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A thesis submitted to the Nanyang Technological University, Singapore

in fulfilment of the requirements for the degree of

Doctor of Philosophy

2018

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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This thesis contains material from 1 paper(s) published in the following peer-reviewed journal(s) where I was the first and/or corresponding author.

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- I collected the data from literature. The model calibration and evaluation were done by me. I prepared the manuscript drafts.
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ABSTRACT

The relationship between water content and soil suction, referred to as the soil-water characteristic curve (SWCC), plays a central role in understanding the behaviour of an unsaturated soil. The SWCC is used to estimate the water coefficient of permeability, shear strength, volume change and aqueous diffusion of unsaturated soils. However, there are still gaps in the current understanding of the SWCC. This thesis investigates the effects of grain-size distribution and hysteresis on the SWCC.

Pore size distribution (PSD) governs the SWCC. There is a correlation between PSD and grain size distribution (GSD). Hence, GSD has often been used to estimate SWCC. To obtain the parameters of the GSD, it is more convenient if the GSD can be described by a mathematical equation. In this study, the equations for GSD were first reviewed and an improved GSD equation was proposed and used in this study.

Sigmoidal or unimodal SWCC is common. More recently, it is recognised that bimodal SWCC is present for some soils. The SWCC is usually determined by laboratory tests at discrete suction levels. If some important suction levels are not tested, the SWCC data could be wrongly interpreted as a unimodal SWCC when it should be a bimodal SWCC. A bimodal SWCC is corollary to dual porosity. Dual porosity can arise due to bimodal GSD. Bimodal SWCC can be due to bimodal GSD or soil aggregations due to reconstitution. This study only addresses bimodal GSD.. This study developed a classification tree to distinguish bimodal GSD soils with bimodal SWCC from bimodal GSD soils with unimodal SWCC. The classification tree can serve as a guide to determine the discrete suction levels in laboratory determination of SWCC so

that critical suction levels are not missed from the test to avoid wrong interpretation of the SWCC. Based on the study, recommendations were made to change the suction levels proposed in the standard for determination of SWCC. It is recommended that suction levels for unimodal and bimodal SWCCs are different with bimodal SWCC having more suction levels than unimodal SWCC.

The estimation of SWCC from other easily, routinely, or cheaply measured properties, such as GSD, index properties and dry density or void ratio, is performed with pedotransfer functions (PTFs). The PTFs for estimating SWCC can be divided into two types: point-estimation and parametric-estimation. A number of point-estimation PTFs has been proposed to estimate the water contents for suctions of 4(or 3), 10, 33, 100 and 1500 kPa for soils of a specific region soil. It is "expected" that the PTF will perform poorly when applied to soils of other regions. This thesis examines the relevancy and usefulness of PTFs in unsaturated soil mechanics by evaluating numerous PTFs using data collated from the literature. The data covers a wide region so that specificity bias of the PTFs is removed in the evaluation. Based on the evaluation, a simple estimation method for unimodal SWCC was proposed. The simple estimation method was demonstrated to work well.

In unsaturated soil mechanics, researchers have developed one-point methods to estimate SWCC. One-point methods are actually parametric-estimation PTFs used together with one-point measurement of the SWCC. One-point methods reduce the wait time to obtain an estimate of the SWCC. However, one-point measurement of the SWCC is expensive and time-consuming. This study developed a zero experimental point method by eliminating the need of using a one-point measurement of the SWCC.

The one-point measurement of SWCC is substituted using two point-estimation PTFs. The zero experimental point method was demonstrated to work as well as the one-point method but is more advantageous as no one point measurement of the SWCC is needed.

It is widely recognised that the water content of a soil at a certain suction is not unique. Water content of a soil on the wetting path is always lower than on the drying path for the same suction. This is referred to as hysteresis of SWCC. The hysteretic nature of SWCCs has been known for a long time, but in many routine engineering applications the drying SWCC is often used since the measurement of a complete hysteretic SWCC is extremely time consuming and costly. Few hysteresis models have been proposed for SWCC. But these models require data on the wetting SWCC for model calibration, which increases the difficulty of applying unsaturated soils in engineering practice. In this study, a relatively simple model, which requires limited data and no data on the wetting SWCC for estimating the hysteretic SWCC is proposed. The proposed SWCC hysteresis model outperformed the other SWCC hysteresis models.

Bimodal SWCCs are associated with dual-porosity soils. Determination of bimodal SWCC in the laboratory is costly and time-consuming. Few bimodal SWCC estimation parametric-estimation PTFs have been proposed but the existing bimodal SWCC estimation PTFs are very complicated. In this research, a parametric-estimation PTF was proposed to estimate bimodal SWCC and was shown to provide good agreement with experimental data from the literature.

No hysteresis model has been proposed for bimodal SWCC. In this research, a model to estimate the hysteresis of bimodal SWCC was proposed based on the

hysteresis model proposed for unimodal SWCC in this research. Good agreement of the estimation with experimental data was shown for the limited experimental data available in the literature.

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LIST OF SYMBOLS

a	Curve-fitting parameter for SWCC (kPa)
a_0, a_i	Fitting parameter for SWCC (kPa)
$a_1, a_2, a_3, a_4, a_5, a_6, a_7$	Constant
a_g	Curve-fitting parameter related to AEV (kPa)
a_{GSD}	Fitting parameter for the cumulative GSD function
a_{gr}, a_{bi}, j_{bi}	Fitting parameter related to unimodal curve initial breaking point, bimodal curve initial break point and bimodal curve second break point, respectively (kPa)
a_{id}, a_{bw}, a_{bd}	Fredlund and Xing (1994) equation fitting parameters for initial drying curve/ boundary wetting curve/ boundary drying curve (kPa)
A	Area (mm ²)
b	Curve-fitting parameter for SWCC
b_1, b_2	Constant
b_d/ b_w	Parameters for drying /wetting curves for Feng and Fredlund (1999) equation
c'	Effective cohesion of soils (kPa)

C	Shape constant of flow system
$c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7$	Regression parameters
C_c	Coefficient of uniformity of soil
c_d/ c_w	Parameters for drying /wetting curves for Feng and Fredlund (1999) equation
c_i	The curvature parameters for the joint of segment i and segment $i-1$
C_u	Coefficient of curvature of soil
d	Size of soil grains (mm) / index of agreement
d_1	Largest size of the coarsest fraction of the soil (mm)
d_d/ d_w	Parameters for drying /wetting curves for Feng and Fredlund (1999) equation
d_e	Effective grain diameter of soil (mm)
d_i	Grain size corresponding to the joint between GSD curve i^{th} and $(i-1)^{th}$ segments (mm)
d_m	The minimum allowable size particle (mm)
d_{rgr}, d_{rbi}	Parameter related to the number of fines in soil
D	Parameter of Pham et al. (2005) hysteresis model

$D_{10}, D_{30}, D_{50},$ D_{60}, D_{90}	Soil particles sizes corresponding to 10, 30, 50, 60 and 90 percent passing on cumulative GSD curve, respectively (mm)
D_{mean}	Mean grain size of soil (mm)
D_{median}	Median grain size (mm)
$\frac{d\phi}{dx}$	Hydraulic gradient
e	Void ratio or Euler's number
f	Grain-size distribution index
g	Acceleration due to gravity (m/s^2)
G_s	Specific gravity of soil solids (g/cm^3)
h	Pressure head/ hydraulic head (cm or m)
H_d	Degree of hysteresis
L	Length (m)
k	Water coefficient of permeability (m/s)
k_w	Water coefficient of permeability of unsaturated soil (m/s)
k_r	Relative water coefficient of permeability, k_w/k_s
k_s	Water coefficient of permeability of soils at saturated state (m/s)
K	Intrinsic Permeability (m/s)
K_r	Relative hydraulic conductivity

$m/ m_1/ m_2$	SWCC fitting parameter
m_{gr}, m_{bi}, l_{bi}	Fitting parameter related to shape of the curve
m_{id}, m_{bw}, m_{bd}	Fredlund and Xing (1994) equation fitting parameters for initial drying curve/ boundary wetting curve/ boundary drying curve
M_s	Mass of soil solids (g)
M_w	Mass of water (g)
n	SWCC fitting parameter/ Soil porosity
$n_1/n_2/n_i$	SWCC fitting parameter
n_g	Curve-fitting parameter related to curve slope at inflection point on SWCC
n_{GSD}	Fitting parameter for cumulative GSD function
n_{gr}, n_{bi}, k_{bi}	Fitting parameter related to unimodal curve's steepest slope, bimodal curve's steepest slope and bimodal curve's second steepest slope, respectively
n_i	The number of spherical particles at i^{th} fraction
n_{id}, n_{bw}, n_{bd}	Fredlund and Xing (1994) equation fitting parameters for initial drying curve/ boundary wetting curve/ boundary drying curve
nm	Nanometer
n_s	Structural porosity

p	Constance
P	Pressure (kPa)
P_{200}	Percentage of soil particles passing 200 μm sieve (%)
P_c	Capillary pressure (kPa)
P_d	Cumulative grain-size distribution function
Q	Volume of water charged (mm^3)
r	Pore radius of soils (mm)
$\bar{r}$	Normalised neck pore size (mm)
R	Grain size radius of soils (mm) / Parameter for Pham et al. (2005) hysteresis model
R^2	Coefficient of determination
R_{max}	The maximum pore size in the domain (mm)
R_{min}	The minimum pore size in the domain (mm)
R_i	Ramp function for i^{th} segment of SWCC
S	Degree of saturation
S_i	Slopes of the i^{th} segment of the SWCC
S_e	Effective degree of saturation
S_r	Residual degree of saturation

t	Time (s)
T_m	Surface tension of the air-mercury interface (mN/m)
T_s	Surface tension at the air-water interface (mN/m)
$ T_v^A $	The cardinality of the set of instances for which the attribute A has the value of v
u_a	Pore-air pressure (kPa)
u_m	Mercury pressure (kPa)
u_w	Pore-water pressure (kPa)
$u_a - u_w$	Matric suction (kPa)
$u_m - u_a$	Net mercury pressure (kPa)
$\bar{v}$	Average flow velocity (m/s)
V	Total volume of soil specimen or original volume of soil specimen (mm ³)
V_s	Volume of soil solids (mm ³)
V_v	Volume of voids (mm ³)
V_w	Volume of water (mm ³)
w	Gravimetric water content or sub-curve weighting factors in GSD curve-fitting (g/g)

$w_{s,i}$	Soil particle weight at i^{th} fraction
w_r	Gravimetric residual water content (g/g)
w_s	Gravimetric water content at 1 kPa soil suction (g/g)
w_b	Gravimetric water content at air-entry value (bubble pressure) (g/g)
x	Variable
y	The value of the polynomial estimated
$Y/Y/$	The logarithmic scale span between the major dominant grain size and the minor dominant grain size at PSD/ absolute value of Y
α	Scaling parameter
α_d, α_w	Parameters for drying and wetting curves for van Genuchten (1980) equation (kPa^{-1})
α_1	Contact angle for soil and water (degree)
α_m	Contact angle for mercury and soil (degree)
Δg_1	The coarsest fraction weightage percentage
$\Delta \theta_{max}$	Maximum difference of water content for drying and wetting curve (mm^3/mm^3)
θ	Volumetric water content (mm^3/mm^3)
$\bar{\theta}$	Average value of the volumetric water content (mm^3/mm^3)

θ'	Derivative of the SWCC equation
θ_a	Residual volumetric air content (mm ³ /mm ³)
θ_d	Volumetric water content for drying curve (mm ³ /mm ³)
θ_r	Residual volumetric water content (mm ³ /mm ³)
θ_s	Saturated volumetric water content (mm ³ /mm ³)
$\theta_{s,bw}$	Volumetric water content at zero suction at boundary wetting curve (mm ³ /mm ³)
θ_w	Volumetric water content for wetting curve (mm ³ /mm ³)
θ	Normalised volumetric water content
θ_d	Dimensionless volumetric water content, equal to θ/θ_s
λ	Pore-size distribution index
v	Related to shape of grain constant
π	Number, 3.14159265357982
ρ	Density (g/cm ³)
$\bar{\rho}$	Normalised body pore size
ρ_d	Dry density of soil (g/cm ³)
ρ_w	Density of water (g/cm ³)
ρ_s	Density of soil particles (g/cm ³)

σ	Total stress
σ'	Effective stress
σ_1	Major principle stress
σ_3	Minor principle stress
σ_r	The standard deviation of $\ln(r)$
ν	Kinematic coefficient of viscosity (Pa·m)
ψ	Soil suction (kPa)
ψ_a	Air-entry value (kPa)
ψ_d, ψ_w	Soil suction for drying/ wetting curve (kPa)
ψ_{in}	Soil suction at inflection point of SWCC (kPa)
ψ_m	Median soil suction (kPa)
ψ_{mean}	Mean soil suction (kPa)
ψ_r	Residual soil suction (kPa)
$\psi_{r,d}, \psi_{r,w}$	Residual soil suction for boundary drying/ wetting curve (kPa)

LIST OF ABBREVIATIONS

AASHTO	American Association of State Highway and Transportation Officials
AEV	Air-entry value (kPa)
AE	Air entrapment
AIC	The Akaike's information criterion (Akaike, 1973)
ASTM	American Society for Testing and Materials
BDC	Boundary drying curve
BS	British Standard
BWC	Boundary wetting curve
CA	Carducci et al. (2011) bimodal SWCC equation
CART	The method of classification and regression tree
Cl	Clay content (%)
COC	Coefficient of correlation
GDI	Gini's Diversity Index
GR	Grain ratio
GSD	Grain-size distribution
IDC	Initial drying curve
LI	Li et al. (2014) bimodal SWCC equation
LI ^s	Simplified Li et al. (2014) bimodal SWCC equation
MAE	Mean absolute error

MaP	Major peak of grain-size by percentage
MiP	Minor peak of grain-size by percentage
MIP	Mercury Intrusion Porosimetry
MIT	Massachusetts Institute of Technology, USA
MLR	Multiple linear regression
MSE	Mean of squared error
NTU	Nanyang Technological University, Singapore
OM	Organic content (%) in soil
PDF	Probability density function for soil grain-size distribution
PI	Plasticity index of soil
PSD	Pore-size distribution
PTF	Pedotransfer function
RMSE	Root mean squared error
SD	Standard deviation
Sa	Sand content (%)
SA	Satyanaga et al., (2013) equation
Si	Silt content (%)
SSE	Sum of squares errors
SSR	Sum of the squared residuals
SST	Sum of squares total
SWCC	Soil-water characteristic curve
RMSE	Root mean squared error

UNSODA	Unsaturated Soil Hydraulic Database (Nemes, et al., 2001)
USBR	United States Bureau of Reclamation
USCS	Unified soil classification system
USDA	United States Department of Agriculture
WL	Wijaya and Leong (2016) bimodal SWCC equation
ZC	Zhang and Chen (2005) bimodal SWCC equation

CHAPTER 1

INTRODUCTION

1.1 Background

Unsaturated soil mechanics has received widespread attention across geotechnical engineering communities over the past few decades. The soil-water characteristic curve (SWCC) is a very important constitutive relationship in unsaturated soil mechanics. Laboratory studies showed that there is a relationship between SWCC and the properties of unsaturated soils (Fredlund and Rahardjo, 1993a; Leong and Rahardjo, 1997a). The SWCC is related to the shear strength of unsaturated soils (Fredlund et al., 1978; Vanapalli et al., 1996; Wulfsohn et al., 1998; Goh et al., 2010). The unsaturated permeability function of soil is often estimated using the SWCC and the saturated water coefficient of permeability (Wessolek et al., 1994; Leong and Rahardjo, 1997a; Børgesen and Schaap, 2005; Fodor et al., 2011). The volume change and aqueous diffusion of unsaturated soils are related to the SWCC (Fredlund and Morgenstern, 1976; Pham and Fredlund, 2011; Choudhry et al., 2014; Bea et al., 2011). Hence, SWCC plays a central role in understanding the behaviour of an unsaturated soil.

The SWCC is a relationship of water content of soil with matric suction. The water content can be expressed in terms of either gravimetric water content, volumetric water content or degree of saturation (Fredlund, 2006). Typical SWCCs are shown in Figure 1.1. Recently, research attention is also placed on bimodal SWCC as shown in Figure 1.2.

Figure 1.1 Typical soil-water characteristic curves for sandy soil, silty soil and clayey soil (modified from Fredlund and Xing, 1994)

Figure 1.2 Bimodal SWCC

The SWCC is usually determined by laboratory tests. Often, SWCCs are determined from such tests at discrete suction levels. Missing some important suction levels may mean that the SWCC could be wrongly interpreted as a unimodal SWCC when it should be a bimodal SWCC.

For unimodal SWCC, the continuous SWCC is determined by using a SWCC equation to fit the measured discrete SWCC points. Leong and Rahardjo (1997b) have reviewed the commonly used unimodal SWCC equations and concluded that the Fredlund and Xing (1994) and van Genuchten (1980) equations performed best among the reviewed unimodal SWCC equations.

Bimodal SWCCs are associated with the dual-porosity soils (Satyanaga et al., 2013; Miguel and Bonder, 2012; Zhang and Chen, 2005; Li et al., 2014). Several bimodal SWCC equations have been proposed (Zhang and Chen, 2005; Carducci et al., 2011; Satyanaga et al., 2013; Li et al., 2014; and Wijaya and Leong, 2016). Compared with unimodal SWCC equations, bimodal SWCC equations have more unknown parameters. Hence, the determination of the parameters for bimodal SWCC equations is more complicated and time-consuming. Sometimes, numerical problem may occur during the optimising process for curve fitting as the equation has too many unknown parameters. It is necessary to constrain the parameters of bimodal SWCC equations to avoid numerical problem during curve fitting and to provide reasonable fit to the bimodal SWCC data.

Laboratory determination of SWCC is time-consuming and labour intensive. Hence, researchers have developed procedures to indirectly estimate the SWCC using soil parameters that can be easily determined in the laboratory, such as grain-size

distribution, index properties and dry density or void ratio. In soil science, these are known as pedotransfer functions (PTFs). Generally, there are two major types of PTF: point-estimation (determining the water content at one suction level), and parametric-estimation (determining the parameters of a SWCC equation by regression, neural networks or other methods). The major problem of these two approaches is that each estimation method was developed for soils of a specific region. It is "expected" that the estimation method will perform poorly when applied to soils of other regions.

Existing parametric-estimation PTFs focus on unimodal SWCC. Hence, parametric-estimation PTFs for unimodal SWCC are not capable of estimating bimodal SWCCs (Li et al., 2014). Zhang and Chen, (2005) and Satyanaga et al. (2013) proposed parametric-estimation PTFs to estimate the bimodal SWCC from grain-size distribution (GSD) and other routinely measured soil properties. But the proposed PTFs are very complicated to use.

A complete SWCC includes drying, wetting and scanning curves. The drying curve is different from the wetting curve, giving rise to hysteresis. Commonly, only the drying curve of the SWCC is determined. However, many problems (e.g., slope failure) are initiated by a soil wetting process (e.g., infiltration of rainwater). The wetting curves of SWCCs are more difficult to determine than the drying curves. Hence, indirect estimation of the wetting SWCC is desirable. As soils undergo many drying and wetting cycles due to climatic condition, the study of the hysteretic behaviour of SWCC is also very important. Current available hysteresis models require SWCC measurement of the wetting curve, which is both challenging and costly.

Existing hysteresis models of SWCC mainly focused on unimodal SWCC. Currently, there is no hysteresis model for the bimodal SWCC. It is necessary to develop the hysteresis model for bimodal SWCC.

1.2 Objective and Scope of Research

The objective of this research is to study the effects of grain-size distribution and hysteresis on SWCC.

The scope of the research includes:

- (i) Reviewing the general behaviour of unsaturated soils as well as the background concepts of unsaturated soils.
- (ii) Reviewing the existing grain-size distribution (GSD) equations and proposing formulations for unimodal and bimodal GSD.
- (iii) Distinguishing unimodal SWCC from bimodal SWCC based on GSD for planning of the SWCC test.
- (iv) Reviewing point-estimation pedotransfer functions (PTFs) to determine the usefulness of the point-estimation PTFs in unsaturated soil mechanics.
- (v) Comparing and evaluating commonly used parametric-estimation pedotransfer functions (PTFs) using data collated from a wide region to remove the specificity bias of the PTFs.
- (vi) Developing a model for estimating unimodal SWCC from basic soil properties and comparing the proposed model with other parametric-estimation PTFs.
- (vii) Developing a hysteresis model to estimate unimodal SWCC boundary wetting curve from initial/ boundary drying curve.

- (viii) Comparing and evaluating existing bimodal SWCC equations and developing a model for bimodal SWCC estimation.
- (ix) Developing a hysteresis model to estimate bimodal SWCC boundary wetting curve from initial/ boundary drying curve.

1.3 Structure of the Thesis

Chapter 1 summarises the background, the scope of the research and the structure of the thesis.

Chapter 2 reviews the literature on unsaturated soils, unimodal and bimodal SWCC equations, PTFs using GSD to estimate SWCC, relationship between GSD and PSD, and hysteresis of SWCC.

Chapter 3 develops equations to best fit the bimodal and unimodal GSD and to compare with other GSD equations.

Chapter 4 develops a classification tree to distinguish bimodal GSD soils with bimodal SWCC from bimodal GSD soils with unimodal SWCC. Recommendation on suction levels for SWCC tests to obtain unimodal and bimodal SWCCs is made for the test methods in ASTM 6836-16 (2016).

Chapter 5 evaluates point-estimation PTFs for various suctions and recommends an ensemble of point-estimation PTFs as a simple method of estimating unimodal SWCC that can be applied in unsaturated soil mechanics.

Chapter 6 proposes a zero experimental point method based on Chin et al. (2010) one-point method to estimate unimodal SWCC. A hysteresis model for unimodal SWCC is also developed in this chapter.

Chapter 7 evaluates the existing bimodal SWCC equations, develops a method to estimate bimodal SWCC and a model to estimate the hysteresis of bimodal SWCC.

Chapter 8 concludes the research and provides recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Unsaturated soils

2.1.1 Unsaturated soils in nature

The soil condition near the ground surface is affected by the climate. The climate and the variation of the groundwater table position directly control the moisture condition of the soils. Generally, soils below the groundwater table are in saturated condition while soils above the groundwater table are in unsaturated condition.

2.1.2 Phases of unsaturated soil

Unsaturated soil is commonly referred to as a three-phase system (i.e., soil solid, air and water). However, Fredlund and Morgenstern (1977) suggested that the air-water interface (also known as contractile skin) can be treated as the fourth independent phase in an unsaturated soil system. The air-water interface plays an important role from the standpoint of stress state consideration (Terzaghi, 1936). The air-water interface, whose properties are different from ordinary water properties, is like a very thin membrane. Adopting the air-water surface as the fourth phase assists in understanding the stress state variables for unsaturated soil (Fredlund and Morgenstern, 1977). Considering that the volume of the air-water interface is small, it is possible to ignore the fourth phase and to consider the unsaturated soil as a three-phase system from the standpoint of the volume-mass relation (Fredlund, 2006).

The air-water interface possesses a property called surface tension. The surface tension, which causes the air-water surface to behave like an elastic membrane, is the result of the intermolecular forces acting on molecules in the air-water interface. Using Kelvin's capillary model (Equation 2.1), capillary pressure (also known as matric suction) can be calculated from surface tension and radius of curvature of air-water surface:

$$u_a - u_w = \frac{T_s}{R_1 + R_2} \quad (2.1)$$

where u_w = water pressure; u_a = air pressure; T_s = surface tension; and R_1, R_2 = in each of the axes that are parallel to air-water surface.

2.1.3 Stress-state variables for unsaturated soil

The single-valued effective stress-state variable, σ' , controls the mechanical behaviour of saturated soils (Terzaghi, 1936). The effective stress σ' is the difference between total stress σ and pore-water pressure u_w . The effective stress concept provides a fundamental basis for understanding saturated soil mechanics and leads to similar formulations for unsaturated soil using stress-state variables to describe the mechanical behaviour of unsaturated soils.

Biot (1941) proposed to use two stress-state variables, effective stress ($\sigma - u_w$) and pore-water pressure u_w , to model the stress-strain relationship for unsaturated soil consolidation. He recognises that the effects for total stress change and pore-water pressure change should be separated to describe the constitutive behaviour of unsaturated soil. Coleman (1962) suggested using three stress-state variables: net normal pressure ($\sigma_1 - u_a$), net confining pressure ($\sigma_3 - u_a$) and matric suction ($u_w - u_a$) to formulate the relations for volume change of unsaturated soils. Bishop and

Blight (1963), Matyas and Radhakrishna (1968) and Barden et al. (1969) adopted the use of two independent stress-state variables, net normal stress ($\sigma - u_a$) and matric suction ($u_a - u_w$), for unsaturated soils. Fredlund and Morgenstern (1977) established the theoretical basis and provided the justification to use two independent stress-state variables for unsaturated soils. They concluded that the combination of ($\sigma - u_a$) and ($u_a - u_w$) separate the effects caused by normal stress and pore-water pressure, and should be used to characterise the properties of shear strength and volume change of unsaturated soils.

2.2 Soil-water Characteristic Curve (SWCC)

2.2.1 Introduction

The soil-water characteristic curve (SWCC) is also known as soil-water retention curve, soil-water release curve, soil-moisture retention curve or capillary pressure curve. It is an essential function to model hydraulic and mechanical properties of unsaturated soils.

The SWCC describes the relationship between suction and water content of the soil. The suction used for SWCC is usually the matric suction, $\psi = u_a - u_w$, but occasionally, total suction is used as well. But at suctions greater than 1500 kPa, matric suction and total suction are generally equivalent (Fredlund and Xing, 1994) if there is no significant salt content in the pore water. For water content, either volumetric water content θ_w , gravimetric water content w , or degree of saturation S , as defined in Equations 2.2, 2.3 and 2.4, respectively, can be used.

$$\theta_w = \frac{V_w}{V_v + V_s} \text{ or } \theta_w = \frac{V_w}{V} \quad (2.2)$$

where θ_w = volumetric water content; V_w = volume of water; V_v = volume of voids; V_s = volume of soil solids; and V = total volume of soil specimen or original volume of soil specimen.

$$w = \frac{M_w}{M_s} \quad (2.3)$$

where w = gravimetric water content; M_w = mass of water; and M_s = mass of soil solids.

$$S = \frac{V_w}{V_v} \quad (2.4)$$

where S = degree of saturation.

Volumetric water content θ_w is most commonly used in soil science and gravimetric water content w is generally used in geotechnical engineering. The advantages and disadvantages are summarised in Table 2.1. In this thesis, water content is assumed to be volumetric water content unless otherwise indicated. The choice of using volumetric water content is because pedotransfer functions developed in soil science are a major part of this thesis. Researchers proposed many equations for SWCCs which will be discussed in Section 2.2.2.

A complete SWCC includes drying, wetting and scanning curves. The drying curve is different from the wetting curve, which may be explained by hysteresis. Hysteresis of SWCC will be presented in Section 2.5. The hysteretic nature of SWCCs has been known for a long time but in many routine engineering applications, the SWCC is often assumed to be non-hysteretic since the measurement of a complete hysteretic SWCC is extremely time consuming and costly. Most models proposed in the literature require,

directly or indirectly, at least two boundary curves (i.e., boundary drying curve and boundary wetting curve) to estimate the scanning curves. Consequently, it is important to establish an approach to estimate the boundary wetting curve from the boundary drying curve, and vice versa. A relatively simple model, which requires limited data and no data on the boundary wetting curve to estimate a reasonably accurate boundary wetting curve will be developed.

Table 2.1 Advantages and disadvantages of variable of water content designation used in SWCC (modified from Fredlund, 2006)

Variables	Advantages	Disadvantages
volumetric water content, θ_w	Commonly used in databases of soil science and agronomy; is the basic form that emerges in the derivation of transient seepage and water storage in unsaturated soils.	Requires volume measurement at each soil suction; not widely adopted in engineering practice. Does not yield correct air-entry value (AEV) when the volume of the soil changes during drying
Gravimetric water content, w	Commonly used in geotechnical engineering practice; requires only weight measurement to calculate.	Does not yield the correct air-entry value (AEV) when the soil changes volume during drying; does not allow differentiation between change in volume and degree of saturation.
Degree of saturation, S	Clearly defines the AEV; is the variable which most controls unsaturated soil property functions.	Requires volume change to calculate.

Typical drying and wetting curves are illustrated in Figure 2.1. In the drying process, the air-entry value (AEV) is the matric suction where air starts to enter the largest pores in the soil (Brooks and Corey, 1966). When the water phase starts to become

discontinuous, the SWCC reaches the residual state (Yang et al., 2004). The water content and soil suction at the residual state are residual water content θ_r and residual soil suction ψ_r , respectively. In the wetting process, the water-entry value (WEV) is defined as the matric suction at which water first enters the pores of the soils. After the WEV, as matric suction decreases, the water content increases significantly until water content reaches the air entrapment (AE), which is defined as the air content trapped in the soil in an occluded form and not replaceable by water. At each suction level, the wetting curve's water content is lower than the drying curve's water content.

Figure 2.1 Idealised complete SWCC

Generally, most measured SWCCs only show the drying curve as the wetting curve is more difficult to obtain. Soil suction can be matric suction, osmotic suction or total suction (Fredlund, 2006). Soil suction can range from 0 to 1 GPa (Fredlund and Xing, 1994), and hence the laboratory results for SWCCs are plotted on a logarithmic scale of

soil suction. The 1 GPa is from consideration of thermodynamic principles (Fredlund and Xing, 1994), and suctions smaller than 1 GPa have also been suggested (See Lu and Khorshidi, 2015).

Typical drying SWCCs for soils ranging from sands to clays are sigmoid as shown in Figure 1.1. The saturated water content and AEV generally increase with the plasticity of the soil and the shape of the SWCC is also affected by the stress history and the soil structure (Fredlund, 2006).

Recently, it is recognised that besides sigmoidal SWCCs, which are unimodal, there are other forms of SWCCs namely bimodal SWCCs and multimodal SWCCs, which are shown in Figure 2.2 and Figure 2.3, respectively. In this study, multimodal SWCCs are grouped as bimodal SWCCs. Bimodal SWCC is a consequence of dual-porosity soils. Pores in dual-porosity soils are governed by coarse grains and fine grains, which form large pores (macro-pores) and small pores (micro-pores), respectively (Burger and Shackelford, 2001; Zhang and Chen, 2005). Dual porosity soils occur as the pores formed by coarse grains are not completely filled by the fine grains (Zhang and Chen, 2005). A bimodal SWCC is corollary to dual porosity. Dual porosity can arise due to bimodal GSD. Bimodal SWCC can be due to bimodal GSD or soil aggregations due to reconstitution. This study only addresses bimodal GSD. The bimodal SWCC discussed in the literature is generally a drying bimodal SWCC, as a wetting bimodal SWCC is seldom found in the literature.

Figure 2.2 Bimodal SWCC for Soil 276 (data from Andersson and Wiklert, 1972)

Figure 2.3 Multimodal SWCC for Soil 283 (data from Andersson and Wiklert, 1972)

2.2.2 Unimodal SWCC equations

Soil-water characteristic curves (SWCCs) are usually determined from laboratory tests at discrete suction levels. Every suction level and its corresponding water content give one point on the SWCC. To describe the continuous SWCC, a fitting equation is needed. There are two groups of SWCC equations classified by the modality of the SWCC, namely unimodal and bimodal.

Unimodal SWCC equations are more commonly found in the literature. Leong and Rahardjo (1997a) suggested a generic equation shown as Equation 2.5, which can derive almost all proposed unimodal SWCC equations.

$$a_1 \theta^{b_1} + a_2 e^{a_3 \Theta^{b_1}} = a_4 \psi^{b_2} + a_5 e^{a_6 \psi^{b_2}} + a_7 \quad (2.5)$$

where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, b_1$ and b_2 are constants; ψ = soil suction; and Θ = normalised volumetric water content.

Three equations, Brooks and Corey (1964), van Genuchten (1980) and Fredlund and Xing (1994), are commonly used in unsaturated soil mechanics. The Brooks and Corey (1964) equation (Equation 2.6) describes the soil drying process for suction greater than AEV to study moisture movement in agricultural soils:

$$\Theta = \left(\frac{\psi_a}{\psi} \right)^\lambda \quad (2.6)$$

where $\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}$, normalised water content; θ_r = residual water content; ψ_a = air-entry value (or bubbling pressure); ψ = capillary pressure (or soil suction); and λ = pore-size distribution index.

Equation 2.6 introduces a factor, λ , to characterise the pore-size distribution (PSD). The Brooks and Corey (1964) equation is valid only when PSD is close to the distribution $f(r) = \frac{A}{r^{m+1}}$ (Leong and Rahardjo, 1997b), where parameters A and m are constants.

van Genuchten (1980) (Equation 2.7) relates normalised water content and soil suction using a logistic equation:

$$\theta = \left[\frac{1}{1 + (a\psi)^n} \right]^m \quad (2.7)$$

where a , m , n = SWCC fitting parameters.

It gives a sigmoid, which is the general shape of the unimodal SWCC (Leong and Rahardjo, 1997b). The equation is not valid in the high suction region as the water content drops to zero over a narrow suction range (Fredlund and Xing, 1994).

Fredlund and Xing (1994) established Equation 2.8a with correction factor $C(\psi)$ shown in Equation 2.8b:

$$\theta = \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a} \right)^n \right]} \right\}^m C(\psi) \quad (2.8a)$$

$$C(\psi) = \frac{-\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{1,000,000}{\psi_r} \right)} + 1 \quad (2.8b)$$

where a , m , n = SWCC fitting parameters; ψ_r = residual soil suction; e = Euler's number. The correction factor $C(\psi)$ (Equation 2.8b) is to terminate the curve at zero water content corresponding to maximum suction at 1 GPa. The van Genuchten (1980) and Fredlund and Xing (1994) equations are widely adopted in unsaturated soil

mechanics (Yang et al., 2004; Gallage and Uchimura, 2010; Chin, et. al., 2010; Zhai and Rahardjo, 2012). Table 2.2 summarises the other unimodal SWCC equations found in the literature.

Table 2.2 Summary of unimodal SWCC equations

Reference	Equation	Fitting Parameters
Gardner (1958)	$\theta = \frac{1}{1+a_g\psi^{n_g}}$	a_g, n_g, θ_r
Brooks and Corey (1964)	$S_e = \left(\frac{\psi_a}{\psi}\right)^\lambda$	λ, ψ_a, S_r
Brutsaert (1966)	$\theta = \frac{1}{1+\left(\frac{\psi}{a}\right)^n}$	a, n, θ_r
Farrel and Larson (1972)	$\theta = \theta_s - \frac{1}{\alpha} \ln\left(\frac{\psi}{\psi_a}\right)$	α, ψ_a
Campbell (1974)	$\theta = \theta_s \left(\frac{\psi}{\psi_a}\right)^{\frac{1}{b}}$	b, ψ_a
Van Genuchten (1980)	$\theta = \left[\frac{1}{1+(a\psi)^n}\right]^m$	a, m, n, θ_r
Tani (1982)	$\theta = \left(1 + \frac{\psi}{\psi_0}\right) \exp\left(-\frac{\psi}{\psi_0}\right)$	ψ_0, θ_r
Williams et al. (1983)	$\theta = \exp(A + B \ln \psi)$	A, B, θ_r
Mckee and Bumb (1984)	$\theta = e^{\left(\frac{a-\psi}{b}\right)}$	a, b, θ_r
Mckee and Bumb (1987)	$\theta = \frac{1}{1+e^{\left(\frac{\psi-a}{b}\right)}}$	a, b, θ_r
Russo (1988)	$\theta = \theta_r + (\theta_s - \theta_r) [(1 + 0.5\alpha\psi) \exp(-0.5\alpha\psi)]^{\frac{2}{2+m}}$	α, m, θ_r

Reference	Equation	Fitting Parameters
Fredlund and Xing (1994)	$\theta = \theta_s \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a} \right)^n \right]} \right\}^m$ $\theta = \left(\frac{-\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{1,000,000}{\psi_r} \right)} + 1 \right) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m}$	a, m, n, ψ_r
Kosugi (1994)	$\theta = \frac{1}{2} \operatorname{erfc} \left(\frac{\ln \psi - \ln \psi_m}{\sigma_r \sqrt{2}} \right)$	$\psi_m, \sigma_r, \theta_r$
Assouline et al. (1998)	$\theta = \theta_L + (\theta_s - \theta_L) \left\{ 1 - \exp \left[-\xi \left(\frac{1}{\psi} - \frac{1}{\psi_L} \right)^\eta \right] \right\}$	$\xi, \eta, \theta_L, \psi_L$
Feng and Fredlund (1999)	$w = \frac{ab + c\psi^d}{b + \psi^d}$	a, b, c, d
Gitirana and Fredlund (2004)	$S_i = \frac{\tan \theta_{gi}(1+r^2_i) \ln \left(\frac{\psi}{\psi_a} \right)}{(1-r^2_i \tan^2 \theta_{gi})} -$ $(-1)^i \frac{1 + \tan^2 \theta_{gi}}{1 - r^2_i \tan^2 \theta_{gi}} \sqrt{r^2_i \ln^2 \left(\frac{\psi}{\psi_a} \right) + \frac{a^2(1-r^2_i \tan^2 \theta_{gi})}{1 + \tan^2 \theta_{gi}}}$ $S = \frac{S_1 - S_2}{1 + \left(\frac{\psi}{\sqrt{\psi_a \psi_r}} \right)^d} + S_2$	$\psi_r, \psi_a, a, \theta_r$
Pham and Fredlund (2008)	$w_1(\psi) = S_3 \log(10^6) + (S_1 - S_2) \log(\psi_{AEV}) + (S_2 - S_3) \log(\psi_r) - S_1 \log(\psi)$ $w_2(\psi) = S_3 \log(10^6) + (S_2 - S_3) \log(\psi_r) - S_2 \log(\psi)$ $w_3(\psi) = S_3 \log \left(\frac{10^6}{\psi} \right)$	ψ_r, S_1, S_2, S_3
Tarantino (2009)	$S = \left\{ 1 + \left[\left(\frac{e}{a} \right)^{\frac{1}{b}} \psi \right]^n \right\}^{-\frac{b}{n}}$	a, b, n
Pham and Fredlund (2011)	$w(\psi) = \left\{ [w_s - S_1 \log(\psi) - w_r] \frac{a}{(\psi)^{b+a}} + w_r \right\} \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{10^6}{\psi_r} \right)} \right]$	w_r, S_1, a, b

Reference	Equation	Fitting Parameters
Li et al. (2014)	$w(\psi) = w_b \frac{\sqrt{\psi_a \psi_r}^{n/\log(\frac{\psi_r}{\psi_a})}}{\psi^{n/\log(\frac{\psi_r}{\psi_a})} + \sqrt{\psi_a \psi_r}^{n/\log(\frac{\psi_r}{\psi_a})}} + w_r \frac{(l\psi_r)^m}{\psi^m + (l\psi_r)^m}$	m, n, l
Satyanaga (2015)	$\theta = C(\psi) \left\{ (\theta_s - \theta_r) \exp \left[-5 \left(\frac{\psi_a - \psi}{\psi_a - \psi_r} \right) \right] + \theta_r \right\}$	θ_r, ψ_a, ψ_r
Wijaya and Leong (2016)	$\theta = \theta_s - S_1 [\log(\psi) - \log(\psi_i)] - \sum_{i=2}^5 R_i [\log(\psi)] (S_i - S_{i-1})$ $R_i [\log(\psi)] = \frac{1}{2} \left\langle [\log(\psi) - \log(\psi_i)] + \frac{1}{c_i} \ln \left\{ \frac{\cosh[c_i [\log(\psi) - \log(\psi_i)]]}{\cosh[c_i [\log(\psi_i) - \log(\psi_i)]]} \right\} \right\rangle$	S_1, S_2, S_3

2.2.3 Bimodal SWCC equations

The shape of unimodal SWCC is sigmoidal, whereas bimodal SWCC is bi-sigmoidal. Wijaya and Leong (2016) summarised three approaches for obtaining bimodal SWCC equation: piecewise, fraction volume and unique parameter. The piecewise approach is to separate the bimodal SWCC into two unimodal segments joined at a merging point. The merging point affects the parameters of the SWCC equation significantly and the parameters are highly non-unique (Wijaya and Leong, 2016). This approach may not give a smooth transition at the merging point for the bimodal SWCC equation. The fraction volume approach improves the piecewise approach. In this approach, the two fractions of the porosity corresponding to macro- and micro-pores are described by a volume factor. The SWCC equations describing macro- and micro-pores have the same form and the optimisation is done together.

