Propagative plasmons in graphene investigated by simulation method and infrared spectroscopy

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**Propagative plasmons in graphene investigated by
simulation method and infrared spectroscopy**

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2018

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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02/07/2019

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Prof. Tay Beng Kang

Authorship Attribution Statement

This thesis contains material from 1 paper published in the following peer-reviewed journal in which I am listed as an author.

Chapter 4 is partly published as D. Legrand, L. O. Le Cunff, A. Bruyant, R. Salas-Montiel, Z. Liu, B.K. Tay, T. Maurer, and R. Bachelot, "Surface plasmons in suspended graphene: launching with in-plane gold nanoantenna and propagation properties," Opt. Express 25, 17306-17321 (2017).

The contributions of the co-authors are as follows:

- Dr Le Cunff provided the illustrations.
- The manuscript was revised by Dr Le Cunff, Prof. Maurer, Prof. Bachelot and Prof. Tay.
- Prof Bachelot and Prof Maurer assisted in getting licenses of the simulation software Lumerical.
- Dr Le Cunff and Prof Bachelot assisted in the interpretation of the field's analysis.

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Summary

English

Graphene physics and plasmonics are two fields which, once combined, promise a variety of exciting applications. One of those applications is the integration of active nano-optoelectronic devices in electronic systems, using the fact that plasmons in graphene are tunable, highly confined and weakly damped. Before achieving these active device, a crucial challenge remains : finding a platform enabling a high propagation of Graphene Plasmons Polaritons (GPPs).

In this framework, a full study of the propagation of plasmons through suspended graphene is addressed. This study start by reviewing the theoretical and experimental research achievement related to the really recent emerging field of graphene plasmonics. A numerical investigation by FDTD method is presented, and the conditions for a realisation of the study of graphene plasmons in suspended graphene are analysed. A design of the sample with an experimental process is proposed. Finally, an experimental study from the fabrication in clean room, then the characterisation of graphene and plasmonics structure, to the optical measurement through Scanning Near-field Optical Microscopie is presented.

Keywords— Surface plasmon resonance; Graphene; Simulation methods; Near-field microscopy; Infrared spectroscopy

French

La physique du graphène et la plasmonique sont deux domaines qui, une fois combinés, promettent une variété d'applications intéressantes. Une de ces applications est l'intégration de dispositifs nano-optoélectroniques actifs dans des systèmes électroniques, en utilisant le fait que les plasmons dans le graphène sont accordables, fortement confinés et faiblement amortis. Avant de réaliser ces dispositifs actifs, un défi crucial demeure : trouver une plateforme permettant une forte propagation des Polaritons Plasmons Graphène (GPP).

Dans ce cadre, une étude complète de la propagation des plasmons à travers le graphène en suspension est abordée. Cette étude commence par examiner les résultats de la recherche théorique et expérimentale liés au domaine émergent de la plasmonique du graphène. Une étude numérique par méthode FDTD est présentée, et les conditions pour la réalisation de l'étude des plasmons de graphène dans le graphène en suspension sont analysées. Une conception de l'échantillon avec un processus expérimental est proposée. Enfin, une étude expérimentale, de la fabrication en salle blanche, puis la caractérisation de la structure graphène et plasmonique, à la mesure optique par microscopie optique à champ proche à balayage, est présentée.

Mots Clés — Résonance plasmonique de surface; Graphène; Simulation, Méthodes de; Microscopie en champ proche; Spectroscopie infrarouge

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Chapter 1

Introduction

1.1 Motivation

Over the past century, the field of infinitely small has started to be investigated, as the tools were developed and being available in the research centers. Nanoscience have been carrying the promises of revolution in many fields. Among other, huge effort are made in the fields of medicine and information technology, where progress are strongly demanded. Nanoscience is present in our daily life through the electronic devices, where the size of the component has been reduced steadily to the nanoscale. However, the nanoworld does not work with the same physical rules, where others phenomena take place. Many researches are directed toward the resolution of the arising problems, to make nano-device work in the way it works on our scale. Other research are driven in order to take advantage of those new rules and phenomena. This research aims at showing some recent results about electromagnetic waves appearing down to the atomic scale. This thesis work is taking place at the intersection of two fields, plasmonic and graphene.

Plasmonics is part of the general field of plasma. Plasma is a state of the matter, where collective electrons density waves can be manipulated. It exists on several forms, and one of them is the 2D collective oscillation of electrons at the surface of conducting materials. At the interface between a metal and an insulator, surface plasmons can be found. This electromagnetic wave can be coupled with light, and the latter can be confined at the subwavelength scale. The confinement of the light is an important research topic as photonic devices suffer from their size. This plasmonic phenomenon has been studied on 2D material and more particularly graphene.

In the quest of understanding and manipulating the infinitely small, graphene appears as a very promising material. It is in fact infinitely thin: the first single-atom-thick layer discovered. Moreover, the strong carbon-carbon bound allow the scientist to manipulate it with macroscopic tools. On the top of that, it is possible to isolate a graphene layer by the scotch-tape mechanical exfoliation technique. This technique is so simple and cheap that it can be reproduced by any research team all over the world. Finally, its peculiar properties open up new fields of research. In order to have an idea of the importance of the discovery, it took only 6 years to Andrei Geim and Kostya Novoselov to get their Nobel price after their famous publication. In this context appears the field of graphene plasmonics.

1.2 Objectives

The project aimed at tuning the Localized Surface Plasmon Resonance (LSPR) of plasmonic nanostructures on graphene via electrostatic doping. We aimed at fabricating plasmonic nanostructures at the vicinity of suspended graphene and to investigate how plasmon resonances of metallic nanoparticles will be launched and will propagate in graphene with electrostatic doping. We intended to investigate structures with different LSPR wavelengths and more particularly plasmonic structures with resonance close to graphene's plasmon resonance (which depends on the interaction with the substrate). The cou-

pling between the graphene plasmon and the plasmonic metallic structures allow launching of graphene plasmons and the modification of graphene doping allow enhancing or quenching the graphene plasmon propagation.

1.3 Major contribution of the thesis

During the first part of the PhD, numerical results have been gathered in France in order to design a sample which would lead to the efficient launching, propagation and detection of surface plasmons in suspended graphene. The numerical results gave birth to a first paper that has been published. This paper is the first of the kind, studying plasmons in suspended graphene.

Then, samples have been fabricated during the year in Singapore. Many technical issues have been solved, and among other, the remarkable success on the deposition of suspended graphene.

Observation have been carried out both in the far field and the near-field using scattering microscopy and apertureless scanning optical microscope working in the infrared, constituting promising results for further studies.

In parallel, a new collaboration about graphene on diamond have been started based on simulation. A device based on graphene plasmons is proposed in conclusion.

1.4 Organization of the Thesis

The field of graphene plasmonics emerged recently, within few years after the Nobel price. Graphene plasmons have shown their potential through their high confinement and tunability. This work is all about graphene plasmonics, which is introduced through this thesis. In the first chapter, some basic knowledges about plasmonics in general and graphene properties, mostly electronic, are presented. The second chapter is a literature review about graphene and graphene plasmonics, as well as the demonstration of a basic model explaining graphene plasmons properties. In the third chapter, a numerical study about plasmons propagating in suspended graphene is shown. In the fourth chapter, fabrication of samples as well as optical measurements through home-made SNOM are introduced. Finally, the main results of this thesis are summarized, as well as the possible research direction to consider, following this work.

Chapter 2

Theoretical background

This thesis takes place in the framework of a broad and not really intuitive field of science usually called nanotechnology. The name refers directly to the size (nanometres) of the systems studied because of the wide properties arising from this range of length. New properties arise from the confinement of the media in one or more dimensions. As an example, the ratio of surface atoms over core atoms vary dramatically when reducing the dimensions of an object to the nanometric scale. Figure 2.1 shows the ratio of surface atoms over core atoms in the case of a spherical bulk of palladium when the cluster diameter is reduced. In fact, surface atoms exhibit different properties as the forces applied on them differ, as well as chemical terminations. Therefore, surface atoms give to the structure new chemical properties, as they build the physical interface between the solid and its environment. The confinement of the structure also gives physical, mechanical, electromagnetic, and in particular plasmonic, remarkable new properties. That is noticeably the case for the 2 dimensions materials discovered quite recently [119], and among them graphene.

Figure 2.1: Variation of the percentage of surface atoms as a function of the size of a palladium cluster, adapted from [120]

In order to go deeper into the field of graphene plasmonics, the first chapter aims to expound the bases in plasmonics and about graphene. Therefore, in a first part, Maxwell's equations and the Drude model are introduced, which will give the keys for treating the second part of this chapter, i.e. the surface plasmons polaritons (SPPs). Finally, some of the graphene electronics properties will be explained.

2.1 Maxwell's equations and Drude model

Maxwell's equations are the bases of a revolution in physics [56, 71]. These equations couple the magnetic field $\mathbf{H}(\mathbf{r}, t)$ and the electric field $\mathbf{E}(\mathbf{r}, t)$. In space and time domain, the 6 scalar functions of these fields are interrelated by the Maxwell's equations, which give birth to the electromagnetic wave's equations. This set of partial differential equations is a part of the laws of science. There are four equations that it is possible to define depending on the surrounding media. In a medium, two additional vector fields are required, showing up in the Maxwell's equations : the electric flux density $\mathbf{D}(\mathbf{r}, t)$ and the magnetic flux density $\mathbf{B}(\mathbf{r}, t)$. To simplify the notations, the informations about time and space " $(\mathbf{r}, t)$ " will be omitted.

Maxwell's equation, in a medium without (and with) free charges ρ_f and currents density $\mathbf{J}_f$, are given by :

$$\nabla \cdot \mathbf{D} = 0(+\rho_f) \quad \text{Gauss's law for electricity} \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{Gauss's law for magnetism} \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{Faraday's law} \quad (2.3)$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}(+\mathbf{J}_f) \quad \text{Ampère's law} \quad (2.4)$$

The two first Equations (2.1) and (2.2) are known as the Gauss's laws for electricity and magnetism respectively. They are related to the charge and magnetic sources. The two last equations, i.e. the Faraday's law (2.3) and the Ampère's law (2.4), are governing the time evolution of the electromagnetic fields. The relation between the fields and the flux densities are described by :

$$\begin{aligned} \mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{B} &= \mu_0 \mathbf{H} + \mu_0 \mathbf{M} \end{aligned} \quad (2.5)$$

where $\epsilon_0 \approx 8.854187817 \cdot 10^{-12} F/m$, or approximately $1/36\pi \cdot 10^{-9} F/m$, is the vacuum permittivity, $\mu_0 = 4\pi \cdot 10^{-7} V.s.A^{-1}m^{-1}$ is the vacuum permeability. The vectors fields $\mathbf{P}$ and $\mathbf{M}$ are related to the properties of the media. The first one is called the polarization density [C/m^2] that corresponds to a volume density of dipole moments. The second one is the magnetisation density [A/m]. They are describing the reaction of the medium to external fields. In non-magnetic media, we have :

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \epsilon_r \mathbf{E} \quad (2.6)$$

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (2.7)$$

with $\epsilon = \epsilon_0 \epsilon_r$ the absolute permittivity of the medium. The physical quantity ϵ_r refer to the relative permittivity, which can be written as $\epsilon_r(r)$ when highlighting a spacial dependence. ϵ_r is the sqaure of the refractive index of the medium.

2.1.1 Electromagnetic wave equation and dispersion relation

The electromagnetic wave equation is a second order partial equation, arising from the Maxwell's Equations (2.3) and (2.4). In fact, these equations can be combined in non magnetic media which gives, in absence of charge and current :

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} \quad (2.8)$$

Then, using mathematical identities such as $\nabla \times \nabla \times \mathbf{E} \equiv \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ as well as $\nabla \cdot (\epsilon_r(r) \mathbf{E}) \equiv \mathbf{E} \cdot \nabla \epsilon_r(r) + \epsilon_r(r) \nabla \cdot \mathbf{E}$, and inserting Equation (2.1) in Equation (2.8), leads to :

$$\nabla \left(-\frac{1}{\epsilon_r(r)} \mathbf{E} \cdot \nabla \epsilon_r(r) \right) - \nabla^2 \mathbf{E} = -\mu_0 \epsilon_0 \epsilon_r(r) \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (2.9)$$

Latter the last equation will be solved in different media separately. In an homogeneous medium, the dielectric function can be considered as constant in space, which leads to : $\nabla \epsilon_r(r) = \nabla \epsilon_r = 0$. Finally, the well known electromagnetic wave equation, named d'Alembert equation, can be deduced as :

$$\nabla^2 \mathbf{E} - \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (2.10)$$

where $c = 299792458 \text{ m/s}$ is the speed of the light in vacuum. In order to simplify the model, the wave is assumed to be solution of the d'Alembert equation, i.e. this wave is a superposition of progressive harmonic plane waves. Such a wave has a plane wavefront orthogonal to its propagating direction. It is a solution of Maxwell's equations, which has 6 electromagnetic components, with a same pulsation w and wave vector $\mathbf{k}$. It is common to write the harmonic electric and magnetic fields with complex notations so that $\mathbf{E}(r, t) = \mathbf{E}_0 e^{j(\mathbf{k} \cdot \mathbf{r} - wt)}$. Using the partial derivative in time and in space, $\mathbf{E}$ is governed by :

$$\nabla \mathbf{E} = j\mathbf{k} \quad \text{and} \quad \frac{\partial \mathbf{E}}{\partial t} = -jw \quad (2.11)$$

Combining equations (2.10) and (2.11), leads to the dispersion relation :

$$k^2 = \epsilon \frac{w^2}{c^2} \quad \text{i.e.} \quad k = \sqrt{\epsilon} \frac{w}{c} \quad (2.12)$$

This dispersion relation describes the dynamic of a plane wave in a medium. If the medium is said to be dispersive, its dielectric function is function of the light wavelength. The dielectric function is usually noted $\epsilon(w)$. In the vacuum, $\epsilon(w) = 1$ and the well known linear dispersion relation becomes :

$$k = \frac{w}{c} \quad (2.13)$$

2.1.2 Boundary conditions

Maxwell's equations are valid for a continuous medium. However, they can be used in the case of interfaces, where they constrain the behavior of electromagnetic fields leading to the so-called boundary conditions. As a result, a part of the energy is generally reflected, and another transmitted. It is also possible to deduce from it directions and angles of the resultant fields, depending on the incident beam, its polarisation as well as the media properties. More specifically for this study, the dispersion relation of graphene plasmons (see section 3.3.2) arises from the boundary conditions applied on the magnetic field (see below).

The boundary conditions for electromagnetic fields can be derived from integral forms of the Maxwell's equations expounded in the introduction of Section 2.1. Let's consider an interface between two homogeneous media. In the following description, the notation $\mathbf{E}_{i1}$ will be used, with $i = x, y, z$ (cartesian coordinate) or $i = t$ (tangential to the boundary i.e. in the plane (x,y)), for the component of the electric field lying in the medium 1 (see Figure 2.2). Four boundary conditions arise from the 4 Maxwell's equations seen above. They are demonstrated in the four paragraphs below.

From the Ampère's law Starting from Equation (2.4), which is integrated over a surface bounded by a loop (see Figure 2.2), Stokes' theorem is applied :

$$\int_S \nabla \times \mathbf{H} \cdot d\mathbf{S} = \oint_C \mathbf{H} \cdot d\mathbf{l} = \frac{\partial}{\partial t} \int_S \mathbf{D} \cdot d\mathbf{S} \quad (2.14)$$

Following Figure 2.2, where the Roman numbers are defining the edges of a rectangle loop perpendicular to the interface, and assuming a constant electric flux $\mathbf{D}$ over the surface S, it is possible to develop :

$$(I) + (II) + (III) + (IV) = \frac{\partial}{\partial t} \mathbf{D} \cdot \mathbf{e}_y \int_S dS \quad (2.15)$$

with $\mathbf{e}_y$ the unit vector in the y direction. The Roman numbers are directly related to the terms on the left side in the next equation. The electric flux density $\mathbf{D}$ is oriented through the loop, and is directly proportional to the surface involved in the experiment. Therefore, it is possible to re-write Equation (2.15) as:

$$\mathbf{H}_{1x} \cdot d\mathbf{x} + (\mathbf{H}_{1z} + \mathbf{H}_{2z}) \cdot d\mathbf{z} - \mathbf{H}_{2x} \cdot d\mathbf{x} - (\mathbf{H}_{1z} + \mathbf{H}_{2z}) \cdot d\mathbf{z} = \frac{\partial}{\partial t} \mathbf{D} \cdot \mathbf{e}_y (dx dz) \quad (2.16)$$

If the distance dz tends to zero, Equation (2.16) becomes :

$$(\mathbf{H}_{1x} - \mathbf{H}_{2x}) \cdot d\mathbf{x} = 0 \quad (2.17)$$

$$\mathbf{H}_{1x} = \mathbf{H}_{2x} \quad (2.18)$$

Equation (2.18) can be generalized for all the magnetic field compounds in plane as follow :

$$\mathbf{H}_{1t} = \mathbf{H}_{2t} \quad (2.19)$$

As it has been mentioned in introduction, a key equation for obtaining graphene plasmons relation dispersion is lying in the boundary condition applied on the magnetic field. That comes from the fact that graphene is a semi-metal (see Section 2.3.2), which has quasi-free electrons (see Section 2.3). Therefore, taking into account surface currents at the interface of two media, with dz tending to zero and the use of Equations (2.17) and (2.16)), Equation (2.14) becomes :

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = (\mathbf{H}_{1x} - \mathbf{H}_{2x}) \cdot d\mathbf{x} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_f \right) \cdot d\mathbf{S} = dx \mathbf{J}_S \cdot \mathbf{e}_y \quad (2.20)$$

and therefore

$$H_{2x} - H_{1x} = -J_{Sy} \quad (2.21)$$

with J_f the volumetric density of current ($A.m^{-2}$) and J_S the surface density of current ($A.m^{-1}$). When an electric field is applied to the object, electrons will move. The Ohm's law states that :

$$\mathbf{J} = \sigma \mathbf{E} \quad (2.22)$$

Plugging Equation (2.22) in Equation (2.21) leads to :

$$H_{2t} - H_{1t} = -\sigma E_{t\perp} \quad (2.23)$$

with σ the surface conductivity. This key equation will be used in order to obtain the dispersion relation of graphene plasmons in section 3.3.2, in the case of TM (Transverse Magnetic) modes.

From the Faraday's law Using the same development as in the previous paragraph, Equation (2.3) leads to :

$$\mathbf{E}_{1x} = \mathbf{E}_{2x} \quad (2.24)$$

Generalised to all the tangential components, it is possible to write :

$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \quad (2.25)$$

From the Gauss's law (electricity) The schema helping to solve this boundary condition problem is illustrated in Figure 2.3. We can see there an interface, in blue, separating two media (medium 1 and 2). The Roman numbers represent surfaces delimiting a pillbox, in red, which is crossing the interface. (I) represents the surface of the pillbox parallel to the interface in the medium 1. (II) represents the surface of the pillbox parallel to the interface in the medium 2. (III) represents the surface of the pillbox which is perpendicular and crossing the interface. The electric flux density $\mathbf{D}_{i,n}$ normal to the interface is represented in the two media ($i=1,2$) with an arrow. Let's take the Maxwell equation (2.1), integrate over the pillbox and apply Gauss' theorem in the absence of charge :

$$\int_V \nabla \cdot \mathbf{D} \, dV = \oint_S \mathbf{D} \cdot d\mathbf{S} = 0 \quad (2.26)$$

Figure 2.2: Schema describing the interface (in blue) between two media, medium 1 and the medium 2. Two electric field tangential to the interface are displayed. A rectangle loops crossing perpendicularly the interface is also shown in red, used for getting the boundary condition using the Maxwell's Equations (2.3) and (2.4)

where V and S are respectively the volume and the surface of the pillbox. The surface integral can be separated into three parts :

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{S_I} \mathbf{D}_{1,n} \cdot d\mathbf{S}_I + \int_{S_{II}} \mathbf{D}_{2,n} \cdot d\mathbf{S}_{II} + \int_{S_{III}} \mathbf{D}_t \cdot d\mathbf{S}_{III} \quad (2.27)$$

where (I) and (II) are the flat ends, and (III) is the curved surface as shown on Figure 2.3. It can be noticed first of all notice that $S_I = S_{II} = S$. Then, let's take the limit in which the pillbox height tends to zero, to obtain :

$$-\mathbf{D}_{1n} \cdot S + \mathbf{D}_{2n} \cdot S = 0 \quad (2.28)$$

$$\mathbf{D}_{1n} = \mathbf{D}_{2n} \quad (2.29)$$

From the Gauss's law for magnetism Starting from Maxwell's equation (2.2), and using the same development as in the previous paragraph, we have :

$$\mathbf{B}_{1n} = \mathbf{B}_{2n} \quad (2.30)$$

2.1.3 Review of the Drude model and the free electrons gas equations of motion

The Drude model was constructed three years after Thomson's discovery of electrons. In a metal, Drude considers a sea of electrons and uses the theory of gases to describe the electrical and thermal conductions. In this model, it is assumed that electrons are spherical solids identically moving along a straight line. A time average ($t = \tau$) known as the collision time is set delimiting two collisions. The electrons will emerge with a random direction and random speed after each collision (no particular initial condition). There is no force applied on electrons except the one occurring when they collide: no electron-electron interaction (independent electrons approximation), no electron-ion interaction (free electrons approximation).

Case of a uniform and constant perturbation. In the case where a uniform and constant electrical field ($\mathbf{E}$) is applied, electrons feel the electrostatic force $\mathbf{F}_{el}$:

$$\mathbf{F}_{el} = -e\mathbf{E} \quad (2.31)$$

Figure 2.3: Schema describing the interface (in blue) between two media, medium 1 and the medium 2. Two electric flux density normal to the interface are displayed. A pillbox crossing perpendicularly the interface is also shown in red, used for getting the boundary condition using the Maxwell's Equations (2.1) and (2.2)

with the electronic charge $e = 1.6021765 \times 10^{-19}$ C. With m_e the mass of a free electron (9.11×10^{-31} kg), the fundamental principle of dynamic gives:

$$m_e \frac{d\mathbf{v}}{dt} = \mathbf{F}_{el} = -e\mathbf{E} \quad (2.32)$$

Since the electron past move is unknown (direction, velocity), all prior contribution to the electron motion is ignored and the average of the electronic velocity is defined as following :

$$\mathbf{v}_{avg} = \frac{-e\mathbf{E}}{m_e} \tau \quad (2.33)$$

The current density $\mathbf{J}$ is expressed as:

$$\mathbf{J} = -ne\mathbf{v} \quad (2.34)$$

with n the charge carrier density. Combining the last equation with the Ohm law, $\mathbf{E} = \rho\mathbf{J}$, leads to the resistivity ρ :

$$\rho = \frac{m_e}{ne^2\tau} \quad (2.35)$$

This first result is interesting because it gives an order of magnitude of the collision time for metals.

Dielectric function of the electron gas. The optical response of a Drude metal, in absence of drag force, can also be established. Plugging the electron displacement $\mathbf{x}(\omega) = \mathbf{x}_0 e^{-i\omega t}$ in Equation (2.32), and linking the polarisation density $\mathbf{P}(\omega)$ with $\mathbf{x}(\omega)$ through $\mathbf{P}(\omega) = -nex(\omega)$ lead after a little algebra to :

$$\mathbf{P}(\omega) = \frac{-ne^2}{m\omega^2} \mathbf{E} \quad (2.36)$$

The frequency dependent dielectric function of Drude metal can be expressed, using Equation (2.5) as:

$$\epsilon(\omega) = 1 - \frac{ne^2}{\epsilon_0 m \omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \quad (2.37)$$

with ω_p the plasma frequency. At this angular frequency, the collective motion of electrons in a conducting medium, such as a metal, is resonant. Under this frequency, the medium is reflecting the electromagnetic wave, whereas above it, the wave are propagating through the medium.

Equations of motion for a free charge carrier. It is possible to describe the dynamic of a free charge carrier adding a damping term. An electron has a velocity $\mathbf{v}_0(t)$ at time t and the probability $\frac{dt}{\tau}$ to collide at $t+dt$. Therefore $1 - \frac{dt}{\tau}$ is the probability to avoid a collision. While an electron does not experience collision, it moves following Equation (2.32) and will get a velocity:

$$d\mathbf{v} = \frac{\mathbf{F}_{el}dt}{m_e} \quad (2.38)$$

Thus, an electron has a probability of $1 - \frac{dt}{\tau}$ to not collide at a time $t+dt$ and add to its own velocity ($\mathbf{v}_0(t)$ at the time t) a velocity $d\mathbf{v}$. Considering a metal with free electrons, all the electrons which collide can be neglected. Thus, the contribution to the velocity per electron added by all the electrons which survive during a time dt , $\mathbf{v}(t+dt)$, can be calculated as:

$$\mathbf{v}(t+dt) = \left(1 - \frac{dt}{\tau}\right) \left(\mathbf{v}_0(t) + \frac{\mathbf{F}_{el}dt}{m_e}\right) = \left(1 - \frac{dt}{\tau}\right) (\mathbf{v}_0(t) + d\mathbf{v}) \quad (2.39)$$

The second magnitude order (dt^2) in the development of the following equation is not taken into account:

$$\mathbf{v}(t+dt) = \mathbf{v}_0(t) + \frac{\mathbf{F}_{el}dt}{m_e} - \frac{dt}{\tau}\mathbf{v}_0(t) \quad (2.40)$$

To obtain finally the equations of motion for a free charge carrier when $dt \rightarrow 0$:

$$\frac{\mathbf{v}(t+dt) - \mathbf{v}_0(t)}{dt} = \frac{d\mathbf{v}(t)}{dt} = \frac{\mathbf{F}_{el}}{m_e} - \frac{\mathbf{v}(t)}{\tau} \quad (2.41)$$

It is possible to re-write the last equation with $\mathbf{p}$ the momentum of the electron:

$$\frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau} - e\mathbf{E} \quad (2.42)$$

Equation (2.42) is the other significant result of the Drude model, with Equation (2.35). When an external perturbation $\mathbf{E}(\omega)$ oscillates periodically such as $\mathbf{E}(\omega) = \mathbf{E}_0 e^{-i\omega t}$, the solution of Equation (2.42) is of the form $\mathbf{p}(\omega) = \mathbf{p}_0 e^{-i\omega t}$, and we get :

$$\mathbf{p}(w) = -\frac{ie\mathbf{E}(w)}{w + i/\tau} \quad (2.43)$$

Taking Equation (2.22), with $\mathbf{J} = -ne\mathbf{p}/m_e$ the current density and n the carrier density, the optical conductivity $\sigma(w)$ as a function of the pulsation can be deduced as:

$$\sigma(w) = \frac{ine^2}{m(w + i/\tau)} \quad (2.44)$$

From the optical conductivity to the permittivity. Let's take the case where a free current takes place at an interface between two media, characterized by their relative permittivity ϵ_1 and ϵ_2 . $\epsilon_s = (\epsilon_1 + \epsilon_2)/2$ is defined as the surrounding permittivity. Taking Maxwell Ampère's law, it is possible to write :

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_f \quad (2.45)$$

$$= \epsilon_0 \epsilon_s \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} = (-i\omega \epsilon_0 \epsilon_s + \sigma) \mathbf{E} \quad (2.46)$$

Using equation (2.9), it follows :

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} = -\mu_0 \frac{\partial \nabla \times \mathbf{H}}{\partial t} \quad (2.47)$$

$$= -\mu_0 (-i\omega) (-i\omega \epsilon_0 \epsilon_s + \sigma) \mathbf{E} \quad (2.48)$$

After some rearranging, using the partial derivative in space (2.11), the dispersion relation can be found :

$$k^2 = \left(\epsilon_s + \frac{i\sigma}{\omega\epsilon_0} \right) \frac{\omega^2}{c^2} \quad (2.49)$$

The relative permittivity of a metal-like material (Drude conductivity) as a function of the optical conductivity can thus be written as:

$$\epsilon(w) = \epsilon_s + \frac{i\sigma(w)}{\epsilon_0 w} = \epsilon_s - \frac{ne^2}{m\epsilon_0(w^2 + iw/\tau)} = \epsilon_s - \frac{w_p^2}{w^2 + iw/\tau} \quad (2.50)$$

with $w_p = ne^2/m\epsilon_0$ the plasma frequency of the free electron gas. Because the Drude model doesn't take into consideration interband transition in its model, the description is not valid for high frequency, relatively to the plasma frequency w_p and the relaxation time τ . In this case, the sign of the real part of the relative permittivity change and its optical response is not anymore metallic. However, in the scope of this thesis, we would like to work in the infrared range, where no interband transition may appear.

The permittivity is a really interesting intrinsic property of materials because it gives access to the optical answer of a material under perturbations. Unfortunately, this property is convenient only for 3D materials modeling. Graphene being a one atom thick carbon sheet, we will more likely use 2D conductivity. However, as a first approach, the Drude model can convert the 2D conductivity into a 3D permittivity, giving us a good idea about the optical response of graphene in the infrared range. Its optical properties via materials permittivity has been widely used at first for graphene, thus enabling FDTD simulation.

This 3D approach implies to model materials with a finite thickness arbitrary set as $2a$, for the sake of simplicity in the next section. In this model, graphene surroundings, characterized by a permittivity ϵ_s , also plays a crucial role :

$$\epsilon_g(\omega) = \epsilon_s(\omega) + \frac{i\sigma_{2D}(w)}{\omega 2a\epsilon_0} \quad (2.51)$$

In the case of suspended graphene, $\epsilon_1 = \epsilon_2 = \epsilon_{air} = 1$ and thus $\epsilon_s = 1$, and Equation (2.51) leads to :

$$\epsilon_g(\omega) = 1 + \frac{i\sigma_{2D}(w)}{\omega 2a\epsilon_0} \quad (2.52)$$

2.2 Surface plasmon polaritons

In the framework of the study of electromagnetic waves, Surface Plasmons Polaritons (SPPs) can be seen as a particular result of the Maxwell's equations at the interface of two media with specific properties [101, 128, 130]. In order to give a picture about plasmonics, it is interesting to go back to its physical origin. It can be seen as a collective motion of electron reacting to an electric field. However, they tend to travel slightly too far and as a result, a restoring force pulls them back in order to screen the charge disturbance created by their displacement. This first collective motion is repeated such that a weakly damped oscillation occurs. Therefore, a plasmon can be defined as a coherent collective oscillations of free electrons. When the excitation of the plasmon is light, a coupling between plasmons and the electromagnetic wave takes place, resulting in a Surface Plasmon Polariton (SPPs).

2.2.1 Surface plasmon polaritons at a single interface in absence of free current

We can start the description of surface plasmons by solving the problem of a plane wave crossing an interface separating two media defined by their dielectric function ϵ_1 and ϵ_2 .

An arbitrary polarized plane wave $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$ can always be split up into two orthogonally polarized plane waves. Basing our problem on the interface between the two medium ϵ_1 and ϵ_2 , the electric field is moving parallel to the interface for the first case, and in the other case, we set the magnetic field parallel to the interface. In the first case, we speak about a transverse electric (TE or s) polarization. In the second case, we speak about a transverse magnetic polarization (TM or p), which is illustrated in Figure 2.4.

Figure 2.4: Schema of an incident TM polarized wave propagating in the plane xz, crossing an interface (from medium 2 to medium 1). A part of this incident electromagnetic wave is reflected back in the medium 2, and a part is transmitted in the medium 1. The wave is characterized by a wavevector $\mathbf{k}_{i,j}$, with $i=1,2$ stands for the medium, and $j=I,R,T$ in case where we are looking at the Incident, Reflected of Transmitted component. As well, the electric field $\mathbf{E}_{i,j}$ and the magnetic field $\mathbf{B}_{i,j}$ are illustrated.

