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**State Monitoring of Future Power
Distribution Grid using
Micro-PMU and State Estimation**

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2019

State Monitoring of Future Power Distribution Grid using Micro-PMU and State Estimation

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

12-Apr-2019

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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Rusli

Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

Chapter 3 is published as X. Chen, K. J. Tseng, and G. Amaratunga, “State Estimation for Distribution Systems Using Micro-Synchrophasors,” in *2015 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, Nov 2015, pp. 1-5.

The contributions of the co-authors are as follows:

- Prof Tseng provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts and performed all the simulation and analyzed the data.
- Prof Amaratunga revised the manuscript.

Chapter 4 is published as X. Chen, T. Chen, K. J. Tseng, Y. Sun, and G. Amaratunga, “Customized Optimal Micro-PMU Placement Method for Distribution Networks,” in *2016 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, Oct 2016,

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The contributions of the co-authors are as follows:

- Prof Tseng suggested the initial research direction and edited the manuscript drafts.
- I wrote the manuscript drafts and performed all the simulation and analyzed the data.
- Chen Tengpeng prepared the data for simulation.
- Dr Sun Yuhao helped to conduct data evaluation.
- Prof Amaratunga revised and polished the manuscript.

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The contributions of the co-authors are as follows:

- I wrote the drafts of the manuscript and performed all the simulation.
- Dr Sun Lu helped to evaluate the performance of the proposed method and came up with ideas to improve the proposed method.
- Dr Chen Tengpeng and Dr Sun Yuhao helped to revise the manuscript after receiving comments from journal reviewers.
- Prof Rusli proofread the manuscript before submission.
- Prof Tseng, Prof Prof Ling, Prof Ho and Prof Amaratunga provided valuable comments on the proposed method.

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Abstract

In recent years, distributed energy resources (DER) systems have increasingly played an important role in the electric power distribution system. Nevertheless, they also concurrently introduce new characteristics to the distribution system, such as bi-directional power flows, unstable power supply, unintentional islanding, and voltage profile issues. The evolvement of distribution networks necessitates rapid and accurate distribution network monitoring. Thanks to the high accuracy and synchronization of Phasor Measurement Unit (PMU) measurements, a more reliable and accurate state estimation (SE) is expected if the system is monitored using only PMU. However, so far PMUs have been used almost exclusively in transmission systems monitoring.

This thesis discusses the application of μ PMUs in distribution networks, with a focus on their optimal placement (OPP). Many OPP algorithms have been well documented. However, most of them are designed and tested on transmission networks, and very few studies have been conducted on distribution networks. Distribution networks have very different topology from transmission networks, and hence there is a need to study how the μ PMUs can be optimally placed in radial topology networks. This thesis fully considers the characteristics of the distribution system and proposes two customization steps to improve the efficiency of OPP for the distribution system. Customization steps are applied to exhaustive search and simulated annealing, and the proposed customized OPP methods have been tested on several networks. Besides the above mentioned exhaustive method, application of integer linear programming (ILP) on OPP has been investigated in this thesis. Compared to exhaustive algorithm, integer linear programming is much

faster in solving the OPP problem. However, common solver can only give limited solutions, while in practice it is possible that there are multiple solutions that can satisfy the given constraints. Obtaining all the alternative solutions efficiently will allow the decision maker to examine and select the overall best solution, taking into consideration the factors that are mathematically difficult to express, without suffering any deterioration in the objective function. An ILP based OPP method is proposed in this thesis to identify all alternative optimal placements that satisfy certain constraints. This method is efficient in determining the solution when applied to large-scale power system.

This thesis also discusses distribution system state estimation (DSSE). The methods and techniques used in transmission system state estimation may not be valid when applied directly to distribution system due to its distinct characteristics. Several state estimators including weighted least squares (WLS), multiple-segment (MS), quadratic-linear (QL) and square-root (SR) estimators are implemented and tested on IEEE 13 node test feeder. A full phase linear state estimator is developed for three phase unbalanced distribution system. Considering distribution systems usually have limited number of measurements, state estimation has been carried out with different level of measurement redundancy and the performance was evaluated in terms of mean square error (MSE). Simulation shows that the introduction of μ PMUs makes those mentioned SE better suit DSSE. It is also indicated that the performance of all the estimators deteriorates with the decrease of measurement redundancy, which gives us the guidance to have more measurement redundancy when considering placement of μ PMUs in order to improve DSSE accuracy.

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Chapter 1

Introduction

This chapter first introduces the background of power system state estimation and Phasor Measurement Unit (PMU), as well as methods of incorporating PMU in state estimation and their optimal placement. It then continues with the presentation of the motivation, objectives and major contributions of the work carried out in this research. Following that the outline of this thesis will be covered.

1.1 Background

1.1.1 Electric Power System and State Estimation

The beginnings of commercial use of electricity date back to late the 18th century, and the standard "power system" then was made up of an individual generator and an appropriately matched load. Soon as electricity was becoming widespread, the isolated systems were interconnected with each other since the early 1900s [1]. After continuing development and enhancement over the course of the last century, the electric power system has now become one of the critical infrastructure systems.

The modern power system typically is composed of generation, transmission and distribution systems, as illustrated in Fig.1.1 [2]. The generation system produces electricity in power plants using different types of sources such as fossil fuels, wind energy and solar energy. The generated power is at the generation voltage level. Transformers are then used to increase the voltage level to minimize copper losses as the power is transmitted over long distances. The transmission system is responsible for delivering electricity to load centers through cables or overhead transmission lines. At this point, the transmitted power is at extra high voltage (EHV) (transmission network) or high voltage (HV) (sub-transmission network) level. Following that comes the distribution system, where the voltage is stepped down at the substations and electricity is dispatched to consumers through distribution lines or cables [1,2].

Figure 1.1: Modern power system.

A power system has three possible states depending on the operating conditions, namely normal, emergency and restorative states. System operators continually

plan, monitor, analyze and control the operation of the power grid using special tools such as load forecasting, state estimation, security assessment and others, in order to maintain the system in the normal secure state, so as to provide reliable power and satisfy the needs of consumers. Continuous monitoring of the system conditions is the very first step of security analysis which aims to maintain the system in the normal secure state [3,4]. In 1970, Schweppe introduced the concept and definition of power system state estimation (SE), and formulated the state estimation problem [5–7]. The state of an electric power system is defined as the vector of voltage magnitudes and angles at all network buses. State estimation is the process that provides the optimal estimate of the current state of the a power system using measurements data and network topology information [3]. The system state can be used in conjunction with the network topology and impedances to calculate the power flow in transmission lines and power injections at each node. Hence, state estimation is crucial for the secure operation of power systems, and system operators need the state estimation solution to know how the system is behaving. In most historical examples, the system operator could have reduced the scale of the blackout if they were provided with more accurate and up-to-date information of the system’s operating condition [3].

The state estimator typically includes several functions: topology processor, observability analysis, state estimation solution and bad data processing. It processes raw measurement data collected from instruments and produces data base of current state of system for application functions [3,4]. Conventional measurements used in state estimation are voltage and current magnitudes, and real and reactive powers. Most state estimators in practice use a group of non-linear equations because these measurements are related to the system states through non-linear relationships. The estimation equations are solved though Weighted Least Squares (WLS) method in an iterative manner. The measurement data

are assigned a weight based on their accuracy and the objective function is the error between the state estimates and the measurement results. Taylor expansion is normally used to linearize the non-linear state estimation equations, and the equations are updated at each iteration with the estimated value to minimize the error between the estimate and true values [5–7]. The solution of this minimization process is the system state, which is used by operators to control and operate their power systems.

1.1.2 Phasor Measurement Unit (PMU)

Developed in the mid 1980s, phasor measurement unit (PMU) is an advanced measurement device which is able to provide synchronized and accurate voltage and current phasors measurements in power system [8]. PMUs convert the measured parameters into phasors, at a rate of typically 30 or more values per second. Integration of GPS receiver enables PMUs to add a precise time stamp to these phasors, turning them into synchrophasors. With the help of the time stamp, these phasors that are provided by PMUs in different locations that could be situated far away, and across different power industry organizations, can be correlated, time-aligned and integrated. The resulting information provides power system operators a high-resolution "picture" of the conditions throughout the grid in real time [4, 8]. Fig. 1.2 illustrates a collection of geographically dispersed PMUs forming a Wide Area Monitoring System (WAMS), which delivers GPS time-tagged measurements to a Phasor Data Concentrator (PDC) [9].

Power system operators are always concerned with phase angle measurement, as the phase angle between the voltage and current phasors are utilized to monitor the condition of power networks. Theoretically, real power flow in a transmission line is proportional to the sine of the angle difference between the voltage phasors

at the two terminals of the line [10]. Therefore, PMU's ability to provide accurate and synchronized phasors of voltages and currents has rendered it a very useful tool for power networks monitoring, and increasingly more PMUs are deployed in power systems throughout the world.

Figure 1.2: PMUs layout with GPS time stamped signal

1.1.3 Distribution System State Estimation and μ PMU

In contrary to state estimation of transmission networks which has been well studied since the early 1970s, the application of state estimation in a distribution system has not attracted significant interest until recently. Conventionally, distribution systems have been configured to operate in a radial configuration, where power flow is along one direction and hence it is relatively easy to predict and manage. In recent years, there is a tendency of having more distributed generation (DG) systems connected to the distribution system as a result of proliferation of renewable energy, increased environmental awareness and liberalization of the energy market [11].

The increasing penetration of renewable energy resources brings new characteristics into distribution system such as bi-directional power flows and voltage profile issues, which have necessitated more continuous monitoring of the power network at the distribution level [12]. Compared to transmission systems, distribution systems have relatively limited measurements of their characteristics, such that the state estimation will have to be carried out with generally lower redundancy. Besides the issue of low redundancy, the three phases of the distribution system are not balanced, which further differentiate the estimator for distribution system from that of a transmission system. As mentioned earlier, most of the distribution systems have a radial topology, which is another difference compared to transmission system. In addition, distribution systems have high resistance reactance ratio (r/x) in lines, and also a very large number of nodes, which brings about additional challenges to conventional estimators used for transmission system.

Phasor measurement units that have been widely applied in transmission networks can also be used for the monitoring and state estimation of distribution networks. However, conventional PMUs are not applicable in distribution networks because distribution system applications are more challenging [13,14]. For example, the voltage angle differences between different locations on a distribution circuit can be up to two orders of magnitude smaller than those in transmission networks, and hence cannot be precisely measured using conventional PMUs. This arises because distribution networks usually transfer a limited amount of power, in contrast to transmission networks that transfer a much large amount of power. In addition, the cost of conventional PMU is too expensive to make a business case for distribution networks applications considering the less power delivered and lower profit in distribution networks.

Arising from the above considerations, micro-phasor measurement unit (μ PMU) has been developed to measure the angles between voltage and current phasors

in distribution networks [13, 15]. The μ PMUs provide measurements with ultra-precision and at relatively low cost, which distinguish them from traditional PMU used on transmission networks.

1.1.4 Optimal PMU Placement

As mentioned earlier, a PMU installed at a bus can make direct measurements of the voltage phasor of the installed bus and the current phasors of all the branches incident to the bus, as long as the PMU has sufficient number of channels. Since the system states (bus voltage phasors) have linear relationship with PMU measurements, it is expected that the entire network can be monitored by PMUs so that a linear state estimator can be formulated [16]. Nonetheless, deployment of PMUs at all locations is normally not feasible as they are costly, and furthermore the installation cost involved is also very high, and may be more than the device cost.

It can be shown that it is possible to estimate the entire system state by just deploying PMUs to a subset of a minimum number of network buses [9, 17–19]. This is because PMUs can measure not only the voltage phasors at the installed buses, but also all the incident currents phasors [20]. With the network topology information known, the adjacent buses voltage phasors can be calculated and hence the entire system state can be estimated. This possibility has inspired plenty of research on how best to optimally place a minimum number of PMUs in a given system such that it is observable using the measurements from the PMUs [19]. The optimal PMU placement (OPP) is essentially an optimization problem which aims to determine a minimum set of PMUs such that the whole system is observable.

In the case of distribution systems to be monitored using μ PMU, although the cost of μ PMU is relatively lower compared to conventional PMU installed in

transmission system, the large amount of nodes involved render it not feasible to deploy μ PMUs at all locations in a distribution system. Therefore, choices have to be made regarding the locations to optimally place a minimum set of μ PMUs so that a given distribution network can be observed. In contrast to the OPP problem for transmission systems that has been extensively studied, it is noted that the OPP problem for distribution system has not been widely investigated, as the study of application of μ PMUs in distribution network is in its infancy [10, 17, 20–30].

1.2 Motivation and Contributions

Phasor Measurement Units (PMUs) have been used in power system to enhance the performance of state estimators, and their contribution to transmission networks monitoring has been well proven and documented [8]. At present, PMUs are mainly deployed in transmission networks because the monitoring of transmission networks has been prioritized over distribution networks. However, in recent years, the increasing penetration of renewable energy sources brings new characteristics into the distribution system such as bi-directional power flows, unstable power supply, unintentional islanding and voltage profile issues, which necessitate faster and more accurate distribution networks state estimation.

The advent of μ PMU which suits distribution system application provides a new possible solution to distribution system state estimation using synchronized phasor measurements. Note that the existing approach to state estimation in transmission networks cannot be directly transferred to distributions networks, due to their very different characteristics. For example, in contrast to transmission systems, distribution systems have many different features, including radial topology, high resistance to reactance ratio and three-phase unbalanced system.

The first objective of this thesis is to develop a full phase state estimator for distribution system state estimation. The estimator is expected to achieve good accuracy even when the system has unbalanced three phase loads. Assuming that the distribution system is fully observed by μ PMU, we aim to develop and test a linear three phase state estimator. Given the large scale of distribution system, computation burden is a concern. In this work, based on symmetrical components, we target to decouple three phase state estimator into three separate single phase state estimators so that the dimension of the problem is reduced and the computation burden involved is relieved.

Another objective of this thesis is on the study of optimal μ PMU placement (OPP) for distribution systems. Although a lot of literature has been dedicated to this research for transmission networks since 1993 [17, 19, 25, 31–33], however, to date there is only very little work focused on OPP problem for distribution networks. Although the placement methods proposed for transmission networks may be applied to distribution networks, nevertheless, they are far from perfect because of the very different features between distribution networks and transmission networks, as explained earlier. Furthermore, as distribution networks tend to change their topology to transfer loads among the feeders, the reconfiguration also has to be considered in deciding the optimal placement of μ PMU [4].

In this thesis, by analyzing the radial topology of distribution networks, we propose specially tailored customization steps which can reduce the search space and accelerate the process of determining the OPP in distribution systems. The customization steps include the identification of candidate locations and determination of the upper bound of number of μ PMU required to render the network observable. The first customization step of identification of candidate locations serves to reduce the number of candidate locations dramatically by preassigning some μ PMUs to specific locations and hence removing two kinds of buses from candidate locations.

Since distribution systems have a considerable number of radial buses, this step is proven to be able to improve the OPP method significantly in terms of the time required to derive the optimal placement of μ PMUs. So far the identification of candidate location has not gained much attention in literatures, because most OPP methods are designed for transmission networks where there are limited number of radial buses. The second customization step of determining the upper bound of the minimum number of μ PMU that can make the system observable helps to narrow down the search range. By carefully analyzing the characteristics of distribution networks, we propose an upper bound that is lower than the theoretical upper bound given in the literature [17]. With a smaller upper bound proposed for the minimum number of μ PMU required, it will reduce the search space and greatly accelerate the solving of the OPP problem. The customization steps proposed in this thesis have been incorporated into the exhaustive search and simulated annealing optimization methods to customize them for the study of OPP in distribution systems. The methods have been tested on various systems including IEEE 13, IEEE 34 and IEEE 37 node test feeders. The simulation results clearly indicate that the customized optimization methods are more efficient in deriving the OPP, as compared to the corresponding optimization methods without the customization steps. The time required to determine the optimal placement of μ PMU has been reduced from more than 30 minutes to just several seconds or less.

The OPP problem in general has multiple objectives, such as the minimization of the PMU number needed for full observability, maximization of measurement redundancy, good performance of state estimation and meeting other planning constraints. Some of these objective or constraints cannot be easily incorporated into the optimization process. Hence, it is often desirable to have multiple optimal solutions taking some objectives and constraints into consideration, and leave it to the decision maker to examine them and choose the best solution overall. To date,

the issue of multiple solutions to OPP problem has not drawn much attention.

In this thesis, we propose a hybrid OPP method that is able to identify all the alternative optimal placements that have the same objective function value. The proposed hybrid OPP method is based on integer linear programming (ILP) algorithm and combined with exhaustive search. In the proposed method, the integer linear programming is first used to solve for the minimum number of PMU required in a distribution network. After that, the objective function is modified by adding a small fraction to each entry of the coefficient to generate multiple solutions having the same minimum number of PMU, and maximum system observability redundancy index (SORI). Generally not all the optimal solutions can be identified by this step, which means there are some optimal solutions not included in these multiple solutions. Further steps are conducted to make sure that no solution is missing. The network buses are classified into three categories, i.e. primary buses, trivial buses and uncertain buses based on the multiple solutions obtained, followed by the grouping of uncertain buses and forming of subnetworks corresponding to groups. Each subnetwork is then optimized in an exhaustive manner. Finally, all the optimal placements for the whole system can be consolidated.

Our hybrid ILP based algorithm has been shown to be efficient at locating all the optimal placements without resorting to exhaustive search. It takes only a few seconds to identify all the optimal placements of IEEE 123 node test feeder, while regular exhaustive search may take more than 3 hours and regular integer linear programming can only locate one optimal placement at a time. Compared with the existing OPP methods [24–26, 28], this hybrid method combines the efficiency of the integer linear programming method and the ability of the exhaustive search method to locate all the optimal solutions.

1.3 Outline

Chapter 1 introduces the general background of power system state estimation and PMU, as well as incorporating PMU in state estimation and optimal PMU placement. The motivation, objectives and contributions of this thesis are also presented in this chapter.

Chapter 2 presents the background of PMU and its basic working principle. The mathematical formulations of algorithms employed in traditional state estimation are introduced in detail in this chapter. The methods of including PMU in state estimation are also presented. This chapter also compares transmission system and distribution system and addresses the challenges of state estimation in distribution systems. Following that μ PMU techniques are introduced and several typical optimal PMU placement algorithms are presented in detail, which form the foundation of the materials to be covered in Chapter 4 and 5.

In Chapter 3 investigates the performance of non-robust and robust estimator on distribution system. Weighted least squares (WLS), multiple-segment (MS), quadratic-linear (QL) and square-root (SR) estimators are introduced and tested on IEEE 13 node test feeder. A unified approach is developed to construct three phase linear state estimator.

Chapter 4 is concerned with the optimal placement of μ PMUs. It begins with the motivation and definition of optimal placement, followed by the description of the proposed customized optimal placement methods. The proposed methods are then applied to IEEE standard systems and the simulation results are presented and analyzed.

Chapter 5 develops an algorithm to locate all alternative optimal solutions that satisfy certain constraints using integer linear programming combined with

exhaustive search. The procedures to improve integer linear programming are described in detail, after which the proposed method is tested on several test feeders.

Chapter 6 concludes the work completed in this thesis and recommends some future studies with regards to the optimal placement of μ PMU and state estimation in distribution systems.

Chapter 2

Literature Review

This chapter presents the history of phasor measurement unit (PMU) and its basic working principle, following that the advent, development and application of μ PMU is discussed. The mathematical formulations of algorithms employed in conventional state estimation techniques are then introduced in detail, followed by the presentation of methods of including phasor measurement in state estimation. Finally, three popular optimal PMU placement (OPP) algorithms are introduced, which form the foundation of the materials to be covered in Chapters 4 and 5.

2.1 Phasor Measurement Unit

2.1.1 History of PMU

Phasor Measurement Units (PMUs) can be regarded as a direct descendant of the symmetrical component distance relay. In 1970s, the calculation burden of transmission line relaying functions exceeded the available computer power. Under the circumstances, a new relaying technique was developed to avoid fault types

examination by solving six fault loop equations. This new relaying technique introduced efficient algorithms to compute symmetrical components of three-phase voltages and currents. Positive-sequence voltages at each node of a network represent the state of a power system, and are important for power system analysis. Therefore, the development of efficient algorithms for positive-sequence voltages calculation motivated modern phasor measurement systems. On the other hand, the deployment of the Global Positioning System(GPS) provided the most effective way for the synchronization of power system measurements [8].

The prototype PMU units were built at the Power Systems Research Laboratory in Virginia Tech in early 1980s. In 1991, Macrodyne started the first commercial manufacture of PMUs in collaboration with Virginia Tech [8]. A standard was published and revised by IEEE in 1991 and 2005 respectively to govern the format of data files created and transmitted by the PMU, as PMU has since been manufactured by a number of manufacturers and deployed on power system around the world.