Both piecewise approach and fraction volume approach can be described by Equation 2.9 (modified from Wijaya and Leong, 2016).

$$\theta = a_1 f_1(\psi) + a_2 f_2(\psi) + a_3 \quad (2.9)$$

where $f_1(\psi)$ and $f_2(\psi)$ are the unimodal SWCC equations, such as van Genuchten (1980) equation or Fredlund and Xing (1994) equation. For piecewise approach, $a_2 = 0$ for $\psi \leq \psi_m$; $a_1 = 0$ for $\psi > \psi_m$. where ψ_m is the suction at the merging point. For fraction volume approach, $a_1 + a_2 = \theta_{sat}$. Generally, piecewise approach and fraction volume approach extend a unimodal SWCC equation to a bimodal SWCC equation. The unique parameter approach incorporates some graphically determined parameters to reduce the number of the parameters needed in the bimodal SWCC equation. The third approach is more attractive as fewer parameters need to be determined (Wijaya and Leong, 2016).

The bimodal SWCC equations proposed in the literature are summarised in Table 2.3. This study will focus on bimodal SWCC equations based on the unique parameter approach.

2.2.4 Summary

The basic theory of SWCC has been introduced. The SWCC shows the relationship between soil suction and water content (volumetric, gravimetric or degree of saturation). The complete SWCC includes one drying curve and one wetting curve with multiple scanning curves. The drying curve is different from the wetting curve, giving rise to hysteresis. The modality of SWCC can be classified as unimodal and bimodal. The commonly used equations for unimodal and bimodal SWCC are summarised.

Table 2.3 Bimodal SWCC equations

Reference/ Acronyms	Equation	Number of unknown parameters	Parameters
Zhang and Chen (2005)/ ZC	$\theta = p_l n_{pl} \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_{rl}} \right)}{\ln \left(1 + \frac{10^6}{\psi_{rl}} \right)} \right] \left[\frac{1}{\ln \left[e + \left(\frac{\psi}{a_l} \right)^{n_l} \right]} \right]^{m_l}$ $+ p_s n_{ps} \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_{rs}} \right)}{\ln \left(1 + \frac{10^6}{\psi_{rs}} \right)} \right] \left[\frac{1}{\ln \left[e + \left(\frac{\psi}{a_s} \right)^{n_s} \right]} \right]^{m_s}$	12	<p>p_l, p_s= percentage of volume for macro- and micro- pore series</p> <p>n_{pl}, n_{ps}= porosity for macro- and micro pore series</p> <p>ψ_{rl}, ψ_{rs} = suctions at residual conditions corresponding to macro- and micro- pore</p> <p>$a_l, n_l, m_l, a_s, n_s, m_s$ = fitting parameters for SWCC corresponding to macro- and micro- pore</p>
Carducci et al. (2011)/ CA	$\theta = \theta_r + \frac{\theta_s - \theta_m}{\left[1 + (\alpha_1 \psi)^{n_1} \right]^{m_1}} + \frac{\theta_m - \theta_r}{\left[1 + (\alpha_2 \psi)^{n_2} \right]^{m_2}}$	8	<p>θ_r=residual water content</p> <p>θ_m = water content corresponding to divided two sub-curves</p> <p>$\alpha_1, n_1, m_1, \alpha_2, n_2, m_2$ = Fitting parameter for two sub-curves</p>
Satyanaga et al. (2013)/ SA	$\theta = C_r \left\{ \theta_r + (\theta_{s1} - \theta_{s2}) \left[1 - \operatorname{erfc} \frac{\ln \left(\frac{\psi_{a1} - \psi}{\psi_{a1} - \psi_{m1}} \right)}{s_1} \right] \right\}$ $+ (\theta_{s2} - \theta_r) \left[1 - \operatorname{erfc} \frac{\ln \left(\frac{\psi_{a2} - \psi}{\psi_{a2} - \psi_{m2}} \right)}{s_2} \right]$	9	<p>θ_{s2} = second sub-curve water content at air-entry level</p> <p>ψ_{a1}, ψ_{a2} = air-entry value for first and second sub-curve</p> <p>ψ_{m1}, ψ_{m2} = suction at inflection point for first and second sub-curve, respectively</p> <p>s_1, s_2= geometric standard deviation of first and</p>

Reference/ Acronyms	Equation	Number of unknown parameters	Parameters
	$C_r = 1 - \frac{\ln(1 + \psi / \psi_r)}{\ln(1 + 10^6 / \psi_r)}$ $erfc = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)} dx$		<p>second sub-curve, respectively</p> <p>ψ_r = suction at residual condition; θ_r = residual water content</p>
Li et al. (2014)/ LI	$\theta = \lambda \left(\frac{\theta_s}{\lambda + 1} - \theta_r \right) \frac{\sqrt{\psi_t \psi_{a1}} \frac{n}{\log(\psi_t / \psi_{a1})}}{\psi^{\frac{n}{\log(\psi_t / \psi_{a1})}} + \sqrt{\psi_t \psi_{a1}} \frac{n}{\log(\psi_t / \psi_{a1})}}$ $+ \lambda \theta_r \frac{\sqrt{\psi_t \psi_{a2}} \frac{n}{\log(\psi_t / \psi_{a2})}}{\psi^{\frac{n}{\log(\psi_t / \psi_{a2})}} + \sqrt{\psi_t \psi_{a2}} \frac{n}{\log(\psi_t / \psi_{a2})}} + \theta_r \frac{(l \psi_r)^m}{(l \psi_r)^m + \psi^m}$ $+ \left(\frac{\theta_s}{\lambda + 1} - \theta_r \right) \frac{(l \psi_t)^m}{(l \psi_t)^m + \psi^m}$ $\lambda = \frac{\theta_s - \theta_r}{\theta_r}$	8	<p>θ_r = residual water content</p> <p>ψ_{a1}, ψ_{a2} = air-entry value for first and second sub-curve</p> <p>ψ_r = suction at residual condition</p> <p>ψ_t = suction at residual condition for first sub-curve</p> <p>m, l, n = fitting parameters</p>
Wijaya and Leong (2016)/ WL	$\theta = \theta_s - S_1 [\log(\psi) - \log(\psi_i)]$ $- \sum_{i=2}^5 R_i [\log(\psi)] (S_i - S_{i-1})$ $R_i [\log(\psi)] = \frac{1}{2} \left\langle [\log(\psi) - \log(\psi_i)] + \frac{1}{c_i} \ln \left\{ \frac{\cosh[c_i (\log(\psi) - \log(\psi_i))]}{\cosh[c_i (\log(\psi_i) - \log(\psi_1))]} \right\} \right\rangle$	13	<p>S_i = curve slope for the segment i; ψ_i = suction corresponding the intersection of segment i and $i-1$; c_i = curvature parameter corresponding the intersection of segment of i and $i-1$</p>

2.3 Soil-water Characteristic Curve (SWCC) Estimated from Grain-size Distribution (GSD)

2.3.1 Grain-size distribution (GSD)

Grain-size distribution is also referred to as particle-size distribution (Chan and Govindaraju, 2004; Hwang and Choi, 2006), or as aggregate size distribution (Wagner and Ding, 1994; Fredlund et al., 2000). The grain-size distribution (GSD) is commonly used in soil classification, and it has been used to estimate unsaturated soil properties (Fredlund et al., 2000). The GSD for a soil is defined as the relationship between percent passing by weight and the particle size (Fredlund et al., 2000).

In practice, there are five soil classification systems: USCS (1974), AASHTO (1978), ASTM (2007), USDA (1993), and BS/MIT (Taylor, 1948) shown in Figure 2.4. There is not much differences between the classification methods for the finer grain sizes (e.g., sand, silt and clay), which are more critical for SWCC estimation. In this thesis, only the USDA (1993) and the BS/MIT classification system are adopted.

There are three primary types of GSD: well-graded soils, uniform soils and gap-graded soils (Holtz and Kovacs, 1981). The three types of GSD are illustrated in Figure 2.5. This thesis covers all three types of GSD, but gap-graded soils are renamed as bimodal GSD soils, and uniform and well-graded soils are renamed as unimodal GSD soils in this thesis. For gap-graded soils, the GSD may be bimodal or multimodal, but in this thesis, all bimodal and multimodal GSD will be classified as bimodal GSD, which will be explained in Chapter 3.

Figure 2.4 Grain size ranges according to soil classification systems (modified from Holtz and Kovacs, 1981)

The GSD provides information on the percentage and dominant sizes of grains present in a soil. Grain-size PDF can be used to represent the distribution of grain sizes. The integral of the PDF for all sizes of grains should be equal to 100%. In reality, it is impossible to determine every size of grains; hence, in this thesis, grain-size frequency histogram plot will replace grain-size PDF. The frequency histogram plot and the cumulative frequency GSD are shown in Figure 2.6. The grain size bin width adopted is based on the BS/ MIT classification as shown in Figure 2.6. The width between the

two neighbouring sizes is about half log-cycle, which is more suitable for logarithmic scale plotting.

Figure 2.5 Three primary types of GSD curves (modified from Holtz and Kovacs, 1981)

Figure 2.6 Cumulative frequency GSD and grain-size frequency histogram for Soil 153 from Andersson and Wiklert (1972)

The GSD tests determine a series of discrete points plotted on a semi-logarithmic graph. Hence a mathematical equation is needed to provide a continuous description of the GSD data. A number of GSD equations have been proposed for unimodal and bimodal GSD. The parameters of existing GSD equations have been commonly determined by an optimisation method, where the GSD data are curve-fitted with the GSD equation and the error minimised by iterating the parameters. The parameters of the GSD equation obtained using the curve-fitting techniques, may not be unique (Wijaya and Leong, 2016). Correlations of these non-unique GSD parameters to parameters of the SWCC become problematic. In this research, unimodal and bimodal GSD equations are proposed. The proposed GSD equations use parameters that can be determined directly from the GSD plot. Hence, a unique GSD equation can be established.

2.3.2 Using GSD to estimate SWCC

Since cost, time and labour for direct determination of SWCC are expensive, SWCC has been frequently estimated from routinely measured physical properties, such as GSD, densities, soil texture and/ or organic content. A number of SWCC estimation methods use GSD.

Although the indirect methods for determining SWCC are attractive, the associated assumptions and limitations should be noted. For example, Hwang and Hong (2006) assumed that the GSDs are of lognormal distribution, but Buchan (1989) found that only about half of the GSD of the soils from database UNSODA are of lognormal distribution. Fredlund et al. (2000) found that the lognormal distribution assumption gives symmetrical PDFs, which is not always true. Both Gupta and Larson (1979) and

Rawls et al. (1982) methods have suction range limitation (up to 1500 kPa). If the estimated suction is more than 1500 kPa, the methods are invalid.

McBratney et al. (2002) defined pedotransfer function (PTF) as estimative functions of certain soil properties from other easily, routinely, or cheaply measured properties. According to Minasny et al. (1999), the PTFs for estimating SWCC can be divided into three types: point-estimation, parametric-estimation and physico-empirical PTFs. However, physico-empirical PTFs can be classified as point-estimation PTFs, as the physico-empirical PTFs also give water contents at a specific suction. Hence, point-estimation and parametric-estimation PTFs will be discussed in this research.

Point-estimation PTFs estimate water content at one suction level using GSD or other physical properties. The most frequently used suction levels are at 10, 33 kPa (corresponding to field capacity) and 1500 kPa (corresponding to permanent wilting point), which are needed to determine plant available water content (Minasny et al., 1999). The most common method used in developing point-estimation PTF is to employ multiple linear regression (Minasny et al., 1999).

Besides multiple linear regression, researchers developed exponential point-estimation PTFs (e.g., Arruda et al., 1987; Minasny et al., 1999). Both PTFs assumed the water content at the selected suction trend exponentially for fine-grained soils (e.g., silt or clay). Minasny et al. (1999) also highlighted that the water contents at suctions of 10, 33 and 1500 kPa have a decreasing trend with sand content.

Besides the above point-estimation PTFs, Arya and Paris (1981) developed the first physico-empirical PTF. The model uses the information from the GSD curve and the

dry density. Water content is calculated based on the estimation of the pore sizes, which was assumed to be strongly related to GSD. The pore size and the grain size are related by Equation 2.10.

$$r_i = 0.816R_i \sqrt{en_i^{(1-\alpha)}} \quad (2.10a)$$

$$\psi_i = \frac{2T_s \cos \alpha_1}{\rho_w g r_i} \quad (2.10b)$$

where r_i = mean pore radius for the i^{th} fraction of soils; R_i = mean grain radius for the i^{th} fraction of soils; e = void ratio; $n_i = \frac{3w_{s,i}}{4\pi\rho_s R_i^3}$, the number of spherical particles for the i^{th} fraction; T_s = surface tension at air-water interface; α_1 = contact angle; α = scaling parameter; ψ_i = soil suction; ρ_w = water density; g = acceleration due to gravity; $w_{s,i}$ = soil particle weight at i^{th} fraction; and ρ_s = soil solids density.

The method converts pore radii to soil suctions using capillary theory (Taylor, 1948) as shown by Equation 2.10. The PSD is estimated from GSD. For the PSD estimation, spherical particles and cylindrical pores were assumed. The GSD is divided into some size fractions and every fraction's solid particles form a hypothetical, cubic close-packed structure consisting of mono-sized spherical particles.

Other than regression methods, neural network methods are also used to develop point-estimation PTFs. The point-estimation PTFs for the neural network method are generally complicated as a computer program is required to use the PTF. In this thesis, PTFs from neural network methods will not be considered due to the inaccessibility of the PTFs.

For the point-estimation PTFs, the relationship between the input variables and water content at selected soil suction is determined. Many point-estimation PTFs (Pidgeon, 1972; Lal, 1979; Aina and Perisawamy, 1985; Oliveira et al., 2002; Tomasella and Hodnett, 1998; Reichert et al., 2009) show that the fines (clay and silt) content has positive relation with water content and sand content has negative relation with water content.

Another group of PTFs is the parametric-estimation PTFs. The parametric-estimation PTFs assumed that the SWCC can be adequately described by a SWCC equation (such as, Brooks and Corey, 1964; van Genuchten, 1980; and Fredlund and Xing, 1994) and built relationships between parameters of the SWCC equations and GSD, index properties and dry density or void ratio. The most commonly used SWCC equations are Brooks and Corey (1964) equation (Equation 2.6), van Genuchten (1980) equation (Equation 2.7) and Fredlund and Xing (1994) equation (Equation 2.8).

The Brooks and Corey (1964) equation has three fitting parameters, but some researchers (Bloemen, 1977; Saxton et al., 1986; Mayr and Jarvis, 1999) assumed that the residual water content is zero and hence, the number of fitting parameters is reduced to two. Assuming residual water content to be zero in the Brooks and Corey (1964) equation reduces the Brooks and Corey (1964) equation to the Campbell (1974) equation. The Rawls and Brakensiek (1985), Saxton et al. (1986), Tomasella and Hodnett (1998) and Mayr and Jarvis (1999) parametric-estimation PTFs, which use the Brooks and Corey (1964) equation, are discussed in Chapter 6.

The van Genuchten (1980) equation is the most commonly used equation to fit SWCC. It has four fitting parameters, namely, a , m , n and θ_r . But some researchers

(Varallyay et al., 1982; Rajkai et al., 1996; Wosten et al., 1999; Minasny et al., 1999; Hodnett and Tomasella, 2002; Rajkai et al., 2004) also consider parameter θ_s , which can be estimated by soil texture, as a fitting parameter. Regression methods (Varallyay et al., 1982; Vereecken et al., 1989; Rajkai et al., 1996; Scheinost et al., 1997; Wosten et al., 1999; Minasny et al., 1999; Hodnett and Tomasella, 2002; Rajkai et al., 2004; Ganjian et al., 2007; Lee and Ro, 2014) and neural network methods (Pachepsky et al., 1996; Schaap et al., 1998; Schaap and Leij, 1998; Wosten et al., 2001; Schaap and Wosten, 2003; Tomasella et al., 2003; Nemes et al., 2003; Minasny and McBratney, 2007; Matula et al., 2007) are commonly used for establishing the parametric-estimation PTFs based on the van Genuchten (1980) equation.

Another commonly used SWCC equation is the Fredlund and Xing (1994) equation, which contains four parameters, namely, a , m , n , and ψ_r , for equation with correction factor and three parameters, namely, a , m , and n , for equation without correction factor (Leong and Rahardjo, 1997b). Researchers proposed a number of parametric-estimation PTFs based on the Fredlund and Xing (1994) equation (Vanapalli and Catana, et al., 2005; Houston et al., 2006; Perera et al., 2005; Chin et al., 2010), which require one-point SWCC measurement for the PTFs calibration. The one-point SWCC measurement can improve the performance of the parametric-estimation PTFs significantly (Perera et al., 2005). However, the one-point SWCC measurement increases the time and cost of the SWCC determination.

The PTFs which use GSD are reviewed. The point-estimation and parametric-estimation PTFs are summarised in Figure 2.7 and Figure 2.8, respectively.

The previously mentioned SWCC equations are all for unimodal SWCC. But many residual and colluvial soils show bimodal SWCCs (Satyanaga et al., 2013). Hence, the bimodal SWCC equation is useful and necessary. As the bimodal SWCC equation need more parameters (8 to 10 parameters), the determination of the parameters is more complex. Satyanaga et al. (2013) used the shape similarity of GSD and SWCC to develop a GSD equation to fit the GSD data and subsequently to use the parameters of the GSD equation to estimate the parameters of the bimodal SWCC equation. Li et al. (2014) directly estimates the parameters of their proposed bimodal SWCC equation from GSD and other physical parameters. Both Satyanaga et al. (2013) and Li et al. (2014) PTFs for bimodal SWCC estimation can be classified as parametric-estimation PTFs.

2.3.3 Summary

Many point-estimation and parametric-estimation PTFs have been developed. These two types of PTFs are rarely compared. Minasny et al. (1999) and Tomasella et al. (2003) have developed the two types of PTFs using the same set of data. A comparison will be done in the thesis for the two types of PTFs but using independent data sets (i.e., data which have not been used for developing the PTFs). According to the literature review, many researchers (Arya and Paris, 1981; Haverkamp and Parlange, 1986; Vereecken et al., 1989; Kosugi, 1994; Rajkai et al., 1996; Schaap and Bouten, 1996; Fredlund et al., 1997; Nimmo, 1997; Scheinost et al., 1997; Minasny et al., 1999; Fredlund et al., 2002; Hwang and Powers, 2003; Chan and Govindaraju, 2004; Mohanmmadi and Meskini-Vishkaee, 2013; Satyanaga et al., 2013; Lee and Ro, 2014) assumed that the shape of the GSD is strongly related to the SWCC. The assumption

will be investigated in Chapter 3. Generally, it is agreed that the GSD critically affects the SWCC, therefore validating the assumption is possible.

Figure 2.7 Summary of point-estimation PTFs

Figure 2.8 Summary of parametric-estimation PTFs for unimodal SWCC

2.4 Pore-size Distribution (PSD) and Grain-size Distribution (GSD)

2.4.1 Pore-size distribution (PSD)

Pore-size distribution (PSD) is also referred to as void-size distribution (Hwang and Powers, 2003). Soil does not contain discrete particles with obvious boundaries that could be called individual pores. The PSD characterises the pore space, which is the portion of the soil volume that is not occupied or isolated by soil materials (Nimmo, 2004). Generally, the definition of the pore space should exclude the voids that are totally enclosed by soil materials (isolated voids). The PSD is the relative abundance of each pore size in a representative volume of soil.

Many researchers believe that the PSD controls the shape of the SWCC (Arya and Paris, 1981; Vereecken et al., 1989; Kosugi, 1994; Fredlund et al., 1997; Nimmo, 1997; Fredlund et al., 2002; Simms and Yanful, 2002; Hwang and Powers, 2003; Chan and Govindaraju, 2004), and permeability properties (Leong and Rahardjo, 1997b; Fredlund et al., 1997; Simms and Yanful, 2002; Nimmo, 2004). But some researchers also pointed out that relating PSD to SWCC or water coefficient of permeability linearly have some discrepancies (Leong and Rahardjo, 1997b; Simms and Yanful, 2004; Beckett and Augarde, 2013). The assumptions to linearly correlate PSD to SWCC and water coefficient of permeability are: (i), the PSD is constant during test; and (ii), pore geometry is assumed to be a bundle of cylindrical capillaries (Simms and Yanful, 2002), as shown in Figure 2.9. The bundle of cylindrical capillaries (BCC) approach idealised units of soil pores as cylindrical tubes. There are only two conditions of the BCC if the surface adsorption of water is ignored, either completely filled or completely empty. Soil pores are clearly not simple cylindrical tubes. The

pores are not only either completely filled or completely empty. Despite the limitations, the BCC approach is still useful and simple to use (Or and Tuller, 1999).

Miller and Miller (1956) described the physical theory for the capillary phenomenon by assuming that: (i), the soil particles are homogenous, isotropic and permanent; (ii), the water is uniform in its surface tension, contact angle, density and viscosity; (iii), the atmosphere is constant; and (iv), no isolated drops of water and isolated bubbles of air exists. Miller and Miller (1956) concluded that similar media would give similar flow system of the media.

Figure 2.9 Bundle of cylindrical capillary tubes of pore geometry of soils

For natural soils, especially for fine-grained soils, the PSD is changing during the SWCC tests or hydraulic tests (Simms and Yanful, 2004; Leong and Wijaya, 2015). For the geometry of the pores, there is some disagreement with the BCC assumption (Tuller et al., 1999; Or and Tuller, 1999; Hueckel et al., 2001; Frydman and Weisberg,

1999). Baker and Frydman (2009) summarised two alternative models of the pore geometry: (i) Tuller et al. (1999) model of capillary in an angular space (shown in Figure 2.10), where it is assumed that all surfaces of the pore are screened by adsorbed water, the capillary water concentrates at the corners and the gas bubble occupies the centre of the pore; and (ii), double-porosity model which classifies the pores in soil into macro-pores, which is the pore space between clay pods, and micro-pores, which is the pore space between the soil platelets within the pods.

In the two proposed models, water adsorbed on the effective surface is greater than the BCC model, which corresponds to higher suction (Baker and Frydman, 2009). Hence, the two models will cause the estimated SWCC to shift horizontally compared to the estimated SWCC using BCC model. Even with the disagreement of the pore model, researchers agreed that the PSD has a high correlation with SWCC and hydraulic properties of the soil.

The PSD is generally determined indirectly from GSD (Arya and Paris, 1981; Kosugi, 1994; Fredlund et al., 1997; Nimmo, 1997; Fredlund et al., 2002; Hwang and Powers, 2003) by curve-fitting. But the accuracy in determination is significantly affected by the size of the experimental data and the soil type.

There are some direct measurement methods to determine PSD. The most common PSD measurement method is called the Mercury Intrusion Porosimetry (MIP) method, which is standardised in ASTM D4404-10 (2010). The non-wetting and high surface tension property of mercury is used to probe the soil pores. The applied force pushes the mercury into soil pores and the corresponding volume of the mercury filling the soil pores are monitored. Thus, an applied pressure, P , corresponds to a pore size (as shown

in Equation 2.11), and the filling volume of mercury represents the volume of the pore size. Thus, the pore size and the volume of the pores can be used to produce the PSD.

$$P = \frac{4T_m \cos \alpha_m}{r} \quad (2.11)$$

where P = pressure applied for mercury injection; T_m = surface tension of the mercury; α_m = contact angle for mercury and soil; and r = pore size.

As highlighted in ASTM D4404-10 (2010), the MIP method can only measure the open pores, which means that the pores that are completely enclosed by soil particles (isolated pores) are not measurable. Furthermore, the MIP procedure involves high pressure, which may cause irreversible change in the pore geometry of the soil.

Figure 2.10 Unsaturated angular capillary model for soils (modified from Tuller et al. 1999)

Lawrence (1977) reviewed other methods besides the MIP method. Non-polar liquid desorption method dries the soil sample and then saturates the soil sample with a non-polar liquid. But the method is invalid for measurement of pore sizes smaller than 3 *nm* and the measurement time is often very long. Nitrogen sorption method uses nitrogen to determine the PSD at the temperature of -196°C for pore sizes smaller than 20 *nm* but there are doubts on the validity of the method. Lawrence (1977) concluded that the MIP method is fast, most convenient, and gives highest accuracy with the widest applicable pore size range, but it is expensive.

Besides direct measurement of the pores with the filling methods, there are some image-based techniques, which use various types of microscopy on thin sections or flat soil surfaces or tomography (Romero and Simms, 2008; Warner et al., 1989). The techniques require enough pores to be analysed to give an adequate statistical representation. Recently, some electron microscopy scanning techniques were also adopted in soil science (e.g., Romero and Simms, 2008; Lourenco et al., 2012).

2.4.2 GSD and PSD

Researchers using GSD to estimate SWCC through PSD showed that there is a relationship between GSD and PSD (Arya and Paris, 1981; Kosugi, 1994; Fredlund et al., 1997; Fredlund et al., 2002; Hwang and Powers, 2003; Chan and Govindaraju, 2004).

Arya and Paris (1981) established a model to estimate PSD from GSD for SWCC determination of undisturbed soils. They first divided the GSD into several size fractions and for each fraction, the size of the soil particles was assumed to be equal. Then, each fraction's pore size was calculated from the fraction of soil mass and void

ratio, beginning with the smallest grain size fraction. Finally, the void volumes were determined from the smallest pore size to build the PSD and then the PSD was used to estimate the SWCC using the capillary theory. But Arya and Paris (1981) assumed that the pores between soil particles are hexahedra, which rarely is the case (Reboul et al. 2008). The assumed natural pore length is also not easy to determine. Later, Arya et al. (2008) modified the model to use two parameters, the radius of the grains and the number of grains. Even with these limitations, the model still achieved good accuracy in estimating SWCC.

Haverkamp and Parlange (1986) proposed a linear model for the relationship between GSD and PSD to estimate SWCC. To estimate pore size, a packing parameter, which is assumed to be constant for each texture class, was introduced to factor the grain size to give the pore size.

Assouline et al. (1998) proposed a power law relationship between PSD and GSD as shown in Equation 2.12.

$$r = \alpha d^v \quad (2.12)$$

where r = the radius of the pores; α = the parameter related to the packing condition; d = the grain size radius; and v = the parameter related to the shape of the soil solids. The soil particles are assumed to be multi-sized spheres with lognormal and normal distributions. This assumption, of course, has limitations as it ignored the shape effects of the soil particles, and did not consider soils with non-lognormal and non-normal distributions GSD. Assouline et al. (1998) estimated the parameters α and v to be 1.37

and 1.72, and 1.33 and 1.63 for normal and lognormal distributions of GSD, respectively.

Hwang and Powers (2003) derived a mathematical expression to estimate SWCC from GSD by applying the Kosugi (1996) model for GSD and PSD for sandy soils. The PSDs and GSDs are assumed to be lognormally distributed. They used a linear relationship (Haverkamp and Parlange, 1986) between grain size radius and equivalent pore radius and a non-linear relationship between grain size radius and equivalent pore radius (Assouline et al., 1998) to give two models. They concluded that the non-linear model performed better than the linear model. The relationship between the equivalent pore radius and grain size radius is given by Equation 2.12, where α ranges from 0.991 to 1.618, and ν is approximately equal to 0.69. The values of α and ν are quite different from those given by Assouline et al. (1998). Hwang and Powers (2003) explained that this is because of the medium characteristics. Assouline et al. (1998) worked on spheres while Hwang and Powers (2003) used natural soils.

Arthur et al. (2012) showed that the PSD of a soil, which was fitted with an equation similar in form to Campbell (1974) SWCC equation (see Table 2.2), is highly correlated with the GSD of the soil which was fitted with the Rosin and Rammler (1933) distribution function shown in Equation 2.13.

$$P(X < x) = 100 - 100\exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right] \quad (2.13)$$

where $P(X < x)$ = the percentage by weight of particles less than size x ; and α, β = adjustable parameters related to the characteristic size and spread of the particle sizes, respectively. Arthur et al. (2012) found that only parameter b , which could be related to

parameters α and β with a linear function, has high correlation with clay content. Arthur et al. (2012) also assumed that the SWCC equation has a linear relationship with PSD. Hence, an empirical linear relationship was built between PSD and GSD.

2.4.3 Summary

In this section, PSD is introduced. The assumptions to linearly correlate PSD to SWCC and permeability properties are discussed. Various PSD determination methods are reviewed. Often, PSD is determined indirectly using GSD and directly using MIP method. Several models for estimation of PSD from GSD are reviewed. The point-to-point relation method between PSD and GSD is widely adopted, which means one point of GSD can be mapped on to a unique point of the PSD.

2.5 Hysteresis of Soil-water Characteristic Curve (SWCC)

2.5.1 Hysteresis of SWCC

It is widely recognised that the water content of a soil at a certain suction is not unique. Water content on the wetting curve is always lower than on the drying curve for a known suction (see Figure 2.1), which means that the drying SWCC and wetting SWCC follow different paths. This phenomenon is called hysteresis of SWCC. The magnitude of the hysteresis, described by degree of hysteresis, H_d , shown in Equation 2.26 (Konyai et al., 2009), is mainly dependent on the soil type. Generally, soil with more clay content has more significant hysteresis in the SWCC.

$$H_d = \frac{\Delta\theta_{max}}{\theta_s - \theta_r} \quad (2.14)$$

where H_d = degree of hysteresis; $\Delta\theta_{max}$ = maximum difference of water content for drying and wetting curves; θ_s = saturated water content for initial drying curve; and θ_r = residual water content.

In nature, soils are exposed to cyclical drying and wetting due to climatic conditions, e.g., dry and wet periods or seasons. During the dry period, water is desorbed from soil as the matric suction increases. During the wet period, the water is absorbed into the soil and the matric suction decreases. Goh et al. (2010) summarised the four specific mechanisms causing the hysteresis of the SWCC: (i), the “ink bottle effect” (Taylor, 1948); (ii), the swelling and shrinkage of soil (Hillel and Mottas, 1966); (iii), the contact-angle effect (Hillel, 1971; Lu and Likos, 2004); and (iv), the presence of entrapped air in soil (Hillel, 1971).

Figure 2.11 was used to explain the “ink bottle effect” (Taylor, 1948). Generally, it is assumed that the pores in soils consist of a bundle of cylindrical capillary tubes, (Figure 2.11a). When the tube is put into water, the water will rise in the tube above the water table until it reaches a certain height according to capillary theory. But soil pores are not like the tubes shown in Figure 2.11(a), but more like the tube shown in Figure 2.11(c) and (d) as there are small size pores connecting large size pores (variations of the pore size). As shown in Figure 2.11(c) and (d), the wetting and drying processes are different because of the expanded space, which is represented as the large size pores. During the soil wetting or drying, the soil’s structure, PSD or fabric may change (Hillel and Mottas, 1966; Lu and Likos, 2004), which also causes hysteresis. The contact angle and radius of curvature are significantly different during wetting and drying as shown

in Figure 2.12. Therefore, for known water content, soil suction for the drying process is higher than for the wetting process, which is reflected in the SWCC as hysteresis.

Hysteresis increases the difficulty of applying unsaturated soil mechanics in practice. In the field, partial wetting or drying occurs in several cycles, which results in drying and wetting scanning curves between the main drying and wetting curves, as shown in Figure 2.13. There are infinite number of intermediate drying and wetting scanning curves that lie between the boundaries of the main drying and wetting curves. This means that there is a range instead of a unique value of soil suction at certain water content. Slight changes in nature can also cause the matric suction to shift laterally with small change in water content, i.e., movement along a scanning curve. Hence, it is difficult to use the SWCC to determine the soil suction, other than to obtain a wide range of approximate values (Fredlund et al., 2012). As the SWCC is fundamental to estimate unsaturated soil properties, it is important to study the hysteretic behaviour of the SWCC. As the wetting SWCC is more difficult to determine, an indirect estimation is attractive.

Figure 2.11 Height and radius on capillarity (modified from Taylor, 1948)

Figure 2.12 Illustration of difference between drying and wetting contact angles of water droplet on inclined surface (from Lu and Likos, 2004)

Figure 2.13 Bounding and scanning curves for drying and wetting process of unsaturated soils (modified from Yang et al., 2012)

2.5.2 Estimating hysteresis of SWCC

Pham et al. (2005) summarised the approaches to account for hysteresis in the SWCC, which include numerous physics-based approaches and empirical approaches. The physics-based approaches are generally formed based on domain theory (Everett, 1954, 1955; Enderby, 1955; Poulovassilis, 1962; Mualem, 1973), or similarity theory (Philip, 1964; Mualem, 1974). The empirical approaches include two types: (i), using the same curve-fitting equation for both wetting and drying curves and adjusting the values of the fitted parameters separately (Scott et al., 1983; Feng and Fredlund, 1999); and (ii), correlating the drying and wetting curves parameters at specified points (Jaynes, 1984; Pham et al., 2005).

Pham et al. (2005) categorised the domain models into four types: (i), independent models with the similarity hypothesis applied; (ii), dependent models with the similarity hypothesis applied; (iii), independent models without the similarity hypothesis applied; and (iv), dependent models without the similarity hypothesis applied. The development of the domain models is summarised by Pham et al. (2005) in a flow chart as shown in Figure 2.14.

The independent domain models are widely adopted to calculate soil hysteretic behaviour by many researchers (Neel, 1942, 1943; Everett, 1954, 1955; Enderby, 1955; Poulovassilis, 1962; Mualem, 1973). The independent domain models assumed that wetting and drying of each pore was determined strictly by that pore's geometry, regardless of the connection of that pore to the surrounding pores (thus the term “independent”). The dependent domain models account for the effect of the

surrounding pores on the entry of water or air into the pore, which means that the independent domain models are simplified dependent domain models.

Figure 2.14 Summary of domain models of SWCC hysteresis (modified from Pham et al., 2005)

The similarity hypothesis, initially proposed by Philip (1964) and later improved by Mualem (1973), was applied in a series of independent domain models proposed by Mualem (1973, 1974, 1977, 1984b), Parlange (1976, 1980), Hogarth, et al. (1988), and Liu et al. (1995), and dependent domain models proposed by Mualem and Dagan (1975), Mualem and Miller (1979), and Mualem (1984a). The similarity hypothesis significantly improved the application of the domain models (Pham et al., 2005).

Mualem (1974) proposed an independent domain model with similarity hypothesis, similar to the Neel (1942, 1943) model. Mualem (1974) used two variables, normalised

neck pore size, $\bar{r}$, and normalised body pore size, $\bar{\rho}$, which can be calculated by Equations 2.15a and 2.15b, respectively.

$$\bar{r} = \frac{r - R_{min}}{R_{max} - R_{min}} \quad (2.15a)$$

$$\bar{\rho} = \frac{\rho - R_{min}}{R_{max} - R_{min}} \quad (2.15b)$$

where $\bar{r}$, $\bar{\rho}$ = normalised neck pore size, body pore size, respectively; r, ρ = neck pore; and R_{max}, R_{min} = the maximum, and minimum of the pore size in the domain, respectively. The ranges of $\bar{r}$ and $\bar{\rho}$ are between 0 and 1. The model uses two independent functions of $\bar{r}$ and $\bar{\rho}$ to express the pore distribution function, $f(\bar{r}, \bar{\rho})$, as shown in Equation 2.16a. The drying boundary and wetting boundary are calculated by Equations 2.16b and 2.16c, respectively, which are also illustrated in Figure 2.15.

$$f(\bar{r}, \bar{\rho}) = h(\bar{r})l(\bar{\rho}) \quad (2.16a)$$

$$\theta_d(\bar{R}) = \int_0^{\bar{R}} l(\bar{\rho}) d\bar{\rho} \int_0^1 h(\bar{r}) d\bar{r} + \int_{\bar{R}}^1 l(\bar{\rho}) d\bar{\rho} \int_0^{\bar{R}} h(\bar{r}) d\bar{r} + \theta_{min} \quad (2.16b)$$

$$\theta_w(\bar{R}) = \int_0^{\bar{R}} l(\bar{\rho}) d\bar{\rho} \int_0^1 h(\bar{r}) d\bar{r} + \theta_{min} \quad (2.16c)$$

The existing independent domain models with similarity hypothesis can describe the hysteresis of the SWCC well. But for model calibration, the pore-water distribution function, $f(\psi_d, \psi_w)$, and a pair of soil suctions, (ψ_d, ψ_w) are required. Philip (1964) proposed an independent domain model with similarity hypothesis to minimise the calibration data required. The model assumed that the pore-water distribution function, $f(\psi_d, \psi_w)$, between the drying and the wetting processes is independent of the pore size, which is expressed as:

$$f(\psi_d, \psi_w) = \frac{l(\psi_w)h(\frac{\psi_d}{\psi_w})}{\psi_w} \quad (2.17)$$

Parlange (1976, 1980) proposed an independent domain model with similarity hypothesis by assuming that the pore-water distribution function $f(\psi_d, \psi_w)$, which describes the relative volume of pore space that is filled for a change of suction from ψ_w to $\psi_w + d\psi_w$ and drained for a change of suction from ψ_d to $\psi_d + d\psi_d$, is independent of ψ_w , which means $f(\psi_d, \psi_w) = f(\psi_d)$. Parlange (1976, 1980) model can be expressed as Equation 2.18, which is developed by applying and simplifying Mualem (1973) similarity hypothesis assumption.

$$\theta_d(\psi_1, \psi) = \theta_d(\psi) - (\psi - \psi_1) \left(\frac{d\theta_w}{d\psi} \right)_\psi \quad (2.18)$$

Later, Hogarth et al. (1988) proposed a new independent domain model with similarity hypothesis, based on Parlange (1976, 1980) model, by fitting the Brooks and Corey (1964) SWCC equation. Main, primary and secondary wetting and drying curves were derived. Hogarth et al. (1988) improved Parlange (1976, 1980) model by reducing the data required for the model calibration. Hogarth et al. (1988) model requires four parameters to be determined or the wetting curve can be obtained with the known drying curve and the water-entry value. Liu et al. (1995) further improved Parlange (1976, 1980) model by using data from only one boundary curve to estimate scanning curves.

(a), Schematic representation of SWCC hysteresis

(b), Boundary drying curve drying process

(c), Boundary wetting curve wetting process

Figure 2.15 Boundary drying and boundary wetting processes in soils for Mualem (1974) independent domain model (Modified from Mualem, 1974)

The Neel (1942, 1943) model (Figure 2.16), which was extended by Everett (1954, 1955), Enderby (1955), Poullovassilis (1962) and Mualem (1973), was originally used

to describe the shape of magnetic hysteresis curves in low strength field (Everett, 1955; Pham et al. 2005). The Neel (1942, 1943) model assumed that there are only two states in the soil pores: fully filled with water or completely empty. Two soil suction values, called drying soil suction, ψ_d , and wetting soil suction, ψ_w , are used to characterise the pore states. When the soil suction increases to the pore's ψ_d , the pore will be completely empty. When the soil suction decreases to the pore's ψ_w , the pore will be fully filled.

