Ultra-wideband-based navigation for unmanned aerial vehicles

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Ultra-wideband-based Navigation for Unmanned Aerial Vehicles

Kexin Guo

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2018

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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郭克信

.....
Kexin Guo

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Lihua Xie

Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following peer-reviewed journal(s) where I was the first and/or corresponding author.

Chapter 3 is published as K. Guo, Z. Qiu, C. Miao, A. H. Zaini, C.-L. Chen, W. Meng, and L. Xie, “Ultra-wideband-based Localization for Quadcopter Navigation,” *Unmanned Systems*, vol. 4, no. 01, pp. 23-34, 2016.

The contributions of the co-authors are as follows:

- Prof. L. Xie provided the initial project direction and edited the manuscript drafts.
- Dr. Z. Qiu assisted in the analysis of the theory and carried out the outdoor tests.
- I performed all the laboratory work at the School of Electrical and Electronic Engineering and co-designed the experiment with Dr. C. Miao. I also analyzed the data.
- Mr. A. H. Zaini assisted in the flight experiments for safety and repaired quadcopters.
- Dr. C.-L. Chen assisted in the debugging of codes.
- I prepared the manuscript drafts. The manuscript was revised by Dr. W. Meng.

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The contributions of the co-authors are as follows:

- Prof Xie suggested the research topic and edited the manuscript drafts.
- I wrote the drafts of the manuscript. The manuscript was revised together with Dr. Z. Qiu and Dr. W. Meng.
- I performed all the real-world tests, collected data and conducted data evaluation.

- Dr. Z. Qiu conducted the first part of simulation and provided the idea of motion constraints.
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- I wrote the drafts of the manuscript. The manuscript was revised together with Dr. X. Li.
- Dr. X. Li assisted in the proof of the stabilization of the control system.

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 Date

郭克信

 Kexin Guo

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Abstract

Micro unmanned aerial vehicles (UAVs) play more and more important roles in both civilian and military applications. Currently, the navigation and control of UAVs is critically dependent on the localization service provided by the Global Positioning System (GPS), which suffers from the multipath effect and blockage of line-of-sight, and fails to work in an indoor, forest or urban environment. Therefore, how to achieve positioning in a spatially restricted area or dense environment, say woods, multi-functional office or urban canyons, with sufficient localization accuracy that can support autonomous flight is a worthy research topic. This thesis concentrates on GPS-denied localization for UAVs by leveraging a radio frequency (RF) technology - ultra-wideband (UWB) signal. We shall devise algorithms and develop systems for localization and apply the location estimates to multi-UAV navigation and formation control.

In the first part of this thesis, we establish a localization system for UAVs based on UWB ranging measurements to enable each UAV to estimate its own global position in a customized reference frame. To achieve the localization, a UWB module is installed on the UAV to actively send ranging requests to some fixed UWB modules at known positions (a.k.a., anchor nodes). Once a distance is obtained, it is calibrated first and then goes through outlier detection before being fed to a localization algorithm. The localization algorithm is initialized by trilateration and sustained by the extended Kalman filter (EKF). Afterwards, the position and velocity estimates produced by the algorithm will be further fed to the control loop to aid the navigation of the UAV. Flight tests in different environments have been conducted to validate the performance of our UWB-based localization system with anchor nodes.

However, in many operation environments, it is impractical to have fixed anchor nodes due to large operational scale and area. Moreover, the accuracy of anchors' positions will greatly affect the positioning accuracy of UAVs. To reduce human labor and human-induced errors, in the second part of this thesis, we study an active relative localization (RL) with single landmark problem by using UAV's self-displacements and UWB ranging measurements. Note that in practice, a UAV may only be allowed to travel a small distance to fulfill the RL due to the environment or task constraint. To achieve an accurate RL estimate, we find a lower bound of mean square error in terms of this small distance and the sample size. As revealed from the lower bound, UAV only needs to enlarge the ranging span from the starting point to the ending point and increase the number of rangings to reduce the estimation error of the initial relative position, which is related to and extends the existing results of unconstrained optimal sensor placement. In this light, we design an algorithm to actively reduce the localization error by incrementally enlarging the ranging span, and apply EKF to account for the noise in the displacement measurements. Simulations and flight experiments have been conducted to validate our proposed RL strategy.

With the booming development of low-cost micro UAV such as quadcopter, multi-UAV systems and relevant applications have been extensively studied recently, and most of the proposed multi-agent controllers rely on inter-agent relative positions which are assumed to be measurable. However, in practice, the relative position is difficult to obtain and so far there exists no commercial product for UAV to measure this information. In addition, most of existing formation experiment results including both simulation and actual flight still depend on some external infrastructure for positioning. Therefore, the third and fourth part of this thesis puts forth a simultaneous infrastructure-free cooperative RL and distributed formation control strategy for UAVs in GPS-denied environments. Instead of estimating relative coordinates by detecting specific patterns using image processing methods, an on-board UWB ranging and communication (RCM) network is adopted to sense both the inter-UAV distance and exchange information for RL estimation. Without any

external infrastructures prepositioned, each UAV cooperatively performs the proposed consensus-based fusion method, which fuses the developed direct and indirect RL estimates, to generate the relative positions to its neighbors in real time despite the fact that some UAVs may not have direct range measurements to their neighbors. The RL estimates together with the relative velocity and inter-UAV distance measurements are used to control a UAV swarm. Both the cooperative RL and the formation control are implemented in a distributed fashion. Extensive real-world flight tests corroborate the merits of the developed simultaneous RL and formation control system.

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List of Symbols

$\mathbb{N}$	the set of natural numbers
$\mathbb{N}^+$	the set of positive natural numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^n$	the set of n -dimensional real column vectors
$\mathbb{R}^{m \times n}$	the set of $m \times n$ -dimensional real matrices
$\mathbb{C}$	the set of complex numbers
$\mathbb{Z}^+$	the set of positive integer numbers
$ \mathcal{S} $	the cardinality of set $\mathcal{S}$
$\mathbf{0}_{m \times n}$	an $m \times n$ matrix with all entries being zero
$\mathbf{I}_N$	the N -by- N identity matrix
$\ \mathbf{A}\ $	the Euclidean norm of matrix $\mathbf{A}$
$\mathbf{A}^\top$	the transpose of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	the inverse of matrix $\mathbf{A}$
$\text{null}(\mathbf{A})$	the null space of matrix $\mathbf{A}$
$\mathbb{E}\{\cdot\}$	the expectation operator
$\otimes$	the Kronecker product
$\underline{d}$	the measurement of d

List of Acronyms

UWB	ultra-wideband
UAV	unmanned aerial vehicle
UGV	unmanned ground vehicle
TW-ToF	two-way time of flight
RL	relative localization
TDMA	time division multiple
NLoS	non-line of sight
MSE	mean square error
RMS	root mean square
NLS	nonlinear least squares
RCM	ranging and communication

Chapter 1

Introduction

1.1 Motivation and Objective

Recent years have witnessed the surge of research interest on micro unmanned aerial vehicles (UAVs). Compared with mobile ground robots, UAVs have the aerial maneuvering ability to easily avoid obstacles and acquire an excellent bird's eye view, which provides a great convenience in mapping and monitoring tasks [1]. At a small size, micro UAVs are able to navigate through spatially restricted areas, such as forests [2], urban canyons [3] and indoor environments [4], while still maintain the lowest disturbance to environment and the inhabitants therein. Thus, micro UAVs play crucial roles for both civilian and military purposes [5], like reconnaissance for search and rescue [6], environment monitoring [7], security surveillance [8], inspection [9], law enforcement [10], etc. Specifically, quadcopters are selected to be the universal research platform due to its low cost and availability of various sizes, as well as easy maintenance.

Precise localization is critical in achieving autonomous navigation of micro UAVs. In an outdoor environment, space-based satellite navigation systems such as GPS are able to provide an absolute position for the UAV with an approximate error of 2 - 5m in civilian applications. With a ground-based reference station, Differential-GPS is able to achieve less than 1m accuracy in general. Although GPS is the

primary choice for position estimations in outdoor environments, it is not able to provide sufficient localization accuracy in certain GPS-denied environments, say urban canyons, forests or multi-function offices, which suffer from non-line of sight (NLoS) transmission channels between satellites and a receiver [11]. Thus, the ability to locate UAVs in GPS-denied environments remains a substantial challenge, posing the major bottleneck for UAV extensions. In this case, some other localization systems [12] are needed to replace GPS for the navigation of UAVs, and their localization accuracy needs to reach a certain level in order to support autonomous flight in required environments.

Over the recent decade there has been recurring interest in multi-UAV systems and relevant applications. Compared with a single UAV, a UAV team is able to provide better sensing and actuation capabilities under various situations and environments, and carry out complex tasks that require cooperation. Therefore, multi-UAV systems are promising and of great significance in cooperative surveillance, cooperative search, cooperative map building, forest monitoring and disaster assistance. In these cooperative tasks, relative positions between UAVs are pre-requisite for most of formation control methods [13–18]. Moreover, the UAVs applied for formation should be designed to be as efficient as possible. This means that payload, sensing and computational capabilities are limited. Therefore, how to simultaneously achieve relative position estimates with less resources and perform formation flight is a worthy research topic.

One simple method to obtain relative positions is to construct a communication network in which each UAV knows its own global position in a shared frame and communicates it with others. The UAVs can infer the relative estimates to their neighbors based on the shared data. Formation experiments in this manner, particularly those involving quadcopter UAVs, already exist in both simulation and actual flight [14, 19–22], but UAVs therein must depend on some external infrastructure for positioning. Examples include GPS [19, 20], motion tracking systems [14, 21], and radio-based positioning with anchors such as ultra-wideband (UWB) networks [22].

However, infrastructure dependence is disadvantageous. GPS is unreliable in cluttered urban environments, and the aforementioned alternatives require careful setup in the operation area and have limited ranges. In addition, the accuracy of the infrastructures (e.g., positions of anchors, camera calibration) seriously affects the positioning accuracy. Especially, in some unknown environments such as forests, it is difficult to set up such infrastructure. Therefore, these infrastructure-dependent positioning systems are unsuitable for exploration tasks in an unknown environment. Even in [19], where the formation is distance-based and thus intended for use with inter-UAV range measurements without infrastructure, the experiment reported was still GPS dependent.

A straightforward solution to realize RL is to extract the distance and bearing information by using cameras [23]. However, cameras can only operate within a limited range and suffer greatly from occlusion and unfavorable conditions. On the other hand, distance measurement can be obtained from different types of sensing information over a much longer range, e.g., UWB signals and received signal strength indicator (RSSI). In particular, the UWB technology stands out in accurate ranging due to its ability to alleviate multi-path effect. Hence, it is clear that distance-based RL can be applied in a more general setting and deserves more attention.

To sum up, our research interests focus on UWB-based navigation for UAVs, where a distributed formation algorithm will be investigated followed by the exploration of UWB-based self and relative localization for multiple UAVs.

1.2 Major Contributions of the Thesis

The main contributions of the thesis are as follows:

- UWB-based self-localization with anchor nodes. Firstly, a commercial UWB module is selected for the application of UAV autonomous flight in GPS-denied environments. To study the specification of the UWB module and

achieve accurate range measurements, we conduct numerous calibration experiments and collect thousands of data in our laboratory, corridor, garden and forest environments. Calibration results will be illustrated in Section 3.5. With the integration of UWB and UAVs, each UAV is able to actively send ranging requests to UWB anchor nodes at known positions to obtain distance measurements. Then, two real-time localization algorithms are proposed and applied on UAVs. Considering the asynchronism of distance measurements from different anchor nodes, trilateration method is utilized during the initialization phase of flights. Once an initial estimate is obtained by the proposed trilateration, an extended Kalman filter (EKF) based method is proposed to estimate the position and velocity of a UAV. The position and velocity estimates will be updated by single calibrated distance in each measurement step and further fed to the control loop to navigate the UAV. Various experiments in different environments are conducted to demonstrate the performance of the proposed UWB-based self-localization through single and multi-UAV flights, see Chapter 3.

- UWB-based relative localization with single landmark. Firstly, we focus on the active relative localization problem of a moving UAV to a static UAV on 2D plane, using the self-displacements and distance measurements without any priori knowledge. Then, some ranging path is designed to effectively reduce the localization error. In practice, since energy saving is of critical importance, the initial estimation process should be as short as possible especially for a multi-UAV system. The case of a moving UAV under motion constraint is also considered, namely that the UAV can only move in a limited range. Given the maximum allowed moving distance, we manage to find a lower bound of mean square error (MSE) in terms of this maximum moving distance and the sample size. In the presence of displacement noise, we use EKF to deal with the relative localization (RL) problem, which can also be applied to a disturbed beacon such as a hovering UAV. A series of simulation tests

and flight experiments are conducted to demonstrate our proposed relative localization approach, see Chapter 4.

- Infrastructure-free cooperative relative localization. Firstly, an onboard UWB ranging and communication (RCM) network is designed for both distance measurement and inter-UAV communication. Without any external infrastructures prepositioned, a persistent excitation-based RL estimator is proposed to estimate the relative positions between UAVs and their neighbors. Then, a consensus-based RL fusion method is proposed to increase the robustness of the RL estimation. By fusing the direct and indirect RL estimates, UAVs can obtain continual and reliable RL estimates in real time despite the fact that some UAVs may not have direct range measurements to their neighbors. In addition, both noise-free and noise-corrupted cases are considered for the proposed RL methods, where the stability of RL estimation and boundedness of RL error are discussed accordingly. Numerous simulations and flight experiments are carried out to corroborate the merits of the developed infrastructure-free cooperative RL system, see Chapter 5.
- Distributed formation control using RL estimates. Firstly, a simultaneous infrastructure-free cooperative RL and distributed formation control strategy is proposed. Based on the proposed RL methods in Chapter 5, each UAV is able to obtain the relative positions to its neighbors without any external infrastructures. Combined with the inter-UAV distances and relative velocity estimates, these RL estimates are directly fed into our designed distributed formation control law to control a UAV swarm to achieve formation flights. Secondly, by taking into account the measurement noises and RL estimation errors, the stability of formation control is studied. Both the cooperative RL and formation control are implemented in a distributed fashion. Extensive simulations and real-world flight tests validate the effectiveness of our proposed system, see Chapter 6.

1.3 Organization of the Thesis

The remainder of this thesis is organized as follows:

- *Chapter 2* presents a literature review on GPS-denied localization techniques and algorithms.
- *Chapter 3* describes our proposed UWB-based self-localization with anchor nodes.
- *Chapter 4* introduces our proposed UWB-based relative localization with single landmark under motion constraint.
- *Chapter 5* presents our proposed UWB-based infrastructure-free cooperative relative localization.
- *Chapter 6* introduces our proposed distributed formation control using RL estimates.
- *Chapter 7* presents conclusions and future work.

Chapter 2

Literature Review

2.1 Overview of GPS-denied Localization Technologies

There is a large body of work in the GPS-denied localization field, ranging from infrared (IR), radio frequency identification (RFID) [24, 25], wireless local area networks (WiFi) [26, 27] and motion capture system (e.g., VICON^{®1}) to UWB [28]. Here we discuss the most applicable works for UAV navigation in consideration of high localization accuracy, real-time update rate, dynamic response, applicability to complex environments and simplicity of setting up.

2.1.1 Infrared

The wavelength of IR ($700nm - 1mm$) is between that of visible light ($390nm - 700nm$) and terahertz radiation. Hence, one advantage of IR technology is less intrusive than a vision-based positioning system. IR signals can be exploited in three general methods: one is the use of active beacon [29]; another is IR imaging

¹<https://www.vicon.com/>

based on natural radiation such as thermal [30]; the other one is IR imaging using artificial light sources [31].

Active beacons or natural radiation based systems can be used for rough position estimation or detecting the presence of a person. By contrast, the systems which are capable of detecting artificial IR light sources can reach sub-*mm* accuracy with range of 4*m*. Using reflecting IR light for communication and distance sensing between agents, a collective of 1024 Kilobot [32] robots executed self-assembly² at the swarm level [33].

2.1.2 Wireless Local Area Networks (WiFi)

Wireless Local Area Networks (i.e., WiFi) have been widely available in numerous commercial and residential buildings, and a WiFi receiver is essentially embedded in every mobile device. Therefore, it is low-cost and practical to exploit and develop a WiFi based positioning system to localize a mobile target carrying a WiFi receiver in GPS-denied environments.

By deploying specially designed indoor system Chronos, the MIT CSAIL group can achieve *dm*-level localization with a single WiFi access point and apply it to indoor personal quadcopter hovering³ within a safe distance radius to the controller in laboratory environment [34]. However, this WiFi system's signal processing part should be redesigned and reflushed with special techniques. Moreover it can only be applied indoor with pre-configuration.

2.1.3 Laser Scanner

Simultaneous localization and mapping (SLAM) technology is widely adopted for UAV flights in GPS-denied environment. Two steps are constituted: position estimation and map building. The key challenge to enable a UAV flight in GPS-denied

²<https://www.youtube.com/watch?v=JmyTJSYw77g>

³<https://www.youtube.com/watch?v=cJx7ewEyuzo>

environment is how to estimate its position and velocity by sensing unknown surroundings with enough update rate and sufficient accuracy to stably control the platform. Using a Hokuyo[®] 2D laser scanner⁴ cooperating with a laser-deflecting mirror and stereo cameras, the UAV in [35] autonomously explored and mapped unstructured and unknown environments⁵.

Generally, in the basic layer of a SLAM-based UAV system, the main function is to stabilize the UAV's pitch and roll through a very tight feedback of onboard IMU. At the navigation level, the UAV's relative motion is first calculated based on a high-resolution and efficient scan-matching algorithm and then these motion estimates will be fused with the IMU readings to generate UAV's states such as local position and velocity. An obstacle avoidance module can be involved in this system level to ensure that the UAV flies outside the dangerous region. The highest layer provides path planning based on an updated consistent global map of the world to navigate a UAV toward unexplored regions. This constructed map helps correct real-time state estimates by improving the data fusion filter and thus makes the estimates remain globally consistent. However, there still exist some fundamental limitations of using a 2D laser scanner for UAV's motion estimation: a) slanted walls in indoor environment; b) featureless environments, say featureless hallways (with no doors, windows, shelves, etc.); c) leaves and branches of trees (as opposed to vertical tree trunks); d) open areas that are beyond the maximum range of the sensor. Therefore, an integration of other sensing modalities is required to enable a system to operate in such environments. Currently camera sensors serve as an aiding sensor to reduce the chance where the system would fail.

2.1.4 Cameras

In recent years, vision-based research results have proliferated, which makes cameras become a widely used technique for positioning since the basis for a large variety

⁴<https://www.hokuyo-aut.jp/>

⁵<https://www.youtube.com/watch?v=bviYpMkzuSY>

of UAV navigation tasks is the availability of a self-motion estimate as well as a dense environment map representation for obstacle avoidance [36] and trajectory planning⁶.

Optical indoor positioning systems can be categorized into static camera system and self-motion system, where the first one aims to locate moving objects in images such as VICON[®] system and the second one is to locate the mobile camera itself. With the advent of 3D cameras (e.g., Intel RealSense R200 [37]), depth information can be obtained indirectly by making use of the motion of a monocular camera or through a stereo camera. Typically, vision-based technologies are sensitive to light conditions and dynamic environments. These limits make the vision-based UAVs can only operate in the restricted environments, and the heavy computing requirement makes it difficult to miniaturize the platform.

2.1.5 Ultra-wideband (UWB)

UWB is a short duration, pulsed RF technology which aims to achieve as the highest bandwidth at the lowest center frequency as possible. Benefitting from its high bandwidth, the UWB technology can be used for ranging, radar and communication applications. UWB sensors allow high accuracy of range measurements. Nowadays, because of its small size and light weight, a low-power UWB module could easily be installed on a UAV. This makes UWB a promising alternative for autonomous navigation in an unknown environment.

Basically, the more bandwidth a system achieves the better it performs. Due to UWB short pulses and increased bandwidth, UWB modules hold the properties of strong multipath resistance and to some extent penetrability for building material. With centimeter-level ranging accuracy, UWB technique is able to be applied for localization.

⁶<https://www.youtube.com/watch?v=iksfHQkkq88>

The UWB radio modules can estimate inter-module distance by measuring the transmission and reception time of UWB pulses [38–40]. In contrast with other RF systems, UWB pulses can be transmitted between 3.5 and 5.3 GHz and have an RF bandwidth of 1.4 GHz. The increased bandwidth not only avoids the interference with other types of RF signals, e.g., remote control signal and WiFi signal, but also allows the UWB signal to easily go through walls and other obstacles. Besides, the ultra-short duration pulses permit an easy filtering method to deal with the multipath effect, as well as an accurate determination of Time of Flight (ToF). Some commercial UWB systems exploit a peer-to-peer Two-Way Time of Flight (TW-ToF) to simultaneously achieve precise ranging and data exchange, which potentially provides more information for localization.

In terms of the advantages for implementation, UWB device is a capable system with an agile timing module which can provide system clock accuracy less than $10ps$ and this clock can be distributed to other UWB nodes wirelessly. This enables precise range measurements (PRM) and communications among UWB units. This advantage boosts the development of distance-based collaborative control that relies on accurate range measurements and desired communication graph. Fusing UWB signals with other sensors, say gyroscopes, accelerometers and altimeters, can improve the accuracy of GPS-denied localization [41] and support for multi-UAV localization using one-way UWB communication [42]. From the localization perspective, we summarize the different wireless technologies in maximum range, power consumption, advantages and disadvantages areas as shown in Table 2.1.

2.2 Overview of Distance-based Localization Algorithms

In a multi-UAV system, less computation, small and low cost platforms are generally favored. Thus, we focus on the problem of making each UAV in a multi-UAV

Table 2.1: Summary of different wireless technologies for localization [43]

Technology	Maximum Range	Power Consumption	Advantages	Disadvantages
Acoustics	Couple of meters	Low-moderate	Can be used for proprietary applications, can provide high accuracy	Affected by sound pollution, requires extra anchor points or hardware
RFID [44]	200m	Low	Consumes low power, has wide range	Localization accuracy is low
Bluetooth [45]	100m	Low	Low energy consumption, high throughput, low cost	Prone to noise, low localization accuracy
Ultrasound [46]	Couple-tens of meters	Low-moderate	Comparatively less absorption	High dependence on sensor placement
Visible Light [47]	1.4km	Relatively higher	Wide-scale availability, potential to provide high accuracy, multipath-free	Comparatively higher power consumption, range is affected by obstacles, primarily requires line-of-sight
IEEE 802.11n [48]	250m outdoor	Moderate	Widely available, high accuracy, does not require complex extra hardware	Prone to noise, requires complex processing algorithms
UWB [49]	10 – 100m	Moderate	Immune to interference, provides high accuracy	Requires extra hardware on different user devices, relative high cost

system estimate the local or relative position of itself depending only on distance measurements to pre-arranged anchors or their neighbors.

2.2.1 Self-localization with Anchor Nodes

In a spatially restricted area or dense environment, say woods or urban canyons, GPS based localization accuracy degrades to a level that it can no longer support autonomous flight. In this case, some other anchor-based localization systems [12] are needed to replace the GPS for the navigation of UAV. Typically, for a UAV team, the self-localization problem is to estimate the Euclidean coordinates of mobile UAVs by making use of the relative measurements (e.g., bearing, RSSI and distance) to several anchor nodes with known positions. Table 2.2 provides a summary of different GPS-denied localization techniques and discusses the advantages and disadvantages of these techniques.

Table 2.2: Challenges of GPS-denied localization and suggested solutions [43]

Technique	Advantages	Disadvantages
RSSI	Easy to implement, cost efficient, can be used with a number of technologies	Prone to multipath fading and environmental noise, lower localization accuracy, require fingerprinting
CSI : channel state information	More robust to multipath and indoor noise	Might require powerful computer and specially-tailored system
TDoA : time difference of arrival	Fingerprinting-free, does not require clock synchronization between the device and all of the reference nodes	Requires clock synchronization among all of the reference nodes, might require time stamps, prone to clock drift
AoA : angle of arrival	Can provide high localization accuracy, does not require any fingerprinting	Performance deteriorates with increase in distance between the transmitter and receiver, might require directional antennas, requires comparatively complex hardware and algorithms
ToF	High localization accuracy, fingerprinting-free	Requires time synchronization between the transmitters and receivers, might require time stamps and multiple antennas at the transmitter and receiver

Considering the accuracy and reliability of localization, most of self-localization for mobile agents adopt distance sensors [28, 50]. Different kinds of ranging techniques

have been proposed for the distance estimation [51] [52]: Time of Flight (ToF), Time of Arrival (ToA), and RSSI. In the ultrasound-based Cricket Location-Support System [53], once range measurements to the landmarks with known locations are obtained, it is able to estimate the locations of mobile agents. Furthermore, [54] shows how to obtain a finer grained position estimate by extending a similar setup. Without any knowledge of the fixed stationary anchor nodes, the localization problem has been investigated in [55, 56]. More importantly, unlike ground robots moving in 2D plane, the localization for UAVs requires high-precision and real time positioning. Drift of position estimates is more of concern, since the information is meant to be used for precise position control and perform formation without collisions over longer time intervals.

2.2.2 Localization with Single Landmark

In the case of a static beacon, the relative localization (RL) problem can be viewed as a source localization problem. Regarding the source location in 3D space as parameters to be estimated and assuming exact distance and self-position measurements, [57] proposed a continuous-time adaptive method to achieve the exponential convergence of the location estimation, given that the mobile agent regularly avoids the planar motion.

Due to limited sensing capability and various environments, the topology of measurements and communications among the agents and the landmark is time-varying. [58] developed a consensus-like fusion scheme together with a continuous-time estimator to calculate the relative position of mobile agents with respect to a single landmark. The distances and their change rates to close-by agents, as well as each agent's own velocity are necessary for estimate update. However, the relationship between the accuracy of localization and the trajectories of mobile agents was not discussed. [59] proposed a constrained moving horizon estimation (MHE) based localization algorithm and analyzed how various constraints would impact computational complexity and estimator accuracy. Besides, a semidefinite programming (SDP) based

initial pose estimation was developed by using convex relaxation. From the perspective of Fisher Information Matrix (FIM), constrained optimization was examined to evaluate why and how multi-robot information and constraints were able to reduce uncertainties.

By including the distance measurement as an augmented state, [60] transformed the original nonlinear system of the source location into a linear time-varying system and applied Kalman filter for its location estimation. The necessary motion to guarantee a convergent estimate was given by analyzing the observability of the time-varying system and Kalman filter has been widely studied by researchers [61, 62]. A similar method was also adopted in the discrete-time case [63]. Based on the observability analysis of the linearized system, [64] designed control laws to maintain a full rank observability matrix facilitating the RL, and two common cases of dive and translation were specifically studied. With a periodic motion of the mobile agent, a recursive least squares fading memory filter was applied to the squared distance measurements in [65]. However, note that the above approaches require the agent to move a long distance before getting a convergent solution, and the design of a ranging path to effectively reduce error is not considered. Also, the squaring of distance measurements may result in a larger estimation error.

Some results on optimal sensor placement for a static target can be taken into account when designing the ranging path. Martinez [66] examined the determinant of FIM and characterized the global minima in 2D case, and a motion coordination algorithm was designed accordingly to steer the mobile sensors to the optimal deployment. The case of a partially controllable sensor network, namely when only a part of nodes can be maneuverable, was studied in [67], with the full analysis of the global minima found in [68]. The optimal placement in 3D space has also been explored. [69] considered the optimal configuration of a surface sensor network to localize an underwater target with distance-dependent measurement noise. A more general case was studied in [70], where sensors are grouped by the path-loss exponent (which is incorporated in the measurement noise) and different groups are located at different surfaces.

2.2.3 Infrastructure-free Cooperative Relative Localization

Based on rigidity theory, a robust quadrilaterals algorithm [71] was proposed to cope with landmark-free localization using distance measurements between agents. This method seems close in spirit to our work. However, this algorithm aims primarily at static sensor networks. [72] proposed a global optimal solution to this problem by formulating localization as a weighted least squares estimation problem. An early cooperative localization [73] required bearing and/or range sensors, and advocated for an approach where robots are grouped as mobile robots and stationary landmarks according to their roles. A similar approach was proposed in [74] using range sensors but requiring global all-to-all communication and sensing towards the landmarks. However, these approaches neglect the selection mechanism between mobile robots and stationary landmarks.

Two distributed localization algorithms were proposed in [75]. The first algorithm relies on a nonlinear least squares (NLS) based strategy to enable robots to estimate the relative pose of neighbor robots without restrictions on the motion whatsoever. Without enforcing motion coordination, distance-only based algorithm cannot recover the relative pose of neighboring agents in degenerate motions which cause flip or rotation ambiguity. The author claims that, in practice, agents' motions are random, not precisely degenerate motions, so the degenerate motions are rare indeed. The second algorithm leverages distributed computing theory to coordinate which robots are stationary and which robots are mobile, rotating robots between these roles in a fair and distributed fashion. This algorithm requires cooperation between motion-control strategy and standard trilateration techniques to estimate the relative position of close-by robots. To resolve the flip ambiguity, an agent needs to repeat executing cosine law and check for consistency of the predicted position and orientation, as well as avoid collinear motions. Compared to the first algorithm in the case that distance measurements and odometry estimates are subject to noise, the second algorithm is less sensitive to noise and eases computation.

A convex optimization based RL was formulated in [59]. However, the method has

high computational complexity and requires centralized implementation. Source localization with sampled range measurements was proposed in [76], where all the mobile agents cooperatively estimate their relative positions to a static source with unknown position. However, it is still an infrastructure-dependent method and no practical tests were conducted to validate their approach. An integrated RL and leader-follower formation control in continuous time was proposed in [77]. Numerical simulations were carried out but there is no discussion on the influence of measurement noises. In [78], simultaneous velocity and relative position estimation with application to formation control is studied, but it requires that each agent must conduct a combined linear and circular motion during the entire formation period. Accurate angular velocity required in the circular motion, makes it impractical for application in a real platform. All these observations motivate us to investigate a combined system of RL and formation control, which should be practical and efficient for real-world applications.

2.3 Conclusion

In this chapter, we have surveyed the current major GPS-denied localization technologies and algorithms. Among various GPS-denied localization techniques, IR, WiFi, Cameras and UWB have been introduced for their successful application on mobile agents. The corresponding distance-based localization algorithms can be classified into three categories: self-localization with anchor nodes, localization with single landmark and infrastructure-free cooperative relative localization. The challenges to localization along with the proposed solutions are summarized in Table 2.3.

Table 2.3: Challenges of GPS-denied localization and suggested solutions [43]

Challenge	Description	Suggested Solution
Multipath & Noise	The presence of obstacles and different interfering signals can affect the performance of the GPS-denied localization systems	Utilize schemes that are energy efficient, less complex and robust enough to minimize the adverse effects of multipath and noise. Chirp spread spectrum is one of the probably techniques robust to multipath and noise
Environment Dynamics	The change in the environment where the localization system is used such as the number of people, the presence of different equipment such as cupboards, shelves etc. makes it much more challenging to precisely obtain user's position	Do not rely on approaches such as fingerprinting, but rather use the improved signal processing and superior computing power of the user devices and servers to obtain metrics such as CSI that are not highly influenced by environment dynamics. Furthermore, always design the system keeping in mind the worst case
Energy Efficiency	Real-time and highly accurate localization might drain user device and reference node's power. However, for wide-scale adoption of any indoor localization system, the system must be highly energy efficient	Use less complex and energy efficient algorithms. Offloading complex algorithms to some servers or cloud based platform is also more energy efficient when compared with using user device
Cost	The use of extra hardware or proprietary systems for GPS-denied localization is a major hurdle to its adoption particularly when it comes to small business which might not be able to afford them	Use existing architecture for providing localization services without requiring any extra hardware
Lack of Standardization	Currently there is no standard that can govern GPS-denied localization research. Therefore, there are a number of orthogonal solutions	There is a need to define a standard for future localization. The standard should take into account different applications and requirements and accordingly set the benchmark or minimum requirements

Chapter 3

UWB-based Self-localization with Anchor Nodes

3.1 Introduction

In this chapter, we shall study and explore the potential of using the UWB technology for UAV localization in GPS-denied environments. As introduced in Section 2.1.5, several advantages of the UWB technology, such as high accuracy of range measurements, light weight, low-power, and portable size of UWB modules, as well as the properties of strong multipath resistance, make it superior to other wireless communication technologies for UAV positioning. In order to achieve high-precision distance estimation using UWB in various environments, we carry out a series of calibration experiments in indoor, semi-outdoor and forest environments. Based on this accurate range information, we propose a trilateration and EKF combined localization algorithm. As presented in Section 3.3.1, an initial position and velocity estimate is first calculated through trilateration method and then the dynamic positioning estimate is sustained by EKF in Section 3.3.2.

This chapter is organized as follows. The Two-Way Time-of-Flight (TW-ToF) ranging technique and calibration of the UWB rangings are introduced in Section 3.2.

After that, a UWB localization algorithm is proposed in Section 3.3. The hardware configuration of the navigation system is shown in Section 3.4, and experiment evaluations of the UWB localization in different environments, as well as related autonomous quadcopter flights in indoor and outdoor environments, are presented in Section 3.6. We conclude our work in Section 3.7.

3.2 TW-ToF Ranging and Calibration of UWB Measurement

In this section we shall introduce the TW-ToF ranging technique implemented on the UWB module for the localization in GPS-denied environments. This ranging technique works in an active way, namely a mobile UWB sends range requests actively to a responder for ranging estimation. Besides, we shall also discuss the calibration of UWB measurement.

3.2.1 TW-ToF Ranging

Unlike ultrasonic or laser-based signals, UWB pulses can travel through walls and dense foliage. Compared with conventional narrow-band RF, these pulses do not suffer from errant multi-path reflections, since the most direct path through the environment is measured based on TW-ToF technique (Fig. 3.1).

