Improving optical coherence tomography theories and techniques for advanced performance and reduced cost

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Improving Optical Coherence Tomography
Theories and Techniques for Advanced
Performance and Reduced Cost

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School of Electrical & Electronic Engineering
2019

**Improving Optical Coherence Tomography
Theories and Techniques for Advanced
Performance and Reduced Cost**

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

25 Jan 2019

Date

Wang Nanshuo

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Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

25 Jan 2019

Date

Liu Linbo

Authorship Attribution Statement

This thesis contains material from 1 paper published in the following peer-reviewed journal where I was the first author.

Chapter 4 was published as N. Wang, X. Liu, Q. Xiong, J. Xie, S. Chen, and L. Liu. “Polarization management to mitigate misalignment-induced fringe fading in fiber-based optical coherence tomography.”

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The contributions of the co-authors are as follows:

- Asst. Prof. Liu Linbo suggested the initial direction of the project and edited the manuscript.
- I built the optical system and carried out the validation experiment. I drew the figures in the manuscript.
- Mr. Liu Xinyu characterized the polarization mode dispersion of the system and conducted the numerical simulation.
- Mr. Liu Xinyu and I wrote the first draft of the paper, and did several rounds of revision.

- Mr. Xiong Qiaozhou, Mr. Xie Jun and Ms. Chen Shi helped review the draft.

25 Jan 2019

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Summary

Optical coherence tomography (OCT) has been developed as a high-resolution three-dimensional imaging technique for clinical diagnosis. This thesis proposes several theoretical discoveries and technical improvements to the current OCT systems aiming to drastically reduce system costs while achieving the state-of-the-art performance.

Polarization-sensitive OCT (PS-OCT) is a functional extension of conventional OCT which offers the depth-resolved birefringence imaging capability. To resolve local retardation in tissue samples, the current PS-OCT system employs two multiplexed polarization states to illuminate the sample under investigation and detects the backscattered light with two detection channels. We observed that the polarization state of light after round-trip propagation through a birefringent medium frequently aligns with the employed input polarization state but “mirrored” by the horizontal plane of the Poincaré sphere. We explore the predisposition for this mirror state and demonstrate how it constrains the evolution of polarization states as a function of the round-trip depth into weakly scattering birefringent samples, as measured with PS-OCT. The constraint enables measurements of depth-resolved sample birefringence with PS-OCT using only one input polarization state, which offers a critical simplification and cost reduction compared to the use of multiple input states. We demonstrated the capability of polarization mirror state in local birefringence restoration with birefringent phantom imaging and swine retina imaging *ex vivo*.

If we take transpose for both sides of the Jones transmission equation of a PS-OCT system with two input states and two detection channels, the mirror state constraint still holds while the input and output states exhibit interchangeability, which implies that depth-resolved birefringence imaging is possible with only one detection channel, if the sample is illuminated with two orthogonal polarization

states. Considering the high cost of spectrometers and the difficulty of pixel alignment between two detection channels, the transposed mirror state constraint helps to cut the cost and complexity of spectral domain PS-OCT system.

Inspired by the mirror state constraint, we realized that the polarization states of detected sample light, after propagating through a stochastically moving reciprocal optical path, would not distribute evenly on the Poincaré sphere, but tend to aggregate around the mirror state. This can be used to solve a long-standing problem that the detected interferograms of fiber-based OCT suffer from the fringe attenuation effect due to the polarization state mismatch between the light from the two interference arms which results in sensitivity deterioration and intensity fluctuation. This problem was conventionally solved by expensive polarization diverse detection. The polarization mirror states implies the existence of an optimal polarization state for the reference light. If the reference light is aligned with the polarization mirror state of the input light, a 3.5 dB signal-to-noise ratio improvement is demonstrated without additional hardware costs.

The axial resolution, being a critical parameter to the resolving power of OCT system, is determined by the light spectral shape and bandwidth. The pursuit of higher axial resolution leads to exponentially increasing system cost, and sometimes is even impossible because of the unavoidable gaps in the wavelength range of illumination, transmission, and detection. We demonstrate that the axial resolution deteriorated by gaps in OCT spectra can be restored by adopting the gapped amplitude and phase estimation (GAPES) method. The algorithm estimates the missing parts between separated spectral bands and obtains a tissue axial profile with reduced sidelobe artifacts and significantly improved axial resolution over the individual bands. This technique may make it possible to combine spectrally separated sources and detectors to improve axial resolution in OCT images, hence greatly reduce the cost of ultra-high-resolution OCT system.

In conclusion, this thesis proposed methods to reduce the cost of current OCT system with advanced performances in terms of state-of-the-art contrast, sensitivity and resolution, which has been demonstrated with phantom imaging and biological tissue imaging.

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Symbols and Acronyms

Symbols

$ s\rangle, t\rangle$	the input/output Jones vector in $\mathbb{C}^2$ space: $ s\rangle = [s_x, s_y]^\top$, ($s_x, s_y \in \mathbb{C}$)
$\langle s , \langle t $	the conjugate transpose of the input/output Jones vector: $\langle s = [s_x^*, s_y^*]$
$\langle s_1 s_2\rangle$	the inner product of two Jones vectors $ s_1\rangle$ and $ s_2\rangle$
$\mathbf{s}, \mathbf{t}$	the input/output Stokes vector in $\mathbb{R}^3$ space
$\bar{\mathbf{s}}, \bar{\mathbf{t}}$	the input/output full Stokes vector in $\mathbb{R}^4$ space
$\mathbf{m}$	the polarization mirror state in Stokes space
$\mathbf{J}$	the Jones transmission matrix in $\mathbb{C}^{2 \times 2}$
$\mathbf{M}$	the Mueller transmission matrix in $\mathbb{R}^{3 \times 3}$
$\bar{\mathbf{M}}$	the Mueller transmission matrix in $\mathbb{R}^{4 \times 4}$
ω	the angular frequency
k	the variable in wavenumber domain
z	the variable in (spatial) depth domain
δz	the axial resolution of optical coherence tomography
$\gamma(z)$	the reflectivity profile along depth of the sample
λ_0	the center wavelength of the light spectrum
$\Delta\lambda$	the wavelength bandwidth of the light spectrum
Δn	the refractive index difference
$\dagger$	the conjugate transpose of a matrix
$\top$	the transpose of a matrix
$*$	the complex conjugate of a number or a matrix
$[\cdot, \cdot]$	the commutator operator
$\text{SO}(3)$	the 3D rotation group
α	the azimuth angle in the $\mathbb{R}^3$ Stokes space with respect to +Q axis
ε	the elevation angle in the $\mathbb{R}^3$ Stokes space with respect to the QU -plane
ρ	the responsivity of the photodetector

$\otimes$ the Kronecker product

Acronyms

AOM	acousto-optical modulator
APES	amplitude and phase estimation
AR	autoregressive
CCD	charge-coupled device
DGD	differential group delay
DOP	degree of polarization
EOM	electro-optic modulator
FD-OCT	Fourier domain optical coherence tomography
FWHM	full width at half maximum
GAPES	gapped data amplitude and phase estimation
HWP	half-wave plate
OCT	optical coherence tomography
OFDI	optical frequency domain imaging
PDD	polarization diverse detection
PDF	probability density function
PDU	polarization delay unit
PMD	polarization mode dispersion
PMF	polarization maintaining fiber
PSF	point spread function
PS-OCT	polarization sensitive optical coherence tomography
QWP	quarter-wave plate
RNFL	retinal nerve fiber layer
SD-OCT	spectral domain optical coherence tomography
SLD	superluminescent diode
SMF	single mode fiber
SNR	signal-to-noise ratio
SS-OCT	swept source optical coherence tomography

Chapter 1

Introduction

1.1 Motivation

Optical coherence tomography (OCT) has been developed as a well-established clinical diagnostic device since its first introduction as a non-invasive biomedical imaging technique at the beginning of the 1990s [1].

However, the high cost of OCT devices has been a limiting factor for its widespread research and application. Efforts have been made to reduce the cost of OCT system in recent years. Z. Wang *et al.* demonstrated a miniaturized swept-source OCT (SS-OCT) with polarization sensitive capability [2]. The design employed the silicon photonic integrated circuit as a major part of the system which featured compact size and reduced manufacturing cost. P. Pande *et al.* presented a low-cost handheld probe using linear OCT built with standard inexpensive optical components, but came with the cost of reduced signal-to-noise ratio (SNR) and imaging depth [3]. S. Kim *et al.* demonstrated a low-cost spectral domain OCT (SD-OCT) with an overall material cost of under \$7200, achieving 5-fold cost reduction compared to commercial OCT systems with similar performance [4].

In conclusion, the existing exploration of cost reduction in OCT mainly focused on building the system with off-the-shelf low-price optical or electronic components while trying to maintain comparable performance as the commercial systems. Based on a physical constraint in polarization optics, this thesis trying to

cut expensive components from prevailing PS-OCT system which is considered indispensable for a fully-functioning system and hence intrinsically cut the cost.

We then realized that the newly discovered constraint can be applied to mitigate the fringe attenuation due to polarization state mismatch in fiber-based OCT. This issue was previously solved by polarization diverse detection or common path designed probe, which requires extra cost. With proper reference polarization state management, we could achieve statistical improvement of the system SNR.

The axial resolution of OCT is determined by the spectral bandwidth and shape of the detected light, which are limited by the gaps in the wavelength range of illumination, transmission, and detection. To achieve higher resolution, the cost of the laser source and detector would increase enormously. If the gaps could be estimated by the available spectrum, we could either use two combined spectrum to achieve ultra-high-resolution OCT, or link two completely separate current bands to further extend the current axial resolution.

1.2 Organization of the thesis

The following chapters of this thesis are organized as follows:

Chapter 2 provides a review of the foundations, characteristics and different modalities of OCT system. We focus on the Fourier-domain (FD-OCT) system, and discussed its improvement from the original time-domain configuration (TD-OCT). Besides, a functional extension of traditional OCT, the polarization-sensitive OCT (PS-OCT) and the Jones and Mueller-Stokes calculus for polarization analysis are introduced. The common depth-resolved birefringence estimation method with two input state PS-OCT is explained.

Chapter 3 describes the polarization mirror state constraint in reciprocal backscattering system. Thanks to the discovery of this physical constraint, depth-resolved birefringence imaging with single input PS-OCT becomes possible, which greatly reduces the cost and complexity of the system. A thorough theoretical and experimental validation of the discovery is presented. We also demonstrate the application of polarization mirror state in biological tissue imaging. The swine retina was imaged with a customized single input state free space PS-OCT system.

Chapter 4 demonstrates that by taking transpose for both sides of the Jones transmission equation of PS-OCT system, the mirror state constraint still holds while the input and output states exhibit interchangeability, which implies that depth-resolved birefringence imaging is possible with only one detection channel, if the sample is illuminated with two orthogonal polarization states. Considering the high cost of spectrometers and the difficulty of pixel alignment between the two detection channels, the transposed mirror state constraint helps to further cut the cost and complexity of spectral-domain PS-OCT system.

Chapter 5 investigates fringe attenuation effect in fiber-based OCT system which is caused by polarization state misalignment between the reference and sample light, due to the randomization of sample polarization state caused by the rotation and twisting of the optical fiber. In the validation experiment of the polarization mirror state constraint, we have found the polarization state of the backscattered sample light tends to concentrate to the mirror state. By presetting the polarization state of the detected reference light to the mirror state, fringe attenuation effect can be mitigated, hence statistically increase the system signal-to-noise ratio by 3.5 dB.

Chapter 6 proposes a method to further improve the axial resolution of OCT system by fusing two separate spectral bands together, thus breaks the limitation of the gaps between current available spectral bands in the source, transmission and detection of OCT systems. This method estimates the missing parts between separated spectral bands and obtains the axial profile of tissue with reduced side-lobe artifacts compared to the gapped spectra and significantly improved axial resolution over the individual bands. This technique may make it possible to combine spectrally separated sources and detectors to improve axial resolution in OCT images.

Chapter 7 summarizes the overall study and highlights the potential impact of the discoveries. Limitations and future exploration directions are also presented.

Chapter 2

Optical coherence tomography and its functional extensions

2.1 Introduction

Based on low coherent interferometry, OCT is a non-invasive, high-resolution imaging tool that is able to generate three dimensional image up to about 2 mm below the surface of biological tissues. Since its introduction in 1991, OCT has been established and adopted as a gold standard in ophthalmology, because of its micrometer level volumetric imaging capability. In a typical ophthalmic OCT setup, the sample beam is focused onto an intermediate image plane with a relay lens and then delivered to the retina by a Volk lens and the lens of the human eye. The axial scanning is obtained by measuring the echo delay of the light signal interferometrically. The transverse position of the beam is scanned by a pair of scanning mirrors perpendicular with each other, enabling three-dimensional imaging of the retina.

Integrated with flexible probes like fiber-based catheters, balloons, capsules and endoscopes, OCT has evolved as a promising imaging tool for internal luminal structures such as the arterial wall [5, 6], airway mucosa [7–10], esophageal mucosa [11, 12], urinary tract wall [13, 14], and colonic mucosa [15, 16]. The light is directed to the distal end of the probe by a single mode fiber, focused by grin lens, reflected by a prism radially away from the probe to the tissue. By a combination of rotation and pull-back movement of the probe, the three-dimensional

data containing volumetric structural information of the hollow organs is recorded. This makes OCT probe particularly suitable for the diagnosis of diseases originates from the surface of the luminal structures, such as atherosclerosis [17–21], Barrett’s esophagus [22–25] and gastrointestinal carcinoma [26–28].

OCT was originally introduced in its time domain configuration where the depth profile of the sample in the axial dimension was obtained by the linear scanning of the reference reflector. Over the recent twenty years, one of the most important improvements of OCT devices is the introduction of Fourier domain OCT (FD-OCT) [29], where the interference fringe in the wavenumber domain is recorded and the depth profile is retrieved using inverse Fourier transform. FD-OCT provides more than 20 dB sensitivity boost over that of the TD-OCT due to the wavelength-division detection using a large number of detectors involved [30–32]. Another improvement of FD-OCT over the TD-OCT is the imaging speed. The recording happens in the wavelength domain, thus no moving part for spatial scanning is needed. With the help of high speed swept sources or line scan cameras, FD-OCT is capable to reach an A-line rate of more than 100 kHz [33, 34], even up to 20 MHz [35], providing the possibility of fast *in vivo* imaging. As the new generation of OCT technology, FD-OCT has better performance over TD-OCT in nearly all respects. Therefore, all analysis and experiments in this thesis are based on FD-OCT.

2.2 Fourier domain OCT

2.2.1 Theory of FD-OCT

Let us consider a simplified OCT system illustrated in Figure 2.1, which ensembles a Michaelson interferometer where the input beam (assume as plain wave) is split into the reference and sample arms by a beam splitter or a fiber coupler. The reference light is reflected back by a mirror, while the sample light is backscattered by the scatterers in the sample, containing reflectivity information along the axial direction.

FIGURE 2.1: Basic model of OCT detection.

The electrical field of the input light can be expressed as $E_0 = s(k, \omega) \cdot \exp[i(kz - \omega t)]$, where $s(k, \omega)$ is the amplitude of the electrical field with respect to the wave number k and angular frequency ω . The input light is split by a 50:50 wavelength-independent beam splitter to the reference and sample arms.

The reference light is back-reflected by a mirror with an electrical field reflectivity of r_R (the corresponding power reflectivity is $R_R = |r_R|^2$) located at the distance of z_R to the beamsplitter, the reflected electrical field would be $E_R = \frac{E_0}{\sqrt{2}} r_R e^{i2kz_R}$, where the factor of 2 before z_R indicates round-trip propagation through the reference arm.

For the convenience of analysis, we assume the sample under investigation is composed of a series of discrete reflecting surfaces located at different axial positions along the propagating direction of the sample light, each with an amplitude reflectivity of r_{Sn} ($n = 1, 2, \dots, N$). Taking the axial position of each sample reflector z_{Sn} into account, the reflectivity of the whole sample can be modeled as the summation of a series of axially spaced delta functions: $r_S(z_S) = \sum_{n=1}^N r_{Sn} \delta(z - z_{Sn})$, namely the depth profile of the sample, which is the main target of measurement in OCT. Thus the light returning from the sample can be denoted as $E_S = \frac{E_0}{\sqrt{2}} e^{i2kz_S} \otimes r_S(z_S)$.

The reference and sample light then loses half of their intensity after passing the beam splitter again and interfere at the photodetector, whose responsivity is denoted as ρ , hence the detected photocurrent can be written as

$$\begin{aligned}
 i_D(k) &= \frac{\rho}{2} \langle (E_R + E_S) (E_R + E_S)^* \rangle \\
 &= \frac{\rho}{2} \left\langle \left| \frac{s(k)}{\sqrt{2}} r_R e^{i2kz_R} + \frac{s(k)}{\sqrt{2}} \sum_{n=1}^N r_{Sn} e^{i2kz_{Sn}} \right|^2 \right\rangle \\
 &= \frac{\rho S(k)}{4} \left[R_R + \sum_{n=1}^N R_{Sn} \right] \\
 &\quad + \frac{\rho S(k)}{4} \sum_{n=1}^N \left[2\sqrt{R_R R_{Sn}} \cos(2k(z_R - z_{Sn})) \right] \\
 &\quad + \frac{\rho S(k)}{4} \sum_{n=1}^N \sum_{m \neq n}^N \left[2\sqrt{R_{Sn} R_{Sm}} \cos(2k(z_{Sn} - z_{Sm})) \right]
 \end{aligned} \tag{2.1}$$

where $S(k) = \langle |s(k)|^2 \rangle$ denotes the detected power spectrum. The final results are composed of three components: (1) a pathlength-independent DC offset; (2) a cross-correlation term indicating the interference between the light reflected from the reference mirror and the sample reflectors. (3) an autocorrelation component indicating the interference between different sample reflectors.

In practice, the reflectivity of the reference reflected is much higher than that of the reflectors in the tissue ($R_R \gg R_{Sn}$), making the autocorrelation term negligible. Therefore the summation of reflectivity of sample reflectors in the DC term could be ignored with the presence of R_R . The DC offset can be eliminated by subtracting the reference background signal obtained when the sample arm is blocked.

After background subtraction and ignorance of autocorrelation term, the cross-correlation term is the signal left:

$$i_X(k) \propto \frac{\rho S(k)}{2} \sum_{n=1}^N \left[\sqrt{R_R R_{Sn}} \cos(2k(z_R - z_{Sn})) \right] \tag{2.2}$$

Fourier transform exerted on both sides of the equation maps the wavenumber k domain onto the spatial depth domain z . Applying the convolution property of

Fourier transform and a Fourier transform pair $\left(\cos(kz_0) \xleftrightarrow{\mathcal{F}} \frac{1}{2} [\delta(z + z_0) + \delta(z - z_0)] \right)$, we have

$$i_X(z) = \mathcal{F}[i_X(k)] = \frac{\rho}{4} \gamma(z) \otimes \sum_{n=1}^N \sqrt{R_R R_{Sn}} \cdot \delta(z \pm 2(z_R - z_{Sn})) \quad (2.3)$$

As revealed by Eq. (2.3), the interferometer detects the pathlength difference between the two interference arms, thus the zeros position is better to be set at the position of the reference mirror at z_R . The sample reflectivity profile $\sqrt{R_{Sn}}$ is amplified by the reflectivity of the reference reflector $\sqrt{R_R}$ and then sampled by a delta function located at position $2\Delta z_{Sn}$ (the doubled path length difference is due to the detection of round-trip propagation), then convoluted with the Fourier transform of the power spectrum of the light source. The source spectrum is often modeled to be Gaussian shaped, in which case the normalized power spectrum $S(k)$ and the corresponding coherent function $\gamma(z)$ form a Fourier transform pair:

$$S(k) = \frac{1}{\Delta k \sqrt{\pi}} \exp \left[- \left(\frac{k - k_0}{\Delta k} \right)^2 \right] \xleftrightarrow{\mathcal{F}} \gamma(z) = \exp(-z^2 \Delta k^2) \quad (2.4)$$

where k_0 is the central wavenumber and Δk is the bandwidth. The perfect pulse function is broadened by the convolution with $\gamma(z)$, whose full width at half maximum (FWHM), namely the coherence length of the light source, determines the axial resolution of OCT imaging. For a standard normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{(x-x_0)^2}{2\sigma^2} \right]$, its FWHM is represented by the standard deviation as $2\sqrt{2 \ln 2} \sigma$. Thus the FWHM of $\gamma(z)$ can be written as:

$$\delta z = \frac{2\sqrt{\ln 2}}{\Delta k} = \frac{2 \ln 2}{\pi} \frac{\lambda_0^2}{n \Delta \lambda} \quad (2.5)$$

where λ_0 is the center wavelength, n is the tissue's refractive index, $\Delta \lambda$ is the wavelength bandwidth defined as the FWHM of the wavelength spectrum ($\Delta k = \frac{\pi}{\sqrt{\ln 2}} \frac{\Delta \lambda}{\lambda_0^2}$).

From Eq. (2.3), there exists a mirrored version of depth profile symmetric about the zero pathlength, termed as the complex conjugate artifact. Because the interferogram is composed of real numbers, the corresponding Fourier transform is

Hermitian symmetric, the Fourier domain signal is symmetric about DC, and the positive and negative signals are complex conjugate to each other. If the sample image can be located completely at the positive side of the axis, this symmetry would not cause any bad consequences. If the image crossed the zero pathlength boundary, the mirror image would appear from the negative side and superimposed on the normal image, making the details difficult to distinguish. The Hermitian symmetry reduces the already insufficient imaging range by a factor of 2, and attracts a lot of research on removing the resulted artifact [36–43].

2.2.2 Swept source and spectral domain configurations

The FD-OCT can be implemented using either a broadband low coherent source together with a spectrometer, or a wavelength swept source together with a photodiode detector, which are termed as the spectral domain (SD-) OCT and the swept source (SS-) OCT, respectively.

FIGURE 2.2: A comparison of simplified schematics of (a) SD-OCT and (b) SS-OCT (OFDI).

The SD-OCT [29, 44, 45], illustrated in Figure 2.2(a), illuminates the sample with a broadband light source and detects the backscattered light with a spectrometer composed of grating and line scan camera. The frequency of the detected interference fringes is proportional to the path length difference Δz between the two arms.