There are some features that promote PMU persistent propagation in power system throughout the world. One of the most important features is about GPS time. The one pulse-per-second provided by GPS is coincident with all other received pulses to within 1 microsecond. The accuracy of synchronization can even reach a few hundred nanoseconds in practice [8]. The state of the power system at a specified instant can be estimated using all measurements with the same time stamp corresponding to the specified instant. The time-tags associated with the phasor data provide an indexing tool which helps create a coherent picture of the power system out of such data.

Another important feature is its ability to measure voltage and current phase angles. In power system theory, real power flow transmitted in a power line is

proportional to the sine of angle difference between voltages at two terminals of the line. Since the flow of real power is an important reference for many of power system planning and operation, power engineers have always paid special attention to phase angles of bus voltages in power system. These features together make PMU one of most important measuring devices for power system.

2.1.2 Generic PMU

Figure 2.1: Configuration of the modern PMU

Fig. 2.1 shows the major elements of the modern PMU: anti-aliasing filters, analog-digital converters, GPS receiver, modem and phasor micro-processor. The current and voltages cannot be used directly as input because they usually exceed the measuring range of the analog-to-digital converters. Therefore, the analog inputs of currents and voltages signals are first converted with appropriate transformers. The step-down signals are then sampled through analog-to digital converters after passing through the anti-aliasing filters. Based on Nyquist criterion, the sampling rate of the A/D converter should be at least twice the frequency response of the anti-aliasing filter.

2.1.3 Micro Phasor Measurement Unit

It has been decades since PMU was applied on power system to improve the power system state estimation. They are almost exclusively used on transmission systems because of two reasons: it was not necessary to continually observe the operating state in passive distribution systems and PMU cannot be directly used in distribution systems.

Distribution application is more challenging in both technical and economic aspects: Due to the short transmission lines and low power flow, the voltage angles between different locations on a distribution network will be up to two orders of magnitude smaller than those on the transmission network, in the range of tenths of a degree, rather than tens of degrees [13]. This cannot be achieved using the measurement resolution and accuracy of conventional PMU [34], which are shown in Table 2.1.

Table 2.1: Measurement Accuracy of PMU

Amplitude resolution	0.025 %
Amplitude Accuracy	$\pm 0.1\%$
Angle resolution	0.1°
Angle Accuracy	$\pm 0.1^\circ$

Moreover, considering lower profit and importance as compared to the transmission setting, the costs must be far lower to make a business case for the installation of multiple PMUs on a distribution circuit. However, the situation is changing as qualified devices for distribution system application, known as μ PMU, become available lately. μ PMU developed by Power Standards Lab is a high precision phasor measurement unit which is able to discern angle differences down to $\pm 0.01^\circ$, contextualize these phase angle data with a detailed power quality recording, and

do so at a low installed cost. Table 2.2 gives the specification of a commercial μ PMU product.

Table 2.2: Measurement Accuracy of μ PMU

Amplitude resolution	0.0002 % (2PPM)		
Amplitude Accuracy	Typical	±0.010% (120V - 600VAC L-N)	
	Factory pass/fail	±0.025%	
	Guaranteed	±0.05% (10VAC-750VAC L-N)	
Angle resolution	0.001°		
Angle Accuracy	Typical	± 0.003° (120V - 600VAC L-N)	
	Factory pass/fail	± 0.005°	
	Guaranteed	± 0.010° 1 Standard Deviation	

Considering its low cost and high precision measurements, μ PMUs can be used by system operators to better understand the present or past condition of a distribution system. Some diagnostic applications are introduced as follows:

1. Topology status verification

The topology of distribution networks changes faster than transmission networks, but majority of switching devices in distribution network are not remotely monitored. Implementation of μ PMU can help detect or confirm the actual status of field switches.

2. Phase identification and balancing

With the help of μ PMUs, it will be simple to identify the connection of single-phase loads and laterals to phases A, B or C to facilitate proper balancing.

3. State estimation

Conventional transmission level state estimation cannot be applied directly to distribution system due to different characteristics of distribution networks. Employment of μ PMUs potentially can address the problem. This part will be explained in the next section.

4. Fault location

Existing measurements on distribution circuits are often insufficient to locate faults. It is expected that fault identification can achieve greater precision with the voltage angle measurements provided by μ PMU.

5. Characterization of distributed generation

μ PMU measurements can observe inverter real and reactive power output at very small time scale relative to line voltage frequency and angle, thus help qualify and quantify the behavior of inverters in relation to stabilizing system frequency and damping disturbances in power angle or frequency.

There are more applications of μ PMU not listed here, such as unintentional island detection, reverse power flow detection, oscillation detection and unmasking loads from net metered distributed generation(DG) . This thesis mainly considers the potential contribution of μ PMU to distribution system state estimation.

2.2 Power System State Estimation

Power system state estimation is the process that provides the best estimates of current state of the power system using imperfect measurement data and network topology information. The state of an electric power system is the collection of voltage magnitudes and angles at all the network buses. Power system state estimation was proposed and introduced by Schweppe in 1970 [5–7]. It is able to

take advantage of the measurement redundancy and contribute to broadening the capabilities of supervisory control and data acquisition (SCADA) systems. The measurement data used in state estimation utilized real and reactive power flows and injections with some voltage magnitude. As part of the development of PMU, synchronized phasor measurements are also used in state estimation [8, 35–38].

State estimation is of great importance in secure operation of power systems. State estimation solution are necessary for operators to perceive the operating conditions of the system, thus achieve efficient and optimum economic operation and planning [4, 37]. The core of state estimator involves solving state estimation equations, which will be discussed in detail in next section. Besides state estimation solution, there are some additional functions in state estimator such as topology processor, observability analysis and bad data processing. The main processes and information flows of power system state estimation is shown in Fig. 2.2 [11].

Figure 2.2: Graphical overview of main processes of power system state estimation

In order to provide accurate system model for state estimation, the topology

processor checks the status of the circuit breakers and switches, and provides the configuration of the system. The state estimator is not able to give state estimates of the entire system without sufficient measurement data. Therefore, observability analysis has to be performed before state estimation. This function can determine the observable area with the available measurement data. Bad data processing is further used to detect measurement errors, and identify and rectify them. Weighted Least Square (WLS) estimation method is the most commonly used technique in state estimation of power system. Next the basic formulation of a WLS estimator is presented.

Suppose the measurements are denoted by a vector z , while the number of measurements is m , the state vector is x , and the number of state variables is n .

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} = h(x) = \begin{bmatrix} h_1(x_1, x_2, \dots, x_n) \\ h_2(x_1, x_2, \dots, x_n) \\ \vdots \\ h_m(x_1, x_2, \dots, x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \quad (2.1)$$

in this equation $h^T = [h_1(x), h_2(x), \dots, h_m(x)]$

$h_i(x)$ is the nonlinear function relating measurement i to the state vector x

$x_T = [x_1, x_2, \dots, x_n]$ is the system state vector

$e_T = [e_1, e_2, \dots, e_m]$ is the measurement errors vector.

It is commonly assumed that the measurement errors are independent and have a Gaussian distribution. The objective function of the WLS estimator will be:

$$\begin{aligned} J(x) &= \sum_{i=1}^m (z_i - h_i(x))^2 / R_{ii} \\ &= [z - h(x)]^T R^{-1} [z - h(x)] \end{aligned} \quad (2.2)$$

The first order partial derivative of the objective function is equal to zeros at

the minimum point as follows:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x)R^{-1}[z - h(x)] = 0 \quad (2.3)$$

where $H(x) = [\frac{\partial h(x)}{\partial x}]$

Since $g(x)$ is a non-linear function, it is expanded into its Taylor series around the state vector x^k .

$$g(x) = g(x^k) + G(x^k)(x - x^k) + \dots = 0$$

The higher order terms are neglected so that an iterative solution scheme can be obtained as follows:

$$x^{k+1} = x^k - [G(x^k)]^{-1} \cdot g(x^k) \quad (2.4)$$

where k is the iteration index, x^k is the solution vector at the k iteration,

$$\begin{aligned} G(x^k) &= \frac{\partial g(x^k)}{\partial x} = H^T(x^k) \cdot R^{-1} \cdot H(x^k) \\ g(x^k) &= -H^T(x^k) \cdot R^{-1} \cdot (z - h(x^k)) \end{aligned}$$

$G(x)$ is called the gain matrix. When the system is fully observable, the gain matrix will be sparse, positive definite and symmetric. The state vector at $k + 1$ iteration can be solved by the following equation:

$$[G(x^k)]\Delta x^{k+1} = H^T(x^k)R^{-1}[z - h(x^k)] \quad (2.5)$$

where $\Delta x^{k+1} = x^{k+1} - x^k$. In this equation, the inverse of matrix $G(x)$ is avoided by premultiplying $G(x)$ to both side of the equation. This is for simplicity of the calculation, considering that $G(x)$ is a sparse matrix and the inverse will be a full matrix.

According to the equations above, R , $h(x)$ and $H(x)$ are necessary to solve this optimization problem. R is the covariance matrix of measurement errors and $H(x)$

is the measurement Jacobian, which is the derivative of the measurement function with respect to the state vector. $H(x)$ can be calculated given information about system model and measurement data [39].

WLS state estimation is an iterative method, therefore an initial state vector x^0 is necessary. Usually the initial state vector is set to be 1.0 per unit and in phase with each other. In conjunction with the measurement data, the next iteration of the state vector is calculated again until a desired tolerance is reached. The measurement function $h(x^k)$ is the relationship between measurements and system states. There are different types of measurements such as voltage and current magnitudes, power flows and injections. Equations to relate state vector to each of these types of measurements are derived based on a two-port π -model branch which is illustrated in Fig. 2.3.

Figure 2.3: Two-port π -model of a network branch

- Real and reactive power injection at bus i :

$$P_i = V_i \sum_{j=1}^N V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (2.6)$$

$$Q_i = V_i \sum_{j=1}^N V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (2.7)$$

- Real and reactive power flow from bus i to bus j :

$$P_{ij} = V_i^2(g_{si} + g_{ij}) - V_i V_j(g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (2.8)$$

$$Q_{ij} = -V_i^2(b_{si} + b_{ij}) - V_i V_j(g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (2.9)$$

- Line current flow magnitude from bus i to bus j :

$$I_{ij} = \frac{\sqrt{P_{ij}^2 + Q_{ij}^2}}{V_i} \quad (2.10)$$

With the relationship of each type of measurements and system states, the measurement Jacobian can be obtained. The structure of measurement Jacobian H will be as follows. The rows in the matrix correspond to measurement vector, whereas the columns correspond to state vector.

$$H = \begin{bmatrix} \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \\ \frac{\partial I_{mag}}{\partial \theta} & \frac{\partial I_{mag}}{\partial V} \\ 0 & \frac{\partial V_{mag}}{\partial V} \end{bmatrix} \quad (2.11)$$

2.3 Challenges in Distribution System State Estimation

Traditionally, distribution systems have been designed and operated as passive systems, where power flows are unidirectional and relatively easy to predict and manage. So it is only necessary to evaluate the envelope of design conditions, e.g. peak loads or fault currents. As a result, state estimation has not been widely implemented in distribution grid to date [11, 13].

However, in recent years, new technologies have emerged which bring about new characteristics to distribution systems. For example, the penetration of distributed generation (DG) is increasing rapidly in recent years, motivated by the provision of clean and efficient energy, which helps reduce green house gas emissions. Since DGs are placed near to the load centre, appropriately placed DGs in the distribution network will contribute to reduction of power losses [40]. As DG increasingly plays an important role in power system, it also changes many aspects of distribution system operation, design and implementation [41]. For example, power flow in distribution systems used to be unidirectional, but it may be bidirectional in the presence of distributed generators in distribution networks. The power quality and reliability may also be affected, and utility protection and DG protection measures must be coordinated. Some important effects of DG on distribution systems are as follows:

1. Unintentional islanding.

There is a lot of circuit breakers and switches in distribution systems and majority of them are not monitored. Therefore, it is possible that a circuit breaker motion can result in an islanding of a DG unit. The DG unit will continue to operate if it does not detect the loss-of-mains. If the DG unit does

not match the load in the islanded system, frequency or voltage variations will occur. The variations could be severe if there is great disparity between DG unit output and the load in the islanded system.

2. Voltage profile issues.

Distribution systems are passive and radial, so the voltage drops are predictable and they are taken into account in the design of the network. However, the current flows are changed or even reversed when a DG unit is connected. The voltage will generally increase in a way that is not easy to predict.

3. Unbalancing.

Various DGs supply to network in single-phase, and their integration may cause unbalancing in distribution systems.

As a result, distribution system state estimation (DSSE) has been receiving significant research interest. However, due to the characteristics of distribution grids, DSSE presents a number of unique challenges. The methods and assumptions used in transmission-level SE are often not valid when considering distribution systems, and many of well-established techniques used in “conventional” SE cannot be applied directly. There are distinctions between the two networks:

1. Transmission networks transfer large amounts of power through mesh topologies, whereas distribution systems typically transfer limited amounts of power through a radial topology
2. Transmission networks have high reactance to resistance ratios transmission line, whereas distribution systems have low reactance to resistance ratio feeders
3. Transmission systems are usually assumed as three phase balanced. In

contrast, distribution systems usually have untransposed lines, and unbalanced loads.

4. In contrary to sufficient measurement redundancy in transmission networks, distribution systems have very limited amount of network measurements, resulting in little measurement redundancy.

Distribution networks usually have a large amount of nodes and radial topology, which means more measurement devices are needed to make the entire system observable. However, measurement redundancy for distribution system is generally low. The absence of adequate network measurements is one of the greatest challenges to developing distribution system state estimation. For example, due of the sparsity of measurements, pseudo measurements are employed to make the state estimation programs work efficiently, but the pseudo measurements are likely to be treated as bad data and there is no redundancy to eliminate them. Moreover, as there is a large number of current magnitude measurements in a distribution system, their inclusion in state estimation algorithms may lead to multiple solutions. In addition, the fast decoupled methods and DC approximations are often applied in conventional state estimation (SE) simply do not work when applied to distribution system state estimation (DSSE) because radial feeders in distribution systems have low reactance-resistance ratio [4].

This situation will be alleviated because of the advent of μ PMUs. Considering its relatively low cost and high resolution, μ PMUs can provide additional measurements with high precision, which will contribute to a more robust state estimator, thus more accurate monitoring of distribution systems. Moreover, μ PMUs are able to provide voltage and current phasor measurements which have a linear relation with the system states. Therefore, it is possible to formulate a linear state estimator for distribution system as long as there are sufficient μ PMUs to observe the entire

system. Thus, it is anticipated that more and more μ PMU will be employed in distribution systems in the future.

2.4 State Estimation with Sychrophasor Measurements

Conventional measurements such as real and reactive power injection have nonlinear relationship with the system states, which result in nonlinear state estimation equation, need to be solved iteratively and introduce computational burden. With the proliferation of synchronous phasor measurements, new types of measurement devices such as PMU become available, where the measurements are linearly related to the system states. Consequently, state estimation is a natural application of PMU measurements. In this section the linearity of PMU measurement will be introduced first, which is the most important characteristic of PMU, followed by the introduction of different approaches to include PMU measurement into state estimation.

2.4.1 Linearity of PMU measurement

In the recent years the number of PMUs deployed has rapidly increased and it is predicted that PMU will observe the entire power system in future. On the condition that the system is fully observable by PMU, linear state estimator can be developed, considering that phasor measurements are related to the system states linearly. The linear relationship is revealed using a two port π -model as shown in Fig. 2.3. Generally system states are bus voltage magnitude and phase angle and the states in this simple system are the voltage phasors V_i and V_j .

Assuming a PMU is installed at the bus i , the PMU measurements will be the voltage phasor at bus i , V_i , and the current phasor I_{ij} that flows from bus i to bus j . The system state vectors and measurement vectors are:

$$x = \begin{bmatrix} V_i \\ V_j \end{bmatrix} \quad (2.12)$$

$$z = \begin{bmatrix} V_i \\ I_{ij} \end{bmatrix} \quad (2.13)$$

The voltage phasor measurement V_i is one of the system states, so V_i is related to one of system states identically. The current measurement I_{ij} can be related to system states by the following equation:

$$[I_{ij}] = \begin{bmatrix} y_{ij} + y_{i0} & -y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \quad (2.14)$$

where the impedance is defined as:

$$y_{ij} = (r_{ij} + jx_{ij})^{-1} = g_{ij} + jb_{ij} \quad (2.15)$$

$$y_{i0} = g_{si} + jb_{si} \quad (2.16)$$

In case the estimation solver is not able to deal with complex equations directly, the following equations express the linear relationship of current phasor measurement and system states without involving complex number. The measurements and system states are expressed in rectangular coordinate, because the linearity relationship does not hold in polar coordinate, i.e. current phase angle is not linearly related to voltage phase angle or magnitude.

$$I_{ij}^R = g_{ij}(V_i^R - V_j^R) - b_{ij}(V_i^I - V_j^I) - b_{ii}V_i^I \quad (2.17)$$

$$I_{ij}^I = g_{ij}(V_i^I - V_j^I) + b_{ij}(V_i^R - V_j^R) + b_{ii}V_i^R \quad (2.18)$$

The measurement equation $z = Hx$ can then be expressed as:

$$\begin{bmatrix} V_i \\ I_{ij} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ y_{ij} + y_{i0} & -y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \quad (2.19)$$

If the PMU is placed at bus j , the equations can be derived likewise as shown in Eq.2.20. From this equation, it can be seen that when a PMU is deployed at one end of a transmission line, the voltage at the other end can be calculated.

$$\begin{bmatrix} V_j \\ I_{ji} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -y_{ij} & y_{ij} + y_{j0} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \quad (2.20)$$

2.4.2 Hybrid State Estimation with SCADA and PMU measurements

Conventional measurements such as real and reactive power injection have nonlinear relationship with the system states, which result in nonlinear state estimation equation, need to be solved iteratively and introduce computational burden. With the proliferation of synchronous phasor measurements, new types of measurement devices such as PMU become available, where the measurements are linearly related to the system states. Consequently, state estimation is a natural application of PMU measurements [8]. Considering the implementation cost of a single PMU and large scale of modern power system, traditional SCADA cannot be replaced by PMU overnight. Thus, hybrid SE is proposed to integrate SCADA and PMU measurements before the power systems become fully observed by PMUs [42, 43].

Hybrid SE faces special challenges because PMU measurements have quite different characteristics compared to conventional SCADA measurements such as the different refresh rates and different accuracy level [44, 45]. The refresh rate of PMU measurements is typically 30 times per second, whereas SCADA systems capture one snapshot every 2-6 seconds. Different accuracy level may result in

ill-conditioning and convergence problem [42]. Two types of solutions have been proposed in literatures, indirect and direct approaches [42, 43].

In indirect approaches, the system is assumed to be fully observed by conventional SCADA measurements with which the conventional SE is carried out before the PMU measurements are incorporated. Zhou et al. proposed a linear post-processing step to combine PMU measurements with estimation results of the conventional state estimator [35]. The two-stage method is also used by Asprout et al. to track for track the dynamics of the power system states during a fault. To account for the different refresh rate of PMU and SCADA, an hybrid SE is executed as the SCADA measurements update and a linear SE is run with the PMU measurements and pseudomeasurements between two consecutive SCADA update [46]. Fusion techniques are adopted in [47, 48] to combine the results obtained from two estimators that processing SCADA and PMU measurements separately. Multistage-based methods are also utilized by Nuqui et al. and Cheng et al. [49, 50]. The biggest advantage of indirect approaches is that the internal structure of existing SE is kept unchanged. PMU measurements can be processed by an additional estimation module [47]. However, this approach is only feasible when the system is observable with SCADA measurements.

The direct methods, on the contrary, process PMU and SCADA measurements simultaneously instead of in a staged manner. The disparity refresh rates of PMU and SCADA systems can be handled by the use of buffering which means selecting the subset of the PMU time series of measurements corresponding to the instant at which SE is executed [51, 52]. Glavic et al. propose a method to reconstruct and track network state when there are limited number of PMU measurements, which is not sufficient to make the entire system observable. Pseudo-measurements are calculated using the last reconstructed state in a recursive manner [53]. The weighted least absolute value (LAV) is adopted to estimate the system states at

instants when only PMU measurements are updated [54]. Kabiri et al. propose a method to decouple the SCADA and PMU measurements and transform the hybrid SE into two separate problems to circumvent the ill-conditioning caused by disparity of accuracy level of PMU and SCADA measurements [42]. The disadvantage of direct method is that it requires change to the current SCADA based SE software package.

2.4.3 Post-processing Step to Integrate SCADA and PMU Measurements

This subsection presents a method to integrate PMU and SCADA measurements by a linear post-processing step is developed to include phasor measurements [35]. This method relies on the estimates of the system state from the conventional state estimator, and enhances them with a linear calculation using PMU measurements.

Note that the post-processing step has linear characteristic when the state estimation equations are expressed in rectangular coordinates.