The vertical axis in Figure 2.16 is the water content distribution function. The function of $f(\psi_d^1, \psi_w^1)$ represents the volume of pores in the soil that has soil suction of ψ_d^1 on the drying curve and ψ_w^1 on the wetting curve. Because ψ_d is always greater than ψ_w , all domains should be located in the shaded triangle in the suction plane, meaning that no pores are located outside the triangle as no pores can have ψ_w greater than ψ_d . The maximum water content after the initial drying process, θ_u , can be expressed as:

$$\theta_u = \theta_{min} + \int_{\psi_{min}}^{\psi_{max}} \int_{\psi_{min}}^{\psi_{max}} f(\psi_d, \psi_w) d\psi_d d\psi_w \quad (2.19)$$

where the parameters are defined in Figure 2.16.

Figure 2.17(a) illustrates the process of the primary drying and primary wetting scanning curves based on Neel (1942, 1943) model. The primary drying scanning curve from suction ψ_1 at the boundary wetting curve, is shown in Figure 2.17(b). The primary wetting scanning curve for suction ψ_2 is shown in Figure 2.17(c). For the primary wetting process, the pores, whose sizes correspond to the suction between ψ_2 and ψ_{max} , are filled with water. Hence the water content for primary wetting scanning

curve is always higher than the boundary wetting curve at the same suction. For primary drying process, pores whose size correspond to the suction between ψ_{min} and ψ_1 are completely empty, and will not be filled with water during the drying process. Hence, the water content for primary drying scanning curve is always lower than the boundary drying curve at the same suction.

Figure 2.16 Water content distribution function for the Neel (1942, 1943) model (from Pham et al., 2005)

The model has been shown to be able to account for the SWCC hysteresis fairly accurately (Poulovassilis and Kargas, 2000). But some researchers (Topp and Miller, 1966; Poulovassilis and Childs, 1971; Vachaud and Thony, 1971) found that the Neel (1942, 1943) model did not perform well for the calculation of the hysteretic behaviour of pore water properties.

For the revision of Neel (1942, 1943) model, the PSD can be used to determine the hysteresis of SWCC directly if the capillary theory is valid for the SWCC. The pore size corresponds to soil suction according to the capillary theory. With ψ_{min} and ψ_{max} determined, the boundary wetting and drying curves, the scanning curves between two boundaries can be determined by the PSD.

Everett (1954, 1955) developed another independent model without similarity hypothesis based on the Neel (1942, 1943) model, which can be summarised in four equations (Equation 2.20 a-d). The model assumed that there is no hysteresis if the suctions are either higher than ψ_{max} or lower than ψ_{min} .

$$\theta_d(\psi) = \theta_{min} + \int_{\psi_{min}}^{\psi_{max}} \int_{\psi}^{\psi_{max}} f(\psi_d, \psi_w) d\psi_d d\psi_w \quad (2.20a)$$

$$\theta_w(\psi) = \theta_{min} + \int_{\psi}^{\psi_{max}} \int_{\psi}^{\psi_{max}} f(\psi_d, \psi_w) d\psi_d d\psi_w \quad (2.20b)$$

$$\theta_d(\psi_1, \psi) = \theta_d(\psi_1) - \int_{\psi_1}^{\psi} \int_{\psi_1}^{\psi} f(\psi_d, \psi_w) d\psi_d d\psi_w \quad (2.20c)$$

$$\theta_w(\psi_2, \psi) = \theta_w(\psi_2) - \int_{\psi}^{\psi_2} \int_{\psi}^{\psi_2} f(\psi_d, \psi_w) d\psi_d d\psi_w \quad (2.20d)$$

where $\theta_d(\psi), \theta_w(\psi)$ = the water content on the drying and wetting curves, respectively, at the soil suction ψ ; $\theta_d(\psi_1, \psi), \theta_w(\psi_2, \psi)$ = the water content at suction of ψ on the primary drying and wetting scanning curves, respectively, starting at a suction of ψ_1 on the boundary drying or wetting curve.

(a), Primary drying scanning curve and primary wetting scanning curve

(b) Pore draining diagram corresponding to the primary drying scanning curve

(c), Pore filling diagram corresponding to the primary wetting scanning curve

Figure 2.17 Illustration of pore filling and draining for primary drying and wetting scanning curve (modified from Mualem, 1973)

A series of dependent domain models were developed. The dependent domain models consider for the effects of pore blockage by adding two pore blockage functions, P_d and P_w , for drying and wetting process, respectively. Additional scanning curves are required for calibrating these models. There are two types of dependent models. Dependent models with similarity hypothesis, which include Mualem and Dagan (1975) (model-III), Mualem and Miller (1979), and Mualem (1984b), use Equation 2.16 or 2.17 to estimate scanning curves. Dependent models without similarity hypothesis, which include Everett (1967), Topp (1971), Poulouvassilis and Childs (1971), and Poulouvassilis and El-Ghamry (1978), requires additional measurement data for the model's calibration.

Besides the domain models to calculate the hysteresis of SWCC, there is another series of models called empirical models. The empirical models which can be grouped either using curve fitting techniques or correlation of two boundary curves are summarised in Figure 2.18.

Scott et al. (1983) developed an empirical model to describe SWCC hysteresis by using a curve-fitting technique. The model adopted van Genuchten (1980) SWCC equation (Equation 2.7). As shown in Figure 2.19, the scanning loop is not closed. The drying scanning curve does not meet at a point on the wetting scanning curve but departs from the boundary drying curve. Consequently, a pumping effect occurs when the suction undergoes a cyclic change (Jaynes, 1984). Kool and Parker (1987) simplified Scott et al. (1983) model and reduced the data requirements for the model calibration using a numerical technique. Huang et al. (2005) further improved Kool and Parker (1987) model by eliminating the pumping effect. Scott et al. (1983), Kool and

Parker (1987) and Huang et al. (2005) models were compared by Huang et al. (2005), and the results showed that the three models performed equally well in the estimation of the primary scanning curves, but the Kool and Parker (1987) model gave the worst accuracy for the scanning curves and Scott et al. (1983) model showed pumping effect for cyclic pressure variation. Huang et al. (2005) overcame the disadvantages of the two models by eliminating the pumping effect and achieving perfect closure at the reversal point of the scanning curve.

Figure 2.18 Summary of empirical models of SWCC hysteresis (modified from Pham, 2005)

Feng and Fredlund (1999) developed a hysteresis model by using a curve-fitting technique for the water retention curve equation shown here as Equation 2.21.

$$\theta(\psi) = \frac{ab+c\psi^d}{b+\psi^d} \quad (2.21)$$

where a , c = constants; and b , d = curve-fitting parameters, where b is related to AEV of boundary curves and d is related to the slope of the curve at the inflexion point.

Figure 2.19 Primary scanning SWCC loop estimated by Scott et al. (1983) model

Equation 2.21 is based on measurements of the hysteresis associated with ceramics used in thermal conductivity sensor for soil suction measurement. The parameter, a , was assumed to have the same value for both drying and wetting boundary curves. Parameter c is called the ceramic water content or the sensor reading at the ceramic tip at drying condition. Both a and c are constants. When the boundary drying curve is measured, parameters, a , and c , are known. Hence, there are only two unknown parameters, b and d , that need to be determined using two points on the boundary wetting curve. The two points on the boundary wetting curve are not specified. Pham et

al. (2003) recommended an approach to locate the two points on the boundary wetting curve, which are shown in Equations 2.22a and b.

$$\psi_{1,w} = \left(\frac{b_d}{10}\right)^{\frac{1}{d_d}} \quad (2.22a)$$

$$\psi_{2,w} = \psi_{1,w} - 2 \left\{ \left[\frac{b_d(a - \theta_{1,w})}{\theta_{1,w} - c} \right]^{\frac{1}{d_d}} - b_d^{\frac{1}{d_d}} \right\} \quad (2.22b)$$

where b_d, d_d = curve-fitting parameters for boundary drying curve; $\psi_{1,w}, \psi_{2,w}$ = two points on boundary wetting curve recommended soil suction; and $\theta_{1,w}$ = water content at point 1 on boundary wetting curve.

At the same time, Pham et al. (2003) also proposed to estimate the boundary drying curve from the boundary wetting curve and two points on the boundary drying curve. The Pham et al. (2003) model is a simplified version of the Feng and Fredlund (1999) model, which assumed that the boundary wetting and boundary drying curves are parallel at the inflexion point when the SWCCs are plotted on a semi-log plot. The curve-fitting parameter d in Feng and Fredlund (1999) model which controls the slopes of the curves in the model has the same value for both the boundary wetting curve and the boundary drying curve. The calibration of the model requires only one point on the boundary wetting curve when the boundary drying curve is known.

Pham et al. (2005) improved Feng and Fredlund (1999) model by correlating the parameters associated with the SWCCs by statistical analysis. The model only requires the boundary drying curve for calibration. The boundary wetting curve is estimated by the boundary drying curve. The ratio of the slopes of the boundary drying curve and the boundary wetting curve as well as the logarithmic suction difference between the

inflexion points of the boundary drying curve and the boundary wetting curve were studied and values were suggested.

Hanks et al. (1969) developed a hysteresis model by using a numerical empirical approach. Hanks et al. (1969) model is generally referred to as a linear model as the model assumes that the scanning curves between boundary drying and wetting curves are straight lines (linear relationship between water content and soil suction). Wheeler et al. (2003) used the same assumption of a linear relationship between soil suction and water content for the scanning curves.

Jaynes (1984) compared four numerical empirical hysteresis models (point method, linear method, slope method and Model II proposed by Mualem, 1974). Dane and Wierenga (1975) model is generally referred to as a point model as the model calculates the water content at a certain soil suction on the drying and wetting boundary curves. It introduces a weighting factor to describe the effects of drying and wetting boundaries on the drying and wetting scanning curves. The slope method (Jaynes, 1984) was developed from the point method (Dane and Wierenga, 1975) and uses the slopes of drying and wetting boundary curves instead of water content to estimate the slopes of the scanning curves. Jaynes (1984) concluded that the linear model (Hanks et al., 1969) and Model II (Mualem, 1974) are constructed to avoid pumping problems during the cyclic change of soil suction, and the point method (Dane and Wierenga, 1975) and slope method (Jaynes, 1984) are significantly affected by the numerous soil wetting-drying cycles. For the scanning curves estimation, the differences in existing scanning models are not significant (Jaynes, 1984; Yang et al., 2012).

Most of the domain models (Everett, 1954, 1955; Enderby, 1955, 1956; Poulovassilis, 1962; Philip, 1964; Poulovassilis and Childs, 1971; Topp 1971; Mualem, 1973; Mualem and Dagan, 1975; Poulovassilis and El-Ghamry, 1978; Mualem, 1984a and 1984b) and empirical models (Hanks et al., 1969; Dane and Wierenga, 1975; Scott et al., 1983; Kool and Parker, 1987) require boundary hysteresis curves to be measured. But a complete dataset, including drying and wetting boundary curves, and scanning curves, is seldom available, and the wetting curve measurement is difficult and time-consuming (Nimmo, 1992). To overcome the shortcoming of the models, Nimmo (1992) proposed a model that requires less measurement data. The model ignores the contact angle and hysteresis considers the “ink-bottle effect” only. The model introduced two parameters, v which describes the hysteretic pores fraction and, β which represents the body-to-neck size ratio of the largest water filled pores. The model applied the v and β into the PSD and used capillary theory to relate PSD and SWCC, to estimate the main wetting curve. The model requires the entire boundary drying curve and two measured points on the boundary wetting curve to determine the hysteresis loop and to calculate the scanning curves.

2.5.3 Summary

The reviewed hysteresis models have practical advantages. But all the models have the same disadvantage, which is calibration of the model requiring at least one boundary curve and some points on the other boundary curve. But the wetting curve measurement is difficult and time-consuming by most techniques (Nimmo, 1992), and hence, the boundary wetting curve is seldom available for calibration. In this research, a hysteresis model will be proposed to overcome the limitation of the common

disadvantage of the existing hysteresis models. Existing scanning curves models seem to work well and have no significant differences. Hence, scanning curves models are not considered in the research.

2.6 Research Gaps

The following research gaps have been identified through the literature review:

(i) Grain-size distribution (GSD) has a close relationship to SWCC. However, no explicit study has been done to show the relationship.

(ii) A number of GSD equations have been proposed for unimodal and bimodal GSD.

The existing GSD equations have been commonly determined by an optimisation method, where the GSD data are curve-fitted with a GSD equation using an optimisation technique. The GSD equation parameters obtained through curve-fitting may not be unique and are dependent on the optimisation technique (Wijaya and Leong, 2016). Correlations of these non-unique GSD parameters to SWCC become problematic.

(iii) Bimodal SWCC is a consequence of dual-porosity soil which arises mainly due to bimodal GSD (Zhang and Chen 2005; Satyanaga et al. 2013). In addition, compaction or aggregation may give rise to dual porosity as well (Li and Zhang, 2009; Satyanaga et al., 2013; Li et al., 2014). Pores in dual-porosity soils are largely governed by the arrangement of coarse grains and fine grains, which cause large pores (macro-pores) and small pores (micro-pores), respectively (Burger and Shackelford, 2001; Zhang and Chen, 2005). A bimodal SWCC is corollary to dual porosity. Dual porosity can arise due to bimodal particle size distribution. Bimodal GSD can be due to bimodal GSD or soil aggregations due to reconstitution. This

study only addresses bimodal GSD. Simple criteria to identify bimodal GSD soils which have bimodal SWCC have been proposed by Satyanaga et al. (2013) and Li et al. (2014). This research will evaluate the adequacy of these criteria and if necessary, to propose a new method to identify bimodal GSD soils that have bimodal SWCC using parameters of the GSD.

- (iv) A number of point-estimation PTFs have been proposed to estimate the water contents for suctions at 10, 33, 100 and 1500 kPa for soils of a specific region soil. It is "expected" that the PTF is not applicable to soils of other regions. Hence, it is necessary to examine the relevancy and usefulness of PTFs in unsaturated soil mechanics by evaluating numerous PTFs using data collated from literature. The data need to cover a wide region so that specificity bias of the PTFs is removed in the evaluation. Based on the evaluation, a simple method of estimating unimodal SWCC will be proposed.
- (v) A number of SWCC equations for unimodal SWCC have been evaluated by Leong and Rahardjo (1997a). Several bimodal SWCC equations have been proposed (Zhang and Chen, 2005; Carducci et al., 2011; Satyanaga et al., 2013; Li et al., 2014; and Wijaya and Leong, 2016; Zhang et al., 2017). Compared to the unimodal SWCC equations, bimodal SWCC equations have more unknown parameters. Hence, the determination of the parameters for bimodal SWCC equations is more complicated and time-consuming. Sometimes, numerical problem may occur during the optimisation process for curve fitting as the equation has too many unknown parameters. Bimodal SWCC equations will be evaluated to examine the difficulty of the curve fitting and to suggest improvement.

- (vi) A number of models for the hysteretic SWCC have been proposed. But most of the models require some measurement data of the wetting SWCC to calibrate the model. The laboratory determination of the SWCC is time-consuming, labour-intensive and expensive, and even more so for the wetting curve. This research aims to provide a simpler method to estimate the boundary wetting curve, which requires no measurement of the wetting SWCC measurement.
- (vii) All hysteresis models focused on unimodal SWCC. Currently, there is no hysteresis model for the bimodal SWCC. A new model to estimate the hysteretic bimodal SWCC will be proposed.

CHAPTER 3

GRAIN-SIZE DISTRIBUTION AND SOIL-WATER CHARACTERISTIC CURVE

3.1 Introduction

The grain-size distribution (GSD) is routinely measured in laboratory tests by sieve analysis and/or sedimentation analysis, which are standardised in ASTM D6913M-17 (ASTM, 2017a) and ASTM D7928-17 (ASTM, 2017b), respectively. The GSD is a fundamental physical property of soils. The GSD for a soil is defined as the relationship between percent passing by weight and the particle size (Fredlund et al., 2000). It is commonly used for soil classification. However, in recent decades, it is widely used for SWCC estimation (e.g., Gupta and Larson, 1979; Arya and Paris, 1981; Chin et al., 2010).

The GSD tests only provide a series of discrete points plotted on a semi-logarithmic graph. Hence a mathematical equation is needed to provide a continuous description of the GSD data. Fredlund et al. (2000) gave three benefits for a continuous GSD: (i), the fitting parameters of the GSD equations can be used for soil classification; (ii), the GSD equation can be used for the SWCC estimation (e.g., Haverkamp and Parlange, 1986); and (iii), the GSD equation can be used to represent the entire GSD curve from the measured data points.

Gardner (1956) proposed a two-parameter GSD equation which assumed that the grain-size PDF of soil is lognormal. Many researchers have subsequently assumed that

the grain-size PDF of soils is approximately lognormal (Shirazi and Boersma, 1984; Campbell, 1985; Buchuan, 1989; Wagner and Ding, 1994; Hwang and Choi, 2006; Satyanaga et al., 2013). The assumption of a lognormal grain-size PDF implies that the PDF is always symmetrical. In reality, the PDF is often not symmetrical (Fredlund et al., 2000).

It is believed that the shape of the GSD is similar to the SWCC (Arya and Paris, 1981; Fredlund et al., 2000a). Based on this similarity, researchers use the form of the SWCC equations for GSD equations. For example, Haverkamp and Parlange (1986) used the van Genuchten (1980) SWCC equation for the GSD equation. According to Leong and Rahardjo (1997a), the van Genuchten (1980) and Fredlund and Xing (1994) equations performed the best among the unimodal SWCC equations. It can be expected that GSD equations based on these two forms of SWCC equations will perform better than GSD equations based on forms of other SWCC equations. Hwang et al. (2002) reviewed seven GSD equations, which includes five equations based on lognormal grain-size PDF assumption. Huang et al. (2002) concluded that Fredlund et al. (2000) equation provided the best fit among the seven reviewed GSD equations. Meskini-Vishkaee and Davatgar (2018) compared Fredlund et al. (2000) GSD equation with three other GSD equations (Skaggs et al., 2001; Fooladmand et al., 2004; and Wu et al., 2009) using 263 soils. Meskini-Vishkaee and Davatgar (2018) found that Fredlund et al. (2000) GSD equation performed the best for all soil textural classes.

Generally, GSDs are categorised into three types: well-graded soils, uniform soils and gap-graded soils (Holtz and Kovacs, 1981). The well-graded or uniform soil GSD

can be represented by a unimodal GSD equation and the gap-graded soil GSD can be represented by a bimodal GSD equation.

This chapter proposes a GSD equation based on the Wijaya and Leong (2016) SWCC equation for both bimodal and unimodal soils. The evaluation of commonly used GSD equations is conducted in this chapter as well. Since many SWCC estimation methods are established based on the assumption that soils with similar GSD have similar SWCCs. In this chapter, the validity of this assumption is investigated.

3.2 Unimodal Grain-size Distribution Equations

3.2.1 Proposed unimodal grain-size distribution equation

Wijaya and Leong (2016) proposed a unimodal SWCC equation which allows all of the parameters to be determined graphically instead of using curve-fitting procedures. In this section, a GSD equation is proposed based on the form of Wijaya and Leong (2016) SWCC equation. Wijaya and Leong (2016) SWCC equation assumes that the unimodal SWCC curves are joined by three linear segments. Hence, the proposed GSD equation is shown below and the parameters are as defined in Figure 3.1.

$$P = 100 - S_1(D_{max} - D) - R_2(D)(S_2 - S_1) - R_3(D)(S_3 - S_2) \quad (3.1)$$

where $R_i(D) = \frac{1}{2} \langle (D_{max} - D) + \frac{1}{C_i} \ln \left\{ \frac{\cosh[C_i(D_i - D)]}{\cosh[C_i(D_{max} - D_i)]} \right\} \rangle$, $C_i = 2 / \log \left(\frac{d_i^-}{d_i^+} \right)$, d_i^+ and d_i^- are the grain size next to the two linear segments merging point, D is log value of d , d_{max} equals to the grain-size passing at 100%, P is the percentage, by mass, of particles passing a particular size, d is the sieve size in mm. The slopes can be determined directly from the graph.

Since all of the parameters for the proposed GSD equation can be obtained graphically, no curve-fitting procedure is needed and non-uniqueness of the parameters due to the optimisation algorithm is eliminated. However, curve-fitting procedure can still be employed, if desired, for the proposed equation by treating the slope parameters, S_1 to S_3 , as the unknown parameters.

Figure 3.1 Unimodal GSD parameters

3.2.2 Unimodal grain-size distribution equations

Hwang et al. (2002) compared seven GSD equations and concluded that the Fredlund et al. (2000) equation performed best among the evaluated GSD equations. It is not necessary to repeat their work in this thesis. In this section, only Fredlund et al.

(2000) GSD equation (FR equation), is compared with other GSD equations. The Fredlund et al. (2000) GSD equation is shown below:

$$P = \frac{1}{\left\{ \ln \left[e + \left(\frac{a_{gr}}{d} \right)^{n_{gr}} \right] \right\}^{m_{gr}}} \left\{ 1 - \left[\frac{\ln \left(1 + \frac{d_{rgr}}{d} \right)}{\ln \left(1 + \frac{d_{rgr}}{d_{min}} \right)} \right]^7 \right\} \quad (3.2)$$

where a_{gr} , n_{gr} , m_{gr} and d_{rgr} are fitting parameters, and d_{min} is the minimum grain size. In most cases, $d_{min} = 0.0001$ mm. The FR equation can be further simplified by considering $d_{rgr} = 100$ mm as suggested by Fredlund et al. (2000) to avoid curve over-fitting. In this study, d_{rgr} is assumed to be 100 mm.

Haverkamp and Parlange (1986) used the van Genuchten (1980) SWCC equation as the form for the GSD equation. The Haverkamp and Parlange-van Genuchten GSD equation (HV equation) is given by:

$$P = \frac{1}{\left[1 + \left(\frac{d_a}{d} \right)^n \right]^m} \quad (3.3)$$

where $m = 1 - \frac{1}{n}$, d_a and n are fitting parameters.

Recently, Satyanaga et al. (2013) proposed another GSD equation (SA equation) with parameters based on lognormal grain-size PDF assumption. The SA equation is given by:

$$P = \left\{ 1 - \left[0.15 \ln \left(1 + \frac{0.075}{d} \right) \right] \right\} \left[\operatorname{erfc} \frac{\ln \left(\frac{d_{max} - d_m}{d_{max} - d} \right)}{s_d} \right] \quad (3.4)$$

where $\operatorname{erfc} = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2} \right) dx$, d_{max} = the maximum grain size, d_m = the mean grain size, and s_d = the standard deviation of the grain size. The SA equation has

three parameters, d_m , d_{max} and s_d . If d_{max} is assumed known, the equation has only two parameters.

The proposed equation (Equation 3.1) will be compared with FR, HV, and SA equations using GSD of 33 soils from the UNSODA dataset (Nemes et al., 2001). The UNSODA database provides grain-size distribution, texture, dry density, specific gravity, porosity, organic content, saturated volumetric water content, and saturated and unsaturated coefficient of permeability for soils from 790 horizons all over the world. The soils are selected based on the following criteria: (i) only soils with data containing at least seven points for GSD are selected; (ii) only soils with data containing dry density are selected.

3.2.3 Evaluation of unimodal grain-size distribution equations

There are a number of performance indices to measure the performance of a model. The three most commonly used indices are the coefficient of determination, R^2 , the mean of the squared errors, MSE , and the root mean squared error, $RMSE$, as defined by Equations 3.5 to 3.7:

$$MSE = \frac{1}{N} \sum_{i=1}^N (\theta_i - \theta_i^{est})^2 \quad (3.5)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\theta_i - \theta_i^{est})^2} \quad (3.6)$$

$$R^2 = 1 - \frac{SSE}{SST} \quad (3.7)$$

where θ_i = measured volumetric water content; θ_i^{est} = estimated volumetric water content by PTFs; $\bar{\theta}_i$ = average of the measured volumetric water content; N = the number of the data; $SSE = \sum_{i=1}^N (\theta_i - \theta_i^{est})^2$; $SST = \sum_{i=1}^N (\theta_i - \bar{\theta}_i)^2$. For MSE and RMSE, a smaller value infers a better performance, whereas for R^2 , a value closer to 1 infers a better performance.

Another index named mean absolute error, *MAE*, is used by Meskini-Vishkaee and Davatgar (2018), which is defined by:

$$MAE = \frac{1}{N} |\theta_i - \theta_i^{est}| \quad (3.8)$$

The Akaike's information criterion (Akaike, 1973), *AIC*, defined by Equation 3.9, imposed penalties for additional parameters and is also a well-used performance index (e.g., Russo, 1988).

$$AIC = N \ln (SSE) + p \quad (3.9)$$

where p is the number of the equation parameters.

The index of agreement d which reflects the degree to which observations are accurately estimated (Willmott, 1981) is also included for a complete assessment of the model performance (Legates and McCabe, 1999; Khodaverdiloo et al., 2011). The index of agreement d is given as:

$$d = 1 - \frac{\sum_{i=1}^N (\theta_i^{est} - \theta_i)^2}{\sum_{i=1}^N [|\theta_i^{est} - \bar{\theta}| + |\theta_i - \bar{\theta}|]^2} \quad (3.10)$$

Each of the criteria has specific pros and cons, which have to be considered during model evaluation. Compared to *MAE*, R^2 , *MSE*, *RMSE*, and d , are preferred as the use

of squared errors accentuates the effects of larger errors, and are more suitable for unsaturated soil models' evaluation. Chai and Draxler (2014) also highlighted that *RMSE*, compared to *MAE*, avoid the use of absolute value which is highly undesirable in many situations. Only *AIC* considers the number of parameters in a model and is biased towards model with less parameters. Since R^2 , *MSE*, and *RMSE*, are correlated, only *RMSE*, *AIC* and *d* are used as the evaluation criteria.

Thirty-three soils from UNSODA database were used to evaluate the proposed, FR, HV and SA GSD equations. The soils used are listed in Table 3.1. The textures of the soils are presented in Figure 3.2.

Table 3.1 Soils used for evaluation of unimodal GSD equations

Soils from UNSODA (Nemes et al., 2001)	Sand/Silt/Clay content (%)
1210, 1212, 1213, 1260, 1241, 1342, 1360, 1380, 1400, 1430, 1460, 2211, 2220, 2221, 2492, 2532, 3020, 3030, 3050, 3160, 3191, 3194, 3202, 3210, 3214, 4010, 4020, 4110, 4120, 4180, 4251, 4283, 4284	Minimum: 1/1/1 Maximum: 98/88/58 Mean: 39/40/11

The FR, HV and SA equations were curve-fitted by minimising the value of *SSE*. For FR equation, as the parameter d_{rgr} has insignificant effect on the GSD curve, d_{rgr} is assumed to be 100mm as recommended by Fredlund et al. (2000). Hence, p in the *AIC* criterion is 3 instead of 4 for FR equation. For proposed equation, all five parameters can be directly estimated using graphical approach and $p = 5$ in the *AIC* criterion for the proposed GSD equation.

The performance of the four equations (FR, HV, SA and proposed equation) are presented in Table 3.2. Table 3.3 uses Leong and Rahardjo (1997a) ranking approach to carry out the evaluation. A smaller value of ranking indicates better overall

performance. It can be observed that the proposed GSD equation performed the best followed by FR, HV and then SA for all performance indices. Because the SA equation assumes that the grain-size PDF has a lognormal distribution, it can be expected that SA equation will not perform as well as other equations. For the proposed equation, the maximum grain size equals to the grain size at 100% passing. All the five parameters can be obtained graphically as described in Wijaya and Leong (2016). The performances of FR, HV, SA and proposed equation are illustrated in Figure 3.3 for soil 2532 from UNSODA.

Figure 3.2 Texture of soils shown in USDA textural triangle used for evaluation of the unimodal GSD equation

Figure 3.3 GSD equations fitting for Soil 2532 from UNSODA (Nemes et al., 2001)

3.2.4 Summary

Based on Wijaya and Leong (2016) unimodal SWCC equation, a unimodal GSD equation is proposed. The proposed GSD equation was compared with three other GSD equations, which are Fredlund et al. (2000) (FR), Haverkamp and Parlange (1986) -van Genuchten (HV) and Satyanaga et al. (2013) (SA) GSD equations, using 33 soils from UNSODA database (Nemes, et al., 2001). The results showed that the proposed model performs the best with the Fredlund et al. (2000) equation in a close second. But the proposed equation has some advantages over Fredlund et al. (2000) equation: (i), no curve-fitting technique is required as the parameters can be determined directly from the graph; and (ii) all parameters have clear physical meaning.

Table 3.2 Summary of the performance indices for GSD equations

Soil	AIC				RMSE				d			
	FR	HV	SA	P*	FR	HV	SA	P*	FR	HV	SA	P*
1210	-62	-29	-52	-49	0.004	0.042	0.007	0.008	1.00	1.00	1.00	1.00
1212	-65	-26	-41	-51	0.003	0.051	0.016	0.007	1.00	0.99	1.00	1.00
1213	-75	-33	-27	-56	0.001	0.030	0.043	0.005	1.00	1.00	0.99	1.00
1260	-96	-72	-78	-94	0.010	0.022	0.018	0.009	1.00	1.00	1.00	1.00
1241	-102	-67	-46	-93	0.008	0.026	0.051	0.010	1.00	1.00	0.99	1.00
1342	-40	-26	-34	-43	0.017	0.053	0.027	0.012	1.00	0.99	1.00	1.00
1360	-65	-40	-24	-33	0.003	0.019	0.056	0.025	1.00	1.00	0.98	1.00
1380	-31	-30	-21	-33	0.034	0.038	0.068	0.025	1.00	1.00	0.99	1.00
1400	-45	-47	-15	-41	0.012	0.012	0.106	0.014	1.00	1.00	0.92	1.00
1430	-47	-42	-26	-65	0.011	0.016	0.047	0.002	1.00	1.00	1.00	1.00
1460	-47	-40	-63	-80	0.016	0.026	0.006	0.002	1.00	1.00	1.00	1.00
2211	-62	-43	-47	-59	0.009	0.027	0.020	0.009	1.00	1.00	1.00	1.00
2220	-72	-54	-34	-58	0.024	0.044	0.079	0.035	1.00	0.99	0.98	1.00
2221	-61	-57	-35	-44	0.034	0.040	0.077	0.054	1.00	0.99	0.98	0.99
2492	-80	-70	-54	-85	0.007	0.011	0.022	0.005	1.00	1.00	1.00	1.00
2532	-54	-47	-39	-57	0.014	0.022	0.032	0.011	1.00	1.00	1.00	1.00
3020	-38	-39	-40	-43	0.028	0.028	0.024	0.017	1.00	1.00	1.00	1.00
3030	-47	-46	-12	-24	0.006	0.007	0.116	0.037	1.00	1.00	0.93	0.99
3050	-87	-84	-45	-85	0.015	0.017	0.055	0.015	1.00	1.00	0.99	1.00
3160	-52	-39	-44	-59	0.011	0.027	0.019	0.007	1.00	1.00	1.00	1.00
3191	-48	-46	-37	-66	0.015	0.018	0.028	0.004	1.00	1.00	1.00	1.00
3194	-46	-53	-30	-69	0.017	0.012	0.046	0.003	1.00	1.00	0.99	1.00
3202	-39	-23	-32	-94	0.025	0.072	0.039	0.001	1.00	0.99	1.00	1.00
3210	-58	-39	-58	-61	0.008	0.028	0.008	0.006	1.00	1.00	1.00	1.00
3214	-67	-45	-28	-87	0.004	0.019	0.052	0.001	1.00	1.00	0.99	1.00
4010	-61	-39	-57	-71	0.009	0.035	0.012	0.005	1.00	1.00	1.00	1.00
4020	-64	-46	-12	-63	0.008	0.024	0.142	0.008	1.00	1.00	0.98	1.00
4110	-78	-31	-46	-57	0.004	0.052	0.022	0.011	1.00	1.00	1.00	1.00
4120	-54	-53	-41	-57	0.014	0.016	0.029	0.011	1.00	1.00	0.99	1.00
4180	-47	-40	-48	-56	0.021	0.032	0.020	0.011	1.00	1.00	1.00	1.00
4251	-69	-41	-54	-51	0.004	0.024	0.010	0.011	1.00	1.00	1.00	1.00
4283	-36	-38	-40	-44	0.030	0.030	0.023	0.017	1.00	1.00	1.00	1.00
4284	-36	-36	-28	-78	0.030	0.033	0.050	0.002	0.99	0.99	0.99	1.00

P*=proposed GSD equation.

Table 3.3 Summary of the ranking of the GSD equations

Soil	AIC				RMSE				d				Soil Type ²
	FR	HV	SA	P ¹	FR	HV	SA	P ¹	FR	HV	SA	P ¹	
1210	1	4	2	3	1	4	2	3	1	4	2	3	Fine
1212	1	4	3	2	1	4	3	2	1	4	3	2	Fine
1213	1	3	4	2	1	3	4	2	1	3	4	2	Fine
1260	1	4	3	2	2	4	3	1	2	4	3	1	Fine
1241	1	3	4	2	1	3	4	2	1	3	4	2	Coarse
1342	2	4	3	1	2	4	3	1	2	4	3	1	Fine
1360	1	2	4	3	1	2	4	3	1	2	4	3	Fine
1380	2	3	4	1	2	3	4	1	2	3	4	1	Fine
1400	2	1	4	3	2	1	4	3	2	1	4	3	Fine
1430	2	3	4	1	2	3	4	1	2	3	4	1	Fine
1460	3	4	2	1	3	4	2	1	3	4	2	1	Coarse
2211	1	4	3	2	1	4	3	2	1	4	3	2	Coarse
2220	1	3	4	2	1	3	4	2	1	3	4	2	Fine
2221	1	2	4	3	1	2	4	3	1	2	4	3	Coarse
2492	2	3	4	1	2	3	4	1	2	3	4	1	Fine
2532	2	3	4	1	2	3	4	1	2	3	4	1	Fine
3020	4	3	2	1	3	4	2	1	3	4	2	1	Fine
3030	1	2	4	3	1	2	4	3	1	2	4	3	Fine
3050	1	3	4	2	1	3	4	2	1	3	4	2	Coarse
3160	2	4	3	1	2	4	3	1	2	4	3	1	Coarse
3191	2	3	4	1	2	3	4	1	2	3	4	1	Fine
3194	3	2	4	1	3	2	4	1	3	2	4	1	Fine
3202	2	4	3	1	2	4	3	1	2	4	3	1	Fine
3210	2	4	3	1	2	4	3	1	2	4	3	1	Fine
3214	2	3	4	1	2	3	4	1	2	3	4	1	Fine
4010	2	4	3	1	2	4	3	1	2	4	3	1	Fine
4020	1	3	4	2	2	3	4	1	2	3	4	1	Coarse
4110	1	4	3	2	1	4	3	2	1	4	3	2	Fine
4120	2	3	4	1	2	3	4	1	2	3	4	1	Fine
4180	3	4	2	1	3	4	2	1	3	4	2	1	Fine
4251	1	4	2	3	1	4	2	3	1	4	2	3	Coarse
4283	4	3	2	1	4	3	2	1	4	3	2	1	Fine
4284	2	3	4	1	2	3	4	1	2	3	4	1	Fine
Sum	59	106	111	54	60	107	111	52	60	107	111	52	

Note: P¹=proposed GSD equation; Soil Type²: based on Chin et al. (2010) classification.

3.3 Bimodal Grain-size Distribution Equations

3.3.1 Proposed bimodal grain-size distribution equation

Wijaya and Leong (2016) proposed a bimodal SWCC equation, which allows all of the parameters to be determined graphically instead of using curve-fitting procedures. This is most important for bimodal SWCCs, as bimodal SWCC equations have more curve-fitting parameters, which may require more time and have higher potential for numerical error in the curve-fitting procedure. Another problem for existing bimodal SWCC equations is the non-uniqueness of parameters from curve-fitting, which means that different combinations of the parameters may produce the same SWCC (Wijaya and Leong, 2016; Zhai and Rahardjo, 2012). The non-uniqueness gives problems when correlating parameters with other soils properties. Wijaya and Leong (2016) bimodal SWCC equation overcomes these problems by using graphical approach to determine the bimodal SWCC equation parameters.

In this section, the form of Wijaya and Leong (2016) bimodal SWCC equation is used for a new bimodal GSD equation. Wijaya and Leong (2016) SWCC equation assumes that the bimodal SWCC consists of five linear segments. Similarly, the proposed GSD equation is shown below and the parameters are defined in Figure 3.4:

$$P = 100 - S_1(D_{max} - D) - R_2(D)(S_2 - S_1) - R_3(D)(S_3 - S_2) - R_4(D)(S_4 - S_3) - R_5(D)(S_5 - S_4) \quad (3.11)$$

where $R_i(D) = \frac{1}{2} \langle (D_{max} - D) + \frac{1}{C_i} \ln \left\{ \frac{\cosh[C_i(D_i - D)]}{\cosh[C_i(D_{max} - D_i)]} \right\} \rangle$, $C_i = 2 / \log \left(\frac{d_i^-}{d_i^+} \right)$, d_i^+ and d_i^- are the grain size next to the two linear segments merging point, D is log value of d ,

d_{max} equals to the size passing percentage is 100%. The slopes, S_i , can be determined directly from the graph.

All of the parameters for proposed GSD equation can be obtained graphically from the GSD plot. Therefore, the parameters do not rely on the optimisation algorithm. However, curve-fitting technique can still be used for the proposed equation by considering the slopes as the unknown parameters if desired.

Figure 3.4 Bimodal GSD parameters

3.3.2 Bimodal grain-size distribution equations

Several bimodal SWCC equations have been proposed and are summarised in Table 2.3. The bimodal SWCC equations will be evaluated later. In this section, bimodal

GSD equations based on FR and SA bimodal SWCC equations shown in Equations 3.12 and 3.13, respectively, will be discussed.

$$P = \left\langle \frac{w}{\left\{ \ln \left[\exp(1) + \left(\frac{a_{gr1}}{d} \right)^{n_{gr1}} \right] \right\}^{m_{gr1}}} + \frac{1-w}{\left\{ \ln \left[\exp(1) + \left(\frac{a_{gr2}}{d} \right)^{n_{gr2}} \right] \right\}^{m_{gr2}}} \right\rangle \left\{ 1 - \left[\frac{\ln \left(1 + \frac{d_{rgr}}{d} \right)}{\ln \left(1 + \frac{d_{rgr}}{d_{min}} \right)} \right]^7 \right\} \quad (3.12)$$

where a_{gr1} , n_{gr1} and m_{gr1} are fitting parameters for coarse soils particle sub-curve; w is the weightage of the larger soil particle sub-curve; a_{gr2} , n_{gr2} and m_{gr2} are fitting parameters for finer soils particle sub-curve. In this study, d_{rgr} and d_{min} are assumed to be 100 mm and 0.0001 mm, respectively, as recommended by Fredlund et al. (2000). Hence, FR bimodal GSD equation has seven unknown parameters.

$$P = \left\{ 1 - \left[0.15 \ln \left(1 + \frac{0.075}{d} \right) \right] \right\} \left[w \operatorname{erfc} \frac{\ln \left(\frac{d_{max1} - d_{m1}}{d_{max1} - d} \right)}{s_{d1}} + (1 - w) \operatorname{erfc} \frac{\ln \left(\frac{d_{max2} - d_{m2}}{d_{max2} - d} \right)}{s_{d2}} \right] \quad (3.13)$$

where $\operatorname{erfc} = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{x^2}{2} \right) dx$, d_{max1} = the maximum grain size for coarse soils particle sub-curve, d_{m1} = the mean grain size for coarse soils particle sub-curve, s_{d1} = the standard deviation of the grain size for coarse soils particle sub-curve; d_{max2} = the maximum grain size for finer soils particle sub-curve, d_{m2} = the mean grain size for finer soils particle sub-curve, and s_{d2} = the standard deviation of the grain size for finer soils particle sub-curve. The SA bimodal GSD equation has seven unknown parameters and assumes that the bimodal GSD curve is made up of two sub-curves. It can be expected that SA bimodal GSD equation will not perform as well as the FR

bimodal GSD equation mentioned in Section 3.2. Hence, in this study, only the FR bimodal GSD equation will be compared with the proposed equation.

