechanics*, Vol. 96, No. pp. 570-587.
- [47] Zhang, Y.M., Xiao, Z.M. and Zhang, W.G., (2013), "On 3-D crack problems in offshore pipeline with large plastic deformation," *Theoretical and Applied Fracture Mechanics*, Vol. 67-68, No. pp. 22-28.
- [48] Zhang, Y.M., Yi, D.K., Xiao, Z.M., Huang, Z.H. and Kumar, S.B., (2013), "Elastic–plastic fracture analyses for pipeline girth welds with 3D semi-elliptical surface cracks subjected to large plastic bending," *International Journal of Pressure Vessels and Piping*, Vol. 105-106, No. pp. 90-102.
- [49] Souza, R.F., Ruggieri, C., (2017), "Development of a strain based fracture assessment procedure for undermatched pipe girth welds subjected to bending," *Theoretical and Applied Fracture Mechanics*, Vol. 92, No. pp. 381-393.
- [50] Macdonald, K.A., Cheaitani, M., (2010), "Engineering critical assessment in the complex girth welds of clad and lined linepipe materials," In: Proceedings of the 8th international pipeline conference (IPC 2010), Calgary, Alberta, Canada, pp. 823-843.
- [51] ABAQUS, (2011), "Standard user's manual, version 6.11," Hibbett, Karlsson & Sorensen Inc, USA.
- [52] Chiodo, M.S.G., Ruggieri, C., (2010), "*J* and CTOD estimation procedure for circumferential surface cracks in pipes under bending," *Engineering Fracture Mechanics*, Vol. 77, No. 3, pp. 415-436.
- [53] Lotsberg, I., (1998), "Stress concentration factors at circumferential welds in tubulars," *Marine Structures*, Vol. 11, No. pp. 203-230.
- [54] Lotsberg, I., (2008), "Stress concentration factors at welds in pipelines and tanks subjected to internal pressure and axial force," *Marine Structures*, Vol. 21, No. 2-3, pp. 138-159.
- [55] DNV-RP-C203, (2011), "Fatigue design of offshore steel structures," Det Norske Veritas, Recommended Practice DNV-RP-C203, Høvik.
- [56] Ferreira, J.M., Branco, C.M., (1991), "Influence of misalignment on the fatigue strength of butt welds," *International Journal of Fatigue*, Vol. 13, No. 5, pp. 405-409.
- [57] Wylde, J.G., Maddox, S.J., (1979), "Effect of misalignment on fatigue strength of transverse butt welded joints," Proceedings, Conference on Significance of Deviations from Design Shapes. Institute of Mechanical Engineers, London, pp.
- [58] Lie, S.T., Lan, S., (1998), "A boundary element analysis of misaligned load-carrying cruciform welded joints," *International Journal of Fatigue*, Vol. 20, No. pp. 433-439.
- [59] AWS D1.1:2000, (2000), "Structural welding code: Steel," American Welding Society (AWS), USA.
- [60] Newman, J.C., Raju, I.S., (1981), "An empirical stress-intensity factor equation for the surface crack," *Engineering Fracture Mechanics*, Vol. 15, No. 1/2, pp. 185-192.
-

-
- [61] Maddox, S.J., (1975), "An analysis of fatigue cracks in fillet welded joints," *International Journal of Fracture*, Vol. 11, No. 2, pp. 221-243.
- [62] Lie, S.T., Zhao, Z. and Yan, S.H., (2000), "Two-dimensional and three-dimensional magnification factors, M_k , for non-load-carrying fillet welds cruciform joints," *Engineering Fracture Mechanics*, Vol. 65, No. 4, pp. 435-453.
- [63] Fu, B., Haswell, J.V. and Bettess, P., (1993), "Weld magnification factors for semi-elliptical surface cracks in fillet welded T-butt joint models," *International Journal of Fracture*, Vol. 63, No. 2, pp. 155-171.
- [64] Bowness, D., Lee, M.M.K., (2000), "Prediction of weld toe magnification factors for semi-elliptical cracks in T-butt joints," *International Journal of Fatigue*, Vol. 22, No. 5, pp. 369-387.
- [65] Lie, S.T., Vipin, S.P. and Li, T., (2015), "New weld toe magnification factors for semi-elliptical cracks in double-sided T-butt joints and cruciform X-joints," *International Journal of Fatigue*, Vol. 80, No. pp. 178-191.
- [66] Lie, S.T., Lee, C.K. and Wong, S.M., (2003), "Model and mesh generation of cracked tubular Y-joints," *Engineering Fracture Mechanics*, Vol. 70, No. 2, pp. 161-184.
- [67] Lie, S.T., Lee, C.K., Chiew, S.P. and Shao, Y.B., (2005), "Mesh modelling and analysis of cracked uni-planar tubular K-joints," *Journal of Constructional Steel Research*, Vol. 61, No. 2, pp. 235-264.
- [68] Lie, S.T., Lee, C.K., Chiew, S.P., ASCE, M. and Yang, Z.M., (2006), "Static strength of cracked square hollow section T joints under axial loads. II: numerical," *Journal of Structural Engineering*, Vol. 132, No. 3, pp. 378-386.
- [69] Lie, S.T., Vipin, S.P. and Li, T., (2012), "A new mesh modeling technique for elastic-plastic analyses of cracked plate-to-plate and circular hollow section welded joints," *Proceeding of 11th International Conference on Steel, Space and Composite Structures*, Qingdao, China, pp. 259-268.
- [70] Lie, S.T., Li, T. and Shao, Y.B., (2014), "Plastic collapse load prediction and failure assessment diagram analysis of cracked circular hollow section T-joint and Y-joint," *Fatigue & Fracture of Engineering Materials & Structures*, Vol. 37, No. 3, pp. 314-324.
- [71] Zhao, H.S., Lie, S.T., (2017), "Determination of dimensionless stress intensity factor of plate-to-plate butt welds between axially aligned members of different thickness," *Engineering Fracture Mechanics*, Vol. 172, No. pp. 90-105.
- [72] Motarjemi, A.K., Kokabi, A.H., Ziaie, A.A., Manteghi, S. and Burdekin, F.M., (2000), "Comparison of the stress intensity factor of T and cruciform welded joints with different main and attachment plate thickness," *Engineering Fracture Mechanics*, Vol. 65, No. pp. 55-66.
- [73] Bowness, D., Lee, M.M.K., (2000), "Fracture mechanics assessment of fatigue cracks in offshore tubular structures," *Offshore Technology Report 077*, Health and Safety Executive, UK.
- [74] Lie, S.T., Zhao, H.S. and Vipin, S.P., (2017), "New weld toe magnification factors for semi-elliptical cracks in plate-to-plate butt-welded joints," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 40, No. 2, pp. 207-220.
-