The measurements used in the post-processing step are traditional state estimation results $\begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix}_{SE}$, bus voltage phasors $\begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix}_{PMU}$ and branch current phasors $\begin{bmatrix} I_{real} \\ I_{imag} \end{bmatrix}_{PMU}$. The state estimation equations for the post-processing step becomes:

$$z = Hx + v \Rightarrow \begin{bmatrix} \begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix}_{SE} \\ \begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix}_{PMU} \\ \begin{bmatrix} I_{real} \\ I_{imag} \end{bmatrix}_{PMU} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \\ I' & 0 \\ 0 & I' \\ C_1 & C_2 \\ C_3 & C_4 \end{bmatrix} \begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix} + \begin{bmatrix} e \end{bmatrix} \quad (2.21)$$

where e is the measurement error, I is a unit matrix, I' is a matrix that has only one nonzero entry (equals to 1) in each row and the rest of the entries are zeros. C_1, C_2, C_3, C_4 are matrices that made up the branches admittance. H is a constant matrix, such that this equation can be solved directly. Note that the covariance matrix should be transformed to rectangular coordinates because all quantities are expressed in rectangular coordinates.

In the above linear model, the coefficient matrix H is not easy to construct because of the existence of current phasor measurements. There is another model to perform the post-processing step, where the entry of the matrix H is either one or zero.

This model uses the estimation results from conventional state estimator, voltage phasor measurements at PMU buses, and the voltage phasor at buses observed by PMU. The estimation results are in polar coordinate, in terms of voltage amplitude and angle of nodes. The voltage phasor at buses observed by PMU are calculated using current phasor measurement together with voltage phasor measurements as follows:

$$\begin{bmatrix} \begin{bmatrix} V \\ \theta \end{bmatrix}_{SE} \\ \begin{bmatrix} V \\ \theta \end{bmatrix}_{PMU_V} \\ \begin{bmatrix} V \\ \theta \end{bmatrix}_{PMU_I} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \\ I' & 0 \\ 0 & I' \\ I'' & 0 \\ 0 & I'' \end{bmatrix} \begin{bmatrix} V \\ \theta \end{bmatrix} + \begin{bmatrix} e \end{bmatrix} \quad (2.22)$$

where V and θ are the voltage amplitude and angle respectively, I is a unit matrix, I' and I'' are matrices that have only one nonzero entry (equals to 1) in each row and the rest of the entries are zeros. I' indicates the relation between direct voltage phasor measurements to the system states, whereas I'' indicates the relationship between calculated voltage phasor on adjacent buses to system states.

2.5 Optimal Placement of PMU

OPP is essentially an optimization problem on the placement of minimum number of PMUs to achieve total network observability. The input of OPP is the topology of a network and the output is a subset of the network buses where PMUs should be installed. Algorithms that have been applied to solve the OPP problem have been well documented [10, 17, 20–30]. They can be divided into three categories: mathematical programming, heuristic optimization and meta-heuristic optimization. Mathematic programming methods consist of linear programming (LP), nonlinear programming (NLP) and dynamic programming etc, while heuristic methods consist of depth first search and minimum spanning tree, and meta-heuristic methods include simulated annealing (SA), genetic algorithm (GA), tabu search (TS), differential evolution (DE) etc [9, 18, 19].

Simulated annealing and integer linear programming will be next introduced in

detail in this section. Note that simulated annealing and integer linear programming will form the foundation of the materials to be covered in Chapters 4 and 5.

2.5.1 Dual Search Method

The dual search method is a hybrid method that uses bisecting search to adjust the search range and simulated annealing to determine the observability of system [17]. It is among the earlier methods used in OPP research and it consists of two modules:

1. Fix the number of PMUs n_p using bisecting search method.
2. The fixed number n_p is then applied to simulated annealing-based procedure to minimize the size of the unobservable region. The simulated annealing method helps to avoid local minima in the search for global minimum.

In this method, the minimization of n_p and the determination of network observability are executed separately to result in reduced size of the search space for evaluating the PMU coverage at each iteration. In order to narrow down the search range of optimal number of PMUs, the upper bound of the minimum number of PMUs is determined theoretically as follows:

$$N_{PMU}^{UB} = \left\lceil \frac{N + \frac{k}{2}}{3} \right\rceil \quad (2.23)$$

where N is the number of buses in the network and k is the number of unknown power injection. Assuming a worst case system, that is $k = N$, this equation yields $N_{PMU}^{UB} = \frac{N}{2}$.

Graph Theoretic Procedure

Besides these two procedures, the graph theoretic procedure is used in this method to look for a good initial guess of PMU placement, which can help accelerate the

solution process [17]. Graph theoretic procedure can identify a placement set that makes the system observable using the following three steps:

1. Identify the number of incident lines of each bus in the network. A PMU is then assigned to the bus with the most number of incident lines in the unobservable region.
2. Determine the observability of the network and identify the unobservable region.
3. Stop when the unobservable region is an empty set, otherwise go to step 2.

Fig. 2.4 shows the flow-chart of the graph theoretic procedure.

Figure 2.4: Flow Chart of Graph Theoretic Procedure

Bisection Search Method

Bisection search method is used to narrow the search interval of the number of PMU at each iteration. The upper and lower limits of the interval consists of the number of PMUs which make the system observable and unobservable, respectively. In each iteration, a point is selected as a candidate to become a new upper or lower limit. Fig. 2.5 shows the flow chart of the modified bisection search method, where ν is the number of PMUs, and $\Psi(\nu)$ is defined as the number of buses of the smallest unobservable sub-network for a given number of placed PMUs ν .

The flowchart can be summarized into three procedures

1. Initialization. The graph theoretic-based procedure is applied to determine the upper limit of the initial search range. The low limit is set to zero.
2. Test point selection. Generally, the middle point of the interval is selected as the test point. However, the finally results is typically 85% to 95% of the graph theoretic guess according to the author in [17]. Therefore the test point is selected as 85% of the upper limit if the low limit of the interval is zero.
3. Termination. The process ends when the length of the interval equals to one, and the upper limit is able to make the system observable whereas the low limit is not.

Simulated Annealing

For a given number of PMUs deployed, there may be a large number of location combinations where they can be placed . Although exhaustive enumeration is a straightforward method that can be used to check all the combinations one by one and find the optimal one, however, it may take a long time to derive the solution

Figure 2.5: The Dual Search Method

due to the large number of combinations. On the other hand, simulated annealing (SA) algorithm is an efficient method that can be applied in the dual search method to solve the global optimization problem and determine the observability of network given a specific number of PMU.

To construct this problem using SA, some definitions are introduced as follows and the flowchart is presented in Fig. 2.6:

1. An objective function E is selected to be the number of unobservable buses. The objective function is minimized over the search space of PMU placement set. The objective function is analogous to energy.
2. A random generator function randomly chooses a PMU in the present placement set, and move it to a randomly chosen new bus, to generate a new placement set. The new set is a neighboring set of the present placement set. The objective function E is re-evaluated for the new placement set.
3. A monotonically decreasing function T is analogous to a cooling temperature. The new placement set is accepted if the incremental change of the objective function is negative. Otherwise, it is accepted with a probability. The probability is modeled by the modified Boltzman function.

$$\text{Prob}(\Delta E) = e^{-\frac{\Delta E}{T}} \quad (2.24)$$

Figure 2.6: Flowchart of Simulated Annealing

2.5.2 Integer Linear Programming

Integer linear programming has been used to solve OPP problem by many researchers [20, 26, 30, 55, 56]. The OPP problem is formulated as an integer linear programming problem in Eq.(2.25).

$$\begin{aligned} \min \sum_i^n w_i x_i \\ \text{subject to } \mathbf{Ax} \geq \mathbf{b} \end{aligned} \quad (2.25)$$

where $\mathbf{x}$ is defined as a binary decision variable vector indicating whether a PMU is placed at a bus. The entries in $\mathbf{x}$ is defined as follows:

$$x_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{if otherwise} \end{cases} \quad (2.26)$$

The coefficient w_i could be the installation cost of PMUs at different location. Matrix $\mathbf{A}$ is the connectivity matrix of the system topology. The formulation of $\mathbf{A}$ and $\mathbf{b}$ are given as follows: which is built in this way:

$$A_{k,m} = \begin{cases} 1, & \text{if } k = m \text{ or } k \text{ and } m \text{ are connected} \\ 0, & \text{if otherwise} \end{cases} \quad (2.27)$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{N \times 1} \quad (2.28)$$

A 7-bus example with the topology shown in Fig. 2.7 illustrates the formulation of the constraint equations.

Figure 2.7: The topology of a 7 bus system

$$\mathbf{Ax} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (2.29)$$

2.5.3 OPP With Conventional Measurements

In practice, PMUs are supposed to be installed in systems which are already equipped with conventional measurements which contribute to the observability of the system [56]. Taking into consideration these conventional measurements helps to reduce the minimum number of PMUs, and various methods have been proposed to integrate existing measurements into the mathematical formulation of OPP problem [20,26,27,55,56]. Xu et al. propose a method to modify the observability constraint to account for the conventional power flow and power injection measurements, which is referred to as the Individual Bus Merging (IBM) method [20]. However,

the method proposed to account for existing measurements requires manually simplification of inequation constraints. In addition, this method gives different solutions for different selection of the bus merged with the injection bus [56]. The Augmented Bus Merging (ABM) Method is proposed by Abbasy et al. to overcome the drawback of the IBM method. A permutation matrix $\mathbf{P}$ is constructed to generate a new connectivity matrix. A similar method can be found in [26], where the permutation matrix is utilized for incomplete observability analysis. Xia et al. investigate on how to find the placement that can improve the local redundancy of the conventional measurements the best under the assumption that the system is fully observable with conventional measurements [30].

Next the 7 bus system is used to illustrate how to use the IBM and ABM methods to account for zero-injection buses and power flow measurements. Assuming that bus 3 in Fig.2.7 is a zero injection bus, and there is a flow measurement in branch 1-2. If IBM method is used, the zero injection bus is merged into one of its incident buses. The following equation gives the modified connectivity matrix after merging.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (2.30)$$

The power flow measurement in branch 1-2 indicates that if either one of bus 1 or 2 is observable, the other one will become observable as well. Hence the connectivity

matrix is further modified to:

$$\mathbf{A}^{new} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (2.31)$$

Note that the $\mathbf{b}$ remains as a unit vector, but the dimension should be modified accordingly. Besides the above modification, the IBM could also lead to other connectivity matrix, depending on which bus is selected to merge with bus 3.

If ABM method is adopted for the same scenario, the modification of $\mathbf{A}$ will be quite different. First, the buses that are incident to the conventional measurements are listed, $\mathbf{S}_i \mathbf{n} = 1, 2, 3, 4, 6$, and the buses not incident conventional measurements are $\mathbf{S}_u \mathbf{n} = 5, 7$. Two conventional measurements correspond to two augmented buses $1^{aug}, 2^{aug}$. The new bus order and permutation matrix $\mathbf{P}$ will be:

$$\mathbf{b}_o = \begin{bmatrix} 5 & 7 & 1^{aug} & 2^{aug} \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.32)$$

Hence, the new connectivity matrix $\mathbf{A}^{new}$ and $\mathbf{b}^{new}$ can be obtained.

$$\mathbf{A}^{new} = \mathbf{P}\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 3 & 4 & 2 & 1 & 3 & 2 \\ 2 & 2 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (2.33)$$

$$\mathbf{b}^{new} = \begin{bmatrix} 1 & 1 & 3 & 1 \end{bmatrix}^T$$

2.6 Conclusion

In this chapter, the background of PMU and its basic working principle are presented, followed by the introduction of μ PMU. The main processes and information flows of power system state estimation are also introduced, including network topology processing, observability analysis, state estimation algorithm and bad data identification and processing. Challenges in distribution system state estimation are also discussed. On one hand, the increasing penetration of distributed generation in power distribution brings along new characteristics which necessitate close distribution system monitoring. On the other hand, distribution systems differ from transmission networks in terms of construction, redundancy, measurement types, scale and complexity. Following that, the inclusion of phasor measurement in state estimation equations, i.e. mixed with SCADA measurement in different ways or PMU measurement exclusively, to form linear state estimator is presented. Finally, several typical optimal PMU placement algorithms, simulated annealing, genetic algorithm and integer programming, are introduced.

Chapter 3

State Estimation with μ PMU for Distribution System

3.1 Introduction

Distribution system state estimation (DSSE) has been receiving significant research interest in recent years, due to the increasing penetrations of distributed energy resources in distribution networks, and hence a need for improved observability in distribution systems [11, 57–60]. The distinct characteristics of distribution systems bring challenges to DSSE as compared to transmission system state estimation. For example, distribution systems usually have unbalanced three phase load, which makes the positive sequence state estimator used in transmission-level SE not valid. Another challenge of DSSE is the low measurement redundancy, since for technical and economic reasons distribution networks have limited number of measurement points. Moreover, distribution system measurements are fraught with much more noise, which necessitates the deployment of robust state estimator [3, 4, 13].

Besides the plenty of research that has been conducted to address the challenges

of DSSE, new technologies have been invented to deal with the distribution system applications. μ PMU is such a device designed for distribution system application and is able to provide synchronized phasor measurement, which may bring DSSE more options [13]. As the study of the application of μ PMUs in the distribution network is in its infancy, there are limited works study how the use of μ PMU will affect DSSE. In this chapter, a linear state estimator for distribution system is formulated based on the assumption that the system is fully observed by μ PMUs. The state estimator adopts three phase formulation to deal with unbalanced loads, and a unified approach to constructing three phase linear state estimator is developed in this chapter. Various state estimation (SE) algorithms including weighted least squares (WLS), multiple-segment (MS), quadratic-linear (QL) and square-root (SR) estimators are implemented and the SE accuracy is evaluated in terms of mean square error (MSE). The impact of measurement redundancy on SE is investigated by controlling the number of input measurements.

3.2 Measurement Model and State Equation

Assuming a distribution system is fully observed by μ PMUs, a linear state estimator can be formulated as:

$$z_t = Hx_t + v_t, \quad (3.1)$$

where t is the time step and $z \in \mathbb{R}^m$ is the measurement vector composed of the real and imaginary components of bus voltage (or the line current) phasors. The state vector x includes the real and imaginary parts of the voltage phasors. v is assumed to be measurement noise with zero mean. Specially the j -th measurement is given by

$$z_{j,t} = H_j x_t + v_{j,t}, \quad (3.2)$$

where j is the measurement index and v_j is uncorrelated between different measurements. We assume that the matrix $G = \sum_{j=1}^m H_j^T H_j$ is full rank in this work.

A traditional power system may be considered as a quasi-static system [61, 62] because load demands change slowly and hence the state changes slowly. The oscillations in the state variables are thus assumed small, and hence the following simplified process model is considered for the state estimation problem:

$$x_{t+1} = Ax_t + w_t \quad (3.3)$$

where A is an identity matrix [62, 63] and w_t represents the zero-mean disturbance with variance $Q > 0$.

3.3 Robust Estimators

Different types of robust estimators are presented in this section including Multiple-Segment (MS), Quadratic-Linear (QL) and Square-Root (SR). We assume the load demands change slowly and may be considered as a quasi-static system [62] i.e. $x_{t-N} = \dots = x_t = x$. Given $N + 1$ sets of measurements $z_{j,k}$, $k = t - N, \dots, t$ collected from $j = 1, \dots, m$ measurements, the state x can be estimated by minimizing the following cost function:

$$J = \sum_{j=1}^m \sum_{k=t-N}^t \rho(e_{j,k}), \quad (3.4)$$

where $e_{j,k}$ is the measurement residual,

$$e_{j,k} = z_{j,k} - H_j \hat{x}. \quad (3.5)$$

Eq. (3.5) gives $\frac{\partial e_{j,k}}{\partial \hat{x}} = -(H_j)^T$. Differentiating the above cost function (3.4)

with respect to $\hat{x}$,

$$\begin{aligned}
\frac{\partial J}{\partial \hat{x}} &= \Psi(E) = \frac{\partial J}{\partial e_{j,k}} \frac{\partial e_{j,k}}{\partial \hat{x}} \\
&= \sum_{j=1}^m \sum_{k=t-N}^t \frac{\partial \rho(e_{j,k})}{\partial e_{j,k}} \frac{1}{e_{j,k}} e_{j,k} \frac{\partial e_{j,k}}{\partial \hat{x}_k} \\
&= - \sum_{j=1}^m \sum_{k=t-N}^t W_{j,k} e_{j,k} (H_j)^T,
\end{aligned} \tag{3.6}$$

where

$$W_{j,k} = \sum_{j=1}^m \sum_{k=t-N}^t \frac{\partial \rho(e_{j,k})}{\partial e_{j,k}} \frac{1}{e_{j,k}}. \tag{3.7}$$

Using Eq. (3.5), $\Psi(E)$ can also be written by

$$\begin{aligned}
\Psi(E) &= - \sum_{j=1}^m \sum_{k=t-N}^t W_{j,k} (z_{j,k} - H_j \hat{x}) (H_j)^T \\
&= - \bar{H}^T W E \\
&= - \bar{H}^T W (Z - \bar{H} \hat{x}),
\end{aligned} \tag{3.8}$$

where

$$\begin{aligned}
\bar{H} &= [H^T \dots H^T]^T \in \mathbb{R}^{(N+1)m \times n}, \\
Z &= [z_{t-N}^T \dots z_t^T]^T \in \mathbb{R}^{(N+1)m}, \\
E &= [e_{t-N}^T \dots e_t^T]^T \in \mathbb{R}^{(N+1)m}, \\
W &= \text{diag}(W_{1,t-N}, \dots, W_{m,t-N}, \dots, W_{1,t}, \dots, W_{m,t}) \in \mathbb{R}^{(N+1)m \times (N+1)m}.
\end{aligned}$$

Since we assume the matrix $G = \sum_{j=1}^m H_j^T H_j$ is full rank, the matrix $\bar{H}^T W \bar{H}$ is also full rank and invertible. Set $\Psi(E) = 0$, then from Eq.(3.8) the estimation of x_t is given by

$$\hat{x}_t = (\bar{H}^T W \bar{H})^{-1} \bar{H}^T W Z. \tag{3.9}$$

The configuration of the diagonal matrix W for the MS, QL, SR estimators are described in following subsections.

3.3.1 The Multiple-Segment, Quadratic-Linear and Weighted-Least-Squares Estimators

The cost function of the MS estimator, sometimes known as the Hampel estimator [64], is given as

$$\rho(e_{j,k}) = \begin{cases} \frac{(e_{j,k})^2}{2\sigma_j^2} & |e_{j,k}| \leq a_j\sigma_j \\ \frac{a_j|e_{j,k}|}{\sigma_j} - \frac{a_j^2}{2} & a_j\sigma_j < |e_{j,k}| < b_j\sigma_j \\ -\frac{a_j(r_j\sigma_j - |e_{j,k}|)^2}{2(r_j - b_j)\sigma_j^2} \\ + \frac{1}{2}a_jr_j + \frac{1}{2}a_jb_j - \frac{1}{2}a_j^2 & b_j\sigma_j \leq |e_{j,k}| < r_j\sigma_j \\ \frac{1}{2}(a_jr_j + a_jb_j - a_j^2) & r_j\sigma_j \leq |e_{j,k}| \end{cases}$$

where a_j, b_j and r_j are the thresholds selected by the user and σ_j is the variance of measurement $z_{j,t}$.

Differentiating the above cost function with respect to $e_{j,k}$ and substituting into Eq.(3.7) gives

$$W_{j,k} = \begin{cases} \frac{1}{\sigma_j^2} & |e_{j,k}| \leq a_j\sigma_j \\ \frac{a_j}{\sigma_j|e_{j,k}|} & a_j\sigma_j < |e_{j,k}| < b_j\sigma_j \\ \frac{a_j(r_j\sigma_j - |e_{j,k}|)}{(r_j - b_j)\sigma_j^2|e_{j,k}|} & b_j\sigma_j \leq |e_{j,k}| < r_j\sigma_j \\ 0 & r_j\sigma_j \leq |e_{j,k}| \end{cases} \quad (3.10)$$

The MS estimator is reduced to the QL estimator when $b_j \rightarrow \infty, r_j \rightarrow \infty$ and the weighted-least-squares (WLS) estimator when $a_j \rightarrow \infty, b_j \rightarrow \infty, r_j \rightarrow \infty$. The cost functions of QL and WLS are not listed separately for simplicity.

3.3.2 The Square-Root Estimator

The M-estimator using Square-Root (SR) function is given by

$$\rho(e_{j,k}) = \begin{cases} \frac{(e_{j,k})^2}{2\sigma_j^2} & |e_{j,k}| \leq a_j\sigma_j \\ 2a_j^{3/2}\sqrt{\frac{|e_{j,k}|}{\sigma_j}} - \frac{3}{2}a_j^2 & \text{otherwise} \end{cases}$$

Differentiating the above cost function with respect to $e_{j,k}$ and substituting into Eq.(3.7) gives

$$W_{j,k} = \begin{cases} \frac{1}{\sigma_j^2} & |e_{j,k}| \leq a_j\sigma_j \\ \sqrt{\frac{a_j^3}{\sigma_j|e_{j,k}|^3}} & \text{otherwise} \end{cases} \quad (3.11)$$

The cost functions for different estimators and their derivatives are respectively shown in Figs. 3.1 and 3.2, where the thresholds are set as $a_j = 3, b_j = 4$ and $r_j = 5$.

