Multi band microwave circuits with metamaterial effects

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**NANYANG
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UNIVERSITY**

SINGAPORE

**MULTI BAND MICROWAVE CIRCUITS WITH
METAMATERIAL EFFECTS**

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**SCHOOL OF ELECTRICAL & ELECTRONIC
ENGINEERING**

2019

MULTI BAND MICROWAVE CIRCUITS WITH METAMATERIAL EFFECTS

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarized materials, and has not been submitted for a higher degree to any other University or Institution.

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Authorship Attribution Statement

This thesis contains material from 1 published paper and 1 under review paper published in the following peer-reviewed journal(s) and from 2 published and 1 yet to be published International conference papers where I was the first and/or corresponding author.

Chapter 3 contents are from

1. Manoj Prabhakar Mohan, A. Alphones and M. F. Karim, " Design of SIW Resonator Using Epsilon Negative Transmission Line Analysis" (accepted in EUCAP 2019).
2. Manoj Prabhakar Mohan, A. Alphones and F. Karim, "Triple band SIW cavity backed slot antenna," 2017 IEEE Asia Pacific Microwave Conference (APMC), Kuala Lumpur, 2017, pp. 698-701.

The contributions of the co-authors are as follows:

- Assoc. Prof Arokiaswami Alphones gave the initial direction of using SIW for designing multi band components. He verified my derivations and results and helped in decision making and gave some technical suggestions. He also helped me in revising the manuscripts.
- I came up with the idea of using SIW open circuited resonators. I did all the derivations, simulations and measurements of the resonator and antenna. I wrote the manuscripts.
- Dr. Muhammad Faeyz Karim helped in revising the manuscripts and gave some technical suggestions

Chapter 4 contents are from

1. Manoj Prabhakar Mohan and Arokiaswami Alphones, "Double Periodic CRLH Transmission line for Wideband Performance," 2016 Asia-Pacific Microwave Conference (APMC), New Delhi, 2016, pp.1-4.
2. Manoj Prabhakar Mohan, A. Alphones and M. F. Karim, "Triple Band Filter Based on Double Periodic CRLH Resonator," in IEEE Microwave and Wireless Components Letters, vol. 28, no. 3, pp. 212-214, March 2018.

The contributions of the co-authors are as follows:

- Assoc. Prof Arokiaswami Alphones gave the initial direction of using CRLH concepts to design of multi band components. He verified my derivations and results and helped in decision making and some technical suggestions. He also helped me in revising the manuscripts.
- I came up with the idea of using symmetric DPCRLH unit cells. I did all the derivations, simulations and measurements of the symmetric unit cells, resonators and filter. I wrote the manuscripts.
- Dr. Muhammad Faeyz Karim helped in revising the manuscripts and gave some technical suggestions.

Chapter 5 contents are from

1. Manoj Prabhakar Mohan, A Alphones and MF Karim, "CPW Triple Band Antenna based on DPCRLH resonator" (submitted to IEEE Antennas and Wireless Propagation Letters).

The contributions of the co-authors are as follows:

- Assoc. Prof Arokiaswami Alphones gave the initial direction of using CRLH concepts in the design of multi band components. He verified my derivations and results and helped in decision making. He also helped me in revising the manuscripts.
- I came up with the idea of using CPW symmetric DPCRLH unit cells. I did all the derivations, simulations and measurements of the symmetric unit cell, resonator and antenna. I wrote the manuscripts.
- Dr. Muhammad Faeyz Karim helped in revising the manuscripts.

08.05.2019

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Mohan Manoj Prabhakar

Dedicated to my mother Gandhimathi

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Summary

In this thesis, multi band components are realized by exploiting the electromagnetic metamaterial effects. At first, single negative material (a type of electromagnetic metamaterial) design is discussed. Substrate integrated waveguide (SIW) is usually visualized as planar implementation of conventional metallic waveguide, but it can also be visualized as negative permittivity or epsilon negative (ENG) transmission line. The unit cell of epsilon negative TL is similar to CRLH TL except the former do not have series capacitor in the unit cell. The phase constant obtained using both the theories (waveguide and negative epsilon) are compared. The effective epsilon found out using unit cell parameters and wave guide theory are compared with the simulation. SIW resonator is designed and its resonant frequencies using waveguide and epsilon negative theory are compared with the measured resonant frequencies. A triple band antenna is designed using open circuited SIW. The antenna is designed by including two slots to have three bands at 3.3 GHz, 4 GHz and 4.9 GHz. The first band is formed by merging two resonant modes. So, the first band has higher bandwidth. The second and third bands are from single resonant modes. The 10dB impedance bandwidth of antenna in the three bands are 1.49% (3.3 GHz band), 0.72% (4 GHz band) and 0.56% (4.9 GHz band). The measured realized gains of the antenna are 5.81 dBi (3.3 GHz), 4 dBi (4 GHz) and 2.43 dBi (4.9 GHz).

After seeing the single negative material design, next double negative material design is focused. The double negative material considered here is Double periodic CRLH (DPCRLH) TL. DPCRLH unit cell is formed by concatenating two different CRLH unit cells. Two symmetric unit cells of DPCRLH TL are proposed. The DPCRLH symmetric unit cells are analysed using ABCD matrix. The dispersion diagram of

DPCRLH has two left and two right handed regions. The important frequency points in the dispersion diagram are analysed. The closing condition of attenuation gap at $\beta d = 0$ in the dispersion diagram is derived. A DPCRLH symmetric unit cell is designed and fabricated with closing condition of attenuation gap at $\beta d = 0$ at 3.39 GHz. The measured results are compared with the simulated results. The DPCRLH symmetric unit cell is converted to resonator by open circuiting the ends. The open circuited resonator has five resonant points which is confirmed by simulation and measurement. The measured resonant points are at around 1.574 GHz, 2.05 GHz, 3.24 GHz, 6 GHz, and 9 GHz. The last three resonant points are selected to design a triple band filter at 3.6 GHz, 6 GHz and 9 GHz. The filter has different fractional bandwidths on all three bands. The order of the filter is three and it is designed using coupling coefficient and external quality factor. The filter is designed and fabricated with microstrip as host line.

In order to avoid via holes, coplanar waveguide (CPW) is chosen as host line to create DPCRLH structure in the next design. A DPCRLH symmetric unit cell is designed in CPW and its characteristics are analysed. The measured and simulated results of the DPCRLH unit cell are compared. The open circuited resonator is formed from DPCRLH unit cell. As in the previous case it has five resonant points. The simulated resonant points are at 1.5494 GHz, 2.1968 GHz, 3.5072 GHz, 5.0438 GHz and 6.089 GHz. The relation between the quality factor at resonant points and DPCRLH unit cell parameters is discussed. A triple band antenna is designed using last three resonant points at 3.66 GHz, 5.095 GHz and 6.1188 GHz and fabricated. The 10dB measured impedance bandwidth at the three bands are 1.67%, 11.08%, and 2%. The measured Co-polarization realized gain of the antenna in the three bands are 2.1 dBi (3.66 GHz), 3.19 dBi (5.0119 GHz) and 2.24 dBi (6.1188 GHz).

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Chapter 1

Introduction

1.1 Background and Motivation

The first hand held devices for wireless applications were introduced in 1980s. The antennas for the first generation hand held devices were operated at single frequency band [1]. Subsequently several generations were introduced, and several useful frequency bands came up. Fig.1.1 shows the developments in the wireless technology in recent years [2]. It is observed from Fig.1.1 under hardware platforms, recently there has been a rapid development of technologies like 2G, 3G, 4G, Wi-Fi, Bluetooth, Zigbee, RFID, software defined radio. In many cases there is a need to access more than one frequency bands. For example, smartphone should need cellular, Wi-Fi, Bluetooth, NFC (Near field communication), GPS (Global positioning system), and FM (Frequency modulation) antennas [1]. The modern-day vehicles for intelligent transportation and smart mobility need the wireless services like GPS, radio and TV broad casting, remote controls, and traffic information [3] is another example for multi

band transceiver requirements. As most of the wireless applications have been pushing the frequency to a higher value for broad bandwidth requirement, it is necessary to design a device which can function over all the desired bands. So, to make the devices simple and compact there is a need for multi band components. In this thesis, few multi band circuits are designed using metamaterials. The main reason of choosing metamaterials is the nonlinearity of the dispersion diagram (details are given in Chapter 2) so the resonance frequencies of the resonators obtained from the metamaterials are not in harmonic multiples.

Figure 1.1 Developments in wireless technology in recent years [2]

The electromagnetic metamaterials are artificial structures with abnormal characteristics. Since the metamaterials are created artificially, they need to satisfy the effective homogeneous condition. The effective homogeneous condition makes metamaterials to act as real materials for the electromagnetic waves in the operating bandwidth. The effective homogeneity is achieved usually by making the cell size less than quarter of the guided wavelength $\lambda_g/4$. The effective homogeneity makes the

refraction phenomenon dominant over the scattering and diffraction [4]. The refractive index n is related to permeability μ and permittivity ε as

$$n = \pm\sqrt{\varepsilon_r\mu_r} \quad (1.1)$$

where ε_r and μ_r are the relative permittivity and permeability. They are related to free space permittivity and permeability as $\varepsilon_r = \varepsilon/\varepsilon_0$ and $\mu_r = \mu/\mu_0$. The values of free space permittivity and permeability are given as $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Figure 1.2 All possible properties of isotropic materials in $\varepsilon - \mu$ domain

The material classification based on ε and μ values is shown in Fig.1.2 [4-6]

The first quadrant where both ε and μ are positive i.e $\varepsilon > 0$ and $\mu > 0$ represents the right handed materials with $n = +\sqrt{\varepsilon_r\mu_r} \in \mathbb{R}, n > 0$ where $\mathbb{R}$ denotes purely real. Most

naturally available materials fall under this category. The electric field, magnetic field and wave vectors form the right handed system so they are called right handed materials.

The second quadrant represent the materials with $\varepsilon < 0$ and $\mu > 0$. Since ε is negative these materials are called epsilon negative (ENG) materials. Here $n = \sqrt{\varepsilon_r \mu_r} \in \mathbb{I}$ where $\mathbb{I}$ denotes purely imaginary, so no wave propagation is possible. Plasmas exhibit this behavior in certain frequencies.

The fourth quadrant represent the materials with $\varepsilon > 0$ and $\mu < 0$. These materials are called mu negative (MNG) materials. In certain frequencies ferrimagnetic materials exhibit this behavior. Here also wave propagation is not possible since $n = \sqrt{\varepsilon_r \mu_r} \in \mathbb{I}$.

The third quadrant denotes the left handed materials i.e. $\varepsilon < 0$ and $\mu < 0$. These materials are also called double negative (DNG) materials with $n = -\sqrt{\varepsilon_r \mu_r} \in \mathbb{R}, n < 0$. The naturally occurring DNG materials have not been discovered yet, so they should be artificially made. The electric field, magnetic field and wave vectors form the left handed system.

In the Fig.1.2 there are few important lines and points with some distinguished features. The line $\mu = \varepsilon$ in right handed and left handed regions represent the impedance matching materials. These materials are perfectly matched with air. The point having $\varepsilon = -\varepsilon_0$ and $\mu = -\mu_0$ is anti-air which can be used in the design of perfect lens. The point with $\varepsilon = 0$ and $\mu = 0$ represents nihility with perfect tunneling effect. The material near $\varepsilon = 0$ is ε near zero (ENZ) material and in the vicinity of $\mu = 0$ is μ near (MNZ) material [5].

1.2 Objectives:

The main objective of the research is to design multiband microwave circuits using metamaterial open circuited resonators. The requirements for the multi band circuits in guided wave and radiated wave applications are different. Also, the design guidelines need to be changed when the host medium is changed. The size of the resonators used to design the multi band circuits are also restricted to be as small as possible. In order to complete the main objective of the research few goals need to be achieved which are summarised as follows.

1. An extensive literature review about metamaterial especially CRLH characteristics and its applications is the first goal of this research. This gives information about the basic operation of CRLH transmission line, its advantages, and disadvantages. The details about how the characteristics of CRLH are changed to suit for each and every application, are also obtained. The literature review also provides the research gaps in the DPCRLH transmission line.
2. The metamaterial implementation in different host lines like microstrip, coplanar waveguide and SIW are studied. This gives the information about the difference in the implementation on the different host lines. Because in this research the metamaterial design is going to be implemented in different host lines. This also provides the information about the different parameter extractions suitable for different host lines.
3. The information about the SIW resonators and CRLH implementation in SIW gives the details to design SIW open circuited resonator. The details about different modes of SIW resonators and their characteristics gives the enough information to design triple band antenna using SIW open circuited resonator.

4. The literature review about CRLH transmission lines provide details about symmetric unit cells of CRLH. This provides the information required to propose the symmetric unit cells of DPCRLH. The proposal of symmetric unit cells and the analysis of their characteristics based on the dispersion diagram is next goal of this study.
5. Based on the reviews about the CRLH resonators and its applications, the design of DPCRLH resonators are to be proposed. The study about design of filters and other guided wave applications using CRLH gives the required information about the design of multiband guided wave circuits using DPCRLH. The DPCRLH open circuited resonator analysis and its application in design of triple band filter are to be discussed.
6. The CRLH structure in CPW provides enough details to design DPCRLH structure and DPCRLH open circuited resonators in CPW. The CRLH resonators in the design of various antennas gives the necessary details about structural parameters to be changed in the design of resonators for antenna. The design of triple band antenna in CPW using DPCRLH open circuited resonator is the final goal of this thesis.

1.3 Major contributions of the thesis

The thesis gives the information required to design multi band microwave circuits using metamaterial open circuited resonators. The analysis of SIW as epsilon negative structure is presented. The design and analysis of DPCRLH symmetric unit cells and the open circuit resonator using DPCRLH unit cell are discussed. The major contributions of the thesis are as follows

1. The analysis of SIW and its open circuited resonator using waveguide theory and also epsilon negative theory are presented. The various modes of SIW open circuited resonator are analysed and the design of triple band antenna using the SIW resonator is described. The triple band antenna is designed at 3.3 GHz, 4 GHz and 4.9 GHz.
2. The symmetric unit cells of the DPCRLH structure are proposed. The symmetric unit cells are analysed using ABCD parameters. The dispersion diagram is analysed based on the important frequency points $\beta d = 0$ and $\beta d = \pi$. The expressions for these points in terms of unit cell parameters are derived. The condition for the closing of the attenuation gap between $\beta d = 0$ points in the dispersion diagram is found out. A symmetric unit cell is designed and fabricated with attenuation gap closed at 3.3 GHz ($\beta d = 0$). The electric field is uniform at this point (3.3 GHz) because $\beta d = 0$ which means the wavelength is infinite at this point.
3. The design of open circuited resonator from DPCRLH symmetric unit cell is presented. The DPCRLH open circuited resonator has five resonant points and are found out using the input impedance of the resonator. The DPCRLH open circuited resonator is designed and it is proved to have five resonant points. The design of triple band filter using DPCRLH open circuited resonator is presented. The triple band filter has different fractional bandwidth at three bands by using a coupling mechanism which gives separate paths to three band signals. The triple band filter is designed at 3.3 GHz, 6 GHz, and 9 GHz.
4. The design of DPCRLH symmetric unit cell in CPW is proposed. The parameter extraction based on Y and Z parameters of DPCRLH unit cell is proposed. The unit cell is designed and fabricated. This unit cell has the attenuation gap

between two $\beta d = 0$ points in the dispersion diagram. The DPCRLH open circuited resonator is formed from the unit cell. To analyse the radiation characteristics of the DPCRLH resonator, a triple band antenna is designed based on the DPCRLH open circuited resonator. The antenna is designed at 3.66 GHz, 5.095 GHz, and 6.1188 GHz.