In our case, we are interested in surface plasmons, only appearing in TM mode (for the TE case, see [101]). In TM mode, the non-zero components are H_y , E_x and E_z . Let's write the fields in the media (1) and (2) as follows :

$$\begin{aligned}
\text{For } z > 0 \quad \mathbf{H}_2 &= (0, \mathbf{H}_{y2}, 0) e^{i(k_{x2}x + k_{z2}z - wt)} \\
\mathbf{E}_2 &= (\mathbf{E}_{x2}, 0, \mathbf{E}_{z2}) e^{i(k_{x2}x + k_{z2}z - wt)} \\
\text{For } z < 0 \quad \mathbf{H}_1 &= (0, \mathbf{H}_{y1}, 0) e^{i(k_{x1}x - k_{z1}z - wt)} \\
\mathbf{E}_1 &= (\mathbf{E}_{x1}, 0, \mathbf{E}_{z1}) e^{i(k_{x1}x - k_{z1}z - wt)}
\end{aligned} \tag{2.53}$$

At the interface between two media, in the absence of free charges and currents, the tangential components of the electric and magnetic fields, $\mathbf{E}_t$ and $\mathbf{H}_t$, as well as the normal components of the electric and magnetic flux densities, $\mathbf{D}_n$ and $\mathbf{B}_n$, must be continuous (see sub-section 2.1.2) :

$$\begin{cases} \mathbf{E}_{1t} = \mathbf{E}_{2t} & \text{for the electric field} \\ \mathbf{H}_{1t} = \mathbf{H}_{2t} & \text{for the magnetic field} \\ \mathbf{D}_{1n} = \mathbf{D}_{2n} & \text{for the electric flux density} \\ \mathbf{B}_{1n} = \mathbf{B}_{2n} & \text{for the magnetic flux density} \end{cases} \tag{2.54}$$

Let's start from the fact that the boundary conditions must be satisfied everywhere on the interface. Let us consider a point $p(x, y, 0)$ such as it belongs to the plane $z=0$. It means that the electric field can be written :

$$\mathbf{E}_{x1} e^{i(k_{x1}x - wt)} = \mathbf{E}_{x2} e^{i(k_{x2}x - wt)} \text{ so } \mathbf{E}_{x1} = \mathbf{E}_{x2} = \mathbf{E}_x \text{ and } k_{x1} = k_{x2} = k_p \tag{2.55}$$

And the magnetic field :

$$\mathbf{H}_{y1} e^{i(k_p x - wt)} = \mathbf{H}_{y2} e^{i(k_p x - wt)} \text{ so } \mathbf{H}_{y1} = \mathbf{H}_{y2} \tag{2.56}$$

The time dependence is assumed to be harmonic ($\frac{\partial}{\partial t} = -iw$). The harmonic time dependence, applied to the magnetic field via the d'Alembert Equation (2.10) thus yields to the Helmholtz equation :

$$\nabla^2 \mathbf{H}_i + k_0^2 \epsilon \mathbf{H}_i = \frac{\partial^2 \mathbf{H}_i}{\partial z^2} + (\epsilon k_0^2 - k_p^2) \mathbf{H}_i = 0 \tag{2.57}$$

If this equation is applied to the media (1) and (2), the following relations are obtained:

$$(-ik_{z1})^2 + (\epsilon_1 k_0^2 - k_p^2) = 0 \text{ and } (ik_{z2})^2 + (\epsilon_2 k_0^2 - k_p^2) = 0 \tag{2.58}$$

which yield to the general equation :

$$k_{zi} = \sqrt{\epsilon_i k_0^2 - k_p^2} \tag{2.59}$$

It is important to keep in mind that the wave is polarized TM ($E_y = H_x = H_z = 0$) and that homogeneity is maintained in the y-direction, such that ($\frac{\partial}{\partial y} = 0$). Maxwell's Equation (2.4) can now be applied to obtain the electric field :

$$iw\epsilon_0\epsilon_i E_{xi} = \frac{\partial H_{yi}}{\partial z} \tag{2.60}$$

either

$$E_{xi} = \frac{-i}{w\epsilon_0\epsilon_i} \frac{\partial H_{yi}}{\partial z} \tag{2.61}$$

for each media. Applying Equation (2.61) at the boundary, plugging the Equations (2.53) in and using the results found in the Equations (2.55) and (2.56) lead to:

$$\frac{k_{z1}}{\epsilon_1} = \frac{-k_{z2}}{\epsilon_2} \text{ or } \frac{\epsilon_2}{\epsilon_1} = -\frac{k_{z2}}{k_{z1}} \tag{2.62}$$

Inserting Equation (2.59) into Equation (2.62) leads to the dispersion relation of the SPs in a single interface system :

Figure 2.5: Dispersion relation and magnetic field profile of SPPs at a planar interface between air and a Drude-metal with no damping.

$$k_x = k_p = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}} \quad (2.63)$$

k_p is the surface plasmon wavevector. The component k_{zi} can also be developed combining Equation (2.59) and Equation (2.63), to find :

$$k_{zi} = k_0 \sqrt{\frac{\epsilon_i^2}{\epsilon_1 + \epsilon_2}} \quad (2.64)$$

From these results, it is possible to describe the appropriate environment for surface plasmon mode at the interface between two media. Indeed, surface plasmon propagate parallel to the surface, vanishing in the z -direction. Hence, as it has been seen previously, k_{zi} should be purely imaginary. Therefore, following Equation (2.64), the condition becomes $\epsilon_1 + \epsilon_2 < 0$. Then, considering a wave that propagates along the interface, k_p should be real. Therefore, following Equation (2.63), we shall have $\epsilon_1 \epsilon_2 < 0$.

This means that one medium must exhibit a negative dielectric function, with an absolute value exceeding that of the other. Such materials can be found among noble metals, which exhibit a large negative real part. It is also the case of graphene in the infrared range (see Figure 3.13), using the model described Section 4.2. The second medium should have a positive real dielectric value, not too large. Dielectric media such as SiO_2 or air are good candidates.

In the case of a system presented in Figure 2.4, with a medium 2 which is air, $\epsilon_2 = 1$. Defining the medium 1 as a Drude metal, neglecting the damping (Equation (2.37)), Equation (2.63) becomes :

$$\frac{k_p c}{\omega_p} = \frac{\omega}{\omega_p} \sqrt{\frac{\epsilon_1(\omega/\omega_p)}{\epsilon_1(\omega/\omega_p) + 1}} \quad (2.65)$$

Equation (2.65) is plotted in Figure 2.5a. For real k_p (in blue) and $\omega < \omega_p$ (half bottom part), $\omega/\omega_p \rightarrow 1/\sqrt{2}$. The black dashed curve represents the limit for the incident photon dispersion relation. The lack of overlapping between the incident photon relation dispersion and the SPPs relation dispersion poses problems regarding to the excitation of SPPs, where a third structure is needed for the coupling. For k_p pure imaginary, in red, the plasmons are localized. For $\omega > \omega_p$, so-called Brewsters modes are supported.

Going further, Figure 2.5b displays the magnetic field profile at this interface, using Equation (2.64) and $H_y = H_0 e^{-ik_{zi}z}$, with $H_0 = 1$. We can observe that SPPs are more confined to the interface as we approach $\omega_{SP} = \omega_p/\sqrt{2}$. As ω tends to ω_{SP} , the group velocity $v_g = \partial\omega/\partial k$ of the SPPs decreases.

Finally, coming back to Equation (2.65), the plasmons wavevector k_p can give interesting information such as the propagation length :

$$L = \frac{1}{2\text{Im}[k_p]} \quad (2.66)$$

as well as the effective surface plasmon wavelength and the penetration depth in the medium :

$$\lambda_p = \frac{2\pi}{\text{Re}[k_p]} \quad (2.67)$$

$$\delta = \frac{1}{\text{Im}[k_{zi}]} = \frac{1}{|k_{zi}|} \quad (2.68)$$

The propagation length is the length at which the intensity, proportional to the square of the electromagnetic field modulus, fall to $1/e$. The penetration depth gives a direct information about the confinement of the electromagnetic field in its surrounding. Finally, the wavelength of the plasmons is the effective wavelength that can be measured, for example, by Scanning Near-Field Optical Microscopy (SNOM). This last physical quantity can be normalized by the excitation field and is called in this case the localization factor. It gives also, indirectly, a good idea of the confinement of the electromagnetic field in the materials.

2.2.2 Surface plasmon polariton in a multilayer system

In the multi-layer model (illustrated in Figure 2.6), a thin metallic layer (in blue) sandwiched between two (infinitely) thick dielectric claddings (media 1 and 2). This system is often refereed as an insulator/metal/insulator (IMI) heterostructure. Here the simplest case is considered, such as the same insulator above and below is sandwiching the metallic layer.

A multilayer system is defined when an interaction/coupling between the surface plasmon of different interfaces takes place. This means that the thickness of the metal is comparable to or smaller than the decay of the surface plasmon.

Figure 2.6: Geometry of a two-interface system consisting of a thin layer (metallic medium with a thickness of $\Delta = 2a$) sandwiched between two infinite half spaces (dielectric medium).

The vectorial base being centred in the middle of the metallic thin layer (with a thickness of $\Delta = 2a$), we will distinguish three parts of the system : for $z > a$, a medium with a permittivity ϵ_2 , for $a > z > -a$, a medium with a permittivity ϵ_m and for $z < -a$, a medium with a permittivity ϵ_1 .

Following the same path as in Section 2.2.1, and taking into account the symmetry of the system, the boundary conditions imply that $k_{x1} = k_{x2} = k_p$.

Let's assume that for $z > a$, the magnetic field can be simplified as follow :

$$H_{y_2} = Ae^{ik_{z2}z}e^{ik_px} \quad (2.69)$$

with an harmonic time dependence, and A the amplitude of the $\mathbf{H}_{y2}$ field. It must be noticed below that the notation for the wavevector in the z direction is the following : $k_i \equiv k_{zi}$ for $i=1,2$ in the dielectric medium and m for the metallic one. Using Equation (2.61), it is possible to write :

$$E_{x_2} = \frac{k_2}{w\epsilon_0\epsilon_2} Ae^{ik_{z2}z}e^{ik_px} \quad (2.70)$$

In the same way, we can write for $z < -a$:

$$H_{y_1} = Be^{-ik_1z}e^{ik_px} \quad (2.71)$$

$$E_{x_1} = -\frac{k_1}{w\epsilon_0\epsilon_1} Be^{-ik_1z}e^{ik_px} \quad (2.72)$$

with B the amplitude of $\mathbf{H}_{y1}$. While for $-a < z < a$, we have a combination of the modes from the media upon and below and we can write, with C^+ and C^- the amplitude of the field for respectively z positive and z negative :

$$\begin{aligned} H_{y_m} &= C^+ e^{i(k_px + k_mz)} + C^- e^{i(k_px - k_mz)} \\ &= (C^+ e^{ik_mz} + C^- e^{-ik_mz}) e^{ik_px} \end{aligned} \quad (2.73)$$

$$\begin{aligned} E_{x_m} &= \frac{-i(ik_m)}{w\epsilon_0\epsilon_m} C^+ e^{ik_mz} e^{ik_px} + \frac{-i(-ik_m)}{w\epsilon_0\epsilon_m} C^- e^{-ik_mz} e^{ik_px} \\ &= \frac{k_m}{w\epsilon_0\epsilon_m} (C^+ e^{ik_mz} - C^- e^{-ik_mz}) e^{ik_px} \end{aligned} \quad (2.74)$$

H_y has to fulfill the wave equation which gives again (see Equation (2.59)) :

$$k_i = \sqrt{\epsilon_i k_0^2 - k_p^2} \quad (2.75)$$

Here, we have $k_i \equiv k_{zi}$ for $i=1,2$ and m, and $k_p \equiv k_x$. The boundary continuity requirement seen in the subsection 2.1.2 implies that $H_{y_2} = H_{y_m}$ at $z = a$ as well as $H_{y_1} = H_{y_m}$ at $z = -a$:

$$Ae^{ik_2a} = C^+ e^{ik_ma} + C^- e^{-ik_ma} \quad \text{at } z=a \quad (2.76)$$

$$Be^{ik_1a} = C^+ e^{-ik_ma} + C^- e^{ik_ma} \quad \text{at } z=-a \quad (2.77)$$

While for E_x , we have :

$$\frac{k_2}{\epsilon_2} Ae^{ik_2a} = \frac{k_m}{\epsilon_m} (C^+ e^{ik_ma} - C^- e^{-ik_ma}) \quad \text{at } z=a \quad (2.78)$$

$$-\frac{k_1}{\epsilon_1} Be^{ik_1a} = \frac{k_m}{\epsilon_m} (C^+ e^{-ik_ma} - C^- e^{ik_ma}) \quad \text{at } z=-a \quad (2.79)$$

Those four equations can be seen as a linear system. Let's take first Equations (2.76) and (2.78), which lead to :

$$C^- = C^+ e^{2ik_ma} \frac{k_m/\epsilon_m - k_2/\epsilon_2}{k_m/\epsilon_m + k_2/\epsilon_2} \quad (2.80)$$

Then, doing the same for Equations (2.76) and (2.78) results in :

$$C^+ = C^- e^{2ik_ma} \frac{k_m/\epsilon_m - k_1/\epsilon_1}{k_m/\epsilon_m + k_1/\epsilon_1} \quad (2.81)$$

Finally, multiplying Equation (2.80) by Equation (2.81) yields to the general equation :

$$e^{i4k_m a} = \frac{k_m/\epsilon_m + k_2/\epsilon_2}{k_m/\epsilon_m - k_2/\epsilon_2} \frac{k_m/\epsilon_m + k_1/\epsilon_1}{k_m/\epsilon_m - k_1/\epsilon_1} \quad (2.82)$$

This equation is a general equation for a two interfaces system. It is interesting to notice that if the thickness a is infinite, Equation (2.62) for a single interface is retrieved (keeping in mind that k_m is pure imaginary). In the case of this thesis, the study of suspended graphene requires to set the two media above and below the graphene as the same dielectric one, with a permittivity ϵ_d . Therefore, the simplest case of a symmetric structure will be solved and Equation (2.82) becomes :

$$e^{2ik_m a} = \pm \frac{k_m/\epsilon_m + k_d/\epsilon_d}{k_m/\epsilon_m - k_d/\epsilon_d} \quad (2.83)$$

Here, it is really interesting to realize that two solutions arise and therefore two branches on the dispersion relation will appear. Going back to Equations (2.81) and (2.80), and replacing k_1/ϵ_1 and k_2/ϵ_2 by k_d/ϵ_d , yield to $C^- = \pm C^+$. This result can be plugged in Equation (2.73) which becomes :

$$H_{y_m} = (e^{ik_m z} \pm e^{-ik_m z}) C e^{i(k_p x - \omega t)} \quad (2.84)$$

Figure 2.7: The magnitude of the field H for the symmetric (left) and anti-symmetric (right) modes while surface plasmon is propagating in a symmetric structure

It is possible to solve Equation (2.83) knowing that $\tanh(z) = (e^{2z} - 1)/(e^{2z} + 1)$ which leads to the two following solutions :

$$\tanh(ik_m a) = \frac{\epsilon_d k_m}{\epsilon_m k_d} \quad \text{for the symmetric mode where } C^- = -C^+ \quad (2.85)$$

$$\tanh(ik_m a) = \frac{\epsilon_m k_d}{\epsilon_d k_m} \quad \text{for the anti-symmetric mode where } C^- = C^+ \quad (2.86)$$

These results are shown in Figure 2.7 where the magnetic field profile for the symmetric modes is shown on the left and for the anti-symmetric modes on the right. Using the result of Section 2.2.1, i.e. k_{zi} should be a pure imaginary, it is possible to replace it as $k_{zi} = iq_{zi}$ with q_{zi} real positive and $q_{zi} = \sqrt{k_p^2 - \epsilon_i k_0^2}$. Finally, the dispersion relations for the symmetric modes and anti-symmetric modes in a thin slab of metal m sandwiched between a dielectric d turn into their most known forms :

$$\tanh(a\sqrt{k_p^2 - \epsilon_m k_0^2}) = -\frac{\epsilon_d \sqrt{k_p^2 - \epsilon_m k_0^2}}{\epsilon_m \sqrt{k_p^2 - \epsilon_d k_0^2}} \quad (2.87)$$

$$\tanh(a\sqrt{k_p^2 - \epsilon_m k_0^2}) = -\frac{\epsilon_m \sqrt{k_p^2 - \epsilon_d k_0^2}}{\epsilon_d \sqrt{k_p^2 - \epsilon_m k_0^2}} \quad (2.88)$$

When d tends to infinity, Equation (2.85) and Equation (2.86) are reduced to the equation of dispersion of surface plasmons at a single interface, Equation (2.63), since $\tanh$ tends to 1.

(a) The thickness of the core is chosen as $a = 0.5 \cdot c/\omega_p$. For typical value of $f_p = 2.10^{15} \text{ Hz}$ (silver and gold), we get a thickness around 12 nm.

(b) The thickness of the core is chosen as $a = 0.012 \cdot c/\omega_p$. For typical value of $f_p = 2.10^{15} \text{ Hz}$ (silver and gold), we get a thickness around 0.3 nm.

Figure 2.8: Dispersion relations of symmetric and antisymmetric modes in a Metal-Insulator-Metal waveguide. The metal is modeled as a lossless Drude dispersion. The black dotted lines indicate the dispersion relation of the light in air, whereas the dashed line represent decoupled SPPs.

The two branches of the dispersion relations of plasmons for a IMI hetero-structure are shown in Figure 2.8, for two different thicknesses. It is interesting to notice that the anti-symmetric modes vanish as the thickness of the metal gets atomically thin. It can be explained as follow. When the metallic core becomes thinner and tends to atomic thickness, we can express the hyperbolic tangent as Taylor serie as :

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} + \mathcal{O}(x^7) \quad (2.89)$$

Taking Equations 2.85 and 2.86, we find :

$$-\frac{\epsilon_d}{\epsilon_m k_d} \rightarrow 0 \quad \text{for the symmetric mode} \quad (2.90)$$

$$-\frac{\epsilon_m k_d}{\epsilon_d k_m^2} \rightarrow 0 \quad \text{for the anti-symmetric mode} \quad (2.91)$$

For the symmetric modes, k_d tends to infinity which means that $k_p \gg k_0$. For the anti-symmetric modes, k_d tends to 0 and therefore $k_p \rightarrow k_0$. Therefore, the antisymmetric modes tend to become radiative, as it follows the dispersion relation of the light in air. Therefore, for graphene which is a one atom thick material, only the symmetric dispersion relation remains. As it can be observed in Figure 2.8b, a significant mismatch between the light wave-vector (black dotted line) and the plasmons wave-vector takes place. In fact, at a certain frequency, the mismatch is higher than in conventional noble material, as it can be observed in Figure 3.19b. As $q_{zi} = \sqrt{k_p^2 - \epsilon_i k_0^2}$ is really high, the penetration depth get small (Equation (2.68)), which leads to very high confinement.

Coming back to Equation (2.84), the magnetic field for the symmetric modes and for the anti-symmetric modes can be obtained applying the boundary conditions at $z = \pm a$:

(a) Y component of the magnetic field H_y as a function of z , the normal space axis, in the case of the symmetric modes

(b) Y component of the magnetic field H_y as a function of z , the normal space axis, in the case of the anti-symmetric modes

Figure 2.9: Magnetic field profile in a Metal-Insulator-Metal waveguide. The thickness of the core is chosen as $a = 0.5 \cdot c/\omega_p$. For typical value of $f_p = 2.10^{15} \text{ Hz}$ (silver and gold), we get a thickness around 12 nm. The metal is modeled as a lossless Drude dispersion. The black lines indicate the interface between the air and the metal.

$$H_y^s(x, z) = \begin{cases} A \frac{\cosh(k_m z)}{\cosh(k_m a)} e^{ik_p x}, & \text{where } |x| < a \\ A e^{-k_d(|z|-a)} e^{ik_p x}, & \text{where } |x| > a \end{cases}$$

$$H_y^a(x, z) = \begin{cases} A \frac{\sinh(k_m z)}{\sinh(k_m a)} e^{ik_p x}, & \text{where } |x| < a \\ \frac{z}{|z|} A e^{-k_d(|z|-a)} e^{ik_p x}, & \text{where } |x| > a \end{cases}$$

, which gives modes with the shape drawn in Figure 2.9.

The study done in this chapter have been conducted toward plasmonics in extremely thin metal. Graphene, which is a semi-metal, has plasmonics behaviour remarkably close to the SPPs arising from extremely thin layer of common noble metal. However, its 2D crystalline structure as well as its electrons dynamic make Graphene Plasmons (GPs) a bit different. In the next section, we will therefore focus on graphene crystalline and electronics properties, before going deeper into graphene plasmonics, in Chapter 3.

2.3 Properties of Graphene

2.3.1 A sheet of carbon atom

Graphene is the mother of all graphitic forms: it is a sheet of carbon atoms in a honeycomb lattice. It possesses unique physical properties: graphene is the strongest and the most stretchable known material, it has the record of thermal conductivity, and is completely impermeable [50, 8]. Moreover, from the opto-electronic point of view, its carriers (both electrons and holes) exhibit ultra-high mobility and long mean free path. Graphene has gate-tunable carrier densities, anomalous quantum Hall effects, fine structure constant defining optical transmission, and so on. Also, this two dimensional one atom thick membrane is a perfect example of a two-dimensional electron system for a physicist. According to [62], "When the quantum theory of solids was developed, graphite was one of the earliest materials to which it was applied. The first calculations of the energy band structure of graphite appears to have been made by Hund and Mrowska in 1937 as an academic exercise."

Figure 2.10: Artistic representation of graphene

In order to study graphene opto-electronic properties in detail, it is important to look at its crystalline structure. It is a monolayer of sp_2 bounded carbon atoms in a honeycomb lattice. Each carbon atom is attached to three neighbours by covalent bounds with a length of about $a_0 = 0.142nm$. This lattice configuration comes from the atomic orbitals interferences between neighbouring carbon atoms (orbital hybridisation).

Let's first zoom on the smallest unit of its structure: the carbon atom. This chemical element has six electrons, four of the electrons are in its valence shell (outershell). Carbon's ground state configuration is $1S^2 2S^2 2P^2$ which can also be written in order to show the valence electrons $[He]2S^2 2P^2$, or more easily read :

$$\boxed{\uparrow\downarrow}_{1S} \quad \boxed{\uparrow\downarrow}_{2S} \quad \boxed{\uparrow}\boxed{\uparrow}\boxed{}_{2P} \quad (2.92)$$

Here we can see the six electrons represented by arrows (the direction of the arrow represents the electron spin). The electrons located the closest to the nucleus are found in the 1s orbital. These electrons are usually considered as "frozen" in the pseudo-potential approximation, which takes place in many models. The pseudopotential model is a way of replacing the complicated effects of the motion of the core (i.e. non-valence) electrons of an atom and its nucleus with an effective potential, or pseudopotential. Those electrons are not involved in bounding. The next two will go into the 2s orbital. The remaining ones will be in two separate 2p orbitals. The p orbitals have the same energy and the electrons would rather be in separate orbitals.

In graphene, carbon atom rearranges its valence electrons so as to create bounds with its three neighbours. In fact, creating a bound decreases the energy of the system. The first step is to describe the atom in one of its excited state, which can be displayed as

$$\left\{ \begin{array}{c} s \\ p_x \\ p_y \\ p_z \end{array} \right\} \quad \text{with a electronic configuration : } \boxed{\uparrow}_{2S} \quad \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}_{2P} \quad (2.93)$$

Then, these orbitals will mix in a special manner in order to create three covalent bounds. This concept of mixing atomic orbitals into new hybrid orbitals is called hybridisation. A more intuitive way

to look at the carbon atom and its hybridisation is to use an energy level graph shown in Figure 2.11. One s orbital is hybridized with two p orbitals. The orbital p_z remains unchanged.

Figure 2.11: Illustration of the three hybridised orbitals : as an energy diagram (a) and as a picture on the right (red sphere represent Carbon atoms).

The three orbitals sp^2 form a triangular base with 120° angle in the hexagonal plane and will create σ bounds. The last orbital p_z is the only one electron which does not belong to the core neither to the covalent bounds and form a π bound perpendicular to the plane. Its interaction with the lattice will give the main electronic properties of the graphene at relatively low energy, such as the massless fermion behavior (see section 2.3.2)

Crystal structure in the real space and in the reciprocal space

Carbon atoms in a graphene plane are located at the vertices of a hexagonal lattice (Figure 2.12a). This triangular lattice has two atoms per unit cell (in green), A atom (in red) and B atom (in blue). The atoms A form a sublattice, and each atom A is surrounded by three atoms B, and vice versa. The basis vectors of this honeycomb lattice are $\mathbf{a}_1$ and $\mathbf{a}_2$, defined with the unit vector $\mathbf{x}, \mathbf{y}$ by :

$$\mathbf{a}_1 = a_0 \sqrt{3} \left(\frac{\sqrt{3}}{2}, \frac{1}{2} \right) \quad \text{and} \quad \mathbf{a}_2 = a_0 \sqrt{3} \left(\frac{\sqrt{3}}{2}, -\frac{1}{2} \right) \quad (2.94)$$

with $a_0 = 1.42 \text{\AA}$ the nearest-neighbour distance.

Figure 2.12: Graphene lattice (a) in the real space and (b) in the reciprocal space with the main parameters described in the text.

It is also interesting to describe the sub-lattice B starting from one A atom. We can build in this case three vectors :

$$\delta_1 = a_0 \left(-\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \quad \text{and} \quad \delta_2 = a_0 \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2} \right) \quad \text{and} \quad \delta_3 = a_0 (1, 0) \quad (2.95)$$

Then, it is interesting to go a bit further into the description of graphene by defining the concept of reciprocal space. This is an important concept because every crystalline structure have a particular reciprocal network. This signature of a structure can be recorded experimentally with diffraction pattern that give valuable informations about the reciprocal space .

We define the reciprocal space vectors by :

$$\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi \delta_{ij} \quad \text{therefore we get} \quad \begin{matrix} \mathbf{b}_1 \cdot \mathbf{a}_1 = 2\pi & \mathbf{b}_1 \cdot \mathbf{a}_2 = 0 \\ \mathbf{b}_2 \cdot \mathbf{a}_2 = 2\pi & \mathbf{b}_2 \cdot \mathbf{a}_1 = 0 \end{matrix} \quad (2.96)$$

We can then deduce the reciprocal vectors, drawn on the Figure 2.12b :

$$\mathbf{b}_1 = \frac{2\pi}{3a_0} \left(1, \sqrt{3} \right) \quad \text{and} \quad \mathbf{b}_2 = \frac{2\pi}{3a_0} \left(1, -\sqrt{3} \right) \quad (2.97)$$

In Figure 2.12b, we have drawn in green the Brillouin zone, which is the the primitive cell of the reciprocal space. It is a really interesting notion because it makes possible to study the lattice vibration as well as the electronic properties of the crystal by its symmetries. The three high symmetry points of the Brillouin zone, Γ , K and M are the center, the corner, and the center of the edge of the hexagon, respectively. More in detail, there are two non-equivalent corners, here noted K_1 and K_2 , like in the real space :

$$\mathbf{K}_1 = \frac{2\pi}{3a_0} \left(1, \frac{1}{\sqrt{3}} \right) \quad \text{and} \quad \mathbf{K}_2 = \frac{2\pi}{3a_0} \left(1, -\frac{1}{\sqrt{3}} \right) \quad (2.98)$$

This results will be important for the next section, where the main electronic properties of graphene are expounded through the tight binding model. This model will give the so peculiar band structure of graphene in the reciprocal space. In between a semi-conductor and a metallic behavior, graphene can be considered as a zero-gap semi-conductor, or a semi-metal.

2.3.2 The tight binding model : a study about the dynamics of the electrons

Modelling the dynamics of electrons, a set of hypothesis

In order to study in detail the opto-electronic properties of graphene, giving rise to plasmons, it is important to have a look at the electron dynamics in this 2D material.

This problem is well known and can be solved by mixing solid states physics with quantum mechanics. The first point is to realize that such a n-body system cannot be solved exactly, due to the huge interactions and degrees of freedom which take place. In fact, electrons and holes are charge carriers, therefore their dynamic depends on all the others particles (electrons, holes, nuclei and defects) in the media. Exact computation on such systems requires power of calculus that is not available nowadays. This is the reason why physicists set up methods of calculus based on approximations which enable to get results close to the experimental ones.

Here, the goal is to understand the electronic properties of graphene via the tight binding model which gives a result in a form of band energy structures [23, 54]. In this approach, electrons are moving through a periodic potential that is very strong in the vicinity of the the lattice atoms (see Figure 2.13). In general, the tight-binding approximation [10, 82] can handle the case where the overlapping of atomic wave functions is significant enough to require corrections to the representation of isolated atoms, but not enough to make inappropriate atomic description.

Looking at the dynamics of electrons in a lattice is somehow looking at the delocalization of the electrons caused by the neighboring atoms. This system is a many-electron problem which requires us

to make the following assumptions to solve it.

The π -band approximation. In a single atom, electrons belong to atomic orbitals. The atomic orbitals can be seen as a mathematical function which describes the wave-like behaviour of an electron in an atom. In a crystalline solid, the atomic orbitals interact with each others to form bands. As seen before, the electrons which are involved in the low energy range of electronic properties, are those that can be found in the π band. The p_z orbitals do not overlaps with the sp_2 orbitals (see Figure 2.11b), therefore, other electrons belonging to the σ band should not be taken into account. This unsaturated orbital p_z can be described by a wave-function that is noted $P_z^s(\mathbf{r})$ with s being the index for the two sub-lattice ($s = A$ or B). s can also be seen as the number of bands or the number of atomic wave-functions used in a unit cell.

The frozen heart. We will consider the approximation of "frozen heart", which assumes that all the electrons belonging to the core shell (1S), which are not involved in bounding, are not disrupting. As well, we consider the nuclei as entities positively charged (cations) and immobile (in reality, the atoms vibrate). This approximation gives rise to a positive pseudo-potential emerging from the lattice nucleus.

The perfect crystal. The perfect crystal will not suffer any defects. In order to build a description of the behaviour of electrons delocalizing from π to π bands, we need to take into account all the π -bonded electrons of the graphene lattice. We will first consider one unit cell (in green in Figure 2.12a), which is composed of two carbon atoms, A and B. Then, the process will be duplicated in order to recover a perfect lattice.

The single and independent electron approximation. In this approximation, the electron-electron interaction is not taken into account. Indeed, the electron-electron interaction does not give rise to a periodic potential, which could complexify the model. The tight-binding model is a one-electron model, which is not suitable for electron-electron interaction. Fortunately, electron-electron interaction is usually weaker than ion-electron interactions.

The time independent periodic potential. From the previous approximations, we can now assume that electrons will be subjected to a periodic and static potential. Therefore, the wave functions Ψ describing the dynamics of electrons form standing waves called stationary states, which are the physical quantities defining orbitals. They are solutions of the time independent Schrödinger equation :

$$\hat{H}\Psi = E\Psi \quad (2.99)$$

$\hat{H}$ is the Hamiltonian, which can be seen as a matrix which gives us the possible energy of the system when it is operating on the wave-function of the graphene lattice. E is the energy eigen-values of the system, which constitute a discrete set of solutions that would be found if the energy of the system was measured. In order to get the energy of delocalized electrons in the honeycomb lattice of graphene, the Schrödinger equation should be solved for all the π -bonded electrons of the 2D lattice. In our perfect lattice, we will sum the orbitals wave-function of the atoms belonging to each sub-lattice $\Phi_s(\mathbf{r}, \mathbf{k}) \propto \sum P_z^s(\mathbf{r})$, and then build the total wave-functions $\Psi(\mathbf{r}, \mathbf{k}) \propto \Phi_A(\mathbf{r}, \mathbf{k}) + \Phi_B(\mathbf{r}, \mathbf{k})$ describing the whole lattice. $\mathbf{k}$ is the wave vector, which is related to the quantized crystal momentum $\mathbf{p}$, by $\mathbf{p} = \hbar\mathbf{k}$.

