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UNIVERSITY**

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**OPTIMAL DESIGN OF SHARED PUBLIC
TRANSPORT SERVICES**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2018

Optimal Design of Shared Public Transport Services

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School of Civil and Environmental Engineering

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in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

04/05/2019

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I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

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This thesis contains material from [x number] paper(s) published in the following peer-reviewed journal(s) / from papers accepted at conferences in which I am listed as an author.

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The contributions of the co-authors are as follows:

- A/Prof Wang provided the initial direction and edited the manuscript drafts.
- I prepared the manuscript drafts, built the model, wrote the codes and conducted numerical analysis
- Dr Meng revised the manuscript, assisted in model building and numerical analysis.

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The contributions of the co-authors are as follows:

- Prof Wang suggested the area and edited the manuscript drafts.
- I wrote the drafts of the manuscript, built the traffic network and model, solved model and conducted numerical analysis.
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The contributions of the co-authors are as follows:

- A/Prof Wang provided the initial direction and edited the manuscript drafts.
- I prepared the manuscript drafts, built the model, wrote the codes and analysed the numerical results.
- Dr Meng revised the manuscript, assisted in the derivation and numerical analysis.

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ABSTRACT

Public transport services are always recognised as one of the most efficient ways to solve traffic problems. While it has been reaching a bottleneck in the most metropolis as the traditional public transport could not meet the increasing requirements from diversified traffic demand. Those demands are hardly switched to public transport services. As a result, shared public transport services have been launched to improve the public transport usages. Among them, dockless bike sharing service and on-demand public bus (ODPB) service have emerged as alternative options to attract more passengers with diverse travel requirements. Dockless bike sharing system is a self-service public bike rental scheme, which enables users to unlock a public bike through scanning a smartphone application and lock it after using it anywhere. ODPB is a demand responsive public transport service, which offers users the public transport services according to their requests. Due to the recent emergence of dockless bike sharing and ODPB service, the research works in the literature are still very few, and most of the existing works are very descriptive using statistical approaches.

This research aims to develop a set of methodologies based on analytical approaches, rather than statistical approaches, to determine the optimal strategies for dockless bike sharing and ODPB service's planning and design. In line with this aim, five research tasks have been defined. These are initial bike allocation problem; bike relocation problem; feasibility analysis of ODPB service; applicability analysis of ODPB service; optimisation stop spacing design.

In this regard, this research proposes five sets of models and algorithms to address the planning and design issues. The first set of models and algorithms is to guide the market leader and follower to allocate the bikes. Two market scenarios are designed including both monopoly and competitive situation. The initial bike allocation position, coverage area, and allocation method are determined based on community structure and diffusion theory. A new dynamic pricing strategy with negative price is introduced in dockless bike sharing system in the second set of model and algorithm to achieve bike sharing relocation through travellers' behaviour. In a normal situation, the user pays a positive price to the operator for using a bike. When bikes are imbalanced distributed in the system compared to the real-time demand, the user who cycles from the oversupply area to undersupply area will get paid from the operator.

A user equilibrium dynamic traffic assignment formulation is developed to capture travellers' mode-path choice in response to the proposed dynamic pricing strategy. The third set of model and algorithm is to provide a systematic account of the impact of ODPB service on a multimodal transport corridor by using a traffic equilibrium formulation. Six alternative travel modes: auto, park-and-ride (P&R), taxi, metro, conventional bus, and ODPB are considered, while the generalised travel utility functions for each mode are developed with incorporating travel time, monetary cost and travel discomfort. To investigate the applicable condition of ODPB service, the fourth set of model and algorithm is to maximise the social welfare after introducing the ODPB service by analysing different scenarios. The length and fare of ODPB service will be considered, together with the size of the residential area and population density along the service. The fifth set of model and algorithm is to optimise the bus stop spacing of ODPB service with the objective to minimise the total travel time of all the passengers. To capture the unique characteristics of ODPB service, the model constraints will include the length of the segment, the configuration of shed line, minimum stop spacing, vehicle capacity limitation, and nonnegative constraints.

Preliminary results have found that suitable bike allocation strategies could cover maximum users and service area. The dynamic pricing strategy with negative price could efficiently achieve bike relocation when the resources are limited in the initial promotion stage. The introduction of ODPB service can bring a positive impact to mode shift, especially for the long-distance trips. The mode shift performance depends greatly on the fare, usage of the bus lane and the out-of-vehicle travel time of ODPB service. Users with different VOT have different mode choice preferences. The optimal bus stop spacing for an on-demand public bus is very different from a conventional bus. Passenger demand density, travel time perception weight ratio, access/egress walking speed, and the time loss near/at the stop will affect the on-demand public bus stop spacing significantly.

Future work could extend the proposed methodologies to other situations. To the best knowledge of the author, this is the first work on systemically exploring the planning and design issues for dockless bike sharing and ODPB services.

LIST OF PUBLICATIONS

Journal Papers:

1. **Zhang, J.**, Meng, M., & Wang, Z.W. David (2018). A dynamic pricing strategy with negative price in dockless bike sharing system. *Transportation Research Part B: Methodological*, under review.
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4. **Zhang, J.**, Wang, Z.W. David, & Meng, M. (2018). Which service is better on a linear transport corridor, park & ride or on-demand public bus?. *Transportation Research Part A: Policy and Practice*, 118, 803-818.
5. **Zhang, J.**, Wang, Z.W. David, & Meng, M. (2018). Analyzing Customized Bus Service on a Multimodal Travel Corridor: An Analytical Modeling Approach. *Journal of Transportation Engineering, Part A: Systems*, 143(11), 04017057.
6. **Zhang, J.**, Koh, P. P., Meng, M., Leow, B. W., & Wong, Y. D. (2017, July). Feasibility study of bike sharing in Singapore. *Proceedings of the Institution of Civil Engineers-Engineering Sustainability* (pp. 1-7). Thomas Telford Ltd.
7. Meng, M., **Zhang, J.**, Wong, Y.D., & Au, P.H. (2016). Effect of weather conditions and weather forecast on cycling behaviour in Singapore. *International Journal of Sustainable Transportation*, 10(9), 773-780.
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9. Meng, M., **Zhang, J.**, & Wong, Y.D. (2016). Integrated foresight urban planning in Singapore. *Proceedings of the ICE - Urban Design and Planning*, 169(1), 1-13.

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LIST OF SYMBOLS

BPR	Bureau of Public Roads
CBD	central business district
DCV	directly connected vertex
DRT	demand responsive transport
F-N	Fast Newman
FP	free price
GPS	global positioning system
ICV	indirectly connected vertex
LRT	light rail transit
MSA	method of successive averages
MSWA	method of successive weighted average
NP	negative price
OD	origin & destination
ODPB	on-demand public bus
P&R	park-and-ride
PP	positive price
TAZ	traffic analysis zone
VI	variational inequality
VOT	value-of-time

CHAPTER 1 INTRODUCTION

1.1 Background

Urbanisation and motorisation have placed an ever-increasing amount of pressure on the current transportation system, resulting in such widespread traffic problems as traffic congestion, traffic fatalities and injuries, traffic pollution and increased energy consumption. Public transport services have been recognised as one of the most efficient ways to solve the problems as of its social and environmental sustainability. Governments have been launched various strategies to encourage travellers to use public transport over the decades. Although significant increment has been achieved, public transport mode share has been reaching a bottleneck in the most metropolis. It is because the traditional public transport philosophy designing for maximum travellers could not meet the increasing requirements from diversified traffic demand. Travellers are more and more sensitive on punctuality, speed, comfort level, monetary cost and so on. Those demands that are beyond the supply from conventional public transport services will mostly choose personal travel modes (e.g. private vehicle, taxi) and are hardly switched to public transport services.

Recently, various shared public transport services have been rapidly popular due to the distinguishing features of flexibility and efficiency. Shared public transport services are derived by demand with similarity, wherein travellers share a vehicle either simultaneously (e.g. ride-sharing) or over time (e.g. car sharing or bike sharing). In this way, those travellers with high requirements might choose the shared public transport services than personal travel modes. Therefore, the development of share transport services could contribute to the promotion of public transport mode share.

This thesis aims to develop a framework to guide the optimal design of shared public transport services. There are many kinds of shared public transport services, wherein the dominant modes are bike sharing, on-demand public bus (ODPB), car sharing and carpooling. As car sharing and carpooling utilise private vehicles, it might stimulate the potential usage of private vehicles in the long run. Therefore, this thesis focuses on the study of bike sharing and ODPB.

Bike sharing system, also known as public bike system or bicycle-share system is a self-service public bike rental scheme, which enables a user to rent a public bike for a short period. Its many benefits such as offer alternative public transport, encourage

more cycling, improve city image, provide complimentary service linking transit stations, reduce congestion and increase accessibility. Bike sharing was first introduced in Amsterdam in 1965 but went to decline due to the shortage technical support in operation. In recent years, a new type of bike sharing system, wildly known as dockless bike sharing system, has already taken over in many cities around the world such as Beijing, Singapore, London, and Washington DC. The idea is that riders do not have to return the bike to a set place and can leave the bike anywhere. Everything is controlled through Global Positioning System (GPS) sensors in the bikes and an app in the rider's phone. Due to recent emergence, the research on dockless bike sharing planning has not caught up the rapid growth in practice, which leads to several issues in real operation. Therefore, one of the research tasks in this thesis is the optimal design of dockless bike sharing system.

ODPB is a type of demand-responsive public transport service, which provides a high level of service to specific passenger groups, especially those who have the similar long-distance travel patterns regarding origin, destination, and expected departure and arrival time. In the planning stage, passengers could submit their requests of service through online information platforms (e.g. smartphone and website). Then the planners almost get all the trip information of potential passengers (e.g. origin, destination, departure and expected arrival time). They finalise the detailed service operation design (e.g. bus stop location, timetable, fare ticket) for those high-demand corridors according to the requests and then publish the information online. After having sufficient subscribers, this service could be operated. In operation, the scheduled bus collects the passengers from a certain group of stops, then drives on bus lane or expressway directly to the destination without stopping in the middle of the trip. Thus, ODPB has the advantages of reliability, high-speed and convenience, which attracts patronage from the conventional public bus passengers who are not satisfied with the bus delay and the overcrowding. Compared to conventional bus and metro service, ODPB has the advantages of personalization, flexibility, attractiveness, reliability, and speediness, with no need for transfers. Similarly, despite the recent emergence of ODPB, little research has been carried out to study its planning strategies. Since some ODPB projects have been failed after launching in a short time, besides the research on the optimal design of ODPB, this thesis shall also investigate the feasibility and applicability of ODPB in different situations.

1.2 Objectives

Based on the discussion above, this study aims to provide optimal models for the planning and design of shared public transport services. Considering the similarities with other public transport services and unique features of public transport services, a systematic framework will be proposed to guide the shared public transport services planning and design. Specifically, the tasks of this study are listed below:

1) Initial bike allocation problem in dockless bike sharing system

As dockless bike sharing system does not require solid docking stations, the network and stations planning for the traditional station-based bike sharing are not required for dockless bike sharing system. A set of initial bike allocation model will be developed to distribute bikes at different operational stages.

2) Bike relocation problem in dockless bike sharing system

Due to the one-way usage characteristics, the bikes are easily distributed unbalanced in the system. A dynamic pricing strategy will be designed to guide the travellers to use the bike spontaneously for bike relocation.

3) Feasibility analysis of ODPB service in the urban multimodal transportation system

An analytical model will be developed to assess the feasibility of ODPB in the multimodal transportation network and investigate influence factors. Different scenarios will be discussed to test the effects by considering ODPB service into a multimodal transportation system. The research also takes into account the cost of ODPB service from several stakeholders.

4) Applicability analysis of ODPB service in a multimodal transportation network

If the local situation is suitable to develop ODPB, the setting of critical planning parameters in ODPB, especially for comparing to the other modes (P&R) with similar characteristics, will be given by an analytical model.

5) Optimisation stop spacing design

An optimisation model for bus stop spacing of ODPB service will be proposed by using a spatial-temporal analysis of the transport demand. To capture the unique characteristics of ODPB service, the model constraints will include the length of the segment, the configuration of shed line, minimum stop spacing, vehicle capacity limitation and so on.

1.3 Outlines

The rest of this thesis is organised as follows:

Chapter 2 reviews the development shared public transport services and relevant research studies. The history of the bike sharing system and ODPB is introduced, as well as the research outcomes from traditional public transport planning.

Chapter 3 proposes an overall framework to lead the public transport service planning and design. The essential components and special requirements are defined according to the characteristic of the bike sharing system and ODPB.

Chapter 4 develops a set of methods to allocate the bikes in different market scenarios. Three situations have been considered with both monopoly and competition conditions. An improved community structure detection algorithm is developed to determine the boundaries of the service area for each allocation position. A typical benchmark network Sioux Falls network is applied to test the performance of the proposed algorithms. A real large-scale network in Singapore is used to test the practicability of the proposed methods.

Chapter 5 builds a new pricing strategy with the negative price for dockless bike sharing service. Traveller behaviour is modelled by a user equilibrium dynamic traffic assignment model, in which travellers are allowed to use single or combined travel mode to complete a first/last mile trip from the walk, bike, and bus. Two numerical examples are designed to test the effect of introducing the proposed pricing strategy compared to the traditional positive price and free price. Two numerical examples are given to illustrate the effect of the proposed pricing strategy for bike relocation, travel behaviour, and network performance.

Chapter 6 provides a systematic account of the impact of ODPB service on a multimodal transport corridor by using a traffic equilibrium formulation. The choices of mode and route/service are taken into account simultaneously in a logit-type formula with a discrete set of value-of-times (VOTs) for multi-class commuters. The resultant traffic equilibrium condition is expressed as a variational inequality (VI) problem. One ideal corridor is used to test the sensitivity of the model responses to the numerical value of the parameters. Two numerical examples are used to

investigate the performance of ODPB service: one is a simple and idealised corridor, and the other one is a realistic travel corridor in Beijing.

Chapter 7 develops a model to investigate the service selection problem between P&R and ODPB with considering the following three questions: first, are there any factors that significantly affect the commuter's service preference on a macroscopic level, like urban city structure, population density? Second, what is the suitable condition to develop P&R or ODPB? Third, how does the operator operate the service to ensure profit while satisfying the requirements of the users at the same time? A set of numerical results have been given to illustrate the model's effectiveness.

Chapter 8 proposes an optimisation model for ODPB stop location. The objective is to minimise the total travel time of all the passengers. According to the unique characteristic of ODPB service, the model constraints include the length of the segment, relation of the shed line and the bus stop location, minimum stop spacing, vehicle capacity and nonnegative constraints. A set of numerical results are presented.

Chapter 9 summarises the thesis and points out the plan.

CHAPTER 2 LITERATURE REVIEW

2.1 Shared Public Transport Services

2.1.1 Bike Sharing

Bike sharing or the public bicycle system has been flourishing over the past decades since its first introduction in Amsterdam in 1965. It is one mobility strategy to share the use of a bicycle fleet. Its many benefits such as offer alternative public transport, encourage more cycling, improve city image, provide complimentary service linking transit stations, reduce congestion and increase accessibility. Bike sharing went through an evolution from first-generation to the fourth generation. Although there is often variability, a bike share program can be designated “first-generation” if there is a fleet of bikes, usually recovered from abandonment and painted the same colour, which is fixed up and distributed throughout an area available for free use. The second generation applied locking system with coin access at a dedicated locking location, which tried to correct for the problems of disorganisation and theft experienced by the first generation. The third generation of bike share programs is the current popular scheme of first 30 minutes of bike use for free yet harnesses current technology “smart card” in order to make the bike shares even more efficient. The fourth generation realised real-time tracking of a bike, which enables people to check with their cell phones for when and where the specially designated bikes are available (DeMaio, 2009). The fourth generation has been improved and now is widely known as dockless bike sharing. This system has already taken over in many cities around the world such as Beijing, Singapore, London, and Washington DC. The idea is that riders do not have to return the bike to a set place and can leave the bike anywhere. Everything is controlled through GPS sensors in the bikes and an app in the rider’s phone. Lots of cities are introducing these on a large scale with one or several operators, in which ofo is the largest bike sharing firms in the world. Till Feb 2018, ofo has connected more than 10 million bikes to over 200 million users in more than 20 countries (e.g. Australia, China, Singapore) (ofo, 2018). The platform generates more than 32 million daily transactions and has provided global users in 250+ cities with over 6 billion efficient, convenient, green rides.

Great research attentions have been paid to bike sharing schemes in these ten years. At present, most of the modelling studies focus on station-based bike sharing issues,

such as station location selection, demand prediction, inventory rebalancing, vehicle routing, and pricing analysis (Forma et al., 2015; Bordagaray et al., 2016; Faghih-Imani et al., 2017; Pal and Zhang, 2017; Datner et al., 2017). Lots of valuable approaches have been developed in these topics, and excellent results have been obtained in applications. Reviews of the state of the art of these studies are given in Fishman (2016) and Pucher and Buehler (2017).

Significant advances have been realised recently in the analysis of dockless bike sharing problems. Shen et al. (2018) analysed the spatiotemporal patterns of bike usage through an autoregressive model using the dockless bikes' GPS data in Singapore. The models explored the impact of bike fleet size, surrounding built environment, access to public transportation, bicycle infrastructure, and weather conditions on the usage of dockless bikes. Ai et al. (2018) measured the transfer tolerance for dockless bike sharing mode. s. Results indicated that dockless bike-sharing users concern more about the time cost of finding an available bike and the delay in traffic signals. Zhou and Ni (2018) investigated the modal shift from walking and bus to dockless bike sharing through a binary logistic model. Based on marginal effects, the time of walking or bus riding is the most critical factor. The above studies are all about the usage pattern analysis for a dockless bike sharing system. Recently, Pan et al. (2018) modelled the bike rebalancing problem in dockless bike sharing system as a Markov decision process. To the best knowledge of the author, this is the first work that has been published in the area of planning and operation of the dockless bike sharing scheme. Dockless bike sharing has different unique characteristics than station-based bike sharing. The research outcomes from station-based bike sharing could not be directly applied in dockless bike sharing planning.

2.1.2 ODPB

Generally, ODPB service is a special kind of demand responsive transport (DRT). This concept was introduced in the 1960s to improve the transportation accessibility for the elderly and persons with disabilities as well as to those who live in small rural areas where transit demand is almost non-existent. The most intensive academic research into on-demand transit was at MIT starting in 1970, and the work resulted in heuristic algorithms and a demonstration project by Wilson and Colvin (1977). Some numerous studies have been conducted on this subject (Hall 1970; Louviere

1979; Miller 1989). However, due to low demand requests at that time and the deficiencies in communication and computer technology to effectively manage such systems, providing on-demand service to the general public is too expensive. The service is always operated as a complementary paratransit service to a particular group of the population such as the elderly and disabled who have difficulties to access regular public transit system. Different from regular transit, paratransit commonly uses small to medium sized vehicles to provide shared-ride, door-to-door services. Due to its particular applicability, recent years the researchers developed mainly in terms of system design, (Fu 2003; Aldaihani and Dessouky 2003; Sandlin and Anderson 2004; Karlaftis 2005; Quadrifoglio et al. 2008) contributed their work to model the service frequencies, vehicle types, system capacity, and service quality problem. Horn (2002a) proposed a public transport performance evaluation system with particular attention to DRT's performance in a multimodal transport network. Horn (2002b) tested this framework by considering DRT fleet scheduling and dispatching issues. Horn (2004) also described journey-planning procedures designed for use in a traveller information system covering DRT services, where alternative fare structures were considered. Edwards and Watkins (2013) compared the fixed-route transport system and the DRT by using passenger survey data, published transit schedules, and optimal routing techniques. Results showed that DRT could be used to improve transit service levels in low-demand areas. Davison et al. (2014) surveyed to examine and assess the design and performance of DRT schemes in Britain. Results showed that funding or the commercial potential for DRT continued to be the factor which required the most attention from practitioners and policy-makers. Furuhata et al. (2015) determined properties of cost-sharing mechanisms that make DRT systems attractive to both operators and passengers, namely proportional online cost sharing, and examined this mechanism in theory and computational experiments. Amirgholy and Gonzales (2016) proposed an analytical model to approximate the agency's operating cost for running a DRT system, where dynamic demand is considered. This study compared several optimisation strategies using a numerical example. Jokinen (2016) presented a model addressing optimal welfare policies, and unique cost structures of DRT, where three optimal pricing structures were discussed. The development of technologies in cloud computing, smartphone, GPS and traffic control provides the basis for the communication between travellers and operators.

Within a short span of five years, ODPB has witnessed substantial growth in the number of service providers across the globe. Due to its emerging status, the research on ODPB is quite limited. Tsubouchi et al. (2009) programmed the vehicle-choosing algorithm and routing algorithm to design route and schedule for the vehicle chosen to serve the request. Tsubouchi et al. (2010) developed a cloud computing system from the cost viewpoint, and the result of field tests shows that the system is valid for different city types. Raymond et al. (2011) built a location recommendation system that predicts data would help riders during the reservation process and help target potential riders when buses are idle. Kameda et al. (2011) assumed the on-demand bus use electric vehicles and used taxi probe data to optimise the placement of charging stations in a service area. Hamilton and Sankaranarayanan (2013) built an intelligent agent-based radio-frequency identification system for bus scheduling and ticketing. Liu et al. (2016) applied a methodological analysis framework to quantify operational performance measures that enable the comparison of the different travel modes. Cao and Wang (2017) proposed an optimisation method of passenger assignment for ODPB, which guarantees benefits to passengers by balancing the elements of travel time, waiting time, delay, and economic cost. Tong et al. (2017) developed a model for jointly optimising passenger-to-vehicle assignment and vehicle routing.

2.2 Relevant Bike Sharing Studies

2.2.1 Bike Allocation

Bike sharing, or the public bicycle system has been flourishing over the past decades since its first introduction in Amsterdam in 1965 (Zhang et al, 2017). It is one mobility strategy to share use of a bicycle fleet. Its many benefits include offer alternative public transport, encourage more cycling, improve city image, provide complimentary service linking transit stations, reduce congestion and increase accessibility. Bike sharing went through an evolution from first-generation to fourth generation. The four generation now is wildly known as dockless bike sharing. The idea is that riders do not have to return the bike to a set place and can leave the bike anywhere – and the city is saturated enough with these bikes so that riders are always easy to find one. This system has already taken over in many cities around the world

such as Beijing, Singapore, London and Washington DC. It has provided global users in 250+ cities with over 6 billion efficient, convenient, green rides.

Great research attentions have been paid to bike sharing schemes in these ten years. At present, most of the modelling studies focus on station-based bike sharing issues, such as station location selection, demand prediction, inventory rebalancing, vehicle routing, and pricing analysis (Ho and Szeto, 2014; Forma et al., 2015; Bordagaray et al., 2016; Szeto et al., 2016; Faghih-Imani et al., 2017; Pal and Zhang, 2017; Datner et al., 2017; Ho and Szeto, 2014). Lots of valuable approaches have been developed in these topics and excellent results have been obtained in applications. Reviews of the state of the art of these studies are given in Fishman (2016) and Pucher and Buehler (2017).

Important advances have been realised recently in the analysis of dockless bike sharing problems. Shen et al. (2018) analysed the spatiotemporal patterns of bike usage through an autoregressive model using the dockless bikes' Global Positioning System data in Singapore. The models explored the impact of bike fleet size, surrounding built environment, access to public transportation, bicycle infrastructure, and weather conditions on the usage of dockless bikes. Ai et al. (2018) measured the transfer tolerance for dockless bike sharing mode. s. Results indicated that dockless bike-sharing users concern more about the time cost of finding available bike and the delay in traffic signals. Zhou and Ni (2018) investigated the modal shift from walking and bus to dockless bike sharing through a binary logistic model. Based on marginal effects, time of walking or bus riding is the most critical factor. The above studies are all about the usage pattern analysis for a dockless bike sharing system. Recently, Pan et al. (2018) modelled the bike rebalancing problem in dockless bike sharing system as a Markov decision process. Dockless bike sharing has different unique characteristics than station-based bike sharing. The research outcomes from station-based bike sharing could not be directly applied in dockless bike sharing planning.

As dockless bike sharing is a new and growing industry, there is only one or few limited operators in the market. For example, in China, there are two main dockless bike sharing operators, Ofo and Mobike, which takes a total market share of more than 90 percent (Ouyang, 2018). The dockless bike sharing service will soon be launched in many cities with one or several operators. Since it is a new service in the

market, the demand is hardly perfectly predicted. Moreover, oversupply leads to a waste of resources in terms of money and road space. Considering the limited budget and unpredictable demand, how to allocate the bikes efficiently to obtain maximum benefits? Where and how many bikes should the operator be planned? How large area could these bikes be covered? Such research findings can be helpful in provide guidance in planning dockless bike sharing schemes.

Complex network is a graph (network) with non-trivial topological features that occur in graphs modelling of real systems. Complex network includes lots of characteristics, such as small-world property, heavy-tailed degree distributions, spreading and immunization, community structure, robustness, vulnerability and so on. Researchers have applied complex network theory to study real networks in many fields, such as social network, informational network, neural network, transport network and so on (Huang et al., 2005; Kaluza et al., 2010; Rubinov and Sporns, 2010; Sporns, 2011). However, the research work in the field of transport is limited, mainly focused on finding methods to represent the transport system and evaluate transport networks (Sen et al., 2003; Chen et al., 2007; Tang et al., 2014; Yang et al., 2014; Zhu and Luo, 2016; Murali et al., 2016). Few studies have applied the complex network theory in practice.

This paper shall utilise the community structure theory to design the bike allocation strategies in different scenarios. Community structure is one common characteristic of complex network. A group of vertices in a network is easily to cluster together with high concentrations of edges and generate a structure in a hierarchical way. This phenomenon in real network is defined as community structure or clustering (Girvan and Newman, 2002). Community, also known as cluster or module, is a group of vertices which have highly similarities in the network. The community structure makes the generation and evolution of networked system much quicker and more stable than if the system is unstructured (Zhou et al, 2012).

The formation of community structure will impede the distribution of bikes, as the bikes are likely distributed inside of each community. To effectively cover the network using limited number of bikes, allocation position of bikes must have the strongest effect in the network, which are the key vertices in the community. For transport system, it could be abstracted as a complex network, wherein the vertices

(nodes) represent the traffic analysis zones (TAZs) and the edges (links) represent the relationships among TAZs. In a realistic transportation network, the edges are distributed homogeneously among the vertices. For example, the degree which is the number of neighbours of a vertex is binomial so that most vertices have the same or similar degree. In reality, transportation network reflects significant inhomogeneities due to high level of order and organisation. The degree distribution is broad, with a tail that often follows a power law: therefore, many vertices with low degree coexist with some vertices with large degree (Fortunato, 2010). Dockless bike sharing system exists in urban transport system, which also has the feature of community structure.

2.2.2 Bike Relocation

There are two types of approaches to solve the bike relocation problem: user-based approach and operator-based approach. The user-based approach is to guide the users to leave their bikes at specific areas to balance the bike distribution in the system. The operator-based approach is to relocate the bikes by operators using relocation vehicles (e.g. small truck). The operator-based approach is commonly used in practice, which is indeed the vehicle routing problem or pickup and delivery problem. Various models and algorithms have been developed to describe and solve the operator-based problem (Raviv et al., 2013; Chemla et al., 2013a; Dell'Amico et al., 2014; Forma et al., 2015; Erdoğan et al., 2015; Schuijbroek et al., 2017; Shui and Szeto, 2018; Szeto and Shui, 2018; Ho and Szeto, 2014; Ho and Szeto, 2017, Kadri et al., 2016, Szeto et al., 2016). Most of the existing studies dealt with the station-based bike sharing system. Nevertheless, very few researches worked on the relocation problem for the recently emerging dockless bike sharing system; meanwhile, they all applied the operator-based relocation approach. For example, Reiss and Bogenberger (2015 and 2016) used GPS data to obtain an optimal bike distribution model and an operator-based relocation strategy base on it is given. Pal and Zhang (2017) presented a Novel Mixed Integer Linear Program for tackling static rebalancing with single and multiple vehicles. Caggiani et al. (2018) developed a comprehensive dynamic bike relocation framework that includes the prediction and relocation Decision Support System.

In the literature, very few studies have been carried out for bike relocation using user-based approach, neither for station-based bike sharing system nor dockless bike

sharing system. The Paris Vélib' system provides static incentives to users who return their bicycle to given stations (Laporte et al., 2015). Chemla et al. (2013b) and Pfrommer et al. (2014) proposed dynamic pricing strategies to encourage users to return bikes to empty nodes. Singla et al. (2015) extended the model of Pfrommer et al. (2014) by incorporating a crowdsourcing mechanism by providing users with alternate choices to pick or return bikes in exchange for monetary incentives. Dötterl et al. (2017) developed an event-driven agent architecture to predict the future demand with situation-aware incentives. Reiss and Bogenberger (2017) found that the user-based approach could save cost for the operator and supply a balanced bike distribution in the system. However, existing studies have shown that the effect of user-based relocation approach is not as significant as the operator-based relocation approach in real operation (Pfrommer et al., 2014; Reiss and Bogenberger, 2017). The reason is that the traditional user-based approach applies low cost pricing strategy, which may not be sufficient to induce the travellers to alter their travel behaviour. Therefore, this study introduces a new concept of negative price to achieve the objective of balanced bike relocation.

