Studies of imperfections in roller bending steel hollow section members

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**STUDIES OF IMPERFECTIONS IN ROLLER
BENDING STEEL HOLLOW SECTION
MEMBERS**

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2018

STUDIES OF IMPERFECTIONS IN ROLLER BENDING STEEL HOLLOW SECTION MEMBERS

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ABSTRACT

Structurally, the action of curving a steel hollow section member makes it possible to cover significant longer spans, since the load is carried largely in compression instead of through bending action. Due to the excellent mechanical properties and geometric tolerances, the curved steel hollow section members are often employed for the construction of high load-bearing roof structures.

Cold bending by means of a rolling machine with three adjustable rollers in a triangle or pyramid type is suitable for the large bends required for construction purposes. Roller bending involves progressive bending of a section by passing the member through a set of bending rolls. During the cold-bending process of steel hollow section member, with either square or rectangular section, relatively high loads are applied to achieve a permanent member curvature. These could cause imperfections such as sectional deformations and residual stresses in the member. Such loads impart large permanent local deformations in the four relatively thin hollow section member faces and sectional deformation in the cross section of the member. The sectional deformations in the cross section are caused by web crippling coupled with compression flange buckling. As a result, there is a reduction in the section moment of inertia. The permanent sectional deformation of the cross section may affect its aesthetic appeal or integrity to carry loads when used after rolling. The cold roller bending process also induces residual stress in the section walls. The residual stress magnitude and distribution could have sufficient influence on the member's stability and buckling resistance.

Therefore, a certain acceptable level of sectional deformation should be established. In this paper, the experimental, numerical and analytical methods were used to investigate the relationship between the cross-sectional sectional deformation and the member's potential curvature. Furthermore, the yield line mechanism and energy minimization theory are employed to predict the sectional deformation of steel rectangular hollow section after cold roller bending. The yield line models were developed based on the actual deformation observed from experimental results and numerical simulations. Based on the theoretical analysis as well as the

experimental results, simple analytical models are proposed to predict the relationships between the bending radius of the curved member and the resulting deformations of the cross section under different deformation modes.

While few studies have paid attention to the residual stress distribution caused by roller bending, a proper numerical modelling procedure is employed to simulate the rolling process and reproduce the residual stress in this paper. In addition, a small scale parametric study is conducted to investigate the effects of some key roller bending parameters, including the rolling boundary conditions, the bending ratio, the steel yield stress, the thickness ratio and the shape factor, on the resulted residual stress distribution of the bent sections. Based on the results obtained from the parametric studies, a simple residual stresses model which could predict the residual stress distribution of the curved member is proposed.

Contents

ACKNOWLEDGEMENTS	0
ABSTRACT.....	II
Contents	IV
List of Figure.....	VI
List of Table	IX
List of Symbols	X
Chapter 1 Introduction	1
1.1 Research Background.....	1
1.2 Methods of Bending Steel Beams	2
1.3 Literature Review	5
1.4 Objectives.....	8
Chapter 2 Experimental Investigation of Sectional Deformation for Roller Bent Hollow Sections	11
2.1 Introduction	11
2.1 Material test.....	11
2.2 Specimen Details.....	14
2.3 Roller Bending Process	14
2.4 Test Results and Discussion	17
2.5 Summary	25
Chapter 3 Experimental Investigation of Residual Stress for Roller Bent Steel Hollow Sections	26
3.1 Introduction	26
3.2 Hole-drilling method	26
3.3 Strain gauge scheme and specimens	29
3.4 Measurement procedure	32
3.5 Experimental Results and Discussion	35
3.6 Summary	39

Chapter 4 Numerical Study of Imperfections for Roller Bent Steel Hollow Sections	40
4.1. Finite Element Modelling.....	40
4.2 Numerical Simulation of Roller bending Process	43
4.3 Residual Stresses Forming	44
4.4 Numerical Results and Validation with Sectional Deformation	47
4.5 Validation with Residual Stress	51
4.6 Summary	60
Chapter 5 Sensitivity Study	61
5.1 Sensitivity Analysis on Deformation	61
5.2 Sensitivity Analysis on Residual Stress	66
5.3 Proposed Residual Stresses model	78
5.4 Validation with Proposed Model.....	79
5.5 Summary	83
Chapter 6 Theoretical Analysis of Deformation Caused by Roller Bending.....	84
6.1 Roller Bending Process	84
6.2 Yield Line Mechanism	84
6.3 Energy Minimization Theory	89
6.4 Analytical Model.....	90
6.5 Results and Verification	94
6.6 Summary	97
Chapter 7 Conclusions and Recommendations.....	98
7.1 Conclusions	98
7.2 Recommendations for future research work	99
Reference	101

List of Figure

Figure 1.1 Curved steel elements in building	2
Figure 1.2 Induction bending equipment	4
Figure 1.3 Rotary-draw bending equipment	4
Figure 1.4 Roller bending machine.....	5
Figure 1.5 Force diagram of roller bending	5
Figure 2.1 The dimensions of the coupon.....	12
Figure 2.2 Coupon test machine	13
Figure 2.3 Details of specimen after roller bending	17
Figure 2.4 Roller bending by pyramid-type bending machine	15
Figure 2.5 Specimen under roller bending.....	15
Figure 2.6 Web concave of section after roller bending	18
Figure 2.7 Measurement of deformation of cross-section	18
Figure 2.8 Section before and after roller bending	20
Figure 2.9 Plots of concave ratio against bending ratio.....	21
Figure 2.10 Plots of expansion ratio against bending ratio.....	21
Figure 2.11 Plots of concave ratio against shape factor.....	23
Figure 2.12 Plots of expansion ratio against shape factor	23
Figure 2.13 Plots of concave ratio against thickness ratio.....	24
Figure 2.14 Plots of expansion ratio against thickness ratio.....	24
Figure 3.1 Stress state near the drilling point	28
Figure 3.2 Schematic Diagram of strain gauge for residual stress measurement ...	29
Figure 3.3 Strain gauge rosette types	30
Figure 3.4 Arrangements of strain gauge rosettes	31
Figure 3.5 Arrangements of strain gauge rosettes in specimen	32
Figure 3.6 RS-200 milling apparatus	34
Figure 3.7 Hole-drilling method measuring residual stress	34
Figure 3.8 Measured residual stress distribution in RHS 150×100×6mm.....	35
Figure 3.9 Measured residual stress distribution in RHS 200×100×6mm.....	37
Figure 3.10 Measured residual stress distribution in RHS 200×150×6mm.....	37
Figure 3.11 Measured residual stress distribution in RHS 200×200×5mm.....	38

Figure 4.1 Typical roller bending process	41
Figure 4.2 Material behavior.....	41
Figure 4.3 Typical mesh used in FE modelling	41
Figure 4.4 Description of curvature along beam	43
Figure 4.5 Sectional deformation patterns after roller bending	44
Figure 4.6 Material behaviour under loading	45
Figure 4.7 Stress history of one web at the mid-span section of a $203 \times 154 \times 6.35$ mm SHS during cold-rolling process.....	47
Figure 4.8 Final residual stresses distribution of the section at mid-span and plot of the residual stress variation along the free web (Compressive stress is negative)...	47
Figure 4.9 Comparison of deformation pattern between experimental and numerical results	48
Figure 4.10 Experimental and numerical deformation results.....	50
Figure 4.11 Longitudinal and Von-Mises residual stress distributions along section circumference.....	53
Figure 4.12 Residual stress layering distribution.....	55
Figure 4.13 Tested and numerical residual stress along RHS150×100×6mm.....	56
Figure 4.14 Tested and numerical residual stress along RHS200×100×6mm.....	57
Figure 4.15 Tested and numerical residual stress along RHS200×150×6mm.....	58
Figure 4.16 Tested and numerical residual stress along RHS200×200×5mm.....	59
Figure 5.1 Plots of concave value against bending ratio	62
Figure 5.2 Plots of difference of height against bending ratio.....	62
Figure 5.3 Plots of web concave against shape factor	63
Figure 5.4 Plots of section depth difference against shape factor.....	64
Figure 5.5 Plots of concave value against bending ratio R/h	65
Figure 5.6 Plots of concave value against thickness ratio b/t	65
Figure 5.7 Variation of $\sigma_{cfl,c}/f_y$ with bending ratio R/h	68
Figure 5.8 Variation of $\sigma_{web,m}/f_y$ with bending ratio R/h	68
Figure 5.9 Longitudinal residual stress distribution (in N/mm^2) of a $254 \times 152 \times 9.53$ mm hollow section with different bending ratios.....	69
Figure 5.10 Variation of $\sigma_{cfl,m}/f_y$ with bending ratio R/h	69
Figure 5.11 Variation of $\sigma_{tfl,m}/f_y$ with bending ratio R/h	69

Figure 5.12 Suggested simplified trends for $\sigma_{cfl,m}/f_y$, $\sigma_{cfl,c}/f_y$, $\sigma_{web,m}/f_y$ and $\sigma_{tfl,m}/f_y$ against R/h	70
Figure 5.13 Plots of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ against f_y	71
Figure 5.14 Plots of $\sigma_{cfl,m}/f_y$ against f_y for two SHS with bending ratio $16 \leq R/h \leq 124$	72
Figure 5.15 Plots of $\sigma_{tfl,m}/f_y$ against f_y two SHS with bending ratio $16 \leq R/h \leq 124$	72
Figure 5.16 Plot of $\sigma_{cfl,c}/f_y$ against R/h for SHS with different shape factor h/b	73
Figure 5.17 Plot of $\sigma_{cfl,c}/f_y$ against h/b for SHS with $20 \leq R/h \leq 150$	73
Figure 5.18 Variations of $\sigma_{cfl,c}/f_y$, $\sigma_{web,m}/f_y$ and $\sigma_{tfl,c}/f_y$ with shape factor h/b with $20 \leq R/h \leq 150$	75
Figure 5.19 Variations of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ with thickness ratio h/t with $20 \leq R/h \leq 150$	76
Figure 5.20 Variations of $\sigma_{cfl,m}/f_y$ and $\sigma_{tfl,m}/f_y$ with thickness ratio h/t with $20 \leq R/h \leq 150$	77
Figure 5.21 Proposed residual stresses distribution	79
Figure 5.22 Verification of the proposed residual stresses model for square hollow sections	80
Figure 5.23 Verification of proposed residual stresses model for rectangular hollow sections	81
Figure 5.24 Verification of proposed residual stresses model for RHS $150 \times 100 \times 6$ mm	81
Figure 5.25 Verification of proposed model for RHS $200 \times 100 \times 6$ mm	82
Figure 5.26 Verification of proposed model for RHS $200 \times 150 \times 6$ mm	82
Figure 5.27 Verification of proposed model for RHS $200 \times 200 \times 5$ mm	83
Figure 6.1 Curved member with distorted walls	85
Figure 6.2 Yield line models	86
Figure 6.3 The assumed plastic hinge	89
Figure 6.4 Comparison of theoretical and experimental flange deformation in Model 1	96
Figure 6.5 Comparison of theoretical and experimental deformation in Model 2 ...	97

List of Table

Table 2.1 Material properties	13
Table 2.2 Specimen details	16
Table 2.3 Sectional deformation obtained from test	19
Table 3.1 Specimen details	31
Table 4.1 Dimensions of roller system used in FE models.....	42
Table 4.2 Comparison between test and numerical results.....	49
Table 4.3 Changes in section properties after rolling	51

List of Symbols

h	Height of the hollow section
b	Width of the flange of a hollow section
t_f	Thickness of the flange of the steel section
t_w	Thickness of the web of the steel section
e	The concave value of the compressed flange
h_1	The height of the section after roller bending
b_1	The width of the section after roller bending
f_y	Yield strength of the steel
R	Bending radius of the member
e_u	Percentage elongation
E_0	Young's modulus
L	Length of the specimen
s	Space between the two outer rollers
ν	Poisson's ratio
n	Roller bending pass number
I	Inertia of moment
K	Stiffness factor of the member
W_{el}	Elastic section modulus
W_{pl}	Plastic section modulus
M	Bending moment
M_p	Moment dissipate the plastic energy
M_b	Moment to distort the sectional walls
M_{pd}	Plastic bending moment change

y_n	Distance from neutral axis to bottom flange of the section
k	Adjust factor
<i>SHS</i>	Steel Hollow Section
σ_x	Normal stress in X-axis
σ_y	Tested material's yield strength/ Normal stress in Y-axis
σ_u	Tested material's ultimate strength
τ_{xy}	Shear stress
ε	Relieved strain by hole-drilling
$\sigma_{cfl,m}$	Residual stress at middle of compressive flange
$\sigma_{cfl,c}$	Residual stress at compressive corner
$\sigma_{tfl,m}$	Residual stress at middle of tension flange
$\sigma_{web,m}$	Residual stress at middle of web
μ	Fiction coefficient
φ	Bending angle
θ	Rotation angle of plastic hinge
Δ	Generic deflection
Δ_f	Deflection in the compression flange
Δ_w	Deflection in the webs
ΔE	Total energy change
σ_{zb}	Average stress value in compression flange
λ_f	Normalized geometric factor about flange
λ_w	Normalized geometric factor about web
$\bar{a}$	Calibration constant for isotropic stress
$\bar{b}$	Calibration constant for shear stress

Chapter 1 Introduction

1.1 Research Background

Due to their excellent mechanical properties and geometric tolerances, hot finished steel rectangular structural hollow sections are often employed for the construction of high load-bearing structures. In addition, curved steel hollow section members are required in many applications in structural engineering such as the construction of domes, arched roofs, bridges and aesthetic structures. Curved steel structures are often designed to provide the structure with natural light and a sense of spaciousness and grander in public facilities. This has led to forms of structures in which relatively light curved steel trusses or arched frames support substantial areas.

Structurally, the function of a curved member makes it possible to cover longer spans as the action is carried largely in compression instead of bending moment. For certain curved member geometries, supporting conditions and load distributions, the effects of the imposed forces translate into uniform compression within the whole member. Some of the individual members are curved about the strong axis of the shapes; others are curved about the weak axis. These forms of curved elements may employed in buildings individually or mutually (See Figure 1.1).

The curved members that are applied in buildings and bridges impose lower deformation demands on the structure. The radius of curvature is generally very large. The early arch structures were made from build-up elements by welding or bolted connection. The shape is usually created by cutting the steel to form in smaller elements, connecting the curved elements to develop the curvature, and then building the completed arch on the construction site. For the structures required relatively low accuracy of curvature and the processing equipment is limited, this method is commonly used. However, the stringent assembling specification should be fulfilled during in-site construction. Any tiny installing inaccuracy will be accumulated and leads to the structural internal force a huge difference. Commonly, the change of structural mechanical diagram is harmful for the structure. The shaping process for large span arches requires the steel only to undergo small

plastic deformations during the fabrication. In-service conditions entail only elastic response demands for the curvature, as is the case for most structures.

Figure 1.1 Curved steel elements in building

1.2 Methods of Bending Steel Members

There are a multiple methods used to bend steel members. Three of the most common types of bending are induction bending, rotary-draw bending, and roller bending or pyramid bending. The heat induction process generally cost much than the latter two cold processes, although smaller radii can be achieved. The different processes produce different finishes, depending on its sectional and material properties, the intentional bending radius, and the bending machines (boundary conditions).