The SS-OCT [46–49], also known as optical frequency domain imaging (OFDI), is shown in Figure 2.2(b). The light source sweeps across the wavelength repeatedly,

thus the spectral interferogram is captured as a function of time by the photodetector.

2.3 Polarization-sensitive OCT (PS-OCT)

2.3.1 Principles of PS-OCT

Traditional intensity OCT image lacks tissue specificity. PS-OCT is a functional extension of traditional OCT to detect birefringence information in the sample. Biological tissues with organized fibrous structures exhibit structural birefringence, such as tendons, muscle, nerve, bone, skin, teeth and cartilage [50–56]. The birefringence is characterized by the refractive index difference (Δn) for electromagnetic wave vibrating along and perpendicular to the long axis of the fibrous structures. After propagating through the tissue for a distance z , the orthogonally polarized components of the light exhibit phase retardation with respect to each other, given by

$$\delta = \frac{2\pi z \cdot \Delta n}{\lambda} \quad (2.6)$$

PS-OCT provides an extra contrast and specificity by recording the change of polarization state of light backscattered from the sample. A variety of application of polarization contrast has been demonstrated such as the evaluation of atherosclerotic plaques [57], burned skin [58] and thickness of retinal nerve fiber layer [59, 60].

PS-OCT was first introduced by Hee *et al.* in 1992 in bulk optics configuration [61]. An important development to measure simultaneously the full interference fringes in two orthogonal polarization detection channels enabled the measurement of the full reflected stokes vector in a single depth profile measurement, and became the standard in nearly all PS-OCT systems [62]. Over the past decades, fiber-based PS-OCT has been developed, and system configuration has evolved from the time domain to Fourier domain together with the evolution of OCT system. More polarization related parameters can be detected besides the amount of birefringence,

including optic axis orientation [62, 63], degree of polarization [64], sample transmission matrix [65–67]. The Jones and Mueller-Stokes formalism provide PS-OCT with a theoretical foundation.

FIGURE 2.3: A typical schematic of bulk-optic PS-OCT. BS: beam splitter; PBS: polarizing beam splitter; Pol: polarizer; QWP: quarter-wave plate.

A generalized bulk-optic based PS-OCT system is illustrated in Figure 2.3. The input light is transformed into linearly polarized in the horizontal direction by a polarizer (Pol) and split into the reference and sample arms by a non-polarizing beam splitter (BS). The reference light passes a round trip through a quarter-wave plate (QWP) oriented at 22.5° with respect to the horizontal direction, becomes 45° linear polarization which ensures identical amplitude and phase in the two orthogonal detection channels. Another QWP oriented at 45° in the sample arm ensures circularly polarized illumination light hitting the surface of the sample. The light backscattered from the sample shows arbitrary polarization state containing the birefringence properties of the tissue under investigation. The sample light recombines with the reference light, then is split into two orthogonal polarization

components by a polarizing beam splitter (PBS) and received by two identical detectors.

2.3.2 System analysis in Jones formalism

The linearly polarized input light $\mathbf{E}(z) = E(z) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is split by the beam splitter and becomes identical input light of the reference and sample arms $\mathbf{E}_{\text{Ri}}(z) = \mathbf{E}_{\text{Si}}(z) = \frac{E(z)}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The effect of the polarization changing elements can be described by 2×2 Jones Matrix. For example, the Jones matrix of a linear retarder with phase retardation δ oriented at angle θ can be denoted as the multiplication of two rotation matrices and a diagonal matrix:

$$\mathbf{J}(\delta, \theta) = \mathbf{R}(-\theta) \mathbf{P}(\delta) \mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} e^{i\delta} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (2.7)$$

If light passes through a cascade of the polarization changing elements whose Jones matrices are $\mathbf{J}_1, \mathbf{J}_2, \dots, \mathbf{J}_N$, respectively, the overall transmission matrix is the multiplication of all the matrices: $\mathbf{J} = \mathbf{J}_N \cdots \mathbf{J}_2 \mathbf{J}_1$. For light passing the optical element in the reverse direction, its Jones matrix needs to be transposed. Therefore, the back-reflected reference light after the second passing through the beam splitter can be described as

$$\mathbf{E}_{\text{R}} = \frac{r_{\text{R}}}{\sqrt{2}} \cdot \mathbf{J}^{\top}(\frac{\pi}{2}, \frac{\pi}{8}) \cdot \mathbf{J}(\frac{\pi}{2}, \frac{\pi}{8}) \cdot \mathbf{E}_{\text{Ri}} = \frac{E(2z_{\text{R}})}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (2.8)$$

where $\mathbf{J}(\frac{\pi}{2}, \frac{\pi}{8})$ is the Jones matrix of the QWP oriented at 22.5° , the matrix transpose operation is necessary when light propagates in the reverse direction. z_{R} is the pathlength from the beam splitter to the reference reflector.

We can simply model the overall birefringence effect of the sample as a linear retarder $\mathbf{J}_{\text{S}}$, since the circular retardance will be cancelled out for round-trip propagation. Thus the Jones matrix describing the round-trip in the sample can be written as

$$\mathbf{J}_S = e^{i2kz\bar{n}} \mathbf{J}(2kz\Delta n, \phi) = e^{i2kz\bar{n}} \cdot \mathbf{R}(-\theta) \cdot \begin{bmatrix} e^{i2kz\delta n} & 0 \\ 0 & 1 \end{bmatrix} \cdot \mathbf{R}(\theta) \quad (2.9)$$

where $\bar{n}$ and Δn are the average and difference of refractive index of ordinary and extraordinary beams, respectively. θ denotes the orientation of the fast optic axis with respect to the horizontal direction. Thus the Jones vector of sample beam backscattered from the tissue can be denoted as

$$\mathbf{E}_S = \frac{r_S}{\sqrt{2}} \mathbf{J}^\top\left(\frac{\pi}{2}, \frac{\pi}{4}\right) \cdot \mathbf{J}_S \cdot \mathbf{J}\left(\frac{\pi}{2}, \frac{\pi}{4}\right) \cdot \mathbf{E}_{Si} = \frac{r_S E(k) e^{i2k(z_S + \bar{n}z)}}{2} e^{-ikz\Delta n} \begin{bmatrix} e^{i2\phi} \sin(kz\Delta n) \\ \cos(kz\Delta n) \end{bmatrix} \quad (2.10)$$

where z_S is the pathlength from the beam splitter to the sample surface, z is the optical path along which the light travels within the sample. The interference pattern captured in the two detection channels can be obtained as

$$\begin{aligned} i_H(z) &\propto E_{Rx}^* E_{Sx} + E_{Rx} E_{Sx}^* \propto \sqrt{R_R R_S} \int \sin(kz\Delta n) \cos(2k\Delta z + 2\phi) S(k) dk \\ i_V(z) &\propto E_{Ry}^* E_{Sy} + E_{Ry} E_{Sy}^* \propto \sqrt{R_R R_S} \int \cos(kz\Delta n) \cos(2k\Delta z) S(k) dk \end{aligned} \quad (2.11)$$

where z is the depth in the tissue and $\Delta z = z_R - z_S - \bar{n}z$ is the optical pathlength difference between the two interference arms. We assume the source power spectral density $S(k)$ is Gaussian shaped $S(k) \propto \exp\left(-\left(\frac{k - k_0}{\kappa}\right)^2\right)$. The FWHM bandwidth is $2\sqrt{\ln 2}\kappa$. With the approximation $\kappa z\Delta n \ll 1$, Eq. 2.11 can be integrated as

$$\begin{aligned} i_H(z) &\propto \sqrt{R_R R_S} \sin(k_0 z \Delta n) \cos(2k_0 \Delta z + 2\phi) e^{-(\Delta z / \Delta l)^2} \\ i_V(z) &\propto \sqrt{R_R R_S} \cos(k_0 z \Delta n) \cos(2k_0 \Delta z) e^{-(\Delta z / \Delta l)^2} \end{aligned} \quad (2.12)$$

where $\Delta l = \frac{1}{\kappa} = \frac{\sqrt{\ln 2} \lambda_0^2}{\pi \Delta \lambda}$ and $\Delta \lambda$ is the FWHM of the source in the wavelength domain. Demodulation of the signals eliminates the $\cos(2k_0 \Delta z)$ terms in Eq. 2.12 to

yield intensities in the two orthogonal polarization detection channels proportional to

$$\begin{aligned} I_H(z) &= |i_H(z)|^2 \propto R_R R_S \sin^2(k_0 z \Delta n) \\ I_V(z) &= |i_V(z)|^2 \propto R_R R_S \cos^2(k_0 z \Delta n) \end{aligned} \quad (2.13)$$

The information of tissue birefringence can be obtained from the interference fringes of the two channels. The phase retardation can be retrieved by

$$\delta(z) = k_0 z \Delta n = \tan^{-1} \left(\sqrt{\frac{I_H(z)}{I_V(z)}} \right) \quad (2.14)$$

2.3.3 The Mueller-Stokes formalism

2.3.3.1 The Stokes vector

The Jones formalism can only describe fully polarized light and non-depolarizing behaviors of optical system, The Stokes vector, composed of four components, I , Q , U , and V (sometimes the four components are also referred to as s_0 , s_1 , s_2 , and s_3), is defined by a set of irradiance measurements conducted by a photodetector and several perfect polarizers. Suppose the light under investigation can be described as $|s\rangle = [s_x, s_y]^\top$, the irradiance of the beam can be directly measured as $I = s_x s_x^* + s_y s_y^*$. With a linear polarizer, we can measure the irradiance difference of the horizontal and vertical polarization components as $Q = s_x s_x^* - s_y s_y^*$. With a linear polarizer oriented at $+45^\circ$ and -45° and measure the irradiance difference, we have $U = s_+ s_+^* - s_- s_-^*$, where $s_+ = \frac{\sqrt{2}}{2}(s_x + s_y)$, $s_- = \frac{\sqrt{2}}{2}(s_x - s_y)$. Lastly, with circular polarizers, we measure $V = s_R s_R^* - s_L s_L^*$, where $s_R = \frac{\sqrt{2}}{2}(s_x - i \cdot s_y)$, $s_L = \frac{\sqrt{2}}{2}(s_x + i \cdot s_y)$. These measurements compose a complete Stokes vector shown as follows [68]:

$$\begin{aligned}
I &= s_x s_x^* + s_y s_y^* \\
Q &= s_x s_x^* - s_y s_y^* \\
U &= s_+ s_+^* - s_- s_-^* = s_x s_y^* - s_y s_x^* \\
V &= s_R s_R^* - s_L s_L^* = i (s_x s_y^* - s_y s_x^*)
\end{aligned} \tag{2.15}$$

If the light is fully polarized, $I^2 = Q^2 + U^2 + V^2$; if the light is completely unpolarized, $Q = U = V = 0$. Thus the degree of polarization (DOP) can be defined as

$$\text{DOP} = \sqrt{\frac{Q^2 + U^2 + V^2}{I^2}} \tag{2.16}$$

which ranges from zero for unpolarized light to unity for fully polarized light.

The one-to-one mapping from Jones vector to Stokes vector can be established with a set of 2×2 Pauli spin matrices:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{2.17}$$

Then the components s_i of the Stokes vector can be expressed as [62]

$$s_i = \langle s | \sigma_i | s \rangle, \quad (i = 0, 1, 2, 3) \tag{2.18}$$

where $|s\rangle$ is the Jones vector and $\langle s|$ denotes its complex conjugate. If we denote the Pauli spin vector in Stokes space as $\vec{\sigma} = (\sigma_0, \sigma_1, \sigma_2, \sigma_3)^\top$, the element-wise equation (2.18) can be written as

$$\bar{s} = \langle s | \vec{\sigma} | s \rangle, \tag{2.19}$$

where $\bar{s}$ is the full Stokes vector in $\mathbb{R}^4$ space.

2.3.3.2 The Mueller matrix

Under the Stokes representation, the transmission matrix of an optical system can be described as the Mueller matrix $\overline{\mathbf{M}} \in \mathbb{R}^{4 \times 4}$, which links the incident Stokes vector $\overline{\mathbf{s}}$ to the transmitted Stokes vector $\overline{\mathbf{t}}$ as

$$\overline{\mathbf{t}} = \begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \overline{\mathbf{M}} \cdot \overline{\mathbf{s}} \quad (2.20)$$

A straight forward way to transfer a Jones matrix to its corresponding Mueller counterpart is listed below [69]:

$$\overline{\mathbf{M}} = \mathbf{U}(\mathbf{J} \otimes \mathbf{J}^*)\mathbf{U}^\dagger$$

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{bmatrix} \quad (2.21)$$

where $\mathbf{J}$ is the Jones matrix, $*$ denotes the complex conjugate operation, and $\otimes$ is the tensor (Kronecker) product.

2.3.3.3 The Poincaré sphere representation

As the absolute irradiance is not necessary to identify the polarization state, we can normalize the Stokes vector with the light irradiance I and use only the last three components to represent the Stokes vector:

$$\mathbf{s} = \begin{bmatrix} Q/I \\ U/I \\ V/I \end{bmatrix} \in \mathbb{R}^3 \quad (2.22)$$

The $\mathbb{R}^3$ Stokes vectors are still able to represent both fully and partially polarized light, more importantly, they can be visualized in a Cartesian coordinate system where the Q , U , and V components are mapped to x -, y - and z - coordinates, respectively. Particularly, all fully polarized Stokes vectors, whose degree of polarization ($\|\mathbf{s}\|_2 = 1$) is unity, forms the surface of a unit sphere termed as the Poincaré sphere. The partially polarized Stokes vectors, whose degree of polarization is smaller than one ($\|\mathbf{s}\|_2 < 1$), are mapped inside the Poincaré sphere.

2.3.3.4 Birefringence analysis on the Poincaré sphere

In a PS-OCT system, since the diattenuation effect is negligible in biological tissues [54, 70], we will focus on resolving of birefringence parameters from the Stokes evolution. The Poincaré sphere offers an intuitive geometric representation of the polarization states and the change upon propagating through the polarization changing medium. This geometric reasoning has originally been elaborated by B. H. Park et al. in detail [34, 71].

FIGURE 2.4: The effect of birefringence visualized on the Poincaré sphere. The polarization state $\mathbf{s}$ is rotated to $\mathbf{t}$ around rotation axis $\mathbf{r}_0$ by a rotation angle of φ .

The effect of birefringence, when presented geometrically in QUV -coordinate (shown in Figure 2.4), is the rotation of the input polarization state around a rotation vector $\mathbf{r}$, where the direction of the rotation vector indicates the optic axis orientation

($\mathbf{r}_0$) and the angle of rotation (φ) indicates the amount of phase retardation. The rotation vector is defined by $\mathbf{r} = \varphi \cdot \mathbf{r}_0$. The rotation vector $\mathbf{r}$ can be converted to its rotation matrix representation by [72]

$$\mathbf{R} = \exp(\varphi \cdot \mathbf{K}) \quad (2.23)$$

where $\mathbf{K}$ is the cross product matrix of $\mathbf{r}$, which means $\mathbf{K} \cdot \mathbf{s} = \mathbf{r} \times \mathbf{s}$ holds true for all vectors $\mathbf{s} \in \mathbb{R}^3$. If the rotation axis $\mathbf{r}_0$ (normalized vector of $\mathbf{r}$) is denoted as $[r_1, r_2, r_3]^\top$, the skew-symmetric matrix $\mathbf{K}$ is obtained by

$$\mathbf{K} = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \quad (2.24)$$

With the rotation matrix $\mathbf{R}$, the relation between the input and output polarization states is expressed as $\mathbf{t} = \mathbf{R} \cdot \mathbf{s}$

However, for a homogeneous birefringent layer under investigation of PS-OCT, knowing the polarization state of light backscattered from the upper and lower boundary of the layer is insufficient for the calculation of the amount of birefringence within the layer. For the fiber-optic systems, the birefringence induced by the fiber is unknown and unstable, which invalidate the restriction that the optic axis should lie in the QU -plane [73]. In this case, as shown in Figure 2.5(a), the rotation vector around which $\mathbf{s}$ is rotated to $\mathbf{t}$ can not be uniquely determined, all eligible rotation axes fall in a plane spanned by vectors $\mathbf{s}_{\text{plus}}$ ($= \mathbf{s} + \mathbf{t}$) and $\mathbf{s}_{\text{cross}}$ ($= \mathbf{s} \times \mathbf{t}$). Starting from the origin, for each direction on the plane a different rotation trace and rotation angle can be solved. For example, if $\mathbf{s}_{\text{plus}}$ is considered as the direction of the rotation vector, $\mathbf{s}$ will be rotated to $\mathbf{t}$ along the smallest circle possible, results in the largest rotation angle π ; if $\mathbf{s}_{\text{cross}}$ is considered as rotation axis, the rotation will go along the largest circle on the sphere (centered at the origin O), and the rotation angle is the angle between vectors $\mathbf{s}$ and $\mathbf{t}$ ($\cos^{-1}(\langle \mathbf{s}, \mathbf{t} \rangle)$). Except for these two exceptions, every direction on the spanned plane leads to a possible solution, making the calculation of the birefringence ambiguous.

This problem can be addressed by introducing a second input polarization state. Two input polarization states ensures the solvability of the rotation vector. In

FIGURE 2.5: The illustration of geometric birefringence analysis on the Poincaré sphere. (a) With only one polarization state, the rotation vectors that rotate s to t cannot be uniquely determined, every direction in the shaded plane spanned by s_{plus} (shown as normalized vector) and s_{cross} is a possible solution. (b) With two different launching polarization states s_1 and s_2 , and the corresponding detected states t_1 and t_2 after propagating through the sample, the rotation axis can be uniquely defined.

Figure 2.5(b), with two different input polarization states s_1 and s_2 , and the corresponding detected Stokes vectors t_1 and t_2 , as the rotation axis should be perpendicular to both rotation circles, its direction can be uniquely determined by the cross product of $(t_1 - s_1)$ and $(t_2 - s_2)$.

The two input polarization states are best arranged if they are perpendicular with each other in the Stokes space. If one input aligns with the optic axis and exhibit no rotation at all, the orthogonal input state would rotate along the largest possible circumference of the sphere, which maximizes reliability of measurement.

The two input states can be multiplexed in a time-division manner or a depth-division manner. The time-division multiplexing can be done by quickly switching the input polarization state between adjacent A-lines, which is usually implemented by the electro-optic modulator (EOM) [74]. The depth-division multiplexing can be implemented by a polarization delay unit (PDU) inserted in the sample arm [75]. The depth-division method has the advantage of ensuring the phase stability of the two input polarization states (especially for wavelength swept source) but sacrifices

half of the ranging depth, because two copies of images must be accommodated side by side axially.

Chapter 3

The polarization mirror state constraint

We observed that the polarization state of light after round-trip propagation through a birefringent medium frequently aligns with the employed input polarization state ‘mirrored’ by the horizontal plane of the Poincaré sphere. We explore the predisposition for this mirror state and demonstrate how it constrains the evolution of polarization states as a function of the round-trip depth into weakly scattering birefringent samples, as measured with PS-OCT. The constraint enables measurements of depth-resolved sample birefringence with PS-OCT using only a single input polarization state, which offers a critical simplification compared to the use of multiple input states. More importantly, the transposed version of this constraint also holds true, making it possible to get local birefringence with two input polarization states and only one detection channel, which would greatly reduce the cost of polarization-sensitive SD-OCT system.

3.1 Observation of the polarization mirror state

Polarization offers access to unique, distinguishing signatures of samples for diverse applications from remote sensing [76, 77] to biomedical optics [78–80]. Conventionally, multiple input polarization states are required in addition to polarization-diverse detection to fully characterize the polarization properties of a sample [62],

prompting complex instrumentation. Alleviating these hardware requirements would enable more widespread exploration of this compelling contrast mechanism.

In previous experiments, we observed that the polarization state of backscattered or reflected light, when measured through identical illumination and detection paths, frequently evolved through the employed input polarization state but with reversed handedness, corresponding to the input state mirrored by the horizontal plane of the Poincaré sphere [81]. Earlier investigations of the polarization properties of single mode fibers reported on aspects of the polarization mirror state [82–84], yet without elucidating its manifestation. To examine the polarization mirror state, we measured the round-trip signal through a 1.5 m long single-mode optical fiber. Instead of using a conventional polarimeter, we employed interferometric measurements for later coherence gating experiments, as depicted in Figure 3.1(a). Light from a super-continuum source was linearly polarized, prepared with an achromatic quarter-wave plate (QWP) to different input polarization states, and split into reference and sample arms. A linear polarizer oriented at 45° in the reference arm defined the reference polarization state independent of the input polarization. For polarization diverse detection, the sample and reference light was combined in a polarization-maintaining fiber to then direct each of the fibers two linear eigenstates towards a grating-based spectrometer (760~920 nm)[85]. The recorded fringe signals reveal the amplitude and relative phase of the two orthogonal electromagnetic field components in the sample arm and hence the polarization state of the sample light.

Employing a polarization controller to alter the birefringence of the fiber, we measured the time-varying polarization state resulting from randomly moving the paddle positions of the controller. Visualized in Figure 3.1(b) as the normalized Stokes vectors of the center wavelength (840 nm) in the Q , U , and V -coordinates of the Poincaré sphere, the polarization mirror state manifests by the repeated crossing of the polarization state evolution in a specific state $\mathbf{m}$, highlighted by the red arrow in Figure 3.1(b). Repeated with different launching polarization states $\mathbf{s}$ (indicated by the blue arrows in Figure 3.1(c,d)), we recognize that $\mathbf{m} = \mathbf{D} \cdot \mathbf{s}$, where $\mathbf{D} = \text{diag}(1, 1, -1)$. $\mathbf{m}$ corresponds to the input state mirrored by the horizontal QU -plane, explaining its designation as the polarization mirror state. The input states were determined by reflecting the light to the detector in free space, without the fiber in place. All measurements were performed in the fixed coordinates

FIGURE 3.1: Demonstration of the polarization mirror state. (a) Schematic drawing of the optical system employed throughout all experiments. QWP: quarter-wave plate; BS: beamsplitter. (b)-(d) Polarization state evolution on the Poincaré sphere as a result of moving the three paddles of the PC when using circularly, elliptically and linearly polarized input states, respectively, indicated by the blue arrows.

of the receiver and are independent of the orientation of the coordinates in the illumination path.