Generally, travellers need to pay a positive price for using a bike. Traveller is willing to pay as long as the positive utility from the received service could offset the disutility of making the payment. From the operators' point of view, the price floor is determined by the marginal cost, that is, the price can at least make a positive contribution. Normally, the marginal cost for a mature transport service (e.g. bus, metro) is greater than zero, i.e., the price of using this service is positive. The phrase of "negative price" was first introduced in the studies of good storage (Working, 1949; Wright and Williams, 1989). The price of certain good will decrease after it has been stored for some time. People still choose to store them although they might suffer loss. Researchers concluded that the storage leads to a negative price. Different from its initial definition of price drop after storage, negative price now has been defined as the phenomenon that the customers acquire a good or service with extra payment from sellers. Unlike below-cost price or free price, customers do not need to pay anything; instead, they receive the good or service as well as payment from seller. Some early studies also pointed out that the price of the good or service could be zero or negative from theoretical point of view (Dantzig et al., 1979; Brihaye et al., 2014). In application, the negative price strategy was firstly and mainly applied in energy

field. In Germany, parts of the electricity production stemmed from wind power. As wind is an intermitting energy source, the generated power reacts strongly to the stochastic wind power infeed (Nicolosi, 2010). As the power could not be stored, when the demand is lower, the power supplier suffered from huge cost to quickly adjust the status of equipment to reduce the power generation. Until 2008, power supplier started to pay negative price to customers who consume the power when the power generation exceeds the demand. It has been verified that negative price strategy is a valid method to reduce power supplier's loss (Brandstätt et al., 2011; Simon, 2016). Another application area of negative price is the internet. Microsoft launched a Bing Cashback programme in 2008 that allowed advertisers to bid for search advertisements by offering a percentage of sales back to users (Fried, 2010). However, this programme was terminated in 2010 as the contributions from users do not achieve the desired range of vision. In this study, we investigate the feasibility of negative price concept in dockless bike sharing system. Facing unbalanced distribution in the system, operators could consider offering rewards to the users who cycle from oversupplied area to undersupplied area. This strategy could achieve bike relocation as well as operation cost reduction, and thus bring a positive contribution to the operation of dockless bike sharing service.

Dynamic traffic assignment (DTA) model is one of the key methods to support the evaluation of different pricing strategies in a network context. DTA was first proposed by Merchant and Nemhauser (1976) in considering time varying flows. Various DTA methods (e.g. analytical models, simulation models, and cell models) have been developed and applied in transportation analysis (Lo and Szeto, 2004; Szeto and Lo, 2006; Wang and Du, 2016). One major application of DTA is the evaluation of dynamic road pricing strategies (Lu et al., 2008; Zheng et al., 2012; Zhang et al., 2013). Some researchers also test the effect of different pricing strategies in taxi market (Ciari et al., 2015; Qian et al., 2017; Long et al., 2017). Recently, Li et al. (2018) examined the different pricing strategies in free-floating car-sharing scheme and found that rental price has significant implications for travel demand management. Xu et al., (2018) also pointed out the demand for car-sharing service is elastic with respect to the trip price. These studies all followed the traditional pricing strategy with positive price at any time. A negative price will significantly affect the travelers' behavior and thus lead to a different flow distribution. This thesis would

find out whether the proposed pricing strategy is feasible and effective in dockless bike sharing system.

2.3 Relevant ODPB Studies

2.3.1 Feasibility Analysis of New Service

Many studies have been done to examine the impact of new public transport services using mathematical programming approach and simulation-based approach (Liu et al. 2009; Memon et al. 2014; Meng et al. 2014). One group of previous studies on the topic of Bus Rapid Transit (BRT) investigated its impact on the transit ridership and overall system performance (Godavarthi et al. 2014; Wang et al. 2012; Zheng and Peeta 2015). Another major group of existing studies modelled the P&R behaviour to examine the effects on the mode shift, network performance, social welfare, and facility planning (Du and Wang 2014; Li et al. 2007; Wang and Du 2013; Wang et al. 2004). Several studies applied the network models to analyse novel taxi-based services, e.g. dial-a-ride, shared-taxi, and e-hailing, with focuses on the spatiotemporal relationship between taxi supply and demand (He and Shen 2015; Marković et al. 2015; Parragh 2011; Santi et al. 2014). With the recent resurgence of bike usage, many cities have piloted bike sharing services to improve first and last mile connectivity to public transport services (Meng et al., 2016). Studies regarding the bike sharing repositioning issue and the integration planning of public transport and bike sharing are getting more and more attention (Forma et al., 2015; Ho and Szeto, 2014; Pucher and Buehler, 2009). Recently, the integration of conventional and flexible bus services has also been considered (Chandra and Quadrioglio 2013; Kim and Schonfeld 2014; Quadrioglio and Li 2009), wherein headway and buffers were optimised to coordinate bus arrivals and minimise the transfer delay. In summary, the studies mentioned above provide potential directions for further operation of innovative public transport services. However, due to the ODPB service's recent emergence, research in this area is still in its description and conceptual level (Cao and Wang 2016; Liu et al. 2016), while the analytical or numerical investigation is very limited.

2.3.2 Service Selection

Various public transport modes have been successfully operated in urban cities, like metro, light rail transit (LRT), bus rapid transit (BRT), tram, bus, and streetcars. In

the presence of emerging technologies, many innovative public transport services also attract the public's attention, such as carpool, autonomous feeder, P&R, ODPB, bike-sharing. When more than one mode could satisfy the transport demand, transport authorities will compare different alternatives and select one for practical implementation. How to select the suitable service under a specific condition has been discussed through different aspects. The conventional approach is to compare the cost or operational efficiency (Allport, 1981; Bruun, 2005; Stutsman, 2002; Tirachini et al., 2010a; Jara-Díaz and Gschwender, 2009). Recently, some researchers started to explore this issue in detail with optimised parameters of transit lines. Parajuli and Wirasinghe (2001) proposed an analytic decision model which considered the users', operators' and community requirements roughly equally and had identical level of comfort, convenience and other nonquantifiable attributes of performance measures. Tirachini et al. (2010b) compared analytically and numerically the optimised performance of different urban public transport modes including bus, LRT, BRT and metro regarding three objectives: total cost minimisation, profit maximisation, and welfare maximisation. Chen et al. (2015) and Li et al. (2015) aimed to maximise the social welfare of the transit system during the service selection by determining the optimal combination of transit line length, number of stations, station location (or spacing), headway, and fare. Moccia and Laporte (2016) considered optimal stop spacing, train length, crowding cost, and multiple periods in their service selection model, and tested this model in an illustrative example. Barabino and Di Francesco (2016) developed a framework for the involvement of all stakeholders in the characterisation, measurement, and management of the stages of quality monitoring, which was jointly analysed at different planning levels. Sun et al. (2017) explored how the selection of public service can be optimised over a planning horizon. A series of static models, namely total cost minimisation models for uncoordinated bus and rail operations, rail only, bus only, and coordinated bus and rail operations were formulated and solved for a commuter corridor.

However, these previous studies mostly focused on operational strategies, like station spacing, fare, and headway, rather than strategic factors from a macroscopic level, such as population density, distribution of the demand density, residential area size and so on. Also, the previous studies concerned with the comparison among existing

and traditional public transport service, which cannot be applied to consider emerging public transport service modes.

2.3.3 Bus Stop Spacing

Bus stop spacing problem has been attracting much research attention from public transport system planners and researchers. Vuchic and Newell (1968) first systematically summarised the findings from notable publications about this topic between 1913 and 1966. These studies mostly assumed that the population distribution is uniform along the bus service line, and the interstation spacing is constant. To relax this assumption to a more realistic situation, Vuchic and Newell (1968) then proposed a general model to fit all types of population distribution and inconstant interstation spacing. The objective function is to minimise the total passengers' travel time while considering the impact of the operational factors (e.g. access walking time, bus acceleration and deceleration time). Furthermore, they obtained some interesting conclusions, such as the density of stations depends on the number of passengers along the line and on the vehicle.

Vuchic and Newell's seminal paper on this topic has been extended by many following researches, where various objective functions have been proposed, such as minimizing total cost, minimizing user cost, maximizing social welfare, maximizing private profit, maximizing area coverage, maximizing ridership, with consideration of more practical factors, such as network design, bus frequency, route density and bus size, access cost, waiting cost and so on. Mohring (1972) proposed the first microeconomic model, in which the bus schedule and bus stop spacing problems were taken into account simultaneously. Wirasinghe and Ghoneim (1981) initially developed the optimal spacing model with non-uniform many to many travel demand. Ceder et al. (1983) concentrated their efforts on a model aiming to minimise the number of bus stops where the passenger could reach the public bus stop within a certain distance. Kuah and Perl (1988) presented an analytic model for a feeder bus network and found that the optimal route spacing and headway were shown to be cubic root functions of the various parameters. The models proposed in this decade usually assumed that the passenger demand follows a continuous and uniform distribution along a route or over an urban area. The continuous model could analyse and investigate the optimal bus spacing and the effect caused by related parameters.

The resurgence of this topic in this century tended to build the model to select the optimal bus stop locations from limited numbers of potential bus stop locations, which is easier to implement in practice (Furth and Rahbee, 2000; Furth et al., 2007). Moreover, most researchers considered the discrete demand and system situation and would like to make use of auxiliary tools (e.g. GIS) to solve the problem. Murray (2003) proposed a hybrid set covering model which was applied in GIS to decide the location of bus stops under the objection of maximising transit demand coverage over a study area in the city of Brisbane, Australia. In the same year, Sanka et al. (2003) considered various parameters and used the multi-criteria analysis in GIS to decide the optimal bus stop locations in an Indian city. Chien and Qin (2004) developed a more realistic mathematical model where the demand was not distributed uniformly along the route. Furth and Mekuria (2005) developed a software package based on GIS to evaluate the effect of bus stop spacing. The proposed model was tested by analysing two lines in Boston and New York. Alterkawi (2006) also developed a computer program to calculate optimal bus stop spacing and applied in Saudi Arabia. Furth (2007) proposed a parcel-level model base on street network and discrete demand area in GIS, which can relocate the stops and be used to evaluate the influence of removing stops for a Boston line. Ibeas et al. (2010) presented a bi-level optimisation for locating bus stops which considered possible changes under different bus stop configurations. The technique was applied in a Spanish city to analyse the variables influence.

More recently, Medina et al. (2013) developed a simultaneous optimisation model for stop locations and headways with multiperiod demand. The demand was assigned at each point along the corridor, while two variables were considered in the model: the density of stops in the vicinity of each point of the corridor and the frequency of operation in each period. The model was solved by a two-phase solution method and applied in a Greater Santiago corridor. Zheng et al. (2015) proposed a model based on game theory to get the optimal stop spacing, where the Thiessen polygon method was applied to separate the service areas according to the number of stations. Ceder et al. (2015) developed a mathematical modelling approach to place bus stop which considered the uneven topography. Stewart and El-Geneidy (2016) proposed a bus-stop consolidation methodology to remove unnecessary stops, where the objective is whether the operating cost savings will come at the expense of passengers. Moccia

and Laporte (2016) considered the bus stop spacing for optimisation of a transit line with fixed demand. Results showed that faster modes require longer stop spacing, but the ratio of optimal stop spacings among different modes follows a square root formula.

2.4 Summary

This chapter first reviews the development history of two types of shared public transport services, then discusses the existing research outcomes in some key issues. These studies provide the research direction and foundation for the thesis.

CHAPTER 3 PLANNING AND DESIGN FRAMEWORK

In public transport service design, the core question is how to design the service to attract more users. Generally, the public transport service planning and design framework include these basic activities: (1) network route design, (2) timetable and scheduling design and (3) operation strategies design. The systematic decision sequence of these activities is outlined in Figure 3.1. According to the different operational characteristics of different traffic modes, the detailed inputs and outputs are different.

Figure 3.1 The systematic decision sequence of basic design activities

3.1 Network Route Design

In traditional public transport system, the network route-design activity aims at designing a new network or redesigning an existing network. The goal is to define a set of routes for a planning area, with each route associated with a sequence of stops.

Bike mode is entirely different from other modes because it only can be used for one passenger once. Dockless bike sharing operators do not need to design route for cyclists. The primary issue for dockless bike sharing design is to determine the bike allocations places that can make maximum contributions, which is further discussed in Chapter 4.

With the integrated information platform, ODPB operators can easily collect user demand data. User demands that have similar origin area, destination area, departure time and arrival time are aggregated. A route is then designed to serve users between origin and destination areas. For such an ODPB route, stops are generally located only in the origin and destination areas. These stops are designed to be located close to users' residential and workplaces so that passengers can enjoy a door-to-door transport service. After collecting the passengers, ODPB vehicle does not need to stop until reaching the destination. ODPB drivers are also allowed to change the route after collecting the passenger to avoid congestion area. Therefore, the critical issue for ODPB network and route design is the stop location determination in origin and destination area, which is investigated in Chapter 8.

3.2 Timetable and Scheduling Design

Timetable of public transport service should be designed to meet travel demand and reduce the waiting time. As bikes are directly ridden by travellers, there is no timetable and scheduling issue in bike sharing system. Timetable design for ODPB is different from the traditional public transport services. Unlike the traditional public transport services wherein the demand is estimated from historical data, ODPB is designed for fixed traffic demand, in which the travellers have paid the services before operation. Therefore, operator could directly arrange the service based on the demand distribution. Moreover, as ODPB targets the travellers with similar departure and arrival time with same origin and destinations, timetable could be determined precisely fitting with the demand requirement, as well as the crew and vehicle schedule.

3.3 Operation Strategies Design

Due to the one-way use characteristics in bike sharing system, there is an operational issue of imbalanced distribution of bikes over time and space, especially during peak period. For example, most of the users cycle from residential area to the nearest transit stations in first-mile trip stage in the morning. Bikes in residential area are quickly out of stock, and the subsequent potential users could not have access to bikes. In this case, the level of user satisfaction may drop as the users cannot get the service on time. Operator usually utilises a dedicated relocation vehicle (e.g. light truck) to move the bikes from oversupply area to undersupply area. Because of limited resources (e.g.

workforce, budget, vehicle capacity) and high relocation cost (\$3 per bike (DeMaio, 2009)), which could not fundamentally solve the imbalance problem. A dynamic reposition model with negative price strategy for dockless bike sharing system is given in Chapter 5.

The ODPB operation strategy mainly includes operation control and fare design. The operation control aims at improving the reliability of ODPB service, which could utilise the exiting operation control strategies in traditional public transport system. Most ODPB schemes are equipped with GPS devices and monitoring system. ODPB control centre could obtain the real-time location, speed and direction information of all vehicles. These data are used as inputs for future operational planning. The real-time communication between ODPB control centre and driver ensures the flexibility in particular cases. A reasonable fare system plays a vital role in the success of an ODPB system, which could be designed either in a fixed fare structure or a distance-based fare structure. Since ODPB is mostly operated by local public transport agencies, which are usually state-owned enterprises and subsidised by local governments, the primary goal of ODPB fare design is to maximise the total social benefits other than pure fare revenue.

3.4 Summary

Planning and design of a shared public transport service could follow the general public transport system planning process, but with unique features. In the network route design phase, dockless bike sharing system design shall mainly deal with the initial bike allocation, while ODPB system will mainly focus on the stop location determination. In timetable and scheduling design phase, there are no relevant issues in dockless bike sharing system, while ODPB service will be provided precisely to meet the requirements from real demand. In the last operation strategies design phase, dockless bike sharing system shall solve the bike relocation issue while ODPB service can utilise the exiting outcomes from traditional public transport services.

CHAPTER 4 DETECTING THE INITIAL BIKE ALLOCATION AND COVERAGE OF DOCKLESS BIKE SHARING SYSTEM

4.1 Introduction

Dockless bike sharing has different unique characteristics than station-based bike sharing. The research outcomes from station-based bike sharing could not be directly applied in dockless bike sharing planning.

As dockless bike sharing is a new and growing industry, there are only one or few limited operators in the market. For example, in China, there are two main dockless bike sharing operators, ofo and Mobike, which takes a total market share of more than 90 percent (Ouyang, 2018). The dockless bike sharing service will soon be launched in many cities with one or several operators. Since it is a new service in the market, the demand is hardly perfectly predicted. Moreover, oversupply leads to a waste of resources regarding money and road space. Considering the limited budget and unpredictable demand, how to allocate the bikes efficiently to obtain maximum benefits? Where and how many bikes should the operator be planned? How large area could these bikes be covered? Such research findings can be helpful in providing guidance in planning dockless bike sharing schemes.

Based on the issues above, the principal objective of this study is to determine the bike allocation strategy in a dockless bike sharing market. Individually, two market scenarios have been considered. In scenario A, the market leader has potential competition pressure from the forthcoming market follower. In scenario B, the market leader has no potential competitor. To achieve the objective, three research questions need to be discussed through the modelling analysis, which are: firstly, where to allocate the bikes; secondly, how large is the coverage area for each bike allocation point; thirdly, how frequently to allocate the bikes.

This chapter shall utilise the community structure theory to design the bike allocation strategies in different scenarios. The formation of community structure will impede the distribution of bikes, as the bikes are likely distributed inside of each community. To adequately cover the network using a limited number of bikes, allocation position of bikes must have the most substantial effect in the network, which are the critical vertices in the community. For transport system, it could be abstracted as a complex

network, wherein the vertices (nodes) represent the TAZs, and the edges (links) represent the relationships among TAZs. In a realistic transportation network, the edges are distributed homogeneously among the vertices. For example, the degree which is the number of neighbours of a vertex is binomial so that most vertices have the same or similar degree. In reality, transportation network reflects significant inhomogeneities due to a high level of order and organisation. The degree distribution is broad, with a tail that often follows a power law: therefore, many vertices with low degree coexist with some vertices with a large degree (Fortunato, 2010). Dockless bike sharing system exists in urban transport system, which also has the feature of community structure.

4.2 Basic Definition

As shown in Figure 4.1, vertices within each community are densely connected; meanwhile, the three communities are less connected among each other. In dockless bike sharing system, when the bikes are placed at one vertex, due to the stochastic characteristic of demand, these public bikes shall be frequently re-distributed to other vertices. Different local features (e.g. demand level, land use characteristics, route direction and geographical constraints) will lead to different coupling degrees among vertices. Thus, community structure will be formed by clustering the vertices with similar coupling degree. Vertices in the same community are densely connected with high traffic volume, while vertices in different communities are sparsely connected with low or even zero traffic volume.

There are various types of methods to detect the community structure. Among them, modularity-based method has been widely adopted in large-size network due to its fast and efficient computation. The most typical modularity-based method is the greedy method proposed by Newman (2004), which is an agglomerative hierarchical clustering method. Applying it into a dockless bike sharing system, one can represent the network by a complex network $G = (V, E, W)$, where V, E, W represent the set of vertex, edge and weight, respectively. There are N number of vertices and E number of edge in G . Several key concepts are defined as follows in a dockless bike sharing network.

Figure 4.1 A sketch of a small network with three communities

Weight of edge: w_{uv} is the weight of direct edge from vertex u to v . The higher value of w_{uv} reflects the tie between vertices u and v is stronger. Conversely, the lower value of w_{uv} indicates the tie between vertices u and v is weaker.

Weight of vertex: ζ_u is the weight of vertex u , which is the sum of the weight of the connected edges, that is $\zeta_u = \sum_v w_{uv}$. Higher weight of vertex means that this vertex has a significant influence onto the connectivity of the network. In an unweighted network, $w_{uv}=1$, the weight of vertex is equal to the degree of vertex.

Degree of vertex: k_u is the degree of vertex u . Each edge in the network connects two vertices, which are neighbours of each other. All the neighbours of one vertex is called the vertex's neighbourhood. The number of neighbours in the neighbourhood is the degree of the vertex. Larger degree of vertex means that the vertex has more neighbours and therefore it has a higher level of importance towards the connectivity of the network.

Modularity: M is a quality function to test whether clustering is meaningful in community structure theory. Higher modularity of the network means that there is a definite community structure in the network. The concept of modularity is proposed based on coordination theory which is firstly proposed by Newman (2004). The philosophy is to compare the network after partition with a null model, which has the

same configuration as the original network but contains no community structure because of random placement of its edges (Chang et al., 2012).

In an unweighted network, set A_{uv} is an element in the adjacency matrix in the network. If vertex u is linked with vertex v , $A_{uv} = 1$; otherwise, $A_{uv} = 0$. It is assumed that vertices u and v belong to the communities of c_u and c_v . The ratio of number of edges in the community to the number of edges in the network is calculated by:

$$\frac{\sum_{uv} A_{uv} \delta(c_u, c_v)}{\sum_{uv} A_{uv}} = \frac{1}{2m} \sum_{uv} A_{uv} \delta(c_u, c_v) \quad (4.1)$$

where $\delta(c_u, c_v)$ is the relationship function of c_u and c_v . $\delta(c_u, c_v)$ is defined as follows: if vertices u and v belong to the same community, that is $c_u = c_v$ then $\delta(c_u, c_v) = 1$. Otherwise, $\delta(c_u, c_v) = 0$. m is the total number of edges in the network. Modularity M is defined as the fraction of edges within community minus the expected fraction of such edges in a randomised null model of the network (Yan and Gerstein, 2017). Mathematically, the modularity is calculated as:

$$M = \frac{1}{2m} \sum_{uv} \left[A_{uv} - \frac{k_u k_v}{2m} \right] \delta(c_u, c_v) \quad (4.2)$$

where k_u is the degree of vertex u .

$$k_u = \sum_v A_{uv} \quad (4.3)$$

Let e_{ij} be the ratio of the total number of edges in communities i and j to the total number of edges in the network, a_i be the ratio of number of edges connected with all the vertexes in community i to the total number of edges in the network. Then, we have,

$$e_{ij} = \frac{1}{2m} \sum_{uv} A_{uv} \delta(c_u, i) \delta(c_v, j) \quad (4.4)$$

$$a_i = \frac{1}{2m} \sum_u k_u \delta(c_u, i) \quad (4.5)$$

To simplify the calculation of the modularity M, we assume that the network has been divided into n ($i = 1, 2, \dots, n$) number of communities. Eq.(4.2) could be expressed as:

$$\begin{aligned}
M &= \frac{1}{2m} \sum_{uv} \left[A_{uv} - \frac{k_u k_v}{2m} \right] \delta(c_u, c_v) \\
&= \frac{1}{2m} \sum_{uv} \left[A_{uv} - \frac{k_u k_v}{2m} \right] \sum_i \delta(c_u, i) \delta(c_v, i) \\
&= \sum_{i=1}^n \left[\frac{1}{2m} \sum_{uv} A_{uv} \delta(c_u, i) \delta(c_v, i) - \frac{1}{2m} \sum_u k_u \delta(c_u, i) \frac{1}{2m} \sum_v k_v \delta(c_v, i) \right] \\
&= \sum_{i=1}^n (e_{ii} - a_i^2)
\end{aligned} \tag{4.6}$$

4.3 Community Structure-Based Framework: Obtaining Optimal Bike Allocation Strategy

Some assumptions are made as follows:

A1: Operators seek to minimise the number of bikes to achieve the optimal resource allocation.

A2: Operators are not able to predict precisely the demand and will allocate the bikes in batches.

A3: Operators only have a small number of new bikes available for each allocation as they need to test the market before placing a new order of bikes.

A4: Operators will allocate the bikes repeatedly when the market is in the status of undersupply.

A5: Riders will distribute bikes via trips. The repositioning issue is not considered in this study.

A6: Two scenarios are considered in this study as follows: scenario A: The market leader has potential competitor, which shall enter the market soon. scenario B: The market leader has no potential competitor shortly. The time of competitive market entry for the two scenarios is illustrated in Figure 4.2.

Figure 4.2 *The time of competitive market entry for the two scenarios*

Since dockless bike sharing is an emerging market, the market leader may not be able to precisely predict the rider demand. Besides, considering the production time and the operation budget, the most efficient way for bikes allocation is to allocate the bikes in small batches to test the market. Generally, an allocation strategy framework could be designed in an iterative process:

Step 1: initialise the system and set the necessary parameters. Once the planning area is confirmed, a certain number of TAZs need to be classified. The total demand for the study area and each TAZ is predicted according to the land use pattern.

Step 2: detect the community structure in the network and find the critical vertex in each community. The operator defines the features of the critical vertex according to its market objective. The community area with boundaries is the service coverage area of this vertex.

Step 3: allocate a certain number of bikes to the vital vertex in each community.

Step 4: check whether the number of bikes could meet the demand requirement based on the ratio of zero inventory duration to the whole day duration. If the ratio is below a certain point, it means that the bike supply is near optimal. The allocation is finished. Otherwise, the demand is updated and go to step 1 to allocate new bikes.

Since each bike is equipped with GPS, operator could easily locate each individual bike and analyse its utilization. The updated demand is set to be equal to the difference between the total demand of each TAZ minus the available number of bikes at TAZ. Apparently, when a vertex always has available bikes during the whole day, it reflects that the bikes at this vertex is oversupplied.

4.3.1 Scenario A

Considering the potential competitor, the primary goal of the market leader is to quickly dominate the market before the competitor enters into the market. Therefore,

the objective of the market leader is to attract more riders in a short time period. In this circumstance, the best strategy for the market leader is to allocate the bikes at the positions which could distribute the bikes in a fast pace. The bike distribution speed relies on the demand level of the vertex, which is the weight of the vertex. From a network (physical) perspective, the public bikes will be distributed to the whole area in the community with the fastest speed only when the bikes are placed on the vertex with the highest weight in the network. As shown in Figure 4.1, if the network is unweighted, the allocation points are vertex 5, 9 and 15 in community c1, c2 and c3 respectively. In this scenario, the vertex with highest weight would be set to be the allocation point, and the associated community is its catchment area.

Employing this objective in the allocation strategy design framework, we propose a modified community structure detection algorithm. As is well known, it is a NP hard problem to find the optimal solution to a network partitioning problem in a large-scale transport network (Brandes et al., 2008). Aiming to tackle the high complexity in computation, many researches have been conducted to ease the procedure and improve the accuracy of solution. One of the most classical algorithms is the Fast Newman (F-N) algorithm, which is indeed a bottom-up agglomerative algorithm. The main steps are listed as follows:

Step 1: Network initialisation. Each vertex represents one community;

Step 2: Community combination. Two communities are combined together only if it could result in the greatest increase in modularity;

Step 3: Repeat Step 2 until the whole network is jointed as one community;

Step 4: Optimal result selection. The optimal result of community structure is the best cut with the largest modularity in the process.

However, the F-N algorithm is proposed for unweighted network. To adopt it in the bike sharing network which has weightage on each edge, some adjustments have been made. First, e_{ij} is redefined as:

$$e_{ij} = \frac{\sum_{uv} w_{uv} A_{uv} \delta(c_u, i) \delta(c_v, j)}{\sum_{uv} w_{uv}} \quad (4.7)$$

where w_{uv} is the edge weight. Similar with the formulation of Gravity model, w_{uv} is proportional to the passenger demand between vertices u and v , and inversely proportional to the square of the distance between vertices u and v . Meanwhile, cycling is not suitable for overly short trips (e.g. distance less than 0.1km) or overly long trips (e.g. distance more than 3km). Thus, the edge weight is defined as:

$$w_{uv} = \begin{cases} (p_u p_v) / (d_{uv}^2), & \forall d_{uv} \in [0.1, 3] \\ 0, & \text{otherwise} \end{cases} \quad (4.8)$$

where p_u and p_v are the passenger demand of vertex u and v , d_{uv} is the shortest distance between vertex u and v . In bike sharing network, an edge with higher weight means there is more substantial bike sharing flow on this edge.

As the weight of vertex and degree of vertex are both the indicators from the perspective of vertex, which is only one-side of the network evaluation, an improvement is achieved by taking the weight of edge, that is:

$$a_i = \gamma \sum_j e_{ij} + (1 - \gamma) \cdot k_i / (2m) \quad (4.9)$$

The detailed procedures of finding the optimal bikes allocation strategy under scenario A are listed as follows:

Step 1: divide the whole study area into n number of TAZs;

Step 2: number all TAZs and convert the study area into an undirected network composed of vertices and edges;

Step 3: calculate the correlation weight w_{uv} of two adjacent TAZs u and v by using the method proposed in Eq.(4.8) as the weighted value of each edge in the network, and then construct the corresponding adjacency matrix;

Step 4: initialise the network with a state in which each vertex is one community;

Step 5: combine communities together in pairs sequentially and compute the variation of modularity. ΔM is given by the merging of any two communities of the running partition as follows:

$$\Delta M = e_{ij} + e_{ji} - 2a_i a_j = 2(e_{ij} - a_i a_j) \quad (4.10)$$

Step 6: choose the combination that results in the highest increase or the smallest decrease in M by greedy algorithm, and then compute the current modularity;

Step 7: update the elements e_{ij} ;

Step 8: execute Steps 5–7 repeatedly until all communities merge into one community.

This algorithm would carry out $n-1$ merging operations until completion, and hence, the entire computational complexity is $O((m+n)n)$, or $O(n^2)$ on a sparse graph (Newman, 2004). This algorithm is simple to obtain the optimal decision of the bikes allocation.

4.3.2 Scenario B

If there is no potential competitor in the market in the near future, the market leader has sufficient time to allocate bikes in the network. In this case, with limited number of bikes and available budget, operator would allocate bikes at a minimum number of locations while ensuring to cover the maximum service area. It could be regarded as a problem of finding the best k vertices which could affect the most vertices in the network.

As mentioned before, according to the characteristic of community structure, bikes are mostly used inside the community, and infrequently used among different communities. For example, as illustrated in Figure 4.1, there are three communities in the small unweighted network. Assume that only one public bike allocation vertex is required. If the operator still follows the method in scenario A, vertex 9 will be selected as vertex 9 is the one with the greatest degree in the network. In this case, most of the bikes will be circulated in community c2 while few bikes could go to other communities. On the contrary, if vertex 6 is selected, bikes could be easily distributed to all the three communities so that the catchment area of the bike sharing service could be expanded.

A similar problem has been investigated by Domingos and Domingos (2001, 2002), and be further extended in social network algorithms (Palla and Derenti, 2005; Han et al., 2012; Morone and Makse, 2015). To extend this idea into the application of transportation network, an improved algorithm is proposed in this study.