3.3.3 Evaluation of bimodal grain-size distribution equations

Thirty-three soils from literature were used to evaluate the proposed and FR bimodal GSD equations. The soils are listed in Table 3.4. The texture of the soils is presented in Figure 3.5.

The FR bimodal GSD equation is curve fitted by minimising the value of SSE as defined in Equation 3.7. As the parameters d_{rgr} and d_{min} have insignificant effect on the FR bimodal GSD equation, it is assumed that d_{rgr} and d_{min} equal to 100 mm and 0.0001 mm, respectively, as recommended by Fredlund et al. (2000). Hence, p in the AIC criterion is 7 instead of 9 for the FR bimodal GSD equation. For proposed bimodal GSD equation, all parameters are directly estimated from the plot. These include five slopes and four particle sizes of the intersection points. Hence, p in the AIC criterion is 9 for proposed bimodal GSD equation.

The performances of FR and proposed bimodal GSD equations are summarised in Table 3.5. The ranking approach (Leong and Rahardjo, 1997a) was used to compare the two equations. It can be observed that the proposed equation performed better than FR equation for all three performance indices of AIC , $RMSE$ and d . On top of the performance, the proposed equation requires no curve-fitting technique to determine the parameters and hence, and avoids the difficulty of fitting large number of parameters as well as non-uniqueness of the parameters. All the parameters in the proposed equation have easily understood physical meaning as well. The FR and

proposed bimodal GSD equations are illustrated in Figure 3.6 for soil SLR-1-b from Agus et al. (2001).

Table 3.4 Soil used for evaluation of bimodal GSD equation

Soils	Sand/Silt/Clay content (%)	Reference
181, 359, 189, 303, 349, 74, 134, 140, 253, 254, 255, 320, 340, 98, 360, 237, 175, 145, 326, 250	Max: 90/58/74	Andersson and Wiklert, (1972)
Sandy ML, SC with gravel; SM with gravel	Min: 15/2/1	Li, 2009
M1, M2, M3, RS	Mean: 56/20/15	Rahardjo et al., 2012
NTU-1-a, NTU-1-b, NTU-3-a, NTU-3-b, SLR-1-a, SLR-1-b		Agus et al., 2001

Figure 3.5 Texture of soils shown in USDA textural triangle used for evaluation of the bimodal GSD equation

Table 3.5 Results of bimodal GSD equation comparison

S/N	Soil	Performance Indices						Ranking					
		AIC		RMSE		d		AIC		RMSE		d	
		FR	P*	FR	P*	FR	P*	FR	P*	FR	P*	FR	P*
1	181	-55	-67	0.011	0.005	1.00	1.00	2	1	2	1	2	1
2	359	-57	-69	0.009	0.004	1.00	1.00	2	1	2	1	2	1
3	189	-52	-58	0.013	0.008	1.00	1.00	2	1	2	1	2	1
4	303	-57	-58	0.010	0.008	1.00	1.00	2	1	2	1	2	1
5	349	-49	-77	0.015	0.003	1.00	1.00	2	1	2	1	2	1
6	74	-58	-62	0.009	0.007	1.00	1.00	2	1	2	1	2	1
7	134	-47	-70	0.017	0.004	1.00	1.00	2	1	2	1	2	1
8	140	-60	-75	0.008	0.003	1.00	1.00	2	1	2	1	2	1
9	253	-57	-76	0.010	0.003	1.00	1.00	2	1	2	1	2	1
10	254	-68	-67	0.005	0.005	1.00	1.00	1	2	2	1	2	1
11	255	-67	-78	0.006	0.003	1.00	1.00	2	1	2	1	2	1
12	320	-37	-50	0.029	0.012	1.00	1.00	2	1	2	1	2	1
13	340	-44	-55	0.020	0.009	1.00	1.00	2	1	2	1	2	1
14	98	-40	-60	0.025	0.007	1.00	1.00	2	1	2	1	2	1
15	360	-65	-73	0.006	0.003	1.00	1.00	2	1	2	1	2	1
16	237	-51	-63	0.014	0.006	1.00	1.00	2	1	2	1	2	1
17	175	-41	-33	0.024	0.032	1.00	1.00	1	2	1	2	1	2
18	145	-47	-38	0.017	0.025	1.00	1.00	1	2	1	2	1	2
19	326	-48	-63	0.015	0.006	1.00	1.00	2	1	2	1	2	1
20	250	-42	-53	0.022	0.011	1.00	1.00	2	1	2	1	2	1
21	Sandy ML	-116	-116	0.015	0.014	1.00	1.00	2	1	2	1	2	1
22	SC with	-48	-121	0.059	0.010	0.99	1.00	2	1	2	1	2	1
23	gravel	-87	-119	0.023	0.010	1.00	1.00	2	1	2	1	2	1
24	SM with	-158	-167	0.007	0.005	1.00	1.00	2	1	2	1	2	1
25	gravel	-110	-118	0.012	0.009	1.00	1.00	2	1	2	1	2	1
26	M1	-138	-136	0.008	0.008	1.00	1.00	1	2	1	2	1	2
27	M2	-155	-105	0.003	0.012	1.00	1.00	1	2	1	2	1	2
28	M3	-102	-102	0.010	0.009	1.00	1.00	2	1	2	1	2	1
29	RS	-97	-128	0.011	0.004	1.00	1.00	2	1	2	1	2	1
30	NTU-1-a	-83	-85	0.015	0.013	1.00	1.00	2	1	2	1	2	1
31	NTU-1-b	-114	-145	0.010	0.004	1.00	1.00	2	1	2	1	2	1
32	NTU-3-a	-94	-58	0.007	0.025	1.00	1.00	1	2	1	2	1	2
33	NTU-3-b	-60	-93	0.015	0.003	1.00	1.00	2	1	2	1	2	1
34	SLR-1-a												
35	SLR-1-b												
		Sum						60	39	61	38	61	38

P*=proposed bimodal GSD equation.

Figure 3.6 Bimodal GSD equation fitting soil SLR-1-b (data from Agus et al., 2001)

3.3.4 Summary

A bimodal GSD equation based on Wijaya and Leong (2016) bimodal SWCC equation was proposed. The proposed bimodal GSD equation was compared with Fredlund et al. (2000) bimodal GSD equation using 33 soils from the literature. The results showed that the proposed equation performs better than Fredlund et al. (2000) equation. The proposed equation has several advantages over Fredlund et al. (2000) equation: (i), no curve-fitting technique is required as all parameters in the proposed equation can be determined from the plot; and (ii) all parameters of the proposed equation have clear physical meaning.

3.4 Grain-size Distribution and Soil-water Characteristic Curve

3.4.1 Introduction

The soil-water characteristic curve (SWCC) can be used as a means of deriving and linking soil behaviours, such as permeability, shear strength and volume change, as reviewed in Chapter 2. The SWCC controls the state at which each engineering behaviour is calculated, which is important for modelling more than one aspect of soil behaviour in a single analysis (Fredlund et al., 1996). There are several methods available for the SWCC determination. The most accurate method to determine a SWCC is through laboratory test. Because of the cost, time and effort in conducting the SWCC experiment, indirect methods are often preferred.

As reviewed in Section 2.3.2, a number of methods to estimate SWCC from GSD have been proposed. All the methods premised that the soils with similar GSD have similar soil structure, and hence should present similar SWCCs. Based on the assumption, researchers (e.g., Fredlund et al., 1996; Minasny and McBratney, 2006) proposed SWCC estimation method by using databases, such as, UNSODA (Nemes et al., 2001), SoilVision (SoilVision, 2002), HYPRES (Wosten et al., 1999), WISE (Batjes, 1996), GRIZZLY (Haverkamp et al., 1997) and the Australian soil hydraulic properties database (Minasny and McBratney, 2002). In this section, the validity of the assumption will be examined based on soil classification and GSD using data from Andersson and Wiklert (1972) and UNSODA (Nemes et al., 2001).

3.4.2 SWCCs for soils with similar GSD

As reviewed in Section 2.3.1, there are five commonly used engineering soil classification systems for GSD. In this chapter, USDA classification system is used.

Seven soil textures and 15 groups each consisting of four soils from Andersson and Wiklert, (1972) and UNSODA (Nemes et al., 2001) were used for the study. The soils used in this study are listed in Table 3.6.

Table 3.6 Soils used for the study of SWCCs for soils with similar soil classification

Group	Soil Type	Soil	ρ_d Mg/m ³	Sand %	Silt %	Clay %	Reference
1a	Sand	141	1.48	87	9	4	Andersson and Wiklert (1972)
		142*	1.47	87	10	3	
		12	1.46	90	9	1	
		348	1.59	90	6	4	
1b	Sand	160	1.42	97	3	0	Andersson and Wiklert (1972)
		161	1.42	97	3	0	
		280*	1.72	98	2	0	
		281	1.69	100	0	0	
1c	Sand	1063	1.55	93	3	4	UNSODA (Nemes, et al., 2001)
		1061	1.59	91	5	4	
		1100	1.74	89	6	5	
		1060*	1.55	93	5	2	
2a	Sandy Loam	1010	1.64	83	14	3	UNSODA (Nemes, et al., 2001)
		1011	1.52	82	15	3	
		1012*	1.4	82	15	3	
		1013	1.49	85	13	2	
2b	Sandy Loam	29	1.43	60	32	8	Andersson and Wiklert (1972)
		35*	1.39	59	37	4	
		250	1.2	58	33	9	
		38	1.53	56	38	6	
2c	Sandy Loam	138*	1.4	71	15	14	Andersson and Wiklert (1972)
		139	1.53	74	13	13	
		332	1.6	73	15	12	
		347	1.49	78	11	11	
3a	Silt Loam	6*	1.48	24	70	6	Andersson and Wiklert (1972)
		230	1.58	25	68	7	
		18	1.32	22	70	8	
		15	1.29	23	73	4	

Group	Soil Type	Soil	ρ_d Mg/m ³	Sand %	Silt %	Clay %	Reference
3b	Silt Loam	60	1.36	24	65	11	Andersson and Wiklert (1972)
		61	1.36	22	66	12	
		18*	1.32	22	70	8	
		334	1.72	22	69	9	
3c	Silt Loam	76*	1.25	28	61	11	Andersson and Wiklert (1972)
		77	1.26	28	62	10	
		79	1.64	26	63	11	
		63	1.22	25	66	9	
4	Sandy Clay Loam	1113	1.49	64	9	27	UNSODA (Nemes, et al., 2001)
		1114*	1.58	62	11	27	
		1116	1.7	60	12	28	
		1117	1.75	66	10	24	
5a	Loam	157*	1.6	56	23	21	Andersson and Wiklert (1972)
		186	1.62	52	25	23	
		187	1.48	53	24	23	
		188	1.58	59	22	19	
5b	Loam	2401	1.25	43	40	17	UNSODA (Nemes, et al., 2001)
		2400*	1.08	46	38	16	
		2450	1.11	45	39	16	
		2531	1.46	41	43	16	
6	Clay Loam	24	1.12	20	48	32	Andersson and Wiklert (1972)
		25	1.26	22	46	32	
		112	1.5	24	41	35	
		111*	1.36	24	40	36	
7a	Clay	123*	1.16	5	9	86	Andersson and Wiklert (1972)
		124	1.38	5	9	86	
		151	1.34	5	8	87	
		240	1.34	6	9	85	
7b	Clay	240	1.34	6	9	85	Andersson and Wiklert (1972)
		241	1.27	5	12	83	
		242*	1.29	5	13	82	
		249	1.16	6	12	82	

Note: * indicates reference soil used in Figure 3.7.

Each group of soils have similar sand, silt and clay contents. The SWCCs for the soils in the same textural group are compared to study the assumption that soils with similar GSD have similar SWCC. As the dry densities of the soils in the same textural group are different, it is more convenient to use degree of saturation as the water content for the SWCC comparison. The soils are selected based on two criteria: (i) the differences of the sand, silt and clay contents of the soils in the same textural group are less than 5%; (ii) the SWCCs of the soils contain at least five data points. One soil in each group is selected as the reference soil. The fitted SWCC for the reference soil is used to evaluate the fitted SWCCs for the other three soils in the group. In other word, the deviations of the fitted SWCC of the other three soils at the measured suction levels from the fitted SWCC of the reference soil are computed in term of R^2 . The comparison is summarised in Figure 3.7.

Figure 3.7 SWCC for soils with similar soil classification

Based on Figure 3.7, the SWCCs for each groups of soils are similar with $R^2 > 0.9$. Hence, it can be concluded that soils with similar GSD have similar SWCC when plotted with respect to degree of saturation. However, based on Figure 3.7, the GSD is not the only factor influencing SWCC. For example, as shown in Figure 3.7(a), Soil 141 is the one closest to the Soil 142 in GSD and dry density in Group 1(a), but the SWCC of Soil 141 is the most different from the SWCC of Soil 142 in Group 1(a). These two soils are from different soil horizons and they may have different stress history according to Andersson and Wiklert (1972). Similarly in Figure 3.7(m) for Group 6 soils, Soil 112 have the closest GSD with Soil 111, but the SWCC of Soil 111 is the most different from the SWCC of Soil 112 in Group 6. The difference in the SWCCs may be caused by the different stress history or particle shape. However, the GSD is still the main factor influencing the SWCC.

3.5 Conclusion

Unimodal and bimodal grain-size distribution (GSD) equations were proposed based on Wijaya and Leong (2016) unimodal and bimodal SWCC equations. The proposed GSD equations used parameters that can be determined directly from the GSD plot. Hence, a unique GSD equation can be established. All the parameters in the proposed equations have clear physical meaning. The proposed unimodal GSD equation was compared with three other unimodal GSD equation using 33 soils from UNSODA database (Nemes, et al., 2001). It was observed that the proposed unimodal GSD equation performed the best while Fredlund et al. (2000) GSD equation is a close second. The proposed bimodal GSD equation was compared with Fredlund et al. (2000) bimodal GSD equation using 33 soils from the literature. The results showed that the

proposed bimodal GSD equation performed better than Fredlund et al. (2000) bimodal GSD equation.

Soils from Andersson and Wiklert (1972) and UNSODA (Nemes, et al., 2001) database were used to evaluate the assumption that the SWCC for soils with similar GSD are similar. The results showed that the soils with similar GSD have similar SWCC which was plotted with respect to degree of saturation. Hence, the GSD is uniquely related to SWCC and can be used to estimate the SWCC.

CHAPTER 4

METHOD TO PREDICT MODALITY OF SOIL-WATER CHARACTERISTIC CURVES FOR SOILS WITH BIMODAL GRAIN-SIZE DISTRIBUTION

4.1 Introduction

Determination of the SWCC has been standardised in ASTM D6836-16 (2016). According to ASTM D6836-16 (2016), there are five laboratory test methods to determine SWCCs: hanging column (Method A), pressure chamber with volumetric measurements (Method B), pressure chamber with gravimetric measurements (Method C), chilled mirror hygrometer (Method D) and centrifuge (Method E). These five methods determine the water content of the soils at various suction levels. A summary of the five methods together with applicable suction range and suction levels are shown in Table 4.1. Methods A and E are used for lower suctions (0 to 80 and 0 to 120 kPa, respectively), Methods B and C are used for intermediate suctions (100 to 1500 kPa), and Method D is used for higher suctions (>1000 kPa) of the SWCC. However, the recommended suction levels in ASTM D6836-16 (2016) are too many for unimodal SWCC and too few for bimodal SWCC. Using the SWCC data for BLOCO 4 from Mendes (2008) as shown in Figure 4.1, the SWCC determined using Methods A, B or C and E gave the points indicated by the triangle, square and circle markers, respectively. Thus, the SWCC for BLOCO 4 may be erroneously interpreted as a

unimodal SWCC if only one test method is used. Using Methods B or C to determine the SWCC for BLOCO 4 will lead to erroneously interpreting the SWCC as unimodal or incorrect air-entry value for the first sub-curve (Figure 1.1(b)). The SWCC for BLOCO 4 can be obtained correctly if either Method A is used with Method B or C, or Method E is used with Method B or C. However, this entails the use of two separate apparatuses and the number of suction levels recommended in Method A is excessive. Due to cost and time constraints, it is important to conduct SWCC tests at only the critical suction levels so that the “correct” SWCC can be obtained in the shortest possible time.

(a) Using Method A

(b) Using Method B or C

(c) Using Method E

Figure 4.1 Expected SWCC using suggested suction levels for test methods in ASTM D6836-16 (2016) for soil BLOCO 4 from Mendes (2008)

Table 4.1 Summary of ASTM D6836-16 (2016) methods for determining SWCC

	Method A ¹	Methods B & C	Method D	Method E ²
Applicable Suction Range	0 to 80 kPa	0 to 1500 kPa	500 kPa to 100 MPa	0 to 120 kPa
	0.05	10		0.5
	0.2	50		2
	0.4	100		8.5
	1	300		34
	2	500		120
Suction Level Applied (kPa)	4	1000	Not specified	
	6	1500		
	10			
	15			
	20			
	40			

Note: 1, Converted from 5, 20, 40, 100, 200, 400, 600, 1000, 1500, 2000, and 4000 mm of water. 2, Estimated from angular velocities of 100, 200, 400, 800 and 1500 rpm and may vary with centrifuge.

The relationship between SWCC and grain-size distribution (GSD) has been demonstrated in the previous chapter. Bimodal SWCC is a consequence of dual-porosity soils which arises mainly due to bimodal GSD (Zhang and Chen 2005; Satyanaga et al. 2013). In addition, compaction or other features such as cracks in the soil may give rise to dual porosity as well (Li and Zhang, 2009; Satyanaga et al., 2013; Li et al., 2014). Pores in dual-porosity soils are largely governed by the arrangement of coarse grains and fine grains, which cause large pores (macro-pores) and small pores

(micro-pores), respectively (Burger and Shackelford, 2001; Zhang and Chen, 2005). This study only addresses bimodal GSD. Simple criteria to identify bimodal GSD soils which have bimodal SWCCs have been proposed by Satyanaga et al. (2013) and Li et al. (2014). The inadequacy of these criteria is demonstrated in this chapter. The objective of this chapter is to propose an improved method to identify bimodal GSD soils that have bimodal SWCC using parameters of the GSD. This method can be used as a guide to reduce the number of suction levels needed in a SWCC test according to the test methods in ASTM D6836-16 (2016).

4.2 Existing Criteria

Condappa et al. (2008) found that the soils in tropical and subtropical regions have bimodal SWCC when they contain high sand (Sa) and clay (Cl) size fractions compared to silt (Si) fraction such that

$$Sa \geq Si$$

$$Cl \geq Si \tag{4.1}$$

The bimodal SWCC zone in the soil textural triangle identified by Equation 4.1 is given by the shaded zone in Figure 4.2. Following the shape similarity of GSD and SWCC, such soils are expected to exhibit bimodal hydraulic properties (Condappa et al., 2008). However, this is shown to be not the case for all bimodal GSD soils (Satyanaga et al., 2013; Li et al., 2014).

Figure 4.2 Bimodal soil zone shown as shaded region of the USDA textural triangle (after Condappa et al. 2008)

Satyanaga et al. (2013) suggested that bimodal GSD soils which satisfy the following criteria exhibit bimodal SWCC:

$$\rho_d \leq 1.44 \text{ Mg/m}^3, \text{ or}$$

$$\rho_d \geq 1.44 \frac{\text{Mg}}{\text{m}^3} \text{ and } w_{sat} < 18\% \text{ or } w_{sat} > 33\% \quad (4.2)$$

where ρ_d = dry density; and w_{sat} = saturated gravimetric water content. The above criteria can be better illustrated using Figure 4.3.

Figure 4.3 Criteria for unimodal and bimodal SWCCs by Satyanaga et al. (2013)

Li et al. (2014) suggested that bimodal GSD soils have a bimodal SWCC characterised by $\psi_{a2}/\psi_t > 3$ where ψ_{a2} is the air-entry value for water stored in micro pores and ψ_t is the residual suction for water stored in macro pores. Li et al. (2014) found regression equations for ψ_{a2} and ψ_t with D_{10} , D_{30} , D_{60} and e as shown in Equations 4.3 and 4.4, respectively, where D_{10} , D_{30} , D_{60} are the grain size diameters at 10%, 30% and 60% passing, respectively, and e is void ratio.

$$\psi_{a2} = \frac{0.44D_{60}^{0.4}}{D_{10}^{0.9}D_{30}^{1.2}} \quad (4.3)$$

$$\psi_t = \frac{1.4^e D_{60}^{0.14}}{2.118D_{10}^{0.14}D_{30}} \quad (4.4)$$

The criteria that Li et al. (2014) proposes for bimodal GSD soils having bimodal SWCC can be simplified to the following expression:

$$\frac{D_{60}^{0.26}}{D_{10}^{0.13}D_{30}^{0.2}1.4^e} > 3.22 \quad (4.5)$$

4.3 Development of Classification Tree

Separation of bimodal GSD soils into those with unimodal and bimodal SWCC can be treated as a classification problem. To develop the classification tree, the soil database from Andersson and Wiklert (1972) was used. The database consists of 385 soils from different geographical regions of Sweden. The soils were from 82 soil profiles and were subdivided into top soils from depths of 0 to 200 mm and subsoils from depths of 20 to 100 cm. The soil type ranges from sands to clays with clay content as high as 80%. However, only 226 soils in the database which have bimodal grain-size distribution (bimodal GSD soils) were used to develop the classification tree. The data in the database includes dry density ρ_d , soil particle density ρ_s , grain-size distribution (GSD), and SWCC. The SWCCs of the soils were determined using the hanging column method from 0 to 10 kPa suctions, pressure extractor method from 10 to 1500 kPa suction and relative humidity chamber for suctions greater than 1500 kPa. In general, the SWCC tests were conducted on 100 mm long and 70 mm diameter soil cores sampled with the same sampling method as Lewan and Jansson (1996) and Rowell (1996) for 0 to 500 kPa suction and on disturbed samples for suctions greater than 500 kPa. The soil properties are summarised in Table 4.2.

Table 4.2 Summary of soil properties selected from the database

Total Number of Soils	226	
Range of Soil Properties (Min~Max/Mean)	e	0.53~1.91/0.92
	ρ_d (Mg/m ³)	0.85~1.77/1.42
	Sand Content (%)	0~100/23.6
	Silt Content (%)	0~87.5/34.5
	Clay Content (%)	0~87/38.3

A soil can exhibit bimodal GSD as long as there is a gap in its GSD curve. The gap suggested by Condappa et al. (2008) is only for one particular group of soils with bimodal GSD that is dominant in the tropical and subtropical regions. The soil texture of the 226 soils used in this study is shown in Figure 4.4. Square markers are for bimodal GSD soils with unimodal SWCC, while circle markers are for soils with bimodal SWCC. Figure 4.4 shows the disagreement of the results suggested by Condappa et al. (2008) which is presented in Figure 4.2. The 226 soils are uniformly distributed in the USDA soil textural triangle. A better descriptor for the gap in grain-size distribution is defined using the frequency plot rather than the cumulative frequency GSD plot. For the frequency plot, the grain-size bin width is set according to the grain size intervals in BS/MIT classification system, i.e., 0.0006-0.002-0.006-0.02-0.06-0.2-0.6-2-6-20 mm. The grain sizes corresponding to the two peaks of the grain size frequency plot are defined as major and minor peak grain sizes, respectively, as illustrated in Figure 4.5. The percentages of grains at the major and minor peak grain sizes are denoted as MaP and MiP , respectively. The span Y between the major and minor peak grain sizes is defined as:

$$Y = \log_{10} \left(\frac{MaP}{MiP} \right) \quad (4.6)$$

where Y is positive if the major peak grain size is greater than the minor peak particle size, and negative vice versa. As Y becomes greater, the possibility of the existence of macro-pores and micro-pores increases. The existence of macro-pores and micro-pores also depends on the relative proportions of grains with major and minor peak grain sizes. Bimodal SWCCs are attributed to the existence of macro-pores and micro-pores. It has been suggested that macro-pores are formed by large grains and

micro-pores are formed from small grains (Burger and Shackelford, 2001; Miguel and Bonder, 2012). Bimodal SWCC is the consequence if the major peak grain size is coarser than the minor peak grain size and there are insufficient fine grains to fill the macro-pores created by the coarse grains completely. On the contrary, if the major peak grain size is finer than the minor peak grain size, the finer grains are more likely to fill the macro-pores created by the fewer coarse grains and the soil will most likely exhibit a unimodal SWCC.

Figure 4.4 Texture of 226 bimodal GSD soils in the USDA textural triangle used to develop classification tree (circle marker: soil having unimodal SWCC; square marker: soil having bimodal SWCC)

Figure 4.5 Definition of Y, major and minor peak particle sizes in a frequency grain-size distribution plot

Figure 4.6 Flowchart for the classification tree implemented in Matlab

Besides Y , MaP and MiP , other grain-size parameters used to develop the classification tree includes clay, silt, and sand (Cl , Si , and Sa) contents, D_{10} , D_{30} , D_{60} and void ratio e as adopted by others. The classification tree was developed using the method of classification and regression trees (CART) (Clark and Pregibon, 1992; Wosten, et al., 2001; Pachepsky et al., 2006). More specifically, the CART algorithm based on Breiman et al. (1984) was adopted and programmed in software MATLAB Statistics Toolbox (The MathWorks, Inc., 2015) to develop the classification tree. The flowchart for the CART algorithm is shown in Figure 4.6. The split criterion adopted is the measure of node impurity, Gini's Diversity Index (GDI):

$$GDI = 1 - \sum_i [p(i)]^2 \quad (4.7)$$

where the sum is over the classes i at the node, and $p(i)$ is the observed fraction of classes with class i that reaches the node (The MathWorks, Inc., 2015). A node with just one class has $GDI = 0$; otherwise, $GDI > 0$. For each of the two child nodes, the binary split of the remaining predictors is examined. Splitting stops when one of the following conditions is satisfied:

- (i) The node is pure, which means that it only contains observations of one class, i.e., $GDI = 0$;
- (ii) There are less than 10 observations in the data;
- (iii) A split imposed on the node produces children with less than 1 observation; or
- (iv) The number of nodes exceeds five. Boschi and Rodrigues (2014) recommended that the number of nodes should not exceed five for ease of explanation and implementation.

The quality of the classification tree is determined using the k -fold cross-validation approach (Burman, 1989). In the k -fold cross-validation approach, the data are subdivided into k sample populations of roughly equal sizes and the modelling process is repeated k times, leaving one sample population out each time for validation purposes. The use of k -fold cross-validation approach is to avoid establishing a biased model from the database (Twarakavi et al., 2009). A reliable classification tree can be obtained with $k = 10$ when the data is greater than 100 (Borra and Ciaccio, 2010). Hence, a $k = 10$ cross-validation approach was selected in this study. Briefly, the procedure to establish the classification tree is as follows:

- (i) Classify all the SWCCs in the database as either bimodal or unimodal.
- (ii) Determine the parameters, D_{10} , D_{30} , D_{60} , Cl , Si and Sa from the cumulative grain-size distribution curve. In this study, only soils with $MiP \geq 5\%$ are considered as bimodal GSD soils.
- (iii) Re-plot the cumulative grain-size distribution curve as a frequency distribution plot and determine the major peak grain size percentage MaP , minor peak grain size percentage MiP , and Y .
- (iv) Use variables MaP , MiP , Y , D_{10} , D_{30} , D_{60} , Cl , Si , Sa and void ratio (e) as tentative predictors to develop the classification tree (Model 1).
- (v) Evaluate accuracy of the classification tree as ratio of total number of correct classification and total number of data.
- (vi) Steps (iv) and (v) are repeated using $/Y/$ instead of Y to check if using $/Y/$ results in an improved classification tree (Model 2).

For each set of data, three models, which are “Simple Tree”, “Medium Tree” and “Complex Tree”, can be developed. The maximum split numbers for these three types of tree are 4, 20, and 100, respectively. As recommended by Boschi and Rodrigues (2014), the number of nodes should not exceed five for ease of explanation and implementation. Hence, only the “Simple Tree” is developed in this study.

In developing the classification tree, there is a possibility of class imbalance which is related to the number of observations in one class being more than that of the other class (Batista et al., 2004). As a result, the developed classification tree may nearly always predict the class with the greater number of observations. In this study, of the 226 bimodal GSD soils 30.5% have bimodal SWCC and the remaining 69.5% have unimodal SWCC. Hence, the data is imbalanced. To overcome class imbalance, an over-sampling approach called SMOTE (Chawla et al., 2002) was used to create synthetic over-sampling of the minority class which in this study is the soils with bimodal SWCCs.

Models 1 and 2 of the classification trees are shown in Figure 4.7. The performance of the two models is summarised in Table 4.3. Model 2 showed better performance and gave good accuracy. Hence, Model 2 was selected as the classification tree to determine the type of SWCC for bimodal GSD soils.

Table 4.3 Summary of the performance of the two models

Model	Overall Accuracy	Unimodal Accuracy	Bimodal Accuracy
1	71.70%	78.30%	65.80%
2	83.20%	80.20%	85.80%

(a) Model 1

(b) Model 2

Figure 4.7 Models 1 and 2 of classification trees for bimodal GSD soils

The proposed classification tree (Model 2) uses three parameters ($|Y|$, e and MaP) compared to two parameters (ρ_d and w_s) in Satyanaga et al. (2013) and four parameters (C_c , C_u , D_{10} and e) in Li et al. (2014). The proposed classification tree has only three nodes which means that the classification tree is simple and easy to use. From Figure 6, it can be observed that when $|Y|$ is less than 1.74, the type of SWCC is determined by

void ratio (soil structure), otherwise the SWCC is determined by *MaP* (soil texture). A bimodal SWCC may still be possible if the soil has a large void ratio (>1.085). For bimodal GSD soils with $|Y|$ greater than 1.74, a bimodal SWCC is possible if the *MaP* is less than 68.5%. When *MaP* is greater than 68.5%, there are insufficient grains with the minor grain size (*MiP*) to form the micro-pores and hence, the SWCC is unimodal.

4.4 Evaluation of Proposed Classification Tree against Criteria Proposed by Others

Sixty soils, which include 30 bimodal GSD soils with unimodal SWCC and 30 bimodal GSD soils with bimodal SWCC from the literature, were used to evaluate the proposed classification tree. The textures of the 60 soils in the USDA textural triangle are shown in Figure 4.8. These 60 soils are from various countries. The 60 soils were also used to evaluate Satyanaga et al. (2013) and Li et al. (2014) criteria for comparison. The GSDs of the 60 soils are used together with the proposed bimodal GSD equation (Equation 3.12) described in Chapter 3 to plot the frequency grain-size distribution plot.

The results are summarised in Table 4.4. The proposed model correctly classified 24 of the 30 bimodal GSD soils with unimodal SWCC (80%) while Satyanaga et al. (2013) only correctly classified 13 of the 30 bimodal GSD soils (43.3%) and Li et al. (2014) correctly classified 22 of the 30 bimodal GSD soils with unimodal SWCC (73.3%). The proposed classification tree correctly classified 25 of the 30 bimodal GSD soils with bimodal SWCC (83.3%) while Satyanaga et al. (2013) correctly classified 23 of the 30 bimodal GSD soils (76.7%) and Li et al. (2014) correctly classified 7 of the 30 bimodal GSD soils (23.3%). Overall, the accuracies of the proposed classification tree, Satyanaga et al. (2013) and Li et al. (2014) are 81.7%, 60.0% and 48.3%,

respectively. Hence, the proposed classification tree showed a better performance than Satyanaga et al. (2013) and Li et al. (2014) to determine the type of SWCC for bimodal GSD soils. Bimodal SWCC requires more SWCC data to define, hence the more critical consequence is the wrong classification of bimodal GSD soils with bimodal SWCC as unimodal SWCC. Nonetheless, the proposed model performed equally well for classification of bimodal GSD soils with unimodal and bimodal SWCCs.

Figure 4.8 Texture of 60 bimodal GSD soils in the USDA textural triangle for evaluation (circle marker: soil having bimodal SWCC; square marker: soil having unimodal SWCC)

Table 4.4 Evaluation of proposed classification tree, Satyanaga et al. (2013) and Li et al. (2014) criteria

S/N	Soil No.	Reference	SWCC type	Predicted *			S/N	Soil No.	Reference	SWCC type	Predicted *		
				M1	M2	M3					M1	M2	M3
1	Edosaki Sand	Gallage and Uchimura (2010)	U	U	B	U	31	ND-Z=1.5 m	Miguel and Bonder (2011)	B	B	B	U
2	1022	Nemes et al., 2001	U	U	B	U	32	ND-Z=2.5 m	Miguel and Bonder (2011)	B	B	B	U
3	1023	Nemes et al., 2001	U	U	B	U	33	ND-Z=4.3 m	Miguel and Bonder (2011)	B	B	B	U
4	1131	Nemes et al., 2001	U	U	B	U	34	P1C650	Mendes (2008)	B	B	B	U
5	1450	Nemes et al., 2001	U	U	B	U	35	P1C250	Mendes (2008)	B	B	B	U
6	2340	Nemes et al., 2001	U	U	B	B	36	M1C600	Mendes (2008)	B	U	U	U
7	2351	Nemes et al., 2001	U	U	B	U	37	Bloco 4	Mendes (2008)	B	B	B	U
8	3033	Nemes et al., 2001	U	U	B	B	38	Bloco 3	Mendes (2008)	B	B	B	B
9	3260	Nemes et al., 2001	U	U	B	U	39	P1A30	Mendes (2008)	B	B	B	B
10	P1C500b	Mendes (2008)	U	U	U	B	40	M2A30	Mendes (2008)	B	B	B	B
11	2-1-VT-A	Jauhiainen (2004)	U	U	B	U	41	Bloco 1	Mendes (2008)	B	B	B	U
12	2-1-VT-B1	Jauhiainen (2004)	U	B	B	B	42	M2A50	Mendes (2008)	B	B	B	U
13	3-1-OMT-A	Jauhiainen (2004)	U	U	B	U	43	M1C800	Mendes (2008)	B	B	B	U
14	3-3-OMT-C	Jauhiainen (2004)	U	U	U	U	44	Bloco 2	Mendes (2008)	B	B	B	U
15	16-1-MT-C	Jauhiainen (2004)	U	U	U	U	45	M2A30	Mendes (2008)	B	B	U	B
16	25-1-MT-A	Jauhiainen (2004)	U	U	U	U	46	M2B500	Mendes (2008)	B	U	B	U
17	RS	Indrawan et al. (2006)	U	B	U	B	47	P1C350	Mendes (2008)	B	B	B	B
18	RM-75	Indrawan et al. (2006)	U	B	U	B	48	NTU-4-b	Agus (2001)	B	B	U	U
19	RM-50	Indrawan et al. (2006)	U	U	U	U	49	NTU-4-c	Agus (2001)	B	B	U	U
20	BG-8.5m	Aung et al. (2001)	U	B	B	U	50	NTU-1-a	Agus (2001)	B	B	U	U
21	BG-13m	Aung et al. (2001)	U	U	U	U	51	NTU-1-b	Agus (2001)	B	B	U	U
22	NTU-2-b	Agus (2001)	U	U	B	U	52	NTU-3-b	Agus (2001)	B	B	U	B
23	SLR-1-a	Agus (2001)	U	U	B	U	53	SLR-1-a	Agus (2001)	B	B	B	U
24	NTU-3-a	Agus (2001)	U	U	U	U	54	SLR-1-b	Agus (2001)	B	U	B	U
25	SLR-2-b	Agus (2001)	U	B	B	B	55	SLR-2-a	Agus (2001)	B	B	B	B
26	LA-a	Agus (2001)	U	B	U	B	56	SLR-2-c	Agus (2001)	B	U	B	U
27	YS-a	Agus (2001)	U	U	U	U	57	LA-b	Agus (2001)	B	B	B	U
28	SK-5	Goh (2012)	U	U	U	U	58	LA-c	Agus (2001)	B	B	B	U
29	SK-10	Goh (2012)	U	U	U	U	59	LA-d	Agus (2001)	B	U	B	U
30	SK-17	Goh (2012)	U	U	B	U	60	YS-b	Agus (2001)	B	B	B	U
Accuracy for bimodal soils with unimodal SWCC (%):				80.0	43.3	73.3	Accuracy for bimodal soils with bimodal SWCC (%):				83.3	76.7	23.3

Note: M1: Proposed classification tree; M2: Satyanaga et al. (2013); M3: Li et al. (2014)

4.5 Number of Suction Levels in A SWCC Test

Compared to a bimodal SWCC, a unimodal SWCC can be obtained using fewer suction levels. The number of suction levels recommended in ASTM D6836-16 (2016) is excessive for unimodal SWCC and insufficient for bimodal SWCC. The number of suction levels required for bimodal SWCC is more than that required for unimodal SWCC. The minimum suction ranges for coarse-grained and fine-grained soils are about two and four log cycles, respectively. The minimum number of data points in a SWCC can be estimated to be at least equal to M , the number of parameters in a SWCC equation. Most unimodal SWCC equations have M between 3 and 5 (Leong and Rahardjo, 1997a) while bimodal SWCC equations have M between 8 and 10 (e.g., Gitirana and Fredlund, 2004; Zhang and Chen, 2005; Satyanaga et al. 2013). Considering the minimum number of data points and suction range, it is reasonable that two and three suction levels per log cycle are sufficient to obtain unimodal and bimodal SWCCs, respectively. Therefore, it is recommended that the suggested suction levels in ASTM D6836-16 (2016) be modified and separated into unimodal and bimodal SWCCs as shown in Table 4.5. To illustrate the application of Table 4.5, Soil BLOCO 4 from Mendes (2008) is used. Soil BLOCO 4 has $|Y| = 2.00$, $e = 1.414$ and $\text{MaP} = 21\%$. According to the classification tree in Figure 4.7(b), it has a bimodal SWCC. Hence, the SWCC data points for soil BLOCO 4 should be obtained using the recommended suction levels for a soil with bimodal SWCC in Table 4.5 using Method B or C. The resulting SWCC is shown in Figure 4.9 and the correct type of SWCC can be obtained using only one test method.

Table 4.5 Recommended suction levels for unimodal and bimodal SWCCs following ASTM D6836-16 (2016) SWCC test methods

Method	A		B & C		D		E ¹	
Applicable Suction Range	0 to 80 kPa		0 to 1500 kPa		500 kPa to 100 MPa		0 to 120 kPa	
SWCC Type	U	B	U	B	U	B	U	B
Suction Level Applied (kPa)	0.2	0.2	1	1	500	500	0.2	0.2
	1	0.5	10	4	1000	1000	1	0.5
	10	1	100	10	10000	2000	10	1
	30	2	500	40	50000	5000	40	2
	80	5	1500	100	100000	10000	120	5
		10		200		20000		10
		30		500		50000		50
		80		1500		100000		120

Note: 1, Suction levels are suggestions only and depend on centrifuge; 2, U=Unimodal SWCC; 3, B=Bimodal SWCC.