The Bloch's theorem. We know that the eigenstates of an electron in a periodic potential can be written as Bloch waves, according to the Bloch theorem. Those eigenstates are characterized by a

Figure 2.13: Representation of the potential for the three models described here

wave-vector $\mathbf{k}$ that belongs to the first Brillouin zone. Assuming a perfect pristine graphene sheet, an orbital wave-function $P_z^s(\mathbf{r})$ should satisfy the Bloch's theorem, which gives us :

$$T_R P_z^s(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{R}} P_z^s(\mathbf{r}) \quad (2.100)$$

with $\mathbf{R}$ the periodicity of the underlying lattice and T_R the translation operator along the periodicity. This means that we have a translational symmetry of the unit cells in the direction of a lattice vector. We can notice that the eigenfunctions assessed at two different lattice locations, 1 and 2 associated to respective position vectors r_1 and r_2 , differ from each other in just a phase factor, $e^{i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}$. Finally, it is possible to write the Bloch function, which is a sum of the Bloch wave-function over the lattice, for one sub-lattice :

$$\Phi_s(\mathbf{r}, \mathbf{k}) = \frac{1}{\sqrt{N}} \sum_{R_s} e^{i\mathbf{k} \cdot \mathbf{R}_s} P_z^s(\mathbf{r} - \mathbf{R}_s) \quad (2.101)$$

with N the number of cells that are taken into account in the model.

Therefore, we can write the total wave-function for the lattice as a sum of the Bloch wave-functions of the lattice A and B, which will give :

$$\Psi(\mathbf{r}, \mathbf{k}) = \sum_s C_s \Phi_s(\mathbf{r}, \mathbf{k}) = C_A \Phi_A(\mathbf{r}, \mathbf{k}) + C_B \Phi_B(\mathbf{r}, \mathbf{k}) \quad (2.102)$$

with C_s the weight associated to sum of the atomic orbitals ($\Phi_s(\mathbf{r}, \mathbf{k})$) of the sub-lattice s . With the Dirac notations, the wave-function Ψ can finally be described by :

$$|\Psi\rangle = \sum_s C_s |\Phi_s\rangle \quad (2.103)$$

The tight binding model : solving the Schrödinger time independent equation

Let's come back on the time independent Schrödinger equation (Equation (2.99)), and insert the equation describing the wavefunction seen in Equation (2.103). We obtain using bra and ket notation :

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \quad (2.104)$$

$$\sum_s C_s \hat{H} |\Phi_s\rangle = E \sum_s C_s |\Phi_s\rangle \quad (2.105)$$

In order to work with a scalar equation, the last equation is multiplied by its conjugate, which leads to :

$$\sum_{s,s'} C_s C_{s'}^* \langle \Phi_{s'} | \hat{H} | \Phi_s \rangle = E \sum_{s,s'} C_s C_{s'}^* \langle \Phi_{s'} | \Phi_s \rangle \quad (2.106)$$

$$\sum_{s,s'} C_s C_{s'}^* H_{s's} = E \sum_{s,s'} C_s C_{s'}^* S_{s's} \quad (2.107)$$

where $H_{s's}$ is the matrix representation of the Hamiltonian in the orbitals $|\Phi_s\rangle$ base. $S_{s's}$ is the overlapping orbitals matrix, which is a quantitative measurement of the overlap of two atomic orbitals belonging to two different atoms. The last equation can be re-written more compactly in matrix form as :

$$HC = ESC \quad (2.108)$$

This matrix equation leads to the following condition for non-trivial solutions:

$$\det[H - ES] = 0 \quad (2.109)$$

This equation will give us the dispersion relation of the graphene and provides n energy eigen-value as a function of $\mathbf{k}$, with n the number of bands for n orbitals in the unit cell. Since the unit cell contains two atoms, $n=2$ and the Hamiltonian matrix will have a dimension of 2×2 .

Diagonal matrix elements. The Hamiltonian $H_{\alpha,\beta}$, with $(\alpha, \beta \in A, B)$, when $\alpha = \beta = A$ is given as follows:

$$H_{AA} = \frac{1}{N} \sum_i^N \sum_j^N e^{i\mathbf{k} \cdot (\mathbf{R}_{A,j} - \mathbf{R}_{A,i})} \langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | \hat{H} | P_z^A(\mathbf{r} - \mathbf{R}_{A,j}) \rangle \quad (2.110)$$

with i and j defining different unit cell. Here we will only consider the nearest neighbour ($i=j$), the others being neglected so that the equation becomes :

$$H_{AA} = \frac{1}{N} \sum_i^N \langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | \hat{H} | P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) \rangle = \frac{1}{N} \sum_i^N \epsilon_{2P} = E_{2P} \quad (2.111)$$

with E_{2P} being the reference energy. The reference energy is usually set to zero, in order to have the junction (Dirac point) between the upper band and the lower band of the p_z orbital at 0. The atoms of the second sub-lattice being chemically identical, it is possible to write :

$$H_{AA} = H_{BB} = E_{2P} \quad (2.112)$$

For the overlap matrix, regarding the A sublattice, it is assumed that there is no contribution from the other A atoms, which leads to :

$$S_{AA} = \frac{1}{N} \sum_i^N \langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) \rangle = 1 \quad (2.113)$$

This means that the wave-function does not overlap any other wave-function of another same sub-lattice atom.

Off-diagonal matrix elements. The Hamiltonian $H_{\alpha,\beta}$, with $(\alpha, \beta \in A, B)$, when $\alpha \neq \beta$ is given as follows:

$$H_{AB} = \frac{1}{N} \sum_i^N \sum_j^N e^{i\mathbf{k} \cdot (\mathbf{R}_{B,j} - \mathbf{R}_{A,i})} \langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | \hat{H} | P_z^B(\mathbf{r} - \mathbf{R}_{B,j}) \rangle \quad (2.114)$$

In the approximation of the nearest-neighbours, only the three nearest atoms will be taken into account, belonging to the other sub-lattice. In this case of an atom A belonging to the unit cell i , the three neighbours atoms B will be considered and they will be indexed with $l=1,2,3$. This can be written as:

$$H_{AB} = \frac{1}{N} \sum_i^N \sum_l^3 e^{i\mathbf{k} \cdot (\mathbf{R}_{B,l} - \mathbf{R}_{A,i})} \langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | \hat{H} | P_z^B(\mathbf{r} - \mathbf{R}_{B,l}) \rangle \quad (2.115)$$

For each neighbouring pair (atoms A-B), we will have the same matrix element that we can note:

$$\gamma_0 = -\langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | \hat{H} | P_z^B(\mathbf{r} - \mathbf{R}_{B,l}) \rangle \quad (2.116)$$

Therefore

$$H_{AB} = -\frac{\gamma_0}{N} \sum_i^N \sum_l^3 e^{i\mathbf{k} \cdot \delta_1} \equiv -\gamma_0 f(\mathbf{k}) \quad (2.117)$$

and since the Hamiltonian is also hermitian,

$$H_{BA} \equiv -\gamma_0 f^*(\mathbf{k}) \quad (2.118)$$

with

$$\begin{aligned} f(\mathbf{k}) &= \frac{1}{N} \sum_i^N \sum_l^3 e^{i\mathbf{k} \cdot \delta_1} = \sum_l^3 e^{i\mathbf{k} \cdot \delta_1} = e^{ia_0 k_x} + e^{-ia_0 \frac{1}{2} k_x} e^{-ia_0 \frac{\sqrt{3}}{2} k_y} + e^{-ia_0 \frac{1}{2} k_x} e^{ia_0 \frac{\sqrt{3}}{2} k_y} \\ &= e^{ia_0 k_x} + 2 \cos \left(a_0 \frac{\sqrt{3}}{2} k_y \right) e^{-ia_0 \frac{1}{2} k_x} \end{aligned} \quad (2.119)$$

using Equations (2.95). The overlap matrix will be treated in the same way, and for the neighbouring atoms pair (A-B), we expect the same matrix element:

$$s_0 = -\langle P_z^A(\mathbf{r} - \mathbf{R}_{A,i}) | P_z^B(\mathbf{r} - \mathbf{R}_{B,i}) \rangle \quad (2.120)$$

which leads to :

$$S_{AB} = -s_0 f(\mathbf{k}) \quad \text{and} \quad S_{BA} = -s_0 f^*(\mathbf{k}) \quad (2.121)$$

To summarize, the Hamiltonian matrix and the overlapping matrix become:

$$H = \begin{pmatrix} E_{2P} & -\gamma_0 f(\mathbf{k}) \\ -\gamma_0 f^*(\mathbf{k}) & E_{2P} \end{pmatrix} \quad (2.122)$$

$$S = \begin{pmatrix} 1 & -s_0 f(\mathbf{k}) \\ -s_0 f^*(\mathbf{k}) & 1 \end{pmatrix} \quad (2.123)$$

which can be pugged in Equation (2.109) :

$$\det[H - ES] = \det \begin{pmatrix} E_{2P} - E & -(\gamma_0 + Es_0)f(\mathbf{k}) \\ -(\gamma_0 + Es_0)f^*(\mathbf{k}) & E_{2P} - E \end{pmatrix} = 0 \quad (2.124)$$

to obtain:

$$E_{2P} - E = \pm(\gamma_0 + Es_0)|f(\mathbf{k})| \quad (2.125)$$

which leads to the final dispersion relation for the dynamic of electrons in graphene, taking into account the three nearest neighbour:

$$E^\pm = \frac{E_{2P} \pm \gamma_0 |f(\mathbf{k})|}{1 \mp s_0 |f(\mathbf{k})|} \quad (2.126)$$

with (+) for the π band and (-) for the π^* , and

$$|f(\mathbf{k})| = \sqrt{4\cos^2\left(\frac{a_0\sqrt{3}}{2}k_y\right) + 4\cos\left(\frac{a_0\sqrt{3}}{2}k_y\right)\cos\left(\frac{3a_0}{2}k_x\right) + 1} \quad (2.127)$$

E_{2P} , γ_0 and s_0 are three parameters used to fit the experiments. E_{2P} is usually set to 0 in order to have the Fermi energy at 0eV. Indeed, taking the point of high symmetry K_1 and K_2 seen in Equation (2.98), the calculation gives $E^\pm=0$: graphene is a semi-conductor with a bandgap null. According to [136], the parameters $E_{2P}=0$, $\gamma_0=3.033\text{eV}$ and $s_0=0.129\text{eV}$ should be used in order to get the best fit of the first principles calculations. The plot is displayed in Figure 2.14a as well as zoom on the high symmetry point K_1/K_2 in Figure 2.14b.

It is also possible to make approximations in order to simplify Equation (2.126). For the overlap matrix of the neighbouring atoms pair (A-B), it can be reasonably supposed that the overlap of the A and B wavefunctions is very low and can be neglected. As a matter of fact, the electrons which account for transport are in p_z orbitals which exhibit a small spatial extension in the x,y plane. Therefore, we can ignore non-diagonal terms and set $s_0 = 0$. It is interesting to note that by setting $s_0 = 0$, the π and π^* bands will become symmetrical. Hence, we find the widely spread equation describing the band structure of the graphene layer :

$$E^\pm = \pm\gamma_0 \sqrt{4\cos^2\left(\frac{a_0\sqrt{3}}{2}k_y\right) + 4\cos\left(\frac{a_0\sqrt{3}}{2}k_y\right)\cos\left(\frac{3a_0}{2}k_x\right) + 1} \quad (2.128)$$

The band structure. The orbitals p_z being half filled, each energy band is either full or empty. The valence band and the conduction band touch at two points in the Brillouin zone (Figure 2.14a), K_1 and K_2 in our model (Figure 2.12a). As a consequence, graphene is often refereed to as a zero-gap semi-conductor or a semi-metal. It is interesting to observe that at low energy, the dispersion relation around the K points is linear (Figure 2.14b), which gives to graphene electronic band structure this peculiar shape of cone at the vicinity of the Dirac point.

Figure 2.14: Illustration of the dispersion relation of graphene electrons (a) in the first brillouin zone and (b) at the vicinity of the high symmetry point K.

The Dirac equation for low energy. The dispersion relation of graphene tends to be linear for low energy, around the Dirac cones. In the model developed before, it happens therefore at the points K_1 and K_2 on Figure 2.12a (see Equations (2.98)). The best fit at low energy is given with $E_{2P}=0$, $\gamma_0=2.7\text{eV}$ and $s_0=0$ according to [132], which gives $E^\pm = \pm\gamma_0|f(\mathbf{k})|$. In order to demonstrate it, let's go back to Equation 2.119 and insert $\mathbf{q} = \mathbf{K}_1 + \delta\mathbf{k}$. Using Taylor serie for trigonometric functions and keeping only the first order term, it is possible to write:

$$\begin{aligned}
 f(\mathbf{q}) &= e^{ia_0q_x} + 2\cos\left(a_0\frac{\sqrt{3}}{2}q_y\right)e^{-ia_0\frac{1}{2}q_x} \\
 &= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(1 + ia_0\delta k_x) + \left(1 - \frac{3}{2}a_0\delta k_y\right)\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)\left(1 + ia_0\frac{1}{2}\delta k_x\right) + \mathcal{O}(\delta k^2) \\
 &= \frac{3a_0}{2}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)(i\delta k_x + \delta k_y) + \mathcal{O}(\delta k^2) \\
 &= \frac{3a_0}{2}e^{i\frac{2\pi}{3}}(i\delta k_x + \delta k_y) + \mathcal{O}(\delta k^2)
 \end{aligned} \tag{2.129}$$

Therefore, in the vicinity of the K points, the dispersion relation can be approximated as

$$E^\pm = \pm\gamma_0|f(\mathbf{q})| = \pm\gamma_0\frac{3a_0}{2}|\delta\mathbf{k}| = \pm\hbar v_F|\delta\mathbf{k}| \tag{2.130}$$

where the Fermi velocity has been defined as $v_F = \gamma_0\frac{3a_0}{2\hbar} \approx 10^6\text{m.s}^{-1}$. Moreover, the Fermi speed does not depend on the mass. We will therefore speak about the electron as massless Fermions of Dirac in graphene. One of the consequence is the ultra-high mobility of the carrier in graphene, leading to a high conductivity of this material even at room temperature. It is also a really interesting behaviour to study in quantum mechanics.

2.3.3 Graphene electron-hole excitation spectrum

The electron hole excitations in the low energy range can be explained looking carefully at the cone-like band structure (Figure 2.14b). Those excitations are really important in the study of graphene plasmonics because there is a possibility of an energy transfer between plasmons and single particle excitation resulting in plasmons damping. This damping is called the Landau damping, which is a loss of collective electronic motion from the plasmons to the excitation of electron-hole pairs, a single particle excitation.

Figure 2.15: Illustration of the construction of the intraband excitation zone in graphene (on the right), from its electronic band structure (on the left). Here it has been considered a n-doped graphene (conduction band half filled in orange on the electronic band structure). Each arrow on the band structure represent a couple energy-wavevector of a specific electronic transition. The crosses on the corresponding map represent the different transitions illustrated by the arrows on the band structure.

Figure 2.15 illustrates the construction of the single particle excitation zone $\mathbf{SPE}_{\text{intra}}$ (on the right in orange on the map $E(k_{\parallel})$) where intraband excitation takes place. In this case, both the electrons and holes taking place in the excitations of electron-hole pairs, are located in the conduction band (for E positive, upper cone, on the left of Figure 2.15). The different possible excitations are represented by the coloured arrows on a graphene n-doped electronic band structure, on the left of Figure 2.15. Each arrow is defined by a particular energy/wavenumber couple $(E, k_{\parallel})$. Picking up all the possible couple $(E, k_{\parallel})$ of intraband electron-hole excitations, it is possible to plot a map displaying the wavenumber and the energy, in x and y axis respectively (on the right Figure 2.15). A region in orange appears on the dispersion relation graphic, where each coloured arrow has been replaced by a corresponding cross. This region is called the intraband single particle excitation zone ($\mathbf{SPE}_{\text{intra}}$).

Figure 2.16: Possible interband single particle excitation (electron-hole-excitations) in graphene n-doped and illustration of the interband transition zone. The conduction band is illustrated by the orange zone, whereas the valence band is illustrated in green. The arrow represent the couple energy-wavevector of a specific transition. The cross represent the different transitions illustrated on the band structure.

Figure 2.16 illustrates the interband excitation, in which electrons participating to the excitations are located in the valence band, whereas holes are located in the conduction band. The electron is promoted from the valence band to the conduction band when the condition $(E, k_{\parallel})$ is met. In the same way than above, a region in green appears, which is called the interband single particle excitation zone ($\text{SPE}_{\text{inter}}$). The interband transition are only possible above an energy threshold of $\mathbf{E}_{\text{F}}$, where an electron at the dirac crossing can get promoted with a couple of energy-wavevector $(\mathbf{E}_{\text{F}}, \mathbf{k}_{\text{F}})$. An interesting point is to realize that a photon can excite an electron only at the energy of $2\mathbf{E}_{\text{F}}$ and above. This $2\mathbf{E}_{\text{F}}$ is usually called the Pauli-blocking threshold. Therefore, at energy below this threshold, the interband contribution to the optical response of graphene becomes insignificant as compare to the intraband one. This will be see section 3.2.1, where the optical conductivity of graphene is demonstrated.

Figure 2.17: Illustration of the zone where there is no Landau damping in the ideal case, in yellow. The orange zone on the spectrum illustrates the intraband excitation, whereas the green zone illustrates the interband single particle excitation.

Figure 2.17 summarize the figure 2.15 and 2.16. In yellow is highlighted the zone in which there should be a weak Landau damping for the surface plasmons to propagate. However, outside this triangular zone, plasmons experience high dampings due to electron-hole excitations. Therefore, when studying graphene plasmons, it is important to carefully choose the energy of excitation.

2.4 Conclusion

In this first chapter, the fundamental principles have been presented in order to cover both fields, plasmonics and graphene. The physics principles of electromagnetism have been used to understand the basis of plasmonics, in the general case as well as in the case of thin films. Then, the main properties of graphene are introduced. Particularly, the tight binding model provides an understanding on the behavior of electrons in graphene. It leads to the construction of a graphic representing the possible excitation of electrons-holes as a function of a given energy and momentum. This latter is a necessity in order to work on a long propagation of plasmons in graphene.

In the next chapter, the knowledge gathered here will be utilized to study the field of graphene plasmonics.

Chapter 3

Graphene plasmonics

Electronic devices, used in telecommunication and information processing, exhibit inherent limitations due to materials electronic losses and noises which have consequences on their conductivity and associated energy consumption. To remove this barrier and keep on increasing electronic devices performances, the technological trend is to find another information carrier that is able to replace or complete electrons, among which photons appear to be the ideal candidate. Until now, however, despite recent promising demonstrations of strong near-field confinement in hybrid Silicon on Insulator structures, e.g [99], photonic devices have been diffraction-limited by the light wavelength which leads to some difficulties when components are scaled down to nanoscale.

Recently, graphene [50] has been proposed as a good candidate to bridge the gap between electrons and photons for a new generation of optoelectronic devices [14]. While many promising properties like its ultimate thickness, transparency, ultra-high electronic mobility or mechanical strength have concentrated the efforts of numerous groups, a few pioneering groups have recently started considering and studying graphene for its optoelectronic [20] and plasmonic properties [57], opening new routes to ultimate optoelectronic nanodevices based on electron-photon interactions.

From all the pioneering theoretical [159, 58, 66, 59] and experimental [117, 114] studies, it now appears feasible to use light as an information carrier by manipulating it through Graphene Plasmons Polaritons (GPPs).

The promises of graphene for plasmonics lie in the high confinement and weak damping [49] of sustained plasmons as well as in the possibility to electrostatically tune them simply by applying a voltage. The main asset would be to enable miniaturization of photonic component without facing usual thermal limitations thanks to low graphene electronic losses. The applications could therefore be numerous.

This chapter aims to expound the literature review of graphene and graphene plasmonics, as well as the properties of graphene plasmons. At first, the discovery of graphene will be traced as well as its limits in term of transport properties. Then, the optical properties of graphene will be introduced and an overview of graphene plasmonics state of the art will be presented. Subsequently, the dispersion relation of graphene plasmons will be demonstrated. Afterwards, the graphene plasmonics properties will be reviewed in details and finally, a conclusion through the objectives of this study will be made.

3.1 Literature review on graphene and its transport properties

3.1.1 The discovery of graphene and the synthesising process

In october 2004, two papers were published [17, 117] and showed some properties of few layers of carbon on different substrates. Despite the common thought [105], such extremely thin films are thermodynamically stable. Two different ways of fabrication were used : on silicon carbide by thermal desorption of Si [17] (Figure 3.1a, STM image), and by mechanical exfoliation [117] (Figure 3.1b, AFM image).

(a) STM image of few layers graphene grown by carbon segregation on SiC [17]

(b) AFM image of a single layer graphene mechanically exfoliated [117]

Figure 3.1: The two first papers published in October 2004 reporting graphene monolayer with different fabrication process

The behavior of 2D electron gas properties in such thin film have been demonstrated, opening wild area of research in domain such as quantum mechanics. It has also been shown that graphene is a semi-metal and can sustain huge current. The transport of electrons in this medium shows small electrical resistivity (ballistic transport), limited by scattering effect, and a significant mobility. Finally, the most important feature discovered is the possibility to inject carriers in graphene (doping) which changes its conductivity/resistivity and exhibits an ambipolar electric field effect (conductivity carried by electron or hole depending on the applied voltage). This discovery led to a Physics Nobel price and many promising applications. In the next paragraphs, the different ways of producing graphene will be reviewed.

Mechanical exfoliation. This method [117] consists in a repeated peeling of highly oriented pyrolytic graphite with the use of clean room tape (Figure 3.2). The advantage of this method is a high quality of single layer graphene deposited on various substrate. The mono-cristaline flake can be up to millimetre scale and shows excellent electronic properties. Although this method have been widely used for fundamental studies, it is impractical for large scale application.

Carbon segregation. It is possible to get graphene from silicon carbide [17] or other metal substrates, through carbon segregation. At high temperature, on SiC, silicon desorption occurs leaving on the substrate few to one layer of graphene. It is possible to obtain free standing graphene of high quality via, among others, hydrogen treatment [134].

Chemical Vapour Deposition. Another way for graphene synthesis emerged in 2008/2009 [166, 93]. This process has drawn a lot of interest in the industry because of the possibility to grow it on a large scale basis at low cost. Hydrocarbon species are introduced in a high temperature chamber, where is lying a metal substrate. Two type of metals are widely used : Nickel (Figure 3.3a) and Copper (Figure 3.3b). The choice of metal is important because it determines the growth mechanism and eventually the quality of the graphene.

It has been demonstrated [169] that the graphene growth on Ni (Figure 3.3a) is a segregation process, where carbon is first dissolved in the metal substrate at high temperature. Then, the substrate is cooled down to a temperature for which the carbon is less soluble in Nickel, forming at the surface multi layer and polycrystalline graphene.

Figure 3.2: Illustration of Micro exfoliation (a) A pressure is applied to stick the tape to the surface of the material. (b) Few layers of the material are attached to the tape. (c) The tape is then pressed against the substrate. (d) One or few layers remain on the substrate once the tape is withdrawn [118]

On Cu substrate (Figure 3.3b), the metal acts as a catalyst and graphene is formed at its surface from the decomposition of the hydrocarbon precursor. This comes from the fact that carbon solubility in copper is low. Two consequences arise: only one layer can be formed, because the metal underlying is the catalyst, and once it is covered by a first layer, no other reaction can take place. The second consequence is that it is possible to get large monocrystalline flakes of graphene. The quality of the copper substrate is crucial in order to get low defects in CVD graphene [171].

Figure 3.3: Growth of graphene by Chemical Vapour Deposition on different substrate [169]

Chemical methods and chemically modified graphene. It is possible to fabricate graphene through colloidal suspension of graphite [31]. In the presence of strong acids and oxidants, oxidation of graphite occurs with the intercalation of epoxide and hydroxyl groups. The hydrophilic properties of these chemical end points facilitate the introduction of water and the dispersion of graphene layers in solution. This method is inexpensive and produce large flakes but the remaining graphene have a poor conductivity with significant amount of defects. Research groups [126] have been trying to reduce the graphene oxide (Figure 3.4) but obtaining pristine graphene properties remains a challenge.

Liquid phase exfoliation of graphene. By using different types of solvent (aqueous or not), it is possible to disperse graphene flakes from powdered graphite via ultrasonication. In that way, unoxidised,

Figure 3.4: Illustration of a flake of graphene oxide with epoxide (-O-) and hydroxyl (-OH) groups [126]

high quality, scalable graphene has been obtained [61], which can be for example deposited by spray coating. This inexpensive method, as compared to CVD graphene, and the high quality of graphene obtained, as compared to the chemically modified graphene, attract wide interest in the scientific community.

Transfer of graphene on a targeted area. The potential possibilities of devices offered by graphene through its opto-electronic properties require the ability to deposit it on a targeted area where electrodes are located. However, the manipulation of an atomic layer is not trivial. The main challenge is to keep the sheet undamaged and its opto-electronic properties intact. Several ways have been studied [20], and the most popular one is based on the use of PMMA layer [133] (see Figure 3.5).

Figure 3.5: Illustration of the transfer of a CVD graphene layer by the use of PMMA. (a) and (b), the graphene is grown on metal substrate. (c) A thin film of PMMA is deposited on the metal+graphene sample. (d) The metal is then etched, and graphene+PMMA is remaining. (e) Graphene+PMMA is transferred on the chosen substrate and (f) the PMMA layer is removed. [118]

One of the most challenging step is then to clean the graphene. Acetone is widely used to remove the PMMA layer, but it also contaminates the graphene layer [68]. In general, any chemical component in contact with graphene can potentially affect its opto-electronic, and thus transport properties. Therefore, many groups have been working on the measurement, the identification and the quantification of the limit of graphene transport properties. This issue will be discussed in the next section.

3.1.2 Limits on the transport properties of graphene

The transport properties of graphene are limited by different scattering mechanisms. The magnitude of the carrier scattering can be expressed in term of mobility μ , which links the average drift velocity of a charge carrier to an applied electric field. Graphene on SiO_2 substrate exhibits a mobility of more than $1\text{m}^2\text{V}^{-1}\text{s}^{-1}$, one order of magnitude higher than the silicon that is the most common electronic material [57]. However, the scattering rate remains an obstacle and limits potential application in opto-electronic, as well as in fundamental physics [19].

Scattering can be divided into two categories. The first one concerns intrinsic scattering mechanisms, such as graphene defects, grain boundaries, intrinsic ripples or graphene phonons, emerging from the

graphene itself. The second one involves the extrinsic mechanisms, arise from the surrounding materials, like charged impurities in the substrate or trapped under/above the graphene, corrugations and strains induced by the substrate, and remote interfacial phonon. All these scattering sources influence the mobility of the charged carriers and we are going to enumerate and quantify their weight through a literature review.

Figure 3.6: Charged impurities as an explanation of the limitation of experimental transport properties of graphene

Charged impurities. Graphene being confined ultimately in one dimension, it is natural to speak about two-dimensional electron systems. As other two-dimensional systems, the electrons in graphene do not move as independent particles, but rather by pair, where the distance between two electrons have a central role [57]. Electrons are strongly interacting with each other, and we speak about an interaction potential $u(r) = e^2/\epsilon_s r$. Their motion are therefore correlated and the potential $u(r)$ created by their separation distance $\mathbf{r}$ is really sensitive to the surrounding media, $\epsilon_s = (\epsilon_1 + \epsilon_2)/2$ [57] (see section 2.1.3). Therefore, the effects of the substrate and any chemical component in the vicinity of the carbon sheet should be taken into account carefully.

Scattering from charged impurities by the substrate [7, 116, 173] was pointed out as the limiting factor of the transport properties. Located near the interface between graphene and substrate (Figure 3.6a), or in the substrate (Figure 3.8), charged impurities generate charged puddles in the graphene layer which influence the transport properties, being a dominant source of scattering [65]. At high Fermi level, the density of impurities impact linearly graphene carrier transport, whereas it could explain why there is a finite conductivity near the Dirac point [4]. Although the local density of charges from the substrate have been imaged (see Figure 3.6b) using a scanning single-electron transistor [104], the study emphasis that it is not sufficient to explain the emerging electron-holes puddles on graphene. It would rather come from the pollution by resist and other chemical post treatments exposing graphene. The resist residue have been imaged [68] by scanning tunnelling microscopy (STM) which showed a layer of photo-resist remaining despite a careful treatment of the sample. On the other hand, molecules from air, such as H_2O or N_2 trapped in between the substrate and graphene during the transfer of the carbon layer have been suggested [170] as potential charged impurities creating charged puddles.

By adding potassium atoms as impurity centers, a study [27] showed that the conductivity gets lower as the density of impurities gets higher. It could be possible, according to [28], to get a mobility as high as $4m^2V^{-1}s^{-1}$, provided one gets rid of charged impurities. However, an alternative explanation have been pointed out through the study of suspended graphene [105]: corrugation of graphene layer (see section below).

Figure 3.7: Artistic illustration of what a real graphene should look like, with the presence of ripples [105]

Corrugation. Using Transmission Electronic Microscope (TEM) technique, it was shown that graphene is not flat [105], in agreement with the theory predicting that perfect 2D material are unstable under ambient condition. Suspended graphene exhibits microscopic corrugation in the three dimensions as illustrated on Figure 3.7. On SiO_2 substrate, perturbations and fractures of the hexagonal lattice symmetry have been imaged [68] by scanning tunnelling microscopy (STM). The authors demonstrated a correlation between the graphene lattice perturbation and the underlying substrate. However, it was suggested that in their study, there was no independent intrinsic graphene corrugations, but rather extrinsic corrugations caused by the lattice mismatch with the substrate. The presence of corrugations partially matching the underlying SiO_2 substrate, inducing stiffness in the graphene sheet, are more likely to create strain-induced disorder, and dramatically affect the electronic transport properties in graphene. Both extrinsic and intrinsic ripples have been observed on SiO_2 substrate [52], meaning that the graphene would be partially suspended in between the sample surface relief (see Figure 3.8).

Figure 3.8: Schema drawn from STM and AFM data, showing how graphene wraps a substrate. Graphene is represented in orange, charged impurities in light grey dots [9].

Following these investigations, studies demonstrated that intrinsic [78] as well as extrinsic [80] corrugations could be an alternative explanation to the scattering rate observed experimentally in graphene. In fact, such scattering mechanisms produce similar effects on the conductivity, as compared to the charges impurities, and could be confused with charge impurities effect. However, it was shown, by STM-based dI/dV mapping, that there is no correlation between corrugations and scattering centers.[170]

Lattice disorder. The structure of graphene, made of sp^2 covalent bond in between carbon (see Figure 2.11), make lattice defects relatively rare, according to [19]. However, several types of defect can still be found, such as grain boundaries (Figure 3.9b, 3.9c), vacancies or atoms substitution(point defect, Figure 3.9a), mechanical distortion of the lattice, etc... Studies have allowed one to analyze the influence of

Figure 3.9: (a) STM image of point defects, (b) TEM image of a grain boundary and (c) TEM image of a polycrystalline CVD graphene.