Three types of vertices are defined: the first type is the directly connected vertex (DCV) to the vertex where the bikes are initially allocated; the second type is the indirectly connected vertex (ICV) to the bike-allocation vertex which is in the same community with DCV; all of the rest vertices fall into the third type. Take Figure 4.1 as an example: if vertex 9 is the allocation node, the vertices that are directly connected to vertex 9, i.e., the DCVs, include nodes 6,7,9,10,11,12,13. The vertices that are directly connected with DCVs and located in the same community, i.e., the ICVs, include vertices 1,2,3,4,5,8. A new indicator Q is defined to measure the importance of vertex in affecting the coverage area as

$$Q_u = \omega_1 \sum_{i \in \Delta N_{DC}^u} \zeta_i + \omega_2 \sum_{i \in \Delta N_{IC}^u} \zeta_i, \forall \omega_1 + \omega_2 = 1 \quad (4.11)$$

$$\Delta N_{DC}^u = N_{DC}^u \cup N_{DC} - N_{DC} \quad (4.12)$$

$$\Delta N_{IC}^u = N_{IC}^u - (N_{DC}^u \cup N_{DC} \cup N_{IC}) \cap N_{IC}^u \quad (4.13)$$

where N_{DC}^u and N_{IC}^u are sets of DCV and ICV of vertex u . N_{DC} and N_{IC} are the sets of all direct and indirect vertices. ω_1 and ω_2 are weight coefficients, which reflect the importance of DCV and ICV. When $\omega_1=1$ and N_{DC}^u and N_{IC}^u are not updated during calculation, and the results are same with the results from the algorithm for Scenario A.

Step1: Obtain the community structure following the steps in scenario A;

Step2: Initialise the vertices set by setting $N_{DC} = \emptyset$ and $N_{IC} = \emptyset$;

Step3: Enumerate all the vertices and get N_{DC} and N_{IC} for each vertex;

Step4: Calculate the value of Q for each vertex u and select the vertex with maximum Q ;

Step5: Update the N_{DC} and N_{IC} ;

Step6: Repeat Steps 3-5 until the total number of selected vertices is equal to a predefined value k or the number of vertices in N_{DC} is equal to the number of vertices in the network.

Scenario A

Applying the algorithm presented in section 4.3.1, we obtain the result of modularity calculation, which is shown in Figure 4.4 and the dendrogram of the clustering result is shown Figure 4.5. Figure 4.4 shows how the modularity changes after clustering. When there are 5 communities in the network, the modularity reaches the highest value. Therefore, to achieve the market objective, 5 vertices need to be selected in the network. Figure 4.5 shows how the vertices are grouped in each community. The numbers at the bottom of the dendrogram represent the vertices in the network. The dendrogram depicts how the vertices are merged together to form communities during the process at different levels. A cross-section of the dendrogram at any level gives the division of the subnetworks at that level. As displayed in Figure 4.4, $M = 0.465$ is the maximum modularity corresponding to a configuration with 5 communities. Thus, the dendrogram needs to be cut for the split of 5 groups at the dashed line depicted in Figure 4.5.

Figure 4.4 Modularity of partition results

Figure 4.5 Dendrogram of clustering results

Figure 4.6 shows the detailed allocation position and coverage area based on the community structure. The optimal allocation position is the vertex with the highest weight in each community. Specifically, vertex 1 itself is a single community with no connection with other vertices. Based on Table 4.1, it could be found that the demand level on vertex 1 is so small that very few trips are generated from or to this vertex. Hence, vertex 1 could be removed from the realistic allocation strategy.

Figure 4.6 First allocation strategy in scenario A (the number on edge denotes the weight of the edge)

The bikes are usually allocated in batches. The bike usage information could be obtained through GPS data in real application, or using Markov chain to simulate, while this study estimated the usage based on the weight of edge. Assume that 150 bikes have been allocated at the selected 4 vertices (6, 10, 12, 18), and each bike will be used twice per day. The number of trips on each edge is estimated by multiplying the total trips by the edge selection probability, which is determined based on the weight of edge. The updated unsatisfied demand is the difference between the total demands and the number of finished trips that we have just estimated. Applying the algorithm in section 4.3.1 again, we can obtain the best allocation strategy for the second time bike allocation, whose results are shown in Figure 4.7(a). Similarly, the

third allocation strategy is determined, as depicted in Figure 4.7(b). Comparing Figure 4.7 with Figure 4.6, one can easily find that, as more and more bikes are allocated into the service market, the unsatisfied demands across this area change and thus lead to different solutions of allocation position and coverage area. For example, in Figure 4.7(b), one can observe that a new community structure with single vertex (4.8) is obtained. It means that the supply of bikes could meet the demand requirement at this vertex, which results in less connectivity relationship of this vertex with other vertices.

Figure 4.7 Second and third allocation strategy in scenario A (the numbers on edge denotes the edge weight)

Scenario B

Now, assuming scenario B is prevalent in the market, we apply the proposed solution algorithm in section 3.2 to find the best bike allocation strategy. It is assumed that k allocation positions are to be found. Table 2 lists the best allocation positions and coverage areas when $k=1,2,3$. When $k=1$, only vertex 10 is selected as allocation position. If the number of bikes provided at vertex 10 is sufficiently large, the demand covered by the DCV set of vertexes 10 is 24.84% of the total demand. Considering ICV set of vertexes 10, the total demand coverage by both DCV and ICV sets could increase to 88.28%. When $k=2$, an additional new allocation position vertex 7 is

selected other than vertex 10, in which case the total demand coverage goes to 99.92%. When $k=3$, although the demand coverage by DCV increases to 59.87%, the total demand coverage remains the same with the case when $k=2$. That is, $k=2$ could almost satisfy all the demand requirements in this case. In principle, if k is sufficiently large, the total demand coverage can achieve 100%. However, it would not be a cost-effective operation strategy. The results the demand coverage are also illustrated in Figure 4.8, where the vertices with dark background are the vertices in the set of DCV. For example, vertex 10 could cover three communities in the bottom of the network. The addition of vertex 7 for bike allocation could extend the coverage area to right-upper corner in the network. However, if vertex 20 is also included as a position for bike allocation, the total demand coverage area remains the same.

Table 4.2 Allocation positions and coverage areas when $k=1, 2, 3$

k	Key vertex	DCV	Demand Coverage by DCV	ICV	Demand Coverage by ICV	Total Demand Coverage
$k=1$	10	9,10,11,15,16,17	24.84%	3,4,7,12,13,14,18,19,20,21,22,23,24	63.44%	88.28%
$k=2$	10,7	7,8,9,10,11,15,16,17,18	43.69%	2,3,4,5,6,12,13,14,19,20,21,22,23,24	56.23%	99.92%
$k=3$	10,7,20	7,8,9,10,11,15,16,17,18,19,20,21,22	59.87%	2,3,4,5,6,12,13,14,23,24	40.05%	99.92%

Figure 4.8 Demand coverage area when $k=1, 2, 3$

From Table 4.2 and Figure 4.8, one can conclude that, selecting only two allocation positions (i.e., vertices 10 and 7) to allocate sufficient bikes could meet the demand

requirements for this case. Along with demand updating, the allocation strategy can follow the same procedure in Scenario A.

4.4.2 Large Scale Network In Singapore

A real large-scale network in Singapore as shown in Figure 4.9 is used to test the validity of the proposed methods in practical applications. Assume the dockless bike sharing demand is in direct proportion to the population size in each TAZ, and the population size in each TAZ is in direct proportion to the maximum plot ratio, which is defined to be equal to the maximum permissible plot ratio marked in the master plan times the area of TAZ. The distance matrix is extracted from google map by using the minimum walking distance. The total running time for Scenario A and B is 0.037 seconds.

Figure 4.10 shows the first allocation position and service area for scenario A. Compared with Figure 4.9, it could be found that lines 1 to 3 is all rivers, which increase the trip distance between vertices on both sides of the river. The long trip distance decreases the connections between vertices on both side of the river and thus forms the boundaries of the community structure. Although the expressway of line 4 increases the trip distance between vertices on both sides, the vertices are still in the same community. It is due to the vertices on the upper side of the expressway are mostly in business or commercial area. Riders could not cross the building like in residential area to the other side to seek for the shortest distance. Riders from vertices like 52, 85 89 to vertex 81 must follow the road so that the real trip distance is larger than the linear distance between coordinates. Metro stations (e.g. vertices 54, 77 and 81), shopping mall (e.g. vertices 54, 77 and 81) and other vertices with high plot ratio (e.g. vertices 58 and 92) have all been selected as allocation positions, which is reasonable in real. It is because they are with high demand level and play the crucial role in the network.

Figure 4.9 Study area in Jurong Lake District in Singapore
(<https://www.ur.gov.sg/maps2/?service=mp#>)

Figure 4.11 shows the results for scenario B when $k=3$. In real network, vertices are connected densely than Sioux Falls network. Although the study area is divided into 100 number of TAZs, only three vertices could cover the 100% demand.

Figure 4.10 Allocation strategy for scenario A

Figure 4.11 Allocation strategy for scenario B when $k=3$

4.5 Summary

This chapter proposes a framework to determine the bike allocation strategy in a dockless bike sharing market considering two scenarios of market regimes. An improved community structure detection algorithm is developed to determine the boundaries of the service area for each allocation position. Based on the market objective, two types of key vertices have been defined, vertices with the maximum weight and the vertices with maximum effect in coverage. For the operators as market leaders, as the demand level is not clear at the beginning, the allocation should be made in small batches. If there is potential competitor entering the market soon, the vertices with maximum weight shall be selected to allocate bikes. This strategy could allow the bikes to be distributed across the service area with the fastest speed. On the contrary, if there is no potential competitor, the bikes should be initially allocated to the vertices with maximum catchment area so that minimum number of bikes are required to cover all the service area.

CHAPTER 5 REPOSITION OF DOCKLESS BIKE SHARING USING NEGATIVE PRICING STRATEGY

5.1 Introduction

Due to the one-way use characteristic, there is an issue of imbalanced distribution of bikes over time and space, especially during peak period. For example, most of the users will cycle from residential area to the nearest transit stations in the first-mile trip stage in the morning. Subsequently, bikes in residential area are quickly out of stock and the latter arrival users could not have the access to bikes. In this case, the level of user satisfaction may drop significantly as they cannot receive the service on time. Operators usually utilise dedicated relocation vehicles (e.g. light trucks) to move the bikes from oversupply area to undersupply area. Due to the reasons of limited resources (e.g. manpower, budget, vehicle capacity) and high relocation cost (\$3 per bike (DeMaio, 2009)), this approach could not fundamentally solve the imbalance problem for dockless bike sharing system.

In this study, we propose a dynamic pricing strategy to guide the users' cycling behaviour so as to achieve the objective of balanced bikes' relocation. Normally, users need to pay a positive price to the operator for using the shared bikes. However, if the users park the bikes at an undersupplied area, wherein the ratio of the available bikes to the real-time demand is below certain threshold value, a radical negative price is adopted, i.e., the operator will even offer the users monetary incentives. With such a negative pricing scheme, existing users might adjust their routing choice while new users might be attracted from their original travel modes to use bike. Both will contribute to the objective of balanced bike relocation. To test the performance of the proposed pricing strategy, a dynamic traffic assignment model is developed with user equilibrium principle to reflect the users' behaviour choice and estimate network flow distribution. The flow assignment results could be used for impact assessment of the proposed pricing strategy from both profit and network performance aspects. This study has the following novel significant contributions:

- 1) We introduce the dynamic pricing strategy with negative price into bike sharing system. Indeed, this would be the first attempt to theoretically investigate the application of negative pricing scheme in an urban transportation problem.

- 2) To the best of our knowledge, this is the first study to deal with the bike relocation problem using user-based approach in dockless bike sharing system while most the existing solutions to bike relocation applied operator-based approaches.
- 3) A dynamic traffic assignment model and its solution algorithm are developed to capture travellers' mode-path choice behaviour in response to the proposed dynamic pricing strategy.
- 4) A systematic comparison among the traditional positive price strategy, free price strategy and the proposed strategy is analysed.

5.2 Basic Considerations

5.2.1 Assumptions

Some assumptions are made before the modelling as follows:

A1: The study time horizon $[0, T]$ is discretised into a finite set of equal time intervals, i.e., $\{1, 2, 3, \dots, \bar{T}\}$. Let Δ be the duration of each time interval $\Delta \cdot \bar{T} = T$.

A2: The study focuses on first/last mile trip, where three common travel modes are considered including walk, bike and bus. Travellers all have access to dockless public bikes and do not own their private bikes.

A3: Travellers may use one or up to three modes in a trip. In other words, maximum two transfers are allowed.

A4: Traveller heterogeneity is considered regarding a discrete set of value of time.

A5: Bus runs on the dedicated bus lane, so that bus travel time is fixed according to the timetable and is not affected by other modes.

A6: Bidirectional flow on walk and bike links are considered separately. There is no mixed-traffic flow on all links.

A7: All travellers will check the bike availability and price before making a mode-path choice. They can reserve any available bike regardless the distance. The reservation follows first-reserve-first-use principle.

A8: There is a limited number of bikes provided in the network. If there are no bikes available, the travellers at this location place could only choose to walk or take bus.

A9. All travellers who take the negative price reward will follow the rules to return the bikes on time.

5.2.2 Notations

Primary notations used in this study are defined in Table 5.1 as follows:

Table 5.1 Notations

Sets	
N	Set of nodes
T	Set of total time
W	Set of Origin-Destination (OD) pairs
P	Set of paths
L	Set of links
$A(i)$	Set of links going out of node i , $A(i) = \{(i, j) \in A\}$
$B(i)$	Set of incoming links of node i , $B(i) = \{(j, i) \in A\}$
M	Set of travel mode, $\{M M = walk, bike, bus\}$
Indices	
w, w'	OD pair, $w \in W$
$p, p', \bar{p}$	Path $p, p', \bar{p} \in P$
l, l', l_+, l_-	Travel link, $l, l', l_+, l_- \in L$, $+$, $-$ represent the direction
i, j	Nodes
r, s	Origin and destination nodes of w
r_{bike}, s_{bike}	Origin and destination nodes of bike stage
m, m'	Travel mode, $m, m' \in M$
t, t'	Time intervals
k	Passenger groups according to value of time
Parameters	
ξ^k	Value of time for group k
λ	Comfort loss-time conversion coefficient
η_m	Comfort loss of mode m per time
g_{bike}^{bv}	Large value, to impose extremely high cost for the case with no available bikes
g_{bike}^{pos}	Normal positive price of renting bike
Q_{bike}	Supply of bikes
ε	Tolerance level
Variables	
$PU^{w,p,k}(t)$	Disutility of w via p for group k at time t
$LU^{l,k}(t)$	Disutility of group k on link l at time t
$\delta^{w,p,l,t'}(t)$	Indicator variable: traveller of w depart at t via p and entry link l at t' , equal to 1; otherwise 0

$T_m^{l,k}(t)$	Time cost of mode m for group k on link l at time t
$G_m^{l,k}(t)$	Monetary cost of mode m for group k on link l at time t
$C_m^{l,k}(t)$	Comfort cost of mode m for group k on link l at time t
$t_{0,m}^l$	Free flow time of mode m on link l
$x_m^l(t)$	Flow of mode m on link l at time t
$\Phi_{th,1}^i(t)$	threshold value of bike supply at node i at time t below which undersupply is defined
$\Phi_{th,2}^i(t)$	threshold value of bike supply at node i at time t above which oversupply is defined
$\Phi_{bike}^i(t)$	value of bike supply at node i at time t
$f^{w,p,k}(t)$	Flow for group k of w via p at time t
$\Gamma^{w,p,k*}(t)$	Minimum disutility for group k of w via p at time t
$u^{w,p,l}(t)$	Inflow of w via p on link l at time t
$v^{w,p,l}(t)$	Outflow of w via p on link l at time t
$f^w(t)$	Flow of w departing at time t
$e^w(t)$	Flow of w arriving at time t
$E_m^{w,p,k}(t)$	Cumulative flow of mode m for group k of w via p by time t
$\tau_m^l(t)$	Travel time of mode m on link l depart at time t

5.2.3 Multimodal Supernetwork

Supernetwork is abstracted from a normal network, wherein the hierarchy of real network and the virtual connection among the network are both included. In this study, a multimodal transportation supernetwork is constructed. Firstly, the whole study area is divided into certain number of traffic analysis zones (TAZs), where each TAZ is connected with real road links. As users can park the bikes at any location in the dockless bike sharing system, the size of one TAZ is usually selected to encapsulate one or several buildings within the field of human vision. Secondly, supplementary virtual links are added to the real transportation network to support the full description of travel behaviour, such as access link, transfer link as well as the weight on each link. Accordingly, an illustrative transportation network as in Figure 5.1 could be expanded and represented by a supernetwork as shown in Figure 5.2, wherein the solid lines represent the running links and the virtual links represent the access and

transfer links. The weight on each link could be various attributes, such as travel time, monetary cost, comfort loss.

A path in supernetwork is called superpath. Not all superpaths can describe actual travel routes. For example, travellers will not have unlimited transfers in one trip. Based on the assumptions in this study, a superpath is defined to be feasible only if it has maximum two transfers and no two continuous transfer links between one OD pair is allowed. For example, in Figure 5.2, path 1-4-24-17-18-19 is a feasible superpath between OD pair (1,10) while path 1-4-26-29-17-18-19 is not a feasible superpath between OD pair (1,10) as there are two continuous transfer links 26 and 29.

Figure 5.1 Study Area

Figure 5.2 Super Network

5.2.4 Negative Price Scheme

As mentioned above, negative price strategy in this study means that user could get monetary reward from operator. The negative price scheme is only applicable when the users cycle from oversupply area to undersupply area so that the bike relocation is achieved in the system. Before introducing the negative price scheme, three statuses are defined for each TAZ in the study area, i.e., undersupply, normal and oversupply:

$$status(i, t) = \begin{cases} \text{undersupply} & , \text{if } \Phi_{bike}^i(t) \leq \Phi_{th,1}^i(t) \\ \text{oversupply} & , \text{if } \Phi_{bike}^i(t) > \Phi_{th,2}^i(t) \\ \text{normal} & , \text{otherwise} \end{cases}, \forall \Phi_{th,1}^i(t) < \Phi_{th,2}^i(t), i \in N \quad (5.1)$$

Therefore, the dynamic pricing scheme in this study is proposed as: user will pay a positive price if the status of the origin TAZ or destination TAZ is normal; Negative price applies when the status of the origin TAZ is oversupply and the destination TAZ is undersupply. To illustrate the performance of the proposed strategy, two common pricing strategies are also considered in this study as the comparison benchmarks: traditional fixed pricing strategy where the price is a constant positive value, and the free pricing strategy. For simplification purpose, we abbreviate the proposed negative price strategy as NP, the traditional positive price strategy as PP, and the free price strategy as FP in the rest of the paper.

The negative price value is assumed to be dependent on the level of undersupply or scarcity degree of bikes at the destination nodes. Hereby, we propose a simple linear relationship between the negative price and the level of undersupply of bikes. The structure of the pricing scheme can be schematically depicted in Figure 5.3. Basically, if the bike trip starts from an oversupplied location to an undersupplied one, the negative price will apply; otherwise, the normal positive price will be adopted. Bounded by a maximum allowable value, the negative price is set to be linear with respect to the level of undersupply.

Figure 5.3 Negative price scheme

Based on the above analysis, the dynamic pricing scheme with negative price can be proposed as follows:

$$G_{bike}(t) = \begin{cases} g_{bike}^{neg}(t) \cdot \left[\frac{\Phi_{bike}^{s_{bike}}(t) - \Phi_{th,1}^{s_{bike}}(t)}{\Phi_{th,1}^{s_{bike}}(t)} \right], & \forall \begin{cases} \Phi_{bike}^{r_{bike}}(t) > \Phi_{th,2}^{r_{bike}}(t) \\ \Phi_{bike}^{s_{bike}}(t) \leq \Phi_{th,1}^{s_{bike}}(t) \end{cases} \\ g_{bike}^{pos}, & otherwise \end{cases} \quad (5.2)$$

Here, when users cycling bikes from an oversupplied area to an undersupplied area, a negative price applies, wherein the value of negative price is set to be determined by the level of undersupply at the destination TAZ (i.e., $\frac{\Phi_{bike}^{s_{bike}}(t) - \Phi_{th,1}^{s_{bike}}(t)}{\Phi_{th,1}^{s_{bike}}(t)}$) and the

allowable most negative price $g_{bike}^{neg}(t)$. That is, to determine the negative price, it is required to know the supply of bikes $\Phi_{bike}^{s_{bike}}(t)$, as well as the defined threshold value of undersupply $\Phi_{th,1}^{s_{bike}}(t)$. It is noted that, the real-time information of bike supply can

be easily obtained in practice from the GPS data. On the other hand, the key is to determine how to set the threshold value for undersupply. It is not hard to understand that the threshold value for undersupply is highly related to the bike travel demand. In practice, the bike demands at different locations could be estimated by adopting prediction methods, such as the Auto Regressive Conditional Heteroscedasticity (ARCH) model, based on the historical and real-time data. In the proposed negative pricing scheme in this study, we set the threshold value for undersupply by assuming a simple linear relationship with the predicted demand value of $Q(i,t)$, i.e.,

$\Phi_{th}^i(t) = Q(i,t) + \phi$. Herein, the parameter ϕ could be determined based on the specific objectives to be achieved, e.g., attracting more users or maximizing fare revenue. In practice, other than applying rigorous optimization modeling approach,

one can obtain the optimal value of the parameter ϕ by conducting a simple sensitivity analysis approach.

Proposition 1 With a sufficient number of bikes in market, NP will reduce to PP.

Proof. From Eq.(5.2), it is known that negative price occurs when the relationship between bike supply and the threshold values at the origin and destination area satisfies $\Phi_{bike}^{r_{bike}}(t) > \Phi_{th,2}^{r_{bike}}(t)$, $\Phi_{bike}^{s_{bike}}(t) \leq \Phi_{th,1}^{s_{bike}}(t)$. The threshold values $\Phi_{th,2}^{r_{bike}}(t)$ and $\Phi_{th,1}^{s_{bike}}(t)$ are highly relevant to dynamic bike travel demand at certain location. As the bike supply level in the market goes up, $\Phi_{bike}^{s_{bike}}(t)$ will increase and therefore the chance of fulfilling negative price condition $\Phi_{bike}^{s_{bike}}(t) \leq \Phi_{th,1}^{s_{bike}}(t)$ will reduce. If the bike supply in the market is sufficiently large, the negative price conditions would not be satisfied, i.e., the proposed NP scheme defined in (5.2) will reduce to PP.

Essentially, two types of behaviour changes might happen if NP is adopted: One is the change of travel mode, that is, the non-cycling travel mode users would use bikes and end the trips at the undersupply TAZ. The other one is the change of travel path, that is, the existing bike users would use another path and end the trip at the undersupply TAZ. For example, in Figure 5.4, node 1 is the origin node and node 3 is the destination node. For the travellers who usually take bus from node 1 to node 3, after NP is adopted, they might choose to cycle from node 1 to node 2 and then take bus from node 2 to node 3, as the reward from negative price scheme might offset the incurred extra travel disutility. Similarly, for an existing bike user from node 1 to node 3, he/she might park the bike at node 2 and then walk or take bus from node 2 to node 3, if reward from negative price could offset the added disutility.

Figure 5.4 A super path with three nodes

As can be seen, both of these travel behaviour changes mentioned above can contribute to the bike relocation. In this case, operators are able to reduce the operation cost for bike relocation. The achieved real-time relocation through

travellers' spontaneous travel behaviour could also improve the level of service, and thus bring new users and increase revenue.

Proposition 2 Under NP, for the first type of behaviour change, i.e., the switch from non-cycling mode to cycling mode, one bike will at least serve two users. If the maximum NP value is set to be not greater than PP fare, non-negative fare revenue could be obtained.

Proof. Under NP, if a traveller changes travel mode, he/she will relocate a bike from an oversupplied area to an undersupplied area where demand is larger than supply. Then, another user will be able to ride this bike to other places. In this case, this bike will be used by at least two travellers. If the traveller selects his/her original non-cycling travel mode, the bike sharing service operator can achieve zero fare revenue. If modal switch occurs, from Eq(5.2), the fare revenue for the first trip is subject to NP scheme ($g_{bike}^{neg}(t) \cdot [\Phi_{th,1}^{S_{bike}}(t) - \Phi_{bike}^{S_{bike}}(t)] / \Phi_{th,1}^{S_{bike}}(t)$), while the second trip follows PP (g_{bike}^{pos}). In this case, the total direct fare revenue can be rewritten as $-g_{bike}^{neg}(t) \cdot \Phi_{bike}^{S_{bike}}(t) / \Phi_{th,1}^{S_{bike}}(t) + g_{bike}^{pos} + g_{bike}^{neg}(t)$. If $-g_{bike,max}^{neg} \leq g_{bike}^{pos}$, the fare revenue must be positive.

Proposition 3 Under NP, for the second type of behaviour change, i.e., the travel path adjustment, more users would be served and positive fare revenue will be obtained with nevertheless smaller amount of fare revenue.

Proof. Under NP, if a traveller decides to change path, he/she must switch from a path with positive price to a path with negative price. The original fare revenue is g_{bike}^{pos} . The new trip after changing would apply negative pricing, which means that the new destination is undersupplied. In this case, the bike will be used by another user with positive price, and more users would be served as compared with PP. The new fare revenue is $g_{bike}^{pos} + g_{bike}^{neg}(t) \cdot \left[\frac{\Phi_{bike}^{S_{bike}}(t) - \Phi_{th,1}^{S_{bike}}(t)}{\Phi_{th,1}^{S_{bike}}(t)} \right]$. As $\Phi_{bike}^{S_{bike}}(t) < \Phi_{th,1}^{S_{bike}}(t)$, we have $g_{bike,max}^{neg} \leq g_{bike}^{neg}(t) \cdot [\Phi_{bike}^{S_{bike}}(t) - \Phi_{th,1}^{S_{bike}}(t)] / \Phi_{th,1}^{S_{bike}}(t) < 0$. If it is assumed $-g_{bike,max}^{neg} \leq g_{bike}^{pos}$, fare revenue would be positive, but it is still less than original fare revenue.

A set of illustrative examples with three scenarios is given to demonstrate the effectiveness of NP as compared to traditional PP scheme and FP scheme. Assume the original travel modes in three scenarios are all under PP, where traveller travels from node 1 to node 3 passing node 2 as shown in Figure 5.4. Node 1 is in the status of oversupply, while node 2 undersupply and node 3 normal. Two links share the same distance in three scenarios. Three strategies are given for three scenarios: FP(1-2-3) represent the scenario that the whole network is under FP, i.e., with free charge; FP (1-2) is that link connecting node 1 and node 2 is under FP while the other link is under PP; NP (1-2) is that link connecting node 1 and node 2 is under NP while the rest link is still under PP. The highly possible travel decisions under different pricing strategies are listed in Table 5.2. From Table 5.2, it could be concluded that FP (1-2-3) cannot achieve bike relocation while FP (1-2) and NP (1-2) are able to do so. The behaviour adjustment in Scenario A belongs to the second type change, i.e., path changing, while Scenarios B and C apply modal switching. In Scenario B, the original travel mode is walking from node 1 to node 3. When FP is offered from node 1 to node 2, travellers are not likely to choose to use bike as they need to take extra effort (disutility) to cycle. However, if extra reward is given, travellers may choose cycling from node 1 to node 2 as the reward might offset their added disutility from using bike. Therefore, NP (1-2) is more effective than FP (1-2-3) and FP (1-2) in terms of attracting travellers to cycle and thus achieving balanced relocation. In reality, NP and FP could directly attract large number of users in a short time and indirectly reduce the advertisement cost. With the huge number of users, ancillary revenue other than fare revenue could be generated. Unlike below-cost price, free price and negative price have highly likelihood to let users overvalue the service because humans naturally fear of loss. Free price and negative price will not lead to any loss while below cost price still have a risk of loss.

Table 5.2 *Highly possible travel behaviour under different pricing strategies*

Scenario	PP(original)	FP (1-2-3)	FP (1-2)	NP (1-2)
A	bike:1-2-3	bike: 1-2-3	bike:1-2-3 or bike:1-2 bus 2-3 or bike 1-2 walk 2-3	bike:1-2-3 or bike:1-2 bus 2-3 or bike 1-2 walk 2-3
B	walk:1-2-3	walk:1-2-3 or bike:1-2-3	walk:1-2-3	walk:1-2-3 or bike:1-2 walk 2-3
C	bus:1-2-3	bus:1-2-3	bus:1-2-3	bus:1-2-3

5.3 Path Travel Disutility

Based on the assumption that the travel disutility on each link is independent, the generalised path travel disutility could be calculated by summing up the corresponding link disutility on this path as follows:

$$PU^{w,p,k}(t) = \sum_l \sum_t LU^{l,k}(t) \cdot \delta^{w,p,l,t,k}(t), \forall w, p, l, k, t, t' \quad (5.3)$$

For each link, the link travel disutility includes three components, travel time, monetary cost and comfort loss (discomfort). Travel time consists of walking time, cycling time, in-vehicle time and waiting time. Considering the weight of each component, the link travel disutility could be expressed as:

$$LU^{l,k}(t) = \omega_T T^{l,k}(t) + \omega_G G^{l,k}(t) + \omega_C C^{l,k}(t), \forall l, k \quad (5.4)$$

where ω_T , ω_G and ω_C are weight parameters, $\omega_T + \omega_G + \omega_C = 1$.

In a multimodal network with consideration of the combined travel mode, there are three types of travel links, i.e., running links, transfer links and access links. Running links include walk links, bike links and bus links. For simplification purpose, the monetary cost on bike and bus links is moved to the connected access link.