Figure 4.9 Expected SWCC for soil BLOCO 4 from Mendes (2008) using recommended suction levels in Table 4.5 for Method B or C

4.6 Conclusion

Bimodal GSD soils may not always exhibit bimodal SWCC. In this chapter, a classification tree to determine the type of SWCC for bimodal GSD soils was developed based on 226 bimodal GSD soils from Sweden. The proposed classification tree only needs three predictors $|Y|$, MaP and e . The classification tree was evaluated using an independent data set of 60 bimodal GSD soils from a number of countries. The classification tree was shown to outperform existing criteria to distinguish bimodal GSD soils with bimodal SWCC from bimodal GSD soils with unimodal SWCC. More importantly, the classification tree provides a simple guide for planning the SWCC test. The recommended suction levels for determining SWCC in ASTM 6835-16 (2016) are excessive for unimodal SWCC and too few for bimodal SWCC. Hence, it is possible to use a more suitable number of suction levels for SWCC tests with the aid of the classification tree. The number of suction levels in the SWCC tests is recommended to be different for unimodal and bimodal SWCC tests.

CHAPTER 5

POINT ESTIMATION PEDOTRANSFER FUNCTIONS FOR THE SOIL-WATER CHARACTERISTIC CURVE

5.1 Introduction

The SWCC can be either directly determined by laboratory tests or indirectly estimated by pedotransfer function (PTF) using routinely measured soil properties, such as grain-size distribution (GSD), index properties and dry density or void ratio.

The tests that directly measure the SWCCs are not easily accessible and are often extremely time-consuming and labour intensive. Many indirect methods of estimating the SWCC using routinely measured physical properties, such as, GSD, index properties and dry density or void ratio, have been proposed (e.g., Arya and Paris, 1981; Rawls et al., 1982; Saxton et al., 1986; Tomasella and Hodnett, 1998; Scheinost et al., 1997; Mayr and Jarvis, 1999; Minasny et al., 1999). Such methods are commonly referred to as pedotransfer functions (PTFs). There is another group of indirect SWCC estimation approach named data mining approach. This approach normally requires a very large data set for the model calibration. This approach is not considered in this study.

According to Minasny et al. (1999), the PTFs for estimating SWCC can be divided into three types: point, parametric and physico-empirical PTFs. The point-estimation PTFs estimate water content at one suction level using GSD or other physical

properties of the soil. The parametric-estimation PTFs assume that the SWCC can be adequately described by a SWCC equation and relate the parameters in the SWCC equation with GSD and other physical properties of the soil. The physico-empirical PTFs can be considered as a subset of point-estimation PTFs as physico-empirical PTFs also estimate the water content at one suction level. The most commonly used SWCC equations in parametric-estimation PTFs are Brooks and Corey (1964) equation, van Genuchten (1980) equation and Fredlund and Xing (1994) equation (Chin et al., 2010). The accuracy of parametric estimation PTFs can be improved greatly if a single point measurement of the SWCC is incorporated. Such one-point methods have been proposed by Vanapalli and Catana (2005), Catana et al. (2006), Houston et al. (2006) and Chin et al. (2010). The approaches that can be used to determine the SWCCs are summarised in Figure 5.1.

Figure 5.1 Approaches used to determine SWCC (modified from Fredlund, 2006)

Point-estimation PTFs estimate the soil water content at one suction level. Point-estimation PTFs were mainly developed to determine the available soil water for plants and crops (Minasny et al., 1999). The most common suction levels used in point-

estimation PTFs are at 10, 33 kPa (corresponding to field capacity) and at 1500 kPa (corresponding to permanent wilting point). In some point-estimation PTFs, 4 (or 3) kPa and 100 kPa were also used. For coarse-grained soils, the water content at 4 (or 3) kPa is very important. Hence, point-estimation PTFs at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa will be discussed in this chapter.

5.2 Point-estimation Pedotransfer Functions

Point-estimation PTFs assume that the water content at a particular suction is dependent on the soil texture (sand, silt, clay and organic contents) and/or soil structure (dry density or void ratio). The most common method used to develop point-estimation PTFs is to employ multiple linear regression to determine the relationship of water content at one particular suction with other soil properties (Minasny et al., 1999). Table 5.1 to Table 5.5 summarize the regression coefficients of point-estimation PTFs for point-estimation PTFs at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa, respectively. In Table 5.1 to Table 5.5, Sa, Si, Cl and OC represent sand, silt, clay and organic contents, respectively, and ρ_d is dry density, which is defined as the ratio of the mass of the soil solids, M_s , to the total volume of soil, V :

$$\rho_d = \frac{M_s}{V} \quad (5.1)$$

The number of point-estimation PTFs at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa reviewed are 3, 11, 18, 5 and 22, respectively. Table 5.1 to Table 5.5 are non-exhaustive but the number of PTFs at each suction is indicative that the more common PTFs were developed for suctions at 33 and 1500 kPa which correspond to the field capacity and the permanent wilting point, respectively.

Several observations can be made on Table 5.1 to Table 5.5. The coefficient for Cl is narrowly bounded between 0.3 and 0.4, while coefficients for Sa , Si and OC showed greater variation. The coefficient for ρ_d increases as soil suction increases for the same PTF. Almost all the PTFs have negative coefficient for Sa which implies that water content decreases as Sa increases.

5.3 Evaluation of Point-estimation PTFs

The point-estimation PTFs at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa were evaluated using a soil database comprising of 250 soils which includes 60 soils from Andersson and Wiklert (1972), 60 soils from Jauhiainen (2004) and 130 soils from UNSODA (Nemes, et al., 2001). The data collated from the literature covers a wide region so that specificity bias of the PTFs is eliminated in the evaluation. The soil properties of the database are summarised in Table 5.6. As soils in Jauhiainen (2004) did not include the soil particle density and the water content at 33 kPa, the soil particle density for all of Jauhiainen (2004) soils was assumed to be 2.7 Mg/m^3 and was excluded from the evaluation of the point-estimation PTFs at 33 kPa. The textures of the 250 soils in the USDA textural triangle are shown in Figure 5.2. All the soil types are covered by this evaluation dataset. Only some soils contain negligible organic content. Hence, the organic content was assumed to be zero for all the soils.

Table 5.1 Summary of the regression coefficients of point-estimation PTFS suction of 4 (or 3) kPa

S/N	Reference	Regression Coefficients						Soil Source
		Sa	Si	Cl	OC	ρ_d	Intercept	
		(%)	(%)	(%)	(%)	(Mg/m ³)	(-)	
1	Gupta and Larson (1979)	0.71	1.024	1.01	0.633	-32.12	-	America
2	Rawls et al. (1982)	-0.4	-	-	1.000	-13.15	78.99	America
3	Tomasella and Hodnett (1998)*	-	0.552	0.26	-		18.50	Brazil

*Point-estimation PTF estimated water content at suction of 3 kPa

Table 5.2 Summary of the regression coefficients of point-estimation PTFS suction of 10 kPa

S/N	Reference	Regression Coefficients						Intercept	Soil Source
		Sa	Si	Cl	OC	ρ_d	ρ_d^2		
		(%)	(%)	(%)	(%)	(Mg/m ³)	(Mg ² /m ⁶)	(-)	
1	Pidgeon (1972)*^		0.18	0.34	1.65	-	-	5.32	Uganda
2	Hall et al. (1977)	-	0.15	0.41	-	-8.32	-	27.87	UK
3	Lal (1979)*	-0.30	-	-	-	-	-	36.40	Nigeria
		-	-	0.30	-	-	-	10.20	
4	Gupta and Larson (1979)	0.50	0.85	0.89	0.50	-24.23	-	-	America
5	Rawls et al. (1982)	-0.30	-	0.23	3.17	-	-	41.18	America
6	Batjes (1996)	-	0.40	0.53	3.18	-	-	-	-
7	van den Berg et al. (1997)	-	0.21	0.35	1.76	-	-	10.88	America, Africa and Asia
8	Tomasella and Hodnett (1998)	-	0.54	0.32	-	-	-	9.81	Brazil
9	Dashtaki et al. (2010)	-0.38	-	-	-	12.4	-	34.3	Iran

Note: (1) Sand: 50-2000 μm (* 20-2000 μm); Silt: 2-50 μm (* 2-20 μm); Clay: < 2 μm .

(2) ^estimated water content is gravimetric water content.

Table 5.3 Summary of the regression coefficients of point-estimation PTFs at suction of 33 kPa

S/N	Reference	Regression Coefficients						Intercept (-)	Soil Source
		Sa (%)	Si (%)	Cl (%)	OC (%)	ρ_d (Mg/m ³)	Si* ρ_d (%*Mg/m ³)		
1	Rawls et al. (1982)	-0.20	-	0.36	2.99	-	-	25.76	America
2	Pidgeon (1972)*^	-	0.17	0.32	1.58	-	-	3.80	Uganda
3	Hall et al. (1977)	-	0.12	0.45	-	-5.95	-	20.81	UK
4	Lal (1979)*	-	-	0.40	-	-	-	6.50	Nigeria
		-0.30	-	-	-	-	-	33.40	
5	Gupta and Larson (1979)	0.31	0.59	0.80	0.22	-14.34	-	-	America
6	Aina and Perisawamy (1985)	-0.55	-	-	-	-0.13	-	57.88	Nigeria
7	Beke and MacCormick (1985)	-0.50	-	-	-	-	-0.23	64.37	Canada
8	Dijkerman (1988)	-0.35	-	-	-	-	-	36.97	West Africa
9	Manrique et al. (1991)^	-	-	0.23	-	-28.44	-	57.84	America and
									Other Countries
10	Batjes (1996)	-	0.46	0.30	2.07	-	-	-	-
11	Tomasella and Hodnett (1998)	-	0.43	0.40	-	-	-	4.05	Brazil
12	Oliveira et al. (2002)*^	-	0.33	0.39	-	-	-		Brazil
13	Mohamed and Ali (2006)	-0.14	-	-	-	-	-	37.68	Tunisia
14	Reichert et al. (2009)*^	-	0.29	0.29	0.93	-4.80	-	10.60	Brazil
15	Dashtaki et al. (2010)	-0.283	-	-	-	17.1	-	14.1	Iran

Note: (1) Sand: 50-2000 μm (* 20-2000 μm); Silt: 2-50 μm (* 2-20 μm); Clay: < 2 μm .

(2) ^estimated water content is gravimetric water content.

Table 5.4 Summary of the regression coefficients of point-estimation PTFs at suction of 100 kPa

S/N	Reference	Regression Coefficients					Intercept	Soil Source
		Sa (%)	Si (%)	Cl (%)	OC (%)	ρ_d (Mg/m ³)		
1	Gupta and Larson (1979)	0.16	0.36	0.72	2.39	-5.76	-	America
2	Rawls et al. (1982)	-	0.14	0.55	2.51	-	2.81	America
3	Tomasella and Hodnett (1998)	-	0.37	0.35	-	-	3.20	Brazil
4	Reichert et al. (2009)*^	-0.08	0.15	0.23	1.08	-	10.20	Brazil
5	Dashtaki et al. (2010)	-0.31	-	-	-	14.30	12.20	Iran

Note: (1) Sand: 50-2000 μm (* 20-2000 μm); Silt: 2-50 μm (* 2-20 μm); Clay: < 2 μm .

(2) ^estimated water content is gravimetric water content.

Table 5.5 Summary of the regression coefficients of point-estimation PTFs at suction of 1500 kPa

S/N	Reference	Regression Coefficient						Intercept (-)	Soil Source
		Sa (%)	Si (%)	Cl (%)	OC (%)	ρ_d (Mg/m ³)	Other Term		
1	Petersen et al. (1968)	-	-	0.08	-	-	0.01 Cl ²	1.74	America
2	Pidgeon (1972)*^	-	0.19	0.39	0.90	-	-	-4.19	Uganda
3	Hall et al. (1977)	-	-	0.84	-	-	-0.01 Cl ²	1.48	UK
4	Lal (1979)*	-	-	0.30	-	-	-	0.60	Nigeria
		-0.24	-	-	-	-	-	28.40	
5	Gupta and Larson (1979)	-0.01	0.11	0.58	0.22	2.67	-	-	America
6	Rawls et al. (1982)	-	-	0.50	1.58	-	-	2.60	America
7	Aina and Perisawamy (1985)	-	-	0.31	-	-	-	0.21	Nigeria
8	Beke and MacCormick (1985)	-	-	0.66	-	-	1.07 OC* ρ_b	4.41	Canada
9	Dijkerman (1988)^	-	-	0.39	-	-	-	0.74	West Africa
		-0.35	-	-	-	-	-	36.97	
10	Manrique et al. (1991)^	-	-	0.37	-	-	-	2.41	America and Other Countries
11	Rajkai and Varallyay (1992)	-	-	0.36	0.22	-	-	1.39	Hungary
12	Batjes (1996)	-	0.12	0.36	1.61	-	-	-	-
13	van den Berg et al. (1997)	-	0.21	0.27	1.76	-	-	3.83	America, Africa and Asia
14	Tomasella and Hodnett (1998)	-	0.15	0.40	-	-	-	0.91	Brazil
15	Oliveira et al. (2002)*^	0.04	0.15	0.34		3.09	-	-	Brazil
16	Mohamed and Ali (2006)	-	-	0.23	-	-	-	10.61	Tunisia
17	Reichert et al. (2009)*^	-	0.17	0.32	0.91	2.60	-	-4.00	Brazil
18	Dashtaki et al. (2010)	-	-	0.33	-	-	-	6.20	Iran

Note: (1) Sand: 50-2000 μm (* 20-2000 μm); Silt: 2-50 μm (* 2-20 μm); Clay: < 2 μm .

(2) ^Estimated water content is gravimetric water content.

To evaluate the performance of the PTFs, as discussed in Section 3.2.3, the performance indices *RMSE* (Equation 3.6) and index agreement *d* (Equation 3.10) are chosen to evaluate the point-estimation PTFs. The values of *RMSE* and *d* for the evaluated soils are shown in Table 5.7 to Table 5.11 for suctions of 4 (or 3), 10, 33, 100 and 1500 kPa, respectively. to present the measured water content versus estimated water content at suction of 4 (or 3) kPa, 10 kPa, 33 kPa, 100 kPa and 1500 kPa, respectively.

Table 5.6 Summary of soil properties for the database to evaluate the point-estimation PTFs

	Void Ratio	ρ_d (Mg/m ³)	Sa (%)	Si (%)	Cl (%)
Min	0.490	1.050	0	0	0
Max	2.511	1.812	100	88	85
Average	0.914	1.440	57	26	17

Table 5.7 Evaluation indices for point-estimation PTFs at suction of 4 (or 3) kPa

S/N	Point-estimation PTF	<i>RMSE</i>	<i>d</i>
1	Gupta and Larson (1979)	0.070	0.885
2	Rawls et al. (1982)	0.083	0.864
3	Tomasella and Hodnett (1998)*	0.083	0.854

*Point-estimation PTF estimated water content at suction of 3 kPa

Table 5.8 Evaluation indices for point-estimation PTFs at suction of 10 kPa

S/N	Point-estimation PTF	<i>RMSE</i>	<i>d</i>
1	Pidgeon (1972)	0.098	0.851
2	Lal (1979) (Sand)	0.111	0.803
3	Lal (1979) (Clay)	0.103	0.723
4	Rawls et al. (1982)	0.065	0.939
5	Batjes (1996)	0.116	0.845
6	van den Berg et al. (1997)	0.093	0.841
7	Tomasella and Hodnett (1998)	0.071	0.928
8	Gupta and Larson (1979)	0.053	0.956
9	Hall et al. (1977)	0.069	0.907
10	Minasny et al. (1999)	0.091	0.824
11	Dashtaki et al. (2010)	0.071	0.916

Table 5.9 Evaluation indices for point-estimation PTFs at suction of 33 kPa

S/N	Point-estimation PTF	<i>RMSE</i>	<i>d</i>
1	Rawls et al. (1982)	0.074	0.940
2	Pidgeon (1972)	0.105	0.862
3	Lal (1979) Clay	0.107	0.810
4	Lal (1979) Sand	0.111	0.848
5	Dijkerman (1988)	0.081	0.936
6	Batjes (1996)	0.103	0.883
7	Tomasella and Hodnett (1998)	0.058	0.963
8	Oliveira et al. (2002)	0.084	0.933
9	Mohamed and Ali (2006)	0.121	0.671
10	Hall et al. (1977)	0.077	0.920
11	Gupta and Larson (1979)	0.052	0.971
12	Aina and Perisawamy (1985)	0.062	0.968
13	Beke and MacCormick (1985)	0.072	0.939
14	Manrique et al. (1991)	0.106	0.803
15	Reichert et al. (2009)	0.077	0.930
16	Arruda et al. (1987)	0.097	0.888
17	Minasny et al. (1999)	0.094	0.876
18	Dashtaki et al. (2010)	0.101	0.837

Table 5.10 Evaluation indices for point-estimation PTFs at suction of 100 kPa

S/N	PTF	<i>RMSE</i>	<i>d</i>
1	Gupta and Larson (1979)	0.056	0.962
2	Rawls et al. (1982)	0.064	0.946
3	Reichert et al. (2009)	0.080	0.910
4	Tomasella and Hodnett (1998)	0.079	0.910
5	Dashtaki et al. (2010)	0.102	0.834

Table 5.11 Evaluation indices for point-estimation PTFs at suction of 1500 kPa

S/N	Point-estimation PTF	<i>RMSE</i>	<i>d</i>
1	Petersen et al. (1968)	0.072	0.865
2	Rawls et al. (1982)	0.036	0.966
3	Pidgeon (1972)	0.077	0.886
4	Hall et al. (1977)	0.039	0.958
5	Lal (1979) Clay	0.080	0.793
6	Lal (1979) Sand	0.081	0.790
7	Aina and Perisawamy (1985)	0.080	0.794
8	Dijkerman (1988) Clay	0.042	0.958
9	Manrique et al. (1991)	0.039	0.962
10	Rajkai and Varallyay (1992)	0.060	0.886
11	Batjes (1996)	0.050	0.921
12	van den Berg et al. (1997)	0.070	0.837
13	Tomasella and Hodnett (1998)	0.047	0.936
14	Mohamed and Ali (2006)	0.060	0.828
15	Gupta and Larson (1979)	0.073	0.898
16	Beke and MacCormick (1985)	0.068	0.915
17	Oliveira et al. (2002)	0.135	0.689
18	Reichert et al. (2009)	0.111	0.763
19	Minasny et al. (1999)	0.083	0.827
20	Arruda et al. (1987) (1987)	0.125	0.709
21	Dashtaki et al. (2010)	0.041	0.936

Figure 5.2 Texture of soils shown in USDA textural triangle to evaluate point-estimation PTFs

Figure 5.3 Measured and estimated volumetric water contents using point-estimation PTFs at suction of 4 (or 3) kPa: (a), Gupta and Larson (1979); (b), Rawls et al. (1982); and (c), Tomasella and Hodnett (1998)

Figure 5.4 Measured and estimated volumetric water contents using point-estimation PTFs at suction of 10 kPa: (a), Pidgeon (1972); (b), Lal (1979) (Sand); (c), Lal (1979) (Clay); (d), Rawls et al. (1982); (e), Batjes (1996); (f), van den Berg et al. (1997); (g), Tomasella and Hodnett (1998); (h), Gupta and Larson (1979); (i), Hall et al. (1977); (j), Minasny et al. (1999); and (k), Dashtaki et al. (2010)

According to Table 5.7 and Figure 5.3, Gupta and Larson (1979) PTF performed the best among the three point-estimation PTF for suction of 4 (or 3) kPa. Rawls et al.

(1982) and Tomasella and Hodnett (1998) PTFs over-estimated water content for the high clay content soils. For very high coarse content soils, i.e., $S_a > 95\%$, all three PTFs over-estimated the water content.

According to Table 5.8 and Figure 5.4, Gupta and Larson (1979) performed best among the 11 evaluated point-estimation PTFs at 10 kPa. Pidgeon (1972), Lal (1979) (Sand) and Batjes (1996) PTFs under-estimated water content for all soil types. Lal (1979) (Clay) PTF is not applicable for coarse-grained soils and under-estimated water content for the fine-grained soils. Rawls et al. (1982), Gupta and Larson (1979) and Hall et al. (1977) PTFs performed better than other PTFs for suction of 10 kPa, but these three point-estimation PTFs over-estimated the water content for high-sand content soils (i.e., $S_a > 80\%$). Tomasella and Hodnett (1998) PTF over-estimated water content for high-clay content soils (i.e., $Cl > 50\%$).

Figure 5.5 Measured and estimated volumetric water contents using point-estimation PTFs at suction of 33 kPa: (a), Rawls et al. (1982); (b), Pidgeon (1972); (c), Lal (1979) Clay; (d), Lal (1979) Sand; (e), Dijkerman (1988); (f), Batjes (1996); (g), Tomasella and Hodnett (1998); (h), Oliveira et al. (2002); (i), Mohamed and Ali (2006); (j), Hall et al. (1977); (k), Gupta and Larson (1979); (l), Aina and Perisawamy (1985); (m), Beke and MacCormick (1985); (n), Manrique et al. (1991); (o), Reichert et al. (2009); (p), Arruda et al. (1987); (q), Minasny et al. (1999); and (r), Dashtaki et al. (2010)

According to Table 5.9 and Figure 5.5, Gupta and Larson (1979) PTF gave the best performance among the 18 evaluated point-estimation PTFs for the suction of 33 kPa. Rawls et al. (1982), Pidgeon (1972), Lal (1979) Clay, Hall et al. (1977), Manrique et al. (1991), and Minasny et al. (1999) PTFs slightly over-estimated water content for coarse-grained soils and under-estimated water content for fine-grained soils. Aina and Perisawamy (1985) and Reichert et al. (2009) PTFs performed well for coarse-grained soils but they under-estimated the water content for fine-grained soils.

Figure 5.6 Measured and estimated volumetric water contents using point-estimation PTFs at suction of 100 kPa: (a), Gupta and Larson (1979); (b), Rawls et al. (1982); (c), Reichert et al. (2009); (d), Tomasella and Hodnett (1998); and (e), Dashtaki et al. (2010)

According to Table 5.10 and Figure 5.6, Gupta and Larson (1979) PTF performed the best among the five evaluated point-estimation PTFs at 100 kPa. Rawls et al. (1982) PTF slightly under-estimated the water content at suction of 100 kPa. The other three PTFs did not perform as well as Gupta and Larson (1979) and Rawls et al. (1982) PTFs.

Figure 5.7 Measured and estimated volumetric water contents using point-estimation PTFs at suction of 1500 kPa: (a), Petersen et al. (1968); (b), Rawls et al. (1982); (c), Pidgeon (1972); (d), Hall et al. (1977); (e), Lal (1979) Clay; (f), Lal (1979) Sand; (g), Aina and Perisawamy (1985); (h), Dijkerman (1988) Clay; (i), Manrique et al. (1991); (j), Rajkai and Varallyay (1992); (k), Batjes (1996); (l), van den Berg et al. (1997); (m), Tomasella and Hodnett (1998); (n), Mohamed and Ali (2006); (o), Gupta and Larson (1979); (p), Beke and MacCormick (1985); (q), Oliveira et al. (2002); (r), Reichert et al. (2009); (s), Minasny et al. (1999); (t), Arruda et al. (1987) (1987); and (u), Dashtaki et al. (2010)

According to Table 5.11 and Figure 5.7, Rawls et al. (1982) performed the best among the 21 evaluated PTFs. Hall et al. (1977), Dijkerman (1988) Clay, Manrique et al. (1991), Tomasella and Hodnett (1998) and Dashtaki et al. (2010) PTFs predicted the water content at suction 1500 kPa well. Petersen et al. (1968), Lal (1979) Clay, Aina and Perisawamy (1985), and Rajkai and Varallyay (1992) PTFs tend to under-estimate the water content. Gupta and Larson (1979), Oliveira et al. (2002), Reichert et al. (2009) and Arruda et al. (1987) PTFs over-estimated the water content.

The best point-estimation PTFs for each suction are summarised in Table 5.12. Gupta and Larson (1979) PTFs performed best at 4, 10, 33, and 100 kPa and Rawls et al. (1982) PTF performed best at 1500 kPa.

5.4 Estimating SWCC Using Ensemble PTFs

Point-estimation pedotransfer functions (PTFs) are commonly used in soil science and agriculture to estimate the water content. However, point-estimation PTFs have not been commonly used in unsaturated soil mechanics (Zou and Leong, 2017b). Using these point-estimation PTFs as an ensemble PTF, a quick estimate of SWCC can be obtained for suctions at 4, 10, 33, 100 and 1500 kPa. Five points SWCC data is enough to determine a continuous unimodal SWCC by fitting a unimodal SWCC equation, such as Fredlund and Xing (1994) equation. As bimodal SWCC equations normally have more than five unknown parameters, it is not sufficient to determine a bimodal SWCC using the five SWCC data points estimated by the point-estimation PTFs.

Three soils from UNSODA database (Nemes, et al., 2001) are used to demonstrate the SWCC estimated from point-estimation PTFs. Soils 2361 and 1360 are fine-grained soils while soil 4560 is a coarse-grained soil. The soil properties are summarised in Table 5.13. The SWCCs are presented in Figure 5.8. The PTFs provide good estimates of the water contents at 4, 10, 33, 100 and 1500 kPa, and hence, good estimation of the SWCC can be obtained. However, the estimated water content for soil 4560 seems slightly higher than measured water content at 4, 10, 33 and 100 kPa. It is mainly because the chosen Gupta and Larson (1979) PTFs over-estimated water contents at suction levels 4, 10, 33 and 100 kPa for the extremely high sand content soils as shown in Figure 5.3(a), 5.4(h), 5.5(k), and 5.6(a), respectively.

Table 5.12 Summary of the best point-estimation PTFs at each suction

Suction (kPa)	Selected PTF	PTF Equation
4	Gupta and Larson (1979)	$\theta_4 = (0.7053Sa + 1.0242Si + 1.007Cl - 32.12\rho_d) \times 10^{-2}$
10	Gupta and Larson (1979)	$\theta_{10} = (0.5018Sa + 0.8548Si + 0.8853Cl - 24.239\rho_d) \times 10^{-2}$
33	Gupta and Larson (1979)	$\theta_{33} = (0.3075Sa + 0.5886Si + 0.8039Cl - 14.23\rho_d) \times 10^{-2}$
100	Gupta and Larson (1979)	$\theta_{100} = (0.1563Sa + 0.3620Si + 0.7154Cl - 5.759\rho_d) \times 10^{-2}$
1500	Rawls et al. (1982)	$\theta_{1500} = (2.6 + 0.5Cl) \times 10^{-2}$

Table 5.13 Properties of three soils from UNSODA (Nemes et al., 2001) used for estimation of SWCC with four SWCC points from PTFs

Soil	Void Ratio	ρ_d (Mg/m ³)	Sa (%)	Si (%)	Cl (%)
2361	1.257	1.280	8.0	36.0	56.0
1360	0.813	1.500	15.7	40.8	43.5
4650	0.613	1.622	92.0	7.0	1.0

Figure 5.8 Estimation of SWCC using five estimated SWCC points from PTFs and Fredlund and Xing (1994) equation (solid line) and van Genuchten (1980) equation (dotted line)

5.5 Conclusion

Point-estimation pedotransfer functions (PTFs) are commonly used in soil science and agriculture to estimate the water content at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa. However, point-estimation PTFs are not commonly used in unsaturated soil mechanics. In this study, a number of point-estimation PTFs (3 PTFs for 4 (or 3) kPa, 11 PTFs for 10 kPa, 18 PTFs for 33 kPa, 5 PTFs for 100 kPa and 21 PTFs for 1500 kPa)

was evaluated using 60 soils from Andersson and Wiklert (1972), 60 soils from Jauhiainen (2004) and 130 soils from UNSODA (Nemes, et al., 2001). The evaluation results show that Gupta and Larson (1979) PTF performed well at suctions of 4, 10, 33 and 100 kPa while Rawls et al. (1982) PTF performed well at suction of 1500 kPa. Using these point-estimation PTFs, a quick estimate of SWCC can be obtained using Gupta and Larson (1979) PTF to estimate the water contents at suctions of 4, 10, 33 and 100 kPa and Rawls et al. (1982) PTF to estimate the water content at suction of 1500 kPa as illustrated for three soils. The estimated SWCCs show a good agreement with the experimental SWCC for suctions between 4 and 1500 kPa. Such point-estimation PTFs provide an alternative method to estimate unimodal SWCC and should be further explored in unsaturated soil mechanics.

CHAPTER 6

UNIMODAL SOIL-WATER CHARACTERISTIC CURVE

6.1 Introduction

Soil-water characteristic curves (SWCCs) can be classified into unimodal and bimodal SWCCs. The sigmodal SWCCs, called unimodal SWCCs, which are illustrated in Figure 1.1, are more commonly used in practice. Well-graded soils and uniform soils often give unimodal SWCCs (Fredlund et al., 2000; Zhai and Rahardjo, 2012; Li et al., 2014). Unimodal SWCC can be directly determined by laboratory tests at discrete suction levels. The tests that directly measure the SWCCs are not easily accessible and are often extremely time-consuming and labour intensive. Point-estimation PTFs, which estimate water content at one suction level, have been studied in Chapter 5. To describe the continuous SWCC, a fitting equation is usually needed. A list of equations for unimodal SWCCs is summarised in Table 2.2. The most commonly used unimodal SWCC equations are the Brooks and Corey (1964), van Genuchten (1980) and Fredlund and Xing (1994) equations.

In this chapter, parametric-estimation PTFs that provide estimation of the parameters in a SWCC equation is first investigated. Next, a more attractive method of estimating unimodal SWCC using point-estimation PTFs and a parametric-estimation PTF is suggested. Lastly, a relatively simple model which requires limited data but

provides reasonably accurate boundary curves for estimating a hysteretic SWCC is developed.

6.2 Parametric-estimation PTFs for Unimodal SWCC

6.2.1 Introduction

The approaches to determine SWCC are summarised in Figure 5.1. Point-estimation PTFs have been investigated in Chapter 5. In this section, parametric-estimation PTFs are investigated. The parametric-estimation PTFs assumed that the SWCC can be adequately described by a SWCC equation. The most commonly used SWCC equations are Brooks and Corey (1964) equation, van Genuchten (1980) equation and Fredlund and Xing (1994) equation, summarised in Table 6.1.

The objective of this section is to evaluate numerous parametric-estimation PTFs using data collated from literature.

Table 6.1 SWCC equations used for parametric-estimation PTFs

Reference	Equation	Parameters
Brooks and Corey (1964)	$\theta = (\theta_s - \theta_r) \left(\frac{\psi_a}{\psi} \right)^\lambda + \theta_r$	$\lambda, \psi_a, \theta_r$
van Genuchten (1980)	$\theta = (\theta_s - \theta_r) \left[\frac{1}{1 + (a\psi)^n} \right]^m + \theta_r$	a, m, n, θ_r
Fredlund and Xing (1994)	$\theta = \left[1 - \frac{\ln \left(1 + \psi / \psi_r \right)}{\ln \left(1 + 1,000,000 / \psi_r \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m}$	a, m, n, ψ_r

6.2.2 Parametric-estimation PTFs

Four parametric-estimation PTFs (Rawls and Brakensiek, 1985, Saxton et al., 1986, Tomasella and Hodnett, 1998, and Mayr and Jarvis, 1999) using Brooks and Corey (1964) SWCC equation, seven parametric-estimation PTFs (Varallyay et al., 1982, Vereecken et al., 1989, Rajkai et al., 1996, Scheinost et al., 1997, Minasny et al., 1999, Rajkai et al., 2004, and Mohanmmadi and Meskini-Vishakae, 2013) using van Genuchten (1980) SWCC equation and one parametric-estimation PTF (Chin et al., 2010) using Fredlund and Xing (1994) SWCC equation are evaluated in this study. A summary of the PTFs are given in Table 6.2.

The objective of this section is to examine the relevancy and usefulness of parametric-estimation PTFs in unsaturated soil mechanics by evaluating numerous parametric-estimation PTFs using data collated from the literature

Among the evaluated PTFs, Saxton et al. (1986), Tomasella and Hodnett (1998) and Mayr and Jarvis (1999), Varallyay et al. (1982), Vereecken et al. (1989), Rajkai et al. (1996), Scheinost et al. (1997), Minasny et al. (1999), and Rajkai et al. (2004) PTFs estimate the saturated water content. To ensure a fair evaluation, saturated water content which is equal to porosity of soil will be used for all the PTFs in the evaluation.

For the residual water content, Varallyay et al. (1982), Rajkai et al. (1996), Rajkai et al. (2004), and Ganjian et al. (2007) PTFs assumed it as zero while Vereecken et al. (1989) and Scheinost et al. (1997) PTFs used clay and organic content to estimate the residual water content. In Vereecken et al., (1989) and Scheinost et al. (1997), the residual water content increases as clay and organic content increase.

Table 6.2 Parametric-estimation PTFs

SWCC Equation	Reference	PTF
Brooks and Corey (1964)	Rawls and Brakensiek (1985)	$\psi_a = \exp \left(\begin{aligned} &5.340 + 0.185 Cl - 0.002 Cl^2 - \\ &0.044 San + 0.001 Sa^2 n^2 - 0.009 Cl^2 n^2 \\ &- 0.00001 Sa^2 Cl + 0.009 Cl^2 Sa \\ &- 0.0007 Sa^2 n + 0.000005 Cl^2 Sa - 0.5 n^2 Cl \end{aligned} \right)$ $\lambda = \exp \left(\begin{aligned} &-0.784 + 0.018 Sa - 1.062 n \\ &-0.00005 Sa^2 + 0.003 Cl^2 + 1.111 n^2 \\ &-0.031 n Sa + 0.0003 Sa^2 n^2 - 0.006 Cl^2 n^2 \\ &-0.000002 Sa^2 Cl + 0.008 n Cl^2 - 0.007 n^2 Cl \end{aligned} \right)$ $\theta_r = -0.018 + 0.009 Sa + 0.005 Cl + 0.029 n - 0.0002 Cl^2$ $-0.001 n Sa - 0.0002 Cl^2 n^2 + 0.0003 Cl^2 n - 0.002 n^2 Cl$
	Saxton et al. (1986)	$\psi_a \theta_s^{-\frac{1}{\lambda}} = 100 \exp \left(\begin{aligned} &-4.396 - 0.0715 Cl \\ &-4.880 \times 10^{-4} Sa^2 - 4.285 \times Sa^2 Cl \end{aligned} \right)$ $\frac{1}{\lambda} = -3.140 - 0.00222 Cl^2 - 3.484 \times 10^{-5} Sa^2 Cl$
	Tomasella and Hodnett (1998)	$\psi_a = 0.285 + 7.33 \times 10^{-4} Si^2$ $-1.3 \times 10^{-4} Si Cl + 3.6 \times 10^{-6} Si^2 Cl$ $\frac{1}{\lambda} = \exp \left(\begin{aligned} &(1.197 + 4.17 \times 10^{-3} Si - 4.50 \times 10^{-3} Cl \\ &+ 8.94 \times 10^{-4} Si Cl - 1.00 \times 10^{-5} Si^2 Cl) \end{aligned} \right)$ $\theta_r = -2.094 + 0.047 Si + 0.431 Cl - 8.27 \times 10^{-3} Si Cl$
	Mayr and Jarvis (1999)	$\psi_a = 10^{0.0509226283 Sa + 0.1575152771 Si + 0.1240901644 \rho_b - 0.0021767278 Si^2 + 1.438224 \times 10^{-5} Si^3 + 8.040715 \times 10^{-4} Cl^2}$ $\text{Log } \lambda = 0.0046806123 Sa - 0.0092463819 Si - 0.1240901644 \rho_b - 3.294687 \times 10^{-4} Sa^2 + 1.689056 \times 10^{-6} Sa^3$

SWCC Equation	Reference	PTF
van Genuchten (1980)	Varallyay et al. (1982)	$n = 0.336\rho_b - 0.053$ $a = 10^{[0.417 - 0.04271\rho_b Cl - 1.513\rho_b]}$
	Vereecken et al. (1989)	$a = \exp(-2.486 + 0.025 \times Sa - 2.617 \times \rho_b - 0.023 \times Cl)$ $n = \exp(0.053 - 0.009 \times Sa - 0.013 \times Cl + 0.00015 \times Cl^2)$ $\theta_r = 1.5 + 0.5Cl$
	Rajkai et al. (1996)	$a = \exp(-7.3 - 3.5\rho_b - 0.8 \ln(Cl) + 0.102Sa + 0.037Si)$ $n = 0.36 + 0.35n_{GSD} - 0.002a_{GSD} - 0.0037Cl$
	Scheinost et al. (1997)	$a = a_0 + a_1 d_g$ $n = n_0 + n_1 \sigma_g^{-1}, \theta_r = r_1 Cl$
	Minasny et al. (1999)	$a = 0.1361 + 1.6929d_g, n = 1.4062 - 0.0050\sigma_g$ $\theta_r = -0.733 + 0.427Cl + 0.267Si$
	Rajkai et al. (2004)	$\ln a = 27.18 + 0.15\rho_b Si + 0.18Cl - 13.32 \ln(Si) - 0.024\rho_b^2 Cl + 0.314Si - 0.0027Cl^2 - \frac{0.75Sa}{Si} - 0.0013\rho_b^2 Si^2$ $\ln n = -0.287 + 0.47\rho_b - 0.008OM - 0.00007Cl^2 + 0.06 \ln(Cl) - 0.00046\rho_b Sa - 0.01\rho_b^2 Cl - \frac{0.0068Sa}{Si} + 0.00015\rho_b^2 Cl^2$
	Mohammadi and Meskini-Vishakae (2013)	$\frac{p_d - w_r}{1 - w_r} = \left[\frac{1}{1 + \left(a \frac{0.543 \times 10^{-4}}{d} \zeta \right)^n} \right]^m$
Fredlund and Xing (1994)	Chin et al. (2010)	<p>Fine-grained soils: $a = -2.4x + 722, n = 0.07x^{0.4}$ $m = 0.015x^{0.7}, \psi_r = 914 \exp(-0.002x)$</p> <p>Coarse-grained soils: $a = 0.53D_{50}^{-0.96}, n = x$ $m = -0.23 \ln x + 1.13, \psi_r = 100$</p>

For the PTFs using van Genuchten (1980) equation, Varallyay et al. (1982), Vereecken et al. (1989), Rajkai et al. (1996), Scheinost et al. (1997), and Weynants et al. (2009) assumed parameter m as 1, Vereecken et al. (1989), Schaap and Leij (1998), Schaap et al. (1998), Wosten et al. (1999); Minasny et al. (1999), Rajkai et al. (2004), and Lee and Ro (2014) assumed parameter m as $(1-1/n)$, while Mohammadi and Meskini-Vishakae (2013) and Ganjian et al. (2007) assumed that the parameter m is independent from parameter n . The parameter a of van Genuchten (1980) equation is related to the air-entry value (AEV). Generally, AEV increases as fine content increases. However, the value of a decreases as clay content increases in Varallyay et al. (1982), Vereecken et al. (1989), and Rajkai et al. (1996). Hence, these three PTFs provide an inconsistent relationship between parameter a of van Genuchten (1980) equation and clay content.