It is clear from Fig. 3.1 that the two UWB modules involved in ranging are asymmetrical. Specifically in our localization context, the module installed on the quadcopter (denoted as Q), will actively send its ranging request to the anchor module at known position (denoted as A). The anchor will automatically respond to any ranging request it receives from the requester. As shown in Fig. 3.1, the on-board UWB module Q initiates the TW-ToF by transmitting a request packet at $Q_{M_0}^{T_x}$. Anchor A receives the pulse at $A_{M_0}^{R_x}$ after a delay of f . After calculating the leading edge offset with a known delay δ_A , anchor A responds by transmitting a pulse at

Figure 3.1: UWB range measurement based on TW-ToF protocol

$A_{M_1}^{Tx}$. Q receives it after a delay of f at $Q_{M_1}^{Rx}$. The pulse transmission time f between Q and A is calculated by

$$f = (Q_{M_1}^{Rx} - Q_{M_0}^{Tx} - \delta_A)/2.$$

Furthermore, the range d_{QA} between Q and A is given by

$$d_{QA} = f * c_0,$$

where c_0 is the speed of light in vacuum with the approximate constant of $3 \times 10^8 m/s$.

Using TW-ToF the time taken by the signal to travel from a transmitter Q to a receiver A and back is measured. TW-ToF avoids the need for time synchronization between the two modules, allowing its application in uncoordinated mesh networks with the advantage of low complexity and cost. As a drawback of this method, range measurements to multiple UAVs need to be carried out sequentially which may cause critical latencies, collisions or dropouts for applications where UAVs move quickly.

3.2.2 Calibration of UWB Measurement

Here we discuss a methodology to calibrate the UWB but defer the experiment results to Section 3.5. Assume that a UWB range r has been obtained, which can

be modeled as a simple linear function of the true distance d . More exactly, if we take into account the zero-mean random error ϵ , then the following holds:

$$r = ad + b + \epsilon. \quad (3.1)$$

The calibration problem is to estimate d from the range r .

A classical approach is adopted for the calibration, which firstly regresses the instrumental response r on the manipulated variable d and then solves for d . Specifically, given n samples (d_i, r_i) , $i = 1, \dots, n$ collected from different locations, the calibration can be conducted as follows [79]. Linear regression is firstly applied to n samples to obtain an estimate $\hat{a}$, $\hat{b}$ respectively for scaling factor a and bias b in the measurement model (3.1), as well as an estimate $\hat{\sigma}$ of the standard deviation of the error ϵ :

$$\begin{aligned} \hat{a} &= \frac{\sum_{i=1}^n (d_i - \bar{d})(r_i - \bar{r})}{\sum_{i=1}^n (d_i - \bar{d})^2}, \\ \hat{b} &= \bar{r} - \hat{a}\bar{d}, \\ \hat{\sigma} &= \frac{\sum_{i=1}^n (r_i - \hat{a}d_i - \hat{b})^2}{n - 2}, \end{aligned} \quad (3.2)$$

where $\bar{d}$ and $\bar{r}$ are respectively the mean of d_i 's and r_i 's, $i = 1, \dots, n$. After that, the ranging estimate $\underline{d}$ can be obtained as

$$\underline{d} = \tilde{a}\hat{r} + \tilde{b}, \quad (3.3)$$

with $\tilde{a} = \frac{1}{\hat{a}}$ and $\tilde{b} = -\frac{\hat{b}}{\hat{a}}$. It can be shown that [80] the estimate is asymptotically unbiased and normal in the case of a normal error ϵ , and the asymptotic mean square estimation error is approximated by

$$R = (\hat{\sigma}/\hat{a})^2. \quad (3.4)$$

Note that in the following sections, we will use the calibrated distance measurements

Figure 3.2: UWB-based self-localization workflow

from (3.3) to design our localization and control methods. The *underline* of each variable throughout this thesis represents its corresponding measurement.

3.3 UWB-based Self-localization

The workflow of the UWB localization system is shown in Fig. 3.2. To achieve the range-based localization, anchor nodes equipped with UWB module are firstly installed at known positions. Following the TW-ToF ranging method described in Section 3.2.1, the UWB module on the quadcopter is to actively send ranging requests to anchor nodes for distance measurement.

Once a distance is obtained, it will be calibrated by (3.3) where the calibration parameters $\tilde{a}$ and $\tilde{b}$ are determined by experiments as shown later in Section 3.5. The calibrated range then goes through the outlier detection before it is fed into

a localization algorithm to produce a position estimate. Note that trilateration is used to start the algorithm, whose output will serve as the initial state estimate for EKF. The follow-up localization is sustained by EKF in a recursive way.

Below we shall elaborate each part of our UWB-based localization system. The trilateration algorithm for the initialization phase is discussed firstly. In specific, the kinematic and measurement model is presented subsequently, followed by a comparison of two major recursive Gaussian filters, namely EKF and Unscented Kalman Filter (UKF). The equations of EKF are introduced later, based on which the outlier detection is mentioned.

3.3.1 Trilateration

Theoretically, trilateration method utilizes simultaneous distance measurements from several anchor nodes to determine the location of the mobile node. However, with the UWB ranging technique, these distances can only be retrieved in a cyclic fashion, which entails unsynchronous measurements and results in large localization error, especially when the mobile node is moving at a high speed. Hopefully, trilateration is still able to generate a rough estimate to initialize or restart the UWB localization. As is known, it is necessary to obtain ranges from at least 4 non-coplanar anchors as well as their coordinates to uniquely determine a target position in a 3D space. Denote the known position of the i -th anchor node as $\mathbf{p}_i^a = [x_i^a, y_i^a, z_i^a]^\top$ and the corresponding calibrated distance as d_i , then we have

$$d_i = \sqrt{(x - x_i^a)^2 + (y - y_i^a)^2 + (z - z_i^a)^2}, \quad i = 1, \dots, n \quad (3.5)$$

where (x, y, z) is the target location to be found.

Many different methods have been proposed to solve (3.5) [81–83]. Typically, it can be solved by a combination of linear and nonlinear least square methods [84]. In fact, by squaring both sides of (3.5) and subtracting

$$d_1^2 = (x - x_1^a)^2 + (y - y_1^a)^2 + (z - z_1^a)^2,$$

we get $n - 1$ linear equations as

$$d_i^2 - d_1^2 - d_{i1}^2 = 2[(x_1^a - x_i^a)x + (y_1^a - y_i^a)y + (z_1^a - z_i^a)z] \quad (3.6)$$

with $d_{i1}^2 = (x_i^a - x_1^a)^2 + (y_i^a - y_1^a)^2 + (z_i^a - z_1^a)^2$, $i = 2, \dots, n$, which can be solved by least squares. On the other hand, the nonlinear least squares problem related with (3.5), i.e.,

$$\min_{(x,y,z)} \sum_{i=1}^n [d_i - \sqrt{(x - x_i^a)^2 + (y - y_i^a)^2 + (z - z_i^a)^2}]^2,$$

can be efficiently solved by Gauss-Newton method [85] with an acceptable initial guess provided by solving the linear least squares problem (3.6).

3.3.2 Kinematic and Measurement Models

For the UWB localization, the position $\mathbf{p} = [p_x, p_y, p_z]^\top$ and velocity $\mathbf{v} = [v_x, v_y, v_z]^\top$ of the mobile UAV are to be estimated. Assume that the acceleration $\mathbf{a} = [a_x, a_y, a_z]^\top$ remains constant in $[t_{k-1}, t_k]$, we can model the discrete-time dynamics of the mobile UAV as

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix}_{k+1} = \begin{bmatrix} \mathbf{I} & \Delta t_k \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix}_k + \begin{bmatrix} \frac{1}{2} \mathbf{a}_k (\Delta t_k)^2 \\ \mathbf{a}_k \Delta t_k \end{bmatrix}. \quad (3.7)$$

In the case when the acceleration is not available, $\{\mathbf{a}_k\}$ can be treated as a white noise sequence with the covariance $\mathbb{E}(\mathbf{a}_k \mathbf{a}_k^\top) = \text{diag}(\sigma_x^2, \sigma_y^2, \sigma_z^2)$. Note that σ_* can be set as the maximum acceleration in the $*$ -direction ($*$ = x, y, z) for a conservative but consistent estimation.

On the other hand, the measurement equation is given by

$$\underline{d}_k = \|\mathbf{p}_k - \mathbf{p}^a\| + \epsilon_{d,k}, \quad (3.8)$$

where $\underline{d}_k$ is the calibrated distance from the UWB ranging r_k by (3.3), $\mathbf{p}^a = [x^a, y^a, z^a]^\top$ is the anchor's position and $\epsilon_{d,k}$ is the measurement noise whose vari-

ance R is given by (3.4). Note that the linearization around $\hat{\mathbf{p}}_k$ can be written as $\underline{d}_k \approx \hat{d}_k + \frac{1}{\hat{d}_k}(\hat{\mathbf{p}}_k - \mathbf{p}^a)^\top (\mathbf{p} - \hat{\mathbf{p}}_k) + \epsilon_{d,k}$ and $\hat{d}_k = \|\hat{\mathbf{p}}_k - \mathbf{p}^a\|$.

To sum up, the process dynamics and the measurement equation for the UWB localization can be found as

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \boldsymbol{\epsilon}_k, & \mathbb{E}(\boldsymbol{\epsilon}_k \boldsymbol{\epsilon}_k^\top) = \mathbf{Q}_k, \\ \underline{d}_k = \|\mathbf{p}_k - \mathbf{p}^a\| + \epsilon_{d,k}, & \mathbb{E}(\epsilon_{d,k} \epsilon_{d,k}^\top) = R, \end{cases} \quad (3.9)$$

where $\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \end{bmatrix}$, $\mathbf{A}_k = \begin{bmatrix} \mathbf{I} & \Delta t_k \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$, and $\mathbf{Q}_k = \begin{bmatrix} \frac{1}{4}(\Delta t_k)^4 & \frac{1}{2}(\Delta t_k)^3 \\ \frac{1}{2}(\Delta t_k)^3 & (\Delta t_k)^2 \end{bmatrix} \otimes \text{diag}(\sigma_x^2, \sigma_y^2, \sigma_z^2)$.

3.3.3 Range-based Localization using EKF and UKF

The localization of moving objects is usually achieved by recursive Bayesian filters. Since the seminal paper of Rudolph Kalman [86], the Kalman filter has been widely used in the state estimation of discrete-time linear systems with Gaussian noise. It provides a recursive state estimation from measurements, and is also able to fuse asynchronous measurements from different sensors as well.

For a general nonlinear dynamics, two major types of extension of Kalman filter can be applied: EKF and UKF. EKF is obtained by implementing the Kalman filter for a linearized system, and it similarly consists of the prediction from the process equation and correction from the measurement, as shown later in (3.10a) – (3.11e). These two phases are also comprised in UKF, however, UKF employs the unscented transform to choose a set of sample points (sigma points) for the state and observation prediction, as well as the covariance propagation. Compared with EKF, UKF is capable in dealing with highly nonlinear dynamics due to the unscented transform; nonetheless, the selection of sigma points relies on finding the matrix square root of the covariance, and hence entails more computational burden. In addition, the weights of sigma points have to be selected carefully and cannot be applied universally. The EKF and UKF algorithms for range-based localization are elaborated below.

3.3.3.1 EKF

Applying the EKF to the system (3.9) yields the localization equations as in (3.10a) – (3.11e), with $\hat{\mathbf{x}}_k^-$ and $\hat{\mathbf{x}}_k$ respectively being a priori and a posteriori estimate of the state at the time t_k , $\mathbf{P}_k^-$ and $\mathbf{P}_k$ the corresponding covariance of the estimation error.

Process update (“Prediction”)

$$\hat{\mathbf{x}}_k^- = \mathbf{A}_{k-1} \hat{\mathbf{x}}_{k-1}, \quad (3.10a)$$

$$\mathbf{P}_k^- = \mathbf{A}_{k-1} \mathbf{P}_{k-1} \mathbf{A}_{k-1}^\top + \mathbf{Q}_{k-1}. \quad (3.10b)$$

Measurement update (“Correction”)

$$\hat{d}_k = \|\hat{\mathbf{p}}_k^- - \mathbf{p}^a\|, \quad (3.11a)$$

$$\mathbf{H}_k = \frac{1}{\hat{d}_k} [(\hat{\mathbf{p}}_k^- - \mathbf{p}^a)^\top \quad \mathbf{0}]_{1 \times 6}, \quad (3.11b)$$

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^\top (\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^\top + R)^{-1}, \quad (3.11c)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\hat{d}_k - \hat{d}_k), \quad (3.11d)$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-. \quad (3.11e)$$

It’s clear that the implementation of the EKF above only requires a minimal amount of computation. Most of the calculation comes from the matrix multiplication and addition. Although the Kalman gain update (3.11c) involves a matrix inversion, it’s actually a scalar division due to the scalar distance measurement. Therefore the real time localization is readily achieved.

3.3.3.2 UKF

The UKF addresses the approximation issues of EKF. An unscented transformation (UT) [87] is adopted to calculate the statistics of the state $\mathbf{x}$ (seen as a random

variable, dimension L with mean $\hat{\mathbf{x}}$ and covariance $\mathbf{P}$) which undergoes a nonlinear transformation. To calculate the statistics of the outputs through this nonlinear transformation, $2L + 1$ *sigma vectors* $\mathcal{X}_{i,k}$ (with corresponding weights W_i) at time t_k are formed as following.

Sigma vectors

$$\mathcal{X}_{0,k} = \hat{\mathbf{x}}_k, \quad (3.12a)$$

$$\mathcal{X}_{i,k} = \hat{\mathbf{x}}_k + \left(\sqrt{(L + \lambda)\mathbf{P}_k} \right)_i^\top \quad i = 1, \dots, L, \quad (3.12b)$$

$$\mathcal{X}_{i,k} = \hat{\mathbf{x}}_k - \left(\sqrt{(L + \lambda)\mathbf{P}_k} \right)_{i-L}^\top \quad i = L + 1, \dots, 2L, \quad (3.12c)$$

$$W_0^{(m)} = \lambda / (L + \lambda), \quad (3.12d)$$

$$W_0^{(c)} = \lambda / (L + \lambda) + (1 - \alpha^2 + \beta), \quad (3.12e)$$

$$W_i^{(m)} = W_i^{(c)} = 1 / [2(L + \lambda)] \quad i = 1, \dots, 2L, \quad (3.12f)$$

where $\left(\sqrt{(L + \lambda)\mathbf{P}_k} \right)_i$ is the i th row of the matrix square root and $\lambda = \alpha^2(L + \kappa) - L$ is a scaling parameter. α determines the spread of the sigma points around $\hat{\mathbf{x}}$ and generally it is set to a small positive value (e.g., 10^{-3} adopted in our simulations in Section 3.3.4). The secondary scaling parameter κ is set to 0 and β is equal to 2 under the assumption of $\mathbf{x}$ corrupted by Gaussian white noise (see [88] for a detailed discussion of the UT).

Calculate sigma points

$$\mathcal{X}_{k-1} = [\mathcal{X}_{0,k-1}, \mathcal{X}_{1,k-1}, \dots, \mathcal{X}_{2L,k-1}]. \quad (3.13)$$

Process update

$$\mathcal{X}_k^- = [\mathbf{A}_{k-1}\mathcal{X}_{0,k-1}, \dots, \mathbf{A}_{k-1}\mathcal{X}_{2L,k-1}], \quad (3.14a)$$

$$\hat{\mathbf{x}}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{X}_{i,k}^-, \quad (3.14b)$$

$$\hat{\mathbf{P}}_k^- = \sum_{i=0}^{2L} W_i^{(c)} [\boldsymbol{\mathcal{X}}_{i,k}^- - \hat{\mathbf{x}}_k^-] [\boldsymbol{\mathcal{X}}_{i,k}^- - \hat{\mathbf{x}}_k^-]^\top + \mathbf{Q}_k, \quad (3.14c)$$

$$\mathcal{D}_k^- = [\|[\mathbf{I}_{3 \times 3}, \mathbf{0}_{3 \times 3}] \boldsymbol{\mathcal{X}}_{0,k}^- - \mathbf{p}^a\|, \dots, \|[\mathbf{I}_{3 \times 3}, \mathbf{0}_{3 \times 3}] \boldsymbol{\mathcal{X}}_{2L,k}^- - \mathbf{p}^a\|], \quad (3.14d)$$

$$\hat{d}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{D}_{i,k}^-. \quad (3.14e)$$

Measurement update

$$\mathbf{P}_{d_k, d_k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{D}_{i,k}^- - \hat{d}_k^-] [\mathcal{D}_{i,k}^- - \hat{d}_k^-]^\top + R, \quad (3.15a)$$

$$\mathbf{P}_{x_k, d_k} = \sum_{i=0}^{2L} W_i^{(c)} [\boldsymbol{\mathcal{X}}_{i,k}^- - \hat{\mathbf{x}}_k^-] [\mathcal{D}_{i,k}^- - \hat{d}_k^-]^\top, \quad (3.15b)$$

$$\mathbf{K} = \mathbf{P}_{x_k, d_k} \mathbf{P}_{d_k, d_k}^{-1}, \quad (3.15c)$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^- + \mathbf{K}(\underline{d}_k - \hat{d}_k^-), \quad (3.15d)$$

$$\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{K} \mathbf{P}_{d_k, d_k} \mathbf{K}^\top. \quad (3.15e)$$

Note that UKF equations require no explicit calculation of Jacobians for implementation. However, $2L + 1$ times of UT are calculated in each time slot and the weights of sigma points seem sensitive to different anchor configurations (see discussions in Section 3.3.4). Matrix square root of the covariance is also required in (3.12).

3.3.4 Comparison between EKF and UKF

In this section, we compare the localization performance of EKF and UKF for the system (3.9). In detail, two different sets of anchors are set up in the simulation, one with the configuration as in an indoor environment (Section 3.6.1), and the other one as in an outdoor environment (Section 3.6.2). The sampling interval of UWB ranging is set as $0.1s$ in both cases. In the UKF algorithm, the weights of sigma points are chosen according to Section 3.3.3.2. In the indoor simulation, the tracked object is assumed to move along a circle with radius of $2m$ at the height of $1.5m$ and uniform linear velocity of $0.8m/s$; in the outdoor case, it is assumed to move

Figure 3.3: Simulation results of EKF and UKF under indoor configurations

along a circle with radius of $25m$ at the same height and a uniform linear velocity of $2m/s$.

Fig. 3.3 and Fig. 3.4 respectively show the simulation results of these two scenarios over 1000 steps of rangings, where UKF adopts the same process and measurement noise covariances as EKF interpreted in Section 3.3.2. Clearly in both cases EKF is able to track the object accurately in its x and y coordinates, along with some variation in the z coordinate which is due to the anchor configuration. Note that in system (3.9) only the measurement equation is nonlinear. With a small initial estimation error, EKF is expected to provide a consistent estimate [89]. In contrast, UKF can only perform similarly in the indoor simulation, but yields an inconsistent estimation in the outdoor case. It reveals the sensitivity of weights of sigma points of UKF to different anchor configurations. From the application perspective, we hope UAVs are able to fly in various sizes of fields with as *simple* configurations and *convenient* debugging as possible. Whereas, UKF requires careful tuning of parameters according to different environments although a good performance can be achieved. As a result, EKF is preferred for the UWB-based localization system, and the corresponding algorithm is detailed in Section 3.3.3.

Figure 3.4: Simulation results of EKF and UKF under outdoor configurations using indoor parameters

3.3.5 Outlier Detection

For UWB ranging modules, although the range measurement is accurate even in close proximity of strong reflectors like walls, there exist unreasonable data (e.g., negative distances or dramatic change between two consecutive samples) especially in NLoS environment like forest (Fig. 3.10(a)), as can be seen from Fig. 3.5. On the other hand, it is noted that the Kalman filter is not robust to measurement outliers, which is also true for EKF and results in erratic estimation. Therefore, it's necessary to let measurements go through outlier detection before applying them, which is completed by calculating the Mahalanobis distance related with the innovation covariance. Specifically, from (3.11c) and (3.11d) the innovation is given by $\underline{d}_k - \hat{d}_k$ and its approximate covariance $\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^\top + R$. As a result, the Mahalanobis distance [90] of the innovation is found as

$$D = (\underline{d}_k - \hat{d}_k)(\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^\top + R)^{-1}(\underline{d}_k - \hat{d}_k) = \frac{(\underline{d}_k - \hat{d}_k)^2}{\mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^\top + R}.$$

The larger the normalized distance D , the more likely the measurement is an outlier. Some threshold can be set for the detection.

Figure 3.5: Raw range measurements obtained in forest environment

3.4 Hardware Configuration

In this section we shall have an overview of the hardware configuration of the quadcopter used in experiments. The range calibration and UWB-based localization tests are carried out on a commercial low-cost frame of DJI F450 radio-controlled (RC) quadcopter, with its four motors replaced by the small and powerful AXI 2217/20 for endurance extension. All of the airborne modules are shown in Fig. 3.6(a). Three main electronic components constitute the on-board avionics (Fig. 3.6(b)): UWB module, BeagleBone Black[®] (BBB)¹ micro computer, and Pixhawk[®]² Autopilot [91].

The UWB platform for the experiment is Time Domain's PulsON 410[®] (Fig. 3.7) with operating frequency from 3.1 to 5.3 GHz. Within the range of 125m, it is able to provide precise measurement at an update rate of around 40 Hz. Its dimension ($7.6 \times 8.0 \times 1.6\text{cm}$) and weight (58g) are suitable for micro unmanned systems. The RF emission is compliant with the U.S. Federal Communications Commission (FCC) limits.

The BBB is a low-cost credit-card-sized development platform with good support including 512-MB RAM, 2GB eMMC flash and ARM Cortex-A8 processor with

¹<https://beagleboard.org/black>

²<https://pixhawk.org/>

(a) System block diagrams

(b) Connection between components

Figure 3.6: Hardware architecture

Figure 3.7: PulsON 410 ultra-wideband platform

speed of 1 GHz. The maximum power consumption is around 2 watts. This BBB on-board computer performs three functions: as a host, it signals the UWB module to send a ranging request; it receives the distance measurement from the UWB module; it communicates the localization information as well as the path planning messages with Pixhawk[®] through the interface board as shown in Fig. 3.6(b).

The Pixhawk[®] Autopilot is a high-performance autopilot-on-module suitable for different types of vehicles. It integrates the IMU and flight controller on-board. By default it is connected to the GPS module for absolute position, while in our case it receives the localization message from BBB.

In addition, a micro lidar light sensor is installed to measure the relative altitude to the ground. The power of all the electronic components comes from a 3-cell LiPo battery with capacity of $5200mAh$.

3.5 Range Calibration

Considering that the calibration parameters in (3.3) may depend on the specific environment, we conduct the calibration of UWB measurements in different environments. Typically, 3 kinds of environments were chosen: indoor environment, semi-outdoor environment (corridor), and forest environment.

3.5.1 Indoor Calibration

For the indoor calibration, experiments were carried out in the test bed of Internet of Things Lab (Fig. 3.8(a)), which covers an approximate area of $7m \times 7m$. Samples were collected every $0.5m$ within the range of $8m$, with the ground truth provided by a VICON^{®3} system which has a mm -level accuracy. We found that the accuracy of UWB measurement decreases significantly within $1.5m$, and hence separated the calibration into 2 intervals, $0 - 1.5m$ and $1.5 - 8m$. As mentioned in Section 3.2.2,

³<https://www.vicon.com/>

(a) IoT lab indoor environment

(b) IoT lab calibration < 1.5m

(c) IoT lab calibration 1.5-8m

Figure 3.8: Range calibration results in indoor environment

we first did the linear regression for the samples (Fig. 3.8(b) and Fig. 3.8(c)) and then got the calibration equation of (3.3). The parameters of the calibration are listed in Table 3.1.

Table 3.1: Indoor Range Calibration (unit: m)

Range	$\tilde{a}$	$\tilde{b}$	STD after calibration	STD before calibration
0-1.5m	1.0447	-0.1932	0.032	0.156
1.5-10m	1.0029	-0.0829	0.010	0.069

3.5.2 Semi-outdoor Calibration

For the semi-outdoor calibration, experiments were carried out over the corridor of S1 building, level B4 in Nanyang Technological University (Fig. 3.9(a)), which consists of a long corridor of about 100m. Samples were collected every 5m within the range of 100m, with the ground truth provided by a laser distance meter (Fluke 424D[®]) which has a mm-level accuracy. We found that the accuracy of UWB measurement degrades after 50m, and hence separated the calibration into 2 intervals, 10 – 50m and 50 – 100m. As mentioned in Section 3.2.2, we first did the linear regression for the samples (Fig. 3.9(b) and Fig. 3.9(c)) and then got the calibration equation of (3.3). The parameters of the calibration are listed in Table 3.2.

Table 3.2: Semi-outdoor Range Calibration (unit: m)

Range	$\tilde{a}$	$\tilde{b}$	STD after calibration	STD before calibration
10-50m	0.9976	-0.0511	0.061	0.675
50-100m	1.0007	-0.3121	0.046	0.259

(a) Corridor semi-indoor environment (b) Corridor calibration 10-50m (c) Corridor calibration 50-100m

Figure 3.9: Range calibration results in semi-outdoor environment

3.5.3 Outdoor Calibration

For the outdoor calibration, experiments were carried out inside a woods in Nanyang Technological University (Fig. 3.10(a)), which covers an approximate area of $40m \times 30m$. Samples were collected at different locations along a 50m long line-of-sight, with the ground truth provided by the laser distance meter. The parameters of the calibration are listed in Table 3.3.

Table 3.3: Outdoor Range Calibration (unit: m)

Range	$\tilde{a}$	$\tilde{b}$	STD after calibration	STD before calibration
0-50m	0.9989	-0.0847	0.054	0.576

From the Tables 3.1, 3.2 and 3.3 we can see that the UWB ranging is robust to the change of environments; actually, the largest standard deviation of error in 3 different environments is $0.061m$. However, the data loss and outliers increase with the distance, and vary in different environments. As expected, the data loss and outliers

(a) Forest outdoor environment

(b) Forest calibration 0-50m

Figure 3.10: Range calibration results in forest outdoor environment

occur most in the forest environment. The prediction based on EKF can be used to continue operating for a system in the case of short-term measurement failures. The outliers can result in erratic estimation followed by large maneuvers and even crash during autonomous flight. Moreover, imprecise measurements without calibration will reduce the localization accuracy and influence the performance of trajectory tracking. Thus, the calibration is necessary before applying raw range measurement. Fig. 3.11 illustrates that the outlier-removed and calibrated distance measurements are much closer to the reference than the raw distances at $9.84m$, $14.85m$ and $24.70m$ respectively. Compared with the raw range measurements with the mean error of $15cm$, the mean error of calibrated measurements is around $2cm$.

3.6 Experimental Results and Performance Evaluation

To validate the proposed UWB-based localization approach, various experiments have been conducted in different environments. Below, we shall discuss the experiments in details. Particularly, the discussion is divided into two parts: indoor localization and outdoor localization with applications to single UAV autonomous flight and multiple UAVs formation.

(a) Range tests at 9.84m

(b) Range tests at 14.85m

(c) Range tests at 24.7m

Figure 3.11: Comparison of UWB range measurements before and after calibration

Figure 3.12: Indoor autonomous flight workflow

3.6.1 UWB-based Localization in Indoor Environment

3.6.1.1 Performance of UWB-based localization

To validate the performance of UWB localization in indoor environment, we conducted an autonomous flight based on UWB localization in the same test bed as shown in Fig. 3.8(a). The quadcopter controllers can be modeled as a set of nested control loops as shown in Fig. 3.12. Each control loop has a reference set point and the current position state as input. For the position control loop, the current position of the UAV is provided by the UWB-based localization module. The inner loop and innermost loop are carried out on Pixhawk[®] Autopilot as shown in Fig. 3.6(a). The complete description of this design scheme can be seen in [92].

The UWB estimate is compared with the VICON[®] measurements for the x - y coordinate and the corresponding velocity in Fig. 3.13. The commercial motion capture system (VICON[®]) detects the pose of marked objects on the quadcopter and generates attitude and position information. All measurements are relative to a local coordinate frame. The system is capable of producing measurements at a rate of $200Hz$ with position accuracy of millimeters. Similar as the set-up in outdoor environment, four UWB modules were installed at the 4 corners of the test bed, with

the coordinates measured as $(-3.0, -3.0, -1.78)$, $(3.0, -3.0, -1.17)$, $(3.0, 3.0, -1.31)$ and $(-3.0, 3.0, -1.31)$ respectively, where the z -axis is taken downward.

The quadcopter takes off autonomously and flies around the center of the test bed three times in a squarish trajectory. The position and velocity accuracy is affected by the Proportional-Integral-Derivative (PID) parameters and the precision of on-board magnetic sensor measurements. It is noted that the position and velocity estimates based on UWB are quite close to those of VICON[®] as shown in Fig. 3.13. The average position error in x - y plane is $0.071m$ and in the worst case the accuracy still falls within $0.2m$. For the velocity estimates in Fig. 3.13(b) and Fig. 3.13(c), the UWB estimation can track the dynamic movement of the quadcopter accurately. The estimation accuracy decreases at the turning points of trajectory, but it still falls within a reasonable range around the true value.

(a) Flight trajectory in x - y plane

(b) Velocity x

(c) Velocity y

Figure 3.13: Position and velocity estimates obtained by UWB-based system and VICON[®] system in indoor environment

3.6.1.2 Multi-UAV formation flight

To further validate the performance of our proposed UWB-based localization, multi-UAV formation flights were carried out in the same IoT lab environment as seen in Fig. 3.14. Since UWB distance estimation is based on the TW-ToF ranging protocol as described in Section 3.2, before conducting localization, a ranging sequence of 4 UAVs to 4 UWB anchors is designed first to reduce the conflict rate of UWB signal transmission. The proposed UWB signal communication can be seen in Fig. 3.15.

Figure 3.14: 4 UAVs form a square formation in a laboratory environment.

Figure 3.15: Ranging sequence of 4 UAVs to 4 anchors, the lighter shade indicates a past instance.

The indoor formation flight test was carried out using VICON[®] as ground truth. 4 UAVs were loaded with predefined trajectories. The trajectories were chosen to be

on a $4m \times 4m$ square for the outer loop and a $2.5 \times 2.5m$ square for the inner loop. The anchors are positioned at the corners of a $6m \times 6m$ area with varying heights. A common RC signal is used to trigger the autonomous flight mode, and each UAV will register the trigger time using its local clock. Based on the predefined trajectory, each UAV generates its current target point first, then it flies to this point and stays there until the local clock exceeds the set time, finally, it proceeds to the next point. The ground truth was processed by a central computer and sent to each UAV to be stored locally. By using time stamps and assuming that the clocks of UWB modules, VICON[®], high-level and low-level controller boards have negligible drift during the experiment period, all of the flight data is referenced to the VICON[®] system time in Fig. 3.16⁴.

Figure 3.16: Trajectories of 4 UAVs loaded on each UAV

The high level board computes the location (EKF outputs) and sends the position

⁴Flight video can be checked in: <https://youtu.be/DR7KW0MqYLE>

estimates (blue curves in Fig. 3.16) to the low-level board (Pixhawk[®] built-in estimator) which fuses this location estimate with the measurements from the high-rate IMU and barometer sensors (green pathes in Fig. 3.16). The output is then sent back to the high level board at a lower rate for logging and analysis. The RMS errors of the experiment data are summarized in Table 3.4. From Table 3.4, we can

Table 3.4: RMS error of the indoor flight test for each UAV (unit: m).

UAV	EKF		FUSED	
	x, y	z	x, y	z
1	0.124	0.353	0.127	0.354
2	0.111	0.343	0.126	0.349
3	0.150	0.335	0.162	0.338
4	0.107	0.348	0.110	0.347

see that the proposed technique can yield accuracy at centimetre level. The fused estimate appears to have slightly larger error but this is negligible. By comparing the x and y coordinates with VICON[®] data among all UAVs, the maximum RMS error is $0.162m$.

Moreover, we repeated the experiment by using the calibrated distance measurements and the results are listed in Table 3.5.

Table 3.5: RMS error of EKF and fused results after calibration (unit: m).

UAV	EKF		FUSED	
	x, y	z	x, y	z
1	0.130	0.065	0.129	0.056
2	0.118	0.060	0.115	0.048
3	0.156	0.067	0.152	0.059
4	0.111	0.066	0.108	0.046

The greatest improvement to accuracy comes from the calibration of distance measurement from UWB as can be shown in Table 3.5 with a maximum RMS altitude error of $0.067m$ from the result of running EKF with the calibrated distance on UAV1. Since the altitude estimation is quite important for the safety of UAV flight, we did not attempt to fly with uncalibrated distance estimates.

In addition, our proposed UWB-based localization system has been successfully demonstrated at Airshow Singapore 2016. Supported by our localization system, two UAVs cooperated with one unmanned ground vehicle (UGV) in disaster response, e.g., surveillance and rescue. A video screenshot during the show time can be seen in Fig. 3.17⁵.

Figure 3.17: 2 UAVs and 1 UGV in disaster response for Airshow Singapore 2016

3.6.2 UWB-based Localization in Outdoor Environment

3.6.2.1 Performance of UWB-based localization

To further validate the UWB localization algorithm in outdoor environment, flight tests were conducted in a sports field (Fig. 3.18(a)). Four UWB modules were installed in the center of the field as anchors, marked as 4 green stars. The ground anchors share the same local frame and their Cartesian coordinates are given by $(0.0, 0.0, 0.32)$, $(0.0, 30.6, 1.26)$, $(55.85, 30.6, 0.29)$ and $(55.85, 0.0, 1.75)$ respectively from anchor 1 to 4. We used the same laser meter in Section 3.5 to determine the coordinates.

To validate the performance of the proposed UWB-based localization, two tests were carried out. In the first experiment, the quadcopter was manually held overhead to move along the rectangle connecting the 4 anchors three times. Fig. 3.18(b)

⁵The full video is available in: <https://youtu.be/5QxEx9M0zDU>

illustrates the UWB-based localization results, which almost overlap the rectangle. The average x - y error is $0.175m$ with the largest error of $0.32m$ at a corner of the testing area, where the quadcopter was just above an anchor.