1.4 Organisation of the thesis

The thesis has been organised into six chapters. The chapter 1 gives the background about the metamaterial structures and the objectives of this study. The motivation to use the electromagnetic metamaterial in the design of various multiband circuits is discussed. The objectives and contribution of the thesis are explained.

Chapter 2 gives the extensive literature review of theory and analysis of CRLH and its applications. The CRLH resonators and the concept of zero order resonator (ZOR) are discussed. This gives the difference between conventional resonators and metamaterial resonators. CRLH implementations in three different host lines namely microstrip, CPW, and SIW are studied using previously reported examples. The epsilon negative resonators are also explained. The literature review concludes by introducing the DPCRLH and multi periodic CRLH transmission lines.

Chapter 3 presents the analysis of SIW as a waveguide and also as an epsilon negative transmission line. The SIW open circuited resonator is also analysed by these two theories. The various modes of the resonator are discussed. The design of triple band antenna using SIW resonator is also explained.

Chapter 4 proposes the symmetric unit cells of DPCRLH structure. The symmetric unit cells are analysed using ABCD parameters. The dispersion diagram is analysed and the expressions for the important frequency points in it are derived. The condition for

closing of the attenuation gap between two $\beta d = 0$ in the dispersion diagram is also derived. The parameter extraction method for finding the unit cell parameters of DPCRLH structure is discussed. The unit cell based on the above analysis is designed and measured. The design of open circuited resonator based on the DPCRLH unit cell is presented. The design of triple band filter using the open circuited resonator along with its coupling mechanism are explained.

Chapter 5 deals with the design of DPCRLH unit cell in CPW host line. The parameter extraction method suitable for CPW host line is proposed. The design of open circuited DPCRLH resonator in CPW is explained. The resonator has five resonant points. The triple band antenna designed using open circuited resonator is explained.

Chapter 6 concludes by summarizing the research findings in the previous chapters. The advantages and disadvantages of the circuits designed are explained. Few possible ways to extend the works proposed are also presented.

Chapter 2

Literature Review

The metamaterial concepts have been introduced in Chapter 1. In this chapter, the changes in fundamental phenomenon due to the LH medium is first studied. The demonstrations of single and double negative materials are discussed. The reason for choosing metamaterials in the design of resonators is the nonlinearity of the dispersion diagram. The transmission line approach yields relatively wider bandwidth so multi band components can be designed easily using this approach. So, in this chapter the transmission line approach is discussed in detail. The transmission line implementation of double negative metamaterial i.e. Composite right and left handed structure (which has relatively wider bandwidth) is focused. The theoretical analysis and important terminologies of CRLH transmission line (TL) are explained. Bloch- Floquet's Theorem and dispersion relation are explained. The formation and analysis of CRLH resonators are discussed. The special properties of ZOR resonators are mentioned. The applications of CRLH TL in guided wave components and in resonant antennas are discussed. The implementation of CRLH TL in coplanar waveguide along with the applications are presented. Substrate integrated waveguide and their applications in cavity backed slot

antennas are also discussed. The application of CRLH TL in leaky wave antennas is beyond the scope of this chapter. Further the extensions of CRLH TL like Double periodic CRLH and multi periodic CRLH are also explained.

2.1 Theoretical prediction of left handed materials

In 1967, Viktor Veselago theoretically predicted [7] the existence of the substance with negative permittivity and permeability. He also expressed these substance supports the wave propagation and electric field, magnetic field and phase constant vector triad form a left handed (LH) triad rather the right handed system the latter is not uncommon in conventional materials.

Several fundamental phenomena due to the existence of negative permittivity and permeability in a LH medium are different from the conventional materials. They are

1. The simultaneous negative permittivity and permeability is possible only in dispersive medium until it satisfies the entropy condition given in [8]. The simultaneous negative ϵ and μ are impossible to occur in non dispersive medium.

2. Reversal of Doppler effect [9]: In the RH medium, for a receding source the frequency is downshifted or red shifted and for preceding source the frequency is up shifted. But in the LH medium for the receding source it is blue shifted and for the preceding source it is red shifted.

3. Reversal of Vavilov-Cerenkov radiation: when the solids and liquids are encountered by fast moving electrons having high velocity, they emit the radiations in visible spectrum. These radiations are called Vavilov-Cerenkov radiation. In the RH medium the Vavilov-Cerenkov radiation occurs in the positive θ angles in the half plane in the direction of motion, but in LH medium the radiation occurs in the negative θ angles in

the same half plane in the direction of motion (but opposite to the group velocity). The reversal of Vavilov-Cerenkov radiation was theoretically proved in [10]

4. Because of the negative ε and μ the normal components of the electric and magnetic fields at the interface between a conventional right handed (RH) medium and a left handed (LH) medium are anti parallel to each other.

5. Reversal of Snell's law[11, 12]: The wave at the interface of mediums having same handedness sees positive refraction. On the other hand, the wave at the interface of the mediums having different handedness experiences the negative refraction characterized by a negative refraction angle.

2.2 Demonstrations of single negative and left handed materials

After 30 years the prediction of left handedness by Viktor Veselago, Pendry et al designed metal thin wire metamaterial structure with negative ε and positive μ [13, 14] with electric plasma frequency in the GHz range. Later metal split ring resonator structure with positive ε and negative μ was proposed by Pendry et al [15]. This structure has magnetic plasma frequency in the microwave range.

Figure 2.1 (a) Thin wire structure with negative ϵ and positive μ if $\vec{E} \parallel z$ [14] (b) split ring resonator with negative μ and positive ϵ if $\vec{H} \perp y$ [15]

In [16] smith et al. designed thin wire structure and split ring resonator structure with overlapping negative ϵ and μ and combined the two structures to form the first LH material. An intense experimental demonstration of bidimensional LH material was done in [11].

Figure 2.2 (a) Single dimensional LH structure [16] (b) Bidimensional LH structure [11]

After the experimental demonstration by Smith several groups verified the LH Properties numerically [17-21], theoretically (EM theory [22-25] and TL theory [26-29]) and experimentally (Thin wire and split ring resonator [11, 30-32], planar TL [33, 34])

2.2.1 Transmission Line approach

The method of achieving LH characteristics using split ring resonator and thin wire is called resonant approach. The planar implementation of resonant type LH structures and its applications in microwave range are given in [35]. But the resonant type LH structure suffers from the disadvantage of narrow bandwidth. An alternate approach called transmission line approach to design LH structures was proposed by three groups (Eleftheriades [36], Oliner [29], and Caloz-Itoh [28]). The conventional transmission line has series inductor and shunt capacitor so by duality a transmission line with series capacitor and shunt inductor should support the backward propagation. Fig.2.3 shows the hypothetical model of LH transmission line. Z' and Y' are the per unit length impedance and admittance.

Figure 2.3 Incremental equivalent circuit of hypothetical LH transmission line

The characteristics of the LH transmission line can be derived as follows.

The propagation constant γ and phase constant β are given as,

$$\gamma = j\beta = \sqrt{Z'Y'} = \frac{1}{j\omega\sqrt{L'_L C'_L}} \quad (2.1)$$

$$\beta = -\frac{1}{\omega\sqrt{L'_L C'_L}} \quad (2.2)$$

The characteristic impedance is given as,

$$Z_c = \sqrt{\frac{Z'}{Y'}} = \sqrt{\frac{L'_L}{C'_L}} \quad (2.3)$$

The phase velocity v_p and group velocity v_g derived using (2.2) are given as

$$v_p = \frac{\omega}{\beta} = -\omega^2\sqrt{L'_L C'_L} \quad (2.4)$$

$$v_g = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} = \omega^2\sqrt{L'_L C'_L} \quad (2.5)$$

From (2.4) and (2.5) it can be found out, phase velocity v_p and group velocity v_g are antiparallel to each other. But when this transmission line is implemented in a host medium like microstrip, the host medium introduces the right handed properties. So pure LH transmission line cannot be achieved practically, but a composite right left handed transmission line (CRLH TL) can be implemented. The detailed analysis of CRLH is presented in section 2.3. Since the metamaterial structure achieved using CRLH TL is non-resonant, the bandwidth of CRLH TL is relatively wider than resonant type metamaterial.

Figure 2.4 CRLH TL in microstrip host line.[37] [35]

Fig.2.4 shows the CRLH implementation in microstrip host line where C_L is implemented as interdigital capacitor and L_L by short circuited stub. The CRLH unit cell in CPW host line is shown in Fig.2.5. Here also interdigital capacitor and short circuited stub are used to implement C_L and L_L respectively.

Figure 2.5 CRLH unit cell in CPW with $W=1.4$ mm, $S=0.4$ mm, $W_d=1.35$ mm, $W_e=0.475$ mm, $L_a=0.1$ mm, $L_b=1.3$ mm, $L_c=1.15$ mm and $l=2.8$ mm. All strip and slot widths=0.2 mm.[38]

The CRLH structure implementation in substrate integrated waveguide (SIW) is shown in Fig.2.6. L_L is provided by via holes at the sides of SIW and interdigital capacitor implemented in the top of SIW provides C_L . The CRLH structures shown in the figures from Fig. 2.4 to Fig. 2.6 are one dimensional CRLH. The CRLH structures can be implemented two dimensionally and also three dimensionally.

Figure 2.6 CRLH in Substrate integrated waveguide [39]

From the Fig.2.6 it can be found out CRLH is implemented in SIW by inserting interdigital capacitor on the top of the SIW structure. So, it can be visualized that SIW is also an epsilon negative transmission line because of the periodic via holes at the sides of SIW. The SIW as epsilon negative transmission line is already discussed in [40, 41]. But analysis was done for the first two resonant frequencies of SIW open circuited resonator. The analysis of SIW as epsilon negative transmission line can be extended to wider bandwidth. The design of multi band antennas using SIW can be explored.

2.3 CRLH transmission line and its theoretical analysis

The ideal homogeneous CRLH Transmission line (CRLH TL) is not readily available in nature unlike the conventional RH transmission line. So, the unit cell of CRLH TL is artificially created as shown in Fig.2.7 where C_L and L_L are the left handed parameters inserted manually. As the wave propagates, the host medium begins to show its own right handed properties through the parameters L_R and C_R .

Figure 2.7: Unit cell of CRLH TL

Since the CRLH unit cell is artificially created it will occupy some physical length say 'p'. This unit cell is repeated periodically to form CRLH Transmission line (CRLH TL). This artificially created transmission line is not fully homogeneous, but it can be effectively homogenous within certain bandwidth. The condition for the unit cell to be effectively homogeneous is $p < \lambda_g/4$ or $|\Delta\phi| < \pi/2$, where λ_g is the guided wavelength and $\Delta\phi$ is the phase shift produced by the unit cell.

The impedance and the admittance parameters of CRLH unit cell are given by

$$Z = j\omega L_R + \frac{1}{j\omega C_L} = j \frac{\left(\frac{\omega}{\omega_{se}}\right)^2 - 1}{\omega C_L} \quad (2.6)$$

$$Y = j\omega C_R + \frac{1}{j\omega L_L} = j \frac{\left(\frac{\omega}{\omega_{sh}}\right)^2 - 1}{\omega L_L} \quad (2.7)$$

The series and shunt resonance frequencies are given by

$$\omega_{se} = \frac{1}{\sqrt{L_R C_L}} \quad (2.8)$$

$$\omega_{sh} = \frac{1}{\sqrt{L_L C_R}} \quad (2.9)$$

The ABCD matrix is used to analyse the CRLH TL. Since CRLH TL is constructed by repeating the single unit cell, the analysis become easy if ABCD matrix of a single unit cell is known then the ABCD matrix of the N cell CRLH TL can be found out by taking the Nth power of the ABCD matrix of the unit cell.

Figure 2.8: General CRLH TL

The ABCD matrix of the unit cell given in Fig.2.7 is given as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 + ZY & Z \\ Y & 1 \end{bmatrix} \quad (2.10)$$

The unit cell shown in the Fig. 2.7 is asymmetric. The impedances seen from the two sides of the unit cell in Fig.2.7 are not equal. So, in order to have same impedances on both sides, symmetric unit cells are needed. Figure 2.9 shows two types of the symmetric unit cells of CRLH TL, viz. T-symmetric unit cell and π - symmetric cell.

Figure 2.9 (a) T-symmetric unit cell (b) π - symmetric cell.

The analysis of both T-symmetric cell and π - symmetric cell are similar, so the analysis of T-symmetric cell is given here. The ABCD matrix of the T- symmetric unit cell can be found out as follows

$$\begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} 1 & Z/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z/2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 + ZY/2 & Z/2 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z/2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 + \frac{ZY}{2} & Z \left(1 + \frac{ZY}{4} \right) \\ Y & 1 + \frac{ZY}{2} \end{bmatrix} \end{aligned} \quad (2.11)$$

$$= \begin{bmatrix} 1 + \frac{ZY}{2} & \frac{j(\omega/\omega_{se})^2 - j}{\omega C_L} \left(1 + \frac{ZY}{4} \right) \\ \frac{j(\omega/\omega_{sh})^2 - j}{\omega L_L} & 1 + \frac{ZY}{2} \end{bmatrix} \quad (2.12)$$

If the CRLH TL is made up of N unit cells then ABCD matrix of the CRLH TL is Nth power of (2.12). ABCD parameters is then converted to S parameters, the phase constant β and the attenuation constant α of the CRLH TL of N unit cells are obtained from S-parameter of N unit cells $S_{21,N}$ as

$$\beta = -\frac{\varphi^{unwrapped}(S_{21,N})}{l} + \xi \quad (2.13)$$

$$\alpha = -\frac{\ln|S_{21,N}|}{l} \quad (2.14)$$

Where l is the physical length of the transmission line and $l = N * p$, p is the length of the unit cell. The phase origin used for unwrapping β is ω_0 , where

$$\omega_0 = \sqrt{\omega_{se}\omega_{sh}} = \frac{1}{\sqrt[4]{L_R C_R L_L C_L}} = \sqrt{\omega_R \omega_L} \quad (2.15)$$

and

$$\omega_R = \frac{1}{\sqrt{L_R C_R}} \quad (2.16)$$

$$\omega_L = \frac{1}{\sqrt{L_L C_L}} \quad (2.17)$$

Further if $\omega_{se} = \omega_{sh}$ for a unit cell then it is called a balanced cell, if $\omega_{se} \neq \omega_{sh}$ then unit cell is unbalanced.