There is only one parametric-estimation PTF using Fredlund and Xing (1994) SWCC equation (Chin et al., 2010) evaluated in this study. This PTF is different from the other PTFs as it requires one measurement point of the SWCC.

6.2.3 Evaluation of Parametric-estimation PTFs

Forty soils from SoilVision (2002) and Andersson and Wiklert (1972), summarised in Table 6.3, were used to evaluate the parametric-estimation PTFs. The soil textures of the evaluation soil dataset are shown in Figure 6.1.

To evaluate the performance of the parametric-estimation PTFs, the performance indices, $RMSE$ (Equation 3.6), d (Equation 3.10) and R^2 (Equation 3.7) discussed in Section 3.2.3 are chosen.

Table 6.3 Soil used for evaluation of the parametric-estimation PTFs

Soil	Reference	Sand/silt/clay (%)	Organic content	ρ_d (Mg/m ³)
1030, 1032, 1090, 1091, 1092, 1130, 1132, 1134, 1160, 1161, 1163, 1182, 1370, 1464, 1465, 1466, 1467, 2530, 2531, 2661, 2682, 2691, 2723, 2761, 3281, 4681	SoilVision (2002)	Min: 3/6/1 Max: 93/90/84	0	Min: 0.95 Max: 1.81
15, 7, 66, 45, 81, 85, 292, 23, 284, 345, 264, 171, 315, 243	Andersson and Wiklert (1972)	Mean: 38/32/29		Mean: 1.48

The measured and estimated water contents from the SWCCs using the parametric-estimation PTFs are shown in Figure 6.2 to Figure 6.4. The performance indices of the PTFs are summarised in Table 6.4.

For the Brooks and Corey (1964) SWCC equation, it was observed that Saxton et al. (1986) PTF has the best performance among the four evaluated PTFs (see Figure 6.2 and Table 6.4). Rawls and Brakensiek (1985) and Mayr and Jarvis (1999) PTFs tend to overestimate the water contents for all soils (Figure 6.2 a and d, respectively) while Tomasella and Hodnett (1998) PTF underestimated the water content (Figure 6.2c).

For the van Genuchten (1980) SWCC equation, Rajkai et al. (1996) PTF has the best performance among the seven evaluated parametric-estimation PTFs followed closely by Vereecken et al. (1989) PTF (See Figure 6.3 and Table 6.4). Scheinost et al. (1997) and Minasny et al. (1999) PTFs overestimated the residual water content and hence overestimated the water content in the high suction range, especially for high clay content soils (Figure 6.3d and e, respectively). Rajkai et al. (2004) PTF tends to always overestimate the water content for sandy soils (Figure 6.3f) while Mohammadi and

Meskini-Vishakae (2013) PTF underestimated the water contents for all soils (Figure 6.3g).

Figure 6.1 Texture of soils shown in USDA textural triangle for soils used to evaluate parametric-estimation PTFs

For Fredlund and Xing (1994) SWCC equation, Chin et al. (2010) PTF performed well for all soils. But Chin et al. (2010) PTF requires SWCC measurement at one suction, 10 kPa for coarse-grained soils and 500 kPa for fine-grained soils. Because of this requirement, Chin et al. (2010) PTF has the best performance among all the evaluated PTFs. Although there are similar PTFs as Chin et al. (2010) using Fredlund and Xing (1994) SWCC equation, they are not included here due to obvious reasons as explained later in Section 6.3.1.

Figure 6.2 Parametric-estimation PTFs used Brooks and Corey (1964) equation estimated and measured volumetric water contents comparison: (a), Rawls and Brakensiek (1985) PTF; (b), Saxton et al. (1986) PTF; (c), Tomasella and Hodnett (1998) PTF; and (d), Mayr and Jarvis (1999) PTF

Table 6.4 Performance indices of evaluated parametric estimation PTFs

SWCC Equation	PTF	<i>RMSE</i>	R^2	<i>d</i>
Brooks and Corey (1964)	Rawls and Brakensiek (1985)	0.096	0.591	0.884
	Saxton et al. (1986)	0.067	0.800	0.945
	Tomasella and Hodnett (1998)	0.078	0.732	0.923
	Mayr and Jarvis (1999)	0.108	0.477	0.842
van Genuchten (1980)	Varallyay et al. (1982)	0.091	0.629	0.871
	Vereecken et al. (1989)	0.064	0.817	0.947
	Rajkai et al. (1996)	0.061	0.835	0.955
	Scheinost et al. (1997)	0.070	0.778	0.931
	Minasny et al. (1999)	0.076	0.742	0.917
	Rajkai et al. (2004)	0.072	0.765	0.936
	Mohammadi and Meskini-Vishakae (2013)	0.077	0.736	0.939
Fredlund and Xing (1994)	Chin et al. (2010)	0.039	0.931	0.982

Figure 6.3 Parametric-estimation PTFs used van Genuchten (1980) equation estimated and measured volumetric water contents comparison: (a), Varallyay et al. (1982) PTF; (b), Vereecken et al. (1989) PTF; (c), Rajkai et al. (1996) PTF; (d), Scheinost et al. (1997) PTF; (e), Minasny et al. (1999) PTF; (f), Rajkai et al. (2004) PTF; (g), Mohammadi and Meskini-Vishakae (2013) PTF

Figure 6.4 Parametric-estimation PTFs used Fredlund and Xing (1994) equation estimated and measured volumetric water contents comparison: Chin et al. (2010) PTF.

Generally, parametric-estimation PTFs using van Genuchten (1980) and Fredlund and Xing (1994) SWCC equations performed better than parametric-estimation PTFs using Brooks and Corey (1964) equation.

6.2.4 Summary

Four PTFs using Brooks and Corey (1964) SWCC equation, seven PTFs using van Genuchten (1980) SWCC equation and one PTF using Fredlund and Xing (1994) SWCC equation were evaluated using 40 soils collated from the literature. The soils cover a wide region. The evaluation showed that generally, PTFs using van Genuchten (1980) and Fredlund and Xing (1994) SWCC equations performed better than the PTFs using Brooks and Corey (1964) equation. For the Brooks and Corey (1964) SWCC equation, Saxton et al. (1986) PTF has the best performance. For the van Genuchten (1980) SWCC equation, Rajkai et al. (1996) PTF has the best performance. The best performance amongst all the evaluated parametric-estimation PTFs was given by Chin

et al. (2010) PTF using Fredlund and Xing (1994) SWCC equation. However, this is at the expense of performing one-point measurement of the SWCC. For a quick estimate of the SWCC, Saxton et al. (1986), Rajkai et al. (1996) and Chin et al. (2010) PTFs could be used in unsaturated soil mechanics.

6.3 Zero Experimental Point Method to Estimate Soil-water Characteristic Curve

6.3.1 Introduction

One-point methods to estimate SWCC have been proposed by Vanapalli and Catana (2005), Catana et al. (2006), Houston et al. (2006) and Chin et al. (2010). Chin et al. (2010) developed a one-point method PTFs that outperformed Vanapalli and Catana (2005), Catana et al. (2006), Houston et al. (2006) one-point methods. One-point methods are actually parametric-estimation PTFs used together with one-point measurement of SWCC.

One-point methods reduce the wait time to obtain an estimate of the SWCC. Vanapalli and Catana (2005) one-point method estimates the SWCC of coarse-grained soils using one-point SWCC measurement in the suction range of 0.1 to 10 kPa and basic soil parameters (dominant grain size diameter d_e , grain size at 10% passing D_{10} , grain size at 60% passing D_{60} , and void ratio e). Essentially, Vanapalli and Catana (2005) uses a parametric-estimation PTF based on Fredlund and Xing (1994) SWCC equation. For fine-grained soils, Catana et al. (2006) one-point method proposed using one-point SWCC measurement in the range of 50-500 kPa and suction capacity, C , which is correlated with the product of liquid limit and clay fraction. Catana et al. (2006) one-point method uses the Brutsaert (1966) SWCC equation. Houston et al. (2006) one-point method estimates SWCC of non-plastic soils using soil parameters

grain size at 10, 20, 30, 60 and 90% passing (D_{10} , D_{20} , D_{30} , D_{60} , D_{90} , respectively) and fines content less than 200 μm (P_{200}) with one-point SWCC measurement. For plastic soils, Houston et al. (2006) one-point method proposes to use a product of plasticity index (PI) and fines content less than 200 μm (P_{200}) coupled with one-point SWCC measurement to estimate the SWCC. The product of PI and P_{200} is adjusted until the SWCC passes through the one-point SWCC measurement. Houston et al (2006) one-point method uses the Fredlund and Xing (1994) SWCC equation and does not specify the suction for the one-point SWCC measurement. Chin et al. (2010) proposed a one-point method based on Fredlund and Xing (1994) SWCC equation. Chin et al. (2010) one-point method separated the parametric-estimation PTFs for coarse-grained and fine-grained soils and recommend that the one-point SWCC to be measured at 10 kPa and 500 kPa for coarse-grained and fine-grained soils, respectively.

Chin et al. (2010) conducted an evaluation using SWCCs of 31 coarse-grained soils and 31 fine-grained soils and showed that their method outperformed the other one-point methods (Vanapalli and Catana, 2005; Catana et al. 2006; Houston et al., 2006). Separately the good performance of Chin et al. (2010) one-point method has been corroborated by Zou and Leong (2017a), Tarantino and El Mountassir (2013) and Maqsoud et al. (2016). Hence Chin et al. (2010) one-point method was adopted in this study.

The objective of this section is to develop a zero experimental point method by eliminating the need of using a one-point measurement of the SWCC. The one-point measurement of SWCC is substituted using point-estimation PTFs.

6.3.2 Point-estimation PTFs

Point-estimation PTFs at suction of 4 (or 3), 10, 33, 100 and 1500 kPa were evaluated in Chapter 5. The evaluation results show that Gupta and Larson (1979) PTF performed the best at suctions of 4, 10, 33 and 100 kPa while Rawls et al. (1982) PTF performed the best at suction of 1500 kPa.

According to Dodds (1980) and Maier et al. (1999), when the fines content (clay and silt) reaches a certain percentage (e.g., 20 to 35%), the pore geometry will be controlled by the fines content. In the one-point methods, soils are separated into coarse-grained soils and fine-grained soils (Vanapalli and Catana, 2005; Catana et al., 2006; Chin et al., 2010) or plastic and non-plastic soils (Houston et al., 2006). Plastic and non-plastic soils are differentiated by the presence of clay minerals in the former. Hence, it is reasonable to suggest that the performance of one-point method may be improved if coarse-grained and fine-grained soils or plastic and non-plastic soils are separated. In this study, coarse-grained and fine-grained soils separation is used as plastic and non-plastic soils require Atterberg's limits to be determined. In addition, Atterberg's limits are not usually determined in agriculture and soil science. Chin et al. (2010) defined coarse-grained soils as soils where percent passing no. 200 Sieve (P_{200}) is less than 30% and fine-grained soils as soils where P_{200} is more or equal to 30%.

The same dataset as used in Chapter 5 is used to re-evaluate the point-estimation PTFs separated into fine-grained and coarse-grained soils. The evaluation results are presented in Table 6.5 to Table 6.9. The best performing point-estimation PTFs for coarse-grained and fine-grained soils for various suction levels are summarised in Table 6.10.

At suction of 4 (or 3) kPa, Gupta and Larson (1979) point-estimation PTF performed the best for both coarse- and fine-grained soils. This is unchanged from the conclusion arrived at Chapter 5.

At suction of 10 kPa, Gupta and Larson (1979) point-estimation PTF performed the best for coarse-grained soils. For fine-grained soils, in terms of RMSE, Hall et al. (1977) point-estimation PTF performed the best and in terms of d , Gupta and Larson (1979) point-estimation PTF performed the best.

At suction of 33 kPa, Gupta and Larson (1979) point-estimation PTF performed the best for fine-grained soils. for coarse-grained soils, Tomasella and Hodnett (1998) point-estimation PTF performed the best, but in terms of d , Aina and Perisawamy (1985) performed as well as Tomasella and Hodnett (1998).

At suction of 100 kPa, Gupta and Larson (1979) point-estimation PTF performed the best for fine-grained soils. For coarse-grained soils, Gupta and Larson (1979) point-estimation PTF performed the best in terms of RMSE but Rawls et al. (1982) point-estimation PTF performed the best in terms of d .

At suction of 1500 kPa, Rawls et al. (1982) point-estimation PTF performed the best for both coarse-grained and fine-grained soils. But in terms of RMSE, Dashtaki et al. (2010) point-estimation PTF performed as well as Rawls et al. (1982) point-estimation PTF for coarse-grained soils and Hall et al. (1977) point-estimation PTF performed as well as Rawls et al. (1982) for fine-grained soils.

Table 6.5 Point-estimation PTFs at suction of 4 (or 3) kPa evaluation for coarse-grained and fine-grained soils

S/N	PTF	coarse-grained		Fine-grained	
		<i>RMSE</i>	<i>d</i>	<i>RMSE</i>	<i>d</i>
1	Gupta and Larson (1979)	0.074	0.856	0.056	0.945
2	Rawls et al. (1982)	0.080	0.843	0.089	0.899
3	Tomasella and Hodnett (1998)	0.086	0.842	0.074	0.892

Table 6.6 Point-estimation PTFs at suction of 10 kPa evaluation for coarse-grained and fine-grained soils

S/N	PTF	coarse-grained		Fine-grained	
		<i>RMSE</i>	<i>d</i>	<i>RMSE</i>	<i>d</i>
1	Pidgeon (1972)	0.102	0.831	0.084	0.901
2	Lal (1979) (Sand)	0.105	0.840	0.128	0.638
3	Lal (1979) (Clay)	0.107	0.655	0.091	0.847
4	Rawls et al. (1982)	0.066	0.914	0.063	0.968
5	Batjes (1996)	0.127	0.821	0.075	0.928
6	van den Berg et al. (1997)	0.092	0.843	0.095	0.834
7	Tomasella and Hodnett (1998)	0.072	0.919	0.068	0.947
8	Gupta and Larson (1979)	0.056	0.936	0.044	0.982
9	Hall et al. (1977)	0.076	0.854	0.042	0.980
10	Minasny et al. (1999)	0.100	0.745	0.058	0.952
11	Dashtaki et al. (2010)	0.075	0.879	0.060	0.965

Table 6.7 Point-estimation PTFs at suction of 33 kPa evaluation for coarse-grained and fine-grained soils

S/N	PTF	coarse-grained		fine-grained	
		<i>RMSE</i>	<i>d</i>	<i>RMSE</i>	<i>d</i>
1	Rawls et al. (1982)	0.081	0.921	0.059	0.968
2	Pidgeon (1972)	0.087	0.909	0.133	0.752
3	Lal (1979) Clay	0.115	0.743	0.091	0.896
4	Lal (1979) Sand	0.086	0.923	0.147	0.566
5	Dijkerman (1988)	0.069	0.950	0.100	0.914
6	Batjes (1996)	0.077	0.945	0.141	0.640
7	Tomasella and Hodnett (1998)	0.052	0.970	0.068	0.949
8	Oliveira et al. (2002)	0.073	0.949	0.101	0.902
9	Mohamed and Ali (2006)	0.133	0.449	0.092	0.877
10	Hall et al. (1977)	0.084	0.885	0.059	0.964
11	Gupta and Larson (1979)	0.054	0.959	0.049	0.983
12	Aina and Perisawamy (1985)	0.056	0.970	0.073	0.967
13	Beke and MacCormick (1985)	0.080	0.879	0.053	0.982
14	Manrique et al. (1991)	0.123	0.595	0.060	0.962
15	Reichert et al. (2009)	0.061	0.957	0.102	0.874
16	Arruda et al. (1987)	0.088	0.900	0.113	0.870
17	Minasny et al. (1999)	0.100	0.852	0.080	0.916
18	Dashtaki et al. (2010)	0.085	0.886	0.126	0.735

Table 6.8 Point-estimation PTFs at suction of 100 kPa evaluation for coarse-grained and fine-grained soils

S/N	PTF	coarse-grained		fine-grained	
		<i>RMSE</i>	<i>d</i>	<i>RMSE</i>	<i>d</i>
1	Gupta and Larson (1979)	0.056	0.934	0.054	0.979
2	Rawls et al. (1982)	0.063	0.938	0.067	0.955
3	Reichert et al. (2009)	0.073	0.914	0.092	0.906
4	Tomasella and Hodnett (1998)	0.073	0.913	0.089	0.905
5	Dashtaki et al. (2010)	0.075	0.916	0.136	0.666

Table 6.9 Point-estimation PTFs at suction of 1500 kPa evaluation for coarse-grained and fine-grained soils

S/N	PTF	coarse-grained		fine-grained	
		<i>RMSE</i>	<i>d</i>	<i>RMSE</i>	<i>d</i>
1	Petersen et al. (1968)	0.059	0.867	0.097	0.863
2	Rawls et al. (1982)	0.036	0.926	0.038	0.984
3	Pidgeon (1972)	0.082	0.780	0.063	0.961
4	Hall et al. (1977)	0.040	0.906	0.038	0.983
5	Lal (1979) Clay	0.057	0.876	0.116	0.669
6	Lal (1979) Sand	0.085	0.536	0.071	0.927
7	Aina and Perisawamy (1985)	0.060	0.869	0.115	0.680
8	Dijkerman (1988) Clay	0.043	0.910	0.041	0.982
9	Manrique et al. (1991)	0.038	0.906	0.042	0.982
10	Rajkai and Varallyay (1992)	0.047	0.901	0.082	0.871
11	Batjes (1996)	0.044	0.902	0.062	0.935
12	van den Berg et al. (1997)	0.073	0.649	0.064	0.938
13	Tomasella and Hodnett (1998)	0.049	0.862	0.042	0.977
14	Mohamed and Ali (2006)	0.059	0.407	0.062	0.932
15	Gupta and Larson (1979)	0.062	0.730	0.094	0.938
16	Beke and MacCormick (1985)	0.042	0.860	0.107	0.926
17	Oliveira et al. (2002)	0.138	0.039	0.126	0.893
18	Reichert et al. (2009)	0.113	0.312	0.108	0.911
19	Minasny et al. (1999)	0.085	0.588	0.078	0.933
20	Arruda et al. (1987) (1987)	0.131	0.381	0.111	0.893
21	Dashtaki et al. (2010)	0.036	0.890	0.052	0.956

Table 6.10 Summary of best performing point-estimation PTFs for coarse-grained and fine-grained soils

Soils	Suction	PTF
Coarse-grained	4 kPa	Gupta and Larson (1979)
	10 kPa	Gupta and Larson (1979)
	33 kPa	Tomasella and Hodnett (1998) and Aina and Perisawamy (1985)
	100 kPa	Gupta and Larson (1979) and Rawls et al. (1982)
	1500 kPa	Rawls et al. (1982) and Dashtaki et al. (2010)
Fine-grained	4 kPa	Gupta and Larson (1979)
	10 kPa	Gupta and Larson (1979) and Hall et al. (1977)
	33 kPa	Gupta and Larson (1979)
	100 kPa	Gupta and Larson (1979)
	1500 kPa	Rawls et al. (1982) and Hall et al. (1977)

6.3.3 Methodology

Chin et al. (2010) recommended that the one-point SWCC measurement be conducted at 10 and 500 kPa for coarse-grained and fine-grained soils, respectively. In addition, Chin et al. (2010) also found that one-point SWCC measurement at 100 kPa instead of 500 kPa may still be satisfactory for fine-grained soils. Point-estimation PTFs are available at suctions of 4, 10, 33, 100 and 1500 kPa. Hence, it appears that point-estimation PTFs for 10 kPa and 100 kPa can be used with Chin et al. (2010) one-point method for coarse-grained and fine-grained soils, respectively. Fan and Leong (2016) suggested improving accuracy of Chin et al. (2010) method by using two points of SWCC measurement at 2 kPa and 10 kPa for coarse-grained soils and at 20 kPa and 500 kPa for fine-grained soils. Cresswell and Paydar (1996) proposed a two-point method based on Campbell (1974) SWCC equation. The two-point SWCC

measurements were recommended to be at 5 and 1500 kPa. With the above considerations, point-estimation PTFs at suctions of 4, 10 and 33 kPa can be potentially used for coarse-grained soils and point-estimation PTFs at suctions of 100 and 1500 kPa can be potentially used for fine-grained soil to replace the experimental measurement of the SWCC at these suctions.

Chin et al. (2010) one-point method is based on Fredlund and Xing (1994) SWCC equation given as:

$$\theta = \theta_s \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a} \right)^n \right]} \right\}^m C(\psi) \quad (\text{viz. 2.8a})$$

$$C(\psi) = \frac{-\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{1,000,000}{\psi_r} \right)} + 1 \quad (\text{viz. 2.8b})$$

where e is Euler's number (i.e., 2.718281); a , n and m are the curve-fitting parameters; and ψ_r is the matric suction corresponding to the residual volumetric water content, θ_r .

For coarse-grained soils, Chin et al. (2010) recommended a , n , m and ψ_r as follows:

$$a = 0.53 D_{50}^{-0.96} \text{ kPa} \quad (6.1a)$$

$$n = x \quad (6.1b)$$

$$m = -0.23 \ln x + 1.13 \quad (6.1c)$$

$$\psi_r = 100 \text{ kPa} \quad (6.1d)$$

For fine-grained soils, Chin et al. (2010) recommended a , n , m and ψ_r as follows:

$$a = -2.4x + 722 \text{ kPa} \quad (6.2a)$$

$$n = 0.07x^{0.4} \quad (6.2b)$$

$$m = 0.015x^{0.7} \quad (6.2c)$$

$$\psi_r = 914 \exp(-0.002x) \text{ kPa} \quad (6.2d)$$

In Equations 6.1 and 6.2, x is an adjustable variable based on the one-point SWCC measurement.

In this study, it is assumed that the adjustable variable x for Chin et al. (2010) one-point method for coarse-grained soils is constrained between 0.5 and 15 as suggested by Fan and Leong (2016). The estimation procedures for the proposed zero experimental point method and Chin et al. (2010) one-point method are summarised in Figure 6.5. Several zero experimental point procedures are possible with the point-estimation PTFs at suction of 4, 10, 33, 100 and 1500 kPa as follows:

- (i) For coarse-grained soils, one point-estimation PTF at suction of 4 kPa (4) or 10 kPa (10), and two point-estimation PTFs at suctions of 4 and 10 kPa (4+10), 4 and 33 kPa (4+33), 10 and 33 kPa (10+33), or 10 and 100 kPa (10+100) and three point-estimation PTFs at suctions of 4, 10 and 33 kPa (4+10+33) can be used with Chin et al. (2010) one-point method (Equation 6.2), and
- (ii) For fine-grained soils, one point-estimation PTF at suction of 100 kPa (100) or 1500 kPa (1500), two point-estimation PTFs at suctions of 33 and 100 kPa (33+100), 33 and 1500 kPa (33+1500), or 100 and 1500 kPa (100+1500), and three point-estimation PTFs at suctions of 33, 100 and 1500 kPa (33+100+1500) can be used with Chin et al. (2010) one-point method (Equations 6.2).

6.3.4 Development of zero experimental point method

To develop the zero experimental point method, 65 coarse-grained soils and 65 fine-grained soils from the literature as tabulated in Table 6.11 and Table 6.12, respectively, were collated. For the coarse-grained soils, the properties range of the soils are: S_a : 70.5%-97.0% (mean = 87.0%), S_i : 1%-22% (mean = 9.4%), Cl : 0.4%-14% (mean = 3.4%), OM : 0%-3.9% (mean=0.5%), D_{50} : 0.077-0.444 mm (mean = 0.138 mm), and ρ_d : 1.26-1.83 Mg/m³ (mean = 1.57 Mg/m³). For the fine-grained soils, the properties range of the soils are: S_a : 0-57% (mean = 19.0%), S_i : 14.0%-79.3% (mean = 60.0%), Cl : 8.8%-63% (mean = 24.2%), OM : 0.0%-6.36% (mean=0.49%), and ρ_b : 0.71-1.72 Mg/m³ (mean = 1.41 Mg/m³).

The performance of the zero experimental point method is assessed by the performance indices, $RMSE$, R^2 and d tabulated in Table 6.13 and Table 6.14 for coarse-grained soils and fine-grained soils, respectively.

Table 6.11 Coarse-grained soils used for development of zero experimental point method

Reference	Soil
SoilVision (2002)	11134, 11250, 11254, 11275, 11276, 11277, 11286, 11287, 11288, 11289, 11290, 11291, 11308, 11309, 11310, 11311, 11312, 11313, 11316, 10700, 10701, 10705, 10706, 10767, 10768, 10769, 10770, 10771, 10771
UNSODA (Nemes et al., 2001)	2203, 2373, 3141, 3142, 3143, 3144, 3150, 3151, 3152, 3153, 3454, 3455, 3160, 3163, 3164, 3165, 3171, 3172, 3173, 4011, 4020, 4021, 4050, 4052, 4060, 4061, 4062, 4140, 4141, 4142, 4150, 4151, 4152, 4650, 4651, 4660

Table 6.12 Fine-grained soils used for development of zero experimental point method

Reference	Soil
Lu et al. (2008)	Soil 4, Soil 5, Soil 7, Soil 8
Campbell and Shiozawa (1992)	Salkum, Walla Walla, Palouse, Palouse B
Prebble (1991)	Grey Clay Gilgai Bank Depression, Beauraba Black Earth, Charlton Black Earth, Knapdale Black Earth, Black Earth, Mywybilla Black earth, Kenmuir clay
SoilVision (2002)	11075, 11083, 11084, 11086, 11087, 11088, 11089, 11100, 11101, 11126, 11218, 11219, 11222, 11223, 11225, 11226, 11227, 11228, 11278, 11282, 11283, 11284, 11285, 11293, 11294, 11295, 11296, 11297, 11298, 11299, 11484, 11487, 11488
UNSODA (Nemes et al., 2001)	1282, 1360, 1361, 1362, 1372, 2360, 2361, 2461, 2462, 2712, 2713, 3241, 3242, 4041, 4040, 4530

Table 6.13 Performance of Chin et al. (2010) one-point method and proposed zero experimental point methods for coarse-grained soils

Method		RMSE	R2	d
One-point	Chin et al. (2010)	0.028	0.949	0.987
	4	0.036	0.918	0.979
	10	0.034	0.925	0.981
	4+10	0.032	0.935	0.984
	4+33(TH) ¹	0.035	0.924	0.981
	4+33(AP) ²	0.034	0.926	0.982
Zero-point	10+33(TH) ¹	0.031	0.938	0.985
	10+33(AP) ²	0.031	0.938	0.985
	10+100(GL) ³	0.031	0.938	0.984
	10+100(RW) ⁴	0.029	0.946	0.987
	4+10+33(TH) ¹	0.030	0.943	0.986
	4+10+33(AG) ²	0.030	0.942	0.986

Note: 1, SWCC data point at 33 kPa estimated by Tomasella and Hodnett (1998) point-estimation PTF; 2, SWCC data point at 33 kPa estimated by Aina and Perisawamy (1985) point-estimation PTF; 3, SWCC data point at 100 kPa estimated by Gupta and Larson (1979) point-estimation PTF; 4, SWCC data point at 100 kPa estimated by Rawls et al. (1982) point-estimation PTF; other methods without superscript SWCC data point estimated by Gupta and Larson (1979) PTFs at 4 and 10 kPa.

Figure 6.5 SWCC estimation procedures for Chin et al. (2010) one-point method and proposed zero experimental point method

For coarse-grained soils, Table 6.13 shows that the zero experimental point method using two point-estimation PTFs at suctions of 10 and 100 kPa performed the best (10+100(RW)) and its performance is comparable with Chin et al. (2010) one-point method for coarse-grained soils.

Table 6.14 Performance of Chin et al. (2010) one-point method and proposed zero experimental point methods for fine-grained soils

	Method	RMSE	R ²	d
One-Point	Chin et al. (2010)	0.041	0.924	0.980
Zero-point	100	0.047	0.897	0.972
	1500(HL) ¹	0.052	0.875	0.967
	1500(RW) ²	0.060	0.835	0.957
	33+100	0.052	0.879	0.967
	33+1500(HL) ¹	0.048	0.897	0.972
	33+1500(RW) ²	0.043	0.914	0.977
	100+1500(HL) ¹	0.044	0.913	0.976
	100+1500(RW) ²	0.040	0.927	0.980
	33+100+1500(HL) ¹	0.047	0.899	0.972
	33+100+1500(RW) ²	0.044	0.910	0.975

Note: 1, SWCC data point at 1500 kPa estimated by Hall et al. (1977) point-estimation PTF; 2, SWCC data point at 1500 kPa estimated by Rawls et al. (1982) point-estimation PTF; other method without superscript SWCC data point at 33 and 100 kPa estimated by Gupta and Larson (1979) PTFs.

For fine-grained soils, Table 6.14 shows that the zero experimental point method using two point-estimation PTFs at suctions of 100 and 1500 kPa (100+1500(RW)) performed the best and its performance is slightly better than Chin et al. (2010) method for fine-grained soils. The performance of Chin et al. (2010) one-point method and the proposed zero experimental point method in terms of the measured and estimated volumetric water contents are shown in Figure 6.6 and Figure 6.7 for coarse-grained soils and fine-grained soils, respectively.

Considering that the proposed zero experimental point method does not require a one-point SWCC measurement, it is an improvement over Chin et al. (2010) one-point method. Finally, typical cases of best, intermediate and worst estimations of SWCC by the proposed zero experimental point methods are shown in Figure 6.8 and Figure 6.9 for coarse-grained and fine-grained soils, respectively. The SWCCs by Chin et al. (2010) method are also shown in Figure 6.8 and Figure 6.9 for comparison.

(a) Chin et al. (2010)

(b) Proposed zero experimental point method

Figure 6.6 Comparison of Chin et al. (2010) one-point method and proposed zero experimental point methods for coarse-grained soils.

(a) Chin et al. (2010)

(b) Proposed zero experimental point method

Figure 6.7 Comparison of Chin et al. (2010) one-point method and proposed zero experimental point method for fine-grained soils.

(a) Soil 11290 from SoilVision (2002) - best

(b) Soil 4651 from UNSODA (Nemes et al., 2001) - intermediate

(c) Soil 11289 from SoilVision (2002) – worst

Figure 6.8 SWCC estimation for coarse-grained soils

(a) Soil 11488 from SoilVision (2002) – best

(b) Soil 11295 from SoilVision (2002) – intermediate

(c) 11227 from SoilVision (2002) – worst

Figure 6.9 SWCC estimation for fine-grained soils

6.3.5 Summary

A zero experimental point method was developed based on Chin et al. (2010) one-point method and point-estimation PTFs. For coarse-grained soils, the point-estimation PTFs at suctions of 10 and 100 kPa which are Gupta and Larson (1979) and Rawls et al. (1982) PTFs, respectively, are used. For fine-grained soils, the point-estimation PTFs at suctions of 100 and 1500 kPa which are Gupta and Larson (1979) and Rawls et al. (1982) PTFs, respectively, are used. The zero experimental point method was compared with Chin et al. (2010) one-point method using 65 coarse-grained and 65 fine-grained soils from the literature. The comparison shows that the zero experimental point method performance is comparable with Chin et al. (2010) one-point method for both coarse-grained soils and fine-grained soils. In general, the proposed zero experimental point method is more advantageous as no experimental measurement of the SWCC is needed to estimate the SWCC.

6.4 Hysteresis for Unimodal SWCC

6.4.1 Introduction

It is widely recognised that the water content of a soil at a certain suction level is not unique. Water content on the wetting curve is always lower than on the drying curve for a known matric suction, which means that the drying and wetting processes trace different SWCC paths. This phenomenon is called hysteresis. The hysteretic nature of SWCCs has been known for a long time but often the initial drying SWCC is used since the measurement of a complete hysteretic SWCC is extremely time consuming and costly. Hysteresis increases the difficulty of applying unsaturated soil mechanics in practice. In the field, partial drying or wetting occurs in several cycles, which results in

drying and wetting scanning curves between the main or boundary drying curve (BDC) and boundary wetting curve (BWC). The initial drying curve (IDC) of the SWCC is different from the BDC. There is infinite number of drying and wetting scanning curves that span between the BDC and BWC. This means that there is a range instead of a unique soil suction value at a certain water content. Slight changes in environmental condition can also cause the SWCC to be on the scanning curves instead of on the BDC or the BWC. Hence, it is difficult to use the SWCC to determine the soil suction, other than to obtain a range of approximate values (Fredlund et al., 2012). A number of models for the hysteretic SWCC have been proposed as reviewed in Chapter 2. But most of the models require some measurement data of the wetting SWCC to calibrate the model. The laboratory determination of the SWCC is time-consuming, labour-intensive and expensive, and even more so for the wetting curve.

The hysteretic SWCC is important to model unsaturated soil behaviour under climatic conditions. To overcome the difficulty of obtaining both the drying and wetting SWCCs, a relatively simple model which requires limited data but provides reasonably accurate BDC and BWC is developed in this study. The existing hysteretic SWCC models will be reviewed.

6.4.2 Existing hysteretic SWCC models

The review on the possible mechanism causing hysteresis in SWCC and different approaches to model hysteresis has been given in Chapter 2. Pham et al. (2005) summarised 28 hysteresis models. Compared with the domain models, empirical models require less data for the model calibration. In this study, an empirical hysteresis

SWCC model is proposed. Hence, only empirical models, namely, Pham et al., (2005) model and Likos et al, (2013) model are compared with the proposed model.

Pham et al. (2005) model

Pham et al. (2005) combined Feng and Fredlund (1999) water characteristic equation for thermal conductivity sensor with Fredlund and Xing (1994) correction factor $C(\psi)$ to give Equation 6.3a. Equation 6.3a contains four fitting parameters, b , c , d and ψ_r . Pham et al. (2005) correlated the parameters b , c , d (with subscript w) for BWC with the corresponding parameters (with subscript d) for BDC as presented in Equations 6.3b to 6.3e.

$$\theta = C(\psi) \frac{\theta_s b + c \psi^d}{b + \psi^d} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{10^6}{\psi_r} \right)} \right] \frac{\theta_s b + c \psi^d}{b + \psi^d} \quad (6.3a)$$

$$c_w = c_d \quad (6.3b)$$

$$b_w = \left[\frac{b_d}{(10^D)^{d_d}} \right]^{\frac{1}{R}} \quad (6.3c)$$

$$d_w = \frac{d_d}{R} \quad (6.3d)$$

$$\psi_{r,d} = (2.7b_d)^{\frac{1}{d_d}} \text{ and } \psi_{r,w} = (2.7b_w)^{\frac{1}{d_w}} \quad (6.3e)$$

where θ is volumetric water content, ψ is matric suction and ψ_r is the residual matric suction. Parameters, D and R , are the distance and slope ratio, respectively, between the

BWC and BDC on a semi-logarithmic SWCC plot. Thus, the BDC and BWC have different slopes instead of being parallel. Suggested values for D and R for different soil types (e.g., sand, sandy loam, silt loam and clay loam, and compacted silt and compacted sand) were given in Pham et al. (2005) and summarised in Table 6.15. Pham et al. (2005) suggested that the volumetric water content at zero matric suction for the boundary wetting curve $\theta_{s,iw}$ as $0.9\theta_{s,id}$, where $\theta_{s,id}$ is the saturated volumetric water content at zero matric suction for the IDC, and the BDC and BWC have the same volumetric water content at zero matric suction.

Table 6.15 Recommended R and D values between two boundary curves for different soil types from Pham et al. (2005)

Soil Type	R	D (log-scale)
Sand	2.0	0.20
Sandy Loam	2.5	0.25
Silt loam and Clay Loam	1.5	0.50
Compacted Silt and Compacted Sand	1.0	0.35

Pham et al. (2005) model is relatively accurate in estimating the hysteretic SWCC, but the model introduces two parameters, D and R , to correlate the BDC and BWC, which requires several calculation steps. In addition, Pham et al. (2005) used Feng and Fredlund (1999) equation instead of the more popularly used Fredlund and Xing (1994) or van Genuchten (1980) SWCC equation.

Likos et al. (2013) model

Likos et al. (2013) model uses van Genuchten (1980) equation (Equation 6.4a) with $m=1-1/n$. It assumed that the BDC and BWC have the same shape and only the air entry values are different. The parameter α is often assumed to be equivalent to the inverse of the air-entry value (Likos et al., 2013). It also assumed that the BWC air-

entry value is equal to half of the BDC air-entry value (Equation 6.4b). Likos et al. (2013) observed that assuming the volumetric water content at zero matric suction for the boundary wetting curve $\theta_{s,iw}$ as $0.85\theta_{s,id}$ (Equation 6.4c) did not perform as well as the regression relationship given in Equation 6.4d.

$$\theta = \frac{\theta_s - \theta_r}{[1 + (\alpha\psi)^n]^m} + \theta_r \quad (6.4a)$$

$$\alpha_w = 2\alpha_d \quad (6.4b)$$

$$\theta_{s,iw} = 0.85\theta_{s,id} \quad (6.4c)$$

$$\theta_{s,iw} = 2.21\theta_{s,id} - 0.58 \quad (6.4d)$$

However, the model does not consider the effect of soil types although Likos et al. (2013) observed that soil type has a significant effect on the model.

In this study, an empirical hysteresis SWCC model will be developed using Fredlund and Xing (1994) equation, which is a more common SWCC equation than Feng and Fredlund (1999) equation. The proposed model will also consider the effect of soil types.

6.4.3 Model development

The soils used in Pham et al. (2005) and Likos et al. (2013) were used to develop the hysteresis SWCC model. The soils are summarised in Table 6.16. The soils are classified according to the USDA classification system. Soil classification for S19-S25 from Likos et al. (2013) is not given but based on the plastic limits given for these soils can be assumed to be either clayey loam, loam or silty loam.

Fredlund and Xing (1994) equation (Equation 2.8) is used to develop the model. Fredlund and Xing (1994) equation has four unknown parameters: a , n , m and ψ_r .

$$\theta = \theta_s \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a} \right)^n \right]} \right\}^m C(\psi) \quad (\text{viz. 2.8a})$$

$$C(\psi) = \frac{-\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{1,000,000}{\psi_r} \right)} + 1 \quad (\text{viz. 2.8b})$$

In this study, it is assumed that a direct relationship exists between Fredlund and Xing (1994) SWCC equation parameters used to define the BWC and corresponding parameters of the IDC or BDC. The general form of the relationships can be written as:

$$a_{bw} = f_1(a_{id}) \quad (6.5a)$$

$$m_{bw} = f_2(m_{id}) \quad (6.5b)$$

$$n_{bw} = f_3(n_{id}) \quad (6.5c)$$

$$\psi_{r,bw} = f_4(\psi_{r,id}) \quad (6.5d)$$

where f_1 to f_4 denote functions. Equation 6.5a describes the relationship between AEVs of the IDC (a_{id}) and the BDC (a_{bw}). For the 59 soils summarised in Table 6.16, curve-fitting is done using Fredlund and Xing (1994) equation to obtain a_{id} , m_{id} and n_{id} if the IDC is available. If only the BDC is available, the BDC is curve fitted by Fredlund and Xing (1994) equation to obtain a_{bd} , m_{bd} and n_{bd} .