Figure 3.1: The cost function $\rho(e_{j,k})$ of the MS, QL and SR estimator.

$W_{j,k}$ is re-weighted according to the measurement residuals $e_{j,k}$, which is calculated after each iteration of the estimation. The steps to implement the robust estimators can be summarized in the following pseudo-code:

Figure 3.2: The derivative of the cost function $\rho(e_{j,k})$ for the MS, QL and SR estimator.

1. Initialization: Choose an initial estimate $\hat{x}_0$;

2. Main procedure:

 Compute the state estimation $\hat{x}_t$.

while new measurement exists **do**

 (a) **if** $t \leq N$, set $t - N = t$, $\hat{x}_t = (H^T W H)^{-1} H^T W z_t$;

 (b) **else**

 (c) Set $q = 0$;

 (d) Set $\hat{x}_t^q = \hat{x}_{t-1}$;

 (e) **repeat**

 i. Set $q = q + 1$;

 ii. Calculate $e_{j,k}$ using (3.5);

 iii. Renew $W_{j,k}$ using (3.10) or (3.11);

 iv. Calculate $\hat{x}^q$ using (3.9);

 (f) **until** $(\max(|\hat{x}_t^q - \hat{x}_t^{q-1}|) \leq \delta)$ or $q = q_{max}$.

- (g) **end if**
- (h) Set $t = t + 1$;
- end while**

The iteration process will be terminated until the difference $(\hat{x}_t^q - \hat{x}_t^{q-1})$ reduces to a threshold parameter δ or the iterative index q reaches a fixed number q_{max} [65, 66].

3.4 Characteristics of Distribution Lines

In order to formulate three phase state estimator to take care of the unbalanced loads in distribution networks, the three phase representation of a distribution line is necessary. Usually, the impedance will be a 3×3 matrix. The diagonal elements are impedance of each branch, and the off-diagonal elements represent the mutual inductance of lines.

$$\begin{bmatrix} Z_D & Z_O & Z_O \\ Z_O & Z_D & Z_O \\ Z_O & Z_O & Z_D \end{bmatrix} \quad (3.12)$$

For a ideal distribution line, the diagonal elements are the same Z_D because three conductors are identical; the off-diagonal elements should also be the same Z_O if spacing between every two conductors are the same.

However, desired balanced impedances are difficult to be realized in practice. Three conductors of a distribution line will be slightly different from each other, and the spacing cannot be balanced due to physical constraints of supporting structures. The three phase representation of a branch with unequal conductor spacing can

will be:

$$\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \quad (3.13)$$

In this matrix, the diagonal elements have three different values considering the impedance and self-inductance of each phase is different. The mutual inductances between every two phases are also different. Therefore, there are six different values for this matrix.

Then the model of a distribution line is represented through a three phase π model, as shown in figure 3.3. The shunt admittance is shown as a group in the figure for simplicity.

Figure 3.3: Simplified distribution line segment model

The input current to the line segment at node i is:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_i = \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_{i0} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i \quad (3.14)$$

Applying Kirchhoff's Voltage Law to the model gives:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_j + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} \quad (3.15)$$

Substituting Eq.(3.15) into Eq.(3.14):

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_i = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}^{-1} \cdot \left\{ \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i - \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_j \right\} + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_{i0} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i \quad (3.16)$$

Collecting terms, we obtain the relationship between input current at node i and voltages at node i and j .

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_i = \left\{ \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}^{-1} + \frac{1}{2} \cdot \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}_{i0} \right\} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i - \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}^{-1} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_j \quad (3.17)$$

Actually, for distribution system, the shunt admittance of an overhead line is so small that it can be neglected [67]. When the shunt admittance is neglected, the relationship becomes:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_i = \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}^{-1} \cdot \left\{ \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_i - \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_j \right\} \quad (3.18)$$

However, the shunt admittance of underground line cannot be neglected. Therefore, Eq.(3.18) is used for overhead line and Eq.3.17 is used for underground cable.

3.5 Unified Approach to Construct Three Phase Linear State Estimator

This section discusses how to construct the matrix H , which relates the measurements vector to the system states vector. H can be constructed by putting impedance on the corresponding position. However, this method is prone to error especially when the scale of network increases, and is not suitable for programming. Therefore, a unified approach is developed to construct the matrix H in this section. It will be broken down into several individual matrices, and these individual matrices are formed separately, and then combined to form the matrix H .

In Section 2.4.1 we have listed the linear measurement equation:

$$\begin{bmatrix} V_i \\ V_j \\ I_{ij} \\ I_{ji} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ y_{ij} + y_{i0} & -y_{ij} \\ -y_{ij} & y_{ij} + y_{j0} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \quad (3.19)$$

This equation shows that the matrix H is made up of two parts: one part relates the voltage measurements to the system states and the other part relates current measurements to the system states. In the following, we will elaborate on the formulation of matrix H .

1. Voltage measurement coefficient matrix II

This matrix indicates the locations of voltage measurements in the network. We assume that this matrix is an $m_V \times n$ matrix, where m_V is the number of voltage measurements, and n is the number of system states. Obviously the sum of each row equals to 1 in this matrix. This matrix is denoted by II and can be formed using the following rules:

- If measurement X corresponds to voltage at bus Y , then a 1 is assigned to the matrix element (X,Y) .
- All remaining elements are assigned with zero.

2. Current measurement-bus incidence matrix A

The current measurement coefficient matrix M is the matrix that relate the current measurement to the system states. This matrix is determined by the location of the current phasor measurement and the network parameters. We assume this matrix is an $m_I \times n$ matrix, where m_I is the number of current measurements, and n is the number of system states. Note that usually the number of current measurements m_I does not equal to the number of voltage measurements m_V . This matrix can be obtained by multiplying current measurement-bus incidence matrix by series admittance matrix, and then add the shunt admittance matrix.

The following rules can be used to construct the current measurement-bus incidence matrix, which is denoted by A . In this matrix, the sum of each row equal to zero.

- If current measurement X leaves bus Y , a 1 will be assigned to the matrix element (X,Y) .
- If current measurement X leaves bus Y heading towards bus Z , a -1 will be assigned to the matrix element (X,Z) .
- All remaining elements are assigned with zero.

3. The series admittance matrix y

This is a diagonal matrix made up of admittance of lines being measured. It is an $m_I \times m_I$ matrix, and there is only one rule to form this matrix:

- The matrix entry (X, X) is the admittance of the branch that is measured by current measurement X .

4. The shunt admittance matrix y_s

This is an $m_I \times n$ matrix. This matrix is determined by the location of current measurements and the shunt admittance of the line. The rule to form this matrix is:

- If current measurement X leaving bus Y , the matrix element (X, Y) will be the shunt admittance of the line being measured.

Using the matrices mentioned above, the linear state estimation equation can be written as follows:

$$z = Hx + e \Rightarrow \begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} II \\ M \end{bmatrix} [x] + [e] \quad (3.20)$$

In this equation, $[z]$ is made up of voltage and current phasor measurements vectors. The system states $[x]$ is related to voltage measurements E by voltage measurement coefficient matrix II . The current measurement coefficient matrix M relates the system states to current measurements vector I , and it can be obtained by following equation:

$$[M] = [y][A] + [y_s] \quad (3.21)$$

Therefore the state estimation equation becomes:

$$[z] = \begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} II \\ yA + y_s \end{bmatrix} [x] + [e] \quad (3.22)$$

There are some characteristics of matrix H to be noted: (i) no zero rows, because every measurement must be related to at least one of system states; (ii)

no zero column, which means that all of the systems states should be linked to at least one of measurements, otherwise the system is not fully observable; (iii) tall matrix, as there should be more measurements than system states to ensure the system is observable, and taller matrix H means there is more redundancy.

In the linear state estimation equation, the matrix H is a tall matrix, so a direct inverse is not achievable. The system state can be solved using pseudo-inverse:

$$[x] = [(H^T H)^{-1} H^T][z] \quad (3.23)$$

In practice, the measurements contain errors, so a covariance matrix is included in the solution which becomes:

$$[x] = [(H^T W^{-1} H)^{-1} H^T W^{-1}][z] \quad (3.24)$$

Since distribution systems usually have unbalanced phases, a three phase state estimator is necessary. To obtain the three phase version of the matrix, firstly, three phase voltages are designated as system states.

$$x = \left[\begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_1 \quad \cdots \quad \begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_N \right]^T \quad (3.25)$$

The measurements z will then be switched to three phase.

$$z = \left[\begin{bmatrix} V_a & V_b & V_c \end{bmatrix}_1 \quad \cdots \quad \begin{bmatrix} I_a & I_b & I_c \end{bmatrix}_m \right]^T \quad (3.26)$$

Lastly, the matrix H needs to be switched to three phase representation. The formulation of three phase H follows the same rule as single phase generally. However, there are some differences which will be explained below:

1. Voltage measurement coefficient matrix $\mathbf{II}$

Identity matrix will replace all 1's, and all 0's are replaced by a null 3×3 matrix

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.27)$$

2. Current measurement-bus incidence matrix $\mathbf{A}$

This matrix is changed in the same way. All 1's and -1's are replaced by $\mathbf{I}$ and $-\mathbf{I}$ respectively, and all 0's are replaced by $\mathbf{0}$.

3. The series admittance matrix $\mathbf{y}$

Branch impedance is no longer a complex number. The three phase representation of a branch is a 3×3 matrix. Therefore, each impedance on the diagonal is replaced by the corresponding impedance matrix and the dimension becomes $3m_I \times 3m_I$.

4. The shunt admittance matrix $\mathbf{y}_s$

The shunt admittance matrix is changed in the same way with series admittance matrix.

After each of the matrices has been changed, the system matrix for the state equation is calculated in the exact same way as the single phase system.

$$\mathbf{z} = \begin{bmatrix} \mathbf{E} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{II} \\ \mathbf{yA} + \mathbf{y}_s \end{bmatrix} \mathbf{x} + \mathbf{e} \quad (3.28)$$

3.6 Simulation Results

In this section different estimators, including WLS, MS, QL and SR estimator, are tested on distribution system. The mean square error (MSE) is used to evaluate

the performance of each estimator. A set of measurement is taken at each time instance k and 3 sets of measurements i.e. $k = t-2, t-1, t$ are used to give one set of estimates. State estimation is executed for 100 time steps. The impact of measurement redundancy is evaluated by removing some measurements while maintaining the observability of the whole system. The state estimation codes were written using MATLAB and implemented on a computer(Intel Core i7)with processor speed of 2.4GHz.

3.6.1 Test Feeder and Estimation Parameters

The IEEE 13 node test feeder was selected to be the test feeder and it was modeled using Simulink. Fig. 3.4 shows the topology of the selected test feeder, which contains overhead and underground lines with variety of phasing. Unbalanced spot and distributed loads can be found in this feeder and are tabulated in Table 3.1.

Figure 3.4: IEEE 13 node test feeder

In order to develop linear three phase state estimator, the network must be

Table 3.1: Unbalanced Spot Load Data

Node	Ph-1	Ph-1	Ph-2	Ph-2	Ph-3	Ph-3
	kW	kVAr	kW	kVAr	kW	kVAr
4	160	110	120	90	120	90
5	0	0	170	125	0	0
6	0	0	230	132	0	0
7	385	220	385	220	385	220
8	0	0	0	0	170	151
9	485	190	68	60	290	212
11	0	0	0	0	170	80
13	128	86	0	0	0	0

fully observable by μ PMUs. Based on the topology of this feeder, five μ PMUs are placed at bus 2, 3, 5, 7(8), 10. The true values were obtained by carrying out a load flow using Simulink and saved in MATLAB workspace. Fig.3.5 shows the IEEE 13 node feeder incorporated with the μ PMUs. The current and voltage measurements positions are also indicated in this figure. The blue boxes denote phasor measurements, and reflect the placement of the μ PMUs. Current measurements are located at incident lines and denoted by the red boxes. Some incident lines connected to the buses installed with μ PMU but are not measured for various reasons. The current flows from bus 2 to bus 7 is not measured because of the presence of distributed load on this line. The current flows from bus 7 to bus 2 is not measured for the same reason. Since there is no load at bus 12, the current flows from bus 7 to bus 12 is considered to be zero and the voltage at bus 12 is removed from the state vector.

The estimator parameters and noise are different for voltage and current mea-

Figure 3.5: IEEE 13 node test feeder with μ PMUs

surements. For the voltage measurements, the parameters for the QL and SR estimators are chosen as $a_j = 2.5$ while the parameters for the MS estimator are $a_j = 2.5$, $b_j = 3.5$, $r_j = 4.5$ [68,69]. The noise v_j is associated with the probability density function:

$$f_j(v_j) = \frac{0.99}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{v_j^2}{2\sigma_j^2}\right) + \frac{0.01}{\sqrt{2\pi(10\sigma_j)^2}} \exp\left(-\frac{v_j^2}{2(10\sigma_j)^2}\right) \quad (3.29)$$

where $\sigma_j = 0.009$.

The first term accounted for 99% in $f_j(v_j)$ is the normal noise and the second part with larger standard deviations is assumed to be the outlier. The uniform distribution is useful for modeling initial conditions, disturbances, and measurement errors that are equally likely to occur anywhere within a given interval [70].

For the current measurements, the parameters for the QL and SR estimators are chosen as $a_j = 3$ while the parameters for the MS estimator are $a_j = 3$, $b_j = 4$,

$r_j = 5$. The noise v_j is associated with the probability density function:

$$f_j(v_j) = \begin{cases} \frac{0.99}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{v_j^2}{2\sigma_j^2}\right) + \frac{0.01}{2 \times 10\sigma_j} & |v_j| \leq 10\sigma_j \\ \frac{0.99}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{v_j^2}{2\sigma_j^2}\right) & \text{otherwise} \end{cases} \quad (3.30)$$

where $\sigma_j = 0.01$.

With the above settings, the state estimation process is conducted. We define the Mean Square Error (MSE) for the estimation results at time step k as

$$MSE_k^{[i]} = \frac{1}{n_i} \|\hat{x}_{k|k}^{[i]} - x_k^{[i]}\|^2, \quad i = 1, \dots, \ell \quad (3.31)$$

As in Fig. 3.5, there are 5 blue boxes and 11 red boxes representing voltage and current phasor measurements respectively. Since each box may involve more than one phase, in total there are 13 voltage and 27 current phasor measurements. Considering bus 8 is connected to bus 7 through a breaker and the breaker is closed. We assume that voltage at bus 8 is the same with bus 7, giving us three additional voltage phasor measurements.

WLS, MS, QL and SR estimations were carried out and the MSE of different estimation is compared in Fig. 3.6. In the first few time steps, the MSE is relatively high, reaching 11×10^{-3} . After several steps, the MSE drops to around 4×10^{-3} . This is because 3 sets of measurements i.e. $k = t-2, t-1, t$ are used to give one set of estimates and at the first two steps, only one or two sets of measurements are available. Fig. 3.6 also indicates that the WLS has higher MSE than other estimators at some time steps, this is because the WLS is not a robust estimator and some bad data may present at these time steps. In contrary, MS, QL and SR remain unaffected due to their robustness.

Figure 3.6: Mean square errors of WLS, MS, QL, SR in IEEE 13 bus feeder.

3.6.2 Case Study with Lower Redundancy

In order to observe the impact of redundancy on state estimation, some measurements are removed to reduce the measurement redundancy. These include the current measurement from bus 5 to bus 2, current measurement from bus 3 to bus 2 and current measurement from bus 10 to bus 7. It is validated that removing these measurements does not affect the observability of the system. We have 16 voltage phasor measurements and 20 current phasor measurements remaining, which are shown in Fig.3.7a. The comparison of MSE is shown in Fig.3.7b.

Next, three voltage measurements are removed to reduce the measurement redundancy further. Specifically, the voltage measurements at buses 3, 5, 10 are moved and it is validated that the system is still observable. In this case there are only 9 voltage and 20 current phasor measurements as shown in Fig.3.8a. Note that there is no redundancy in this case. The number of measurements is the same with the number of system states. Fig.3.8b shows the comparison of MSE of different estimators.

Figure 3.7: State estimation with 36 measurements. (a) IEEE 13 bus feeder with 36 measurements. (b) Mean square errors of WLS, MS, QL, SR with 36 measurements.

Figure 3.8: State estimation with 29 measurements. (a) IEEE 13 bus feeder with 29 measurements. (b) Mean square errors of WLS, MS, QL, SR with 29 measurements.

Similarly, the MSE of all the estimators is high initially and drops after a few time steps. When the redundancy is low, it is noted that the MSE for all the estimators have increased. For non-robust estimator, the impact of lower redundancy is stronger. When the number of measurements is reduced to 36, the MSE of WLS estimation can be as high as 12×10^{-3} . The MSE is further increased to 16×10^{-3} as the number of measurements reduces to 29.

3.7 Conclusion

In this chapter, weighted least squares (WLS), multiple-segment (MS), quadratic-linear (QL) and square-root (SR) estimators have been introduced, implemented and tested on the IEEE 13 node test feeder. A linear state estimation is formulated based on the assumption that the distribution feeder is fully observed by μ PMUs. A unified approach is developed to construct three phase linear state estimator efficiently. Simulation shows that the introduction of μ PMUs makes those mentioned SE better suit distribution state estimation (DSSE). The comparison of the mean square error (MSE) of each estimator indicates that the WLS is vulnerable to bad data and the impact of bad data is even stronger in the case of low redundancy. It is also indicated that the performance of all the estimators deteriorates with the decrease of measurement redundancy, which gives us the guidance to have more measurement redundancy when considering placement of μ PMUs in order to improve DSSE accuracy. Our simulation results also indicate that the MSE is lower when more than one set of measurements are used to execute the state estimation. Hence, rather than relying only on the present “snapshot” of input measurements, state estimation should also involve some immediate past measurement data.

Chapter 4

Customized Optimal PMU Placement Methods for Distribution System

4.1 Introduction

In this chapter, two customized optimal phasor measurement unit (PMU) placement (OPP) methods proposed for distribution systems will be presented. The customization steps are designed specifically to tailor to distribution systems, and are inspired by their high ratio of radial buses, which arises because of their radial topology, in contrast to transmission systems that possess mesh topology. The increasing penetrations of distributed energy resources in distribution networks have necessitated close monitoring in distribution systems in recent years [11]. On the other hand, the advent of μ PMU provides affordable high-resolution measurement device for distribution systems. It can be anticipated that the distribution system will be fully observed by μ PMUs in future [13]. Therefore, we need to

consider optimal placement of μ PMU for distribution systems. Since the optimal placement of μ PMU is the same problem with the optimal placement of PMU essentially except for the application networks, we use the same term OPP for optimal placement of μ PMU rather than introducing another abbreviation. In contrary to the vast literature of OPP on transmission network, there is little OPP designed for distribution network which has a quite different topology from transmission networks [10, 17, 20–30]. In this chapter, two customization steps are proposed based on the topology of the distribution system and applied to two commonly applied OPP algorithms: exhaustive search and simulated annealing. The two customization steps consist of pre-assigning some μ PMUs to buses connected to radial buses and determination of upper bound of the search range, both of which contributed to reducing the search space. These two customized methods are tested on various IEEE distribution systems including IEEE 13 node test feeder, IEEE 34 node test feeder and IEEE 37 node test feeder. The simulation results obtained show that the customization steps are helpful to reduce the search space and significantly alleviate the computational burden in the OPP of distribution networks.

4.2 Fundamental of Optimal PMU Placement

By receiving real-time PMU measurement information over wide locations, operators in a central control room can monitor and analyse the quality of the distribution network under static and dynamic operating conditions. It is expected that the entire network is monitored by PMU, so that a linear state estimator can be developed. Nonetheless, deployment of PMUs at all locations is normally not feasible as they are costly, and furthermore the installation cost involved is also very high, and may be more than the device cost. Alternatively, it is possible

that the entire system state can be estimated with a subset of a minimum number of network buses that are deployed with PMUs, because PMUs can measure not only the voltage phasors at the installed buses, but also all the incident currents [20]. Thus, based on the PMU measurements, and given the network topology information, all the adjacent buses voltages can be calculated. Therefore, to reduce the installation and maintenance cost of PMUs, optimal PMU placement (OPP) should be implemented to minimize the number of PMUs needed to be installed at selected buses, to achieve the entire degree of observability [10]. OPP is essentially an optimization problem on the placement of minimum number of PMUs to achieve total network observability. The input of OPP is the topology of a network and the output is a subset of the network buses where PMUs should be installed.

Algorithms that have been applied to solve the OPP problem have been well documented [10,17,20–30]. They can be divided into three categories: mathematical programming, heuristic optimization and meta-heuristic optimization. Mathematic programming methods consist of linear programming (LP), nonlinear programming (NLP) and dynamic programming etc, while heuristic methods consist of depth first search and minimum spanning tree, and meta-heuristic methods include simulated annealing (SA), genetic algorithm (GA), tabu search (TS), differential evolution (DE) etc [9,18,19].