5.3.1 Walk Link Disutility

Pedestrian walks on the walk link without being affected by other modes. The interaction of bi-direction pedestrian flow on walk link is considered. Travel time for walking is calculated as in Wu and Lam (2003). There is no monetary cost on walk link. Comfort loss (discomfort) on walk link is directly related to the travel time. Thus, three components in walk link disutility are calculated by:

$$T_{walk}^{l_{\pm}}(t) = t_{0,walk}^{l_{\pm}} + A_{walk} \left[\frac{x_{walk}^{l_{\pm}}(t)}{x_{walk}^{l_{\pm}}(t) + v_{walk}^{l_{\pm}}(t)} \right]^{\alpha_{walk}} \left[\frac{x_{walk}^{l_{\pm}}(t)}{B_{walk}^{l_{\pm}}(t)} \right]^{\beta_{walk}}, \forall l, l_{\pm} \in L_{walk} \quad (5.5)$$

where l_{+} and l_{-} represent the two directions on the link l . A_{walk} , $B_{walk}^{l_{\pm}}$, α_{walk} and β_{walk} are parameters to be calibrated with observed data.

$$G_{walk}^l(t) = 0, \forall l \in L_{walk} \quad (5.6)$$

$$C_{walk}^l(t) = \lambda \eta_{walk} T_{walk}^l(t), \forall l \in L_{walk} \quad (5.7)$$

5.3.2 Bike Link Disutility

Similarly, cyclists do not share the bike link with pedestrians or bus passengers. The interaction of bi-direction cyclist flow on bike link is also considered in the travel time calculation. The monetary cost is assumed to be dependent on the supply and demand interaction at the departure time, and not relevant with the travel time. Thus, the monetary cost on bike link can be transferred to the connected access link. As cycling is a human-powered travel mode, comfort loss on bike link is closely related to the travel time. Then, the three components in bike link disutility are given by (Meng et al., 2014):

$$T_{bike}^{l_+}(t) = t_{0,bike}^{l_+} + A_{bike} \left[\frac{x_{bike}^{l_+}(t)}{x_{bike}^{l_+}(t) + x_{bike}^{l_-}(t)} \right]^{\alpha_{bike}} \left[\frac{x_{bike}^{l_+}(t)}{B_{bike}^{l_+}} \right]^{\beta_{bike}}, \forall l_+, l_- \in L_{bike} \quad (5.8)$$

$$G_{bike}^l(t) = 0, \forall l \in L_{bike} \quad (5.9)$$

$$C_{bike}^l(t) = \lambda \eta_{bike} T_{bike}^l(t), \forall l \in L_{bike} \quad (5.10)$$

5.3.3 Bus Link Disutility

Bus service is assumed to follow strictly the running time on timetable, which is not affected by other modes. Assuming that a flat fare ticket structure applies, one can move the monetary cost on bus link to the connected access link. Besides the travel time, passengers may feel discomfort on the bus due to the crowded environment in peak period. Then, the three components in bus link disutility could be obtained as follows:

$$T_{bus}^l(t) = t_{0,bus}^l, \forall l \in L_{bus} \quad (5.11)$$

$$G_{bus}^l(t) = 0, \forall l \in L_{bus} \quad (5.12)$$

$$C_{bus}^l(t) = \lambda \eta_{bus} T_{bus}^l(t) + \left[\frac{v_{bus}^l(t)}{D_{bus} Y_{bus}^l} \right], \forall l \in L_{bus} \quad (5.13)$$

where D_{bus} is bus capacity and Y_{bus}^l is capacity of link l .

5.3.4 Transfer Link Disutility

Travel time on transfer link includes transfer walking time and transfer waiting time. Transfer walking time is approximately equal to the ratio of transfer walking distance

to the average walking speed. Transfer waiting time is incurred only for transfer from walk or bike link to bus link, which is equal to half of bus headway. There is no monetary cost on transfer link. Comfort loss on transfer link is decided by the transfer walking time. The three components in transfer link disutility are expressed as follows:

$$T_{m,m'}^l(t) = t_{m,m'}^l, \forall m, m' \in M, l \in L_{transfer} \quad (5.14)$$

$$G_{m,m'}^l(t) = 0, \forall m, m' \in M, l \in L_{transfer} \quad (5.15)$$

$$C_{m,m'}^l(t) = \lambda \eta_{walk} t_{m,m'}^l, \forall m, m' \in M, l \in L_{transfer} \quad (5.16)$$

5.3.5 Access Link Disutility

Travel time on access link is zero if the access link is connected to walk or bike link, and equal to half of the bus headway if the access link is connected to bus link. Monetary cost on access link could be bike rental fee or bus fare cost. There is no comfort loss on access link. The three components in access link disutility are expressed as follows:

$$T_{ae}^l(t) = T_m^{wait}(t), \forall l \in L_{ae} \quad (5.17)$$

$$G_{ae}^{l,k}(t) = \zeta^k G_m^k(t), \forall l \in L_{ae} \quad (5.18)$$

$$C_{ae}^l(t) = 0, \forall l \in L_{ae} \quad (5.19)$$

where $T_{walk}^{wait} = T_{bike}^{wait} = 0$, $T_{bus}^{wait} = H / 2$.

Bike rental fee follows the dynamic pricing structure, which can be expressed as follows:

$$G_{bike}(t) = \begin{cases} g_{bike}^{bv}, \forall \Phi_{bike}^{r_{bike}}(t) = 0 \\ \chi g_{bike}^{neg}(t) \cdot \left[\frac{\Phi_{bike}^{s_{bike}}(t) - \Phi_{th,1}^{s_{bike}}(t)}{\Phi_{th,1}^{s_{bike}}(t)} \right], \forall \begin{cases} \Phi_{bike}^{r_{bike}}(t) > \Phi_{th,2}^{r_{bike}}(t) \\ \Phi_{bike}^{s_{bike}}(t) \leq \Phi_{th,1}^{s_{bike}}(t) \end{cases} \\ g_{bike}^{pos}, otherwise \end{cases} \quad (5.20)$$

where χ is a constant coefficient to represent contrast preference from negative price. According to the theory of loss aversion, when people face the same amount of gains and losses, losses are more intolerant than gains. The disutility that comes from losses is 2.25 times of the utility from gains (Tversky and Kahneman, 1992).

5.4 Model and Algorithm

5.4.1 Dynamic User Equilibrium Principle

According to assumption A4, we consider k classes of travellers with different values of time. The dynamic user equilibrium condition is defined as: for each class k and each OD pair at each time interval, the total travel disutility for all superpaths that are being used equal to the minimal total super path travel disutility. It could be formulated as:

$$PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t) \geq 0, \quad \forall w, p, k \quad (5.21)$$

$$f^{w,p,k*}(t) \cdot [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] = 0, \quad \forall w, p, k \quad (5.22)$$

$$f^{w,p,k}(t) \geq 0, \quad \forall w, p, k \quad (5.23)$$

5.4.2 Dynamic Network Constraints

Five types of constraints are given according to the model requirements. First, the necessary definitional constraints are expressed as:

$$\sum_w \sum_p \sum_k x_m^{w,p,l,k}(t) = x_m^l(t), \quad \forall w, p, l, k, m \quad (5.24)$$

$$\frac{dx_m^{w,p,l,k}(t)}{dt} = u_m^{w,p,l,k}(t) - v_m^{w,p,l,k}(t), \quad \forall w, p, l, k, m \quad (5.25)$$

$$\sum_w \sum_p \sum_k u_m^{w,p,l,k}(t) = u_m^l(t), \quad \forall w, p, l, k, m \quad (5.26)$$

$$\sum_w \sum_p \sum_k v_m^{w,p,l,k}(t) = v_m^l(t), \quad \forall w, p, l, k, m \quad (5.27)$$

Second, at the origin and destination nodes, flow conservation constraints satisfy:

$$f_m^{w,k}(t) = \sum_{l \in A(r)} \sum_p u_m^{w,p,l,k}(t), \quad \forall w, p, k, m \quad (5.28)$$

$$\sum_{l \in B(s)} \sum_p v_m^{w,p,l,k}(t) = e_m^{w,k}(t), \quad \forall w, p, k, m \quad (5.29)$$

where $f_m^{w,k}(t)$ denote the m mode flow of class k departing at origin node to destination node at time t , and $e_m^{w,k}(t)$ denote the m mode flow of class k arriving at destination from origin at time t . For all the other nodes, flow conservation constraints can be written as:

$$\sum_{l \in A(i)} v_m^{w,p,l,k}(t) = \sum_{l \in B(i)} u_m^{w,p,l,k}(t), \forall w, p, k, m; i \notin \{r, s\} \quad (5.30)$$

Third, flow propagation constraints entail:

$$x_m^{w,p,l,k}(t) = E_m^{w,p,k}(t + \tau_m^l(t)) - E_m^{w,p,k}(t) + \sum_{l' \in \tilde{p}} \{x_m^{w,p,l',k}(t + \tau_m^{l'}(t)) - x_m^{w,p,l',k}(t)\} \quad (5.31)$$

$\forall l \in B(i); i \neq r; w, p, k, m$

$$\frac{dE_m^{w,p,k}(t)}{dt} = e_m^{w,p,k}(t), \forall w, p, k, m \quad (5.32)$$

Fourth, nonnegativity conditions are:

$$x_m^{w,p,l,k}(t) \geq 0, \forall w, p, l, k, m \quad (5.33)$$

$$u_m^{w,p,l,k}(t) \geq 0, \forall w, p, l, k, m \quad (5.34)$$

$$v_m^{w,p,l,k}(t) \geq 0, \forall w, p, l, k, m \quad (5.35)$$

$$e_m^{w,p,k}(t) \geq 0, \forall w, p, k, m \quad (5.36)$$

$$E_m^{w,p,k}(t) \geq 0, \forall w, p, k, m \quad (5.37)$$

Last, boundary conditions include:

$$E_m^{w,p,k}(0) = 0, \forall w, p, k, m \quad (5.38)$$

$$x_m^{w,p,l,k}(0) = 0, \forall w, p, l, k, m \quad (5.39)$$

$$\sum_i \Phi_{bike}^i(0) = Q_{bike} \quad (5.40)$$

$$\Phi_{bike}^i(t) = e_{bike}^{w,r}(t) - f_{bike}^{w,s}(t) \quad (5.41)$$

$$\Phi_{bike}^i(t) \geq 0 \quad (5.42)$$

5.4.3 Variational Inequality Model

The dynamic traffic flow pattern satisfying network constraint set Eqs. (5.24)-(5.40) is indeed a multi-class generalised travel-disutility-based dynamic user equilibrium superpath choice state as described in the variational inequality problem Eq.(5.41).

$$\sum_k \sum_w \sum_p \sum_t PU^{w,p,k*}(t) \cdot (f^{w,p,k}(t) - f^{w,p,k*}(t)) \geq 0 \quad (5.43)$$

where the feasible region is $\Omega = \{(5.24)-(5.40)\}$. The necessity and sufficiency of the equivalence of variational inequality is given in Appendix.

Theorem 1 There is at least one solution to variational inequality Eq.(5.43).

Proof. A standard theorem in the theory of variational inequalities indicates that: if the feasible region Ω is compact and the mapping function of the problem are continuous, there exists at least one solution to VI formulation Eq.(5.43).

First, as the feasible region $\Omega=\{(5.24)-(5.40)\}$ of VI formulation Eq.(5.43) consists of a set of linear constraints, thus the region is compact. Second, due to the imbalanced distribution of bikes at each TAZ, the travel disutility value during calculation is unsmooth, but the travel disutility function is still continuous. If the interval length is infinitely close to zero (i.e., continuous-time), the link inflows will be continuous with path inflows vector. Then the existence of the dynamic user equilibrium solution can be guaranteed in principle.

Due to the inclusion of 0-1 integer variables $\delta^{w,p,l,t',k}(t)$, the disutility calculated by Eq. (5.3) are non-linear and non-convex. In addition, strict monotonicity of the path disutility is not ensured as it is highly dependent on the structure of supernetwork. Therefore, the solution uniqueness of VI formulation Eq.(5.43) cannot be guaranteed. In real situation, the time interval length is hardly to be infinitely close to zero. The violation of the solution existence requirement might lead to the non-existence of solution. Moreover, the unbalanced distribution of demand and supply will lead to the unsmooth property of travel disutility function. As of the discrete time assumption, the disutility value may also not be small enough to ensure the existence of the proposed dynamic user equilibrium. Thus, we adopt the tolerance-based dynamic user equilibrium principle proposed by Szeto (2003), and therefore Eqs.(5.21)-(5.23) and Eq.(5.43) could be converted as:

$$\begin{cases} PU^{w,p,k}(t) \leq (1+\varepsilon) \cdot \Gamma^{w,p,k*}(t), \text{ if } f^{w,p,k}(t) > 0 \\ PU^{w,p,k}(t) \geq (1+\varepsilon) \cdot \Gamma^{w,p,k*}(t), \text{ if } f^{w,p,k}(t) = 0 \end{cases}, \forall w, p, k, t \quad (5.44)$$

$$\sum_k \sum_w \sum_p \sum_t \max \{ PU^{w,p,k*}(t), (1+\varepsilon) \cdot \Gamma^{w,p,k*}(t) \} \cdot (f^{w,p,k}(t) - f^{w,p,k*}(t)) \geq 0 \quad (5.45)$$

where the feasible region is $\Omega=\{(5.24)-(5.40)\}$. When $\varepsilon = 0$, Eq.(5.45) is equivalent to Eq. (5.43). When $\varepsilon > 0$, it is reasonable in real situation, where the travel disutility of all used paths between the OD pair are within an acceptable tolerance from the minimum OD path disutility.

Inference 1 There is at least one solution to variational inequality Eq.(5.45).

Proof. Szeto (2003) has proved that the tolerance-based principle is a generalisation of the traditional dynamic user equilibrium principle. That is, if $\varepsilon=0$, Eq.(5.45) will be reduced to Eq.(5.43). It is a relaxation of Eq.(5.43), so the solution exists.

5.4.4 Solution Algorithm

A path-flow swapping algorithm is developed to solve the proposed model. The solution algorithm starts with initialised flows on the feasible path and available bikes in each TAZ. The detailed steps are described as follows:

Step 0: Set up the parameters and load the multi-state super networks for all modes, $t=1$.

Step 1: Load the demand and other parameters at time t on the network.

Step 2: Identify the feasible path sets.

Step 3: Assign the demand stochastically on feasible paths.

Step 4: Consider the volume already on the path, calculate the real time traffic volume on each link.

Step 5: Calculate the travel time, monetary cost and comfort loss on each link.

Step 6: Calculate the travel time, monetary cost and comfort loss on each feasible path.

Step 7: Calculate the travel disutility on each feasible path.

Step 8: Convergence test. If the following convergence condition $\max(\mu^a \cdot (\mathbf{PU}^a - \mathbf{PU}^{*a})) \leq \varepsilon \cdot \mathbf{PU}^{*a}$ is satisfied, the algorithm stops, record the traffic volume on the network at time t' , and update $t, t=t+1$, return to Step 1; otherwise, continue. Herein, μ^a and $\mathbf{PU}^a$ denote the vectors of traffic flow indicator and path disutility at iteration a respectively, $\mathbf{PU}^{*a}$ is the minimum path disutility at iteration a .

Step 9: Update current traffic flows, using:

$$\mathbf{f}^{a+1} = \begin{cases} \max(0, \mathbf{f}^a - \rho^a \cdot \mathbf{f}^a \cdot (\mathbf{PU}^a - \mathbf{PU}^{*a})), & \text{if } w, p, t, t' \notin \Lambda^a \\ \mathbf{f}^a + \frac{\sum_{w, p, t, t' \in \Lambda^a} (\mathbf{f}^a - \mathbf{f}^{a+1})}{|\Lambda^a|}, & \text{if } w, p, t, t' \in \Lambda^a \end{cases}$$

where $\Lambda^a = \{w, p, t : PU^{w,p,k,t'}(t)^a \leq (1+\varepsilon) \cdot PU^{*a}\}$, and $\rho^a = \rho^0 / (a/\gamma^0)$. ρ^a and γ^0 are given as flow adjustment parameters; f^a is the path flow vector at iteration a . $a=a+1$, return to Step 4.

During the path-flow adjustment process, the manipulated conditions, $\lim_{a \rightarrow \infty} \rho^a = 0$, $\lim_{a \rightarrow \infty} \rho^a \cdot f^{w,p,k,t'}(t)^a = 0$ and $\sum_a \rho^a \cdot f^{w,p,k,t'}(t)^a = +\infty$ (Nagurney and Zhang, 1997; Huang and Lam, 2002) hold and channel the process to a convergence state with tolerance.

5.4.5 Profit Analysis

One intrinsic question arises is whether the operator can still gain profits or not after introducing the proposed pricing strategy. We consider a general profit definition, which includes not only the fare revenue, but also the user value. User value is important when a new service enters the market. Operators might be willing to lose money at the initial stage of service operation if they can attract more users and dominate the market. After the initial promotion stage, the operator can adjust the operational goal to pursue fare revenue maximisation. Therefore, a set of comprehensive profit optimisation model is developed for both initial promotion stage and the normal operation stage. At the initial promotion stage, the main decision variable is the total bikes investment, i.e., what is optimal bike supply to the market, while the main operational goal is to attract maximum number of users with limited resources. Therefore, the maximisation profit objective function could be expressed as:

$$\max \Upsilon(Q_{bike}) = \varpi Q_{user} + \pi Q_{user} - \psi Q_{bike} \quad (5.46)$$

where Υ is the overall profit, and Q_{bike} is the optimal number of bikes, $Q_{bike} = \sum_i \Phi_{bike}^i(0)$. Q_{user} is the number of registered users who have paid the deposit. The first item on the right-hand side describes the total user value in the market, where ϖ is the unit user value. The second item represents the total deposit revenue, where π is the deposit per user. The third item depicts the total bike cost, where ψ is the cost per bike. The constraints of model Eq.(46) are Eqs.(3)-(42).

After the initial promotion stage, there will be a huge number of stable existing users in the market. Few potential new users will be attracted, even though the price promotion continues. Operators could adjust the operational goal to achieve fare revenue maximisation. At this normal operation stage, the bike investment and new deposit income are nearly zero due to the few new users. In this case, the operators seek to find out the optimal pricing scheme and bike supplies so as to maximize the total profit, wherein the objective function could be expressed as:

$$\max \Upsilon(\Phi_{bike}^i(0), G_{bike}(t)) = \bar{\pi}Q_{user} + G_{fare} - \bar{\psi}Q_{bike} \quad (5.47)$$

The first item on the right side is the profit from user deposit, where $\bar{\pi}$ is the interest from one user deposit. Q_{user} is the number of registered users who have paid the deposit. The second item is the fare revenue, where $G_{fare} = \sum_t \sum_w \sum_p \sum_k f_{bike}^{w,p,k}(t) \cdot G_{bike}(t)$ and $f_{bike}^{w,p,k}(t)$ is the dynamic traffic equilibrium solution of (42) that is related to $G_{bike}(t)$ and $\Phi_{bike}^i(t)$. The third item covers the bike depreciation expense and operation cost, which is assumed to be positively related to the number of bikes, $Q_{bike} = \sum_i \Phi_{bike}^i(0)$. The constraints of Eq.(47) are Eqs.(3)-(42).

The primary objective of the above analysis is to present model formulation for the dockless bike sharing service providers to find the optimal operation strategy if the proposed negative pricing scheme is applied. Two formulated models are introduced for different stages of service operation as the service providers' objective changes as time advances. One can notice that, the formulated models are indeed in the format of bi-level programming formulation, which is very hard to be solved to its global optimal solutions. In this study, while we propose this model formulation on finding optimal operation strategy in presence of negative pricing scheme, the development of the efficient solution algorithms to obtain quality solutions is left to be addressed in the future studies. In practice, when high solution quality is not required, one can be recommended to apply sensitivity analysis approach to obtain a good solution result.

5.5 Numerical Examples

Two numerical examples are presented to illustrate the effect of the proposed negative pricing strategy on bike relocation, travel behaviour and network performance.

5.5.1 Numerical Results for Test Example 1

Figure 5.5 Example 1 network

The first example is designed to show the effects of the proposed pricing strategy onto achieving balanced bike relocation. For illustration purpose, we conduct the numerical example in a simple network with three nodes and links as shown in Figure 5.5. The number on each link represents the travel distance. Traffic flow over time is uniformly in the network as shown in Table 5.3, where the node in business area only has inflow demand, the node in residential area only has outflow demand, and the rest node at metro station has both inflow and outflow demand. Average speed, bus ticket price, headway information and a series of discomfort conversion parameters for different modes are set to be the same as in Meng et al. (2014). VOT parameters are adopted as the same in Zhang et al. (2017). Bi-directional flow travel time equation is adopted from Wu and Lam (2003). Other parameters are set as follows: $k=1$, $VOT=10$, $\varepsilon=0$, $\Phi_{bike}^i(0)=50$, $\Phi_{th,1}^i(t)=3$, $\Phi_{th,2}^i(t)=5$, $g_{bike}^{neg}(t)=0.5$, $g_{bike}^{pos}=0.5$, $T=70$ min and $\Delta=1$ min. Traffic departures from 0 to 60 min.

Table 5.3 OD demand of example 1

D \ O	Business Area (B)	Residential Area (R)	Metro Station (M)
Business Area	0	0	0
Residential Area	1 traveller /min	0	2 travellers /min
Metro Station	1 traveller /min	0	0

Figure 5.6 shows the number of bike trips and bike trip accumulation at metro station over time. Three different pricing strategies, i.e., negative pricing, free pricing and normal positive pricing, are tested and compared in this example. One can observe that the number of bike trips under three pricing strategies are identical, as well as the bike trip accumulation. It is because the travel disutility of bike is the lowest as compared to other alternative modes. In this study, while the travel disutility is determined by many factors including travel time, monetary cost, and comfort loss, comfort loss, the differential monetary cost under the three pricing strategies would not change the fact that cycling is the most favourite travel mode in this tested example. However, the dynamic and spatial bike departures and arrivals, as well as the bike supply in the network, are significantly different under differential pricing strategies, as will be analysed in the following discussions.

Figure 5.6 *The number of bike trips and bike trip accumulation departure from metro station over time*

The detailed supply, demand and actual bike trip numbers are given in Figure 5.7. One can observe that the supply is always above the real demand. However, the supply lines under three different pricing strategies are different. The number of available bikes continuously decreases at the beginning stage. As the time advances, the number of inflow bike arrivals would exceed the outflow bike departures and the number of bikes starts to increase. The arrival rates at the metro station under different pricing strategies are displayed in Figure 5.8. As the initial number of bikes in residential area is limited, the bikes that flow from residential area to metro station would reduce to zero and the number of bikes at metro station would decrease. It

should be noted that, the time points when the bike supply starts to switch from increasing to decreasing for the three pricing strategies are different. The switching time under NP or PP is later than that with FP. According to Figure 5.8, the number of bike arrivals under FP is less than that for NP and PP. Specifically, the number of bike arrivals under NP does not continuously decrease as under PP, due to the extra bike arrivals stimulated by NP.

Figure 5.7 The detailed supply, demand and actual trip numbers at metro station over time

Figure 5.8 The number of bike trips and bike trip accumulation arrival at metro station over time

Next, we want to further analyse the extra bike arrivals when negative pricing scheme is applied. As in Table 5.3, the demand from node B to node M is 0. The new bike arrivals must be from node R. Figure 5.9 shows the departure and arrival information at node R. One can notice that the bike departure from node R is still positive, while at the same time there are no available bikes under FP and PP at node R. The total bike departures are even higher than the initial supply of 50 bikes. As R has been set as an outflow node, the extra bikes at node R demonstrate that some travellers have changed their travel behaviour to park bike at node R under NP and the bike is later used by another traveller from node R to node M. All of these have clearly illustrated the effectiveness of the negative pricing in terms of a more balanced bike relocation.

Figure 5.9 The number of bike trips and bike trip accumulation at residential area

It seems that free pricing scheme could be a good substitute of negative pricing scheme as free charging is able to attract bike users and requires no additional cost from operators. The results from this numerical example can provide some direct comparisons between the two pricing strategies.

Figure 5.9 can explain why the number of bike arrivals under FP at node M is smaller than that under NP. Under FP, the number of travellers using bike sharing is more than it under NP. Travellers who originally walk from node R to node B will change to use bike under FP. Under NP, as the node B is in the status of oversupply, there is no monetary reward from node R to B. Those travellers will remain to walk from node R to node B. However, according to Table 5.3, there are positive travel demand from node M to node B. The supply at node M is sufficient at the very beginning. Thus, there is positive bike flow from node M to node B under FP. However, under NP there is no bike arrival at node B at certain time intervals (e.g., from 35 to 45 and from 57 to 60). This is because traveller has changed his/her path to park the bike at node R, according to Figure 5.9 and Figure 5.10. In other words, under NP, traveller will cycle from M to R and change to other modes to arrive at B, due to triggered negative price from node M to node R. For travellers, although the total trip distance and total travel time increase, the monetary reward from NP can offset the increased disutility. For operators, travellers contribute to achieve balanced bike relocation, which can save the operation cost substantially.

Figure 5.10 The number of bike trips and bike trip accumulation arrival at business area over time

Figure 5.11 shows the bike flow evolution on each link over time. First, under PP, there are bike flows only on link M-B and link R-M, which is equal to the demand according to Table 5.3. The demand from node R to node B does not select to cycle because the short trip distance is more suitable for walking. As of the sufficient supply at node M, the bike flow on link M-B is stable. Since the supply at node R is not sufficient, when all bikes are used, the link flow R-B becomes to zero. Second, under FP, bike flow on link M-B does not change over time. It is also because of the sufficient supply at node M. The bike flow on link R-M is similar to the situation under PP, which first increases to peak value, then stabilizes for a while before it decreases in the end. The stable period at peak value in Figure 5.11(d) under FP is shorter than that under PP due to the bike flow on link R-B as shown in Figure 5.11(c). This is in line with the above conclusion obtained from Figures 5.9 and 5.10. It reflects that when the number of shared bikes is limited, FP could only speed up the bike usage rate but further enlarge the gap between supply and demand at the same time, which indeed cannot improve the service quality of the bike sharing system. Furthermore, FP attracts the short-distance travel demand, which is more suitable for walking. It is indeed a waste of resources and can even deteriorate the level of service. Last, when NP implemented, bike flow on link M-B fluctuates over time as shown in Figure 5.11(a), which indicates the successful bike relocation and repeated use of bikes within the peak hour. Basically, NP attracts and diverts some bike flow on link M-B to link M-R to support the bike supply at node R. Compared to FP, NP induces higher utilization rate for the shared bikes through a balanced relocation achieved by the bike users. Also, NP does not increase the traffic flow in the network.

(a) Link M-B

(b) Link M-R

(c) Link R-B

(d) Link R-M

Figure 5.11 The bike flow changes on each link over time

In addition, we conduct the sensitivity analysis of the threshold values and the results are illustrated in Figure 5.12(a) and (b). Firstly, we assume a fixed the threshold value at origin node and seek to know how the threshold value at destination node affects the system performance in terms of total trip number and fare revenue. From Figure 5.12, one can observe that, as threshold value at destination node increases, the NP condition tends to be fulfilled more easily. Thus, the total number of trips goes up while the total fare revenue grows up first and then reduces due to the large amount of negative reward. If maximum fare revenue is to be achieved, the optimal threshold value at destination node could be easily obtained from the sensitivity analysis. In addition, we want to find how threshold value at trip origin node would affect the service operation performance. To do so, we assume fixed threshold value at destination node. It can be observed that, when threshold value at origin node increases, the NP condition becomes more difficult to be satisfied, and thus the total bike trips and the total fare revenue reduce.

Figure 5.12 The influence of threshold values on bike trips and fare revenue

The previous analysis is based on the parameter setting that VOT equals to 10 dollars per hour for all travellers. Figure 5.13 is depicted to show how the total amount of bike trips and fare revenue change after different VOT values. As VOT reduces, more travellers prefer to use bike. The increasing bike demands would result in more negative reward and thus renders the total fare revenue first increases before it goes down. In example 1, when all the travellers have the same VOT (VOT=10), the total bike trips are 230 units and the total fare revenue is 78.7 units. When we consider different classes of travellers, as categorized by different VOT values (VOT=20, 10, 7), we can obtain the model results that the total amount of bike trips is 236 and the total fare revenue is 79.6.

Figure 5.13 The influence of VOT on bike trips and fare revenue

5.5.2 Numerical Results for Test Example 2

The second example is designed to illustrate the performance of NP in more complicated network. A benchmark network Sioux Falls network is applied. The network parameter and population data are extracted from Chakirov and Fourie (2014). Three type of land use characteristics is classified: residential area, business area, and metro station as shown in Figure 5.14. The traffic flow among OD is calculated based on Gravity model in Table 5.4. Assume the departure time of each traveller randomly distributes in $[1,60]$ minutes, $T=75$, $\varepsilon=0.05$, $k=3$. Three types of

VOTs are considered with the value of 20, 10 and 7. Other parameters are same as the setting in example 1.

Figure 5.14 Simplified Sioux Falls network (the numbers on link denotes the link distance)

Table 5.4 OD demand of example 2

O \ D	Business Area	Residential Area	Metro Station
Business Area	0	0	0
Residential Area	2140 travellers	0	4291 travellers
Metro Station	3541 travellers	0	0

The computational experiments are run in MatlabR2017b on a acer notebook with an Intel Core i5-4210U, 1.7 GHz under Windows 10 Professional with 4 GB memory. When the program is complete, the running time for example 2 is 1943.07 seconds. Figure 5.15 shows the convergence of the solution algorithm.

Figure 5.15 The convergence of the solution algorithm

Figure 5.16 shows the number of bike trips and bike trip accumulation in the network over time. At the beginning, the number of bike trips are quite high under all three strategies. When time advances, the bikes run out at high demand area, and the number of bike trips quickly drop under PP and FP. Consequently, the bike trip accumulation become stable under PP and FP. However, due to relocation feature of NP, bikes can quickly be re-supplied in the undersupply area. The number of bike trips is always at a high level when NP is applied. The bike trip accumulation is even two times more than that under FP and PP.