-
- [75] Zhao, H.S., Lie, S.T. and Zhang, Y., (2017), "Stress intensity factors for semi-elliptical surface cracks in plate-to-plate butt welds with parallel misalignment of same thickness," *Marine Structures*, Vol. 53, No. pp. 148-163.
- [76] Fisher, J.W., Frank, K.H., (1979), "Fatigue strength of fillet welded cruciform joints," *Journal of the Structural Division*, Vol. 105, No. pp. 1727-1740.
- [77] Guha, B., (1995), "A new fracture mechanics method to predict the fatigue life of welded cruciform joints," *Engineering Fracture Mechanics*, Vol. 52, No. pp. 215-229.
- [78] Kainuma, S., Kim, I., (2005), "Fatigue strength evaluation of load-carrying cruciform fillet-welded joints made with mild steel plates of different thickness," *International Journal of Fatigue*, Vol. 27, No. 7, pp. 810-816.
- [79] Kainuma, S., Mori, T., (2006), "A fatigue strength evaluation method for load-carrying fillet welded cruciform joints," *International Journal of Fatigue*, Vol. 28, No. 8, pp. 864-872.
- [80] Smith, I.F.C., Smith, R.A., (1983), "Fatigue crack growth in a fillet welded joint," *Engineering Fracture Mechanics*, Vol. 18, No. pp. 861-869.
- [81] Ferreira, J.M., Branco, C.M., (1991), "Influence of fillet weld joint geometry on fatigue crack growth," *Theoretical and Applied Fracture Mechanics*, Vol. 15, No. pp. 131-142.
- [82] Nykänen, T., Li, X., Björk, T. and Marquis, G., (2005), "A parametric fracture mechanics study of welded joints with toe cracks and lack of penetration," *Engineering Fracture Mechanics*, Vol. 72, No. 10, pp. 1580-1609.
- [83] Bowness, D., Lee, M.M.K., (1996), "Stress intensity factor solutions for semi-elliptical weld-toe cracks in t-butt geometries," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 19, No. pp. 787-797.
- [84] Pang, J.H.L., Tsang, K.S. and Hoh, H.J., (2016), "3D stress intensity factors for weld toe semi-elliptical surface cracks using XFEM," *Marine Structures*, Vol. 48, No. pp. 1-14.
- [85] Han, J.W., Han, D.K. and Han, S.H., (2014), "Stress intensity factors for three-dimensional weld toe cracks using weld toe magnification factors," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 37, No. 2, pp. 146-156.
- [86] FEACrack™, (2010), "User's manual, Version 3.2," Quest Integrity Group LLC, USA.
- [87] Japanese Society of Steel Construction, (1995), "Fatigue design recommendations for steel structures," JSSC Technical Report, No. 32.
- [88] Xu, K., Lu, Y. and Lie, S.T., (2005), "Loading-carrying welds joints analysis using boundary element method," *International Journal of Solids and Structures*, Vol. 42, No. pp. 2965-2975.
- [89] Kim, I.-T., Kainuma, S., (2005), "Fatigue life assessment of load-carrying fillet-welded cruciform joints inclined to uniaxial cyclic loading," *International Journal of Pressure Vessels and Piping*, Vol. 82, No. 11, pp. 807-813.
- [90] Rice, J.R., (1968), "A path independent integral and the approximate analysis of strain concentration by notches and cracks," *Journal of Applied Mechanics*, Vol. 35, No. pp. 379-386.
-

-
- [91] Lie, S.T., Bian, C., (1997), "Load-carrying fillet welds using dual boundary element method," *Journal of Structural Engineering*, Vol. 123, No. pp. 1603-1613.
- [92] Fricke, W., (2013), "IIW guideline for the assessment of weld root fatigue," *Welding in the World*, Vol. 57, No. 6, pp. 753-791.
- [93] Kainuma, S., Mori, T., (2008), "A study on fatigue crack initiation point of load-carrying fillet welded cruciform joints," *International Journal of Fatigue*, Vol. 30, No. 9, pp. 1669-1677.
- [94] Saiprasertkit, K., Sasaki, E. and Miki, C., (2014), "Fatigue crack initiation point of load carrying cruciform joints in low and high cycle fatigue regions," *International Journal of Fatigue*, Vol. 59, No. pp. 153-158.
- [95] Lie, S.T., Zhao, H.S. and Wai, M.H., (2017), "Prediction of stress intensity magnification factors of a semi-elliptical weld toe crack in load-carrying cruciform fillet-welded joints," Vol. No.
- [96] Lee, M.M.K., Bowness, D., (2002), "Estimation of stress intensity factor solutions for weld toe cracks in offshore tubular joints," *International Journal of Fatigue*, Vol. 24, No. pp. 861-875.
- [97] Ayatollahi, M.R., Khoramishad, H., (2010), "Stress intensity factors for an axially oriented internal crack embedded in a buried pipe," *International Journal of Pressure Vessels and Piping*, Vol. 87, No. 4, pp. 165-169.
- [98] Bergman, M., (1995), "Stress intensity factors for circumferential surface cracks in pipes," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 18, No. pp. 1155-1172.
- [99] Hoh, H.J., Pang, J.H.L. and Tsang, K.S., (2016), "Stress intensity factors for fatigue analysis of weld toe cracks in a girth-welded pipe," *International Journal of Fatigue*, Vol. 87, No. pp. 279-287.
- [100] Zhao, H.S., Lie, S.T. and Zhang, Y., (2017), "Elastic-plastic fracture analyses for misaligned clad pipeline containing a canoe shape surface crack subjected to large plastic deformation," *Ocean Engineering*, Vol. 146, No. pp. 87-100.
- [101] Poette, C., Albaladejo, S., (1991), "Stress intensity factors and influence functions for circumferential surface cracks in pipes," *Engineering Fracture Mechanics*, Vol. 39, No. pp. 641-650.
- [102] Lie, S.-T., Zhao, H.S., (2018), "Fracture analysis of load-carrying cruciform fillet welded joints with multiple cracks," *Engineering Fracture Mechanics*, Vol. 193, No. pp. 32-46.
- [103] Wetzal, R.M., (1977), "Fatigue under complex loading: analysis and experiments," Society of Automotive Engineers, USA.
- [104] Bažant, Z.P., Estenssoro, L.F., (1979), "Surface singularity and crack propagation," *International Journal of Solids and Structures*, Vol. 15, No. 5, pp. 405-426.
- [105] Pook, L.P., (1994), "Some implications of corner point singularities," *Engineering Fracture Mechanics*, Vol. 48, No. pp. 367-378.
-