4.2.1 Network observability

Observability is defined as the ability of the system model to be solved for a state estimate. A network being fully observable means that all of the system states can be calculated using measurements and network topology information. In OPP, checking network observability is an important element, which is often achieved by

calculating the times a bus is observed by PMU.

Since PMU measures not only the voltage phasor at the installed bus but also all incident currents, some rules can be applied to analyze the network to ensure that it is fully observable.

1. For a bus installed with a PMU, the voltage phasor at this bus and the current phasors of all its incident branches are known.
2. The voltage phasor at one end of a branch can be calculated if the voltage and current phasors at the other end of the branch are known.
3. The current phasor of a branch can be obtained if the voltage phasors at both ends of this branch are known.
4. When there is only one incident branch of a zero-injection bus in which the current phasor is not known, the current phasor can be calculated using Kirchhoff's Current Law (KCL) and the current phasors in the other incident branches.
5. The voltage phasor of a zero-injection bus without PMU can be calculated using node voltage equations if the voltage phasors of all adjacent buses are known.

Note that rule 1 provides direct measurements, since these measurements are directly obtained by PMUs without additional calculation. On the other hand, the measurements obtained from rules 2 and 3 are called pseudo-measurements, while those from rules 4 and 5 are called extension-measurements [19].

4.2.2 Redundancy

In OPP, a multiple number of optimum solutions may be available after applying the optimization algorithm. In this case, two indexes are proposed to evaluate the quality of these solutions. The bus observability index (BOI) refers to the number of PMUs observing a given bus, and the system observability redundancy index (SORI) is the sum of all BOI for the system [10].

In the BOI, β_i is defined as the times a given bus i is observed, and equals the number of PMUs observing the bus i . Note that the maximum BOI is limited to the maximum connectivity η_i of a bus plus one, which happens only when all the adjacent buses and the bus itself are equipped with PMUs.

$$\beta_i \leq \eta_i + 1 \quad (4.1)$$

SORI can be obtained by simply adding up the BOI OF all the buses in the system. A higher SORI value indicates that the PMU-based monitoring system is more reliable. The SORI can be calculated:

$$\gamma = \sum_{i=1}^n \beta_i \quad (4.2)$$

Therefore, the OPP problem can be formulated as:

$$\begin{aligned} & \min_{n_P} \{ \max \gamma(n_P, S(n_P)) \} \\ & \text{subject to } O_{bs}(n_P, S(n_P)) = 1 \end{aligned} \quad (4.3)$$

where n_p is the minimum number of PMUs, $S(n_p)$ is the optimal location of the n_p PMUs and O_{bs} is the observability evaluation logical function. The mathematical formulation may be different when different algorithms are applied to solve the OPP problem, however, the problem definition is the same.

4.2.3 Graph theory

Graph theory is extensively applied in the analysis of network observability [4]. In graph theory, the elements of a network are represented by lines called edges and their connecting points are called nodes. The graph of a physical system is made up of a set of nodes and edges, where each edge is connected to two different nodes. For a given network, the geometrical interconnection of the elements can be depicted by a line graph. A graph is said to be fully connected if all the nodes are connected by edges and any node can be reached from any other node by tracing the edges of the graph. This graph is known as undirected graph. On the other hand, a directed graph is a graph where all the edges are assigned directions. The direction of the edge is identified by an arrow from the sending node to the receiving node. In the field of optimal PMU placement, the networks are supposed to be undirected graph.

In graph theory, the terms Tree and Cut-set are defined as follows. Tree refers to a connected graph containing all the nodes but no closed path or loops. If n is the number of nodes, then the number of branches (edges) in the tree is $(n - 1)$. Cut-set is the minimum number of elements in the connected graph that if removed, will divide the graph into only two connected subgraphs. Basic cut-sets are those that contain only one branch.

Graphs can be represented and effectively stored in computers using matrices. The following types of matrices are used to represent graphs:

1. Incidence matrix: If a network is represented by a graph G with n nodes and e elements(branches), then the matrix A with n rows and e columns is known as node to branch incidence matrix. The matrix elements are given by:

$$A_{ij} = 1, \text{ if the } j\text{th element is incident to but directed away from the node } i$$

$A_{ij} = -1$, if the j th element is incident to but directed towards node i

$A_{ij} = 0$, if the j th element is not incident to node i

2. Reduced node to branch incidence matrix: Any node of the network graph can be selected as a reference node, and the matrix obtained by deleting the row corresponding to the reference node is known as reduced node to bus incidence matrix (A_r) for a power system network. Here the nodes are termed as buses and the elements as branches. A loop-free subgraph F of G is called a forest.

4.3 Customization steps

Transmission systems usually have mesh topology, where large number of substations are interconnected by transmission lines, transformers and other devices. In contrast, distribution systems are typically configured to operate in a radial configuration, where feeders stretch from distribution substations and form a tree structure with their roots at the substation and branches spreading over the distribution area [3]. If we name those single connection buses as radial buses, there will be a large number of radial buses in distribution systems. Traditional OPP methods are designed for transmission systems which have few radial buses. When we consider the application of OPP to distribution systems, some improvement could be achieved by taking advantage of their radial topology. By analyzing the topology of distribution systems, two customization steps are proposed to render the OPP algorithms better suited and more efficient when applied to distribution systems. The customized steps are shown to greatly accelerate the optimization process, and reduce the time needed to solve the optimal placement problem from more than 30 minutes to just several seconds or less.

4.3.1 Reduction of Candidate Locations

In determining the candidate locations for PMU placement, two kinds of buses, namely radial buses (single connection buses) and buses connected to radial buses, can be eliminated for the following reasons. Radial buses are not considered because these buses will definitely not be assigned with μ PMUs, whereas buses that cover radial buses are omitted because they are preassigned with μ PMUs [10, 25]. This is because μ PMUs placed at radial buses can only observe two buses, the radial buses and buses connected to them, while μ PMUs placed at buses connected to radial buses can observe at least three buses, including the radial buses and buses connected to them. In other words, if a μ PMUs is assigned to a radial bus in a placement strategy, this placement can be improved by moving the μ PMUs from the radial bus to the bus connected to it. Therefore, the assignment status of radial buses as well as the corresponding adjacent buses are no longer uncertain. When searching for optimal placement, no more effort is needed to determine the assignment for these two kinds of buses. Hence, radial buses and the corresponding adjacent buses are not included in candidate location set. Note that in so doing, the observability will not be affected, and the measurement redundancy will be increased by at least one. Considering that there are many radial buses in distribution networks, this step can help reduce the number of candidate locations greatly and hence relieve the computational burden in next step dramatically.

4.3.2 Proposed Upper Bound

In order to narrow down the search range of optimal number of μ PMUs required, a upper bound of the minimum number of μ PMUs is determined and used as the start point before the searching [17, 25, 71–73]. Baldwin et. al. proposed a

theoretical upper bound and the formula is given in Eq.(4.4) [17].

$$N_P^{UB} = \left\lceil \frac{N + \frac{k}{2}}{3} \right\rceil \quad (4.4)$$

where N is the number of buses in the network and k is the number of unknown power injection. Considering the worst case system, that is $k = N$, this equation yields $N_P^{UB} = \frac{N}{2}$. This is easy to understand. Since one μ PMU can observe at least two buses, the maximum number of μ PMU is reached when every μ PMU placed in a system observes only two buses. In such a case, the number of μ PMUs required equals to half of the number of system buses, i.e. $\frac{N}{2}$. In practice, as one μ PMU usually observes more than two buses, the number of μ PMUs required must be less than $\frac{N}{2}$. $N_P^{UB} = \frac{N}{2}$ can also be found in other works [25, 73]. Mesbah et. al. utilize $\frac{N}{3}$ as the initial upper bound [71]. But $\frac{N}{3}$ may not be applicable to random topology networks. For example, more than $\frac{N}{3}$ μ PMUs are required to achieve full observability in the worst-case network which is sketched in Fig.4.1. Zhu et. al. adopt a branch-and bound algorithm to iteratively update upper and lower bound until they approach each other and the result show that the initial upper bound calculated is over 90% [72].

Figure 4.1: Topology of worst-case network

The theoretic upper bound can help to narrow down the search range of optimal number of μ PMUs required significantly. In this work, another upper bound is proposed considering that there are many radial buses in distribution networks.

In the reduction of candidate locations, those buses that are connected to radial buses are preassigned with μ PMUs. These μ PMUs will observe all installed buses and all radial buses. Besides the radial buses, there are buses that are connected

to the preassigned buses. These buses also become observable after some buses are preassigned with μ PMUs. The remaining unobserved buses that are needed to be assigned with μ PMUs defined the maximum number of additional μ PMUs required. Therefore, the theoretic upper bound should be the summation of the number of preassigned μ PMU and the number of unobserved buses with placement of preassigned μ PMUs.

$$N_P^{UB} = N_{pre} + N_{unobs} \quad (4.5)$$

Up to this step, the upper bound of the minimum number of μ PMUs is reduced in distribution networks, which helps to reduce the search space further. A comparison of the theoretical and proposed upper bound for different IEEE standard test systems is shown in Table 4.1. It is observed that the proposed upper bounds are lower than the theoretical upper bounds.

Table 4.1: Comparison of theoretical and proposed upper bounds for the number of μ PMUs needed in various IEEE systems

Systems	IEEE 13 bus	IEEE 34 bus	IEEE 37 bus	IEEE 123 bus
Theoretical upper bound	6	17	18	61
Proposed upper bound	5	13	13	49

4.4 Customized Exhaustive Search Method

The exhaustive search method is the most straightforward method to solve the OPP problem. It is able to give the optimal solution after searching for all the possible combinations of candidate locations. However, the exhaustive nature leads to consumption of a huge amount of computational time and computer memory space. In this section, the two customization steps we proposed in this work are

applied to improve the exhaustive search method. In contrast to the conventional exhaustive search method, the customized exhaustive search method is able to solve for the global optimal solution in a much shorter time, and yet alleviate significantly the computational resources needed.

The customized exhaustive search method consists of three main steps. First of all, the candidate locations are refined by the customization steps to reduce the number of potential locations. After that, a binary search algorithm is applied to determine the minimum number of μ PMU required. When applying the binary search algorithm, our proposed upper bound for the number of μ PMUs needed is adopted instead of the theoretical upper bound. This has helped to reduce the search space and improve the efficiency of the process. Finally, the optimal solution is selected by ranking the solutions by their SORI.

4.4.1 Search for Minimum Number of μ PMUs

Firstly, two kinds of buses are removed from the candidate locations: radial buses and buses connected to the radial buses. The binary search algorithm is then used to determine the minimum number of μ PMUs [25, 28]. The proposed upper bound is adopted here to speed up the iteration. The modified binary search algorithm is shown in Fig. 4.2 [25].

This process can be summarized as follows:

1. A test point N_P is selected, which is initialized as the proposed upper bound for the number of μ PMUs needed. We examine whether the system can be fully observable if the number of μ PMU equals to N_P .
2. For the chosen test point N_P , all combinations of locations are generated. If one of these combinations can render the system observable, the number

Figure 4.2: The flow chart of the modified binary search algorithm.

of test point N_P is decreased by one, i.e. $N_P = N_P - 1$ and this process is repeated.

3. When N_P is decreased to a point that none of its combinations of locations makes the system observable, the search process is terminated.
4. The minimum number of μ PMU is then obtained by increasing the number of test point by one, that is, $N_P^{min} = N_P + 1$.

4.4.2 Redundancy Comparison

In the previous step, the minimum number of μ PMUs required, N_P^{min} , has been determined. In this step, the locations of these μ PMUs are determined as follows to achieve maximum redundancy.

1. Generate all possible combinations of locations.

$$N_{cbm} = \binom{P}{N_P} = \frac{P!}{N_P!(P - N_P!)} \quad (4.6)$$

Check the observability of every combination to find the feasible solutions.

2. Assume that the number of candidate solutions is N_{cad} . For each candidate, we calculate the bus observability index (BOI) vector β_i and the SORI γ_i .
3. Choose the candidate having the maximum redundancy as the optimal solution(s).

The above three steps use a hybrid approach to solve the optimal placement method. The exhaustive manner ensures sure that the solution obtained is the global optimum, whereas the computational burden is significantly alleviated by the reduction of the candidate locations and further reduced by applying our proposed upper bound for the number of μ PMUs required. This approach is also very straightforward and easy to implement without any additional toolbox required.

4.5 Customized Simulated Annealing

Simulated annealing (SA) is a generic heuristic optimization method inspired from annealing in metallurgy. It finds a good solution to an optimization problem by trying random variations of the current solution. A better variation is immediately accepted as a new solution. On the other hand, instead of being rejected directly, a worse variation is accepted as the new solution with a probability that decreases as the computation proceeds. The slower the cooling schedule, or rate of decrease, the more likely the algorithm is able to find an optimal or near-optimal solution [19]. This algorithm can be used to determine the observability of a network given the number of μ PMU available. This method is faster than the exhaustive search method because it does not check all possible placement sets exhaustively. The search space is sequentially explored using the random generator function whose responsibility is to generate the next placement set based on the current placement set. However, it will also consume long time when the size of the network increases.

Considering that our proposed customization steps contribute to the reduction of the search space and hence acceleration of the solving process in the exhaustive search method, it is expected that the SA method can be similarly improved by our proposed customization steps. In this section, the customized simulated annealing method is presented. It is different from the conventional simulated annealing in two aspects. Firstly, the initialization placement is determined by the customization steps as presented in Section 4.3.1 to reduce the number of possible candidate locations. Secondly, the generator function utilizes the set of reduced candidate locations rather than arbitrarily select the next placement set. This will help narrow down the search space and accelerate the process of solving for the solutions. The customized simulated annealing method is described in detail as follows.

1. Initialization of placement and search range. The initial placement set is made up of preassigned buses (buses connected to radial buses) and unobservable buses. The initial search range is $[0, N_P^{UB}]$.
2. Select the test point N_P as the mid-point of the search interval. Check the system observability using the simulated annealing algorithm.
3. Adjust the search range interval based on the system observability. If the system is observable, $N_l = N_P$. Otherwise, $N_u = N_P$.
4. When the interval length is one, the process is terminated, and the solution is N_u . Otherwise go to Step 2.

The check of the system observability with N_P μ PMUs in Step 2 is the most complex process. It is carried out using the following steps.

1. Given the test point N_P , randomly generate a placement having N_P μ PMUs.
2. Randomly select a μ PMU and a non- μ PMU bus. Note that the selected μ PMU should not be one of the preassigned μ PMUs, and the non- μ PMU bus should not be one of the radial buses.
3. Move the μ PMU to the new bus and evaluate the coverage of the modified placement set E_{new} .
4. If $E_{new}=0$, the system can be observed with N_P μ PMUs. The check of system observability is terminated. If $E_{new} \neq 0$ go to next step.
5. Accept the new placement based on the incremental change of the objective function ΔE and the probability function which is a function of ΔE and temperature T . Go to Step 2.
6. The temperature is reduced after a number of steps (M) have been attempted.

7. The above steps are repeated until one of the stopping criteria is satisfied.

There are some functions to be defined in the simulated annealing process. The first one is the objective function E to be minimized in Step 3. It is defined as the number of buses in the unobservable region for the given placement. The function is minimized over the search space of μ PMU placement sets of size N_P .

$$E = N_{unobs} \quad (4.7)$$

In Step 5, the acceptance of new placement is based on the incremental change of the objective function ΔE . If there is a decrease in the number of unobservable buses ($\Delta E < 0$), the new placement set replaces the old one. Otherwise, the new set will replace the old set according to probability function which is controlled by T , a function that is analogous to a cooling temperature. The probability function is defined as:

$$Prob(\Delta E) = e^{-\frac{\Delta E}{T}} \quad (4.8)$$

while the temperature function is given as:

$$T_i = T_0 \beta^i \quad (4.9)$$

The value of the two parameters T_0 and β have an effect on the overall solution time. In this work, we have chosen the following parameter, $T_0 = 15, \beta = 0.879$, referring to [17]. When customization steps are adopted, these two parameters remain the same so that we can credit any improvement to customization steps.

In Step 6, the number of μ PMUs placement sets M that have to be evaluated at a given temperature is based on the size of the search space. The search space has $\binom{N}{N_P}$ placement sets, where N is the number of buses in the system and N_P is the test point. It has been observed that a good choice for M is.

$$M = 0.002 \binom{N}{N_P} \quad (4.10)$$

After applying our customization steps, the number of candidate locations is reduced compared to the total number of all buses in a system, i.e. it is reduced from N , to N_{cad} . The test point is N_P , but among some μ PMUs these N_P μ PMUs have been preassigned to some locations. Therefore, the number of μ PMUs that have not yet be assigned is $N_P - N_{pre}$. Hence, the number of iteration in Step 6 becomes

$$M = 0.002 \left(\frac{N_{cad}}{N_P - N_{pre}} \right) \quad (4.11)$$

There are two stopping criteria. Either of them being satisfied will trigger the termination of the simulated annealing algorithm. If the objective function E reaches zero, which means the system can be fully observable with N_P μ PMUs provided, the check of system observability is terminated. Alternatively, if the lowest allowed control temperature T_L is reached and there is no placement set that makes the system observable, the process is terminated and return with 'system unobservable'. Note that T_L is related to the probability P_{acc} of accepting a new placement set that increases the value of the objective function through

$$T_L = \frac{-1}{\ln(P_{acc})} \quad (4.12)$$

In our work, we have set $P_{acc} \leq 0.0001$ referring to literature [17], which yields $T_L = 0.0869$.

4.6 Case Studies

In this section the customized exhaustive search method and customized simulated annealing algorithm are tested on various IEEE standard systems. The codes were written using MATLAB and implemented on a computer (Intel Core i7) with a processor speed of 2.4GHz. The impact of the customization steps on the various systems are presented first, and after which the customized methods are

applied to these systems. Finally, the customized methods are compared with the corresponding conventional methods without the customization steps, in terms of their optimal solutions obtained and the time incurred to obtain the solutions.

4.6.1 Customization Steps for Various Systems

The systems tested in this work include IEEE 13, 34 and 37 node test feeders. Figs. 4.3, 4.4, 4.5 show the topology of these standard test feeders respectively. For the IEEE 13 node test feeder, there is a switch connecting bus 7 and bus 8. It is assumed to be closed when conducting optimal placement of μ PMU. So, there are actually 12 buses rather than 13 buses in the system.

Figure 4.3: IEEE 13 node test feeder

The first customization step serves identify all the candidate locations. In this step, all the radial buses (single connection buses) are removed from the candidate locations, as well as all the buses that are connected to radial buses. As distribution systems have a lot of radial buses, this step is very important in the customization of the OPP method to reduce the search space dramatically. Table 4.2 lists the numbers of different kinds of buses for the test systems. It is observed that the number of candidate locations is much smaller than the number of buses

Figure 4.4: IEEE 34 node test feeder

Figure 4.5: IEEE 37 node test feeder

in the system. The significant reduction in the number of candidate locations greatly alleviates the computational burden. The other customization step is to determine the upper bound using Eq. (4.5). The comparison of our proposed and the theoretical upper bound is presented in Table 4.3. For these three test systems, it can be seen that the proposed upper bound is lower than the theoretical upper bound.

Table 4.2: Identification of candidate locations

System	Number of radial buses	Number of buses cover radial buses	Number of candidate locations
IEEE 13	7	5	0
IEEE 34	10	10	4
IEEE 37	16	10	11

Table 4.3: Proposed Upper Bound of minimum number of μ PMUs

System	Proposed Upper Bound	Theoretical Upper Bound
IEEE 13	5	6
IEEE 34	13	17
IEEE 37	13	18

4.6.2 Customized Exhaustive Search method

In this section the customized exhaustive search method is applied to the IEEE 13, 34 and 37 standard test feeders to solve for the optimal placement.

IEEE 13 node test feeder

Carrying out the first step as explained in Section 4.3.1, the radial and preassigned buses which will be removed from the candidate locations are listed in Table 4.4.

Table 4.4: Radial buses and preassigned buses

System	Radial buses	Preassigned buses
13 Node	1, 4, 6, 9, 11, 12, 13	2, 3, 5, 7(8), 10

It is observed that all buses of this networks are included in this table. Therefore, the candidate locations for this network is a null set, because of the radial topology of the test feeder. That means that the preassigned μ PMUs are able to observe the entire system. Therefore, the optimal placement for the IEEE 13 node test feeder is the locations of the preassigned buses, as shown in Table 4.5.

Table 4.5: PMU placement for 13 node test feeder

System	Minimal Number of PMUs	Locations
13 Node	5	2, 3, 5, 7(8), 10

IEEE 34 node test feeder

The customized exhaustive search method is next tested on the IEEE 34 node test feeder shown in Fig. 4.4. Similarly, the first step is to identify the radial buses and buses that cover radial buses as listed in Table 4.6.