The second experiment was carried out in the same area with the same configuration of anchors. The quadcopter was maneuvered by a pilot to firstly fly along the same rectangle, and then along the $400m$ running track. Fig. 3.18(c) shows a comparison between the UWB-based estimate and the GPS estimate. Note that here the coordinate frame is taken as North-East plane, thus the coordinates of the anchors have been rotated compared to those in Fig. 3.18(b). As seen from the figure, the position estimates of the UWB-based localization system are closer to those projected from the GPS data on the North-East plane. In the outdoor case it is difficult to retrieve the ground truth, especially when the UAV is moving, and hence we are unable to conclude which estimate is more accurate than the other. But if we examine the rectangular path carefully, we can find that the deviation of the GPS estimate from the UWB-based estimate is different on the two opposite widths on north-west direction. Since the UWB-based localization is quite accurate on the rectangular path as seen in Fig. 3.18(b), this deviation implies a better accuracy of the UWB-based localization than the GPS. On the other hand, we should also note that the UWB-based estimation degrades much on two arches, where the protective net for hammer-throw has been installed nearby. These metal materials result in more data loss and outliers, as well as a decreased accuracy of UWB localization.

3.6.2.2 Multi-UAV formation flight

For the outdoor formation test, we reallocated the anchors to the corners of a rectangular area of size $18m \times 10m$, which is five times larger than the indoor area. The formation performance was very similar with the indoor test (as seen in Fig. 3.19⁶).

⁶Flight video can be seen in: https://youtu.be/jpr1jBwf_Qo

(a) Outdoor flight tests in SRC

(b) Flight trajectory in North-East plane

(c) Flight trajectory in North-East plane

Figure 3.18: Performance of the UWB-based localization and on-board GPS

Figure 3.19: 4 UAVs at the closest set points (roughly $2.12m$ apart from each other) in the outdoor test. The anchors were put at the corners of a $18m \times 10m$ rectangular area.

In addition, based on the proposed localization system, we also demonstrated formation flights in other GPS-denied environments such as courtyard and tunnel. The video screen shots can be found in Fig. 3.20⁷ and Fig. 3.21⁸, respectively.

3.7 Conclusion

To achieve the localization in GPS-denied environments, UWB radios have been utilized to localize mobile UAVs. The range is measured in an active way that the mobile UWB module sends range requests to the anchors on the ground and calculates distance once receives response signals. This range measurement is calibrated and goes through outlier detector first for reliability and better accuracy, then fed into the localization algorithm. The initialization is conducted in the static state using trilateration algorithm and then EKF will provide precise position and velocity for a mobile UAV.

The performance of the proposed UWB-based localization has been validated through a series of experiments carried out in indoor and outdoor environments. In cooperation with ST Engineering, our proposed localization method was implemented for 2 UAVs and 1 UGV cooperation in disaster response, and demonstrated at Airshow Singapore 2016.

⁷Flight video can be seen in: <https://youtu.be/U7HZQIfSf5c>

⁸Flight video can be seen in: <https://youtu.be/CKnbM192d6E>

(a) 4 UAVs formation in courtyard (side view)

(b) 4 UAVs formation in courtyard (top view)

Figure 3.20: 4 UAVs formation flight in the courtyard surrounded by S2.1 and S2.2 building in NTU, where GPS signal is faint.

Figure 3.21: 10 UAVs formation flight in the tunnel environment

Chapter 4

UWB-based Relative Localization with Single Landmark

4.1 Introduction

In the last chapter, we proposed UWB-based active self-localization with four anchor nodes and evaluated its performance on our quadcopters. As stated in Section 3.3, distances between mobile UAVs and fixed UWB modules are fed into our proposed algorithm for self-localization. Nevertheless, setting up fixed anchor nodes is impractical in some complex environments such as forest and large flight area. Besides, the accuracy of our proposed self-localization heavily relies on the configuration of the fixed anchor nodes including the accuracy of their positions and the directions of their antennas.

To mitigate the influence coming from the aforementioned configuration, in this chapter, we study an active RL with single landmark problem. In consideration of the environment and task constraints, a UAV is only allowed to fly a small distance to undertake the RL. Thus, we adopt a similar approach as in the optimal sensor placement problem to solve the active RL problem under the constraint of the UAV's motion. Actually, assuming an accurate self-displacement measurement, solving RL

problem is equivalent to the estimation of the relative position at the starting point, and the motion constraint can be projected on the sensor placement. As a result, the active RL problem is reformulated as a constrained optimal sensor placement problem and we seek the optimal configuration to minimize the error bound under the constraint. Given by the maximum allowed moving distance D and the sample size n , as shown in Theorem 4.3.1, the lower error bound is dependent on D and n , and hence one only needs to enlarge the ranging span to effectively reduce the estimation error. Following this idea, a novel design of ranging path is proposed in Section 4.3.3.

This chapter is organized as follows. We firstly formulate the problem in Section 4.2. In Section 4.3, we analyze the error and discuss the corresponding lower bound of the RL at the starting point, based on which we respectively design the RL algorithm in the cases of noise-free and noise-corrupted displacements under the motion constraint. Section 4.4 presents a strategy for relative localization between UAVs when all the UAVs move simultaneously. Simulations and experiments are conducted respectively in Section 4.5 and 4.6 to validate our approach, and we conclude our work in Section 4.7.

4.2 Preliminaries and Problem Formulation

For convention, we use vector $\mathbf{p}$ and $\boldsymbol{\chi}$ to represent the global position and relative position respectively. For instance, $\boldsymbol{\chi}^{ij} = \overrightarrow{P_i P_j} = [x_j - x_i, y_j - y_i]^\top$ represents the relative coordinate between UAV i and UAV j in an agreed frame (e.g., North-East plane) where (x_i, y_i) is the global position of UAV i . $\hat{\boldsymbol{\chi}}$ denotes an estimate of $\boldsymbol{\chi}$ and $\|\boldsymbol{\chi}\|$ represents the relative distance.

Before delving into details, it is necessary to state the UWB-based indirect cooperative relative localization problem. We consider a team of mobile UAVs labeled $1, 2, \dots, N$ in Fig. 4.1(a). The problem is to estimate the relative position of each

UAV to their neighbors. Suppose that each UAV i is able to access its own displacement at time step k , i.e., $\Delta \mathbf{p}_k^i = [\Delta x_k^i, \Delta y_k^i]^\top$, and has distance measurements with respect to its neighbor UAV j , i.e., $d^{ij} = \|\chi^{ij}\|$ as shown in Fig. 4.1(b). The goal is to estimate the inter-UAV relative coordinate χ^{ij} based on distance measurements and displacement information exchanged with neighbors using UWB sensors which are installed on each UAV. For each UAV i , denote the set of its neighbors by $\mathcal{N}_i$, which means $j \in \mathcal{N}_i$ if UAV i is able to range with UAV j and UAV j can transmit its displacement $(\Delta x^j, \Delta y^j)$ and height (for transforming a 3D distance measurement into 2D) to UAV i through UWB radio.

Figure 4.1: Relative distances, self-displacements and relative positions among 3 UAVs

4.3 Relative Localization with Single Beacon

4.3.1 Initial Relative Position Estimation

As seen in Fig. 4.1(b), to achieve an accurate RL estimate, we first assign one static UAV (e.g., UAV i) as beacon while other UAVs (e.g., UAV j and UAV r) conduct range measurement and communication with it. In this section we shall formulate the problem in the perspective of nonlinear least squares (NLS).

As shown in Fig. 4.2, O is the location of the static beacon and the mobile UAV starts from P_0 and takes the distance measurement $\underline{d}_k$ at each point P_k , $k = 0, \dots, n$.

Figure 4.2: RL estimation to a static beacon

RL aims to estimate the vector $\overrightarrow{OP_0}$ from the displacements $\overrightarrow{P_0P_k}$, as well as the distance measurement $\underline{d}_k$.

Denote $\mathbf{p}_0 = \overrightarrow{OP_0} = [x_0, y_0]^\top$, $\Delta \mathbf{p}_k = \overrightarrow{P_0P_k} = [\Delta x_k, \Delta y_k]^\top$, and $d_k = f_k(x_0, y_0) = \|\overrightarrow{OP_k}\| = \sqrt{(x_0 + \Delta x_k)^2 + (y_0 + \Delta y_k)^2}$. To simplify the analysis and help us design a path to reduce the RL estimation error under a motion constraint, we first assume that the displacement measurements $\Delta \mathbf{p}_k$ are free of noise and the distance measurements $\underline{d}_k$ are corrupted by independently and identically distributed noises, i.e., $\underline{d}_k = d_k + \epsilon_{d,k}$, where $\mathbb{E}\{\epsilon_{d,k}\} = 0$, $\mathbb{E}\{\epsilon_{d,i}\epsilon_{d,j}\} = 0$ for all $i \neq j$ and $\mathbb{E}\{\epsilon_{d,k}^2\} = \sigma^2$ (σ is the standard deviation of distance measurement). Consequently, we only need to estimate $\mathbf{p}_0$ by solving the NLS problem, i.e.,

$$\min_{x_0, y_0} \sum_{k=0}^n (\underline{d}_k - f_k)^2. \quad (4.1)$$

Denote $\hat{\mathbf{p}}_0 = [\hat{x}_0, \hat{y}_0]^\top$ as the position estimate, $\hat{f}_k = f_k(\hat{x}_0, \hat{y}_0)$, $\hat{\mathbf{p}}_k = [\hat{x}_k, \hat{y}_k]^\top = \hat{\mathbf{p}}_0 + \Delta \mathbf{p}_k$ and $\cdot$ as the inner product of two vectors. The solution to the above problem can be obtained by using Gauss-Newton method [93], which is based on the linear approximation of f_k around the current estimate $(\hat{x}_0, \hat{y}_0)$ as

$$f_k \approx \hat{f}_k + \frac{1}{\hat{f}_k} (\hat{x}_0 + \Delta x_k, \hat{y}_0 + \Delta y_k) \cdot (x_0 - \hat{x}_0, y_0 - \hat{y}_0), \quad (4.2)$$

and solving the corresponding normal equation

$$\underbrace{\begin{bmatrix} f_0 - \hat{f}_0 \\ f_1 - \hat{f}_1 \\ \vdots \\ f_n - \hat{f}_n \end{bmatrix}}_{\Delta \mathbf{r}} = \underbrace{\begin{bmatrix} \hat{x}_0/\hat{f}_0 & \hat{y}_0/\hat{f}_0 \\ \hat{x}_1/\hat{f}_1 & \hat{y}_1/\hat{f}_1 \\ \vdots & \vdots \\ \hat{x}_n/\hat{f}_n & \hat{y}_n/\hat{f}_n \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_0 - \hat{x}_0 \\ y_0 - \hat{y}_0 \end{bmatrix}}_{\Delta \boldsymbol{\kappa}}. \quad (4.3)$$

The above equation can be rewritten as $\Delta \mathbf{r}(\hat{\mathbf{p}}_0) = \mathbf{H}(\hat{\mathbf{p}}_0)\Delta \boldsymbol{\kappa}$ and $\Delta \boldsymbol{\kappa}$ is found as

$$\Delta \boldsymbol{\kappa} = (\mathbf{H}^\top \mathbf{H})^{-1} \mathbf{H}^\top \Delta \mathbf{r}. \quad (4.4)$$

If $\mathbf{H}$ is of full column rank, the newly obtained estimate $\hat{\mathbf{p}}_0 \leftarrow \hat{\mathbf{p}}_0 + \Delta \boldsymbol{\kappa}$ can be used to update $\mathbf{H}$ and $\Delta \mathbf{r}$ respectively for the next iteration until $\Delta \boldsymbol{\kappa}$ is sufficiently small.

Remark 4.3.1. *An initial guess is needed to solve (4.1), which comes from the solution of a related linear least squares problem with the squared distances [84]. To be exact, the distance measurements $\underline{d}_k$ is corrupted by noise. However, to achieve an initial guess, we temporarily assume that $\underline{d}_k$ is equal to d_k regardless of noise, i.e., $\underline{d}_k = \sqrt{(x_0 + \Delta x_k)^2 + (y_0 + \Delta y_k)^2}$. Consequently, by squaring both sides of that and subtracting the first equation $\underline{d}_0^2 = x_0^2 + y_0^2$, we get the following linear system*

$$2 \underbrace{\begin{bmatrix} \Delta x_1 & \Delta y_1 \\ \Delta x_2 & \Delta y_2 \\ \vdots & \vdots \\ \Delta x_n & \Delta y_n \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}}_{\mathbf{p}_0} = \underbrace{\begin{bmatrix} \underline{d}_1^2 - \underline{d}_0^2 - (\Delta x_1^2 + \Delta y_1^2) \\ \underline{d}_2^2 - \underline{d}_0^2 - (\Delta x_2^2 + \Delta y_2^2) \\ \vdots \\ \underline{d}_n^2 - \underline{d}_0^2 - (\Delta x_n^2 + \Delta y_n^2) \end{bmatrix}}_{\Delta \mathbf{d}} \quad (4.5)$$

or $2\mathbf{A}\mathbf{p}_0 = \Delta \mathbf{d}$. Now an initial guess of $\mathbf{p}_0$ is given by

$$\hat{\mathbf{p}}_0 = \frac{1}{2}(\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \Delta \mathbf{d}, \quad (4.6)$$

Algorithm 4.1: Pseudocode for NLS-GN based Initial RL Estimation

Procedure: Initial Relative Estimation

Collect rangings and produce relative localization:

Starting from P_0 , the UAV moves along some nonlinear path as described in Section 4.3.2 and collects n distance measurements $\underline{d}_k$.

Calculate rough initial estimate:

Form the linear equations in (4.5) based on the overall available ranging measurements $\underline{d}_k$ and $\Delta \mathbf{p}_k$.

calculate an initial rough estimate $\hat{\mathbf{p}}_0 = [\hat{x}_0, \hat{y}_0]^\top$ using (4.6).

Improve rough initial estimate:

Set the maximum number of iteration steps as S and the positive threshold of estimation error as ϵ .

for $k \leftarrow 1$ **to** S **do**

 Calculate GN iteration (4.4) $\rightarrow \Delta \boldsymbol{\kappa}$

$\hat{\mathbf{p}}_0 \leftarrow \hat{\mathbf{p}}_0 + \Delta \boldsymbol{\kappa}$

if $|\Delta \boldsymbol{\kappa}| \leq \epsilon$ **then**

 break;

end

end

Each UAV stores the initial RL estimates

as long as $\mathbf{A}$ has full column rank, or equally the n points P_k 's ($k = 1, \dots, n$) are not along the same line.

Remark 4.3.2. Although the equation (4.5) is able to give an exact solution when there is no ranging error, the introduction of squared distances would increase the error of calculation and only achieves a rough estimate in the presence of measurement noises. In fact, assume that the range measurement $\underline{d} \sim N(\bar{r}, \sigma^2)$, then the variance of $\underline{d}^2$ is given by $4\bar{r}^2\sigma^2 + 2\sigma^4$ which is dependent on $\bar{r}^2$. In contrast, (4.3) employs the original measurements as the input and obtains a refined estimate by Gauss-Newton (GN) method. This rough initial guess is able to guarantee the convergence to a more accurate solution in just a few iterations. Hence we employ the rough estimate of (4.6) as the initial guess for solving (4.1). The process of solving the rough initial RL guess is summarized in Algorithm 4.1.

4.3.2 Error Analysis

In the sequel, we seek to find a lower bound of the estimation error of the problem (4.1) as well as a desirable path configuration, given that the mobile UAV can only move by a maximum distance of D in the whole duration of RL. The optimum configuration then serves to guide the design of the ranging path, in an effort to effectively reduce the estimation error under the motion constraint.

In this section we shall first analyze the RL error based on (4.4) to find its lower bound of MSE, and then propose an effective sampling strategy to reduce the error. Two different cases are studied, depending on whether or not the displacement measurements are corrupted by noise.

By (4.4), the error covariance matrix can be approximated by

$$\mathbf{E} = \sigma^2(\mathbf{H}^\top \mathbf{H})^{-1}. \quad (4.7)$$

Rewrite $\mathbf{H} = [\mathbf{c} \ \mathbf{s}]$ with $\mathbf{c}$ and $\mathbf{s}$ being two column vectors of dimension $(n+1)$ and denote $[\hat{x}_i/\hat{f}_i \ \hat{y}_i/\hat{f}_i] = [\cos \theta_i \ \sin \theta_i]$ for each row in $\mathbf{H}$, where θ_i is shown in Fig. 4.2. Then $(\mathbf{H}^\top \mathbf{H})^{-1}$ is obtained as

$$(\mathbf{H}^\top \mathbf{H})^{-1} = \frac{1}{\det(\mathbf{H}^\top \mathbf{H})} \begin{bmatrix} \mathbf{s}^\top \mathbf{s} & -\mathbf{c}^\top \mathbf{s} \\ -\mathbf{c}^\top \mathbf{s} & \mathbf{c}^\top \mathbf{c} \end{bmatrix}.$$

Now the MSE is given by the trace of $\mathbf{E}$ as

$$MSE = \frac{\sigma^2(n+1)}{\det(\mathbf{H}^\top \mathbf{H})}, \quad (4.8)$$

where $n+1$ is the number of distance measurements. Direct computation shows

that

$$\begin{aligned}
\det(\mathbf{H}^\top \mathbf{H}) &= \sum_{i \neq j} \cos^2 \theta_i \sin^2 \theta_j - \sum_{i \neq j} \cos \theta_i \sin \theta_i \cos \theta_j \sin \theta_j \\
&= \sum_{i \neq j} (\cos^2 \theta_i \sin^2 \theta_j - \cos \theta_i \sin \theta_i \cos \theta_j \sin \theta_j) \\
&= \sum_{i \neq j} (\cos \theta_i \sin \theta_j - \cos \theta_j \sin \theta_i)^2 \\
&= 2 \sum_{0 \leq i < j \leq n} \sin^2(\theta_i - \theta_j).
\end{aligned} \tag{4.9}$$

Remark 4.3.3. Note that the error covariance matrix $\mathbf{E}$ is also the Cramer-Rao lower bound, or the inverse of the Fisher Information Matrix (FIM). In the GPS literature, the squared root of the MSE is defined as the geometric dilution of precision (GDOP) [94].

Given a fixed number of range measurements, to minimize MSE is to maximize the cost function

$$J = 2 \sum_{0 \leq i < j \leq n} \sin^2(\theta_i - \theta_j). \tag{4.10}$$

Denote $\Theta = \arcsin(D/d_0)$ as the largest angle spanned by the mobile UAV as shown in Fig. 4.3(b) and assume that $\Theta \leq \frac{\pi}{4}$, or equivalently $D \leq d_0 \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}d_0$. Obviously we can always sort θ_i 's in an ascending order, and θ_0 can be selected to be 0 (as shown in Fig. 4.3(a)) after proper rotation without changing the determinant. Moreover, when $0 \leq \theta_i \leq \Theta \leq \frac{\pi}{4}$, it can be seen that the maximum of J dictates the largest angle $\theta_n = \Theta$. To summarize, we try to solve the problem

$$\max_{\theta_i \in G} \sum_{0 \leq i < j \leq n} \sin^2(\theta_i - \theta_j), \tag{4.11}$$

where $G = \{(\theta_0, \theta_1, \dots, \theta_n) : 0 = \theta_0 \leq \theta_1 \leq \dots \leq \theta_n = \Theta \leq \frac{\pi}{4}\}$ is a closed convex region. We have the following result:

Theorem 4.3.1. Assume that the maximum allowed moving distance $D \leq \frac{\sqrt{2}}{2}d_0$. Then (4.11) is solved if $\theta_0 = \dots = \theta_{\lfloor \frac{n}{2} \rfloor} = 0$ and $\theta_{\lfloor \frac{n}{2} \rfloor + 1} = \dots = \theta_n = \Theta$, where $\lfloor \frac{n}{2} \rfloor$

(a) Illustration of θ_i and θ_j

(b) Maximum ranging span

Figure 4.3: Angle tags and maximum ranging span within the moving radius

denotes the biggest integer no greater than $\frac{n}{2}$. Correspondingly,

$$\max J = \begin{cases} \frac{(n+1)^2}{2} \left(\frac{D}{d_0}\right)^2 & n \text{ odd,} \\ n\left(\frac{n}{2} + 1\right) \left(\frac{D}{d_0}\right)^2 & n \text{ even.} \end{cases} \quad (4.12)$$

Proof. Notice that each individual term is the squared sine of a linear function of $(\theta_0, \theta_1, \dots, \theta_n)$, and $y = \sin^2 \theta$ is increasing and convex over $[0, \frac{\pi}{4}]$. Therefore, each individual term $\sin^2(\theta_j - \theta_i)$ is a convex function for $0 \leq \theta_i \leq \theta_j \leq \frac{\pi}{4}$ [95], and so does the summation J . As a result, the maximum of J on the closed convex region G is obtained at the extreme points of G , namely those with $\theta_i = 0$ or Θ for $i = 1, \dots, n-1$. The rest follows naturally. ■

Remark 4.3.4. From the above result we can see that the localization error is dependent on the maximum spanning angle Θ , as well as the sample size. Intuitively, a larger field of view and more distance measurements help reduce the localization

error. By (4.12) and (4.8) we can see that

$$GDOP = \begin{cases} \sigma \sqrt{\frac{2}{n+1} \frac{d_0}{D}} & n \text{ odd}, \\ \sigma \sqrt{\frac{2(n+1)}{n(n+2)} \frac{d_0}{D}} & n \text{ even}, \end{cases} \quad (4.13)$$

which extends the result of lowest $GDOP = \sigma \sqrt{\frac{2}{n+1}}$ when there is no restriction on the sensor placement [94].

4.3.3 Relative Localization Algorithm with Single Beacon

4.3.3.1 Noise-free case

From Remark 4.3.4 we know that the key point of reducing localization error is to enlarge the spanning angle between $\overrightarrow{OP_0}$ and $\overrightarrow{OP_n}$. Clearly if the relative position $\mathbf{p}_0$ can be known exactly, then the UAV only needs to move in the straight line along the optimal direction to the peripheral in an effort to maximize the span. However, due to the inaccurate position estimate, we need to adaptively determine the heading to enlarge the span as much as possible. Besides, as mentioned in Remark 4.3.1, a nonlinear path is also needed to initialize the estimation. Bearing these in mind, we propose a two-phase algorithm to actively reduce the localization error as follows, which consists of a nonlinear path of length D_1 and a piecewise linear path of length D_2 with the total traveling distance $D = D_1 + D_2$. Also see Fig. 4.4 for clarification.

Remark 4.3.5. *Several points have to be noted from the above algorithm. The first is the choice of length of the nonlinear path. Although an initial guess can be generated from 3 samples on a nonlinear path as seen from (4.5), the estimation error may be so large that a refined estimate in phase II may have a counter direction to that of the rough estimate due to the flip ambiguity, thus reduce the ranging span as a whole. Therefore, the nonlinear path should be long enough to guarantee a relatively accurate initial estimate. The second point is with regard to the choice of the shape of the nonlinear path. Note that we assume no prior knowledge of the relative position*

Algorithm 4.2: Two-phase Active RL (TPARL)**Procedure:** TPARL**Phase I (Nonlinear):**

Starting from P_0 , the UAV moves along some nonlinear path of length D_1 to collect distance measurements, and produce an initial rough estimate $\hat{\mathbf{p}}_0 = [\hat{x}_0, \hat{y}_0]^\top$ following Algorithm 4.1 in Section 4.3.

Phase II (Piecewise linear):

Assume that the mobile UAV is to move N steps of linear path with fixed length l and $Nl \leq D_2$.

for $i \leftarrow 1$ **to** N **do** $\hat{\theta} \leftarrow \arcsin(l/r_0)$ $\hat{\lambda} \leftarrow \arctan 2(\hat{y}_0, \hat{x}_0)$ $\alpha \leftarrow \hat{\lambda} - \hat{\theta} - \frac{\pi}{2}$ (in North-East frame) Move along a straight line with the heading α Collect new rangings and produce new estimate $\hat{\mathbf{p}}_0 = (\hat{x}_0, \hat{y}_0)$ by GN method (Algorithm 4.1) from the overall rangings and displacements**end**

$\mathbf{p}_0$, the nonlinear path should avoid the case where the path is relatively monotonous when viewed along the direction of $\mathbf{p}_0$ (such as circling around $\mathbf{p}_0$ with small changes in distance) and incurs a loss of ranging span. Hence, in the follow-up simulation and experiment we take a square path. Thirdly, by Theorem 4.3.1 and Fig. 4.3 we note that samples should be taken at the extreme points to reduce error. However, considering that such a maneuver incurs frequent stops which not only increases the RL duration but also consumes extra energy, and the sampling at the extreme points for a short path will only make a slight improvement to RL estimation, we do not apply such a strategy in the piecewise linear phase. Instead, we only focus on how to enlarge the ranging span from the baseline $\mathbf{p}_0$, as seen from the for-loop in phase II.

Remark 4.3.6. It should be noted that each change of heading actually serves to maximize the span from the initial position to the next waypoint based on the latest relative position estimate. For simplicity, here we set a fixed length movement for each step, but it can be variable in practice.

Figure 4.4: Two-phase active RL with single beacon

4.3.3.2 Noise-corrupted case

In this subsection we consider the design of EKF for the same strategy as that of the case without noise to take the displacement noise into account and the initial RL estimate from Algorithm 4.2 will be fed into the proposed EKF for the first step calculation. The sampling algorithm remains the same, while the estimate is provided by EKF.

For the EKF design, we need to determine the state equation and observation equation. The state space representation consists of the initial relative position $\mathbf{p}_{0,k}$ and the displacement $\Delta \mathbf{p}_k$, with the system dynamics given as follows:

$$\begin{cases} \mathbf{p}_{0,k+1} = \mathbf{p}_{0,k} + \boldsymbol{\xi}_{\mathbf{p}_{0,k}}, \\ \Delta \mathbf{p}_{k+1} = \Delta \mathbf{p}_k + \mathbf{u}_k + \boldsymbol{\xi}_{\mathbf{u},k}, \end{cases} \quad (4.14)$$

where $\mathbf{u}_k$ is the position increment from time step k to $k+1$ provided by the on-board navigation system, and $\boldsymbol{\xi}_{\mathbf{u},k}$ is the related white noise. Note that the term $\boldsymbol{\xi}_{\mathbf{p}_{0,k}}$ denotes the possible perturbation of the beacon, e.g., a hovering UAV or a floating buoy, which can be seen as white noise, and $\boldsymbol{\xi}_{\mathbf{p}_{0,k}} \equiv \mathbf{0}$ if the beacon keeps static.

On the other hand, the observation consists of the relative range $\underline{d}_k$, and the displacement $\underline{z}_k$ from the starting P_0 as below:

$$\begin{cases} \underline{d}_k = \|\mathbf{p}_{0,k} + \Delta\mathbf{p}_k\| + \epsilon_{d,k}, \\ \underline{z}_k = \Delta\mathbf{p}_k + \boldsymbol{\epsilon}_{z,k}, \end{cases} \quad (4.15)$$

where $\epsilon_{d,k}$ and $\boldsymbol{\epsilon}_{z,k}$ are the corresponding white noises. Note that the only nonlinear part of the system comes from the relative range $\underline{d}_k$, which needs to be linearized in the EKF.

Remark 4.3.7. *In the state equation (4.14) we split the relative position into the initial relative position $\mathbf{p}_0$ and the displacement $\Delta\mathbf{p}$. For one thing, we need the estimate of $\mathbf{p}_0$ to determine the heading at the next step as seen from the algorithm; for the other, the inclusion of $\Delta\mathbf{p}$ in the filter helps improve the navigation of the mobile UAV.*

4.4 Cooperative Relative Localization without Beacon

In this section, we consider the case of a dynamic team as depicted in Fig. 6.1, where the UAV serving as a “beacon” in Section 4.3.3 is also mobile. For any two team members, say UAV i and UAV j , their k th relative position estimate $\boldsymbol{\chi}_k^{ij}$ is to be estimated, $k \in \mathbb{N}$ at time $t = t_k$ according to $\boldsymbol{\chi}_k^{ij} = \overrightarrow{P_{i,k}P_{j,k}} = [x_{j,k} - x_{i,k}, y_{j,k} - y_{i,k}]^\top$. For brevity, we denote $\boldsymbol{\chi}_k^{ij} = [X_k, Y_k]^\top = [X_0 + \Delta X_k, Y_0 + \Delta Y_k]^\top$, $\boldsymbol{\chi}_0^{ij} = [X_0, Y_0]^\top$ and $\Delta\boldsymbol{\chi}_k^{ij} = [\Delta X_k, \Delta Y_k]^\top$ which is the vector of the difference of the displacement from their respective launching point, namely $\Delta\boldsymbol{\chi}_k^{ij} = \overrightarrow{\Delta P_{i,k}\Delta P_{j,k}} = [\Delta x_{j,k} - \Delta x_{i,k}, \Delta y_{j,k} - \Delta y_{i,k}]^\top$. Then the distance d_k can be described as $d_k = \|\boldsymbol{\chi}_k^{ij}\| = \|\boldsymbol{\chi}_0^{ij} + \Delta\boldsymbol{\chi}_k^{ij}\|$. A similar EKF structure as that in Section 4.3.3.2 is proposed to account for the noise of the difference of the increment of UAVs’ displacements and their relative distances. The state space representation consists of the initial relative position

$\mathbf{x}_{0,k}^{ij}$ and the relative variation of two displacements $\Delta\mathbf{x}_k^{ij}$ with the system dynamics given as follows:

$$\begin{cases} \mathbf{x}_{0,k+1}^{ij} = \mathbf{x}_{0,k}^{ij} + \boldsymbol{\zeta}_{\mathbf{x}_0^{ij},k}, \\ \Delta\mathbf{x}_{k+1}^{ij} = \Delta\mathbf{x}_k^{ij} + \boldsymbol{\varsigma}_k + \boldsymbol{\zeta}_{\boldsymbol{\varsigma},k}, \end{cases} \quad (4.16)$$

where $\boldsymbol{\varsigma}_k$ is the difference of the displacements' increment of UAV*i* and UAV*j* from time step k to $k+1$. Note that $\boldsymbol{\zeta}_{\mathbf{x}_0^{ij},k}$ denotes the possible perturbation of the initial relative position from UAV*i* to UAV*j*, which can be seen as white noise, and $\boldsymbol{\zeta}_{\mathbf{x}_0^{ij},k} \equiv \mathbf{0}$ if UAV*i* and UAV*j* remain absolutely stationary in the initial phase. Assume the measurement noise of each UAV's displacement $(\Delta x, \Delta y)$ is Gaussian with the distribution $N(0, \sigma^2)$ and independent with neighboring UAVs' displacement measurements. Then, we can get $\boldsymbol{\zeta}_{\boldsymbol{\varsigma},k} \sim N(0, 2\sigma^2)$.

The observation consists of the relative range $\underline{d}_k$, and the displacement difference $\underline{\boldsymbol{\rho}}_k$ from the starting $P_{i,0}$ and $P_{j,0}$ as below:

$$\begin{cases} \underline{d}_k = \|\mathbf{x}_{0,k}^{ij} + \Delta\mathbf{x}_k^{ij}\| + \epsilon_{d,k}, \\ \underline{\boldsymbol{\rho}}_k = \Delta\mathbf{x}_k^{ij} + \boldsymbol{\epsilon}_{\boldsymbol{\rho},k}, \end{cases} \quad (4.17)$$

where the displacement difference $\underline{\boldsymbol{\rho}}_k$ of UAV*i* and UAV*j* is different from (4.15), which can be obtained by inter-UAV communication since each UAV stores the relative position to its starting point.

Combined with the initial RL algorithm described in Algorithm 4.1 and Algorithm 4.2, the proposed cooperative relative localization is summarized in Algorithm 4.3.

4.5 Simulation Results

4.5.1 Relative Localization with Single Beacon

In this section, the performance of the proposed active RL Algorithm 4.2 is evaluated by simulations. Two cases of noise-free and noise-corrupted self-displacements are

Algorithm 4.3: Pseudocode for Cooperative Localization without Beacon**Procedure:** Relative Localization**Data communication:** UAV i sends range request to its neighbor UAV j and receive this neighbor's displacement and height information (which helps transfer a 3D distance measurement into 2D).**Data Matching:** The collected UAV j 's information and measured distance are stored together with UAV i 's displacement and height measurement by querying the closest UAV i 's sensor data in the buffer. Based on the initial relative estimate (Algorithm 4.2) and selected historical data (e.g., m samples), execute m steps EKF in (4.14) and (4.15) to initialize the estimation of the states of UAVs for the simultaneous movement.**while** *CooperativeFlight* **do**

$$\hat{\mathbf{x}}_{0,k+1|k}^{ij}, \Delta\hat{\mathbf{x}}_{k+1|k}^{ij} \leftarrow (4.16)$$

$$\mathbf{H1} = \frac{[(\hat{\mathbf{x}}_{0,k+1|k}^{ij} + \Delta\hat{\mathbf{x}}_{k+1|k}^{ij})^\top, (\hat{\mathbf{x}}_{0,k+1|k}^{ij} + \Delta\hat{\mathbf{x}}_{k+1|k}^{ij})^\top]}{\hat{d}_{k+1|k}}$$

$$\mathbf{H2} = [\mathbf{1}_{2 \times 2}, \mathbf{0}_{2 \times 2}]$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H1} \\ \mathbf{H2} \end{bmatrix}$$

carry out EKF

update $\hat{\mathbf{x}}_{0,k+1}^{ij}$ and $\Delta\hat{\mathbf{x}}_{k+1}^{ij}$

calculate $\hat{\mathbf{x}}_{k+1}^{ij} = \hat{\mathbf{x}}_{0,k+1}^{ij} + \Delta\hat{\mathbf{x}}_{k+1}^{ij}$

end

simulated, respectively. Specifically, in the former case, we compare the performance of the proposed algorithm with that of a constant heading in the piecewise linear phase, in order to show the necessity of adaptive heading. In addition, we also compare the performance of our approach with that of other two methods in the literature.