-
- [106] Pook, L.P., Berto, F., Campagnolo, A. and Lazzarin, P., (2014), "Coupled fracture mode of a cracked disc under anti-plane loading," *Engineering Fracture Mechanics*, Vol. 128, No. pp. 22-36.
- [107] Pook, L.P., Campagnolo, A., Berto, F. and Lazzarin, P., (2015), "Coupled fracture mode of a cracked plate under anti-plane loading," *Engineering Fracture Mechanics*, Vol. 134, No. pp. 391-403.
- [108] Pook, L.P., Campagnolo, A. and Berto, F., (2016), "Coupled fracture modes of discs and plates under anti-plane loading and a disc under in-plane shear loading," *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 39, No. 8, pp. 924-938.
- [109] Pook, L.P., Berto, F. and Campagnolo, A., (2017), "State of the art of corner point singularities under in-plane and out-of-plane loading," *Engineering Fracture Mechanics*, Vol. 174, No. pp. 2-9.
- [110] Hilberink, A., (2011), "Mechanical behaviour of lined pipe," Ph.D. thesis, Delft University of Technology.
- [111] Brazier, L.G., (1927), "On the flexure of thin cylindrical shells and other "thin" sections," *Proceedings of the Royal Society A-Mathematical Physical and Engineering Sciences*, Vol. 116, No. pp. 104-114.
- [112] Ades, C.S., (1957), "Bending strength of tubing in the plastic range," *Journal of the Aeronautical Sciences*, Vol. 24, No. pp. 605-610.
- [113] Lie, S.T., Zhang, Y. and Zhao, H.S., (2017), "Fracture response of girth-welded pipeline with canoe shape embedded crack subjected to large plastic deformation," *Journal of Pressure Vessel Technology*, Vol. 139, No. pp. 021406.
- [114] Cicero, S., Ainsworth, R.A. and Gutiérrez-Solana, F., (2010), "Engineering approaches for the assessment of low constraint fracture conditions: A critical review," *Engineering Fracture Mechanics*, Vol. 77, No. 8, pp. 1360-1374.
- [115] Nourpanah, N., Taheri, F., (2010), "Development of a reference strain approach for assessment of fracture response of reeled pipelines," *Engineering Fracture Mechanics*, Vol. 77, No. 12, pp. 2337-2353.
- [116] Kim, Y.J., Kim, J.S., Lee, Y.Z. and Kim, Y.J., (2002), "Non-linear fracture mechanics analyses of part circumferential surface cracked pipes," *International Journal of Fracture*, Vol. 116, No. pp. 347-375.
- [117] Kumar, V., German, M.D. and Shih, C.F., (1981), "An engineering approach to elastic-plastic fracture analysis," Tech rep. EPRI NP-1931, Electric Power Research Institute, Palo Alto, CA.
- [118] Lei, Y., Ainsworth, R.A., (1997), "A J integral estimation method for cracks in welds with mismatched mechanical properties," *International Journal of Pressure Vessels and Piping*, Vol. 70, No. pp. 237-245.
- [119] Souza, R.F., Ruggieri, C. and Zhang, Z., (2016), "A framework for fracture assessments of dissimilar girth welds in offshore pipelines under bending," *Engineering Fracture Mechanics*, Vol. 163, No. pp. 66-88.
-

-
- [120] Christo Michael, T., Veerappan, A.R. and Shanmugam, S., (2012), "Effect of ovality and variable wall thickness on collapse loads in pipe bends subjected to in-plane bending closing moment," *Engineering Fracture Mechanics*, Vol. 79, No. pp. 138-148.
- [121] Dadfar, B., Hesham El Naggar, M. and Nastev, M., (2015), "Ovalization of steel energy pipelines buried in saturated sands during ground deformations," *Computers and Geotechnics*, Vol. 69, No. pp. 105-113.
- [122] Rice, J.R., Rosengren, G.F., (1968), "Plane strain deformation near a crack tip in a power-law hardening material," *Journal of the Mechanics and Physics of Solids*, Vol. 16, No. pp. 1-12.
- [123] Hutchinson, J.W., (1968), "Singular behavior at the end of a tensile crack tip in a hardening material," *Journal of the Mechanics and Physics of Solids*, Vol. 16, No. pp. 13-31.
- [124] Zahoor, A., (1989), "Ductile fracture handbook," Tech rep. EPRI NP-6301-D, Electric Power Research Institute, Palo Alto, CA.
- [125] Hertel, S., Waele, W.D., Denys, R., Verstraete, M. and Minnebruggen, K.V., (2013), "Weld Strength Mismatch in Strain Based Flaw Assessment: Which Definition to Use?," *Journal of Pressure Vessel Technology*, Vol. 135, No. pp. 061402.
- [126] Miller, A.G., (1988), "Review of limit loads of structures containing defects," *International Journal of Pressure Vessels and Piping*, Vol. 32, No. pp. 191-327.
- [127] Kim, S.H., Han, J.J. and Kim, Y.J., (2014), "Limit Load Solutions of V-groove Welded Pipes with a Circumferential Crack at the Centre of Weld," *Procedia Materials Science*, Vol. 3, No. pp. 706-713.
- [128] American Lifelines Alliance, (2005), "Guideline for the design of buried steel pipe," FEMA and ASCE.
- [129] R6, (2005), "Assessment of the integrity of structures containing defects," Revision 4, Amendment 4, British Energy, Gloucester.
- [130] Lie, S.T., Zhang, Y. and Zhao, H.S., (2017), "Fracture Response of Girth-Welded Pipeline With Canoe Shape Embedded Crack Subjected to Large Plastic Deformation," *Journal of Pressure Vessel Technology*, Vol. 139, No. 2, pp. 021406.
- [131] Jia, P., Jing, H., Xu, L., Han, Y. and Zhao, L., (2016), "A modified reference strain method for engineering critical assessment of reeled pipelines," *International Journal of Mechanical Sciences*, Vol. 105, No. pp. 23-31.
- [132] Jia, P., Jing, H., Xu, L., Han, Y. and zhao, L., (2017), "A modified engineering critical assessment method for deeply-embedded cracks in metallic pipelines subjected to large plastic strain," *Engineering Fracture Mechanics*, Vol. No.
- [133] Jia, P., Jing, H., Xu, L., Han, Y. and Zhao, L., (2017), "A modified fracture assessment method for pipelines under combined inner pressure and large-scale axial plastic strain," *Theoretical and Applied Fracture Mechanics*, Vol. 87, No. pp. 91-98.
- [134] Zhang, Y., Lie, S.T. and Zhao, H., (2018), "Fracture behavior of clad pipeline containing a canoe shape surface crack subjected to large bending moment," *Marine Structures*, Vol. 58, No. pp. 92-108.
-