2.3.1 Bloch Floquet's Theorem and dispersion relation of CRLH TL

Consider an infinite periodic structure with period p . The waves at point z and at the point $z+p$ are related by the complex constant $C = e^{-\gamma p}$ where $\gamma = \gamma(\omega) = \alpha(\omega) + j\beta(\omega)$, $\alpha(\omega)$ is the attenuation coefficient and $\beta(\omega)$ is the phase constant. So, this can be mathematically written as

$$\frac{\psi(z+p)}{\psi(z)} = \frac{\psi(z+2p)}{\psi(z+p)} = \frac{\psi(z+np)}{\psi[z+(n-1)p]} = C = e^{-\gamma p}, \forall n \quad (2.18)$$

From the above expression it can be written as $\psi(z + p) = C\psi(z)$, $\psi(z + 2p) = C\psi(z + p) = C^2\psi(z)$. So, in general

$$\psi(z + np) = C^n\psi(z) = e^{-\gamma np}\psi(z) \quad \forall n \quad (2.19)$$

Equation (2.19) gives the periodic boundary condition (PBCs) existing in periodic structures. From (2.19) the wave function $\psi(z)$ can be written as

$$\psi(z) = \psi(z + np)e^{+\gamma np} \quad (2.20)$$

Multiplying by $e^{+\gamma z}$ on both sides of (2.20)

$$\psi(z)e^{+\gamma z} = \psi(z + np)e^{+\gamma(z+np)} \quad (2.21)$$

The above expression is represented as

$$\xi_\gamma(z) = \psi(z)e^{+\gamma p} \quad (2.22)$$

Equation (2.22) is a periodic function with period p for any γ . Since it is a periodic function, (2.22) can be expanded in Fourier series as

$$\xi_\gamma(z) = \sum_{n=-\infty}^{\infty} \xi_{\gamma n} e^{-j(2n\pi/p)z} \quad (2.23)$$

The Fourier coefficients of (2.23) are given as

$$\xi_{\gamma n} = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \xi_\gamma(z) e^{+j(2n\pi/p)z} dz \quad (2.24)$$

Substituting (2.23) in (2.22), the wave function $\psi_\gamma(z)$ can be found out as

$$\psi_\gamma(z) = e^{-\alpha(\omega)z} \sum_{n=-\infty}^{\infty} \xi_{\gamma n} e^{-j[\beta(\omega)+2n\pi/p]z} = e^{-\alpha(\omega)z} \sum_{n=-\infty}^{\infty} \xi_{\gamma n} e^{-j\beta_n z} \quad (2.25)$$

where

$$\beta_n = \beta(\omega) + \frac{2n\pi}{p} \quad (2.26)$$

The attenuation constant $\alpha(\omega)$ and the propagation constant $\beta(\omega)$ in (2.25) represent the attenuation and the dispersion relation of the periodic structure. Also, in (2.25) only the positive going wave $C^+ = e^{-\gamma p}$ was only represented. If there are reflections in the periodic structure, then both positive going $C^+ = e^{-\gamma p}$ and negative going $C^- = e^{+\gamma p}$ wave exists which results in the general expression

$$\psi_\gamma(z) = e^{-\alpha(\omega)z} \sum_{n=-\infty}^{\infty} \xi_{\gamma n}^+ e^{-j\beta_n z} + e^{+\alpha(\omega)z} \sum_{n=-\infty}^{\infty} \xi_{\gamma n}^- e^{+j\beta_n z} \quad (2.27)$$

Equation (2.27) is the mathematical representation of Bloch-Floquet's theorem. This theorem states in periodic structure, the wave is superposition of infinite number of plane waves called Bloch-Floquet's waves or space harmonics. The fundamental is $\beta_0 = \beta(\omega)$. The waves β_n with $n > 0$ are positive space harmonics and with $n < 0$ are negative space harmonics.

Consider the CRLH unit cell represented as [ABCD] matrix in Fig.2.10. The periodic boundary conditions in (2.19) is applied to find the dispersion relation of CRLH transmission line. So, the output voltage and current in Fig.2.10 are related to the input voltage and current by $e^{-\gamma p}$ where $\gamma = \gamma(\omega) = \alpha(\omega) + j\beta(\omega)$ as discussed previously. So, mathematically the relation between the input and output voltages are represented as

Figure 2.10 Unit cell of CRLH represented as two port network

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \psi \begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix}, \psi = e^{+\gamma p} \quad (2.28)$$

Equation (2.28) is an eigen system with eigen values $\psi_n = e^{+\gamma_n p}$, $n = 1, 2$. γ_n can be found out from ψ_n as $\gamma_n = \ln(\psi_n(\omega))/p$. $\alpha(\omega)$ and $\beta(\omega)$ are found out as $\alpha_n(\omega) = \Re(\gamma_n(\omega))$ and $\beta_n(\omega) = \Im(\gamma_n(\omega))$. Rewriting (2.28) in the form of homogeneous linear system

$$\begin{bmatrix} A - e^{\gamma p} & B \\ C & D - e^{\gamma p} \end{bmatrix} \begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.29)$$

For the non trivial case the determinant must be zero. So, the determinant is given as

$$AD - (A + D)e^{\gamma p} + e^{2\gamma p} - BC = 0 \quad (2.30)$$

$$1 - (A + D)e^{\gamma p} + e^{2\gamma p} = 0$$

$$e^{-\gamma p} - (A + D) + e^{\gamma p} = 0$$

$$2\cosh \gamma p = (A + D)$$

So, the propagation constant γ can be given as [37, 42]

$$\cosh \gamma p = \frac{(A + D)}{2} \quad (2.31)$$

The dispersion relation of the CRLH TL is given by (2.31). Equation (2.31) is solved for different values of β in the Brillouin zone to find eigen frequencies $\omega_n(\beta)$.

In the stop band (2.31) becomes,

$$\alpha = \frac{1}{p} \cosh^{-1} \left(\frac{A + D}{2} \right) \quad (2.32)$$

In the pass band (2.32) is converted as,

$$\beta = \frac{1}{p} \cos^{-1} \left(\frac{A + D}{2} \right) \quad (2.33)$$

Substituting (2.11) in (2.33)

$$\beta = \frac{1}{p} \cos^{-1} \left(1 + \frac{ZY}{2} \right) \quad (2.34)$$

Usually in order to satisfy the homogeneity condition the phase shift produced by the unit cell is made very small. So, $|\Delta\Phi| = |\beta p| \ll 1$. So, $\cos(\beta p)$ can be approximated using Taylor's series expansion up to second order

$$\cos(\beta p) = 1 - \frac{(\beta p)^2}{2} \quad (2.35)$$

From (2.34) and (2.35)

$$1 - \frac{(\beta p)^2}{2} = 1 + \frac{ZY}{2}$$

$$\beta = \frac{s(\omega)}{p} \sqrt{\left(\frac{\omega}{\omega_R} \right)^2 + \left(\frac{\omega_L}{\omega} \right)^2 - \kappa \omega_L^2} \quad (2.36)$$

where $\kappa = L_R C_L + L_L C_R$ and $s(\omega)$ is the sign function. Under balanced condition $\omega_{se} = \omega_{sh}$, (2.36) becomes much simplified as

$$\beta = \frac{1}{p} \left(\frac{\omega}{\omega_R} + \frac{\omega_L}{\omega} \right) \quad (2.37)$$

The scattering parameter responses and dispersion diagram of balanced and unbalanced CRLH TLs are shown in Fig.2.11 and 2.12 respectively. Figures 2.11 and 2.12 contain the frequencies f_{cL} and f_{cR} which are the LH high pass and RH low pass cut-off frequencies.

Figure 2.11: Balanced CRLH TL (symmetric unit cell) response with parameters $N=10$, $L_R=2.5$ nH, $C_R=1$ pF, $L_L=2.5$ nH, $C_L=1$ pF (a) Magnitude response (b) dispersion diagram obtained using equation (2.8) [37]

Figure 2.12: Unbalanced CRLH TL (symmetric unit cell) response with parameters $N=10$, $L_R=2$ nH, $C_R=1$ pF, $L_L=2.5$ nH, $C_L=0.75$ pF (a) Magnitude response (b) dispersion diagram obtained using equation (2.8) [37].

If the unit cell is balanced then the cut-off frequencies can be given in the simplified form as [37]

$$f_{cl}^{bal} = f_R \left| 1 - \sqrt{1 + \frac{f_L}{f_R}} \right| \quad (2.38)$$

$$f_{cr}^{bal} = f_R \left(1 + \sqrt{1 + \frac{f_L}{f_R}} \right) \quad (2.39)$$

2.3.2 Bloch Impedance

When the CRLH TL is implemented in the practice it is effectively homogenous so there will be discontinuities in the network. If the CRLH TL is implemented in a periodic manner, then ratio of voltage to current is constant at the end of each unit cell. This impedance is called Bloch impedance and it is given as [37, 42]

$$Z_{B\pm} = \frac{-2B}{A - D \mp \sqrt{(A + D)^2 - 4}} = \frac{(A - D) \pm \sqrt{(A + D)^2 - 4}}{2C} \quad (2.40)$$

For symmetrical unit cells $A=D$, (2.40) becomes

$$Z_{B\pm} = \frac{\pm B}{\sqrt{A^2 - 1}} = \frac{\pm \sqrt{D^2 - 1}}{C} \quad (2.41)$$

Substituting (2.11) in (2.41)

$$Z_B = \frac{Z \left(1 + \frac{ZY}{4} \right)}{\sqrt{(ZY/2)^2 + ZY}} = \frac{\sqrt{(ZY/2)^2 + ZY}}{Y} \quad (2.42)$$

In the CRLH TL the physical length of unit cell is very small to satisfy the homogeneity condition, so in the limiting case $p \rightarrow 0$, the immittances become very small $Z, Y \rightarrow 0$. So removing square terms in (2.42) leaves the bloch impedance as

$$\lim_{p \rightarrow 0} Z_B = \sqrt{\frac{Z}{Y}} \quad (2.43)$$

2.4 CRLH resonators

The CRLH TL can be converted to resonator by open or short circuiting the ends. Due to the open or short circuited ends the standing waves are produced and the CRLH TL is converted into resonator [43]. Since CRLH TL is not readily available in nature, it is created by repeating unit cell of period ‘p’. The balanced CRLH TL of length l with ‘N’ unit cells resonates when the following condition is satisfied [43, 44]

$$\beta_n \cdot p = \frac{n \cdot \pi \cdot p}{l} = \frac{n\pi}{N} \quad \text{with} \quad n = 0, \pm 1, \pm 2, \dots \dots \pm N - 1 \quad (2.44)$$

Where β is the phase constant.

The positive and negative values correspond to the RH and LH regions of the CRLH respectively. The value $n=0$ is at the transition of LH and RH regions where $\lambda=\infty$. So, at this frequency point there is no field variation in the CRLH structure. If CRLH resonates at this point it is called zeroth order resonator (ZOR). Since the CRLH structure is implemented in the form of LC unit cell, the response is bounded by LH high pass and RH low pass cut off frequencies. So, CRLH resonator has only finite number of resonances unlike the RH TLs which has infinite resonance frequencies. The region between the cutoff frequencies is called the Brillouin zone. This is shown in the dispersion diagram in Fig.2.13. The CRLH resonator has N-1 resonances in the RH side and N-1 resonances in the LH side and one zeroth order resonance, all within the

Brillouin zone. The resonances are separated by the gap of π/Np in the dispersion diagram. The frequencies at the end of the Brillouin zone are called Bragg frequencies and are represented as empty circles in Fig.2.13 [43]. At this Bragg frequencies, only unit cell resonates not the whole structure. The Bragg frequencies can be used if the CRLH structure has only single unit cell. In [45] where ZOR and Bragg frequencies are used to design dual band filter.

Figure 2.13: Dispersion diagram of 4 cell CRLH resonator showing resonances. [43]

CRLH resonators have some special properties compared to conventional RH resonators. They are

- Negative Resonance: The resonance modes in the LH region of CRLH resonators.
- Every mode 'm' (except mode m=0) in the RH region of CRLH resonator has a mode '- m' with identical field distribution in the LH region.
- The mode m=0 called Zeroth order mode is not related to the physical size of the resonator.

- From Fig.2.13 it can be found, the resonance frequencies are not in harmonic ratios due to the nonlinear property of dispersion diagram. This property is important for the design of multi band circuits using CRLH resonators. This property applies to other metamaterial structures also. This property is used in the design of various multi band circuits in this thesis.

2.4.1 Zeroth Order Resonator (ZOR):

The zeroth order resonance of CRLH resonator has several special characteristics. The field variation is zero at this frequency is already mentioned. The resonant frequency of ZOR in unbalanced ($\omega_{sh} \neq \omega_{se}$) condition and balanced condition ($\omega_{sh} = \omega_{se}$) are given as follows.

If the CRLH in unbalanced condition is open circuited on both ends, input impedance at zeroth order frequency point is given by

$$Z_{in} = \frac{1}{N * Y} \quad (2.45)$$

and the resonant frequency is given by

$$\omega_{res}^{open} = \omega_{sh} = \frac{1}{\sqrt{L_L C_R}} \quad (2.46)$$

If the unbalanced CRLH is short circuited at both ends, the input impedance at zeroth order frequency is given by

$$Z_{in} = N * Z \quad (2.47)$$

and the resonance frequency is given as

$$\omega_{res}^{short} = \omega_{se} = \frac{1}{\sqrt{L_R C_L}} \quad (2.48)$$

If the CRLH is balanced then $\omega_{sh} = \omega_{se}$ then resonance frequency is given as

$$\omega_0 = \omega_{sh} = \omega_{se} = \frac{1}{\sqrt{L_L C_R}} = \frac{1}{\sqrt{L_R C_L}} \quad (2.49)$$

From (2.46), (2.48), and (2.49) it can be found out that ZOR's resonant frequency does not depend on length of the resonator. The resonant frequency depends only on the unit cell parameter values. The equivalent circuits of ZOR at zeroth order resonant frequency is found out from (2.46) for open circuited case and from (2.48) for short circuited case. The equivalent circuits are shown in Fig.2.14

Figure 2.14 Equivalent circuit of ZOR (a) open ended case (b) short ended case

The unloaded quality factor for open circuited case is given as [44, 46]

$$Q_0^{open} = \frac{1/NG}{\omega_{sh} \left(\frac{L_L}{N} \right)} = \frac{1/G}{\omega_{sh} L_L} = \omega_{sh} (1/G) C_R \quad (2.50)$$

The unloaded quality factor for short circuited case is given as follows [44]

$$Q_0^{short} = \frac{NR}{\omega_{se} NL_R} = \frac{R}{\omega_{se} L_R} = \omega_{se} R C_L \quad (2.51)$$

The other ZOR's special properties can be concluded from (2.50) and (2.51). In the open circuited case, the unloaded quality factor is independent of series resistance R and in short circuited case it is independent of shunt loss factor G. The number of unit cells (which implies physical length of the resonator) does not affect the unloaded quality

factor [44, 46]. Zeroth order resonance is also not affected by adding series resistance in open circuited case even though other resonances are affected [44, 46].