Table 6.16 Soils used for SWCC hysteresis SWCC model development

S/N	Soil ¹	Reference	Available SWCCs ²	Soil Classification
1	ADS	Talsma, (1970)	D,W	Sandy Loam
2	ACL	Watson et al., (1975)	D,W	Clay Loam
3	CSL	Topp, (1971)	D,W	Silt Loam
4	C1	Feng and Fredlund, (1999)	I,D,W	Sand
5	C-Sand	Viaene et al., (1994)	D,W	Sand
6	D-Sand	Gillham et al., (1976)	D,W	Sand
7	GB(4D)	Nimmo and Miller, (1986)	D,W	Sand
8	GB(20D)	Nimmo and Miller, (1986)	D,W	Sand
9	GB(35D)	Nimmo and Miller, (1986)	D,W	Sand
10	GB(50D)	Nimmo and Miller, (1986)	D,W	Sand
11	GBs	Bomba and Miller, (1967)	D,W	Sand
12	GBB	Poulovassilis, (1962)	I,D,W	Sand
13	MSF-I	Poulovassilis, (1970b)	D,W	Sand
14	MSF-II	Poulovassilis and Ghamry, (1978)	D,W	Sand
15	MRS	Talsma, (1970)	D,W	Sand
16	NSL(15D)	Hopmans and Dane, (1986)	D,W	Sandy Loam
17	NSL(32D)	Hopmans and Dane, (1986)	D,W	Sandy Loam
18	PSa	Vachaud and Thony, (1971)	D,W	Sand
19	PSaL(20D)	Nimmo and Miller, (1986)	D,W	Sandy Loam
20	PSaL(35D)	Nimmo and Miller, (1986)	D,W	Sandy Loam
21	PSaL(50D)	Nimmo and Miller, (1986)	D,W	Sandy Loam
22	PSiL(4D)	Nimmo and Miller, (1986)	D,W	Silt Loam
23	PSiL(35D)	Nimmo and Miller, (1986)	D,W	Silt Loam
24	PSiL(50D)	Nimmo and Miller, (1986)	D,W	Silt Loam
25	PB-I	Poulovassilis, (1970a)	I, D,W	Sand
26	PB-II	Poulovassilis, (1970a)	I, D,W	Sand
27	RCL	Topp, (1971)	D,W	Clay Loam
28	RSL	Topp, (1969)	D,W	Sandy Loam
29	Sand	Poulovassilis and Childs, (1971)	I, D,W	Sand
30	Sand-17	Perrens and Watson, (1977)	D,W	Sand
31	Sand-R8	Ayers and Watson, (1977)	D,W	Sand
32	WDS	Mualem and Klute, (1984)	I, D,W	Sand
33	BCS	Pham et al., (2003)	I, D,W	Sand (CP ³)
34	PSi	Pham et al., (2003)	I, D,W	Silt Loam (CP ³)
35	A20	Lu et al., (2013)	I,W	Loamy Sand
36	A35	Lu et al., (2013)	I,W	Loam
37	A55	Lu et al., (2013)	I,W	Sand

S/N	Soil ¹	Reference	Available SWCCs ²	Soil Classification
38	B20	Lu et al., (2013)	I,W	Sandy Loam
39	B37	Lu et al., (2013)	I,W	Loam
40	B62	Lu et al., (2013)	I,W	Loamy Sand
41	S7	Likos et al., (2013)	I,W	Sand
42	S8	Likos et al., (2013)	I,W	Sand
43	S9	Likos et al., (2013)	I,W	Sand
44	S10	Likos et al., (2013)	I,W	Sand
45	S11	Likos et al., (2013)	I,W	Sand
46	S12	Likos et al., (2013)	I,W	Loamy Sand
47	S13	Likos et al., (2013)	I,W	Loamy Sand
48	S14	Likos et al., (2013)	I,W	Sandy Loam
49	S15	Likos et al., (2013)	I,W	Sandy Loam
50	S16	Likos et al., (2013)	I,W	Sandy Loam
51	S17	Likos et al., (2013)	I,W	Sandy Loam
52	S18	Likos et al., (2013)	I,W	Loam
53	S19	Likos et al., (2013)	I,W	Loam
54	S20	Likos et al., (2013)	I,W	Loam
55	S21	Likos et al., (2013)	I,W	Loam
56	S22	Likos et al., (2013)	I,W	Loam
57	S23	Likos et al., (2013)	I,W	Loam
58	S24	Likos et al., (2013)	I,W	Loam
59	S25	Likos et al., (2013)	I,W	Loam

Note: 1: Soils 1 to 34 from Pham et al. (2003) and soils 35 to 59 from Likos et al. (2013); 2: I = Initial drying curve; D = boundary drying curve; W = boundary wetting curve; 3: CP = compacted soil.

Pham et al. (2005) suggested that the AEVs for IDC and BDC are the same ($a_{id} = a_{bd}$), which is adopted in this study. Feng and Fredlund (1999) assumed that BDC and BWC have similar shape and only the air-entry value is different, which means that the BWC can be obtained by changing the AEV or shifting the BDC laterally. Various lateral shifts ranging from 0.25 to 0.5 log cycles are suggested by Pham (2002). Likos et al. (2013) suggested that the AEV for BWC equals to half of the AEV of either the IDC or the BDC. For the 59 soils in Table 6.16, the AEV of the BWC is not equal to

half of the AEV of the IDC or the BDC. The mean and standard deviation (SD) of the ratio for the AEV-related parameters for the 34 soils used by Pham et al. (2005) and 25 soils used by Likos et al. (2013) are presented in Table 6.17. The mean ratios of AEV-related parameters are 2.32 and 2.24 for Pham et al. (2005) and Likos et al. (2013) datasets, respectively, and the values of the standard deviation (SD) are both greater than 50%. Nielsen and Luckner (1992) suggested that during the drying and wetting processes, the AEV of the drying curve corresponds to the radii of the bottlenecks and the AEV of the wetting curve corresponds to the radii of the bottle bellies based on capillary theory and ink-bottle effect. Thus, the AEV is affected by soil structure. Hence, Equation 6.5a needs to account for soil structure effect.

Table 6.17 Mean and standard deviation of the ratio of AEV-related parameters for Pham et al. (2005) and Likos et al. (2013) datasets

	Pham et al. (2005), a_{bd}/a_{bw}	Likos et al. (2013), α_{bw}/α_{bd}
Mean Value	2.32	2.24
SD	1.21	1.23
SD%	52.1%	54.8%

Equations 6.5b and 6.5c describe the relationship between slopes of drying and wetting curves. The slope of the SWCC is correlated with the slope of the grain-size distribution, which is independent of wetting direction unless there is significant fabric rearrangement and air entrapment (Likos et al., 2013). Kool and Parker (1987) examined eight soils' BDCs and BWCs and concluded that by assuming that the BDC and BWC are parallel does not lead to an unacceptable loss of accuracy. Table 6.18 summarised the mean ratios of the slope-related parameters for BDC and BWC. The mean ratios for the slope-related parameters are 0.89 and 1.00 for Pham et al. (2005)

and Likos et al. (2013) datasets, respectively. The values of SD are 39.6% and 10.4%, respectively. For Pham et al. (2005) dataset, the mean ratio of the slope-related parameters between drying and wetting curves is less than 1 but the SD is relatively large. Adopting the mean ratio of the slope-related parameters of 1 as found for Likos et al. (2013) dataset is reasonable as it is still within one SD. Thus, it can be assumed that IDC and BWC have the same values for parameters m and n .

Table 6.18 Mean and standard deviation of the ratio of the slope-related parameters of the SWCC for Pham et al. (2005) and Likos et al. (2013) datasets

	Pham et al. (2005), S_{bd}/S_{bw} *	Likos et al. (2013), n_{bd}/n_{bw}
Mean	0.89	1.00
SD	0.35	0.10
SD%	39.6%	10.4%

Note: * S=slope of SWCC at inflection point as used in Pham et al. (2005).

Equation 6.5d describes the relationship between the residual soil suctions ψ_r for drying and wetting SWCCs. The residual suctions for IDC, BDC and BWC are recommended to be equal to 100 kPa for coarse-grained soils and 1500 kPa for fine-grained soils as suggested by Chin et al. (2010) and Lu (2016) to simplify the SWCC determination without significantly affecting the accuracy of the SWCC equation.

The results of the study are presented in Table 6.19. For all soils, the mean ratio for the parameter a for IDC or BDC and BWC, a_{bw}/a_{bd} , equals to 1.980, which is similar with Likos et al. (2013) finding. For sand, the mean a_{bw}/a_{bd} is 2.164, which is greater than Pham et al. (2005) recommended value of 1.585. This is mainly because several soils in Likos et al. (2013) dataset gave greater a_{bw}/a_{bd} , such as A55, S8, and S11 which gave 4.69, 6.17 and 3.88, respectively. For sandy loam and loamy sand, the mean a_{bw}/a_{bd} is 1.806, which is very close to the mean a_{bw}/a_{bd} for silt loam, loam and clay

loam soils. For compacted soils, the mean a_{bw}/a_{bd} is 2.242. Hence, it is recommended that the a_{bw}/a_{bd} in the proposed model is equal to the mean a_{bw}/a_{bd} for the respective soil types.

Table 6.19 Mean and standard deviation of a_{bw}/a_{bd}

Soil Type	Mean	SD	SD%
Sand	2.164	1.128	52.12%
Sandy Loam and Loamy Sand	1.806	0.733	40.58%
Silt Loam, Loam, and Clay Loam	1.832	0.765	41.73%
Compacted Soils	2.242	0.287	12.79%
All Soil	1.980	0.936	47.30%

The difference between $\theta_{s,bw}$ and $\theta_{s,id}$ normalised by $\theta_{s,id}$ corresponds to the air entrapment, AE. For the 59 soils in Table 6.16, only 33 soils have IDC. For the 33 soils, the range of AE is from 2.2% to 35.0% and the mean AE is 15.4%, which is consistent with Fayer and Hillel (1986), who found that AE typically ranges from 0 to 20%. For sand, the mean AE is 19.5%. For sandy loam and loamy sand, the mean AE is 14.5%, which is slightly greater than the findings of Fayer and Hillel (1986) and Sakaguchi et al., (2005), which are both 13%. For loam, silt loam and clay loam, the mean AE is 8.8%, which agrees with Faybishenko (1995) (10%), Seymour (2000) (between 3 to 15%) and Sakaguchi et al. (2005) (9%). For compacted soils, the mean AE is 7.3%. The mean AE for different soil types as summarised in Table 6.20 is adopted for the proposed model. Table 6.20 shows that the finer the soils, the smaller the AE.

Table 6.20 Mean and standard deviation of AE in soils

Soil Type	Mean	SD	SD%
Sand	19.5%	6.8%	34.6%
Loamy Sand and Sandy Loam	14.5%	6.8%	46.7%
Loam, Silt Loam, and Clay Loam	8.8%	7.7%	88.2%
Compacted Soil	7.3%	2.1%	29.3%
All Soils	15.4%	9.2%	60.1%

In summary, the equations for the proposed model are as follows:

$$(\theta_{s,id} - \theta_{s,bw}) / \theta_{s,id} = \text{mean AE from Table 6.20 for respective soil types} \quad (6.5a)$$

$$\frac{a_{bw}}{a_{bd}} = \text{mean value from Table 6.19 for respective soil types} \quad (6.5b)$$

$$\frac{n_{bw}}{n_{id}} = 1 \quad (6.5c)$$

$$\frac{m_{bw}}{m_{id}} = 1 \quad (6.5d)$$

$$\psi_{r,bw} = \psi_{r,id} = \begin{cases} 100 \text{ kPa for coarse – grained soils} \\ 1500 \text{ kPa for fine – grained soils} \end{cases} \quad (6.5e)$$

The proposed model can use either a known IDC to estimate BDC and BWC or, a known BDC to estimate BWC.

6.4.4 Evaluation of proposed hysteresis SWCC model

Forty soils from the the literature not used in the calibration of the hysteresis model were used to evaluate the proposed model. Comparison is also made with Likos et al. (2013) and Pham et al. (2005) models. The soils used for the evaluation are summarised in Table 6.21. The estimated BWC was used to determine the water contents at the suctions for the measured BWC. The measured and estimated water contents were then compared using three performance indices, $RMSE$, R^2 and d , to compare the three models. For the models' comparison, the water content at zero suction for IDC was assumed known when both the IDC and BDC are available and the water content at zero matric suction for the BDC was assumed known when only the BDC is available.

The evaluation results are summarised in Table 6.22. The measured versus estimated water contents for BWC for proposed, Pham et al. (2005) and Likos et al. (2013) models are presented in Figure 6.10 to Figure 6.12. According to Table 6.22, all the three models performed well for the BWC estimation. But the proposed model has the lowest *RMSE* and the highest R^2 and d according to the results. Hence, this suggests that the proposed model performed better than the other two models.

Table 6.21 Soils used for SWCC hysteresis SWCC model evaluation

S/N	Soil	Reference	Available SWCCs	Soil Type
1	DV	Chiu and Ng, (2003)	I,W	Silt Loam
2	FS	Yang et al., (2004)	I,W	Sand
3	CSI	Yang et al., (2004)	I,W	Sandy Loam
4	CSII	Yang et al., (2004)	I,W	Sandy Loam
5	CS	Viaene, et al., (1994)	I,W	Sand
6	NW1	Baker, (2001)	I,D,W	Sand
7	NW2	Baker, (2001)	I,D,W	Sand
8	NW7	Baker, (2001)	I,D,W	Loamy Sand
9	NW8	Baker, (2001)	I,D,W	Loamy Sand
10	NW9	Baker, (2001)	I,D,W	Loamy Sand
11	NW10	Baker, (2001)	I,D,W	Loamy Sand
12	NW11	Baker, (2001)	I,D,W	Sandy Loam
13	NW12	Baker, (2001)	I,D,W	Sandy Loam
14	NW16	Baker, (2001)	I,D,W	Loam
15	NW17	Baker, (2001)	I,D,W	Sandy Loam
16	NW18	Baker, (2001)	I,D,W	Sandy Loam
17	NW19	Baker, (2001)	I,D,W	Silt Loam
18	NW20	Baker, (2001)	I,D,W	Sandy Loam
19	NW21	Baker, (2001)	I,D,W	Sandy Loam
20	NW23	Baker, (2001)	I,D,W	Loamy Sand
21	NW24	Baker, (2001)	I,D,W	Loamy Sand
22	NW25	Baker, (2001)	I,D,W	Sand
23	NW27	Baker, (2001)	I,D,W	Sand
24	ES1	Gallage and Uchimura, (2010)	I,W	Loamy Sand
25	ES2	Gallage and Uchimura, (2010)	I,W	Loamy Sand
26	ES3	Gallage and Uchimura, (2010)	I,W	Loamy Sand
27	IS	Gallage and Uchimura, (2010)	I,W	Loamy Sand

S/N	Soil	Reference	Available SWCCs	Soil Type
28	TS	Gallage and Uchimura, (2010)	I,W	Sand
29	CS1	Gallage and Uchimura, (2010)	I,W	Sandy Loam
30	CS2	Gallage and Uchimura, (2010)	I,W	Sandy Loam
31	SK-5	Goh, (2012)	I,D,W	Silt Loam
32	SK-10	Goh, (2012)	I,D,W	Loam
33	SK-17-1	Goh, (2012)	I,D,W	Sandy Loam
34	SK-17-2	Goh, (2012)	I,D,W	Sandy Loam
35	Sand IV	Goh, (2012)	I,D,W	Sand
36	Sand V-1	Goh, (2012)	I,W	Sand
37	Sand V-2	Goh, (2012)	I,W	Sand
38	Sand VI	Goh, (2012)	I,D,W	Sand
39	SCS	Khoury, (2010)	I,W	Sandy Loam
40	CK	Meilani, (2004)	I,W	Silt Loam

Table 6.22 Performance comparison for SWCC hysteresis SWCC models

	Proposed Model	Pham et al. (2005)	Likos et al. (2013)
R^2	0.976	0.894	0.963
$RMSE$	0.019	0.040	0.023
d	0.994	0.969	0.991

Figure 6.10 Estimated water content versus measured water content for boundary wetting curves for proposed model

Figure 6.11 Estimated water content versus measured water content for boundary wetting curves for Pham et al. (2005) model

Figure 6.12 Estimated water content versus measured water content for boundary wetting curves for Likos et al. (2013) model

Figure 6.13 to Figure 6.15 present the worst, average and best estimations of the BWC, respectively, from the 40 soils evaluation dataset based on R^2 and $RMSE$. Proposed model gives worst estimation for Soil NW12 in terms of both R^2 and $RMSE$, which is presented in Figure 6.13. Figure 6.14(a) and (b) present the average estimation of the BWC for soils NW18 and soil FS based on R^2 and $RMSE$, respectively. Figure 6.15(a) and (b) present the best estimation of the BWC for soils Sand VI and NW2 based on R^2 and $RMSE$, respectively.

Figure 6.16(a) compares the estimation of BWC for Soil NW12 where the proposed model gave the worst estimation in terms of both R^2 and $RMSE$ with the BWC estimated by Pham et al. (2005) and Likos et al. (2013) models. A summary of the R^2 and $RMSE$ is also shown in the plot. Pham et al. (2005) model predicted the best BWC but fails to predict the correct AEV of BWC, while the proposed and Likos et al. (2013) models gave good prediction of the AEV. Both the proposed and Pham et al. (2005) models gave good estimation of the BDC. The proposed model did not perform well for soil NW12 mainly because that there is no data beyond 10 kPa for the IDC. It is recommended that for coarse-grained soils and fine-grained soils, the IDC should have the SWCC data till 100 kPa and 1500 kPa, respectively, which are near the residual matric suctions for coarse-grained and fine-grained soils as suggested by Chin et al. (2010) and Lu (2016), respectively. If it is assumed that the measured volumetric water content for the IDC at 100 kPa is equal to the volumetric water content of the BDC at 100 kPa, it can be observed that the performance of the proposed model improved significantly as illustrated in Figure 6.16(b). The $RMSE$ for the proposed model decreased from 0.045 to 0.017 for the BWC estimation.

Figure 6.17 compares the estimated $\theta_{s,iw}$ from the proposed model and measured $\theta_{s,iw}$. The proposed relationship between $\theta_{s,iw}$ and $\theta_{s,id}$ (or $\theta_{s,bd}$) for the proposed model given in Table 6.20 gives good prediction of $\theta_{s,iw}$ with $R^2=0.934$ and $RMSE=0.023$. The comparison shows that the assumption for AE using Table 6.20 is reasonable.

Figure 6.13 Worst estimation for boundary wetting curve of soil NW12 from Baker (2001)

Figure 6.14 Average estimation for boundary wetting curve

Figure 6.15 Best estimation for boundary wetting curve

Figure 6.16 Comparison of estimated boundary wetting for proposed model, Pham et al. (2005) and Likos et al. (2013) for soil NW12 from Baker (2001): (a), estimation using original IDC, which has no data beyond 10 kPa; (b) estimation using original measured IDC and add one measured data at 100 kPa

Figure 6.17 Estimated and measured $\theta_{s,iw}$ comparison

6.4.5 Summary

An empirical model is developed in this study to estimate the boundary wetting SWCC from the initial drying SWCC or boundary drying SWCC. The proposed model was developed using 59 soils from Pham et al. (2005) and Likos et al. (2013). The mean value for the slope-related parameter ratios was found to be about 1. Hence, the proposed model suggests shifting the initial drying curve or boundary drying curve laterally to estimate the boundary wetting curve. In the proposed model the AEV-related parameter, a , and air entrapment for the initial volumetric water content of the boundary drying or wetting curve are dependent on the soil types. The proposed model was evaluated and compared with two other hysteresis SWCC models, Pham et al. (2005) and Likos et al. (2013), using 40 soils from the literature. The evaluation shows that the proposed model performed better than the other two models. Compared to

Pham et al. (2005) model, the proposed model uses the Fredlund and Xing (1994) SWCC equation and uses less parameters. Compared to the Likos et al. (2013) model, the proposed model can use either the initial drying curve or the boundary drying curve as input and accounts for air entrapment based on soil types. The model proposed is only applicable to unimodal SWCC.

6.5 Conclusion

Four parametric-estimation PTFs using Brooks and Corey (1964) SWCC equation, seven parametric-estimation PTFs using van Genuchten (1980) SWCC equation and one parametric-estimation PTF using Fredlund and Xing (1994) SWCC equation were evaluated using 40 soils collated from the literature in this chapter. The evaluation showed that generally, PTFs using van Genuchten (1980) and Fredlund and Xing (1994) SWCC equations performed better than the PTFs using Brooks and Corey (1964) equation. The best performance amongst all the evaluated parametric-estimation PTFs was given by Chin et al. (2010) PTF using Fredlund and Xing (1994) SWCC equation. However, this is at the expense of performing one-point measurement of the SWCC.

A zero experimental point method was established based on Chin et al. (2010) one-point method and point-estimation PTFs. The proposed zero experimental point method used two point-estimation PTFs at suctions of 10 and 100 kPa, which are Gupta and Larson (1979) and Rawls et al. (1982) point-estimation PTFs, respectively, for coarse-grained soils and 100 and 1500 kPa, which are Gupta and Larson (1979) and Rawls et al. (1982) PTFs, respectively, for fine-grained soils. The proposed zero experimental point method was evaluated and compared with Chin et al. (2010) one-point method by using 65 coarse-grained and 65 fine-grained soils from the literature. The comparison

showed that the zero experimental point method performance is comparable with Chin et al. (2010) one-point method for both coarse-grained soils and fine-grained soils. In general, the proposed zero experimental point method is more advantageous as no actual measurement of the SWCC is needed to provide an estimation of the SWCC.

A SWCC hysteresis model is developed in this chapter using 59 soils from Pham et al. (2005) and Likos et al. (2013). The proposed SWCC hysteresis model suggests shifting the initial drying curve or boundary drying curve laterally to estimate the boundary wetting curve. In the proposed SWCC hysteresis model, the AEV-related parameter, a , and air entrapment for the initial volumetric water content of the boundary drying or wetting curve are dependent on the soil types. The proposed SWCC hysteresis model was evaluated and compared with two other SWCC hysteresis models, Pham et al. (2005) and Likos et al. (2013), using 40 soils from the literature. The evaluation shows that the proposed hysteresis SWCC model performed better than the other two models. The proposed SWCC hysteresis model described in this chapter is only applicable to unimodal SWCC.

CHAPTER 7

BIMODAL SOIL-WATER

CHARACTERISTIC CURVE

7.1 Introduction

Bimodal SWCCs are associated with the dual-porosity soils (Satyanaga et al., 2013; Miguel and Bonder, 2012; Zhang and Chen, 2005; Li et al., 2014), such as gap-graded soils (Satyanaga et al., 2013), soils compacted dry of optimum (Sivakumar and Wheeler, 2000), and coarse colluvial soils (Li and Zhang, 2009; Miguel and Bonder, 2012).

The SWCC is usually determined by laboratory test at discrete suction levels. A SWCC equation is needed to fit the discrete SWCC data to obtain a continuous SWCC. A number of SWCC equations for unimodal SWCC have been evaluated by Leong and Rahardjo (1997a). Leong and Rahardjo (1997a) concluded that Fredlund and Xing (1994), and van Genuchten (1980) equations gave the best fit among the unimodal SWCC equations evaluated. Recently, it was observed that soils with dual porosity structure have associated bimodal SWCC (Burger and Shackelford, 2001, Li et al., 2014). The typical bimodal SWCC is shown in Figure 7.1. Bimodal SWCC usually consists of five segments. Segment 3 joins two unimodal SWCC. Several bimodal SWCC equations have been proposed (Zhang and Chen, 2005; Carducci et al., 2011; Satyanaga et al., 2013; Li et al., 2014; and Wijaya and Leong, 2016). This chapter will evaluate various bimodal SWCC equations.

Figure 7.1 Typical bimodal SWCC

Determination of bimodal SWCC in the laboratory is costly and time-consuming. Zhang and Chen, (2005) and Satyanaga et al. (2013) proposed parametric-estimation PTFs to estimate the bimodal SWCC from grain-size distribution (GSD) or other routinely measured soil properties. But the proposed PTFs are very complicated to use. In this study, a relatively simple PTF will be proposed for the bimodal SWCC determination.

It is widely recognised that the water content of a soil at a certain suction level is not unique. This phenomenon is called hysteresis of SWCC. A relatively simple hysteresis model for unimodal SWCC was proposed in Chapter 6. In this chapter, the hysteresis of bimodal SWCC will also be investigated.

7.2 Evaluation of Bimodal SWCC Equations

7.2.1 Bimodal SWCC equations

Wijaya and Leong (2016) summarised three approaches: piecewise, fraction volume and unique parameter. The piecewise approach separates the bimodal SWCC into two unimodal segments joined at a merging point. The location of the merging point affects the parameters of the SWCC equation significantly and the parameters are highly non-unique (Wijaya and Leong, 2016). This approach may not give a smooth transition at the merging point. The fraction volume approach improved the piecewise approach. In this approach, the two fractions of the porosity corresponding to macro- and micro-pores are described by a volume. The SWCC equations describing macro- and micro-pores have the same form and the optimisation is done together. Generally, piecewise and fraction volume approaches extend a unimodal SWCC equation to a bimodal SWCC equation. The unique parameter approach incorporates some graphically determined parameters to reduce the number of curve-fitting parameters. The third approach is more attractive as less parameters need to be determined (Wijaya and Leong, 2016). This study concentrates on bimodal SWCC equations using unique parameter approach.

A number of SWCC equations for unimodal SWCC have been evaluated by Leong and Rahardjo (1997a). Several bimodal SWCC equations have been proposed (Zhang and Chen, 2005; Carducci et al., 2011; Satyanaga et al., 2013; Li et al., 2014; and Wijaya and Leong, 2016, Zhang et al., 2017). Compared to the unimodal SWCC equations, bimodal SWCC equations have more unknown parameters. Hence, the determination of the parameters for bimodal SWCC equations is more complicated and

time-consuming. Sometimes, numerical problem may occur during the optimisation process for curve fitting as the equation has too many unknown parameters. Bimodal SWCC equations will be evaluated to examine the difficulty of the curve fitting and to suggest improvement.

The bimodal SWCC equations evaluated in this study are summarised in .

Zhang and Chen (2005) (ZC) and Carducci et al. (2011) (CA) bimodal SWCC equations are extensions of Fredlund and Xing (1994) and van Genuchten (1980) unimodal SWCC equations, respectively. The effects of the parameters of ZC and CA on the bimodal SWCC are similar as the effects of the parameters on Fredlund and Xing (1994) and van Genuchten (1980) unimodal SWCC equations, respectively. The ZC equation parameters p_b and n_{pl} , p_s and n_{ps} can be combined together as θ_l and θ_s (Zhang et al., 2017), respectively, and $\theta_s + \theta_l = \theta_{sat}$, where θ_{sat} is assumed known. Hence, the number of unknown parameters in the ZC equation can be reduced from 12 to 9 and the ZC equation can be rewritten as Equation 7.1 .

$$\theta = \theta_l \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_{rl}} \right)}{\ln \left(1 + \frac{10^6}{\psi_{rl}} \right)} \right] \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a_l} \right)^{n_l} \right]} \right\}^{m_l} + (\theta_s - \theta_l) \left[1 - \frac{\ln \left(1 + \frac{\psi}{\psi_{rs}} \right)}{\ln \left(1 + \frac{10^6}{\psi_{rs}} \right)} \right] \left\{ \frac{1}{\ln \left[e + \left(\frac{\psi}{a_s} \right)^{n_s} \right]} \right\}^{m_s} \quad (7.1)$$

Satyanaga et al. (2013) proposed a bimodal SWCC equation (SA) based on Fredlund and Xing (1994) SWCC equation by assuming that the shape of the two unimodal SWCC sub-curves follows lognormal distribution, which can be found in Table 2.3. The SA contains nine unknown parameters. Figure 7.2 illustrates the effects of each of the nine parameters. Parameters ψ_{a1} , ψ_{m1} , s_1 , and θ_{s2} control the shape of the

first sub-curve (Figure 7.2 a to d, respectively) and parameters ψ_{a2} , ψ_{m2} , S_2 , and θ_r control the shape of the second sub-curve (Figures 7.2 e to h, respectively). It is observed that parameter ψ_r has insignificant affects for the bimodal SWCC (Figure 7.2 i). Zhai and Rahardjo (2012) recommended ψ_r as 1500 kPa for unimodal SWCC. For bimodal SWCC, Li et al. (2014) recommended ψ_r to be 3000 kPa considering the constraint of $\psi_r > \psi_{a2} > \psi_{m2}$, to simplify the equation.

Li et al. (2014) extended their unimodal SWCC equation to bimodal SWCC equation. Li et al. (2014) bimodal SWCC equation (LI) has nine unknown parameters. Four parameters, λ , l , m , and n were assumed to be equal to 3, 4, 0.8 and 2, respectively, to simplify the equation to Equation 7.2. Equation 7.2 has five unknown parameters. Figure 7.3 illustrated the effects of each of the five parameters on the shape of the bimodal SWCC. It is observed that ψ_{a1} , ψ_t , θ_r , and ψ_{a2} , ψ_r , θ_r control the shapes of first and second sub-curves, respectively.

$$\theta = \frac{(0.75\theta_s - 3\theta_r)\sqrt{\psi_a\psi_t}^{2/\log(\frac{\psi_t}{\psi_a})}}{\psi^{2/\log(\frac{\psi_t}{\psi_a})} + \sqrt{\psi_a\psi_t}^{2/\log(\frac{\psi_t}{\psi_a})}} + \frac{(0.25\theta_s - \theta_r)(4\psi_t)^{0.8}}{\psi^{0.8} + (4\psi_t)^{0.8}} + \frac{3\theta_r\sqrt{\psi_{a2}\psi_r}^{2/\log(\frac{\psi_r}{\psi_{a2}})}}{\psi^{2/\log(\frac{\psi_r}{\psi_{a2}})} + \sqrt{\psi_{a2}\psi_r}^{2/\log(\frac{\psi_r}{\psi_{a2}})}} + \frac{\theta_r(4\psi_t)^{0.8}}{\psi^{0.8} + (4\psi_t)^{0.8}} \quad (7.2)$$

Wijaya and Leong (2016) proposed a bimodal SWCC equation (WL) using graphical approach. The WL equation divides the bimodal SWCC into five segments and describes the SWCC by a continuous equation using parameters of each segments' slope, intersections and curvatures between segments. Besides determining the parameters graphically from the plot, the parameters of the WL equation can also be determined by curve-fitting.

(a), Effect of ψ_{a1} (b), Effect of ψ_{m1}

(c), Effect of S_1 (d), Effect of θ_{s2}

(e), Effect of ψ_{a2} (f), Effect of ψ_{m2}

(g), Effect of S_2 (h), Effect of θ_r

(i), Effect of ψ_r Figure 7.2 Effect of parameters for SA equation (assumed $\theta_s=0.55$) (a), Effect of ψ_a

(b), Effect of ψ_t (c), Effect of ψ_{a2}

(d), Effect of ψ_r (e), Effect of θ_r Figure 7.3 Effect of parameters for LI equation (assumed $\theta_s = 0.5$)

7.2.2 Evaluation of bimodal SWCC equations

Most of the time, saturated water content can be treated as known (Leong and Rahardjo, 1997a). Compared with unimodal SWCC equations, bimodal SWCC equations have more unknown parameters. Hence, the determination of the parameters for bimodal SWCC equations is more complicated and time-consuming. Sometimes, numerical problem may occur during the optimising process as the equation has too many unknown parameters. The numbers of unknown parameters for bimodal SWCC equations are summarised in . The number of unknown parameters for these equations range from 8 to 13.

Thirty-one soils from the literature, summarised in Table 7.1, were used to evaluate the bimodal SWCC equations. The textures of the soils are shown in Figure 7.4. Microsoft Excel Solver was used to perform the curve fit. The best fit for each curve is the one giving the highest coefficient of determination, R^2 .

Table 7.1 Soils used for bimodal SWCC equations evaluation

Reference	Soil	Max/Min/Mean			
		Sa (%)	Si (%)	Cl (%)	ρ_d (Mg/m ³)
Andersson and Wiklert, (1972)	276, 277, 296, 297, 299, 300, 302, 326, 327, 328, 116, 166, 113, 114, 183, 201, 202, 203, 204	96/	55/	87/	1.71/
Miguel and Bonder, (2012)	1.5, 2.5, 3.5, 4.5, 5.5, 6.5	1/47	2/20	2/19	1.01/1.38
UNSODA (Nemes et al., 2001)	2530, 2601, 2604, 2750, 2751, 2753

Figure 7.4 Texture for soils shown in USDA textural triangle used for evaluating bimodal SWCC equation

Three performance indices, AIC (Equation 3.9), $RMSE$ (Equation 3.6), and d (Equation 3.10), were used to evaluate the performance of the bimodal SWCC equations. For $RMSE$ and AIC , a smaller value infers a better performance. For d , a value of 1 indicate perfect agreement (Willmott, 1981). AIC considers the difficulty of parameters determination by including the number of unknown parameters in the computation of AIC . Leong and Rahardjo (1997a) ranking approach was employed to carry out the evaluation. A smaller ranking means better performance.

Both LI equation and its simplified form (Equation 7.3), LI^s , were evaluated in this study. The WL equation is evaluated using optimisation approach to be consistent with the other equations. The SA equation is evaluated by constraining the value of ψ_r to be 3000 kPa (Fredlund and Xing, 1994; Li et al., 2014). The ZC equation with reduced parameters was evaluated. The initial values for the optimisation process of the various equations are summarised in Table 7.2. These initial values were chosen such that corresponding parameters of the various equations have the same initial values for the optimisation process for consistency.

Table 7.2 Summary of the initial values of parameters in the bimodal SWCC equations for the optimisation process

Equation	Initial values of parameters	
	First sub-curve	Second sub-curve
ZC	$\theta_l=0.25$, $a_l=10$ kPa, $n_l=2$, $m_l=2$, $\psi_{rl}=100$ kPa	$a_s=1000$ kPa, $n_s=2$, $m_s=2$, $\psi_{rs}=3000$ kPa
CA	$\alpha_l=0.1$ kPa ⁻¹ , $n_l=2$, $m_l=2$, $\theta_m=0.25$	$\alpha_s=0.0001$ kPa ⁻¹ , $n_s=2$, $m_s=2$, $\theta_r=0.05$
SA	$\psi_{a1}=1$ kPa, $\psi_{m1}=10$ kPa, $s_1=1.5$, $\theta_{s2}=0.25$	$\psi_{a2}=100$ kPa, $\psi_{m2}=1000$ kPa, $s_1=1.5$, $\theta_r=0.05$, $\psi_r=3000$ kPa
LI	$\psi_a=10$ kPa, $\psi_t=100$ kPa, $l=4$, $m=0.8$, $n=2$	$\psi_{a2}=1000$ kPa, $\psi_r=3000$ kPa, $\theta_r=0.05$
LI^s	$\psi_a=10$ kPa, $\psi_t=100$ kPa	$\psi_{a2}=1000$ kPa, $\psi_r=3000$ kPa, $\theta_r=0.05$
WL	$S_1=S_3=0.05$, $S_2=0.5$, $c_2=c_3=6$, $\psi_2=5$ kPa, $\psi_3=50$ kPa	$S_5=0.05$, $S_4=0.5$, $c_4=c_5=6$, $\psi_4=1000$ kPa, $\psi_5=3000$ kPa

7.2.3 Results and discussion

A summary of the evaluation is presented in Table 7.3. It is observed that, based on all three performance indices, the WL equation performed the best among the six bimodal SWCC equations. The LI equation performed better than the LI^s equation.

All equations except the WL equation assumed that the SWCC before the first air-entry value is a horizontal line. The WL equation does not always give zero water content at suction of 10^6 kPa and tends to fit an approximately horizontal line for segment 5,

especially for SWCCs without data in the high suction range (i.e., greater than residual suction of the second sub-curve).

The ZC equation performed well for all soils except soils with weak bimodal SWCC. If the slope of segment 3 of the curve (as shown in Figure 7.1), sl_3 , has similar slope as segment 2 or 4, (i.e., $sl_2/sl_3 \leq 2$ or $sl_4/sl_3 \leq 2$), or length of segment 3 is less than one log-cycle, or the pore volume of any sub-curve is less than $\frac{1}{4}$ of total soil pore volume, the SWCC is considered to be a weak bimodal SWCC (Zhao and Zhang, 2014). The ZC equation tends to fit the weak bimodal SWCC as a unimodal SWCC.

Figure 7.5 Bimodal SWCC equations fitting for Soil 327 from Andersson and Wiklert, (1972)

Table 7.3 Bimodal SWCC equations evaluation performance summary using Leong and Rahardjo (1997a) ranking approach

Soil	<i>d</i>						<i>AIC</i>						<i>RMSE</i>					
	ZC	CA	SA	LI	LI ^s	WL	ZC	CA	SA	LI	LI ^s	WL	ZC	CA	SA	LI	LI ^s	WL
276	2	6	5	3	4	1	3	6	5	2	4	1	2	6	5	3	4	1
277	3	6	2	4	5	1	3	6	2	4	5	1	3	6	2	4	5	1
328	2	1	5	4	6	3	5	1	6	4	2	3	2	1	5	4	6	3
327	2	3	5	4	6	1	6	2	5	4	3	1	2	3	5	4	6	1
326	3	2	5	4	6	1	5	2	6	3	4	1	3	2	5	4	6	1
300	5	4	2	3	6	1	6	4	3	2	5	1	5	4	2	3	6	1
299	2	4	3	5	6	1	3	4	2	5	6	1	2	4	3	5	6	1
297	2	3	4	5	6	1	4	2	5	6	3	1	2	3	4	5	6	1
296	4	5	6	2	3	1	5	4	6	3	2	1	4	5	6	2	3	1
302	3	5	6	2	4	1	4	5	6	2	3	1	3	5	6	2	4	1
116	1	2	4	5	6	3	1	2	4	5	6	3	1	2	4	5	6	3
166	3	5	4	1	6	2	3	5	4	1	6	2	3	5	4	1	6	2
113	4	5	3	2	6	1	4	5	3	2	6	1	4	5	3	2	6	1
114	2	4	1	5	6	3	2	4	1	5	6	3	2	4	1	5	6	3
183	1	6	4	3	5	2	1	6	4	2	5	3	1	6	4	3	5	2
201	2	5	4	1	6	3	5	3	4	1	6	2	2	5	4	1	6	3
202	4	5	3	1	6	2	4	5	3	1	6	2	4	5	3	1	6	2
203	2	5	3	4	6	1	2	5	3	4	6	1	2	5	3	4	6	1
204	1	5	3	4	6	2	3	6	2	4	5	1	1	5	3	4	6	2
1.5	2	5	1	4	6	3	2	6	1	4	3	5	2	5	1	4	6	3
2.5	1	4	6	3	5	2	2	5	6	3	4	1	1	5	6	3	4	2
3.5	1	5	3	4	6	2	1	6	4	3	5	2	1	5	3	4	6	2
4.5	1	6	3	4	5	2	2	6	3	4	5	1	1	6	3	4	5	2
5.5	1	5	3	4	6	2	2	6	4	3	5	1	1	6	3	4	5	2
6.5	1	3	2	5	6	4	1	2	3	6	4	5	1	3	2	5	6	4
2530	5	6	4	1	3	2	5	6	4	2	1	3	5	6	4	1	3	2
2601	2	1	3	4	5	6	2	1	4	3	5	6	2	1	3	4	5	6
2604	2	6	5	1	4	3	2	6	5	1	3	4	2	6	5	1	4	3
2750	2	4	3	5	6	1	3	4	2	6	5	1	2	4	3	5	6	1
2751	3	4	1	5	6	2	6	4	1	5	2	3	3	4	1	5	6	2
2753	5	3	1	4	6	2	5	3	1	4	6	2	5	3	1	4	6	2
Total Rank	67	120	95	95	154	57	102	132	112	104	137	64	67	122	95	95	152	57

The SA equation performed well for all SWCCs except SWCCs having a slope before the first AEV as the SA equation assumed that the slope of this segment is zero (Figure 7.1). For this type of SWCCs, SA equation tends to fit with $\psi_{al} = 0$ kPa.