- [135] Parise, L.F.S., Ruggieri, C. and O'Dowd, N.P., (2015), "Fully-plastic strain-based J estimation scheme for circumferential surface cracks in pipes subjected to reeling," *Journal of Pressure Vessel Technology*, Vol. 137, No. 4, pp. 041204.
- [136] Anderson, T.L., (2005), "Fracture mechanics: fundamentals and applications. 3rd ed.," Boca Raton, CRC Press.
- [137] Kim, Y.J., Schwalbe, K.H., (2001), "Mismatch effect on plastic yield loads in idealised weldments: part II-HAZ cracks," *Engineering Fracture Mechanics*, Vol. 68, No. pp. 183-199.
- [138] Gong, S., Xu, P., (2016), "The influence of sea state on dynamic behaviour of offshore pipelines for deepwater S-lay," *Ocean Engineering*, Vol. 111, No. pp. 398-413.
- [139] Yun-dong, L., Yi-ren, Y., (2017), "Vibration analysis of conveying fluid pipe via He's variational iteration method," *Applied Mathematical Modelling*, Vol. 43, No. pp. 409-420.
- [140] Wang, L., Yang, Y., Liu, C., Li, Y. and Hu, Q., (2018), "Numerical investigation of dynamic response of a pipeline-riser system caused by severe slugging flow," *International Journal of Pressure Vessels and Piping*, Vol. 159, No. pp. 15-27.

APPENDIX A

Deepest point and axial loading

$$M_{k,ma} = f_1\left(\frac{a}{T}, \frac{a}{c}\right) + f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) + f_3\left(\frac{a}{T}, \theta, \frac{L}{T}\right)$$

where

$$f_1\left(\frac{a}{T}, \frac{a}{c}\right) = 0.018591\left(\frac{a}{T}\right)^{\left[A_1 + \left(A_2\left(\frac{a}{T}\right)\right)^{A_3}\right]} + 0.318665\exp\left[\left(\frac{a}{T}\right)^{A_4}\right] + A_5$$

$$A_1 = -0.300072\left(\frac{a}{c}\right)^2 + 0.854833\left(\frac{a}{c}\right) - 1.647902$$

$$A_2 = 6.292003\exp\left[-12.010142\left(\frac{a}{c}\right)\right] + 0.021331$$

$$A_3 = 0.030954\left(\frac{a}{c}\right)^2 + 0.060738\left(\frac{a}{c}\right) + 0.056515$$

$$A_4 = 0.013722\left(\frac{a}{c}\right)^3 - 0.042872\left(\frac{a}{c}\right)^2 + 0.041921\left(\frac{a}{c}\right) - 0.000254$$

$$A_5 = 0.018613\left(\frac{a}{c}\right)^2 - 0.038751\left(\frac{a}{c}\right) + 0.106211$$

$$f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) = \left[A_6\left(\frac{a}{c}\right)^2 + A_7\left(\frac{a}{c}\right) + A_8\right]\left[1 - \left(\frac{a}{T}\right)\right]^{A_9} + A_{10}\left(\frac{a}{T}\right)^{A_{11}}$$

$$A_6 = 0.006478\theta^2 + 0.071648\theta - 0.019019$$

$$A_7 = 0.044226\theta^2 - 0.186262\theta + 0.045583$$

$$A_8 = -0.052941\theta^3 + 0.033686\theta^2 - 0.067654\theta + 0.016364$$

$$A_9 = -0.480374\theta^2 + 0.924563\theta - 0.003431$$

$$A_{10} = 0.038421\theta^3 - 0.040712\theta^2 + 0.035804\theta - 0.007276$$

$$A_{11} = 0.386852\theta^3 + 0.030445\theta - 0.967961$$

$$f_3\left(\frac{a}{T}, \theta, \frac{L}{T}\right) = (A_{12}\theta^2 + A_{13}\theta + A_{14})\left(\frac{a}{T}\right)^{-0.828234} + (A_{15}\theta^2 + A_{16}\theta + A_{17})\left(1 - \frac{a}{T}\right)^{(A_{18}\theta^2 + A_{19}\theta + A_{20})}$$

$$A_{12} = 0.015408\left(\frac{L}{T}\right)^2 - 0.054164\left(\frac{L}{T}\right) + 0.032102$$

$$A_{13} = -0.026715\left(\frac{L}{T}\right)^2 + 0.104212\left(\frac{L}{T}\right) - 0.065706$$

$$A_{14} = 0.000089\left(\frac{L}{T}\right)^3 - 0.000352\left(\frac{L}{T}\right)^2 + 0.000436\left(\frac{L}{T}\right) - 0.000169$$

$$A_{15} = -0.058682\left(\frac{L}{T}\right)^3 + 0.196813\left(\frac{L}{T}\right)^2 - 0.124954\left(\frac{L}{T}\right) + 0.004049$$

$$A_{16} = 0.045924\left(\frac{L}{T}\right)^2 - 0.268432\left(\frac{L}{T}\right) + 0.178434$$

$$A_{17} = 0.000724\left(\frac{L}{T}\right)^2 - 0.001963\left(\frac{L}{T}\right) + 0.001144$$

$$A_{18} = -0.416211\left(\frac{L}{T}\right)^3 + 1.526304\left(\frac{L}{T}\right)^2 - 1.179002\left(\frac{L}{T}\right) + 0.179473$$

$$A_{19} = 0.469224\left(\frac{L}{T}\right)^3 - 1.493311\left(\frac{L}{T}\right)^2 + 0.464082\left(\frac{L}{T}\right) + 0.344185$$

$$A_{20} = 0.008762\left(\frac{L}{T}\right)^3 - 0.028206\left(\frac{L}{T}\right)^2 + 0.020814\left(\frac{L}{T}\right) + 0.078849$$