3.2 Existence of polarization mirror state

3.2.1 Proof based on the inevitable crossing of rotation vector trace

To appreciate the mirror state phenomenon, we consider a general retarder $\mathbf{R}(x)$ with its retardation varying as a function of x , e.g. the polarization controller's paddle positions. It may be preceded by a static element $\mathbf{P}$. The combined system, illustrated in Fig. 3.2(a), transforms the input polarization state $\mathbf{s}$ into the output state $\mathbf{t}$:

$$\mathbf{t} = \mathbf{D} \cdot \mathbf{P}^\top \cdot \mathbf{R}^\top(x) \cdot \mathbf{D} \cdot \mathbf{R}(x) \cdot \mathbf{P} \cdot \mathbf{s} = \mathbf{T}(x) \cdot \mathbf{s} \quad (3.1)$$

Here, $\mathbf{T} = \mathbf{D} \cdot \mathbf{P}^\top \cdot \mathbf{R}^\top \cdot \mathbf{D} \cdot \mathbf{R} \cdot \mathbf{P}$, and all vectors and matrices are in the rotation group $\text{SO}(3)$. We chose to follow the convention of maintaining the orientation of the spatial xy -coordinates irrespective of the light's propagation direction [86, 87]. In reciprocal media, the reverse transmission through element $\mathbf{R}$ is described by $\mathbf{D} \cdot \mathbf{R}^\top \cdot \mathbf{D}$ [88, 89] (see Appendix I). It is important to note that the round-trip transmission $\mathbf{T}$ is D -transpose symmetric $\mathbf{T} = \mathbf{D} \cdot \mathbf{T}^\top \cdot \mathbf{D}$, which makes $\mathbf{T}$ a linear retarder. The round-trip propagation effectively cancels any optical activity of circular retardation. The effect of $\mathbf{T}$ on the input state can be described by a rotation vector $\boldsymbol{\tau}(x)$ lying in the QU -plane of the Poincaré sphere, with its direction indicating the rotation axis, and its length defining the amount of rotation. Considering their 2π -ambiguity, the rotation vectors of all possible linear retarders are confined to a circle with a radius of π within the QU -plane (Fig. 3.2(b)). When moving the polarization controller paddles, $\boldsymbol{\tau}(x)$ traces out an intricate path in the QU -plane, as shown by the green line in Fig. 3.2(b) for simulating a synchronous movement of the three paddles.

There exists only a single rotation vector within the QU -plane that rotates a given input state $\mathbf{s}$ onto an arbitrary output state $\mathbf{t}$. This rotation vector is defined by the intersection of the QU -plane and the plane bisecting $\mathbf{s}$ and $\mathbf{t}$. In order for $\mathbf{s}$ to pass through $\mathbf{t}$, $\boldsymbol{\tau}(x)$ has to evolve through this specific point within the π -circle of the QU -plane. The only exception, there exists a continuum of rotation vectors

FIGURE 3.2: Theoretical explanation of the polarization mirror state. (a) Model of the round-trip propagation through a reciprocal sample, comprising $\mathbf{R}(x)$ with varying birefringence and a static element $\mathbf{P}$. (b) The rotation vectors of all linear retarders localize within a circle of radius π within the QU -plane of the Poincaré sphere. The orange curve represents the end points of the rotation vectors mapping the randomly chosen input polarization state $[\cos(-\pi/6) \cdot \cos(\pi/3), \cos(-\pi/6) \cdot \sin(\pi/3), \sin(-\pi/6)]^\top$ exactly to its mirror point. The green trace represents the simulated rotation vector evolution for a synchronous movement of the polarization controller paddles. The purple curves represent the rotation vectors of $\mathbf{D} \cdot \mathbf{P}^\top \cdot \mathbf{D} \cdot \mathbf{R}(x) \cdot \mathbf{P}$ for $\mathbf{R}(x)$ with linearly increasing retardation, for three representative sets of distinct $\mathbf{P}$ and $\mathbf{R}(x)$.

that map $\mathbf{s}$ onto its mirror state $\mathbf{m} = \mathbf{D} \cdot \mathbf{s}$, because the QU -plane coincides with the bisecting plane in this case. These rotation vectors are located on a curve $\boldsymbol{\tau}_{\mathbf{m}}$ within the QU -plane (orange curve in Fig. 3.2(b)). Every intersection of $\boldsymbol{\tau}(x)$ with $\boldsymbol{\tau}_{\mathbf{m}}$ corresponds to $\mathbf{s}$ evolving through the mirror state $\mathbf{m}$, explaining its frequent realization.

The effect of a linear retarder can be expressed as a rotation vector $\boldsymbol{\tau} = \varphi \cdot \boldsymbol{\tau}_0 = \varphi \cdot [\cos \alpha, \sin \alpha, 0]^\top$, where $\boldsymbol{\tau}_0$ has unitary length, α is the azimuth angle with respect to the Q -axis and φ is the rotation angle. The corresponding $\text{SO}(3)$ rotation matrix $\mathbf{T}$ is [72]:

$$\mathbf{T} = \mathbf{I} + \sin \varphi \cdot \mathbf{K} + (1 - \cos \varphi) \mathbf{K}^2 = \exp(\varphi \mathbf{K})$$

$$\mathbf{K} = \begin{bmatrix} 0 & \tau_3 & -\tau_2 \\ -\tau_3 & 0 & \tau_1 \\ \tau_2 & -\tau_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\sin \alpha \\ 0 & 0 & \cos \alpha \\ \sin \alpha & -\cos \alpha & 0 \end{bmatrix} \quad (3.2)$$

Here, we are looking for the linear retarder $\mathbf{T}(\varphi, \alpha)$ that maps an input state $\mathbf{s} = [s_1, s_2, s_3]^\top$ to some state $\mathbf{t} = [t_1, t_2, t_3]^\top$. Its rotation axis is given by the intersection of the QU -plane and the plane bisecting $\mathbf{s}$ and $\mathbf{t}$:

$$(\mathbf{t} - \mathbf{s}) \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} t_2 - s_2 \\ s_1 - t_1 \\ 0 \end{bmatrix} \quad (3.3)$$

from where

$$\alpha = \tan^{-1} \left(\frac{s_1 - t_1}{t_2 - s_2} \right) \quad (3.4)$$

In general, $\mathbf{s}$ and $\mathbf{t}$ clearly define a single rotation axis. Only for the mirror polarization state, in which case both the nominator and denominator vanish, α is undefined and can take any value. To find the amount of rotation around this axis required to map $\mathbf{s}$ onto $\mathbf{t}$, we project both vectors onto the plane orthogonal to the rotation axis and evaluate their relative angle:

$$\begin{aligned} \varphi &= \cos^{-1} \left(\frac{(\mathbf{s} - (\boldsymbol{\tau}_0^\top \cdot \mathbf{s}) \boldsymbol{\tau}_0)^\top \cdot (\mathbf{t} - (\boldsymbol{\tau}_0^\top \cdot \mathbf{t}) \boldsymbol{\tau}_0)}{\sqrt{1 - (\boldsymbol{\tau}_0^\top \cdot \mathbf{s})^2} \sqrt{1 - (\boldsymbol{\tau}_0^\top \cdot \mathbf{t})^2}} \right) \\ &= \cos^{-1} \left(\frac{\mathbf{s}^\top \cdot \mathbf{t} - (\boldsymbol{\tau}_0^\top \cdot \mathbf{s})^2}{1 - (\boldsymbol{\tau}_0^\top \cdot \mathbf{s})^2} \right) \end{aligned} \quad (3.5)$$

where $\boldsymbol{\tau}_0 = [\cos \alpha, \sin \alpha, 0]^\top$, and $\boldsymbol{\tau}_0^\top \cdot \mathbf{s} = \boldsymbol{\tau}_0^\top \cdot \mathbf{t}$ by construction. A defined rotation vector $\boldsymbol{\tau}$ within the QU -plane maps $\mathbf{s}$ onto a general $\mathbf{t}$. Only for the mirror state $\mathbf{s} = \mathbf{m}$, there exists a rotation for any azimuth direction within the QU -plane,

tracing out a continuous curve in the QU -plane when wrapped to $\varphi \in (-\pi, \pi)$ (the red curve in Figure 3.3(b)).

FIGURE 3.3: The trace of the possible rotation vectors that map the input state $\mathbf{s}$ to its mirror state $\mathbf{m}$ is defined by the input state. (a) The maximum and minimum rotation angle φ_A , φ_B and the corresponding rotation axes $\boldsymbol{\tau}_{A0}$, $\boldsymbol{\tau}_{B0}$ shown on the Poincaré sphere. (b) The corresponding rotation vectors $\boldsymbol{\tau}_A$ and $\boldsymbol{\tau}_B$ define the orientation and curvature of the trace of the possible rotation vectors (red line).

The shape and orientation of this trace is determined by $\mathbf{s}$. Assuming the azimuth and elevation angles of $\mathbf{s}$ are α_s and ε_s , respectively ($\alpha_s \in [0, 2\pi)$, $\varepsilon_s \in [-\pi/2, \pi/2]$), then the largest rotation angle $\varphi_A = \pi$ appears when the rotation vector aligns with the projection of $\mathbf{s}$ onto the QU -plane, indicated by $\boldsymbol{\tau}_{A0}$ in Figure 3.3(a). The smallest rotation angle appears when the rotation axis $\boldsymbol{\tau}_{B0}$ is perpendicular to $\boldsymbol{\tau}_{A0}$. In this case the rotation trajectory coincides with a longitude of the sphere and corresponds to a rotation angle of $\varphi_B = 2|\varepsilon_s|$. As visualized in Figure 3.3(b), the long axis of the trace aligns with the azimuth angle of input state $\mathbf{s}$, and the curvature of the trace is determined by the elevation angle of $\mathbf{s}$.

Importantly, the presence of diattenuation that induces polarization-dependent loss would skew the measured polarization states and frustrate the repeated evolution through the mirror state. The mirror state only manifests in systems that can be accurately modeled with unitary transmission matrices.

We next used PS-OCT to measure the polarization state of light backscattered within a scattering sample as a function of its round-trip depth [61, 90]. At the scale

of the axial resolution of OCT, tissue can be modeled as a sequence of homogeneous linearly birefringent layers with distinct optic axis orientations. $\mathbf{R}(x)$ describes in this case a linear retarder with a retardance that linearly increases with depth x , resulting in $\mathbf{D} \cdot \mathbf{R}^\top \cdot \mathbf{D} = \mathbf{R}$. $\mathbf{P}$ contains the combined effect of system components and preceding tissue layers. The resulting rotation vectors $\boldsymbol{\tau}(x)$ form regular curves across the π -circle (purple curves in Fig. 3.2(b)). All possible traces intersect the curve $\boldsymbol{\tau}_{\mathbf{m}}$ precisely once, ensuring periodic crossing of $\mathbf{m}$. To inspect in more detail the evolution of $\mathbf{t}$, we take its derivative with respect to x , and substitute $\mathbf{s} = \mathbf{T}^\top(x) \cdot \mathbf{t}$:

$$\frac{\partial \mathbf{t}}{\partial x} = \frac{\partial \mathbf{T}(x)}{\partial x} \cdot \mathbf{T}^\top(x) \cdot \mathbf{t} = \boldsymbol{\beta}(x) \times \mathbf{t} \quad (3.6)$$

Because $\mathbf{T}^\top \cdot \mathbf{T}$ is the identity matrix, $(\partial \mathbf{T} / \partial x) \cdot \mathbf{T}^\top = -\mathbf{T} \cdot (\partial \mathbf{T}^\top / \partial x)$ is skew-symmetric and can be expressed as the cross-product operator $\boldsymbol{\beta} \times$, which is constant for a retardance that linearly increases with x . Accordingly, within a single sample layer, $\mathbf{t}$ evolves on the Poincaré sphere with constant speed rotating around the apparent optic axis $\boldsymbol{\beta}$ on a circle constrained to pass through $\mathbf{m}$.

According to Eq. (3.2) the SO(3) rotation matrix $\mathbf{T}$ can be written as $\mathbf{T} = \exp(\varphi \mathbf{K})$ [72]. From Eq. (3.6), we know that $\partial \mathbf{t} / \partial x = \boldsymbol{\beta} \times \mathbf{t}$, where

$$\boldsymbol{\beta}(x) \times = \frac{\partial \mathbf{T}(x)}{\partial x} \cdot \mathbf{T}^\top(x) = \frac{d(\varphi \mathbf{K})}{x} + \frac{1}{2!} \left[\varphi \mathbf{K}, \frac{d(\varphi \mathbf{K})}{x} \right] + \frac{1}{3!} \left[\varphi \mathbf{K}, \left[\varphi \mathbf{K}, \frac{d(\varphi \mathbf{K})}{x} \right] \right] + \dots \quad (3.7)$$

Here, the Hadamard's lemma has been used to write the derivative of the matrix exponential and $[\mathbf{A}, \mathbf{B}] = \mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}$ is the commutator [91]. The evolution of $\mathbf{t}(x)$ can always be described locally as a rotation around $\boldsymbol{\beta}(x)$. However, the relation of $\boldsymbol{\beta}$ with φ and $\mathbf{K}$ is non-trivial, in general, and results in a varying rotation axis if both $\varphi(x)$ and $\mathbf{K}(x)$ depend on x .

The round-trip transmission through a stack of linearly birefringent layers can be described by

$$\mathbf{T} = \mathbf{P}' \cdot \exp(2z \cdot \delta \cdot \boldsymbol{\Gamma}) \cdot \mathbf{D} \cdot \mathbf{P}'^\top \cdot \mathbf{D} \quad (3.8)$$

Here, z is the depth in the last layer, defined by the cross-product operator $\mathbf{\Gamma} = \boldsymbol{\gamma} \times$ with unitary-length rotation vector, and local phase retardation δ . $\mathbf{P}'$ describes the reverse transmission through all previous layers. Plugging Eq. (3.8) into Eq. (3.7), and setting $z = x$, simplifies to

$$\boldsymbol{\beta} \times = \mathbf{P}' \cdot \frac{\partial \exp(2z \cdot \delta \cdot \mathbf{\Gamma})}{\partial z} \cdot \exp(-2z \cdot \delta \cdot \mathbf{\Gamma}) \cdot \mathbf{P}'^\top = \mathbf{P}' \cdot 2\delta \cdot \mathbf{\Gamma} \cdot \mathbf{P}'^\top \quad (3.9)$$

The apparent rotation vector $\boldsymbol{\beta}$ is now constant within this layer, causing the evolution of $\mathbf{t}$ on a circle passing through the mirror state. Of note, the apparent rotation axis $\boldsymbol{\beta}$ is related to the true sample optic axis $\boldsymbol{\gamma} \times$ through a similarity transform with $\mathbf{P}'$.

3.2.2 Proof based on geometry

We are interested in the variation of the output Stokes vector $\mathbf{t}(z)$ as a function of depth of reflection z within a homogeneous linearly retarding layer characterized by its double-pass $\text{SO}(3)$ matrix $\mathbf{L}(z)$ (as shown in Figure 3.4(a)) in response to a launching state $\mathbf{s}$. Before entering and after reflection within this layer of interest, the light travels through additional layers. Without loss of generalization, these preceding layers can be expressed as a linear retarder $\mathbf{Q}$, and the overall transmission becomes:

$$\mathbf{t}(z) = \mathbf{Q} \cdot \mathbf{L}(z) \cdot \mathbf{Q} \cdot \mathbf{s} = \mathbf{Q} \cdot \mathbf{L}(z) \cdot \underbrace{\mathbf{Q}^{-1} \cdot \mathbf{Q}}_{\mathbf{I}} \cdot \mathbf{Q} \cdot \mathbf{s} = \mathbf{L}'(z) \cdot \mathbf{Q} \cdot \mathbf{Q} \cdot \mathbf{s} = \mathbf{L}'(z) \cdot \mathbf{t}(0) \quad (3.10)$$

The reflection from the interface between $\mathbf{Q}$ and $\mathbf{L}$, at depth $z = 0$, transforms the input state $\mathbf{s}$ by the double action of $\mathbf{Q}$ onto the state $\mathbf{t}(0)$. Interpreting $\mathbf{Q}$ as a homogenous retarding element, it rotates $\mathbf{s}$ along a circle having a horizontal rotation axis within the QU -plane, and hence guaranteed to pass through the mirror point $\mathbf{m}$ which is the symmetric point of input state $\mathbf{s}$. To reach $\mathbf{m}$ would take a rotation by an angle 2η , but when reaching $\mathbf{t}(0)$, an offset angle of 2ζ remains. Each $\mathbf{Q}$ contributes a rotation of $\eta - \zeta$.

FIGURE 3.4: Illustration of geometric proof of existence of the mirror state. (a) Model of the round-trip propagation through a reciprocal sample. (b) Side view of the polarization state evolution on the Poincaré sphere. (c) Front view of the of the polarization state evolution on the Poincaré sphere, when observing through the rotation vector of Q .

The effect of increased penetration z into the layer L on the output state $t(z)$ can be described by multiplication of $t(0)$ with L' . L' is the similarity transform of the linear retarder L . Similarity-transforming L with the linear retarder Q simply rotates the rotation axis of L by Q . If previously the rotation axis of L was located within the QU -plane, it now lies in a plane rotated by an angle $\eta - \zeta$ around the axis of Q . This plane precisely bisects the states $t(0)$ and m . It would take a rotation of η to rotate the horizontal plane to include m , and only $\eta - 2\zeta$ to reach $t(0)$. Intrinsically, any rotation with an axis lying within this bisecting plane will result in the evolution of the $t(0)$ through m . And since it also holds true for any Q , it proves that the evolution of the polarization state reflected at increasing depth within a homogenous retarding layer is confined to a circle passing through the mirror point m .

3.2.3 Experimental validation

For experimental validation, we prepared a scattering phantom consisting of three linearly birefringent layers with distinct optic axis orientations [92] (Fig. 3.5(a)). Without the fiber segment in the sample arm, we focused the light with a 30 mm focal length lens into the sample, achieving a full-width at half maximum (FWHM) spot diameter of $\sim 8 \mu\text{m}$, and scanned with galvanometric mirrors in the lateral direction. The spectrometer's bandwidth offers an axial resolution of $\sim 2.4 \mu\text{m}$ in

air. At each scanning location, using PS-OCT, we constructed the Stokes vector as a function of depth in the sample. To remove speckle and improve the signal, we spatially filtered the original Stokes vectors with a two-dimensional Gaussian kernel of $20 \mu\text{m } 1/e^2$ width in the axial direction and $80 \mu\text{m}$ in the lateral direction. Finally, we computed the normalized three-component Stokes vector $\mathbf{r}(z)$ as a function of depth, shown in Figs. 3.5(b-d) for three distinct input polarization states at one lateral sample location. We then fitted circles to the polarization state evolution within each layer. The circles (in purple color) demonstrate a close match with the measured polarization states and all circles evolve through the polarization mirror state $\mathbf{m}$ (indicated by red arrows), as expected.

3.3 Resolving local birefringence with mirror state

Using a single input polarization state for PS-OCT, it is straightforward to compute the cumulative retardation that propagation through the sample to a given depth and back imparts on the input polarization state [93]. Yet, cumulative retardation can be difficult to interpret in samples with a layered architecture, and it is more insightful to compute local retardation, i.e. the derivative of the retardance of $\mathbf{T}(x)$ with depth, which is given by the norm of $\boldsymbol{\beta}$ and is proportional to the sample birefringence [55, 67, 94] at that depth location. Following Eq. (3.6) we have

$$\left| \frac{\partial \mathbf{t}}{\partial x} \right| = |\boldsymbol{\beta}| \sin \theta \quad (3.11)$$

where we used $|\mathbf{t}| = 1$. θ is the angle between the rotation vector $\boldsymbol{\beta}$ and the polarization state $\mathbf{t}$ and is needed to deduce local retardation. With only a single input state this angle is generally unknown. Using, instead, two input polarization states oriented at 90° to each other on the Poincaré sphere reveals the orientation of the apparent optic axis. However, recognizing that the evolution of $\mathbf{t}$ is constrained to go through $\mathbf{m}$, it is possible to recover the orientation and magnitude of $\boldsymbol{\beta}$ from measurements with only a single input polarization state. Owing to this constraint, both $\partial \mathbf{t} / \partial x$ and $(\mathbf{t} - \mathbf{m})$ lie within the same plane orthogonal to $\boldsymbol{\beta}$. Hence, the direction of $\boldsymbol{\beta}$ can be obtained by the cross-product $\boldsymbol{\beta}_0 = (\partial \mathbf{t} / \partial x) \times (\mathbf{m} - \mathbf{t})$, and $\sin \theta = |\boldsymbol{\beta}_0 \times \mathbf{t}| / |\boldsymbol{\beta}_0|$, allowing to calculate, after some algebraic manipulations:

FIGURE 3.5: Evolution of coherence-gated polarization states in a three-layer birefringence phantom. (a) Schematic sketch of the phantom consisting of three layers with distinct optic axis orientations. (b)-(d) Polarization state evolution (color-coded corresponding to the layers in (a)) for a circularly polarized (b), elliptically polarized (c), and linearly polarized input state (d).

$$\boldsymbol{\beta} = \frac{\frac{\partial \mathbf{t}}{\partial x} \times (\mathbf{m} - \mathbf{t})}{1 - \mathbf{t}^\top \cdot \mathbf{m}} \quad (3.12)$$

To validate the ability of the polarization mirror state to reconstruct local retardation, we imaged a tissue-like phantom consisting of a long birefringent band followed by four parallel elements with distinct birefringence levels and an optic

FIGURE 3.6: Local retardation imaging of birefringence phantom using the polarization mirror state constraint. (a,b) Schematic drawings of the two-layer phantom in either orientation. (c,d) Corresponding cumulative retardation and (e,f) local retardation images, respectively. Scale bars in (c) measure $100\ \mu\text{m}$ (vertical) and $400\ \mu\text{m}$ (horizontal).

axis orientation different from the long band [92] (Fig. 3.6(a) and 3.6(b)) and the gap was filled with non-birefringent matrix.