In induction heat bending, the sections (pipes, hollow sections or open sections) to be curved are passed through an electric coil. In this process, the heating coil is placed around the bend point to warm up the entire cross section over a narrow range. Once the steel reaches a certain temperature range, pressure is applied to the front end of the member in order to bend it to the proper radius. When bending is completed, air or liquid (water) is used to cool down the material immediately adjacent to the heated zone. The leading end of the section is clamped to a pivoted radius arm (see Figure 1.2). Heat induction bending relaxes the steel during the bending process, which helps achieve tight radii as well as minimizes deformations.

Rotary-draw bending is mainly used for complicated bends in the machine and parts industry. Rotary draw bending involves clamping on the outside diameter of a tube

and drawing it over a form whose radius matches the desired bend radius. This bending method often employs an internal supporting mandrel and a wiper die to prevent wrinkling on the inside wall of a tight bend, as shown in Figure 1.3. Some rotary draw machines can perform both push bending and rotary bending with a single tooling setup. The bending process can cause the material to get slight thicker where the metal is under compression and thinner where it is under tension.

Cold bending by means of a rolling machine has three adjustable rollers in a triangle or pyramid type (see Figure 1.4) was named as roller bending in this paper, which is suitable for the large bends required for construction purposes (King and Brown 2001). Roller bending method involves progressive bending of an open or hollow section by passing the member through a set of bending rollers. Before starting the bending process, the three rollers were adjusted to the proper spacing. The process is repeated until the required curvature is achieved.

Rolling machines are commonly used to bend plates or beams with solid sections. During the roller bending process, the straight hot finished steel hollow section member could be considered as a simple beam which is supported by two outer rollers and subjected to loading from the moving center roller, as shown in Figure 1.5. High loads are required to achieve a permanent radius of curvature. Such loading together with the changing position of the center roller eventually produce the required permanent radius of curvature of the final curved section. However, the loads applied during the rolling process also induce large permanent local deformations of the four walls of the steel hollow section and generate residual stresses in different parts of the section (King and Brown 2001).

The sectional deformations in the cross section are caused by web crippling coupled with compression flange buckling. As a result, there is a reduction in the section moment of inertia as a beam. The amount of deformation in a steel hollow section member depends on the factors of section dimensions, material properties, radius of bend imposed, dimensions of the machine rollers and the distance between the two outer rollers.

The permanent deformation of the cross-section may affect its aesthetic appeal or structural integrity to carry loads when used after roller bending. Therefore, a certain acceptable level of sectional deformation should be established. Based on this acceptable level, the minimum radius of bending of a steel hollow section member can be determined.

Figure 1.2 Induction bending equipment

Figure 1.3 Rotary-draw bending equipment

Figure 1.4 Roller bending machine

Figure 1.5 Force diagram of roller bending

1.3 Literature Review

Most of the studies carried out on the rolling of structural elements were directed towards studying the different aspects of rolling of plates and beams with solid sections. Hanson and Jannerup (Hanson and Jannerup 1979) developed a method to study the geometry of the deflected shape of beams with solid sections during rolling process. Bjorhovde (Bjorhovde 2006) developed a basic curving criterion to evaluate the relationship between the radius of curvature, the depth of the section, the yield stress of steel and the maximum strain factor in wide flange sections.

Some experimental and theoretical studies were conducted to study the relationship between the radius of stretch bending and the permanent deformations of the

bended SHS. Paulsen and Welo (Paulsen and Welo 2001 & 2002) have investigated the deformation of rectangular hollow section members in stretch bending by experimental and analytical methods. Based on the analytical models, a design method for evaluation of bendability of sections in industrial forming operations is being proposed.

Brady (Brady 1978) has carried out the experimental test to determine the radii to which rectangular and square HSS can be cold bent using the pyramid type three-roller method. Based on the results by Brady (Brady 1978), Kennedy and Seddeik (Kennedy and Seddeik 1985 & 1988) developed an analytical approach to calculate the relationship between the radius of curvature of roller bending hollow section members and the resulting deformation in the cross section. Based on different conditions that specify an acceptable level of sectional deformation, the minimum radius for the different HSS members can be calculated. However, one obvious limitation of these studies is the asymmetric deformation of the cross section (Chiew et al 2016). As the transverse deformation of one web was constrained by the rolling machine in an actual cold bending process, the out-of-plane displacement degree of freedom of this web is defined as zero, while other three walls were allowed to deform. Hence, in this paper the sectional deformation caused by roller bending has been investigated.

In general, the final residual stress is closely related to the member radius after roller bending. The effects of residual stress on the behaviour and stability of straight hot finished steel hollow section member has been investigated and reflected by the inclusion of buckling curves in design standards such as the Eurocode 3 (European Standards 2005). For the members subjected to bending moment only, the design guide (King and Brown 2001) suggests that the influence of residual stress on curved members could be more important than straight members. Simple assumptions of residual stress distribution based on the bilinear material law were adopted to calculate the resistance of curved members. However, some of the simple assumptions adopted, including no stress variation over both tension and compression flanges and no deformation of plane sections, may violate the real situation during the rolling process. Hence, a more comprehensive study on

the residual stress distribution is worth investigating. Giorgi (Giorgi 2011) conducted an Experimental analysis to access the residual stress generation in cold-formed steel materials. The results showed that the yield stress could play an important role in the residual stress field evolution. For open steel sections, experimental study was conducted by Spoorenberg (Spoorenberg et al 2010 & 2011) to measure the residual stress distributions in both straight and curved members and the results were validated by finite element modelling. Based on the analysis results, a residual stress model was proposed for cold-rolled wide flange sections (Spoorenberg et al 2011). The effects of residual stress on in-plane inelastic buckling and lateral buckling strength of open sectional steel arches were studied by Pi and Trahair (Pi and Trahair 1996 & 2000). In their study, groups of arches with varying slenderness are subjected to different kinds of loads. The results showed that the residual stress could reduce the buckling strength significantly. For the straight steel hollow section members, the main focus in many previous studies was to investigate the magnitude and distribution of residual stress due to cold forming process (Li and Zeng 2009, Tong and Hou 2012). Jandera (Jandera et al. 2008 & 2014) explored the presence and influence of residual stress in cold-formed stainless steel box sections using experimental and numerical techniques. It was found that the influence of residual stress on the column buckling behaviours of square hollow section varies with the non-dimensional slenderness of the section.

When forces and moments are applied to a curved steel member, a number of effects that are absent in straight members should be considered. For steel wide-flange shapes, the Basic Curving Criterion which following some fundamental moment-curvature relationships and aiming for a certain maximum strain in the extreme fibres of the cross section has been developed by Reidar (Reidar 2006). The Steel Construction Institute design guide (King and Brown 2001) covers the design of common types of curved members used in building structures. This guide takes the curvature into account and it is a beneficial supplement to the current structural design standards. While the detailed analysis models of open sections are described in the guide, the behaviour of hollow structural sections is relatively less studied.

An acceptable level of sectional deformation in section and the minimum bending ratio was recommended based on the experimental results. Seddeik and Kennedy (Seddeik and Kennedy 1987) applied energy methods to solve the distortion problem of a plastically bent thin-walled rectangular tube. Based on the numerical results, empirical equations were proposed to express the relationship between the member's bending ratio and the sectional deformations. It should be noted that the empirical expressions suggested are only suitable for the HSS members with one web kept flat during the roller bending process; while the deformation are allowed in the compressed flange and the free web.

Two analytical approaches, namely the yield line mechanism and the energy minimization theory, are commonly employed to investigate the deformation of hollow sections. Zhao (Zhao and Gregory 2005) proposed a design model for square and rectangular hollow sections under end-bearing forces based on the yield line theory. Zhou and Young (Zhou and Young 2006) investigated the web crippling of cold forming stainless hollow section using yield line mechanism analysis. The proposed web crippling design equations were derived through a combination of theoretical and empirical analysis to predict the sections' crippling strength. Pan (Pan and Stelson 1995) used the energy method to determine the distorted shape of plastically bent thin-walled tubes. Stelson (Stelson and Kramer 1999) and Hong (Hong and Stelson 2002) employed energy minimization theory to deduce the expressions for the sectional deformation magnitude of the U-channel and rectangular sections in pure plastic bending. The expressions are purely geometrical and this indicates that the sectional deformation should not depend on material properties.

1.4 Objectives

During the roller bending process, the transverse movement of center roller (towards outer rollers) contributes to formation of permanent curvature of the final curved section. However, the applied loads inevitably induce local deformations as well as cross-sectional deformation on the member. As the deformation of the hollow structural member may affect its stability and reduce its cross-sectional and

buckling resistance, a reliable and effective method that could predict the magnitude of deformation after roller bending will be an important tool for the application of the curved member in structural design. The experimental and numerical methods were employed to investigate the imperfections caused by the roller bending process in this paper.

Apart from the experimental and numerical methods, the aim of the present study is to develop an analytical approach by combining the yield line mechanism and the energy minimization theory to predict the relationship between the desired bending radius of the curved member and the resulting sectional deformation of the hollow section. This is because none of the previous studies was intended to obtain simple analytical expressions for the cross sectional deformation caused by roller bending on hollow steel section (HSS) member. The analytical approach is more practical and theoretical.

As it is found that very few previous studies were focused on the residual stress of curved steel hollow section members produced by cold roller bending, the objective of this study is also to investigate the residual stress of curved steel hollow section. Based on the experimental results and accuracy of the numerical models, a numerical parametric study will also be conducted to evaluate the effects of some key parameters of the roller bending process. Based on the results of the parametric study, a simple residual stress model will be suggested. This model would allow engineers to estimate the residual stress distribution of the steel hollow section which could then be used as the initial stress state for further investigation of the steel hollow section's structural behaviours (e.g. buckling strength analysis).

Therefore, the objectives of the present study are:

1. To investigate the influence of the key factors like bending ratio(R/h), shape factor (h/b), and section thickness (t) as well boundary conditions on the imperfections (sectional deformation and residual stress) of rectangular (square) steel hollow sections caused by roller bending by experimental methods.
2. To develop a numerical model to simulate the roller bending process as well as the resulted sectional deformation and residual stress. The numerical model

would be verified by the experimental results in terms of sectional deformation and residual stress along the section circumference.

3. To propose a computing expression to predict the distribution and magnitude of the residual stresses resulting from the roller bending process in the cross section. The computing model was induced by the parameter studies.
4. To develop a calculation model to predict the relationship between the curvature radiuses of roller bent hollow section members and the resulting sectional deformation in the cross-section. The key sectional factors were considered in the analytical model. .

Chapter 2 Experimental Study of Sectional Deformation in Roller Bent Steel Hollow Section Members

2.1 Introduction

The experimental investigation deals with deformations of cross-sectional members as flanges and webs in bending of rectangular hollow sections. The hollow sections were bent by means of the three rollers pyramid type rolling machine into a number of different bend radii. During the roller bending process, high loads induced by the roller are required to achieve a permanent radius of curvature; such loads incur large local deformations in the sectional walls. The amounts of deformation resulting in a hollow section member depend on its dimensions, material properties and the imposed bending radius.

The deformation of the cross-section may affect its structural integrity to carry loads in practical engineering. Therefore, the relationship between the inevitable sectional deformation and the bending factors (sectional properties, bending radius and imposed load) must be investigated. Measurements of curvatures, deflections and sectional deformation of the specimens were taken to study the influence of such factors on the members' deformation after roller bending in this chapter.

2.1 Material Test

The curved specimens were made from steel of grades S275 and S355 respectively, which conformed to the European delivery standard EN 10025-2 (Technical Committee ECISS/TC10 2004). In theory, steel S275 have the nominal yield stress of 275 MPa, ultimate strength between 410MPa to 560MPa, and elongation 18%; steel S355 have the nominal yield stress of 355 MPa, ultimate strength between 510MPa to 580MPa and elongation capacity of 17%.

The specific material mechanical properties at ambient temperature were acquired from tensile coupon test. The coupons (see Figure 2.1) were prepared and tested according to BS EN 10002-5 (BSI 1992) to determine the initial Young's modulus (E_0), the yield stress (σ_y), the ultimate tensile strength (σ_u), and the percentage

elongation (e_u) at fracture. Total 14 coupons have been prepared, in which two coupons were cut from adjacent walls of each type of hollow section in longitudinal direction. The tensile specimens were tested by the servo-hydraulic testing Machine with a load cell of 100kN capacity (see Figure 2.2). The measured values of E_0 , σ_y (0.2% proof stress), σ_u , and elongation (e_u) are shown in Table 2.1. It can be seen from Table 2.1 that the average ratio of the measured $\sigma_u/\sigma_y = 1.16$ for steel S275, and 1.24 for steel S355; the average percentage elongation $e_u = 25\%$ and 31% for steel S275 and S355 respectively.

Figure 2.1 The dimensions of the coupon

Figure 2.2 Coupon test machine

Table 2.1 Material mechanical properties

Coupons	Thickness (mm)	E0 (MPa)	σ_y (MPa)	σ_u (MPa)	σ_u / σ_y	e_u (%)
RHS1	6	194.9	372	436	1.16	26
RHS2	6	196.2	374	446	1.19	24
RHS3	6	192.8	395	458	1.16	18
RHS4	5	199.5	356	408	1.15	30
RHS5	6	193.6	449	529	1.18	28
RHS6	6.3	193.5	419	507	1.21	30
RHS7	10	191.2	398	534	1.34	34

2.2 Roller Bending Process

A typical pyramid-type bending machine consists of three rollers: two fixed outer ones and one removable roller in the middle as shown in Figure 2.3. The movement of the middle roller produces the curvature in the member between the three supporting points. Furthermore, rolling of these three rollers induces permanent curvature to the steel hollow section as it passes above the middle roller. At any instant of the cold rolling process, the steel member under rolling can be modelled as a beam simply supported by the two outer rollers and subjected to a concentrated loading from the middle roller.

The steel hollow sections were bent in 2 to 6 passes to a radius at which some minor distortion was apparent. A section near end of the specimen was then marked and this section was subjected to no further rolling. The remainder of the section was then bent tighter with several additional passes until the curvature of the member was considered acceptable. After the specimen was removed from the machine, accurate measurements of bending radii were then made. Based upon these, a decision was made as to what further radii the second length of the same profile should be rolled to. The rolling procedure was then repeated with the second length thus giving an indication of the level of deformation obtained at different radii for each profile. Cross sections were then cut from sections bent to each of the four radii for each profile and these were subsequently accurately measured, thus at the completion of the test program detailed information had been obtained for separate bend radii for each of 32 different profiles.

During the bending process, the potential lateral deformation of both section webs was partly restricted by the rollers flange, as shown in Figure 2.4. For the deep sections, the fact of fewer parts of both flanges has been restricted leads to crippling or convex of the flanges. By contrast, the potential distortion of shallow profiles has been blocked by the roller flanges. While the tensile web usually keeps flat, the sectional deformation was just apparent in the compressed web. The common pattern of sectional deformation is concave of the web.

Figure 2.3 Roller bending by pyramid-type bending machine

Figure 2.4 Specimen under roller bending

2.3 Specimen Details

Roller bending of the steel hollow section was carried out on a three-roller bending machine of the pyramid type similar to that shown in Figure 2.4. This bending method was chosen due to its practicality and suitability for large bends required from construction purposes. A series of 32 tests were conducted on 7 different sizes of square and rectangular hollow sections. Profiles of the specimens varied from 150×100×6mm to 300×200×10mm, which completely listed in Table 2.2.