Deepest point and bending loading

$$M_{k,ba} = f_1\left(\frac{a}{T}, \frac{a}{c}\right) + f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) + f_3\left(\frac{a}{T}, \theta, \frac{L}{T}\right)$$

where

$$f_1\left(\frac{a}{T}, \frac{a}{c}\right) = 0.014868\left(\frac{a}{T}\right)^{\left[A_1 + \left(A_2\left(\frac{a}{T}\right)\right)^{A_3}\right]} + 0.311059 \exp\left[\left(\frac{a}{T}\right)^{A_4}\right] + A_5$$

$$A_1 = 0.287551\left(\frac{a}{c}\right)^2 + 0.320180\left(\frac{a}{c}\right) - 1.587701$$

$$A_2 = 5.473002 \exp\left[-11.440124\left(\frac{a}{c}\right)\right] + 0.165711$$

$$A_3 = 0.289432 \left(\frac{a}{c} \right)^2 - 0.080521 \left(\frac{a}{c} \right) + 0.085385$$

$$A_4 = 0.032757 \left(\frac{a}{c} \right)^3 - 0.071563 \left(\frac{a}{c} \right)^2 + 0.042176 \left(\frac{a}{c} \right) + 0.002832$$

$$A_5 = 0.021968 \left(\frac{a}{c} \right)^2 - 0.059168 \left(\frac{a}{c} \right) + 0.105291$$

$$f_2 \left(\frac{a}{T}, \frac{a}{c}, \theta \right) = \left[A_6 \left(\frac{a}{c} \right)^2 + A_7 \left(\frac{a}{c} \right) + A_8 \right] \left[1 - \left(\frac{a}{T} \right) \right]^{A_9} + A_{10} \left(\frac{a}{T} \right)^{A_{11}}$$

$$A_6 = -0.014433\theta^2 + 0.087139\theta - 0.018015$$

$$A_7 = 0.110391\theta^2 - 0.238322\theta + 0.052413$$

$$A_8 = 0.041088\theta^3 + 0.045213\theta^2 - 0.172351\theta + 0.041313$$

$$A_9 = -1.880332\theta^2 + 1.593042\theta + 0.026762$$

$$A_{10} = 0.001910\theta^3 - 0.015049\theta^2 + 0.019178\theta - 0.004026$$

$$A_{11} = -0.088578\theta^3 - 0.056898\theta - 1.091022$$

$$f_3 \left(\frac{a}{T}, \theta, \frac{L}{T} \right) = (A_{12}\theta^2 + A_{13}\theta + A_{14}) \left(\frac{a}{T} \right)^{-1.031556} + (A_{15}\theta^2 + A_{16}\theta + A_{17}) \left(1 - \frac{a}{T} \right)^{(A_{18}\theta^2 + A_{19}\theta + A_{20})}$$

$$A_{12} = 0.008167 \left(\frac{L}{T} \right)^2 - 0.027438 \left(\frac{L}{T} \right) + 0.017692$$

$$A_{13} = -0.013229 \left(\frac{L}{T} \right)^2 + 0.046042 \left(\frac{L}{T} \right) - 0.029593$$

$$A_{14} = 0.000090 \left(\frac{L}{T} \right)^3 - 0.000306 \left(\frac{L}{T} \right)^2 + 0.000323 \left(\frac{L}{T} \right) - 0.000109$$

$$A_{15} = 0.006379 \left(\frac{L}{T} \right)^3 - 0.075891 \left(\frac{L}{T} \right)^2 + 0.238632 \left(\frac{L}{T} \right) - 0.145604$$

$$A_{16} = 0.133072 \left(\frac{L}{T} \right)^2 - 0.497163 \left(\frac{L}{T} \right) + 0.317941$$

$$A_{17} = 0.000340 \left(\frac{L}{T} \right)^2 - 0.001618 \left(\frac{L}{T} \right) + 0.001543$$

$$A_{18} = 0.259711\left(\frac{L}{T}\right)^3 - 0.626033\left(\frac{L}{T}\right)^2 + 0.649982\left(\frac{L}{T}\right) - 0.252292$$

$$A_{19} = -0.155562\left(\frac{L}{T}\right)^3 + 0.355093\left(\frac{L}{T}\right)^2 - 0.796604\left(\frac{L}{T}\right) + 0.489671$$

$$A_{20} = -0.003530\left(\frac{L}{T}\right)^3 + 0.009893\left(\frac{L}{T}\right)^2 - 0.040271\left(\frac{L}{T}\right) + 0.067436$$

Crack ends and axial loading

$$M_{k,mc} = f_1\left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right) f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right)$$

where

$$f_1\left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right) = A_1\left(\frac{a}{T}\right)^{\left[A_2\left(\frac{a}{c}\right)^2 + A_3\left(\frac{a}{c}\right) + A_4\right]} + \left[A_5\left(\frac{a}{c}\right)^2 + A_6\left(\frac{a}{c}\right) + A_7\right] \left[1 - \left(\frac{a}{T}\right)\right]^{A_8}$$

$$A_1 = 0.618891\left(\frac{L}{T}\right)^3 - 2.555901\left(\frac{L}{T}\right)^2 + 3.775001\left(\frac{L}{T}\right) - 0.620191$$

$$A_2 = 0.258511\left(\frac{L}{T}\right)^3 - 1.082302\left(\frac{L}{T}\right)^2 + 1.442903\left(\frac{L}{T}\right) - 0.648541$$

$$A_3 = -0.385603\left(\frac{L}{T}\right)^3 + 1.619112\left(\frac{L}{T}\right)^2 - 2.168701\left(\frac{L}{T}\right) + 0.981631$$

$$A_4 = 0.384482\left(\frac{L}{T}\right)^3 - 1.616204\left(\frac{L}{T}\right)^2 + 2.154005\left(\frac{L}{T}\right) - 0.984921$$

$$A_5 = -0.016285\left(\frac{L}{T}\right)^2 + 0.189341\left(\frac{L}{T}\right) + 0.062376$$

$$A_6 = 0.030528\left(\frac{L}{T}\right)^2 - 0.272301\left(\frac{L}{T}\right) - 0.084979$$