For reconstruction of local retardation, we employed the pre-calibrated polarization mirror state $\mathbf{m}$, and implemented Eq. (3.12) by approximating $\mathbf{t} = \frac{1}{2} (\mathbf{r}[p+1] + \mathbf{r}[p])$ and $\partial \mathbf{t} / \partial x = (\mathbf{r}[p+1] - \mathbf{r}[p]) / \Delta z$, where p is the pixel index along depth $z = p \cdot \Delta z$, and Δz is the axial sampling distance. To avoid high-frequency noise introduced by taking the difference between adjacent points, we axially averaged the reconstructed rotation vector $\boldsymbol{\beta}(z)$ with a Gaussian window of the same axial size as used to filter the Stokes vectors. The norm of $\boldsymbol{\beta}$, scaled to degrees of retardation per depth ($^\circ/\mu\text{m}$), reveals the sample's local retardation, imaged with either side of the sample facing up (Figs. 3.6(e) and 3.6(f)). For comparison, the cumulative retardation of $\mathbf{T}(x)$ was computed by evaluating the angle between $\mathbf{r}(z)$ at each depth and $\mathbf{r}(z_{\text{surf}})$, where z_{surf} is the axial location of the sample surface within each depth profile (Figs. 3.6(c) and 3.6(d)). Whereas cumulative retardation is difficult to interpret, the local retardation clearly reveals the individual sample segments with their distinct levels of birefringence and is recovered irrespective of the sample orientation [67, 95].

Previous strategies to reconstruct local birefringence from single-input-state PS-OCT rely on the intrinsic symmetry of the imaging system [94, 96] and assume that the optical elements in the illumination and detection paths have no impact on

the polarization states. Most OCT instruments for clinical applications, however, use fiber-based optical components with distinct illumination and detection paths, which breaks the intrinsic D -transpose symmetry [97]. Crucially, the evolution of $\mathbf{t}$ through the mirror state persists also in systems with distinct retardation in the illumination and detection optics. This is equivalent to left-multiplying Eq. (3.1) with an additional matrix $\mathbf{B}$. Although the apparent cumulative retarder that maps the input state onto the measured output state is no longer a linear retarder in this case, $\mathbf{B}$ simply alters the location of the circular evolution of the polarization states on the Poincaré sphere to go through the actual mirror state to $\mathbf{B} \cdot \mathbf{m}$.

3.4 Removal of mirror state artifact

A remaining challenge manifests whenever $\mathbf{t}$ aligns with $\mathbf{m}$, which impairs the reconstruction of local retardation (cyan arrows in Fig. 3.6(f)). This corresponds to the effective polarization state in the target layer to orient along one of that layers optic axes, and even prevents the cumulative retardation from accumulating retardance. Using circularly polarized input light requires a half wave of retardation to realize this alignment, which is uncommon in many biological samples. Yet, some tissues feature substantial birefringence and controlling the input state is not necessarily possible. The resulting artifact can be avoided by introducing a modest amount of polarization mode dispersion (PMD) into the system and using spectral binning for reconstruction [98]. Because PMD disperses the input polarization state across the spectral bins, simultaneous alignment of $\mathbf{t}$ with $\mathbf{m}$ in all bins is very unlikely.

Coupling the sample light through the 1.5-m-long single mode fiber twisted around the polarization controller paddles provided sufficient PMD for our broad-bandwidth source. For spectral binning, we multiplied the spectral fringe signals with Hanning windows $h(k, n)$ of width $\Delta k/N$ centred on $n \cdot \Delta k / (2N)$ within the available k -support, Δk , $n \in [1, 2N - 1]$, $N = 5$, resulting in 9 spectral bins, to compute the binned Stokes vectors $\mathbf{r}(z, n)$. We also evaluated the degree of polarization $\text{DOP} = \langle (Q^2 + U^2 + V^2)^{1/2} / I \rangle$, where $\langle \rangle$ indicates averaging over the spectral bins, and Q , U , V and I are the spatially filtered Stokes components before normalization. Following the identical processing for local retardation for each bin as described above, we obtained the rotation vectors $\boldsymbol{\beta}(z, n)$. Fig. 3.7(a-d) illustrates

FIGURE 3.7: Inaccurate estimation of local retardation can be avoided with a small amount of polarization mode dispersion (PMD) in combination with spectral binning. (a)-(f) Cross-sectional images of a two-layer phantom. (a) Local retardation reconstructed using only the 1st spectral bin. (b) Local retardation reconstructed using only the 9th spectral bin. (c,d) Reliability metric maps of the 1st and 9th spectral bin, respectively. (e) Local retardation reconstructed using the entire spectrum without spectral binning. (i) Local retardation image reconstructed with spectral binning combining all bins. Scale bars in (f) measure $100\ \mu\text{m}$ (vertical) and $400\ \mu\text{m}$ (horizontal).

the local retardation of bins 1 and 9, together with a map $w(z, l) = |\mathbf{t} - \mathbf{m}|$ expressing the reliability of the given Stokes vector by the distance from its mirror state, for a tissue-like birefringence phantom. Bin 9 results in high local retardation values but with little reliability, unlike bin 1, which indicates more modest local retardation yet with higher reliability. The $\boldsymbol{\beta}(z, n)$ with high reliability of all bins describe the same sample retardation but may be offset in their relative orientation due to system PMD. The required rotation $\mathbf{G}(n)$ to align the vectors of each bin to the central bin N in the least-square sense is given by:

$$\max_{\mathbf{G}(n)} \text{Tr} \left(\mathbf{G}(n) \cdot \sum_{z,l} \boldsymbol{\beta}(z, l, N) \cdot \boldsymbol{\beta}^\top(z, l, N) \cdot w(z, l) \right) \quad (3.13)$$

where z and l are point indices in the axial and lateral directions, respectively, $\mathbf{G}(n)$ is assumed constant within an entire B-scan, and the sum is taken over all points with sufficient $\text{DOP} > 0.8$ and signal intensity $\text{SNR} > 5\ \text{dB}$. From

the singular value decomposition of the 3×3 matrix defined by the summation $\sum \boldsymbol{\beta} \cdot \boldsymbol{\beta}^\top \cdot w = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^\dagger$, where † denotes conjugate transpose, the solution to Eq. (3.13) is obtained by $\mathbf{G} = \mathbf{V} \cdot \mathbf{U}^\dagger$. Lastly, the aligned rotation vectors are averaged among the spectral bins considering their weights $w(z, l)$, and then axially filtered, as previously, to obtain the final local retardation image, free from artifacts, as demonstrated in Fig. 3.7(f).

3.5 Swine retina imaging with single input free space PS-OCT

3.5.1 System construction and calibration

3.5.1.1 System description

The swine retina imaging was based on a spectral domain PS-OCT system illustrated in Figure 3.8. The output of a super-continuum light source (SCL: SC-50EM, YSL Photonics) was clipped by a dichroic mirror (DMSP950 Thorlabs Inc., not shown in Figure 3.8(a)) and collimated into the fiber coupler (90/10, TW850R2A2, Thorlabs Inc.). The input light was transformed into a circularly polarized light using a linear polarizer (P1: LPNIRE100-B, Thorlabs Inc.) and a QWP (AQWP10M-980, Thorlabs Inc.) before split into the sample and reference arms by a non-polarizing beam splitter (50/50, BS014, Thorlabs Inc.). Another linear polarizer (P2) in the reference arm was used to produce reference light with the balanced amplitude and phase detected by the two spectrometers. The dispersion caused by polarizer P2 was balanced by a compensator (DC) made of polarization independent BK7 glass in the sample arm. In the detection arm, a half-wave plate (HWP: AHWP10M-980, Thorlabs Inc.) was employed to align the polarization coordinates of the free-space interference light to match with the polarization axes of the polarization-maintaining fiber (PMF: P3-780PM-FC-2, Thorlabs Inc.). The polarization diverse detection unit consists of a polarizing beam splitter and two identical spectrometers, each of which was made up of a transmission grating (Wasatch TG: 1200 line/mm), a camera lens (L9: Nikon AF Nikkor 85 mm f/1.8D), and a line scan CCD (E2V, EV71YEM4CL2010-BA9). The operating bandwidth

was ~ 160 nm centered at 840 nm. The software for image acquisition and reconstruction was implemented with LabVIEW 2015 (National Instrument Inc.) and MATLAB 2016b (MathWorks Inc.), respectively.

FIGURE 3.8: (a) Schematic of single input polarization state PS-OCT system. BS, beam splitter; DC, dispersion compensator; HWP, half-wave plate; L1-9, lenses; LS, linear stage; M1-2, mirrors; P1-2, polarizers; PBS, polarizing beam splitter; PM, power meter; PMF, polarization-maintaining fiber; QWP, quarter-wave plate; SCL, supercontinuum laser; SMF, single-mode fiber; SP, spectrometer; TG, transmission grating. (b)-(e) Process of k -space linearization. (b) the mapping function $k = \Phi(n)$ from CCD pixel index n to the k -pace. (c) Raw interferogram. (d) Remapped interferogram in linear k -space. (e) Normalized point spread function (PSF) with a full width at half maximum (FWHM) of $2.3 \mu\text{m}$ in air after dispersion calibration. (f)-(h) Spectral alignment process after k -space resampling. Normalized profiles of the horizontal channel (red line) and vertical channel (blue line) with reflector optical path length difference of Δz_1 (dashed line), Δz_2 (solid line) before (f) and after (g) the spectral alignment, respectively. The spectrum rescaling factor a was determined by the ratio of the distances between the signal peaks' positions of each channel. After spectrum rescaling, the peak positions of reflection surfaces of both channels were aligned together. (h) Determination of k -space pixel shift by drawing the functions of cross-correlation between interferograms of two channels against amount of k -space shift at different path length difference. The point at which all cross-correlation functions of different path length difference reach their own maximum value simultaneously is the amount of shift b we are looking for.

3.5.1.2 Polarizing component alignment

With the help of a power meter located at the PM port of the input fiber coupler, we were able to precisely adjust the angular position of each polarization-changing element (polarizers, QWP, HWP) following the steps below:

1. Place polarizer P1 in the input arm. The angle of the polarizer is arbitrary since the match between the interference light coordinate and the detection coordinate can be achieved later by rotating the HWP.
2. Block the reference arm and place a reflector (e.g. a mirror) at the focus of the objective lens in the sample arm. Precise axial position and pitch angle of the reflector is achieved by the maximum reading of the power meter.
3. Rotate the HWP to align the horizontal and vertical polarization axes of the interference light with that of the PMF until the extinction ratio between the two detection channels is maximized.
4. Install the QWP in the input arm (between polarizer P1 and the BS) and rotate the QWP at angle 45° with respect to the transmission axis of polarizer P1, when the power at PM port reaches its minimum. This ensures a circularly polarized illumination light.
5. Unblock the reference light and rotate polarizer P2 to balance the amplitude of interference fringes between the two spectrometers.

3.5.1.3 k -space linearization and spectral alignment

To remap the interferograms into evenly sampled k -space and compensate for dispersion, we used a method reported previously by our group (the linearization process is illustrated in Figure 3.8) [99]. Briefly speaking, the compensation is since the interference fringes generated with a specular sample is ideally sinusoidal. Note that we added a polarizer between the beam splitter and HWP to make sure the interference fringes captured by the two spectrometers were in-phase, which was removed after the calibration was completed. And these interference fringes recorded for dispersion compensation were also used in next step, namely the spectral alignment.

After the interference fringes were linearly resampled in k -space respectively, we aligned the index of wavenumber between spectra of the two spectrometers. We modeled the relation of the wavenumber indices between the spectra from the two spectrometers as a linear function $k_2 = a \cdot k_1 + b$, where k_1 , k_2 stands for wavenumber indices of the two spectra, respectively, a and b are the stretching coefficient and the k -space shift. If the two fringes are well aligned, the stretching

factor a should be equal to 1, corresponding to the aligned peaks position in depth z domain. According to scaling properties of Fourier transform, the effects of stretching coefficient a on the wavenumber domain and the Fourier transformed depth domain can be revealed by Eq. (3.14) where I is the spectrum, k is the wavenumber, $\mathcal{F}$ stands for the Fourier transform, γ is the depth profile, z is the depth in the sample.

$$\mathcal{F}(I(a \cdot k)) = \frac{1}{|a|} \mathcal{F}\left(I\left(\frac{k}{a}\right)\right) = \frac{1}{|a|} \gamma\left(\frac{z}{a}\right) \quad (3.14)$$

Figure 3.8(f)-(h) shows the procedures to determine coefficients a and b for spectral alignment. The stretching coefficient a could be determined via the linear regression on the peaks' position of the PSFs of the two channels. (shown in Figure 3.8(f)-(g)). The k -space shift b was determined by cross-correlation between the interference fringes of the two channels. The point at which all cross-correlation functions from different optical path length difference reach their maximum simultaneously locates the proper k -space pixel shift number b (shown in Figure 3.8(h)).

3.5.2 Imaging result

Swine eyes were collected from a local slaughterhouse within one hour after harvesting. We cut the eyeball along the equator, removed the lens, emptied the vitreous chamber, and instilled just enough saline to prevent specular reflection while scanning the retina. The scanning area centered on the optical nerve head, spanning a dimension of 6 mm by 6 mm. Figure 3.9 demonstrates local retardation imaging of excised swine retina. The intensity image visualizes the layered structure of the retina (Figure 3.9(a)). The local retardation reveals the birefringence of the retinal nerve fiber layer (RNFL), clearly differentiating it from the less birefringent subsequent layers (Figure 3.9(b)). We have recorded 1024 A-scans for each frame and 1024 frames in total. After manually segmented the upper and lower boundary of RNFL in each frame, we averaged the intensity and retardation values within the range of RNFL along depth direction and concatenated them as the en face projections shown in Figure 3.9(c) and (d). RNFL birefringence is being investigated for its potential in diagnosing early degeneration of the RNFL associated with glaucoma [100, 101].

FIGURE 3.9: Swine retinal retardation imaging *ex vivo*. (a) Intensity image. (b) retardation image. (c) intensity en face projection. (d) retardation image en face projection. Both scale bars in (b) measure $400\ \mu\text{m}$ and apply to (c, d).

3.6 Conclusion

In the chapter, we demonstrated the peculiar properties of the mirror polarization state that manifest when measuring backscattered light along identical illumination and detection paths free of polarization-dependent loss. In PS-OCT, the mirror state constrains the evolution of the depth-dependent polarization states and enables local retardation imaging, which previously has not been available to PS-OCT without substantially more complex measurements using multiple input states.

Chapter 4

The transposed mirror state constraint

The mirror state constraint introduced in the previous chapter already exhibit potential in reducing system cost and complexity. By looking at the system transmission equation in Jones calculus, it can be noticed that as the transmission matrices are symmetric, they would remain symmetric after taking tranpose of both side of the equation, which means that the transposed version of mirror state constraint is still valid. This is useful because the column of the input/output matrix represents different input polarization states, while the rows represents the two orthogonal detection channels. The tranposed constraint implies the interchangeability of input states and detection channels. To obtain the depth-resolved birefringence information of the sample, the single input state system with two detection channels should be equivalent to a two input states system with only one detection channel, if the mirror state constraint is employed.

4.1 Theoretical analysis

Let us consider a fiber-based system illustrated in Figure 4.1. The Jones transmission of the sample arm optics is $\mathbf{J}_{\text{fib}}$ for the forward propagation, hence is $\mathbf{J}_{\text{fib}}^{\text{T}}$ for the reverse propagation. Assume the round-trip Jones matrix of the sample is $\mathbf{J}_{\text{sam}}$, and the output arm is denoted as $\mathbf{B}$. The input polarization state at the fiber coupler is $\mathbf{s}$ and the detected polarization state is $\mathbf{t}$.

FIGURE 4.1: Schematic of a simplified fiber-based system. L1-6: lenses; M: mirror.

For two input states PS-OCT system with polarization diverse detection, the transmission equation is written as

$$\begin{bmatrix} |t_1\rangle & |t_2\rangle \end{bmatrix} = \mathbf{B} \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot \begin{bmatrix} |s_1\rangle & |s_2\rangle \end{bmatrix} \quad (4.1)$$

where the input states $|s_1\rangle = [s_{1x}, s_{1y}]^\top$ and $|s_2\rangle = [s_{2x}, s_{2y}]^\top$ form an input matrix, so do the output states. If single input state is employed, the equation can be accommodated by right-multiplying a matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$\begin{bmatrix} |t_1\rangle & |t_2\rangle \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \mathbf{B} \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot \begin{bmatrix} |s_1\rangle & |s_2\rangle \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (4.2)$$

In this case, only the first column of the input and output matrix is selected, $|s_2\rangle$ and $|t_2\rangle$ are set as zeros, hence can be ignored in the equation:

$$|t\rangle = \mathbf{B} \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot |s\rangle \quad (4.3)$$

With the conclusion we have drawn previously, the mirror state is the complex conjugate of the input polarization state $|s\rangle$, but rotated by the asymmetric part of the system $\mathbf{B}$:

$$|m\rangle = \mathbf{B} \cdot |s\rangle^* \quad (4.4)$$

If we use two orthogonal input states, but receive only one of the detection channels, it is equivalent to left multiplication of matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} |t_1\rangle & |t_2\rangle \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \mathbf{B} \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot \begin{bmatrix} |s_1\rangle & |s_2\rangle \end{bmatrix} \\ \begin{bmatrix} t_{1x} & t_{2x} \end{bmatrix} &= \begin{bmatrix} B_{1x} & B_{2x} \end{bmatrix} \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot \mathbf{S} \end{aligned} \quad (4.5)$$

where $\mathbf{S} = \begin{bmatrix} |s_1\rangle & |s_2\rangle \end{bmatrix}$ is the input Jones matrix. Taking transpose for both sides of the equation, we have

$$|t_x\rangle = \mathbf{S}^\top \cdot \mathbf{J}_{\text{fib}}^\top \cdot \mathbf{J}_{\text{sam}} \cdot \mathbf{J}_{\text{fib}} \cdot |B_{\text{row}1}\rangle \quad (4.6)$$

Where $|t_x\rangle = \begin{bmatrix} t_{1x} & t_{2x} \end{bmatrix}^\top$, $|B_{\text{row}1}\rangle = \begin{bmatrix} B_{11} & B_{12} \end{bmatrix}$. Here the system reciprocity still holds because all the system matrices are still unitary matrix, so the mirror state should exist under such condition. By comparing Eq. (4.3) and Eq. (4.6), the mirror state can be expressed as

$$|m'\rangle = \mathbf{S}^\top \cdot |B_{\text{row}1}\rangle^* \quad (4.7)$$

From Eq. (4.7) we know that the mirror state in this case is the conjugate state of the first row of the asymmetric rotation matrix of the system $\mathbf{B}$, but rotated by the transposed input Jones matrix $\mathbf{S}^\top$. We term the mirror state under such condition as the row mirror state.

There may be concerns that the mirror state in this case is difficult to locate. By definition (Eq. (4.7)) the Jones transmission matrix of the asymmetric part of the system needs to be measured which already seems to be a very cubersome or even impossible task. This is actually not a problem because in practice the mirror state is not determined through its definition equation. By averaging the acquired Stokes vectors of several cross sections of the sample, excluding points with low degree of polarization, the mirror state will be automatically revealed as the most concentrated point.

4.2 Experimental validation

For experimental validation of the existence of row mirror state, we used a polarization sensitive OFDI system as shown in Figure 4.2 [102]. The two input polarization states are multiplexed using a polarization delay unit (PDU). The ranging depth of the system is enough for depth-division multiplexing for different input polarization states, more importantly, unlike the time-division multiplexing method, this multiplexing implementation ensures phase stability between the two input polarization states. The two input polarization states, which are linearly polarized in the horizontal and vertical directions, respectively, happen to be orthogonal with each other in the Jones space ($\langle s_1 | s_2 \rangle = 0$), hence satisfies the condition that the input Jones matrix $\mathbf{S} = \begin{bmatrix} |s_1\rangle & |s_2\rangle \end{bmatrix}$ should be a unitary matrix (as illustrated in Eq. (4.6)) to ensure the existence of the row mirror state.

FIGURE 4.2: Schematic of the polarization sensitive optical frequency domain imaging system. A custom polygon scanning laser swept at 50 kHz across a 110 nm wavelength range, centered at $1.3 \mu\text{m}$. Two polarization states that are orthogonal in Jones space are generated and multiplexed to illuminate the sample simultaneously. The polarization states are carried to center about the beating frequencies ($2\Omega \pm \Omega$) modulated by three acoustic optical modulators, and hence are separated with each other in the depth direction in OCT image. AOM: acoustic optical modulator; BS: beam splitter; D1-2: dispersion compensator; M: mirror; Obj: objective lens; PBS: polarizing beam splitter; PC: polarization controller; PD: photodetector; Pol: linear polarizer.

We first placed a specular reflector at the focal point of the objective lens as the sample. A polarization controller with three paddles installed in the sample arm

was used to change the birefringence of the fiber. We recorded the changing polarization state caused by randomly moving the paddles of the polarization controller. The polarization-diverse balanced receiver is able to measure the full Jones matrix. For all the Jones matrix obtained over time, we extracted the first row and the first column to form two sets of Jones vectors and converted them into the corresponding Stokes representation, and then draw the normalized Stokes vectors on the Poincaré sphere, as shown in Figure 4.3. It can be clearly seen that for each case, the evolution of the polarization state passes a specific point multiple times, which ensures the existence of the row mirror state.

FIGURE 4.3: Polarization state evolution on the Poincaré sphere as a result of moving the three paddles of the polarization controller. (a) The polarization state of the first column of the detected Jones matrix. (b) The polarization state of the first row of the detected Jones matrix.

To further validate the phenomenon in a scene closer to its actual application, namely the variation of polarization state caused by the birefringence of the sample in which the light propagates axially, we imaged a phantom composed of three layers of birefringent scattering polycarbonate sheets fabricated following the steps of [92]. The three pieces of polycarbonate sheets were clamped together and submerged in water to reduce surface reflections. The acquired Stokes vectors of a single cross section are averaged by a Gaussian kernel, and normalized to unit length. One A-line was selected and the evolution of Stokes vectors along depth was plotted in Figure 4.4. For each layer, the detected Stokes vectors are involved

for a fitted evolving circle on the sphere plotted in purple. The three fitted evolving circles crossed the same point, both for row and column vectors.

FIGURE 4.4: Evolution of polarization states in a three-layer birefringence phantom. The three layers, from the top to bottom, are color-coded as blue, green and orange, respectively. (a) Polarization states converted from the first column of the detected Jones matrix. (b) Polarization states converted from the first row of the detected Jones matrix.