Figure 5.16 The number of bike trips and bike trip accumulation departure over time

Figure 5.17 presents the number of bike trips and bike trip accumulation in each type of area. As shown in Figure 5.17(a), the number of bike trips and bike trip

accumulation in residential area have the same trend with Figure 5.16. Specifically, under FP, the number of bike trips is extremely high in the beginning and reduces to 0 after certain time period. Under NP, there are more bikes relocated from residential area, as can be observed in Figure 5.17(b). NP obviously attracts more travellers to park bikes at the undersupplied residential area. More specifically, there are 11 bikes arriving at residential area under PP. The number is 56 under FP, and 593 under NP. According to Figure 5.16, the total bike usage is about 1000 under PP or FP. The 593 bikes will be reused by travellers from residential area to other places. The number of bike trips under NP has increased by about 50%. Figure 5.17(c)(d) shows the specific information at node 15 in residential area. 50 bikes run out quickly under FP or PP. However, about 230 bikes depart from node 15 under NP in the whole period, where 180 of them come from the relocation effect according to Figure 5.17(d).

Figure 5.17(e)(f) gives the number of bike trips and bike trip accumulation in business area and Figure 5.17(g)(h) presents the detailed information at one specific node in business area. According to Table 5.4, the demand in business area is 0. The bike departure number under PP and FP are both zero. Under NP, there are about 600 bikes out of business area while the total bikes in the network are 1200. It reflects that NP can achieve more than 50% bike relocation.

(a) Residential area departure

(b) Residential area arrival

(c) Residential node 15 departure

(d) Residential node 15 arrival

Figure 5.17 The number of bike trips and bike trip accumulation over time

To describe the bike flow status in entire network, the dynamic bike flow rates on each link are calculated. The number of bike flow are divided into six groups: 1-40, 41-80, 81-120, 121-160, 161-200, and 201+. After classification, the results are shown in Figure 5.18. One can observe that FP easily leads to traffic congestion on bike links as it has the highest flow group (201+), as FP can quickly attract huge number of travellers to use bike. Although the total number of bike trips is similar with it under PP, the departure time of bike trips under FP is relatively concentrated. Under NP, though the total number of bike trips is greater than it under PP or FP, the trips are more evenly distributed in both time and space dimensions. Figure 5.19 shows the network performance under three pricing strategies. The average travel speed under FP is the slowest while the average travel speed under PP and NP are similar. The average travel distance under PP is the longest as the travellers who have paid the bike sharing service will fully utilise the bike to achieve door to door transport. Consequently, the number of average used links is maximum. Travellers might choose to cycle even if the trip distance is quite small under FP. Therefore, the average trip distance under FP is shorter than that under PP. Travellers will change their paths and modes under NP to relocate the bikes, which decreases the average

trip distance. Indeed, the number of used links and the average trip distance are both the smallest for NP implementation.

Figure 5.18 Link-time base on flow groups

Figure 5.19 Network performance under three pricing strategies

Next, we would like to investigate the performances of the three pricing strategies from operators' perspective. Assuming there are 50 bikes at each node provided at the beginning, we plot the bike usage and price revenue under different pricing strategies in Figure 5.20. Under FP, the total number of bike trips is only half of that under NP. Besides, the price revenue is the least for FP., while the number of used bikes is maximum, which requires highest initial investment on bikes supply from operators. In this example, NP has better performance than FP in any performance indicator. Compared to PP, NP also performs better in terms of total bike trips. However, the price revenue under NP is less than that under PP. It should be noted that the total price revenue is parameter sensitive. In the proposed dynamic pricing strategy, the threshold value of negative price is determined by the real time ratio of

supply to demand, which is a time related function Eq.(5.22). In this example, the threshold value is set at a fixed value $\Phi_{th,2}^{r_{bike}}(t)=4$. If the parameter $\Phi_{th,2}^{r_{bike}}(t)$ is increased, higher price revenue can be achieved, up to the requirement of the operators.

Next, we seek to find out the optimal number of bikes and fare revenue in initial promotion stage when different pricing strategies are applied. Some other parameters are set as follows: user value is 1000, deposit fee is 200/user, bike cost is 1500/bike (CEIBS, 2017). The relationship between the number of bikes and fare revenue is shown in Figure 5.21(a). In this example, the fare revenue from FP is always 0, which is the minimum among the three pricing schemes. The fare revenue from NP is less than PP in this example. The maximum objective function value under NP is indeed very close to that under FP. However, the required number of bikes is only 1127 with the budget of 1.17 million, which is only one third of the budget under FP.

Figure 5.20 Bike usage and price revenue under different pricing strategies

Figure 5.21 The relationship between the number of bikes and revenue and value

5.6 Summary

In this chapter, a new pricing strategy is proposed with negative price for dockless bike sharing service. Operator will pay the users who cycle from the oversupplied area to undersupplied area. This pricing strategy could achieve bike relocation through stimulating travellers' behaviour changes. Travellers' choice behaviour is formulated by a user equilibrium dynamic traffic assignment model, in which traveller are allowed to use single or combined travel mode to complete a first/last mile trip by using walk, bike or bus. Two numerical examples are designed to test the effect of introducing the proposed pricing strategy as compared to the traditional PP and FP.

The first example illustrates that FP and NP can both attract more users due to the low cost compared to PP. FP will increase the supply and demand gap when bike supply is insufficient. NP can guide the travellers to change their modes or paths to achieve a more balanced bike relocation. It also demonstrates that the proposed dynamic traffic assignment model can capture the dynamic supply-demand interactions in bike sharing system. The second example is conducted to illustrate the operation performance under different pricing strategies. With limited resources in initial promotion stage, NP performs better than FP and PP in various aspects, such as user attraction and price revenue.

CHAPTER 6 FEASIBILITY ANALYSIS OF ODPB SERVICE IN A MULTI-MODAL TRANSPORTATION NETWORK

6.1 Introduction

In reality, as of February 2016, 76 cities in China have ODPB service in operation and some other cities ODPB service in the phase of planning or construction. However, very few research papers have explicitly analysed the impact of offering ODPB in a multimodal transport system. Besides, the results from these works are conceptual and descriptive, which were mainly derived by applying statistical approaches. The primary objective of this study is to fill in the research gap by applying analytical approaches to predict the travel demand for ODPB service, evaluate how ODPB service would affect the existing multimodal transport system and investigate how ODPB service should be operated. More specifically, this study applies a simple demand analysis and network modelling approach to capture the possible adoption of ODPB service, wherein four key aspects are specially considered. First, because of ODPB's unique feature that travellers can utilise their travel time in a friendly and relaxed environment, the discomfort value is added to the travel disutility function. It has been found from existing studies that the discomfort value considerably affected the usage of public transport (Cirillo and Axhausen 2006; Ettema et al. 2012; Lyons et al. 2007). Usually, the discomfort value comes from the travel fatigue resulting from long travel time for all traffic modes users. For public transport travellers, crowdedness is an extra reason for feeling uncomfortable. Study of willingness-to-pay on the ODPB service in Beijing showed that 61.7% of the ODPB subscribers choose ODPB service is mainly because of the comfort seat in the vehicle (An et al. 2015). Second, the out-of-vehicle travel time is also considered in the travel disutility function. Out-of-vehicle travel time, including the access/egress travel time and the transfer time, is often neglected in the existing multimodal transport network modelling studies. One of the most critical advantages of auto trip is the convenient door-to-door connection, which profoundly affects the mode shift rates, especially from public transport mode to private transport mode. Third, this study inspects the crucial competitors of ODPB service, taxi, and P&R, in the multimodal transport network, as well as the common modes: auto, conventional bus and metro. Fourth, multi-class users with different VOT are taken into account in the

system. Different travellers make different trade-offs on time, cost and discomfort thus may react differently towards different changes.

6.2 Basic Consideration

6.2.1 Assumptions

Several assumptions are made before the simple model formulation as follows:

A1: This study focuses on the long-distance single-direction commuter trips on a multimodal transport corridor from suburb area to central business district (CBD) during peak hour. This multimodal transport corridor is a generally linear area that is comprised by expressway, major arterial road, and railroad.

A2: The travel demand is assumed to be uniformly and continuously distributed along the corridor, which is exogenously given.

A3: All commuters have six alternative modes: auto, P&R, taxi, metro, conventional bus and ODPB. Parking facilities are available near the origin, destination and metro station. Vehicle for auto, P&R (auto), and taxi mode is occupied by one commuter. Bus and metro could carry many commuters at the same time. ODPB users have the fixed seats in ODPB service.

A4: Metro runs on an exclusive track, where the travel time is fixed. Auto and taxi share the street in a mixed condition. Conventional bus and ODPB could use either the bus lane or the mixed-traffic street with auto and taxi, depending on the provision of bus lane. Meanwhile, considering that ODPB users usually board and alight at the two ends of the corridor, ODPB service usually uses the bus lane in the middle of the corridor, if bus lane is provided. If the conventional bus or ODPB runs on the bus lane, the travel time is measured based on the travel distance and speed. If no bus lane is provided, conventional bus and ODPB share the street with other travel modes (e.g. auto, taxi), where the travel time is calculated based on BPR (Bureau of Public Roads) function.

A5: All modes have several alternative services/routes to be selected. Commuters do not have the sufficient travel information on their mode and route choices, thus the mode and route choices follow the stochastic user equilibrium (SUE) principle and can be described by multinomial logit formulations.

A6: Commuters value the travel time differently, i.e., they have different levels of VOTs. Multi-class users are considered in this study regarding a discrete set of VOTs.

6.2.2 Notations

A multimodal travel corridor with six travel modes is schematically described in Figure 6.1, where O is the origin and D is the destination. Let m represent the travel mode, $m=a, b, c, d, e, f$ represents mode of auto, P&R, taxi, metro, conventional bus and ODPB, respectively. Let k represent the user class regarding VOT, v^m be the driving speed of mode m , p_m represent one service/route of mode m . Set L represent the length of the corridor, x is the distance from the origin to one node on the corridor, l_r is the distance from the origin to P&R facility, l_b is the distance that bus and ODPB do not use bus lane and share the street with auto and taxi at one end of the corridor. Thus, the bus lane usage proportion is $(L - 2l_b)/L$. The exogenous vehicles influence density of mode m at location x is $q^m(y)$. The exogenous passengers influence density of mode m at location x is $q^m(z)$.

Figure 6.1 A multimodal transport corridor

6.3 Travel Disutility

The travel disutility of service/route p_m is defined as a generalised cost function, which includes travel time, monetary cost, and travel discomfort. Meanwhile, travel time is comprised of in-vehicle travel time and out-of-vehicle travel time (e.g. access/egress travel time, transfer time). Therefore, the generalised travel disutility of service/route p_m by user class k denoted as $U_{p_m}^k$ could be expressed as

$$U_{p_m}^k = \omega_{ig}^k (IT_{p_m} + OT_{p_m}) + G_{p_m} + \omega_{hg} H_{p_m} \quad (6.1)$$

where IT_{p_m} and OT_{p_m} are the in-vehicle travel time and out-of-vehicle travel time of service/route p_m , respectively. G_{p_m} is the monetary cost of service/route p_m . H_{p_m} is the travel discomfort of service/route p_m . ω_{ig}^k and ω_{hg} are the conversion parameters that convert travel time of user of class k and travel discomfort to monetary cost. The cost component of each mode is listed in Table 6.1, where the detailed calculation is given in the subsections.

Table 6.1 The cost component of each mode

Mode	Time Cost	Monetary Cost	Comfort Loss
Auto	$IT_{p_a}^L + ta_{p_a} + tp_{p_a}$	$ga_{p_a} L + pa_{p_a}$	$\lambda^a IT_{p_a}^L$
P&R	$IT_{p_a}^{l_r} + ta_{p_a} + tp_{p_a} + (L - l_r)/v^d + ta_{p_d} + te_{p_d}$	$ga_{p_a} l_r + pa_{p_a} + ga_{p_d} (L - l_r)$	$\lambda^a IT_{p_a}^{l_r} + \lambda^a (L - l_r)/v^d + B^{d,L-l_r}$
Taxi	$IT_{p_e}^L + ta_{p_e}$	$ga_{p_e} L$	$\lambda^c IT_{p_e}^L$
Metro	$L/v^d + ta_{p_d} + te_{p_d}$	$ga_{p_d} L$	$\lambda^d L/v^d + B^{d,L}$
Bus	$(L - 2l_b)v^e + IT_{p_e}^{l_b} + IT_{p_e}^{L-l_b} + ta_{p_e} + te_{p_e}$	$ga_{p_e} L$	$\lambda^e \left((L - 2l_b)v^e + IT_{p_e}^{l_b} + IT_{p_e}^{L-l_b} \right) + B^{e,L}$
ODPB	$\frac{L - 2l_b}{v^f} + IT_{p_f}^{l_b} + IT_{p_f}^{L-l_b} + ta_{p_f} + te_{p_f}$	$ga_{p_f} L$	$\lambda^f \left(\frac{L - 2l_b}{v^f} + IT_{p_f}^{l_b} + IT_{p_f}^{L-l_b} \right)$

Based on vehicle travel characteristic, the in-vehicle travel time can be calculated from two different approaches. The first approach is that the in-vehicle travel time is equal to the distance travelled divided by the speed. The second approach is to use BPR function. Specifically, the in-vehicle travel time from $X = \{0, l_b, l_r, L - l_b\}$ to location x could be calculated by

$$IT_{p_m}^x = \int_{xx \in X}^x t_{p_m}^0 \left[1 + \alpha (V_{p_m}^{veh,x} / C_{p_m})^\beta \right] dx \quad (6.2)$$

where α and β are calibrated parameters. $t_{p_m}^0$ represents the free flow travel time per unit distance on the route p_m , which could be expressed as $t_{p_m}^0 = 1/v^m \cdot C_{p_m}$

denotes the traffic capacity of route p_m . $V_{p_m}^{veh,x}$ is the on-street traffic volume on route p_m at location x .

$$V_{p_m}^{veh,x} = \sum_m V_m^{veh,x} + \int_0^x \left(\sum_m q^m(y) \right) dy, \forall m = \begin{cases} \{a, b, c, e, f\}, & \text{if } 0 < x \leq l_b \\ \{a, b, c\} & , \text{if } l_b < x \leq l_r \\ \{a, c\} & , \text{if } l_r < x \leq L - l_b \\ \{a, c, e, f\} & , \text{if } L - l_b < x \leq L \end{cases} \quad (6.3)$$

6.3.1 Auto

The in-vehicle travel time by auto on route p_m could be calculated by BPR function. The out-of-vehicle travel time by auto mainly includes the access time from home to the origin node ta_{p_a} and searching time for an available parking space tp_{p_a} . Monetary cost by auto includes the distance-based cost ga_{p_a} (e.g. petrol cost) and the parking cost pa_{p_a} . Travel discomfort by auto comes from the driver fatigue, which is closely related with the travel time. Then the generalised travel disutility of route p_a by user class k could be calculated by

$$U_{p_a}^k = \omega_{ig}^k \left(IT_{p_a}^L + ta_{p_a} + tp_{p_a} \right) + \left(ga_{p_a} L + pa_{p_a} \right) + \omega_{hg} \left(\lambda^a IT_{p_a}^L \right) \quad (6.4)$$

where λ^a is comfort loss per unit of time for auto mode. ga_{p_a} is the average monetary cost per unit travel distance.

6.3.2 P&R

The generalised travel disutility by P&R mode includes two parts: the auto mode part $U_{p_b,auto}^k$ and the metro part $U_{p_b,metro}^k$. Similar to Eq. (6.4), the auto mode part here could be described as

$$U_{p_b,auto}^k = \omega_{ig}^k \left(IT_{p_a}^{l_r} + ta_{p_a} + tp_{p_a} \right) + \left(ga_{p_a} l_r + pa_{p_a} \right) + \omega_{hg} \left(\lambda^a IT_{p_a}^{l_r} \right) \quad (6.5)$$

where the only difference between the Eq. (6.4) and Eq. (6.5) is the travel distance. In Eq. (6.5), the travel distance is l_r while in Eq. (6.4) is L .

The in-vehicle travel time by metro mode is calculated based on the travel distance $L - l_r$ and the metro speed v^d . The out-of-vehicle travel time by metro mode is decided by two components: access time to metro station ta_{p_b} and the egress time

from metro station to destination te_{p_b} . Monetary cost by metro mode is mainly the ticket fare, which is a distance-based cost and the average monetary cost per unit travel distance by metro is ga_{p_d} . Travel discomfort by metro mode comes from long distance travelling discomfort and the crowdedness. Therefore, the auto mode part here could be described as

$$U_{p_b, metro}^k = \omega_{tg}^k \left[(L - l_r) / v^d + ta_{p_d} + te_{p_d} \right] + ga_{p_d} (L - l_r) + \omega_{hg} \left[\lambda^a (L - l_r) / v^d + B^{d, L-l_r} \right] \quad (6.6)$$

where $B^{d, L-l_r}$ is the crowding cost on the metro. As is done in Huang (2000), a simple linear function concerning traffic volume on the train is applied to describe the crowding cost as

$$B = a + b\zeta \quad (6.7)$$

where a and b are the calibrated parameters. ζ represents the number of passengers by metro at location x . Therefore, the crowding cost travelled from origin to location x should be

$$B^{d, x} = a + b \left[\sum_{m \in \{b, d\}} V_m^{per1, x} + \int_0^x q^b(z) dz + \int_{l_r}^x q^d(z) dz \right] \quad (6.8)$$

6.3.3 Taxi

In-vehicle travel time by taxi is the same as the in-vehicle travel time by auto. The out-of-vehicle travel time by taxi is the access time before getting the taxi. Monetary cost by taxi is a distance-based function. Travel discomfort by taxi comes from long distance travelling

$$U_{p_c}^k = \omega_{tg}^k \left(IT_{p_c}^L + ta_{p_c} \right) + ga_{p_c} L + \omega_{hg} \left(\lambda^c IT_{p_c}^L \right) \quad (6.9)$$

6.3.4 Metro

In-vehicle travel time by metro is defined by the timetable, which is highly related with the distance and the speed. Out-of-vehicle travel time by metro includes the access time ta_{p_d} and the egress time te_{p_d} . Travel discomfort by metro comes from long distance travelling and the crowdedness.

$$U_{p_d}^k = \omega_{tg}^k \left(L / v^d + ta_{p_d} + te_{p_d} \right) + ga_{p_d} L + \omega_{hg} \left(\lambda^d L / v^d + B^{d, L} \right) \quad (6.10)$$

where $B^{d, L}$ is the crowding cost on the metro from origin to location L .

6.3.5 Conventional Bus

In-vehicle travel time by conventional bus is significantly affected by the usage of bus lane. When conventional bus runs on the bus lane, the travel time could be calculated by the distance and speed. When the conventional bus runs on the street mixed with other modes, the travel time is measured by the current traffic condition. The out-of-vehicle travel time by conventional bus includes the access time ta_{p_e} and the egress time te_{p_e} . Monetary cost by conventional bus is also calculated by a distance-based function, where the unit distance charge is ga_{pe} . Travel discomfort by conventional bus comes from long distance travelling and the crowdedness. The generalised travel disutility by conventional bus can be described by

$$U_{p_e}^k = \omega_{ig}^k \left[(L - 2l_b)v^e + IT_{p_e}^{l_b} + IT_{p_e}^{L-l_b} + ta_{p_e} + te_{p_e} \right] + ga_{pe}L + \omega_{hg} \left[\lambda^e \left((L - 2l_b)v^e + IT_{p_e}^{l_b} + IT_{p_e}^{L-l_b} \right) + B^{e,L} \right] \quad (6.11)$$

where $B^{e,L}$ is the crowding cost on the bus from origin to the location L .

6.3.6 ODPB

The generalised travel disutility by conventional bus is similar to conventional bus, where the only difference is that ODPB's travel discomfort only comes from long distance travelling.

$$U_{p_f}^k = \omega_{ig}^k \left(\frac{L - 2l_b}{v^f} + IT_{p_f}^{l_b} + IT_{p_f}^{L-l_b} + ta_{p_f} + te_{p_f} \right) + ga_{p_f}L + \omega_{hg} \lambda^f \left(\frac{L - 2l_b}{v^f} + IT_{p_f}^{l_b} + IT_{p_f}^{L-l_b} \right) \quad (6.12)$$

6.4 Model and Algorithm

6.4.1 Equilibrium Condition

Let Q^k denote the total travel demand of user of class k from origin to destination on this corridor, q_m^k be the travel demand of user of class k by mode m , f_{pm}^k represent traffic volume on service p_m of user of class k . When the multimodal transport network achieves equilibrium condition, no commuter could change his/her mode and route to reduce the expected travel disutility by using other modes or routes, and the mode and route choices are governed by multinomial logit formula as follows

$$q_m^k = Q^k \cdot \exp\left[-\left(g_m^k + \theta_{\varphi m}^k\right)\right] / \sum_m \exp\left[-\left(g_m^k + \theta_{\varphi m}^k\right)\right] \quad (6.13)$$

$$f_{pm}^k = q_m^k \cdot \exp(-\mu U_{pm}^k) / \sum_{pm} \exp(-\mu U_{pm}^k) \quad (6.14)$$

where g_m^k is the mode m choice preference of user of class k ; θ and μ describe the importance of the generalised travel disutility perception in the mode and route choice decisions; φ_m^k is the expected travel disutility of user of class k by mode m , which is measured by

$$\varphi_m^k = -(1/\mu) \cdot \ln \sum_p \exp(-\mu U_{pm}^k) \quad (6.15)$$

Other than the mode choice and route choice equilibrium conditions, OD demand and traffic flow in the multimodal network are subject to the following constraints. The left items of the equations are the corresponding dual variables.

$$\sum_p f_{pm}^k = q_m^k \quad (6.16)$$

$$\sum_m q_m^k = Q^k \quad (6.17)$$

$$f_{pm}^k \geq 0 \quad (6.18)$$

$$V_i^{per,m} = \sum_k \sum_{pm} f_{pm}^k \delta_{ipm} \quad (6.19)$$

$$q_m^k > 0 \quad (6.20)$$

6.4.2 VI Model

The equivalent VI formulations for the network equilibrium Equations (6.13) -(6.14) is expressed as (Wu and Lam, 2003) :

$$\sum_k \sum_m \left\{ \sum_p \left[U_{pm}^k(f^*, q^*) \cdot (f_{pm}^k - f_{pm}^{k*}) \right] + (1/\theta) \cdot \ln(q_m^{k*}/Q^k) \cdot (q_m^k - q_m^{k*}) \right\} \geq 0 \quad (6.21)$$

where $U_{pm}^k(f^*, q^*) = U_{pm}^k(f^*, q^*) + (1/\mu) \cdot \ln(f_{pm}^{k*}/q_m^{k*})$, and the feasible region is $\Omega = \{(6.16) - (6.20)\}$.

From the Karush-Kuhn-Tucker conditions, it could be proved that the proposed model Eq. (6.21) is equivalent to the multimodal network equilibrium condition Eqs. (6.16) -(6.20). Based on Brouwer fixed point theorem, the proposed model Eq. (6.21) has at least one solution. Because the generalised travel disutility in this mode could not satisfy the monotonic condition, the unique solution of Eq. (6.21) could not be ensured. Therefore, there is a possibility of multiple solutions for Eq. (6.21).

6.4.3 Solution Algorithm

The above VI problem could be divided into two sub-problems alternately. Therefore, the solution procedure includes two phases, wherein each phase corresponds to one sub-problem. For a given q_m^k , the equilibrium model is equivalent to the mathematical problem Eq. (6.22), which is a Logit-based stochastic user equilibrium model (sub-problem 1). The commonly used solution method is diagonalisation algorithm, in which it starts from an initial feasible solution by solving a sequence of separable cost function problems that approximate the problem.

$$\min Z(f_{pm}^k) = \sum_m \sum_i \sum_k \int_0^{V_i^{per,m}} U_i^{a,k,m}(w) dw + (1/\theta) \cdot \sum_m \sum_i f_{pm}^k \cdot \ln f_{pm}^k \quad (6.22)$$

The second sub-problem is to determine the q_m^k based on the auxiliary demand f_m^k obtained from the subproblem 1. The updated q_m^k is obtained by using Method of Successive Weighted Average (MSWA), which is improved from the traditional Method of Successive Average (MSA) algorithm (Liu et al., 2009). The difference between MSWA and MSA is that the iterative step size of MSWA is not a fixed value, but gives more weight to more recent iteration points which will speed up the convergence. Therefore, the detailed solution steps are given as follows:

Step 1: Initialisation. Calculate the route generalised travel disutility for each type of user at the original situation with only considering the exogenous travel demand on the corridor. Assign all the commuters on the corridor and obtain the original demand for each type of user and the feasible path flow.

Step2: Update the path flow:

- a. Calculate the auxiliary flow by using the diagonalisation algorithm to solve the Eq. (6.22);
- b. Update the path flow by MSWA method;
- c. Repeat Steps 2.a and 2.b until the path flows are satisfied.

Step3: Calculate the expected generalised travel disutility of each path, as well as the expected travel disutility of each mode.

Step4: Calculate the auxiliary demand by logit model and update the path flow.

Step5: Convergence. Repeat Step 2-4 until the path flows are satisfied.

6.5 Numerical Example

Two numerical examples are used to investigate the performance of ODPB service: one is a simple and idealised corridor, and the other one is a realistic travel corridor in Beijing.

6.5.1 Idealised Corridor

The idealised linear corridor is shown in Figure 6.2, wherein six alternative modes exist at the same time. Each mode has only one route/service to be offered.

There are three types of parameters in this model:

First type of parameters comes from model assumption, including the location of P&R station, the weights of in-vehicle travel time and out-of-vehicle time, and the traveller demand. These parameters are simplified in the test with a given specific value. In this network, $l_r = 0.5L$, $\zeta_{it} = \zeta_{ot} = 1$, $Q^1 = Q^2 = Q^3 = Q^4 = Q^5 = 1000$.

Second type of parameters is used to describe demand and supply characteristics, which adopted from government statistical reports and existing studies outcomes. According to Beijing Statistical Year Book (2015), the parameters for value of time could be set according to the average income and transportation spending cost: $k = 5$,

$\omega_{ig}^1 = 1$, $\omega_{ig}^2 = 3$, $\omega_{ig}^3 = 4$, $\omega_{ig}^4 = 5$, $\omega_{ig}^5 = 6$, $g_a^1 = 0.8$, $g_a^2 = 0.6$, $g_a^3 = 0.7$, $g_a^4 = 0.8$, $g_a^5 = 0.9$, $g_b^1 = 0.9$, $g_b^2 = 0.5$, $g_b^3 = 0.3$, $g_b^4 = 0.7$, $g_b^5 = 0.9$, $g_c^1 = 0.9$, $g_c^2 = 0.6$, $g_c^3 = 0.3$, $g_c^4 = 0.2$, $g_c^5 = 0.1$, $g_d^1 = 0$, $g_d^2 = 0.3$, $g_d^3 = 0.5$, $g_d^4 = 0.7$, $g_d^5 = 1$, $g_e^1 = 0.4$, $g_e^2 = 0.3$, $g_e^3 = 0.2$, $g_e^4 = 0.1$, $g_e^5 = 0$, $g_f^1 = 0.6$, $g_f^2 = 0.5$, $g_f^3 = 0.4$, $g_f^4 = 0.7$, $g_f^5 = 1$.

According to Beijing Transport Annual Report (2016), the parameters relevant to trip information (e.g. average vehicle speed, public transport ticket price and headway information) are set as $v^a = v^c = 50$, $v^d = 33$, $v^e = 28$, $v^f = 32$, $ta_{pa} = 5$, $ta_{pc} = 3$, $ta_{pd} = ta_{pe} = 10$, $ta_{pf} = 6$, $tp_{pa} = 10$, $ga_{pa} = 0.8$, $ga_{pc} = 2$, $ga_{pd} = 0.25$, $ga_{pe} = 0.15$, $ga_{pf} = 0.2$, $te_{pe} = te_{pf} = 6$, $te_{pd} = 10$. As was done in Meng et al. (2014), a series of discomfort conversion parameters for different transit modes and vehicle information

in Beijing are applied here: $\omega_{hg} = 0.1$, $\lambda^a = \lambda^c = \lambda^f = 1$, $\lambda^d = 4$, $\lambda^e = 6$, $C_i^a = C_i^c = 4000$, $C_i^d = 20$, $C_i^e = 150$, $B^d = 1200$, $B^e = 80$.

Third types of parameters are those that need to be calibrated before using, including α, β, θ and μ . Using the data from real corridor in Beijing introduced in next section (as shown in Figure 6.7), a series of calibration test is conducted. The actual travel time is estimated based on the official websites (Baidu Map, 2017; Beijing Public Transport, 2017; Beijing Subway, 2017). The average travel time is selected as the calibration indicator using the percent error formula. After several tests, the parameters are finally set as $\alpha = 0.1$, $\beta = 4$, $\theta = 3$, $\mu = 0.5$, where the calibration results are shown in Table 6.2.

Table 6.2 Output data comparison based on the calibrated model

Travel type	Model result (min)	Actual result (min)	Error (%)
Auto	44.6	41.0	8.8
ODPB	53.2	50.0	6.4
Bus	96.8	100.0	3.3
Bus-Metro	68.0	85.0	20.0
P&R	59.2	70.0	15.4
Metro (section)	43.7	47.0	7.1

Figure 6.2 Corridor network

Figure 6.3 shows the effect of introducing ODPB service on mode choice on the idealised corridor, where only auto, taxi and conventional bus modes are existing for travellers. The initial mode choice in Figure 6.3(a) indicates that, with the increase in travel distance, the number of conventional bus users decreases while the number of auto users grows rapidly. It reflects that travellers prefer to use auto mode for long distance trip as compared with taxi and conventional bus. After introducing ODPB

service on the corridor, the mode choice in Figure 6.3(b) changes significantly, wherein a large portion of auto and taxi users switch to ODPB service. ODPB service demonstrates superior advantage for long distance trips, while the number of auto user is almost stable along with distance. The variation of conventional bus user is not obvious. From this comparison, it could be concluded that ODPB service has the great potential to attract auto users to use public transport service.