2.5 CRLH in filter design

The CRLH structure under balanced condition ($\omega_{sh} = \omega_{se}$) has wider bandwidth response from its lower cutoff point f_{cl} to upper cutoff point f_{cr} . This property can be utilized to design UWB filters which are required to have a wide bandwidth. The UWB filter using single π unit cell of CRLH was reported in [47]. The other characteristic improvements were also done to this CRLH unit cell based UWB filter. They are size reduction of the filter by changing short circuited stub to Shorted Open Gap Ring Resonator [48] and to get sharp response by adding defected ground structure (DGS) low pass filter to CRLH structure[49].

The unbalanced CRLH structure has no propagation between ω_{sh} and ω_{se} . This characteristic can be exploited to design bandstop filters. A band stop filter at 2.4 GHz using unbalanced CRLH structure was examined in [50]. The dual band bandstop filter at 5.2 GHz and 5.8 GHz was designed by cascading two different unbalanced CRLH structures [51].

Traditionally band pass and band stop filters are designed using quarter wavelength shunt stubs as resonators (open circuited for band stop and short circuited for band pass filter) with quarter wavelength transmission lines as admittance inverters in between resonators. These resonators have resonance at $3f_0$ in addition to f_0 . So, they cannot be used to design dual band filters. CRLH transmission line has nonlinear phase response as shown in Fig.2.15. If the RH resonators and the stubs in these filters are replaced by CRLH TLs, then due to the nonlinear phase response of the CRLH TLs, the resonances can be obtained at arbitrary frequencies f_1 and f_2 . This principle was used to

design dual band bandpass and dual band bandstop filters[52]. The same principle was used for half wavelength stubs in [53] and in conventional capacitor gap coupled filter [54] to replace the half wavelength resonator.

Figure 2.15 Phase response of CRLH TL in comparison with RH TL [43]

CRLH TL can be used to reduce the size of the filter for example in [55] CRLH TL was used in place of 270° transmission lines which is usually larger in size. CRLH TL can also be used to improve specific characteristics of the filters. The selectivity of the filter was made higher using unbalanced CRLH TL and 0° feed technique [56], super wide band pass filter using compact meta structure for implementing CRLH TL [57] were reported.

The CRLH resonators have been used to design various single band and multi band filters. A single band filter at 3.8 GHz was designed using two cell mushroom type CRLH resonators in [58]. The CRLH resonator with two unit cells produce 3 resonances

(-1 , 0 and $+1$ modes) and the filter in [58] was designed by coupling the -1 mode and 0 mode. The coupling was done by reducing C_L and also the spurious resonance due to $+1$ mode was moved to higher frequency by reducing L_R . The left handed filters i.e. filters designed using left handed modes (like -1 , -2 etc.) are also possible. In [59] a left handed filter based on -1 mode was designed at 2.4 GHz. The smaller size characteristic of ZOR compared to conventional resonators was used in the design of UHF band filter in [60]. A similar filter based on ZOR at 900 MHz with source load coupling to introduce three transmission zeros and size reduction by using triangular bend was reported in [61]. The dual band filters designed using ZORs and inductive coupling between them were also reported [62-64].

2.5.1 Design of multiband filters

Multiband filters have been designed by several techniques in the recent times. In the previous section the design of multiband filters using CRLH is described, here few general techniques to design multi band filters are reported. Stub loaded resonators can be used to design multiband filters. Two dual band filters were reported in [65] based on open stub loaded half wavelength resonator. The odd and even mode analysis was used to find the characteristics of this structure. The odd mode resonance is due to half wavelength resonator and even mode resonance is due to both stub and resonator, so by adjusting the length of the stub the even mode resonant frequency can be adjusted. The first filter was with three transmission zeros and spur line was introduced in the second filter to get four transmission zeros. This principle was extended to design triple band filter by using open and short stub loaded half wave length resonator [66]. In this case, there are two even mode resonances and one odd mode resonances. By changing the length of the stubs, and the half wavelength resonator, the resonant frequencies can be adjusted. Multi band filters can be designed using Stepped impedance resonators

(SIR). A dual band filter designed using first two resonant frequencies of SIR was reported in [67]. Since SIR has two different impedances the first two resonant frequencies can be controlled by adjusting the two impedances. Triple band filter designed using Trisection stepped impedance resonator was reported in [68]. Trisection stepped impedance resonator has three different impedances. The design of various dual and triple band filter using SIR was reported in [69]. Multiband filters can also be designed using coupling matrix method [70]. In all the cases reported above, the central frequencies of the bands were closely spaced. Designing filters with widely spaced central frequencies are difficult (although dual band filter with widely spaced central frequencies can be designed using SIR but it is difficult for triple band filter) using the above methods. A five pole slab-cube-slab (SCS) filter with spurious suppression was designed using silver coated ceramic cube was reported in [71]. Further a seven pole SSCSS filter with improved spurious suppression was also reported in [71]. A triple band filter designed using dielectric resonators was proposed in [72]. Here, cuboid shaped dielectric (ceramic) resonator with four slots was used to design the triple band filter. The layout of the triple band filter is shown in Fig.2.16. The filters using dielectric resonators are not planar.

Figure 2.16 Triple band filter using dielectric (ceramic) resonator [72].

2.6 CRLH in bandwidth extension of components

The dispersion diagram of CRLH TL can be engineered by changing the parameter values and this property has been used in the bandwidth extension of components like hybrid ring [73] where CRLH TL is used along with other RH TLs. The principle of bandwidth extension is to reduce the difference between the slope of phases of the balanced CRLH TL and the RH TL used in the component [43]. If the difference is low, then bandwidth extension can be achieved. Fig.2.17 shows the phase difference between two RH TLs and phase difference between dispersion engineered CRLH TL and RH TL. The balanced CRLH TL was designed to make the difference between the slopes of phases of the CRLH and RL TLs minimum.

Figure 2.17 (a) Phase difference between two RH lines with $\phi_{diff}=900$ at $f_s=5$ GHz (b) Phase difference between CRLH and a RH line with $\phi_{diff}=900$ at $f_s=5$ GHz [43].

In [73] the conventional 270° RH TL of the hybrid ring was replaced by -90° pure LH TL and -90° balanced CRLH TL. The bandwidth extension by CRLH TL was higher than LH TL because of less frequency dependence. Since CRLH TL is of small size compared to ordinary RH transmission line the size of the hybrid rings was also reduced. The hybrid ring using CRLH showed the bandwidth increase of 49% in the in-phase operation and 58% in the 180° out of phase operation [73].

A broadband via free Balun was reported in [74]. Here two rows of CRLH structures were joined together so that the two rows were mirror image of each other that is the short circuited stubs of the two rows were joined together and the via was removed. In the odd mode virtual ground was formed at the center between the two rows and the CRLH passband became the Balun passband. In the even mode the central line separating the two rows became open so the CRLH passband became stopband. In [74] it was also reported the performance of the balun can be increased by increasing the number of unit cells of CRLH structure.

2.7 CRLH in multi band applications

The role of CRLH TL in the design of dual band filters is already mentioned in section 2.5. The same principle can be used to design other microwave components. From the Fig.2.15, it can be seen due to nonlinear phase response of CRLH structure phase shifts of -90° and -270° can be obtained at arbitrary frequencies say f_1 and f_2 . CRLH structure was used to make quarter wave short circuit and open circuit transmission lines as dual band components [43, 75]. The quadrature hybrid was also made to operate in dual bands [43, 75]. All the conventional RH transmission lines of phase shifts -90° in the Quadrature hybrid were replaced by CRLH TLs. CRLH TLs were engineered to give phase of -90° and -270° at the desired frequencies. The same principle was used in the design of the dual band rat race coupler and dual band Wilkinson power divider. In the rat race coupler case, all the RH TLs of phase shift -90° were replaced by CRLH TLs of phase shift -90° and RH TL of 270° was replaced by three -90° CRLH TLs [43, 75]. In Wilkinson power divider the two RH TLs of phase shift -90° were replaced by the CRLH TLs having phase shifts of 90° (phase advance)

and -90° (phase delay) at the two desired frequencies to make the power divider to work in dual bands.

2.8 CRLH in design of resonant antennas

The conversion of CRLH TL into resonators by open/short circuiting the ends and the resonance condition has been already discussed in section 2.4. These CRLH resonators can be used in the design of antennas. A zero order resonating antenna designed using four unit cell with via-less configuration was reported in [76]. The via-less configuration was achieved using large patch which is coupled to ground through virtual ground capacitance. The virtual ground capacitance is of very high value and should be above certain limit to support LH region propagation. The overall size of the zero order resonating antenna is very small compared to conventional patch antennas (Fig.2.18). Due to this directivity and gain are reduced. The dual band ring antenna based on ± 2 modes was also described in [76]. The antennas based on the ring structure can support only even modes. The resonant modes with same number but opposite magnitude have same characteristics so it is easy to design dual band antennas based on these modes [76]. Monopolar resonant antennas based on CRLH and inductive loaded TL were reported in [77]. These antennas achieved the monopolar radiation due to mushroom like unit cell implementation. The effects of increasing the number of the unit cells in the resonant and non-resonant directions for these antennas were also reported. Since the ZOR antenna does not depend on the length, high directive antennas can be designed using ZOR by increasing the length [78]. A zero order resonator antenna, short circuited at one end was designed and verified experimentally in [78]. Also in [78] the characteristics of ZOR antenna and half wavelength resonant antenna

using CRLH (CRLH antenna operating in ± 1) were compared with conventional patch antenna.

Figure 2.18 (a) Conventional patch antenna at 4.9 GHz (b) ZOR antenna at 4.88 GHz [76]

2.9 Design of multi band antennas

The dual band and triple band antennas formed from the broad band antennas by the use of U slots were reported in [79, 80]. The U slots produces a notch and converts the broad band antenna into dual or triple band antenna. The frequency ratios of the bands are very low in this method of design. The antennas using U slots are shown in Fig.2.19. A triple band antenna using two triangular rings was reported in [81]. The antenna has different polarization in all the three bands. Each triangular ring acts as dual polarized dual band antenna. The triple band was formed by the interaction between the triangular rings. Here also the frequency ratio is small. In [82], a dual polarized triple band antenna was designed. The triple band is achieved by using the stack of three patches. Two slots are used in the lower and middle patch to reduce the size of the patch and improve the axial ratio of the first two bands. The last band is linearly polarized and other two are circularly polarized. A dual band cavity backed slot antenna at 0.9 GHz and 1.8 GHz was reported in [83]. The dual band is achieved using U shaped slot and a ridged slot above the cavity. A dual band circularly polarized antenna was discussed in

[84]. The antenna has two square annular slots backed by a cavity fed by a microstrip line with two stubs.

Two multi band antennas fed by Coplanar waveguide were reported in [85]. The dual band antenna has three strips. Two strips for each band and the third strip is for bandwidth control and suppression of the spurious band. The triple band antenna was formed from this antenna by adding an additional strip. A triple band antenna fed by microstrip line with two strips was reported in [86]. The meandered strip produces two bands and L shaped strip produces the middle band.

Figure 2.19 Multi band antennas using U slots (a) Dual band antenna (b) Triple band antenna [80].

2.10 CRLH in Coplanar waveguide

CRLH TL in microstrip requires via hole for implementation of short circuited stub which is a difficult process during fabrication. This can be avoided by implementing CRLH in coplanar waveguide (CPW). A new type of CRLH unit cell implemented in both microstrip and CPW configuration was proposed in [87]. Here the C_L was implemented using MIM capacitor with one plate in the top microstrip layer and other in bottom CPW layer. The L_L was implemented using short circuited stub in CPW layer at the bottom. This unit cell can be viewed as the link between CRLH unit cell implementations in microstrip and coplanar waveguide.

2.10.1 CPW CRLH in filter designs

A CPW CRLH unit cell was proposed in [38] and parameter extraction based on effective homogeneity was also discussed for CRLH CPW. The CRLH unit cell from [38] is shown in Fig.2.5. The CPW CRLH resonator was formed from the CRLH unit cell by using short circuited stubs at ends of the unit cell in [88]. This CPW CRLH resonator was 49.1% less in size compared to conventional half wavelength resonator. A two pole Band pass filter was also designed using two CPW CRLH resonators and it showed 51.4% reduction in size compared to conventional inductively coupled band pass filter. Finally, CPW Electronic band gap (EBG) structures were added to suppress the harmonics. CPW CRLH short circuited stub was formed from CPW CRLH unit cell by short circuiting it on one side in [89]. The uniform CPW of length l_{in} and l_{end} were added to CRLH unit cell at the feeding side and short circuited side respectively. A single band filter with three cascaded bilateral CPW CRLH short circuited shunt stubs with $l_{end}=0$ was designed at 4.64 GHz and dual band filter was designed using three cascaded bilateral CPW CRLH shunt stubs with $l_{in}=1.4\text{mm}$ and $l_{end}=7\text{mm}$. The above two types of CPW CRLH short circuited stubs (without and with l_{end}) were explained with equivalent circuits. The single and dual band filters using the above two stubs as resonators along with admittance inverters were reported in [90]. A dual band filter designed by periodically loading the above CPW CRLH short circuited stubs and quarter wave open circuited stubs with dual band inverter in between them was reported in [91]. A single band filter using CPW CRLH zero order resonator was demonstrated in [92]. The three ZORs along with gap capacitors as admittance inverters were used to design the bandpass filter at 5.8 GHz.

2.10.2 CPW CRLH and epsilon negative resonators in antenna design

Generally, CRLH TL resonates at ω_{sh} when open circuited and resonates at ω_{se} when short circuited. In [93], novel terminations for CRLH TL was proposed to make it resonate at both ω_{sh} and ω_{se} simultaneously. The proposed pseudo open termination has shunt LC resonator in series with capacitor C_s for $f_{se} < f_{sh}$ or with inductor L_s for $f_{se} > f_{sh}$. The proposed pseudo short circuit termination has capacitor C_p for $f_{se} < f_{sh}$ or inductor L_p for $f_{se} > f_{sh}$ in shunt with series LC resonator. Two dual band CPW CRLH antennas were designed and verified experimentally in [93] for two pseudo open termination cases. The structure of the dual band CPW CRLH antennas with two pseudo open termination cases are shown in Fig.2.20.