It can be noticed that the column mirror state and the launching polarization state (the start point of the layer with blue color, corresponding to the polarization state back-reflected from the surface of the sample) were not symmetrical about the QU -plane of the Poincaré sphere (as shown in the validation experiments of the column mirror state), this is because the OFDI system contains the asymmetric part, which is equivalent to applying a rigid rotation to the coordinate system of the entire Stokes space.

Then we imaged a two-layer phantom composed of a long band and four small bands on the top. The optic axis orientations of the two layers are set to be 45° with each other, and were cured in solidified resin doped with TiO_2 powder which was used as non-birefringent scatterers. The fiber of the system had not been moved since the last experiment, which means both the row and the column mirror states should remain the same. We recorded the cross-sectional scans of the phantom and resolved the local birefringence image with two input states as the ground truth (Figure 4.5(a)). Then we tried to recover the depth-resolved birefringence information of the sample either using only the first column or the

FIGURE 4.5: Local retardation imaging of birefringence phantom. (a-c) local retardation images retrieved using (a) two input states, (b) the first column of the detected Jones matrix with the mirror state and (c) the first row of the detected Jones matrix with the row mirror state. Scale bars in (c) measure $250\ \mu\text{m}$ (vertical) and $2\ \text{mm}$ (horizontal).

first row of the detected Jones matrix, with the knowledge of the corresponding mirror state located in the previous experiments (Figure 4.3 and Figure 4.4). The row Jones vectors, with the help of the row mirror state, offers almost the same result as the column mirror state, which is comparable to the image generated by the two input states ‘ground truth’, judging from the magnitude and uniformity of the recovered birefringence.

4.3 Conclusion

In this chapter, by taking the transpose of the whole Jones transmission equation, we found that the transposed version of the mirror state constraint also holds true, and the constraint point was termed as the row mirror state. We validated this phenomenon by fiber twisting experiment and phantom imaging with a frequency multiplexing PS-OFDI system. Therefore, detecting the depth-resolved birefringence information of the sample with two orthogonal polarization states and only one detection channel became possible. This simplification greatly reduced the cost and complexity of the system, especially for spectral domain PS-OCT, where one of the two spectrometers can be omitted.

Chapter 5

Mitigation of fringe attenuation in fiber-based OCT

In fiber-based OCT systems, the polarization state of the sample light is continuously changing because of the varying birefringence caused by the rotation and twisting of sample optical fiber. The polarization state mismatch between the reference and sample light causes attenuation of the interference fringes hence degrade the detection signal-to-noise ratio. By assuming the detected polarization state of the sample light to be completely random, we modeled the relation between the polarization fading and polarization state mismatch. Based on the phenomenon of polarization mirror state introduced in the previous chapters, it has been recognized that the polarization state of backscattered light randomly varied due to fiber twisting will not be evenly distributed on the Poincaré sphere, but tends to concentrate around the mirror state. We demonstrated the optimal SNR could be achieved if the polarization state of the reference light is carefully handled, and that led to a 3.5 dB SNR improvement compared to a system without any proper management.

5.1 Motivation

Flexible fiber-based probes, such as endoscopes, catheters and tethered capsules, have greatly broadened the clinical diagnosis possibilities of OCT by enabling three-dimensional high-resolution imaging inside the luminal structures of the human

body. To achieve a high degree of flexibility, the imaging probe is composed of a piece of single mode fiber (SMF) with the length of several meters, serving as the medium for the propagation of light to the area of interest. With the rotary joint or translation stage installed at the proximal end of the probe, circumferential or linear scanning movements can be achieved and conveyed to the distal end through the probe [103].

Light traveling in single mode fiber can be decomposed of two orthogonal polarization modes [72]. Due to the asymmetries of fiber geometry and stress, the two polarization modes of the light propagate at different phase velocities through the fiber, resulting in the birefringence effect of the SMF. As the OCT probe is continuously rotating and twisting, the polarization state of the backscattered light from the sample arm is randomly changing and unpredictable [104]. The reference arm usually remains stationary. The amplitude of the interference of light would decrease, when the polarization states of light from the sample arm and the reference arm do not match with each other. This fringe attenuation effect would result in signal-to-noise ratio degradation of the images acquired [105].

To compensate for the birefringence of the single mode fiber, Faraday rotator could be employed in bulk-optic based OCT system [106], but is not applicable for fiber-based probe due to the size restriction of the distal end of the probe. The attenuation can be completely eliminated by two methods. One solution is to use the polarization diverse detection (PDD), where the interference light is split by a polarizing beamsplitter into two orthogonal polarization directions and then received by two identical detectors. However, the polarization diverse detection greatly added up to the overall complexity and cost of the system. Especially for SD-OCT, the diverse detection requires two spectrometers, which demands the cumbersome work like camera synchronization and pixel alignment. The extra set of spectrometer brings huge cost pressure to the entire system.

A second solution is to use the common path design of the probe [107]. The reference light and sample light share the same propagation path in the single mode fiber until being split apart by a miniature beamsplitter located at the distal end of the probe. As they travel all the way through the same birefringent fiber, their polarization state is always aligned, eliminating the attenuation problem. The

common path probe requires advanced manufacturing techniques for the micro-optics, the probe design and post adjustment are also limited by the fixed reference arm.

In the validation experiment of the polarization mirror state, it can be noticed that the polarization state of light propagating a round trip through a reciprocal medium, whose birefringence is continuously and randomly changing, would not be uniformly distributed on the Poincar sphere, but tend to concentrate around the mirror state [83]. Inspired by the observation of this physical constraint, we realized that if the polarization state of reference light could be set to coincide with the mirror state, the polarization state of the two arms would have a higher probability to be aligned, hence mitigate the fringe attenuation problem.

In this study, we analyzed and modeled the intensity variation and average SNR degradation caused by the randomized sample light polarization state. By adjusting the polarization state of the reference light to the optimal position (coincide with the mirror state), a 1.5 times amplitude increase of the interference fringes can be expected, which is equivalent to 3.5 dB SNR improvement.

5.2 System modeling

Figure 5.1(a) shows the schematic of a fiber-based OCT system. Assuming that the light source is purely polarized and the system polarization mode dispersion (PMD) is negligible, we can write the transmission equation of the reference arm and the sample arm as $|t_r\rangle = \mathbf{J}_o \cdot \mathbf{J}_r^\top \cdot \mathbf{J}_r \cdot \mathbf{J}_i \cdot |s\rangle$, and $|t_s\rangle = \mathbf{J}_o \cdot \mathbf{J}_s^\top \cdot \mathbf{J}_t \cdot \mathbf{J}_s \cdot \mathbf{J}_i \cdot |s\rangle$, respectively, where the superscript $^\top$ denotes matrix transpose, $|s\rangle$, $|t_r\rangle$ and $|t_s\rangle$ are the Jones vectors of the input light, the detected reference light and sample light, respectively. $\mathbf{J}_i$, $\mathbf{J}_r$, $\mathbf{J}_s$ and $\mathbf{J}_o$ are the single-trip Jones transmission matrices of the fiber pieces of the fiber coupler, as illustrated in Figure 5.1(a). $\mathbf{J}_t$ is the double-pass Jones matrix of the tissue. The sample Jones matrix can be written as $\mathbf{J}_t = \int \chi(z) \cdot \left[\begin{bmatrix} e^{i2kn_o z} & 0 \\ 0 & e^{i2kn_e z} \end{bmatrix} \right]^\top dz$, where $\chi(z)$ is the reflectivity profile along depth z [108]. As we are analyzing a traditional OCT without polarization sensitive capability, the birefringence of the tissue can be ignored. Hence we can assume that the refractive indices of the ordinary beam and the extraordinary beam

are equal, $\mathbf{J}_t$ then becomes an identity matrix with a factor $F = \int \chi(z) e^{i2knz} dz$. We can express the recorded complex interference fringe $f(k)$ as

FIGURE 5.1: (a) Schematic of a typical fiber-based OCT system. PBS: polarizing beamsplitter; SP: spectrometer. (b) Stokes representation of polarization state mismatch on the Poincaré sphere. (c)-(e) Different fringe attenuation cases with different values of angle A , where A is the angle between the Stokes vectors of reference light t_r and sample $t_{s(m)}$ (sample (mirror)). F_x and F_y were simulated fringes obtained from two orthogonal polarization channels, $F_x + F_y$ was the simulated fringe when using only one detector.

$$f(k) = \langle t_r | t_s \rangle = (\mathbf{J}_o \cdot \mathbf{J}_r^\top \cdot \mathbf{J}_r \cdot \mathbf{J}_i \cdot |s\rangle)^\dagger \cdot (\mathbf{J}_o \cdot \mathbf{J}_s^\top \cdot (F(k) \cdot \mathbf{I}) \cdot \mathbf{J}_s \cdot \mathbf{J}_i \cdot |s\rangle) \quad (5.1)$$

$$= \langle t_r | t_{s(m)} \rangle \cdot F(k)$$

where $|t_{s(m)}\rangle$ is the Jones vector of light back-reflected from the sample arm when a specular reflector is used as the sample. The superscript † denotes the conjugate

transpose. $F(k)$ contains the information of the depth reflectivity profile which can be extracted by the inverse Fourier transform. $\langle t_r | t_{s(m)} \rangle$ is a complex scalar and can be written as $P \cdot \exp(i\psi)$, thus Eq. 5.1 can be written in the form of

$$f(k) = P \cdot \exp(i\psi) \cdot F(k) \quad (5.2)$$

From Eq. (5.2), the polarization mismatch between the reference and sample light exerts an amplitude attenuation coefficient P and a common phase delay ψ to the complex interference fringe. By converting the Jones vectors $|t_r\rangle$ and $|t_{s(m)}\rangle$ to the corresponding Stokes vectors $\mathbf{t}_r$, and $\mathbf{t}_{s(m)}$, we have

$$P = \cos(A/2) \quad (5.3)$$

where A is the angle between Stokes vectors $\mathbf{t}_r$ and $\mathbf{t}_{s(m)}$, illustrated in Figure 5.1(b). As it is intuitive to relate the amplitude attenuation coefficient P with the polarization state mismatch angle A in the Stokes space, we exhibit three special examples (Figure 5.1(b)-(e)) where the amplitude of the interference fringes is not attenuated (Figure 5.1(c)), half attenuated (Figure 5.1(d)) and completely attenuated (Figure 5.1(e)), corresponding to the mismatch angle set to be 0° , 120° and 180° .

In a practical system, $\mathbf{t}_r$ fixed in the Stokes space because the reference arm often remains motionless during the imaging process, while $\mathbf{t}_s$ is continuously moving due to the rotation and twisting of the sample arm fiber. The stationary $\mathbf{t}_r$ and changing $\mathbf{t}_s$ results in a randomly altering attenuation coefficient P throughout the scanning process. Statistically, the sensitivity degradation is related to the distribution of the attenuation coefficient.

To characterize the distribution of fringe attenuation, we used a beamsplitter based OCT system with polarization diverse detection illustrated in Figure 5.2(a). Light generated by the supercontinuum laser was turned to be circularly polarized after going through a polarizer (P1) and a quarter-wave plate (QWP). The input light was then split into the reference arm and sample arm and went through identical 1.5 m single mode fiber, reflected by a mirror, went through the fiber in the reverse direction, reunited at the beamsplitter and received by the polarization diverse detectors. A polarizer oriented at 45° was installed in the reference arm before

the single mode fiber, providing reference light with identical amplitude and phase for the two detection channels. A three paddle inline polarization controller (PC, Thorlabs, FPC030) was installed in the sample arm fiber to manually alter the birefringence of the fiber. The two orthogonal polarization channels were composed of two sets of identical transmission grating-based spectrometers, the detected interference fringes from denoted as $F_x(k)$ and $F_y(k)$. By directly summation of the fringes from the two channels, we obtained the single detected by a traditional OCT without polarization sensitive detection $F_{\text{OCT}}(k) = F_x(k) + F_y(k)$. The sensitivity degradation was quantified as the attenuation ratio between the peak amplitude corresponding to the sample mirror obtained by the inverse Fourier transform of $F_{\text{OCT}}(k)$ and the sum of individual detection channels $F_x(k)$ and $F_y(k)$.

FIGURE 5.2: (a) Schematic of the system used to measure the polarization state randomization after round-trip propagation through single mode fiber. BS: beamsplitter; P1-2: polarizers; PC: polarization controller; QWP: quarter-wave plate; PBS: polarizing beamsplitter; SP: spectrometer; L1-6: lenses; SMF: single mode fiber; M1-3: mirrors. (b) Measured fringe amplitude attenuation coefficient P against $\cos(A/2)$; (c) Histogram of the measured degree of polarization (DOP); (d) Measured differential group delay (DGD) distribution and corresponding Rayleigh probability density function (PDF) fitting.

One factor that must be considered is the polarization mode dispersion (PMD) of the fiber. Because the twisting of fiber would introduce wavelength-dependent

birefringence, which decreases the degree of polarization of light and influences the reliability of the measurement. By extracting the amplitude and phase information of $F_x(k)$ and $F_y(k)$ with Hilbert transform, we obtained the Jones vector with respect to the wavenumber $|t_{s(m)}(k)\rangle$, and were able to calculate the differential group delay (DGD) which characterized the magnitude of PMD [104].

Two hundred interference fringes were recorded while the paddles of polarization controller were randomly changing. We drew the relation between the measured attenuation P and the value $\cos(A/2)$, where A is the angle between Stokes vectors $\mathbf{t}_r$ and $\mathbf{t}_{s(m)}$. The points in the scatter plot are basically consistent with the theoretical relation we have derived in Eq. (5.3) (indicated by the red line in Figure 5.2(b)). But for some measurements, deviation from the theoretical line is noticeable. One possible explanation to the inconsistency is the polarization mode dispersion (PMD) of the twisted fiber. The PMD made the detected polarization state $|t_{s(m)}(k)\rangle$ to be wavelength-dependent, implying that the attenuation factor P is not consistent across the wavelength, which led to measurement error. The fiber PMD can be quantified by the degree of polarization (DOP) of the detected polarization state. The points in Figure 5.2(b) were color-coded by their degree of polarization, it can be noticed that higher DOP measurements located closer to the theoretical line.

We calculated the differential group delay (DGD) with the definition $\text{DGD} = 2\sqrt{\det(d(|t_s\rangle)/d\lambda)}$ ($\det$ denotes the matrix determinant) and plotted its distribution in Figure 5.2(d). Before that, the single mode fiber was modeled as pure retarder [11] and the Jones matrix $|t_s\rangle$ of selected wavelength was retrieved for each measured interference fringe. The distribution of DGD was tested to be a good fit ($p = 0.46$ for the chi-squared test) to a Rayleigh distribution, which is the theoretical probability density function of DGD [11]. In addition, the values of DOP of all measurements were counted and plotted as the histogram in Figure 5.2(c). As the degree of polarization was mostly close to unity, it is reasonable to neglect the PMD effect in the following analysis.

5.3 Simulation validation of the mitigation performance

As explained in the previous chapters and other reports [75][77], the polarization state of light propagating a round-trip through a reciprocal optical path whose birefringence is continuously changing, tends to pass the mirror state of the launching state in the Stokes space, which makes the polarization states not uniformly distributed on the Poincaré sphere, but concentrate around the mirror state. We calculated the Stokes vectors from the previously recorded fringes and plotted them on the Poincaré sphere (Figure 5.3(a)). Considering the existence of PMD, the Stokes vectors have been averaged among the vectors obtained across the wavelength domain. It is clear that the polarization states tend to concentrate.

Based on the existence of the mirror state, together with the relation between the fringe amplitude attenuation and angle A , it can be imagined that since the polarization state of the sample light cannot be controlled, we can set the polarization state of the reference light to be the mirror of the launching state, which maximizes the probability of alignment of polarization states of the reference and sample light. We noted this optimized reference polarization state as $\mathbf{t}_{r(\text{opt})}$.

FIGURE 5.3: (a) Normalized Stokes vectors of light after traveling a round-trip through the birefringence varying single mode fiber. The input polarization state was $\mathbf{s}$. The detected Stokes vectors distributed more around $\mathbf{t}_{r(\text{opt})}$; (b)-(d) The empirical probability density functions obtained from simulations drawn together with histograms of the measured attenuation coefficient P , with $A_{\text{ref}} = 0^\circ$, 90° and 180° , respectively. A_{ref} is the angle between the actual reference polarization state $\mathbf{t}_r$ and the optimal state $\mathbf{t}_{r(\text{opt})}$.

A simulation was conducted to validate the optimal reference polarization state. The transmission matrix of the single mode fiber was modeled as random unitary

Jones matrix based on [105] to simulate its movements. For the three different conditions of the simulation, the polarization state of the reference light was altered so that the angle between $\mathbf{t}_r$ and $\mathbf{t}_{r(\text{opt})}$, noted as A_{ref} , was 0° , 90° and 180° , respectively. For each case, the empirical probability density function of the fringe amplitude attenuation coefficient P was plotted with the orange curve in Figure 5.3(b)-(d). For the best case where $\mathbf{t}_r$ was set to align with $\mathbf{t}_{r(\text{opt})}$ (Figure 5.3(b)), the average signal-to-noise ratio is 3.97 dB higher than the worst case where $\mathbf{t}_r$ was set to be $-\mathbf{t}_{r(\text{opt})}$ (Figure 5.3(d)).

To validate the PDFs obtained from the simulation, we revisited the previously acquired two hundred fringes. The $\mathbf{t}_{r(\text{opt})}$ was measured to be $[-0.54, -0.23, -0.8]^\top$. In order to be consistent with the conditions of the simulations, we calculated the angles between the detected Stokes vectors and three polarization states of $[-0.54, -0.23, -0.8]^\top$ ($A_{\text{ref}} = 0^\circ$), $[0.58, 0.58, -0.56]^\top$ ($A_{\text{ref}} = 90^\circ$), and $[0.54, 0.23, 0.8]^\top$ ($A_{\text{ref}} = 180^\circ$). The histograms were plotted together with the empirical PDFs in Figure 5.3(b)-(d).

Sometimes unpolarized light source may be used to illuminate the OCT system. For example, the super-continuum laser shines unpolarized light due to its inherent random generation process. A polarized source like superluminescent diodes (SLD) may be depolarized after propagating through a long single mode fiber. To characterize unpolarized light, we model it as a summation of two cross-polarized components [106]. For a Jones vector $|s\rangle = [s_x, s_y]^\top$, its cross-polarization state is $[-s_y^*, s_x^*]^\top$, where superscript $*$ denotes complex conjugate. For the unpolarized light illumination case, Eq. (5.1) can be expressed as

$$\begin{aligned} f_{\text{unpol}}(k) &= \frac{\langle t_r | t_{s(m)} \rangle}{2} + \frac{\begin{bmatrix} -t_{r-y}^* \\ t_{r-x}^* \end{bmatrix} \begin{bmatrix} -t_{s(m)-y}^* \\ t_{s(m)-x}^* \end{bmatrix}}{2} \\ &= \frac{\langle t_r | t_{s(m)} \rangle}{2} + \frac{\langle t_{s(m)} | t_r \rangle}{2} \\ &= \text{Re} [\langle t_r | t_{s(m)} \rangle] \cdot F(k) \end{aligned} \tag{5.4}$$

where $\text{Re}[\cdot]$ denotes the real part of complex scalar, $f_{\text{unpol}}(k)$ is the detected complex fringes after background subtraction. A succinct form of the attenuation coefficient P cannot be found in this case. It can be noticed from $P = \text{Re} [\langle t_r | t_{s(m)} \rangle]$

that the coefficient is still related to the reference polarization state, which can be manipulated by twisting the reference fiber. But under unpolarized illumination, there is no obvious way to maximize P , whose distribution would always symmetric about a mean value of 0.5.

5.4 Experimental validation of the mitigation performance

To validate the improvement of the optimized reference polarization state, A beam splitter-based OCT system was constructed as illustrated in Figure 5.4(a). A super-continuum laser with a center wavelength of 800 nm was used as the light source. A linear polarizer (P1) installed at the input arm was used to transform the input light to be fully polarized. Another polarizer (P2) was used to control the polarization state reflected from the reference arm. To conveniently alter the polarization state of light, the light went through a free space beamsplitter and before being coupled into the single mode fiber. Like the previous experiment, a polarization controller installed on the sample arm fiber was used to be moved to simulate the probe scanning movement.

If the input light was set to be linearly polarized by polarizer P1, the optimal reference state $\mathbf{t}_{r(\text{opt})}$ would coincide with the input state $\mathbf{s}$. In this case, if the polarization axis of P2 is aligned with P1, the detected reference polarization state would be optimized. Under such configuration, 1000 fringes were acquired while randomly twisting the paddle position of the polarization controller.

For comparison, we conducted the experiment with another two configurations. We rotated polarizer P2 so that the angle between P1 and P2 was roughly 45° , meaning $A_{\text{ref}} = 90^\circ$. Lastly, the two polarizers were removed from the system to test the unpolarized source case. For all three cases, the detected intensity of light from the reference arm and sample arm were adjusted to remain constant. The distribution of the measured fringe amplitude attenuation coefficient was plotted in Figure 5.4(b). Place the polarization state of the reference light at $\mathbf{t}_{r(\text{opt})}$ led to optimized signal-to-noise ratio, which is 1.5 times as the worst case (OCT with an unpolarized light source), that is equivalent to 3.5 dB sensitivity improvement.

FIGURE 5.4: (a) Schematic of a fiber-based OCT with free space beamsplitter and single detection channel; (b) Histogram of Measured attenuation coefficient with different reference output polarization states settings and unpolarized light source. L1-4: lenses; PC: polarization controller; P1-2: polarizers.

5.5 Conclusion

In this work, we characterized the sensitivity degradation caused by polarization mismatch between the two arms in fiber-based OCT quantitatively. Based on the

observation that the polarization state of light propagating a round-trip through a reciprocal optical path is not uniformly distributed on the Poincaré sphere but concentrate to the mirror state, we brought up the solution that by presetting the polarization state of the reference light to the mirror state, an optimized signal-to-noise ratio could be achieved. Before this work, this problem is usually addressed by using polarization diverse detection, or common-path probe design, which are expensive and complicated solutions. The method we have suggested adds negligible burden to the system and offers immediate SNR improvement, and is recommended to be taken into account in the design of the fiber-based OCT system.