Applying the modified binary search method yields the minimum number of μ PMUs that can make the system observable as 12. All the combinations of placement having 12 μ PMUs are generated and it is found that there are 6

Table 4.6: Radial buses and preassigned buses

System	Radial buses	Preassigned buses
34 Node	1, 5, 13, 14, 18, 22, 24, 29, 32, 34	2, 4, 10, 12, 17, 21, 23, 28, 31, 33

placement sets that make the system observable. All the solutions obtained are listed in Table 4.7. Since there are several solutions, these placement sets are further ranked by calculating their SORI. Comparing their SORI, two sets that achieved maximum redundancy are obtained and highlighted in red in Table 4.7.

Table 4.7: μ PMU placement for 34 node test feeder

System	Minimal Number	Location for placing PMUs	SORI
34 Node	12	2, 4, 7, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
		2, 4, 7, 10, 12, 17, 21, 23, 26, 28, 31, 33	41
		2, 4, 7, 10, 12, 17, 21, 23, 27, 28, 31, 33	41
		2, 4, 8, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
		2, 4, 8, 10, 12, 17, 21, 23, 26, 28, 31, 33	41
		2, 4, 8, 10, 12, 17, 21, 23, 27, 28, 31, 33	41

IEEE 37 node test feeder

Fig. 4.5 shows the topology of IEEE 37 node test feeder. As before, the radial buses and preassigned buses numbers are listed in Table 4.8. After applying the binary search method, it is found that as least 12 μ PMUs are needed to make the system fully observable. There are three placement sets that can achieve full observability as listed in Table 4.9. By comparing their SORI, we deduced the set that achieved maximum redundancy with a SORI of 47, as highlighted in red. The optimal placement sets for these three IEEE test systems determined using the customized exhaustive search method are summarized in Table 4.10.

Table 4.8: Radial buses and preassigned buses

System	Radial buses	Preassigned buses
37 Node	1, 7, 9, 10, 14, 17, 19, 20, 23, 24, 26, 27, 31, 33, 36, 37	2, 6, 8, 13, 16, 18, 22, 25, 29, 32, 35

Table 4.9: PMU placement for 37 node test feeder

System	Minimal Number	Location for placing PMUs	SORI
37 Node	12	2, 3, 6, 8, 13, 16, 18, 22, 25, 29, 32, 35;	47
		2, 3, 6, 8, 13, 16, 18, 22, 25, 30, 32, 35;	46
		2, 3, 6, 8, 13, 16, 18, 22, 25, 32, 34, 35;	46

Table 4.10: Optimal placement found by Customized Exhaustive Search

System	Optimal Locations	SORI
IEEE 13	2, 3, 5, 7, 9	20
IEEE 34	2, 4, 7, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
	2, 4, 8, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
IEEE 37	2, 3, 6, 8, 13, 16, 18, 22, 25, 29, 32, 35	47

4.6.3 Customized Simulated Annealing

In order to evaluate the impact of our customization steps on more algorithms, simulated annealing is also coupled with the customization steps. The customized simulated annealing is tested on the IEEE 13, IEEE 34 and IEEE 37 standard test feeders to solve for the optimal placement. Since simulated annealing algorithm is not able to find all the optimal solutions, only one set is identified for the IEEE 34 feeder. The optimal placement sets for these three test systems are summarized in Table 4.11.

Table 4.11: Optimal placement found by Customized Simulated Annealing

System	Optimal Locations	SORI
IEEE 13	2, 3, 5, 7, 9	20
IEEE 34	2, 4, 7, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
IEEE 37	2, 3, 6, 8, 13, 16, 18, 22, 25, 29, 32, 35	47

4.7 Compariosn of Customized Versus Conventional Search Methods

Besides the customized OPP methods, the original exhaustive search method and simulated annealing algorithm have also been tested on the three IEEE test systems. The optimal placements found by customized algorithms are the same as the original algorithms without the customization steps. In other word, the proposed customization steps will not affect the optimal placement solution. More importantly, the customized steps have greatly accelerated the solving process, as can be seen from the time needed to obtain the optimal placement solution tabulated in Table 4.12.

Table 4.12: Simulation Time

System	No. of PMU	Exhaustive	Customized Exhaustive	Simulated Annealing	Customized SA
IEEE 13	5	0.393s	0.016s	0.688s	0.035s
IEEE 34	12	>30min	0.295s	>30min	3.543s
IEEE 37	12	>30min	0.303s	>30min	4.418s

It is noted that without the customization steps, both the conventional exhaustive search and simulated annealing methods will take more than 30 minutes to find the optimal placement for IEEE 34 and IEEE 37 node test feeders. However, the simulation time is dramatically dropped to several seconds or less when using the customized algorithms. When customization steps are adopted for simulated annealing, the values of the parameters T_0 , β and T_L remain the same so that we can credit any improvement to customization steps. Therefore, there is significant improvement in the efficiency in deriving the optimal placement when applying our

proposed customized algorithms, especially for more complex distribution systems.

The customized exhaustive search method has also been tested on IEEE 14 bus system and IEEE 30 bus system to compare its efficiency when applied to transmission networks and distribution networks. The time required to execute the OPP for these networks are compared in Table 4.13.

Table 4.13: Time needed to obtain the optimal placement solution for various transmission and distribution networks.

Transmission networks	Time consumed	Distribution networks	Time consumed
IEEE 14 bus	0.51s	IEEE 13	0.09s
IEEE 30 bus	104.64s	IEEE 34	0.35s
—		IEEE 37	0.34s

The results show that the customized exhaustive search method is able to solve the OPP problem efficiently for the smaller scale IEEE 14 transmission networks. However, when the scale of the network is increased to 30 buses, the computational time becomes long and unacceptable. On the other hand, the customized exhaustive search method is very efficient for distribution networks, even for the more complex IEEE 37 network. Therefore, for similar scale networks, it is found that distribution networks require much less time to deduce the optimal placement of μ PMUs than transmission networks.

4.8 Conclusion

This chapter introduces customized optimal μ PMUs placement methods for distribution systems. The customization includes the identification of candidate locations and determination of the upper bound of the minimum number of μ PMUs required to render the network observable. Optimal placement of μ PMUs usually consumes extensive time especially when the networks size increases. The proposed customization steps take advantage of the unique topology of distribution system to reduce the search space. Two customization steps have been proposed based on the radial topology of distribution networks, and are proved to be helpful to reduce the search space, resulting in dramatically reduced computational burden. The simulated annealing and exhaustive search methods are coupled with our proposed customized steps and tested on various IEEE test feeders. The simulation results obtained show that the customization is able to reduce the solving time significantly without affecting the optimal solution. The improvement in terms of the solving time is dramatic especially when the system scale increases. For example, for the IEEE 34 and IEEE 37 distribution networks, the time needed to deduce the optimal placement of μ PMUs is reduced from more than 30 minutes to just several seconds when the customization steps are applied to the optimization algorithms. Considering the fast development of distribution energy resources and the commercialization of μ PMUs, it is envisaged that distribution networks will be increasingly monitored by the fast and ultra precise μ PMUs in the near future. In this context, our proposed highly efficient customized optimal μ PMUs placement methods will be an important integral part in the development of μ PMUs application in distribution networks.

Chapter 5

Multiple Solutions of Optimal μ PMU Placement Problems

5.1 Introduction

This chapter introduces a hybrid method based on the integer linear programming algorithm to determine the optimal placement of micro phasor measurement unit (μ PMU). Integer linear programming (ILP) is a fast and easy to implement algorithm, and has long been used to solve the problem of optimal placement of PMU (OPP). Xu et al. used a numerical implementation of integer programming for network analysis and cost of PMU installation with mixed measurement sets which included conventional power flow and injection measurements [20]. It dealt with zero injection in an approximate way. An ILP formulation of the OPP problem was also proposed previously [23], which improved the bad data processing capability of the state estimator, assuming that any bad data appearing in a single measurement will be detectable. It showed how the extra PMUs improved the bad data detection and identification capability of a given system. The description

of a simple modified ILP based optimal placement method, ensuring complete topological observability of the system under intact and critical contingency cases, was also reported [29]. On the other hand, Gou used an integer programming technique accounting for power networks with and without conventional power flow and injection measurements [26]. This concept was further extended to consider zero-injection buses, incomplete observability and measurement redundancy [27]. So far, the aforementioned literatures have not considered multiple solutions in the OPP problem. This issue was partially addressed by proposing two indices to rank multiple solutions in a multistage procedure PMU placement method using an ILP framework [24]. The two indices were the bus observability index (BOI) and system observability redundancy index (SORI). The BOI was determined to give a measure of the number of PMUs observing a given bus and the SORI was deduced by summing of all the BOI of the system. However, it is possible that multiple solutions exist even after the ranking using SORI [24].

In practice, it is possible that two solutions are equally optimal in some aspects but can be further distinguished by other criteria that cannot be integrated in the optimization directly. In field of OPP, it is likely that two placements strategies having the same number of PMUs and SORI have different performance in other aspects such as robustness or state estimation (SE) variance [74]. And these additional objectives cannot be easily implemented in the objective function. In addition, different power system applications have different objectives, multiple solutions offer more flexibility for the decision maker [73]. Hence, obtaining all the alternative solutions efficiently will allow the decision maker to examine and select the overall best solution, taking into consideration the factors that are mathematically difficult to express, without suffering any deterioration in the objective function.

Obtaining all alternative solutions using the ILP is always challenging and a

number of papers have been published to address this problem in other fields [75–77]. In contrast, little attention is paid to the multiple solutions problem in the field of OPP. Maji et al. developed an effective exponential binary particle swarm optimization algorithm to provide multiple solutions for OPP problem [73]. A post-processing step is applied to generate multiple solutions based on the solution obtained using integer linear programming algorithm by Xia et al. [55]. However, neither of those works considers the full coverage of optimal solutions. It is desirable that the decision maker is given all the possible optimal PMU placement choices, but typical output from such an optimization solver is usually just a single optimal solution.

Heretofore, there is not such a method that is able to identify all the alternative optimal solutions to the OPP problem besides the exhaustive search method which however is extremely time consuming and not practical for large scale systems. In this chapter, a hybrid method based on integer linear programming algorithm and exhaustive search is proposed to identify all alternative optimal solutions that can achieve full observability of a system with minimum number of PMU and maximum SORI. In this work, we innovatively design a hybrid method that first generates multiple optimal solutions using ILP by changing the objective function in a certain range, followed by searching for all the optimal solutions based on observation of these multiple solutions. The proposed hybrid method is tested on various IEEE systems and our case study results show that the proposed hybrid method is able to give the same results with exhaustive search method, while consuming much less time. Therefore, the proposed hybrid method is a useful and powerful tool when considering OPP for large scale distribution systems.

5.2 Integer Linear Programming for OPP

This section explains the basic procedures involved in the integer linear programming method to solve the OPP problem. As mentioned in Chapter 4, the OPP problem is formulated as:

$$\begin{aligned} & \min_{N_P} \{ \max \gamma(N_P, S(N_P)) \} \\ & \text{subject to } O_{bs}(N_P, S(N_P)) = 1 \end{aligned} \quad (5.1)$$

where N_P is the minimum number of PMUs, $S(N_P)$ is the optimal location of the N_P PMUs, $\gamma(N_P, S(N_P))$ is the redundancy measurement index and O_{bs} is the observability evaluation logical function [19].

Integer programming is a mathematical programming method of solving an optimization problem in which some or all of the variables are restricted to integers. while the objective function and the constraints are linear, nonlinear, or quadratic, thus leading to integer linear programming (ILP), integer nonlinear programming (INLP) and integer quadratic programming (IQP) algorithms, respectively. [18]. OPP problem is usually formulated as a integer linear programming problem, and ILP is efficient in minimizing the number of PMUs required, while ensuring system observability. Besides, it is efficient in maximizing SORI given a specific number of PMUs. We have earlier on presented on Section 2.5.2 on the modelling of the objective function and constraints when applying the integer linear programming method. There are two steps involved when the integer linear programming method is applied to solve the OPP problem formulated in Eq. (5.1):

1. Minimizing the number of PMU while ensuring system observability.
2. Maximizing the SORI using limited number of PMUs, with the number of PMU as determined in Step 1.

Step 1 can be summarized in the following equation.

$$\min_x \mathbf{f}\mathbf{x} \quad \text{subject to } \mathbf{A}\mathbf{x} \geq \mathbf{b} \quad (5.2)$$

where $\mathbf{f} = [1, 1, \dots, 1]$ is set such that the objective function $\mathbf{f}\mathbf{x}$ is the number of PMUs allocated. $\mathbf{x}$ is a binary decision variable vector indicating whether a PMU is placed at a bus; $\mathbf{A}$ is the system connectivity matrix, so $\mathbf{A}\mathbf{x}$ indicates how many PMUs are observing a bus; $\mathbf{b}$ is a unit vector (all '1' vector); $\mathbf{A}\mathbf{x} \geq \mathbf{b}$ is the system observability constraint. This above Step 1 will lead to a solution for $\mathbf{x}$. The number of '1's in the solution is the minimum number of PMUs ensuring the system observability. This result will be included in Step 2 as an additional constraint.

Step 2 is summarized in the following equation. This step aims at maximizing the system observability redundancy index.

$$\max_x \mathbf{g}\mathbf{x} \quad \text{subject to } \begin{cases} \mathbf{A}\mathbf{x} \geq \mathbf{b} \\ \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq} \end{cases} \quad (5.3)$$

where $\mathbf{A}_{eq} = [1, 1, \dots, 1]$, $\mathbf{g} = [g_1, g_2, \dots, g_n]$, g_i equals to the number of incident lines of bus i plus 1, which is the BOI of each bus β_i . The objective function $\mathbf{g}\mathbf{x}$ indicates the SORI, while $\mathbf{b}_{eq}$ is the number of PMU as determined in Step 1. After these two steps, an optimal placement strategy can be deduced.

5.3 Multiple solutions

Many OPP algorithms have been well documented [9, 17, 19, 21, 22, 25, 28], such as exhaustive search, integer programming, etc. Exhaustive search can provide all solutions that maximize the system observability redundancy index (SORI) while guarantee the observability of the system and minimum number of PMU

needed. However, exhaustive search may consume extensive computational time for large-scale systems because of huge search space. In contrast, the integer linear programming method is fast, but however, can only provide limited number of solutions. In practice, it is possible that there are multiple solutions satisfying the given constraints.

It is always of interest to identify all alternative solutions using the integer programming method so that the decision maker can examine them, and choose the overall best solution, i.e., the one that also best meets the other criteria that could not be expressed in the integer programming model. This arises because some optimization criteria are inherently subjective and difficult to quantify, while some criteria are difficult to express as linear expressions or they render the model hard to solve [75–77]. There may be two different optimal placement solutions having the same number of PMU and same SORI, but with different other performance indices, such as the state estimation variance or robustness.

For example, Sun et al. evaluated the placement based on the sum of variance (SV) of the robust estimators [74]. The placements evaluated were 'optimal' in terms of the number of PMU and SORI, but they had different performance when considering the SV. As it is difficult to express variance as a constraint or objective function, the authors needed to obtain all the placements that satisfied the given constraints, that is, minimum number of PMU and maximum SORI, before carrying out the SV comparison. Thus, it is meaningful to provide the decision maker with alternative optimum solutions to choose from the multiple solutions, taking into consideration other criteria not factored in the model. Toward this end, an efficient OPP algorithm needs to be developed to identify all the alternative optimal solutions.

5.4 Proposed Method

In this work a hybrid method is proposed which is based on integer linear programming algorithm and exhaustive search to identify all optimal solutions that provide full observability of a system with minimum number of PMU and maximum SORI. The proposed method incorporates several steps to identify all optimal solutions after two steps are applied to minimize the number of PMU and maximize SORI successively. The complete process is summarized as follows:

1. Minimization of number of PMU using integer linear programming. This step has been explained in detail in Section 2.5.2. The minimum number of PMU obtained in this step is denoted as N and included in following step as the second constraint as shown in Eq. (5.3).
2. Generating multiple solutions that maximize the SORI. These multiple solutions may not cover all alternative optimal solutions that maximizes the SORI, so the next few steps are designed to locate all alternative optimal solutions.
3. Classification of network buses according to multiple solutions obtained in Step 2
4. Grouping of uncertain buses and forming subnetworks
5. Optimization of each subnetwork in an exhaustive manner
6. Summarize the placement strategy and list all alternative optimal solutions.

Following subsections will elaborate on Step 2 to Step 6.

5.4.1 Generating Multiple Solutions

Step 2 in the proposed method serves to generate multiple solutions that maximize the SORI. In this step, a small fraction is added to each entry of the coefficient $\mathbf{g}$. The original g equals the BOI of bus i β_i and is given by $\mathbf{g} = [g_1, g_2, \dots, g_n]$. The fraction added is chosen to be 0.1, which is so small that when it is removed, these multiple solutions will lead to the same objective value. This means that these different solutions are alternative optimal solutions for the original problem. Eq. (5.4) lists a series of modified coefficient $\mathbf{g}$ and how they are constructed. The overline above the symbol $\mathbf{g}$ indicates that it is a modified coefficient, and the superscript indicates which entry of $\mathbf{g}$ is modified.

$$\begin{aligned}\bar{\mathbf{g}}^1 &= [g_1 + \Delta, g_2, \dots, g_n] \\ \bar{\mathbf{g}}^2 &= [g_1, g_2 + \Delta, \dots, g_n] \\ &\vdots \\ \bar{\mathbf{g}}^n &= [g_1, g_2, \dots, g_n + \Delta]\end{aligned}\tag{5.4}$$

In order to avoid missing any solutions, an opposite sign disturbance is also added to the coefficient as follows.

$$\begin{aligned}\bar{\mathbf{g}}^{n+1} &= [g_1 - \Delta, g_2, \dots, g_n] \\ \bar{\mathbf{g}}^{n+2} &= [g_1, g_2 - \Delta, \dots, g_n] \\ &\vdots \\ \bar{\mathbf{g}}^{2n} &= [g_1, g_2, \dots, g_n - \Delta]\end{aligned}\tag{5.5}$$

These modified coefficients are used to maximize the SORI as shown in Eq. (5.6), where the two constraints stand for system observability and the number of PMU as determined in Step 1.

$$\max_x \bar{\mathbf{g}}\mathbf{x} \quad \text{subject to} \quad \begin{cases} \mathbf{A}\mathbf{x} \geq \mathbf{b} \\ \mathbf{A}_{eq}\mathbf{x} = \mathbf{b}_{eq} \end{cases}\tag{5.6}$$

By solving Eq. (5.6), we will then obtain $2n$ solutions corresponding to the $2n$ different coefficients $\bar{\mathbf{g}}\mathbf{x}$. Note that boldface letter with a superscript indicates a solution which is generated by adding a small disturbance to the corresponding entry of $\mathbf{g}$, while the subscript denotes an entry in the solution.

$$\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^n, \mathbf{x}^{n+1}, \mathbf{x}^{n+2}, \dots, \mathbf{x}^{2n}$$

These $2n$ solutions have following features:

- It is possible that some solutions are identical with some other, which means that the number of actual solutions may be less than $2n$.
- If multiple solutions exist for the original problem, $\mathbf{x}^1$ to $\mathbf{x}^{2n}$ would NOT be identical. In other words, more than one distinct solution will be obtained by Step 2 as long as multiple solutions exist for this problem.
- $\mathbf{x}^1$ to $\mathbf{x}^{2n}$, irrespective of whether they are identical, are solutions to the original optimization problem which has the original coefficient $\mathbf{g}$, and that they share the same maximized value of SORI.
- $\mathbf{x}^1$ to $\mathbf{x}^{2n}$ may not cover all the multiple solutions to the original problem. In other words, some alternative optimal solutions may be missing from the collection of $\mathbf{x}^1$ to $\mathbf{x}^{2n}$.

So far we have located some optimal solutions, however, our aim is to find all alternative optimal solutions. These multiple solutions obtained can provide indication on how the other missing optimal solutions can be located. Additional steps are proposed, as outlined in the next section, to help identify all other missing optimal solutions.

5.4.2 Classification of Network Buses

From the obtained multiple solutions, some patterns can be identified. For example, some locations are never assigned with a PMU no matter how the coefficient g changes, while some locations that are always assigned with a PMU irrespective of the coefficient g . On the other hand, there are some locations that are assigned with a PMU in some solutions but not in other solutions. We classify the buses in the network into three categories based on the above observations:

- The first category includes those buses that are allocated with PMUs in each of these multiple solutions. These buses are called primary buses and they form the set γ .
- The second category includes those buses that are not allocated with PMUs in each of these multiple solutions. These buses are called trivial buses and they form the set ξ .
- The rest of the system buses are classified into the third category. These buses are allocated PMUs in some of these multiple solutions, but are not allocated PMUs in the other multiple solutions. These buses are called uncertain buses and they form the set λ .