Figure 2.20 CPW CRLH antenna with novel terminations (a) for the case $f_{se} < f_{sh}$ with $G = 0.7$ mm, $l_m = 1$ mm, $l_f = 0.6$ mm, $w_f = 2.05$ mm, $g_f = 0.2$ mm (b) for the case $f_{se} > f_{sh}$ with $l_t = 1$ mm, $g_t = 0.2$ mm, $l_m = 0.2$ mm, $G_f = 0.1$ mm, $w_f = 4.6$ mm, $g_f = 0.2$ mm [93].

Instead of CRLH structure inductor loaded and capacitor loaded TLs were used to produce ZOR in [94]. These inductor loaded and capacitor loaded ZORs were used to design antenna in CPW host line. Three configurations of CPW CRLH ZOR antennas were reported in [95]. The three configurations were symmetric, asymmetric and chip loaded antennas. The impedance matching was achieved by adding bottom patch to the CRLH structure. The bandwidth extension by reducing the quality factor (given in (2.45)) was also discussed. A wide band antenna was designed by combining ZOR and FOR (first order resonance) in [96]. The ZOR and FOR were obtained by open circuiting the negative epsilon structure i.e. CRLH structure with $C_L=0$. The bandwidth of ZOR and FOR were increased by increasing L_L and decreasing C_R and frequencies of ZOR and FOR were designed so close to merge both resonances. As a result, a wide band antenna of 70.5% was designed. The same principle was used to design dual band filter using ZOR and FOR in [97] (epsilon negative) and [98] (CRLH). Asymmetric CPW with one of the ground planes placed on the bottom side was used in [97] but in [98] asymmetric CPW was realized by moving one of the ground planes away from the signal line but both grounds are in top side. Asymmetric CPW with one of the two ground planes moved to bottom side was used to design a single band antenna at 1.94 GHz in [99]. A wide fractional bandwidth of 10.3% was obtained by increasing L_L and decreasing C_R of ZOR which was formed from open circuited CRLH structure. A wide band antenna of fractional bandwidth 109.1% was designed using CRLH resonator in asymmetric CPW [100]. The asymmetric CPW was formed with one of the two grounds moved to bottom and cut in the form of U shape. The wide bandwidth was achieved from two resonances from CRLH resonator and two resonances from the ground plane at the bottom. Two ZOR antennas using double negative (DNG) and epsilon negative (ENG) structures were reported in [101]. DNG structure was realized by mushroom

structure and ENG was by via holes only and no slots in the top layer as in mushroom structure. Both ZOR antennas were designed at 10 GHz and showed similar characteristics.

A short circuited CRLH resonator in CPW was used to design bandwidth enhanced antenna in [102]. The interdigital capacitor was adjusted to decrease L_R and increase C_L to attain the maximum bandwidth in this case it was reported as 15.1%. In [103], the bandwidth was enhanced by bisecting the ZOR formed from the short circuited capacitor loaded TL. A CPW dual band antenna utilizing ZOR and +1 frequencies was reported in [104]. The linear polarization at ZOR frequency and circular polarization at FOR frequency were achieved. The circular polarization was obtained by using square complementary split ring resonator at bottom side which excited TM_{01} mode orthogonal to FOR mode in CRLH structure.

2.11 SIW and its cavity resonator

Substrate integrated waveguide (SIW) has been used in the design of various microwave and millimeter wave components. A review of SIW active and passive circuits and antennas can be found in [105]. SIW resonator is formed from the SIW structure by closing the remaining two sides of SIW by electric walls [106]. The via holes are used to implement these electric walls.

2.11.1 SIW in cavity backed slot antenna

One of the important applications of SIW is cavity backed antennas. The ordinary $\lambda/2$ slot in a metal screen radiates in both directions. To get the radiation in one direction usually the slot is backed by the cavity [107]. SIW cavity resonator can be used as cavity for the cavity backed antennas. A planar slot antenna with SIW cavity

backed was reported in [108]. The mode TE_{120} of SIW cavity was responsible for the radiation and slot was in the bottom side of cavity. This SIW cavity backed slot antenna is shown in Fig.2.21. The antenna in [108] was converted to wide band antenna of 6.3% by using rectangular cavity and slot of length higher than half resonant wavelength in [109]. The increased bandwidth was due to two hybrid modes formed due to the combination of modes TE_{110} and TE_{120} . Three antennas based on SIW cavity backed cross slots were reported in [110]. The first antenna was dual band linear polarized implemented by cross slots of different lengths on the top of rectangular SIW cavity. The other two antennas were circularly polarized with different handedness, implemented using cross slots of different length and square SIW cavity. The right handed and left handed circular polarizations were produced by having crossed slots on the top and bottom of SIW cavity in the antennas II and III respectively. In [111] a dual mode antenna was demonstrated using ring slot above the SIW cavity. In the lower band the radiation is due to TE_{101} mode which resulted in the radial electric field on the ring slot. So, monopole like radiation was formed in the first band. The second band was due to TE_{102} mode which resulted in the broad side radiation pattern.

Figure 2.21 SIW cavity backed slot antenna (a) front side (b) back side [108]

The dual band antenna with 45° linear polarization using triangular ring slot backed by SIW cavity was reported in [112]. The SIW cavity was made to operate in TE_{110} mode which excited the slot mode and patch (which was inside the slot) mode. These two modes were responsible for dual band radiation. Further a left handed circular polarization antenna was created by combining two SIW backed triangular ring slot antennas and SIW coupler.

Antenna having two circular polarizations at 5.9 GHz was demonstrated in [113]. Here, a square cavity SIW having four slots parallel to four edges of the cavity were used. Grounded CPW was used to excite two orthogonal modes TE_{120} and TE_{210} simultaneously. Left handed circular polarization was formed when excited with port 1 and with port 2 right handed polarization was formed. In [114], a spoon shaped slot along with SIW cavity was used to design a circular polarized antenna. The SIW cavity was formed by merging a rectangular cavity and circular cavity. The spoon shaped slot excited the two orthogonal electric fields to radiate the circularly polarized wave. In order to reduce the size and increase the bandwidth, half mode SIW and semicircular slot were used to generate the same right handed circularly polarized wave. In addition to half mode SIW antennas quarter mode [115, 116] and eighth mode [117-119] SIW cavities were also used to design antennas of smaller sizes.

2.11.2 CRLH in SIW

The open ended and short ended SIW antennas based on negative resonance were demonstrated in [120]. The negative resonance was obtained using CRLH structure by etching interdigital capacitor on top of SIW. Single stage and two stage open ended and short ended antennas were reported. These open ended and short ended SIW CRLH slot antennas are shown in Fig.2.22. It was shown open ended antennas were smaller in

size while short ended antennas achieved better radiation efficiency and gain. Frequency reconfigurable SIW CRLH short ended antenna was reported in [121]. The varactor diode along with the biasing circuit was placed in the center of the cavity where the electric field was maximum to get the maximum tuning range. Antenna operating at 2.1 GHz with a miniaturization factor of 87% was reported in [122]. Antenna switchable from negative to positive resonance by reactance loading was reported in [123]. As usual the negative resonance was achieved by inserting interdigital capacitor on top metal layer of the SIW cavity. The biasing circuit was isolated from the antenna by using second dielectric layer above the cavity top metal layer. The miniaturization was achieved in three stages. In the first stage ramp shaped non radiating slots are added to reduce the width of the SIW cavity backed slot antenna then ramp shaped interdigital slot was added to make SIW to operate in -1 resonance mode. In the final stage, floating metal slot was added in the middle layer. The first order resonances i.e. ± 1 resonances of CRLH resonator decreases with increase in value of L_R [124]. This property was used to design miniaturized antenna using series inductance flexible (SIF) CRLH structure on the SIW cavity. SIF CRLH structure was achieved by etching a slot in addition to interdigital capacitor on the top metal layer of SIW cavity. The slot increased the inductance L_R .

Figure 2.22 CRLH SIW slot antennas [120]

2.12 Double periodic and multi periodic CRLH:

Double periodic CRLH (DPCRLH) is formed by concatenating two different CRLH unit-cells to form a new unit cell. If more than two unit cells are concatenated, then it becomes multi-periodic CRLH. Double periodic CRLH TL was reported in [125-128]. The DPCRLH structure was analyzed by singular perturbation method and verified experimentally in [125]. The leaky wave antenna by DPCRLH structure has additional leaky wave band compared to CRLH structure. The leaky wave antennas by DPCRLH and CRLH TLs were compared in [126]. DPCRLH TL used in [126] is shown in Fig.2.23. DPCRLH leaky wave antenna in substrate integrated waveguide (SIW) was reported in [127]. The comparison between DPCRLH and CRLH leaky wave antennas in Substrate Integrated Waveguide implementation was reported in [128].

Figure 2.23 DPCRLH structure in microstrip host line [126]

This double periodic CRLH is further extended to multi periodic CRLH in [129] and [130]. Multi periodic CRLH was introduced in [129]. The relation between the periodicity and number of stop bands of multi periodic CRLH was also reported in [129]. In [130] the analysis of multi periodicity was extended to both dispersive and non-dispersive cases and multi periodic phase reversal antenna was given as an example. Fig. 2.24 shows a multi periodic CRLH TL (triple periodic) along with its equivalent circuit.

Figure 2.24: A triple periodic CRLH structure [129]

2.13 Conclusion

This chapter provides extensive review on analysis and the design of metamaterial and CRLH structure. Several fundamental phenomenon changes in LH medium are mentioned. The demonstrations of single negative and LH medium are studied. The analysis of the CRLH and its implementation in microstrip, CPW and SIW have been studied. To a large account CRLH has been used in bandwidth extension and dual band applications due to its nonlinear dispersion diagram. The resonator which is independent of its length i.e., zero order resonator is also examined along with other resonance characteristics of CRLH. The applications of CRLH resonators in filters and antennas has been studied in microstrip and CPW host mediums. SIW resonators and cavity backed slot antennas have been examined. The design of CRLH based antennas in SIW has also been reviewed. Finally, the DPCRLH and multiperiodic CRLH structures are studied. The study about SIW resonators and CRLH implementation in SIW gives details to the design triple band antenna in chapter 3. The study of CRLH characteristics and its applications in various microwave circuits and DPCRLH and multi periodic CRLH provides a lot of information to design triple band filter in chapter 4 and triple band antenna in chapter 5 using DPCRLH resonators

Chapter 3

Substrate Integrated Waveguide resonator based triple band antenna

In chapter 2, several applications of CRLH transmission line in microstrip, CPW and SIW host lines have been discussed. Substrate Integrated Waveguide (SIW) is formed by having metal plates on the top and bottom and via holes on the two sides. Usually SIW is regarded as planar implementation of conventional rectangular waveguide but there are some differences too [131]. SIW is formed by periodic repetition of via holes so it is considered as a periodic structure. SIW has periodic gaps which leads to leakages so there is a difference in wave propagation and modes between SIW and conventional waveguides. During analysis, SIW is visualized as a planar implementation of conventional metallic waveguide, but it can also be visualized as an epsilon negative transmission line [40, 41]. In this chapter SIW is analyzed using waveguide theory and also epsilon negative transmission line theory. The phase constant using both the methods are compared. SIW is converted to resonator by open circuiting the ends and a triple band antenna is designed by opening two slots on top of SIW. First

the analysis of SIW by using waveguide theory will be seen and then followed by epsilon negative theory.

3.1 SIW as Waveguide

Substrate Integrated Waveguide (SIW) of width w can be considered as equivalent to rectangular waveguide of width W_{eff} [131, 132]. The detailed analysis of the SIW characteristics can be found in [131-133]. The relation between the width of SIW and rectangular waveguide is given in [131] as

$$W_{eff} = w - 1.08 \frac{d^2}{s} + 0.1 \frac{d^2}{w} \quad (3.1)$$

With the conditions $\frac{s}{d} < 2$ and $\frac{d}{w} < \frac{1}{5}$. Where ‘ d ’ is the diameter of the via hole and ‘ s ’ is the distance between the centers of the two adjacent via holes (Fig.3.1).

Figure 3.1 SIW with width $w = 57.6$ mm, $d = 2.32$ mm, and $s = 3.6$ mm.

Since SIW is equivalent to rectangular waveguide of width W_{eff} all the characteristic equations of rectangular waveguide can be used. The characteristic equations of the rectangular waveguide with width ‘ a ’ and thickness ‘ b ’ ($a > b$) are as follows

The phase constant,

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad (3.2)$$

which is positive during propagation

$$k > k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad (3.3)$$

the cut off frequency for each mode is given as

$$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad (3.4)$$

Dominant mode cut off frequency TE₁₀

$$f_{c_{10}} = \frac{1}{2a\sqrt{\mu\epsilon}} \quad (3.5)$$

Consider the SIW with width $w = 57.6$ mm, $d = 2.32$ mm, and $s = 3.6$ mm (Fig.3.1). Using (3.1) the effective width of rectangular waveguide can be found out as 55.99 mm. So, equations from (3.2) to (3.4) can be used to find the characteristics of the SIW in Fig.3.1. But in SIW the thickness is relatively very small so there is no variation in Z direction (Fig.3.1). So, the second term in the equations from (3.2) to (3.4) can be neglected.

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{W_{eff}}\right)^2} \quad (3.6)$$

and the cut off frequency is given as

$$f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{W_{eff}}\right)^2} \quad (3.7)$$

With $k = 2\pi f\sqrt{\mu\epsilon}$, (3.6) can be rewritten as

$$\beta = k \sqrt{1 - \left(\frac{f_{c_{mn}}}{f}\right)^2} \quad (3.8)$$

The cut off frequency of the dominant mode TE₁₀ mode for the SIW in Fig. 3.1 can be calculated using (3.7) as 1.4219 GHz and the phase constant β calculated using (3.8) is shown in Fig.3.2.

Figure 3.2 Phase constant for SIW in Fig. 3.1

3.1.1 SIW open circuited cavity resonator

SIW cavity resonator usually has four electric walls implemented by via holes on four sides and metal layers on top and bottom which also act as electric walls. SIW resonator considered here is open circuited at two sides and via holes on other two sides with metals at top and bottom.