For a comprehensive evaluation, the initial starting point P_0 is placed at different circles centered at O , over a span of radius from 20 to 50m with an interval of 5m, as seen from the abscissa in Fig. 4.5 and Fig. 4.6. On each circle P_0 will be randomly generated by the uniform distribution and 50 Monte–Carlo simulations are repeated, with the whole absolute localization error distribution on each direction represented in the form of box plot. Note that the blue box contains an approximate 95% of the data in case of Gaussian distribution, and the red one covers the data around the sample mean within one unit of standard deviation.

Some common parameters in the simulation are listed as follows. The UAV moves at the speed of $1m/s$, the ranging frequency is given by 5 Hz, and the ranging noise $\epsilon_{d,k} \sim N(0, 0.1^2)$. In both cases the nonlinear path (phase I) is taken as a square of $1.5 \times 1.5m^2$, and the piecewise linear path (phase II) consists of 6 movements along each segment of $1m$. Therefore, the UAV will move $12m$ in total, and it is noted that $12 < \frac{\sqrt{2}}{2}d_{min}$ with $d_{min} = 20m$ as the smallest distance in the simulation. In other words, the assumption of Theorem 4.3.1 is satisfied.

4.5.1.1 Noise-free case

With the noise-free displacement measurement, the RL problem is equivalent to estimating $\mathbf{p}_0$ and the estimation error is the difference between $\mathbf{p}_0$ and $\hat{\mathbf{p}}_0$. Fig. 4.5(a) and 4.5(b) show the absolute estimation error on each direction in the cases of switching and constant heading, respectively. In the latter case, the heading of the mobile UAV in phase II is fixed based on the estimate $\hat{\mathbf{p}}_0$ at the end of phase I. From Fig. 4.5(a), it can be seen that the estimation error tends to increase with the distance, with most of the estimation error falling within the bound of $0.4m$ on each direction. In the worst case, the estimation error is still within $0.6m$. In comparison, we observe from Fig. 4.5(b) that the sizes of error boxes are all larger than their counterparts in the switching heading case, ranging from $0.4m$ to $0.8m$, and in some extreme cases the estimation error can be as large as $2.5m$. Obviously, the performance of Algorithm 4.2 is satisfactory if we consider the distance of $50m$ and accuracy of $0.6m$ in the worst case, with only a movement of $12m$. The comparison between Fig. 4.5(a) and 4.5(b) also justify the necessity of changing heading to effectively reduce the localization error.

4.5.1.2 Noise-corrupted case

In this subsection we evaluate the performance of EKF in Section 4.3.3.2 when the displacement measurement is corrupted by noise. Under the assumption of $\xi_{u,k} \sim N(0, 0.1^2)$ and $\epsilon_{z,k} \sim N(0, 0.7^2)$ (see (4.14) and (4.15) for the noise definition

(a) Absolute estimation error with switching heading in noise-free case (b) Absolute estimation error with constant heading in noise-free case

Figure 4.5: Comparison of the absolute error between switching heading and constant heading in noise-free case

with the standard deviations summarized from experiments), we redo the simulation under the same setting in Section 4.5.1.1 and plot the result in Fig. 4.6(a). Note that the absolute estimation error in this case is the average absolute error of relative localization over the whole ranging path. It can be seen that on each sampling circle, all the mean absolute estimation errors are bounded within $1m$ on each direction, and most of the estimation falls within the error bound of $1.5m$.

4.5.1.3 Comparison with other methods

Here we compare the performance of our approach with that of [63] and [96] under the same setting in Section 4.5.1.2, as respectively shown in Fig. 4.6(b) and Fig. 4.6(c). We can see that the absolute estimation error in Fig. 4.6(b) generally ranges from 2 to $5m$, much larger than that in Fig. 4.6(a), while the error in Fig. 4.6(c) can be as large as dozens of meters. Such a poor performance when compared with the approach in this chapter can be expected, if we note that the augmented system in [63] requires more movement and samples to produce a convergent estimate, and the squared distance measurements result in an enlarged error and a poor estimate when the ranging path adopted in this chapter is not so nonlinear as that in [96].

(a) Absolute estimation error of the proposed RL

(b) Absolute estimation error by including augmented state [63]

(c) Absolute estimation error by recursive least squares method [96]

Figure 4.6: Comparison of the absolute error between the proposed method and reference methods in noise-corrupted case

4.5.2 Cooperative Relative Localization without Beacon

In this section, we will demonstrate the accuracy of the cooperative relative localization for 3 UAVs. In the simulation, the initial RL (with single beacon) and its corresponding covariance are fed into the cooperative RL algorithm (without beacon) directly for the first step estimation. The noise of displacement difference $\zeta_{s,k} \sim N(0, 2 \times 0.25^2)$ and distance noise $\epsilon_{d,k} \sim N(0, 0.1^2)$ are added into our simulation. For consistency, all the parameters for initial RL estimation keep the same as those in Section 4.5.1. The flight time of initial RL is 12s (phase I and phase II are set as 6s and 6s respectively), and that of simultaneous movement (without

beacon) is 600s.

In practice, the relative positions among UAVs are leveraged for achieving multi-UAV cooperation such as formation shape control, flocking and coverage control, where the trajectory of each UAV is controlled by their behavior manager. To validate the proposed RL algorithm independently (without using any multi-agent controller), a serpentine path of UAV1 is applied while UAV2 and UAV3 just fly straight and keep the same heading as that at the end of phase II (Algorithm 4.2). Note that no control laws (e.g., obstacle avoidance, formation control and path planning) are applied in the simulation.

The flight trajectories of 3 UAVs are depicted in Fig. 4.7 and the statistic accuracy (350 tests as introduced in Section 4.5.1) of RL estimation during the simultaneous flight phase is shown in Fig. 4.8. Fig. 4.9 shows the indirect RL estimate of UAV3 to UAV2 ($\hat{\chi}^{32}$) generated by the RL estimates of UAV3 to UAV1 ($\hat{\chi}^{31}$) and UAV2 to UAV1 ($\hat{\chi}^{21}$).

Figure 4.7: Flight trajectories of 3 UAVs for 600s in noise-corrupted case

From Fig. 4.7, it can be seen that the UAV2 and UAV3 fly a short nonlinear and linear path (RL initialization) before UAV1 starts to move. Fig. 4.8 depicts that

Figure 4.8: Absolute error of RL estimation with measurement noise during the simultaneous flight phase (600s) in Fig. 4.7

the estimation errors of $\hat{\chi}^{21}$ and $\hat{\chi}^{31}$ fall within the bound of $0.2m$. In comparison, we observe that the estimation error of $\hat{\chi}^{32}$ tends to increase, especially for the interval distance from $35m$ to $50m$. Despite this, in the worst case, the mean of estimation error is still within $0.8m$. Note that the distance between UAV2 and UAV3 increases as time goes on (Fig. 4.7) which may lead to the decreased RL accuracy. Actually, in real multi-UAV applications, control strategy will let UAVs cooperate with each other and the spacing gap of inter-UAV may not differ greatly in general. For instance, in the process of forming an equilateral triangle formation or flocking, the relative positions among 3 UAVs are more or less fixed and not different greatly. An alternative indirect way to obtain $\hat{\chi}^{32}$ is to calculate $\hat{\chi}^{31} - \hat{\chi}^{21}$ through inter-UAV communications and the estimation error reduces to be within $0.2m$ as seen in Fig. 4.9 due to more accurate $\hat{\chi}^{31}$ and $\hat{\chi}^{21}$ estimates.

Remark 4.5.1. *From the preceding simulations, the statistical errors of RL estimation for a relative long-term flight (612s) can be acceptable. Despite this, there still exists a possibility of RL estimation drift over a long time due to various unreasonable measurements or long-term data dropout. For safety issue, Algorithm 4.1 can be operated concurrently as a backup of RL estimate or for fusion.*

Figure 4.9: Absolute error of indirect RL estimate $\hat{\chi}^{32}$, $\hat{\chi}^{31} - \hat{\chi}^{21}$, during the simultaneous flight

Figure 4.10: Test environment

4.6 Experiments with UAVs

4.6.1 Methodology

We implemented the active RL algorithm on quadcopters and conducted tests in a sports field and a woods environment respectively as shown in Fig. 4.10 and Fig. 4.16(b), where the moving UAVs navigate to their relative targets based on the beacon's position and their RL estimates to this beacon. UWB modules, PulsON 410[®], are installed on each UAV to obtain reliable long distance measurements. Pixhawk[®] is adopted for low-level flight control, which is also connected to a GPS module for safety and displacement measurement purpose. It should be noted that although the distance measurement provided by UWB is a range in 3D space, the

projected distance on 2D plane will be almost the same if the 3D distance is much larger than the height, say in our case the mobile UAV always moved at a distance further than $20m$ from the beacon, and at a height of no greater than $2m$.

To evaluate the active RL Algorithm 4.2, flight tests for 2 UAVs were carried out first as shown in Fig. 4.10, where the moving UAV and the beacon are respectively circled in yellow and red. UWB modules were installed on both quadcopters, and one of them worked as a stationary beacon. 10 different starting (launching) points were chosen for 10 different vectors $\overrightarrow{OP_0}$ as seen in Section 4.6.3.1. The norms of these vectors were randomly generated from 20 through $50m$, with the corresponding bearings in NE frame from 15° through 95° . The ground truth of the starting points were determined by measuring distance and bearing respectively. During the experiment, the moving UAV will first take off from the starting point, then fly along the nonlinear and piecewise linear paths as in the simulation before landing. The vector connecting the static beacon and the landing point can be measured similarly as in the case of launching points. In addition, flight experiments for 3 UAVs were conducted to further corroborate the effectiveness of our proposed RL method, where two UAVs moved simultaneously and one UAV hovered in the sky. The details can be seen in Section 4.6.3.2.

4.6.2 UWB-based Relative Localization Workflow

Fig. 4.11(a) illustrates the workflow of the UWB-based relative localization system. The UWB module on a hovering quadcopter actively sends ranging requests to neighboring UAVs for distance measurement and communication based on the two-way time of flight ranging method described in Section 3.2. Once a distance is obtained by UAV2 and UAV3, it will be calibrated by linear regression where the calibration parameters are determined by a series of experiments in different environments. This calibrated range then goes through the outlier detection before it is stored in the database. To reduce the computation and avoid excessive repetition of similar data, only selected distance measurements and neighbor's information are

recorded and stored. For example, only the distance measurements that vary more than $0.05m$ from the previous stored measurement can be selected for storage, and this operation is done automatically in real application. Note that RL initialization algorithm is executed first and its output serves as the initial state estimate for EKF. The follow-up localization is sustained by EKF in a recursive way. Finally, the update of RL estimation will be fed into the flight controller for quadcopter navigation as depicted in Fig. 4.11(b).

4.6.3 Evaluation

4.6.3.1 Flight tests for 2 UAVs

Table 4.1 and Table 4.2 show the absolute estimation errors of 10 tests in each direction at the launching and landing points, respectively. The flight trajectories are shown in Fig. 4.12(a) where each test includes a $1.5 \times 1.5m^2$ square nonlinear path (*red*, phase I), a $6m$ linear path (*green*, phase II) and landing path (*blue*). Note that the green curves tend to maximize the ranging span relative to the static quadcopter for reducing RL errors as mentioned in Section 4.3.2. The corresponding ground truth and estimate are illustrated in Fig. 4.12(b), with circles denoting the error bound of $1m$ and $1.5m$ respectively around the launching and landing points. As can be seen from the tables and Fig. 4.12(b), most of the estimates at the launching points fall within the error bound of $1m$, with the largest one no larger than $1.5m$. Since the focus of this chapter is relative localization to the beacon, compared to the range ($20 - 50m$), this error can be considered small. The estimates at the landing points degrade a little, but most still fall within the error bound of $1.5m$. Such a performance is acceptable if we consider that the landing occurs after $12m$ of movement, and inevitably the undesirable drift during landing leads to a deviation from each RL estimate. The linearization in EKF may also contribute to the increased error. Nonetheless, a better initial estimate tends to improve the convergence of EKF [89].

(a) UWB-based relative localization workflow

(b) Diagram of flight control workflow

Figure 4.11: System structure

We also examine the variation of the estimation error of the starting point during the whole test, as shown in Fig. 4.13. Note that totally seven different estimates¹ are

¹Seven measurements are selected by setting the variation between two samples larger than

Table 4.1: Absolute estimation error at launching points.

Test	range (m)	angle ($^{\circ}$)	$ x $ error (m)	$ y $ error (m)
1	20	24.5	0.267	0.920
2	24	15.5	0.096	1.039
3	27	94.2	0.045	1.187
4	30	51.5	0.498	1.338
5	34	87.5	0.632	0.210
6	37	42.5	0.150	0.858
7	40	78.5	0.755	0.662
8	45	33.5	1.166	0.951
9	47	69.5	0.690	1.106
10	50	60.5	0.103	0.023
mean			0.440	0.830

Table 4.2: Absolute estimation error at landing points.

Test	range (m)	angle ($^{\circ}$)	$ x $ error (m)	$ y $ error (m)
1	20	40.2	1.138	1.779
2	24	31.7	1.0	1.062
3	27	108.9	1.417	1.294
4	30	63.7	0.02	0.967
5	34	97.6	2.30	0.355
6	37	51.6	1.174	0.798
7	40	88.0	0.612	0.476
8	45	42.5	1.192	1.094
9	47	76.2	0.186	0.676
10	50	70.3	0.822	0.929
mean			0.986	0.943

involved in the whole duration of RL, corresponding to seven EKF updates. From Fig. 4.13 we observe that the error keeps decreasing with the increasing traveling distance, and the ranging along the piecewise linear path drastically reduces the error of the initial estimation. Such a result demonstrates the efficacy of reducing error by enlarging the ranging span as revealed in Theorem 4.3.1. On the other hand, the decrease of error after each change of heading suggests the necessity of changing heading. We also note that after some steps the estimation error tends to stabilize, implying that the increase of ranging span is no longer able to counter the

0.5m

(a) Trajectories of 10 flight tests

(b) Comparison of relative position estimate and ground truth

Figure 4.12: Evolutions of the flight trajectories and estimation errors

displacement error.

(a) $|x|$ error (b) $|y|$ error

Figure 4.13: Evolutions of the RL estimation errors at the launching point

4.6.3.2 Flight tests for 3 UAVs

To further validate the performance of the proposed RL method, flight tests of 3 UAVs were conducted at Dover in Singapore with an area of $30 \times 40m^2$ as seen in Fig. 4.15(b). Firstly, to support RL estimation among 3 UAVs, we had to design a new UWB ranging and communication topology which is different from that utilized for 2 UAVs. Without clock synchronization among UWB modules, a master-slave ranging mode is adopted to avoid sensing conflicts as shown in Fig.

4.14. Specifically in our relative localization context, the module installed on the UAV1 (denoted as M), will actively send its ranging request and odometry data to the module on UAV2 and UAV3 (denoted as S , assume that they are equivalent). The responder module (UAV2 or UAV3) will automatically respond to the ranging request it receives from UAV1 and at almost the same time, collect the ranging and odometry data which has been synchronized by the master node one time slot afore.

Figure 4.14: Master-slave ranging mode

Subsequently, the UWB module M installed on UAV1 transmits and receives ranging command signal as well as odometry information from UAV3 at $M_{t_2}^{Tx}$ and $M_{t_3}^{Rx}$ respectively. And the distance to UAV3 can be achieved by $(M_{t_3}^{Rx} - M_{t_2}^{Tx} - \delta_S) \times c/2$. UAV1 switches ranging ID between UAV2 and UAV3 alternately in each loop with a fixed ranging rate of 20Hz.

Since the reference of our relative position estimate can be any customized coordinates, a baseline herein, which is parallel to a ditch nearby with 233° relative to the north, was first selected for calibrating the ground truth. UAV2 and UAV3 are desired to navigate to their destinations based on the gap between the preconfigured destinations and their real time RL estimates to UAV1. In this test, the destination

points relative to UAV1 were set preliminarily based on the method in Section 4.6.1 and marked by yellow flags as shown in Fig. 4.16(a). UAVs will hover over their destination points once they reach them. Such performance is able to demonstrate the practicability of our UWB-based relative localization and the accuracy of landing reveals the applicability of the proposed RL method. The following gives a detailed description on the flight experiment.

Initially, UAV1 was set in the middle of UAV2 and UAV3 with $20m$ interval as shown in Fig. 4.15(a). Once all of the UAVs were powered on, UAV1 started automatically to range and communicate with UAV2 and UAV3. After taking off, UAV1 hovered over its launching point while UAV2 and UAV3 flew along a preset nonlinear trajectory for collecting distance data and calculating the initial relative position, namely phase I. The RL estimates were improved when UAV2 and UAV3 flew along a piecewise linear path during phase II. Based on the difference between the on-line RL estimates and their preset destinations, in phase III, UAV2 and UAV3 navigated directly to their desired target positions (the red stars in Fig. 4.15(a)).

The flight video shot is shown in Fig. 4.15(b). Finally all of these 3 UAVs hovered over their destination as seen in Fig. 4.16(a) with the position error less than $1m$ (Fig. 4.16(b)). More details concerning the trajectories of 3 UAVs are depicted in Fig. 4.17 where UAV2 and UAV3 eventually hovered over their relative targets to UAV1, $(0, 30)$ and $(0, 15)$, respectively, and the RL estimation errors both fell within the $1m$ error bound (cyan circle). Note that the RL error was measured after UAVs landed and in fact there always exists an inevitable drift during landing (e.g., avoiding to land on the marker, wind disturbance, near-ground effect and control accuracy). Such drift would cause an undesirable deviation from each RL estimate. Despite this, a RL accuracy with less than $1m$ can be acceptable when considering the maximum inter-UAV distance reaches $30m$ in this scenario².

²The flight video can be found in: <https://youtu.be/ye1B6M6yHSU>

(a) Demo scenario at Dover

(b) Flight video shot of 3 UAVs at Dover

Figure 4.15: Flight demo of 3 UAVs at Dover

4.7 Conclusion

In this chapter we studied the UWB-based RL problem for a UAV team. An active RL problem of a moving UAV with respect to a static UAV under motion constraint was introduced first. Specifically, the UAV is only allowed to move by a small distance D and we aim to find a ranging path to reduce the RL error as much as possible. We considered the problem under the assumption of noise-free self-displacements, and reformulated it as an optimal sensor placement under constraint. A lower bound of estimation error was obtained in terms of D and the sample size n , and it was found that one only needs to enlarge the ranging span and increase the number of rangings to reduce the error. In this light, we designed an active

(a) Performance of the relative localization for 3 UAVs

(b) Accuracy of the relative localization measured after landing

Figure 4.16: Evaluation of the UWB-based RL estimates with 3 UAVs flight

RL algorithm to enlarge the ranging span within the distance D , and applied the EKF to account for the noise of the displacement measurements. Simulations and experiments were conducted to validate the algorithm.

Secondly, we extended the EKF-based RL method to a dynamic UAV group where the beacon UAV starts to move and the RL estimate from the first stage is fed into the proposed cooperative RL algorithm for initialization. To validate the accuracy of the cooperative RL estimate during UAVs' simultaneous flight, a RL simulation was conducted and the error bound falls within $0.8m$ on average.

Figure 4.17: Flight paths of 3 UAVs

Chapter 5

Infrastructure-free Cooperative Relative Localization

5.1 Introduction

Recent progress in multi-agent control using some extra infrastructures such as GPS [19, 20], motion tracking system (e.g., VICON[®]) [14, 97], and UWB anchor nodes [22], has enabled the formation flight of quadcopters in indoor and outdoor environments. In theory, to realize the autonomous and distributed formation control, each UAV should independently measure the relative positions to its neighbors. Unfortunately, there is no commercialized system for quadcopters to measure the relative positions to its neighbors without extra infrastructures. Moreover, the implementation of an infrastructure-free RL for real-world flights is still a hard problem. As a result, the researchers focusing on the design of control laws will generally leave the measurement problem as an unsolved problem and assume that the relative position measurements can be obtained indirectly by communicating their global coordinates. However, the global coordinates are not generally available, for example, in urban or forest environments. To realize on-board relative position estimation of UAVs, an infrastructure-free cooperative relative localization scheme is proposed in this chapter.

Existing RL works mainly apply computer vision [98] to assist with agents maneuvers, however, visual localization requires heavy computation, has limited angle of view and may not provide accurate RL estimates in dark or constantly changing environments. Another onboard Bluetooth-based RL method proposed in [99] was leveraged for collision avoidance in UAV swarms. However, the flight results show that the UAVs can only fly in some special environments. [59] formulated the RL estimation as a convex optimization problem and solved the problem in a centralized manner and required a high computational cost.

In Chapter 3 and Chapter 4, we have developed single and multiple landmarks based UAV localization and navigation systems, leveraging on UWB technology, which can achieve a sub-metre accuracy. The centimetre level of ranging accuracy within the radius of $150m$, light weight, small size and low power consumption make UWB a reliable and feasible ranging technique that can be applied for UAVs. However, in many operation environments, it is impractical to have fixed landmarks due to large operational scale and area. Therefore, this chapter will propose another relative localization scheme without any known references and the proposed system can be applied in a variety of environments, indoor and outdoor, none-line of sight, with/without occlusion thanks to the respective advantages of UWB and optical flow sensors.

However, in a large group, not all UAVs are able to sense the relative distance to their neighbors due to physical constraints of onboard sensors (e.g., out of measuring range) or complex environments (e.g., urban or forest environments). Thus, RL estimation in a cooperative way among UAVs is adopted in this chapter to obtain continual relative estimates in real time. Another reason for cooperative relative localization is that cooperation among UAVs can improve the localization performance and ensures robustness in responding to measurement dropouts. Thus, this chapter falls into infrastructure-free cooperative RL problem in noise-contaminated case and the major contributions are summarized as follows:

- 1) RL estimators are proposed for cooperative relative localization among UAVs by taking into account the effects of measurement noises;

- 2) Consensus-based RL fusion is designed to enhance the robustness of RL estimation in case of communication or sensing loss;
- 3) By designing an on-board UWB ranging and communication network, the proposed cooperative RL method is demonstrated with three UAVs flying.

The rest of this chapter is organized as follows. We first formulate the problem in Section 5.2. Section 5.3 proposes a cooperative RL method and consensus-based RL fusion estimation, followed by a UWB network-based RL initialization strategy. Section 5.4 discusses the sensor specifications for real-world experiments. Simulation and flight experiments are conducted in Section 5.5 and Section 5.6, respectively. Finally, conclusion remarks are drawn in Section 5.7.

5.2 Problem Formulation and Background

5.2.1 Problem of Interest

Consider a team of N UAVs (labeled consecutively from 1 to N) working collaboratively to localize themselves with respect to their neighbors. Meanwhile, each UAV employs these relative estimates to jointly perform formation.

Let $\mathcal{N}_i$ be UAV i 's neighbor set, in which UAV i is able to obtain the distance measurement d^{ij} and receive data packets from UAV j for $j \in \mathcal{N}_i$. That is, when performing a formation, neighboring UAV j is actually within an area with a perceiving radius $\bar{d}$, i.e., $d^{ij} \leq \bar{d}$. Leveraging the obtained sensing information, UAV i aims to estimate the relative position of UAV j , $\mathbf{x}_{i,k}^{ij}$, at $t = t_{i,k}, k \in \mathbb{N}$ in UAV i 's own frame $\mathcal{F}_i$ as depicted in Fig. 5.1. Then, based on the relative position estimates to its neighbors, each UAV generates a corresponding formation control command. In this chapter, two scenarios are considered: (1) UAV i is able to sense UAV j and conduct direct RL estimation, i.e., UAV j is a primary node for UAV i , and (2) UAV i is only able to range and communicate with UAV r which is a neighbor of UAV j , but fails to sense UAV j directly, and UAV j herein serves as a secondary node.

Figure 5.1: Infrastructure-free cooperative RL among UAVs

Assume that each UAV i is able to access its own velocity $\mathbf{v}_i$ in its local frame $\mathcal{F}_i$. The orientations of reference frames $\mathcal{F}_i$, $i = 1, \dots, N$, are the same. This can be satisfied if the UAVs carry compasses. Moreover, we assume that each UAV i is equipped with an onboard UWB module, which can measure the range d_k^{ij} to UAV j and obtain its corresponding change rate $\dot{d}_k^{ij}$ at $t = t_k, k \in \mathbb{N}$. In this chapter, two-dimensional case is considered. Consequently, it follows that $d_k^{ij} = d_k^{ji} = \|\boldsymbol{\chi}_k^{ij}\|$ and $\dot{\boldsymbol{\chi}}_k^{ij} = \boldsymbol{\nu}_k^{ij}$, where $\boldsymbol{\chi}_k^{ij} = [x_{j,k} - x_{i,k}, y_{j,k} - y_{i,k}]^\top$ and $\boldsymbol{\nu}_k^{ij} = \mathbf{v}_{j,k} - \mathbf{v}_{i,k}$ are the relative coordinates and relative velocity of UAV j in UAV i 's local frame respectively, and (x_i, y_i) denote the global coordinates of UAV i . The first objective is to develop a RL estimator such that each UAV i can estimate $\boldsymbol{\chi}_{i,k}^{ij}$, $j \in \mathcal{N}_i$. With these RL estimates and inter-UAV distance measurements, the next objective is to design a distributed formation control for real-world UAV swarm applications. This chapter focuses on the first objective and the second one will be discussed in the next chapter. To proceed, we make the following standard assumption.

Assumption 5.2.1. *The velocity $\mathbf{v}_i$ of each UAV i , $i = 1, \dots, N$, is continuously differentiable and bounded.*

Assumption 5.2.1 ensures that the motions of the UAVs are driven by finite forces. For any $i, j \in \mathcal{N}_i$, the continuous differentiability of $d_k^{ij} : \mathbb{R}_{>0} \mapsto \mathbb{R}$ can be inferred

from Assumption 5.2.1 as well. Suppose there exists a constant $\bar{v} > 0$ such that $\|\mathbf{v}_k\| \leq \bar{v}$. Then UAVs' real velocities are upper bounded, from which it follows $\|\boldsymbol{\nu}_k^{ij}\| \leq 2\bar{v}$. Next, graph theory is introduced in Section 5.2.2.

5.2.2 Graph Theory

If each UAV is regarded as a node, then their coupling topology can be described by a graph. Let $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ denote a *directed graph* consisting of a node set $\bar{\mathcal{V}} = \{1, 2, \dots, N\}$ and arc set $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. To study the relative localization problem (e.g., estimate the relative positions to UAV j), we also introduce another *weighted digraph* graph $\mathcal{G}^j = (\mathcal{V}^j, \mathcal{E}^j, \mathcal{A}^j)$ which is composed of a set of nodes $\mathcal{V}^j = \bar{\mathcal{V}} \setminus \{j\}$, a directed edge set $\mathcal{E}^j \subseteq \mathcal{V}^j \times \mathcal{V}^j$, and a weighted adjacency matrix $\mathcal{A}^j = [\kappa_{ir}^I] \in \mathbb{R}^{(N-1) \times (N-1)}$. $\mathcal{V}^j$ is the set of $N-1$ nodes excluding j , and if $r \in \mathcal{N}_i \setminus \{j\}$, there is an edge $(i, r) \in \mathcal{E}^j$. Moreover, $\forall i, r \in \mathcal{V}^j$, $\kappa_{ii}^I = 0$, $\kappa_{ir}^I > 0$ if $(i, r) \in \mathcal{E}^j$, and $\kappa_{ir}^I = 0$ otherwise.

Denote $\mathcal{L}^j$ the Laplacian matrix of $\mathcal{G}^j$, whose diagonal elements $l_{ii} = \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I$ and for $i \neq r$, there holds $l_{ir} = -\kappa_{ir}^I$. For $\bar{\mathcal{G}}$, if there is a path from every UAV in $\bar{\mathcal{G}}$ to UAV j , we say that UAV j is *globally reachable* in $\bar{\mathcal{G}}$, which is much weaker than *strong connectedness* (every node is reachable from every other node). Define $\mathcal{B}^j \in \mathbb{R}^{(N-1) \times (N-1)}$ to be a j adjacency matrix associated with $\bar{\mathcal{G}}$ with diagonal elements κ_{ij}^D for $i \in \mathcal{V}^j$, where $\kappa_{ij}^D > 0$ if UAV j is a neighbor of UAV i , that is, UAV i has a direct distance measurement of UAV j and $\kappa_{ij}^D = 0$ otherwise.

5.3 Cooperative RL Estimation

In this section, the cooperative RL estimators based on UWB RCM network is presented. To enhance the robustness, a consensus-based RL fusion estimation method is then pursued. Finally, a RL initialization strategy for real implementations is introduced.

5.3.1 Persistent Excitation-based RL Estimation

5.3.1.1 Incorporating local measurements

Suppose each UAV i is able to measure the relative distance d_k^{ij} to its neighbor j at sampling time instant $t_{i,k}$. Meanwhile, UAV i will transmit certain information, including $\mathbf{v}_{i,k}$, local RL estimates $\hat{\mathbf{x}}_{i,k}^{ij}$, etc., to its neighbors (e.g., UAV j) by encoding the data packet into one ranging request command. For the listener's side, although UAV j can acquire its own velocity $\mathbf{v}_{j,t}$ continuously, $\mathbf{v}_{j,k}$ is actually sampled for calculation at $t = t_{j,k}$ once UAV i 's information is received. A time-synchronization signal has been entrained in our UWB RCM¹ protocol for each cycle, thus, without consideration of the transmission time, we assume $t_{i,k} = t_{j,k}$. Since $j \in \mathcal{N}_i$, UAV i can receive UAV j 's information as well. Assume that all of the clocks have been synchronized, based on the obtained information on UAV i , we aim to design an estimator for each UAV i in a team to localize the relative position $\mathbf{x}_{i,k}^{ij}$ in its own frame $\mathcal{F}_i$ at $t = t_{i,k}, j \in \mathcal{N}_i, k \in \mathbb{N}$.

With the assumption in Section 5.2 that the orientations of reference frames $\mathcal{F}_i$ and $\mathcal{F}_j$ for $i \neq j$ are consistent, it follows that $\mathbf{x}_{k+1}^{ij} = \mathbf{x}_k^{ij} + T\boldsymbol{\nu}_k^{ij}$ and $\mathbf{x}_{i,k}^{ij} = -\mathbf{x}_{j,k}^{ji}$. Taking the derivative of both sides of $d_t^{ij2} = \|\mathbf{x}_t^{ij}\|^2$ with respect to time in continuous-time version and we can get $d_t^{ij} \dot{d}_t^{ij} = \boldsymbol{\nu}_t^{ij\top} \cdot \mathbf{x}_t^{ij}$. By discretizing it, one obtains $d_k^{ij} \dot{d}_k^{ij} = \boldsymbol{\nu}_k^{ij\top} \cdot \mathbf{x}_k^{ij}$. By taking into consideration of the sensor noises, then the RL estimation on UAV i at $t = t_{i,k}$ is given by

$$\begin{aligned} \hat{\mathbf{x}}_{i,k+1}^{ij} &= \hat{\mathbf{x}}_{i,k}^{ij} + T(\boldsymbol{\nu}_{i,k}^{ij} + \boldsymbol{\epsilon}_k) + \gamma T(\boldsymbol{\nu}_{i,k}^{ij} + \boldsymbol{\epsilon}_k) \\ &\quad \times \left[(d_k^{ij} + \epsilon_k^d) (\dot{d}_k^{ij} + \epsilon_k^{\dot{d}}) - (\boldsymbol{\nu}_{i,k}^{ij} + \boldsymbol{\epsilon}_k)^\top \hat{\mathbf{x}}_{i,k}^{ij} \right], \end{aligned} \quad (5.1)$$

where $\boldsymbol{\epsilon}_k$, ϵ_k^d and $\epsilon_k^{\dot{d}}$ are the measurement noises of $\boldsymbol{\nu}_{i,k}^{ij}$, d_k^{ij} and $\dot{d}_k^{ij}$ respectively at $t_{i,k}$. $\gamma \in \mathbb{R}^+$ is a tuneable constant gain. Next, we will give the stability analysis

¹Each UAV has both transmitter and listener roles in our UWB RCM protocol. Note that the larger the data packet the longer the time duration of one ranging slot and thus, practically, a trade-off between ranging update rate and the quantity of data transmission should be considered for a UWB RCM protocol design.

of RL estimation in noise-free case first, then we will show that the RL estimation error is bounded in noise-contaminated case.

5.3.1.2 Stability analysis of RL estimation in noise-free case

The RL estimator in noise-free case can be written as

$$\hat{\mathbf{x}}_{i,k+1}^{ij} = \hat{\mathbf{x}}_{i,k}^{ij} + T\boldsymbol{\nu}_{i,k}^{ij} + \gamma T\boldsymbol{\nu}_{i,k}^{ij} \left[d_k^{ij} \dot{d}_k^{ij} - \boldsymbol{\nu}_{i,k}^{ij\top} \hat{\mathbf{x}}_{i,k}^{ij} \right]. \quad (5.2)$$

For $i = 1, \dots, N$, we let $\tilde{\mathbf{x}}_i^{ij} = \hat{\mathbf{x}}_i^{ij} - \mathbf{x}^{ij}$. Since $d_k^{ij} \dot{d}_k^{ij} = \boldsymbol{\nu}_k^{ij\top} \cdot \mathbf{x}_k^{ij}$, the error dynamic of (5.2) can be obtained as follows

$$\tilde{\mathbf{x}}_{i,k+1}^{ij} = \mathbf{A}_{i,k} \tilde{\mathbf{x}}_{i,k}^{ij}, \quad (5.3)$$

where $\mathbf{A}_{i,k} := \mathbf{I} - \gamma T \boldsymbol{\nu}_{i,k}^{ij} \boldsymbol{\nu}_{i,k}^{ij\top}$.

Before moving on, it is necessary to make the following assumption:

Assumption 5.3.1 (Persistent excitation condition). *There exist $m \in \mathbb{Z}^+$ and $\mu > 0$ such that, $\forall k \in \mathbb{Z}^+$,*

$$\sum_{l=k}^{k+m-1} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \geq \varpi \mathbf{I}. \quad (5.4)$$

Now, we are in a position to give the following result.