The length of each steel hollow section member is taken as 2.5m. The bending ratio equals to curvature radius R divided by the nominal section height h , which reflects the curvature degree of the specimen. The decrease of bending ratio represents the increase of the magnitude of member has been made. The range of bending ratio is $20 \leq R/h \leq 70$ in this study.

Table 2.2 Specimen details

Specimens	Sections (mm)	Material	R(m)	R/h	h/b	b/t
RHS1-1	150×100×6	S275	3.0	20	1.5	16.7
RHS1-2			3.75	25		
RHS1-3			4.5	30		
RHS2-1	200×100×6	S275	4.5	22.5	2	16.7
RHS2-2			6.0	30		
RHS2-3			8.0	40		
RHS3-1	200×150×6	S275	4.5	22.5	1.33	25
RHS3-2			6.0	30		
RHS3-3			8.0	40		
RHS3-4			10.0	50		
RHS3-5			12.0	60		
RHS4-1	200×200×5	S275	4.5	22.5	1	40
RHS4-2			6.0	30		
RHS4-3			8.0	40		
RHS4-4			10.0	50		
RHS4-5			12.0	60		
RHS5-1	300×100×6	S355	7.5	25	3	16.7
RHS5-2			9.0	30		
RHS5-3			12.0	40		
RHS5-4			15.0	50		
RHS5-5			18.0	60		
RHS5-6			21.0	70		
RHS6-1	300×200×6.3	S355	7.5	25	1.5	31.7
RHS6-2			10.5	35		
RHS6-3			12.0	40		
RHS6-4			15.0	50		
RHS6-5			18.0	60		
RHS6-6			21.0	70		
RHS6-7			24.0	80		
RHS7-1	300×200×10	S355	10.5	35	1.5	20
RHS7-2			14.5	50		
RHS7-3			22.3	75

Figure 2.5 Details of specimen after roller bending

2.4 Test Results and Discussion

2.4.1 Test Results

After roller bending process, the sectional deformation appeared in most of the specimens. In all cases, the deformations resulting from the rolling process has almost the same pattern, i.e., major deformations in both the compression flange and web with less restriction while minor deformations resulted in the tension flange. In details, for sections with thin wall or subjected to low bending ratio (R/h), the concave appeared in compressed web and slight convex shown in both flanges. On the contrary, the specimens with high bending ratio and thick section walls are less affected by the roller bending process. The high value of b/t is susceptible to local buckling. The pattern of concave in compressed web is shown in Figure 2.6.

The section's original properties (h , b and t), final bending radius (R) and detailed sectional deformations (e , h_1 and b_1) have been measured (see Figure 2.7). The complete explanation of these terms is shown in Figure 2.8. The complete list of test results is given in Table 2.3. The results from a total number of 32 sections were analysed.

In order to conduct an objective and fair comparison, 32 steel hollow sections with depth ranging from 150mm to 300mm that are bent to various curvatures are

analysed in this section. Two geometrical parameters, the percentage increase in width after bending P_b and the percentage convex in the compressed flange after bending P_e are calculated to express the deformation of the cross section after the roller bending process.

$$P_e = \frac{e}{h} \times 100 \quad (2.1a)$$

$$P_b = \frac{b_1 - b}{b} \times 100 \quad (2.1b)$$

where, h and b stand for the original section's depth and width;

b_1 stands for the profile's width after bending;

e stands for the concave value of compressed flange.

Figure 2.6 Web concave of section after roller bending

Figure 2.7 Measurement of deformation of cross-section

Table 2.3 Sectional deformation obtained from test

Specimens	R/h	h/b	b/t	e	h ₁	b ₁	P _e	P _b
RHS1-1	20			1	149	102	0.67	2
RHS1-2	25	1.5	16.7	1	149	101	0.67	1
RHS1-3	30			0	150	100	0	0
RHS2-1	22.5			1.5	198	104	1	4
RHS2-2	30	2	16.7	1	199	102	0.5	2
RHS2-3	40			0	199	102	0.5	2
RHS3-1	22.5			15	196	208	7.5	5.3
RHS3-2	30			14	196	206	7	4
RHS3-3	40	1.33	25	12	197	203	6	2
RHS3-4	50			6	197	202	3	1.3
RHS3-5	60			3	198	201	1.5	0.5
RHS4-1	22.5			25	190	212	12.5	6
RHS4-2	30			24	190	209	12	4.5
RHS4-3	40	1	40	15	192	205	7.5	2.5
RHS4-4	50			10	195	204	5	2
RHS4-5	60			6	196	202	3	1
RHS5-1	25			10	288	106	3.33	6
RHS5-2	30			6	292	105	2	5
RHS5-3	40			8	293	102	2.67	2
RHS5-4	50	3	16.7	5	295	101	1.67	1
RHS5-5	60			3	294	101	1	1
RHS5-6	70			2	295	100	0.67	0
RHS6-1	25			30	284	212	10	6
RHS6-2	35			23	284	209	7.67	4.5
RHS6-3	40			22	286	208	7.33	4
RHS6-4	50	1.5	31.7	22	286	204	7.33	2
RHS6-5	60			20	287	203	6.67	1.5
RHS6-6	70			12	290	203	4	1.5
RHS6-7	80			14	290	204	4.67	0.5
RHS7-1	35			2.5	295	205	0.83	2.5
RHS7-2	50	1.5	20	1	296	203	0.33	1.5
RHS7-3	75			0	298	200	0	0

Figure 2.8 Section before and after roller bending

2.4.2 Discussion

The effects of some key bending parameters (the bending ratio, R/h ; the shape factor, h/b ; and the thickness-to-depth (width) ratio, h/t (b/t)) on the sectional deformation of the rectangular hollow sections have been investigated. Figure 2.9 shows the relationship between sectional concave ratio (Pe) and the bending ratio (R/h). It can be seen that the magnitude of sectional deformation will be closely related to the radius of bending R of the section. In addition to points shown in lower left circular, Figure 2.9 obviously shows that a downward trend can be observed with the increase in R/h (i.e. the magnitude of the concave value decreases as R/h increases) over the range $20 \leq R/h \leq 80$. A feasible explanation for the points in the red circular is that the higher thickness ratio (t/b) leads the section to be insensitive to the bending ratio.

The relationship between the expansion ratio (P_b) and the bending ratio (R/h) is shown in Figure 2.10. The overall trend of the points' distribution is similar with the Pe - R/h plots, which indicates that the potential deformation of the hollow section after roller bending will aggravate with the increasing of bending ratio (within the range of $20 \leq R/h \leq 80$).

Figure 2.9 Plots of concave ratio against bending ratio

Figure 2.10 Plots of expansion ratio against bending ratio

The ratio between section depth h and width b with equal thickness is defined as shape factor (h/b). The shape factor h/b expresses the bending rigidity of the members with same cross-sectional area under roller bending. The higher value of shape factor means the higher bending resistance of the section. Figure 2.11 shows the concave ratio (P_e) of the compressed flange varies from the bending ratio in the range of $20 \leq R/h \leq 80$ for the 32 tested specimens. It's obviously seen that the sectional deformation (concave appeared in compressed flange) decrease with the increasing of shape factor. On other words, for a section with same thickness and depth, the width has positive correlation with the section sectional deformation caused by roller bending. The relation between the expansion ratio and shape factor is obscure, as shown in Figure 2.12. The random distribution of the points indicates that the influence of shape factor on webs' deformation is weak. For a section (with same h/b), the convex of webs are mainly determined by the bending radius.

The thickness of section wall (t) affects the bending caused deformation of a hollow section member greatly. The normalized thickness ratio b/t and h/t were employed to study the influence of thickness on sectional deformation after roller bending. Total 32 measured section distortion results (P_e and P_b) are plotted in Figure 2.13 and 2.14. The sensible variation of concave ratio for different thickness ratio (b/t) can be observed from Figure 2.13. In short, the potential deformation of the compressed flange increase with the increasing of width-to-thickness ratio. This implies that thinner section tends to result in serious sectional deformation after roller bending. The influence of height-to-thickness ratio (h/t) on web's deformation are plotted in Figure 2.14. The distribution of the tested points share the same feature with the above figure. It indicates that the possibility of the webs suffering deformation is positively correlated to the thickness of the web.

Figure 2.11 Plots of concave ratio against shape factor

Figure 2.12 Plots of expansion ratio against shape factor

Figure 2.13 Plots of concave ratio against thickness ratio

Figure 2.14 Plots of expansion ratio against thickness ratio

2.5 Summary

Through the experimental studies, the relationship between the sectional distortion resulted by roller bending and the geometrical factors has been investigated. Total 32 specimens with grade S275 and S355 steel have been curved by the roller bending method. After the bending, the sectional deformation of the hollow sections have been measured and recorded for further study.

Two geometrical parameters, the percentage increase in width after bending P_b (named as expansion ratio) and the percentage convex in the compressed flange after roller bending P_e (named as concave ratio) are calculated to express the deformation of the cross section after the roller bending process. The relationships between the two parameters and the geometric factors are also investigated. The results show that as the decreasing of bending radius (R) for a certain section depth, the potential deformation caused by roller bending would increase. For different section types, the effect of shape factor (h/b) on sectional deformation is also investigated; and the results show that the deformation will decrease as the increase of shape factor. For influence of thickness t , the test results imply that thinner section tends to produce higher value of distortion under the same bending ratio.

Chapter 3 Experimental Study of Residual Stress in Roller Bent Steel Hollow Section Members

3.1 Introduction

When a structure is loaded beyond its elastic limit, some parts of the structure are permanently deformed while the others are still elastic. If the applied load is removed, the plastically deformed parts prevent the remaining elastic parts from returning to their initial positions causing some residual stresses. The magnitude of these residual stresses depends on the stress-strain relation of the material as well as on the stresses in the material before the release of the load. During the roller bending process, the applied load caused by the moved roller introduces the deformation and residual stress on the hollow section. In this chapter, an experimental investigation on the residual stress distributions along the cross section is carried out.

Hole-drilling technique (ASTM 2008), X-ray and neutron diffraction (Park, Yang et al 2004) are efficient approaches for experimental investigation. Though X-ray diffraction technique offers a non-destructive alternative, it has severe limitations such as bulking and complexity of the equipment and measurement for only very shallow surface layers. The hole-drilling method is a common method for measuring residual stress by removing localized stress and measuring strain-relief in the adjacent material. The hole-drilling method is to measure residual stress near the metal surface by stress relaxation when a hole is drilled into the center of a rosette strain gauge. It has advantages of reliability, simplicity, convenient practical implementation, and therefore it is broadly used in civil and mechanical engineering.

3.2 Hole-Drilling Method

The hole-drilling method was employed to measure the bent caused residual stresses in the out surface of the section. This method includes the procedures of affixing strain gauge rosettes to the surface, drilling a standard hole in the vicinity of the rosette, and measuring and recording the relieved strains. The measured

strains are then calculated and figured out the relieved principal stresses through the expresses presented in ASTM (ASTM 2008).

Rendler and Vigness (Rendler and Vigness 1966) developed the hole-drilling method into a systematic procedure thereby formed the basis of ASTM E837 standard. Beaney and Procter (Beaney and Procter 1974) improved the application of air abrasive machine to eliminate introduction of additional residual stress in the process of drilling. Flaman (1982) develop another practical stress-free drilling method with ultra-high speed. Schajar (1988) reviewed the history and progress of hole-drilling method, and carried out the primary finite element analysis of hole-drilling method to list calibration coefficients in the formula of calculation.

The ASTM hole-drilling method (ASTM 2008) is a common way for measuring residual stress by relieving localized stress and measuring strain relief in the adjacent material. It causes relatively little damage to the specimen and allows localized residual stress measurements. The principle of this approach is to release the localized stress by introducing a small hole into a residually stressed structure and thus changing the local strain on the surface of the testing. By comparing the strain at this drilled point before and after hole-drilling, stress relaxation due to hole drilling can be determined.

Figure 3.1 shows the stress near the drilled point O. attaching strain gauge rosette at the point P in a radius r from the point O and drill one small hole at the point O. the radius of the hole is a and σ_x and σ_y are the principal residual stresses at the point O. at point P, σ_r and σ_t are the radial and the tangential stress caused by hole-drilling stresses release at point O, and is the angle between σ_r and σ_x . On the assumption that the material of the plate is homogenous and isotropic and its stress-strain curve is linear, the relieved strain at a point P(r, a) can be obtained by substituting the stress relaxation into the Hooke's law:

$$\varepsilon = \frac{1+\nu}{E} \cdot \left(\bar{a}\right) \cdot \frac{\sigma_x + \sigma_y}{2} + \frac{1}{E} \cdot \left(\bar{b}\right) \cdot \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \frac{1}{2} \cdot \left(\bar{b}\right) \cdot \tau_{vy} \cdot \sin 2\alpha \quad (3.1)$$

where, E is the Young's modulus, ν is the Poisson's ratio and ε is the relieved strain, α is the angle from the x -axis to the maximum principle stress, $\bar{a}$ and $\bar{b}$ are

calibration constants for isotropic and shear stress respectively, which can be determined corresponding to the hole diameter and type of rosette used from Table in ASTM E837-08. In measuring residual stress, after the three set of relieved strains are obtained by a strain rosette, σ_x , σ_y and α can be determined.

Figure 3.1 Stress state near the drilling point

The combination stresses for the measured strains ε_1 , ε_2 , ε_3 can be expressed as the formula following:

$$\begin{cases} P = \frac{\sigma_x + \sigma_y}{2} = -\frac{Ep}{a(1+\nu)} \\ Q = \frac{\sigma_y - \sigma_x}{2} = -\frac{Eq}{b} \\ T = \tau_{xy} = -\frac{Et}{a(1+\nu)} \end{cases} \quad (3.2)$$

where, p, q and t are the three strain combinations with expressions of

$$\begin{cases} p = \frac{\varepsilon_3 + \varepsilon_1}{2} \\ q = \frac{\varepsilon_3 - \varepsilon_1}{2} \\ t = \frac{\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2}{2} \end{cases} \quad (3.3)$$

E is Young's modulus, ν is Poisson's ratio. In this case, the in-plane Cartesian stresses can be expressed:

$$\begin{cases} \sigma_x = P - Q \\ \sigma_y = P + Q \\ \tau_{xy} = T \end{cases} \quad (3.4)$$

Figure 3.2 Schematic Diagram of strain gauge for residual stress measurement

3.3 Strain Gauge Scheme and Specimens

3.3.1 Strain gauge rosettes

Three types of strain gauge rosette are introduced by ASTM E837, as shown in Figure 3.3. The type A is available in several different conditions. The special pattern of Type C has three pairs of opposite grids that is useful where large strain sensitivity and high thermal stability are required. The strain gauge grids of Type B rosette locate on one side. It is useful where measurements need to be made near an obstacle. Type B rosette (as shown in Figure 3.3) was employed in the experimental text in this study.

Figure 3.3 Strain gauge rosette types

3.3.2 Specimen preparation

The measurements were performed on the roller bending curved rectangular/square hollow sections. Since for hot-rolled section, the magnitude of residual stress produced during their manufacturing (hot-rolling) process is much lower than the magnitude of residual stress produced during the cold roller bending process, the initial residual stress of the section before the cold roller bending process is ignored and not considered in the numerical study. A total of four specimens were prepared to test and to study the influence of roller bending on the residual stress magnitude and distribution. The details of the tested specimens are listed in Table 3.1. The nomination of the specimen is same as shown in Chapter 2. The sections were produced with steel in grade S275. The different factor values (R/h , h/b and h/t , b/t) were involved in the chosen specimens. The surfaces of the chosen specimens were intact without physical damage.