Figure 3.10: Intrinsic phonon of a monolayer graphene and extrinsic phonon-plasmons coupling at the interface between graphene and SiO₂ substrate

such defects on the opto-electronic properties of graphene [127]. They have been imaged through various techniques. As an example, bilayer graphene grown epitaxially on SiC have been investigated by STM technique, where it is possible to see different perturbation and point defect of the lattice structure [135]. It puts into evidence the critical influence of the substrate and the substrate quality on the transport properties of graphene.

More specifically, CVD graphene, which has been used in this study, presents many defect types, depending on the type and the quality of the substrate [171], as well as on the process of growth and transfer. One of the most common defects is grain boundaries (Figure 3.9b), that have been imaged through STM [33], TEM [81, 64], but also Raman spectroscopy [75] and later with SNOM [43] (Figure 3.18b). In general, grain boundaries tend to reduce the electrical conductivity of polycrystalline graphene [75], even if larger grain does not coincide with higher conductivity [64]. Comparing the influence on the conductivity of three types of defects (point defect, surface contamination and line defects), grain boundary seems to be the main factor [142] limiting the electronic transport in CVD graphene.

Electron-Phonon interactions. Several types of phonon can interact with electrons in graphene, which are as many possible energetic channel leading to the scattering of charge carrier.

The intrinsic phonons, three optical and three acoustic phonons, are present in this 2D material (see Figure 3.10a). The optical phonons are energetically too high for IR-SNOM to be taken into account, and are usually studied by spectroscopy. The intrinsic acoustic phonons have been studied for their unusual heat transport properties [115] as well as for their impact on the conductivity. They are nowadays pointed to be one of the main graphene plasmons scattering mechanism, limiting their propagation in h-BN/graphene heterostructure [153, 114].

A first study analysing the transport in high quality graphene [110] predicted an extremely low electrons-phonons scattering, even at high temperature. The limitation of the mobility would stand as high as $20m^2V^{-1}s^{-1}$ if extrinsic scattering are eliminated. A second one discussed theoretically in plane acoustic phonons [67]. The authors show that those phonons would limit the mobility of graphene at more than $10m^2V^{-1}s^{-1}$. Therefore, according to these studies, in plane phonon would not be a significant scattering mechanism. However, flexural phonon could be significant in suspended membrane [102]. These out-of-plane quasi-particles would be present in any samples due to intrinsic ripples [78], and more particularly in suspended graphene [122].

This scattering process involves two flexural phonons [22], excited simultaneously, and limits the mobility in suspended graphene around $1m^2V^{-1}s^{-1}$. However, strain induced mechanically or by back gate potential could reduce the influence of this scattering in suspended graphene, to be analogous to the in plane one [103]. Further experimental data agreed [88] with the theory for a value of mobility in suspended graphene at room temperature around $1.5m^2V^{-1}s^{-1}$ [37]. However, recently, it has been recorded a mobility as high as $24m^2V^{-1}s^{-1}$ in CVD suspended graphene at room temperature. In bilayer graphene, the electron-phonon coupling mechanism is different. As a consequence, the mobility could be found as high as $10 - 20m^2V^{-1}s^{-1}$ [121].

The Remote Interfacial Phonon (RIP) is a phonon of the substrate excited by the charge carriers in graphene. In the presence of a polar substrate, such as SiO_2 or SiC, the carriers in graphene couple with the polarisation field at the interface graphene-substrate. This scattering mechanism is known to limit electronic transport in 2D-like Si structure [48]. Plasmons-phonon coupling have been observed experimentally on SiC substrate [95] as well as on SiO_2 substrate ([100], Figure 3.10b). Others high-k dielectric substrates have been investigated, studying mobility in presence of charged impurities [83]. The advantage of dielectric screening effect of impurities, thanks to high-k substrate, is lost by RIP arising. So it has been predicted not much improvement of the mobility at room temperature. A numerical study [123] of graphene on SiO_2 and HfO_2 (high-k substrate) at room temperature predicted a limitation for the mobility between 0.55 to 2.4 $m^2V^{-1}s^{-1}$ due to RIP (with E_F from approximately 0.1eV to 0.3eV).

Dc scattering approximation. As it has been seen in this section, the sources of scattering are multiple. Usually, the mobility is evaluated for a Direct Current, which is a good starting point. Following this rule, the mobility can be expressed as $\mu = \frac{\tau_{DC}ev_F^2}{E_F}$. Here, τ_{DC} is the relaxation time of the electrons in the DC approximation. However, it has been observed values relatively different when coming to optical frequency. At first, electrons-electrons scattering has been pointed [70] to be responsible for unexpected increased of scattering rate [42]. In the case of the study of plasmons, authors in ref. [57] emphasized that "The plasmon lifetime, τ_{pl} , however, should not be confused with the d.c. transport scattering time, τ_{DC} ".

In another study [129] focusing on the plasmons losses in h-BN/graphene heterostructures, the authors compare the contributions of graphene's acoustic phonons and h-BN optical phonons to GPs damping. In the introduction, the authors reminded the Boltzman-transport theory, which demonstrates that the mobility μ is inversely proportional to the charge carrier density n . The Fermi level E_F being proportional to the square roots of the density of charge carrier, $\tau_{DC} \propto \frac{1}{\sqrt{n}}$. However, it was claimed that the plasmons lifetime should be increasing for increasing carrier density. τ_{pl} would therefore be anticorrelated with the mobility. However, recent imaging of plasmons in encapsulated h-BN, heterostructure known to enhance greatly the mobility, gives the best results [153]. In any case, increasing the Fermi level should increase the plasmons lifetime. Working with high-k dielectric material, in order to gate electrostatically the graphene and obtain high Fermi level, is one direction where research teams have been looking at recently.

Relationship between Fermi level and mobility. As a conclusion of what was seen above, the mobility, which aims to measure scattering mechanism, is limited by numerous factors. To summarize mobility, we can divide the different contribution in two categories [158], the intrinsic mechanisms or short range scattering (μ_{short} in the Figure 3.11 : lattice disorder e.g. vacancies, adatoms, absorbed molecules, grain boundaries) and the extrinsic mechanisms or coulomb scattering ($\mu_{coulumb}$ in the Figure 3.11 : charged impurities from substrate, interfacial phonons). According to [158], the former depends on carrier concentration in graphene. The dependence of the carrier concentration/Fermi level versus the mobility ($\mu_{coulumb}$, μ_{short} and μ_{total}) is plotted Figure 3.11. The chemical doping, with H_3PO_4 , increase the carrier concentration (Fermi level), but increase also the short range scattering.

Maximizing the propagation length L_p of plasmons in graphene is a complexe task. In one hand, a high Fermi level seems to be a rather good target for getting long propagation, which is in agreement with recent experiment (will be seen section 3.3). In the other hand, plasmons being a collective oscillation of free charge carriers, a high mobility seems to be necessary. Moreover, h-BN, which is the support for the longest propagation, maximized the mobility [153]. However, high Fermi energy does not always match with high mobility, and uncertainties remain [129].

Beside the electronic transport, the optical behavior of graphene is of the first importance in order to understand graphene plasmons properties. Therefore, in the following section, the optical properties of graphene will be reviewed.

3.2 Literature review on optics and plasmonics graphene properties

As it has been seen in Section 2.3 , graphene exhibits exotic properties. Among others, the massless dirac particule behavior of electrons leads to a really high mobility of the charged carriers (see Section 3.1.2). Moreover, the possibility to tune the Fermi level of graphene, and thus its optical properties, leads to a wide interest in the scientific community for graphene plasmonics. In fact, charge carriers can be injected up to 0.01 per atom of carbone for back gate system ($1 \times 10^{13}cm^{-2}$), and up to 0.1 per atoms for solid gel electrolyte [97]. Although much below the doping concentration that can be achieve in noble metal (up to 1 per atom), the injection of charges in graphene results in a significant change of optical properties of graphene.

The tunability of optical properties of graphene compared to a bulk of metal one can be understood

Figure 3.11: Mobility as a function of carrier concentration for graphene deposited on SiO_2 when graphene is doped chemically with H_3PO_4 [158]

as follow. When a bulk of metal stands in a strong electric field, the electrons accumulate along the surface and shift the state of all the electrons inside the bulk. However, the total Fermi level is not changed and therefore, the property of the metal is not changed. At the contrary, in a single layer of carbon atoms, there is an effective change of the electrons/holes density in the material, which changes the conductivity and then the optical response [69]. Therefore, it is possible to modulate the dispersion relation of plasmons (see Section 3.3), which attracts considerable interest in the scientific world.

In the following section, the theory of the optical properties of graphene will be introduced, and then, the predicted opto-electronic devices emerging subsequently through simulation will be expounded. In a second part, an overview of what have been experimentally done in the graphene plasmonics field will be presented.

3.2.1 Theory of the optical properties and simulation of graphene-based devices

Graphene is a promising material for opto-electronics, but remain challenging to manipulate and control, because of its one atom thickness. However, the possible applications coming out have attracted wide interest and many groups worked on the theory and simulated devices based on graphene to design future opto-electronic platform. In this sub-section, a semi-classical way to treat the graphene optical properties will be given, based on the Kubo formula [59]. Subsequently, devices based on graphene, predicted through different simulation methods, will be enumerated.

The optical properties of graphene

The needs of understanding graphene peculiar electro-optics behaviour through the theory have been motivated early by the discovery and the characterisations of a monolayer graphene sheet [17, 117]. The tight binding approach (see Section 2.3.2) applied to this 2D honeycomb lattice enables to understand

most of the unusual electronic properties [54]. However, the interaction of graphene with light is not trivial and many research groups have been working on different approaches to model it [58, 40, 59, 106]. Here, a first look will be taken at the semi-classical formalism, starting from the Kubo formula [59], which represents graphene as a conductive surface. Ryogo Kubo introduced an electronic transport formalism in order to describe the conductivity, based on linear response theory. It gives the conductivity depending on the electric field applied. To express the intrinsic conductivity of the graphene, the magnetic effect will be ignored. It is then possible to obtain the conductivity as a function of the radian frequency ω , the chemical potential μ , the scattering rate Γ and the temperature T [41]:

$$\sigma(\omega, \mu, \Gamma, T) = -\frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \left[\frac{1}{(\omega + i2\Gamma)^2} \int_0^\infty E \left(\frac{\partial f(E)}{\partial E} - \frac{\partial f(-E)}{\partial E} \right) dE \right. \\ \left. - \int_0^\infty \frac{f(-E) - f(E)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} dE \right] \quad (3.1)$$

where

$$f(E) = \frac{1}{1 + e^{\frac{E-\mu}{k_B T}}} \quad (3.2)$$

is the Fermi distribution. Equation (3.1) can be expressed as a sum of the intraband term (first integral term) and the interband term (second integral term), i.e $\sigma = \sigma_{intra} + \sigma_{inter}$.

$$\sigma_{intra}(\omega, \Gamma, T, \mu_c) = \frac{-ie^2}{\pi\hbar^2(\omega + i2\Gamma)} \int_0^\infty E \left(\frac{\partial f(E)}{\partial E} - \frac{\partial f(-E)}{\partial E} \right) dE \quad (3.3)$$

$$\sigma_{inter}(\omega, \Gamma, T, \mu_c) = \frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \int_0^\infty \frac{f(-E) - f(E)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} dE \quad (3.4)$$

It is interesting to notice that σ_{intra} (Equation (3.3)) is responsible for the intraband transitions that can be seen as the electron-phonon scattering process [98], whereas the second term (Equation (3.4)) is about the interband electron transitions. The scattering rate Γ can also be expressed as a function of relaxation time τ , by $\Gamma = 1/2\tau$.

It is possible to solve analytically the intraband term which gives :

$$\sigma_{Intra}(w, E_F, \tau, T) = \frac{ie^2 E_F}{\pi\hbar^2(w + i/\tau)} \left[1 + 2 \frac{k_B T}{E_F} \ln(e^{-E_F/k_B T} + 1) \right] \quad (3.5)$$

, where the chemical potential μ has been replaced by the Fermi energy. It is possible to do so when the chemical potential is much higher than $k_B T$ [98]. It is also interesting to note that the intra-band term turns to a Drude like form when $E_F \gg k_B T$:

$$\sigma_{intra}(w, E_F, \tau, T) = \frac{ie^2 E_F}{\pi\hbar^2(w + i/\tau)} \quad (3.6)$$

However, the interband contribution is more difficult to evaluate analytically. At finite, but low temperature ($E_F, \omega \gg k_B T$), several interband contribution part of the conductivity have been proposed. According to [40]:

$$\sigma_{inter}(w, E_F, \tau, T) = \sigma_0 \left[\frac{1}{2} + \frac{1}{\pi} \arctan \left(\frac{\hbar w - 2E_F}{2k_B T} \right) - \frac{i}{2\pi} \ln \left(\frac{(\hbar w + 2E_F)^2}{(\hbar w - 2E_F)^2 + 4(k_B T)^2} \right) \right] \quad (3.7)$$

with $\sigma_0 = e^2/4\hbar$. According to [84], the electron disorder scattering processes $i\hbar/\tau$ can be added to the second part of the equation to get at any temperature :

$$\sigma_{inter}(w, E_F, \tau, T) = \sigma_0 \left[\frac{1}{2} + \frac{1}{\pi} \arctan \left(\frac{\hbar w - 2E_F}{2k_B T} \right) - \frac{i}{2\pi} \ln \left(\frac{(\hbar w + 2E_F + i\hbar/\tau)^2}{(\hbar w - 2E_F + i\hbar/\tau)^2 + 2(k_B T)^2} \right) \right] \quad (3.8)$$

According to [24], the inter band term of the optical conductivity in a single layer of graphene can be written as :

$$\sigma_{inter}(w, E_F, \tau, T) = \frac{\sigma_0}{2} \left[\tanh \left(\frac{\hbar w + 2E_F}{4k_B T} \right) + \tanh \left(\frac{\hbar w - 2E_F}{4k_B T} \right) - \frac{i}{\pi} \ln \left(\frac{(\hbar w + 2E_F)^2}{(\hbar w - 2E_F)^2 + 4(k_B T)^2} \right) \right] \quad (3.9)$$

(a) σ/σ_0 of graphene details term by term

(b) σ/σ_0 of graphene for two different temperature

Figure 3.12: Optical conductivity of graphene using the Equations (3.9) and the full intraband term, Equation (3.5) as a function of the pulsation ω . The mobility of graphene (unit $e^2/4\hbar$) is set at $1m^2V^{-1}s^{-1}$.

Plots of the optical conductivity σ/σ_0 is shown Figures 3.12, where σ_{inter} has been taken from Equation (3.9) and σ_{intra} is computed through the Equation (3.5). The conductivity is plotted as a function of the energy of the electromagnetic wave, normalized by the Fermi energy. In Figure 3.12a, the different terms of the conductivity are plotted separately. It is interesting to observe that the intraband terms of the conductivity (in red) approach the total term (in black) when $\hbar\omega \ll E_F$. Therefore, the intra band term is dominating in the infrared region, while in the near-infrared and the visible regions, the interband term dominates. In figure 3.12b, as expected from Equations (3.7), (3.8), (3.9), the real part of the interband contribution displays a step at $\hbar\omega = 2E_F$, which represents the losses due to the interband excitation of carriers. Two electronic band structures of graphene are shown in inset. The Fermi level is illustrated by the interface between the orange and the white. The black arrow represents the interband transition, not possible for $\hbar\omega < 2E_F$, and possible above the threshold of $2E_F$.

Finally, it is worth speaking about the 3D approach based on a convenient 3D permittivity for graphene. As seen in the Section 2.1.3, Equation (2.51) gives a conversion of the 2D optical conductivity into a 3D electrical permittivity, to represent graphene as a 3D material with a finite thickness. Let's write the optical conductivity of the graphene as a sum of a Real part and an Imaginary part, $\sigma = \sigma_R + i\sigma_I$, which leads to :

$$\epsilon(\omega) = \epsilon_s(w) - \frac{\sigma_I(\omega)}{\omega 2a\epsilon_0} + i \frac{\sigma_R(\omega)}{\omega 2a\epsilon_0} \quad (3.10)$$

It is interesting to notice that the positive imaginary part of the optical conductivity at low frequency (see Figure 3.12b) will give a negative real part of the graphene permittivity when $\epsilon_s(w) - \frac{\sigma_I(\omega)}{\omega 2a\epsilon_0} < 0$. In this case, graphene layer has a metallic behavior and plasmons can be supported. The positive real part of the conductivity corresponds to the imaginary part of the permittivity, which is related to losses

experienced by an electromagnetic field. This last term will be determinant for the dissipation losses of surface plasmons in graphene.

Figure 3.13: Permittivity of graphene ($E_f = 0.6\text{eV}$, $\mu = 1m^2V^{-1}s^{-1}$) compare to Gold in the infrared range, using Equation (2.51) and the Drude-like conductivity Equation (3.6), valid under these conditions (see Section 4.2.1)

Simulations of devices based on graphene plasmonics

Early, the power of graphene local tunability has shown the possibility to create exotic devices. As an example, at a p-n junction of graphene layers, it is theoretically possible to get a Veselago lens. This type of lens act like a slab of material with negative refractive index, giving counter intuitive properties. [25]. Therefore, graphene based device could lead to electronic lenses, but also graphene-based transistors. Soon after, the p-n junction applied for graphene plasmonics have been studied [108] and a plasmon transistor has been envisaged.

Figure 3.14: Two snapshot in time showing the y component of the electric field at low temperature ($T=3\text{K}$) on free standing graphene. From [148]

An interesting study has shown [148] that the local modulation of optical properties in a single layer of suspended graphene could lead to complex photonic function and eventually transformative optics device (Figures 3.14). It is due to the low-loss plasmonic properties and to its intrinsic two dimensional structure. Playing with local electrostatic potential applied to graphene, they demonstrated total reflection

of plasmons (Figure 3.14a) and plasmonics waveguide (Figure 3.14b). From this starting point emerged a lots of new device proposals based on the control of the Fermi level in graphene (electrostatic gating). Plasmonic switches and transistors [3, 32, 55, 26], light modulators [94, 163, 35, 167], photodetectors [172], ring filters [91, 92], plasmonic lens [149], logic gates [125, 164] and waveguides [124, 86] have been predicted numerically.

In most of the cases cited above, the studies suppose to have an excellent control over the amount of charge carrier located in different part of the layer, by applying electric field. Unfortunately, it remains a big challenge since the electric field needed locally is really important. Therefore, many research groups are working on finding an experimental system that could lead to variation of graphene optical and electronic properties on micrometric scale on demand. In fact, the substrate plays a key role (see section 3.1.2) in order to get optimal eletro-optic properties [57].

3.2.2 Experimental observation of plasmons in graphene

Several techniques enable researchers to access to a direct proof of the existence of plasmons in graphene. The first experimental evidence showing the phenomenon has been performed by Electron Energy Loss Spectroscopy (EELS) [111, 39, 96], where electrons are used to probe the graphene properties. In this method, an incident beam of electrons strikes the carbon sheet. Some of the electrons will transfer a part of their energy, through inelastic scattering, to the graphene plasmons. Their energy will be eventually recorded after the reflection or the transmission through the sample, and an electrons energy loss spectrum is obtained (see Figure 3.15). In graphene, it is known that there are two kinds of valences electrons : the π electrons responsible for the covalent bounds between carbone atoms, and the σ electrons, quasi free, which are creating bounds over p_z orbitals (see Figure 2.11b). Therefore, several types of plasmons may exist [36]:

- The low energy plasmons, usually refereed as 2D plasmons, with energy lower than 3 eV, which is the field of interest of this thesis.
- The high energy plasmons, corresponding to the π (around 4.7 eV) and $\pi + \sigma$ (around 14.6 eV) plasmons.

In Figure 3.15, the signature of two different plasmons can be observed : one of high energy ((b) on the Figure), corresponding to a $\pi + \sigma$ plasmon, and one with lower energy ((a) on the Figure), corresponding to the 2D plasmons [111]. However, 2D plasmons are difficult to probe because they appear with relatively high doping (Fermi level), beeing highly dependent to it. They are also dependent on other parameters like the substrate and the temperatures [98].

It is also possible to record plasmonic effects in graphene by confining the carbon sheet in a second dimension, which leads to structures so-called ribbons. As for noble materials, a resonance peak can be observed with an infrared light polarized perpendicularly to the ribbon axis, function of its width. The resonance in graphene ribbons is characteristic of 2D electrons gas (Figure 3.16b). It is worth mentioning that this property is also present at room temperature, compared to other 2D materials [76]. But the most interesting feature in graphene as a plasmonic material is, one more time, the possibility to tune its optical properties, and therefore to tune the absorption peak of the device through electrostatic gating (see Figure

Figure 3.15: Electrons energy loss spectra obtained in 1992 for graphene grown on TiC surface [111]

(a) Tuning of the transmission of an array of graphene ribbons on SiO_2 through electrostatic gating [76]. $\Delta T = T - T_{CNP}$ with CNP Charge Neutral Point. Inset the transmission for a parallel polarisation. Ribbons width of 4 μm.

(b) Tuning of the transmission of an array of graphene ribbons on SiO_2 varying the width of the ribbons [161].

Figure 3.16: Intrinsic graphene plasmons confined by designing ribbons

3.16a).

Confining the graphene in 3 dimensions gives also interesting results. Micro-disk of graphene+insulators, deposited on quartz for its transparency in the infrared region, have been engineered in order to create plasmonics resonators. Stacks of several layers have been fabricated enhancing the plasmonics resonance frequency and magnitude, and far-infrared notch filters as well as polarisers are demonstrated [160].

Figure 3.17: Illustration of the experimental configuration describing the SNOM tips launching and detecting simultaneously the Graphene plasmon in (a), and the obtained near field amplitude on graphene nanoribbon on top of 6H-SiC. [29]

In 2012, two papers [42, 29] have been published simultaneously describing the first real-space imaging of surface plasmons propagating on a substrate Si/SiO_2 for the first one (Fig 3.18a), and on graphene nanoribbon on the top of 6H-SiC substrate for the second (Fig 3.17). It has been a challenge to record the signature of such a wave because of its nonradiative nature. In order to visualize those evanescent waves, a Scanning Near-field Optical Microscope has been used (SNOM, Fig. 3.17) in the infrared region.

An infrared beam is focused on an Atomic Force Microscope (AFM) metal tip (apertureless). The tip scatters the light and a fraction of the resulting angular spectrum will excite a plasmon in graphene with the correct wave-vector. The plasmon will propagate in the 2D sheet of carbone and be reflected by a grain boundary, or graphene edge (Figure 3.18b). This reflected plasmon will interfere with the incoming one, resulting in an interference pattern, creating a standing wave, (Figure 3.18a). It is pos-

sible to extract from the standing wave the effective wavelength of the graphene plasmons, which agree with the theory to be much smaller than the excitation one. These two papers are the starting point of nano-imaging of plasmons in graphene by SNOM technique.

Following the previous studies, in 2013, a study [43] demonstrates that the SNOM technique is able to highlight the grain boundary (Figure 3.18b) in CVD graphene, and could be applied in order to characterize graphene crystallinity. The key point here is that the grain boundaries, that were challenging to image, are scattering centers that obstruct the electronic transport as well as the plasmon propagation. In the meanwhile [30], nano-defects in graphene on silicon carbide have been revealed by the same technique and surface roughness of the substrate such as SiC terraces have been studied. It is shown that plasmons in graphene are highly disturbed by substrate and substrate discontinuities.

Figure 3.18: Interferometric image of graphene layer using SNOM technique at graphene edges and grain boundary

The next step was the development of h-BN/graphene structure, predicted to be one of the best platform for optimizing transport properties of graphene [19, 174]. Low plasmons damping as well as high confinement have been shown [153]. Intriguing behaviour of this heterostructure have been studied [34] showing hybrid plasmons-phonons polariton phenomenon with low losses. The properties of infrared plasmons have been shown by pump-probe technique [114]. It must be underlined that the high quality of such structure is able to sustain high propagation of plasmons, which can be switched by electrostatic gating.

3.3 Properties of Graphene Plasmons

It is interesting to notice two things when speaking about plasmons in graphene. They show a lot of similarities with the surface plasmons at an interface between a dielectric and a metal (see section 2.2.1). However, two properties of the system imply different characteristics of plasmon in graphene. The first one is that electrons are frozen perpendicular to the surface, and the second is about the dispersion relation of electrons in graphene (see Section 2.3.2), as compared to the parabolic one in metals [70]. Those two major differences lead to different dispersion relations in between those two systems, that will be studied below.

This study aims to focus on longitudinal modes, where the electric field associated with the plasmons are in the plane, parallel to the wave vector. Those modes are known as Transverse Magnetic modes. They appear at a frequency below the Pauli-blocking energy. A transverse electric mode of plasmon exists also in graphene, where a collective oscillation of electrons transverse to the graphene plane can be found [15]. However, the dispersion curve of those modes seems to be too close to the one of the light.

As a results, on the energy confinement point of view, it seems less interesting than the TM modes are[70].

In 2007, it has been proposed several ways to study the transverse electric and magnetic plasmonic modes on graphene. [66, 106]. In the following sections two semi-classical approaches will be proposed in order to get the dispersion relation of plasmon in graphene. One is based on a volume approach, and the other one consists of a surface approach. Then the different parameters that matter in the propagation of graphene plasmons will be analysed.

3.3.1 Dispersion relation by the volume approach

In this section, graphene will be considered as a material with a finite thickness. Therefore, graphene will be described as an extremely thin metallic slab with a complex permittivity ϵ_m . The results that have been found in section 2.2.2 will be extensively used, where the system can be described as a multilayer and graphene is embedded in a dielectric media with dielectric permittivity ϵ_d . Graphene being a material which is one-carbone atom thick, the thickness $\Delta = 2a$ can be approximated as tending towards 0. Introducing Equation (2.87) and using a finite expansion of $\tanh$ when z tends to 0, $\tanh(z) = z + o(z^2)$, it is possible to write :

$$\tanh(a\sqrt{k_p^2 - \epsilon_m k_0^2}) = -\frac{\epsilon_d \sqrt{k_p^2 - \epsilon_m k_0^2}}{\epsilon_m \sqrt{k_p^2 - \epsilon_d k_0^2}} \approx a\sqrt{k_p^2 - \epsilon_m k_0^2} \quad (3.11)$$

$$-\frac{\epsilon_d}{a\epsilon_m} \approx \sqrt{k_p^2 - \epsilon_d k_0^2} \quad (3.12)$$

$$k_p^2 \approx \epsilon_d k_0^2 + \frac{\epsilon_d^2}{a^2 \epsilon_m^2} \quad (3.13)$$

In order to further simplify this equation, let's suppose that the graphene sheet is suspended, and therefore $\epsilon_d = 1$. Let's insert Equation (2.52), in the last equation to find :

$$k_p^2 \approx k_0^2 + \frac{1}{a^2} \frac{(\epsilon_0 w 2a)^2}{(\epsilon_0 w 2a)^2 - \sigma_{2D}^2 + 2i w 2a \epsilon_0 \sigma_{2D}} \quad (3.14)$$

In the vanishing thickness limit, we finally get :

$$k_p^2 \approx k_0^2 - \left(\frac{2\epsilon_0 w}{\sigma_{2D}} \right)^2 \quad (3.15)$$

$$k_p \approx k_0 \sqrt{1 - \left(\frac{2\epsilon_0 w}{k_0 \sigma_{2D}} \right)^2} \quad (3.16)$$

Knowing that $k_0 = w/c$ and inserting the impedance of the surrounding medium $\eta = \sqrt{\mu_0/\epsilon_0}$, it leads to :

$$k_p \approx k_0 \sqrt{1 - \left(\frac{2}{\eta \sigma_{2D}} \right)^2} \quad (3.17)$$

In section 4, an introduction to the simulation of material-light interaction will be given. In order to study graphene via numerical tools, a material model is needed. As a first approach, the results obtained above has been implemented in Lumerical FDTD software, where graphene was treated as a 3D material (3D permittivity, Equation (3.10)), with finite thickness. However, the meshing required is very small compare to the infrared wavelength, and some aberration may occur (see Section 4.2.2). Therefore, a second model had been developped, with graphene seen as a 2D material, defined by its 2D conductivity. This second approach is demonstrated hereafter.

3.3.2 Dispersion relation by the surface approach

Another way to get the dispersion relation of plasmons in graphene is to describe the system as an interface of two dielectric media, with a free current laying in between. Therefore, the same method as in Section 2.2.1 will be used. The same TM mode, using Equations (2.53), will be considered. The boundary conditions on the electric field lead to $\mathbf{E}_{x1} = \mathbf{E}_{x2} = \mathbf{E}_x$ and $k_{x1} = k_{x2} = k_p$. Let's take Equation (2.61) and apply it in the two media, which leads in media 1:

$$\frac{E_x w \epsilon_0 \epsilon_1}{k_{z1}} = -H_{y1} \quad (3.18)$$

and in media 2:

$$\frac{E_x w \epsilon_0 \epsilon_2}{k_{z2}} = H_{y2} \quad (3.19)$$

Combining with Equation (2.23), the boundary equation for $\mathbf{H}_x$ in the case of surface current, it becomes :

$$E_x w \epsilon_0 \left(\frac{\epsilon_2}{k_{z2}} + \frac{\epsilon_1}{k_{z1}} \right) = H_{y2} - H_{y1} = -\sigma E_x \quad (3.20)$$

And the dispersion relation is finally obtained for TM mode:

$$\frac{\epsilon_2}{k_{z2}} + \frac{\epsilon_1}{k_{z1}} = -\frac{\sigma}{w \epsilon_0} \quad (3.21)$$

It has been seen previously that k_{z_i} should be pure imaginary. It is therefore possible to replace it as $k_{z_i} = i q_{z_i}$ with q_{z_i} real positive and $q_{z_i} = \sqrt{k_p^2 - \epsilon_i k_0^2}$. The dispersion relation is revealed in its most known form :

$$\frac{\epsilon_2}{\sqrt{k_p^2 - \epsilon_2 k_0^2}} + \frac{\epsilon_1}{\sqrt{k_p^2 - \epsilon_1 k_0^2}} = -\frac{i\sigma}{w \epsilon_0} \quad (3.22)$$

If graphene is sandwiched in air, $\epsilon_1 = \epsilon_2 = 1$, the dispersion relation of plasmons in graphene seen in Equation (3.15) can be found easily.

(a) Plot of the full dispersion relation of graphene plasmons (Equation (3.22)) Vs the non-retarded approximation for graphene (Equation (3.23))

(b) Plot of the dispersion relation of plasmons for a single interface Gold/air (in red, Equation (2.63)), for a thin film of gold (1nm) sandwiched in air (in blue, Equation (2.87)) and graphene sandwiched in air (in green, Equation (3.23)). Frequency is taken from 8.3 microns to 50 microns. Gold is set as a Drude metal with $\omega_p = 2, 15 \cdot 10^{15} \text{ Hz}$ and $\Gamma = 17, 14 \cdot 10^{12} \text{ Hz}$

Figure 3.19: Dispersion relation of graphene plasmons without and with non-retarded approximation, and comparison with Gold. The substrate is set as a pure dielectric with $\epsilon = 5$. Graphene is set with $E_F = 0.3 \text{ eV}$ and a mobility of $\mu = 1 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$.