$$A_7 = -0.645332\left(\frac{L}{T}\right)^3 + 2.640602\left(\frac{L}{T}\right)^2 - 3.783913\left(\frac{L}{T}\right) + 1.612812$$

$$A_8 = 0.210511\left(\frac{L}{T}\right)^3 - 0.755932\left(\frac{L}{T}\right)^2 + 0.725541\left(\frac{L}{T}\right) - 0.122863$$

$$f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) = \left[A_9 \left(\frac{a}{c}\right)^2 + A_{10} \left(\frac{a}{c}\right) + A_{11} \right] \left(\frac{a}{T}\right)^{\left[A_{12} \left(\frac{a}{c}\right)^2 + A_{13} \left(\frac{a}{c}\right) + A_{14} \right]} + A_{15} \left[1 - \left(\frac{a}{T}\right) \right]^{A_{16}}$$

$$A_9 = -0.6933910^3 + 0.1453320^2 + 0.7126540 - 0.184223$$

$$A_{10} = 0.8504520^3 - 0.0243320^2 - 1.0756050 + 0.268172$$

$$A_{11} = 1.0391020^2 - 1.3530150 + 0.856011$$

$$A_{12} = 0.5156440^3 - 0.6341220^2 - 0.0685370 + 0.052212$$

$$A_{13} = -0.6621420^3 + 0.7860810^2 + 0.1407220 - 0.078922$$

$$A_{14} = 0.1905640^2 - 0.6032410 + 0.131393$$

$$A_{15} = -1.1373040^2 + 1.4888020 + 0.116554$$

$$A_{16} = -0.3106020^3 + 0.5847110^2 - 0.3609440 + 0.060015$$

$$f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right) = \left(A_{17} \theta^2 + A_{18} \theta + A_{19} \right) \left(\frac{a}{T}\right)^{\left(A_{20} \theta^2 + A_{21} \theta + A_{22} \right)} + \left[A_{23} \left(\frac{L}{T}\right)^2 + A_{24} \left(\frac{L}{T}\right) + A_{25} \right] \exp \left[\left(\frac{a}{T}\right)^{\left(A_{26} \theta^2 + A_{27} \theta + A_{28} \right)} \right]$$

$$A_{17} = 0.069567 \left(\frac{L}{T}\right)^2 - 0.232402 \left(\frac{L}{T}\right) + 0.131471$$

$$A_{18} = -0.061544 \left(\frac{L}{T}\right)^2 + 0.198201 \left(\frac{L}{T}\right) - 0.104263$$

$$A_{19} = 0.035552 \left(\frac{L}{T}\right)^2 - 0.157712 \left(\frac{L}{T}\right) + 0.252584$$

$$A_{20} = 0.329033 \left(\frac{L}{T}\right)^2 - 1.775411 \left(\frac{L}{T}\right) + 0.351504$$

$$A_{21} = -0.710651 \left(\frac{L}{T}\right)^3 + 2.497912 \left(\frac{L}{T}\right)^2 - 2.470604 \left(\frac{L}{T}\right) + 1.759511$$

$$A_{22} = -0.054719 \left(\frac{L}{T}\right)^2 + 0.354112 \left(\frac{L}{T}\right) - 0.263754$$

$$A_{23} = -0.006395 \left(\frac{a}{c}\right)^2 + 0.003646 \left(\frac{a}{c}\right) - 0.008501$$

$$A_{24} = -0.099852 \left(\frac{a}{c}\right)^3 + 0.176802 \left(\frac{a}{c}\right)^2 - 0.080224 \left(\frac{a}{c}\right) + 0.050058$$

$$A_{25} = -0.011362\left(\frac{a}{c}\right)^2 + 0.011613\left(\frac{a}{c}\right) + 0.283922$$

$$A_{26} = 0.091841\left(\frac{a}{c}\right)^2 - 0.093043\left(\frac{a}{c}\right) + 0.093268$$

$$A_{27} = 0.591981\left(\frac{a}{c}\right)^3 - 1.085602\left(\frac{a}{c}\right)^2 + 0.544943\left(\frac{a}{c}\right) - 0.164171$$

$$A_{28} = -0.129862\left(\frac{a}{c}\right)^3 + 0.218673\left(\frac{a}{c}\right)^2 - 0.099574\left(\frac{a}{c}\right) + 0.001715$$

Crack ends and bending loading

$$M_{k,bc} = f_1\left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right) f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right)$$

where

$$f_1\left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right) = A_1\left(\frac{a}{T}\right)^{\left[A_2\left(\frac{a}{c}\right)^2 + A_3\left(\frac{a}{c}\right) + A_4\right]} + \left[A_5\left(\frac{a}{c}\right)^2 + A_6\left(\frac{a}{c}\right) + A_7\right] \left[1 - \left(\frac{a}{T}\right)\right]^{A_8}$$

$$A_1 = 0.458302\left(\frac{L}{T}\right)^3 - 2.022114\left(\frac{L}{T}\right)^2 + 2.943704\left(\frac{L}{T}\right) - 0.432281$$

$$A_2 = 0.127791\left(\frac{L}{T}\right)^3 - 0.534512\left(\frac{L}{T}\right)^2 + 0.722951\left(\frac{L}{T}\right) - 0.377214$$

$$A_3 = -0.221293\left(\frac{L}{T}\right)^3 + 0.918844\left(\frac{L}{T}\right)^2 - 1.225802\left(\frac{L}{T}\right) + 0.623011$$

$$A_4 = 0.251933\left(\frac{L}{T}\right)^3 - 1.063602\left(\frac{L}{T}\right)^2 + 1.423611\left(\frac{L}{T}\right) - 0.713012$$

$$A_5 = 0.013401\left(\frac{L}{T}\right)^2 + 0.032429\left(\frac{L}{T}\right) + 0.048154$$

$$A_6 = -0.024901\left(\frac{L}{T}\right)^2 - 0.033321\left(\frac{L}{T}\right) - 0.077792$$

$$A_7 = -0.477242\left(\frac{L}{T}\right)^3 + 2.101822\left(\frac{L}{T}\right)^2 - 3.017314\left(\frac{L}{T}\right) + 1.429803$$

$$A_8 = -0.004042 \left(\frac{L}{T}\right)^3 + 0.044753 \left(\frac{L}{T}\right)^2 - 0.133422 \left(\frac{L}{T}\right) + 0.018403$$