(a) Basic corridor with three alternative modes

(b) Corridor with four alternative modes

Figure 6.3 Influence of introducing ODPB service on mode choice

When the metro and P&R are taken into account on the corridor, the effect of introducing ODPB service on the mode choice is shown in Figure 6.4. After providing ODPB service to all travellers, the usages of auto, taxi, metro and conventional bus

decrease, especially for metro and auto usages. On the contrary, the usages of conventional bus have slight changed. Compared with the results from Figure 6.3, it could be found that auto users and metro users who have high requirement on comfort and reliability are more likely to switch their modes to ODPB service. Meanwhile, ODPB service hardly attracts the conventional bus users who are sensitive to monetary cost. It reflects that ODPB service can reduce auto and taxi usage and promote the public transport usage, especially for long-distance trip (e.g. more than 15 km). Previous studies have recognised that P&R scheme could encourage mode switch from private mode to public transport mode. Results from Figure 6.3 and 6.4 indicate that ODPB service could exert the similar function of P&R scheme regarding attracting travellers to use public transport and reducing reliance on private car.

(a) Corridor with five alternative modes

(b) Corridor with six alternative modes

Figure 6.4 Changes of mode choice before & after introducing ODPB

To further evaluate whether ODPB could have better performance than P&R, Figure 6.5 depicts the changes in mode choice when only P&R or ODPB is available for travellers on the idealised corridor with four modes. The mode choice results from the basic scenario with auto, taxi, metro and conventional bus are given in Figure 6.5(a), and the results after introducing P&R service only and ODPB service only are given in Figure 6.5(b) and Figure 6.5(c). In general, the effects of introducing P&R or ODPB on the mode choice are similar, where most of the new service users come from metro mode users. However, comparing Figures 6.5(b) and 6.5(c), one can find that the attracted patronage of ODPB is greater than P&R, especially for the users shifting from auto mode. From the operator's perspective, P&R scheme demands a large amount of investment on infrastructure construction, while ODPB service is much cheaper, with no need of new infrastructures. The investment cost for ODPB service could be recovered quickly if reasonable operation strategies are designed, as the bus for ODPB service could be rented from other usages during off-peak hours (e.g. tourist bus). Meanwhile, from the travellers' perspective, choosing ODPB service does not need to purchase a car and spend another maintenance cost, which is cheaper and environmental-friendly. Therefore, ODPB service is more competitive than P&R mode in future sustainable development from the aspects of both operator and travellers.

(a) Basic corridor with four alternative modes

(b) Basic corridor after introducing P&R scheme

(c) Basic scenario after introducing ODPB service

Figure 6.5 Changes of mode choice when only P&R scheme or ODPB service is introduced

Figure 6.6 shows the influence of road capacity on the mode choice changes. It is also found that ODPB is more attractive than P&R for any road capacity situations in this example. It is straightforward that with a fixed total demand, the road becomes crowded along with the decreasing road capacity. However, one interesting observation from Figure 6.6 is that this congestion effect does not affect ODPB users' travel decisions significantly. With decreasing road capacity, the number of ODPB users first slightly grows, and then drops a bit when the road capacity is less than 1,400 vehicle/h. It is because ODPB mostly runs on the bus lane, which is not affected by road congestion. This benefit attracts users from other modes to use ODPB. ODPB service does not operate purely on the bus lane; thus, its travel time will also increase with road congestion, which reduces its attractiveness. ODPB users may also switch to metro mode in even worse situations.

Figure 6.6 Influence of road capacity on the mode choice changes

6.5.2 Realistic Corridor

Figure 6.7 One example ODPB service in Beijing (map data © 2018 Google)

From the analysis of the idealised corridor example, it could be found that the introduction of ODPB service could attract more users from using the private cars. In this section, a realistic corridor in Beijing is selected as an example to discuss how to design operation strategies when running the ODPB service. As shown in Figure 6.7, this corridor starts from Liangxiang in Fangshan District to Liuliqiao in Fengtai District, with one OD pair (1,8) and 8 nodes. Node 1 is a residential area in Liangxiang, Fangshan District, which is one of the suburban areas in Beijing. Node 8 is business area in west part of Beijing. The distance between nodes 1 and 8 is about

30km. Nodes 2, 3 and 4 represent the metro stations, where parking lots are sufficient for P&R trips.

In reality, the travel demand is not fixed, which could be affected by various exogenous factors. To test the network performance after introducing the ODPB service, demand elasticity is considered in this example. The travel demand is set as a monotonically decreasing function of the generalised travel disutility. The responses could be maintained/change their travel mode and route, or even not travel. According to the assumption, the elastic travel demand $\hat{Q}^k$ is a monotonically decreasing function $D(\cdot)$ of the minimum expected travel disutility S^k (Li et al., 2007)

$$\hat{Q}^k = D(S^k) = Q^k \exp(-\mu S^k) \quad (6.23)$$

where Q^k is potential maximum travel demand by user of class k , S^k is the minimum expected travel disutility by user of class k . S^k is used as the index of the level of service provided by the multimodal network (Li et al., 2007), which can be calculated by

$$S^k = -\frac{1}{\theta} \ln \sum_m \exp[-(\mathcal{G}_m^k + \theta \varphi_m^k)] \quad (6.24)$$

To assess the impact of ODPB service, the change of social welfare (SW) of the multimodal transport system before and after introducing the ODPB service is used and defined as

$$\text{Relative gain of social welfare} = \frac{SW^{\text{after}} - SW^{\text{before}}}{SW^{\text{before}}} * 100\% \quad (6.25)$$

where the superscript “after” and “before” refer to the cases of “after” and “before” introducing the ODPB service, respectively (Li et al., 2007). The SW is defined as the sum of travellers’ and operators’ surplus, that is

$$SW = \left[\omega_{tg}^k \sum_k \int_0^{Q^k} D^{-1}(w) dw - \omega_{tg}^k \sum_k S^k Q^k \right] + \left[\sum_k \sum_{p_f} ga_{p_f} f_{p_f}^k - \sum_{p_f} Con_{p_f} \right] \quad (6.26)$$

where the term in the first square brackets represents the travellers’ surplus, which is the difference between the total net benefit and the total social cost incurred by all travellers (Li et al., 2007). The term in the second square brackets represents the operators’ surplus, which is the difference of the total revenues received from fare charges minus the total cost for constructing the ODPB services. Where Con_{p_f} is the

average operational cost of one ODPB vehicle, $Q^1 = Q^2 = Q^3 = Q^4 = Q^5 = 2000$, $Con_{pf} = 600$, $L=27\text{km}$, $k=5$, other user parameters in this example are set as identical to the above corridor parameters.

(a) influence of bus lane usage proportion and fare (out-of-vehicle time: 10 min)

(b) influence of out-of-vehicle time and ODPB fare (bus lane proportion: 70%)

Figure 6.8 Total realised demand and relative gain of SW resulting from ODPB services

Figure 6.8 demonstrates the influence of bus lane usage proportion, ODPB ticket price and out-of-vehicle travel time on the total realised travel demand and the SW. For this corridor, the high bus lane usage proportion, low ODPB ticket price and the short out-of-vehicle travel time will lead to positive SW and high total realised travel

demand. For example, the bus lane usage proportion is planned at about 70%, the ODPB fare should be set to less than 15 yuan to achieve positive SW according to Figure 6.8(a). Consequently, the out-of-vehicle travel time needs to be less than 10 minutes if the ODPB fare is up to 15 yuan according to Figure 6.8(b). Therefore, for the situation that bus lane usage proportion is fixed at about 70%, to ensure the positive SW, the ODPB fare should be less than 15 yuan and the out-of-vehicle time should be less than 10 minutes.

From Figure 6.8, one can also observe that the trends of SW change and total demand (TD) change are different. Total demand increases along the tendency from Q_{min} to Q_{max} while SW goes up from the minimum point W_{min} to outside around. It is because SW is affected by various factors (according to the Eq.(6.26)), where the two most significant factors are potential demand and travel disutility. When the travel disutility of ODPB service is much lower than other modes, the influence of disutility change is stronger than that of potential travel demand change. When travel disutility of ODPB service increases and becomes very close to other modes, the sensitivity of disutility change is lower than it of potential travel demand change; therefore, the total SW shows the tendency that first decreases and then increases with the increase of ODPB ticket (e.g. along with the increase of ODPB ticket price where the proportion of bus lane is 70% in Figure 6.8(a)).

(a) Influence of bus lane usage proportion and fare (out-of-vehicle time: 10 minutes)

(b) Influence of out-of-vehicle time and ODPB fare (bus lane proportion: 70%)

Figure 6.9 Average comfort loss, average fare and average time resulting from ODPB services

Meanwhile, we also noticed that W_{min} is not on the origin point of the coordinate. It is because the model includes the mode choice preference and the value of time, therefore a positive x-axis offset is founded. This offset could be explained according to Figure 6.9 where average comfort loss, average fare and average time resulting from ODPB services are described. The decrease of ODPB fare price, increase of bus lane proportion and the decrease of out-of-vehicle travel time could reduce the generalised travel disutility. Although the change tendencies of travel time, money cost and discomfort value are different, the total change is to become smaller. From Figure 6.9(a), when the ticket is cheap, the travel time, money cost and discomfort value decrease along with the increase of bus lane proportion. However, when the ticket is expensive, the graphs for the above three factors indicate significant changes due to the introduction of value of time and mode choice preference under elastic demand. Travellers with different value of time have different mode preferences according to their willingness to pay for the comfort value, travel time-saving and ticket afford. From Figure 6.9(b), along with the decrease of out-of-vehicle time, the travel time cost drops gradually. Meanwhile, due to the influence of mode choice preference and the value of time, the money cost and discomfort cost also present as a curve. All those changes lead to the offset of W_{min} .

(a) Influence of bus lane usage proportion and fare (out-of-vehicle time: 10 minutes)

(b) Influence of out-of-vehicle time and ODPB fare (bus lane proportion: 70%)

Figure 6.10 The number of auto and ODPB trips resulting from ODPB services

Figure 6.10 investigates the numbers of auto and ODPB trips resulting from ODPB services. It can be observed that although the total demand grows along with the changes of the above three influencing factors (increase of bus lane proportion, decrease of ODPB ticket price, and decrease of out-of-vehicle travel time), the changes of each mode choice are different. The result shows that ODPB trips increase, but auto trips decrease. It reflects that ODPB has the greater attraction for auto users. Meanwhile, compared to the contour line in Figures 6.8(a) and 6.8(b), it could be found that if the fare structure is fixed and bus lane usage proportion is low, to obtain the positive SW, it is better to reduce the out-of-vehicle travel time to ensure the

origin and destination of the ODPB service are close to the travellers' expectation. On the contrary, when the ODPB service can use high proportion bus lane, the effect of out-of-vehicle time is somehow not substantial.

(a) Influence of bus lane proportion and fare (out-of-vehicle time: 10 minutes)

(b) Influence of out-of-vehicle and ODPB fare (bus lane proportion: 70%)

Figure 6.11 Relative gain of SW and operators' surplus resulting from ODPB services

From the perspective of operation, it is more important to focus on the operators' surplus. Figure 6.11 shows that the increase of bus lane proportion and ODPB ticket price will both increase the operators' surplus. For Figure 6.11(a), if the out-of-vehicle travel time is 10 min, the operators' surplus is positive when the ODPB ticket

is at least 15. Considering the social surplus, the bus lane proportion should be more than 70%. For Figure 6.11(b), when the ODPB ticket is 15, bus lane proportion is 70%, the out-of-vehicle should be less than 10 minutes to obtain the positive social and operators' surplus.

6.6 Summary

In this chapter, we apply an analytical approach, rather than statistical method, to measure the ODPB service's performance and evaluate the ODPB service's impact in a multimodal travel corridor. Travellers could complete their trips by various modes like auto, taxi, P&R, conventional bus and ODPB. Travel time, monetary cost, and travel discomfort are considered in the generalised travel disutility function. The choices of mode and route/service are taken into account simultaneously in a logit-type model formulation, and the heterogeneity of travellers regarding VOTs is considered. The travel demand forecast is formulated as a VI problem and solved by MSWA-based solution algorithm.

Numerical examples from idealised corridor have demonstrated: ODPB service has the positive effect on mode switch from private car to public transport service, which is similar to P&R scheme. As compared with P&R scheme, the ODPB service has the apparent advantage of lower investment. ODPB service has superior advantage for long-distance trip as compared with other modes. Travellers with high requirement on the comfort and reliability are more likely to switch their modes to ODPB service. The usage of conventional bus is not significantly reduced after introducing ODPB service. Therefore, ODPB service could be an effective alternative public transport mode choice in future sustainable urban development.

An extension model with elastic traffic demand is also proposed and applied in the realistic corridor in Beijing. Results have shown that the bus lane usage proportion, ODPB fare and out-of-vehicle time could affect the total realised demand and relative gain of social welfare significantly. If the usage of bus lane is about 70% in the trip, ODPB fare should be set as 15 yuan per trip and the out-of-vehicle time should be less than 10 minutes to achieve the positive social and operators' surplus. Well-planned ODPB service could achieve a win-win situation between social welfare and operator surplus, which can ensure a sustainable ODPB service.

CHAPTER 7 APPLICABILITY ANALYSIS OF ODPB SERVICE IN A MULTIMODAL TRANSPORTATION NETWORK

7.1 Introduction

From Chapter 6, it can be concluded that: ODPB has some similarities with P&R in functionality and performs better than P&R at certain condition. In a sense, ODPB and P&R both focus the similar type of user group. However, it is surprised to see that most of the P&R schemes received successfully in Europe, America and Australia, while some P&R schemes failed or did not obtain satisfactory effect, especially in Asia like China and Singapore. Moreover, ODPB could be successfully operated in China, but P&R not. Three questions naturally arise in term of this situation: first, are there any factors that significantly affect the commuter's service preference on a macroscopic level, like urban city structure, population density? Second, what is the suitable condition to develop P&R or ODPB? Third, how does the operator operate the service to ensure profit while satisfying the requirements of the users at the same time?

This chapter aims to investigate the service selection problem between P&R and ODPB with considering the above mentioned three questions. The proposed model is expected to provide the scientific supporting for service selection for agencies and operators.

7.2 Basic Consideration

7.2.1 Assumptions

Several basic assumptions are made before the modelling as follows:

A1: A linear urban transportation corridor that connects the city centre and suburb (satellite city) are considered. Previous studies have widely adopted this assumption (Liu et al., 2009; Qu et al., 2014; Chen et al., 2015; Meng et al., 2016).

A2: Passengers will choose either P&R service or ODPB service.

A3: All the ODPB passengers are charged with the same fare regardless of the distance of their trips.

A4: The demand is considered as elastic, which is influenced by adjustments in disutility cost.

A5: The social welfare and operator's net profit of non-ODPB or non-P&R passengers are zero.

A6: All the routes in the residential area are projected along the horizontal and/or vertical direction. Only one road connects the residential area and CBD which is along the corridor. Vehicles start to collect the passengers in the residential area and enter the corridor at the same point. The station of P&R is single which locates on the terminal of the metro.

A7: The parking and transfer time of all P&R passengers are regarded with the same value.

A8: The access time of all ODPB passengers are the same.

A9: The number of parking space is equal to the number of P&R passengers.

A10: Each bus serves one specific area(band) along the residential corridor, as shown in Figure 7.1.

7.2.2 Notations

Primary notations used in this study are defined in Table 7.1 as follows:

Table 7.1 Notations

$L1$	length of the residential area (km)
$L2$	length from the border of residential area to P&R metro station (km)
$L3$	length of metro from P&R metro station to CBD (km)
W	width of the residential area (km)
$g(x, y)$	population density in the residential area (person/km ²)
$p(x, y)$	potential passenger density at location (x, y) (person/km ²)
ϕ	average number of trips to city centre per person per day
τ	peak hour factor
$q(x, y)$	actual passenger demand density at location (x, y) (person/km ²)
$f_1(x, y)$	access time (hour)
$f_2(x, y)$	waiting time (hour)
$f_3(x, y)$	in-vehicle time (hour)
$f_4(x, y)$	fare (\$)
μ_1	sensitivity parameters for access time
μ_2	sensitivity parameters for waiting time
μ_3	sensitivity parameters for in-vehicle time
μ_4	sensitivity parameters for fare
$t_{parking}$	parking time of P&R (hour)
$t_{transfer}$	transfer time of P&R (hour)

t_{waiting}	waiting time of P&R (hour)
H_{metro}	headway of metro (hour)
v_{auto}	speed of auto (km/hour)
v_{metro}	speed of metro (km/hour)
f_{auto}	auto cost per distance (\$/km)
f_{parking}	parking fee (\$)
f_{metro}	metro ticket fee (\$)
Q^{PR}	total passengers of P&R (passengers)
Q^O	total passengers of ODPB (passengers)
c^O	capacity of one ODPB line (passengers)
SW	social welfare (\$/hour)
C	consumer surplus (\$/hour)
O	operator's net profit (\$/hour)
R	total operating revenue (\$/hour)
Ct	total cost (\$/hour)
M^{PR}	average cost per P&R passenger (\$/hour)
M^O	average cost per ODPB line (\$/hour)
l	growth rate of L_1 (km/year)
w	growth rate of W (km/year)
g	growth rate of population (%/year)

7.2.3 Passenger Demand

Consider a linear transportation corridor of length L ($=L_1 + L_2 + L_3$) connecting city centre and suburb residential area, as shown in Figure 7.1. w and L_1 are the width and length of the residential area, L_2 is the length from the border of residential area to the nearest metro station. L_3 is the length of metro from the P&R metro station to CBD.

Figure 7.1 A Linear Corridor

Let $g(x, y)$ be the population density in the residential area. Then the potential demand density in peak hour can be defined by

$$p(x, y) = \phi \tau g(x, y), \forall x \in [0, L_1], y \in [0, W] \quad (7.1)$$

where ϕ is the average number of trips to city centre per area, τ is the peak hour factor. The distribution of the demand density is same as that of population.

According to A4, sensitivity parameters are considered to model the effects of demand elasticity. Three kinds of time cost components, including access time, waiting time and in-vehicle time, and one fare component is considered as the factors that lead to different passenger perceptibility (Chang and Schonfeld, 1991; Li et al., 2012). The actual passenger demand is represented as

$$q(x, y) = p(x, y) \exp \left[\sum_{n=1}^4 u_n f_n(x, y) \right], \forall x \in [0, L_1], y \in [0, W] \quad (7.2)$$

Where $f_1(x, y)$, $f_2(x, y)$, $f_3(x, y)$ and $f_4(x, y)$ are access time, waiting time, in-vehicle time and fare respectively; μ_1 , μ_2 , μ_3 and μ_4 are the sensitivity parameters for access time, waiting time, in-vehicle time and fare respectively. If the sensitivity parameter equals to zero, the factor has no effects on the travel demand.

7.2.3.1 For P&R

The whole travel time of P&R mode includes in-vehicle time and out-vehicle time. For P&R passengers, out-vehicle time consists of access time (park cars and walk to the metro platform) and waiting time. According to A8, P&R passengers' out-vehicle time can be given by

$$f_1^{PR}(x, y) = t_{parking} + t_{transfer}, \forall x \in [0, L_1], y \in [0, W] \quad (7.3)$$

$$f_2^{PR}(x, y) = t_{waiting} = \frac{H_{metro}}{2}, \forall x \in [0, L_1], y \in [0, W] \quad (7.4)$$

where H_{metro} is the metro service headway, and we assume that the average waiting time is equal to half of the headway.

P&R passengers' in-vehicle travelling time from the origin to destination includes two parts: in-vehicle travel time by car (driving distance/ v_{auto}) and in-vehicle travel time by metro (L_3/v_{metro}). The driving distance for each passenger is different, according to A6 and Figure 7.1, one can obtain the passengers' in-vehicle time at different locations as follows

$$f_3^{PR}(x, y) = \begin{cases} \frac{L_1 + L_2 + \frac{W}{2} - x - y}{v_{auto}} + \frac{L_3}{v_{metro}}, \forall 0 \leq y \leq \frac{W}{2} \\ \frac{L_1 + L_2 - \frac{W}{2} - x + y}{v_{auto}} + \frac{L_3}{v_{metro}}, \forall \frac{W}{2} < y \leq W \end{cases}, \forall x \in [0, L_1], y \in [0, W] \quad (7.5)$$

The monetary cost of P&R passengers includes three parts: vehicle cost, parking fee and metro ticket. The vehicle cost is mainly composed of fuel cost and vehicle depreciation cost which is distance base cost. Parking fee and metro ticket is assumed as a fixed value. Then, the total cost is defined as

$$f_4^{PR}(x, y) = \begin{cases} \left(L_1 + L_2 + \frac{W}{2} - x - y \right) f_{auto} + f_{parking} + f_{metro}, \forall 0 \leq y \leq \frac{W}{2} \\ \left(L_1 + L_2 - \frac{W}{2} - x + y \right) f_{auto} + f_{parking} + f_{metro}, \forall \frac{W}{2} < y \leq W \end{cases}, \forall \begin{matrix} x \in [0, L_1] \\ y \in [0, W] \end{matrix} \quad (7.6)$$

where f_{auto} is the vehicle cost per distance.

Then the total P&R passenger demand can be calculated by

$$Q^{PR} = \int_0^W \int_0^{L_1} q(x, y) dx dy, \forall x \in [0, L_1], y \in [0, W] \quad (7.7)$$

Substituting Eqs.(7.1) -(7.6) into (7.7), one can obtain

$$Q^{PR} = \phi \tau \int_0^{W/2} \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(\frac{W}{2} - x - y \right)} dx dy + \phi \tau \int_{W/2}^W \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(-\frac{W}{2} - x + y \right)} dx dy \quad (7.8)$$

$\forall x \in [0, L_1], y \in [0, W]$

where

$$\lambda_1 = \mu_1 (t_{parking} + t_{transfer}) + \mu_2 \frac{H_{metro}}{2} + \lambda_2 (L_1 + L_2) + \frac{\mu_3}{v_{metro}} L_3 + \mu_4 (f_{parking} + f_{metro}) \quad (7.9)$$

$$\lambda_2 = \mu_4 f_{auto} + \frac{\mu_3}{v_{auto}} \quad (7.10)$$

7.2.3.2 For ODPB

Similarly, according to A6, A10 and Figure 7.1, all routes are along horizontal and/or vertical direction. The ODPB passengers' access time is the time cost from their location (x, y) to nearest ODPB stop (on line i), which can be expressed as follows

$$f_1^O(x, y, i) = \begin{cases} \frac{y - \frac{2i-1}{2k}W}{v_{access}} + t_{access,x}, \forall \frac{2i-1}{2k}W \leq y \leq \frac{i}{k}W \\ \frac{\frac{2i-1}{2k}W - y}{v_{access}} + t_{access,x}, \forall \frac{i-1}{k}W \leq y < \frac{2i-1}{2k}W \end{cases}, \forall \begin{cases} x \in [0, L_1] \\ y \in [0, W] \\ i = 1, 2, \dots, k \end{cases} \quad (7.11)$$

where v_{access} is the passengers' average speed from the location (x, y) to nearest ODPB line, $t_{access,x}$ is the average time cost from passengers' projected points on ODPB line to nearest stop, k is the number of ODPB service lines. Passengers' in-vehicle time is calculated by ODPB trip distance and average speed, as follows

$$f_3^O(x, y, i) = \begin{cases} \frac{L_1 + L_2 + L_3 - x + \frac{2i-1}{2k}W - \frac{W}{2}}{v_O}, \forall 2i \geq k+1 \\ \frac{L_1 + L_2 + L_3 - x + \frac{W}{2} - \frac{2i-1}{2k}W}{v_O}, \forall 2i < k+1 \end{cases}, \forall \begin{cases} x \in [0, L_1] \\ y \in [0, W] \\ i = 1, 2, \dots, k \end{cases} \quad (7.12)$$

ODPB ticket price is fixed and can be given by

$$f_4^O(x, y, i) = f_O, \forall x \in [0, L_1], y \in [0, W], i = 1, 2, \dots, k \quad (7.13)$$

Considering A10, the demand should also be within the bus capacity, as well as meet the minimum bus constraint. Thus, k satisfies the condition of

$$(k-1)c^O < Q^O \leq kc^O \quad \forall k = 1, 2, \dots, N \quad (7.14)$$

where c^O is the capacity of one ODPB service line.

Thus, the ODPB total demand can be calculated by

$$Q^O = \sum_{i=1}^k \int_0^W \int_0^{L_1} p(x, y) \exp \left[\sum_{n=1}^4 \mu_n f_n^O(x, y, i) \right] dx dy, \forall \begin{cases} x \in [0, L_1] \\ y \in [0, W] \end{cases}, i = 1, 2, \dots, k \quad (7.15)$$

7.2.4 Social Welfare

As shown in Figure 7.2, consumer surplus is calculated by analysing the difference between what consumers are willing and able to pay for a good or service relative to its market price, it is the welfare of consumer. Operator's net profit is defined as the difference between the amount the operator is willing to supply goods for and the actual amount received by him when he makes the trade. It is the welfare of operator. So, the social welfare is defined as the sum of the consumer surplus (noted by C) and the operator's net profit (noted by O), that is

$$SW = C + O \quad (7.16)$$

Figure 7.2 A Supply-demand Diagram

7.2.4.1 Consumer Surplus

After converting the demand density function, the fare could be converted as a function of demand density. The total social benefit can be obtained by integrating the inverted function over the demand. Then the consumer surplus can thus be derived as the price that the users are willing to pay minus the price that the users actually pay.

7.2.4.1.1 For P&R

Base on the discussion above, the inverse of the demand density function of P&R can be expressed as

$$q^{PR^{-1}}(x, y) = \frac{1}{\mu_4} \left[\ln \frac{q^{PR}(x, y)}{p(x, y)} - \sum_{n=1}^3 (\mu_n f_n(x, y)) \right], \forall x \in [0, L_1], y \in [0, W] \quad (7.17)$$

The consumer surplus of P&R originating at location (x, y) can be formulated as

$$C^{PR}(x, y) = \int_0^{q^{PR}(x, y)} \frac{1}{\mu_4} \left[\ln \frac{z}{p(x, y)} - \sum_{n=1}^3 (\mu_n f_n(x, y)) \right] dz - q^{PR}(x, y) f_4^{PR}(x, y) \quad (7.18)$$

$$\forall x \in [0, L_1], y \in [0, W]$$

According to the Eqs. (7.8)-(7.10) and (7.18), the consumer surplus for P&R can be calculated by

$$C^{PR} = -\frac{\phi\tau}{\mu_4} \left[\int_0^{W/2} \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(\frac{W}{2} - x - y \right)} dx dy + \int_{W/2}^W \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(-\frac{W}{2} - x + y \right)} dx dy \right] \quad (7.19)$$

7.2.4.1.2 For ODPB

Similarly, we can get the inverse of the demand density function for ODPB as

$$q^{O^{-1}}(x, y, i) = \frac{1}{\mu_4} \left[\ln \frac{q^O(x, y, i)}{p(x, y, i)} - \sum_{n=1}^3 (\mu_n f_n(x, y, i)) \right], \forall i = 1, 2, \dots, k \quad (7.20)$$

Then, the ODPB consumer surplus originating at location (x, y) and boarding on line i can be expressed as

$$C^O(x, y, i) = -\frac{1}{\mu_4} p(x, y) \exp \left[\sum_{n=1}^4 (\mu_n f_n^O(x, y, i)) \right], \forall i = 1, 2, \dots, k \quad (7.21)$$

Thus, according to the Eqs. (7.8)-(7.10) and (7.21), the total consumer surplus of ODPB can be calculated by

$$C^O = -\frac{1}{\mu_4} \sum_{i=1}^k \int_0^W \int_0^{L_1} p(x, y) \exp \left(\sum_{n=1}^4 (\mu_n f_n^O(x, y, i)) \right) dx dy, \forall i = 1, 2, \dots, k \quad (7.22)$$

7.2.4.2 Operator's Net Profit

The net profit of operator can be defined as the total operating revenue (noted by R) minus the total cost (noted by Ct), that is

$$O = R - Ct \quad (7.23)$$

7.2.4.2.1 For P&R

The total revenue is the sum of parking fee and metro price paid by all the P&R passengers, that is

$$R^{PR} = Q^{PR} (f_{parking} + f_{metro}) \quad (7.24)$$

The cost is mainly the parking construction and management fees, and we regard the cost is related to the passenger demand, which is calculated by

$$Ct^{PR} = Q^{PR} M^{PR} \quad (7.25)$$

where M^{PR} is the average cost per passenger.

7.2.4.2.2 For ODPB

Similarly, we can get the total revenue and cost of ODPB

$$R^O = Q^O f_O \quad (7.26)$$

$$Ct^O = k M^O \quad (7.27)$$

where M^O is the cost of one ODPB line.