Theorem 5.3.1. *For UAV i , under Assumption 5.2.1 and Assumption 5.3.1, if the sampling period T satisfies*

$$0 < T < \frac{1}{4\gamma\bar{v}^2}, \quad (5.5)$$

then $\tilde{\mathbf{x}}_{i,k}^{ij}$ in (5.3) exponentially converges to $\mathbf{0}$, $\forall k \in \mathbb{Z}^+$.

Proof. A Lyapunov function $J(\tilde{\mathbf{x}}_{i,k}^{ij}) = \tilde{\mathbf{x}}_{i,k}^{ij\top} \tilde{\mathbf{x}}_{i,k}^{ij}$ is defined, and the difference of this Lyapunov function candidate in m time steps can be written as

$$\Delta J_m^k = J(\tilde{\mathbf{x}}_{i,k+m}^{ij}) - J(\tilde{\mathbf{x}}_{i,k}^{ij})$$

$$\begin{aligned}
&= \tilde{\mathbf{x}}_{i,k+m}^{ij\top} \tilde{\mathbf{x}}_{i,k+m}^{ij} - \tilde{\mathbf{x}}_{i,k}^{ij\top} \tilde{\mathbf{x}}_{i,k}^{ij} \\
&= \sum_{l=k}^{k+m-1} \left(\tilde{\mathbf{x}}_{i,l+1}^{ij\top} \tilde{\mathbf{x}}_{i,l+1}^{ij} - \tilde{\mathbf{x}}_{i,l}^{ij\top} \tilde{\mathbf{x}}_{i,l}^{ij} \right) \\
&= \sum_{l=k}^{k+m-1} (\gamma^2 T^2 \|\boldsymbol{\nu}_{i,l}^{ij}\|^2 - 2\gamma T) \cdot \tilde{\mathbf{x}}_{i,l}^{ij\top} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \tilde{\mathbf{x}}_{i,l}^{ij} \\
&\leq (4\gamma^2 T^2 \bar{v}^2 - 2\gamma T) \left(\sum_{l=k}^{k+m-1} \tilde{\mathbf{x}}_{i,l}^{ij\top} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \tilde{\mathbf{x}}_{i,l}^{ij} \right). \tag{5.6}
\end{aligned}$$

In addition, by using induction method on the system (5.3), RL estimation error at l step is obtained as

$$\tilde{\mathbf{x}}_{i,l}^{ij} = \prod_{t=k}^{l-1} \mathbf{A}_{i,t} \tilde{\mathbf{x}}_{i,k}^{ij} := \mathbf{A}_{i,k:l-1} \tilde{\mathbf{x}}_{i,k}^{ij}, \tag{5.7}$$

where $\mathbf{A}_{i,k:l-1} := \mathbf{I}$ when $l = k$.

To separate $\tilde{\mathbf{x}}_{i,k}^{ij}$ from $\tilde{\mathbf{x}}_{i,l}^{ij}$, next we expand (5.6). According to (5.7), (5.6) can be rewritten as

$$\Delta J_m^k \leq (4\gamma^2 T^2 \bar{v}^2 - 2\gamma T) \left(\sum_{l=k}^{k+m-1} \tilde{\mathbf{x}}_{i,k}^{ij\top} \mathbf{A}_{i,k:l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-1} \tilde{\mathbf{x}}_{i,k}^{ij} \right). \tag{5.6'}$$

To proceed, $\mathbf{A}_{i,k:l-1}$ should be extracted from the summation in (5.6'), and thus we need to prove the following inequality first

$$\mathbf{A}_{i,k:l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-1} \leq \mathbf{A}_{i,k:l-2}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-2}, \tag{5.8}$$

$\forall l = k, k+1, \dots, k+m-1$, where $\mathbf{A}_{i,k:s} := \mathbf{I}$ if $s < k$.

The following is the proof of (5.8). $\forall \mathbf{x} \in \mathbb{R}^2$,

$$\begin{aligned}
&\mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-1} \mathbf{x} \\
&= \underbrace{\mathbf{x}^\top \mathbf{A}_{i,k:l-2}^\top \mathbf{A}_{i,l-1}^\top}_{:= \mathbf{Z}^\top} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,l-1} \underbrace{\mathbf{A}_{i,k:l-2} \mathbf{x}}_{:= \mathbf{Z}} \\
&= \mathbf{Z}^\top \mathbf{A}_{i,l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,l-1} \mathbf{Z}
\end{aligned}$$

$$= \mathbf{Z}^\top (\mathbf{I} - \gamma T \boldsymbol{\nu}_{i,l-1}^{ij} \boldsymbol{\nu}_{i,l-1}^{ij\top})^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} (\mathbf{I} - \gamma T \underbrace{\boldsymbol{\nu}_{i,l-1}^{ij} \boldsymbol{\nu}_{i,l-1}^{ij\top}}_{:=\mathbf{B}_{i,l-1}}) \mathbf{Z}. \quad (5.9)$$

Before proceeding, we denote $\|\boldsymbol{\nu}_{i,l-1}^{ij}\|^2$ by $\lambda_{i,l-1}$, the *null space* of $\mathbf{B}_{i,l-1}$ by

$$\text{Null}(\mathbf{B}_{i,l-1}) = \{\mathbf{v} \in \mathbb{R}^2 | \mathbf{B}_{i,l-1} \mathbf{v} = \mathbf{0}\},$$

and the *eigen space* of $\lambda_{i,l-1}$ by

$$E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})} = \{\mathbf{u} \in \mathbb{R}^2 | \mathbf{B}_{i,l-1} \mathbf{u} = \lambda_{i,l-1} \mathbf{u}\}.$$

It is easy to see that $\|\boldsymbol{\nu}_{i,l-1}^{ij}\|^2$ is the only nonzero eigenvalue of $\mathbf{B}_{i,l-1}$ with eigenvector $\boldsymbol{\nu}_{i,l-1}^{ij}$, which is simple. Therefore, $\lambda_{i,l-1} = \|\boldsymbol{\nu}_{i,l-1}^{ij}\|^2$.

If $\mathbf{Z} \in \text{Null}(\mathbf{B}_{i,l-1})$,

$$\mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-1} \mathbf{x} = \mathbf{Z}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{Z}. \quad (5.10)$$

If $\mathbf{Z} \in E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})}$, without loss of generality, we choose $\mathbf{Z} = \nu \boldsymbol{\nu}_{i,l-1}^{ij}$, $\nu \in \mathbb{R}$, then

$$\mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:l-1} \mathbf{x} = (1 - \gamma T \|\boldsymbol{\nu}_{i,l-1}^{ij}\|^2)^2 \mathbf{Z}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{Z} < \mathbf{Z}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{Z}. \quad (5.11)$$

It is known that for any $\mathbf{Z} \in \mathbb{R}^2$, it has $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2$ for some $\mathbf{Z}_1 \in \text{Null}(\mathbf{B}_{i,l-1})$, $\mathbf{Z}_2 \in E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})}$, together with (5.10) and (5.11), which implies (5.8).

According to (5.8), together with (5.5) and (5.6'), we can get

$$\begin{aligned} \Delta J_m^k &\stackrel{(a)}{\leq} (4\gamma^2 T^2 \bar{v}^2 - 2\gamma T) \sum_{l=k}^{k+m-1} \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij} \\ &= (4\gamma^2 T^2 \bar{v}^2 - 2\gamma T) \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top \left(\sum_{l=k}^{k+m-1} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} \right) \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij} \\ &\stackrel{(b)}{\leq} \varpi (4\gamma^2 T^2 \bar{v}^2 - 2\gamma T) \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij}, \end{aligned} \quad (5.6'')$$

where (a) follows from (5.5) and (5.8); (b) follows from Assumption 5.3.1.

Now we need to prove

$$\mathbf{A}_{i,k:k+m-2}^\top \mathbf{A}_{i,k:k+m-2} \geq (1 - 4\gamma T \bar{v}^2)^{2(m-1)} \mathbf{I}. \quad (5.12)$$

The following is the proof of (5.12).

For $\mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} = (\mathbf{I} - \gamma T \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top})^\top (\mathbf{I} - \gamma T \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top})$, denote $\mathbf{B}_{i,l} := \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top}$ and $\lambda_{i,l} := \|\boldsymbol{\nu}_{i,l}^{ij}\|^2$. Similar with the proof of (5.8), $\forall \mathbf{x} \in \mathbb{R}^2$,

$$\mathbf{x}^\top \mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} \mathbf{x} = \begin{cases} \mathbf{x}^\top \mathbf{x} & \text{if } \mathbf{x} \in \text{Null}(\mathbf{B}_{i,l}), \\ (1 - \gamma T \|\boldsymbol{\nu}_{i,l}^{ij}\|^2)^2 \mathbf{x}^\top \mathbf{x} & \text{if } \mathbf{x} \in E_{\lambda_{i,l}}^{(\mathbf{B}_{i,l})}, \end{cases} \quad (5.13)$$

thus, $\mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} \geq (1 - 4\gamma T \bar{v}^2)^2 \mathbf{I}$. According to (5.5), (5.12) holds.

After combining (5.6'') with (5.12), (5.6) can be presented as follows

$$\Delta J_m^k \leq -2\varpi\gamma T (2\gamma T \bar{v}^2 - 1) (4\gamma T \bar{v}^2 - 1)^{2(m-1)} \|\tilde{\mathbf{x}}_{i,k}^{ij}\|^2 \leq 0,$$

thus $\tilde{\mathbf{x}}_{i,k}^{ij}$ is exponentially stable around $\mathbf{0}$, and then the proof is completed. $\blacksquare$

Remark 5.3.1. Let $\boldsymbol{\nu}_{i,l}^{ij} = [\nu_{ix,l}^{ij}, \nu_{iy,l}^{ij}]^\top$, then

$$\sum_{l=k}^{k+m-1} \boldsymbol{\nu}_{i,l}^{ij} \boldsymbol{\nu}_{i,l}^{ij\top} = \begin{bmatrix} \sum_{l=k}^{k+m-1} (\nu_{ix,l}^{ij})^2 & \sum_{l=k}^{k+m-1} \nu_{ix,l}^{ij} \nu_{iy,l}^{ij} \\ \sum_{l=k}^{k+m-1} \nu_{ix,l}^{ij} \nu_{iy,l}^{ij} & \sum_{l=k}^{k+m-1} (\nu_{iy,l}^{ij})^2 \end{bmatrix}.$$

Abiding by the Cauchy-Schwarz inequality, we have

$$\left(\sum_{l=k}^{k+m-1} \nu_{ix,l}^{ij} \nu_{iy,l}^{ij} \right)^2 \leq \sum_{l=k}^{k+m-1} (\nu_{ix,l}^{ij})^2 \sum_{l=k}^{k+m-1} (\nu_{iy,l}^{ij})^2,$$

for any $k \in \mathbb{Z}^+$. And the two sides of the above inequality are equal if and only if $\nu_{ix,l}^{ij}$ and $\nu_{iy,l}^{ij}$ are linearly dependent for $l \in \{k, \dots, k+m-1\}$. Hence, Assumption 5.3.1 is equivalent to the linear independence of $\nu_{ix,l}^{ij}$ and $\nu_{iy,l}^{ij}$ for $l \in \{k, \dots, k+m-1\}$.

5.3.1.3 RL error analysis in noise-corrupted case

The analysis in the preceding section has demonstrated that the proposed RL estimator can provide unbiased RL estimates in noise-free case. However, in real world applications, there always exist measurement noises which may cause some estimation deviation. In this section, we will take into account the measurement noises and analyze the RL error in noise-corrupted case.

The distance measurement sensor adopted herein (i.e., UWB) is able to achieve a centimeter-level measurement accuracy [28]. Compared with the larger errors (tens of centimeters per second) in velocity estimation (i.e., P4Flow[®]) [100, 101], we simply assume ϵ_k^d and ϵ_k^v equal 0 in (5.1). To proceed, suppose there exists a constant $\bar{\delta} > 0$ such that $\|\epsilon_k\| \leq \bar{\delta}$.

For $i = 1, \dots, N$, we let $\tilde{\chi}_i^{ij} = \hat{\chi}_i^{ij} - \chi_i^{ij}$. Since $d_k^{ij} d_k^{ij} = \nu_k^{ij\top} \cdot \chi_k^{ij}$, the error dynamic of (5.1) can be obtained as follows

$$\tilde{\chi}_{i,k+1}^{ij} = \mathbf{A}_{i,k} \tilde{\chi}_{i,k}^{ij} + \epsilon_{i,k}, \quad (5.14)$$

where $\mathbf{A}_{i,k} := \mathbf{I} - \gamma T(\nu_{i,k}^{ij} + \epsilon_k)(\nu_{i,k}^{ij} + \epsilon_k)^\top$ and $\epsilon_{i,k} := T\epsilon_k - \gamma T(\nu_{i,k}^{ij} + \epsilon_k)\epsilon_k^\top \chi_{i,k}^{ij}$.

Before moving on, it is necessary to make the following assumption:

Assumption 5.3.2 (Persistent excitation condition). *There exist $m \in \mathbb{Z}^+$ and $\mu > 0$ such that, $\forall k \in \mathbb{Z}^+$,*

$$\sum_{l=k}^{k+m-1} (\nu_{i,l}^{ij} + \epsilon_l)(\nu_{i,l}^{ij} + \epsilon_l)^\top \geq \mu \mathbf{I}. \quad (5.15)$$

Now, we are in a position to give the following result.

Theorem 5.3.2. *For UAV i , under Assumption 5.2.1 and Assumption 5.3.2, if the sampling period T satisfies*

$$0 < T < \frac{1}{\gamma(2\bar{v} + \bar{\delta})^2}, \quad (5.16)$$

then the estimate error $\tilde{\mathbf{x}}_{i,k}^{ij}$ is bounded by $c := \frac{\xi_1 + \sqrt{\xi_1^2 - 4\sigma\xi_2}}{2\sigma}$, where

$$\begin{aligned}\sigma &:= \mu\alpha[\gamma T(2\bar{v} + \bar{\delta})^2 - 1]^{2(m-1)}, \\ \xi_1 &:= 2m\xi + \alpha\xi m(m-1)(2\bar{v} + \bar{\delta})^2, \\ \xi_2 &:= \left[m^2 + \frac{1}{6}\alpha m(m-1)(2m-1) \right] \xi^2, \\ \alpha &:= 2\gamma T - \gamma^2 T^2 (2\bar{v} + \bar{\delta})^2, \\ \xi &:= T\bar{\delta} [1 + \gamma (2\bar{v} + \bar{\delta}) \bar{d}].\end{aligned}$$

Remark 5.3.2. Note that if there exists no velocity noise, i.e., $\bar{\delta} = 0$, then $c = 0$ and the RL estimation error will asymptotically converge to zero as seen in Section 5.3.1.2. The effect of ϵ on the RL estimation error and its upper bound c , will be discussed in Section 5.5.2.4.

Proof. A Lyapunov function $J(\tilde{\mathbf{x}}_{i,k}^{ij}) = \tilde{\mathbf{x}}_{i,k}^{ij\top} \tilde{\mathbf{x}}_{i,k}^{ij}$ is defined, and the difference of this Lyapunov function candidate in m time steps can be written as

$$\begin{aligned}\Delta J_m^k &= J(\tilde{\mathbf{x}}_{i,k+m}^{ij}) - J(\tilde{\mathbf{x}}_{i,k}^{ij}) \\ &= \tilde{\mathbf{x}}_{i,k+m}^{ij\top} \tilde{\mathbf{x}}_{i,k+m}^{ij} - \tilde{\mathbf{x}}_{i,k}^{ij\top} \tilde{\mathbf{x}}_{i,k}^{ij} \\ &= \sum_{l=k}^{k+m-1} \left(\tilde{\mathbf{x}}_{i,l+1}^{ij\top} \tilde{\mathbf{x}}_{i,l+1}^{ij} - \tilde{\mathbf{x}}_{i,l}^{ij\top} \tilde{\mathbf{x}}_{i,l}^{ij} \right) \\ &= \sum_{l=k}^{k+m-1} \left(\gamma^2 T^2 \|\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l\|^2 - 2\gamma T \right) \cdot \tilde{\mathbf{x}}_{i,l}^{ij\top} (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \tilde{\mathbf{x}}_{i,l}^{ij} \\ &\quad + 2 \sum_{l=k}^{k+m-1} \boldsymbol{\epsilon}_{i,l}^\top \mathbf{A}_{i,l} \tilde{\mathbf{x}}_{i,l}^{ij} + \sum_{l=k}^{k+m-1} \|\boldsymbol{\epsilon}_{i,l}\|^2 \\ &\leq \left[\gamma^2 T^2 (2\bar{v} + \bar{\delta})^2 - 2\gamma T \right] \left[\sum_{l=k}^{k+m-1} \tilde{\mathbf{x}}_{i,l}^{ij\top} (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \tilde{\mathbf{x}}_{i,l}^{ij} \right] \\ &\quad + 2 \sum_{l=k}^{k+m-1} \boldsymbol{\epsilon}_{i,l}^\top \mathbf{A}_{i,l} \tilde{\mathbf{x}}_{i,l}^{ij} + \sum_{l=k}^{k+m-1} \|\boldsymbol{\epsilon}_{i,l}\|^2.\end{aligned}\tag{5.17}$$

In addition, by using induction method on the system (5.14), RL estimation error

at l step is obtained as

$$\tilde{\chi}_{i,l}^{ij} = \underbrace{\prod_{t=k}^{l-1} \mathbf{A}_{i,t}}_{:=\mathbf{A}_{i,k:l-1}} \tilde{\chi}_{i,k}^{ij} + \sum_{t=0}^{l-k-1} \left(\prod_{s=k+t+1}^{l-1} \mathbf{A}_{i,s} \right) \boldsymbol{\varepsilon}_{i,k+t}, \quad (5.18)$$

where $\mathbf{A}_{i,k:l-1} := \mathbf{I}$ when $l = k$. Due to (5.16), $\|\mathbf{A}_{i,l}\| \leq 1, \forall l \in \mathbb{Z}^+$. Since $\|\boldsymbol{\varepsilon}_{i,l}\| \leq T\bar{\delta} [1 + \gamma(2\bar{v} + \bar{\delta})\bar{d}] = \xi$, then

$$\|\tilde{\chi}_{i,l}^{ij}\| \leq \|\tilde{\chi}_{i,k}^{ij}\| + \sum_{t=0}^{l-k-1} \|\boldsymbol{\varepsilon}_{i,k+t}\| \leq \|\tilde{\chi}_{i,k}^{ij}\| + (l-k)\xi.$$

Thus,

$$2 \sum_{l=k}^{k+m-1} \boldsymbol{\varepsilon}_{i,l}^\top \mathbf{A}_{i,l} \tilde{\chi}_{i,l}^{ij} \leq 2\xi \sum_{l=k}^{k+m-1} [\|\tilde{\chi}_{i,k}^{ij}\| + (l-k)\xi] = 2m\xi \|\tilde{\chi}_{i,k}^{ij}\| + m(m-1)\xi^2.$$

Now, (5.17) can be written as

$$\Delta J_m^k \leq -\alpha \sum_{l=k}^{k+m-1} \tilde{\chi}_{i,l}^{ij\top} (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \tilde{\chi}_{i,l}^{ij} \quad (5.19a)$$

$$+ 2m\xi \|\tilde{\chi}_{i,k}^{ij}\| + m^2\xi^2. \quad (5.19b)$$

To separate $\tilde{\chi}_{i,k}^{ij}$ from $\tilde{\chi}_{i,l}^{ij}$, next we expand (5.19a) individually. According to (5.18), (5.19) can be rewritten as

$$\begin{aligned} \Delta J_m^k &\leq -\alpha \sum_{l=k}^{k+m-1} \tilde{\chi}_{i,k}^{ij\top} \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \tilde{\chi}_{i,k}^{ij} \\ &\quad - \alpha \sum_{l=k}^{k+m-1} \tilde{\chi}_{i,k}^{ij\top} \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \left(\sum_{t=0}^{l-k-1} \mathbf{A}_{i,k+t+1:l-1} \boldsymbol{\varepsilon}_{i,k+t} \right) \\ &\quad - \alpha \sum_{l=k}^{k+m-1} \left(\sum_{t=0}^{l-k-1} \mathbf{A}_{i,k+t+1:l-1} \boldsymbol{\varepsilon}_{i,k+t} \right)^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \tilde{\chi}_{i,k}^{ij} \\ &\quad - \alpha \sum_{l=k}^{k+m-1} \left(\sum_{t=0}^{l-k-1} \mathbf{A}_{i,k+t+1:l-1} \boldsymbol{\varepsilon}_{i,k+t} \right)^\top \left(\sum_{t=0}^{l-k-1} \mathbf{A}_{i,k+t+1:l-1} \boldsymbol{\varepsilon}_{i,k+t} \right) \end{aligned}$$

$$\begin{aligned}
& + 2m\xi \|\tilde{\chi}_{i,k}^{ij}\| + m^2\xi^2 \\
& \leq -\alpha \sum_{l=k}^{k+m-1} \tilde{\chi}_{i,k}^{ij\top} \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \tilde{\chi}_{i,k}^{ij} \\
& \quad + 2\alpha \sum_{l=k}^{k+m-1} [(l-k)\xi(2\bar{v} + \bar{\delta})^2 \|\tilde{\chi}_{i,k}^{ij}\|] \\
& \quad + \alpha \sum_{l=k}^{k+m-1} [(l-k)^2\xi^2] + 2m\xi \|\tilde{\chi}_{i,k}^{ij}\| + m^2\xi^2 \\
& \leq -\alpha \sum_{l=k}^{k+m-1} \tilde{\chi}_{i,k}^{ij\top} \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \tilde{\chi}_{i,k}^{ij} \tag{5.20a}
\end{aligned}$$

$$\begin{aligned}
& + [2m\xi + \alpha\xi m(m-1)(2\bar{v} + \bar{\delta})^2] \|\tilde{\chi}_{i,k}^{ij}\| + \left[m^2 + \frac{1}{6}\alpha m(m-1)(2m-1) \right] \xi^2. \tag{5.20b}
\end{aligned}$$

To proceed, $\mathbf{A}_{i,k:l-1}$ should be extracted from the summation in (5.20a), and thus we need to prove the following inequality first

$$\mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \leq \mathbf{A}_{i,k:l-2}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-2}, \tag{5.21}$$

$\forall l = k, k+1, \dots, k+m-1$, where $\mathbf{A}_{i,k:s} := \mathbf{I}$ if $s < k$.

In the light of the proof of (5.8), the proof of (5.21) is as follows.

$\forall \mathbf{x} \in \mathbb{R}^2$,

$$\begin{aligned}
& \mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \mathbf{x} \\
& = \underbrace{\mathbf{x}^\top \mathbf{A}_{i,k:l-2}^\top}_{:=\mathbf{Z}^\top} \mathbf{A}_{i,l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,l-1} \underbrace{\mathbf{A}_{i,k:l-2}}_{:=\mathbf{Z}} \mathbf{x} \\
& = \mathbf{Z}^\top \mathbf{A}_{i,l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,l-1} \mathbf{Z} \\
& = \mathbf{Z}^\top [\mathbf{I} - \gamma T(\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1})(\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1})^\top]^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) \\
& \quad \cdot (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top [\underbrace{\mathbf{I} - \gamma T(\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1})(\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1})^\top}_{:=\mathbf{B}_{i,l-1}}] \mathbf{Z}. \tag{5.22}
\end{aligned}$$

Before proceeding, we denote $\|\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1}\|^2$ by $\lambda_{i,l-1}$, the *null space* of $\mathbf{B}_{i,l-1}$ by

$$\text{Null}(\mathbf{B}_{i,l-1}) = \{\mathbf{v} \in \mathbb{R}^2 | \mathbf{B}_{i,l-1}\mathbf{v} = \mathbf{0}\},$$

and the *eigen space* of $\lambda_{i,l-1}$ by

$$E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})} = \{\mathbf{u} \in \mathbb{R}^2 | \mathbf{B}_{i,l-1}\mathbf{u} = \lambda_{i,l-1}\mathbf{u}\}.$$

It is easy to see that $\|\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1}\|^2$ is the only nonzero eigenvalue of $\mathbf{B}_{i,l-1}$ with eigenvector $\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1}$, which is simple. Therefore, $\lambda_{i,l-1} = \|\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1}\|^2$.

If $\mathbf{Z} \in \text{Null}(\mathbf{B}_{i,l-1})$,

$$\mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \mathbf{x} = \mathbf{Z}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{Z}. \quad (5.23)$$

If $\mathbf{Z} \in E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})}$, without loss of generality, we choose $\mathbf{Z} = \iota(\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1})$, $\iota \in \mathbb{R}$, then

$$\begin{aligned} & \mathbf{x}^\top \mathbf{A}_{i,k:l-1}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:l-1} \mathbf{x} \\ &= (1 - \gamma T \|\boldsymbol{\nu}_{i,l-1}^{ij} + \boldsymbol{\epsilon}_{l-1}\|^2)^2 \mathbf{Z}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{Z} \\ &< \mathbf{Z}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{Z}. \end{aligned} \quad (5.24)$$

It is known that for any $\mathbf{Z} \in \mathbb{R}^2$, it has $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2$ for some $\mathbf{Z}_1 \in \text{Null}(\mathbf{B}_{i,l-1})$, $\mathbf{Z}_2 \in E_{\lambda_{i,l-1}}^{(\mathbf{B}_{i,l-1})}$, together with (5.23) and (5.24), which implies (5.21).

According to (5.21), together with (5.20), we can get

$$\begin{aligned} & \Delta J_m^k \\ & \leq -\alpha \sum_{l=k}^{k+m-1} \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij} + \xi_1 \|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\| + \xi_2 \\ & = -\alpha \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top \left[\sum_{l=k}^{k+m-1} (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l) (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top \right] \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij} + \xi_1 \|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\| + \xi_2 \\ & \leq -\mu\alpha \tilde{\boldsymbol{\chi}}_{i,k}^{ij\top} \mathbf{A}_{i,k:k+m-2}^\top \mathbf{A}_{i,k:k+m-2} \tilde{\boldsymbol{\chi}}_{i,k}^{ij} + \xi_1 \|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\| + \xi_2. \end{aligned} \quad (5.17')$$

Now we need to prove

$$\mathbf{A}_{i,k:k+m-2}^\top \mathbf{A}_{i,k:k+m-2} \geq [1 - \gamma T(2\bar{v} + \bar{\delta})^2]^{2(m-1)} \mathbf{I}. \quad (5.25)$$

The following is the proof of (5.25) which is similar to the noise-free case in (5.12).

For $\mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} = [\mathbf{I} - \gamma T(\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)(\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top]^\top [\mathbf{I} - \gamma T(\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)(\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top]$, denote $\mathbf{B}_{i,l} := (\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)(\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l)^\top$ and $\lambda_{i,l} := \|\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l\|^2$. Similar with the proof of (5.21), $\forall \mathbf{x} \in \mathbb{R}^2$,

$$\mathbf{x}^\top \mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} \mathbf{x} = \begin{cases} \mathbf{x}^\top \mathbf{x} & \text{if } \mathbf{x} \in \text{Null}(\mathbf{B}_{i,l}), \\ (1 - \gamma T\|\boldsymbol{\nu}_{i,l}^{ij} + \boldsymbol{\epsilon}_l\|^2)^2 \mathbf{x}^\top \mathbf{x} & \text{if } \mathbf{x} \in E_{\lambda_{i,l}}^{(\mathbf{B}_{i,l})}, \end{cases} \quad (5.26)$$

thus, $\mathbf{A}_{i,l}^\top \mathbf{A}_{i,l} \geq [1 - \gamma T(2\bar{v} + \bar{\delta})^2]^2 \mathbf{I}$. According to (5.16), (5.25) holds.

So far the RL estimation error bound can be presented as follows. Combining (5.17') with (5.25), we have

$$\begin{aligned} \Delta J_m &\leq \mu\alpha[\gamma T(2\bar{v} + \bar{\delta})^2 - 1]^{2(m-1)} \|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\|^2 + \xi_1 \|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\| + \xi_2 \\ &= \sigma J(\tilde{\boldsymbol{\chi}}_{i,k}^{ij}) + \xi_1 \sqrt{J(\tilde{\boldsymbol{\chi}}_{i,k}^{ij})} + \xi_2, \end{aligned}$$

where $\sigma < 0$, $\xi_1 > 0$, and $\xi_2 > 0$, therefore, $\sqrt{J(\tilde{\boldsymbol{\chi}}_{i,k}^{ij})}$ is bounded by $c = \frac{\xi_1 + \sqrt{\xi_1^2 - 4\sigma\xi_2}}{2\sigma}$, namely, $\|\tilde{\boldsymbol{\chi}}_{i,k}^{ij}\|$ is bounded by c , and then the proof is completed. $\blacksquare$

5.3.2 Consensus-based RL Fusion Estimation

If UAV j is both a primary node and secondary node for UAV i , that is, UAV j is not only the neighbor of UAV i but also a neighbor of UAV r and $r \in \mathcal{N}_i$, then $\boldsymbol{\chi}^{ij}$ can be estimated indirectly. Let ${}_r\hat{\boldsymbol{\chi}}_{i,k}^{ij}$ be an *indirect* relative estimate of $\boldsymbol{\chi}^{ij}$ obtained by UAV i at $t = t_{i,k}$ through an intermediate UAV r , then, ${}_r\hat{\boldsymbol{\chi}}_{i,k}^{ij} = \hat{\boldsymbol{\chi}}_{i,k}^{ir} + \boldsymbol{\pi}_{r,k}^{rj}$, as shown in Fig. 5.2. Moreover, based on the RL error analysis of (5.1) in Section 5.3.1, we know that $\|\tilde{\boldsymbol{\chi}}_k^{ij}\|$ is bounded for each UAV.

Figure 5.2: Indirect RL estimate ${}_r\hat{\chi}_{i,k}^{ij}$ from UAVi to UAVj through UAVr

Thus, in this section, by combining the *direct* and *indirect* estimates, a consensus-based RL fusion strategy for UAVi in noise-corrupted case can be given by

$$\pi_{i,k+1}^{ij} = \pi_{i,k}^{ij} + T(\nu_{i,k}^{ij} + \epsilon_k) + \kappa_{ij}^D [\hat{\chi}_{i,k}^{ij} - \pi_{i,k}^{ij}] + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I [{}_r\hat{\chi}_{i,k}^{ij} - \pi_{i,k}^{ij}], \quad (5.27)$$

where κ_{ij}^D and κ_{ir}^I are constants representing the *direct* estimation weight and *indirect* estimation weight, respectively. $\kappa_{ij}^D \neq 0$ if UAVi has direct RL estimate to UAVj and $\kappa_{ij}^D = 0$ otherwise, which means that UAVs can obtain reliable and continual RL estimates in real time despite that some UAVs may not have direct range measurements to their neighbors.

5.3.2.1 Stability analysis of RL fusion in noise-free case

Before discussing the noise-corrupted case, we first investigate the stability of the RL fusion estimation and the RL fusion strategy for UAVi will be

$$\pi_{i,k+1}^{ij} = \pi_{i,k}^{ij} + T\nu_{i,k}^{ij} + \kappa_{ij}^D [\hat{\chi}_{i,k}^{ij} - \pi_{i,k}^{ij}] + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I [{}_r\hat{\chi}_{i,k}^{ij} - \pi_{i,k}^{ij}], \quad (5.28)$$

where the direct RL estimate $\hat{\chi}_{i,k}^{ij}$ is generated from (5.2).

Theorem 5.3.3. Suppose the conditions of Theorem 5.3.1 are satisfied and $\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I < 1$. For $(i, j) \in \bar{\mathcal{E}}$ in $\bar{\mathcal{G}}$, the fused RL estimate π_i^{ij} in (5.28) exponen-

tially converges to the relative position χ^{ij} , if Assumption 5.3.1 is satisfied for all UAVs.

Proof. Denote $\tilde{\pi}_i^{ij} = \pi_i^{ij} - \chi^{ij}$, $\tilde{\chi}_i^{ir} = \hat{\chi}_i^{ir} - \chi^{ir}$ and we have ${}_r\tilde{\chi}_i^{ij} = {}_r\hat{\chi}_i^{ij} - \chi^{ij} = \tilde{\chi}_i^{ir} + \tilde{\pi}_r^{rj}$. By these transformations, system (5.28) can be transformed to

$$\tilde{\pi}_{i,k+1}^{ij} = \tilde{\pi}_{i,k}^{ij} + \kappa_{ij}^D [\tilde{\chi}_{i,k}^{ij} - \tilde{\pi}_{i,k}^{ij}] + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I [\tilde{\pi}_{r,k}^{rj} - \tilde{\pi}_{i,k}^{ij}] + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \tilde{\chi}_{i,k}^{ir}. \quad (5.29)$$

This is a typical consensus system of $\tilde{\pi}_{i,k}^{ij}$ for $(i, j) \in \bar{\mathcal{E}}$ with an external reference as $\tilde{\chi}_{i,k}^{ij}$ and an external perturbation as $\sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \tilde{\chi}_{i,k}^{ir}$. By the discrete-time results in [102], the consensus system without the perturbation ensures exponential stability and $\|\tilde{\pi}_{r,k}^{rj} - \tilde{\pi}_{i,k}^{ij}\| \rightarrow 0$ as well as $\|\tilde{\chi}_{i,k}^{ij} - \tilde{\pi}_{i,k}^{ij}\| \rightarrow 0$ as $k \rightarrow \infty$ for all $r \in \mathcal{N}_i \setminus \{j\}$. Note that $\tilde{\chi}_{i,k}^{ij}$ exponentially converges to $\mathbf{0}$ by Theorem 5.3.1. It follows that $\tilde{\pi}_{i,k}^{ij}$ approaches $\mathbf{0}$. On the other hand, the perturbation is the sum of the direct RL estimation errors of UAV i ' neighbors except UAV j with respect to UAV i . It can be easily obtained that the perturbation exponentially asymptotically converges to $\mathbf{0}$. Consequently, system (5.29) ensures exponential stability on $\mathbf{0}$. Then the conclusion follows. $\blacksquare$

Remark 5.3.3. *The proposed relative position scheme fuses the direct estimate and the indirect estimates by a consensus-based method. Obviously, the fused estimate can be considered as a mean value of the direct and indirect estimates. Fusing direct and indirect estimates can speed up the convergence rate of the relative position estimation. It can also provide a better and more robust estimation of relative position in the presence of measurement noises.*

Remark 5.3.4. *Based on Theorem 5.3.1, it can be observed that if the initial direct estimation error $\tilde{\chi}_{i,0}^{ij}$ is $\mathbf{0}$, i.e., $\hat{\chi}_{i,0}^{ij} = \chi_0^{ij}$, the direct estimate $\hat{\chi}_{i,k}^{ij}$ always equals the true relative position χ_k^{ij} , even if $\nu_{ix,k}^{ij}$ and $\nu_{iy,k}^{ij}$ are linearly dependent. Moreover, this result can be extended to the whole system if $\pi_{i,0}^{ij} = \hat{\chi}_{i,0}^{ij} = \chi_0^{ij}$ for all $(i, j) \in \bar{\mathcal{E}}$ in $\bar{\mathcal{G}}$.*

Remark 5.3.5. *Relative localization shares something in common with cooperative source localization studied in [58, 76] which aims to localize a target of common interest (namely, a source or a landmark) by each agent. The common ground is that the objective of both the localization problems is to estimate the coordinates of one or more targets in one's local frame based on available local information. The difference is that the relative localization problem studied here aims to localize all moving neighbors by each agent, while the cooperative source localization problems studied in [58, 76] aim to localize a stationary source by each agent. Therefore, to some extent, the relative localization problem can be considered as a multi-source localization problem in which each agent is expected to localize different moving sources.*

5.3.2.2 Boundedness analysis of fusion error in noise-corrupted case

In noise-corrupted case, the following theorem is proposed for (5.27).