$$f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) = \left[A_9 \left(\frac{a}{c}\right)^2 + A_{10} \left(\frac{a}{c}\right) + A_{11} \right] \left(\frac{a}{T}\right)^{\left[A_{12} \left(\frac{a}{c}\right)^2 + A_{13} \left(\frac{a}{c}\right) + A_{14} \right]} + A_{15} \left[1 - \left(\frac{a}{T}\right) \right]^{A_{16}}$$

$$A_9 = 0.200502\theta^3 - 0.544771\theta^2 + 0.577822\theta - 0.117612$$

$$A_{10} = -0.269671\theta^3 + 0.782733\theta^2 - 0.849852\theta + 0.173781$$

$$A_{11} = 0.392862\theta^2 - 1.238601\theta + 0.882484$$

$$A_{12} = 1.522911\theta^3 - 1.343503\theta^2 + 0.001092\theta + 0.064546$$

$$A_{13} = -2.018804\theta^3 + 1.699712\theta^2 + 0.094563\theta - 0.105144$$

$$A_{14} = -0.356072\theta^2 - 0.377122\theta + 0.130864$$

$$A_{15} = -0.167161\theta^2 + 1.034402\theta + 0.152363$$

$$A_{16} = -0.569322\theta^3 + 1.031023\theta^2 - 0.553011\theta + 0.086576$$

$$f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right) = (A_{17}\theta^2 + A_{18}\theta + A_{19}) \left(\frac{a}{T}\right)^{(A_{20}\theta^2 + A_{21}\theta + A_{22})} + \left[A_{23} \left(\frac{L}{T}\right)^2 + A_{24} \left(\frac{L}{T}\right) + A_{25} \right] \exp \left[\left(\frac{a}{T}\right)^{(A_{26}\theta^2 + A_{27}\theta + A_{28})} \right]$$

$$A_{17} = 0.066072 \left(\frac{L}{T}\right)^3 - 0.191402 \left(\frac{L}{T}\right)^2 + 0.154141 \left(\frac{L}{T}\right) - 0.068214$$

$$A_{18} = -0.066171 \left(\frac{L}{T}\right)^3 + 0.199012 \left(\frac{L}{T}\right)^2 - 0.185811 \left(\frac{L}{T}\right) + 0.081215$$

$$A_{19} = 0.070837 \left(\frac{L}{T}\right)^2 - 0.198582 \left(\frac{L}{T}\right) + 0.171224$$

$$A_{20} = -0.165331 \left(\frac{L}{T}\right)^2 + 1.051502 \left(\frac{L}{T}\right) - 0.265133$$

$$A_{21} = -0.366372 \left(\frac{L}{T}\right)^3 + 1.756404 \left(\frac{L}{T}\right)^2 - 3.417101 \left(\frac{L}{T}\right) + 1.057602$$

$$A_{22} = 0.014953 \left(\frac{L}{T}\right)^2 + 0.099201 \left(\frac{L}{T}\right) - 0.252571$$

$$A_{23} = -0.006141 \left(\frac{a}{c}\right)^2 + 0.010391 \left(\frac{a}{c}\right) - 0.026834$$

$$A_{24} = -0.107572\left(\frac{a}{c}\right)^3 + 0.201764\left(\frac{a}{c}\right)^2 - 0.115550\left(\frac{a}{c}\right) + 0.086581$$

$$A_{25} = -0.001955\left(\frac{a}{c}\right)^2 + 0.006483\left(\frac{a}{c}\right) + 0.300602$$

$$A_{26} = 0.144291\left(\frac{a}{c}\right)^2 - 0.184082\left(\frac{a}{c}\right) - 0.022861$$

$$A_{27} = 0.602053\left(\frac{a}{c}\right)^3 - 1.251604\left(\frac{a}{c}\right)^2 + 0.755411\left(\frac{a}{c}\right) - 0.008257$$

$$A_{28} = -0.183364\left(\frac{a}{c}\right)^3 + 0.326711\left(\frac{a}{c}\right)^2 - 0.160572\left(\frac{a}{c}\right) + 0.033698$$

APPENDIX B

Table B1 Coefficients for the least square fitting of f_1 shown in Eq. (7.14)

M_y	n_w	θ/π	$D_e/t = 10$					$D_e/t = 20$				
			A_1	A_2	A_3	A_4	A_5	A_1	A_2	A_3	A_4	A_5
0.8	6	0.05	25.9664	-2.4434	1.6328	0.6948	0.0986	69.5074	-2.8047	2.0704	1.6044	0.1448
		0.10	73.7508	-2.5561	2.0955	1.4656	0.1666	178.0604	-3.0493	2.5186	2.0545	0.2067
		0.15	128.6505	-2.7276	2.3598	1.6803	0.2055	252.8583	-3.2624	2.716	2.0761	0.2454
		0.20	173.0525	-2.8707	2.5142	1.7516	0.231	270.4205	-3.3526	2.7605	1.9736	0.2615
	9	0.05	36.3515	-2.8601	1.6535	1.5	0.1788	94.6678	-3.3677	2.0485	2.9564	0.2397
		0.10	106.3694	-3.0282	2.1109	2.6102	0.267	191.8493	-3.7777	2.2982	1.4816	0.235
		0.15	200.0094	-3.2628	2.3998	3.0218	0.3227	466.4324	-4.0801	2.8072	4.0549	0.3908
		0.20	294.5795	-3.4772	2.5916	3.1759	0.3645	608.0721	-4.3334	2.9603	4.0084	0.4358
	12	0.05	45.1084	-3.1287	1.568	3.917	0.2665	113.1641	-3.7048	1.904	7.3144	0.322
		0.10	131.7183	-3.1694	1.9781	6.1823	0.3639	357.0244	-3.928	2.3573	10.4029	0.3907
		0.15	257.9342	-3.3307	2.2498	7.4042	0.4246	542.9892	-4.278	2.492	8.0737	0.3369
		0.20	402.2611	-3.5052	2.4376	8.0696	0.4685	546.879	-4.6028	2.4284	1.1162	0.21
1.0	12	0.05	34.7977	-2.8871	1.7353	0.8632	0.1483	100.0262	-3.4969	2.1949	1.929	0.205
		0.10	109.4278	-3.1383	2.2378	1.657	0.2313	315.532	-3.9731	2.7372	2.456	0.2874
		0.15	216.0925	-3.4291	2.5601	1.9043	0.2839	563.4637	-4.4331	3.0592	2.3917	0.3562