Chapter 6

Optical coherence tomography with gapped spectrum

The axial resolution of optical coherence tomography (OCT) is determined by the spectral shape and bandwidth of the detected light, which are limited by the gaps in the wavelength range of illumination, transmission, and detection. In this work, we demonstrate that the axial resolution deteriorated by gaps in OCT spectra can be restored by adopting the gapped amplitude and phase estimation (GAPES) method. GAPES estimates the missing parts between separated spectral bands and obtains the axial profile of tissue with reduced sidelobe artifacts compared to the gapped spectra and significantly improved axial resolution over the individual bands. This technique may make it possible to combine spectrally separated sources and detectors to improve axial resolution in OCT images.

6.1 Introduction

OCT can provide depth-resolved three-dimensional images by the use of coherence gating and confocal gating. In FD-OCT, the axial sample scattering profiles are obtained by inverse Fourier transform of interference spectral fringes. Therefore, the axial point-spread function is determined by the center wavelength and the spectral bandwidth of the low coherence light source. To obtain a PSF with high resolution and insignificant sidelobes, the spectral shape of the light source should be broad and approximately Gaussian shaped [109].

However, one cannot make full use of the near infrared biological imaging window (700–1700 nm), because there exist gaps between the water transmission windows and spectra of available light sources and detectors. Availability of light sources and detectors divides nowadays OCT devices into three wavelength categories: 800 nm, 1000 nm, and 1300 nm [110]. For example, neither Si-based cameras nor InGaAs-based cameras have acceptable response over 900–980 nm. To make things worse, detection efficiency within this spectral band is further degraded by the absorption peak of water at around 970 nm. An OCT system with gapped spectrum cannot provide high-quality images because inverse Fourier transform of spectra with ‘gaps’ would result in significant sidelobe artifacts which compromise the axial imaging performance.

Recently, several methods, rather than inverse Fourier transform, were proposed to retrieve the depth scattering profiles of the sample, such as the maximum entropy method [111], the algebraic reconstruction [112], and autoregressive spectral estimation methods [113, 114]. To make use of a gapped source spectrum, the straightforward idea is to apply a gapped data estimation method to retrieve the depth profiles.

Studies were conducted on the gapped data estimation problem, initially in the field of astronomy and radar, where continuous measurement was difficult because of environmental restrictions. Collected data was always alternated with gaps. Large artifacts would contaminate the spectrum if traditional direct spectral analysis was applied. Many of the data based algorithms, such as least-squares estimation [115], polynomial interpolation [116] and trigonometric polynomial interpolation [117], use the data changing tendency, or derivative, to estimate the missing data. However, these methods are applicable only when tiny gaps scattered throughout the data sequence, in other words, can only be applied to the ‘unevenly sampled data’ case. In our case where there exists one big gap, the spectrum-based estimation algorithms would give more satisfying results.

To address ‘big gaps’ in a data sequence, Fahlman and Ulrych iteratively used the maximum entropy method to predict the missing content [118]. Brown et al. proposed a modification of Fahlman and Ulrych’s method where the data was bandpass filtered before autoregressive (AR) model fitting process. Also, an over-sampled version of the time series was generated aiming to solve the insufficient AR coefficients problem [119]. Broersen et al. presented an approximate maximum

likelihood method which made use of only a limited part of previous data to estimate the likelihood and was usable even if less than 10% of the data was available [120].

Most of the methods are parametric methods based on the assumption that the process through which the data is acquired satisfies a certain parametric model. However, in terms of the OCT fringe data, it is difficult to determine which model, if there is one at all, can be fitted in. Stoica et al. proposed a non-parametric method, termed gapped data amplitude and phase estimation (GAPES) [121]. Without any presuppositions of the data acquiring process, GAPES can be applied in cases where a specific model cannot be derived. In this paper, we present to use the GAPES to estimate the missing part between spectral bands in OCT spectral data and produce images with reduced artifacts and improved axial resolution.

6.2 Theory

GAPES uses available interferogram to estimate the missing part, thus obtains a non-gapped spectrum. With this gap-filled interferogram, an A-line profile along the axial direction of OCT images with little sidelobes can be retrieved. Amplitude and phase estimation (APES) algorithm, instead of inverse FFT, is used to perform the spectral estimation of non-gapped spectra [122]. To treat the spectral gaps, GAPES iteratively finds an estimation of the missing data in the gaps by inversely solving the APES result to obtain the estimated time domain data through least-square optimization. The general procedure of the algorithm is shown in Fig. 6.1.

The recorded raw interferograms must go through the regular pre-processing process: linearly k -space resampling and dispersion calibration. Before applying any gap estimation method, we reshape the fringe data sequence to be uniform in amplitude across the whole spectrum, to fulfill the condition of a wide-sense stationary random sequence. The reshaping can be implemented through pixel-by-pixel dividing the fringe to the source spectrogram. This procedure produces a stationary sequence if the system is shot-noise limited. To avoid introducing too much noise with the spectral reshaping, spectrum at the edges with low power should be trimmed off for light source with limited bandwidth.

FIGURE 6.1: The gapped-spectrum amplitude and phase estimation algorithm flowchart.

If the obtained interferograms $s(k)$, $(k = 0, 1, \dots, N-1)$ does not contain any gaps, APES algorithm can be used to estimate the depth profile $\gamma(z)$. To implement APES algorithm, we define an overlapping data matrix $t(k)$ based on the original interferograms $s(k)$ as follows

$$t(k) = \begin{bmatrix} s(k) \\ \vdots \\ s(k+M-1) \end{bmatrix}, (k = 0, 1, \dots, L-1) \quad (6.1)$$

where $L = N - M + 1$, each row of $t(k)$ is a segment of $s(k)$ with a length of M , and adjacent elements $t(k)$ and $t(k+1)$ have $M-1$ overlapping elements. It was proved in ref [122] that a data sequence collected from a stationary stochastic process can be modeled as $\mathbf{h}^*(z)t(k) = \gamma(z)e^{ikz} + res$, where $\mathbf{h}(z)$ was the impulse response of an M-tap FIR filter, $\gamma(z)$ was the depth profile we want to resolve, and

res was the residual error. The estimation of the filter bank $\hat{\mathbf{h}}(z)$ and the depth profile $\hat{\gamma}(z)$ can be obtained by minimizing res , expressed as

$$\min_{\gamma(z), \mathbf{h}(z)} \sum_{k=0}^{L-1} |\mathbf{h}^*(z) t(k) - \gamma(z) e^{ikz}|^2 \quad (6.2)$$

To deal with interferograms with gaps, APES algorithm is adopted iteratively to estimate the missing part in the gaps. The first step is to give an initial guess of the depth profile and the missing data. Assuming that the missing part of the data generates the same depth profile content as the available part, an initial spectral estimation, noted as $\{\hat{\mathbf{h}}(z_i), \hat{\gamma}(z_i)\}$ can be obtained by directly applying APES to the available data, with the missing part set as zero. Then reversely, with the initial guess of the depth profile, an estimation of whole interferogram s_u without gaps can be obtained by solving a least square problem

$$\min_{s_u} \sum_{i=0}^{Z-1} \sum_{k=0}^{L-1} |\mathbf{h}^*(z_i) t(k) - \gamma(z_i) e^{ikz_i}|^2 \quad (6.3)$$

In this case, the interferogram s_u is the variable to be determined, while minimizing the residual error res . An updated sequence s_u without gaps can be obtained from this step, as a substitution of the original gapped interferogram. Then, by applying APES to this updated gap-filled interferogram s_u , an updated depth profile $\{\hat{\mathbf{h}}(z_i), \hat{\gamma}(z_i)\}$ can be obtained. With this updated depth profile, we repeated updating the interferogram estimation s_u and vice versa, until the iteration converged to the final estimation. The iteration ended when either of the two conditions is met: (1) the change of the criterion $\sum_{i=0}^{Z-1} \sum_{k=0}^{L-1} |\mathbf{h}^*(z_i) t(k) - \gamma(z_i) e^{ikz_i}|^2$ between current iteration and the previous one is smaller than a preset small value; (2) a preset iteration number has been reached.

In experiments, the computation load of the algorithm increased exponentially with the data sequence length. When the data length exceeded 1000 points, the consuming time became unacceptable. So we devised a speed-up method that divided the data sequence into 8 segments in the spatial domain and applied the algorithm individually for each short segment, followed by fusing the spectrum segments into a whole depth profile.

FIGURE 6.2: The segmentation of a data sequence (a) Adjacent bandpass filters; (b) Original gapped data sequence; (c) Spectrum of filtered data; (d) Filtered data segments; (e) Spectrally shifted and downsampled data segments.

To segment the data sequence, we filtered the data with eight adjacent bandpass filters equally distributed along the spatial domain. These finite real filters (Fig. 6.2(a)) were designed to span over half of the entire spatial domain since real data sequences, such as the interference fringes, have symmetric spectral content. To illuminate the process, an A-scan fringe data sequence was shown in Fig. 6.2(b) as an example, and its spatial domain transformation processed by the filters was shown in Fig. 6.2(c). The resultant 8 spatial domain segments were then aligned to the lowest spatial band by multiplying a frequency shifting factor $\left[\exp \left(-j \frac{2\pi(m-1)n}{8N} \right) \right]$ (where $m = 1, 2, \dots, 8$ was the index of filter, n was the index of data sequence and N was the data length). After the shifting, since the spectral information of each shifted spatial segment was constricted in the normalized spatial range $0 \sim 1/8$, we could downsample each segment with a factor of 8 without

FIGURE 6.3: Schematic of the OCT system. BS, beam splitter; GS, galvo scanners; L1-8: lenses; PH, pinhole; RM, reference mirror; SLD: superluminescent diode source; SMF, single mode fiber; TG, transmission grating; PC, personal computer; IMAQ, image acquisition board.

alias (shown in Fig. 6.2(e)). In summary, we segmented a data sequence into 8 segments each with $1/8$ of the original length.

After the segmentation, GAPES algorithm were applied to each filtered and down-sampled interferogram to estimate the non-gapped depth profile. Then we shifted the depth profile segments back to their original position in the spatial domain to form the full depth profile. The segmentation dramatically improved the processing speed of GAPES. In the case of a 1024 points interferogram, the processing speed was improved by a factor of 350 (running time reduced from 25 800 s to 72 s).

6.3 Experiments and results

To verify the efficacy of GAPES method for gapped OCT data restoration, we developed a home-made spectral domain OCT system. A stack of glass slides was imaged to characterize the axial PSF estimation over the entire imaging depth. Beads solution was imaged to demonstrate the method on spherical scatters. To compare the biological tissue images obtained by the gapped spectrum and the full spectrum, we imaged fresh rat cornea with full spectrum and numerically created gapped spectrum.

As shown in Fig. 6.3, a free-space spectral domain OCT setup based on Michelson interferometer was constructed. A superluminescent diode array (SLD, model: Superlum T-850-HP) with a center wavelength of 850 nm and a total bandwidth of 165 nm was used as the illumination source. A 10:90 non-polarizing beam splitter (BS029, Thorlabs Inc.) divided the beam into reference and sample arms. The two beams were focused into diffraction-limited spots by two identical focusing lenses (L4/L5: Mitutoyo Plan Apo NIR, 20 $\times$, NA 0.4). In the sample arm, two galvo mirrors (GVSM002/M, Thorlabs Inc.) were used to scan the spot in orthogonal lateral directions. The backscattered light from the sample and reflected light from the reference mirror was recombined at the beamsplitter, and guided to a home-made spectrometer through a single mode fiber (P3-780A-FC-2, Thorlabs Inc.) and a 50:50 beam splitter (BS014, Thorlabs Inc.). A 10 μ m pinhole (P10S, Thorlabs Inc.) was used to reject stray light from entering the spectrometer. After being dispersed by a transmission grating (1200 lines/mm, 840 nm, Wasatch Photonics Inc.) and focused by a camera lens (Nikon AF-S 85 mm f/1.8D), the beam was directed to the line-scan CCD (E2V, AViiVA EM4) with 2048 pixels operating at an acquisition speed of 20k A-lines per second.

The superluminescent diode array provided a spectrum with a 3dB bandwidth from 760 nm to 925 nm. To create a gap in the spectrum, we imaged the samples with full spectrum and set middle 1/3 part of acquired spectral fringes to zeros, obtaining an illuminating spectrum with two discontinued sub-bands: 760 nm–815 nm and 870 nm–925 nm.

To test the sensitivity ‘roll off’ and imaging depth of the gapped spectrum OCT, we imaged a glass slide stack (Fig. 6.4(a)), which was formed by stacking multiple glass slides together (thickness of each slice was $\sim$ 0.2 mm). As shown in Fig. 6.4, the sensitivity ‘roll off’ and the imaging depth was not affected by the GAPES algorithm. Comparing the full spectrum FFT depth profile (Fig. 6.4(b)) and the GAPES depth profile (Fig. 6.4(d)) it can be confirmed that the layer structure in GAPES is a representation of real sample structure. Besides, we could see significant resolution improvement and sidelobe artifacts suppression provided by the GAPES algorithm compared to the FFT result (Fig. 6.4(c)).

To demonstrate the performance of this method on spherical scatters which resemble biological cells, polystyrene calibration particles (80177, Fluka, diameter 2 μ m) water solution was imaged (Fig. 6.5) with the full spectrum and a gapped spectrum

FIGURE 6.4: Aline profiles of the glass slide stack image. (a) One frame retrieved by FFT using full spectrum light source indicating the phantom structure. The location of depth profile shown in (b-d) was indicated by blue vertical line. AC: autocorrelation artifacts. Scale bar, horizontal: 50 μm ; vertical: 50 μm . (b) FFT with full spectrum. (c) FFT with gapped spectrum. (d) GAPES with the gapped spectrum.

with the middle 1/3 spectral data set to zeros (815 nm to 870 nm in a full spectral range from 760 nm to 925 nm). Figs. 6.5(a)–6.5(c) show images obtained from the Gaussian reshaped full spectrum (Fig. 6.5(d)) using conventional FFT, the Gaussian reshaped gapped spectrum using FFT (Fig. 6.5(e)), and the uniformly reshaped gapped spectrum (Fig. 6.5(f)) using GAPES, respectively. Fig. 6.5(g) reveals the depth profile at the location indicated by color-coded vertical lines in Figs. 6.5(a)–6.5(c). A gap in the middle of the spectrum introduced sidelobe artifacts in axial PSF, totally undermining the image quality. However, by adopting GAPES spectral estimation to retrieve the depth profile, the artifacts were suppressed by ~ 11 dB according to the signal peaks corresponding to the water surface and beads, thereby providing image quality comparable to that of the full spectrum.

To test gapped spectrum OCT in biological tissues, we demonstrated *ex vivo* imaging on fresh rat cornea. Rat eye balls were harvested from a healthy adult female Sprague Dawley rat immediately after the disappearance of vital signs. We imaged

FIGURE 6.5: Images of polystyrene calibration particles in water. (a) Full spectrum image retrieved by FFT; (b) Gapped spectrum image retrieved by FFT. (c) Gapped spectrum image retrieved by GAPES. Scale bar, horizontal: $30\ \mu\text{m}$; vertical: $50\ \mu\text{m}$. (d-f) Gaussian reshaped full spectrum(d), Gaussian reshaped gapped spectrum(e), and uniformly reshaped gapped spectrum(f) from the same A-scan after background subtraction, respectively. (g) Depth profiles at the locations indicated by the red (a), brown (b) and blue (c) vertical lines in the intensity images (a)-(c). For the peaks corresponding to water surface and beads, GAPES achieves ~ 11 dB sidelobe suppression comparing to direct FFT of the gapped spectrum.

FIGURE 6.6: Images of the rat cornea *ex vivo*. (a) Full spectrum image retrieved by FFT, (b) gapped spectrum image retrieved by FFT, and (c) gapped spectrum image retrieved by GAPES. DM: Descemet's membrane; EP: Epithelium; ED: Endothelium; S: Stroma. Scale bar, horizontal: 200 μm ; vertical: 40 μm .

the sample with full spectrum. To simulate a gapped spectrum, we set middle 1/3 part of acquired spectral fringes to zeros. Figs. 6.6(a)–6.6(c) show the images obtained from full spectrum using conventional FFT, gapped spectrum using FFT, and gapped spectrum using GAPES, respectively. The sidelobe artifacts introduced by the spectral gap were well suppressed, and the image quality was improved in the images provided by use of GAPES (Fig. 6.6(c)) compared with the gapped FFT image (Fig. 6.6(b)).

6.4 Discussion and conclusion

The length of the gap significantly influenced the performance of GAPES. We tested this by setting the gap lengths as 20%, 40% and 60% of the total data length, and found that the performance of GAPES decreased as the gap length increased. Larger gaps would result in more artifacts, where the speckle pattern seems like ‘snakes’, degrading the axial resolution and image quality. If the missing part exceeds half of the total data length, matrices in the iteration process will degenerate closely to singular matrices, making the computation of their inversion inaccurate. We tried a 70% gap length but failed because the solution collapsed to an invertible matrix. Empirically, we expected that the performance of GAPES is acceptable with a gap length no larger than 40% of the total sequence length.

Currently, there are still limitations that may hinder the practical application of this algorithm. One is the high computation load due to the large amount of

matrix multiplication, inversion calculation, and iterative procedures adopted by the algorithm. We had adopted an additional digital filtering and recombination procedure to speed up the computation to an acceptable level. To estimate a single A-line profile from 1024 points gapped spectrum, the computing time was about 72 seconds with six iterations running on a Xeon E5 processor computer. However, it was still not fast enough for practical clinical applications.

Another limitation is that the algorithm may introduce additional artifacts to the image. These artifacts turned the axial PSF to thinner peaks, thereby changing the speckle pattern to random ‘snake noise’ pattern, sometimes seriously blurring the image. These artifacts were reported before in the attempts of improving OCT axial resolution without broadening the light source, such as deconvolution [123] and spectral estimation [114]. We believe these artifacts are the result of the assumption that the undetectable fringes (spectral gaps or outside the source window) have the same spectral content with the known fringe data. Since OCT forms a band pass image of the sample in the axial direction [124], this assumption only stands true when the sample is made of Dirac delta functions, whose spectral content are evenly distributed in the frequency domain. Based on this assumption, the GAPES algorithm tends to turn the PSF into thinner peaks like the Dirac delta function and introduces ‘snake noise’ pattern into the image.

To conclude, we present, for the first time to our knowledge, how to retrieve OCT image from a gapped spectrum illuminating light source with significantly suppressed sidelobe artifacts and effective restored axial resolution using GAPES algorithm. In the OCT device, the critical component is the broadband laser source, which rolls up the cost of the whole device [125]. This method may make it possible to improve axial resolution using light sources with gapped spectrum, which is of lower cost than those of supercontinuum sources [126, 127]. What’s more, this method provides a possibility to combine OCT devices in different optical bands, further improving the axial resolution.

Chapter 7

Conclusion and recommendations

7.1 Conclusion

The main purpose of this thesis is to reduce the cost and complexity of OCT system through theoretical and technological innovations.

The discovery of the polarization mirror state makes it possible to resolve depth-resolved birefringence information with only one input polarization state. This not only simplifies the system, but also provides performance improvement in other aspects over the existing method: (1) comparing to EOM-based system, the single input state system method can potentially achieve higher image speed and higher axial resolution, because the bandwidth is no longer limited by the EOM; (2) comparing with the PDU-based system, the single input state system can use the full image range along the depth direction.

More importantly, by taking transpose for both sides of the Jones transmission matrix, we found the transposed version of the constraint is also valid. This makes it possible to resolve local birefringence information using two multiplexed input states and only one detection channel, which further reduced the system cost. Because a set of detection channel requires extra photodetectors, signal acquisition board, and computational resources, which is often the barrier to adding polarization sensitive functionality to the existing system.

Inspired by the polarization state concentration phenomenon observed in the mirror state experiments, we demonstrated the application of the mirror state constraint

in the fiber-based OCT system. By presetting the detected reference polarization state to the mirror state of the input light, a 3.5dB signal-to-noise ratio improvement can be statistically achieved. Before this reference light management method, this issue has either been left unsolved or solved only through costly means. The proposed approach does not completely solve the problem, but made improvement with nearly no extra cost.

The gapped spectrum estimation method offers the possibility to improve the axial resolution by combining two separate spectral bands together. From another perspective, as the light source and detectors with broad spectral band is expensive, combining the source and detection from two sets of cheaper components is now a possible solution which reduced the overall system cost.

7.2 Recommendations for future research

For the polarization state constraint, we only used our single input state system to demonstrate the existence of mirror state and validated its possibility in calculation of local birefringence information. More work should be done to quantitatively comparing the local birefringence of real tissues recovered by prevailing two input states system and the mirror state method, hence validate its performance in practical clinical applications. To avoid the mirror point artifacts, it's better to characterize the performance of artifact mitigation using spectral binning algorithm, with respect to the amount of PMD introduced by the sample arm fiber. In the future, exploring the capability of solving optics axis orientation with the mirror state constraint would be another important work.

We have experimentally validated the row mirror state with a frequency multiplexing PS-OFDI system, where the 'ground truth' of local birefringence is accessible. To further demonstrate the performance of single detection channel polarization sensitive system, it is best to be implemented on a spectral domain PS-OCT system. Because saving a set of spectrometer would potentially make a higher impact to the industry.

The estimation method of gapped spectrum offers the opportunity to break the limits of current available spectral range of OCT system and achieve higher axial resolution, but has some major limitations. (1) Complex matrix operations

and multiple iterations make the algorithm time-consuming and difficult to implement real-time or large-scale data processing; (2) The estimation based on the assumption that the detected signal should have the same spectral content for every wavelength. But in practice, the assumption does not guaranteed to be met in biological tissues. In the experiment, we noticed that the method have better restoration performance for specular surfaces than turbid media. This limits the clinical application of the method.

To summarize, this thesis focused mainly on advancing technical capabilities of OCT. In the field of biomedical imaging, more translational and clinical studies are needed. For the future, we need to investigate how these discoveries and techniques can improve practical diagnostic devices, and how they can better help to diagnose the diseases.