The reason that we make such classification is that the missing alternative optimal solutions can be located by further analyzing the uncertain buses group. It will be proven that the missing alternative optimal solutions also obey the classification in Section 5.5. We have defined sets γ , ξ and λ based on the multiple solutions obtained in the previous steps. Next, another three sets Γ , Ξ and Λ are defined as follows: If a bus is assigned with a PMU in all the alternative optimal solutions, not limited to the optimal solutions found in Step 2 and including

potential missing optimal solutions, it is an element of the set Γ ; if a bus is not assigned with a PMU in any of all the optimal solutions, it is an element of the set Ξ ; if a bus is assigned with a PMU in some, but not all of the optimal solutions, it is an element of the set Λ . It can be proved that the sets γ, ξ and λ are identical to the sets Γ, Ξ and Λ respectively as will be shown in Section 5.5.

Therefore, the sets Γ, Ξ and Λ can be identified using Step 3. The information about the category of buses is then used to further find the solutions exhaustively. After analyzing these uncertain buses, it is found that these buses are usually distributed cluster(s). Following that we will execute Step 4 to group the uncertain buses.

5.4.3 Grouping of Uncertain Buses

Uncertain buses are divided into independent groups that are isolated geographically. In the set Λ , it is very likely that one bus is adjacent to another and these adjacent buses are grouped together. Grouping is accomplished using the system connectivity matrix. We need to next determine the placement of PMUs among those uncertain buses. However, those buses connected to the uncertain buses will also be affected in terms of observability and measurement redundancy. Therefore, a subnetwork is formed for each group, which is made up of the uncertain nodes in the group, as well as the connected nodes. Due to the geographically isolation of each group, no matter how the PMUs are allocated in a group, only the subnetwork will be affected. The remaining part of network will not be affected, in terms of both observability and SORI. If two subnetworks are overlapped, then they are joined together to form a larger subnetwork.

Fig. 5.1 illustrate the grouping of uncertain buses and subnetwork. The red dots represent the uncertain buses, and the blue dots do not belong to Λ , but

are connected to the uncertain buses. The red and blue dots together form a subnetwork.

Figure 5.1: Group and subnetwork

When conducting the optimization of each subnetwork, while the candidate locations are the uncertain buses, but the observability of the whole subnetwork should be considered. Taking the subnetwork 1 shown in Fig. 5.1 as an example, although only two red dots are the candidate locations of the PMUs, however, the observability of the five dots (two red and three blue) need to be considered when searching for optimal placement.

It is possible that two subnetworks overlap. In this case, the overlapped subnetworks are merged together to form a larger subnetwork, without affecting the candidate locations. Fig. 5.2 illustrates the merging process of two subnetworks which have a common node. This merged subnetwork consists of eight buses among which there are four candidate buses.

5.4.4 Optimization of Subnetwork

After we have formed subnetworks, optimization is carried out for each subnetworks. The optimization process for each subnetwork consists of two steps: minimization of PMU number with observability constraint and maximization of SORI

Figure 5.2: Merging overlapped subnetworks

with the minimum number fixed. The minimum number of PMUs required for each subnetwork can be easily determined using the integer linear programming algorithm,

$$\min_x \mathbf{f}_{sub} \mathbf{x} \quad \text{subject to} \quad \mathbf{A}_{sub} \mathbf{x} \geq \mathbf{b}_{sub} \quad (5.7)$$

where $\mathbf{f}_{sub} = [1, 1, \dots, 1]$ is a set such that the objective function $\mathbf{f}_{sub} \mathbf{x}$ is the number of PMUs allocated, $\mathbf{x}$ is a binary decision variable vector indicating whether a PMU is placed at a bus and $\mathbf{A}_{sub}$ is the corresponding matrix that indicates the connection of candidate locations and all nodes in a subnetwork. Since a subnetwork consists of a group of uncertain buses and buses connected to uncertain buses, a subnetwork has more nodes than the number of candidate locations. Therefore, the matrix $\mathbf{A}_{sub}$ usually is not a square matrix.

Note that formulation of $\mathbf{b}_{sub}$ is not the same as the formulation of $\mathbf{b}$ in Eq. (5.2), where $\mathbf{b}$ is a unit vector (all '1' vector) to ensure that every node in a network is observable. As a subnetwork is not a standalone network, it is a part of the whole network and has connection with other part of the network, it is possible that some of the buses in a subnetwork are already observed by a PMU outside this subnetwork. Nonetheless, we will still include those buses in the subnetwork because they have impact on the SORI.

Taking the subnetwork 1 in Fig. 5.3 as an example, the formulation of the optimization for a subnetwork is described. There are five nodes in this subnetwork, among which there are two candidate locations at nodes 3 and 4. The observable condition of the other 3 nodes (1, 2, 5) depends on the placement condition outside of the subnetwork. Let us assume that node 1 is connected to a primary bus and becomes observed and nodes 2 and 5 are not observed. Based on such assumptions, the objective function and the constraints can be determined.

Figure 5.3: Optimization of the subnetwork 1.

$$\begin{aligned}
 \mathbf{f}_{sub} &= \begin{bmatrix} 1 & 1 \end{bmatrix} \\
 \mathbf{A}_{sub} &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \\
 \mathbf{b}_{sub} &= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}'
 \end{aligned}$$

Solving for Eq. (5.7), we can determine the minimum number of PMU required to make a subnetwork fully observable. The next step is to maximize SORI of the

subnetwork in an exhaustive manner. For each subnetwork, given the number of PMU and candidate buses that are the uncertain buses in a group, it is possible to list all the possible combinations of candidate locations. Note that this exhaustive listing approach is possible considering that each subnetwork is typically small.

After applying all these steps, all the alternative optimal placement of the network can be obtained. The minimum number of PMU required to make the system observable is determined as N , and their placements should follow the instructions below to achieve maximum SORI:

1. All the primary buses are assigned PMUs. Since the number of primary buses is less than the minimum number of PMU, there will be some PMUs remain unassigned.
2. The remaining PMUs are allocated to each subnetwork. Based on the optimization results for individual subnetwork, we know how many PMUs should be assigned to each subnetwork and how they should be placed to render the subnetwork observable and achieve the maximum SORI.

One can list all the placement strategies that ensure the minimum number of PMU and maximum SORI following the above instructions. Since all the subnetworks are geographically isolated, the placement in each subnetwork does not affect the other subnetworks. The number of optimal solutions can be calculated using Eq.(5.8).

$$\prod_{i=1}^m K_{sub_i} \quad (5.8)$$

where m denotes the quantity of subnetwork, i is the index of the subnetwork, and K_{sub_i} is the number of optimal placements in each subnetwork.

5.5 Proof of set $\gamma = \Gamma, \xi = \Xi, \lambda = \Lambda$

We have earlier on stated that the proposed hybrid method based on linear integer programming algorithm is able to find all the alternative optimal solutions that satisfy the constraints of achieving the minimum number of PMU and maximizing the SORI. The reasoning is that, on one hand, the classification of network buses is valid not only on multiple solutions, but also those potential missing optimal solutions, and on the other hand, the uncertain buses are grouped into independent groups and each group is optimized separately in an exhaustive manner. Therefore, it is guaranteed that no optimal alternative solution is omitted by our proposed method. In this section, we prove that the classification of network buses also applies to potential missing optimal solutions, which is the most important assertion that underscores our proposed hybrid method.

5.5.1 Proof of $\gamma = \Gamma$

We prove by contrary that $\Gamma \subseteq \gamma$. Assume that a bus k is an element of set Γ but not an element of set γ . That is to say among the $2n$ solutions, at least one of them is

$$\mathbf{x}^k = [\dots, \dots, x_k = 0, \dots]' \quad (5.9)$$

Since all the $2n$ solutions are optimal solution, $\mathbf{x}^k$ must be an optimal solution as well. Therefore, the bus k which equals to zero in one of the optimal solutions cannot be an element of Γ , which however, violates the assumption. Hence, it is impossible that the bus k is an elements of the set Γ , but not an element of the set γ , which therefore indicates that $\Gamma \subseteq \gamma$.

Next, we similarly prove that $\gamma \subseteq \Gamma$ by contradiction. Assume that a bus k is

an element of the set γ but not an element of set Γ . This means that the bus k is assigned with a PMU in every solution among the $2n$ solutions but there is a missing optimal solution in which bus k is not assigned with a PMU. That is to say, there is at least one missing optimal solution with the following structure:

$$\bar{\mathbf{x}} = [\dots, \dots, x_k = 0, \dots]' \quad (5.10)$$

However, note that x_k is always assigned with a PMU (the value of this entry is 1) in all the $2n$ solutions. The following $\bar{\mathbf{g}}^{n+k}$ is used when generating the multiple solutions,

$$\bar{\mathbf{g}}^{n+k} = [g_1, g_2, \dots, g_k - \Delta, \dots, g_n] \quad (5.11)$$

The associated solution, which is one of the $2n$ solutions, is

$$\bar{\mathbf{x}}^{n+k} = [\dots, \dots, x_k = 1, \dots]' \quad (5.12)$$

Since both $\bar{\mathbf{x}}$ and $\bar{\mathbf{x}}^{n+k}$ are the optimal solutions, they ought to share the same objective function value.

$$\mathbf{g}\bar{\mathbf{x}}^{n+k} = \mathbf{g}\bar{\mathbf{x}} \quad (5.13)$$

Note that the only differences between $\bar{\mathbf{g}}^{n+k}$ and $\mathbf{g}$ are their i -th elements, and the i -th element of $\bar{\mathbf{g}}^{n+k}$ is Δ smaller than that of $\mathbf{g}$. Since the i -th element of $\bar{\mathbf{x}}^{n+k}$ is 0, the value of $\mathbf{g}\bar{\mathbf{x}}$ is the same as $\bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}}$. Furthermore, since the i -th element of $\bar{\mathbf{x}}^{n+k}$ is 1, $\bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}}^{n+k}$ is smaller than $\mathbf{g}\bar{\mathbf{x}}^{n+k}$.

$$\mathbf{g}\bar{\mathbf{x}} = \bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}} \quad (5.14)$$

$$\mathbf{g}\bar{\mathbf{x}}^{n+k} > \bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}}^{n+k} \quad (5.15)$$

Using Eqs. (5.13), (5.14) and (5.15), we deduce that

$$\bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}} > \bar{\mathbf{g}}^{n+k}\bar{\mathbf{x}}^{n+k} \quad (5.16)$$

Note that $\bar{\mathbf{x}}^{n+k}$ in Eq. (5.16) is obtained by maximizing the $\bar{\mathbf{g}}^{n+k}\mathbf{x}$ in Eq. (5.6). Therefore, $\bar{\mathbf{x}}$ does not exist such that Eq. (5.16) holds. That is, it is impossible that a bus k is an element of set γ but not an element of set Γ , which means $\gamma \subseteq \Gamma$.

In conclusion, we prove that $\gamma = \Gamma$. This is to say, if a bus is assigned with a PMU in every solution among the $2n$ solutions, it will surely be assigned with a PMU in any potential missing optimal solutions.

5.5.2 Proof of $\xi = \Xi$

We prove by contrary that $\Xi \subseteq \xi$. Assume that a bus k is an element of set Ξ but not an element of set ξ . That is to say, among the $2n$ solutions, at least one of them is

$$\mathbf{x}^k = [\dots, \dots, x_k = 1, \dots]' \quad (5.17)$$

Since all the $2n$ solutions are optimal solutions, $\mathbf{x}^k$ must be an optimal solution as well. Therefore, the bus k which equals to 1 in one of the optimal solutions cannot be an element of Ξ , which thus violates the assumption. Hence, it is impossible that the bus k is an elements of the set Ξ but not an element of the set ξ , which indicates that $\Xi \subseteq \xi$.

Next, we prove that $\xi \subseteq \Xi$ by contradiction. Assume that a bus k is an element of the set ξ but not an element of the set Ξ , which means the bus k is not assigned with a PMU in any of those $2n$ solutions, but assigned with a PMU in a missing optimal solution. That is to say, there is at least one missing optimal solution of the following structure:

$$\bar{\mathbf{x}} = [\dots, \dots, x_k = 1, \dots]' \quad (5.18)$$

However, in all the $2n$ solutions, the x_k is always assigned with 0. In one of the $2n$ solutions, the following $\overline{\mathbf{g}}^k$ is used.

$$\overline{\mathbf{g}}^k = [g_1, g_2, \dots, g_k + \Delta, \dots, g_n] \quad (5.19)$$

The associated solution, which is one of the $2n$ solutions, is

$$\overline{\mathbf{x}}^k = [\dots, \dots, x_k = 0, \dots]' \quad (5.20)$$

Since both $\overline{\mathbf{x}}$ and $\overline{\mathbf{x}}^k$ are optimal solutions, they ought to share the same objective function value.

$$\mathbf{g}\overline{\mathbf{x}}^k = \mathbf{g}\overline{\mathbf{x}} \quad (5.21)$$

Note that the only differences of $\overline{\mathbf{g}}^k$ and $\mathbf{g}$ are their i -th elements, the i -th element of $\overline{\mathbf{g}}^k$ is Δ greater than that of $\mathbf{g}$. Since the i -th element of $\overline{\mathbf{x}}^k$ is 1, the value of $\mathbf{g}\overline{\mathbf{x}}$ is smaller than $\overline{\mathbf{g}}^k\overline{\mathbf{x}}$. Since the i -th element of $\overline{\mathbf{x}}^k$ is 0, $\overline{\mathbf{g}}^k\overline{\mathbf{x}}^k$ is the same as $\mathbf{g}\overline{\mathbf{x}}^k$.

$$\mathbf{g}\overline{\mathbf{x}} < \overline{\mathbf{g}}^k\overline{\mathbf{x}} \quad (5.22)$$

$$\mathbf{g}\overline{\mathbf{x}}^k = \overline{\mathbf{g}}^k\overline{\mathbf{x}}^k \quad (5.23)$$

Using Eqs. (5.21), (5.22) and (5.23)

$$\overline{\mathbf{g}}^k\overline{\mathbf{x}} > \overline{\mathbf{g}}^k\overline{\mathbf{x}}^k \quad (5.24)$$

Note that $\overline{\mathbf{x}}^k$ in Eq. (5.24) is obtained by maximizing $\overline{\mathbf{g}}^k\mathbf{x}$ in Eq. (5.6). Therefore, $\overline{\mathbf{x}}$ does not exist such that Eq. (5.24) holds. That is, it is impossible that a bus k is an element of the set ξ but not an element of the set Ξ , which means $\xi \subseteq \Xi$.

In conclusion, we prove that $\xi = \Xi$. This is to say, if a bus is never assigned with a PMU in any solution among the $2n$ solutions, it will not be assigned with a PMU in any potential missing optimal solution.

5.5.3 Proof of $\lambda = \Lambda$

Since $\gamma = \Gamma$ and $\xi = \Xi$, we can conclude that $\lambda = \Lambda$.

In conclusion, since these three sets are identical, and each subnetwork is searched in an exhaustive manner, we can ascertain that no one optimal solution is missing. Therefore, the proposed hybrid method generates the same solutions to OPP problem with the exhaustive search method.

5.6 Considering PMU Loss and Branch Line Outage

The solutions generated by previous steps guarantee the system observability under normal operating condition. However, in case of branch outage or PMU loss or branch line outage, the system observability is not guaranteed. Although PMU are highly reliable, they are prone to failure. Hence, modification to system observability constraints is required to account for contingency [56, 78, 79].

5.6.1 Considering PMU Loss

In Eq.(5.2) and Eq.(5.3), vector $\mathbf{b}$ is set to 1, which practically means that each bus is observed by at least one PMU. If it is expected that the system remains fully observable in presence of single PMU loss, it is required that each bus is observed by at least two PMUs, which can be achieved by setting the vector $\mathbf{b}$ to 2 [28, 56]. Similarly, if it is expected the system remains fully observable in presence of two PMU loss, the vector $\mathbf{b}$ should be set to 3. Note that it is possible that there is no solution for multiple PMU loss, because the maximum times a bus is observed by PMUs is limited by the number of incident lines connected to the bus. The

observability constraint for single PMU loss is given in Eq.(5.25).

$$\mathbf{A}\mathbf{x} \geq \bar{\mathbf{b}} = [2, 2, 2, \dots, 2]^T \quad (5.25)$$

where $\bar{\mathbf{b}} = [2, 2, 2, \dots, 2]^T$.

5.6.2 Considering Single Branch Outage

Similar modification can be made to account for single branch line outage. The only different is the radial bus, because failure of a line that is connected to a radial bus does not affect the observability of the rest of the system [28]. The modification is given in Eq.(5.26).

$$b_i = \begin{cases} 1, & \text{if } i \text{ is a radial bus} \\ 2, & \text{if } i \text{ is not a radial bus} \end{cases} \quad (5.26)$$

5.7 Case Studies

In this section the proposed hybrid method is tested on various IEEE test systems. The IEEE 37 node test feeder has only one optimal placement that achieves the minimum number of PMU and the maximum SORI, so the optimization for this test feeder is not presented here. The IEEE 34, IEEE 33 and IEEE 123 bus systems have multiple optimal solutions. IEEE 34 has relatively simple topology and only two uncertain nodes which form the only group and subnetwork, and IEEE 33 bus system have six uncertain buses which form three groups but merged into one subnetwork. IEEE 123 has three clusters of uncertain buses which form three independent subnetworks. As will be shown, applying our proposed method to the IEEE 34, IEEE 33 and IEEE 123 node test feeders reveals that there are 2, 4 and 12 optimal placements respectively, that can achieve the maximum SORI with the minimum number of PMU.

5.7.1 IEEE 34 Node Test Feeder

The topology of IEEE 34 node test feeder can be found in Fig. 4.4. The hybrid integer linear programming is applied to this system using the following steps.

1. Minimization of number of PMU. Applying Eq. (5.2) yields the minimum number of PMU required to make the system observable as 12: Therefore, $\mathbf{f}x = 12$
2. Generating multiple solutions that maximize the SORI. In this step, Eq. (5.3) is optimized with the following $\bar{\mathbf{g}}$.

$$\begin{aligned}
 \bar{\mathbf{g}}^1 &= [g_1 + \Delta, g_2, \dots, g_{34}] \\
 \bar{\mathbf{g}}^2 &= [g_1, g_2 + \Delta, \dots, g_{34}] \\
 &\vdots \\
 \bar{\mathbf{g}}^{34} &= [g_1, g_2, \dots, g_{34} + \Delta] \\
 \bar{\mathbf{g}}^{35} &= [g_1 - \Delta, g_2, \dots, g_{34}] \\
 \bar{\mathbf{g}}^{36} &= [g_1, g_2 - \Delta, \dots, g_{34}] \\
 &\vdots \\
 \bar{\mathbf{g}}^{68} &= [g_1, g_2, \dots, g_{34} - \Delta]
 \end{aligned}$$

This step yields 68 solutions, but some of them are identical. There are two distinct solutions among the 68 solutions, and the rest are the same. The two distinct solutions are listed:

- 2, 4, 7, 10, 12, 17, 21, 23, 25, 28, 31, 33
- 2, 4, 8, 10, 12, 17, 21, 23, 25, 28, 31, 33

3. Classification of network buses based on the two solutions obtained from the previous step.

- Primary buses: 2, 4, 10, 12, 17, 21, 23, 25, 28, 31, 33
- Uncertain buses: 7, 8
- Trivial buses: 1, 3, 5, 6, 9, 11, 13, 14, 15, 16, 18, 19, 20, 22, 24, 26, 27, 29, 30, 32, 34

4. Grouping of uncertain buses. As there is only one group, hence only one subnetwork is formed. It consists of these nodes: 6, 7, 8, 9. The group and the subnetwork are shown in Fig. 5.4

Figure 5.4: Group and subnetwork of the IEEE 34 bus system.

5. Minimizing the number of PMU for this subnetwork using Eq. (5.7) gives the minimum number of PMU needed for this subnetwork. The minimization problem is formulated as follows:

$$\min_x \mathbf{f}_{sub} \mathbf{x} \quad \text{subject to } \mathbf{A}_{sub} \mathbf{x} \geq \mathbf{b}_{sub} \quad (5.27)$$

In this equation, we have

$$\mathbf{f}_{sub} = [1, 1]$$

$$\mathbf{x} = [x_7, x_8]'$$

$$\mathbf{A}_{sub} = \begin{bmatrix} 1, 0 \\ 1, 1 \\ 1, 1 \\ 0, 1 \end{bmatrix}$$

$$\mathbf{b}_{sub} = [0, 1, 1, 0]'$$

6. Maximizing the SORI for this subnetwork. From the previous step, the minimum number of PMU for this subnetwork is deduced to be 1. Since there are 2 candidate buses (#7, 8) in this subnetwork, there are 2 possible combinations of candidate locations. These two combinations are then checked in sequence. It is found that both of these two placement strategies can render the subnetwork observable and maximize the measurement redundancy. The optimization result for this subnetwork is that one PMU will be placed on either bus 7 or 8.
7. In summary, the optimal placements for this network can be obtained following instructions below. The results are tabulated in Table 5.1.
 - (a) At least 12 PMUs are needed to make the network observable
 - (b) In order to achieve maximized SORI, these 10 PMUs should be placed as follows:
 - 11 PMUs will be assigned to the primary buses
 - 1 PMU will be assigned to any bus between: 7, 8

Table 5.1: PMU placement for 34 node test feeder using proposed hybrid ILP method

System	Minimum Number	Location for placing PMUs	SORI
IEEE 34	12	2, 4, 7, 10, 12, 17, 21, 23, 25, 28, 31, 33	42
		2, 4, 8, 10, 12, 17, 21, 23, 25, 28, 31, 33	

5.7.2 IEEE 33 Bus Distribution System

The proposed hybrid integer linear programming is next applied to the IEEE 33 bus system. In this case study, we can observe the merging of overlapping subnetworks and how our proposed method deals with the overlap. The topology of IEEE 33 bus system is illustrated in Fig. 5.5.