3.3.3 Graphene plasmons propagation and damping rate

Considering a non retarded regime, in the case where $k_p \gg k_0$, Equation (3.22) can be simplified:

$$k_p \approx \frac{i\omega\epsilon_0}{\sigma}(\epsilon_1 + \epsilon_2) \quad (3.23)$$

This approximation validity is demonstrated in Figure 3.19a. As an example, the plasmonic dispersion relation of a sheet of graphene with a Fermi level $E_F = 0.3\text{eV}$ and a mobility of $\mu = 1\text{m}^2\text{V}^{-1}\text{s}^{-1}$ is computed using the full Equation (3.22), and the simplified Equation (3.23). As we can see, the two curves fit really well and we can safely use the simplified equation (3.23). The next step is to plug in this equation the optical conductivity (seen Section 3.2.1). Let's start with a graphene which is highly doped, $E_F \gg k_B T$ and an excitation energy below the optical phonon in graphene, $\hbar\omega \approx 0.2\text{eV}$. We can therefore use the Drude-like expression of the conductivity, Equation (3.6), and we find:

$$k_p = \frac{1}{\lambda_0} \frac{\pi\hbar}{2\alpha\tau E_F} (\epsilon_1 + \epsilon_2)(\omega\tau + i) \quad (3.24)$$

Therefore, using Equation (2.130) ($E_F = \hbar v_F k_F$), we obtain finally:

$$\frac{\hbar\omega}{E_F} = \sqrt{\frac{c}{v_F} \frac{4\alpha}{(\epsilon_1 + \epsilon_2)(1 + \frac{i}{\omega\tau})} \frac{k_p}{k_F}} \quad (3.25)$$

, where $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ is the fine structure constant. Figure 3.19b shows a comparison between graphene plasmons dispersion relation and gold plasmon relation dispersion. The graphene is free standing (sandwiched in air); it is represented in green in Figure 3.19b. Two cases have been taken into account for gold. The red curve corresponds to a semi infinite system, where surface plasmons take place at an interface between gold and air. The second case, in blue, is a free-standing thin film of gold of 1-nm thickness (sandwiched in air). Comparing dispersion relation of graphene with gold, we can observe that GPs wavevectors k_p are generally much higher in graphene than for the gold, which means that we have a higher spatial confinement.

Dispersion relation of GPs is also calculated via the Random Phase Approximation (RPA) model. This model comes from the wish of researchers to introduce in the theory quantum mechanical interactions between the electrons. It takes into account the weak screening Coulomb interaction, so the electrons are driven by a total potential summing the external potential and the coulomb potential. Several groups [66, 159] worked on doped graphene at 0 temperature in order to get the dielectric function of graphene as a function of q and ω . The graphene plasmons are derived directly from it (Figure 3.20). For small values of q , the dispersion relation $w_{sp}(q)$ is proportional to $\sqrt{q}$ and remains really close to the semi-classical model. But for finite values, it is really interesting to notice that the square roots behaviour vanishes. Instead, the dispersion relation converges to the boundary line between the two SPE region (section 2.3.3, Figure 2.17) without entering in the intraband SPE zone. In any cases, in the mid-infrared region, the dispersion relation lies in the weakly damped area and therefore can theoretically propagate on long distance. Some high-resolution electron energy-loss spectroscopy are fitted well by the RPA theory [139], but the approximation of infinite electron relaxation time coupled with the complicated case of graphene plasmon limits the scope of this model [98].

As it has been seen in Section 2.2.1, the wavelength of the plasmon in the graphene can be obtained using Equation (2.67). It is also possible to get the propagation distance with the Equation (2.66), as well as the penetration depth (Equation (2.68)) of the field in the dielectric medium. All this physical quantities can be derived from the real and the imaginary parts of the wave vector k_p (see Equation (3.24)). Let's take into consideration the case of a graphene layer sandwiched in between a dielectric and the air. Therefore, we can define $\epsilon_1 = \epsilon_{sub} = \epsilon' + i\epsilon''$ and $\epsilon_2 = \epsilon_{air} = 1$. The wavelength of the plasmons λ_p^{sup} can be therefore established, as well as its propagation distance L_p^{sup} and the penetration depth for supported graphene as follow :

$$\lambda_p^{sup} = \lambda_0 \frac{4\alpha\tau E_F}{\hbar((\epsilon' + 1)\omega\tau - \epsilon'')} \quad (3.26)$$

Figure 3.20: Dispersion relation of graphene plasmon calculated by RPA (solid line), and by the classical model (dashed line). The graphene is lying on SiO₂ substrate. The two kind of SPE zone are drawn in background (see Figure 2.17)

$$L_p^{sup} = \lambda_0 \frac{\tau \alpha E_F}{\pi \hbar (\epsilon' + 1 + \epsilon'' \omega \tau)} \quad (3.27)$$

$$\delta_p^{sup} = \frac{1}{Re(k_p)} = \frac{\lambda_p^{sup}}{2\pi} \quad (3.28)$$

From Equation (3.27), it is interesting to notice that the propagation of plasmons in graphene depend on three main factors. The first one is the lifetime of the plasmons τ_{pl} , which is a phenomenology parameter. This factor takes into account mechanism of scattering in graphene seen section 3.1.2. The second interesting parameter is the Fermi energy. The Fermi energy in graphene vary with the process of fabrication and transfer of graphene, but can also be tuned electrostatically. The third parameter is the permittivity of the dielectric medium surrounding graphene. The propagation distance of plasmons in graphene being inversely proportional to those dielectric permittivity, it is interesting to work with substrate having really low permittivity. For example, working in a configuration of suspended graphene, the Equations (3.26) and (3.27) become :

$$\lambda_p^{sup} = \lambda_0^2 \frac{\alpha}{\hbar \pi c} E_F \quad (3.29)$$

$$L_p^{sup} = \lambda_0 \frac{\tau \alpha E_F}{2\pi \hbar} \quad (3.30)$$

It is interesting to notice that the plasmons wavelength in suspended graphene is a direct measurement of the Fermi level in this graphene sheet. An other interesting parameter is the quality factor, also called figure-of-merits, which is usually used to compare a phenomenon applied to different materials [151]. It takes into account the complex permittivity of a substrate at different wavelengths. The imaginary part of the permittivity is responsible for the loss, and the real part of the permittivity is describing the distribution of the field in a material. The inverse of this physical quantity is more common in graphene

Figure 3.21: Plot of the damping rate of graphene plasmons for suspended monolayer graphene

plasmonics field [42] and it is called the damping rate, or propagation loss. Let's take the general case of a monolayer sheet of carbon deposited on a substrate. Therefore, we can define $\epsilon_1 = \epsilon_{sub} = \epsilon' + i\epsilon''$ and $\epsilon_2 = \epsilon_{air} = 1$. The damping rate can be written as :

$$\gamma = \frac{Im(k_p)}{Re(k_p)} = \frac{\epsilon' + 1 + \epsilon''\omega\tau}{(\epsilon' + 1)\omega\tau - \epsilon''} \approx \frac{1}{\omega\tau} + \frac{\epsilon''}{\epsilon' + 1} \quad (3.31)$$

The last approximation made on the right side of the equation above is true as long as we remain far enough from the intrinsic phonon of the substrate. In the case of suspended graphene, $\epsilon' = 1$ and $\epsilon'' = 0$ and it is possible to plot the damping rate for graphene plasmons in the range of interest. Figure 3.21 shows the damping rate of graphene plasmons as a function of the wavelength in the mid-infrared range. The lifetime τ is set as a approximation as the DC relaxation time of electrons (see Section 3.1.2). The different curves correspond to different Fermi level and different mobility. As expected, higher is the mobility/Fermi level, and lower is the damping rate. From this Figure, it is possible to see that the damping rate is increasing as the wavelength does (as long as the wavelength is above the intrinsic phonon in plasmons, around $6.5 \mu m$). However, for high quality of graphene and high doping, the plasmons loss is less and less dependant of the wavelength.

Taking the case of graphene deposited on a SiO_2/Si wafer (Optical constant from Palik Handbook, Figure 3.22a), we can plot the damping rate in the same range, Figure 3.22b. Here again the graphene parameter such as the Fermi level and the mobility have been varied. Figure 3.22b shows a minimum around $11 \mu m$, which correspond to the litterature [42].

However, the values obtained here, are relatively low compared to the literature review. As mention in the papers [42, 43], it is probably due to a many-body interaction at the edge and grain boundary of the graphene sheet, where other scattering mechanisms (electrons-electrons) take place.

(a) Permittivity of SiO_2 substrate using Palik Handbook.

(b) Zoom on the minimum of the damping rate

Figure 3.22: Damping rate of graphene plasmons for supported graphene

3.4 Aim of this study

The initial aim of this study is to lay the foundations of plasmon propagation for the development of future graphene-based optoelectronic devices. Ideally, graphene plasmons in the infrared suffer low metallic losses as compared to noble metals. This is due to its excellent electronic properties. However, substrate implies impurities and defects that affect the transport properties of graphene. Also, the use of a substrate induces a strong damping due to a high optical energy dissipation. In this context, suspended graphene appears to be a promising route for the design of future opto-electronic devices.

The scope of this work is the study of plasmons in free standing graphene. Free standing graphene enable to get rid of the interaction of graphene charges with the substrate. Under special condition, graphene plasmons could reach interesting propagation length. Therefore, a numerical study is detailed in the chapter 3. In the chapter 4, experimental results are presented. The fabrication of samples, the characterizations as well as optical measurements are introduced in the chapter 4. The optical measurement are performed with an home made Scanning Near-field Optical Microscope (SNOM) in the infrared range.

Chapter 4

FDTD Simulation of the propagation of plasmons in suspended graphene

The work of a researcher is to undertake creative investigation on a systemic, organized basis. Therefore, numerical tools have been developed to help scientists to predict interesting roads to explore, before experimental demonstrations. The finite-difference time-domain (FDTD) method is a numerical analysis technique which gives to the community of optics the possibility to simulate various interaction between light and matter. Surface Plasmons excitation is one phenomenon taking place in the scope of those interactions, appearing theoretically after rearranging Maxwell's equations. These equations are the basis of the FDTD method, which aims to solve them in discretized space and time. Therefore, plasmons can be computed and study through this numerical method.

First of all, the basic idea of the FDTD method will be introduced briefly. Then, the modelization of graphene through FDTD will be discussed. Finally, the case of the plasmons propagation in suspended graphene will be detailed: at first, the use of an in plane nano-antenna for Graphene Plasmons (GPs) excitation will be investigated. Then, a system taking into account the constrain induced by the goal of a long propagation is designed. Finally, some critical remarks are addressed.

4.1 Basic idea of the Finite-Difference Time-Domain (FDTD)

FDTD method is based on an algorithm proposed by Kane Yee, who used second-order central difference [146]. The key steps of the algorithm are summarized in the following, with some references to the 1D case that will be developed subsequently.

1. Replace all the time and space derivatives of the Ampère (Equation (2.3)) and Faraday laws (Equation (2.4)) by finite differences (see Equation (4.2)). Doing so, the space and time are discretized and the electric and magnetic fields are staggered in the four dimensions.
2. Solve the different equations (Equation (4.1)), using second-order central differences (Equations (4.6) and (4.5)), and express the future unknown fields in terms of known past fields (Equation (4.7)).
3. Evaluate the magnetic fields one time-step into the future so they are now known (effectively they become past fields).
4. Evaluate the electric fields one time-step into the future so they are now known (effectively they become past fields).
5. Repeat the previous two steps until the fields have been obtained over the desired duration.

Let's have a more detailed look at the simplest case : the one dimension case.

4.1.1 1-D FDTD

From the Maxwell's equations, the wave equation for electromagnetic waves can be obtained (Equation (2.8)). To simplify, the wave equation in free space with no current and source can be expressed in 1D as follow:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad (4.1)$$

with u as a scalar field to be described. A gaussian wave can be seen as a possible solution. Let's assume this wave propagating from left to right (increasing x), so it is possible to draw the following schema (Figure 4.1). It represents snapshots of the field at different time steps t_n and position x_i , where i and n are intergers. $x_i \equiv i\Delta x$ is defined to discretize the space and $t_n \equiv n\Delta t$ is defined for the time.

Figure 4.1: Skech of the discretization in time and space of a gaussian wave propagating towards the right

Then, the new wave equation taking into account these discrete variables is written as :

$$\left. \frac{\partial^2 u}{\partial x^2} \right|_{x_i}^{t_n} - \frac{1}{v^2} \left. \frac{\partial^2 u}{\partial t^2} \right|_{x_i}^{t_n} = 0 \quad (4.2)$$

Let's freeze the time. Taylor series expansion on the term $u_{x_{i+1}}^{t_n} \equiv u_{i+1}^n$ can be written as :

$$u_{i+1}^n = u_i^n + \Delta x \left. \frac{\partial u}{\partial x} \right|_{x_i}^{t_n} + \frac{(\Delta x)^2}{2!} \left. \frac{\partial^2 u}{\partial x^2} \right|_{x_i}^{t_n} + \frac{(\Delta x)^3}{3!} \left. \frac{\partial^3 u}{\partial x^3} \right|_{x_i}^{t_n} + \frac{(\Delta x)^4}{4!} \left. \frac{\partial^4 u}{\partial x^4} \right|_{x_i}^{t_n} + \dots \quad (4.3)$$

From the space-step $i - 1$ to the step i , Equation 4.3 becomes :

$$u_{i-1}^n = u_i^n - \Delta x \left. \frac{\partial u}{\partial x} \right|_{x_i}^{t_n} + \frac{(\Delta x)^2}{2!} \left. \frac{\partial^2 u}{\partial x^2} \right|_{x_i}^{t_n} - \frac{(\Delta x)^3}{3!} \left. \frac{\partial^3 u}{\partial x^3} \right|_{x_i}^{t_n} + \frac{(\Delta x)^4}{4!} \left. \frac{\partial^4 u}{\partial x^4} \right|_{x_i}^{t_n} - \dots \quad (4.4)$$

Adding Equations (4.3) and (4.4) gives :

$$\left. \frac{\partial^2 u}{\partial x^2} \right|_{x_i}^{t_n} = \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} + \mathcal{O}(\Delta x^2) \quad (4.5)$$

The same method can be used to get the partial derivative in time :

$$\left. \frac{\partial^2 u}{\partial t^2} \right|_{x_i}^{t_n} = \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2} + \mathcal{O}(\Delta t^2) \quad (4.6)$$

Substituting Equations (4.5) and (4.6) to Equation (4.2) leads to:

$$u_i^{n+1} \simeq \left(\frac{v\Delta t}{\Delta x} \right)^2 (u_{i+1}^n - 2u_i^n + u_{i-1}^n) + 2u_i^n - u_i^{n-1} \quad (4.7)$$

Equation (4.7) is said to be fully explicit. On the right side of the equation, all the fields values are at the time step n or $n - 1$, therefore already computed because referring to the present and to the past time steps respectively. On the left side, however, the value of the field corresponds to the future, at the time t_{n+1} . The full explicit equation enables to compute the future value of the field from the previous value, already stored in the computer (at the current time step n and the past step time $n-1$).

Numerical dispersion relation In order to have an idea about the numerical accuracy of FDTD, the Maxwell dispersion relation (Equation (4.1)) can be plugged in. Let's write the field as a plane wave:

$$u_i^n = U_0 e^{j(wt_n - kx_i)} \quad (4.8)$$

with $t_n = n\Delta t$ and $x_i = i\Delta x$. Substituting this to the Equation (4.7), and simplifying with the common factor $U_0 e^{j(wn\Delta t - ki\Delta x)}$, it becomes :

$$e^{jw\Delta t} = \left(\frac{v\Delta t}{\Delta x} \right)^2 (e^{-jk\Delta x} - 2 + e^{jk\Delta x}) + 2 - e^{-jw\Delta t} \quad (4.9)$$

It can be also written as :

$$\frac{e^{jw\Delta t} + e^{-jw\Delta t}}{2} - 1 = \left(\frac{v\Delta t}{\Delta x} \right)^2 \left(\frac{e^{jk\Delta x} + e^{-jk\Delta x}}{2} - 1 \right) \quad (4.10)$$

Recognizing the Euler's identity, the so-called numerical dispersion relation appears as :

$$\cos(w\Delta t) - 1 = \left(\frac{v\Delta t}{\Delta x} \right)^2 (\cos(k\Delta x) - 1) \quad (4.11)$$

Two cases have to be considered:

First case : if $\left(\frac{v\Delta t}{\Delta x} \right) \rightarrow 1$, Equation (4.11) becomes $\cos(w\Delta t) = \cos(k\Delta x)$ and finally the exact dispersion relation $w = kv$ will be approached. Unfortunately, it is only occurring in 1-D, but it is not the case in 2D and 3D [146]. Let's therefore look at the second case.

Second case : if $\Delta t \rightarrow 0$ and $\Delta x \rightarrow 0$ Using the Taylor series for cosine expressed as $\cos(x) = 1 - \frac{x^2}{2!} + \mathcal{O}(x^3)$, Equation (4.11) tends to the exact dispersion relation $w = kv$. The discrete dispersion relation in time and space converges to the exact analytical solution for $\Delta t \rightarrow 0$ and $\Delta x \rightarrow 0$, which is intuitively what is expected.

Numerical accuracy in FDTD Two numerical factors are defined. They have to be considered in order to avoid numerical errors and aberrations, which are :

$$S \equiv v \frac{\Delta t}{\Delta x} \quad \text{and} \quad N_\lambda \equiv \frac{\lambda}{\Delta x} \quad (4.12)$$

The first one is the FDTD stability factor, or Courant number. It plays a role in determining the stability of the simulation. In fact, let's assume that the simulation takes place in the vacuum. Therefore, $v=c$, with c the speed of the light. For one time step Δt , a Gaussian pulse (Figure 4.1) will therefore travel over length $L = c \times \Delta t = S\Delta x$. If $S > 1$, the pulse will propagate to more than one spatial step. Therefore, Equation (4.7) which takes into account the first neighbour cells, would not solve the future field accurately. In fact, it produces unstable results and numerical errors. According to [146], when the Courant number is all but 1, the FDTD grid is dispersive, which means that different frequencies travel at different speeds. In practice, the dispersive effect is something to avoid.

The second one is the grid sampling density in the case of free space. According to [2], in order to keep reasonable errors, the size of Δx should be 10 times smaller than the wavelength. Here, it is important to notice that the wavelength of graphene plasmons can be more than 100 times smaller than the excitation in free space [70]. Therefore, the meshing size should be at least 1000 times smaller than the excitation. This leads to extremely small meshing, huge computation resources and possible numerical aberrations (see Section 4.2.2).

Let's come back to the dispersion relation found above. It is possible to re-write the numerical dispersion relation, Equation (4.11), with the two numerical factors introduced above (Equation (4.12)), as follow :

$$k = \frac{N_\lambda}{\lambda} \cos^{-1} \left(1 + \frac{1}{S^2} \left(\cos \left(\frac{2\pi S}{N_\lambda} \right) - 1 \right) \right) \quad (4.13)$$

It is interesting to notice that we recover the exact wave-number when $S=1$. More detailed examples and analysis for different value of S and N_λ can be found in the literature [146], which shows percentages errors and non-physical behavior of waves using not appropriated values of these two parameters. In practice, we will play with these two parameters to minimize the time of simulations whereas converging to a solution which agrees with the theory.

4.1.2 FDTD in 3 dimensions

Figure 4.2: Yee cell with the representation of the different component of the $\mathbf{E}$ and $\mathbf{H}$ field

In 3 dimensions, the FDTD technique, which is a time-domain method, solves the Maxwell's curl equations (Equations (2.3) and (2.4)) in a discretised space. This gives rise to a set of 6 scalar equations for 6 vector field components, $\mathbf{E}_x$, $\mathbf{E}_y$, $\mathbf{E}_z$, $\mathbf{H}_x$, $\mathbf{H}_y$, $\mathbf{H}_z$, that are computed within a Yee cell. The Yee cell, from the name of its inventor, is a staggered and uncollocated grid, which means that $\mathbf{E}$ and $\mathbf{H}$ are not sampled on the same location, and each 3 vectorial components of the two fields are not computed on the same location, respectively.

In 40 years of existence, many other meshing structures have been proposed, but none of them has been able to replace it. Some of the reasons are enumerated subsequently. At first, the Yee algorithm

solves simultaneously the electric field and the magnetic field, rather than solving only one of them through the wave equation (Equation (4.7)). Therefore, it makes the computation more robust and enlarges the field of possible simulations, including singularities near edges and corners.

Secondly, the E-fields and H-fields components are placed particularly, such that each E-field component is surrounded by four circulating H-field components, and inversely. This makes easier to maintain the boundary conditions (see section 2.1.2, Figure 2.2), where the Faraday's law and the Ampère's law are interlinked. The only requirement is to set the optical properties of the different materials at an interface. The interface will be set parallel to a Yee cell face. Therefore, the interface will have a staircase form with a space resolution of the size of the meshing. Finally, in absence of charges and currents, the location of the E-field and H-field, combined with the second-order differences, enforces the Gauss's laws, making the simulations divergence free.

Figure 4.3: Yee Cell with the representation of the $\mathbf{E}$ and $\mathbf{H}$ field, each of them calculated at different time step : the leapfrog arrangement

Finally, the 6 components of the magnetic and electric fields are also centered in time. Figure 4.3 shows the so-called leapfrog arrangement, where the H-field (and E-field) are solved alternatively, in different time steps, using the previous results stored for the E-field (H-field). The time stepping process is centred and second order accurate, as seen Equations (4.5) and (4.6). It is robust because the fields do not dissipate due to the time-stepping algorithm, which could arise from numerical artefacts. It is also called "fully explicit", therefore avoiding again errors and aberrations. For more information, see [146].

For a same amount of error, the Yee cell enables to use a smaller number of cells, as compared to the others possibilities, namely unstaggered-collocated and staggered-collocated cells [73].

The dispersion relation in 3 dimensions is a little bit more complex and it is possible to obtain it as :

$$\left[\frac{1}{c\Delta t} \sin\left(\frac{w\Delta t}{2}\right) \right]^2 = \left[\frac{1}{\Delta x} \sin\left(\frac{k_x\Delta x}{2}\right) \right]^2 + \left[\frac{1}{\Delta y} \sin\left(\frac{k_y\Delta y}{2}\right) \right]^2 + \left[\frac{1}{\Delta z} \sin\left(\frac{k_z\Delta z}{2}\right) \right]^2 \quad (4.14)$$

It is interesting to notice that when Δt , Δx , Δy and Δz tend to zero, we find the analytical dispersion relation :

$$\left(\frac{w}{c}\right)^2 = k_x^2 + k_y^2 + k_z^2 \quad (4.15)$$

Therefore, this means that it is possible to recover any degree of accuracy, as far as the mesh grid and time step are set at the right value. As a rule of thumb [2], the spacial step should be set as :

$$\text{maximum}(\Delta x, \Delta y, \Delta z) \leq \frac{\lambda_{min}}{10n_{max}} \quad \text{or} \quad N_\lambda \geq 10n_{max} \quad (4.16)$$

where n_{max} is the maximum value of the different refractive index found in the system. On the other hand, the time step should be set as :

$$\Delta t \leq \frac{1}{c\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}} \quad (4.17)$$

in the case where the light is propagating through vacuum at the speed c . Again, those parameters are used to minimize the duration of the simulation whereas converging to an acceptable solution. In the next section, the way to deal with graphene as a material is presented.

4.2 Modeling graphene in FDTD

Graphene is a 2D material, and as such it is not trivial to simulate using FDTD. First of all, the theory which gives us the dispersion equation for graphene optical properties is based on the Kubo formula (see Section 3.2.1). The Kubo formula gives two integral terms to be solved, which represent the intraband term and the interband term. The former is solved analytically whereas the second can be accurately solved numerically. Because the interband term can be solved only numerically, several approximations exist.

Secondly, graphene optical properties, usually presented in term of 2D optical conductivity, is not trivial to solve for FDTD 3D numerical simulation. Two methods exist: a 3D approach based on a 3D virtual permittivity, defining graphene with a finite very weak thickness, and a 2D approach using the 2D optical conductivity, using special algorithm for modelling graphene as a 2D material [1]. In fact, in the case of the 3D approach, meshing a layer of graphene with a thickness of the order of magnitude around the Angstrom gives a significant amount of Yee cells to be computed. The electro-magnetic excitation is the infrared, which makes system structures of the order of the micrometer. Coupled with effective wavelength of graphene plasmons which is really small, it makes the computation region huge, relatively to the meshing.

This section will be divided into two sub-sections. At first, the different approximations using different terms of the conductivity are presented and tested. The conditions to apply the different approximations are given. Finally, the two main methods existing to implement graphene as a material in FDTD are shown. The first is to modelize graphene in 3 dimensions via the permittivity. The second, implemented recently, aims to use a special algorithm to modelize graphene as a 2D material, taking place on one face of the cubic Yee cell.

4.2.1 How to deal with the interband term of graphene conductivity

As it has been seen in Section 3.2.1, the optical properties of graphene can be described thanks to its optical conductivity given by the Kubo formula (Equation (3.1)). However, some problems arise when dealing with the interband term that can be solved only numerically.

Let's evaluate the optical conductivity of graphene. The starting point is to express the optical conductivity of graphene, thanks to the Kubo formula (Equation (3.1)). We can therefore express the conductivity as a sum of the intraband term $\sigma_{intra}(\omega, \Gamma, T, \mu_c)$ and the interband term $\sigma_{inter}(\omega, \Gamma, T, \mu_c)$ as follow :

$$\sigma_{intra}(\omega, \Gamma, T, \mu_c) = \frac{-ie^2}{\pi\hbar^2(\omega + i2\Gamma)} \int_0^\infty E \left(\frac{\partial f_d(E)}{\partial E} - \frac{\partial f_d(-E)}{\partial E} \right) \partial E \quad (4.18)$$

$$\sigma_{inter}(\omega, \Gamma, T, \mu_c) = \frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \int_0^\infty \frac{f_d(-E) - f_d(E)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E \quad (4.19)$$

with f_d the Fermi-Dirac distribution, μ_c the chemical potential, e the elementary charge, Γ the scattering rate and ω the excitation pulsation. The scattering rate is a phenomenological physical quantity related to the relaxation time of electron in graphene by $2\Gamma = \tau^{-1}$. The interband term is determined numerically whereas the intraband term of the conductivity is solved analytically as:

$$\sigma_{intra}(\omega, \Gamma, T, \mu_c) = \frac{ie^2\mu_c}{\pi\hbar^2(\omega + i\tau^{-1})} \left[1 + \frac{2k_B T}{\mu_c} \ln(e^{-\mu_c/k_B T} + 1) \right] \quad (4.20)$$

As seen before, the intraband term can be also reduced to a Drude-like expression, in the case where $E_F \gg k_B T$:

$$\sigma_{intra}(\omega, E_F, \tau, T) = \frac{ie^2 E_F}{\pi\hbar^2(\omega + i/\tau)} \quad (4.21)$$

These two equations, Equations (4.20) and (4.21) will be used later on as first approximations of the optical conductivity in the infrared range. In fact, as shown in Figure 3.22b, the ideal wavelength for low plasmons damping lies around 11 microns, in the middle infrared. As seen with Figure 3.12a, in the infrared range, the optical conductivity of graphene tends to the intraband terms, and the interband terms can be neglected (no electron-hole excitation between valence and conduction band, see section 2.3.3). This case would make the analytically solved intraband terms a good approximation for the optical conductivity of graphene.

Let's come back to the Kubo formula and the resolution of its second term, namely the interband term. For numerical evaluation of this term, it is interesting to rearrange the difference of the Fermi function. Using an arbitrary function $G(E)$, it is possible to write [40] :

$$G(E) = \frac{\sinh(E/k_B T)}{\cosh(\mu/k_B T) + \cosh(E/k_B T)} \quad (4.22)$$

Then the interband term can be re-written as :

$$\sigma_{inter} = \frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \int_0^\infty \frac{G(E)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E \quad (4.23)$$

$$= \frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \left[\int_0^\infty \frac{G(E) - G(\omega/2)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E + \int_0^\infty \frac{G(\omega/2)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E \right] \quad (4.24)$$

$$= \frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \left[\int_0^\infty \frac{G(E) - G(\omega/2)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E - iG(\omega/2) \frac{\hbar\pi}{4(\omega + i2\Gamma)} \right] \quad (4.25)$$

$$= \frac{e^2}{4\hbar} \left[G(\omega/2) + \frac{i4(\omega + i2\Gamma)}{\hbar\pi} \int_0^\infty \frac{G(E) - G(\omega/2)}{(\omega + i2\Gamma)^2 - 4(E/\hbar)^2} \partial E \right] \quad (4.26)$$

This mathematical simplification removes the singularity from the integrals. Figure 4.4 shows the real part (solid line) and imaginary part (dashed line) of the optical conductivity of graphene. As it can be seen in Figure 4.4, the conductivity ($\sigma_{intra} + \sigma_{inter}$) plotted with the function $G(E)$ introduced above (in red on the graph) removes the singularity at high energy (low wavelength on the graphic), as compared to the conductivity plotted with the integral (in blue on the figure), Equation (4.19).

It is then possible to compare the optical responses of a graphene sheet for different Fermi level, corresponding to different densities of charge. In Figure 4.5, the optical conductivity of graphene is plotted as function of the wavelength, for different Fermi levels (colour). The dash lines correspond to the imaginary part of the conductivity, whereas the solid lines correspond to the real part. As expected, the wavelength threshold between the interband and intraband domains takes place at $2E_F$, and diminishes as the Fermi level gets higher (Figure 4.5).

Figure 4.4: Conductivity numerically calculated with (in red) and without (in blue) the functions $G(E)$ and $G(\omega/2)$ [40]

Graphene has been implemented in FDTD at first by using only the Drude-like intraband approximation (Equation (4.21)). This approximation is known to be valid at high Fermi level. Figure 4.6 displays four graphs of graphene optical conductivity, corresponding to four different Fermi energies. On each graph, three sets of conductivity obtained in three different ways : the two approximations of the conductivity via the intraband term (Equations (4.20) and (4.21)), compared to the full conductivity

Figure 4.5: Comparison of the optical conductivity for different Fermi energy (0.1,0.2,0.4,0.6,1eV), using numerical calculation for σ_{inter} and Equation (3.5) for σ_{intra} . The conductivity is plotted as a function for the wavelength λ running from 100nm to 15 μm . The optical conductivity of graphene (unit $e^2/4\hbar$) of a graphene layer with a mobility of $1m^2V^{-1}s^{-1}$.

(interband calculated numerically), in the infrared range. In red is shown the Drude-like intraband conductivity approximation, using Equation (4.21). In blue is displayed the complete intraband term approximation, from Equation (4.20). Finally, in green is shown the full conductivity, with the interband term computed numerically with the method shown above. The solid lines correspond to the real part of the optical conductivity, whereas the dashed line+symbol correspond to the imaginary part of the conductivity.

First, let's have a look on the two intraband terms, in red and blue for the partial and complete terms respectively. At low Fermi energy, with $E_F = 0.05eV$ (Figure 4.6a), both approximations (in blue and red) give graphene conductivity with a strong bias (as compared to the green). Unexpectedly, the Drude-like conductivity (in red), which is an approximation only for high E_F as compared to the full intraband term approximation (in blue), tends to be closer to the one obtain with full conductivity (in green). The approximations tend to be correct at a Fermi level $E_F \geq 0.3eV$ (Figures 4.6c and 4.6d). Therefore, the Drude-like approximation can be used in this infrared region for Fermi level higher than 0.3eV. Below 0.3eV, the full conductivity should be considered.

As we will see later (see Section 4.4), an undoped bare suspended graphene sheet can be electrostatically doped only until 0.25eV. For this reason, the intraband approximation is not valid for the study of suspended graphene. We will therefore use the full conductivity when simulating suspended graphene with FDTD method. However, as it will be shown below, the graphene optical properties have been implement at first in FDTD taking only into account Drude-like approximation. Moreover, graphene was only available as a 3D material (permittivity) with finite thickness. We will see in the next section the limit of the 3D approach, as compared to the 2D one.