Theorem 5.3.4. *If the conditions of Theorem 5.3.2 are satisfied and UAV j is globally reachable in $\bar{\mathcal{G}}$ and $\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I < 1$, then the fused RL estimate π_i^{ij} of χ^{ij} in (5.27) has a bounded estimation error.*

This theorem is an extension of [76], but the difference is that here the fused RL estimation error is bounded under bounded direct and indirect RL estimates in the presence of measurement noises. Before proving the result, we present some preliminaries.

Definition 5.3.1. [103] *Consider a nonlinear dynamic system:*

$$\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k)),$$

with the state $\mathbf{x}(k)$ and input $\mathbf{u}(k)$. It is said to be (globally) input-to-state stable (ISS) if for every initial state $\boldsymbol{\xi} = \mathbf{x}(0) \in \mathbb{R}^n$ and each $\mathbf{u}(k) \in \mathbb{R}^m$, there exists a class $\mathcal{KL}$ function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ and a class $\mathcal{K}$ function η satisfying

$$\|\mathbf{x}(k, \boldsymbol{\xi}, \mathbf{u})\| \leq \beta(\|\boldsymbol{\xi}\|, k) + \eta(\|\mathbf{u}\|),$$

for each $k \in \mathbb{N}$.

Lemma 5.3.1. [103] *A linear discrete-time system*

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$

is ISS when $\mathbf{A}$ is a Schur Matrix.

Lemma 5.3.2. [104] *The matrix $\mathcal{L}^j + \mathcal{B}^j$ is positive stable (its eigenvalues have positive real-parts) if and only if UAVj is globally reachable in $\bar{\mathcal{G}}$.*

Proof. For $i = 1, \dots, N$ and $i \neq j$, we let $\tilde{\pi}_i^{ij} = \pi_i^{ij} - \chi^{ij}$. Then, it follows from (5.27) that

$$\tilde{\pi}_{i,k+1}^{ij} = \tilde{\pi}_{i,k}^{ij} - \kappa_{ij}^D \tilde{\pi}_{i,k}^{ij} - \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I [\tilde{\pi}_{i,k}^{ij} - \tilde{\pi}_{r,k}^{rj}] + \mathbf{u}_{i,k}^{ij}, \quad (5.30)$$

where $\mathbf{u}_{i,k}^{ij} = \kappa_{ij}^D \tilde{\chi}_{i,k}^{ij} + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \tilde{\chi}_{i,k}^{ir} + T\epsilon_k$ with $\|\mathbf{u}_{i,k}^{ij}\| \leq c(\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I) + T\bar{\delta}$.

Assume that at $t = t_{k+1}$, UAVj is globally reachable. We aggregate all the system equations (5.30) for $i \in \mathcal{V}^j$, and then we have

$$\tilde{\pi}_{k+1}^{*j} = ((\mathbf{I} - \mathcal{L}^j - \mathcal{B}^j) \otimes \mathbf{I}_2) \tilde{\pi}_k^{*j} + \mathbf{u}_k^{*j}, \quad (5.31)$$

where $\tilde{\pi}_k^{*j} = [\tilde{\pi}_{1,k}^{1j^\top} \dots \tilde{\pi}_{j-1,k}^{j-1j^\top}, \tilde{\pi}_{j+1,k}^{j+1j^\top} \dots \tilde{\pi}_{N,k}^{Nj^\top}]$ and $\mathbf{u}_k^{*j} = [\mathbf{u}_{1,k}^{1j^\top} \dots \mathbf{u}_{j-1,k}^{j-1j^\top}, \mathbf{u}_{j+1,k}^{j+1j^\top} \dots \mathbf{u}_{N,k}^{Nj^\top}]$ with $* \in \mathcal{V}^j$. $\mathcal{L}^j$ is the weighted Laplacian matrix of the graph $\mathcal{G}^j$ and $\mathcal{B}^j \in \mathbb{R}^{(N-1) \times (N-1)}$ is a j adjacency matrix associated with $\bar{\mathcal{G}}$.

Based on Gersgorin disk theorem [105], all the eigenvalues of $\mathcal{L}^j + \mathcal{B}^j$, denoted by λ , are located in the union of discs as follows:

$$Ger(\mathcal{L}^j + \mathcal{B}^j) := \bigcup_{i=1}^{N-1} \left\{ \lambda_i \in \mathbb{C} : \left| \lambda_i - \left(\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \right) \right| \leq \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \right\}. \quad (5.32)$$

Figure 5.3: Eigenvalues of $\mathcal{L}^j + \mathcal{B}^j$

By the condition in Theorem 5.3.4, we know that $\sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \leq 1$ and $\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I \leq 1$. Thus, we can infer that λ_i lies in a union of discs which are inside the unit circle centered at $(1, 0)$ as shown in Fig. 5.3. Moreover, since UAV j is globally reachable in $\bar{\mathcal{G}}$, by Lemma 5.3.2 that all the eigenvalues of $\mathcal{L}^j + \mathcal{B}^j$ are of positive real parts, which indicates that λ_i is strictly inside the unit circle centered at $(1, 0)$. Therefore, all eigenvalues of $\mathcal{L}^j + \mathcal{B}^j$ lie strictly inside the unit circle centered at the origin and accordingly, $\mathcal{L}^j + \mathcal{B}^j$ is a Schur matrix. Based on Lemma 5.3.1, (5.31) is ISS, which means for all $k \geq 0$, $\|\tilde{\pi}_k^{*j}(\tilde{\pi}_0^{*j}, \mathbf{u}^{*j}(k))\| \leq \beta(\|\tilde{\pi}_0^{*j}\|, k) + \eta(\|\mathbf{u}_k^{*j}\|)$.

According to Theorem 5.3.2, for any positive constants κ_{ij}^D , κ_{ir}^I , $\|\mathbf{u}_{i,k}^{ij}\| \leq c(\kappa_{ij}^D + \sum_{r \in \mathcal{N}_i \setminus \{j\}} \kappa_{ir}^I) + T\bar{\delta}$. Since $\eta(\cdot)$ is a class $\mathcal{K}$ function, it follows that $\eta(\|\mathbf{u}_k^{*j}\|) \rightarrow \rho$ as $k \rightarrow \infty$, where ρ is a constant. Because $\|\tilde{\pi}_0^{*j}\|$ is bounded and $\beta(\cdot, \cdot)$ is a class $\mathcal{KL}$ function, $\beta(\|\tilde{\pi}_0^{*j}\|, k) \rightarrow 0$ as $k \rightarrow \infty$. Consequently, $\|\tilde{\pi}_k^{*j}(k, \tilde{\pi}_0^{*j}, \mathbf{u}^{*j}(k))\| \rightarrow \rho$ as $k \rightarrow \infty$. Thus, the proof is completed. ■

Remark 5.3.6. It should be noted that the graph $\mathcal{G}^j$ might be time-varying due to possible failures of range measurements. If a digraph $\bar{\mathcal{G}}$ is strongly connected at $t = t_{k+1}$ (i.e., there exists a path between any two distinct UAVs), besides the relative estimation to UAV j , for example, each UAV can estimate the relative position to UAV q , $q \in \bar{\mathcal{V}} \setminus \{j\}$, accordingly, $\tilde{\pi}_k^{*q}$, $\mathcal{L}^q$ and $\mathcal{B}^q$ can be obtained.

Remark 5.3.7. *Before UAVs take off, a nonlinear regression (NLR) based method [22] in Section 5.3.3 can be applied to estimate the coordinates of the stationary UAVs, and these estimates will serve as an initial guess of the RL estimators in Section 5.3.1 and Section 5.3.4 to improve RL accuracy.*

5.3.3 RL Initialization

In this section a nonlinear regression (NLR) based method [22] is applied to estimate the coordinates of the stationary UAVs before they take off, and these estimates will serve as an initial guess of the RL estimators in Section 5.3 to improve RL accuracy. For a network with N UAVs, each UAV has its own coordinates denoted by $\mathbf{p}_i = [x_i, y_i, z_i]^\top, i = 1, \dots, N$, and we group the coordinates of all the UAVs into one vector as $\mathbf{p} = [\mathbf{p}_1^\top, \mathbf{p}_2^\top, \dots, \mathbf{p}_N^\top]^\top$. A UAV may have none or some of its coordinates known since we designate some UAVs as references or origin beforehand. In the context of a nonlinear regression problem, the known coordinates will be in the set of error-free independent variables $\mathbf{x}$ and the unknown will be in the set of parameters $\boldsymbol{\phi}$ to be estimated.

Let $d_0^{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$ be the distance between UAV i and UAV j before taking off. Our nonlinear model $f(\mathbf{x}, \boldsymbol{\phi})$ is a $N(N-1)/2 \times 1$ vector of all of the distances arranged in a particular way as follows $[d_0^{12}, d_0^{13}, \dots, d_0^{1N}, d_0^{23}, d_0^{24}, \dots, d_0^{2N}, \dots, d_0^{N-1,N}]^\top$.

The observed dependent variables $\underline{d}^{ij}$ are these distances subject to some noise ϵ^d , or $\underline{d}^{ij} = d^{ij} + \epsilon^d$. $\underline{\mathbf{d}}$ is a stack of all distance measurements $\underline{d}^{ij}$. To apply the nonlinear regression method, we start from some initial guess $\boldsymbol{\phi}_0$ and seek to improve our estimate of $\boldsymbol{\phi}$ by the following recursive steps:

$$\mathbf{J}_k = \mathbf{J}_k(\mathbf{x}, \boldsymbol{\phi}_k) = \left. \frac{\partial f(\mathbf{x}, \boldsymbol{\phi})}{\partial \boldsymbol{\phi}} \right|_{\boldsymbol{\phi}=\boldsymbol{\phi}_k}, \quad (5.33a)$$

$$\Delta \underline{\mathbf{d}} = \underline{\mathbf{d}} - f(\mathbf{x}, \boldsymbol{\phi}_k), \quad (5.33b)$$

$$\Delta \boldsymbol{\phi} = (\mathbf{J}_k^\top \mathbf{W} \mathbf{J}_k)^{-1} \mathbf{J}_k^\top \mathbf{W} \Delta \underline{\mathbf{d}}, \quad (5.33c)$$

$$\boldsymbol{\phi}_{k+1} = \boldsymbol{\phi}_k + \Delta \boldsymbol{\phi}, \quad (5.33d)$$

where $k = 0, 1, 2, 3, \dots$, and $\mathbf{W}$ is a weight matrix to signify the comparative importance between the measurements. A time window t_0 to t_m is applied to weaken the role of the very old measurements. For example, m distances, labeled as $t_1, t_2, \dots, t_m$, have been measured within this time window, and $\mathbf{W}$ can be set as: $\mathbf{W} = \frac{1}{\sum_{l=1}^m t_l - mt_0} \text{diag}([\Delta t_1 \Delta t_2 \dots \Delta t_m])$, where $\Delta t_l = t_l - t_0$, $l = 1, 2, \dots, m$.

Remark 5.3.8. *The local referential system is somewhat arbitrary; the polygon can be translated and rotated as needed by the user. Thus, for a three-UAV team, UAV0 can be simply defined as the origin, UAV1 is set along the x -axis (its y value assigned to zero), and all the UAVs' heights z can be set as zero during the initial phase [106]. These mild initial settings constrain the system to three unknowns for regression for a group of 3 UAVs. In our work, this RL initialization method has been simulated and shown to be feasible. To apply this technique, a UWB RCM scheme is developed in Section 5.6.2.*

5.3.4 Filter-based RL Estimation

Since persistent excitation (PE) based method proposed in Section 5.3.1 cannot be updated in the cases of sensing failures and data loss, an extended Kalman filter (EKF) is proposed herein to account for these two scenarios by using its predicted outputs to assist in the RL estimation. The state space consists of relative position $\mathbf{x}_k^{ij}$ and relative velocity $\mathbf{v}_k^{ij}$, with the system dynamics given as follows:

$$\begin{cases} \mathbf{x}_{k+1}^{ij} = \mathbf{x}_k^{ij} + \mathbf{v}_k^{ij} \cdot T + \frac{1}{2} \mathbf{a}_k^{ij} T^2, \\ \mathbf{v}_{k+1}^{ij} = \mathbf{v}_k^{ij} + \mathbf{a}_k^{ij} T, \end{cases} \quad (5.34)$$

where $\mathbf{a}_k^{ij}$ is the acceleration difference between UAV i and UAV j at $t = t_k$. In the case when the acceleration is not available, $\{\mathbf{a}_k^{ij}\}$ can be treated as a white noise sequence with the covariance $\mathbb{E}(\mathbf{a}_k^{ij} \mathbf{a}_k^{ij\top}) = \sigma^2 \mathbf{I}$. Note that σ can be set as the maximum acceleration on the x, y -direction for a conservative but consistent estimation.

The observations are the relative range $\underline{d}_k^{ij}$ and relative velocity $\underline{\nu}_k^{ij}$ between UAV*i* and UAV*j* as below:

$$\begin{cases} \underline{d}_k^{ij} = \|\chi_k^{ij}\| + \epsilon_k^d, \\ \underline{\nu}_k^{ij} = \nu_k^{ij} + \epsilon_k. \end{cases} \quad (5.35)$$

Assume that both the process and measurement noises are Gaussian. Finally, a standard Kalman filter is conducted after linearizing the measurement equation. Note that the *underline* of each variable throughout this chapter represents its corresponding measurement.

Consider a team of N homogeneous UAVs. UAV*i* features $|\mathcal{N}_i| - 1$ EKF estimators to keep track of its neighbors' relative positions and uses their predicted outputs to aid the RL estimation for the further formation control command calculation as seen in Section 6.2.2. Note that the proposed EKF estimator is processed in another thread which is parallel with the consensus-based method, so that its prediction can help update the RL of the consensus-based method when sensing and data communication recovery.

5.4 Sensor Specifications

Before conducting real-world experiments, some on-board sensors used in our system, and their corresponding specifications need to be introduced first.

5.4.1 Derivative of Range Estimation

The derivative of range as a necessary input for consensus-based fusion estimator (5.1) is not able to be measured directly. Some researchers proposed an extended state observer (ESO) [107] to estimate it in real time. However, the accuracy and convergence speed of ESO largely depends on parameter setting. Instead of estimating the derivative of distance by users, we adopt indirect derivative of range measurements which simultaneously come out with range measurement through an

Figure 5.4: Ranging process of node i

EKF in the UWB firmware [108]. The inside filter combines less accurate Coarse Range Estimate (CRE) with very accurate TW-ToF Precision Range Measurements (PRM) as seen in Fig. 5.4.

The Filtered Range Estimate (FRE) and Filtered Radial Velocity (FRV), as well as their associated error estimates FRE Error (FREE) and FRV Error (FRVE), are the result of a recursive optimal (Kalman) estimator in the RCM firmware. This Kalman filter is based on two state variables, range and range velocity². As shown in Fig. 5.4, a unique Kalman filter instance is created and maintained for each unique PRM range target. Whenever a new PRM or valid CRE measurement is generated this filter performs a weighted combination of these measurements with an internal linear motion model of distance. FREE and FRVE are generated and reported as a byproduct of this filter.

To investigate the performance of UWB range rate, we collected four groups of range velocity with respect to four stationary UWB anchor nodes and a 3D random motion trajectory can be seen in Fig. 5.5(a). Fig. 5.5(b) and Fig. 5.5(c) respectively depict the corresponding distance and distance rate measurements. Compared with VICON[®] measurements, the absolute errors fall in $0.15m$ as shown in Fig. 5.5(d).

²<http://www.timedomain.com/wp/wp-content/uploads/2015/12/320-0313D-RangeNet-API-Specification.pdf>.

(a) 3D trajectory

(b) Range measurements

(c) Range rates of Fig. 5.5(b)

(d) Evolutions of the absolute errors

Figure 5.5: 3D trajectory of a UAV which installs a UWB module to sequentially measure the distances and distance rates to four stationary anchors marked as solid blue balls. VICON[®] is the ground truth to compare with.

5.4.2 Optical Flow-based Velocity Estimation

In this chapter, we choose PX4Flow^{®3} integrated with Pixhawk[®] to estimate the self velocity of each UAV and it can work indoors and in low outdoor light conditions without the need for an illumination LED. PX4Flow[®] is an optical flow smart camera which provides the image for setup purposes rather than capture images like a webcam. It has a native resolution of 752×480 pixels and calculates optical flow on a 4×4 binned and cropped area at 400Hz, giving it a very high light sensitivity. The features are summarized as follows:

³<https://pixhawk.org/modules/px4flow>

1. 168MHz Cortex M4F CPU (128 + 64 KB RAM)
2. 752×480 MT9V034 image sensor, L3GD20 3D Gyro
3. 16mm M12 lens (IR block filter)
4. Size $45.5mm \times 35mm$
5. Power consumption $115mA/5V$

A dynamic motion test was conducted to test the velocity estimation through PX4Flow[®] and the results are shown in Fig. 5.6(a) and Fig. 5.6(b). The average absolute error is between $0.2m/s$ to $0.3m/s$, however, in the worst case, namely in the peaks and troughs, the error can be as large as $0.5m/s$ which is a significant error source for RL estimation. Note that in outdoor environment, e.g., grass field (outdoor formation flight in Section 6.4), the error is even larger.

Remark 5.4.1. *Besides PX4Flow[®], other recent velocity estimation techniques and more powerful sensors like day and night workable cameras can also be utilized and more accurate velocity estimation may further improve the RL estimation accuracy. In addition, optical flow-based velocity estimation is not applicable to some certain environments, e.g., fully dark environments, ground in pure color without available features and limited distance to the ground. Moreover, since our proposed RL method requires each UAV's self velocity in consistent coordinate, and thus a velocity transformation from body frame to global frame will add errors to the velocity estimation. Coordinate-free RL estimation can be investigated in the future.*

5.5 Simulation Evaluation

In this section, RL initialization is conducted first, and then we demonstrate the cooperative RL strategy with six mobile UAVs in consideration of sudden change in trajectory.

(a) v_x (b) v_y

Figure 5.6: Raw velocity estimates of PX4Flow[®] sensor (blue) and VICON[®] velcoity measurements in x and y directions respectively.

5.5.1 Simulation of RL Initialization

Firstly, we present a simulation on the use of NLR algorithm as seen in Section 5.3.3, to estimate the initial positions of UAVs which are stationary. In Table 5.1, the true coordinates of the UAVs are listed. We choose to fix the x coordinate of

Table 5.1: True positions of the agents, the numbers in bold are the fixed values x . (unit: m)

UAV	x	y	UAV	x	y	UAV	x	y
01	0	0	05	2.5	0	09	-3	0.1
02	5.2	0	06	-2	-1.75	10	1.2	-5
03	5.3	7.5	07	0.75	1.4	11	0.5	4
04	-5.4	7.8	08	0.85	-3.1			

UAV1, y coordinates of UAV1 and UAV2, and a positive guess of x coordinate of UAV2.

A Gaussian noise with standard deviation of $0.1m$ is added into the distance measurements based on the documented specification of the UWB device by the manufacturer. For 3 UAVs, the coordinates can be initialized as Remark 5.3.7. The position estimates from the beginning to the end are shown in Fig. 5.7(a) and the evolution curves of the 2-norm estimation errors are presented in Fig. 5.7(b). It can be seen that only after 50 iterations the x and y estimates are almost the same with the true values.

5.5.2 Cooperative RL Simulation Results

5.5.2.1 Configuration

Six mobile UAVs are labeled $1, 2, \dots, 6$, respectively. The sensing graph of the six UAVs is depicted in Fig. 5.8. It can be checked that in the sensing graph, UAV1 is *globally reachable*. UAV2, 3, 4 have direct RL estimates to UAV1 while UAV5 and UAV6 only have indirect RL estimates to UAV1. Besides, UAV3 is able to obtain two indirect RL estimates to UAV1 through UAV4 and UAV5, and UAV5 have two indirect RL estimates to UAV1 through UAV3 and UAV4. The six UAVs' movements are as follows:

$$\mathbf{v}_{1,k} = \begin{bmatrix} \cos(\frac{1}{3}k) \\ -\frac{5}{3}\sin(\frac{1}{3}k) \end{bmatrix}, \mathbf{v}_{2,k} = \begin{bmatrix} -2\sin(k) \\ 2\sin(k) \end{bmatrix}, \mathbf{v}_{3,k} = \begin{bmatrix} \cos(\frac{1}{5}k) - \sin(\frac{1}{5}k) \\ \sin(\frac{1}{5}k) \cos(\frac{1}{5}k) \end{bmatrix} \begin{bmatrix} 1 \\ \cos(k) \end{bmatrix},$$

Figure 5.7: **The output of the NLR algorithm to estimate the coordinates of 11 UAVs:** (a) shows the true positions, initial positions and estimated positions respectively in x - y plane; (b) depicts the evolutions of the 2-norm estimation errors with the increasing of iterations.

$$\mathbf{v}_{4,k} = \begin{bmatrix} -3 \sin(k) \\ 3 \cos(k) \end{bmatrix}, \mathbf{v}_{5,k} = \begin{bmatrix} \frac{1}{6} \\ 0 \end{bmatrix}, \mathbf{v}_{6,k} = \begin{bmatrix} -\frac{10}{3} \sin(\frac{1}{3}k) \\ \frac{5}{3} \cos(\frac{1}{3}k) \end{bmatrix},$$

Figure 5.8: The sensing graph

and they are initially located at $(0, 0)$, $(2, -30)$, $(20, -15)$, $(-20, 8)$, $(-14, 8)$ and $(-10, -30)$, respectively.

To demonstrate the effectiveness of the proposed estimation scheme, we set UAV1 as a relative target to estimate. In the first simulation, we let all the UAVs take different continuous motions as seen in Fig. 5.9(a), which include both linear and nonlinear motions. In the second simulation, UAV4 suffers from a sudden deviation of velocity at $t = 100s$ while others remain the same as shown in Fig. 5.9(b). We adopt the estimator proposed in (5.1) and (5.27) to estimate the relative coordinates for every UAV. The following will show the results of the local direct estimation $\hat{\chi}^{ij}$ first, and the results of the fused RL estimation π^{ij} will be discussed later.

5.5.2.2 Evaluation of direct RL estimation $\hat{\chi}^{ij}$

In this section, we first demonstrate the performance of the estimator of (5.2) in noise-free case. Then, the noise-corrupted cases under the conditions of Fig. 5.9(a) and Fig. 5.9(b) are discussed where the distance noise and velocity noise are bounded with $\sup \|\epsilon^d\| \leq 0.1m$ and $\bar{\delta} = 0.5m/s$. Choose $T = 0.05s$ (20Hz)⁴, $\gamma = 0.5$.

Fig. 5.10(d) - Fig. 5.14(d) show that the evolutions of the direct RL estimation errors, i.e., $\|\hat{\chi}_{34} - \chi_{34}\|$, $\|\hat{\chi}_{35} - \chi_{35}\|$, $\|\hat{\chi}_{54} - \chi_{54}\|$, $\|\hat{\chi}_{53} - \chi_{53}\|$, $\|\hat{\chi}_{64} - \chi_{64}\|$. For

⁴UWB module PulsON 440[®] can provide a ranging measurement interval of 25ms (40Hz) by default configuration, which is sufficient for the inner control loop of a UAV in most scenarios.

Figure 5.9: Trajectories of 6 UAVs without/with hopping trajectory of UAV4

the sake of brevity, here we denote $\hat{\chi}_i^{ij}$ by $\hat{\chi}_{ij}$ and its corresponding real RL is χ_{ij} . It is shown that all the direct RL estimation errors are within $0.5m$ in three cases, i.e., noise-free case with the path of Fig. 5.9(a), noise-corrupted case with the path of Fig. 5.9(a) and noise-corrupted case with the path of Fig. 5.9(b). The trajectories of direct RL estimates, $\hat{\chi}_{34}$, $\hat{\chi}_{35}$, $\hat{\chi}_{54}$, $\hat{\chi}_{53}$, $\hat{\chi}_{64}$ together with their true pathes χ_{34} , χ_{35} , χ_{54} , χ_{53} , χ_{64} in these cases are presented in Fig. 5.10(a)(b)(c) - Fig. 5.14(a)(b)(c), respectively. All the trajectories of these direct RL estimates converge to their true pathes with bounded errors.

As seen in Fig. 5.10(d) and Fig. 5.12(d), due to the significant variance of $\mathbf{v}_4$ at $t = 100s$, the estimation error has a transient increment since UAV4 is a neighbor of both UAV3 and UAV5. Note that in Fig. 5.14(d), the RL estimation error in noise-free case is a bit larger than those in noise-corrupted cases. The reason is that the trajectories of UAV4 and UAV6 are both circular which can not fulfill the persistent excitation condition all the time. On the other hand, when involving noises, the persistent excitation condition can be well satisfied. These direct RL estimates will be applied for fusion in the next section.

Figure 5.10: Comparison of direct RL estimate $\hat{\chi}_{34}$ and its truth χ_{34} in three cases

5.5.2.3 Demonstration of fused RL estimation π^{ij}

In this section, the positions of all the UAVs relative to UAV1 are discussed and demonstrated since each UAV is able to *reach* UAV1 as shown in the sensing graph Fig. 5.8. Choose $\kappa_{ij}^D = \frac{\alpha_{ij}}{|\mathcal{N}_i|+1+\alpha_{ij}}$ and $\kappa_{ij}^I = \frac{1}{|\mathcal{N}_i|+1+\alpha_{ij}}$, where $|\mathcal{N}_i|$ is the cardinality of $\mathcal{N}_i$, and $\alpha_{ij} = 1$ if UAV i has a direct measurement of UAV j , and $\alpha_{ij} = 0$ otherwise. Note that the conditions of Theorem 5.3.2 and Theorem 5.3.4 are satisfied. Other parameters are the same as in Section 5.5.2.2. Since each UAV conducts the RL estimation from itself to UAV1 and performs the RL fusion in its own frame, for the sake of brevity, here we denote π_i^{i1} by π_{i1} and its corresponding real RL is χ_{i1} .

Figure 5.11: Comparison of direct RL estimate $\hat{\chi}_{35}$ and its truth χ_{35} in three cases

The evolution curves of the estimation errors $\|\pi_{i1} - \chi_{i1}\|$ in three cases are depicted in Fig. 5.15(d) and Fig. 5.19(d), respectively, which converge to around zero and validate our theoretic results.

In addition, the fused RL estimates π_{i1} , $i = 2, \dots, 6$, and their corresponding true relative coordinates are shown in Fig. 5.15(a)(b)(c) - Fig. 5.19(a)(b)(c), respectively. It is worth noting that, in Fig. 5.17(d) and Fig. 5.19(d), the fused estimates π_{41} and π_{61} fluctuate slightly in the presence of a significant variance of v_4 at $t = 100s$, however, such variance has little influence on π_{31} and π_{51} as seen in Fig. 5.16(d) and Fig. 5.18(d). The reason is that besides an indirect RL estimate ${}_4\hat{\chi}_{31} = \hat{\chi}_{34} + \pi_{41}$ through UAV4, UAV3 has a direct RL estimate $\hat{\chi}_{31}$ and this direct estimate can

Figure 5.12: Comparison of direct RL estimate $\hat{\chi}_{54}$ and its truth χ_{54} in three cases

be used by UAV5 for fusion. This demonstrates that our proposed cooperative RL method is robust to unexpected maneuverings. In the following chapter, we will combine the proposed cooperative RL with UWB-based formation control.

5.5.2.4 The influence of ϵ and γ

According to the analysis of Theorem 5.3.2, the RL estimation error is dominated by the velocity noise ϵ . To show the influence of different velocity noises on the accuracy of the RL estimation, ϵ with different variances of $\bar{\delta} = 0.1 \sim 0.7m/s$ are fed into the relative velocity measurements. Other configurations are the same as the second simulation.

Figure 5.13: Comparison of direct RL estimate $\hat{\chi}_{53}$ and its truth χ_{53} in three cases

For a given γ , without loss of generality, the evolutions of $\|\hat{\chi}_{53} - \chi_{53}\|$ under different conditions of $\bar{\delta}$ are shown in Fig. 5.20(a). It can be seen that the RL estimation error, $\|\hat{\chi}_{53} - \chi_{53}\|$, becomes larger with the increase of $\bar{\delta}$, which is consistent with Theorem 5.3.2.

On the other hand, for given $\bar{\delta}$, γ should be chosen to satisfy (5.16) to ensure the stability of (5.1). A time window of 3s is chosen for Assumption 5.3.2, i.e., $m = 3/T$. Accordingly, μ can be found within this time window. The trajectories of $\|\hat{\chi}_{53} - \chi_{53}\|$ and its upper bound with $\gamma = 0.5, 0.1$ are exhibited in Fig. 5.20(b) and Fig. 5.20(c), respectively. It can be seen that a small γ results in a small error bound but a lower convergence speed while a larger γ reverses. Therefore, a dynamic

Figure 5.14: Comparison of direct RL estimate $\hat{\chi}_{64}$ and its truth χ_{64} in three cases

γ can be adopted, which means at the beginning of RL, a larger γ can be chosen to speed up the convergence of RL, then reduce γ to narrow the RL error. In addition, if μ is obtained at the end of flight, then it should be fixed and c will be a constant. In this simulation, we adopt the data updated in the latest m steps to dynamically calculate μ with the increase of time step k . Thus, the upper bound curve is wavy and we may choose the peak value for reference such as $0.6m$ in Fig. 5.20(c). Next we will combine the proposed cooperative RL with UWB-based formation control.

Figure 5.15: Comparison of fused RL estimate π_{21} and its truth χ_{21} in three cases

5.6 Flight Experimental Results

In this section we present extensive flight experimental evaluations to demonstrate our proposed infrastructure-free cooperative RL system. The performance of cooperative RL is evaluated in terms of average absolute error with comparison of VICON[®] ground truth (millimeter-level positioning accuracy).

5.6.1 System Configurations

Fig. 5.21 illustrates the hardware configuration and signal flow of the UWB-based RL system. UWB modules PulsON 440[®] were installed on three quadcopters. Due

Figure 5.16: Comparison of fused RL estimate π_{31} and its truth χ_{31} in three cases

to the large bandwidth (from 3.1 GHz to 5.3 GHz), UWB is robust to multipath and non-line of sight effects, and provides a reliable long distance ranging with an accuracy of 10cm. All of the algorithms, sensing and communication drivers are executed on a credit-card sized mobile-level Raspberry Pi2[®] with an ultra-low power processor ARM Cortex-A7 running at 900MHz. The quadcopter is equipped with Pixhawk[®], integrating inertial measurement unit and flight controller on board, as well as Px4Flow[®] for onboard velocity estimation. Note that other equivalent velocity or displacement sensing systems can be adopted as well.

The UWB module on each quadcopter actively sends ranging requests to neighboring UAVs for distance measurement and communication, based on the TW-ToF ranging

Figure 5.17: Comparison of fused RL estimate π_{41} and its truth χ_{41} in three cases

method and a customized network protocol introduced in Section 5.6.2. In this UWB RCM network, an instant distance estimate obtained by UAV i will be calibrated through linear regression and its corresponding parameters have been determined by a series of experiments in different environments [28]. This corrected range together with the received information then goes through the outlier detection before it is stored in the database. To reduce the computation and avoid excessive repetition of similar data, only selected distance measurements and neighbor's information are recorded and stored. Note that the RL initialization algorithm is processed first and its output serves as the initial state estimate for PE and EKF-based RL estimators. Finally, the fused RL estimate update will be fed into the flight controller

Figure 5.18: Comparison of fused RL estimate π_{51} and its truth χ_{51} in three cases

for quadcopter navigation as depicted in Fig. 5.22. All the processes above are controlled by Raspberry Pi2[®], while XBee[®] only receives each UAV's global position and velocity from VICON[®] as ground truth.

5.6.2 UWB Ranging and Communication Network Configuration

A contention-free Time Division Multiple Access (TDMA) time-slotted network is used to coordinate communications and range measurements. The basic mechanism of TDMA is a common network clock which is used to drive an airtime “slotmap”. In

Figure 5.19: Comparison of fused RL estimate π_{61} and its truth χ_{61} in three cases

addition, each node transmits its previous range and velocity of range measurement provided by UWB firmware, together with its current velocity estimate as data embedded in the range request packet. All nodes overhear this data and optionally provide it to a co-located host processor as seen in Fig. 5.23.

An example of a slot map supporting three network nodes, with unique identifiers 200 – 202 (i.e., UAV0-2) is given in Table 5.2. Each slot supports a TW-ToF range conversation with associated data package. On completion of slot 3, the process will repeat from slot 1.