Table 3.1 Specimen details

Specimens	Sections (mm)	Material	R(m)	e(mm)	P _e	P _b
RHS1-3	150×100×6	S275	3.75	1	0.67	0
RHS2-1	200×100×6	S275	4.5	1.5	0.5	4
RHS3-1	200×150×6	S275	4.5	15	7.5	5.3
RHS4-1	200×200×5	S275	4.5	25	12.5	6

3.3.3 Strain gauge scheme

The residual strains were measured using stain rosettes that attached onto the outer surface at the appointed places. Total 22 strain rosettes were arranged along the circumference of the specimens, as shown in Figure 3.4. In details, for the distorted section walls (concave flange and double webs), the rosettes were attached at the quarter points along the section wall; for the tensioned flange, the strain gauge was placed at the middle point (residual stress at this flange is plain); for each corner, three rosettes were arranged long the arc. The practical arrangement of the rosettes in the corner and tension flange is shown in Figure 3.5.

Figure 3.4 Arrangements of strain gauge rosettes

(a) Rosette in corner

(b) Rosette in tensioned flange

(c) Rosette in concave flange

Figure 3.5 Arrangements of strain gauge rosettes in specimen

3.4 Measurement Procedure

3.4.1 Drilling equipment

According to ASTM standard (ASTM E837), a device that is equipped to drill a hole in the test workpiece in a controlled manner is required; the device must be able to drill a hole aligned concentric with the strain gauge circle to within wither 0.004D.

The RS-200 milling guide, a high-precision instrument for analysing residual stress by the hole-drilling method, was used for measuring the residual in the specimens. Principal features and components of the milling apparatus assembly are shown in Figure 3.6. When installed on the workpiece, the guide is supported by three

levelling screws footed with swivel mounting pads to facilitate attachment to uneven surfaces. A high-speed air turbine assembly is supplied with the apparatus. The standard milling cutter with 1.6 mm diameter was used for residual stress measurement.

3.4.2 Drilling Procedure

ASTM E837 classifies the tested plate/material as thin and thick workpiece. For a thin plate, where a through-hole is to be used, the plate thickness should not exceed $0.4D$ for Type B rosette. For a thick workpiece, the thickness should be at least $1.2D$ for Type B rosette. D is the diameter of gauges circle that is defined in Figure 3.2. The actual value of D is 4.5mm. hence; the specimens involved in the test were defined as thick plate.

For the residual stress measurement at certain points, careful alignment of the milling guide with the strain gauge hole center with a precision within 0.025mm was accomplished by inserting a special-purpose microscope into the guide's center journal, and then positioning the guide precisely over the center of the rosette. A 1.6mm diameter tungsten carbide cutter was installed in the turbine. The milling cutter was guided precisely during the drilling operation to ensure that it progressed in a straight line during drilling. Finally, a hole was bored by setting the compressed air supply to the high-speed turbine to produce a circular, straight-sided and flat-bottomed hole (see Figure 3.7). The hole should first be advanced slowly to a depth $0.05D$ and the relieved strain was recorded.

The drilling and strain measurement steps were then repeated for seven times with an incremental depth of $0.05D$ until a final hole depth approximately equal to $0.4D$ was achieved. The calculation procedure is followed by ASTM E837-08 that the averaging method is preferred by using the measured 8 groups of data to reduce the effects of random strain measurement.

Figure 3.6 RS-200 milling apparatus

Figure 3.7 Hole-drilling method measuring residual stress

3.5 Experimental Results and Discussion

3.5.1 Experimental Results

The experimental results for all tested specimens are presented in Figure 3.8-3.11. The distribution and magnitude of residual stress are analysed in this section.

Figure 3.8 show the distribution of longitudinal and transverse residual stress on the outside surface for rectangular hollow section $150 \times 100 \times 6 \text{ mm}$. It can be seen that the tension residual stress were found on the middle parts of both webs. The peak value appears in the middle web. In contrast, the compressive residual stress most distribute in both flanges. While the residual stress is plain in the tension flange, the values of residual stress in both compressed corners are higher. The maximum values of tension and compression residual stress reach up to 110 MPa and -118 MPa .

For the transverse residual stress, only tension stress was found along the cross-section. Generally, the residual stress in the compressed flange is higher. This can be partly explained by that the transverse residual stress in out surface is tension; and the stress in inner surface is compression.

(a) Longitudinal residual stress (b) Transverse residual stress
Figure 3.8 Measured residual stress distribution in RHS $150 \times 100 \times 6 \text{ mm}$

The residual stress magnitude and distribution in RHS 200×100×6mm are shown in Figure 3.9. The longitudinal residual stress along the section is similar with specimen RHS1. It's noted that the tension stress in middle of compression flange is obviously smaller than the adjacent corners. The maximum values of tension stress reaches up to 184MPa. The peak value of compressive residual stress was found at the middle web and the value is -103MPa. For the transverse residual stress, while the compressive stress appeared at one corner, the other parts of the section are under tension residual stress.

Compared to specimens RHS1 and RHS2, the sectional deformation of RHS3 and RHS4 are sensible. Hence, the residual stress distributions in the late two specimens are shown some different view. Figure 3.10(a) shows the longitudinal residual stress in RHS 200×150×6mm. while the tension residual stress at both concave corners are higher, the compressive residual stress was found in middle of the concave flange. The residual stress distribution in concave flange can be summarized as an inverse V. And this distribution trend is clearer than above two specimens. In addition, the magnitude of residual stress in RHS3 is bigger than the above two specimens.

For the transverse residual stress, it can be seen from Figure 3.10(b) that the compressive residual stress also appeared in middle of concave flange. It can be explained by that the outside surface of the concave flange is under bigger distortion. Besides the middle concave flange, the other parts of the section are under tension residual stress.

The thickness of RHS4 is 5mm, which is thinner than the other three specimens. The thinner section wall could cause serious sectional deformation under same bending radius. And then affect the residual stress distribution. The longitudinal residual stress for RHS4 is shown in Figure 3.11(a). It can be seen that the compressive residual stress appeared in the middle part of concave flange and both webs. The maximum value is up to -92MPa. The maximum tension residual stress is up to 130MPa, and which was found at the corner. Generally, the magnitude of residual stress for RHS4 is lower than RHS3, which shows that the thickness would influence the stress magnitude. The transverse residual stress in the concave flange

is mostly compressive, which is obviously different from the previous specimens. The rest of the section is under tension, and the distribution of tension residual stress is plain.

(a) Longitudinal residual stress (b) Transverse residual stress
Figure 3.9 Measured residual stress distribution in RHS 200×100×6mm

(a) Longitudinal residual stress (b) Transverse residual stress
Figure 3.10 Measured residual stress distribution in RHS 200×150×6mm

(a) Longitudinal residual stress

(b) Transverse residual stress

Figure 3.11 Measured residual stress distribution in RHS 200×200×5mm

3.5.3 Discussion

The comparison of the longitudinal residual stress of the four specimens show that the tension residual stress in middle of the concave flange decrease as the increasing of sectional distortion. For specimens RHS1 and RHS2, sectional deformation in

compressed flange is indistinct, and the stress are tension. As the deformation increasing, the concave is shown in such flanges, and then the residual stress distribution shown an inverse V distribution.

Summarized the four specimens, it can be seen that the residual stress at the corner of compressed flange are higher than the other parts. Generally the peak value of stress would be found at such corners. This is because during the roller bending process, the compressed corners stand up the higher pressure and squeezing.

For the sections with indistinct deformation, the outside surface of the section is under tension residual stress. Various sectional deformations cause the different residual stress distribution. Generally, the residual stress in the concave surface is common tension, and in the convex surface is compression.

3.6 Summary

In this chapter, a carefully designed experimental study was conducted to investigate the residual stress distribution along the cross-section due to roller bending. The ASTM hole-drilling method was employed to measure the value of residual stress. Four curved hollow section members with various bending ratio and shape factors were experimental measured the residual stress.

The experimental longitudinal and transverse residual stress were recorded and analysed. Based on the test results, it was found that the bending ratio affect the final residual stress magnitude greatly. In addition, it was found that the longitudinal residual stress in the middle of compressed flange varies from the shape factors. In details, as the decrease of shape factor of h/b , the stress in the middle of flange transformed from tension to compression. Furthermore, the residual stresses at the corner of compressed flange are higher than the other parts.

Chapter 4 Numerical Study of Imperfections in Roller Bent Steel Hollow Section Members

4.1. Finite Element Modelling

4.1.1 Model description

In this study, the general propose FE modelling package ABAQUS was employed to simulate the roller bending process. The simulations were conducted on 19 different sizes of square and rectangular hollow structural sections (included the 7 tested profiles), which profiles varied from $101.6 \times 50.8 \times 3.8\text{mm}$ to $300 \times 200 \times 10\text{mm}$. The length of every specimen is taken as 2.5-times outer rollers distance (s) (see Figure 4.1). Two different 3-roller bending systems, System 1 and System 2, were used in the models in accordance with the actual rolling process (Table 4.1).

The FE models employed accounted for both geometrical and material non-linear effects, as well as surface-to-surface contacts and frictional forces between the steel hollow section members and the rollers. The material adopted in the finite element models conformed to the material properties based on coupon test. The material behaviour was approximated by a bilinear relation with a yield stress of 275MPa and 355 MPa and modulus of elasticity of 200GPa. The plastic modulus is set equal to 8000 MPa with an ultimate strength is assumed to be 430MPa and 510MPa which occurs at a strain of 0.019 (Figure 4.2). Geometrical non-linearity was also considered due to the large displacement and rotation occurred during the bending process, especially for the members with small bending radii.

The cross-section geometry of the steel hollow section members were modelled based on the technical standard of EN 10210-2. The radii of four outer corners are all taken as 1.5 times tube thickness, while the inner corner's is 1.0t. The corners are discretized into 5 eight-node brick elements with 4 layers in the thickness direction to reproduce the geometry of the corner accurately. In a typical model for a 2.5 m long section, 45,000 brick elements are employed in the FE analysis. Note that in the previous study (Lee and Xu 2008), it is shown that 4 layers of element in the thickness direction is sufficient to capture the stress variation in the thickness

direction while the computational cost incurred is still within an acceptable limit for the parametric study discussed in Section 4. In order to obtain accurate modelling results but maintain computational efficiency, smaller elements are employed in critical areas like the corners and the free web middle part as shown in Figure 4.3.

Figure 4.1 Typical roller bending process

Figure 4.2 Material behavior

Figure 4.3 Typical mesh used in FE modelling

Table 4.1 Dimensions of roller system used in FE models

	Diameter (mm)		Rollers distance S (mm)	Applicable Members
	Outer Roller	Center Roller		
System 1	385	430	710	$h \leq 150\text{mm}$
System 2	460	515	1015	$150 < h \leq 300\text{mm}$

4.1.2 Curvature and Springback

The curvature of the member after roller bending can be described as the inverse of its plan bending radius. In this study, the changes of nodes coordinates in the center line of section were recorded to express the member's curvature. During the bending process, a typical curvature description of a 2.5m long tube with 152×152×4.78mm rectangular section is shown in Figure 4.4. The vertical axis represents the various curvatures of different parts of whole tube under/after bending and the horizontal axis indicates the bending process companied with member length.

Read the curve from right. The curvature of member begins to increase at point A and peaks at point B, as two points corresponds to initial supporting position of 2nd outer roller and centre roller respectively. The peak value of initial offset as well as the unsteady phase A-C is caused by the one pass assumption in the FE models, which results in excessive bending and shear force applied on the member. The constant curvature presented in the flat phase C-D stands for the expected curvature of the member before bending. Hence the section within this flat region was chose to evaluate the curvatures and residual stresses. The horizontal distance of D-F is equal to the space between two outer rollers (S). A small peak curvature shows at point E, because it is just pressed by the centre roller. The drop of curvature from point E to D due to the elastic release is also known as springback. The reduction rate of curvature due to springback is up to 30% for this tube, which expected radius is 2.4m. The similar curvature pattern included unsteady phase of member subjected to roller bending can be found in previous studies (Yang and Shima 1988 and 1990).

Figure 4.4 Description of curvature along beam

4.2 Numerical Simulation of Roller bending Process

At any moment of the roller bending process, the steel hollow section member being rolled can be treated as a simply supported beam with bending machine and subjected to a concentrated point load. The simulation of such process seems simple, actually is complicated due to the ever-changing contact surfaces between the rollers and hollow sections. The key point of FE modelling is the contact problem which caused by the elements in contact surfaces couldn't share constant nodes.

The fiction force between the rollers and the contact surfaces under rolling enables member movement. It's noted that the fiction force results in antisymmetric bending moment and deflections in rolling process. Therefore, the effect of fiction force was considered by a fiction coefficient (μ) which value of $\mu = 0.2$ is usually assumed for rolling process of steel sections. The imparted curvature leads to the speed difference of flanges (stretched top flange faster), which causes unavoidable interface slip between stretched flange and fixed rollers. To reduce such impact, in present study, a larger value of $\mu = 0.3$ is taken for the compressed flange surface; and $\mu = 0.1$ is taken for the stretched flange.

During test process, the specimen was bent in 2 to 6 passes to a radius at which the minor displacement was apparent. The specimen was then subjected to further rolling passes to obtain the desired radius. To simplify modelling, only one pass simulated in the numerical models, assuming that the residual stresses are identical for single and multiple pass bending. In order to be consistent with the boundary condition of section in test (Brady 1978), the transverse deformation of webs is restricted. Since the specimen's web beyond the roller's flange is less constraint, the actual distortions of sections after roller bending varies from the ideal deformation, as shown in Figure 4.5. For the less constraint members, the deformation pattern can be described as the concavity in compressed flange and the convexity in free web.

Figure 4.5 Sectional deformation patterns after roller bending

4.3 Residual Stresses Forming

The magnitude of residual stress generated during the bending process is depended on the stress-strain relation of the material and the stress in the material before the release of the load (Bjorhovde 2006). A typical material behaviour under loading described by a simple trilinear hardening stress-strain relation is shown in Figure 4.6. In Figure 4.6, it is assumed that if the tension stress goes beyond the yield point a and then the load is removed, unloading will take place along a line ($b-c$) which is parallel to the elastic line $o-a$. This process is equivalent to applying a load at the same position but at an opposite direction. This load will produce compressive

stress to balance the existing tensile stress. The final state of residual stress σ_{RS} shall be depended on the magnitude of the load. That is the final stress state will be defined by either point i or point j on the path $b-c-d$. For point i which is on path $b-c$, the unloading range will be entirely elastic and the residual stresses will be equal to σ_i . For point j , the path $c-d$ stands for the case that the stress resulting from the unloading process is larger than the yield stress of the material in compression. In this case, the unloading process continues until the residual strain ϵ_j (which could be compressive) is reached.

Figure 4.6 Material behaviour under loading

As shown in Figure 4.7, the free web at the mid-span section of a 2.5m long 203×154×6.35mm steel hollow section is selected to show the residual stress generation process during roller bending. The three vertical dotted lines (Lines 1, 2 and 3) represent the time when the three rollers (in the order of the first outer roller, the centre roller and the second outer roller) pass through the section, respectively. The red line in Figure 4.7 shows the stress variation at the compressive corner ($x=y=0$ in Figure 4.8) during the bending process. The compressive stress reached its peak value when the centre roller was pressing the section. The same curve also shows that springback ($b1-c1$) appeared almost immediately when the load was removed, which induces the final tensile residual stress at this point. Figure 4.7 shows that similar mechanism also occurred at the point $y=25\text{mm}$ (the yellow curve).