4.2.2 Two methods to compute graphene in FDTD : 3D vs 2D approach

Graphene is a 2D material, and as such it is not trivial to simulate its properties using FDTD. Its optical properties are usually presented in terms of optical conductivity, which can be expressed using the Kubo formula (Equation 3.1). Two main methods exist to implement it in FDTD.

The first one attempts to use full Yee cells to describe it, but difficulties arise due to their inherent 3D nature. This approach requires converting the 2D conductivity (sections 3.2.1 and 4.2.1) into a volume anisotropic permittivity (Equation (3.10)), assuming that graphene has a finite thickness. A study

Figure 4.6: Comparison of the conductivity $\sigma_{||}$ of the graphene obtained using different approximations from the Kubo formula. In red, $\sigma_{||}$ obtained by the use of the Drude-like intraband term from equation 3.6. In blue, $\sigma_{||}$ obtained by the use of the full intraband conductivity from equation 3.5. In green, the full conductivity obtained by the use of the full intraband conductivity from equation 3.5 and the interband term obtain numerically. The mobility is set at $1m^2V^{-1}s^{-1}$.

[140] has been done in order to know if this procedure is safe or if it will profoundly change the optical properties of the graphene sheet. We first tried following this path, and numerical results confirmed the accuracy of this approach.

A second approach aims at describing graphene as a charged surface boundary condition applied only to a fraction of a Yee cell [112, 59]. This condition is of the general form:

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2) = \sigma \vec{E}_t \quad (4.27)$$

where $\vec{n}$ is the vector normal to the graphene sheet, $\vec{H}_1$ and $\vec{H}_2$ the magnetic fields on each side of the sheet, σ the conductivity and $\vec{E}_t$ the tangent electric field. Through this, there is no need to discretize space (and time) to extreme sizes, and this was found to lead to an increased stability of our simulations. This is the method that will be used in this study.

Let's come back to the 3D method. Using Equation (3.10), it is possible to plug in different models of optical conductivity of graphene. As a first approximation the Drude-like conductivity is used (Equation (3.6)) valid in the infrared range at Fermi level $E_F \geq 0.3eV$ (Section 4.2.1). The permittivity is obtained as follow :

$$\epsilon_{||} = \epsilon_s - \left(\frac{E_F e^2 w}{\pi \hbar^2 (w^2 + \tau^{-2})} \right) \frac{1}{w \epsilon_0 \Delta} + i \left(\frac{E_F e^2}{\pi \hbar^2 (w^2 + \tau^{-2}) \tau} \right) \frac{1}{w \epsilon_0 \Delta} \quad (4.28)$$

$$\epsilon_{\perp} = \epsilon_s \quad (4.29)$$

Equation 4.28 is then implemented directly as an analytical material and several parameter have to be set. At first, the surrounding permittivity, ϵ_s (see Section 2.1.3), which is chosen to be at 2.5 for graphene on SiO_2 [57] and 1 for suspended graphene. Then, the Fermi level E_F and the mobility μ values have to be chosen carefully. In fact, they are function of each others, but they are also function of the system and phenomenon to be studied.

As seen in Section 3.2.1, the Kubo formula which gives the optical conductivity of graphene is function of the relaxation time τ (with $\Gamma = 1/2\tau$). In section 3.1.2, the Boltzmann-transport approximation ($\tau_D C = \mu E_F / e^2 v_f$), linking the relaxation time with the Fermi level and the mobility, has been presented as a good approximation. However, it usually remains too low as compared to the experiment. Different reasons have been put forward. It is possible to enumerate dielectric losses of the substrate, electron-hole scattering and thermal intrinsic phonon of the graphene. The FDTD commercial software used in this study enables user to set graphene parameter only with τ_{DC} .

Therefore, it has been chosen to work with conservative values of the DC mobility. It artificially lows down the relaxation time of graphene, being directly proportional to it. In fact, mobility in suspended graphene has been recently recorded up to 40 times the value of $1m^2V^{-1}s^{-1}$ that has been taken in this study [150]. The graphene mobility limited only by acoustic phonon would be 20 times more important than the value used here. Literature mentions a relaxation time which would be smaller by a factor of 3-4 times as compared to the DC [42].

Finally, it is important to look at the main parameter emerging from this method : the thickness of the graphene Δ . A straight forward way to have an idea about the range of validity of the thickness Δ is to use a mode solver. A mode solver enable to determine all the E-field and H-field components corresponding to the normal mode supported by an optical system. Defining the materials with complex permittivity, it is possible to deal with a wide range of materials, including graphene.

To investigate the effect of the thickness Δ to the TM plasmons mode of graphene in the infrared range, we consider a Fermi level of $E_F = 0.5eV$, in order to safely use Equations (4.28) and (4.29), and a mobility of about $1m^2V^{-1}s^{-1}$, which are realistic properties that have been reported in the literature [158]. That gives a relaxation time $\tau = 50ps$ and a scattering rate $\Gamma = 6,58.10^{-4}eV$ ($\tau_{DC} = \mu E_F / e^2 v_f$). The mode solver computation gives the effective index $n_{eff} = k_p / k_0$ which has a real part, related to

Figure 4.7: Comparison of the (a) real part of the effective index n_{eff} and (b) the imaginary part of the effective index n_{eff} of the plasmons modes for 2D (in blue) vs 3D (in red) graphene (Drude-like approximation) vs theory (in green) using Equation (3.23). The optical conductivity as well as the permittivity of the SiO_2 substrate in the case of the computation of theoretical n_{eff} is taken from the 2D simulation. The mobility of the graphene is set at $1m^2V^{-1}s^{-1}$ whereas the Fermi level is about 0.5eV.

the effective wavelength of the GPs, and an imaginary part, related to the optical loss of the mode.

Figures 4.7 show a comparison of the real part and the imaginary part of the effective index n_{eff} , solved with a mode solver for graphene deposited on SiO_2 , at a wavelength of 11 microns, in the following cases:

1. in red, 3D graphene (Equations (4.28) and (4.29)) where n_{eff} is function of the graphene thickness (x axis)
2. in blue, 2D graphene with the full conductivity (analytical intraband term and numerical interband term)
3. in green, a theoretical n_{eff} based on the dispersion relation of graphene plasmons, Equation (3.23)

We can first observe that the theoretical n_{eff} in green is relatively close to the one obtained from 2D graphene, in blue. We realise also here a strong bias for the simulation using the 3D permittivity for describing graphene. An exact value can be obtained for the real part of the effective index for $\Delta = 2nm$, but not for the imaginary part. The error on the latter one decreases for Δ getting thinner. Therefore, this model, which is a good first approach, is not enough accurate to study graphene plasmons.

Replacing the optical conductivity model (Drude like model) used for 3D graphene by the full conductivity, with the interband term calculated numerically, it is possible to solve the mode for different thickness, following the same process. Figure 4.8 shows a comparison of the normal modes obtained for 3D graphene using the full conductivity, 2D graphene with the full conductivity and using the theoretical equation, as seen above. The comparison shows this time that smaller is the thickness, and higher is the accuracy. This is summarized by the absolute error of the two cases seen Figure 4.7 and Figure 4.8, plotted Figure 4.9.

Figure 4.9 shows the absolute error of the effective index n_{eff} of the TM GPs mode computed for 3D graphene (with the Drude-like approximation Figure 4.7, and the full conductivity Figure 4.8) as compared to the theory (Equation (3.23)). The Drude like approximation gives more accurate results for thickness higher than 2nm, but does not tend to the exact solution, as the errors on the real part of the effective index increase for thinner layer. When using the full optical conductivity, the results seem to converge to the exact solution when the thickness of graphene vanishes. In any case, a really small

(a) Real part of the effective index n_{eff} of the GPs

(b) Imaginary part of the effective index n_{eff} of the GPs

Figure 4.8: Comparison of the (a) real part of the effective index n_{eff} and (b) the imaginary part of the effective index n_{eff} of the plasmons modes for 2D (in blue) vs 3D (in red) graphene (using full conductivity) vs theory (in green) using Equation (3.23). The optical conductivity as well as the permittivity of the SiO_2 substrate in the case of the computation of theoretical n_{eff} is taken from the 2D simulation. The mobility of the graphene is set at $1m^2V^{-1}s^{-1}$ whereas the fermi level is about 0.5eV.

(a) Absolute errors $(n_{eff}(3D)-n_{eff}(theo))/n_{eff}(theo)$ applied for the real part and the imaginary part, of the results obtained Figure 4.7

(b) Absolute errors $(n_{eff}(3D)-n_{eff}(theo))/n_{eff}(theo)$ applied for the real part and the imaginary part, of the results obtained Figure 4.8

Figure 4.9: Absolute value of the errors on the effective index n_{eff} found in Figures 4.7 and 4.8, as compare to the theory

thickness is required for accurate simulation.

Therefore, this 3D graphene method requires a local spatial discretization of the order or thinner than the nanometer, while our simulations involve wavelengths and structures bigger than the micron, leading to huge computational domains and non-uniform meshing. Moreover, the stability criterion of the FDTD then requires an extremely short time discretization, and a huge number of iterations is thus needed to complete a single simulation. Therefore, this method is highly CPU-intensive and numerical aberration may occur.

In order to illustrate a case where numerical errors may occur, Figures 4.10 show an attempt to simulate graphene plasmons launched by a nanorod, using 3D graphene. This approach will be described in the next section. Figure 4.10a is an illustration of the system studied. A 3D graphene layer is lying on the top of a Si/SiO₂ substrate. A gold nanorod is deposited on the top of it and acts as a plasmons launcher. The five other figures, described below, show a serie of snapshots of the electric field recorded inside the graphene layer.

Figure 4.10b shows a half top-view of the simulation illustrated in Figure 4.10a, at $t=0$. Taking advantages of symmetry, we simulate the right-side half of the full 3D system. The light is normal to the plan of the simulation, exciting only the nanorod via Total-Field Scattering-Field source. Half of the gold nanoantenna is also shown. At $t = \Delta t$ (Figure 4.10d), the incident X-polarized light excites typically the dipolar mode of the rod. At $t = 2\Delta t$, the plasmons is spreading in the graphene layer, mainly in the y direction (see next section). At $t = 3\Delta t$ and $t = 4\Delta t$, it is possible to observe a field remaining in the nanoantenna, following the grid which is about $5nm \times 5nm \times 1nm(x, y, z)$. The graphene is set with a thickness of 2nm, using the Drude like approximation. This kind of aberration becomes more serious as the grid becomes smaller. The total energy in the box did not vanish after the time of the simulation. This kind of aberration adds a significant errors when studying the properties of the plasmons, like its effective wavelength λ_p or its propagation length L_p . Therefore, it has been chosen to study Graphene plasmons with the 2D model.

The 2D material modelization, which is the second approach seen above, aims to describe graphene as a charged surface, where the boundary conditions are applied only to a fraction of a Yee cell [6]. As such, there is no need to discretize space (and time) to extreme sizes, and this was found to lead to an increased stability of our simulations. Moreover, it leads to results fitting really well the theory for graphene plasmons, as shown Figures 4.8 and 4.7. This is the method that will be used hereafter. The numerical study of graphene has been made possible thanks to the use of the super-calculator Romeo in Reims, France.

4.3 Use of a plasmonic antenna for GPPs excitation

Originally, excitation of GPPs was achieved by the use of a laser illuminated metal tip in the scanning near-field optical microscopy configuration [42, 29]. Although the resulting scattered angular spectrum was broad, a small part of it was exploited for successfully launching the GPs (see section 3.2.2).

Another way for launching plasmons at metal/dielectric interfaces lies on the use of prisms or gratings [143] to efficiently convert the incident wave-vector into the in plane graphene plasmon one. In our simulations, we use Au nanorod as nanoantenna to study the GPPs launched perpendicularly to the main axis of the nanorod in order to anticipate the possibility to maximize the power of the GPPs launched using a grating. It is indeed possible to place side by side many nanoantenna together, and obtain an enhancement effect thanks to the grating configuration. As a first step, we will only rely on the angular spectrum generated by a single rod. Compared to an out-of-plane tip, the in-plane resonant launchers allow us to envisage future integrated chips. Additionally, the longitudinal resonance enables the enhancement of the angular spectrum components.

As a preliminary study, a gold nano-antennas with a length L acting as a resonant dipole along L

Figure 4.10: Snapshot of the propagation of surface plasmons, resolved in time using an arbitrary Δt , and the 3D graphene model. The monitor is located in the graphene. The launcher is a gold nanorod of $4\ \mu m$. The exciting source is a pulse of light, with a spectrum range from $9\ \mu m$ to $13\ \mu m$, normal to the images. The polarisation of the light follow the big axis. The graphene is lying under the gold nanorod, on the entire surface ($5\ \mu m \times 5\ \mu m$). The substrate is made of 300nm of SiO_2 and $7\ \mu m$ of Si. The graphene is set with a Fermi level of 0.5eV and a mobility of $1m^2V^{-1}s^{-1}$.

is considered. The section of the nano-rods is rectangular, 80nm large and 50nm height. The shape is needle-like, to fit the fabrication via e-beam lithography. The nanorod was illuminated with a pulse that propagates perpendicular to the sample plane, along z (see configuration on Figure 4.10a). The incident light was linearly polarized with the electric field parallel to the rod long axis (x axis). This pulse has a wide-band spectrum going from 8 μm to 12 μm , with a central wavelength around 10 μm .

Figure 4.11: Calculated extinction efficiency of gold nano-rods with a length of 3700 nm, 4000nm and 4300nm, for a width of 80nm and height of 50nm. Nanorods are standing in air.

Figure 4.11 shows the calculated extinction efficiency spectrum of three nanorods without graphene, in air, for different lengths: 3700nm, 4000nm and 4300nm. The extinction efficiency is the sum of the absorption and scattering efficiency. The efficiency are dimensionless properties, and can be directly converted to the cross section by multiplying with the cross-sectional area of the nano-rods [72, 63]. It is well-known [147] that changing the shape of the rods shifts the resonance wavelength, as a result of a change of the oscillation length travelled by the free electrons along the long axis. When we increase the long axis of the nano-rod, we can see a red shift in the optical spectra. We can notice that the resonance is engineered to lie in the mid-infra-red region, in order to avoid plasmons losses and damping when travelling through graphene, at 11 μm .

Simulations have shown a red shift when the nanorods are lying on SiO_2 substrate with a monolayer of graphene deposited on it. This effect results from an increase of the effective surrounding medium. In order to discuss the possible momentum matching between the antenna and GPs, it is interesting to have a look to the Fourier transform of the antenna near field. Figure 4.12 shows this Fourier space at the wavelength of 11 microns both in air and on a SiO_2 substrate.

In the case of suspended graphene, the theory (Equation (3.23)), shows a large wavevector mismatch between the incident light and the Graphene Plasmons in this range of frequencies. At 11 microns, we can calculate the localization parameter λ_0/λ_p , which varies as a function of the graphene layer Fermi level. For $E_F = 0.3\text{eV}$ and a mobility $\mu = 1\text{m}^2\text{V}^{-1}\text{s}^{-1}$, using the DC relaxation time approximation, $n_{eff} \approx 26$.

The momentum of the near field of the nanorods in the air, enhanced by the LSPR, matches the GPs, shown on figure 4.12(a). However, when the rod is deposited on SiO_2 substrate, we expect a lower near field intensity due to the absorption of the energy by the substrate. As seen figure 4.12(b), the system nanorods-substrate is able to excite the GPs. For graphene with $E_F=0.3\text{eV}$, it matches well the

Figure 4.12: Fourier transform of the near field of different systems at 11 microns with the system's illustrations on the top. The red arrows represent the polarisation and the magenta arrow represent the direction of the light. In (a) case of a nanorod in air, in (b) case of a nanorod on a SiO₂ substrate.

angular momentum of the nanorod on substrate. It is also interesting to note that the small width of the nanorods along y results in enhanced wavevector components perpendicular to the nanorod axis and not much along the long axis, which would make it integrable in a grating. To go further, a comparison of the near-field of an antenna with and without graphene is studied.

The local near field enhancement of single rods, with and without graphene, is investigated for a wavelength of 11 microns. It has been shown that the plasmons damping from SiO₂ substrate is the lowest [42] around this wavelength. The configuration with graphene is depicted in Fig. 4.13. A gold nanorod having a rectangular cross section lays on a SiO₂/Si substrate. The nanorod is surrounded by both suspended and supported graphene for comparison. Again, a pulse is exciting the nano-antenna, with a propagation direction normal to the surface of the sample, along z . The incident light was linearly polarized with the electric field parallel to the rod long axis (x axis). This pulse wavelength spectra is going from 8 μm to 12 μm , with a central wavelength around 10 μm . The optical properties of the gold nanoantenna are characterized numerically in order to fit the energy and momentum conditions of GPs.

Following [5, 107], the length L and the width W of the rod are first set to 2.9 μm and 0.6 μm respectively. The nanoantenna section is therefore rectangular, and its thickness is set to 100 nm. Figures 4.14 show maps of electric field intensity around the rod under illumination, without any graphene (Fig. 4.14(a)), and with graphene (Fig. 4.14(b)). The near-field distributions displayed on these figures unambiguously show the dominant mode excited at this wavelength is dipolar in nature. On Figure 4.14(b), two calculated top-view half-maps of the electric field intensity at the graphene layer plane are shown. For $y > 0$, graphene is suspended, meaning that the graphene is surrounded by air. For $y < 0$, graphene is sandwiched between air and the SiO₂ substrate. The rod is localized at $y=0$ nm, acting as a resonant dipole along L . The graphene is defined *via* its 2D conductivity by a Fermi level of 0.2 eV, a temperature of 300 K and a mobility 1 $\text{m}^2\text{s}^{-1}\text{V}^{-1}$. The parameters set for the graphene will be discussed in the next

Figure 4.13: Illustration of a plasmonic antenna lying on a SiO_2/Si substrate + graphene. A trench is etched at the vicinity of the golden rod through the SiO_2/Si substrate. A monolayer of graphene is deposited on the top of the trench, and a gold nanoantenna is deposited on the edge of the trench. A part of the graphene is therefore suspended.

section.

In Figure 4.14(b) the spatial decay of the intensity of propagating plasmons in graphene can be observed. Whereas suspended graphene leads to clear propagation of GPPs (Fig. 4.14(b), top), it is not possible to distinguish this propagation in the case of graphene supported by SiO_2 substrate (Fig. 4.14(b), bottom). In the latter case, we clearly see a dipole-like resonance of the antenna at $11\ \mu m$, comparable to the one observed in Figure 4.14(a), that does not launch any efficient GPP (in terms of propagation length).

It is interesting to notice that the intensity tends to vanish along a vertical line ($x=0$) where destructive interference occurs between the phase shifted graphene plasmons that have been launched by the two out-of-phase antenna extremities. Such effects could open new routes to GPPs designing and engineering.

Over a second phase, the question of the dimension of the nanostructure have been addressed. So far, the geometrical parameters were chosen by considering recent studies [5, 107], where the main parameter used for optimizing the coupling is the electric field average. This factor reflects the significant near-field enhancement, consequence of the Local Surface Plasmons Resonance (LSPR) of the antenna, that is most likely to launch strong plasmons in graphene. As shown before, the current geometry could be used for our purpose, but [5] only optimized the GPPs coupling efficiency through variations of the antenna length. The rod width is another parameter that could be explored to increase the coupling efficiency even further. As such, we followed the same path as [5], but performed a parametric study of the near-field average $|E|/|E_0|$ for different lengths and widths of rods lying on a SiO_2/Si substrate without graphene. The results are shown on Fig. 4.15. The near-field average $|E|/|E_0|$ was taken at a 10 nm distance from the rod surface. The big axis L was set from 1.6 to $3.2\ \mu m$, whereas the width W was chosen from 0.05 to $1.15\ \mu m$. A maximum of the electric near-field average is found for a micro antenna length of $2.5\ \mu m$ and a width of $50\ nm$. Taking advantage of this results, simulations with suspended graphene are done to compare the new geometrical parameters coupling efficiency to the literature, and to study GPPs modes in details.

Figure 4.14: Comparison of the response of a gold nanoantenna, lying on Si/ SiO_2 substrate, without (a) and with (b) graphene, through electric field intensity maps. (b) The graphene is suspended on the half-top, and supported on SiO_2 substrate for the half-bottom. The color scale is clamped in order to see the plasmon propagating through graphene. The $2.9\ \mu m$ long, $600nm$ width, gold nanoantenna is in the center of the maps, at $y=0$, from $x=-1.45$ to $x=1.45\ \mu m$. The wavelength excitation is $11\ \mu m$.

Figure 4.15: Map of the electric near-field average, $|E|/|E_0|$, calculated at the vicinity of rods of length L and width W , without graphene, for an excitation at $\lambda_0 = 11\ \mu m$.

Figure 4.16 shows $\sqrt{|fft(E_x)|^2 + |fft(E_y)|^2 + |fft(E_z)|^2}$, the spatial Fourier Transform of the near-field for the case of two different antenna geometry, with suspended graphene. Figure 4.16(a) shows the case of geometry dimension taken from [5, 107], that is the Fourier transform of the near field plotted in Fig. 4.14(b) top-half, for positive (k_x, k_y) . It turns out that the nanorod launches two main surface plasmons: one in the x direction (peak 2), parallel to the antenna axis, and one in the y direction (peak 1), perpendicular to the nanorod axis. They are illustrated by the two maxima along the quarter-circle of radius $\sqrt{k_x^2 + k_y^2}/k_0 = |k_p|/k_0 = \lambda_0/\lambda_p = 43$. These results were easily predictable from Fig. 4.14(b), where the two main GPPs propagations can also be observed, parallel and perpendicular to the rod long axis.

Figure 4.16(b) is using the geometry found in our parametric study (Fig. 4.15). It can be observed

a strong enhancement of the peak 1, corresponding to GPPs launched in the y direction, whereas the peak 2 disappears (GPPs launched in the x direction). It is concluded that the new dimensions of the rod, following the investigation of the electric near field average (Fig. 4.15), lead to a stronger coupling to the GPPs launched in the y direction, resulting in an increased effective propagation length in this direction. In the future, the coupling efficiency of graphene plasmon propagating along the y direction could be also strongly enhanced by using a set of parallel nanoantennae to form a resonant grating.

However, the coupling may still not be optimal. After all, the optimization was performed without graphene, by using the average field exaltation as a criterion. Yet as shown here, the true value of the coupling efficiency can only be found by actually computing the whole system, and investigating the Fourier transform. As such, we decided to refine our optimisation once again.

Figure 4.16: Maps representing $\sqrt{|fft(E_x)|^2 + |fft(E_y)|^2 + |fft(E_z)|^2}$, the spatial Fourier Transform of the near field in the reciprocal space, with positive (k_x, k_y) , for suspended graphene, in the case of two different rods. (a) The rod length and width are set as $L=2.9\mu m$ and $W=600nm$, according to [5, 107]. (b) A rod of length $L=2.5\mu m$ and width $W=50nm$ is chosen from the parametric study Fig. 4.15. The graphene is set with a Fermi energy about 0.2 eV and a mobility of $1 m^2 s^{-1} V^{-1}$.

For this purpose, spatial Fourier Transforms of the electric fields maps at 11 microns were performed for different lengths and widths of nanorod (the height remained fixed at 100 nm). The results of this final parametric study are shown in Fig. 4.17. Figure 4.17(a) displays the intensity of the Fourier transform peak (Fig. 4.16(a), peak 2) of the graphene plasmons launched towards the x direction and Fig. 4.17(b) is the one for the y direction (figure 4.16(a) and 4.16(b), peak 1). We conclude that the best shape for the gold nanoantenna is to be as thin as possible (needle-like, see white crosses in Fig. 4.17), 50 nm here, with a length around $2.75 \mu m$, in order to efficiently launch a plasmon perpendicular to rod long axis. While the average near field intensity enhancement of the antenna may not be the highest for this shape (see Fig. 4.15), it gives a stronger coupling with the plasmons in y direction. For comparison, the black crosses in Fig. 4.17 indicate the geometry used in the papers [5, 107], Fig. 4.16(a), while the grey crosses correspond to Fig. 4.16(b) and are the result of our first parametric study (Fig. 4.15). As far as plasmon propagation is concerned, the initial geometry (black cross) constitutes an intermediate case where both x and y directions are possible, and the field energy is thus distributed into both.

In the case of suspended graphene, the theory, from Equ. (3.24), shows a large wavevector mismatch between the incident light (characterized by k_0) and the Graphene Plasmons (characterized by k_p) in this range of frequency, illustrating the need for an antenna. At $11 \mu m$, we can calculate the localization parameter, or effective index, λ_0 / λ_p , which varies as a function of the Fermi level of the graphene layer. For $E_F = 0.2eV$ and a mobility of $1 m^2 s^{-1} V^{-1}$, we find $real(n_{eff}) = 42.3$ from the dispersion relation (Equ. (2.65)), whereas the FDTD simulation gives an average value of $43 (\pm 1.1)$ from the Fourier transform maps. Moreover, the propagation length L_p is about 558nm theoretically (from expression

Figure 4.17: Maximum intensity of (a) the peak 2 and (b) the peak 1, the plasmons wavenumber k_p (see Fig. 4.16(a) as an example), in the x direction and in the y direction respectively, as a function of the length and the width of the nanorod. The black cross indicates the dimension of nanoantenna used in papers [5, 107], picked up from Fig.4.16(a). Grey cross corresponds to the antenna dimensions resulting from the parametric study Fig. 4.15. White cross is showing optimum dimensions for launching plasmon in y direction.

(2.65)) whereas we find 532nm in our simulation. It is interesting to notice that we have here already a longer propagation length as compared to SiO_2 supported graphene. Using data from [158], the longest propagation length would be less than 300nm for SiO_2 -supported graphene.

Figure 4.18 shows the optimized system for GPPs launched along the y axis, with the antenna geometry defined in Fig. 4.17 (white crosses). Compared to figure 4.14(b), figure 4.18 shows clearly that a rod with a length of $2.75\mu m$ and a width of 50nm can launch stronger GPPs that propagates on suspended graphene over a distance nearly $3\mu m$ away from the rod, normalized on the incident excitation. Again, supported graphene ($y < 0$) does not enable significant GPP propagation.

As we have shown in this study, the geometry used in [5, 107] does allow for a non-negligible GPP coupling efficiency. Yet, that geometry had a fixed rod width, which could be optimized. Ultimately, our work showed that an accurate optimization of the coupling efficiency requires simulating the full system, and investigating the energy the plasmon actually carries through Fourier Transforms of the near field. Quantitatively speaking, this method is superior to investigating the average field only, as the average near field contains many k components that cannot be coupled to a GPP.

However, the average field criterion needs not to be dismissed entirely. As we showed here, it still leads to a good first order approximation of the optimal coupling efficiency. Unlike our criterion that requires numerical methods to compute the full system, the average field criterion can be used with analytical or pseudo-analytical models, or numerical methods or softwares that are unable to simulate the full system, graphene being fairly hard to simulate.

A third parameter that has not been explored is the nanoantenna thickness, but this parameter also affects the distance between the graphene sheet and the substrate, and can then not only affect the antenna behavior, but also the properties of graphene itself.

4.4 Designed system based on GPP

In equation (3.30), we can notice the importance of the Fermi energy for achieving a long propagation. However, it has been shown that suspended graphene is free of intrinsic doping [16]. Suspended graphene

Figure 4.18: Map of the electric near-field intensity, for an excitation of $11\mu m$, with a limit up on the color scale in order to see the plasmon propagating through graphene. The nanoantenna is on the center of the map. The graphene is set with a fermi energy about 0.2eV and a mobility of $1\text{ m}^2\text{s}^{-1}\text{V}^{-1}$. The dimensions of the rod are optimized for strong plasmons in y-direction : $L=2750\text{nm}$ and $W=50\text{nm}$.

is thus characterized by a limited value for E_F while high mobility constitutes a clear asset that has been discussed in many published papers [37, 109, 150]. The design of a nano-opto-electronic system based on suspended graphene should take into account this issue. Therefore, it is important to engineer a system offering the possibility to inject charges through electrostatic gating. On SOI substrates with 300nm of SiO_2 layer, it has been shown the possibility to apply a difference of potential V_g higher than 100 V [117]. Modeling a capacitor between graphene and a Si-doped substrate, with two dielectric layers in between (SiO_2 300 nm , air 100 nm , see inset of Fig. 4.19), we can estimate a Fermi energy through:

$$E_F = \hbar v_f \sqrt{\pi n} = \hbar v_f \sqrt{\pi \frac{C_g V_g}{e}} \quad (4.30)$$

$$C_g = \left(\frac{1}{C_{air}} + \frac{1}{C_{\text{SiO}_2}} \right)^{-1} \quad (4.31)$$

$$C_{air} = \frac{\epsilon_{air} \epsilon_0}{t_{air}} \quad \text{and} \quad C_{\text{SiO}_2} = \frac{\epsilon_{\text{SiO}_2} \epsilon_0}{t_{\text{SiO}_2}} \quad (4.32)$$

Where t_{air} and t_{SiO_2} are the thicknesses of the air gap and the SiO_2 layer respectively and C_g is the total capacitance. We find $E_F \approx 0.2\text{eV}$, which justifies the value used in the previous section. In addition, the gate potential that is applied between the graphene and the substrate will bend the graphene, resulting in a deflection height h_0 that is a function of C_g , and therefore of the dielectric used. It is also a function of the potential applied V_g , and of the length of suspension L . At high V_g , we can obtain the deflection as [14] :

$$h_0 = \left(\frac{3PL^4}{64Et} \right)^{1/3} \quad (4.33)$$

$$P = \frac{c_g^2 V_g^2}{2\epsilon_0} \quad (4.34)$$

With E the Young's modulus in graphene which is about 1TPa , L the length of the trench, t the thickness of graphene, which is taken as 3.410^{-10}m , and P the electrostatic pressure.

Let us consider the case where the graphene is suspended straight over a 100 nm deep gap, (therefore with a maximum deflection h_0 of 100nm), and a gate voltage V_g of 100V . This enables us to design

Figure 4.19: Illustration of the system studied. A plasmonic antenna is lying on a SiO_2/Si substrate. Gold nano-slots gather and propagate the plasmons on suspended graphene. The slab has also a function of electrode for doping suspended graphene. Gold, Si and SiO_2 are represented in yellow, gray and blue respectively. Inset : a schematic side view along the x axis of the graphene, golden slot and substrate. The deflection height h_0 is also represented.

a trench of $L=1.5\mu m$, using Equ. (4.33). Taking into account the penetration depth of the plasmons in air which is $1/Re(k_p) \approx 41nm$, we have a maximum deflection depth $h_0 = 60 nm$, and a maximum suspension length around $L = 1\mu m$ so that the plasmon experiences damping from the substrate that is as weak as possible.

As a result, we propose the design (see Fig. 4.19) of a realistic sample taking into account the different experimental parameters discussed above. On a $SiO_2(300nm)/Si$ substrate are lying gold structures, which will also play the role of suspension support for graphene, including a nanorod (antenna) for launching GPPs. The thickness of the gold support and the gold antenna is set at 100nm. The gold nanoantenna is embedded in a gold nano-slots system perpendicular to the rod long axis. Gold slots, of 300nm of width, are designed to both suspend graphene and guide GPPs perpendicular to the rod long axis, opening the route to future nanophotonic circuitry based on suspended graphene.

Four slots are designed: two per extremities, in positive and negative y-directions. The extreme left and right parts of the gold constituting the slots are designed to be used as electrodes, in-between which voltage can be applied to generate current in gold. High temperature induced in graphene by high current (joule effect) have shown an improvement of the mobility in suspended graphene, by evaporating the thin layer of pollutants stuck on the carbon sheet after graphene transfer. In fact, electric current is one of the most efficient way to heat the graphene up and clean it, in order to get the best mobility in suspended graphene [150]. Finally, a difference of potential V_g is applied between the Si doped substrate and the graphene, in order to raise the Fermi energy.