Author's Publications

Journal Articles

- **Nanshuo Wang**[†], Xinyu Liu[†], Qiaozhou Xiong, Jun Xie, Shi Chen, and Linbo Liu, “Polarization management to mitigate misalignment-induced fringe fading in fiber-based optical coherence tomography,” *Optics Letters*, 42, 2996-2999 (2017). ([†] These authors contributed equally).
- Qiaozhou Xiong[†], **Nanshuo Wang**[†], Xinyu Liu, Si Chen, Haitao Liang, Shufen Chen, Linbo Liu, “Single input state polarization-sensitive optical coherence tomography with high resolution and polarization distortion correction.” *Optics Express*, 27, 6910-6924 (2019). ([†] These authors contributed equally).
- Qiaozhou Xiong[†], **Nanshuo Wang**[†], Xinyu Liu, Si Chen, Cilywn S. Branganza, Brett E. Bouma, Linbo Liu[‡], Martin Villiger[‡], “Polarization constraints in reciprocal unitary backscattering,” Submitted to *Physics Review Letters*. ([†], [‡] These authors contributed equally).
- **Nanshuo Wang**, Xinyu Liu, Xiaojun Yu, Si Chen, Shi Chen, and Linbo Liu, “Optical coherence tomography with gapped spectrum,” *IEEE Photonics Journal*, doi: 10.1109/JPHOT.2019.2897607.
- Si Chen[†], Xinyu Liu[†], **Nanshuo Wang**, Xianghong Wang, Qiaozhou Xiong, En Bo, Xiaojun Yu, Shufen Chen, and Linbo Liu, “Visualizing Micro-anatomical Structures of the Posterior Cornea with Micro-optical Coherence Tomography,” *Scientific Reports*. 7:10752 (2017). ([†] These authors contributed equally)
- Jinhan Li, Yuemei Luo, Xianghong Wang, **Nanshuo Wang**, En Bo, Si Chen, Shufen Chen, Shi Chen, Meng-Tsan Tsai, and Linbo Liu. “Extending the

depth of focus of fiber-optic optical coherence tomography using a chromatic dual-focus design,” *Applied Optics*, 57.21 (2018): 6040-6046.

- Yuemei Luo, Dongyao Cui, Xiaojun Yu, En Bo, Xianghong Wang, **Nanshuo Wang**, Cilwyn Shalitha Braganza, Shufen Chen, Xinyu Liu, Qiaozhou Xiong, Si Chen, Shi Chen, and Linbo Liu, “Endomicroscopic optical coherence tomography for cellular resolution imaging of gastrointestinal tracts,” *Journal of Biophotonics*, 11(4), p.e201700141(2018).
- En Bo, Yuemei Luo, Si Chen, Xinyu Liu, **Nanshuo Wang**, Xin Ge, Xianghong Wang, Shufen Chen, Shi Chen, Jinhan Li, and Linbo Liu, “Depth of focus extension in optical coherence tomography via multiple aperture synthesis,” *Optica* 4(7), 701-706 (2017).
- Xianghong Wang, Xiaojun Yu, Xinyu Liu, Si Chen, Shufen Chen, **Nanshuo Wang**, and Linbo Liu, “A two-step iteration mechanism for speckle reduction in optical coherence tomography,” *Biomedical Signal Processing and Control*, 43 (2018): 86-95.

Conference Proceedings

- **Nanshuo Wang**, Xinyu Liu, Qiaozhou Xiong, Linbo Liu, “Polarization management to mitigate misalignment induced fringe fading in fiber-based optical coherence tomography,” in *SPIE BIOS 2018*. San Francisco, CA, USA. Oral Presentation.
- **Nanshuo Wang**, Xinyu Liu, Xiaojun Yu, Si Chen, En Bo, Yuemei Luo, Dongyao Cui, Linbo Liu, “Optical coherence tomography with gapped spectrum,” in *SPIE BIOS 2016*. San Francisco, CA, USA. Poster Presentation.
- **Nanshuo Wang**, Xinyu Liu, Qiaozhou Xiong, Linbo Liu, “Local Retardance Determination Using Single Input Polarization Sensitive Optical Coherence Tomography,” in *CLEO-PR, OECC and PGC, 2017*. Singapore. Poster Presentation.
- **Nanshuo Wang**, Xinyu Liu, Xiaojun Yu, Si Chen, Shi Chen, Linbo Liu, “Optical Coherence Tomography with Gapped Spectrum,” in *CLEO-PR, OECC and PGC, 2017*. Singapore. Poster Presentation.

- **Nanshuo Wang**, Xinyu Liu, Xiaojun Yu, Si Chen, Shi Chen, Linbo Liu, “Adaptive filter technique to restore images of SDOCT with high resolution,” in *Photonics Global Conference 2015*. Singapore. Oral Presentation.

Patent and technical disclosure

- Xinyu Liu, **Nanshuo Wang**, Qiaozhou Xiong, Linbo Liu, Martin Villiger, Brett Bouma, “Method And Apparatus For Performing Depth Resolved Tissue Local Birefringence Properties Measurement Using Single Shot Polarization Sensitive Optical Coherence Tomography,” US provisional application 62/615,473 filed on 10 Jan 2018.
- Linbo Liu, Atsushi Tanaka, **Nanshuo Wang**, “Color Optical Coherence Tomography,” Reference number: TD/275/16.

Bibliography

- [1] David Huang, Eric A Swanson, Charles P Lin, Joel S Schuman, William G Stinson, Warren Chang, Michael R Hee, Thomas Flotte, Kenton Gregory, Carmen A Puliafito, and James G Fujimoto. Optical coherence tomography. *Science*, 254(5035):1178–1181, 1991. [1](#)
- [2] Zhao Wang, Hsiang-Chieh Lee, Diedrik Vermeulen, Long Chen, Torben Nielsen, Seo Yeon Park, Allan Ghaemi, Eric Swanson, Chris Doerr, and James Fujimoto. Silicon photonic integrated circuit swept-source optical coherence tomography receiver with dual polarization, dual balanced, in-phase and quadrature detection. *Biomedical optics express*, 6(7):2562–2574, 2015. [1](#)
- [3] Paritosh Pande, Ryan L Shelton, Guillermo L Monroy, Ryan M Nolan, and Stephen A Boppart. Low-cost hand-held probe for depth-resolved low-coherence interferometry. *Biomedical optics express*, 8(1):338–348, 2017. [1](#)
- [4] Sanghoon Kim, Michael Crose, Will J Eldridge, Brian Cox, William J Brown, and Adam Wax. Design and implementation of a low-cost, portable oct system. *Biomedical optics express*, 9(3):1232–1243, 2018. [1](#)
- [5] JG Fujimoto, Stephen A Boppart, GJ Tearney, BE Bouma, C Pitris, and ME Brezinski. High resolution in vivo intra-arterial imaging with optical coherence tomography. *Heart*, 82(2):128–133, 1999. [5](#)
- [6] Teruyoshi Kume, Takashi Akasaka, Takahiro Kawamoto, Nozomi Watanabe, Eiji Toyota, Yoji Neishi, Renan Sukmawan, Yoshito Sadahira, and Kiyoshi Yoshida. Assessment of coronary arterial plaque by optical coherence tomography. *The American journal of cardiology*, 97(8):1172–1175, 2006. [5](#)

- [7] N Hanna, D Saltzman, D Mukai, Z Chen, S Sasse, J Milliken, S Guo, W Jung, H Colt, and M Brenner. Two-dimensional and 3-dimensional optical coherence tomographic imaging of the airway, lung, and pleura. *The Journal of thoracic and cardiovascular surgery*, 129(3):615–622, 2005. [5](#)
- [8] Julian J Armstrong, Matthew S Leigh, Ian D Walton, Andrei V Zvyagin, Sergey A Alexandrov, Stefan Schwer, David D Sampson, David R Hillman, and Peter R Eastwood. In vivo size and shape measurement of the human upper airway using endoscopic long-range optical coherence tomography. *Optics express*, 11(15):1817–1826, 2003.
- [9] Suzanne C Whiteman, Ying Yang, Daniel Gey van Pittius, Mark Stephens, Jitendra Parmer, and Monica A Spiteri. Optical coherence tomography: real-time imaging of bronchial airways microstructure and detection of inflammatory/neoplastic morphologic changes. *Clinical Cancer Research*, 12(3):813–818, 2006.
- [10] Harvey O Coxson, Brendan Quiney, Don D Sin, Li Xing, Annette M McWilliams, John R Mayo, and Stephen Lam. Airway wall thickness assessed using computed tomography and optical coherence tomography. *American journal of respiratory and critical care medicine*, 177(11):1201–1206, 2008. [5](#)
- [11] Kaicheng Liang, Osman O Ahsen, Hsiang-Chieh Lee, Zhao Wang, Benjamin M Potsaid, Marisa Figueiredo, Vijaysekhar Jayaraman, Alex E Cable, Qin Huang, Hiroshi Mashimo, et al. Volumetric mapping of barretts esophagus and dysplasia with en face optical coherence tomography tethered capsule. *The American journal of gastroenterology*, 111(11):1664, 2016. [5](#)
- [12] Michalina J Gora, Jenny S Sauk, Robert W Carruth, Kevin A Gallagher, Melissa J Suter, Norman S Nishioka, Lauren E Kava, Mireille Rosenberg, Brett E Bouma, and Guillermo J Tearney. Tethered capsule endomicroscopy enables less invasive imaging of gastrointestinal tract microstructure. *Nature medicine*, 19(2):238, 2013. [5](#)
- [13] Mieke TJ Bus, Berrend G Muller, Daniel M de Bruin, Dirk J Faber, Guido M Kamphuis, Ton G van Leeuwen, Theo M de Reijke, and Jean JMCH de la Rosette. Volumetric in vivo visualization of upper urinary tract tumors using optical coherence tomography: a pilot study. *The Journal of urology*, 190(6):2236–2242, 2013. [5](#)

- [14] MTJ Bus, DM de Bruin, DJ Faber, GM Kamphuis, PJ Zondervan, MP Laguna-Pes, TG van Leeuwen, Th M de Reijke, and JJMCH de la Rosette. Optical coherence tomography as a tool for in vivo staging and grading of upper urinary tract urothelial carcinoma: a study of diagnostic accuracy. *The Journal of urology*, 196(6):1749–1755, 2016. [5](#)
- [15] Alexandre R Tumlinson, Boris Považay, Lida P Hariri, James B McNally, Angelika Unterhuber, Boris M Hermann, Harald Sattmann, Wolfgang Drexler, and Jennifer Kehlet Barton. In vivo ultrahigh-resolution optical coherence tomography of mouse colon with an achromatized endoscope. *Journal of biomedical optics*, 11(6):064003, 2006. [5](#)
- [16] Desmond C Adler, Chao Zhou, Tsung-Han Tsai, Joe Schmitt, Qin Huang, Hiroshi Mashimo, and James G Fujimoto. Three-dimensional endomicroscopy of the human colon using optical coherence tomography. *Optics Express*, 17(2):784–796, 2009. [5](#)
- [17] Hiroshi Yabushita, Brett E Bouma, Stuart L Houser, H Thomas Aretz, Ik-Kyung Jang, Kelly H Schlendorf, Christopher R Kauffman, Milen Shishkov, Dong-Heon Kang, Elkan F Halpern, et al. Characterization of human atherosclerosis by optical coherence tomography. *Circulation*, 106(13):1640–1645, 2002. [6](#)
- [18] Eveline Regar, JA Schaar, E Mont, Renu Virmani, and PW Serruys. Optical coherence tomography. *Cardiovascular radiation medicine*, 4(4):198–204, 2003.
- [19] Freek J van der Meer, Dirk J Faber, DM Baraznji Sassoon, Maurice C Aalders, Gerard Pasterkamp, and Ton G van Leeuwen. Localized measurement of optical attenuation coefficients of atherosclerotic plaque constituents by quantitative optical coherence tomography. *IEEE transactions on medical imaging*, 24(10):1369–1376, 2005.
- [20] Mark E Brezinski, Guillermo J Tearney, NJ Weissman, SA Boppart, BE Bouma, MR Hee, AE Weyman, EA Swanson, JF Southern, and JG Fujimoto. Assessing atherosclerotic plaque morphology: comparison of optical coherence tomography and high frequency intravascular ultrasound. *Heart*, 77(5):397–403, 1997.

- [21] Ik-Kyung Jang, Brett E Bouma, Dong-Heon Kang, Seung-Jung Park, Seong-Wook Park, Ki-Bae Seung, Kyu-Bo Choi, Milen Shishkov, Kelly Schlendorf, Eugene Pomerantsev, et al. Visualization of coronary atherosclerotic plaques in patients using optical coherence tomography: comparison with intravascular ultrasound. *Journal of the American College of Cardiology*, 39(4):604–609, 2002. [6](#)
- [22] John A Evans, John M Poneris, Brett E Bouma, Jason Bressner, Elkan F Halpern, Milen Shishkov, Gregory Y Lauwers, Mari Mino-Kenudson, Norman S Nishioka, and Guillermo J Tearney. Optical coherence tomography to identify intramucosal carcinoma and high-grade dysplasia in barretts esophagus. *Clinical Gastroenterology and Hepatology*, 4(1):38–43, 2006. [6](#)
- [23] XD Li, SA Boppart, J Van Dam, H Mashimo, M Mutinga, W Drexler, M Klein, C Pitris, ML Krinsky, ME Brezinski, et al. Optical coherence tomography: advanced technology for the endoscopic imaging of barrett’s esophagus. *Endoscopy*, 32(12):921–930, 2000.
- [24] Gerard Isenberg, Michael V Sivak Jr, Amitabh Chak, Richard CK Wong, Joseph E Willis, Brian Wolf, Douglas Y Rowland, Ananya Das, and Andrew Rollins. Accuracy of endoscopic optical coherence tomography in the detection of dysplasia in barrett’s esophagus: a prospective, double-blinded study. *Gastrointestinal endoscopy*, 62(6):825–831, 2005.
- [25] Y Chen, AD Aguirre, P-L Hsiung, S Desai, PR Herz, M Pedrosa, Q Huang, M Figueiredo, S-W Huang, A Koski, et al. Ultrahigh resolution optical coherence tomography of barretts esophagus: preliminary descriptive clinical study correlating images with histology. *Endoscopy*, 39(07):599–605, 2007. [6](#)
- [26] Brett E Bouma, Guillermo J Tearney, Carolyn C Compton, and Norman S Nishioka. High-resolution imaging of the human esophagus and stomach in vivo using optical coherence tomography. *Gastrointestinal endoscopy*, 51(4):467–474, 2000. [6](#)
- [27] S Brand, JM Poneris, BE Bouma, GJ Tearney, CC Compton, and NS Nishioka. Optical coherence tomography in the gastrointestinal tract. *Endoscopy*, 32(10):796–803, 2000.

- [28] Costas Pitris, Christine Jesser, Stephen A Boppart, Debra Stamper, Mark E Brezinski, and James G Fujimoto. Feasibility of optical coherence tomography for high-resolution imaging of human gastrointestinal tract malignancies. *Journal of gastroenterology*, 35(2):87–92, 2000. [6](#)
- [29] Gerd Häusler, Michael Walter Lindner, et al. ” coherence radar” and” spectral radar”-new tools for dermatological diagnosis. *Journal of biomedical optics*, 3(1):21–31, 1998. [6](#), [10](#)
- [30] R Leitgeb, CK Hitzenberger, and Adolf F Fercher. Performance of fourier domain vs. time domain optical coherence tomography. *Optics express*, 11(8):889–894, 2003. [6](#)
- [31] Johannes F De Boer, Barry Cense, B Hyle Park, Mark C Pierce, Guillermo J Tearney, and Brett E Bouma. Improved signal-to-noise ratio in spectral-domain compared with time-domain optical coherence tomography. *Optics letters*, 28(21):2067–2069, 2003.
- [32] Michael A Choma, Marinko V Sarunic, Changhuei Yang, and Joseph A Izatt. Sensitivity advantage of swept source and fourier domain optical coherence tomography. *Optics express*, 11(18):2183–2189, 2003. [6](#)
- [33] Robert Huber, Desmond C Adler, and James G Fujimoto. Buffered fourier domain mode locking: unidirectional swept laser sources for optical coherence tomography imaging at 370,000 lines/s. *Optics letters*, 31(20):2975–2977, 2006. [6](#)
- [34] B Hyle Park, Mark C Pierce, Barry Cense, and Johannes F de Boer. Real-time multi-functional optical coherence tomography. *Optics express*, 11(7):782–793, 2003. [6](#), [18](#)
- [35] Wolfgang Wieser, Benjamin R Biedermann, Thomas Klein, Christoph M Eigenwillig, and Robert Huber. Multi-megahertz oct: High quality 3d imaging at 20 million a-scans and 4.5 gvoxels per second. *Optics express*, 18(14):14685–14704, 2010. [6](#)
- [36] Maciej Wojtkowski, A Kowalczyk, R Leitgeb, and AF Fercher. Full range complex spectral optical coherence tomography technique in eye imaging. *Optics letters*, 27(16):1415–1417, 2002. [10](#)

- [37] Rainer A Leitgeb, Christoph K Hitzenberger, Adolf F Fercher, and Tomasz Bajraszewski. Phase-shifting algorithm to achieve high-speed long-depth-range probing by frequency-domain optical coherence tomography. *Optics letters*, 28(22):2201–2203, 2003.
- [38] Michael A Choma, Changhuei Yang, and Joseph A Izatt. Instantaneous quadrature low-coherence interferometry with 3×3 fiber-optic couplers. *Optics letters*, 28(22):2162–2164, 2003.
- [39] Seok-Hyun Yun, GJ Tearney, JF De Boer, and BE Bouma. Removing the depth-degeneracy in optical frequency domain imaging with frequency shifting. *Optics Express*, 12(20):4822–4828, 2004.
- [40] Erich Götzinger, Michael Pircher, Rainer A Leitgeb, and Christoph K Hitzenberger. High speed full range complex spectral domain optical coherence tomography. *Optics express*, 13(2):583–594, 2005.
- [41] Marinko V Sarunic, Michael A Choma, Changhuei Yang, and Joseph A Izatt. Instantaneous complex conjugate resolved spectral domain and swept-source oct using 3×3 fiber couplers. *Optics express*, 13(3):957–967, 2005.
- [42] Anjul Maheshwari, Michael A Choma, and Joseph A Izatt. Heterodyne swept-source optical coherence tomography for complete complex conjugate ambiguity removal. *Journal of Biomedical Optics*, 10(6):064005, 2005.
- [43] Jun Zhang, J Stuart Nelson, and Zhongping Chen. Removal of a mirror image and enhancement of the signal-to-noise ratio in fourier-domain optical coherence tomography using an electro-optic phase modulator. *Optics letters*, 30(2):147–149, 2005. [10](#)
- [44] Adolph F Fercher, Ch K Hitzenberger, G Kamp, and Sy Y El-Zaiat. Measurement of intraocular distances by backscattering spectral interferometry. *Optics communications*, 117(1-2):43–48, 1995. [10](#)
- [45] Maciej Wojtkowski, Rainer Leitgeb, Andrzej Kowalczyk, Tomasz Bajraszewski, and Adolf Friedrich Fercher. In vivo human retinal imaging by fourier domain optical coherence tomography. *Journal of biomedical optics*, 7(3):457–464, 2002. [10](#)

- [46] SR Chinn, EA Swanson, and JG Fujimoto. Optical coherence tomography using a frequency-tunable optical source. *Optics letters*, 22(5):340–342, 1997. [10](#)
- [47] B Golubovic, BE Bouma, GJ Tearney, and JG Fujimoto. Optical frequency-domain reflectometry using rapid wavelength tuning of a cr 4+: forsterite laser. *Optics letters*, 22(22):1704–1706, 1997.
- [48] FCKH Lexer, Christoph K Hitzenberger, AF Fercher, and M Kulhavy. Wavelength-tuning interferometry of intraocular distances. *Applied Optics*, 36(25):6548–6553, 1997.
- [49] Udo Haberland, Vladimir Blazek, and Hans Juergen Schmitt. Chirp optical coherence tomography of layered scattering media. *Journal of Biomedical Optics*, 3(3):259–267, 1998. [10](#)
- [50] Yueli Chen, Linda Otis, Daqing Piao, and Quing Zhu. Characterization of dentin, enamel, and carious lesions by a polarization-sensitive optical coherence tomography system. *Applied optics*, 44(11):2041–2048, 2005. [11](#)
- [51] Mark C Pierce, John Strasswimmer, Hyle Park, Barry Cense, and Johannes F de Boer. Birefringence measurements in human skin using polarization-sensitive optical coherence tomography. *Journal of biomedical optics*, 9(2):287–292, 2004.
- [52] WOLFGANG Drexler, DEBRA Stamper, CHRISTINE Jesser, XINGDE Li, COSTAS Pitris, KATHLEEN Saunders, SCOTT Martin, MARY BRIGHID Lodge, JAMES G Fujimoto, and MARK E Brezinski. Correlation of collagen organization with polarization sensitive imaging of in vitro cartilage: implications for osteoarthritis. *The Journal of rheumatology*, 28(6):1311–1318, 2001.
- [53] Barry Cense, Teresa C Chen, B Hyle Park, Mark C Pierce, and Johannes F De Boer. Thickness and birefringence of healthy retinal nerve fiber layer tissue measured with polarization-sensitive optical coherence tomography. *Investigative ophthalmology & visual science*, 45(8):2606–2612, 2004.
- [54] B Hyle Park, Mark C Pierce, Barry Cense, and Johannes F de Boer. Jones matrix analysis for a polarization-sensitive optical coherence tomography system using fiber-optic components. *Optics letters*, 29(21):2512–2514, 2004. [18](#)