For the variation of residual stress at the neutral axis in static bending ($y=101\text{mm}$), the thick blue curve in Figure 4.7 shows that it is rather complex. At the starting stage, a small tensile stress appeared at this point. As the centre roller was moved in, the tensile stress is converted into compressive stress and increased gradually. Finally, when the centre roller was moved out, no tensile stress appeared to balance the existing compressive stress. In contrast, the compressive stress increased continually along the line b3-c3. It was caused by stress redistribution in the section in order to achieve an overall force equilibrium. The stress magnitude at the point $y=135\text{mm}$ was low during the whole process. The actual separation point between tensile and compressive residual stresses is between $y=135\text{mm}$ and 170 , and is closer to $y=135\text{mm}$ (as shown in Figure 4.8).

It is worth noting that the residual stress at the top of web ($y=203\text{mm}$) is lower than that at the bottom. Furthermore, the forming process of residual stress at this point is different from others. As shown in Figure 4.7, the tensile stress reached its maximum value when the centre roller was pressing the section and was kept in tension until the second outer roller passed through the section. It was then changed to compressive stress rapidly. After that, the stress was reversed to produce the final small tensile residual stress. The high tensile stresses at both ends of the web between Line 2 and Line 3 explain why the compressive stress increases continually at the neutral axis.

The final residual stress distribution of the section (one web is less constraint, which corresponds to the CIDECT Report) is shown in Figure 4.8 which shows that the final residual stress is not symmetrical. Such result is conflicting with some traditional residual stress theories (Jandera et al 2008), as such theories normally assume that the residual stress is symmetrical. However, with detailed FE modelling it is shown to be not the case. The differences in the residual stress distributions between the two webs are due to the different boundary conditions applied. The variation of residual stresses in the free web is shown in Figure 4.8.

Figure 4.7 Stress history of one web at the mid-span section of a $203 \times 154 \times 6.35\text{mm}$ SHS during cold-rolling process

Note: Compressive stress is negative. The 0 mm line is corresponding to the compressive corner ($x=y=0$) in Fig. 7 while the 101mm line is corresponding to the neutral axis of the SHS section in static bending.

Figure 4.8 Final residual stresses distribution of the section at mid-span and plot of the residual stress variation along the free web (Compressive stress is negative)

4.4 Numerical Results and Validation with Sectional Deformation

To validate the accuracy of the FE simulation and the assumptions adopted, numerical results including deformation and residual stress obtained from numerical models are compared with that obtained by actual rolling processes or from the theories proposed in previous studies (Brady 1978 and Kennedy and Seddeik 1985).

4.4.1 Validation with Tested Sectional-Deformation

The deformation of a section caused by the bending force in numerical model is presented in Figure 4.9 corresponding to the tested sectional deformation figure. It can be seen that the deformation pattern of the two figures as same.

The numerical and the experimental values obtained by actual rolling processes for section concave e and compression of section height h are compared to verify the accuracy and reliability of the numerical models. Total 32 specimens have been chosen to compare the sectional deformation from FE models to the experimental results. Two dimensions of onset and height of section have been recorded and investigated in this part. The details of the experimental and numerical results are listed in Table 4.2. It can be seen from the table that the difference between experimental and numerical results is mostly less than 20%. Due to the error of measurement, the difference is larger for the section with high bending ratio and low value of sectional deformation. An intuitive view with the comparison of experimental and numerical offset values of the specimens is presented in Figure 4.10. The vertical axis in the figure indicates the deformation values and the horizontal axis indicates the No. of the specimens.

Figure 4.9 Comparison of deformation pattern between experimental and numerical results

Table 4.2 Comparison between test and numerical results

Specimens	$e(\text{mm})$			$h_1(\text{mm})$		
	Test	Numerical	Diff.(%)	Test	Numerical	Diff.(%)
RHS1-1	1	2	50	149	148	0.7
RHS1-2	1	2	50	149	148	0.7
RHS1-3	0	1	-	149	149	0
RHS2-1	1	2	100	198	197	1
RHS2-2	1	1.5	50	199	198	0.5
RHS2-3	0	1	-	199	199	0
RHS3-1	15	18	20	196	194	1
RHS3-2	14	16	14	196	194	1
RHS3-3	12	15	25	197	195	1
RHS3-4	6	8	33	197	196	0.5
RHS3-5	3	3.5	16	198	197	0.5
RHS4-1	25	28	12	190	184	3
RHS4-2	24	27	12.5	190	188	1
RHS4-3	15	18	20	191	189	1
RHS4-4	10	12	20	195	192	1.5
RHS4-5	6	7	16.7	196	196	0
RHS5-1	10	8	20	288	285	3
RHS5-2	6	7	16.7	292	283	2
RHS5-3	8	7	12.5	293	286	2
RHS5-4	5	3	40	295	290	3
RHS5-5	3	2	33	294	286	2
RHS5-6	2	1	50	295	290	2
RHS6-1	30	28	7	285	284	0.3
RHS6-2	23	24	4	284	283	0.3
RHS6-3	22	25	13	286	288	0.7
RHS6-4	22	20	9	286	284	1.7
RHS6-5	20	16	20	287	281	2
RHS6-5	12	15	25	290	284	2.4
RHS6-7	14	12	14	290	283	1
RHS7-1	2.5	3.7	48	295	290	1.7
RHS7-2	1	2	50	296	291	2
RHS7-3	0.5	1	50	298	292	2

For the difference of sectional height (h_1) after roller bending, the values obtained from numerical model are almost less than that got from the experimental test. This is partly because the material properties employed in the FE model is different from the actual specimens. The mean and standard deviation of the differences between the experimental and numerical results for e are 0.4% and 12%, respectively. Hence, it can be concluded that reasonable agreements are shown except for a few cases

with higher discrepancies. The numerical modelling method was continuously used to investigate the influence of some key factors on the sectional deformation, which including bending ratio, section shape factor, material yield strength and the thickness ratio.

Figure 4.10 Experimental and numerical deformation results

4.4.2 Validation with Sectional Deformation in CIDECT reports

In order to conduct an objective comparison, 12 steel hollow sections with depth ranging from 101mm to 254mm that are bent to different curvatures are analyzed in this section. The geometrical parameters of expansion ratio P_b and the concave ratio P_e , are employed to express the deformation of the cross section after the roller bending process. The parameters are defined in Chapter 2.

The values of P_b and P_e (Table 4.3) obtained by numerical and the experimental results are compared to verify the accuracy of the FE models. The mean and standard deviation of the differences between the experimental and numerical values for expansion ratio P_b are 0.4% and 12%, respectively. For the compressed flange deformation factor P_e , the corresponding values are 1% and 11.2%, respectively. Hence, it can be concluded that reasonable agreements are shown except for a few cases with higher discrepancies.

Table 4.3 Changes in section properties after rolling

Section	R (m)	P_e		Diff. (%)	P_b		Diff. (%)
		Test	FEM		Test	FEM	
101.6×76.2×6.35	1.52	2.1	2.2	5	3.63	4.2	15.7
101.6×101.6×4.78	2.44	7.02	5.71	-18.7	5.1	5.0	-2
127.0×127.0×4.78	3.05	10	9.13	-9	8.76	8.36	-4.6
127.0×127.0×4.78	3.68	8.76	8.4	-4.1	6.26	6.7	7
152.4×101.6×4.78	2.44	7.82	9.4	20.2	14.1	13.1	-7.1
152.4×152.4×6.35	3.66	10.42	9.3	-10.7	6.25	5.7	-8.8
177.8×127.0×6.35	4.27	7.5	7.1	-5.3	5.0	5.67	13.4
177.8×177.8×7.95	7.32	4.47	4.7	5	5.13	4.7	-8
203.2×101.6×9.53	9.14	10.16	9.76	-4	5.47	6.6	21
203.2×203.2×6.35	17.32	6.25	7.4	18.4	4.89	5.35	9.4
254.0×152.4×6.35	15.24	7.3	6.75	-7.5	10.4	9.1	-12.5
254.0×254.0×9.53	13.72	3.75	4.17	11.2	7.5	6.6	-12

4.5 Validation with Residual Stress

4.5.1 Validation with Residual Stress in CIDECT Report

The magnitude of the residual stresses resulting at different parts of the hollow section member due to the rolling process depends on the geometry of the cross-section, its material properties as well as on the imposed radius of bending. Total 16 points (4 corner points & 12 quarter points) are arranged along the circumference of cross-section. The average values of longitudinal and Von-Mises stresses over the web thickness were recorded to study the residual stress distribution along the cross-section as well as to validate the numerical models.

A computer program has been developed based on the minimum energy theory to predict the residual stresses distribution of the rolled profiles by Kennedy (Kennedy and Seddeik 1985). The residual stresses at the center of the compression flange #1 (point 3) and the quarter point of the web plate #2 (point 6) in 4 different profiles are studied and presented in Figure 4.11. Figure 4.11 shows the longitudinal and Von-Mises residual stresses of certain points, which obtained from numerical models and previous CIDECT report (Brady 1978) respectively. The residual

stresses of the studied 16 points (as shown in horizontal axis) were presented in left vertical axis; meanwhile the right vertical axis stands for the corresponding normalized residual stress. From four sections in Figure 4.11, it can be seen that the maximum longitudinal residual stress occur in both inner corners (point 1 and 5) which under compressive stress from the center roller during rolling process ; and the maximum compressive longitudinal stress can be observed in the center of free web #2 (point 7). The maximum value of Von-Mises stresses also appears in these two inner corners.

(a) Section 203×203×6.35mm

(b) Section 178×178×7.95mm

(c) Section 152×101×4.78mm

(d) Section 127×127×4.78mm

Figure 4.11 Longitudinal and Von-Mises residual stress distributions along section circumference

For the Von-Mises stress of quarter point 6 at web plate, it has been shown that close agreement can be attained between finite element analyses and previous CIDECT results. The residual stresses (compressive) in longitudinal direction resulted from numerical models are slightly larger than that in study. The difference

is caused by the assumption of residual stresses state in study doesn't accord to real thing, which amplify the residual stresses in vertical direction.

For the center of compressive flange (point 3), the discrepancies between EF results and study predicted values could be observed. The difference could be due to two factors: (1) The CIDECT report assumed the uniform residual stress distribution over flange thickness. However, according to the finite element results, the Von-Mises and longitudinal stresses vary greatly in the thickness direction, as shown in Figure 4.12. Longitudinal residual stress are in tension at the outer surface and in compression at the inside surface of the section. (2) The CIDECT report adopt the assumption in study (Spoorenberg et al 2010) that no stress gradient present along flange width. It can be seen that the predicted values at point 3 and the simulated residual stresses at point 5 are the same magnitude. In fact, the maximum residual stresses developed at both inner corners only.

(a) Von-Mises stress

(b) longitudinal stress

Figure 4.12 Residual stress layering distribution

4.5.2 Validation with Tested Residual Stress

The magnitude and distribution of the residual stresses caused by roller bending of the four tested specimens were used to validate the accuracy of the finite element models. For each section, total 22 measured points (see Chapter 3) are arranged along the circumference of cross-section. The average values of longitudinal and transverse residual stress at corresponding points in the numerical models were recorded to study the residual stress distribution as well as to validate the numerical models.

Figure 4.13(a) shows the distribution of experimental and numerical longitudinal residual stresses along the rectangular hollow section 150×100×6mm. In Figure 4.13(a) (the same below), the yellow shade area refers to the measured tension residual stress; the blue shade area refers to the measured compression residual stress; the red dotted line indicates the simulated residual stress. The polar radius of the lines implies the values of the stress. From the numerical results, two points can be drawn: 1) the distribution of residual stresses is symmetrical; 2) the peak value occurs at the middle of the webs or at the compressed corners, which is in accordance with the tested results. The compression residual stress obtained by test and FE model are presented at both middle webs. It's noted that the numerical results are less than the measured results on the whole.

The transverse residual stress obtained by measurement and numerical model are all positive, as shown in Figure 4.13(b). While the measured results in two compressed corners are lower, the residual stress in the compressed flange is higher. This condition is abnormal. The numerical results give a relatively reasonable distribution of residual stress along the section. Generally, the simulated transverse residual stresses agree with the tested results well.

The distributions of measured and simulated residual stresses along the rectangular hollow section $200 \times 100 \times 6 \text{ mm}$ are presented in Figure 4.14. From Figure 4.14(a), a rough barbell shape can be used to describe both experimental and numerical longitudinal residual stress distribution. In details, the stresses at corners are positive and the values are higher; the residual stresses at middle of two webs are compressive and the margin line seems to be slim. The transverse residual stress magnitude and distribution obtained by both test and model are presented in Figure 4.14(b). With the exception of the stress in the left lower corner, the two results share the similar stress pattern and the proximity values. It is obvious that the compressive residual stress at the corner may be caused by the measurement error.

Figure 4.13 Tested and numerical residual stress along RHS $150 \times 100 \times 6 \text{ mm}$

(a) Longitudinal residual stress

(b) Transverse residual stress

Figure 4.14 Tested and numerical residual stress along RHS200×100×6mm

Figure 4.15 shows that comparison of tested and simulated residual stress along the section of 200×150×6mm. As a whole, the both results share the similar residual stress distribution. For the longitudinal residual stress, there are three obvious recesses (compression stress) along the stress margin lines. While the compression stress obtained by test distributed unevenly, the numerical results shows better symmetry. The absolute values of the tested stresses are higher than the calculated ones'. The transverse residual stress pattern determined by both methods is identical. The measured stress values are higher than the numerical results. The possible cause is that the tested steel layer is thin and is closer to the outside surface, whereas the average numerical results involved thicker layer of the section.

Both measured and simulated residual stresses distribution on rectangular hollow section of 200×200×5mm are presented in Figure 4.16. It can be seen that the numerical residual stress agree well with the experimental results in terms of longitudinal and transverse stresses. For the longitudinal stress, the area suffered compression stress was increased. This stress varying pattern can be validated by both experimental and numerical results. For the transverse residual stress, the

numerical results can make up the measurement errors, which can rationalize the final results.

(a) Longitudinal residual stress

(b) Transverse residual stress

Figure 4.15 Tested and numerical residual stress along RHS200×150×6mm

(a) Longitudinal residual stress

(b) Transverse residual stress

Figure 4.16 Tested and numerical residual stress along RHS200×200×5mm

4.6 Summary

The numerical modelling technique was introduced to simulate the roller bending process and to produce residual stresses after roller bending of steel square and rectangular hollow sections. The residual stresses forming process was presented and verified by the material behaviour theory. It can be learned from the stresses forming process that the sequence of stresses releasing and stresses equilibrium in section determine the final residual stresses distribution.

The validity and accuracy of the numerical models were proved by comparing the section deformation and residual stresses magnitude after roller bending to the tested results as well as the results presented in a CIDECT report. The deformed shape and concave/convex values after rolling given by numerical model fit well with the test results. The longitudinal, transverse and Von-Mises residual stresses over the section were recorded to study the residual stress distribution as well as to validate the numerical models. The comparison between the numerical and experimental results shows that the finite element numerical model is a feasible and reliable method to predict the members' potential deformation and residual stress caused by roller bending.