Figure 3.3 Rectangular cavity

Consider the rectangular cavity as shown in Fig. 3.3. For a homogeneous source free region the field satisfies the following equations [134]

$$-\nabla \times \vec{E} = j\omega\mu\vec{H} \quad \nabla \cdot \vec{H} = 0 \quad (3.9a)$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E} \quad \nabla \cdot \vec{E} = 0 \quad (3.9b)$$

The divergence of $\vec{E}$ and $\vec{H}$ are zero, so the fields can be expressed in terms of vector potentials $\vec{A}$ (magnetic vector potential) and $\vec{F}$ (electric vector potential). The general solutions of vector potentials are given as

$$\nabla \times \nabla \times \vec{A} - k^2 \vec{A} = -j\omega\epsilon \nabla \Phi^a \quad (3.10a)$$

$$\nabla \times \nabla \times \vec{F} - k^2 \vec{F} = -j\omega\mu \nabla \Phi^f \quad (3.10b)$$

The electromagnetic fields are expressed in terms of vector potentials as

$$\vec{E} = -\nabla \times \vec{F} + \frac{1}{j\omega\epsilon} \nabla \times \nabla \times \vec{A} \quad (3.11a)$$

$$\vec{H} = \nabla \times \vec{A} + \frac{1}{j\omega\mu} \nabla \times \nabla \times \vec{F} \quad (3.11b)$$

If A and F are chosen as

$$\nabla \cdot \vec{A} = -j\omega\varepsilon\Phi^a \quad \nabla \cdot \vec{F} = -j\omega\mu\Phi^f \quad (3.12)$$

Substituting (3.12) in (3.10) results in

$$\nabla^2 \vec{A} + k^2 \vec{A} = 0 \quad (3.13a)$$

$$\nabla^2 \vec{F} + k^2 \vec{F} = 0 \quad (3.13b)$$

When the conditions in (3.13) are satisfied, equations in (3.11) can be expressed as

$$\vec{E} = -\nabla \times \vec{F} - j\omega\mu\vec{A} + \frac{1}{j\omega\varepsilon} \nabla(\nabla \cdot \vec{A}) \quad (3.14a)$$

$$\vec{H} = \nabla \times \vec{A} - j\omega\varepsilon\vec{F} + \frac{1}{j\omega\mu} \nabla(\nabla \cdot \vec{F}) \quad (3.14b)$$

If $\vec{A} = 0$ and $\vec{F} = \psi \vec{U}_x$ are chosen, then

$$\vec{E} = -\nabla \times \vec{F} \quad (3.15a)$$

$$\vec{H} = -j\omega\varepsilon\vec{F} + \frac{1}{j\omega\mu} \nabla(\nabla \cdot \vec{F}) \quad (3.15b)$$

Expanding,

$$\begin{aligned} E_x &= 0 & H_x &= \frac{1}{j\omega\mu} \left(\frac{\partial^2}{\partial x^2} + k^2 \right) \psi \\ E_y &= -\frac{\partial \psi}{\partial z} & H_y &= \frac{-1}{j\omega\mu} \frac{\partial^2 \psi}{\partial y \partial x} \\ E_z &= -\frac{\partial \psi}{\partial y} & H_z &= \frac{1}{j\omega\mu} \frac{\partial^2 \psi}{\partial z \partial x} \end{aligned} \quad (3.16)$$

From (3.16) it can be noted $E_x = 0$, So $\vec{A} = 0$ and $\vec{F} = \psi \vec{U}_x$ result in TE wave. For

TM wave the vector potentials should be chosen as $\vec{F} = 0$ and $\vec{A} = \psi \vec{U}_x$.

Figure 3.4 SIW open circuited resonator $w=57.6\text{mm}$, length $L=53.2\text{mm}$ diameter of via holes $d=2.32\text{mm}$ and the space between the centers of the adjacent vias $S=3.6\text{mm}$.

The SIW open circuited resonator is also shown in Fig.3.4 whose fields needs to be found out. The SIW resonator in Fig.3.4 is equivalent to cavity resonator (shown in Fig. 3.3) of width W_{eff} and length L_{eff} .

To find the fields in the rectangular cavity ψ is chosen as

$$\psi = A \cos(k_x x) \cos(k_y y) \quad (3.17)$$

In (3.17), no function is chosen in Z direction, since thickness of SIW resonator is very less compared to L_{eff} and W_{eff} .

From Fig.3.4 SIW resonator is open circuited at two sides in X direction. So, tangential magnetic field is zero at these two points i.e. $H_y = 0$ at $x = 0$ and $x = L_{eff}$.

Substituting (3.17) in H_y equation of (3.16),

$$H_y = \frac{-1}{j\omega\mu} \frac{\partial^2 \psi}{\partial y \partial x} = \frac{-1}{j\omega\mu} A k_x k_y \sin(k_x x) \sin(k_y y)$$

At $x = L_{eff}$

$$H_y = \frac{-1}{j\omega\mu} A k_x k_y \sin(k_x L_{eff}) \sin(k_y y) = 0$$

$$\sin(k_x L_{eff}) = 0$$

$$k_x L_{eff} = m\pi$$

$$k_x = \frac{m\pi}{L_{eff}} \quad (3.18)$$

where $m = 0, 1, 2, \dots$

SIW resonator is short circuited at two sides in Y direction (Fig.3.4). So, tangential electric field is zero at these two points i.e. $E_z = 0$ at $y = 0$ and $y = W_{eff}$.

Substituting (3.17) in E_z equation of (3.16),

$$E_z = -\frac{\partial \psi}{\partial y} = A k_y \cos(k_x x) \sin(k_y y)$$

At $y = W_{eff}$,

$$E_z = A k_y \cos(k_x x) \sin(k_y W_{eff}) = 0$$

$$\sin k_y W_{eff} = 0$$

$$k_y W_{eff} = n\pi$$

$$k_y = \frac{n\pi}{W_{eff}} \quad (3.19)$$

where $n = 0, 1, 2, \dots$

Since the SIW resonator thickness is relatively very small compared to its sides and ψ in (3.17) do not have function in Z direction, so E_y and H_z equations in (3.16) becomes

$$E_y = -\frac{\partial \psi}{\partial z} = 0 \quad (3.20)$$

$$H_z = \frac{1}{j\omega\mu} \frac{\partial^2 \psi}{\partial z \partial x} = 0 \quad (3.21)$$

The fields in other directions are given by substituting (3.17), (3.18) and (3.19) in (3.16)

$$\begin{aligned} H_x &= \frac{1}{j\omega\mu} \left(\frac{\partial^2}{\partial x^2} + k^2 \right) \psi = \frac{1}{j\omega\mu} \left(\frac{\partial^2}{\partial x^2} + k^2 \right) A \cos(k_x x) \cos(k_y y) \\ H_x &= \frac{1}{j\omega\mu} (-Ak_x^2 \cos(k_x x) \cos(k_y y) + Ak^2 \cos(k_x x) \cos(k_y y)) \\ H_x &= \frac{A}{j\omega\mu} \left(k^2 - \left(\frac{m\pi}{L_{eff}} \right)^2 \right) \cos\left(\frac{m\pi x}{L_{eff}}\right) \cos\left(\frac{n\pi y}{W_{eff}}\right) \end{aligned} \quad (3.22)$$

$$\begin{aligned} H_y &= \frac{-1}{j\omega\mu} \frac{\partial^2 \psi}{\partial y \partial x} = \frac{-1}{j\omega\mu} Ak_x k_y \sin(k_x x) \sin(k_y y) \\ H_y &= \frac{-1}{j\omega\mu} \frac{Amn\pi^2}{L_{eff} W_{eff}} \sin\left(\frac{m\pi x}{L_{eff}}\right) \sin\left(\frac{n\pi y}{W_{eff}}\right) \end{aligned} \quad (3.23)$$

$$\begin{aligned} E_z &= -\frac{\partial \psi}{\partial y} = Ak_y \cos(k_x x) \sin(k_y y) \\ E_z &= \frac{An\pi}{W_{eff}} \cos\left(\frac{m\pi x}{L_{eff}}\right) \sin\left(\frac{n\pi y}{W_{eff}}\right) \end{aligned} \quad (3.24)$$

Also,

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

But $k_z = 0$, as already mentioned thickness of SIW resonator is very less compared to L_{eff} and W_{eff}

$$\begin{aligned}
k &= \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{m\pi}{L_{eff}}\right)^2 + \left(\frac{n\pi}{W_{eff}}\right)^2} \\
\sqrt{\epsilon_r} k_0 &= \sqrt{\left(\frac{m\pi}{L_{eff}}\right)^2 + \left(\frac{n\pi}{W_{eff}}\right)^2} \\
\sqrt{\epsilon_r} \frac{2\pi}{\lambda_0} &= \frac{\sqrt{\epsilon_r} 2\pi f_{mn0}}{C} = \sqrt{\left(\frac{m\pi}{L_{eff}}\right)^2 + \left(\frac{n\pi}{W_{eff}}\right)^2} \\
f_{mn0} &= \frac{C}{\sqrt{\epsilon_r} 2\pi} \sqrt{\left(\frac{m\pi}{L_{eff}}\right)^2 + \left(\frac{n\pi}{W_{eff}}\right)^2} \\
f_{mn0} &= \frac{C}{2\sqrt{\epsilon_r}} \sqrt{\left(\frac{m}{L_{eff}}\right)^2 + \left(\frac{n}{W_{eff}}\right)^2} \tag{3.25}
\end{aligned}$$

Equation (3.25) gives the resonant frequency of the SIW resonator. The field expressions in SIW resonator are given in (3.20) to (3.24).

3.2 SIW as Epsilon negative transmission line

In the previous section SIW is viewed as a rectangular waveguide, now in this section it is visualized as epsilon negative transmission line (TL). The epsilon negative transmission line has the similar structure as CRLH structure mentioned in Chapter 2, but there is no series capacitor C_L in the unit cell structure. The unit cell of epsilon negative transmission line (SIW) is shown in Fig.3.5(b) along with the equivalent

circuit. So, the SIW in Fig.3.5(a) can be visualized as epsilon negative TL made of 15 unit cells. The unit cell parameters of epsilon negative TL are

$$Z = j\omega L_R \quad (3.26)$$

$$Y = j\omega C_R + \frac{1}{j\omega L_L} = j \frac{\left(\frac{\omega}{\omega_{sh}}\right)^2 - 1}{\omega L_L} \quad (3.27)$$

(a)

Figure 3.5 (a) SIW in Fig.3.1 with $w = 57.6$ mm, $d = 2.32$ mm, and $s = 3.6$ mm (b) unit cell of SIW and its equivalent circuit

The ABCD matrix of the unit cell is given as

$$\begin{aligned}
 \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} 1 & Z/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z/2 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 + \frac{ZY}{2} & Z \left(1 + \frac{ZY}{4} \right) \\ Y & 1 + \frac{ZY}{2} \end{bmatrix} \quad (3.28)
 \end{aligned}$$

The equation (3.28) above is same as (2.6) in chapter 2, except the Z in (3.28) is given by (3.26) whereas Z in (2.6) has series capacitor also. As in CRLH case the propagation constant is given as

The propagation constant γ can be given as [42],[37]

$$\cosh \gamma p = \frac{(A + D)}{2} \quad (3.29)$$

In the stop band (3.29) becomes,

$$\alpha = \frac{1}{p} \cosh^{-1} \left(\frac{A+D}{2} \right) \quad (3.30)$$

In the pass band (3.29) is converted as,

$$\beta = \frac{1}{p} \cos^{-1} \left(\frac{A+D}{2} \right) \quad (3.31)$$

Substituting (3.28) in (3.31)

$$\beta = \frac{1}{p} \cos^{-1} \left(1 + \frac{ZY}{2} \right) \quad (3.32)$$

Substituting the unit cell parameters for Z and Y in $1 + \frac{ZY}{2}$, which can be simplified as

$$\begin{aligned} 1 + \frac{ZY}{2} &= 1 + \frac{(j\omega L_R) \left(j\omega C_R + \frac{1}{j\omega L_L} \right)}{2} \\ &= 1 - \frac{1}{2} \left(\frac{\omega^2}{\omega_R^2} - \frac{L_R}{L_L} \right) \\ &= 1 - \frac{1}{2} \left(\frac{\omega^2}{\omega_R^2} - \frac{\omega_{sh}^2}{\omega_R^2} \right) \\ &= 1 - \frac{1}{2} \left(\frac{\omega^2 - \omega_{sh}^2}{\omega_R^2} \right) \end{aligned}$$

Where $\omega_R^2 = \frac{1}{L_R C_R}$ and $\omega_{sh}^2 = \frac{1}{L_L C_R}$.

So (3.32) becomes,

$$\beta = \frac{1}{p} \cos^{-1} \left(1 - \frac{1}{2} \left(\frac{\omega^2 - \omega_{sh}^2}{\omega_R^2} \right) \right) \quad (3.33)$$

At $\beta p = 0$,

$$1 - \frac{1}{2} \left(\frac{\omega_0^2 - \omega_{sh}^2}{\omega_R^2} \right) = 1$$

$$\omega_0^2 = \omega_{sh}^2$$

$$\omega_0 = \omega_{sh} = \frac{1}{\sqrt{L_L C_R}} \quad (3.34)$$

At $\beta p = \frac{\pi}{3}$,

$$1 - \frac{1}{2} \left(\frac{\left(\frac{\omega\pi}{3} \right)^2 - \omega_{sh}^2}{\omega_R^2} \right) = \frac{1}{2}$$

$$\frac{\left(\frac{\omega\pi}{3} \right)^2 - \omega_{sh}^2}{\omega_R^2} = 1$$

$$\left(\frac{\omega\pi}{3} \right)^2 = \omega_R^2 + \omega_{sh}^2 \quad (3.35)$$

Equation (3.34) is the cutoff frequency above which the propagation begins.

The parameter extraction is done by feeding the unit cell with microstrip transmission line and C_R is found out as 3 pF. The details about the parameter extraction is given in chapter 4. L_R and L_L values depend on the type of feed i.e. with taper ends or without taper. So, they cannot be extracted by method given in chapter 4. The SIW is simulated and S parameters obtained are converted to ABCD parameters and 15th root is taken to find the ABCD parameters of unit cell. The phase constant is found out using (3.31). From the phase constant, the cutoff frequency $\omega_0 = \omega_{sh}$ is found out. Since C_R is known, L_L can be found out from (3.34) as 4.18 nH. $\omega_{\frac{\pi}{3}}$ is found from the phase constant response as 6.5 GHz and it is used in (3.35) to find L_R . L_R is given as 0.2 nH. The phase constant obtained from 15th root of ABCD parameters and from unit cell

parameters are plotted in Fig. 3.6 along with phase constant from waveguide theory. The simulations are done by using CST Microwave Studio.

Figure 3.6 Phase constant by epsilon negative theory and waveguide theory

In order to prove SIW has negative epsilon at certain frequency band the effective epsilon of the SIW is extracted and shown in Fig.3.7. The effective epsilon extracted from the 15th root of ABCD parameter (obtained from the simulated S parameters) is compared with effective epsilon from the waveguide theory and effective epsilon obtained from the unit cell parameters (Z and Y in (3.26) and (3.27)). The effective epsilon is found out in waveguide theory by using the following equation given in [40]

$$\epsilon_{eff} = \epsilon_r \left(1 - \left(\frac{f_c}{f} \right)^2 \right)$$

where f_c is the cut off frequency of the SIW.