1.25	12	0.20	329.9019	-3.6895	2.7791	1.965	0.3241	750.9323	-4.7419	3.2356	2.2271	0.4047
		0.05	24.2198	-2.5064	1.6516	0.4742	0.101	72.1905	-2.9737	2.143	1.3007	0.1488
		0.10	75.0585	-2.64	2.1471	1.1569	0.1686	207.2136	-3.2095	2.6432	1.7992	0.206
		0.15	139.9396	-2.7869	2.4348	1.4138	0.2037	319.8134	-3.4459	2.8868	1.8611	0.2465
		0.20	199.7893	-2.909	2.6137	1.5239	0.2277	352.5346	-3.5618	2.9503	1.7543	0.2655
	15	0.05	27.4631	-2.6246	1.7095	0.538	0.1168	83.6679	-3.1684	2.2115	1.3741	0.1682
		0.10	87.1647	-2.803	2.2182	1.1789	0.1874	253.3084	-3.5051	2.7445	1.7753	0.2367
		0.15	166.0548	-3.0031	2.52	1.38	0.227	409.062	-3.8166	3.0179	1.7893	0.2849
		0.20	244.8391	-3.1869	2.7212	1.4225	0.2591	481.5432	-3.9843	3.123	1.6734	0.3137
	18	0.05	30.3359	-2.7377	1.7538	0.5661	0.1301	94.5801	-3.3816	2.2686	1.3494	0.1884
		0.10	98.4576	-2.9839	2.2753	1.1363	0.2054	310.8643	-3.8714	2.8516	1.617	0.2724
		0.15	197.603	-3.2682	2.6107	1.2506	0.2575	557.3741	-4.2979	3.189	1.5734	0.335
		0.20	302.6124	-3.5125	2.8362	1.2479	0.2962	730.8737	-4.5964	3.3653	1.3713	0.383

Table B2 Coefficients for the least square fitting of f_2 shown in Eq. (7.15)

$D_e/t = 10$								$D_e/t = 20$				
M_y	n_w	θ/π	B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5
0.8	6	0.05	-0.3462	-0.0421	0.0298	0.0031	0.0002	-0.5734	-0.0899	0.0522	0.0127	-0.0001
		0.10	-0.4877	-0.0909	0.0412	0.0111	-0.0001	-0.8892	-0.1515	0.098	0.0264	-0.0007
		0.15	-0.6852	-0.1263	0.0673	0.018	-0.0004	-0.9922	-0.1566	0.1249	0.0292	-0.0009

		0.20	-0.7333	-0.1426	0.0749	0.0216	-0.0006	-0.9506	-0.1425	0.1273	0.0279	-0.0009
	9	0.05	-0.3843	-0.0903	-0.077	0.0032	-0.0001	-1.0719	-0.1686	-0.0581	0.0169	-0.0005
		0.10	-1.062	-0.1824	-0.0331	0.0182	-0.0006	-2.249	-0.3065	0.0415	0.0444	-0.0017
		0.15	-1.6602	-0.2587	0.0157	0.033	-0.0013	-2.8869	-0.3531	0.1101	0.0571	-0.0024
		0.20	-2.0888	-0.3018	0.0574	0.0429	-0.0017	-3.1311	-0.3589	0.1433	0.0608	-0.0026
	12	0.05	0.9784	-0.2038	-0.6112	0.0063	-0.0011	0.5806	-0.3752	-0.8979	0.0268	-0.0017
		0.10	0.0148	-0.3821	-0.6436	0.021	-0.0014	-1.6963	-0.7413	-0.9311	0.0714	-0.0029
		0.15	-1.1138	-0.6038	-0.6299	0.0516	-0.0025	-4.3428	-1.0892	-0.8264	0.1289	-0.005
		0.20	-2.256	-0.7741	-0.5937	0.0759	-0.0033	-6.3674	-1.357	-0.7589	0.1778	-0.0069
1.0	12	0.05	-0.1993	-0.1021	-0.1454	0.0042	-0.0002	-0.9457	-0.201	-0.1537	0.0211	-0.0008
		0.10	-0.9612	-0.2058	-0.102	0.0203	-0.0009	-2.5291	-0.3747	-0.041	0.0537	-0.0023
		0.15	-1.7334	-0.3013	-0.0475	0.0378	-0.0016	-4.3057	-0.4735	0.1327	0.0802	-0.0037
		0.20	-2.3916	-0.3683	0.0074	0.052	-0.0023	-5.2106	-0.5184	0.2235	0.0935	-0.0045
1.25	12	0.05	-0.2834	-0.0768	-0.0506	0.0047	-0.0002	-0.7635	-0.1342	-0.0414	0.0143	-0.0006
		0.10	-0.702	-0.1403	-0.0273	0.0137	-0.0006	-1.4303	-0.2518	0.0127	0.036	-0.0015
		0.15	-1.0445	-0.2124	-0.0006	0.026	-0.0011	-2.0337	-0.3081	0.0793	0.05	-0.0022
		0.20	-1.2285	-0.2645	0.0148	0.0355	-0.0014	-2.1844	-0.3197	0.0997	0.0538	-0.0024
	15	0.05	-0.367	-0.0989	-0.0655	0.0077	-0.0004	-1.004	-0.1744	-0.0561	0.021	-0.0009
		0.10	-0.928	-0.1792	-0.0326	0.02	-0.0008	-2.2396	-0.3142	0.0536	0.0492	-0.0022
		0.15	-1.4212	-0.258	0.003	0.034	-0.0014	-3.0619	-0.3718	0.1379	0.0644	-0.003
		0.20	-1.9914	-0.3094	0.0607	0.0457	-0.002	-3.4469	-0.4025	0.1846	0.073	-0.0035
	18	0.05	-0.4507	-0.1156	-0.0806	0.0096	-0.0004	-1.3417	-0.1997	-0.0643	0.0251	-0.0011

0.10	-1.209	-0.2033	-0.0402	0.0233	-0.001	-3.2993	-0.3656	0.0955	0.0605	-0.0027
0.15	-2.2649	-0.2943	0.0553	0.0424	-0.0019	-4.995	-0.4615	0.2714	0.0872	-0.0042
0.20	-3.0945	-0.3492	0.1369	0.0559	-0.0026	-6.3756	-0.4962	0.429	0.1019	-0.0053