Chapter 5 Sensitivity Study

5.1 Sensitivity Analysis on Sectional deformation

After the validation of the accuracy and reliability of the numerical model, the effects of some key bending parameters on the sectional deformation of the steel hollow sections has been investigated with the experimental results as well as numerical models. In this section, three geometrical parameters, the bending ratio, R/h , the shape factor, h/b , the thickness ratio, b/t , are considered and the results obtained are summarized.

5.1.1 Influence of bending radius R/h

As the sectional deformation is affected by the amount of applied force or energy during the roller bending process, it is expected that the magnitude of sectional deformation will be closely related to the radius of bending R of the section. For the SHS studied, the normalised factor Pe and the flange concave value of e were plotted as a function of the R/h in Figure 5.1 and 5.2. Figure 5.1 shows that a downward trend can be observed with the increase in R/h (i.e. the magnitude of the concave value decreases as R/h increases) over the range $25 \leq R/h \leq 80$. Both profiles (RHS $300 \times 200 \times 6.3\text{mm}$ and $300 \times 100 \times 6.3\text{mm}$) with different bending ratio were plotted in the figure, in which T stands for Test and N stands for Numerical. As the increase of bending radius for a certain section depth, the potential deformation caused by roller bending would decrease. The suggested trend lines were also plotted together with R/h for both selected profiles, which show that the relationship between the bending ratio and sectional concave is not linear.

For the difference of section depth after roller bending, the average value of the change of the web with R/h is shown in Figure 5.2. From Figure 5.2, an obvious decrease trend of point's distribution within the range of $25 \leq R/h \leq 80$ can be observed. The magnitude of the compressive depth change Δh decreases as the bending ratio increase. It can be indicated from the plots that, when the section under the smaller curvature, the web concave is larger; and the depression of the section is also serious. It is noted that, while the trend lines are parallel, the numerical concave values and section depth change of the two profiles are higher than that obtained from the experimental test. It's possible due to the strain

hardening has not considered in the numerical model, which would contribute the stiffness of the section. From both plots, it can be observed that the numerical results fit with the test results well.

Figure 5.1 Plots of concave value against bending ratio

Figure 5.2 Plots of difference of height against bending ratio

5.1.2 Influence of shape factor h/b

Figure 5.3 shows the variation of compressed web concave with R/h in the range of $20 \leq R/h \leq 90$ for four sections which share the same value of $h=200\text{mm}$ and $t=6.3\text{mm}$, but with different values of $h/b=1, 1.33, 1.67$ and 2 ($b=200\text{mm}, 150\text{mm}, 120\text{mm}$

and 100mm). For the values presented in curve of $R/h=22.5$ are obtained from the experimental test. The other tress curves are obtained from the numerical models. It's obviously seen that the web's concave value decrease with the increasing of shape factor. This is to say, for a section with same thickness and depth, the width has positive correlation with the sectional deformation caused by roller bending. This can be explained by the restriction from both flange become weaken as the width is wide. It also can find that the bending ratio affect the section deformation when the shape factor is higher, like $h/b=2.0$. The influence of shape factor also can be seen from the test results shown in Figure 5.1 and 5.2. The points shown in both plots represent the specimens deformation caused by cold bending. It's obvious that the concave and the section depth change of specimen series RHS5 with width of 200mm are higher than specimen series RHS6 with width of 100mm.

Figure 5.4 shows that h/b has strong influence on the value of section depth difference for $20 \leq R/h \leq 90$. Some points shown in the first curve are got from test. In order to further investigate its effect, more sections were modelled and Figure 5.4 plots the variations of section depth difference with h/b .

Figure 5.3 Plots of web concave against shape factor

Figure 5.4 Plots of section depth difference against shape factor

5.1.3 Influence of thickness ratio b/t

The width to section thickness ratio b/t could also affect the sectional distortion due to roller bending. To study the effect of b/t , two types of sections were considered. The first type is that keep the section depth and width as constant and changing the thickness. Second type is: keep the section depth and thickness as constant and changing the width. Four sets of data, which includes experimental and numerical results, were employed to study the influence of thickness factor b/t on the section deformation. For each value of h , different thickness ratios were generated by varying b or t and the section was bent to different R/h from $20 \leq R/h \leq 90$.

The variations of concave and depth change with h/t are plotted in Figure 5.5. In Figure 5.5, individual lines are corresponding to sections bent to different radius of R . It can be seen that both concave value and depth change increase as the factor h/t increasing. The test results and numerical results share the similar pattern. This implies that thinner section tends to produce high value of sectional deformation under the same bending ratio. The influence of thickness ratio on the section deformation is more significant as the bending ratio is low. Figure 5.6 shares the similar curve distribution, which means that sectional deformation on both flange and web are positive related. As all the curves shown in Figure 5.6 are almost

parallel to each other, this means that the effects of b/t and R/h are largely independent.

Figure 5.5 Plots of concave value against bending ratio R/h

Figure 5.6 Plots of concave value against thickness ratio b/t

5.2 Sensitivity Analysis on Residual Stress

In order to study the effect of factors like boundary condition of section, the bending ratio (R/h), material yield stress (f_y), section thickness (t), and shape factor of section (h/b) on the residual stresses pattern of a hollow section member, the parametric analyses were carried out for a total of 146 models with various radii.

5.2.1 Effects of bending radius R/h

As the residual stress distribution is affected by the amount of plastic strain/hardening experienced during the roller bending process, it is expected that the magnitude of residual stress will be closely related to the radius of bending R of the section. In fact, the curving criterion suggested by Reidar (Reidar 2006) for wide flange of sections was based on the bending ratio which is defined as R/h . For the steel hollow sections studied, the normalised residual stress $\sigma_{cfl,c}/f_y$ at the corner of the compression flange obtained were plotted as a function of the R/h in Figure 5.7a. Figure 5.7a shows that the data points scattered in a wide range but generally a downward trend can be observed with the increase in R/h (i.e. the magnitude of the tensile residual stress decreases as R/h increases). Hence, a simple straight trend line with a slope approximately equal to -0.26% (the red dotted line in Figure 5.7b) is selected to approximate the relationship between $\sigma_{cfl,c}/f_y$ and R/h over the range $20 < R/h < 150$. In Figure 5.7b, the suggested trend line is plotted together with $\sigma_{cfl,c}/f_y$ for some selected sections. It can be seen that the trend line is almost parallel to all the curves in the plot. For $\sigma_{web,m}$, the variation of normalised residual stress $\sigma_{web,m}/f_y$ at the middle of the web with R/h are shown in Figure 5.8. From Figure 5.8, an obvious turning point near $R/h = 75$ can be observed and the slopes before this turning point are -0.35% and 0.25%, respectively. The magnitude of the compressive stress $\sigma_{web,m}/f_y$ first increases as R/h increases until the turning point and then that it decreases again. For most sections, the peak values of $\sigma_{web,m}/f_y$ appears in the range of $60 < R/h < 90$. While it seems that it is unusual to have the residual stress reduced as the value of R/h decrease, such phenomenon may be explained by first noting that the maximum compressive residual stress may not always appear exactly at the middle of the web when a hollow section is bent to different R/h .

An example is shown in Figure 5.9 in which the longitudinal residual stresses produced for a 254×152×9.53mm section bent to two different ratios R/h are shown. The red circles in Figure 5.9 indicate the locations of peak (compressive) residual stress. Such observation is directly related to the fact that for a given section, a higher residual stress is produced in the compressive flange when the section is bent to a smaller R/h (Figure 5.7). Due to the self-equilibrium property of residual stress, this implies that the peak (maximum compressive) value of residual stress could be located above the section's neutral axis. Hence, as the value of R/h is increased gradually from a small value, the minimum point on the $\sigma_{web,m}/f_y$ curve will appear when the local maximum compressive residual stress is located at the middle of the web. After that, the location of the maximum compressive residual stress will be located below Point 2 and results in an upward trend of the $\sigma_{web,m}/f_y$ curve (as compressive residual stress is considered as negative). Despite that $\sigma_{web,m}$ is not always the maximum stress of the web, it was also found that the difference between it and the maximum stress is always less 15% for all sections studied. As a result, in order to simplify the proposed residual stress model, $\sigma_{web,m}$ is still selected as the one of the key parameters of the proposed residual stress model.

The variations of the residual stresses $\sigma_{cfl,m}$ (middle of compressive flange) and $\sigma_{tfl,m}$ (middle of tension flange) with R/h are shown in Figure 5.10 and Figure 5.11, respectively. These curves show that both $\sigma_{cfl,m}$ and $\sigma_{tfl,m}$ display similar trends that as R/h increases the initial compressive residual stress decreases and turns into tensile stress and eventually almost remains constant when R/h exceeds 75. Hence, the variations of both $\sigma_{cfl,m}$ and $\sigma_{tfl,m}$ could be simplified into the bilinear dashed red lines shown in Figures 5.10 and 11. The simplified representations for $\sigma_{cfl,m}$, $\sigma_{cfl,c}$, $\sigma_{web,m}$ and $\sigma_{tfl,m}$, are summarized in Figure 5.12 with $\sigma_{cfl,c}$ represented as a simple straight line while all other residual stresses as bilinear curves.

(a) Overall scatter plot

(b) Trend line and variation of different hollow sections
Figure 5.7 Variation of $\sigma_{cf}, \sigma/f_y$ with bending ratio R/h

Figure 5.8 Variation of $\sigma_{web,m}/f_y$ with bending ratio R/h .

Figure 5.9 Longitudinal residual stress distribution (in N/mm^2) of a $254 \times 152 \times 9.53\text{mm}$ hollow section with different bending ratios

Figure 5.12 Suggested simplified trends for $\sigma_{cfl,m}/f_y$, $\sigma_{cfl,c}/f_y$, $\sigma_{web,m}/f_y$ and $\sigma_{tfl,m}/f_y$ against R/h

5.2.2 Influence of yield strength f_y

It is well known that f_y may affect the magnitude of the residual stress. The study by Spoorenberg (Spoorenberg et al 2011) showed that the residual stress in the compression flange of I-sections is correlated positively with f_y . In this study, 8 different square and rectangular sections of sizes from 101.6×101.6×4.78mm to 254×254×9.53mm made from 4 different steel grades (EN 10025-2 S235 to S450) were modelled and bent to different R/h values to explore effects of f_y .

Figure 5.13 shows the variation of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ with different values of f_y for the two sections 203×154×6.35mm and 127×127×4.78mm. Note that as all other sections studied produced similar results so they are not shown here. The curves were generated from the sections bent to different values of R/h . Figure 5.13 shows that sections with higher f_y generally resulted in higher residual stress (tensile for $\sigma_{cfl,c}$ and compressive for $\sigma_{web,m}$) than those with lower f_y . In addition, curves corresponding to different R/h are almost parallel to each another which implies that the effects of f_y is very similar within the range of R/h studied. Hence, simple straight lines are suggested to represent the effect of f_y . From Figure 5.13, the slope of the straight lines obtained from regression analysis for $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ are roughly equal to 0.12% and -0.12%, respectively.

(a) 203x154x6.35mm section with bending ratio $22 \leq R/h \leq 124$

(b) 127x127x4.78mm section with bending ratio $16 \leq R/h \leq 90$

Figure 5.13 Plots of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ against f_y

Regarding the effects of f_y on $\sigma_{cfl,m}$ and $\sigma_{tfl,m}$, it is found that f_y shows no significant influence on the sections studied. Figures 5.14 and 5.15 respectively show the typical results obtained from the sections of 127x127x4.78mm and 203x154x6.35mm bent to different R/h values.

Figure 5.14 Plots of $\sigma_{cf,l,m}/f_y$ against f_y for two SHS with bending ratio $16 \leq R/h \leq 124$

Figure 5.15 Plots of $\sigma_{tf,l,m}/f_y$ against f_y two SHS with bending ratio $16 \leq R/h \leq 124$

5.2.3 Influence of shape factor h/b

In many previous studies (Paulsen and Welo 2001, Spoorenberg 2011, Reidar 2006, and Yong 1975), it was found that for wide flange I sections, the magnitude and distribution of residual stresses due to hot and cold rolling would be positively correlated to the ratio between the flange area and the web area (Galambos 1998, Weng and Pekoz 1990), or the shape factor h/b for section with equal flange and web thicknesses. Figure 5.16 shows the variation of $\sigma_{cf,l,o}/f_y$ with R/h in the range of $20 \leq R/h \leq 160$ for three sections which share the same value of $h=203\text{mm}$ but with different values of $h/b=1, 1.32$ and 2 . Figure 5.16 shows that h/b has strong

influence on the value of $\sigma_{cfl,c}$ for $20 \leq R/h \leq 160$. In order to further investigate its effect, more sections were modelled and Figure 5.17 plots the variations of $\sigma_{cfl,c}/f_y$ with h/b for sections with different R/h . From Figure 5.17, it is suggested that the effect of h/b on $\sigma_{cfl,c}$ can be represented by a simple trend lines with slopes close to -0.09.

For the effects of h/b on $\sigma_{cfl,m}$, $\sigma_{web,m}$ and $\sigma_{tfl,m}$, Figure 5.18 plots their variations with h/b for sections with different R/h . Figure 5.18 indicates that for the range of h/b and R/h studied, the effects of h/b is not affected by the value of R/h . Hence, simple trend lines (dashed red lines in Figures 5.18a to 5.18c) could be employed to represent the effects of h/b on $\sigma_{cfl,m}$, $\sigma_{web,m}$ and $\sigma_{tfl,m}$.

Figure 5.16 Plot of $\sigma_{cfl,c}/f_y$ against R/h for SHS with different shape factor h/b .

Figure 5.17 Plot of $\sigma_{cfl,c}/f_y$ against h/b for SHS with $20 \leq R/h \leq 150$

(a) Plot of $\sigma_{cfl,m}/f_y$ against h/b under different bending ratio R/h

(b) Plot of $\sigma_{web,m}/f_y$ against h/b under different bending ratio R/h

(c) Plot of $\sigma_{tfl,m}/f_y$ against h/b under different bending ratio R/h

Figure 5.18 Variations of $\sigma_{cfl,c}/f_y$, $\sigma_{web,m}/f_y$ and $\sigma_{tfl,c}/f_y$ with shape factor h/b with $20 \leq R/h \leq 150$

5.2.4 Influence of thickness ratio h/t

The depth to section thickness ratio h/t could also affect the residual stress distribution. To study the effect of h/t , three types of sections ($178 \times 178 \times t$ mm, $203 \times 101 \times t$ mm, and $254 \times 254 \times t$ mm, $22 \leq h/t \leq 48$) were considered. For each value of h , different thickness ratios were generated by varying t and the section was bent to different R/h from $20 \leq R/h \leq 150$.

The variations of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ with h/t are plotted in Figure 5.19. In Figure 5.19, individual lines are corresponding to sections bent to different R/h . It can be seen that both $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ increase as h/t increase. This implies that thinner section tends to produce high value of residual stress. As all the curves shown in Fig. 24 are almost parallel to each other, this means that the effects of h/t and R/h are largely independent. As a result, after conducting a regression analysis, the effects of h/t can be simplified as straight lines with slope approximately equal to 0.01 and -0.012 for $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$, respectively. For the residual stresses $\sigma_{cfl,m}$ and $\sigma_{tfl,m}$, the curves in Figure 5.20 indicate that effect of h/t is weak and can be ignored.