Figure 3.7 Effective epsilon of SIW

If the SIW in Fig.3.5(a) is converted to resonator by open circuiting its ends, then resonance occurs when the length of the resonator becomes half wavelength or multiples of it.

$$\theta_m = \beta_m l = M\pi \quad (3.36)$$

$$\beta_m l = \frac{1}{p} \cos^{-1} \left(1 - \frac{1}{2} \left(\frac{\omega^2 - \omega_{sh}^2}{\omega_R^2} \right) \right) \cdot l = M\pi$$

Where l is the length of epsilon negative transmission line, $l = N \cdot p$ and N is the number of unit cells and $M = 0, 1, 2, \dots, N - 1$.

$$1 - \frac{1}{2} \left(\frac{\omega_r^2 - \omega_{sh}^2}{\omega_R^2} \right) = \cos \left(\frac{M\pi}{N} \right)$$

$$\omega_r^2 - \omega_{sh}^2 = 2\omega_R^2 \left(1 - \cos \left(\frac{M\pi}{N} \right) \right)$$

$$\omega_r^2 = \omega_{sh}^2 + 2\omega_R^2 \left(1 - \cos \left(\frac{M\pi}{N} \right) \right)$$

So, the resonant frequencies are given by

$$f_r = \frac{1}{2\pi} \sqrt{\omega_{sh}^2 + 2\omega_R^2 \left(1 - \cos\left(\frac{M\pi}{N}\right)\right)} \quad (3.37)$$

The equation (3.37) can be used to find the resonant frequencies of modes of SIW open circuited resonator in Fig.3.4.

At mode $M = 0$, i.e zeroth order resonance, the resonance frequency is given as

$$f_{r0} = \frac{\omega_{sh}}{2\pi} \quad (3.38)$$

The resonant frequencies of modes of SIW open circuited resonator are calculated using (3.25) (waveguide theory) and (3.37) (epsilon negative theory) and the resonant frequencies of modes within 6 GHz are shown in Table 3.1. The resonant frequencies from both the theories are compared in [40] and [41]. But in these works, only first two resonant frequencies were compared, and these two resonant frequencies were used to design dual band antennas. In this chapter, the comparison is extended to higher order resonant frequencies.

From the Table 3.1 it seems the waveguide theory can find all the resonant frequencies of modes, but epsilon negative theory cannot find all the resonant frequencies. If the harmonics of the ω_{sh} are included in calculation of the resonant frequencies of modes in (3.37) then all the resonant frequencies of modes are found out by epsilon negative theory also. So, (3.37) is modified as

$$f_r = \frac{1}{2\pi} \sqrt{(q\omega_{sh})^2 + 2\omega_R^2 \left(1 - \cos\left(\frac{M\pi}{N}\right)\right)} \quad (3.39)$$

where $q = 1, 2, 3, \dots$. The harmonic order of ω_{sh} are given by $q - 1$.

Table 3.1 Comparison of resonant frequencies of modes of SIW open circuited resonator by waveguide theory and epsilon negative theory

Resonant frequencies of modes by waveguide theory (GHz) (modes TE_{mn0})	Resonant frequencies of modes by epsilon negative theory (GHz) (using (3.28))
1.4219 (TE_{010})	1.421 ($M=0, \omega_{sh}$)
2.0643 (TE_{110})	1.966 ($M=1, \omega_{sh}$)
2.8438 (TE_{020})	3.053 ($M=2, \omega_{sh}$)
3.2135 (TE_{120})	4.266 ($M=3, \omega_{sh}$)
3.3135 (TE_{210})	5.473 ($M=4, \omega_{sh}$)
4.1285 (TE_{220})	
4.2657 (TE_{030})	
4.5206 (TE_{130})	
4.7092 (TE_{310})	
5.2109 (TE_{230})	
5.3143 (TE_{320})	
5.6876 (TE_{040})	
5.8811 (TE_{140})	
6.1524 (TE_{410})	

Table 3.2 shows the comparison of the resonant frequencies of modes by using waveguide theory and epsilon negative theory after including the harmonics of ω_{sh}

below 6 GHz. But there is a slight difference between the resonant frequencies predicted by waveguide theory and epsilon negative theory which is due to the extraction of the epsilon negative theory parameters (C_R , L_L , and L_R) at single frequency ω_{sh} . But usually the parameters of epsilon negative theory vary over the entire bandwidth of the SIW.

Table 3.2 Comparison of resonant frequencies of modes of SIW open circuited resonator by waveguide theory and epsilon negative theory (including harmonics of ω_{sh})

Resonant frequencies of modes by waveguide theory (GHz) (modes TE_{mn0})	Resonant frequencies of modes by epsilon negative theory (GHz) (using (3.30))
1.4219 (TE_{010})	1.4213 ($M=0, \omega_{sh}$)
2.0643 (TE_{110})	1.9660 ($\omega_{sh}, M=1$)
2.8438 (TE_{020})	2.8425 ($M=0, 2\omega_{sh}$)
3.2135 (TE_{120})	3.0528 ($\omega_{sh}, M=2$)
3.3135 (TE_{210})	3.1504 ($2\omega_{sh}, M=1$)
4.1285 (TE_{220})	3.9217 ($2\omega_{sh}, M=2$)
4.2657 (TE_{030})	4.2638 ($M=0, 3\omega_{sh}$)
4.5206 (TE_{130})	4.2598 ($\omega_{sh}, M=3$)
4.7092 (TE_{310})	4.4748 ($3\omega_{sh}, M=1$)
5.2109 (TE_{230})	4.9199 ($2\omega_{sh}, M=3$)
5.3143 (TE_{320})	5.0477 ($3\omega_{sh}, M=2$)
5.6876 (TE_{040})	5.6850 ($M=0, 4\omega_{sh}$)
5.8811 (TE_{140})	5.4733 ($\omega_{sh}, M=4$)
6.1524 (TE_{410})	5.8450 ($4\omega_{sh}, M=1$)

Rogers RO4003C substrate with dielectric constant of 3.55, loss tangent of 0.0027 and thickness of 1.524mm is used to fabricate SIW open circuited resonator in Fig.3.4. The SIW open circuited resonator is fabricated and measured. The photograph of the fabricated resonator is shown in Fig.3.8 and simulated and measured S parameter results are shown in Fig.3.9.

Figure 3.8 Photograph of SIW resonator

Figure 3.9 Simulated and measured $|S_{11}|$ in dB of the SIW open circuited resonator

The simulations are done by using CST Microwave Studio. The measured and simulated S_{11} results match quite well except the resonant frequencies show a slight frequency shift which is due to the fabrication errors. There are eight resonant frequencies below 6 GHz. The number of resonant frequencies is higher than predicted by (3.37). The measured results show the resonant frequencies are at 1.4344 GHz (TE_{010}), 2.0198 GHz (TE_{110}), 3.1595 GHz (TE_{210}), 4.3079 GHz (TE_{030}), 4.4628 GHz (TE_{310}), 4.5709 GHz (TE_{130}), 5.1961 GHz (TE_{230}) and 5.8383 GHz (TE_{410}). The electric field distributions of these resonant frequencies are shown in Fig.3.10.

Figure 3.10 Electric field distributions of the resonant frequencies of SIW open circuited resonator (a) TE_{010} (b) TE_{110} (c) TE_{210} (d) TE_{030} (e) TE_{310} (f) TE_{130} (g) TE_{230} (h) TE_{410}

3.3 Design of SIW triple band antenna

The triple band antenna is formed from the SIW open circuited cavity resonator. The first band of the antenna is obtained by merging the modes TE_{210} and TE_{310} (Fig.3.10). The surface current of the mode TE_{310} is maximum at the center whereas the surface current is minimum at the center for the mode TE_{210} . The surface current distribution of both the modes are shown in Fig.3.11

Figure 3.11 Surface current distribution of modes (a) TE_{210} and (b) TE_{310}

Since the surface current is maximum at the center for mode TE_{310} , opening a vertical slot on top surface of SIW at center decreases the resonant frequency of the mode TE_{310} [135]. So, by increasing the length of this vertical slot shown in Fig.3.12(a), the resonant frequency of mode TE_{310} decreases continuously and finally it merges with mode TE_{210} . The resonant frequency of mode TE_{210} is less affected by this vertical slot since the surface current is minimum at the center.

The length of the slot 'L' is measured from the center of the of the SIW resonator (Fig.3.12(a)). The variation of the resonant frequencies of modes TE_{210} and TE_{310} with respect to the slot length is shown in Fig.3.12(b). It can be found out from Fig.3.12(b), the resonant frequency of mode TE_{210} is less affected by slot. After the slot length $L=19$ mm the two modes merge and the slot length is fixed at $L=20$ mm.

(a)

(b)

Figure 3.12 (a) SIW resonator with vertical slot (b) Resonant frequencies of modes TE₂₁₀ and TE₃₁₀ for different lengths 'L' of vertical slot.

The surface current distribution of the modes TE₀₃₀ and TE₂₃₀ are shown in Fig.3.13 and the surface current is maximum slightly above the feed in the horizontal direction for both the modes. So, a horizontal slot is opened at that location for the radiation from modes TE₀₃₀ and TE₂₃₀ and they form the second and third bands.

Figure 3.13 Surface current distribution of modes (a) TE_{030} (b) TE_{230}

The SIW resonator antenna is fabricated on rogers RO4003C with dielectric constant of 3.55, loss tangent of 0.0027 and thickness of 1.524mm. The diameter of the via holes are 2.32 mm and the distance between the via holes that form the electric walls on the sides of SIW is 3.6mm. The photograph of the fabricated antenna is shown in Fig.3.14. The horizontal slot is 14 mm in length and vertical slot is 20 mm in length both the slots are 1mm wide. The simulated and measured S_{11} responses are shown in Fig.3.15. The simulated and measured responses match well except there is a shift in resonant frequencies which is due to fabrication errors. The return losses at the central frequencies of the three bands are 24.6 dB (3.3 GHz), 34.8 dB (4 GHz) and 31.6 dB (4.9 GHz). The 10dB impedance bandwidths at the three bands are 1.49% (3.3 GHz band), 0.72% (4 GHz band) and 0.56% (4.9 GHz band).

Figure 3.14 Photograph of the fabricated antenna

Figure 3.15 Simulated and measured S_{11} of the antenna

The measured realized gains of the antenna are 5.81 dBi, 4 dBi and 2.43 dBi respectively at 3.3 GHz, 4 GHz and 4.9 GHz. The simulated three dimensional radiation pattern of the antenna at the three bands are given in Fig.3.16. The simulations are done by using CST Microwave Studio. The simulated and measured radiation patterns of the antenna in the planes $\Phi = 0^\circ$ and $\Phi = 90^\circ$ are shown in Fig.3.17.

(a)

(b)

(c)

Figure 3.16 Three dimensional radiation pattern of the antenna at (a) first band (b) second band (c) Third band

(a)

(b)

(c)

Figure 3.17 Measured and simulated radiation pattern of the antenna (a) First band (b) Second band (c) Third band

The comparison of the designed SIW triple band antenna with that of the other that of previously reported SIW antennas is given in Table 3.3. The antenna has triple band compared to the other antennas which have either single or dual bands. The fractional bandwidth of the antenna in the first band is higher than the other two bands. The three bands of the antenna are based on higher order resonant modes and are achieved by using only two slots. Also, the size of the antenna is comparable with that of the previously reported antennas.

Table 3.3 Comparison of the designed SIW antenna with that of previously reported antennas

	Number of bands	Central frequencies (GHz)	Antenna foot print λ_0	Gain (dBi)	Bandwidth %	Radiation Efficiency %
This work	3	3.3, 4, 4.9	0.847×0.77	5.81, 4, 2.43	1.42, 0.72, 0.56	75.7, 55.87, 36.9
[108]	1	10	0.66×0.79	5.4	1.7	86
[110]	2	9.5, 10.5	0.57×0.51 (cavity only)	$\sim 5.8, \sim 5.9$	1.8, 2.1	Not mentioned
[111]	2	10, 12.5	0.83×0.67	1.726, 7.039	Not mentioned	75.75, 84.88
[40]	2	10.05, 17.2	0.35×0.2	4.6, 7.92	Not mentioned	82, 92
[41]	1 (Open resonance) 1 (Short resonance)	8.46 (Open resonance) 8.38 (Short resonance)	0.42×0.85 (Open resonance) 0.42×0.45 (Short resonance)	6.2 (Open resonance) 3.5 (Short resonance)	2.7 (Open resonance) 5.9 (Short resonance)	Not mentioned

3.4 Conclusion

Substrate integrated waveguide has been analyzed based on waveguide theory and epsilon negative (ENG) transmission line theory. The phase constant obtained by both the methods are plotted together for comparison. The open circuited SIW resonator is analyzed and the resonant frequencies of the resonators using both the theories are found out. Various modes and resonant frequencies of the SIW resonator are discussed. The SIW resonator is fabricated and the simulated and measured S parameter responses

are found out. The triple band antenna is designed by using two slots on the top of SIW. The antenna is designed to operate at 3.3 GHz, 4 GHz and 4.9 GHz. The first band is formed by merging of modes TE_{210} and TE_{310} . The second and third bands are formed from the modes TE_{030} and TE_{230} respectively. This chapter gave the introduction to design of single negative (epsilon negative) transmission line and its application in the design of multi band circuits (triple band antenna). In the next two chapters, the double negative metamaterial (Double Periodic CRLH in this case) is discussed and its applications in multi band circuits are verified.

Chapter 4

Design and analysis of DPCRLH structure and design of triple band filter

In this chapter the symmetric unit cells of Double Periodic Composite Right and Left Handed (DPCRLH) structure are introduced. Even though DPCRLH was introduced few years back, the symmetric cells of DPCRLH have not been proposed so far. The symmetric unit cell has an advantage over the asymmetric one, the former has equal impedances on both input and output ends. As already mentioned in Chapter 2, DPCRLH unit cell is formed by concatenation of two unit cells of CRLH. The two symmetric unit cells of DPCRLH are proposed and they are analyzed based on ABCD transmission line parameters. The dispersion diagram and the important resonant frequency points in it are analyzed in terms of the unit cell parameters of DPCRLH. The

period of the unit cell is chosen to be much smaller than the guided wavelength to maintain the homogeneity condition. The formation of DPCLRH open circuited resonator from DPCRLH unit cell and design of triple band filter using DPCRLH resonator are presented next to prove DPCRLH unit cell can be used in the design of multi band circuits. The relation between resonant points of the DPCRLH open circuited resonator and DPCRLH unit cell parameters are discussed. The triple band filter is designed with different fractional bandwidths on three bands. The filter is of order three and designed with the help of coupling coefficient between the resonators and external quality factor.