Figure 5.20: Absolute error of RL estimate $\hat{\chi}_{53}$, $\|\hat{\chi}_{53} - \chi_{53}\|$, and its upper bound c

5.6.3 Experimental Evaluations of RL Estimation

To validate our proposed RL system, on-board tests were carried out on three simultaneous moving UAV platforms in Internet of Things lab as seen in Fig. 5.24 where a VICON[®] system is installed for the ground truth. Each UAV concurrently conducts ranging measurement and communicates its local estimates with its neighbors. Meanwhile, our proposed Algorithm 6.1 will be executed at the end of each communication in a distributed way.

In order to cover distinct situations, 13 tests were conducted with different speeds and travelling patterns including **Static**, **Circle**, **Triangle**, **Line**, random **Walk** and

Figure 5.21: Flight platform and diagram of system workflow

Figure 5.22: Diagram of flight control workflow

Figure 5.23: **Ranging and communication network configuration for 3 UAVs.** The data embedded in range request package is broadcast and overheard by any UWB node within its ranging scope.

Table 5.2: TDMA slotmap configuration for 3 UAVs

Slot	Requester ID	Responder ID	Data package (16 bytes)
00	200	201-202	time-synchronization message
01	200	201	yes
02	201	202	yes
03	202	200	yes

Figure 5.24: Three UAVs fly and conduct RL at IoT lab.

NTU letters. Since the RL performances of these three UAVs are similar, without losing generality, here we show the average absolute RL estimation errors of UAV2 as an example in Table 5.3 where **2S1C_3** means the **3rd** test with **2** UAVs being **S**tationary and **1** moving in a **C**ircle. The statistical results illustrate that the mean of our proposed RL estimation error along one axis is around $0.2m$ in the presence of distance and velocity measurement noises.

Fig. 5.28 illustrates the plot of overhead 2-D trajectory recorded on UAV2 from Test No.13. It can be seen that the average absolute RL estimation errors in x and y directions are less than $0.4m$ in the worst case. The RL estimation is conducted in a distributed way, the evolution curves of RL trajectories and estimation errors on UAV0, UAV1 and UAV2 are shown in Fig. 5.25, Fig. 5.26 and Fig. 5.27, respectively. It can be seen that the RL estimation results on UAV0, UAV1 and

Table 5.3: Average absolute RL estimation errors $\frac{1}{n} \sum_{i=1}^n |\hat{x}_i - x_i^{GT}|$ calculated on UAV2. (unit:m)

Test	$\hat{\chi}_2^{01} - \chi_{GT}^{01}$		$\hat{\chi}_2^{12} - \chi_{GT}^{12}$		$\hat{\chi}_2^{20} - \chi_{GT}^{20}$	
	$ \delta x $	$ \delta y $	$ \delta x $	$ \delta y $	$ \delta x $	$ \delta y $
01_2S1C_1	0.132	0.110	0.141	0.098	0.121	0.111
02_2S1C_2	0.150	0.101	0.136	0.122	0.141	0.118
03_2S1T_1	0.245	0.117	0.230	0.155	0.237	0.122
04_2S1T_2	0.258	0.133	0.249	0.148	0.234	0.161
05_1S2C_1	0.267	0.120	0.233	0.123	0.244	0.131
06_1S2C_2	0.296	0.094	0.261	0.103	0.266	0.139
07_1S2T_1	0.202	0.167	0.215	0.187	0.216	0.151
08_1S2T_2	0.227	0.115	0.212	0.176	0.249	0.133
09_1L2C	0.190	0.186	0.197	0.189	0.201	0.182
10_3W	0.203	0.223	0.210	0.216	0.219	0.206
11_3T	0.217	0.204	0.215	0.216	0.205	0.221
12_3C	0.247	0.220	0.218	0.237	0.232	0.226
13_NTU	0.222	0.190	0.211	0.208	0.238	0.198
mean	0.219	0.152	0.210	0.167	0.215	0.161

UAV2 have the same magnitude even though there exist slight differences between each pair of UAVs. **GT** presents the RL ground truth obtained from VICON[®] system and **Est** is the estimation curve. The solid red and blue circles are the starting points of ground truth and RL estimates, and the diamonds represent the corresponding ending points.

5.7 Conclusion

This chapter proposed a distributed cooperative RL scheme for UAV swarms without infrastructures, global positions and vision-based detecting. Based on the capability of the designed UWB RCM network, each UAV is able to estimate the relative positions of its neighbors by utilizing our proposed consensus-based RL fusion scheme. Simulations and extensive flight experiments validated the effectiveness of our proposed system. The cooperative RL and formation control with application to mobile vehicles will be introduced in the next chapter.

Figure 5.25: Comparison of RL estimates $\hat{\chi}_0^{01}$, $\hat{\chi}_0^{12}$ and $\hat{\chi}_0^{20}$ calculated on UAV0, and their corresponding 2-norm errors $\|\hat{\chi}_0^{01} - \chi_0^{01}\|$, $\|\hat{\chi}_0^{12} - \chi_0^{12}\|$ and $\|\hat{\chi}_0^{20} - \chi_0^{20}\|$.

Figure 5.26: Comparison of RL estimates $\hat{\chi}_1^{01}$, $\hat{\chi}_1^{12}$ and $\hat{\chi}_1^{20}$ calculated on UAV1, and their corresponding 2-norm errors $\|\hat{\chi}_1^{01} - \chi_1^{01}\|$, $\|\hat{\chi}_1^{12} - \chi_1^{12}\|$ and $\|\hat{\chi}_1^{20} - \chi_1^{20}\|$.

Figure 5.27: Comparison of RL estimates $\hat{\chi}_2^{01}$, $\hat{\chi}_2^{12}$ and $\hat{\chi}_2^{20}$ calculated on UAV2, and their corresponding 2-norm errors $\|\hat{\chi}_2^{01} - \chi_2^{01}\|$, $\|\hat{\chi}_2^{12} - \chi_2^{12}\|$ and $\|\hat{\chi}_2^{20} - \chi_2^{20}\|$.

Figure 5.28: Global trajectories and absolute RL estimation errors generated by UAV2 in x and y directions.

Chapter 6

Distributed Formation Control using RL Estimates

6.1 Introduction

In this chapter, we develop a complete multi-UAV system which combines a distributed formation control with infrastructure-free cooperative RL. The proposed formation control is validated by both simulations and experiments on a group of three quadcopters. Unlike [19], the system here is complete, integrating distributed formation control with RL estimates. Moreover, the stability of the proposed distributed formation control is analyzed in consideration of measurement noise and RL estimation error.

The proposed complete system aims to simultaneously estimate the relative positions between UAVs and control a formation to achieve both the objectives of shape control and velocity consensus. Leveraging the availability of both range measurements and relative position estimates, the group motion is governed by the desired formation shape as well as velocity consensus. Numerical simulations and formation flights of real quadcopter UAVs demonstrate the effectiveness of the complete system, which integrates RL into formation control, by utilizing a customized UWB ranging and communication (RCM) network.

Figure 6.1: The simultaneous infrastructure-free cooperative relative localization and distributed formation control strategy is proposed to overcome the shortcomings of widely used global positioning and/or vision-based UAV swarm applications: (a), (b) and (c) depict the distributed formation flights in different scenes; (d) illustrates the relative distances, self-displacements and relative positions of 3 UAVs when they are moving simultaneously. For clarity, all variables shown in (d) are depicted at different instants.

The remainder of this chapter is structured as follows: Section 6.2 discusses a discrete-time formation control algorithm by using the RL estimates followed by algorithm realization. Simulation and flight experiments are conducted in Section 6.3 and Section 6.4, respectively. Finally, conclusion remarks are drawn in Section 6.5.

6.2 Distributed Formation Control

6.2.1 Stability of Formation Control in Noise-corrupted Case

Combining inter-UAV relative localization with distributed formation control is still challenging especially in the presence of noises. Based on global position information, [109] proposed a continuous-time formation control algorithm as

$$\begin{aligned}\dot{\mathbf{p}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= \sum_{j \in \mathcal{N}_i} \boldsymbol{\nu}_i^{ij} + 2 \sum_{j \in \mathcal{N}_i} (d^{ij^2} - d_{ij}^{*2}) \boldsymbol{\chi}_i^{ij},\end{aligned}$$

and its noise-corrupted double-integrator model of the system can be rewritten as

$$\begin{aligned}\dot{\mathbf{p}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= \sum_{j \in \mathcal{N}_i} (\boldsymbol{\nu}_i^{ij} + \boldsymbol{\epsilon}) + 2 \sum_{j \in \mathcal{N}_i} (d^{ij^2} - d_{ij}^{*2}) \boldsymbol{\pi}_i^{ij},\end{aligned}\tag{6.1}$$

where d_{ij}^* is the desired relative distance between UAV i and UAV j . Assume $\boldsymbol{\pi}_i^{ij}$ is a bounded estimate of $\boldsymbol{\chi}_i^{ij}$ with noise $\boldsymbol{\zeta}$, i.e., $\boldsymbol{\pi}_i^{ij} = \boldsymbol{\chi}_i^{ij} + \boldsymbol{\zeta}$. When performing a formation, neighboring UAV j is actually within an area with a perceiving radius $\bar{d}$, i.e., $\|\boldsymbol{\chi}_i^{ij}\| = d^{ij} \leq \bar{d}$, thus we can incorporate these two noises into one item $\mathbf{w}$. Then (6.1) can be rewritten as

$$\begin{aligned}\dot{\mathbf{p}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= \sum_{j \in \mathcal{N}_i} \boldsymbol{\nu}_i^{ij} + 2 \sum_{j \in \mathcal{N}_i} (d^{ij^2} - d_{ij}^{*2}) \boldsymbol{\chi}_i^{ij} + \mathbf{w},\end{aligned}\tag{6.1'}$$

however, this continuous-time version cannot be implemented directly since UAV i ranges to its neighbors at a time period T and then estimates the relative position at the end of each such interval. Thus, this section proposes a discrete-time algorithm which combines the preceding relative position estimate with formation control for multiple UAVs and the corresponding convergence analysis is conducted as well especially with the influence of noises.

The discrete version of (6.1') for UAV i in noise-corrupted case is given by

$$\begin{aligned} \mathbf{p}_{i,k+1} &= \mathbf{p}_{i,k} + T\mathbf{v}_{i,k}, \\ \mathbf{v}_{i,k+1} &= \mathbf{v}_{i,k} + \gamma_1 T \sum_{j \in \mathcal{N}_i} \boldsymbol{\nu}_{i,k}^{ij} + \gamma_2 T \sum_{j \in \mathcal{N}_i} (d_k^{ij^2} - d_{ij}^{*2}) \boldsymbol{\chi}_{i,k}^{ij} + \mathbf{w}_k, \end{aligned} \quad (6.2)$$

where T is the sampling time interval, γ_1 and γ_2 are small positive constants.

To handle the stability of formation control with noise/disturbance, it is helpful to first introduce the theory on Malkin structure in the presence of noise/disturbance seen in Definition 6.2.1. Then Lemma 6.2.1 shows that (6.2) can be transformed to this Malkin structure. After that in Theorem 6.2.1 we demonstrate that a discrete-time version of Malkin structure (6.3) will converge to some small region exponentially. Finally, based on these results, Theorem 6.2.2 shows that the system (6.2) converges to a neighborhood of the desired formation.

Definition 6.2.1. *A Malkin structure for a system in the presence of noise/disturbance is of the form*

$$\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\rho}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\rho} \end{bmatrix} + \begin{bmatrix} \Theta(\boldsymbol{\theta}, \boldsymbol{\rho}) \\ P(\boldsymbol{\theta}, \boldsymbol{\rho}) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{w} \end{bmatrix}, \quad (6.3)$$

where $\mathbf{w}$ is a noise or disturbance, and others are the same as the Malkin structure (18) in [110], i.e., $\mathbf{A}$ is a constant matrix with eigenvalues on the left half plane, $\Theta(\boldsymbol{\theta}, \boldsymbol{\rho})$ and $P(\boldsymbol{\theta}, \boldsymbol{\rho})$ are second order terms where $\Theta(\boldsymbol{\theta}, \mathbf{0}) = \mathbf{0}$ and $P(\boldsymbol{\theta}, \mathbf{0}) = \mathbf{0}$,

and b_1, b_2 are bounded smooth functions with $b_2(\mathbf{0}) = \mathbf{0}$ where

$$b_1 = \begin{cases} \frac{\Theta(\boldsymbol{\theta}, \boldsymbol{\rho})}{\|\boldsymbol{\rho}\|} & \text{if } \boldsymbol{\rho} \neq \mathbf{0}, \\ \lim_{\boldsymbol{\rho} \rightarrow \mathbf{0}} \frac{\Theta(\boldsymbol{\theta}, \boldsymbol{\rho})}{\|\boldsymbol{\rho}\|} & \text{if } \boldsymbol{\rho} = \mathbf{0}, \end{cases} \quad b_2 = \begin{cases} \frac{P(\boldsymbol{\theta}, \boldsymbol{\rho})}{\|\boldsymbol{\rho}\|} & \text{if } \boldsymbol{\rho} \neq \mathbf{0}, \\ \lim_{\boldsymbol{\rho} \rightarrow \mathbf{0}} \frac{P(\boldsymbol{\theta}, \boldsymbol{\rho})}{\|\boldsymbol{\rho}\|} & \text{if } \boldsymbol{\rho} = \mathbf{0}. \end{cases} \quad (6.4)$$

Lemma 6.2.1. *The system (6.1) in the presence of noise/disturbance can be transferred to a noise-corrupted Malkin structure (6.3) through a local diffeomorphism around the equilibrium point of noise-free (6.1).*

Proof. The proof can be achieved by simply extending the proof of Lemma 5 in [110] with consideration of the noise term $\mathbf{w}$ in system (6.1). ■

Similar to Theorem 2 in [110], one can obtain the corresponding result on discrete-time Malkin structure:

$$\begin{bmatrix} \boldsymbol{\theta}_{k+1} \\ \boldsymbol{\rho}_{k+1} \end{bmatrix} = \left(\mathbf{I} + \tau \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \right) \begin{bmatrix} \boldsymbol{\theta}_k \\ \boldsymbol{\rho}_k \end{bmatrix} + \tau \begin{bmatrix} \Theta(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k) \\ P(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k) \end{bmatrix} + \tau \begin{bmatrix} \mathbf{0} \\ \mathbf{w}_k \end{bmatrix}, \quad (6.5)$$

where all parameters satisfy the same conditions as in (6.3). To proceed, define the space $\ell^{1/2} := \{(\mathbf{x}_k) : \mathbf{x}_k \in \mathbb{R}^n, k \in \mathbb{N}, \sum_{k=0}^{\infty} \|\mathbf{x}_k\|^{1/2} \text{ is bounded}\}$. We have the following result which gives the convergence result on discrete-time Malkin structure with noise/disturbance.

Theorem 6.2.1. *Consider (6.5) with sufficient small $\mathbf{w}_k \in \ell^{1/2}$, where $\mathbf{A}$ is a constant matrix with all eigenvalues on the left half plane. Then there exists a sufficiently small sampling time interval τ , and a sufficiently small open ball $\mathcal{V}$ around the origin such that if $(\boldsymbol{\theta}_0, \boldsymbol{\rho}_0)$ lies in this open ball, then $(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)$ lies in the ball for all k .*

Proof. The proof can be adapted from Theorem 2 in [110]. The details are given as follows.

Similar to the proof of Theorem 2 in [110], it can be concluded that there exist sufficiently small $\tau, \lambda > 0$ such that

$$\mathbf{I} - \mathbf{A}_d^\top \mathbf{A}_d \geq \lambda \mathbf{I}, \quad (6.6)$$

where $\mathbf{A}_d := \mathbf{I} + \tau \mathbf{A}$. And for any given $\sigma > 0$, there exist η and a closed ball $\bar{\mathcal{B}}_\eta$ contained in $\mathcal{V}$ such that

$$\|b_2(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)\| \leq \sigma, \quad \forall (\boldsymbol{\rho}_k, \boldsymbol{\theta}_k) \in \bar{\mathcal{B}}_\eta. \quad (6.7)$$

Construct $V(\boldsymbol{\rho}_k) = \boldsymbol{\rho}_k^\top \boldsymbol{\rho}_k$. By similar argument to (49) in [110], it can be obtained that for $(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k) \in \bar{\mathcal{B}}_\eta$

$$\begin{aligned} & V(\boldsymbol{\rho}_{k+1}) - V(\boldsymbol{\rho}_k) \\ &= \boldsymbol{\rho}_k^\top (\mathbf{A}_d^\top \mathbf{A}_d - \mathbf{I}) \boldsymbol{\rho}_k + 2\tau \boldsymbol{\rho}_k^\top \mathbf{A}_d^\top P(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k) + \tau^2 \|P(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)\|^2 \\ &\quad + 2\tau (\boldsymbol{\rho}_k^\top \mathbf{A}_d^\top + \tau P^\top(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)) \mathbf{w}_k + \tau^2 \|\mathbf{w}_k\|^2 \\ &\leq -\lambda \boldsymbol{\rho}_k^\top \boldsymbol{\rho}_k + 2\tau \|\mathbf{A}_d\| \|b_2(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)\| \|\boldsymbol{\rho}_k\|^2 + \tau^2 \|b_2(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)\|^2 \|\boldsymbol{\rho}_k\|^2 \\ &\quad + 2\tau (\|\boldsymbol{\rho}_k\| \|\mathbf{A}_d\| + \tau \|b_2(\boldsymbol{\theta}_k, \boldsymbol{\rho}_k)\| \|\boldsymbol{\rho}_k\|) \|\mathbf{w}_k\| + \tau^2 \|\mathbf{w}_k\|^2 \\ &\leq (-\lambda + 2\sigma + \sigma^2) \|\boldsymbol{\rho}_k\|^2 + 2\eta(1 + \sigma) \|\mathbf{w}_k\| + \|\mathbf{w}_k\|^2. \end{aligned} \quad (6.8)$$

By letting σ small enough such that $2\sigma + \sigma^2 < \lambda/2$, it follows from (6.8) that

$$\|\boldsymbol{\rho}_k\|^2 \leq c^k \|\boldsymbol{\rho}_0\|^2 + a \sum_{i=0}^{k-1} c^i \|\mathbf{w}_{k-1-i}\| + \sum_{i=0}^{k-1} c^i \|\mathbf{w}_{k-1-i}\|^2, \quad (6.9)$$

where $c := 1 - \lambda/2$ and $a := 2\eta(1 + \sigma)$.

Suppose that the function b_1 is continuous with the maximal value $\bar{m}$ on $\bar{\mathcal{B}}_\eta, \forall k \in [0, \infty)$, it has

$$\|\Theta(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k)\| = \|\boldsymbol{\rho}_k\| \|b_1(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k)\|$$

$$\leq \bar{m}\|\boldsymbol{\rho}_0\|c^{k/2} + \bar{m}\sqrt{a} \sum_{i=0}^{k-1} c^{i/2} \|\mathbf{w}_{k-1-i}\|^{1/2} + \bar{m} \sum_{i=0}^{k-1} c^{i/2} \|\mathbf{w}_{k-1-i}\|. \quad (6.10)$$

Note that $\mathbf{w} \in \ell^{1/2}$, thus there exists a constant $M > 0$ such that $\sum_{k=0}^{\infty} \|\mathbf{w}_k\|^{1/2} \leq M$ and $\sum_{k=0}^{\infty} \|\mathbf{w}_k\| \leq M$, together with (6.10) leads to

$$\begin{aligned} \|\boldsymbol{\theta}_k\| &\leq \|\boldsymbol{\theta}_0\| + \frac{\tau\bar{m}\|\boldsymbol{\rho}_0\|}{1-\sqrt{c}} + \frac{\tau\bar{m}\sqrt{a}}{1-\sqrt{c}} \sum_{i=0}^{\infty} \|\mathbf{w}_i\|^{1/2} + \frac{\tau\bar{m}}{1-\sqrt{c}} \sum_{i=0}^{\infty} \|\mathbf{w}_i\| \\ &\leq \|\boldsymbol{\theta}_0\| + \frac{\tau\bar{m}\|\boldsymbol{\rho}_0\|}{1-\sqrt{c}} + \frac{\tau\bar{m}M(\sqrt{a}+1)}{1-\sqrt{c}}. \end{aligned} \quad (6.11)$$

Therefore, it has

$$\begin{aligned} \|(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k)\| &\leq \|\boldsymbol{\rho}_k\| + \|\boldsymbol{\theta}_k\| \\ &\leq \|\boldsymbol{\rho}_0\| + (\sqrt{a}+1)M + \|\boldsymbol{\rho}_0\| + \frac{\tau\bar{m}\|\boldsymbol{\rho}_0\|}{1-\sqrt{c}} + \frac{\tau\bar{m}M(\sqrt{a}+1)}{1-\sqrt{c}}. \end{aligned} \quad (6.12)$$

Now we shall prove the sequence $(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k)$ stays in $\mathcal{B}_\eta$ for all k and here the contradiction method is adopted. Suppose that there exists a finite K satisfying $(\boldsymbol{\rho}_k, \boldsymbol{\theta}_k) \in \mathcal{B}_\eta, \forall k \in [0, K]$ but the claim fails for $k = K + 1$. Specifically, for the initial state $(\boldsymbol{\rho}_0, \boldsymbol{\theta}_0)$ in a ball $\mathcal{B}_{\eta_0}$ with $\eta_0 > 0$, selecting $\eta_0 < \eta$, $\tau > 0$, and $(\mathbf{w}_k)_{k \in \mathbb{N}}$ all sufficiently small such that

$$2\eta_0 + (\sqrt{a}+1)M + \frac{\tau\bar{m}\eta_0}{1-\sqrt{c}} + \frac{\tau\bar{m}M(\sqrt{a}+1)}{1-\sqrt{c}} < \eta. \quad (6.13)$$

Then for $k = K + 1$, from (6.12), we know

$$\begin{aligned} \|(\boldsymbol{\rho}_{K+1}, \boldsymbol{\theta}_{K+1})\| &\leq \|\boldsymbol{\rho}_{K+1}\| + \|\boldsymbol{\theta}_{K+1}\| \\ &\leq 2\eta_0 + (\sqrt{a}+1)M + \frac{\tau\bar{m}\eta_0}{1-\sqrt{c}} + \frac{\tau\bar{m}M(\sqrt{a}+1)}{1-\sqrt{c}} < \eta, \end{aligned} \quad (6.14)$$

which reveals that $(\boldsymbol{\rho}_{K+1}, \boldsymbol{\theta}_{K+1}) \in \mathcal{B}_\eta$. This contradicts the preceding condition.

Hence the claimed result is achieved and the proof is completed. $\blacksquare$

Lemma 6.2.2. Consider a differential equation $\dot{\mathbf{p}} = f(\mathbf{p}) + \mathbf{w}$ where a differen-

tial equation set in Malkin form can be achieved by a coordinate transformation through the diffeomorphism $\mathbf{r} = \phi(\mathbf{p})$ and then a discrete-time Malkin equation can be obtained by discretizing this set. Furthermore, the operations of diffeomorphic coordinate transformation to a Malkin structure and time-discretization commute.

Proof. Inspired by Appendix III of [110], the proof is straightforward with consideration of the noise $\mathbf{w}$. ■

Remark 6.2.1. From Lemma 6.2.2, we know that system (6.2) can be transferred to the Malkin structure (6.5) through time-discretizing (6.1) and diffeomorphic coordinate transformation.

It is now ready to present the main result on the discrete-time system (6.2).

Theorem 6.2.2. Consider system (6.2) with $\mathbf{w}_1, \mathbf{w}_2 \in \ell^{1/2}$ and assume that the communication graph for velocity measurements is undirected and connected. Then, for any $T > 0$, there exist sufficiently small $\gamma_1, \gamma_2 > 0$ such that if the velocity is kept constant in every time interval $(kT, (k+1)T)$, then the system (6.2) converges to a neighborhood of the manifold $d_k^{ij2} - d_{ij}^{*2} = 0, \forall i, j$ as $k \rightarrow \infty$ for sufficiently small $\mathbf{w}_1, \mathbf{w}_2$.

Proof. Lemma 6.2.1 shows that the continuous system (6.1) can be transformed to a noise-corrupted Malkin structure (6.3). Furthermore, based on Theorem 6.2.1, the discretization of this Malkin structure will exponentially converge. In addition, Lemma 6.2.2 demonstrates that the operations of diffeomorphic coordinate transformation to a Malkin structure and discretization commute, and then we conclude that system (6.2) will converge to a neighborhood of the manifold $d_k^{ij2} - d_{ij}^{*2} = 0, \forall i, j$ as $k \rightarrow \infty$. ■

6.2.2 Algorithm Realization Involving RL Estimation

Section 5.3 and Section 6.2 present the proposed distributed cooperative RL estimator and formation control respectively. This section will combine these two parts

and present the realization of the proposed simultaneous RL and formation control algorithm. PE-based RL method in Section 5.3.1 is robust to the initial RL estimates while the EKF-based estimator in Section 5.3.4 is a bit sensitive to the initial guess especially in the case where the initial RL guess is quite far from the true relative position. However, taking account into the sensing failures and packet dropouts, PE-based RL is heavily reliant on data availability and takes a risk of no RL update. Thus, in practical applications, we propose to combine these two methods to make full use of their respective strengths. EKF-based estimator can help enhance the continuity of RL updates in the presence of sensing failures and packetdropouts. On the other hand, PE-based method can aid EKF-based estimator when it significantly deviates from the true value or even diverges. The fused RL estimates from Section 5.3.2 will enhance the robustness of RL estimation, especially in the presence of unexpected maneuverings of a certain UAV or in the case that some UAVs can not have direct range measurement to their neighbors. The details of our proposed RL algorithm are presented in Algorithm 6.1. Here we use left subscripts $_{pe}\hat{\chi}$ and $_{ekf}\hat{\chi}$ to distinguish the RL estimates obtained from Section 5.3.1 and Section 5.3.4.

6.3 Simulation Evaluation

6.3.1 RL Estimation and Formation Control of 3 UAVs

Consider a three-UAV team where each UAV is able to measure the distance to the other two UAVs. The control goal is to form two triangular formations with velocity consensus. In the simulation, we set $T = 0.05s$, $\gamma_1 = 1$, $\gamma_2 = 1.4 \times 10^{-2}$. The parameters of RL estimation (including distance measurement noise ϵ^d , velocity measurement noise ϵ) are identical to Section 5.5.2. In addition, we set the maximum flight velocity is $5m/s$ (for safety), formation error is $1m$ (a formation is achieved if the distance to the desired formation shape is less than $1m$) and the velocity consensus error is $0.5m/s$ (velocity consensus is achieved if the velocity error between

Algorithm 6.1: Pseudocode for Simultaneous RL Estimation and Distributed Formation Control

Procedure: RL Initialization

Carry out ranging and communication, then conduct NLR: **while** $\Delta\phi > \kappa$

do

for $i \leftarrow 1$ to N **do**

 if $i == N$, $j \leftarrow 0$; otherwise $j \leftarrow i + 1$

 UAV i sends range request and data package to UAV j

 Calculate the relative distance $\underline{d}_k^{ij}$

end

$\mathbf{J}_k \leftarrow (5.33a)$, $\Delta\mathbf{d} \leftarrow (5.33b)$, $\Delta\phi \leftarrow (5.33c)$, $\phi_{k+1} \leftarrow (5.33d)$

end

Each UAV stores the initial RL estimates $\hat{\mathbf{x}}_0^{ij} \leftarrow (\mathbf{x}, \phi)$

Procedure: Cooperative RL and Formation Control

while *CooperativeRL* **do**

for $i \leftarrow 1$ to N **do**

 if $i == N$, $j \leftarrow 0$; otherwise $j \leftarrow i + 1$

 UAV i sends range request and data package to UAV j

if *received range response signal or data package* **then**

 Calculate the relative distance $\underline{d}_k^{ij}$, distance rate $\underline{\dot{d}}_k^{ij}$, and indirect relative velocity $\underline{\nu}_k^{ij}$

${}_{pe}\hat{\mathbf{x}}_{i,k+1}^{ij} \leftarrow (5.1)$

 Conduct EKF procedure referring to [28]

 Update ${}_{ekf}\hat{\mathbf{x}}_{i,k+1}^{ij}$

$\hat{\mathbf{x}}_{i,k+1}^{ij} \leftarrow {}_{pe}\hat{\mathbf{x}}_{i,k+1}^{ij}$

end

else

${}_{ekf}\hat{\mathbf{x}}_{i,k+1|k}^{ij}, \hat{\mathbf{v}}_{i,k+1|k}^{ij} \leftarrow (4.16)$

$\hat{\mathbf{x}}_{i,k+1}^{ij} \leftarrow {}_{ekf}\hat{\mathbf{x}}_{i,k+1|k}^{ij}$

end

$\boldsymbol{\pi}_{i,k+1}^{ij} \leftarrow (5.27)$

 Calculate the control command $\leftarrow (6.2)$

 Send command to flight control board

end

end

UAVs is less than $0.5m/s$). Note that the relative position term in (6.2) is generated from Algorithm 6.1 and two equilateral triangular formations are preconfigured with $d_{ij}^* = 15m$ and $30m$ sequentially, $i, j \in \{0, 1, 2\}$.

Compared with the noiseless case (without distance and velocity noises and using true relative positions, see Fig. 6.2(a)), in Fig. 6.2(b), two equilateral triangular

formations are formed successively and sustained by using noisy distance and velocity measurements, and relative position estimates. The desired formations are achieved at 11s and 15s respectively, and the inter-UAV distances are maintained at 30m finally as shown in Fig. 6.2(c). From Fig. 6.2(d), we observe that both of these two cases achieve velocity consensus and note that the fluctuating relative velocity indicates the change of formation. A significant difference in Fig. 6.2(b) is the flight trajectories that are affected by the velocity noise. Note that the intersection of the two tracks of UAV1 and UAV2 is not a collision, but a result of the path overlap (e.g., the follower repeats the leader's path shortly afterwards).

From the above simulation results, 3 UAVs are always able to achieve the desired formation and share a common velocity no matter in noisy or noiseless case. Note that the discrete-time control law (6.2) leverages contaminated distance and velocity measurements as well as our proposed RL estimates, and the simulation results demonstrate that the proposed RL algorithm can provide sufficiently accurate relative position estimates to support UAVs to achieve formation in the presence of both distance and velocity noise.

6.3.2 RL Estimation and Formation Control of 5 UAVs

To further validate our proposed complete formation system, another two simulations with five UAVs were carried out. Similarly, two cases: noiseless measurements with true RL values and noisy measurements with RL estimates, are considered. Fig. 6.3 presents the desired formation shape and sensing graph.

The desired distances between the UAVs are configured as $d_{12}^* = d_{15}^* = \sqrt{10}$, $d_{14}^* = 5\sqrt{2}$, $d_{24}^* = d_{35}^* = 2\sqrt{10}$, $d_{23}^* = d_{25}^* = d_{34}^* = d_{45}^* = 2\sqrt{5}$. The parameters of RL estimation and formation control are the same as the preceding simulation. Fig. 6.4(a) and Fig. 6.4(b) show that both these two cases successfully form the desired formation. It can be checked that a perfect formation is obtained in noise-free case where the relative velocities between UAVs achieve consensus and the formation shape error converges to zero as seen in Fig. 6.4(c). On the other hand,

Figure 6.2: **Simulation results of formation control using RL estimates in noise-corrupted case and the system dynamics are governed by (6.2):** (a), (b) show the flight trajectories of 3 UAVs in two cases: using true RL in noise-free conditions and using RL estimates in noise-corrupted case, respectively; (c) depicts the evolutions of inter-UAV distances; (d) presents the average of 2-norm of inter-UAV relative velocities in two cases: noiseless distance measurements + true RL (red) and noisy distance measurements + RL estimates (blue).

in Fig. 6.4(d), a bounded formation is also achieved. Due to the influence of measurement noises and RL estimation error, the average of inter-UAV relative velocities is bounded by $0.5m/s$ and the average error between inter-UAV distances

Figure 6.3: Formation shape and sensing graph for five UAVs

and the desired distances is bounded by $0.25m$.

Figure 6.4: **Simulation results of integrated RL estimation with formation control in noise-free and noise-contaminated cases:** (a), (b) show the flight trajectories of 5 UAVs using true RL in noise-free case and using RL estimates in noise-corrupted case, respectively; (c), (d) depict the evolutions of inter-UAV relative velocities and formation shape errors.

6.4 Flight Experimental Results

In this section, we implemented the proposed distributed formation control utilizing cooperative RL on quadcopters with the same configuration as described in Section 5.6.1, and conducted a series of real-world flight tests in a grass field in Nanyang Technological University and an abandoned basketball court near Science Center Singapore. These outdoor experiments demonstrate the robustness of the systems against wind disturbances. In both these fields, an equilateral triangle formation of 3 UAVs was achieved successfully¹.

6.4.1 Algorithm Workflow

Based on the analyses in Section 5.3 and Section 6.2, the simultaneous cooperative RL and formation control algorithm workflow is summarized in Fig. 6.5. The proposed workflow will be applied in Section 6.4.2. Note that this workflow excluding multi-UAV formation control part has been applied in Section 5.5.2.

The UWB module mounted on a UAV, actively sends ranging requests with en-trained data packet to neighboring UAVs based on the two-way time of flight (TW-ToF) ranging protocol described in Section 3.2. Once a distance or data packet is obtained by UAV i , as an example, the distance measurement will be calibrated by a linear regression function whose parameters have been determined through a series of experiments in different environments [28]. This calibrated range then goes through the outlier detection before it is stored in the database. To reduce the computation and avoid excessive repetition of similar data, only selected distance measurements and neighbor's information are recorded and stored. Note that RL initialization algorithm is processed first and its output serves as the initial state estimate for EKF and PE-based RL estimators. The follow-up localization is sustained by the consensus-based RL fusion approach. Finally, the fused RL estimates will be fed into the formation controller for UAV navigation as depicted in Fig. 5.22.