(a) Plot of $\sigma_{cfl,c}/f_y$ against h/t under different bending ratio R/h

(b) Plot of $\sigma_{web,m}/f_y$ against h/t under different bending ratio R/h

Figure 5.19 Variations of $\sigma_{cfl,c}/f_y$ and $\sigma_{web,m}/f_y$ with thickness ratio h/t with $20 \leq R/h \leq 150$

(a) Plot of $\sigma_{cfl,m}/f_y$ against h/t under different bending ratio R/h

(b) Plot of $\sigma_{tfl,m}/f_y$ against h/t under different bending ratio R/h

Figure 5.20 Variations of $\sigma_{cfl,m}/f_y$ and $\sigma_{tfl,m}/f_y$ with thickness ratio h/t with $20 \leq R/h \leq 150$

5.3 Proposed Residual Stress Model

Based on the results obtained from the parametric study, the symmetrical residual stresses model shown in Figure 5.21 is proposed to predict the residual stress generated by roller bending. The residual stress distribution is represented by the residual stresses $\sigma_{cfl,m}$, $\sigma_{cfl,c}$, $\sigma_{web,m}$ and $\sigma_{tfl,m}$ at the four selected Points 3, 5, 7 and 11. As the parametric study results indicated that $\sigma_{tfl,m}$ is only slightly affected by the geometrical parameters studied, $\sigma_{tfl,m}$ is simply assumed to be equal to $0.15f_y$. For the webs, it is assumed that compressive stress will be generated at the neutral axis while tensile residual stress will appear at the corners. As shown in Figure 5.21, three triangular stress blocks are employed to represent the web residual stress distribution. The peak compressive stress is assumed to be equal to $\sigma_{web,m}$ while residual stresses at the tension and compression flange corners are equal to $\sigma_{tfl,m}$ and $\sigma_{cfl,c}$, respectively. The two quarter points from the flanges are selected to define where the residual stress is zero. For the compression flange, it is assumed that the middle half part of the flange is under a constant compressive residual stress $\sigma_{cfl,m}$ while the residual stress disappear at a distance of $b/8$ from the corners.

To define the magnitude of the residual stress, based on the trend lines developed in Section 4 and using data fitting method to slightly adjust some of the trend line coefficients to achieve overall optimal results, the suggested expression of $\sigma_{cfl,c}$ is

For $20 \leq R/h \leq 150$, $1 \leq h/b \leq 2$, $20 \leq h/t \leq 50$ and $235 \text{MPa} \leq f_y \leq 450 \text{MPa}$:

$$\sigma_{cfl,c}/f_y = \varepsilon[0.51 - 0.0028(R/h) - 0.09(h/b) + 0.005(h/t)] \quad (5.1)$$

While for $\sigma_{web,m}$ the suggested expressions are

For $1 \leq h/b \leq 2$, $20 \leq h/t \leq 50$ and $235 \text{MPa} \leq f_y \leq 450 \text{MPa}$:

$$20 \leq R/h \leq 75, \sigma_{web,m}/f_y = \varepsilon[-0.12 - 0.0034(R/h) - 0.05(h/b) - 0.004(h/t)] \quad (5.2a)$$

$$75 \leq R/h \leq 150, \sigma_{web,m}/f_y = \varepsilon[-0.43 + 0.0026(R/h) - 0.05(h/b) - 0.004(h/t)] \quad (5.2b)$$

where $\varepsilon = \sqrt{f_y/350}$ is a material parameter that reflects the increasing trend of residual stress as f_y increases. Note that the base value of 350N/mm^2 is employed as grade S350 steel hollow section is widely used roller bending.

The magnitude of stress $\sigma_{cfl,m}$ is determined by the internal stresses equilibrium of the section. From Fig. 26, the internal stresses equilibrium equation can be expressed as

$$b\sigma_{tfl,m} + 2(h\sigma_{tfl,m}/8 + h\sigma_{web,m}/4 + h\sigma_{cfl,c}/8) + 2(b\sigma_{cfl,c}/16) + 5b\sigma_{cfl,m}/8 = 0 \quad (5.3)$$

Rearranging Eqn. 5 then gives the expression of $\sigma_{cfl,m}$ for $20 \leq R/h \leq 150$, $1 \leq h/b \leq 2$, $20 \leq h/t \leq 50$ and $235 \text{ MPa} \leq f_y \leq 450 \text{ MPa}$

$$\sigma_{cfl,m} = -[\sigma_{tfl,m}(1.6 + 0.4(h/b)) + 0.8\sigma_{web,m}(h/b) + \sigma_{cfl,c}(0.2 + 0.4(h/b))] \quad (5.4)$$

where, $\sigma_{tfl,m} = 0.15f_y$ and values of $\sigma_{web,m}$ and $\sigma_{cfl,c}$ could be obtained from Eqns. 5.1 and 5.2.

Figure 5.21 Proposed residual stresses distribution

5.4 Validation of Proposed Model

In order to verify the accuracy of Eqns. 5.1 to 5.4, they are applied to predict the residual stress distributions for both square and rectangular hollow sections with steel grade from S235 to S450. The sizes of the square hollow section tested are

from 101×101×4.78mm to 400×400×12.5mm. While for rectangular hollow sections, the sizes are from 101×76×4.78mm to 500×300×16mm. The results obtained from the proposed model and FE analyses are shown in Figure 5.22 for square hollow sections and Figure 5.23 for rectangular hollow sections. From Figures 5.22 and 5.23, it can be concluded that the predicted stresses distributions match well with the FE results which indicated that the proposed residual stress model and equations could able to give reasonable residual stress predictions.

Furthermore, the measured residual stresses for the four test specimens were used to validate the proposed calculation model. Take the RHS 150×100×6mm with grade S275 steel for example, taking the factors of $R/h=30$, $h/b=1.5$, and $h/t=25$ into Eqns 5.1 to 5.4, the calculated indicative residual stress are: $\sigma_{tfl,m}=41\text{MPa}$, $\sigma_{cfl,c}=101\text{MPa}$, $\sigma_{web,m}=-97\text{MPa}$, and $\sigma_{cfl,m}=54.6\text{MPa}$, while the corresponding tested results are 32MPa, 116MPa, -115MPa, and 72MPa respectively, as shown in Figure 5.24. The Blue line indicates the proposed residual stress model, which is drawn by the four indicative calculated values. In the same way, the residual stress models for the rest three specimens can be calculated and presented in Figures 5.25-5.27.

Figure 5.22 Verification of the proposed residual stresses model for square hollow sections

Figure 5.23 Verification of proposed residual stresses model for rectangular hollow sections

Note: FEM = FE modelling results, Mod = Proposed residual stress model results. The first number after FEM/Mod is the depth of the section in mm while the second number is the yield strength f_y in MPa. R/h of the sections are ranging from 16 to 84.

Figure 5.24 Verification of proposed residual stresses model for RHS 150×100×6mm

Figure 5.25 Verification of proposed model for RHS 200×100×6mm

Figure 5.26 Verification of proposed model for RHS 200×150×6mm

Figure 5.27 Verification of proposed model for RHS 200×200×5mm

5.5 Summary

In this chapter, the relationship between residual stresses and factors included boundary condition of section, the bending ratio (R/h), material yield stress (f_y), section thickness (t), and section shape factor (h/b) were studied to figure out their influences on the residual stresses pattern of a hollow section member. The parametric study found that the peak residual stress values generally increase as the bending ratio decreases and when the yield stress increases. They are also affected by the shape factor and the thickness ratio of the section. Based on the results obtained from the parametric studies and the internal stress equilibrium principle, a simple model which makes use of the stress at four critical locations to describe the residual stress distribution of the section is proposed. The validity of the proposed model is then confirmed by comparing the predicted residual stress distributions with the corresponding numerical modelling and experimental test results for different practical size square and rectangular structural hollow sections with different steel grades. The comparison shows that the proposed residual stress is feasible to predict a member's potential residual stress magnitude and distribution for roller bending.

Chapter 6 Theoretical Study of Sectional Deformation caused by Roller Bending

6.1 Roller Bending Process

A typical pyramid-type bending machine consists of three rollers: two fixed outer ones and one removable roller in the middle as shown in Figure 1.5. At any instant of the roller bending process, the steel hollow section member being rolled can be modelled as a beam simply supported by the two outer rollers and subjected to a deflection from the middle roller. This deflection can be related to the radius of bending of the member by studying its geometry during the roller bending process. The main objective of studying the geometry of the beam model during the bending process is to determine the relation between the plastic radius of curvature R (deflection of the beam) and the local deformation of the cross section. The plastic radius of curvature is equal to the radius of the curved specimen while it is bent inside the machine by ignoring the effect of the specimen springback. While the bending force from the middle roller produces unsymmetrical deflection of the beam, one assumption adopted in this study is that the deflected shape of the beam during bending process is circular.

While the friction forces between the specimen and the rollers assist the rollers-specimen system to run, these forces also affect the deflection as well as the stress distribution of the specimen during roller bending process. According to the previous study by Kennedy [Kennedy 1985], the effect of friction on final sectional deformation can be ignored. Hence, the roller bending process can be seen as the beam is subjected to pure bending moment M .

6.2 Yield Line Mechanism

6.2.1 Yield line models

After the roller bending process is completed, the HSS member is bent into the specified shape with the bending angle ϕ , as shown in Figure 6.1. As assumed above, the member can be treated as a beam subjected to a constant bending

moment of M . The cross section of the member is distorted such that the bottom flange becomes concave, while the upper flange remained flat.

Figure 6.1 Curved member with distorted walls

The deformation of both webs depends on the boundary condition during the roller bending process. As shown in Figure 6.2, three plastic mechanism models are proposed based on experimental and numerical observations. Model 1 describes the case that both webs of the HSS member have been restrained by the bending machine and remained flat after roller bending. Hence, sectional deformation occurred in the compression flange only. The yield line theory was employed to investigate the relation between the local deformation of the compressed flange and the global curvature of the HSS member. As shown in Figure 6.2a, four yield lines (plastic hinges) were assumed. Two formed at the compressed corners, while the other two located at the quarter points.

The asymmetrical deformation shown in Model 2 is caused by the asymmetrical constraint, where the displacement of one web was restrained by the rolling platform. Hence, the yield line Model 2 is proposed obeying the practical conditions that the major deformations occurred in both the compression flange and the free web, as shown in Figure 6.2b. Model 3 shows that the deformation occurred at both webs and the compression flange after bending. This is because the webs could move freely in the lateral direction during the roller bending process. In this model, six yield lines were assumed to simulate the real sectional deformation more realistically, as shown in Figure 6.2c.

(a) Yield Line Model 1

(b) Yield Line Model 2

(c) Yield Line Model 3

Figure 6.2 Yield line models

The yield line length in all models is taken as the whole length of the member, L . The rotation angle of the plastic hinge, θ , can be expressed as follows:

$$\theta = \arcsin\left(\frac{\Delta}{b/4}\right) \quad (6.1)$$

Noted that for simplicity, in Eqn. 1, Δ is the generic deflection corresponding to the deflection Δ_{f1} , Δ_{w2} , Δ_{f2} etc. in Figures 6.2a to 6.2c. If θ is small (less than 15°), one can simplify Eqn. 6.1 to

$$\theta \approx \sin(\theta) = \frac{\Delta}{b/4} \quad (6.2)$$

The total energy associated with the rotation of the plastic hinges is given by:

$$\sum E_i = \sum \int_0^\theta M_b d\theta \quad (6.3)$$

where M_b is the moment required to plastically bend the flange to form a hinge in axial x direction, and

$$M_b = L \cdot \int_{-t/2}^{t/2} \sigma_x y dy \quad (6.4)$$

6.2.2 Material Model

Plane stress conditions on the x-z plane (Figures 6.2 and 6.3) are assumed to simplify the analytical model. The material is assumed to be rigid-plastic with yield stress f_y , obeying the Von-Mises yield criteria. From the plane stress conditions, $\sigma_y = 0$, so the yield criteria becomes:

$$f_y^2 = \sigma_x^2 - \sigma_x \sigma_z + \sigma_z^2 \quad (6.5)$$

The corresponding flow rule is:

$$d\varepsilon_x = \frac{1}{f_y} \left(\sigma_x - \frac{\sigma_z}{2} \right) d\varepsilon \quad (6.6a)$$

$$d\varepsilon_z = \frac{1}{f_y} \left(\sigma_z - \frac{\sigma_x}{2} \right) d\varepsilon \quad (6.6b)$$

In Eqn. 6.6, $d\bar{\varepsilon}$ is the equivalent plastic strain increment. From Figure 6.2 and 6.4, it is easy to obtain the strain in the plastic hinges in the x and z directions as:

$$\varepsilon_x = \frac{y\theta}{k(t/2)} \quad (6.7a)$$

$$\varepsilon_z = \frac{-y_n\phi}{L} \quad (6.7b)$$

where the length of the plastic hinge is taken as $k(t/2)$, as shown in Figure 6.3. Since the wall thickness t is small when comparing with the section depth h , and it is assumed that the section walls of the bent member are in uniaxial tension or compression, the distance from neutral axis of the cross section to bottom flange, y_n , could be approximately taken as $y_n = \frac{h}{2}$. Note that the sign convention of compressed strain in the z direction is considered as negative is used here.

Using Eqns. 6.7a and 6.7b, the strain increments can be expressed as:

$$d\varepsilon_x = \frac{2y}{kt} d\theta \quad (6.8a)$$

$$d\varepsilon_z = \frac{-y_n}{L} d\phi \quad (6.8b)$$

By using the Eqns. 6.6a, 6.6b, 6.8a and 6.8b, the ratio of strain increments is:

$$\frac{d\varepsilon_x}{d\varepsilon_z} = cy = \frac{2\sigma_x - \sigma_z}{2\sigma_z - \sigma_x} \quad (6.9)$$

where, $c = \frac{-4L}{k \cdot t \cdot h} \frac{d\theta}{d\phi}$

Solving for σ_x gives,

$$\sigma_x = \sigma_z \frac{2cy + 1}{2 + cy} \quad (6.10)$$

Substituting the above expression for σ_x into the yield criteria Eqn. 6.5 and using a power series to expand the square root expression, the normal stresses can then be written as

$$\sigma_x = \frac{f_y}{\sqrt{3}} \left(c^2 y^2 + \frac{cy}{14} + 1 \right) \quad (6.11a)$$

$$\sigma_z = \frac{2f_y}{\sqrt{3}} \left(1 - \frac{c^2 y^2}{4} \right) \quad (6.11b)$$

Substituting Eqn. 6.11a into Eqn. 6.4, then the required bending moment is given by

$$M_b = \frac{L^2 t^2 f_y}{42\sqrt{3}kh} \frac{d\theta}{d\phi} \quad (6.12)$$

Figure 6.3 The assumed plastic hinge

6.3 Energy Minimization Theory

The energy minimization theory is based on the principle of stationary potential energy. Oden [40] stated the principle as “*of all the possible displacements which satisfy the boundary condition of a structural system, those corresponding to equilibrium configurations make the total potential energy assume a stationary value.*” Using the principle of minimum potential energy, many structural problems can be expressed mathematically as: *The energy function $E(p)$ is required to evaluate the possible parameters p to obtain its minimum value.* In the present study, the energy function used is the total energy, and the parameters are the coefficients related to the sectional deformation.