The LI equation performed as well as the SA equation. The LI equation tends to fit weak bimodal SWCC (Zhao and Zhang, 2014) as a unimodal SWCC and it may be unreasonable if SWCC data at suctions greater than residual suction are unavailable.

The LI^s and CA equations did not perform as well as the ZC, SA and LI equations, but the fitting results are still reasonable. Compared to the CA equation, the LI^s equation (Li et al., 2014) has less parameters, and, hence, it is easier to use. Similar with the LI equation, the LI^s equation may be unreasonable if SWCC data at suctions greater than residual suction are unavailable and fit weak bimodal SWCC as a unimodal SWCC.

It is recommended to constrain the parameters of bimodal SWCC equations as summarised in Table 7.4 to avoid numerical problem during curve-fitting and to provide reasonable fit to the bimodal SWCC data. The reasons for the values of the constraints are given below.

For the WL equation, parameter c_i is the curvature between segments $i-1$ and i . When c_i increases, the intersection between two segments becomes sharper. To produce a smooth SWCC, it is recommended that c_i should be less than 9. The two sub-curves forming the bimodal SWCC are governed by the macro-pores and the micro-pores, respectively, in a soil. Macro-pores are pores greater than $75 \mu m$ (Soil Science Glossary Terms Committee, 2008), and corresponds to a suction of 200 kPa according to Kelvin's equation. If the first sub-curve suction range is at least one log-cycle, the AEV for first sub-curve is likely to be

less than 20 kPa. Hence, it is recommended that $\psi_2 \leq 20$ kPa and $\psi_3 \leq 200$ kPa for the WL equation. For the ZC equation, it is recommended that $\psi_{rl} \leq 200$ kPa and $a_{rl} \leq 20$ kPa, similar as WL equation. To void numerical problem during curve-fitting for the ZC equation, it is recommended that m_s , n_s , m_l , and $n_l < 15$, as well. For SA equation, ψ_{ml} is the suction of the inflection point of first sub-curve. Hence, it is recommended that $\psi_{ml} \leq 100$ kPa. To avoid numerical problem during curve-fitting for the SA equation, it is recommended that s_l and s_2 should be less than 10 kPa. For LI equation, it is recommended that l , m , n should be less than 10, 3, and 5, respectively. For CA equation, it is assumed that $1/\alpha_l$ is approximately equal to AEV of first sub-curve. Hence, it is recommended that $\alpha_l > 0.05 \text{ kPa}^{-1}$. Similar to the ZC equation, it is recommended that m_1 , m_2 , n_1 , n_2 in the CA equation should be less than 15 to avoid numerical problems during curve-fitting.

Table 7.4 Summary for the bimodal SWCC equation parameters constraints

Equation	Constraints by authors or equation	Additional constraints recommended
ZC	$\psi_{rs} > a_s > \psi_{rl} > a_l$	$\psi_{rl} \leq 200 \text{ kPa}$, $a_l \leq 20 \text{ kPa}$, m_s , n_s , m_l , $n_l < 15$
CA	$\alpha_2 < \alpha_l$	m_1 , m_2 , n_1 , $n_2 < 15$, $\alpha_l > 0.05 \text{ kPa}^{-1}$
SA	$\psi_{m2} > \psi_{a2} > \psi_{m1} > \psi_{a1}$	$\psi_{m1} \leq 100 \text{ kPa}$, $\psi_{a1} \leq 20 \text{ kPa}$, s_1 , $s_2 < 10 \text{ kPa}$
LI	$\psi_a < \psi_t < \psi_{a2} < \psi_r$	$l < 10$, $m < 3$, $n < 5$, $\psi_a \leq 20 \text{ kPa}$
LI ^S	$\psi_a < \psi_t < \psi_{a2} < \psi_r$	$\psi_a \leq 20 \text{ kPa}$
WL	$S_2 > S_1$ and $S_2 > S_3$ for first sub-curve; $S_4 > S_3$ and $S_4 > S_5$ for second sub-curve; $\psi_2 < \psi_3 < \psi_4 < \psi_5$	$c_i < 9$, $\psi_2 < 20 \text{ kPa}$, $\psi_3 < 200 \text{ kPa}$

7.2.4 Summary

Five bimodal SWCC equations: Zhang and Chen (2005), Carducci et al. (2011), Satyanaga et al. (2013), Li et al. (2014), and Wijaya and Leong (2016) equations, were

evaluated using 31 bimodal SWCCs from the literature. The evaluation results showed that Wijaya and Leong (2016) equation outperformed the other equations. Wijaya and Leong (2016) equation, which can use both graphical and numerical optimisation approaches to obtain the parameters performed well for all types of bimodal SWCCs. Zhang and Chen (2005) and Li et al. (2014) equations tend to fit weak bimodal SWCC as unimodal SWCC. To minimise numerical problem during optimisation process, constraints for the parameters were suggested for all the bimodal SWCC equations.

7.3 Estimation of Bimodal Soil-water Characteristic Curve

7.3.1 Introduction

Bimodal SWCC equations have been evaluated in the previous section. Bimodal SWCC has more turning points and thus requires more measurement points compared to unimodal SWCC. To date, parametric-estimation PTFs for unimodal SWCC are not capable of estimating bimodal SWCCs (Li et al., 2014). Based on the evaluation of bimodal SWCC equations in the previous section, Wijaya and Leong (2016) equation performed the best. Hence, it is attractive to extend Wijaya and Leong (2016) bimodal SWCC equation to estimate bimodal SWCC using GSD, and other soil physical properties (i.e., parametric-estimation PTF).

7.3.2 Methodology

Wijaya and Leong (2016) bimodal SWCC equation was chosen not only because of its robust curve-fitting capability, but also because all the parameters of Wijaya and Leong (2016) equation have clear physical meaning and can be determined graphically.

Wijaya and Leong (2016) bimodal SWCC equation contains 13 unknown parameters, which includes five slopes, four intersection and four curvature parameters. The curvature parameters are related to the curvature between two-line segments as illustrated in Figure 7.6. The larger the value of the curvature parameter, the sharper is the intersection. Wijaya and Leong (2016) recommended an average curvature parameter of 9. For most bimodal SWCCs, segment 1 and segment 3 are horizontal lines (i.e.: $S_1 = S_3 = 0$), which are the same assumptions as Santayana et al. (2013) and Li et al. (2014) parametric-estimation PTFs to estimate bimodal SWCC. Hence, there remain seven unknown parameters to be determined.

Thirty-two soils are collated from the literature and summarised in Table 7.5. These soils are different from those in Table 7.3. The GSD data are fitted by the bimodal GSD equation proposed in this study (Equation 3.11). The coefficients of correlation (COC) between e , D_{10} , D_{30} , D_{60} , C_c , C_u , d_2 , d_3 , d_4 , d_5 and ψ_2 , ψ_3 , ψ_4 , ψ_5 , S_2 , S_4 were determined using Microsoft Excel. The COC of exponential relationship between these parameters were also determined and presented in Table 7.6. Multiple-regression analysis was carried out to establish the empirical relationship between different parameters as presented below.

For the intersection between the first and second segments, the matric suction, ψ_2 , is strongly correlated with $\ln(C_u)$, $\ln(d_2)$ and ψ_3 , while $\ln(\psi_2)$ is strongly correlated with $\ln(C_u)$. Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.67$):

$$\psi_2 = 82.403[\ln(C_u)]^{-2.144} \quad (7.3)$$

For the intersection between the second and third segments, the matric suction, ψ_3 is strongly correlated with D_{10} , d_3 , and ψ_2 , while $\ln(\psi_3)$ is strongly correlated to $\ln(C_u)$ and

$\ln(C_c)$. It was found that ψ_3 and ψ_2 are not independent. Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.66$):

$$\psi_3 = \frac{126.5 \exp(0.12\psi_2 + 1.13d_3)}{C_u^{0.35}} \quad (7.4)$$

For the intersection between the third and fourth segments, the matric suction, ψ_4 is strongly correlated with D_{30} , D_{60} and ψ_5 . Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.62$):

$$\psi_4 = 910.8 + 3057D_{60} \quad (7.5)$$

For the intersection between the fourth and fifth segments, the matric suction, ψ_5 , is strongly correlated with C_c and ψ_4 . Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.54$):

$$\psi_5 = 7738.5 + 2.015\psi_4 \quad (7.6)$$

The slope of the second segment, S_2 is strongly correlated with $\ln(D_{30})$ and $\ln(C_u)$. Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.67$):

$$S_2 = \frac{11.542 \exp(6.584D_{30})}{\exp(0.033C_u)} \quad (7.7)$$

The slope of fourth segment, S_4 is strongly correlated with $\ln(D_{60})$ and ψ_3 . Using multiple regression analysis, the following equation gave the highest R^2 (i.e., $R^2 = 0.31$):

$$S_4 = 26.744 \exp(0.620D_{60}) \quad (7.8)$$

The slope of the fifth segment, S_5 is strongly correlated with d_4 and $\ln(C_c)$. Using multiple regression analysis, the following equation give the highest R^2 (i.e., $R^2 = 0.687$).

$$S_5 = 2.975 + 5.684d_4 + 0.312 \ln(C_c) \quad (7.9)$$

Table 7.5 Soils used for bimodal SWCC estimation model calibration

S/N	Soil	Reference	ρ_d (Mg/m ³)	e	$P200(\%)$
1	NTU-1-b	Agus et al., (2001)	1.57	0.68	79
2	NTU-1-a	Agus et al., (2001)	1.52	0.82	87
3	NTU-3-b	Agus et al., (2001)	1.63	0.57	51
4	SLR-1-a	Agus et al., (2001)	1.36	0.92	37
5	SLR-1-b	Agus et al., (2001)	1.38	0.89	27
6	SLR-2-a	Agus et al., (2001)	1.34	0.94	61
7	SLR-2-c	Agus et al., (2001)	1.34	0.91	57
8	LA-b	Agus et al., (2001)	1.42	0.83	29
9	LA-c	Agus et al., (2001)	1.34	0.90	42
10	M2A30	Mendes, (2008)	1.31	1.11	58
11	M2A50	Mendes, (2008)	1.43	0.93	68
12	M1C600	Mendes, (2008)	1.56	0.76	36
13	Bloco2	Mendes, (2008)	1.31	1.13	40
14	M2B500	Mendes, (2008)	1.34	1.05	49
15	P1C250	Mendes, (2008)	1.18	1.34	47
16	P1A30	Mendes, (2008)	1.39	0.91	46
17	Bloco3	Mendes, (2008)	1.43	0.90	50
18	Bloco4	Mendes, (2008)	1.15	1.41	55
19	2530	Nemes et al., (2001)	1.36	0.94	64
20	11494	SoilVision, (2002)	1.31	0.96	29
21	11497	SoilVision, (2002)	1.46	0.85	16
22	11515	SoilVision, (2002)	1.43	0.83	19
23	11523	SoilVision, (2002)	1.27	1.07	30
24	11525	SoilVision, (2002)	1.28	1.05	30
25	11526	SoilVision, (2002)	1.24	1.12	30
26	11545	SoilVision, (2002)	1.38	1.00	33
27	11555	SoilVision, (2002)	1.75	0.51	41
28	11241	SoilVision, (2002)	1.61	0.68	55
29	11248	SoilVision, (2002)	1.50	0.80	46
30	11249	SoilVision, (2002)	1.56	0.75	44
31	11319	SoilVision, (2002)	1.52	0.85	36
32	11322	SoilVision, (2002)	1.59	0.71	35

Table 7.6 Coefficients of correlation between bimodal SWCC parameters and GSD parameters

	e	D ₁₀	D ₃₀	D ₆₀	C _c	C _u	d ₂	d ₃	d ₄	d ₅	ln(D ₁₀)	ln(D ₃₀)	ln(D ₆₀)	ln(C _c)	ln(C _u)	ln(d ₂)	ln(d ₃)	ln(d ₄)	ln(d ₅)	ψ ₂	ψ ₃	ψ ₄	ψ ₅
ψ ₂	-0.35	0.06	-0.18	-0.21	-0.23	-0.39	-0.36	-0.11	-0.27	0.22	0.23	-0.28	-0.44	-0.40	-0.66	-0.56	-0.15	-0.35	0.38	1.00	0.49	-0.11	-0.35
ψ ₃	-0.27	0.55	-0.13	0.26	-0.30	-0.34	0.12	0.57	-0.26	0.46	0.42	-0.05	0.02	-0.44	-0.46	-0.01	0.35	-0.16	0.57	0.49	1.00	0.07	-0.31
ψ ₄	0.19	0.23	0.55	0.60	0.45	-0.19	0.40	0.35	0.55	-0.01	0.30	0.48	0.47	0.32	0.10	0.42	0.38	0.36	0.01	-0.11	0.07	1.00	0.54
ψ ₅	0.24	-0.16	0.22	0.03	0.47	-0.03	-0.04	-0.23	0.11	-0.28	-0.15	0.17	0.06	0.38	0.23	0.06	-0.18	-0.03	-0.39	-0.35	-0.31	0.54	1.00
S ₂	0.07	-0.14	-0.01	0.02	0.13	0.08	-0.11	-0.03	0.14	0.16	0.03	0.44	0.14	0.14	0.40	0.01	0.17	0.23	0.09	0.18	-0.29	-0.13	-0.23
S ₄	0.06	0.43	-0.19	0.15	-0.30	-0.37	0.27	0.30	-0.33	0.32	0.32	-0.17	-0.52	-0.43	-0.38	0.21	0.16	-0.29	0.40	0.36	0.59	0.28	-0.16
S ₅	0.08	-0.21	0.43	0.30	0.38	0.22	0.10	0.14	0.77	-0.14	-0.09	0.48	0.40	0.61	0.45	0.17	0.27	0.62	-0.09	-0.38	-0.44	0.36	0.14
ln(ψ ₂)	-0.26	0.25	-0.19	-0.09	-0.28	-0.48	-0.24	0.06	-0.31	0.38	0.37	-0.24	-0.35	-0.51	-0.74	-0.38	0.01	-0.39	0.58	0.92	0.58	-0.07	-0.34
ln(ψ ₃)	-0.22	0.43	-0.27	-0.02	-0.44	-0.37	0.04	0.33	-0.46	0.43	0.26	-0.29	-0.26	-0.56	-0.54	-0.11	0.07	-0.38	0.51	0.49	0.89	-0.07	-0.25
ln(ψ ₄)	0.19	0.24	0.36	0.46	0.28	-0.30	0.36	0.33	0.38	0.00	0.27	0.25	0.26	0.07	-0.06	0.34	0.29	0.12	0.11	0.04	0.31	0.89	0.48
ln(ψ ₅)	0.07	-0.07	0.13	-0.01	0.14	-0.04	-0.10	-0.18	0.09	-0.19	-0.13	-0.03	0.00	0.06	0.15	0.00	-0.21	-0.12	-0.30	-0.30	-0.18	0.47	0.80
ln(S ₂)	0.09	-0.20	0.06	0.07	0.23	0.19	-0.11	-0.10	0.33	-0.06	-0.01	0.28	0.29	0.32	0.28	0.04	0.14	0.33	-0.13	-0.05	-0.50	-0.01	-0.06
ln(S ₄)	0.29	0.37	-0.12	0.14	-0.19	-0.56	0.34	0.32	-0.19	0.31	0.45	-0.09	-0.02	-0.51	-0.51	0.31	0.25	-0.21	0.51	0.38	0.53	0.30	-0.08
ln(S ₅)	0.02	-0.18	0.36	0.24	0.39	0.31	0.11	0.14	0.59	-0.06	-0.14	0.43	0.36	0.59	0.48	0.14	0.25	0.55	-0.02	-0.41	-0.45	0.19	0.00

Figure 7.6 Illustration of curvature parameters effects of the SWCC for Wijaya and Leong (2016) bimodal SWCC equation

7.3.3 Evaluation

Ten soils with bimodal GSD and bimodal SWCC from Andersson and Wiklert (1972) were used to evaluate the parametric-estimation PTF to estimate bimodal SWCC. The soil properties are summarised in Table 7.8. Performance indices, R^2 , $RMSE$, and d , were employed for the evaluation.

The GSDs of the soils were fitted by the proposed bimodal GSD equation (Equation 3.11) to obtain the GSD parameters. The Wijaya and Leong (2016) bimodal SWCC equation parameters are estimated using Equations 7.4 to 7.10. The estimated SWCC are compared with the measured SWCC and Li et al. (2014) model. Satyanaga et al. (2013) model was not compared as Satyanaga et al. (2013) model wrongly

predicted several soils in Table 7.7 as soils having unimodal SWCCs (i.e., for soils 143, 302, 310, 338 and 287). The evaluation and comparison results are presented in Table 7.8. According to Table 7.8, Li et al. (2014) model performed better than the proposed model for only two soils, 143 and 310. Hence, the proposed model performed better than Li et al. (2014) model for the 10 soils used in the evaluation.

Table 7.7 Soils used for evaluation of the bimodal SWCC estimation model

S/N	Soil	Void Ratio	Cl (%)	Si (%)	Sa +Gr (%)
1	327	0.89	3	6	91
2	276	0.95	6	8	86
3	326	1.12	7	6	87
4	143	0.62	20	27	53
5	302	0.63	23	37	40
6	310	0.59	26	43	31
7	338	0.63	26	61	12
8	164	0.78	31	54	15
9	98	1.05	54	35	11
10	287	0.79	53	40	7

Note: Cl, Si, Sa and Gr mean clay, silt, sand and gravel content of the soil, respectively.

Table 7.8 Bimodal SWCC estimation model evaluation and comparison results

S/N	Soil	Proposed Model			Li et al. (2014)		
		<i>RMSE</i>	R^2	<i>d</i>	<i>RMSE</i>	R^2	<i>d</i>
1	327	0.051	0.891	0.975	0.087	0.682	0.919
2	276	0.051	0.915	0.980	0.075	0.817	0.956
3	326	0.052	0.915	0.980	0.095	0.711	0.929
4	143	0.053	0.816	0.952	0.045	0.869	0.973
5	302	0.042	0.868	0.962	0.095	0.319	0.659
6	310	0.041	0.870	0.962	0.025	0.950	0.989
7	338	0.034	0.908	0.974	0.041	0.866	0.973
8	164	0.036	0.915	0.977	0.067	0.702	0.941
9	98	0.031	0.954	0.988	0.135	0.140	0.671
10	287	0.018	0.980	0.995	0.111	0.201	0.736

7.3.4 Summary

Thirty-two bimodal GSD soils from the literature were used to establish a parametric-estimation PTF based on Wijaya and Leong (2016) bimodal SWCC equation for bimodal SWCC estimation. The parametric-estimation PTF uses GSD parameters and void ratio to determine the parameters of Wijaya and Leong (2016) bimodal SWCC equation. The proposed PTF was evaluated and compared with Li et al. (2014) parametric-estimation PTF for bimodal SWCC using 10 soils from the literature. The results show that the proposed PTF gave better estimation than Li et al. (2014) for 8 out of the 10 soils in the evaluation. The proposed parametric-estimation PTF was developed using only 32 soils and evaluated using only 10 soils due to a lack of bimodal SWCC data in the literature. Further validation of the proposed parametric-estimation PTF for bimodal SWCC using more soils will give greater confidence for its application.

7.4 Hysteresis of Bimodal Soil-water Characteristic Curve

7.4.1 Introduction

In the past several decades, researchers (Poulovassilis, 1962; Poulovassilis and Childs, 1971, Poulovassilis and Ghamry, 1978; Mualem, 1973, 1974, 1984a, 1984b; Mualem and Dagan, 1975; Pham et al., 2003; Lu et al., 2013) developed models to estimate hysteresis of SWCC. But the hysteresis estimation models mainly focused on unimodal SWCC. Currently, there is no hysteresis model for bimodal SWCC. In this study, a model to estimate the hysteresis of bimodal SWCC is proposed based on the findings of the hysteresis model proposed for unimodal SWCC in Chapter 6.

7.4.2 Methodology

For unimodal SWCC, it is believed that boundary drying curve and initial drying curve have similar shape and air-entry value (Pham et al, 2005; Likos et al, 2013), which means that the boundary drying curve and initial drying curve can be interchanged by adjusting the parameter that represents the water content at zero soil suction (i.e., $\theta_{s,id}$ and $\theta_{s,d}$), only. For bimodal SWCC, the assumption of the similarity of drying and wetting curves is adopted.

As evaluated in this chapter, Wijaya and Leong (2016) bimodal SWCC equation gave the best performance among the evaluated bimodal SWCC equations. Hence, Wijaya and Leong (2016) bimodal SWCC equation is used.

In Chapter 6, hysteresis for unimodal SWCC was investigated and it was found that the AEV for IDC is about two times of AEV for BWC. For bimodal SWCC, there are two AEVs for bimodal SWCC, which are for macro- and micro-porosity, respectively. Lu and Korshidi (2015) divided the soil water-characteristic curve into three regimes from fully saturated to oven dry: capillary, adsorbed film and tightly adsorbed. Lu and Korshidi (2015) further concluded that (i), in the capillary regime, the SWCC is controlled by pore-size distribution (PSD); (ii), in adsorbed film regime, the SWCC is controlled by PSD and soil mineralogy; and (iii) in tightly adsorbed regime, the SWCC is controlled by soil mineralogy. Generally, the AEVs are controlled by soil PSD (Fredlund and Xing, 1994). Pham et al. (2005) believed that the SWCC for matric suction greater than residual suction is non-hysteretic. The matric suctions for second sub-curve are greater than first sub-curve's residual suction. Hence, it is reasonable to assume that only the first sub-curve is hysteretic. In this study, due to the scarcity of the

bimodal SWCC hysteresis data, it is assumed that the AEV for the first sub-curve of the BWC is half of the AEV of IDC for the corresponding porosity as illustrated in Figure 7.7. This is the same assumption used for the hysteresis of the unimodal SWCC in Chapter 6. For the second sub-curve, BWC will merge with the IDC. Based on the assumption, the parameters for boundary wetting curve can be summarised as follows:

AEV corresponding to first sub-curve of BWC:

$$\psi_{2,bw} = 0.5\psi_{2,id} \quad (7.10)$$

Water content at first AEV for BWC:

$$\theta_{2,bw} = \theta_{s,bw} - S_{1,bw} \log_{10} \left(\frac{\psi_{2,bw}}{\psi_{min}} \right) \quad (7.11)$$

Water content at ψ_3 for both BWC and IDC is calculated from IDC:

$$\theta_{3,id} = \theta_s - S_{1,id} \log_{10} \left(\frac{\psi_{2,id}}{\psi_{min}} \right) - S_{2,id} \log_{10} \left(\frac{\psi_{3,id}}{\psi_{2,id}} \right) \quad (7.12)$$

The slope of the second segment for BWC can be calculated as:

$$S_{2,bw} = \frac{\theta_{2,bw} - \theta_{3,bw}}{\log_{10} \psi_{3,bw} - \log_{10} \psi_{2,bw}} \quad (7.13)$$

Assuming $S_{1,bw} = S_{1,id}$ and $\theta_{3,bw} = \theta_{3,id}$, and substituting Equation 7.10, 7.11 and 7.12 into 7.13 and rearranging:

$$S_{2,bw} = \frac{0.3S_{1,id} + \theta_{s,bw} + S_{2,id} \log_{10} \left(\frac{\psi_{3,id}}{\psi_{2,id}} \right) - \theta_s}{\log_{10} \left(\frac{2\psi_{3,id}}{\psi_{2,id}} \right)} \quad (7.14)$$

If the measured drying curve is the boundary drying curve, it simply assumes that $\theta_s = \theta_{s,bw}$ as recommended by Pham et al. (2005). Equation 7.13 can be used to determine the S_2 for BWC.

Figure 7.7 Bimodal SWCC hysteresis

7.4.3 Evaluation

Very limited bimodal SWCC data are available in the literature. Only four soils from the literature were found and used to evaluate the proposed hysteresis model for bimodal SWCC. The soils used for the evaluation are summarised in Table 7.9. The evaluation procedures are as follows:

- (i) The drying curve (either IDC or BDC) are fitted using Wijaya and Leong (2016) bimodal SWCC equation to obtain the fitting parameters for the drying curve;
- (ii) The bimodal SWCC equation parameters for BWC are estimated from bimodal SWCC equation parameters for drying curve. The parameters for BWC are equal to the parameters for the drying curves except for ψ_2 and S_2 . The parameters, ψ_2 and S_2 are determined from Equations 7.10 and 7.14, respectively;
- (iii) The estimated BWC is plotted and compared with the measured BWC.

The evaluation results are presented in Figure 7.8. According to Figure 7.8, the performance of the proposed model is acceptable for the four soils.

(a) Soil Tak from Jotisankasa and Mairaing (2010)

(b) GW-GM with sand from Li (2009)

(c) SM with Gravel (Low Density) from Li (2009)

(d) SM with Gravel (Low Density) from Li (2009)

Figure 7.8 Bimodal SWCC hysteresis evaluation

Table 7.9 Soils used for evaluation of bimodal hysteresis model

Soil	Reference	Available SWCCs
Tak	Jotisankasa and Mairaing (2010)	W,D
GW-GM with sand	Li (2009)	I,W
SM with Gravel (Low Density)	Li (2009)	I,W
SM with Gravel (High Density)	Li (2009)	I,W

Note: I=initial drying curve; W=boundary wetting curve; D=boundary drying curve.

7.4.3 Summary

Currently, there is no hysteresis model available to estimate the hysteresis of bimodal SWCC. This section proposed a model to estimate the boundary wetting curve from initial drying curve or boundary drying curve of a hysteretic bimodal SWCC. The model is extended from the hysteresis model for unimodal SWCC established in Chapter 6. The model assumed that hysteresis is present for the first sub-curve only.

Four soils with bimodal SWCC from the literature were used to evaluate the proposed model. The results show that the proposed model can estimate the boundary wetting curve from the initial/ boundary drying curve with good accuracy.

7.5 Conclusion

Bimodal SWCCs are associated with the dual-porosity soils (Satyanaga et al., 2013; Miguel and Bonder, 2012; Zhang and Chen, 2005; Li et al., 2014), such as gap-graded soils (Satyanaga et al., 2013), soils compacted dry of optimum (Sivakumar and Wheeler, 2000), and coarse colluvial soils (Li and Zhang, 2009; Miguel and Bonder, 2012). Equations to describe the bimodal SWCC were proposed by Zhang and Chen, (2005), Carducci et al., (2011), Satyanaga et al., (2013), Li et al., (2014), and Wijaya and Leong, (2016). Five bimodal SWCC equations: Zhang and Chen (2005), Carducci et al. (2011), Satyanaga et al. (2013), Li et al. (2014), Wijaya and Leong (2016) equations, were evaluated using 31 bimodal SWCCs from the literature. The evaluation results showed that Wijaya and Leong (2016) equation outperformed the other equations. Wijaya and Leong (2016) equation which can use both graphical and numerical optimisation approaches to obtain the equation parameters performed well for all types of bimodal SWCCs. To minimise numerical problems during optimisation process, constraints are suggested for the parameters of the bimodal SWCC equations.

Based on the bimodal SWCC equation evaluation results, Wijaya and Leong (2016) bimodal SWCC equation gave the best performance. Based on Wijaya and Leong (2016) bimodal SWCC equation, a parametric-estimation PTF for the bimodal SWCC prediction was developed using 32 soils from the literature. The proposed parametric-estimation PTF uses grain-size distribution and void ratio of the soil to predict the

parameters of Wijaya and Leong (2016) bimodal SWCC equation. The proposed parametric-estimation PTF is evaluated and compared with Li et al. (2014) parametric-estimation PTF using 10 soils from the literature. The results show that proposed PTF performed better than Li et al. (2014) model.

A hysteresis model for bimodal SWCC to estimate the boundary wetting curve from boundary/initial drying curve was proposed by extending the hysteresis model for unimodal SWCC established in Chapter 6. The proposed model uses Wijaya and Leong (2016) bimodal SWCC equation. This model assumed that hysteresis is only present in the first sub-curve of the bimodal SWCC. The model was evaluated with four soils from the literature. The results show that the proposed model can estimate well the boundary wetting curve from the boundary/initial drying curve of the bimodal SWCC.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Introduction

The differentiation between saturated and unsaturated soil becomes necessary due to the basic differences in their nature and engineering behaviour (Fredlund and Rahardjo, 1993a). Unsaturated soil mechanics has received widespread attention across geotechnical engineering communities over the past decades. The soil-water characteristic curve (SWCC) is an essential property in unsaturated soil mechanics. The SWCC is widely used to estimate other unsaturated soil properties, such as permeability, shear strength, volume change and aqueous diffusion coefficient. However, determining the SWCC experimentally is time-consuming and labour intensive. If the SWCC can be obtained more expediently, it may be possible to see unsaturated soil mechanics being applied in practice.

8.2 Conclusions

This thesis investigates the effects of GSD and hysteresis on SWCC. A comprehensive investigation was conducted and the major findings are summarised below:

- (i) Unimodal and bimodal GSD equations are proposed based on the forms of Wijaya and Leong (2016) unimodal and bimodal SWCC equations, respectively. The proposed GSD equations use graphical approach instead of using curve-fitting

techniques to determine the parameters. Hence, a unique GSD equation can be established as the parameters all have clear physical meaning. The proposed unimodal GSD equation was compared with three other unimodal GSD equations using 33 soils from UNSODA database. The proposed unimodal GSD equation outperformed the other three unimodal GSD equation. The proposed bimodal GSD equation was compared with Fredlund et al. (2000) bimodal GSD equation using 33 soils from the literature. The results showed that the proposed bimodal GSD equation performed better than Fredlund et al. (2000) bimodal GSD equation.

- (ii) Bimodal GSD soils may not always exhibit bimodal SWCC. In this study, a classification tree to determine the type of SWCC for bimodal GSD soils was developed based on 226 bimodal GSD soils. The proposed classification tree only needs three predictors $|Y|$, MaP and e . The classification tree was evaluated using an independent data set of 60 bimodal GSD soils. The classification tree was shown to outperform existing criteria to distinguish bimodal GSD soils with bimodal SWCC from bimodal GSD soils with unimodal SWCC. More importantly, the classification tree provides a simple guide for planning the SWCC test. The suggested suction levels for determining SWCC in ASTM 6836-16 (2016) are excessive for unimodal SWCC and too few for bimodal SWCC. Hence, it is possible to use a more suitable number of suction levels for SWCC tests with the aid of the classification tree. It is proposed that the number of suction levels used in SWCC tests be different for unimodal and bimodal SWCC tests in order to reduce the testing duration.

- (iii) Point-estimation pedotransfer functions (PTFs) are commonly used in soil science and agriculture to estimate the water content at suctions of 4 (or 3), 10, 33, 100 and 1500 kPa. However, point-estimation PTFs are not commonly used in unsaturated soil mechanics. In this study, a number of point-estimation PTFs (3 PTFs for 4 (or 3) kPa, 11 PTFs for 10 kPa, 18 PTFs for 33 kPa, 5 PTFs for 100 kPa and 21 PTFs for 1500 kPa) was evaluated using 60 soils from Andersson and Wiklert (1972), 60 soils from Jauhiainen (2004) and 130 soils from UNSODA (Nemes, et al., 2001). The evaluation results show that Gupta and Larson (1979) PTF performed well at suctions of 4, 10, 33 and 100 kPa while Rawls et al. (1982) PTF performed well at suction of 1500 kPa. Using these point-estimation PTFs, a quick estimate of unimodal SWCC can be obtained using Gupta and Larson (1979) to estimate the water contents at suctions of 4, 10, 33 and 100 kPa and Rawls et al. (1982) PTF to estimate the water content at suction of 1500 kPa. Such ensemble point-estimation PTFs can provide an alternative method to obtain the unimodal SWCC and should be further explored in unsaturated soil mechanics.
- (iv) Four parametric-estimation PTFs using Brooks and Corey (1964) SWCC equation, seven parametric-estimation PTFs using van Genuchten (1980) SWCC equation and one parametric-estimation PTF using Fredlund and Xing (1994) SWCC equation were evaluated using 40 soils collated from the literature. The evaluation showed that generally, PTFs based on van Genuchten (1980) and Fredlund and Xing (1994) SWCC equations performed better than the PTFs based on Brooks and Corey (1964) equation. The best performance amongst all the evaluated parametric-estimation PTFs was given by Chin et al. (2010) PTF based on

Fredlund and Xing (1994) SWCC equation. However, this is at the expense of performing one-point measurement of the SWCC.

- (v) A zero experimental point method was established based on Chin et al. (2010) parametric-estimation PTF. The proposed zero experimental point method uses two SWCC points estimated using point-estimation PTFs. For coarse-grained soils, suctions of 10 and 100 kPa given by Gupta and Larson (1979) and Rawls et al. (1982) PTFs, respectively, are used. For fine-grained soils, suctions of 100 and 1500 kPa given by Gupta and Larson (1979) and Rawls et al. (1982) PTFs, respectively, are used. The proposed zero experimental point was evaluated and compared with Chin et al. (2010) method by using 65 coarse-grained and 65 fine-grained soils from the literature. The comparison showed that the performance of the zero experimental point method is comparable with Chin et al. (2010) one-point method for both coarse-grained soils and fine-grained soils for unimodal SWCC estimation. In general, the proposed zero experimental point method is more advantageous as no actual measurement of the SWCC is needed to provide an estimation of the unimodal SWCC.
- (vi) An empirical hysteresis model is proposed in this study to estimate the boundary wetting SWCC from the initial drying SWCC or boundary drying SWCC for unimodal SWCC. The proposed hysteresis model was developed using 59 soils from Pham et al. (2005) and Likos et al. (2013). The mean value for the slope-related parameter ratios was found to be about 1. Hence, the proposed hysteresis model suggests shifting the initial drying curve or boundary drying curve laterally to estimate the boundary wetting curve. In the proposed hysteresis model, the

AEV-related parameter, a , and air entrapment for the initial volumetric water content of the boundary drying or wetting curve are dependent on the soil types. The proposed hysteresis model was evaluated and compared with two other SWCC hysteresis models, Pham et al. (2005) and Likos et al. (2013), using 40 soils from the literature. The evaluation showed that the proposed hysteresis model performed better than the other two hysteresis models. Compared to Pham et al. (2005) hysteresis model, the proposed hysteresis model uses the Fredlund and Xing (1994) SWCC equation and uses less parameters. Compared to the Likos et al. (2013) hysteresis model, the proposed hysteresis model can use either the initial drying curve or the boundary drying curve as input and accounts for air entrapment based on soil types.

(vii) Five bimodal SWCC equations: Zhang and Chen (2005), Carducci et al. (2011), Satyanaga et al. (2013), Li et al. (2014), Wijaya and Leong (2016) equations, were evaluated using 31 bimodal SWCCs from the literature. The evaluation result shown that Wijaya and Leong (2016) bimodal SWCC equation outperformed the other equations. Wijaya and Leong (2016) bimodal SWCC equation, which can use both graphical and numerical optimisation approaches to obtain the equation parameters, performed well for all types of bimodal SWCC. To minimise numerical problem during optimisation process for curve-fitting, constraints for parameters were suggested for all bimodal SWCC equations.

(viii) Based on the evaluation of bimodal SWCC equations, Wijaya and Leong (2016) bimodal SWCC equation was selected to develop an empirical pedotransfer function (PTF) for estimating bimodal SWCC using 32 soils from the literature.

The proposed PTF uses grain-size distribution and void ratio of the soil to predict the parameters of Wijaya and Leong (2016) bimodal SWCC equation. The proposed PTF is evaluated and compared with Li et al. (2014) bimodal SWCC estimation PTF using 10 soils from the literature. The evaluation showed that the proposed PTF performed well for the 10 soils and gave better performance than Li et al. (2014). Due to the limitation of the bimodal SWCC data, the PTF for bimodal SWCC estimation is highly empirical and further validation is needed when more data becomes available.

- (ix) A bimodal SWCC hysteresis model to estimate the boundary wetting curve from boundary/initial drying curve was proposed by extending the findings from the unimodal SWCC hysteresis model. The proposed bimodal SWCC hysteresis model uses Wijaya and Leong (2016) bimodal SWCC equation. This model assumes that the hysteresis is only present for the first sub-curve corresponding to macro-porosity. The proposed bimodal SWCC hysteresis model was evaluated using four soils from the literature. The results showed that the proposed bimodal SWCC hysteresis model can estimate well the boundary wetting curve from boundary/initial drying curve. However, data for hysteretic bimodal SWCC is very scarce and the proposed bimodal SWCC hysteresis model should be further validated when more data becomes available.

In summary, the innovations of this study includes: (i), developing of a simple model to differentiate bimodal SWCC from unimodal SWCC for bimodal GSD soils for planning SWCC test; (ii) bridging the gap between soil science and geotechnical engineering by proposing a simple method to estimate continuous SWCC, which is

usually used in geotechnical engineering, from existing point-estimation PTFs, developed by soil science researchers; (iii), reducing the difficulty to determine the boundary wetting curve of SWCC by proposing a hysteresis estimation model, which requires no measured data on the boundary wetting curve for model calibration; and (iv), developing models to estimate bimodal SWCC and its hysteresis.

8.3 Recommendations

This study has developed a number of models to estimate unimodal and bimodal SWCCs including their hysteresis. However, it is not intended to advocate that experimental determination of SWCC is unnecessary. On the contrary, more tests to determine PSD and SWCC at the same time are needed to better understand the interrelationship of PSD, GSD and SWCC. In addition, validation of the models developed in this study using more data is prudent and will give more confidence for their use in practice.

Besides the above, it is recommended that further extension of the current study is needed in the following areas:

- (i) The proposed hysteresis model for unimodal SWCC requires only the initial/boundary drying curve to obtain the boundary wetting curve. No estimation of the scanning curves is incorporated in the proposed hysteresis model as Jaynes (1984) concluded that the existing SWCC hysteresis models can estimate scanning curves well. The scanning curve estimation from these models can be incorporated into the proposed hysteresis model for unimodal SWCC.

- (ii) The proposed parametric-estimation PTF for bimodal SWCC is highly empirical due to the limitation of the dataset. More bimodal SWCC data is needed for further validation.
- (iii) The proposed bimodal SWCC hysteresis model assumes that hysteresis is only confined to the first sub-curve. This assumption is based on limited data. More tests on soils with bimodal SWCC and studying their hysteresis are needed to confirm the assumption.
- (iv) The proposed hysteresis models for unimodal and bimodal SWCC require only the boundary or initial drying curve for model calibration. The proposed parametric-estimation PTFs for unimodal and bimodal SWCCs can be incorporated into the proposed hysteresis models.
- (v) The SWCC data used in the thesis were based on volumetric water content where volume change is implicit in the data. Hence, the equations developed have accounted for volume change implicitly. It is expected that sandy soils have negligible volume change while clayey soils have some volume changes. It is recommended that the effect of volume change on the SWCC in these equations be addressed in future study.
- (vi) Most of the soil data available for SWCC are from soil science where the USDA classification is used. There is no grain size distribution available and the soils are divided into percentages of sand, silt and clay. Atterberg limits are also not available for these soils although there are some work on shrinkage in soil science. Hence, it is recommended that future work can look into mapping these soils from the USDA classification to the USCS classification or mapping the Atterberg limits

- to the USDA soils for a better understanding by the geotechnical engineering community.
- (vii) Application of proposed models for determination of permeability function and shear strength of unsaturated soil can be done in the future.

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