- [55] Shuguang Guo, Jun Zhang, Lei Wang, J Stuart Nelson, and Zhongping Chen. Depth-resolved birefringence and differential optical axis orientation measurements with fiber-based polarization-sensitive optical coherence tomography. *Optics Letters*, 29(17):2025–2027, 2004. [33](#)
- [56] Nirlep A Patel, Jason Zoeller, Debra L Stamper, James G Fujimoto, and Mark E Brezinski. Monitoring osteoarthritis in the rat model using optical coherence tomography. *IEEE transactions on medical imaging*, 24(2):155–159, 2005. [11](#)
- [57] Seemantini K Nadkarni, Mark C Pierce, B Hyle Park, Johannes F de Boer, Peter Whittaker, Brett E Bouma, Jason E Bressner, Elkan Halpern, Stuart L Houser, and Guillermo J Tearney. Measurement of collagen and smooth muscle cell content in atherosclerotic plaques using polarization-sensitive optical coherence tomography. *Journal of the American College of Cardiology*, 49(13):1474–1481, 2007. [11](#)
- [58] Ki Hean Kim, Sang June Yoon, Mark C Pierce, Gopi N Maguluri, B Hyle Park, Martha Lydon, Robert L Sheridan, and Johannes F de Boer. In vivo imaging of human burn injuries with polarization-sensitive optical coherence tomography. *Journal of biomedical optics*, 17(6):066012, 2012. [11](#)
- [59] Mitsuro Sugita, Michael Pircher, Stefan Zotter, Bernhard Baumann, Philipp Roberts, Tomoyuki Makihira, Nobuhiro Tomatsu, Makoto Sato, Clemens Vass, and Christoph K Hitzenberger. Retinal nerve fiber bundle tracing and analysis in human eye by polarization sensitive oct. *Biomedical optics express*, 6(3):1030–1054, 2015. [11](#)
- [60] Boy Braaf, Koenraad A Vermeer, Mattijs de Groot, Kari V Vienola, and Johannes F de Boer. Fiber-based polarization-sensitive oct of the human retina with correction of system polarization distortions. *Biomedical optics express*, 5(8):2736–2758, 2014. [11](#)
- [61] Michael R Hee, David Huang, Eric A Swanson, and James G Fujimoto. Polarization-sensitive low-coherence reflectometer for birefringence characterization and ranging. *JOSA B*, 9(6):903–908, 1992. [11](#), [29](#)
- [62] Johannes F De Boer, Thomas E Milner, and J Stuart Nelson. Determination of the depth-resolved stokes parameters of light backscattered from turbid

- media by use of polarization-sensitive optical coherence tomography. *Optics letters*, 24(5):300–302, 1999. [11](#), [12](#), [16](#), [23](#)
- [63] Johannes F De Boer, Thomas E Milner, Martin JC Van Gemert, and J Stuart Nelson. Two-dimensional birefringence imaging in biological tissue by polarization-sensitive optical coherence tomography. *Optics letters*, 22(12):934–936, 1997. [12](#)
- [64] Erich Götzinger, Michael Pircher, Wolfgang Geitzenauer, Christian Ahlers, Bernhard Baumann, Stephan Michels, Ursula Schmidt-Erfurth, and Christoph K Hitzenberger. Retinal pigment epithelium segmentation by polarization sensitive optical coherence tomography. *Optics express*, 16(21):16410–16422, 2008. [12](#)
- [65] Gang Yao and Lihong V Wang. Two-dimensional depth-resolved mueller matrix characterization of biological tissue by optical coherence tomography. *Optics Letters*, 24(8):537–539, 1999. [12](#)
- [66] Shuliang Jiao and Lihong V Wang. Jones-matrix imaging of biological tissues with quadruple-channel optical coherence tomography. *Journal of Biomedical Optics*, 7(3):350–359, 2002.
- [67] Shuichi Makita, Masahiro Yamanari, and Yoshiaki Yasuno. Generalized jones matrix optical coherence tomography: performance and local birefringence imaging. *Optics express*, 18(2):854–876, 2010. [12](#), [33](#), [35](#)
- [68] Johannes F De Boer and Thomas E Milner. Review of polarization sensitive optical coherence tomography and stokes vector determination. *Journal of biomedical optics*, 7(3):359–372, 2002. [15](#)
- [69] Craig F Bohren and Donald R Huffman. *Absorption and scattering of light by small particles*. John Wiley & Sons, 2008. [17](#), [99](#)
- [70] Nate J Kemp, Haitham N Zaatari, Jesung Park, H Grady Rylander III, and Thomas E Milner. Form-biattenuance in fibrous tissues measured with polarization-sensitive optical coherence tomography (ps-oct). *Optics express*, 13(12):4611–4628, 2005. [18](#)
- [71] Boris Hyle Park, Christopher E Saxer, Shyam M Srinivas, J Stuart Nelson, and Johannes F de Boer. In vivo burn depth determination by high-speed

- fiber-based polarization sensitive optical coherence tomography. *Journal of biomedical optics*, 6(4):474–480, 2001. [18](#)
- [72] JP Gordon and H Kogelnik. Pmd fundamentals: Polarization mode dispersion in optical fibers. *Proceedings of the National Academy of Sciences*, 97(9):4541–4550, 2000. [19](#), [27](#), [30](#), [52](#)
- [73] B Hyle Park, Mark C Pierce, Barry Cense, and Johannes F de Boer. Optic axis determination accuracy for fiber-based polarization-sensitive optical coherence tomography. *Optics letters*, 30(19):2587–2589, 2005. [19](#)
- [74] Christopher E Saxer, Johannes F de Boer, B Hyle Park, Yonghua Zhao, Zhongping Chen, and J Stuart Nelson. High-speed fiber-based polarization-sensitive optical coherence tomography of in vivo human skin. *Optics letters*, 25(18):1355–1357, 2000. [20](#)
- [75] Bernhard Baumann, WooJhon Choi, Benjamin Potsaid, David Huang, Jay S Duker, and James G Fujimoto. Swept source/fourier domain polarization sensitive optical coherence tomography with a passive polarization delay unit. *Optics express*, 20(9):10229–10241, 2012. [20](#)
- [76] Kenneth Sassen. The polarization lidar technique for cloud research: A review and current assessment. *Bulletin of the American Meteorological Society*, 72(12):1848–1866, 1991. [23](#)
- [77] J Scott Tyo, Dennis L Goldstein, David B Chenault, and Joseph A Shaw. Review of passive imaging polarimetry for remote sensing applications. *Applied optics*, 45(22):5453–5469, 2006. [23](#)
- [78] Nirmalya Ghosh and Alex I Vitkin. Tissue polarimetry: concepts, challenges, applications, and outlook. *Journal of biomedical optics*, 16(11):110801, 2011. [23](#)
- [79] Valery V Tuchin. Polarized light interaction with tissues. *Journal of biomedical optics*, 21(7):071114, 2016.
- [80] Ji Qi and Daniel S Elson. Mueller polarimetric imaging for surgical and diagnostic applications: a review. *Journal of biophotonics*, 10(8):950–982, 2017. [23](#)

- [81] Nanshuo Wang, Xinyu Liu, Qiaozhou Xiong, Jun Xie, Shi Chen, and Linbo Liu. Polarization management to mitigate misalignment-induced fringe fading in fiber-based optical coherence tomography. *Optics letters*, 42(15):2996–2999, 2017. [24](#)
- [82] Ernst Brinkmeyer. Forward-backward transmission in birefringent single-mode fibers: interpretation of polarization-sensitive measurements. *Optics letters*, 6(11):575–577, 1981. [24](#)
- [83] M Oskar Van Deventer. Polarization properties of rayleigh backscattering in single-mode fibers. *Journal of Lightwave Technology*, 11(12):1895–1899, 1993. [53](#)
- [84] Fabrizio Corsi. Polarization mode dispersion characterization of single-mode optical fiber using backscattering technique. *Journal of lightwave technology*, 16(10):1832, 1998. [24](#)
- [85] Michael Pircher, Erich Götzinger, Bernhard Baumann, and Christoph K Hitzenberger. Corneal birefringence compensation for polarization sensitive optical coherence tomography of the human retina. *Journal of Biomedical Optics*, 12(4):041210, 2007. [24](#)
- [86] N Vansteenkiste, P Vignolo, and A Aspect. Optical reversibility theorems for polarization: application to remote control of polarization. *JOSA A*, 10(10):2240–2245, 1993. [26](#), [99](#)
- [87] Shane R Cloude and Eric Pottier. Concept of polarization entropy in optical scattering. *Optical Engineering*, 34(6):1599–1611, 1995. [26](#)
- [88] Richard J Potton. Reciprocity in optics. *Reports on Progress in Physics*, 67(5):717, 2004. [26](#)
- [89] José J Gil. Polarimetric characterization of light and media: physical quantities involved in polarimetric phenomena. *The European Physical Journal-Applied Physics*, 40(1):1–47, 2007. [26](#)
- [90] Johannes F De Boer, Christoph K Hitzenberger, and Yoshiaki Yasuno. Polarization sensitive optical coherence tomography—a review. *Biomedical optics express*, 8(3):1838–1873, 2017. [29](#)

- [91] Brian Hall. *Lie groups, Lie algebras, and representations: an elementary introduction*, volume 222. Springer, 2015. [30](#)
- [92] Xinyu Liu, Kathy Beaudette, Xianghong Wang, Linbo Liu, Brett E Bouma, and Martin Villiger. Tissue-like phantoms for quantitative birefringence imaging. *Biomedical optics express*, 8(10):4454–4465, 2017. [32](#), [35](#), [47](#)
- [93] Christoph K Hitzenberger, Erich Götzinger, Markus Sticker, Michael Pircher, and Adolf F Fercher. Measurement and imaging of birefringence and optic axis orientation by phase resolved polarization sensitive optical coherence tomography. *Optics express*, 9(13):780–790, 2001. [33](#)
- [94] Miloš Todorović, Shuliang Jiao, Lihong V Wang, and George Stoica. Determination of local polarization properties of biological samples in the presence of diattenuation by use of mueller optical coherence tomography. *Optics letters*, 29(20):2402–2404, 2004. [33](#), [35](#)
- [95] Martin Villiger, Dirk Lorenser, Robert A McLaughlin, Bryden C Quirk, Rodney W Kirk, Brett E Bouma, and David D Sampson. Deep tissue volume imaging of birefringence through fibre-optic needle probes for the delineation of breast tumour. *Scientific reports*, 6:28771, 2016. [35](#)
- [96] Chuanmao Fan and Gang Yao. Mapping local optical axis in birefringent samples using polarization-sensitive optical coherence tomography. *Journal of Biomedical Optics*, 17(11):110501, 2012. [35](#)
- [97] Martin Villiger, Boy Braaf, Norman Lippok, Kenichiro Otsuka, Seemantini K Nadkarni, and Brett E Bouma. Optic axis mapping with catheter-based polarization-sensitive optical coherence tomography. *Optica*, 5(10):1329–1337, 2018. [36](#)
- [98] Martin Villiger, Ellen Ziyi Zhang, Seemantini K Nadkarni, Wang-Yuhl Oh, Benjamin J Vakoc, and Brett E Bouma. Spectral binning for mitigation of polarization mode dispersion artifacts in catheter-based optical frequency domain imaging. *Optics express*, 21(14):16353–16369, 2013. [36](#)
- [99] Xiaojun Yu, Xinyu Liu, Si Chen, Yuemei Luo, Xianghong Wang, and Linbo Liu. High-resolution extended source optical coherence tomography. *Optics express*, 23(20):26399–26413, 2015. [40](#)

- [100] Barry Cense, Teresa C Chen, B Hyle Park, Mark C Pierce, and Johannes F de Boer. In vivo depth-resolved birefringence measurements of the human retinal nerve fiber layer by polarization-sensitive optical coherence tomography. *Optics letters*, 27(18):1610–1612, 2002. [41](#)
- [101] Michael Pircher, Christoph K Hitzenberger, and Ursula Schmidt-Erfurth. Polarization sensitive optical coherence tomography in the human eye. *Progress in retinal and eye research*, 30(6):431–451, 2011. [41](#)
- [102] Norman Lippok, Martin Villiger, Alexandre Albanese, Eelco FJ Meijer, Kwanghun Chung, Timothy P Padera, Sangeeta N Bhatia, and Brett E Bouma. Depolarization signatures map gold nanorods within biological tissue. *Nature photonics*, 11(9):583, 2017. [46](#)
- [103] Michalina J Gora, Melissa J Suter, Guillermo J Tearney, and Xingde Li. Endoscopic optical coherence tomography: technologies and clinical applications. *Biomedical optics express*, 8(5):2405–2444, 2017. [52](#)
- [104] Mark C Pierce, Milen Shishkov, B Hyle Park, Nader A Nassif, Brett E Bouma, Guillermo J Tearney, and Johannes F de Boer. Effects of sample arm motion in endoscopic polarization-sensitive optical coherence tomography. *Optics Express*, 13(15):5739–5749, 2005. [52](#)
- [105] Anthony MD Lee, Hamid Pahlevaninezhad, Victor XD Yang, Stephen Lam, Calum MacAulay, and Pierre Lane. Fiber-optic polarization diversity detection for rotary probe optical coherence tomography. *Optics letters*, 39(12):3638–3641, 2014. [52](#)
- [106] GJ Tearney, BE Bouma, SA Boppart, BSEA Golubovic, EA Swanson, and JG Fujimoto. Rapid acquisition of in vivo biological images by use of optical coherence tomography. *Optics letters*, 21(17):1408–1410, 1996. [52](#)
- [107] Dongyao Cui, Kengyeh K Chu, Biwei Yin, Timothy N Ford, Chulho Hyun, Hui Min Leung, Joseph A Gardecki, George M Solomon, Susan E Birket, Linbo Liu, et al. Flexible, high-resolution micro-optical coherence tomography endobronchial probe toward in vivo imaging of cilia. *Optics letters*, 42(4):867–870, 2017. [52](#)
- [108] Martin Villiger, Ellen Ziyi Zhang, Seemantini Nadkarni, Wang-Yuhl Oh, Brett E Bouma, and Benjamin J Vakoc. Artifacts in polarization-sensitive

- optical coherence tomography caused by polarization mode dispersion. *Optics letters*, 38(6):923–925, 2013. [53](#)
- [109] Renu Tripathi, Nader Nassif, J Stuart Nelson, Boris Hyle Park, and Johannes F de Boer. Spectral shaping for non-gaussian source spectra in optical coherence tomography. *Optics Letters*, 27(6):406–408, 2002. [63](#)
- [110] Felix Spöler, Stefan Kray, Patrik Grychtol, Barbara Hermes, Jörg Bornemann, Michael Först, and Heinrich Kurz. Simultaneous dual-band ultra-high resolution optical coherence tomography. *Optics Express*, 15(17):10832–10841, 2007. [64](#)
- [111] Yoshiyuki Takahashi, Yuuki Watanabe, and Manabu Sato. Application of the maximum entropy method to spectral-domain optical coherence tomography for enhancing axial resolution. *Applied optics*, 46(22):5228–5236, 2007. [64](#)
- [112] Hon Luen Seck, Ying Zhang, and Yeng Chai Soh. High resolution optical coherence tomography by ℓ_1 -optimization. *Optics Communications*, 284(7):1752–1759, 2011. [64](#)
- [113] Siavash Yousefi, Jia Qin, and Ruikang K Wang. Super-resolution spectral estimation of optical micro-angiography for quantifying blood flow within microcirculatory tissue beds in vivo. *Biomedical optics express*, 4(7):1214–1228, 2013. [64](#)
- [114] Xinyu Liu, Si Chen, Dongyao Cui, Xiaojun Yu, and Linbo Liu. Spectral estimation optical coherence tomography for axial super-resolution. *Optics express*, 23(20):26521–26532, 2015. [64](#), [74](#)
- [115] Fredericus JM Barning. The numerical analysis of the light-curve of 12 laceratae. *Bulletin of the Astronomical Institutes of the Netherlands*, 17:22, 1963. [64](#)
- [116] EJ Groth. The pulse shape of the crab nebula pulsar, psr 0531+ 21, in the v and i bands. *The Astrophysical Journal*, 200:278–280, 1975. [64](#)
- [117] H-M Adorf. Reconstructing a one-dimensional signal from irregular samples. In *European Southern Observatory Conference and Workshop Proceedings*, volume 47, 1993. [64](#)

- [118] GG Fahlman and TJ Ulrych. A new method for estimating the power spectrum of gapped data. *Monthly Notices of the Royal Astronomical Society*, 199(1):53–65, 1982. [64](#)
- [119] Timothy M Brown and Jorgen Christensen-Dalsgaard. A technique for estimating complicated power spectra from time series with gaps. *The Astrophysical Journal*, 349:667–674, 1990. [64](#)
- [120] Piet MT Broersen, Stijn De Waele, and Robert Bos. Application of autoregressive spectral analysis to missing data problems. *Instrumentation and Measurement, IEEE Transactions on*, 53(4):981–986, 2004. [65](#)
- [121] Petre Stoica, Erik G Larsson, and Jian Li. Adaptive filter-bank approach to restoration and spectral analysis of gapped data. *The Astronomical Journal*, 120(4):2163, 2000. [65](#)
- [122] Jian Li and Petre Stoica. An adaptive filtering approach to spectral estimation and sar imaging. *IEEE Transactions on Signal Processing*, 44(6):1469–1484, 1996. [65](#), [66](#)
- [123] Evgenia Bousi and Constantinos Pitris. Axial resolution improvement by modulated deconvolution in fourier domain optical coherence tomography. *Journal of biomedical optics*, 17(7):071307, 2012. [74](#)
- [124] Yingtian Pan, Reginald Birngruber, Jürgen Rosperich, and Ralf Engelhardt. Low-coherence optical tomography in turbid tissue: theoretical analysis. *Applied optics*, 34(28):6564–6574, 1995. [74](#)
- [125] Angelika Unterhuber, Boris Považay, Boris Hermann, H Sattmann, W Drexler, Vladislav Yakovlev, Gabriel Tempea, Colja Schubert, EM Anger, PK Ahnelt, et al. Compact, low-cost ti: Al₂O₃ laser for in vivo ultrahigh-resolution optical coherence tomography. *Optics letters*, 28(11):905–907, 2003. [74](#)
- [126] Linbo Liu, Joseph A Gardecki, Seemantini K Nadkarni, Jimmy D Tous-saint, Yukako Yagi, Brett E Bouma, and Guillermo J Tearney. Imaging the subcellular structure of human coronary atherosclerosis using micro-optical coherence tomography. *Nature medicine*, 17(8):1010–1014, 2011. [74](#)

- [127] Linbo Liu, Kengyeh K Chu, Grace H Houser, Bradford J Diephuis, Yao Li, Eric J Wilsterman, Suresh Shastry, Gregory Dierksen, Susan E Birket, Marina Mazur, et al. Method for quantitative study of airway functional microanatomy using micro-optical coherence tomography. *PloS one*, 8(1): e54473, 2013. [74](#)

Appendix I.

Muller matrix of reverse transmission

Here we derive the deterministic (i.e. non-depolarizing) Muller matrix of the reverse transmission through a reciprocal element from its corresponding Jones expression. If A and B denote the front and rear interface of this element, the Jones matrix $\mathbf{J}_{AB}$ describes the forward propagation through this element (as shown in Fig. Appd1). The reverse transmission is given by the transpose of this matrix $\mathbf{J}_{BA} = \mathbf{J}_{AB}^T$, when maintaining the orientation of the spatial xy -coordinates irrespective of the propagation direction [86].

FIGURE 1: Schematic diagram of the transmission through an element from A to B and in the reverse direction, described by the Jones and Mueller matrices, respectively.

A matrix $\mathbf{J}$ in the Jones formalism can be converted into its corresponding Mueller matrix $\mathbf{M}$ using [69].

$$\mathbf{M} = \mathbf{U}(\mathbf{J} \otimes \mathbf{J}^*)\mathbf{U}^\dagger$$

$$\mathbf{U} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{bmatrix} \quad (1)$$

where $\otimes$ is the Kronecker tensor product, $\mathbf{J}^*$ is the complex conjugate of $\mathbf{J}$, and $\mathbf{U}^\dagger$ represents the conjugate transpose of $\mathbf{U}$, corresponding also to its inverse. Using the fact that the transpose is distributive for the Kronecker product $((\mathbf{A} \otimes \mathbf{B})^\top = \mathbf{A}^\top \otimes \mathbf{B}^\top)$, we find

$$\begin{aligned} \mathbf{M}_{\text{BA}} &= \mathbf{U} \cdot (\mathbf{J}_{\text{BA}} \otimes \mathbf{J}_{\text{BA}}^*) \cdot \mathbf{U}^\dagger \\ &= \mathbf{U} \cdot (\mathbf{J}_{\text{AB}}^\top \otimes \mathbf{J}_{\text{AB}}^\dagger) \cdot \mathbf{U}^\dagger \\ &= \mathbf{U} \cdot (\mathbf{J}_{\text{AB}} \otimes \mathbf{J}_{\text{AB}}^*)^\top \cdot \mathbf{U}^\dagger \\ &= \mathbf{U} \cdot \mathbf{I} \cdot (\mathbf{J}_{\text{AB}} \otimes \mathbf{J}_{\text{AB}}^*)^\top \cdot \mathbf{I} \cdot \mathbf{U}^\dagger \\ &= \mathbf{U} \cdot (\mathbf{U}^\top \cdot \mathbf{U}^*) \cdot (\mathbf{J}_{\text{AB}} \otimes \mathbf{J}_{\text{AB}}^*)^\top \cdot (\mathbf{U}^\top \cdot \mathbf{U}^*) \cdot \mathbf{U}^\dagger \\ &= (\mathbf{U} \cdot \mathbf{U}^\top) \cdot \mathbf{U}^* \cdot (\mathbf{J}_{\text{AB}} \otimes \mathbf{J}_{\text{AB}}^*)^\top \cdot \mathbf{U}^\top \cdot (\mathbf{U}^* \cdot \mathbf{U}^\dagger) \\ &= (\mathbf{U} \cdot \mathbf{U}^\top) \cdot \mathbf{M}_{\text{AB}}^\top \cdot (\mathbf{U}^* \cdot \mathbf{U}^\dagger) \end{aligned} \quad (2)$$

Evaluating the outermost expressions, we find $\mathbf{U} \cdot \mathbf{U}^\top = \mathbf{U}^* \cdot \mathbf{U}^\dagger = \text{diag}(1, 1, 1, -1)$. The transpose of a Jones matrix corresponds to the transpose of its corresponding Mueller matrix, with the sign of the last row and column, excluding the on-diagonal element, inverted. It represents the reverse transmission through a reciprocal element. For a pure retardation matrix $\mathbf{R}$ in the $\text{SO}(3)$ formalism, this results in $\mathbf{R}_{\text{BA}} = \mathbf{D} \cdot \mathbf{R}_{\text{AB}}^\top \cdot \mathbf{D}$, where $\mathbf{D} = \text{diag}(1, 1, -1)$.