Figure 5.5: Topology of IEEE 33 bus distribution system

All the optimal solutions are identified by carrying out the following steps.

1. Minimization of number of PMU. Applying Eq. (5.2) yields the minimum number of PMU required to make the system observable as 11: Therefore, $\mathbf{f}x = 11$
2. Generating multiple solutions that maximize the SORI. In this step, Eq. (5.3) is optimized with 66 different $\bar{g}$. This step yields 66 solutions. There are four

distinct solutions out of these 66 solutions as shown below, while the rest are identical.

- 2, 4, 8, 11, 14, 17, 21, 24, 26, 29, 32
- 2, 5, 8, 11, 14, 17, 21, 24, 26, 29, 32
- 2, 5, 8, 11, 14, 17, 21, 24, 27, 29, 32
- 2, 5, 8, 11, 14, 17, 21, 24, 27, 30, 32

3. Classification of network buses based on the four distinct solutions obtained from the previous step.

- Primary buses: 2, 8, 11, 14, 17, 21, 24, 32
- Uncertain buses: 4, 5, 26, 27, 29, 30
- Trivial buses: 1, 3, 6, 7, 9, 10, 12, 13, 15, 16, 18, 19, 20, 22, 23, 25, 28, 31, 33

4. Grouping of uncertain buses. After grouping the adjacent uncertain buses, three groups and subnetworks are formed. However, these three subnetworks have overlapping buses. Hence, these subnetworks are merged into one larger subnetwork, as illustrated in Fig.5.6, where green dots indicate the overlapping buses, and the blue box indicates the merged subnetwork. There are ten nodes in this subnetwork, and only six of them are candidate locations.

5. Minimizing the number of PMU for this subnetwork using Eq. (5.7) gives the minimum number of PMU needed as 3. The minimization problem is formulated as follows:

$$\min_x \mathbf{f}_{sub}\mathbf{x} \quad \text{subject to } \mathbf{A}_{sub}\mathbf{x} \geq \mathbf{b}_{sub} \quad (5.28)$$

Figure 5.6: Group and subnetwork of the IEEE 33 bus system.

In this equation, we have

$$\mathbf{f}_{sub} = [1, 1, 1, 1, 1, 1]$$

$$\mathbf{x} = [x_4, x_5, x_{26}, x_{27}, x_{29}, x_{30}]'$$

$$\mathbf{A}_{sub} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{b}_{sub} = [0, 1, 1, 1, 1, 1, 1, 1, 1, 0]'$$

6. Maximizing the SORI for this subnetwork. There are six candidate buses in this subnetwork. The previous step yields the minimum number of PMU in this subnetwork as 3. In total there are 20 possible combinations of candidate locations. After examining the observability and SORI of the subnetwork, four optimal placements are identified as shown below.

- 5, 27, 30
- 5, 27, 29
- 5, 26, 29
- 4, 26, 29

7. In summary, the optimal placements for this network can be obtained following the instructions below. The results are tabulated in Table 5.2.

- (a) At least 11 PMUs are needed to make the network observable.
- (b) In order to achieve maximized SORI, these 11 PMUs should be placed as follows:

- 8 PMUs will be assigned to the primary buses: 2, 8, 11, 14, 17, 21, 24, 32
- The remaining 3 PMUs are to be assigned to uncertain buses: 4, 5, 26, 27, 29, 30. There are four optimal placements that can achieve the maximum SORI and full observability of the subnetwork.
 - 5, 27, 30
 - 5, 27, 29
 - 5, 26, 29
 - 4, 26, 29

5.7.3 IEEE 123 node test feeder

Next, we tested the hybrid integer linear programming method on the IEEE 123 node test feeder. This feeder consists of 123 nodes and several switches. There are 119 nodes in this network and the topology is illustrated in Fig. 5.7.

Table 5.2: PMU placement for the IEEE 33 bus system using the proposed hybrid ILP method

System	Minimum Number	Location for placing PMUs	SORI
IEEE 33	11	2, 8, 11, 14, 17, 21, 24, 32, 4, 26, 29	34
		2, 8, 11, 14, 17, 21, 24, 32, 5, 26, 29	
		2, 8, 11, 14, 17, 21, 24, 32, 5, 27, 29	
		2, 8, 11, 14, 17, 21, 24, 32, 5, 27, 30

Figure 5.7: IEEE 123 node test feeder

1. Using the integer programming method, we deduced that at least 45 PMUs are needed to observe the entire system.
2. Generate the $2n$ (in this case $2n = 238$) solutions, out of which there are 5 distinct solutions and the rest are identical.
3. Classification of the buses yields the following (the bus number are not listed for simplicity)
 - Primary buses: 42 buses
 - Uncertain buses: 7 buses
 - Trivial buses: the rest of the buses (60 buses)
4. Grouping of the uncertain nodes. There are three independent groups for this network, and three subnetworks are formed respectively, which are shown in Fig. 5.8.
5. Optimization for each subnetwork. The minimum number of PMU and placements that maximize SORI are tabulated in Table 5.3.

Table 5.3: Optimization results for each subnetwork

Grouping	Minimum NO. of PMU	Solutions that maximize SORI
52,53	1	52 or 53
62, 63, 64	1	62 or 63 or 64
72, 76	1	72 or 76

6. In summary, the optimal placements for this network can be obtained following instructions below. In total there are 12 optimal placements that achieve minimum number of PMU and maximum SORI.

Figure 5.8: Subnetworks of IEEE 123 node test feeder

- (a) At least 45 PMUs are needed to make the network observable.
- (b) In order to achieve maximized SORI, these 45 PMUs should be placed as follows:

- 42 PMUs will be assigned to the primary buses (the bus number are not listed for simplicity).
- 1 PMU will be assigned to any bus between: 52, 53
- 1 PMU will be assigned to any bus among: 62, 63, 64
- 1 PMU will be assigned to any bus between: 72, 76

5.7.4 Optimal Placements Considering Single PMU Loss

When taking single PMU loss into consideration, a high number of PMU is expected. Table 5.4 compares the minimum number of PMU required in different cases.

Table 5.4: Number of PMU required in different situations

System	IEEE 33	IEEE 34	IEEE 123
Normal	11	12	45
Single PMU loss	24	27	97

Table 5.5 tabulates the optimal solutions for IEEE 34 node test feeder. The optimal solutions for IEEE 33 and 123 node test feeder are too many to be exhaustively listed, so the placement strategies are summarized in Table 5.6 and 5.7 instead.

5.8 Results

The simulation results for the optimal placement obtained for the test systems are summarized in Table 5.8. The objectives of the optimization consist of minimum number of PMUs and maximum SORI. The proposed hybrid ILP based method is able to find 2, 4, and 12 optimal placements for IEEE 34, 33 and 123 bus system respectively. It is validated that the results are the same with those obtained using the exhaustive search method. Therefore, it is concluded that the proposed hybrid ILP based method is able to identify all the alternative optimal solutions, just as the exhaustive search method.

Next, the computational times using the different methods are compared. In Table 5.9 we list the computational time using the customized exhaustive search method which has been introduced in Chapter 4, integer linear programming algorithm and the proposed hybrid ILP based method. The ILP takes less than 1 second to find an optimal solution for these three test systems. It is obviously the fastest method among the three. However, the limitation of this method is that it

Table 5.5: PMU placement for 34 node test feeder using proposed hybrid ILP method considering single PMU loss

System	Min. NO.	Location for placing PMUs
IEEE 34	27	1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,26,6,7
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,26,6,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,26,7,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,27,6,7
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,27,6,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;15,27,7,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,26,6,7
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,26,6,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,26,7,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,27,6,7
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,27,6,8
		1,2,4,5,9,10,12,13,14,17,18,20,21,22,23,24, 25,28,29,31,32,33,34;16,27,7,8

Table 5.6: PMU placement for 33 node test feeder using proposed hybrid ILP method considering single PMU loss

Bus type	Index	Bus Number	NO. of Combi- nation
Primary buses	-	1,2,3,6,8,10,12,14,16,17,18, 21,22,24,25,32,33	-
Groups	1	19,20	2
	2	4,5,26,27,28,29,30,31	9

can only give one solution. In contrast, the customized exhaustive search method is able to identify all the optimal solutions, but however, it takes more than 3 hours to find the solutions for the IEEE 123 system. Therefore its efficiency deteriorates severely with the increase in the scale of the network. The proposed hybrid ILP based method is able to give the same results as the customized exhaustive search method, and yet the computational time for the large scale IEEE 123 system is much less than that needed by the customized exhaustive search method. Therefore, we conclude that the proposed hybrid ILP method encompasses the efficiency of the ILP algorithm and the ability of the exhaustive search method to locate all the optimal solutions. There are some other factors considered in literatures such as the local redundancy, the limitation of measurement channel, the existing measurements and zero-injections buses [30,80]. Considering conventional measurements would help to reduce the minimum number of PMU [20, 26, 27, 55, 56]. The proposed method will be extended to consider the conventional measurements in future work. The local redundancy is addressed in [55] to improve the redundancy distribution. However, this method could result in a lower SORI compared with the proposed method.

Table 5.7: PMU placement for 123 node test feeder using proposed hybrid ILP method considering single PMU loss

Bus type	Index	Bus Number	NO. of Combi- nation
Primary buses	-	1,2,3,4,5,6,8,10,11,12,13,14,15,16,17,19,20, 21,22,23,24,25,27,30,31,32,33,34,37,38,39, 40,41,42,43,45,46,47,48,51,54,55,56,58,59, 60,61,65,66,67,70,71,72,74,75,76,78,79,81, 82,83,84,85,87,88,89,90,91,92,93,94,95,96, 97,100,101,103,104,106,107,108,110,111, 113,114,115,116,117,118,119	-
Groups	1	52, 53	2
	2	28, 29	2
	3	49, 50	2
	4	62, 63, 64	3
	5	68, 69	2
	6	98, 99	2

Table 5.8: Simulation results obtained for the IEEE 34, 33 and IEEE 123 systems.

Systems	Min. NO. of PMU	Max. SORI with Min. PMU	NO. of Subnetworks	NO. of Optimal Placements
IEEE 34	12	42	1	2
IEEE 33	11	34	1	4
IEEE 123	45	162	3	12

Table 5.9: Computational time comparison using three different methods

Systems	Customized exhaustive		Integer programming		Proposed hybrid ILP	
	NO. of solutions	Time	NO. of solutions	Time	NO. of solutions	Time
IEEE 34	2	0.2594s	1	0.09s	2	3.61s
IEEE 33	4	62.13s	1	0.09s	4	3.54s
IEEE 123	12	> 3 hours	1	0.15s	12	12.77s

5.9 Conclusion

In this chapter, we proposed a hybrid ILP method to find all the alternative optimal solutions in the placement of PMU. The constraints considered in the optimization include the system observability, number of PMU and the measurement redundancy. A novel approach is introduced to classify the network buses into three categories, and the uncertain buses are then further analyzed and grouped. It is proven that the proposed method will not omit any optimal solutions that satisfy the given constraints. The proposed method has been tested on the IEEE 34, IEEE 33 and IEEE 123 systems, and the results show that the proposed method is able to give all the alternative solutions efficiently, even for the IEEE 123 large scale systems.

The most important contribution of the proposed hybrid method lies in its ability to locate all the optimal solutions without resorting to exhaustive search. It combines the efficiency of the integer linear programming method and the ability of the exhaustive search method to locate all the optimal solutions. Therefore, it is able to generate all the optimal solutions within a short computational time. Note that obtaining the multiple solutions in the optimal placement of PMU is of great importance, as there are criteria, such as the state estimation variance

or robustness, which cannot be easily implemented in the objective function or constraints. Obtaining all the alternative solutions efficiently based on our proposed hybrid method will allow the decision maker to examine and select the overall best solution, without suffering any deterioration in the objective function.

Chapter 6

Conclusion and Recommendations for Future Work

This chapter summarizes the motivation, objective and outcome of this thesis, and discusses recommendations for future research work.

6.1 Conclusion

This thesis focuses on the application of micro phasor measurement unit (μ PMU) in distribution systems, specifically in terms of the optimal placement of μ PMU in a distribution system, and the integration of the phasor measurements in its state estimation.

The background and development of PMU has been extensively described in Chapter 2, followed by the introduction of μ PMU and power system state estimation. Since distribution system state estimation (DSSE) has not attracted

significant research interest until recently, the characteristic of distribution system and potential challenges of DSSE have been discussed. The integration of phasor measurement in state estimation was also discussed. The linear relationship between phasor measurement and system states makes it possible to establish a linear state estimator, so long as the system is fully observed by PMU. A method to include both conventional measurement and phasor measurement in state estimation was also discussed in this chapter. In addition, the definition and motivation to realize optimal PMU placement (OPP) were introduced, and several commonly applied OPP algorithms in the literature, including integer programming, simulated annealing and genetic algorithm, were presented.

In Chapter 3, a full phase linear state estimator was developed for three phase unbalanced distribution system. Several non-robust and robust state estimators including weighted least squares (WLS), multiple-segment (MS), quadratic-linear (QL) and square-root (SR) estimators were implemented and tested on the IEEE 13 node test feeder. Considering distribution systems usually have limited number of measurements, state estimation was carried out with different level of measurement redundancy and the performance was evaluated in terms of mean square error (MSE). Simulation results indicate that the performance of all estimator deteriorate when the measurement redundancy is reduced, and the deterioration is even severer for non-robust estimator(WLS). The state estimators made use of not only the current measurements but also measurements at previous two time steps. Simulation results show the MSE is smaller compared to state estimation dependent on the current "snapshot" of input measurements. Considering the high sampling rate of μ PMU, it is practical and beneficial to execute state estimation function with multiple sets of measurements.

In Chapter 4, customized OPP methods were proposed to deal with radial topology of distribution system. Since PMU has been mainly used in transmission

systems, existing OPP methods are designed for transmission networks and have not considered the distinct topology of distribution networks. In this chapter, two customization steps were proposed to make the most of radial topology of distribution systems. By preassigning some μ PMUs to buses connected to radial buses, two kinds of buses can be removed from candidate locations. The search space is then reduced significantly, which consequently speed up the solving process. To further accelerate the solving process, the upper bound of search range of the number of μ PMUs is determined. Rather than using the upper bound proposed in literature [17], we proposed a method to calculate the upper bound of search range based on the preassigning of μ PMUs. It is validated that our proposed upper bound is lower in distribution system application than the one used in literature. The simulated annealing and exhaustive search methods are coupled with our proposed customized steps and tested on various IEEE test feeders. Simulation results show that customized methods are able to give the same results with the original methods, while the solving process is accelerated dramatically, e.g. from more than 30 minutes to a few seconds for IEEE 37 node test feeder.

In Chapter 5, a hybrid OPP algorithm based on integer linear programming (ILP) was proposed to identify all the alternative optimal placements that satisfy given constraints. The constraints concerned in this chapter consist of system observability, the minimum number of μ PMUs and maximum system observability redundancy index (SORI). It is possible that there are multiple solutions satisfying these constraints, but no OPP algorithms have ever considered this issue. Providing multiple solutions to decision maker is of great importance because it allows the decision maker to examine and select the overall best solution, without suffering any deterioration in the objective function. On the other hand, some criteria may be difficult to be implemented in the constraints or objective function. Exhaustive search is the only method to obtain all the alternative optimal solutions, but it

is too time-consuming due to its exhaustive nature. In this chapter, an efficient OPP method based on ILP is developed to identify all the optimal placements. Multiple solutions are obtained first by changing each entry of coefficient in the objective function. In order to identify the potential missing optimal solutions, a novel method was proposed to classify the network buses into three categories, and the uncertain buses are grouped and subnetworks are formed accordingly. Each subnetwork is optimized in an exhaustive manner. Finally, all the optimal placements for the entire network can be consolidated. The proposed hybrid OPP method has been tested on IEEE 34, IEEE 33 and IEEE 123 systems. Simulation results indicate the proposed combines the efficiency of the ILP algorithm and the ability of the exhaustive search method to locate all the optimal solutions. It gives the same results with the exhaustive search method, but consumes no more than 20 seconds for IEEE 123 bus system, whereas the exhaustive search method needs more than 3 hours to identify all the solutions.

6.2 Recommendations for Future Work

The optimal placement of PMU or μ PMU (OPP) aims at determining the minimum number of PMU or μ PMU required to be installed in a power system to make the system fully observable so that the cost of implementation of these devices can be lowered. In our study of the optimal placement of μ PMU for distribution systems, the placement methods considered the number of μ PMU as the minimization objective, which is based on the assumption that every μ PMU has the same price. However, it is possible that the price of μ PMU varies with the location to be installed or the number of incident lines needs to be observed. Taking into consideration of cost rather than the number of μ PMU in future work will make the OPP solution more practical. On the other hand, placement methods proposed

have not taken the topology change of a network into account. The topology of the network may change in presence of faults or the status change of switches, which may affect the observability of the system. Feeder reconfiguration for loss reduction or load balancing is a common task in distribution systems, in which process the topology the system will be altered [81]. Therefore, it is suggested to consider the topology change in the future study. Another issue that has not been studied in this thesis is the effect of the zero injection buses. As the consideration of zero injection buses can help to further reduce the minimum number of μ PMUs required to fully observe the network, it can be investigated in future work.

The OPP problem is essentially a topology optimization problem. In this thesis, it has been formulated as a mathematical problem and the optimal placement is found in an optimization search. However, it can also be formulated based on vertex covering or dominating set problems in graphs [18]. If the optimal placement strategy can be directly derived according to the mathematic characteristic such as the rank of the connectivity matrix or transformation matrix of connectivity matrix, the solving process will be sped up significantly. Therefore, it is a promising future work that can be further pursued.

Besides minimization of the number of μ PMU, OPP also aims at the selection of the optimal locations of μ PMU. Careful selection of locations of μ PMU is able to help improve distribution system state estimation (DSSE) accuracy. In this thesis, we have studied to maximize measurement redundancy which to some extent can help improve DSSE accuracy by providing more measurements. However, it is also worthing trying to consider the accuracy of DSSE in OPP in future work.

Author's Publication

1. X. Chen, K. J. Tseng, and G. Amaratunga, "State Estimation for Distribution Systems Using Micro-Synchrophasors," in *2015 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, Nov 2015, pp. 1-5.
2. X. Chen, T. Chen, K. J. Tseng, Y. Sun, and G. Amaratunga, "Customized Optimal Micro-Pmu Placement Method for Distribution Networks," in *2016 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC), Oct 2016*, pp. 135-140.
3. X. Chen, T. Chen, K. J. Tseng, Y. Sun, and G. Amaratunga, "Hybrid Approach Based on Global Search Algorithm for Optimal Placement of Micro-Pmu in Distribution Networks," in *2016 IEEE Innovative Smart Grid Technologies - Asia (ISGT-Asia)*, Nov 2016, pp. 559-563.
4. Y. Sun, X. Chen, S. Yang, Rusli, K. J. Tseng, and G. Amaratunga, "Micro Pmu Based Monitoring System for Active Distribution Networks," in *2017 IEEE 12th International Conference on Power Electronics and Drive Systems (PEDS)*, Dec 2017, pp. 518-522.
5. T. Chen, Y. S. E. Foo, K. V. Ling, and X. Chen, "Distributed State Estimation Using a Modified Partitioned Moving Horizon Strategy for Power Systems," *Sensors*, vol. 17, no. 10, p. 2310, 2017.

6. L. Sun, T. Chen, X. Chen, W. K. Ho, K.-V. Ling, K.-J. Tseng, and G. A. Amaratunga, "Optimum Placement of Phasor Measurement Units in Power Systems," *IEEE Transactions on Instrumentation and Measurement*, no. 99, pp. 1-9, 2018.
7. X. Chen, L. Sun, T. Chen, Y. Sun, Rusli, K. J. Tseng, K. V. Ling, W. K. Ho, and G. A. J. Amaratunga, "Full Coverage of Optimal Phasor Measurement Unit Placement Solutions in Distribution Systems Using Integer Linear Programming," *Energies*, vol. 12, no. 8, p. 1552, 2019.
8. "PMU Placement for Measurement Redundancy Distribution Considering ZIB and Contingencies", under review

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