Suspended graphene should be free of charge impurities, strain, corrugation and remote interfacial phonons induced by the substrate. The mobility, which is the physical value taking into account those effects, would increase up to one order of magnitude [137] in suspended graphene, as compared to the one on Si/SiO_2 substrate. However, studies predicted that the graphene mobility at $T>10K$ could not exceed $4 m^2s^{-1}V^{-1}$ [22, 103] due to flexural phonon. Recent reports measured a mobility of $1.5 m^2s^{-1}V^{-1}$ [37], up to $38 m^2s^{-1}V^{-1}$ [150] in CVD suspended graphene at low Fermi level. Therefore, $1 m^2s^{-1}V^{-1}$ for a Fermi energy of 0.2eV in suspended graphene is reliable.

Figure 4.20: Map of the electric field intensity, for an excitation of $11\mu m$, with a limit up on the color scale in order to see the plasmon propagating through graphene. The nanoantenna is on the center of the map ($y=0$). We have set the graphene with a Fermi energy of 0.2eV and a mobility of $1\text{ m}^2\text{s}^{-1}\text{V}^{-1}$.

Similarly to Fig. 5, we simulated this system, and obtained the map of the electric field intensity shown in Fig. 4.20. In the middle of the map is laying a gold nanorod of $2.75\mu m$ of length, 80 nm of width, and 100nm high which is launching plasmons perpendicular to the long axis (x axis) of the antenna, i.e. along the y axis. As we have seen in Fig. 4.17, the width of the rods should be as small as possible. However, the height of the nanorods is set to 100nm in order to safely suspend graphene. For mechanical stability reasons when the rod is fabricated from e-beam lithography, it is better to consider a width comparable to the height of the structure. Therefore, we choose a width of about 80nm . The length of the rod corresponds to a resonance around 11 microns, where graphene plasmons exhibit an optimal propagation.

Figure 4.21: Maps of the intensity (on the top) and the phase (on the bottom) of the (a) x component of the electric field, and the (b) y component of the electric field of the system described in Fig. 4.19

While figure 4.20 reveals the micronic propagation of the total photonic intensity, Fig. 4.21 provides

more informations. Two modes are appearing in the channel.

Figure 4.21(a) shows a TE-like mode where we have a strong intensity of the x component of the electric field. This mode is partly supported by the edges of the gold slots and is likely to result from coupling between GP and sharp metal edges. Properly speaking, it does not correspond to any pure GP modes.

Figure 4.21(b) shows a TM-like mode, which is the GP mode, with a strong intensity of the y component of the electric field. That is the surface plasmon mode of interest. It is worth noticing that the associated wavefronts, shown in the bottom of Fig. 4.21(b), are clearly defined by the phase giving a GPPs wavelength $\lambda_p \approx 265nm$, that is to say an effective index $n_{eff} \approx 41.5$. Again, the plasmons propagation length can be calculated using the exponential decay of the electric field along the slab. In case of Fig. 4.21(b) we find L_p higher than 525nm. The reduction of the propagation length is due to the confinement of the GPs path by the golden slots. Reducing the width of the slot decrease the propagation length.

As pointed out above, higher propagation length in supported CVD graphene, fabricated on industrial scale, could be achieved with higher Fermi energy. In the case of chemically doped graphene, taking data from [158], we find a propagation length of 1 micron in suspended graphene, as compared to the 300nm with graphene on SiO_2 , using the same data.

Moreover, it is worth noticing that the propagation length is a conservative value. For example, the propagation length for the system described above is about 540nm. However, as we can observe Fig. 4.21(b), the intensity of the electric field decreases to the intensity of the incident light at a distance of 2 microns, that is to say about 4 times the propagation length. This phenomenon results from the field enhancement at the launching resonant nanoantenna.

Finally, it is important to notice that the simulations here do not take into account the interaction between graphene and metal. It is known that metal can transfer charges at the vicinity of graphene, which could lead locally to highly doped graphene, as high as 0.4eV [141]. As we have seen in Equ. (3.30), the propagation length of graphene plasmons is directly proportional to the Fermi energy. Therefore, it could lead to a strong increase of the GPPs propagation.

4.5 Discussion on the applied electric field

An issue uncovered by this study may arise at the experimental step. Indeed, the FDTD does not take into account electro-static phenomenon, such as electrostatic gating. To implement the injection of charge in graphene, the Fermi level parameter has to be changed. However, the electric and magnetic fields in the different materials are not taken into account.

In order to inject charges in graphene, the effective electric field applied to the graphene is huge. Therefore, the dielectric materials in between graphene and the back gate conductive layer should tolerate high electric field amplitude. The property related to the maximum electric field that a material can withstand without breakdown is called the dielectric strength. The dielectric strength of air is about 3MV/m. For a good thin film of SiO_2 , it is possible to have 500MV/m. H-BN dielectric breakdown has been recorded as high as 1GV/m [60, 74]. Diamond can theoretically reach higher value [51, 156]. And finally, ideal vacuum would withstand electric field as high as 10^9 GV/m.

Coming back to the case of the study developed in this chapter, Section 4.4, the system is made of a double layered dielectric sandwiched in between graphene and conductive substrate (Figure 4.22). In order to be effectively accumulating charge in graphene, it has been anticipating breakdown phenomenon. As such, the amplitude of the electric field inside the SiO_2 have been kept away from its dielectric strength, 0.5GV/m. Doing so, the capacitor effect would take place. Unfortunately, the thin layer of air would be able to disrupt the injection of charge in graphene.

When looking at the electric field in the different materials i for such system composed of a serie of capacitor, it is possible to write :

$$\mathbf{E}_i = \frac{C_{tot}}{d_i C_i} V_{tot} \quad (4.35)$$

with $\mathbf{E}_i$ the electric field amplitude in the dielectric i , d_i the thickness of the dielectric layer i , C_{tot} and C_i the capacitance of the total system in Farad per square meter $F.m^{-2}$, and V_{tot} is the difference of potential at the extremities of the multilayer.

$$C_i = \frac{\epsilon_0 \epsilon_i}{d_i} \quad (4.36)$$

Inserting Equation 4.36 into Equation 4.35 leads to :

$$\mathbf{E}_i = \frac{C_{tot}}{\epsilon_0 \epsilon_i} V_{tot} \quad (4.37)$$

The electric field amplitude in the dielectric medium i is therefore given by Equation (4.37). Higher is the static dielectric constant, and lower is the electric field. To come back to our system made of SiO_2 and air, the electric field will be much higher in the air. However, in order to get significant injection of charge in graphene, we stand at the limit of voltage breakdown in the SiO_2 . The disruptive electric field in air being lower than the one in SiO_2 , it will ionized the air. The air will become a plasma that will bring the charges down to the surface of the SiO_2 , instead of staying in the suspended graphene. The graphene will therefore be screened and will not be significantly impacted by the electrostatic gating.

Figure 4.22: Schema illustrating the double layer capacitor used in the study of suspended graphene in Section 4.4. Two thin dielectric layer, air and SiO_2 , are sandwiched in between two electrodes which are the graphene and heavily doped silicon substrate Si^{++} . A potential of 150V is applied in order to preserve the system of breakthrough, in a part of the sample where graphene is supported on SiO_2 . The amplitude of the electric field in the two dielectric layers is displayed on the right.

4.6 Conclusion

In this chapter, a numerical study has been presented. After a brief history of the FDTD method, the solution for a one-dimensional wave problem has been solved, and then applied to the 3D case. The technique to modalize graphene in a 3D simulation has been discussed. Finally, a design is proposed, using an antenna, to study the propagation of plasmons in suspended graphene.

Some concerns about the feasibility of such experiment at ambient condition are giving rise to a final discussion. A set up using a vacuum chamber would be probably required to obtain the measurement expected in laboratory.

Chapter 5

Experimental procedure

In this chapter, the experimental achievements are described. Chronologically, the fabrication of the first samples have been done in parallel with the numerical simulation work presented in Chapter 4. Therefore, a part of the work explained below was in fact a first try in order to record plasmons in suspended graphene and the design may be a bit different than the one shown in the previous chapter. Also, it did not take into some issues evoked in section 4.5, where it is explained why a high vacuum Scanning Near-field Optical Microscope (SNOM) set up is required for recording plasmons on suspended graphene.

Therefore, as a first step, the process to fabricate sample with suspended graphene is described, as well as the different designs that we wanted to study. Then, characterisation of the graphene based on Raman spectroscopy is discussed. Finally, preliminary SNOM data will be shown and commented.

5.1 Fabrication of sample

The samples with suspended graphene have been fabricated in Singapore, at the Center for Advanced 2D Materials.

5.1.1 Fabrication process of suspended graphene samples

Hereafter are presented the patterns fabricated with the goal to observe and measure the propagation distance of Graphene Plasmons.

Figure 5.1: Illustration of the design for the study of graphene plasmons through suspended graphene. The graphene monolayer (red) is on the top of the system. The gold nanostructures are in yellow and the substrate is made of SiO_2 (300nm, light blue)/Si(dark blue) doped wafer.

A preliminary set of simulations have shown the possibility to excite graphene plasmons with plasmonic structures (see Section 4.3). The shape that was theoretically studied is that of the simple rectangular antenna, and the first results gave micrometric length and several tens of nanometers for the width and height. An illustration of the sample is showed Figure 5.1a. The graphene layer is deposited on the top of the system that is made of the substrate, the gold plasmonic nano-structures and the holes.

One of the challenges of fabrication process was to place the antenna really close to the holes in order for plasmonic structures to launch plasmons in suspended graphene (see Figure 5.1b). Gold scatterers were placed on the opposite side of the holes: small gold disks were designed to perturb in propagating graphene plasmons and to scatter them into the far field.

In order to avoid the pollution of the graphene layer, it has been decided to deposit it on the sample at the last step. Therefore, the fabrication process of such samples begins by etching holes and depositing plasmonic structures as close as possible to the edges. Since the structures need to be firmly bounded to the surface when the graphene is added at last, colloidal suspension has been dismissed. The rod being needle-like from the simulations results (section 4.3), e-beam lithography has been used. The holes are micrometric because the propagation length of the plasmons is expected to be micrometric. Therefore, photo-lithography (laser writer) has been used to pattern the holes, as the resolution of this technique is a bit less than a micron.

(a) Optical image of a pattern of holes, designed by laser writer and etched through RIE process. A thin layer of PMMA have been then deposited on the surface in order to perform e-beam lithography. The difference of colors represent gradient of the PMMA layer thickness.

(b) MEB image of a disk shaped hole with gold antenna (rods) and scatterers (dots)

Figure 5.2: Optical (a) and electronic (b) image from microscope of the samples fabricated for the study of Graphene plasmons in suspended Graphene

At first, micrometric markers (crosses with number) and holes have been designed on samples through laser writer technique (see Figure 5.2a). In fact, the resolution of the system LW405-A is about 700nm and has been tested. The wavelength of the writing beam is 405 nm (GaN solid state laser). The markers have been added in order to make possible a nanometric alignment with the e-beam lithography process. This technique is used to deposit the plasmonic nano-antenna close to the holes.

Thanks to a laser interferometer system that guides the stage of the laser writer, the markers are aligned with a resolution below 100 nm. The holes were designed with different shapes, including rectangles, disks and triangles. Finally, the biggest issue was to find out a way to deposit plasmonic structures as close as possible from the hole edge.

As it can be observed in Figure 5.2b, the nanostructures are not present (neither diffusers nor antennas) near the hole edge. They are present few microns away from the holes (red arrows). However, it is possible to observe carefully on the Figure 5.2b those structures left "footprint" near the holes.

This is the first major issue when fabricating the samples. When looking at the PMMA resin before the e-beam lithography process (Figure 5.2a), it is possible to see different color at the vicinity of the holes. It is supposed that the following scenario happened.

When the PMMA is spin coated on the whole pattern, it is deposited on all the surface of the sample. As we get closer to the holes, the thickness of the resin is getting thinner (Figure 5.3(a)). Then, during the development step, the PMMA exposed at the vicinity of the holes is removed (Figure 5.3(b)). Finally, the gold is deposited on the top of the sample (Figure 5.3(c)). But the problem is that the gold layer thickness is of the same height than the PMMA layer at the vicinity of the hole. This is the reason why, during the lift-off step, the structure closer to the holes were removed (Figure 5.3(d)). However, the footprint let by the antenna and diffusers at the end of the process can still be observed in Figure 5.2b.

Therefore, the deposition of the plasmonic structures has to be done at first, and then, the holes have been etched through Reactive Ion Etching (RIE).

In order to reduce the duration of e-beam lithography process, the markers have to be done at first by laser writer. Therefore, the main steps of the process have been plan as follows :

1. Markers designed by photo-lithography (laser writer)
2. Deposition of metal for the markers
3. Nano-structures designed by e-beam lithography
4. Deposition of metal for the nano-antenna
5. Pattern of holes designed by photo-lithography (laser writer)
6. Etching of the holes through RIE

The six steps listed above are detailed in the following pages.

Figure 5.3: Illustration of the main problem that was encountered during the fabrication of the samples dedicated to the study of graphene plasmons.

Step 3 : Etching (see step 11) or
deposition of metal (see step 6)

Step 4 : Preparation of sample for
electron beam lithography

Measurement of the coordinate of the first
marker (top-right)

Using an optical microscope

Deposition of 200 to 300nm of PMMA

Spin coating parameter : 495/950 PMMA A4
at 4000rpm for 70 seconds

Post bake at 180°C for 2 mins

Step 5 : Electron beam lithography of the
plasmonics nano-rods and the diffusers

Load the sample and calibrate the electron
beam lithographer

.gds to .v30, create the jdf files and the sdf
files :

Jdf (marker from the center, 1100 to 1200
microC/cm2,.v30name, ebl mode
(NUS_400pA_AP5))

Sdf(n° cassette, name of jdf, manual
alignment, offset value (first cross
coordinate))

Convert the files and create the file mgn

Execute the .mgn file and align manually to
the cross

MIBK:IPA (1:3) for 30 sec
IPA for 30 secDI water for 30 sec
N2 blow

Development

Step 6 Deposition of metal by thermal evaporation

Vent the vacuum chamber and load the sample

Add metals in the boat(s) and pump down

Edit the deposition parameter

Take back the samples and lift off

Check the result at the optical/electronic microscope

Fix the sample using kapton tap

For Cr/Au deposition
Cr thickness : 0.050 μm
Cr deposition rate : 0.3 $\text{\AA}/\text{sec}$
Au thickness : 0.500 μm
Au deposition rate : 0.8 $\text{\AA}/\text{sec}$
Vacuum : $2\text{e-}6$ to $8\text{e-}7$ Tor

Immersed the samples in acetone for several hours with ultra-sonication about 5-10% at 25kHz

Step 7 : O₂ plasma cleaning with RIE

Load the sample in the vacuum chamber

Pump down and start the exposition

Time : 15 minutes
Power : 50 Watt
Gas flow : 50 sccm

Step 8: Deposit of photoresist (see step 1)

Step 9: Photolithography for etching holes (see step 2)

Align to the cross using the mode A+B

Step 11 : Deep RIE etching for digging holes

Vent the vacuum chamber and fix the samples on the bare Si wafer

Fix the samples using oil for better temperature transfer

Pump down the chamber and edit the duration in order to etch 300nm of SiO₂

Edit : time 02min 36 sec

STEPS	DESCRIPTION	RF	ICP	CHF3	Ar	Press	Time	HC
		W	W	sccm	sccm	mT	min	T
Step 1	Pump down	0	0	0	0	0	1'	0
Step 2	Preheat	0	0	0	40	5	3	8
Step 3	Etch	100	650	25	40	5	xxx	8
Step 4	Pump down	0	0	0	0	0	1'	0

Vent the vacuum chamber and fix the samples on the SiO₂ wafer

Fix the samples using oil for better temperature transfer

Pump down the chamber and edit the number of loops in order to etch 20 μm of Si

Edit : repeat 38

STEPS	DESCRIPTION	RF	ICP	C4F8	SF6	Press	Time	HC
		W	W	sccm	sccm	mT	min	T
Step 1	Pump down	0	0	0	0	0	1	0
Step 2	Temp stable	0	0	50(O ₂)	0	15	3	8
Step 3	Strike	100	1000	50(O ₂)	0	15	5"	8
Step 4	Repeat xx							
Step 5	Dep	3	1300	100	5	40	5"	8
Step 6	Etch	30	1150	5	100	30	7"	8
Step 7	Loop							
Step 8	Pump down	0	0	0	0	0	1	0

Take back the samples and lift off

Immersed the samples in acetone for several hours with ultra-sonication about 5-10% at 25kHz

Check the result at the optical/electronic microscope

5.1.2 Patterns for the study of GPs propagation

Different patterns have been fabricated in order to investigate different platforms for the propagation of graphene plasmons. Several hole designs have been made. The shape of the trench have been made triangular (figures 5.4a and 5.4b), circular (5.4c and 5.4d) and rectangular (5.5a and 5.5b).

(a) Hole with small triangular shape

(b) Hole with large triangular shape

(c) Hole with small circular shape

(d) Hole with large circular shape

Figure 5.4: SEM pictures of the samples fabricated in the clean room following the simulation set up. We can see on the different images gold nano rods of $4 \mu m$ on one side of the holes, and gold diffusers, tubular shape, on the other side of the holes.

These holes are designed to suspend graphene on them. Thanks to a highly accurate alignment between e-beam lithography and photo-lithography, plasmonics nano rods and scatterers are placed very near the edges of the holes. As already emphasized, one of the most challenging part of the fabrication were to deposit the nano structures as close as it is possible from the holes. In fact, once the graphene is deposited on the whole structure, shown in picture 5.1b, it is important that the monolayer does not touch the substrate in between the gold structures and the holes. As we can see on the different SEM pictures (Figures 5.4a, 5.4b, 5.4c, 5.4d, 5.5a, 5.5b, 5.6a, 5.6b), the final results is promising and we can expect graphene to be suspended over the whole structures.

(a) Hole with small rectangular shape

(b) Hole with large rectangular shape

Figure 5.5: SEM pictures of the samples fabricated in the cleanroom following the simulation data. We can see on the different images gold nano rods of $4 \mu m$ on one side of the holes, and gold scatterer, circular shape, on the other side of the holes.

5.1.3 Patterns to study the wavelength of the SPPs

A second set of patterns have been made to study the wavelength of the SPPs through graphene via interferences phenomena.

With the patterns shown Figures 5.6a and 5.6b, we have the possibility to launch two plasmons from two different points.

(a) Small triangular shape of hole

(b) Small triangular shape of hole

Figure 5.6: SEM pictures of the samples fabricated in the clean room following the simulation set up. We can see on the different images gold nanorods of $4 \mu m$ on one side of the holes, and gold scatterers, circular shape, on the other side of the holes.

These two surfaces waves would interfere with each other and an interference pattern may appear at the scatterers. In fact, we would expect to have an interference pattern that would be sampled by the scatterers, showing intense hot spots at the constructive interference location, and weak response at the destructive interference location. Similarly to the double-slit experiment (referred to as Young's experiment), we can describe our experience as shown in Figure 5.7.

Assuming that a plane wave excites the two stacks of nano rods with the same phase, at the same time, two SPPs will be launched as illustrated in Figure 5.7. Simplifying the problem to a two slits problem (see figure 5.7 on the right), the phase of the waves at the position x on the observation screen

Figure 5.7: Illustration of the interference pattern. Left: realistic representation with the fabricated sample. Right: the problem simplified

(at the scatterers plane) relative to the phase at the slit (at the plasmonic nano rods) can be written as :

$$\Phi_{1,2} = kl_{1,2} = \frac{2\pi l_{1,2}}{\lambda_{SPP}} = \frac{2\pi}{\lambda_{SPP}} \sqrt{L^2 + \left(\frac{s}{2} \mp x\right)^2} \quad (5.1)$$

Therefore, the phase difference between the two waves can be written as:

$$\Delta\Phi = \frac{2\pi}{\lambda_{SPP}} \left(\sqrt{L^2 + \left(\frac{s}{2} - x\right)^2} - \sqrt{L^2 + \left(\frac{s}{2} + x\right)^2} \right) \quad (5.2)$$

For a period of $\Delta\Phi = 2\pi$, we should find the inter distance between the interference fringes via the following equation to be solved :

$$\left(\sqrt{L^2 + \left(\frac{s}{2} - x\right)^2} - \sqrt{L^2 + \left(\frac{s}{2} + x\right)^2} \right) = \lambda_{SPP} \quad (5.3)$$

There is no trivial solution to this equation, but we can estimate numerically the results using the experimental design. According to Figure 5.6a, we can take $L=4 \mu m$ and $s=6 \mu m$. According to the numerical studied explained Chapter 4, we can suppose that $\lambda_{SPP} = 0.3 \mu m$ (see also on the phase of the y component of the electric field, in Figure 4.21). These assumptions lead to the plot shown in Fig. 5.1a

From $y=0.3$ ($\lambda_{SPP} \approx 0.3 \mu m$), we get that the period $x \approx 0.25 \mu m$. The distance between diffuser being 100 nm (see Figure 5.8b), we could expect to have 2.5 scatterers, and then 2.5 measurements of the intensity for one period of the diffraction pattern. In vacuum condition, the induced density of charge in suspended graphene could be higher. The wavelength would therefore be higher, according to Equation (3.29). And finally, Equation (5.3) would give a larger period, where the number of diffusers would be more important. For example, for $E_F = 0.4 eV$, $\lambda_{SPP} \approx 0.6 \mu m$ and we get a period of interference about $0.5 \mu m$.

5.1.4 Pattern to study the grating effect

Finally, in order to have the higher plasmons intensity possible, it would be promising to exploit the grating phenomenon, in addition to the resonance phenomenon of the nano rods. For this reason, it has been fabricated for each sample 4 different patterns with different spacings between the nano rods. In fact, the grating period of nano rods can be calculated to have an enhancement of the near field for a particular wave vector, which gives in terms of momentum (see Figure 5.9):

(a) Plot of the function $y = \left(\sqrt{x^2 - 6x + 25} - \sqrt{x^2 - 6x + 25} \right)$

(b) SEM picture of diffusers : small tubular gold structures spaced from each other by 100nm

Figure 5.8: Plot of the Equation (5.3) using experimental data from Figure 5.6a and SEM picture of the scatterers presented Figure 5.1a

$$k_{SPP} = k_0 \sin \theta + n \frac{2\pi}{d} \quad (5.4)$$

As we have seen previously in our simulation, we plan to illuminate our samples perpendicular to it. Therefore, $\sin \theta = 0$ and we find :

$$d = \frac{n2\pi}{k_{SPP}} = n\lambda_{SPP} \approx n300nm \quad (5.5)$$

So for a multiple of 300 nm, we should get a grating effect which should enhance the plasmons strength launched through the graphene. It has been therefore decided to fabricate several samples with different designs changing the spacing between the nanorods which are about 150 nm, 300 nm, 450 nm and finally 600 nm.

Figure 5.9: Illustration of the nano rods grating

5.2 Deposition of graphene and characterisation of the samples

It has been chosen to use CVD graphene which has been proved to sustain graphene plasmons [43].

As it has been explained before, the plasmons of graphene are highly sensitive to the surrounding. One of the main issue is to get really pure and clean graphene for low optical losses and high electrons

mobility. At first, CVD graphene has been deposited in Singapore (see Figure 5.10).

Figure 5.10: Optical image of the first sample where graphene has been deposited

As a first observation, it is interesting to notice that we can observe the presence of graphene on the sample, which appears to be dark blue in Figure 5.10. This apparent color results from the interference of the incident visible light occurring in the multilayer Si/SiO₂/graphene. A second remark is that the graphene is not homogeneously deposited on the surface. Instead, it covers sample with isles. Finally, the samples are covered by pollution chemical rest from the transfer process.

To address this issue, the samples have been chemically washed in order to remove impurities. Three baths have been prepared, one of acetone, one of isopropanol and one of deionized (DI) water.

Unfortunately, the results were not good enough and a lot of pollution remained, even after many washing cycles. A big part of the contamination is most probably coming from the PMMA. In order to have more information about the contaminant, Energy Dispersive X-Ray Spectroscopy (EDS) investigation has been performed.

This method consists in illuminating a sample with an incident x-ray light. Electrons in the inner shell of the atoms gets excited and the subsequent de-excitation of the particle is radiative. The photons emitted by the excited atoms are recorded and give spectra corresponding to a chemical entity. The results of this study is presented Figure 5.11.

Figure 5.11: EDS screen shot where we have : on the left, a SEM picture of the zone studied, on the right, the spectrum obtained.

On the left of Figure 5.11, a SEM picture of the sample with a strong tilt. The tilt is important in order to maximize the response of the impurities. On the right side, the obtained spectrum is displayed, with the different chemical elements indicated above the different peaks. A high peak of carbon is visible, which comes from a carbon tape covering the sample holder. Carbon tape has been used in order to screen the response of the metallic holder where part of the electrons beam could land. Pronounced peaks corresponding to Silicon and Oxygen can be seen too. Both of them come from the substrate made of heavily doped silicon and a thin layer of silicon oxide. Additionally, even if the holder has been protected, it still gives an optical response as the aluminum and the copper are present on the spectrum. Finally, Chlorine is much more unexpected here, and it is not possible to remove it easily.

A washing attempt has been performed, using Reactive Ion Etching (RIE). RIE is a technique that uses a plasma to etch the sample surface. In our case, oxygen plasma has been used. It removed partly the impurities. However, graphene turned out to be removed too. As a backup strategy, new samples have been sent to our partner in Korea (Thanks to Rafael Salas (UTT, France) et Heejun Yang from Sungkyunkwan University (Korea)) for a successful deposition of graphene layer.

Figure 5.12 shows an optical image of a new sample with CVD graphene from Korea on the top of it.

Figure 5.12: Optical microscope image of a pattern for the study of plasmons in suspended graphene with graphene deposited on the top from Korea

Here we can not distinguish any impurities, and graphene (that looks blue) is deposited homogeneously on the top of the sample, in many locations. The next step is to characterize it.

5.2.1 General presentation of Raman spectra in graphene

With Electron-Beam Microscopie (MEB) and Atomic Force Microscopie (AFM), Raman spectroscopy is a routine technique used to characterize graphene [155]. Whereas MEB and AFM may give physical images of the graphene, Raman spectroscopy can provide significantly more information based on the lattice vibration of the material. In fact, Raman spectroscopy is based on inelastic scattering of photons by matter (see Figure 5.13a), the difference of energy being recorded as the vibration of the lattice, or phonons. It is therefore a transfer of energy between the photon and phonon modes in a particular lattice, ie a kind of signature of each crystal. Because Raman spectra provide information about the lattice vibration, it is a technique that is really sensitive to the lattice structure and crystallinity. [47], the number of graphene layers[45], strain, defect and density of charges [89], among others [46]. Moreover, it is a technique that can be performed at room temperature, which is reliable, and non-destructive.

The conservation of the momentum tells us that we cannot probe phonons far from the centre of the Brillouin zone. This is due to the fact that the momentum of a photon is approximately three order of magnitude less important than the one of a phonon (wavelength of the light compared to the lattice

Figure 5.13: Illustration of the Raman effect and the phonon dispersion in graphene

parameter). But the case of graphene is a bit more complicated, and the reason why will be briefly expounded hereafter.

It is not possible to experimentally get the whole phonon dispersion curve of graphene since there are not enough atoms in graphene flakes for inelastic neutron scattering experiment. However, it is possible to calculate it theoretically (see figure 5.13b).

On this figure, we can see six branches. They are called ZA, TA, LA, ZO, LO, TO. A is for acoustic, while O is for optics. Z stands for the flexural modes (out of plane) whereas L and T correspond to the in-plane Longitudinal and Transverse modes respectively. Only some of those modes are Raman active, and the theory predicts that the LO and TO branches are active [154] in a single layer of graphene. Then, looking at the Figure 5.13b, it is possible to predict that we should have a single peak around 1580 cm^{-1} , when performing a Raman experiment (Γ point in abscisse).

This peak, called G (G stands for "Graphite"), is clearly visible in Fig. 5.14. Because of the strong interaction electron-phonon in this zero-gap semiconductor near the centre of the Brillouin zone, the G peak is strongly dependent upon doping.

Figure 5.14: Raman spectra of graphite and graphene probed at a light wavelength of 514 nm [131]

However, the graphene shows a second peak, usually called 2D (also called G' sometime, because always present in graphene), around 2700 cm^{-1} . It has been interpreted [131] as a two phonon-interaction.

In fact, the inelastic scattering of the light is responsible for the emission of two phonon having opposite wavevectors, such as the momentum is conserved. This is the reason why they are always active in any graphene layer. This 2D peak is the second order of a D peak (therefore called 2D), that means that there is a D peak which involve only one phonon, and take place at a frequency $2700/2=1350\text{cm}^{-1}$. This is due to the breathing modes of six-atom rings and requires a defect (D for defect) for its activation. Therefore, this particular peak is really sensitive to defect, grain boundary or edge (not present in the Figure 5.14). The ratio between the intensities of the D peak and the G peak provide a parameter that can be used to quantify the amount of disorder.

Several others peaks exist, mostly for defected graphene sheet. Reading and interpreting properly a Raman spectrum is not trivial, as valuable informations can be deduced from the position, the height, the area, the FWHM (full-width at half-maximum) and the dispersion (shift of energy as a function of the excitation wavelength) of each peak. In the following, we will focus only on the three peaks shown above, the G peak, the 2D peak and the D peak.

Figure 5.15: Results of a Raman spectra on top right part of the pattern represented Figure 5.12

It has been possible to use an hyper-spectral Raman tool (Thermofisher), which makes a Raman mapping of the sample possible. Its high spatial resolution (a few hundreds of nanometers), its fast scanning as well as its image processing tools make it a powerful equipment to identify suspended graphene on the sample. Figure 5.15 shows the results of a Raman mapping of one part of the pattern, using a discretization with four colour. The software categories four different type of Raman response via an algorithm. The four colors represent four Raman spectra illustrated Figure 5.15b, that can overlap each others.

The red color spectrum is relatively close to the Silicon response, with a peak around 520 cm^{-1} , and is present everywhere on the mapping, except in the deep holes (20 microns deep). The blue color is mainly present on the gold structure and in the holes. The green color appears everywhere except in the holes and finally the yellow color appears mainly on the gold structures.

The red Raman response is therefore coming from the substrate, made of silicon and silicon oxide. The blue and the green Raman spectra have a shape which is characteristic of the graphene response. However, when we look at them carefully, it is possible to distinguish them. The position of the blue peaks, related to graphene G and 2D modes, appear at lower energy, as compared to the green spectrum. Finally, the blue response of the 2D mode is much higher than the green one. According to [16], the blue colored Raman spectra corresponds to suspended graphene, whereas green colored Raman spectra corresponds to supported graphene. Therefore, the graphene have successfully been suspended on holes. Other parts of the sample also show suspended graphene.

The next step is to characterize the gold nanoantenna.

5.2.2 Characterisation of the gold nano-antenna

As it has been explained in Section 4.3, the gold nano-antennas are used for launching plasmons through graphene. In section 3.3.3, it has been shown that the lowest plasmons damping on SiO_2 lie around a wavelength of 11 microns. That is also a typical optical frequency used in a home-made Scanning Near-field Optical Microscope (SNOM) that has been developed at UTT. Therefore, as a first try, the nanorods have been designed in order to have a resonance frequency in this range (see Section 4.3). The numerical simulation that we performed shows that rod length should be in the 3.8-4 microns range (Figure 4.11), which is the case for the sample presented above (see Figure 5.4, 5.5, 5.6). Therefore, the last step was to characterize the rods response to an infrared excitation.