¹Flight video can be found in: https://youtu.be/E713k93_Vws

Figure 6.5: UWB-based RL workflow

6.4.2 Flight Evaluations

There are three UAVs involved in the experiments, which comprise of one leader (UAV1) and two followers (UAV0 and UAV2). The control of these UAVs are set in a global coordinate frame with x -axis align with magnetic East and y -axis with magnetic North. In particular, we assign UAV0 as a launching origin and place UAV1 along the East direction with respect to UAV0 in order to keep the flight control coordinate and the RL coordinate system consistent. Meanwhile, UAV2 can be put arbitrarily in North zone.

Upon power-startup, the UWB modules will measure the distances and start communication, and the Raspberry Pi2[®] would then derive initialization configuration and start estimating relative positions. With estimated relative positions, control outputs can be calculated accordingly. Secondly, when onboard control outputs are available for all UAVs, the pilots would then manually takeoff and ascend the UAVs to $1.5m$ altitude, with respect to terrain. This altitude is chosen to minimize ground aerodynamic disturbance while maintaining good visual contact for the optical flow velocity estimator. When the target altitude is reached, two pilots would then switch their UAVs, from manual flight mode to formation flight mode, where the UAV would automatically control its velocity to satisfy formation requirements. One pilot, controlling the leader UAV1, would then manually fly the UAV in predefined trajectories to test the behavior of follower UAVs. If the follower UAVs could automatically maintain the predefined formation against unknown movements from leader UAV, the experiment is considered successful.

A flight video snapshot is shown in Fig. 6.6, where three UAVs reach the desired formation of an equilateral triangle with side length of $3m$ and maintain the formation with the movements of the leader UAV. Online inter-UAV distance measurements and the relative bearing evolutions in the process of forming the desired formation, are depicted in Fig. 6.7(a) and 6.7(b), where three different background colors respectively show the launching, formation keeping and landing phase. After a period

Figure 6.6: **Video snapshot of 3 UAVs simultaneously estimating relative positions and flying as a formation.** The red circled UAV is a designated leader labelled as 1 and it can be controlled by a pilot to prevent the UAVs from flying towards the crowds. The blue and green circles, labelled as 0 and 2 respectively, are two followers to achieve and maintain a desired formation with the movements of UAV1 (yellow arrow indicates the flight direction of the leader at certain time step). Forward, backward, right and left movements of the formation are demonstrated. In fact, these three UAVs run the same code and no leader is assigned beforehand. Here the designated leader is merely for safety considerations, otherwise the formation centroid may be random due to the perturbations and control accuracy.

of adjustment, the desired formation is achieved and kept from 56s to 257s with 3m relative distance and 60° relative azimuth angle.

6.5 Conclusion

This chapter proposed a combined distributed infrastructure-free cooperative RL and distributed formation control scheme for multiple UAVs. Based on the UWB RCM network and our proposed consensus-based RL fusion method, each UAV is able to estimate the relative positions to its neighbors. Meanwhile, these RL estimates combined with the inter-UAV distances and relative velocity estimates are directly fed into our designed discrete-time distributed formation control law to achieve formation flights. Extensive flight experiments demonstrated the effectiveness of our proposed system.

(a) Evolution of inter-UAV distance measurements

(b) Relative bearings of each UAV relative to its neighbors deriving from RL estimates

Figure 6.7: Relative distances and bearings of UAVs during flight

Chapter 7

Conclusions and Future Work

7.1 Conclusions

Micro UAV technology has plenty of potential and is seeing increasing applications in recent years. However, it is challenging to make the system work indoors or in semi-cluttered urban environments and forests due to multipath and NLoS blockage of GPS signals. Hence, a satisfactory localization solution is desired to extend the use of UAV in complex environments. This thesis has focused on investigating GPS-denied localization solutions for UAVs. UWB technology has been chosen to develop our GPS-denied localization for UAVs due to its capability of high ranging accuracy, small size, low power consumption and light weight. The following conclusions can be made:

- First of all, we studied the performance of UWB ranging in different GPS-denied environments and calibrated the distance measurements for reliability and better accuracy. Then, UWB-based self-localization with anchor nodes was introduced where each UAV actively conducts distance measurements to the reference anchor nodes and then estimates their positions and velocities based on the proposed trilateration and EKF algorithm. The performance of our proposed UWB-based localization has been validated through a series of

experiments carried out in indoor and outdoor environments. The system has been successfully demonstrated at Airshow Singapore 2016.

- Taking into account the inconvenience and human-induced errors during the set-up of anchor nodes, UWB-based relative localization with single landmark was studied. Under a motion constraint, we have designed a strategy to actively reduce the localization error along the ranging path and applied EKF to account for the measurement noises. Simulations and flight tests have been conducted to demonstrate our proposed RL method.
- In many unknown environments, it is impractical to configure fixed anchor nodes or landmark. Thus, we further proposed a distributed cooperative RL scheme for UAV swarms without any external infrastructures, global positions and vision-based detecting. Based on the capability of the designed UWB ranging and communication network, each UAV is able to estimate the relative positions of its neighbors by utilizing our proposed persistent excitation-based and consensus-based RL fusion schemes. Simulations and extensive flight experiments validated the effectiveness of our proposed system.
- Combined with the proposed infrastructure-free relative localization method, a complete autonomous multi-UAV system for distributed formation control has been proposed. Each UAV is able to estimate the relative positions to its neighbors. Meanwhile, these RL estimates together with the inter-UAV distances and relative velocity estimates are directly fed into our designed discrete-time distributed formation control law to achieve formation flights. Both simulations and flight tests validated the effectiveness of our proposed system.

7.2 Future Work

There are many issues that deserve future research including:

- In Chapter 3 and Chapter 4, we have studied UWB-based localization with anchor nodes. However, a two-way ranging method was adopted which limits the number of UAVs the system can support. To resolve the problem, one-way UWB ranging method can be studied. The key is to achieve synchronization for wireless and stationary UWB anchors and allow the anchor nodes to localize themselves and construct a coordinate system. Based on the passive reception of the anchors' communications, we can then develop a localization method to enable multiple UAVs flying.
- In Chapter 5, RL estimation error has been studied under the condition of persistent excitation and the assumption of bounded sensor measurement noise. However, in the real world application, there exist data loss and communication delay as well. Thus, in the future, it is worth analyzing the influence of data dropouts and communication delay on the accuracy of localization and exploring some methods to mitigate these effects and improve the localization accuracy.
- In Chapter 6, we have studied the stability of the proposed distributed formation control in the presence of measurement noise and RL estimation error. Further theoretical analysis on the performance of the integrated RL and formation control is needed. Moreover, a complete system including UAV's translational and rotational dynamics, inter-UAV relative information estimation and multi-UAV formation control is worth discussing and investigating.

Author's Publications

Journal papers related to the thesis:

1. **K. Guo**, Z. Qiu, C. Miao, A. H. Zaini, C.-L. Chen, W. Meng, and L. Xie, "Ultra-wideband-based Localization for Quadcopter Navigation," *Unmanned Systems*, vol. 4, no. 01, pp. 23-34, 2016.
2. **K. Guo**, Z. Qiu, W. Meng, L. Xie, and R. Teo, "Ultra-wideband Based Cooperative Relative Localization Algorithm and Experiments for Multiple UAVs in GPS-denied Environments," *International Journal of Micro Air Vehicles*, vol. 9, no. 3, pp. 169-186, 2017.
3. **K. Guo** and L. Xie, "Infrastructure-free Cooperative Relative Localization for Unmanned Aerial Vehicles in GPS-denied Environments," *International Journal of Robotics and Automation*, accepted, 2017.
4. **K. Guo**, X. Li, and L. Xie, "Ultra-wideband-based Cooperative Relative Localization with Application to Multi-UAV Formation Control," under review.
5. **K. Guo**, X. Li and L. Xie, "Simultaneous Cooperative Relative Localization and Distributed Formation Control for Multiple UAVs," *SCIENCE CHINA Information Sciences*, accepted, 2018.
6. Z. Han, **K. Guo**, L. Xie, and Z. Lin, "Integrated Relative Localization & Leader-Follower Formation Control," *IEEE Transactions on Automatic Control*, accepted as Regular Paper, 2018.

Other journal paper:

1. B. Zhu, **K. Guo**, and L. Xie. "A New Distributed Model Predictive Control for Unconstrained Double-Integrator Multi-Agent Systems," *IEEE Transactions on Automatic Control*, accepted for publication, 2018.

Conference papers:

1. **K. Guo** and L. Xie, "Infrastructure-free Cooperative Relative Localization for Unmanned Aerial Vehicles in GPS-denied Environments," *IEEE International Conference on Unmanned Systems (ICUS)*, Beijing, China, Oct. 2017. **Best Paper Award**.
2. **K. Guo**, D. Han, and L. Xie, "Range-Based Cooperative Localization with Single Landmark," in Proceedings of *the 13th IEEE International Conference on Control and Automation (ICCA)*, July 3-6, Ohrid, Macedonia, 2017, pp. 588-593.
3. **K. Guo**, Z. Qiu, W. Meng, and L. Xie, "Relative Localization for Quadcopters using Ultra-wideband Sensors," *International Micro Air Vehicle Competition and Conference (IMAV)*, Beijing, China, Oct. 2016, pp. 243-248.
4. T. M. Nguyen, A. H. Zaini, **K. Guo**, and L. Xie, "An Ultra-wideband based Multi-UAV Localization System in GPS-denied Environments," *International Micro Air Vehicle Competition and Conference (IMAV)*, Beijing, China, Oct. 2016, pp. 56-61. **Best Paper Finalist**.
5. T. M. Nguyen, A. H. Zaini, C. Wang, **K. Guo**, and L. Xie, "Robust Target-relative Localization with Ultra-Wideband Ranging and Communication," *IEEE International Conference on Robotics and Automation (ICRA)*, accepted, 2018.

Bibliography

- [1] P. Liu, A. Y. Chen, Y.-N. Huang, J.-Y. Han, J.-S. Lai, S.-C. Kang, T.-H. Wu, M.-C. Wen, and M.-H. Tsai, “A review of rotorcraft unmanned aerial vehicle (UAV) developments and applications in civil engineering,” *Smart Structures and Systems*, vol. 13, no. 6, pp. 1065–1094, 2014.
- [2] J. Paneque-Gálvez, M. K. McCall, B. M. Napoletano, S. A. Wich, and L. P. Koh, “Small drones for community-based forest monitoring: an assessment of their feasibility and potential in tropical areas,” *Forests*, vol. 5, no. 6, pp. 1481–1507, 2014.
- [3] S. M. Khan, “GNSS multipath interference and mitigation for UAVs in urban canyon environments,” in *Aeronautical Telecommunications Network: Advances, Challenges, and Modeling*. CRC Press, 2015, pp. 137–157.
- [4] C. Teuliere, E. Marchand, and L. Eck, “3-D model-based tracking for UAV indoor localization,” *IEEE Transactions on Cybernetics*, vol. 45, no. 5, pp. 869–879, 2015.
- [5] G. J. Vachtsevanos and K. P. Valavanis, “Military and civilian unmanned aircraft,” in *Handbook of Unmanned Aerial Vehicles*. Springer, 2015, pp. 93–103.
- [6] J. Guo, Z. Wang, M. Zheng, and Y. Wang, “An approach for UAV reconnaissance mission planning problem under uncertain environment,” *International Journal of Imaging and RoboticsTM*, vol. 14, no. 3, pp. 1–15, 2014.

- [7] A. P. Gay, T. P. Stewart, R. Angel, M. Easey, A. J. Eves, N. J. Thomas, D. A. Pearce, and A. I. Kemp, “Developing unmanned aerial vehicles for local and flexible environmental and agricultural monitoring,” vol. 8, no. 11, pp. 471–476, 2009.
- [8] A. Wada, T. Yamashita, M. Maruyama, T. Arai, H. Adachi, and H. Tsuji, “A surveillance system using small unmanned aerial vehicle (UAV) related technologies,” *NEC Technical Journal*, vol. 8, no. 1, pp. 68–72, 2015.
- [9] C. Deng, S. Wang, Z. Huang, Z. Tan, and J. Liu, “Unmanned aerial vehicles for power line inspection: A cooperative way in platforms and communications,” *Journal of Communications*, vol. 9, no. 9, pp. 687–692, 2014.
- [10] J. Straub, “Unmanned aerial systems: Consideration of the use of force for law enforcement applications,” *Technology in Society*, vol. 39, pp. 100–109, 2014.
- [11] B. Buchli, F. Sutton, and J. Beutel, “GPS-equipped wireless sensor network node for high-accuracy positioning applications,” in *Wireless Sensor Networks*. Springer, 2012, pp. 179–195.
- [12] B. Singh, S. K. Sahoo, and S. R. Pradhan, “Performance evaluation of anchor-based range-based localization systems in wireless sensor networks,” *International Journal of Computer Applications*, vol. 52, no. 17, pp. 24–29, 2012.
- [13] Z. Han, L. Wang, Z. Lin, and R. Zheng, “Formation control with size scaling via a complex Laplacian-based approach,” *IEEE Transactions on Cybernetics*, vol. 46, no. 10, pp. 2348–2359, 2016.
- [14] J. A. Preiss, W. Honig, G. S. Sukhatme, and N. Ayanian, “Crazyswarm: A large nano-quadcopter swarm,” in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, May 2017, pp. 3299–3304.
- [15] H. Du, W. Zhu, G. Wen, Z. Duan, and J. Lü, “Distributed formation control of multiple quadrotor aircraft based on nonsmooth consen-

- sus algorithms,” *IEEE Transactions on Cybernetics*, to be published, DOI: 10.1109/TCYB.2017.2777463.
- [16] D. Li, W. Zhang, W. He, C. Li, and S. S. Ge, “Two-layer distributed formation-containment control of multiple Euler–Lagrange systems by output feedback,” *IEEE Transactions on Cybernetics*, to be published, DOI: 10.1109/TCYB.2017.2786318.
- [17] T. Han, Z. Lin, R. Zheng, and M. Fu, “A barycentric coordinate-based approach to formation control under directed and switching sensing graphs,” *IEEE Transactions on Cybernetics*, vol. 48, no. 4, pp. 1202–1215, 2018.
- [18] X. Li and L. Xie, “Dynamic formation control over directed networks using graphical Laplacian approach,” *IEEE Transactions on Automatic Control*, to be published, DOI: 10.1109/TAC.2018.2798808.
- [19] S.-M. Kang, M.-C. Park, and H.-S. Ahn, “Distance-based cycle-free persistent formation: Global convergence and experimental test with a group of quadcopters,” *IEEE Transactions on Industrial Electronics*, vol. 64, no. 1, pp. 380–389, 2017.
- [20] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, “Time-varying formation tracking for second-order multi-agent systems subjected to switching topologies with application to quadrotor formation flying,” *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 5014–5024, 2017.
- [21] W. Jasim and D. Gu, “Robust team formation control for quadrotors,” *IEEE Transactions on Control Systems Technology*, vol. 26, no. 4, pp. 1516–1523, 2017.
- [22] T. M. Nguyen, A. H. Zaini, K. Guo, and L. Xie, “An ultra-wideband-based multi-UAV localization system in GPS-denied environments,” in *Proceedings of International Micro Air Vehicle Competition and Conference (IMAV)*, 2016, Beijing, China, Oct. 2016, pp. 56–61.

- [23] J. Faigl, T. Krajník, J. Chudoba, L. Přeučil, and M. Saska, “Low-cost embedded system for relative localization in robotic swarms,” in *Proceedings of 2013 IEEE International Conference on Robotics and Automation (ICRA)*, 2013, pp. 993–998.
- [24] J. Hightower, R. Want, and G. Borriello, “SpotON: An indoor 3D location sensing technology based on RF signal strength,” *Technical Report, University of Washington, Department of Computer Science and Engineering, Seattle, WA*, 2000. [Online]. Available: https://www.researchgate.net/publication/228541557_SpotON_An_Indoor_3D_Location_Sensing_Technology_Based_on_RF_Signal_Strength
- [25] L. M. Ni, Y. Liu, Y. C. Lau, and A. P. Patil, “LANDMARC: indoor location sensing using active RFID,” *Wireless Networks*, vol. 10, no. 6, pp. 701–710, 2004.
- [26] H. Mehmood, N. K. Tripathi, and T. Tipdecho, “Indoor positioning system using artificial neural network,” *Journal of Computer Science*, vol. 6, no. 10, pp. 1206–1212, 2010.
- [27] A. Kushki, K. N. Plataniotis, and A. N. Venetsanopoulos, “Intelligent dynamic radio tracking in indoor wireless local area networks,” *IEEE Transactions on Mobile Computing*, vol. 9, no. 3, pp. 405–419, 2010.
- [28] K. Guo, Z. Qiu, C. Miao, A. H. Zaini, C.-L. Chen, W. Meng, and L. Xie, “Ultra-wideband-based localization for quadcopter navigation,” *Unmanned Systems*, vol. 4, no. 01, pp. 23–34, 2016.
- [29] K. Atsuumi and M. Sano, “Indoor IR azimuth sensor using a linear polarizer,” in *Proceedings of 2010 IEEE International Conference on Indoor Positioning and Indoor Navigation (IPIN)*, 2010, pp. 1–5.
- [30] *Ambiplex*, 2011 (last accessed 7. April 2016). [Online]. Available: <http://www.ambiplex.com/>

- [31] *Microsoft Kinect*, 2011 (last accessed 7. April 2016). [Online]. Available: <http://www.xbox.com/en-SG/kinect/>
- [32] M. Rubenstein, C. Ahler, and R. Nagpal, “Kilobot: A low cost scalable robot system for collective behaviors,” in *Proceedings of 2012 IEEE International Conference on Robotics and Automation (ICRA)*, 2012, pp. 3293–3298.
- [33] M. Rubenstein, A. Cornejo, and R. Nagpal, “Programmable self-assembly in a thousand-robot swarm,” *Science*, vol. 345, no. 6198, pp. 795–799, 2014.
- [34] D. Vasisht, S. Kumar, and D. Katabi, “Decimeter-level localization with a single WiFi access point,” in *Proceedings of the 13th USENIX Symposium on Networked Systems Design and Implementation (NSDI’16)*, 2016, pp. 165–178.
- [35] A. Bachrach, S. Prentice, R. He, and N. Roy, “RANGE—Robust autonomous navigation in GPS-denied environments,” *Journal of Field Robotics*, vol. 28, no. 5, pp. 644–666, 2011.
- [36] A. J. Barry and R. Tedrake, “Pushbroom stereo for high-speed navigation in cluttered environments,” in *Proceedings of 2015 IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 3046–3052.
- [37] *Intel RealSense*, 2015 (last accessed 7. April 2016). [Online]. Available: <https://software.intel.com/en-us/realsense/r200camera>
- [38] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molisch, H. V. Poor, and Z. Sahinoglu, “Localization via ultra-wideband radios: a look at positioning aspects for future sensor networks,” *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 70–84, 2005.
- [39] M. R. Mahfouz, C. Zhang, B. C. Merkl, M. J. Kuhn, and A. E. Fathy, “Investigation of high-accuracy indoor 3-D positioning using UWB technology,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 56, no. 6, pp. 1316–1330, 2008.

- [40] A. Prorok, A. Arfire, A. Bahr, J. R. Farserotu, and A. Martinoli, "Indoor navigation research with the Khepera III mobile robot: An experimental baseline with a case-study on ultra-wideband positioning," in *Proceedings of 2010 IEEE International Conference on Indoor Positioning and Indoor Navigation (IPIN)*, 2010, pp. 1–9.
- [41] M. W. Mueller, M. Hamer, and R. D'Andrea, "Fusing ultra-wideband range measurements with accelerometers and rate gyroscopes for quadcopter state estimation," in *Proceedings of 2015 IEEE International Conference on Robotics and Automation (ICRA)*, 2015, pp. 1730–1736.
- [42] A. Ledergerber, M. Hamer, and R. D'Andrea, "A robot self-localization system using one-way ultra-wideband communication," in *Proceedings of 2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2015, pp. 3131–3137.
- [43] F. Zafari, A. Gkelias, and K. Leung, "A survey of indoor localization systems and technologies," *arXiv preprint*, 2017. [Online]. Available: <https://arxiv.org/abs/1709.01015>
- [44] S. Nadler, V. Soroka, O. Fuchs, R. Korenshtein, and E. Sonsino, "Presence zones for contextual location based services," *Innovations in Clouds, Internet and Networks*, 2008.
- [45] S. C. Ergen, "ZigBee/IEEE 802.15.4 Summary," *UC Berkeley, September*, vol. 10, pp. 1–37, 2004. [Online]. Available: <http://pages.cs.wisc.edu/~suman/courses/707/papers/zigbee.pdf>
- [46] A. Singer, M. Oelze, and A. Podkova, "Mbps experimental acoustic through-tissue communications: MEAT-COMMS," in *Proceedings of the 17th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2016, pp. 1–4.

- [47] Bemri. Visible Light Communication (VLC/Li-Fi) Systems. Accessed 03-Nov-2016. [Online]. Available: <http://bemri.org/visible-light-communication.html>
- [48] Intel. Wireless LAN Standards Study. [Online]. Available: <http://www.intel.com/content/dam/www/public/us/en/documents/case-studies/802-11-wireless-lan-standards-study.pdf>
- [49] M. Shaik. Ultra Wide-Band vs. Wi-Fi: A Study & Comparison of the Two Technologies. [Online]. Available: <http://www.academia.edu/4810093/Ultra-Wide-Band-vs.-Wi-Fi-A-Study-and-Comparison-of-the-two-technologies>
- [50] J. Luo and Q. Zhang, "Relative distance based localization for mobile sensor networks," in *Proceedings of 2007 IEEE Global Telecommunications Conference (GLOBECOM'07)*, 2007, pp. 1076–1080.
- [51] G. Mao, B. Fidan, and B. D. Anderson, "Wireless sensor network localization techniques," *Computer Networks*, vol. 51, no. 10, pp. 2529–2553, 2007.
- [52] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1067–1080, 2007.
- [53] N. B. Priyantha, A. Chakraborty, and H. Balakrishnan, "The cricket location-support system," in *Proceedings of the 6th ACM Annual International Conference on Mobile Computing and Networking*, 2000, pp. 32–43.
- [54] A. Savvides, C.-C. Han, and M. B. Strivastava, "Dynamic fine-grained localization in ad-hoc networks of sensors," in *Proceedings of the 7th ACM Annual International Conference on Mobile Computing and Networking*, 2001, pp. 166–179.
- [55] J. Djugash, S. Singh, G. Kantor, and W. Zhang, "Range-only SLAM for robots operating cooperatively with sensor networks," in *Proceedings of 2006*

- IEEE International Conference on Robotics and Automation (ICRA)*, 2006, pp. 2078–2084.
- [56] E. Olson, J. J. Leonard, and S. Teller, “Robust range-only beacon localization,” *IEEE Journal of Oceanic Engineering*, vol. 31, no. 4, pp. 949–958, 2006.
- [57] S. H. Dandach, B. Fidan, S. Dasgupta, and B. D. Anderson, “A continuous time linear adaptive source localization algorithm, robust to persistent drift,” *Systems & Control Letters*, vol. 58, no. 1, pp. 7–16, 2009.
- [58] G. Chai, C. Lin, Z. Lin, and M. Fu, “Single landmark based collaborative multi-agent localization with time-varying range measurements and information sharing,” *Systems & Control Letters*, vol. 87, pp. 56–63, 2016.
- [59] S. Wang, D. Gu, L. Chen, and H. Hu, “Single beacon-based localization with constraints and unknown initial poses,” *IEEE Transactions on Industrial Electronics*, vol. 63, no. 4, pp. 2229–2241, 2016.
- [60] P. Batista, C. Silvestre, and P. Oliveira, “Single range aided navigation and source localization: Observability and filter design,” *Systems & Control Letters*, vol. 60, no. 8, pp. 665–673, 2011.
- [61] G. Wang, J. Chen, and J. Sun, “Stochastic stability of extended filtering for non-linear systems with measurement packet losses,” *IET Control Theory & Applications*, vol. 7, no. 17, pp. 2048–2055, 2013.
- [62] J. Chen, G. Wang, and J. Sun, “Power scheduling for Kalman filtering over lossy wireless sensor networks,” *IET Control Theory & Applications*, vol. 11, no. 4, pp. 531–540, 2016.
- [63] P. Batista, C. Silvestre, and P. Oliveira, “GES source localization based on discrete-time position and single range measurements,” in *Proceedings of the 21st IEEE Mediterranean Conference on Control & Automation (MED)*, 2013, pp. 1248–1253.

- [64] G. Antonelli, A. Caiti, V. Calabro, and S. Chiaverini, “Designing behaviors to improve observability for relative localization of AUVs,” in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, 2010, pp. 4270–4275.
- [65] G. Indiveri, P. Pedone, and M. Cuccovillo, “Fixed target 3D localization based on range data only: A recursive least squares approach,” *IFAC Proceedings Volumes*, vol. 45, no. 5, pp. 140–145, 2012.
- [66] S. Martínez and F. Bullo, “Optimal sensor placement and motion coordination for target tracking,” *Automatica*, vol. 42, pp. 661–668, 2006.
- [67] M. Sun and K. Ho, “Optimum sensor placement for fully and partially controllable sensor networks: A unified approach,” *Signal Processing*, vol. 102, pp. 58–63, 2014.
- [68] X. Fang, W. Yan, F. Zhang, and J. Li, “Optimal sensor placement for range-based dynamic random localization,” *IEEE Geoscience and Remote Sensing Letters*, vol. 12, pp. 2393–2397, 2015.
- [69] D. Moreno-Salinas, A. M. Pascoal, and J. Aranda, “Optimal sensor placement for underwater positioning with uncertainty in the target location,” in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, 2011, pp. 2308–2314.
- [70] J. Perez-Ramirez, D. K. Borah, and D. G. Voelz, “Optimal 3-D landmark placement for vehicle localization using heterogeneous sensors,” *IEEE Transactions on Vehicular Technology*, vol. 62, pp. 2987–2999, 2013.
- [71] D. Moore, J. Leonard, D. Rus, and S. Teller, “Robust distributed network localization with noisy range measurements,” in *Proceedings of the 2nd ACM International Conference on Embedded Networked Sensor Systems*, 2004, pp. 50–61.

- [72] N. Trawny and S. I. Roumeliotis, “On the global optimum of planar, range-based robot-to-robot relative pose estimation,” in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, 2010, pp. 3200–3206.
- [73] R. Y. Kurazume, S. Nagata, and S. Hirose, “Cooperative positioning with multiple robots,” in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, 1994, pp. 1250–1257.
- [74] L. E. Navarro-Serment, C. J. Paredis, P. K. Khosla *et al.*, “A beacon system for the localization of distributed robotic teams,” in *Proceedings of the International Conference on Field and Service Robotics*, pp. 232–237.
- [75] A. Cornejo and R. Nagpal, “Distributed range-based relative localization of robot swarms,” in *Algorithmic Foundations of Robotics XI*. Springer, 2015, pp. 91–107.
- [76] G. Chai, C. Lin, Z. Lin, and W. Zhang, “Consensus-based cooperative source localization of multi-agent systems with sampled range measurements,” *Unmanned Systems*, vol. 2, no. 03, pp. 231–241, 2014.
- [77] Z. Han, K. Guo, L. Xie, and Z. Lin, “Integrated relative localization & leader-follower formation control,” *IEEE Transactions on Automatic Control*, to be published, 2018. DOI: 10.1109/TAC.2018.2800790.
- [78] B. Jiang, M. Deghat, and B. D. Anderson, “Simultaneous velocity and position estimation via distance-only measurements with application to multi-agent system control,” *IEEE Transactions on Automatic Control*, vol. 62, no. 2, pp. 869–875, 2017.
- [79] D. C. Montgomery, E. A. Peck, and G. G. Vining, *Introduction to Linear Regression Analysis*. John Wiley & Sons, 2012, vol. 821.
- [80] J. Berkson, “Estimation of a linear function for a calibration line; consideration of a recent proposal,” *Technometrics*, vol. 11, no. 4, pp. 649–660, 1969.

- [81] D. E. Manolakis, "Efficient solution and performance analysis of 3-D position estimation by trilateration," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 4, pp. 1239–1248, 1996.
- [82] Y. Zhou, "An efficient least-squares trilateration algorithm for mobile robot localization," in *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2009, pp. 3474–3479.
- [83] F. Thomas and L. Ros, "Revisiting trilateration for robot localization," *IEEE Transactions on Robotics*, vol. 21, no. 1, pp. 93–101, 2005.
- [84] W. Navidi, W. S. Murphy, and W. Hereman, "Statistical methods in surveying by trilateration," *Computational Statistics & Data Analysis*, vol. 27, no. 2, pp. 209–227, 1998.
- [85] C. T. Kelley, *Iterative Methods for Optimization*. Siam, 1999, vol. 18.
- [86] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Journal of Fluids Engineering*, vol. 82, no. 1, pp. 35–45, 1960.
- [87] E. Wan, R. Van Der Merwe *et al.*, "The unscented kalman filter for nonlinear estimation," in *Proceedings of IEEE Adaptive Systems for Signal Processing, Communications, and Control Symposium (AS-SPCC)*, 2000, pp. 153–158.
- [88] S. J. Julier, "The scaled unscented transformation," in *Proceedings of IEEE American Control Conference*, vol. 6, 2002, pp. 4555–4559.
- [89] M. Boutayeb, H. Rafaralahy, and M. Darouach, "Convergence analysis of the extended kalman filter used as an observer for nonlinear deterministic discrete-time systems," *IEEE Transactions on Automatic Control*, vol. 42, no. 4, pp. 581–586, 1997.
- [90] T. W. Anderson, T. W. Anderson, T. W. Anderson, and T. W. Anderson, *An Introduction to Multivariate Statistical Analysis*. Wiley New York, 1958, vol. 2.

- [91] L. Meier, P. Tanskanen, L. Heng, G. H. Lee, F. Fraundorfer, and M. Pollefeys, "Pixhawk: A micro aerial vehicle design for autonomous flight using onboard computer vision," *Autonomous Robots*, vol. 33, no. 1-2, pp. 21–39, 2012.
- [92] D. Mellinger, N. Michael, and V. Kumar, "Trajectory generation and control for precise aggressive maneuvers with quadrotors," in *Experimental Robotics*. Springer, 2014, pp. 361–373.
- [93] A. Björck, *Numerical Methods for Least Squares Problems*. Siam, 1996.
- [94] N. Levanon, "Lowest GDOP in 2-D scenarios," *IEE Proceedings-Radar, Sonar and Navigation*, vol. 147, no. 3, pp. 149–155, 2000.
- [95] J.-B. Hiriart-Urruty and C. Lemaréchal, *Fundamentals of Convex Analysis*. Springer Science & Business Media, 2012.
- [96] G. Indiveri, P. Pedone, and M. Cuccovillo, "Fixed target 3D localization based on range data only: A recursive least squares approach," *IFAC Proceedings Volumes*, vol. 45, no. 5, pp. 140–145, 2012.
- [97] A. Kushleyev, D. Mellinger, C. Powers, and V. Kumar, "Towards a swarm of agile micro quadrotors," *Autonomous Robots*, vol. 35, no. 4, pp. 287–300, 2013.
- [98] M. Saska, T. Baca, J. Thomas, J. Chudoba, L. Preucil, T. Krajník, J. Faigl, G. Loianno, and V. Kumar, "System for deployment of groups of unmanned micro aerial vehicles in GPS-denied environments using onboard visual relative localization," *Autonomous Robots*, vol. 41, no. 4, pp. 919–944, 2017.
- [99] M. Coppola, K. McGuire, K. Y. Scheper, and G. C. de Croon, "On-board bluetooth-based relative localization for collision avoidance in micro air vehicle swarms," *arXiv preprint*, 2016. [Online]. Available: <https://arxiv.org/abs/1609.08811>

- [100] K. McGuire, G. de Croon, C. De Wagter, K. Tuyls, and H. Kappen, “Efficient optical flow and stereo vision for velocity estimation and obstacle avoidance on an autonomous pocket drone,” *IEEE Robotics and Automation Letters*, vol. 2, no. 2, pp. 1070–1076, 2017.
- [101] Y. Zhang, T. Wang, Z. Cai, Y. Wang, and Z. You, “The use of optical flow for UAV motion estimation in indoor environment,” in *Proceedings of IEEE Chinese Guidance, Navigation and Control Conference (CGNCC)*, Aug 2016, pp. 785–790.
- [102] Y. Su and J. Huang, “Two consensus problems for discrete-time multi-agent systems with switching network topology,” *Automatica*, vol. 48, no. 9, pp. 1988–1997, 2012.
- [103] Z.-P. Jiang and Y. Wang, “Input-to-state stability for discrete-time nonlinear systems,” *Automatica*, vol. 37, no. 6, pp. 857–869, 2001.
- [104] J. Hu and Y. Hong, “Leader-following coordination of multi-agent systems with coupling time delays,” *Physica A: Statistical Mechanics and its Applications*, vol. 374, no. 2, pp. 853–863, 2007.
- [105] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge university press, 2012.
- [106] B. Dewberry and M. Einhorn, “Indoor aerial vehicle navigation using ultra wideband active two-way ranging,” in *Proceedings of XPONENTIAL*, 2016.
- [107] Y. Su, C. Zheng, D. Sun, and B. Y. Duan, “A simple nonlinear velocity estimator for high-performance motion control,” *IEEE Transactions on Industrial Electronics*, vol. 52, no. 4, pp. 1161–1169, 2005.
- [108] B. Dewberry and W. Beeler, “Increased ranging capacity using Ultrawideband direct-path pulse signal strength with dynamic recalibration,” in *Proceedings of IEEE/ION Position Location and Navigation Symposium (PLANS)*. IEEE, 2012, pp. 1013–1017.

- [109] M. Deghat, B. D. Anderson, and Z. Lin, “Combined flocking and distance-based shape control of multi-agent formations,” *IEEE Transactions on Automatic Control*, vol. 61, no. 7, pp. 1824–1837, 2016.
- [110] B. Jiang, M. Deghat, and B. D. O. Anderson, “Simultaneous velocity and position estimation via distance-only measurements with application to multi-agent system control,” *IEEE Transactions on Automatic Control*, *arXiv preprint*, 2015. [Online]. Available: <http://arxiv.org/abs/1411.3841>