In terms of the principle of virtual work, the work done by the externally bending moment is equal to the sum of the plastic energy dissipated by the moment to bend the HSS member, M_p , and the moment to distort the sectional walls of the member, M_b . Hence,

$$E = \int_0^\phi M d\phi = \int_0^\phi M_p d\phi + \sum \int_0^\theta M_b d\theta \quad (6.13)$$

The plastic moment needed to bend the member into the required arc can be expressed as

$$M_p = \int_A \sigma_z \cdot (y - y_n) \cdot dA \quad (6.14)$$

The total energy change associated with the distortion of the cross section can be expressed as

$$\Delta E = E - E_0 \quad (6.15)$$

In Eqn. 6.15, E_0 is the energy needed to bend the member to the desired curvature but without the any sectional deformation. According to Stelson [26], the total energy change for the rectangular hollow section can be calculated as

$$\Delta E = \sum \int_0^\phi M_{pd} \cdot d\phi + \sum \int_0^\theta M_b \cdot d\theta \quad (6.16)$$

where M_{pd} is the plastic bending moment change caused by deformation in the axial z direction. That is, for the Yield Line Model 1 (Figure 6.2a), only the distorted compression flange should be taken into account so that

$$M_{pd} = - \int_{-b/2}^{b/2} \sigma_{zb} t y dx = \frac{-\sqrt{3}}{2} t b f_y \Delta f_1 \quad (6.17)$$

where σ_{zb} in Eqn. 6.17 is the average stress value in the compression flange. By using Eqn. 6.11b, σ_{zb} can be expressed as

$$\sigma_{zb} = \frac{1}{t} \int_{-t/2}^{t/2} \sigma_z dy \approx \frac{2}{\sqrt{3}} f_y \quad (6.18)$$

6.4 Analytical Model

6.4.1 MODEL 1

Assume that $\theta = A\phi^n$, where A is a factor to be determined. Since $L = R\phi$, if the hollow section (yield line) length L is constant, this means that the yield line angle θ increases as the bending radius R decreases or as the bending angle ϕ increases.

This is a reasonable consequence as the smaller the value of R , the larger the sectional deformation. From this, the increment of rotation $d\theta$ is expressed as:

$$d\theta = nA\phi^{n-1}d\phi \quad (6.19)$$

Substituting $d\theta$ into Eqn. 6.13 gives

$$M_b = \frac{-L^2 t^2 f_y}{42\sqrt{3}kh} nA\phi^{n-1} \quad (6.20)$$

The total energy associated with the rotation the plastic hinges can be then calculated by

$$\sum M_b \theta = 4 \cdot \frac{L^2 t^2 f_y}{42\sqrt{3}kh} nA^2 \phi^{2n-1} \quad (6.21)$$

Using Eqns. 6.2 and 6.18, the total energy change caused by sectional deformation is given by:

$$\int_0^\phi M_{pd} \cdot d\phi = \frac{-\sqrt{3}}{8} f_y t b^2 A \frac{\phi^{n+1}}{n+1} \quad (6.22)$$

Hence, the total energy change associated with the deformation can be calculated as

$$\Delta E = -\frac{\sqrt{3}}{8} f_y t b^2 A \frac{\phi^{n+1}}{n+1} + \frac{2L^2 t^2 f_y}{21\sqrt{3}kh} nA^2 \phi^{2n-1} \quad (6.23)$$

Since θ increase as ϕ increases, Eqns. 6.21 and 6.22 should be an increasing function of ϕ . Since the two terms on the RHS of Eqn. 6.23 are of opposite sign, the simplest way to make ΔE decreases as ϕ (or θ) increases is to set $n=2$. By substituting $n=2$ into Eqn. 6.23 and the optimal value of A can be found by setting $\frac{d(\Delta E)}{dA} = 0$. Eventually, the factor A_1 for Model 1 is found to be equal to

$$A_1 = \frac{21}{64} \frac{b^2 kh}{L^2 t} \quad (6.24)$$

From references [10, 41] and empirical observation, the parameter k is taken as $k=6$, which means the plastic hinge length is 3 times the section thickness.

Using $\theta = A\phi^2$ and Eqn. 6.2, the maximum sectional deformation can be expressed as:

$$\Delta_{f1} = 0.49 \frac{b^3 h}{R^2 t} \quad (6.25)$$

It should be noted that Eqn. 6.25 implies that the magnitude of the sectional deformation is a function of the geometry of such section only and is independent of the material properties. Furthermore, it is also reasonable that the sectional deformation of the flange is proportion to the depth (h) and width (b) of the section while it is inversely proportional to the section thickness (t) and the radius of curvature of the member (R).

6.4.2 Model 2 and Model 3

Model 2

For model 2, one web is susceptible to sectional deformation due to the lack of lateral constraints. Four plastic hinges (yield lines) with angles of θ and one with angle of $\theta/3$ are assumed in the model, as shown in Figure 6.3(b). The local coordinate system of the section is defined in Figure 6.3(b), where the y axis is perpendicular to the section walls and the x axis is normal to the y axis and locates on the same plane.

The plastic hinge of the web is located at the first quarter point. From the section geometry shown in Figure 6.3b, the relation between the sectional deformations of the flange and web can be expressed as:

$$\frac{\Delta_{f2}}{b} = \frac{\Delta_{w2}}{h} \quad (6.26)$$

The plastic bending moment change of one web caused by sectional deformation in the z direction can be determined by:

$$M_{pd} = \int_0^h \sigma_{zb} t y dx = \frac{-thf_y \Delta_{w2}}{\sqrt{3}} \quad (6.27)$$

where, $\Delta_{w2} = \frac{h\Delta_{f2}}{b}$

As mentioned above, the total energy change caused by the member curvature can be calculated as

$$\sum \int_0^\phi M_{pd} \cdot d\phi = \frac{f_y t A}{4\sqrt{3}} \left(h^2 + \frac{3}{2} b^2 \right) \frac{\phi^{n+1}}{n+1} \quad (6.28)$$

while the energy used to form the plastic hinge rotation can be expressed as:

$$\sum M_b \theta = 4 \frac{1}{3} \cdot \frac{L^2 t^2 f_y}{42\sqrt{3}kh} n A^2 \phi^{2n-1} \quad (6.29)$$

Substituting Eqns. 6.28 and 6.29 into Eqn. 6.16. By using $n=2$ again and set $\frac{d(\Delta E)}{dA} = 0$, the optimal value of A can be determined as

$$A_2 = \frac{21}{104} \frac{kh}{L^2 t} \left(h^2 + \frac{3}{2} b^2 \right) \quad (6.30)$$

The parameter k is again taken as $k=6$ so that then the maximum sectional deformation of the flange can now be expressed as

$$\Delta_{f2} = 0.3 \frac{bh}{R^2 t} \left(h^2 + \frac{3}{2} b^2 \right) \quad (6.31)$$

Substituting Eqn. 6.31 into Eqn. 6.26 will give the maximum sectional deformation of the web as

$$\Delta_{w2} = 0.3 \frac{h^2}{R^2 t} \left(h^2 + \frac{3}{2} b^2 \right) \quad (6.32)$$

Model 3

For model 3, two webs are susceptible to sectional deformation, and four plastic hinges with rotations equal to θ and two plastic hinges with rotations equal $\theta/3$ are assumed in this model, as shown in Figure 6.2(c). Similar to Model 2, the energy change caused by member curvature can be calculated by

$$\sum \int_0^\phi M_{pd} \cdot d\phi = \frac{f_y t A}{2\sqrt{3}} \left(h^2 + \frac{3}{4} b^2 \right) \frac{\phi^{n+1}}{n+1} \quad (6.33)$$

The energy caused by the sectional deformation can be determined by

$$\sum M_b \theta = 4 \frac{2}{3} \cdot \frac{L^2 t^2 f_y}{42 \sqrt{3} k h} n A^2 \phi^{2n-1} \quad (6.34)$$

Eventually, the factor A can be determined by:

$$A_3 = \frac{3}{8} \frac{k h}{L^2 t} \left(h^2 + \frac{3}{4} b^2 \right) \quad (6.35)$$

The sectional deformation of the flange and web can be calculated by

$$\Delta_{f3} = 0.56 \frac{b h}{R^2 t} \left(h^2 + \frac{3}{4} b^2 \right) \quad (6.36)$$

$$\Delta_{w3} = 0.56 \frac{h^2}{R^2 t} \left(h^2 + \frac{3}{4} b^2 \right) \quad (6.37)$$

6.5 Results and Verification

In order to validate the reliability and accuracy of the theoretical model, the sectional deformation obtained from above theoretical equations are compared to the results obtained by the experimental test. All the results were plotted with a normalized geometric parameter (likes $\lambda_{f1} = \frac{b^3 h}{R^2 t}$) as the horizontal axis and the sectional deformation (e.g. Δ_{f2} and Δ_{w2}) as the vertical axis.

For Model 1, the theoretical and experimental values for the sectional deformation Δ_{f1} were compared for 20 specimens with different bending radius with section dimensions are $h=300\text{mm}$, $b=200\text{mm}$ in Grade S355 steel. During the roller bending process, both webs have been restricted the lateral movement. Hence, the sectional deformation just occurred in the compressed flange and this satisfies the Model 1 assumption.

The comparison is presented in Figure 6.4. From Eqn. 6.25, the deformation Δ_{f1} has linear relation with the geometric parameter λ_{f1} . The experimental results are dispersed above and below the linear line due to random effects like different roller set up conditions and operators' experience, which may account for the discrepancy

between the estimated values and the experimental results. Generally, the theoretical results are in fair agreement with experimental values.

For Model 2, Brady (Brady 1978) tested 27 different hollow sections by bending each member at four different radii so a total 108 deformed sections were formed. 22 deformed sections are chosen here to check the theoretical model, as shown in Figures 6.5(a) and 6.5(b). The deformation of the cross section is expressed in terms of two values of Δ_{f2} and Δ_{w2} , which can be calculated by Eqns. 6.31 and 6.32. The geometric parameters used in the horizontal axis) are defined as

$$\lambda_{f2} = \frac{bh}{R^2_t} \left(h^2 + \frac{3}{2}b^2 \right) \quad (6.38)$$

$$\lambda_{w2} = \frac{h^2}{R^2_t} \left(h^2 + \frac{3}{2}b^2 \right) \quad (6.39)$$

It can be seen from Figure 6.5 that the theoretical model produced reasonable prediction for the relation between the section deformations and the geometric dimensions. While a few discrepancies can be observed, in general, the deformations at both flange and webs increase with the increasing of parameters. It should be noted that the mean and the standard deviation of the differences between the tested and the calculated values for Δ_{f2} are -10.8% and 28.3% respectively, and those for the differences in Δ_{w2} are -2.8% and 30.2%. The similarity of the deviation values shows that it is reasonable to assume that the relation between Δ_{f2} and Δ_{w2} is in proportion to the geometric ratio of h/b (Eqn. 6.26).

Figure 6.4 Comparison of theoretical and experimental flange deformation in Model 1

(a) Comparison of flange deformation

(b) Comparison of webs deformation

Figure 6.5 Comparison of theoretical and experimental deformation in Model 2

6.6 Summary

The deformation of the curved rectangular hollow sections under roller bending has been analysed using the yield line mechanism and the energy minimization theory. An analytical model was proposed based on this approach. Considering the fact that different boundary conditions are applied during the actual roller bending process of the member, three yield line models are assumed to describe the different deformation mode of the sections. The theoretical analysis results in some simple expressions that could predict the deformation magnitude of a section as a function of the geometry of the section and the bending radius. It is also found that the calculated results are in fairly good agreement with the experimental results.

Chapter 7 Conclusions and Recommendations

7.1 Conclusions

In this study, investigations on the imperfections of cross-sectional deformation and residual stress distribution in the curved hollow section members were carried out. Total 32 specimens with grade S275 and S355 steel have been curved by the rolling bending method. After the bending, the sectional deformation have been measured and recorded. Two geometrical parameters, the percentage increase in width after bending P_b (named as expansion ratio) and the percentage bowing in the compression flange after bending P_e (named as concave ratio) are calculated to express the deformation of the cross section after the roller bending process. The relationships between the two parameters and the geometric factors are also investigated. It was found that the factor including bending ratio, shape factor, and section thickness would influence the sectional deformation after roller bending. In the following conditions, member's deformation would deteriorate: a) decreasing of bending radius; b) decreasing of shape factor h/b ; c) decreasing the thickness-to-height ratio.

The standard ASTM hole-drilling method for residual stress measurement was applied to investigate the residual stress deformation on the curved member. Four curved hollow section members with various bending ratio and shape factors were experimental measured the residual stress. The measured residual stress were recorded and analysed. Based on the test results, it was found that the bending ratio affect the final residual stress magnitude greatly. In addition, the longitudinal residual stress in the middle of compressed flange varies as the changing of shape factor of h/b . Furthermore, the residual stresses at the corner of compressed flange are higher than the other parts.

The numerical modelling technique was introduced to simulate the rolling process and to produce residual stresses after roller bending of square and rectangular hollow sections. The residual stresses forming process was presented and verified by the material behaviour theory. The validity and accuracy of the numerical models were proved by comparing the section deformation and residual stresses

magnitude after roller bending to the tested results. The comparison between the numerical and experimental results shows that the finite element numerical model is a feasible and reliable method to predict the members' potential deformation and residual stress caused by roller bending.

The parametric studies were carried out to figure out the influence of some key factors (the bending ratio (R/h), material yield stress (f_y), section thickness (t), and section shape factor (h/b)) on the residual stresses pattern of the hollow section member. Based on the results obtained from the parametric studies and the internal stress equilibrium principle, a simple model which makes use of the stress at four critical locations to describe the residual stress distribution of the section is proposed. The validity of the proposed model is then confirmed by comparing the predicted residual stress distributions with the corresponding numerical modelling and experimental test results for different practical size square and rectangular structural hollow sections with different steel grades.

Finally, the deformation of the curved rectangular hollow sections under roller bending has been analysed using the yield line mechanism and the energy minimization theory. The theoretical analysis results in some simple expressions that could predict the sectional deformation magnitude of a section as a function of the geometry of the section and the bending radius. It is also found that the calculated results are in fairly good agreement with the experimental results.

7.2 Recommendations for future research work

1. Further studies of residual stress measurement by neutron diffraction are highly recommended. In this study, hole-drilling method is used to measure the roller bending caused residual stresses along the section circumference. However, due to the limitation of the setup, it is impossible to measure the residual stress for the inner surface.
2. The bending resistance of the curved hollow section beams would be investigated by experimental and numerical methods. The imperfections of the beam after roller bending would affect the member's sectional and buckling resistance. Further research works should be carried out to analysis the

influence of the imperfections (section deformation and residual stress) on member's bending resistance. Based on the analysis, a reduction factor should be proposed to predict the resistance of curved beam.

3. The influence of the roller bending on the performance and imperfections of circular hollow sections is recommended to investigate. The section properties of circular hollow section vary greatly from the rectangular hollow section. During the bending process, the resistance to deformation of the arc section is stronger than the plain section wall in RHS. Hence the bending caused imperfections for circular hollow section would be different from the RHS.
4. More numerical parameter studies for the residual stress after roller bending are recommended. In this study, the parameters including bending ratio, shape factor, thickness ratio and material yield stress were evaluated for their impact on the magnitude and distribution of residual stress. However, the impact of rollers distance, rollers' diameter and rolling pass numbers are not included in this study. These factors would also impact the final residual stress distribution.

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