7.3 Service Selection Model

Substituting Eqs. (7.8)-(7.10), (7.19) and (7.23)-(7.25), one can obtain the social welfare of P&R as follows

$$SW^{PR} = \left(f_{parking} + f_{metro} - \frac{1}{\mu_4} - M^{PR} \right) Q^{PR}, \quad \forall x \in [0, L_1], y \in [0, W] \quad (7.28)$$

The social welfare maximisation model of P&R service can be formulated as

$$\begin{aligned} & \max SW^{PR} (L_1, L_2, L_3, W, f_{parking}, g(x, y)) \\ & = \phi \tau \left[f_{parking} + f_{metro} - \frac{1}{\mu_4} - M^{PR} \right] \\ & \cdot \left[\int_0^{W/2} \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(\frac{W}{2} - x - y \right)} dx dy + \int_{W/2}^W \int_0^{L_1} g(x, y) e^{\lambda_1 + \lambda_2 \left(-\frac{W}{2} - x + y \right)} dx dy \right] \end{aligned} \quad (7.29)$$

s.t.

$$\lambda_1 = \mu_1 (t_{parking} + t_{transfer}) + \mu_2 \frac{H_{metro}}{2} + \lambda_2 (L_1 + L_2) + \frac{\mu_3}{v_{metro}} L_3 + \mu_4 (f_{parking} + f_{metro}) \quad (7.9)$$

$$\lambda_2 = \mu_4 f_{auto} + \frac{\mu_3}{v_{auto}} \quad (7.10)$$

$$f_{parking} \geq 0$$

Similarly, substituting Eqs. (7.16), (7.22), (7.23), (7.26) and (7.27), one can obtain the social welfare of ODPB as follows

$$SW^O = \left(f_o - \frac{1}{\mu_4} \right) Q^O - k M^O, \quad \forall x \in [0, L_1], y \in [0, W], k = 1, 2, \dots, N \quad (7.30)$$

The social welfare maximisation model of ODPB service can be formulated as

$$\begin{aligned} & \max SW^O (L_1, L_2, L_3, W, f_o, g(x, y)) \\ & = \left(f_o - \frac{1}{\mu_4} \right) \sum_{i=1}^k \int_0^W \int_0^{L_1} \phi \tau g(x, y, i) \exp \left[\sum_{n=1}^4 (\mu_n f_n^O(x, y, i)) \right] dx dy - k M^O \end{aligned} \quad (7.31)$$

$$\text{s.t. } (k-1)c^O < Q^O \leq kc^O$$

$$i = 1, 2, \dots, k$$

$$k = 1, 2, \dots, N$$

$$f_o \geq 0$$

The authority usually chooses the service which can bring higher social welfare. Therefore, the proposed service selection model satisfies

$$SW_{m^*}(L_1, L_2, L_3, W, f_{m^*}, g(x, y)) = \max \{ SW_{m1}(L_1, L_2, L_3, W, f_{m1}, g(x, y)), SW_{m2}(L_1, L_2, L_3, W, f_{m2}, g(x, y)) \} \quad (7.32)$$

where SW is social welfare, $m1, m2$ are the alternative transit modes. This model also works when only consumer surplus or operator's net profit is considered.

As only two transit modes are considered in this study, the mode $m1$ is more attractive if and only if it satisfies

$$SW_{m1}(L_1^*, L_2^*, L_3^*, W^*, f_{m1}^*, g^*(x, y)) \geq SW_{m2}(L_1^*, L_2^*, L_3^*, W^*, f_{m2}^*, g^*(x, y)) \quad (7.33)$$

Other than finding the favourable public transport mode in different scenarios, one can also use the model to acquire more policy implications. For example, from Eq. (7.33), it is easy to find the critical values of different parameters, which satisfies

$$SW_{m1}(L_1^*, L_2^*, L_3^*, W^*, f_{m1}^*, g^*(x, y)) = SW_{m2}(L_1^*, L_2^*, L_3^*, W^*, f_{m2}^*, g^*(x, y)) \quad (7.34)$$

In this case, if the authority has the necessary data and growth rate, they can directly predict the future construction and investment plan. Assuming that the growth rates of $L_1, W, g(x, y)$ per year are l, w, g respectively. Then we can define the simple growth rate models of the parameters for year t as follows

$$L_1(t) = L_1(0)(1+l)^t \quad (7.35)$$

$$W(t) = W(0)(1+w)^t \quad (7.36)$$

$$g(t) = g(0)(1+g)^t \quad (7.37)$$

With the growth of these parameters, the SW of each mode will also change. If they have the critical value (Eq.(7.34)) in the future, the SW lines will intersect with each other. The time of the intersection point of SW lines will be the best investing time for new service mode. Thus, we can calculate the investing time as follows

$$t^* = \left\{ t \left| \begin{aligned} &SW_{m1}(L_1^*(t), L_2^*(t), L_3^*, W^*(t), f_{m1}^*, g^*(t, x, y)) \\ &= SW_{m2}(L_1^*(t), L_2^*(t), L_3^*, W^*(t), f_{m2}^*, g^*(t, x, y)) \end{aligned} \right. \right\} \quad (7.38)$$

7.4 Solution Algorithm

If all the parameters ($L_1, L_2, L_3, W, g(x, y), f_{parking}, f_O$) are considered as unknown variables, and our objective is to search the optimal global value of the function, then

it is a proper approach to develop a heuristic algorithm. However, in practice, the density, distance and residential area size are usually known parameters, the optimal solutions can be obtained by utilising the information of partial derivatives of objective function. In this case, the computational efficiency and accuracy of the heuristic algorithm is not an issue.

7.4.1 For P&R

Firstly, we take the first order partial derivative of the P&R objective function concerning parking fee and set the partial derivatives equal to zero. Then we have

$$\frac{\partial SW^{PR}}{\partial f_{parking}} = \mu_4 (f_{parking} + f_{metro} - M^{PR}) \int_0^W \int_0^{L_1} p(x, y) \exp \left(\sum_{n=1}^4 (\mu_n f_n^{PR}(x, y)) \right) dx dy = 0 \quad (7.39)$$

where the sensitivity parameter of fare μ_{fare} is always negative under the elastic assumption. If the population density is non-zero, the integral item is also positive. We can get the extreme value as follows

$$f_{parking} = M^{PR} - f_{metro} \quad (7.40)$$

Taking second order partial derivative of the P&R objective function concerning parking fee, we can obtain

$$\frac{\partial^2 SW^{PR}}{\partial f_{parking}^2} = \left[1 + \mu_4 (f_{parking} + f_{metro} - M^{PR}) \right] \mu_4 \int_0^W \int_0^{L_1} p(x, y) \exp \left[\sum_{n=1}^4 (\mu_n f_n^{PR}(x, y)) \right] dx dy \quad (7.41)$$

Base on the elastic assumption, if $f_{parking} = M^{PR} - f_{metro}$, the partial derivatives will always be negative, so the extreme value is the maximum value, i.e., the optimal solution of the model. Meanwhile, from Eq.(7.34), we can also find that the optimal parking fee is independent of the other variables ($L_1, L_2, L_3, W, g(x, y)$).

Similarly, we can prove the optimal parking fee for operator's net profit maximisation satisfies

$$f_{parking}^* = M^{PR} - f_{metro} - (1/\mu_4) \quad (7.42)$$

7.4.2 For ODPB

The passenger's demand is elastic. According to Eqs. (7.14) and (7.15), we can notice that if the ODPB ticket price f_o decreases, the total demand Q^o will increase, then more buses are needed, k will increase. It means lower price will lead to larger k , the maximum k exists when $f_o = 0$. As k is integer, the ODPB objective function can be regarded as a piecewise function. For any section, the partial derivative of function could be obtained as follows

$$\frac{\partial SW^o}{\partial f_o} = \mu_4 f_o \sum_{i=1}^k \int_0^W \int_0^{L_1} p(x, y) \exp \left[\sum_{n=1}^4 (\mu_n f_n^o(x, y, i)) \right] dx dy \quad (7.43)$$

where the ticket and the integral item are always positive, the sensitivity parameter of fare is always negative $\mu_4 < 0$, so the partial derivative is negative. It means the ODPB social welfare objective function is a decreasing and convergent function in each section. Thus, the optimal ODPB price is the maximum value of all the sections. It satisfies the following equation

$$SW^o = \max \{ SW^o(f_o^\odot), SW^o(f_o = 0) \} \quad (7.44)$$

where $f_o^\odot$ and $k^\odot$ are bound value, satisfy

$$Q^o(f_o^\odot) = k c^o, \forall k = 1, 2, \dots, k^* - 1 \quad (7.45)$$

$$Q^o(f_o = 0) = k^\odot c^o \quad (7.46)$$

Following the same logic, we obtain the partial derivative function for operator's net profit in the followings

$$\frac{\partial O^o}{\partial f_o} = (1 + \mu_4 f_o) \sum_{i=1}^k \int_0^W \int_0^{L_1} p(x, y) \exp \left[\sum_{n=1}^4 (\mu_n f_n^o(x, y, i)) \right] dx dy \quad (7.47)$$

If $1 + \mu_4 f_o > 0$, the function O^o is increasing in each section; otherwise the function O^o is decreasing in each section. In either case, the maximum is at the boundary. Therefore, the optimal ODPB price for operator's net profit maximization model satisfies the following equation

$$O^o = \max \left\{ O^o(f_o^{\odot 1}), O^o(f_o^{\odot 2}), O^o\left(-\frac{1}{\mu_4}\right) \right\} \quad (7.48)$$

where $f_o^{\odot 1}$ and $f_o^{\odot 2}$ are bound value, satisfy

$$Q^o(f_o^{\odot 1}) = kc^o, \forall 1 + \mu_4 f_o > 0, k = 1, 2, \dots, N \quad (7.49)$$

$$Q^o(f_o^{\odot 2}) = (k-1)c^o, \forall 1 + \mu_4 f_o < 0, k = 1, 2, \dots, N \quad (7.50)$$

7.4.3 For Investing Time

The procedure of searching the investing time in term of Eq.(7.38) is listed as follows:

Step 1: Set the initial values ($L_1, W, g(x, y), l, w, g, t$) and the iterative step Δt ;

Step 2: Solve the social welfare or the profit maximisation models for ODPB and P&R, and compare the social welfare or the profit result and calculate the difference;

Step 3: Update the value of $L_1, W, g(x, y), t$ by Eqs. (7.35)-(7.37), and redo Step 2;

Step 4: If the difference becomes larger, then stop (no critical point). Otherwise, repeat Step 3 until the difference changes its plus-minus.

7.5 Numerical Result

Consider the corridor in Figure 7.1 with two alternative services P&R and ODPB, wherein parameters are set as follows:

The parameters relevant to the size of the residential area and corridor are set to be: $L_1=2, L_2=20, L_3=3, W=1$; the settings of other parameters are referred to some transport statistical reports and previous studies (Beijing Traffic Management Bureau, 2016; Chen et al., 2015; Meng et al., 2017): $\phi=1, \tau=0.1, t_{parking}=1/30, t_{transfer}=1/60, \mu_1=-3.6, \mu_2=-2.3, \mu_3=-1.6, \mu_4=-0.025, H_{metro}=1/30, v_{auto}=54, v_{metro}=43.2, v_O=30, v_{access}=7.2, f_{auto}=1, f_{metro}=5, c^O=100, M^{PR}=30, M^O=1000$.

7.5.1 Influence of Population Distribution

Despite that uniform population density was commonly used in many previous types of research due to its simplicity, it is not very realistic. In this study, aiming to investigate the influence of different types of population distribution onto the proposed model, we consider two types of classic non-uniform distribution patterns (Berry et al., 1963; Alperovich et al., 1994):

a) **Type a:** The density is high in the middle of residential area and reduces exponentially toward the city boundary, which can be expressed as

$$g(x, y) = g_0 e^{\varphi \left[\left(\frac{L_1}{2} \right)^2 + \left(\frac{W}{2} \right)^2 - \left(\frac{L_1}{2} - x \right)^2 - \left(\frac{W}{2} - y \right)^2 \right]} \quad (7.51)$$

where φ is the sensitivity parameter, if it is equal to zero, the density is uniform.

b) **Type b:** The density is highest at the city centre and goes down linearly towards the city boundary, which can be expressed as

$$g(x, y) = g_0 \left(1 + \frac{x}{L_1} \psi \right) \quad (7.52)$$

where ψ is the sensitivity parameter, if it is equal to zero, the density is uniform.

For comparison purpose, the uniform population density is named as **type c**. Figure 7.3 describes the general patterns of the three types of population distribution.

Figure 7.3 Three types of population distribution

For a fixed size and layout corridor, we assume that the highest density is three times higher than that lowest and the total population of different distribution is identical. Then we can calculate the maximum social welfare and operator's net profit that can be achieved in the presence of P&R and ODPB respectively. The numerical results are shown in Table 7.2.

From Table 7.2, it can be seen that if the total population is the same, the population distribution type has little influence on the social welfare and operator's net profit. Therefore, the following tests shall use uniform population density to analyse the model characteristics instead of other distribution patterns.

Table 7.2 Social welfare and operator's net profit of different modes for different density distribution

		Social Welfare				Operator's net profit			
Mode		P&R		ODPB		P&R		ODPB	
Distribution		a	c	b	c	a	c	b	c
Density	1	1565	1564	1464	1459	489	489	-1020	-1022
	2	3130	3129	2834	2826	978	977	-40	-44
	3	4695	4693	4456	4448	1467	1466	940	934
	4	6260	6258	7127	7111	1955	1955	1939	1930
	5	7825	7822	9253	9228	2444	2444	1949	1939
	6	9390	9387	11144	11111	2933	2932	2939	2927
	7	10955	10951	13611	13577	3422	3421	3929	3915
	8	12520	12516	15597	15556	3911	3910	4963	4945
	9	14085	14080	17464	17414	4400	4398	4965	4947
	10	15650	15645	19993	19943	4889	4887	5962	5941
	11	17214	17209	21929	21870	5377	5376	6958	6935
	12	18779	18774	23784	23718	5866	5865	8000	7974

7.5.2 Influence of Population Density

Figures 7.4 and 7.5 demonstrate the relationship between population density and social welfare and operator's net profit respectively. As is well known, high level of social welfare or operator's net profit are the significant factors that determine whether the transport facility project could be eventually be implemented in practice. As the population density increases, both the social welfare and operator's net profit of ODPB and P&R will increase. Specifically, both increasing rates of the ODPB are quicker than P&R. As a result, the social welfare and operator's profit curves of ODPB intersect with those of P&R at points A and B, respectively. The population density at point A is 3240 persons/km², while the population density at point B is 6010 persons/km². It indicates that, if we only consider the social welfare, when the population density is higher than 3240 persons/km², the ODPB is a better choice. If we take the operator's profit into account, the lowest requirement of population density is 6010 persons/km² if ODPB is preferred. These results are very indicative to transport planners when they need to choose between the service of ODPB and P&R. Only when population density reaches a certain minimum requirement, will the ODPB be preferred. One can also observe that, as the population density goes up, the social welfare of ODPB is increasing more rapidly than operator's profit, which is not uncommon for public transport service. Therefore, if the project is invested by

government, the required population density in favour of ODPB is lower. When private sectors are involved in the project investment, and the operator's net profit has to be considered, higher population density is required.

As mentioned above, P&R is usually popular in Europe, Australia and America, while almost no successful case in Asia. The results of Figures 7.4 and 7.5 explain. P&R mode has an advantage only if the population density is low, which features well the cities in western countries. However, it is way different from Asian cities, like Beijing and Singapore, which has very high urban population density. Chen et al. (2015) found that LRT is preferable than bus rapid transit if the population density reached 10400 persons/km². Together with the results from Figures 7.4 and 7.5, it can be concluded that in this case, the ODPB service is a smart choice when the population density is above 3240 persons/km² and below 10400 persons/km².

Figure 7.4 The relationship between population density and social welfare

Figure 7.5 The relationship between population density and operator's net profit

7.5.3 Influence of Residential Area Size and Layout

The size and layout of residential area will also affect travellers' choice behaviour. Figure 7.6 indicates the preferred transit mode for different residential area size. It is evident that when the population density is fixed, increasing residential area means a larger total population size. In this case, ODPB is in a position with evident advantage.

Figure 7.6 The advantage transit mode for different residential area size

7.5.4 Influence of P&R Facility Location Social Welfare

In the literature, the optimal location of P&R facility has been investigated by many researchers (e.g., Du and Wang, 2014). Indeed, the determination of P&R facility

location also in practice needs to take into account many factors, such as land use planning. Therefore, it is necessary to test the influence of corridor distance and P&R facility location onto the model results. Figures 7.7 and 7.8 showed the influence of P&R location on social welfare under high and low population density conditions. In principle, short corridor is not suitable for operating P&R and ODPB services. As compared with other transport modes, these two services could not reduce the travel cost significantly. From Figure 7.7, one can note that when the distance from residential area to P&R station is relatively shorter, P&R service performs better regarding leading to higher level of social welfare. Nevertheless, this finding may have little practical value, because when the distance from residential to P&R station is sufficiently short, most of the travellers would directly use metro to complete their trips rather than using P&R. However, if the distance from P&R station to CBD is extended, the advantage of P&R service will shrink, which is partly due to the fixed metro fare assumption. In reality, many of developing countries with high population density are lack of complete metro system. ODPB, which does not need high investment cost, is an ideal service to support the transition period before a complete transport system is built up. Figure 7.8 depicted that P&R remains the advantage than ODPB service when the population density is low, which further supported the findings in section 7.5.2.

Figure 7.7 The influence of P&R station facility on social welfare for high population density

Figure 7.8 The influence of P&R facility location on social welfare for low population density

7.5.5 Investment Time

From previous analysis, we can conclude that both the residential area and the of total population size have significant effects on the attractiveness of ODPB and P&R services to travellers. Therefore, one fundamental question to be addressed by the policymakers is that when is the best time to invest and provide these two services in the planning horizon. The effect of the changes from some significant factors on the service selection is analysed as follows.

- a) The case when population density increases but residential area size remains unchanged

This situation can be explained based on the results from Figures 7.4 and 7.5. According to the crossing points A and B, we can obtain the population density threshold value depending on the objective of the service selection. If social welfare is set as the primary objective for service selection, then point A will be the critical population density threshold value. When the population density is larger than the value at point A (3240 persons/km²), ODPB is more suitable than P&R for investment. If the operator's net profit is set as the goal during service selection, then point B will be the critical population density threshold point. When the population density is larger than the value at point B (6010 persons/km²), ODPB is more favourable for investment. Figure 7.9 shows a case wherein we assume the population density is

3000 persons/km² and the annual growth rate is 5%. It can be seen that two years later the population density will reach the critical population density threshold point for introducing ODPB if the project will be operated by government. If both social welfare and operator's profit are considered, the proper investment time for ODPB will be 14 years later. If the density continues increasing, then LRT or metro will be the best choice in next phase according to Chen et al. (2015)'s results.

b) The case when residential area size increases but population density remains unchanged

This situation can be explained based on the results from Figure 7.6. The figure shows the social welfare critical curves for ODPB and P&R services. If the residential area size is 2km * 1km, we use the point (2.8, 0) as an example. It means that when the expansion is only along the corridor and the rate is 0.1 km/year, 8 (= (2.8-2)/0.1) years later the ODPB service will be more favourable. Additionally, if the operator's net profit is the objective in this service selection, the investment time of new service is longer, as is similar to the previous examples.

Figure 7.9 Estimation of ODPB investment time from the view of social welfare and profit (growth rate: 5%)

7.6 Summary

In this chapter, a service selection model is proposed to assist the policy makers in selecting one transport service from the two choices: P&R and ODPB. To the authors' best knowledge, the selection of these two services has not been discussed before.

The model aims to achieve maximum social welfare or operator's net profit. The optimal solution is obtained through a heuristic procedure.

A set of numerical results have been given to illustrate the model's effectiveness. The influence from several macroscopic factors have been discussed, such as the level of population density, distribution of population density, size of residential area, distance between the residential area to centre business area, P&R station location, the length of metro line, as well as the changes of residential area layout and population density growth.

The model results indicate that, if the total population is stable in a fixed residential area, the population distribution type has little influence on the social welfare and operator's net profit. Meanwhile, the level of population density, trip distance and other factors considered in this model significantly affect the service selection results. P&R fits for low population density area, while ODPB is more suitable for high population density area. This finding also explains why P&R has been successful in European cities than Asian cities. Achieving positive social welfare is easier than achieving positive operator's net profit as the required urban density is lower. This is one of the reasons why most of the public services are launched and operated by government. ODPB can bring more social welfare than P&R for long distance trip, especially when the total length of metro lines is short. Due to the low cost of investment, ODPB is a smart choice for the cities in developing countries with insufficient infrastructure systems to serve the long trip passengers. At last, the investment time of the service provision in the planning horizon is analysed.

The proposed model could provide guidelines for government agencies and operators in their service selection process. The model can also be extended to the other service selection issues, considering different objectives. However, the model also has many limitations due to the simplified model assumptions. Future work could investigate the service selection issues in a multimodal public transport system where the conventional public transport services are also considered.

CHAPTER 8 OPTIMISATION OF BUS STOP SPACING FOR ODPB SERVICE

8.1 Introduction

Bus stop spacing problem has been studied for several decades. The optimal bus stop spacing could improve the accessibility of the service, maximise the service coverage, and minimise the dwelling time. However, in current practice, determining ODPB stop spacing is simply utilising the existing conventional bus stops. With the increasing development of the system, ODPB stops should be specifically planned to attract more users. It is indeed imperative for the transport planners and bus service operators to understand how to design the optimal bus stop spacing while to the specific attributes of ODPB service are fully considered.

Three unique operational features of ODPB service need to be considered. The primary target users are commuters travelling between suburban area and city centre. Taking the direction from residential area to CBD as an example, firstly, the bus stops are all intensely located at the two ends of the corridor rather than evenly distributed along the corridor. The first few stops in residential area are purely for boarding and the last few stops in CBD are purely for alighting. Secondly, as the passenger demand is massive and concentrated within certain time window, unlike the traditional bus service which is offered the service with high-frequency services from time dimension, ODPB is designed to meet the demand requirement from space dimension where vehicles are dispatched to serve different areas. Thirdly, ODPB service could skip the stops where no passenger is waiting there for boarding or alighting, which could significantly save the total travel time.

This chapter aims to develop an optimisation model to determine ODPB stop spacing based on the specific characteristics of the ODPB services.

8.2 Assumptions

Several assumptions are made before the model formulation, as follows.

A1: The ODPB service is a kind of complementary service which mainly focuses on long-distance commute trip. The direction from residential area to CBD in morning peak hours is considered in this study.

A2: The service area is a linear corridor as shown in Figure 8.1. The left rectangle is the boarding (residential) area, where the bus stops in the boarding area are only for passenger boarding. Likewise, the bus stops in the right rectangle, alighting area (CBD), are only for passenger alighting. After collecting passengers in the residential area, the vehicle drives directly to CBD without stopping in between.

A3: The distance from the passenger's origin point to the boarding station is simplified as the sum of the horizontal and vertical distances between the start point and the boarding station, as shown in Figure 8.1. Similarly, the distance from the alighting stop to the destination point is simplified as the sum of the horizontal and vertical distances between the alighting stop and the destination point.

Figure 8.1 A diagrammatic sketch of the linear corridor

A4: Vehicle operates under the same condition at each stop, e.g. uniform acceleration speed, uniform operating speed, and uniform deceleration speed.

A5: Passenger demand follows a uniform distribution along the corridor, and does not depend on time accumulation.

A6: All the on-demand public buses are the same type.

A7: The queueing and delay times caused by signal control are not considered.

A8: If the demand between one specific origin and destination (OD) pair is over the capacity of one bus, planners shall dispatch the direct bus for this OD pair. This direct bus will not affect the optimisation results of the proposed model.

A9: The access mode to the bus station only takes walking into account.

8.3 Objective Functions

The objective of our model is to minimise the total travel time of all the passengers. Let (x, y) represent the coordinate of origin point, $p(x, y)$ represent the passenger

demand density at point (x, y) , the total travel time TT of all passengers from the origin to destination could be expressed as:

$$TT = \iint_{x,y} p(x, y) t(x, y) dx dy \quad (8.1)$$

where $t(x, y)$ is the shortest travel time from origin to destination. According to assumption A3, $t(x, y)$ could be transformed to one-dimensional expression as:

$$t(x, y) = t(x) + t(y) \quad (8.2)$$

where $t(x)$ and $t(y)$ are the shortest travel time in horizontal and vertical directions from origin to destination, respectively. $t(y)$ is purely passengers' walking time, while $t(x)$ including passengers' walking time and bus travel time.

Substituting Eq.(8.2) to Eq.(8.1), we could obtain:

$$TT = \iint_{x,y} t(x) p(x, y) dx dy + \iint_{x,y} t(y) p(x, y) dx dy \quad (8.3)$$

Since $t(y)$ is purely walking time, it is only dependent on the width of the corridor, it would not influence the optimisation result for bus stop spacing. In this case, we can simplify Eq.(8.3) with merely considering the effect from horizontal direction as follows:

$$TT = \int t(x) p(x) dx \quad (8.4)$$

Meanwhile, the total travel time could also be calculated by summing up the travel time at each trip stage, including the walking time from origin point to the boarding station $\bar{T}_w$, the in-vehicle time in the boarding area $\bar{T}_v$, the main haul (no stops) in-vehicle time $\bar{T}_v$, the in-vehicle time in alighting area $\underline{T}_v$, and the walking time from the alighting station to the destination point $\underline{T}_w$, that is:

$$TT = \omega_w \bar{T}_w + \omega_v \bar{T}_v + \omega_v \bar{T}_v + \omega_v \underline{T}_v + \omega_w \underline{T}_w \quad (8.5)$$

where ω_w and ω_v are the perception weights for walking time and in-vehicle time. Usually, passenger perceived that he spends more time than he actually is during walking than in-vehicle, i.e., $0 < \omega_v < \omega_w$. The main haul in-vehicle time $\bar{T}_v$ is not

affected by the location of the residential area and CBD, which could be expressed as:

$$\bar{T}v = PL_m/v_{veh} \quad (8.6)$$

where P is the total number of the passengers, L_m is the length of the main haul, v_{veh} is the average operation speed of the on-demand public bus. Since the CBD alighting area is small, to calculate the minimum total travel time $TT_{\min}$, we could minimise the travel time in boarding area. Thus the objective function could be simplified as:

$$TT_{\min} = \text{Min}\{\omega_w \bar{T}w + \omega_v \bar{T}v\} \quad (8.7)$$

8.4 Model Constraints

Several constraints are considered in this model based on the outcomes from traditional bus spacing studies: (1) Vehicle and segment constraints. Ideally, more vehicles will improve the level of services, which also means high investment cost. To make profit and meet the demand at the same time, the minimum number of vehicles need to be guaranteed. (2) Shed line constraints. This constraint is from user selection aspect, where users will select the bus stops located on the same side of the shed line. (3) Minimum bus stop spacing constraints. Because of the acceleration and deceleration of bus operation and the budget limitation, the bus stop spacing should be greater than a predefined value.

8.4.1 Vehicle and Segment

Since each vehicle has the same capacity limitation, the required number of vehicles N_{veh} could be calculated by:

$$N_{veh} = \left\lceil \int_0^L \frac{p(x)}{C_{veh}} dx \right\rceil, \forall x \in [0, L] \quad (8.8)$$

where C_{veh} is the capacity of the vehicle, L is the length of the boarding area on the corridor. If $p(x)$ is small, one vehicle is sufficient to service this corridor and

$N_{veh} = 1$. If $p(x)$ becomes larger, more vehicles are needed $N_{veh} > 1$. The first vehicle starts its service at the first stop in the residential area until it becomes fully occupied at a certain stop, then this vehicle will not service the remaining additional passengers and will go directly to the destination. Since the vehicle can start the service at any station in the residential area, the second vehicle will start from the boundary of the previous servicing area and follows the same operational pattern. Thus, we could divide the corridor into multi-segments according to the number of vehicles. With the increase of $p(x)$, the number of segments goes up, and the length of each segment reduces. As the bus stop spacing cannot be reduced without any limit, we assume the minimum bus stop spacing is S_* . When the length of the segment shortens to S_* , this segment is only serviced by one stop. It means that the demand at this stop is precisely equal to the capacity of one vehicle. This stop will dispatch one direct service to destination, where the passenger density $p_*(x)$ in this segment at this moment satisfies the following equation:

$$\int_{S_*} p_*(x) dx = C_{veh}, \forall x \in [0, L] \quad (8.9)$$

According to assumptions A2 and A8, besides the passengers that are serviced by the direct service, the distribution function for the remaining passengers is $p'(x)$ which can be represented as:

$$p'(x) = p(x) - \frac{C_{veh}}{S_*} k, \forall x \in [0, L], k = 0, 1, 2, \dots, N \quad (8.10)$$

where k is the number of the direct vehicles in one segment. Then, the number of non-direct vehicles is:

$$n_{veh} = \left\lceil \int_0^L \frac{p'(x)}{C_{veh}} dx \right\rceil, \forall x \in [0, L] \quad (8.11)$$

Based on the discussion above, one can conclude that each non-direct vehicle services one separate segment alone. The number of segments is equal to the number of the non-direct vehicles:

$$n = n_{veh} \quad (8.12)$$

The capability of the vehicle is limited, thus the length of the segment l_j should meet the following requirement:

$$\int_{l_j} p'(x)dx \leq C_{veh}, \forall x \in [0, L], j = 1, 2, \dots, n \quad (8.13)$$

In real operation, the length of the segment can also be divided by (a) uniformly-spaced calculation, that is $l_j = L/n$. Thus each vehicle will have several empty seats to attract and arrange the casual user to enhance the service quality; (b) The closer to CBD, the longer distance of l_j , that is $l_{j-1} < l_j$. In this situation, long-distance user can save the travel time with less time in the middle of the trip, then reducing the total uncomfortable value; (c) All the segments are fully occupied, except one segment which is servicing the users less than the vehicle's capacity, that is, $l_{j=\eta} < C_{veh}/p'(x)$, $l_{j \neq \eta} = C_{veh}/p'(x)$. This segment can be scheduled with a small capacity vehicle to reduce the operator's cost.

8.4.2 Shed Line

Shed line is the boundary which separates the passenger bus stop selection from two adjacent boarding stops into two parts. In this study, there are two types of shed line: one is the stop shed line within one segment, and the other is the segment shed line.