For this purpose, Fourier Transform InfraRed spectroscopy (FTIR) technique has been used. This technique enables to record simultaneously data on a large range of frequencies. Figure 5.16 shows two spectra. In blue, a spectrum showing the response of SiO_2 in reflectivity, with gold as a reference, as a function of the wave-number. This curve has been inserted on the graph for further discussion (therefore, the corresponding intensity is not correct). In red, the response of the sample substrate+nanorods+graphene as a function of the wave number. The intensity (y axis) is obtained by taking a ratio of : the reflectivity given by the sample with a polarisation along the long axis of the rods, over the reflectivity given by the sample with a polarisation along the width axis of the rods.

Figure 5.16: FTIR spectrum in reflectivity for SiO_2 alone (gold reference) in blue, and substrate+nanorods+graphene with a ratio between two polarisation in red

Let's focus on the blue curve. The response of the SiO_2 is interesting because it is possible to observe its different infrared active modes. According to [138], it is possible to distinguish on the blue curve the optical transverse phonons modes at 807 cm^{-1} and 1084 cm^{-1} , as well as the optical longitudinal phonons modes at 814 cm^{-1} and 1252 cm^{-1} .

Let's now look at the red curve. Two main features appear: one is centered around 900 cm^{-1} , which is the response of the nanoantenna, and one is around 2300 cm^{-1} , which is related to the CO_2 .

Then, the different infrared modes of the SiO_2 are also appearing. This result is a confirmation that the gold structure resonance is around 11 microns, which fits well the available set up. It is therefore possible to analyse the samples in the near-field using the home made SNOM.

5.3 Optical measurement using Scanning Near-field Optical Microscopy

At the beginning of the previous century, it was believed, according to the theory, that the resolution of classical microscopy was limited to about 250 nm [168], corresponding to half light wavelength. In the

eighties, in order to break this diffraction limit, it was proposed to use a little aperture as a light source which would be able to confine the excitation at subwavelength scale. Later, another kind of probes have been proposed, named "apertureless tips", which has been used in this study. Generally speaking, Near-field Scanning Optical Microscopy (SNOM or NSOM) is a local probe microscopy where a tip is placed near the surface of the material ($< \lambda/2$) to interact with matter within the near-field, well before propagation and diffraction of light issued from the sample, which allows one to break the diffraction limit. Therefore, it is possible to get nanometric resolution that is no longer imposed by the wavelength of light. Several types of SNOM exist, classified into two main families : aperture and apertureless probes. For more information about the different systems, see [152].

In this section, the home made apertureless SNOM set up will be introduced. It works in the mid-infrared. Then, the fabrication process of aperturless tips will be presented. Finally, some of the results obtained will be shown.

5.3.1 Introduction to apertureless SNOM and related approaches of signal detection

Apertureless SNOM (also named "scattering-type SNOM: s-SNOM") using a metallic tip is based on the principle that the extremity of a laser-illuminated sharp tip can locally enhance/perturb the electromagnetic field at the sample surface. In general , the near field on the sample interact with the tip. The rather complex local interaction between sample and tip results in a scattered radiative wave that can be detected in the far-field. This radiation carries the local informations (as detected by the tip's extremity) of the sample near field such as its optical phase and amplitude. To some extent, an apertureless scattering-type SNOM picks up the evanescent waves and converts them into propagating waves via a metallic tips that can be considered as an optical nanoantenna. The resulting propagating field is recorded by a detector in far field.

Figure 5.17: Illustration of the different contribution to the field detected by the infrared photo-detector in far field

Figure 5.17 illustrates a SNOM tip probing a sample, and the different electromagnetic fields involved [90]. The excitation field is represented by incident black arrows and noted E_{in} . A scattered background, $E_{scat.bkg}$ represented in blue, occurs on the sample everywhere within the laser spot of the excitation. The scattered field, resulting from the complex interaction between tip, sample and incident field, is represented by $E_{scat.tip}$ with a red color. A specular reflection of the excitation is also represented in black by E_{refl} toward the detector. Therefore, the detector is recording three different contributions. The reflection E_{refl} is stronger than the scattered light $E_{scat.bkg}$, the latter which is stronger than the tip diffusion $E_{scat.tip}$.

First, the specular reflection must be filtered out. A good way to do so is to tilt the sample and to record the signal from the same path than the excitation (back scattering configuration). Higher is the tilt of the sample, lower is the reflection from the sample. For the SNOM in infrared at UTT, the tilt is kept higher than 45° .

Then, a second source of problem come from the strong sample scattering (where tip is not involved) that constitutes a background $E_{scat.bkg}$. It is important to remind that our s-SNOM is working in the MID-infrared. Therefore, the spot illuminating both the tip and the sample is about the size of the wavelength, typically here $10 \mu m$. It is interesting to compare, in term of area, the spot of the laser to the zone of interest where the near field is collected by the tip. This zone corresponds to the dimension of apex of the tip, which is typically around 20 nm . Compared to the size of the spot, it is much smaller and the near field signal is likely to be hidden of the background $E_{scat.bkg}$. It must be filtered.

An efficient way to remove it is to drive periodically the tip perpendicular to the sample so that the tip works in AFM tapping mode [11, 87, 12]. It modulates the field scattered at the tip frequency. As a matter of fact, the near field is strongly confined at the surface of the sample. As the tip get closer, the near field radiated into the far-field gets bigger. Once the tips is withdrawn, the signal gets weaker. The field is therefore modulated at a frequency Ω of the tip. The scattering background being unmodulated (or poorly modulated), it is therefore suppressed post-treating the signal. The signal recorded is demodulated at Ω . The signal coming from the probe is therefore recovered.

Technically, a Lock-in Amplifier (LIA) is used. It acts as a filter that record only the field that is modulated at the oscillation frequency Ω of the tip. The frequency of the fork used in the lab is about $\Omega = 29 kHz$. This approach of detection (ie direct lock-detection of the scattered light at the tip oscillation frequency Ω), is called "homodyne detection and is illustrated Figure 5.18 (a).

Homodyne detection

The homodyne system of detection is the simplest one, where there are only three optical arms in the microscope and an only incident electromagnetic pulsation. The first arm is the laser excitation arm. The second arm is the signal arm involving tip and sample. The third arm is the detection one. A lock-In Amplifier is used to record the signal modulated at the tip frequency (or its harmonics [13]) Ω .

At the detector, interferences occur between the different radiative fields: The back-scattering acts as a background that interferes with the near-field signal, which can be translated into equations by :

$$\begin{aligned} I_{det} &= |E_{scat.bkg} + E_{scat.tip}|^2 \\ &= |E_{scat.bkg}|^2 + |E_{scat.tip}|^2 + |E_{scat.bkg}||E_{scat.tip}|\cos(\omega t + \phi_{scat.bkg} + \phi_{scat.tip}) \end{aligned} \quad (5.6)$$

Here, $|E_{scat.bkg}|^2$ is not modulated at the tip frequency Ω and is therefore filtered. On the other hand, the two others terms, namely $|E_{scat.tip}|^2$ and $|E_{scat.bkg}||E_{scat.tip}|\cos(\omega t + \phi_{scat.bkg} + \phi_{scat.tip})$ are both modulated at Ω and recorded. $|E_{scat.tip}|^2$ being really small, it is the interference term that will be observed. Let us stress that the term $|E_{scat.bkg}||E_{scat.tip}|\cos(\omega t + \phi_{scat.bkg} + \phi_{scat.tip})$ can act as an amplification term with regards to $|E_{scat.tip}|^2$.

However, this amplifier is not controlled because the background diffusion is not homogeneous on the sample. Moreover, this amplitude detection can be perturbed by the respective phases of $E_{scat.bkg}$ and $E_{scat.tip}$ which are not controlled. Fringes generally appear on the image, hiding the signal of interest. Finally, although phase information is contained (due to interferences) within the signal, it is difficult to make a distinction between phase and amplitude informations. In order to recover the phase and get a clean signal, heterodyne detection is used.

Heterodyne detection

In this case, a reference signal is added (see Figure 5.18(b)). The laser beam is split into two arms, one will target the sample+tip, and one will be used as a reference signal. An Acoustic-Optical-Modulators (AOM) is added in the path of the reference beam, which is shifting the signal frequency by a $\Delta\omega$. We

Figure 5.18: Illustration of the different system of detection. Homodyne in (a), Heterodyne in (b) and Pseudo-heterodyne in (c). The black arrows represent the excitation beam path, in red the SNOM signal path, and in green the reference signal path

have then two different electromagnetic frequencies, explaining the term "heterodyne"[144].

We have now three different beam hitting the detector : $|E_{scat.back}|$, $|E_{scat.tip}|$ and $|E_{ref}|$. Each of them will have a different modulation characteristic. $|E_{scat.bkg}|$ is not modulated. $|E_{scat.tip}|$ is modulated at the frequency of oscillation of the tip Ω . Finally, $|E_{ref}|$ is modulated at the frequency of the AOM $\Delta\omega$. All these fields will interfere with each others but only the interference between $|E_{ref}|$ and $|E_{scat.tip}|$ will be modulated at $\Omega \pm \Delta\omega$. This is the field that will be recorded by the LIA system. Hence, the carrier signal becomes E_{ref} , instead of $E_{scat.bkg}$. Unlike $E_{scat.bkg}$, E_{ref} is controlled: both its amplitude and phase can be controlled.

More precisely, similarly to Equation 5.6, the field $|E_{scat.tip}|$ is amplified by a strong signal, here $|E_{ref}|$. The difference is that the reference signal is cleaned, constant, and do not depend on the position of the tip on the sample. The recorded image become therefore clear of background signal and it is possible to have the optical amplitude as well as the optical phase.

However, some experimental problems arise with this method [21]. Acoustic-Optical-Modulators (AOM) are usually expensive, and not always available in a wide range of frequency. Also, the Michelson interferometer layout mentioned above is not well adapted to the implementation of an AOM. A less stable Mach-Zehnder configuration is used. An alternative to this method is the pseudo-heterodyne detection, that is presented below.

Pseudo-heterodyne detection

The last method, illustrated Figure 5.18(c), is not using a frequency shift as it is done for the heterodyne detection. Instead, a periodic oscillation of the reference arm length is performed. The phase of the reference is therefore periodically shifted. The reference beam recombining with the SNOM signal create the interferences which will be exploit for the detection. The main advantage of this method is that it can be applied to any frequency range of excitation.

In fact, mirrors are mounted on a piezo material driven by a periodic signal. A mathematical signal treatment is then performed, including a lock in amplifier process, to treat the signal and recover the phase and the amplitude of the signal. This method is called "General Lock-In Amplifier", and more informations can be found in [21].

5.3.2 Experimental details about the scattering-type SNOM

The experimental set up at UTT uses three lasers that cover a spectral range running from $9.5 \mu m$ to $11.4 \mu m$. At the exit of the lasers, an isolator is placed in order to avoid any reflected beam back to the lasers. A coupler, made of a lense mounted on a stage, drives the beam to the fiber. A collimator is placed at the exit of the fiber to conduct the laser beam to the core of the set up. This core is represented in Figure 5.19.

Figure 5.19: Picture of the SNOM setup. In inset, a simplified schema representing the set up

The core of the set up is composed of four arms detailed in the inset of Figure 5.19 :

1. Both incident and back scattered paths are represented in red.
2. The "signal arms" with the stage, the tip and the sample, a mirror objective and a anti-reflection crystal are hilgited in blue
3. The reference interferometric arm is in green
4. The detection arm with a lens and a detector are indicated in yellow

Finally, another arm, parallel to the detection arm, is added in order to visualize the sample and the tip. It is represented in pink in Figure 5.19.

More in details, the signal arm includes the stages supporting the tip and the sample, and an objective to focus the beam. This is where the near field signal is converted into a far field signal. The sample and the tip are hold by a platform that is made of several mechanical and piezoelectric stages. The mechanical stages enable rough movement, whereas piezoelectric stages enable fine approach and scanning. The tip is placed at the beam focal point and does not move once scanning. Instead, it is the sample which is moving. A anti-reflection crystal is added at the objective entrance to avoid back-reflection to the detector.

The detection arm includes a HgCdTe detector and a lens. The detector is cooled with liquid nitrogen. It is linked to a Generalized Lock-In Amplifier (GLIA). In parallel, it is possible to observe on Figure 5.19 a visualization arm, which is composed of a beam splitter, a camera(webcam), and a lens for a magnification. It is used to focus the beam on the apex of the tip and to visualize the location of the scan.

Finally, the interferometric arm is made of three mirrors. Two of them are mounted on piezo materials and provide a shift in phase of the beam (modulating the length of the arm), whereas the last one is mounted on a mechanical stage, and reflects the beam back to the detector. The latter is used for alignment.

Figure 5.20: Schema representing the electro-chemical set up for the fabrication of SNOM apertureless tips

5.3.3 Fabrication of the tips

Tips are crucial for detecting evanescent waves. In particular, the apex of the tips should be as small as possible in order to act as an electric dipole that picks up the near-field. The incident field from the laser beam can be converted by the tip to local source, and excite surface plasmons. Reciprocally, the evanescent wave of a surface plasmons can be converted into a radiative wave via the tip apex.

In order to accurately record plasmons in graphene, the diameter of the tip extremity should be at least of the same order of magnitude than the expansion of the phenomenon in air. For graphene plasmons, it has been shown [44] that the diameter of the apex should be around few tenths of nanometers.

Figure 5.21: Picture of a SNOM tip taken via an electronic microscope. The apex diameter is around 10nm.

Our probes are home-made probes produced at UTT. The method consists of etching electro-chemically a tungsten wire. It is called the double lamellae dropoff technique, and has been inspired from [85]. It is based on the etching of a tungsten wire through a golden ring used as electrode, as illustrated Figure 5.20. A 2.5 cm long tungsten wire is first glued at the extremity of one arm of a tuning fork. This

tuning fork is used for tapping-mode distance regulation (see sub-section 5.3.1).

Then, the fork is clamped horizontally to a stage, with the glued wire head down. Meanwhile, an electro-chemical circuit is installed. It is made of a generator, where the positive terminal is linked to a multimeter and a beaker, and the negative terminal is linked with a golden ring. The beaker and the golden ring are filled with a solution of NaOH (2.5 mol/L). The ring is attached to a stage and stand on the top of the beaker. The circuit is turned off.

Once the tungsten wire, glued to the fork, is dipped from the ring into the beaker solution, the circuit is turned on. A current pass through the circuit and a chemical reaction occurs between the tungsten and the NaOH solution in the ring. Eventually, the tungsten wire will be electrochemically cut at the ring plane and the current will goes to zero.

It has been a real issue to obtain probe with tenth of nanometers diameter. A study has been performed to find the best parameter toward thinnest tip apex. The influence of the following parameters has been investigated:

1. The applied voltage (V)
2. The solution concentration (NaOH in mol/L)
3. The current (in mA)
4. The length of the tungsten wire (in cm)
5. The distance between the gold ring and the surface of the NaOH solution in the beaker (in cm).

The diameter of the gold ring was 0.8 cm. The applied voltage has been set at 8V. Both parameters have not been varied. The solution concentration has been varied from 1 mol/L to 4.5 mol/L. The current which is going through the system when the generator is turned on is of the first importance. It is related to two main parameters : the concentration of the solution, and the amount of liquid inside the ring. Since the concentration is controlled, the targeted current were reached managing the volume of liquid inside the ring. The liquid inside the ring can be suck out with paper towel in order to set the correct starting current. The length of the tungsten is an important parameter because it is directly related to the gravitational force pulling down the wire. At the etching site, this force will tear apart the two side of the tungsten wire. As well, the distance in between the ring and the NaOH solution in the beaker has to be taken into account. It shows how long tungsten wire is outside the liquid, pulling down the wire stronger. The wire length used were chosen between 0.5 to 4 cm. The distance in between the ring and the beaker solution was chosen as a consequence of the length of the wire.

The concentration has been varied at first and 2.5mol/L gave the best results. Then, the starting current has been investigated, and 3 mA gave the thinnest apex. Finally, the length of tungsten wire giving the best results is about 2cm long and for a distance ring-beaker solution of 1.5cm.

Optimizing these parameters did not lead to good enough distributions of tip size, which were centred around 60 nm. The best achieved tip's diameter was around 30nm. In fact, the quality of the NaOH pallets used for the fabrication of the solution turned out to be the critical factor. Once it has been renew, a tip's distribution centred around 25 nm was obtained and reproduced (see Figure 5.22). The best tip obtained was about 10 nm (see figure 5.21).

5.3.4 First optical measurement

At first, preliminary measurements have been performed to record plasmons in CVD-fabricated graphene. The 2D layer was deposited on the top of a typical wafer of SiO₂/Si. Optical and electronic microscope image of the sample are shown Figure 5.23. We can observe the graphene on both pictures as it appears darker than the bear substrate. Some impurities are also present.

In the optical image, Figure 5.23a, a red cross indicate the location from where a raman spectra have been obtained. The result is shown in the inset. We can see here that the graphene is clean, with really few defects (low defect peak). In the electronic image, Figure 5.23b, a green square have been drawn

Figure 5.22: Two typical distribution of size of tip apex before and after the change of NaOH chemical (12 tips each)

(a) Image of a sample from an optical microscope. In inset, a Raman spectra of the graphene layer taken at the red cross (top left).

(b) Image of the same zone of the sample, taken from an electronic microscope. The green square is delimiting the image obtained by SNOM, Figure 5.24.

Figure 5.23: Optical and electronic images of a sample where graphene CVD layer have been deposited non uniformly on a substrate of SiO_2/Si . The images are taken with two different microscopes at the same location. It is possible to observe the presence of graphene, which appears darker than the substrate.

representing more precisely the place where the optical measurement using SNOM have been made (see Figure 5.24).

Preliminary SNOM images are shown in Figure 5.24. The excitation wavelength was $11\ \mu\text{m}$, which is in the range of low damping for graphene plasmons (see section 3.3.3). A linear polariser was used, as shown on Figure 5.19: the polarisation of the incident beam was horizontal. The long axis of the tip is also horizontal. The polarisation follows the excursion of the oscillation of the tip (tapping mode) too. Therefore, this set up enable a strong excitation of the tip, which is more likely to excite plasmons in the graphene layer.

Using our scanning probe microscope in pseudo-heterodyne mode (see section 5.3.1), it is possible to get different kinds informations simultaneously. The first one is the topography of the sample, as the is working as an AFM (Figure 5.24a). The second information is the optical amplitude that we can observe Figure 5.24b. It is the main information we are looking for in SNOM measurement, and it is really sensitive to the discontinuity of graphene (see section 3.2.2). The third picture, Figure 5.24c, is the mechanical phase of the fork. It is sensitive to the change of the underneath material. In fact, phase contrast can be used for determining physical and chemical differences in scanned materials [79]. Graphene looks in this case darker than SiO₂. The last picture, Figure 5.24d, gives the difference of mechanical amplitude during the scan. This amplitude is expected to be constant during scanning, which is not exactly the case, despite the feedback loop. The feedback may have a delay, related to a physical step encounter by the fork. This contrast in the amplitude image has to be taken into account for interpretation of SNOM images. Indeed, SNOM images can contain artifact corresponding to the AFM error signal [18].

We can observe on Figure 5.24b an optical response of graphene at the edges of different flakes, that is probably related to graphene plasmons. In order to have better resolution, another scan zooming on a different area is presented in Figures 5.25 and 5.26.

On the Figure 5.26b, it is possible to observe oscillations in the graphene that are not present on the substrate. Those oscillations are peak to peak a bit higher than 100nm, which has also been reported in [42]. It correspond to the effective wavelength of graphene plasmons in this condition. The graphene intrinsic doping, deduce from this wavelength, is of the order of what is expected from the CVD synthesis method used here [113]. Unfortunately, due to technical problem, variation of the doping by injection of charge couldn't be perform, and no further evidence are available.

5.4 Conclusion

This chapter presents experimental results as the last step of this research work. It shows the fabrication, characterization and optical measurements with the introduction of Scanning Near Field Optical Microscopy technique. These experimental results are the last part of this study. The obtained results are preliminary and more experiments should be conduct in the future in order to show the true signature of plasmons in graphene. This chapter has also been showing the complexity to manipulate 2D material, and to record such confined phenomenon.

(a) AFM topography (10×10 microns)

(b) Optical amplitude

(c) AFM mechanical phase of the fork

(d) AFM mechanical amplitude of the fork

Figure 5.24: s-SNOM images : topography, optical amplitude, AFM phase and AFM amplitude

(a) Image of a sample from an optical microscope. Graphene CVD layer have been deposited non uniformly on a substrate of SiO₂/Si. In inset, a Raman spectra of the graphene layer taken at the red cross.

(b) Image of the same zone of the sample than Figure 5.23a, taken from an electronic microscope. The green square is delimiting the image obtained by SNOM, Figure 5.24.

Figure 5.25: Optical and electronic image of the sample study. It is possible to observe the presence of graphene, darker than the bear substrate.

(a) AFM topography (1×1 microns)

(b) Optical amplitude

(c) AFM mechanical phase of the fork

(d) AFM mechanical amplitude of the fork

Figure 5.26: s-SNOM images : topography, optical amplitude, AFM phase and AFM amplitude

Chapter 6

Conclusion and recommendations

6.1 Conclusion

In this research work, new studies in the graphene plasmonics field have been done. A particular attention has been paid on the explanation of graphene plasmons phenomena using mostly classical physics.

In chapter one, the basis of plasmonics have been reviewed from the Maxwell's equations. The graphene structures has been discussed and the electrons behavior has been rose up through a tight binding model. This chapter set the foundation necessary to the understanding of Graphene plasmonics physics.

The second chapter has aimed at giving a literature review and analysis covering briefly the discovery of graphene as well as the one of graphene plasmons. It gives some glimpse on the challenge remaining in order to achieve active graphene-based electronic devices. In particular, it introduces the problematic of the propagation of plasmons, and the need to find out a suitable platform in order to get the first active device based on graphene plasmons. Finally, graphene plasmons physics is duscussed through the dispersion curve, providing the necessary key parameters for a further numerical and experimental research work.

The third chapter is dedicated to a full numerical study, based on the Finite Difference Time Domain method. This method is shortly introduced with an history and the resolution of 1D wave problem. The 3D case is then given accordingly with the key parameter to conduct a numerical study. The problem of simulating a 2D material in a 3D grid has been extensively addressed. Finally, a design of sample has been proposed. It uses gold antenna as launcher of graphene plasmons. The propagation in suspended graphene has been shown and propagation length has been discussed. Compared to supported plasmons, surface plasmon progagation length in suspendend graphene turns out to be significant and could be exploited in the near-future in the context of 2-D nanophotonics. A final discussion has given the limits of this experiment proposal, where a vacuum chamber is probably required.

The fourth chapter has been dealing with preliminary experimental achievements. At first, a sample fabrication process has been descrided. First samples have been obtained and characterized. Some issues faced during the process hace been addressed, which could be useful for students and researchers who will carry on this work. Then, the characterization of antenna as well as graphene are given. Finally, Scanning Near field Optical Microscopy technique has been introduced. It is based on a heterodyne apertureless configuration. The home-made set up has been presented and some preliminary results have been shown.

6.2 Recommendations for further research

6.2.1 The case of diamond

Given the challenges met to study plasmons in suspended graphene, an other research path have been explored. As already mentioned, Graphene plasmons lake of propagation length L_p when researcher think about using its properties for applications purposes (section 3.2.1). Although progress have been made recently [114, 38], the propagation length L_p remains too short to be used in active opto-electronic devices. Several parameters [98] must be therefore considered in order to obtain high GPs propagation (see Equation(3.27) for the following). Firstly, L_p is function of the scattering rate of the charge carriers, which should be kept as low as possible. Secondly, L_p is proportional to the squared value of the Fermi energy in the infrared range, which should therefore be as high as possible (following the DC approximation of the relaxation time, section 3.1.2). Finally, L_p is very sensitive to the imaginary part of the permittivity of the substrate ϵ'' , the optical energy dissipation ($\text{Im}(\epsilon)$).

Armed with these parameters, we can draw a picture of the ideal platform for high propagation of GPPs, which should have the following properties:

1. High dielectric strength and k-dielectric constant in order to inject high density of charges into graphene.
2. Low $\text{Im}(\epsilon)$, in order to have low plasmonics losses.
3. Possibility to fabricate within the same process than graphene, using CVD system.
4. Use of sustainable material.

SiO_2 , which is the most common substrate used to study graphene [117], induces external and/or internal scattering [7, 78, 28, 110]. Then, the relatively low k-dielectric constant (3.9) associated to the dielectric strength of 0.5GV/m doesn't enable to reach high enough Fermi energy. Finally, the optical dissipation of SiO_2 induces significant damping in GPPs [42].

Soon after, studies on encapsulated graphene in h-BN, a substrate known to be suitable for graphene [145, 53], showed promising results [34, 114]. h-BN high dielectric strength property (up to 1.2GV/m when exfoliated) [60], as well as its good lattice matching with graphene [162], make it a good candidate. Moreover, thin films of h-BN have been synthesis through CVD process conserving good dielectric properties (0.4 GV/m) [74]. However, h-BN optical properties induce severe damping in graphene plasmons [153], as $\text{Im}(\epsilon)$ is of the same order of magnitude than the one of SiO_2 [157]. Moreover, Boron is rather rare on earth.

Comparatively, Diamond fulfills all the criteria cited above. Its dielectric strength is ideally higher than the h-BN one [51, 156]. It can be synthesis through the unique and same CVD process [165]. It has no optical energy dissipation in infrared region and carbon is a material that is continuously growing on earth surface in stocks, creating environmental issues. It would be also interesting to look at the behavior of plasmons in graphene transferred onto UNCD and SCD by changing the surface termination (H-termination vs O-termination) of underlying diamond. Intercalate nano-spacer between graphene and SiO_2 substrate has been already studied [38], and has shown improvement of the plasmons propagation. Therefore, the combination of diamond and graphene would definitively give important results that could eventually lead to efficient active opto-electronic devices.

6.2.2 Example of application : infrared spectrometer

One of the direct application of graphene plasmons is that it could lead to optoelectronic transistors, already demonstrated experimentally. Others theoretical devices have been also study numerically. An infrared spectrometer concept is presented below.

It is based on the grating effect which is presented Section 5.1.4. Periodic structures have the property to diffract the light. As well as antenna (see Section 4.3), periodic structures can excite graphene

Figure 6.1: Illustration of a golden snail shape structure that is possible to fabricate through e-beam lithography. In the center, golden diffusers are placed.

plasmons. They also show a particularity : they can enhance a near field with a particular angular momentum.

At normal incidence, the momentum of the near field enhanced by the periodic structures is inversely proportional to the spacing between the different periodic structures (see Equation 5.5). Therefore, the wavelength enhanced by the periodicity of the grating is proportional to the distance between two periodic structure.

Hence, it is possible to fabricate a grating with a defined space between the periodic structure which select a wavelength (momentum) to be enhanced. As well, it should be possible theoretically to design a periodic structure with a variable spacing, that could be able to enhance near fields with a pre-defined range of wavelength (pre-defined range of momentum). Figure 6.1 shows a snail shape periodic structure, with a distance in between golden line which vary steadily. This kind of structure can be seen as a many grating, drawing lines from the center of the snail to the edges. In each direction, a different spacing between the rods enhanced a different wavelength. In the center is placed diffusers.

Let's assume a beam of light with a broad frequency spectrum. When this beam is hitting the structure, the different period of the shape would enhanced a range of near field associated with a range of angular momentum, depending on the spacing between the different rod. Therefore, all the plasmons related to those angular momentum would be launched. In the case of graphene, above the intrinsic graphene phonon taking place around $6\mu m$, plasmons are theoretically propagating (in the case we have a suitable platform for propagation of plasmons, with graphene doped electrostatically).

Then, the plasmon will travel toward the center of the snail, exciting the golden diffusers. The diffusers would transfer the energy of plasmons to radiative light out of the plane, that could be eventually recorded. This is general principle of this infrared spectrometer. In the following, we will now take the case where we have a beam made of three frequencies. The spectrum of the beam is shown on the right of the Figure 6.2. The case of a beam hitting the sample with a normal angle (on the left of the Figure 6.2) is taken in this example. Finally, the spectrometer will work thanks to the tunability of graphene plasmons, that is presented below.

Let's start from undoped graphene, which means that no potential is applied to the graphene (Figure 6.3 left). Its Fermi level approach zero (Figure 6.3 center left) and no plasmons would be able to propagate to the diffusers (Figure 6.3 center right), due to high Landau damping (see Section 2.3.3). No signal would be detected (Figure 6.3 right).

Figure 6.2: On the left, a beam of light hit the snail (in yellow) perpendicularly to its plane. The substrate is represented by two different blue color. On the right is shown the spectrum of the beam combining three distinct frequencies.

Figure 6.3: Illustration of the first step of the spectrometer concept. The graphene is undoped (0V applied), no plasmons are propagating, and no signal is broadcast.

Let's now rise slowly the potential (Figure 6.4 left). The graphene is electrostatically doped. Its Fermi energy will rise also slowly (Figure 6.4 center left). Energetically low plasmons (long wavelength) would be able to propagate through graphene. Therefore, the related small angular momentum corresponding to low energy plasmons would be launched and propagate to the diffusers (Figure 6.4 center right). The diffusers would transform plasmons energy into farfield signal, that can be recorded. Intensity increase at the photodetector (Figure 6.4 right). Referring to our three frequency beam, the lower frequency would be then recorded at the photodetector (red small arrow).

Figure 6.4: Illustration of the second step of the spectrometer concept. The graphene is weakly doped by an applied voltage, low energy plasmons are propagating, and a signal is recorded.

Rising again the backgate potential would allowed higher energy plasmons to propagate to the diffusers. In this case, the central frequency of the beam would be recorded (green arrow). That would increase the intensity recorded at the detector (Figure 6.5).

Figure 6.5: As the applied voltage is rising, the Fermi energy in graphene is rising, enabling higher energetical plasmons to propagate to the diffusers. A corresponding energy is transferred to the photodetector.

Eventually, all the permitted graphene plasmons (above $6\mu m$) would be allowed to travel to the diffusers and be recorded to the photodetector (Figure 6.6 left). Because each plasmons correspond to a wavelength, it is possible, from the intensity spectrum recorded, to recover the incident beam spectrum. In fact, each intensity step, corresponding to a distinct applied potential, would be interpreted as a frequency (Figure 6.6 right). A transfer function would actually exist in between the potential and the frequency. As well, the intensity of a peak will depend on the height a a step.

Figure 6.6: On the left, the intensity finally recorded at the photodetector. It is a step function with in x-axis the intensity versus the potential in y-axis. Each step is therefore characterized by a different of intensity, and a potential (ΔI , V). On the right, the recovered frequency spectrum of the beam.

Author's Publications

D. Legrand, L. O. Le Cunff, A. Bruyant, R. Salas-Montiel, Z. Liu, B.K. Tay, T. Maurer, and R. Bachelot, "Surface plasmons in suspended graphene: launching with in-plane gold nanoantenna and propagation properties," *Opt. Express* 25, 17306-17321 (2017).

Yi Huang, David Legrand, Rémi Vincent, Ekoué Athos Dogbe Foli, Derek Nowak, Gilles Lerondel, Renaud Bachelot, Thierry Taliercio, Franziska Barho, Laurent Cerutti, Fernando Gonzalez-Posada, Beng Kang Tay, and Aurelien Bruyant, Spectroscopic Nanoimaging of All-Semiconductor Plasmonic Gratings Using Photoinduced Force and Scattering Type Nanoscopy, *ACS Photonics* 2018 5 (11), 4352-4359.

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