For the stop shed line within one segment, the shed line separates the bus stop spacing into two parts:

$$S_i^j = a_{i+1}^j + b_i^j, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.14)$$

where S_i^j is the bus stop spacing from stop i to stop $i+1$ in segment j , a_{i+1}^j is the distance from the stop shed line to stop $i+1$ in segment j , b_i^j is the distance from the stop i to stop shed line in segment j , $m_{(j)}$ is the number of bus stops in segment j .

Passenger on the stop shed line perceives that the walking time from the origin point to stop $i+1$ is equal to the sum of walking time from origin point to stop i and the in-vehicle time from stop i to stop $i+1$, that is:

$$T_{veh}^{S_i^j} \omega_v + \frac{b_i^j}{v_{walk}} \omega_w = \frac{a_{i+1}^j}{v_{walk}} \omega_w, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.15)$$

where v_{walk} is the average walking speed. $T_{veh}^{S_i^j}$ is the in-vehicle time from stop i to stop $i+1$ in segment j , which includes the average operation time and the loss time near/at the bus stop:

$$T_{veh}^{S_i^j} = \frac{S_i^j}{v_{veh}} + T_{lost,i}^j, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.16)$$

where v_{veh} is the average normal operational speed of the vehicle. $T_{lost,i}^j$ is the loss time near/at the bus stop i in segment j , which includes the deceleration time, acceleration time, time for boarding, and buffer time, as follows:

$$T_{lost,i}^j = \frac{v_{veh}}{2\sigma_1} + \frac{v_{veh}}{2\sigma_2} + \frac{n_{p,i}t_p}{n_{door}} + t_f, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.17)$$

where σ_1 and σ_2 are the deceleration and acceleration speed, $n_{p,i}$ is the number of boarding passengers at stop i , t_p is the average boarding time per passenger, n_{door} is the number of the doors of the vehicle. To ensure the bus can arrive at each stop at the fixed time, we add a buffer time t_f at each stop. Since all the stops in residential are only for boarding and the passenger density is uniform, one can regard the time loss near/at each bus stop is the same, that is:

$$T_{lost,i}^j = T_{lost}, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.18)$$

Assume that the relationship between the weight of perceived walking time and the weight of perceived in-vehicle time meets the following relation:

$$\omega = \omega_w / \omega_v \quad (8.19)$$

Combining Eqs. (8.14)-(8.16) and Eq.(8.19), we get:

$$a_{i+1}^j = \frac{\omega v_{veh} + v_{walk}}{\omega v_{veh} - v_{walk}} b_i^j + \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost,i}^j, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.20)$$

Passenger on the segment shed line perceives that the walking time from the origin point to the first stop in segment $j+1$ is equal to the sum of the walking time from the origin point to the last stop in segment j and the in-vehicle time from the last stop in segment j to the first stop in segment $j+1$, that is

$$\frac{b_{m_{(j)}}^j}{v_{walk}} \omega_w + \frac{b_{m_{(j)}}^j}{v_{veh}} \omega_v + \frac{l^{j+1}}{v_{veh}} \omega_v = \frac{a_1^{j+1}}{v_{walk}} \omega_w + \frac{l^{j+1} - a_1^{j+1}}{v_{veh}} \omega_v + m_{(j+1)} T_{lost} \quad (8.21)$$

$$\forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

where $m_{(j+1)}$ is the number of bus stops in segment $j+1$, $b_{m_{(j)}}^j$ is the distance from the last stop in segment j to the segment j 's shed line, l^{j+1} is the length of the segment

$j+1$, a_1^{j+1} is the distance from the segment j 's shed line to the first stop in segment $j+1$. Similarly, Eq.(8.21) can be simplified as:

$$a_1^{j+1} = \frac{\omega v_{veh} + v_{walk}}{\omega v_{veh} - v_{walk}} b_{m_{(j)}}^j - \frac{v_{veh} v_{walk} T_{lost}}{\omega v_{veh} - v_{walk}} m_{(j+1)}, \forall i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.22)$$

8.4.3 Minimum Stop Spacing

Since $b_i^j \geq 0$, from Eq.(8.20), we can obtain that:

$$a_{i+1}^j \geq \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost}, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.23)$$

Combing with Eq. (8.14), it leads to:

$$S_i^j \geq \frac{v_{veh} v_{walk}}{\omega v_{veh} - v_{walk}} T_{lost}, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.24)$$

According to the assumption A4, the bus stop spacing should meet the requirement of:

$$S_i^j \geq \frac{(v_{veh})^2}{2\sigma_1} + \frac{(v_{veh})^2}{2\sigma_2}, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.25)$$

Therefore, according to Eqs. (8.24) and (8.25), the minimum bus spacing is:

$$S_{\min} = \text{Max} \left\{ \left[\frac{v_{cb} v_{walk}}{\omega v_{cb} - v_{walk}} T_{lost} \right], \left[\frac{(v_{cb})^2}{2\sigma_1} + \frac{(v_{cb})^2}{2\sigma_2} \right] \right\} \quad (8.26)$$

8.5 Optimisation Model

As mentioned, the corridor is divided into n segments, where each segment contains several bus stops. According to the location diagram in Figure 8.2, the distance relationship among the segments can be expressed as:

$$A_i^j = \sum_{\xi=0}^i a_{\xi+1}^j + \sum_{\xi=0}^i b_{\xi}^j, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.27)$$

$$B_i^j = \sum_{\xi=0}^i (a_{\xi}^j + b_{\xi}^j), \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.28)$$

$$l_j = \sum_{i=1}^{m_{(j)}} (a_i^j + b_i^j), \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.29)$$

$$L = \sum_{j=1}^n l_j, \forall i = 0, 1, 2, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.30)$$

Figure 8.2 Diagram of the stop locations distance (a and b are distance between nodes while A and B are the cumulative distance between nodes)

The total travel time includes total walking time from their origin points to the bus stop and total in-vehicle time. As shown in Figure 8.2, the total walking time of all the passengers is the sum of the walking time from all the shed lines to the next adjacent bus stops and the walking time from all the stops to the next adjacent shed lines, as follows:

$$\bar{T}_w = \frac{1}{v_{walk}} \sum_{j=1}^n \sum_{i=0}^{m_{(j)}-1} \left[\int_{B_i^j}^{A_i^j} (A_i^j - x) p'(x) dx + \int_{A_i^j}^{B_{i+1}^j} (x - A_i^j) p'(x) dx \right] \quad (8.31)$$

$$\forall x \in [0, L], i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

The total in-vehicle time consists of three parts: the first part is the total in-vehicle time from the origin point to the last stop in the segment, the second part is the total in-vehicle time from the last stop in the segment to the segment shed line, and the third part is the total in-vehicle time from the segment shed line to the boundary of boarding area, that is:

$$\bar{T}_v = \sum_{j=1}^n \left[\sum_{i=1}^{m_{(j)}-1} T_{veh}^{S_i^j} p'(x) B_i^j + \frac{(B_{m_{(j)}}^j - A_{m_{(j)}-1}^j)}{v_{veh}} p'(x) l_j + \left(L - \sum_{i=1}^j l_i \right) p'(x) l_j \right] \quad (8.32)$$

$$\forall x \in [0, L], i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

The optimisation model could be written as:

$$TT_{min} = Min \left\{ \frac{\omega_w}{v_{walk}} \sum_{j=1}^n \sum_{i=0}^{m_{(j)}-1} \left[\int_{B_i^j}^{A_i^j} (A_i^j - x) p'(x) dx + \int_{A_i^j}^{B_{i+1}^j} (x - A_i^j) p'(x) dx \right] + \right. \\ \left. \omega_v \sum_{j=1}^n \left[\sum_{i=1}^{m_{(j)}-1} T_{veh}^{S_i^j} p'(x) B_i^j + \frac{(B_{m_{(j)}}^j - A_{m_{(j)}-1}^j)}{v_{veh}} p'(x) l_j + \left(L - \sum_{i=1}^j l_i \right) p'(x) l_j \right] \right\} \quad (8.33)$$

Other than the constraints Eqs. (8.10)-(8.30), the following constraints are also needed to be satisfied:

$$S_i^j \geq S_{\min} \quad (8.34)$$

$$a_i^j \geq 0 \quad (8.35)$$

$$b_i^j \geq 0 \quad (8.36)$$

$$a_0^j = 0 \quad (8.37)$$

$$b_0^j = 0 \quad (8.38)$$

8.6 Model Solution Method

One can typically apply dynamic programming approach to solve the proposed model. The detailed derivation process could be found in Vuchic and Newell (1968), where their model is the particular case of the present model when $n_l = 1$, $\omega_w = \omega_v = 1$, $b_m = 1$. We apply the embedded dynamic programming function in the Mathematica software to solve the proposed model.

As the dynamic programming could give the numerical results directly while noting that it is not easy to analyse the characteristics of the solution. Therefore, the method of Lagrange multiplier is introduced. The corresponding Lagrange function of the objective function could be written as:

$$F(a_i^j, b_i^j, \lambda_u, \kappa) = \Gamma - \sum_{u=1}^U \lambda_u g_u(x) - \kappa \left[L - \sum_{j=1}^{n_l} \sum_{i=1}^{n_j} (a_i^j + b_i^j) \right] \quad (8.39)$$

$$\forall i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

The bus stop spacing is limited within $[S_{\min}, L]$. The reduction of bus stop spacing increases the number of bus stops and decreases the total walking time. Due to the increase of the total loss time near and at the bus stops, the total in-vehicle time increases. Therefore, the objective function is convex, and the model has at least one optimal solution, which satisfies the following equation:

$$\frac{\partial F(a_i^j, b_i^j, \lambda_u, \kappa)}{\partial b_i^j} = 0, \forall i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.40)$$

Then,

$$\frac{\partial F(a_i^j, b_i^j, \lambda)}{\partial b_{\xi}^j} - \frac{\partial F(a_i^j, b_i^j, \lambda)}{\partial b_{\xi+1}^j} = 0, \forall \xi = 1, 2, \dots, n_j - 2, i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

(8.41)

Simplifying Eq.(8.40), we have:

$$\frac{2p'\omega_w \left[v_{veh} v_{walk} \omega_v T_{lost} + (v_{walk} \omega_v + v_{veh} \omega_w) b_{\xi}^j - (v_{walk} \omega_v + v_{veh} \omega_w) b_{\xi+1}^j \right]}{v_{walk} (v_{walk} \omega_v - v_{veh} \omega_w)} = 0 \quad (8.42)$$

According to Eqs. (8.14)-(8.15), Eqs. (8.18)-(8.20) and Eq.(8.28), we could obtain that:

$$B_i^j = A_i^j (\omega v_{veh} - v_{walk}) / (2\omega v_{veh}) + A_{i-1}^j (\omega v_{veh} + v_{walk}) / (2\omega v_{veh}) - (v_{walk} T_{lost}) / (2\omega) \quad (8.43)$$

$$\forall i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n$$

Let $(\omega v_{veh} - v_{walk}) / (2\omega v_{veh}) = \alpha$, $(\omega v_{veh} + v_{walk}) / (2\omega v_{veh}) = \beta$, $(v_{walk} T_{lost}) / (2\omega) = \gamma$, Eq.(8.20) can be rewritten as:

$$a_{i+1}^j = (\beta/\alpha) b_i^j + \gamma/\alpha, \forall i = 0, 1, \dots, m_{(j)} - 1, j = 1, 2, \dots, n \quad (8.44)$$

Combining Eq.(8.19), Eq.(8.42) can be simplified as:

$$b_{i+1}^j - b_i^j = \gamma/\beta, \forall i = 0, 1, \dots, m_{(j)} - 2, j = 1, 2, \dots, n \quad (8.45)$$

According to Eq.(8.44) and Eq.(8.45), it leads to:

$$\frac{a_{i+1}^j}{b_{i+1}^j} = \frac{\beta}{\alpha}, \forall i = 0, 1, \dots, m_{(j)} - 2, j = 1, 2, \dots, n \quad (8.46)$$

With Eq.(8.14), we can conclude that:

$$S_{i+1}^j - S_i^j = \frac{\gamma}{\alpha\beta}, \forall i = 0, 1, \dots, m_{(j)} - 2, j = 1, 2, \dots, n \quad (8.47)$$

Therefore, the differences in bus stop spacing in each segment are only related with α , β and γ . As discussed above, if the passenger demand follows a uniform distribution, the bus stop spacing in each segment is an arithmetic progression, and the common difference is $\gamma/(\alpha\beta)$, which is not affected by the length of corridor and the density of passenger. The common difference can be simplified as:

$$\frac{\gamma}{\alpha\beta} = \frac{2\omega T_{lost}}{\omega^2 - (v_{walk}/v_{veh})^2} v_{walk} \quad (8.48)$$

It shows that if the loss time T_{lost} near/at the bus stop is neglected, ($T_{lost} = 0$), the bus stop spacing at each segment is identical. For the conventional bus, passenger can board and alight at each stop, so the passengers in the vehicle maintain balance, the optimal bus stop spacing is identical. This result reflects that the accumulation of the

passengers on the vehicle is the main factor that makes the bus stop spacing to be an arithmetic progression.

8.7 Numerical Results

In this section, a set of numerical results are presented. Parameters in this model are listed in Table 8.1.

Table 8.1 Parameters setting

Parameter	Value	Description
v_{veh}	5.56	Average speed of bus operating (m/s)
v_{walk}	1.39	Average speed of walking (m/s)
ω	2.5	Ratio of perceived walking time to perceived in-vehicle
C_{veh}	40	Capacity of the bus (per)
p	33	Passenger density (per/km)
L	3000	Length of the residential area(m)
T_{lost}	60	Loss time at the bus stop (s)

As concluded before, the bus stop spacing is in an arithmetic progression, where the common difference is not affected by passenger density p . However, the location and the density of the bus stop are influenced by passenger density p . Figure 8.3 depicts the bus stop spacing and the number of bus stops resulting from different passenger densities. If the passenger density is low ($p < 13$), only one bus is needed to service this corridor. With the passenger density increasing, more buses are required, and thus more segments are divided. The length of each segment and the number of bus stops in each segment are correspondingly reduced. With low passenger density ($p < 26$), bus stop spacing is relevant to the number of segments and is not affected by the value of p , which is in line with the results from Vuchic and Newell (1968). The reason can be found in Figure 8.4. The capacity constraint leads to a maximum segment value (C_{veh}/p). If p is small, the optimal distance of each segment would never be longer than C_{veh}/p . If p becomes larger ($p > 26$), the capacity constraint starts to limit the objective function. With the increase of p , the length of the last segment becomes longer, while the length of other segments become shorter and are all equal to C_{veh}/p . If the lengths of all the segments are equal, the total passenger demand meets the equation of $nC_{veh} = pL$. Another conclusion can be found is that if C_{veh}/p remains the same, the locations of bus stops are fixed. It means if the number of

of existing bus stops. For routes A and C, if we extend the corridor from 3000 to 4000 with the total demands unchanged, the total number of segments is unchanged, but the bus stop spacing will increase.

Figure 8.5 Bus stop spacing resulting from passenger density and length

Passengers usually have different time perception in different situations. Usually, passengers perceived that they spend more than the actual time on the trip for both walking and in-vehicle stage. The perception ratio ω is related to the access traffic modes, the weather, geometry, level of service, and even the duration of the travel time. Figure 8.6 shows the optimal bus stop spacing resulting from different time perception weights. Although there is little difference in the length of each segment, the stop density varies significantly. If the weight is small ($\omega < 2.5$), the influence of ω on the bus stop spacing and density is obvious. The increase of the perception ratio ω will increase the bus stop density. If the perception ratio ω increases to a certain number ($\omega > 2.5$ in this case), the influence is not distinct any more. So the influence of different factors should be considered in the planning work.

Figure 8.6 Bus stop spacing resulting from time perception ratio

Some of previous studies also discussed the influence of access walking speed, normal operating speed and the time loss near/at the bus stop. To analyse the influences of those factors in this study, suppose we relax the assumption A9 that more access modes (e.g. cycling, feeder bus) with different speeds are considered. Results from Figure 8.7 show that the increase of the access speed will reduce the bus stop density, which is in line with the previous researches in conventional bus. However, as shown in Figure 8.8, the changes in normal operating speed will not affect the bus stop spacing and density significantly. Only if the speed raises by 50%, the length of the first segment becomes slightly shorter, and the length of the last segment becomes longer a little bit, while the length of the middle segment maintains the same. Moreover, the locations of the stops in different segment do not change too much. The situation of increasing the average operate speed over 50% is rare, we can regard the locations of bus stops are fixed even we change the vehicles.

Figure 8.7 Bus stop spacing resulting from walking speed

Figure 8.8 Bus stop spacing resulting from vehicle speed

Figure 8.9 shows that the optimal bus stop pacing caused by different loss time near/at the bus stop. The increase in the loss time near/at the bus stop does not influence the length of segments, but significantly reduces the number of bus stops in each segment. The density of the bus stop first drops drastically, then becomes stable. The changes in the number of bus stops follow an approximately exponential distribution. If the loss time near/at the bus stop is large (>60), the location of bus stop is relatively fixed. The loss time longer than 1minute is quite common, we can regard the bus stop locations are also fixed in this situation. Meantime, the results from Figure 8.8 are also in line with the conclusion by Lagrange multiplier method. The common difference in bus stop spacing reduces and tends to 0 if the loss time tends to 0.

Figure 8.9 Bus stop spacing resulting from time loss

From Figures 8.6-8.9, it can be noted that if the number of segments is fixed, the time perception weight ratio, access walking speed, normal operating speed and the time loss near/at the stop will not influence the length of the middle segment. Figure 8.10 shows for the case of 3 segments, the effect of the above factors on the length of segment. The increase of time perception weight ratio or the normal operating speed shortens the length of the first segment and elongates the length of the last segment, while the length of the middle segment maintains the same. Another interesting point is that although access walking speed and the time loss near/at the stop affects the stop density and stop spacing in one segment, the two factors do not affect the length of the segments.

Figure 8.10 The length of segment resulting from four factors

An ODPB service line in Beijing is selected as a simple case to illustrate the model application in real. Parameters are same as the setting in Table 8.1. As shown in Figure 8.11, the real stops are denoted as red nodes while the stops from the proposed model are denoted as blue nodes. The bus stop spacing obtained model are similar to the real situation, which can illustrate the effectiveness of the model. However, the locations for the first stop setting are different. The first bus stop in real is utilising an existing bus stop while the location from the model is based on the demand distribution. It is the limitation of the model as the model does not consider the influence of external factors such as bus stop investment, dwell time and topography.

Figure 8.11 One ODPB line in Beijing (map data © 2018 Google) and optimal stop locations

8.8 Summary

An optimisation model for bus stop spacing is proposed for on-demand public bus service. Several unique operational characteristics of on-demand public bus are considered. The model is solved by using the dynamic programming function in the

Mathematica software. To the best of our knowledge, this is the first attempt to systematically model the bus stop spacing problem for on-demand public bus service.

The results of the numerical example show that the on-demand public bus stop spacing should be planned differently from the conventional bus. First, the on-demand public bus service could dispatch the direct service when the demand from specific origin-destination pair is more than the capacity of one vehicle. Each non-direct vehicle services for one separate segment alone. The number of segments is equal to the number of the non-direct vehicles. Second, the bus stop spacing difference in one segment is in an arithmetic progression, and the common differences of the bus stop spacing for all segments are the same. The common difference is affected by the number of passengers on the vehicle. Third, the operation speed of the vehicle does not impact the bus stop spacing. Fourth, the ratio of the walking time perception to the in-vehicle time perception has the significant influence on the bus stop density. Lastly, reducing the time loss near/at the bus stop will decrease the common difference of bus stop spacing and increase the bus stop density. However, the locations for the first stop setting are different. The first bus stop in real is utilising an existing bus stop while the location from the model is based on the demand distribution. It is the limitation of the model as the model does not consider the influence of external factors such as bus stop investment, dwell time and topography.”

CHAPTER 9 CONCLUSIONS AND FUTURE WORK

9.1 Thesis Summary

Planning and design of a shared public transport service could follow the traditional public transport system planning process, but with unique features. With regards to the research objective and tasks, this thesis has obtained the following conclusions:

1) Initial bike allocation problem in dockless bike sharing system

Two market scenarios are designed including both monopoly and competitive situation. The initial bike allocation position, coverage area and allocation method are determined based on community structure and diffusion theory. Numerical examples are tested in a real bike sharing network in Singapore to illustrate the effectiveness. Simulation results show that the proposed algorithms could quickly obtain reasonable and efficient results in a large-scale network using few necessary input data. Moreover, the operational market follower strategies are discussed.

2) Bike relocation problem in dockless bike sharing system

To achieve bike sharing relocation through travellers behaviour, a new dynamic pricing strategy with negative price is introduced in dockless bike sharing system. In normal situation, user pays a positive price to operator for using a bike. When bikes are imbalanced distributed in the system compared to the real-time demand, user who cycles from the oversupply area to undersupply area will get paid from operator. A user equilibrium dynamic traffic assignment model is developed to capture travellers' mode-path choice in response to the proposed dynamic pricing strategy. Travellers can use either pure mode or combined mode to complete the trips selecting from three traffic modes: walk, bike and bus. Heterogeneous travellers with different value of time make their decisions based on the generalised travel disutility consisting of travel time, monetary cost and comfort loss. The user equilibrium principle is formulated as a variational inequality problem then solved by a path-flow swapping algorithm. Two numerical examples demonstrate that the proposed dynamic pricing strategy with negative price displays an excellent ability to attract users as well as relocate bikes, especially when the number of bikes in the system is limited. Compared to free price strategy, the proposed strategy has superior performance from both profit

and network aspects. The relative merits and applicable situations of traditional positive pricing strategy and the proposed strategy are summarised for long-term operation.

3) Feasibility analysis of ODPB service in urban multimodal transportation system

An analytical approach based on demand analysis and network modelling has been developed to analyse the influence of the provision of ODPB. Several alternative travel modes are considered. A logit-type formula captures the choices of mode and route/service and heterogeneous traveller with a discrete set of VOTs are considered. Numerical analysis from one idealised corridor indicates that the introduction of ODPB service could effectively bring a significant impact on mode shift, especially for the long-distance trip. Three factors, including ODPB fare, usage of bus lane and the out-of-vehicle travel time of ODPB service, are identified that significantly affect the mode shift performance. Another numerical example of a realistic travel corridor in Beijing is conducted to investigate the influence of ODPB service on social welfare.

4) Applicability analysis of ODPB service in a multimodal transportation network

An analytical model has been proposed to support the decision-making for selection between P&R and ODPB. The objective of the model is to maximise the total social welfare, which includes consumer surplus and operator's net profit. The model is solved by a heuristic solution procedure and tested on an idealised linear travel corridor. The numerical example considers the effects from population density, density distribution, size of residential area, P&R station location, distance from the residential area to CBD, as well as the changes of residential area layout and population growth. Results show that P&R fits for low population density area while ODPB is more suitable for high population density area. Population distribution type has little influence on the services' social welfare. ODPB is a preferable service for the city which does not have advanced metro network. The investment time for building ODPB service in the planning horizon is discussed at last with consideration of the development of residential area.

5) Optimisation stop spacing design

An optimisation model is developed for bus stop spacing of ODPB services. The objective of this model is to minimise the total travel time of all the passengers. To capture the unique characteristics of ODPB, the model constraints include the

length of segment, the configuration of shed line, minimum stop spacing, vehicle capacity limitation and nonnegative constraints. Numerical results show that the optimal bus stop spacing for ODPB is very different from conventional bus. Passenger density, travel time perception weight ratio, access/egress walking speed, and the time loss near/at the stop will affect the ODPB stop spacing significantly.

9.2 Recommendations for Future Work

As the recent emerging of shared public transport services, there are some issues that require further study. First, an integrated relocation strategy considering operator-based and user-based operation approaches might be more efficient in real operation. Second, the current work shows that ODPB has advantages over modes in given scenario (chapter 6). Further research can carry out for a complete sensitivity analysis concerning area size, demand distribution, operating policies, dispatching strategies and variation in travel time by using the data from survey or an already operating case. Third, although ODPB service is growing popular in some cities, there are still not any policies, laws and regulations which can guide ODPB planning and operation. Meanwhile, ODPB service is mainly run by internet company and government respectively, so how to make most use of the resource is still a challenge. Thus, summarising standard plan procedure will be conducted, which considers both the successful experience of other modes and the special characteristics of on-demand public service. The guide will help the operators to plan and set ODPB service.

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APPENDIX

Proof. We first prove the necessity that the ideal dynamic user equilibrium superpath choice conditions Eqs.(5.21) - (5.23) imply variational inequality Eq.(5.43). For any path p , a feasible inflow at time t is

$$f^{w,p,k}(t) \geq 0, \forall w, p, k \quad (A1)$$

Multiplying ideal dynamic user equilibrium super path choice condition Eq.(5-21) by the Eq.(5.43), it leads to

$$f^{w,p,k}(t) \cdot [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \geq 0, \forall w, p, k \quad (A2)$$

Subtracting Eq.(5.22) from Eq.(A2), one can obtain

$$[f^{w,p,k}(t) - f^{w,p,k*}(t)] \cdot [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \geq 0, \forall w, p, k \quad (A3)$$

Summing up Eq.(A3) for all paths p and all OD pairs, it follows that

$$\sum_w \sum_p \{ [f^{w,p,k}(t) - f^{w,p,k*}(t)] \cdot PU^{w,p,k*}(t) \} \geq 0, \forall w, p, k \quad (A4)$$

where the flow conservation equation

$$\sum_p f^{w,p,k}(t) = \sum_p f^{w,p,k*}(t) = f^{w,k*}(t) \quad (A5)$$

holds for each OD pair at each time interval t . Integrating Eq.(A5) from 0 to T , we can obtain variational inequality Eq.(5.43).

Next, we prove the sufficiency that any solution $f^{w,p,k*}(t)$ to Eq. (5.43) satisfies ideal dynamic user equilibrium super path choice conditions Eqs. (5.21)-(5.23). We know that the first and third ideal dynamic user equilibrium super path choice conditions Eqs. (5.21)-(5.23) hold by definition. Thus, we need to prove that the second ideal dynamic user equilibrium super path choice condition Eq.(5.22) also holds.

Assume that the second ideal dynamic user equilibrium super path choice condition Eq.(5.22) does not hold only for a path p' for OD pair w' during time interval

$$[t_1 - \theta, t_1 + \theta] \in [0, T]$$

$$f^{w',p',k}(t) > 0 \quad \text{and} \quad PU^{w',p',k*}(t) - \Gamma^{w',p',k*}(t) > 0, \forall t \in [t_1 - \theta, t_1 + \theta] \quad (A6)$$

Since the Eq. (5.22) holds for all paths other than path p' for OD pair w' at any time interval t and for OD pair w' at any time interval $t \notin [t_1 - \theta, t_1 + \theta]$, it follows that

$$\begin{aligned} & \sum_w \sum_p \sum_t [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \cdot f^{w,p,k*}(t) \\ &= \sum_{t \in [t_1 - \theta, t_1 + \theta]} [PU^{w',p',k*}(t) - \Gamma^{w',p',k*}(t)] \cdot f^{w',p',k*}(t) > 0 \end{aligned} \quad (A7)$$

Note that all other terms in the above equation vanish because of Eq.(5.22). For each OD pair w , we can always find one minimal actual travel time path $\bar{p}$ for travellers departing origin at time t , where path $\bar{p}$ was evaluated under the optimal inflow pattern $f^{w,p,k*}(t)$. For this path $\bar{p}$, Eq.(5.21) becomes an equality by definition. It follows that

$$PU^{w,\bar{p},k*}(t) - \Gamma^{w,\bar{p},k*}(t) = 0, \forall w, \bar{p}, k \quad (A8)$$

Next, we need to find a set of feasible path inflows $f^{w,p,k}(t)$ satisfying:

$$[PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \cdot f^{w,p,k}(t) = 0, \forall w, p, k \quad (A9)$$

We consider all the departure flows $f^{w,k}(t)$ for all OD pairs at each time interval t . For each OD pair w at each time t , we assign OD departure flow $f^{w,k}(t)$ to the minimal travel time path $\bar{p}$, which was evaluated under the optimal flow pattern $f^{w,p,k*}(t)$. This generates a set of feasible path inflow patterns $f^{w,p,k*}(t)$ in which Eq.(A9) is always satisfied because flows are not assigned to paths with non-minimal travel times evaluated under the optimal path inflow pattern $f^{w,p,k*}(t)$. Summing equations (A9) for all paths p and all OD pairs w , it follows that

$$\sum_w \sum_p [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \cdot f^{w,p,k}(t) = 0, \forall w, p, k \quad (A10)$$

Integrating the above equation for time period $[0, T]$, we have

$$\sum_t \sum_w \sum_p [PU^{w,p,k*}(t) - \Gamma^{w,p,k*}(t)] \cdot f^{w,p,k}(t) = 0, \forall w, p, k, t \quad (A11)$$

We subtract Eq.(A6) from Eq.(A11) and obtain

$$\begin{aligned}
& \sum_t \sum_w \sum_p \left[PU^{w,p,k}(t) - \Gamma^{w,p,k}(t) \right] \cdot \left[f^{w,p,k}(t) - f^{w,p,k*}(t) \right] \\
& = \sum_t \sum_w \sum_p PU^{w,p,k*}(t) \cdot \left[f^{w,p,k}(t) - f^{w,p,k*}(t) \right] < 0
\end{aligned} \tag{A12}$$

where the flow conservation equation

$$\sum_p f^{w,p,k}(t) = \sum_p f^{w,p,k*}(t) = f^{w,k*}(t) \tag{A13}$$

holds for each OD pair w at each time interval t so that the second term vanishes. The above equation contradicts variational inequality problem Eq.(5.43). Therefore, any optimal solution $f^{w,p,k*}(t)$ to variational inequality Eq.(5.43) satisfies Eq.(5.22). Since we proved the necessity and sufficiency of the equivalence of variational inequality Eq.(5.43) to Eqs.(5.21)-(5.23), the proof is complete.