Figure 4.1 DPCRLH unit cell

4.1 Symmetric unit cells of DPCRLH and their analysis

DPCRLH unit cell with period 'd' is shown in Fig.4.1. The parameters of DPCRLH unit cell in Fig.4.1 are given by the following equations

$$Z_1 = j\omega L_{R1} + \frac{1}{j\omega C_{L1}} \quad (4.1)$$

$$Z_2 = j\omega L_{R2} + \frac{1}{j\omega C_{L2}} \quad (4.2)$$

$$Y_1 = j\omega C_{R1} + \frac{1}{j\omega L_{L1}} \quad (4.3)$$

$$Y_2 = j\omega C_{R2} + \frac{1}{j\omega L_{L2}} \quad (4.4)$$

The impedances seen from the two sides of unit cell in Fig.4.1 are not same. The symmetric unit cells can be formed from the DPCRLH unit cell in Fig.4.1. in two ways. First one is by considering two $Z_1/2$ series arms on either side of the unit cell instead of one Z_1 as in Fig.4.1 and making $Y_1=Y_2$. Another method is by splitting Y_2 into two $Y_2/2$ shunt arms and positioning them on the two sides of the sides of the unit cell in Fig.4.1 and equating $Z_1=Z_2$. The symmetric cells of DPCRLH are shown in Fig.4.2.

(a)

(b)

Figure 4.2 DPCRLH symmetric unit cells

The symmetric unit cells are analyzed by using ABCD parameters. The analysis of the two unit cells in Fig.4.2 are similar. So, the detailed analysis of unit cell in Fig.

4.2(a) is given here and brief analysis of unit cell in Fig.4.2(b) is given in section 4.1.2. There are two stages in the analysis of unit cell. They provide the dispersion relation in terms of element values of unit cell and analysis of important resonant frequency points in dispersion diagram and closing condition of attenuation gap at $\beta d = 0$ in the dispersion diagram. ABCD matrix of the DPCRLH unit cell shown in Fig.4.2(a) is given as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{Y_2}{2} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ \frac{Y_2}{2} & 1 \end{bmatrix} \quad (4.5)$$

Simplifying (4.5) gives,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} & 2Z_1 + Z_1^2 Y_1 \\ Y_1 + Y_2 + Z_1 Y_1 Y_2 + \frac{Z_1 Y_2^2}{2} + \frac{Z_1^2 Y_1 Y_2^2}{4} & 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} \end{bmatrix} \quad (4.6)$$

The propagation constant is given as $\cosh \gamma d = \frac{A+D}{2}$ [42]. In pass band, if the attenuation constant is assumed to be zero, then the phase constant is given as $\cos \beta d = \frac{A+D}{2}$ [42]. The values of A and D from (4.6) are substituted to get the phase constant as,

$$\cos \beta d = \frac{A + D}{2} = 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} \quad (4.7)$$

The above equation is the dispersion relation that can be used to draw the dispersion diagram in terms of the parameters of the unit cell. The dispersion diagram of a DPCRLH unit cell is shown in Fig.4.3.

The points $\beta d = 0$ and $\beta d = \pi$ are plotted in the dispersion diagram given in Fig.4.3. These points are considered important because the behaviour of DPCRLH unit cell can be changed by adjusting these points. By adjusting the parameters of DPCRLH

unit cell, the frequency points $\beta d = 0$ and $\beta d = \pi$ are modified which change the shape of the dispersion diagram.

The $\beta d = 0$ points are found out by substituting $\beta d = 0$ in (4.7) which gives

$$Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} = 0 \quad (4.8)$$

Equation (4.8) can also be written as

$$Z_1 \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0 \quad (4.9)$$

Figure 4.3 Dispersion diagram of a DPCRLH unit cell

From (4.9) $Z_1 = 0$ is one of solutions of (4.8) which gives

$$\omega_0^2 = \frac{1}{L_{R1} C_{L1}} \quad (4.10)$$

The other solutions of (4.8) are given by

$$Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} = 0 \quad (4.11)$$

From (4.11), if at some frequency say ω_0 , $Z_1 = 0$ and $Y_1 + Y_2 = 0$ then the attenuation gap between two $\beta d = 0$ points in the dispersion diagram in Fig.4.3 will be closed. Mathematically,

$$Z_1(at \omega_0) = Y_1 + Y_2(at \omega_0) = 0 \quad (4.12)$$

Using (4.1), (4.3) and (4.4) in (4.12), ω_0 is given as

$$\omega_0^2 = \frac{1}{L_{R1}C_{L1}} = \frac{1}{(C_{R1} + C_{R2})\left(\frac{1}{L_{L1}} + \frac{1}{L_{L2}}\right)^{-1}} \quad (4.13)$$

So, the closing condition of attenuation gap of $\beta d = 0$ in the dispersion diagram is given as

$$\frac{1}{L_{R1}C_{L1}} = \frac{1}{(C_{R1} + C_{R2})\left(\frac{1}{L_{L1}} + \frac{1}{L_{L2}}\right)^{-1}} \quad (4.14)$$

If the DPCRLH unit cell satisfies the condition in (4.14), the bandwidth becomes wider and also this wider bandwidth is obtained without attenuation in between two $\beta d = 0$ frequency points, so the DPCRLH unit cell can be used as wide band filter also.

Expanding (4.11) in terms of parameters of unit cell, i.e. substituting (4.1), (4.3) and (4.4) in (4.11), gives the following sixth order equation

$$\begin{aligned} \omega^6 L_{R1} C_{R1} C_{R2} - \omega^4 \left(2C_{R1} + 2C_{R2} + \frac{L_{R1}C_{R2}}{L_{L1}} + \frac{C_{R1}C_{R2}}{C_{L1}} + \frac{L_{R1}C_{R1}}{L_{L2}} \right) \\ + \omega^2 \left(\frac{2}{L_{L1}} + \frac{2}{L_{L2}} + \frac{C_{R2}}{C_{L1}L_{L1}} + \frac{L_{R1}}{L_{L1}L_{L2}} + \frac{C_{R1}}{L_{L2}C_{L1}} \right) - \frac{1}{L_{L1}L_{L2}C_{L1}} \\ = 0 \end{aligned} \quad (4.15)$$

Equation (4.10) and solutions of (4.15) are the solutions of (4.8). Equation (4.15) gives 3 positive roots and 3 negative roots. Out of the 3 positive roots one is same as positive

root of $Z_1 = 0$, if (4.14) is satisfied and other two positive roots of (4.15) give the cut-off frequencies.

The other category of important points in dispersion diagram are $\beta d = \pi$. These points are found out by substituting $\beta d = \pi$, $\cos \beta d = -1$ in (4.7) gives

$$Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} = -2 \quad (4.16)$$

Factorizing (4.16) involves the following steps

$$Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} + 2 = 0$$

$$2Z_1 Y_1 + 2Z_1 Y_2 + Z_1^2 Y_1 Y_2 + 4 = 0$$

$$Z_1 Y_1 (2 + Z_1 Y_2) + 2Z_1 Y_2 + 4 = 0$$

$$Z_1 Y_1 (2 + Z_1 Y_2) + 2(Z_1 Y_2 + 2) = 0$$

$$2(Z_1 Y_1 + 2)(Z_1 Y_2 + 2) = 0$$

So, the solutions of (4.16) are found out as

$$Z_1 Y_1 = -2 \text{ and } Z_1 Y_2 = -2 \quad (4.17)$$

Expanding $Z_1 Y_1 = -2$ and $Z_1 Y_2 = -2$ in terms of parameters of unit cell gives

$$\omega^4 L_{R1} C_{R1} - \omega^2 \left(\frac{L_{R1}}{L_{L1}} + \frac{C_{R1}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L1}} = 0 \quad (4.18)$$

$$\omega^4 L_{R2} C_{R2} - \omega^2 \left(\frac{L_{R2}}{L_{L2}} + \frac{C_{R2}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L2}} = 0 \quad (4.19)$$

Each of the equations (4.18) and (4.19) has two positive and two negative roots. So, total of four positive roots from (4.18) and (4.19) gives $\beta d = \pi$ points in the dispersion diagram.

4.1.1 Bloch impedance

The Bloch impedance can be found out from ABCD parameters using the following equation [37, 42]

$$Z_{B\pm} = \frac{-2B}{A - D \mp \sqrt{(A + D)^2 - 4}} = \frac{(A - D) \pm \sqrt{(A + D)^2 - 4}}{2C} \quad (4.20)$$

For the symmetric unit cells $A = D$, so (4.20) becomes,

$$Z_{B\pm} = \frac{\pm B}{\sqrt{A^2 - 1}} = \frac{\pm \sqrt{D^2 - 1}}{C} \quad (4.21)$$

For DPCRLH case substituting values from (4.6) in (4.21) gives,

$$\begin{aligned} Z_{B\pm} &= \frac{\pm(2Z_1 + Z_1^2 Y_1)}{\sqrt{\left(1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2}\right)^2 - 1}} \\ &= \frac{\pm \sqrt{\left(1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2}\right)^2 - 1}}{Y_1 + Y_2 + Z_1 Y_1 Y_2 + \frac{Z_1 Y_2^2}{2} + \frac{Z_1^2 Y_1 Y_2^2}{4}} \end{aligned} \quad (4.22)$$

4.1.2 Analysis of symmetric unit cell B

ABCD matrix of the symmetric cell in Fig.4.2(b) can be found out in the same way as followed to get (4.6) and it is given below

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & \frac{Z_1}{2} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & \frac{Z_1}{2} \\ 0 & 1 \end{bmatrix} \quad (4.23)$$

On simplifying (4.23),

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + Z_1 Y_1 + Z_2 Y_1 + \frac{Z_1 Z_2 Y_1^2}{2} & Z_1 + Z_2 + Z_1 Z_2 Y_1 + \frac{Z_1^2 Y_1}{2} + \frac{Z_1^2 Z_2 Y_1^2}{4} \\ 2Y_1 + Z_2 Y_1^2 & 1 + Z_1 Y_1 + Z_2 Y_1 + \frac{Z_1 Z_2 Y_1^2}{2} \end{bmatrix} \quad (4.24)$$

ABCD matrix in (4.24) can be obtained from (4.6) by replacing Z_1 by Y_1 and replacing Y_1 and Y_2 by Z_2 and Z_1 respectively and taking the transpose. The analysis of the symmetric cell in Fig.4.2(b) is similar to the analysis of DPCRLH unit cell in Fig.4.2(a).

$$\cos \beta d = \frac{A + D}{2} = 1 + Z_1 Y_1 + Z_2 Y_1 + \frac{Z_1 Z_2 Y_1^2}{2} \quad (4.25)$$

At $\beta d = 0$,

$$Z_1 Y_1 + Z_2 Y_1 + \frac{Z_1 Z_2 Y_1^2}{2} = 0 \quad (4.26)$$

The solutions of (4.26) are

$$Y_1 = 0, \text{ and } Z_1 + Z_2 + \frac{Z_1 Z_2 Y_1}{2} = 0 \quad (4.27)$$

At $\beta d = \pi$,

$$Z_1 Y_1 + Z_2 Y_1 + \frac{Z_1 Z_2 Y_1^2}{2} = -2 \quad (4.28)$$

The solutions of (4.28) are

$$Z_1 Y_1 = -2, \quad \text{and} \quad Z_2 Y_1 = -2 \quad (4.29)$$

As in the case of ABCD matrix in (4.24), the equations from (4.25) to (4.29) which are analysis equations of DPCRLH unit cell B (Fig.4.2(b)) can be directly obtained from analysis equations of DPCRLH unit cell A (Fig.4.2(a)) by replacing Z_1 by Y_1 and replacing Y_1 and Y_2 by Z_2 and Z_1 respectively.

4.2 Self-resonant frequency of interdigital capacitor

The closing condition of the attenuation gap at $\beta d = 0$ increases the bandwidth of the DPCRLH structure. If the bandwidth increases, the self-resonant frequencies of interdigital capacitor (microstrip as host medium) which is used to implement DPCRLH structure affects the response. Fig.4.4 shows the structure of interdigital capacitor and its simulated response affected by its own self resonant frequencies. The simulation is done by using CST Microwave Studio.

(a)

(b)

Figure 4.4 (a) Interdigital capacitor (b) S parameters response of the interdigital capacitor.

Self-resonant frequencies cannot be removed but can be shifted to higher frequencies by connecting the alternate fingers of interdigital capacitors. This can be done either by using wire bonding [136, 137] or by using via holes and defected ground structure [138]. Here the interdigital capacitor is designed based on via holes and defected ground structure. Fig.4.5 shows interdigital capacitor with via holes and defected ground structure to connect alternate fingers of the interdigital capacitor.

(a)

(b)

Figure 4.5 Interdigital capacitor with via and defected ground

The response of interdigital capacitor in Fig.4.5 is shown in Fig.4.6. The self-resonance frequency is moved from 8 GHz (in Fig.4.4 (b)) to 11 GHz (in Fig.4.6) by using via and defected ground structure to connect the alternate fingers of interdigital capacitor.

Figure 4.6 S-parameters response of interdigital capacitor in Fig.4.7

4.3 Parameter Extraction for Microstrip implementation

DPCRLH structure is implemented by using interdigital capacitor and short circuited stub. In this chapter Microstrip is used as the host medium. The equivalent circuits of the interdigital capacitor and the short circuited stubs are shown in Fig.4.7. The simplified equivalent circuits of the interdigital capacitor and short circuited stub are the π and T networks respectively. These π and T networks are shown in Fig.4.8 which are used for derivation purpose only.

(a)

(b)

Figure 4.7 Equivalent circuits (a) Interdigital capacitor (b) Short circuited stub

(a)

(b)

Figure 4.8 (a) π network of interdigital capacitor (b) T network of short circuit stub

The Y matrix of the π network is given as

$$[Y_{\pi}^{ic}] = \begin{bmatrix} Y_{11}^{ic} & Y_{12}^{ic} \\ Y_{21}^{ic} & Y_{22}^{ic} \end{bmatrix} = \begin{bmatrix} Y^{ic} + \frac{1}{Z^{ic}} & -\frac{1}{Z^{ic}} \\ -\frac{1}{Z^{ic}} & Y^{ic} + \frac{1}{Z^{ic}} \end{bmatrix} \quad (4.30)$$

Where $Z^{ic} = j[\omega L_s^{ic} - \frac{1}{\omega C_p^{ic}}]$ and $Y^{ic} = j\omega C_p^{ic}$

The Z matrix of the T network is given as

$$[Z_T^{si}] = \begin{bmatrix} Z_{11}^{si} & Z_{12}^{si} \\ Z_{21}^{si} & Z_{22}^{si} \end{bmatrix} = \begin{bmatrix} Z^{si} + \frac{1}{Y^{si}} & \frac{1}{Y^{si}} \\ \frac{1}{Y^{si}} & Z^{si} + \frac{1}{Y^{si}} \end{bmatrix} \quad (4.31)$$

Where $Y^{si} = j[\omega C_p^{si} - \frac{1}{\omega L_s^{si}}]$ and $Z^{si} = j\omega L_s^{si}$

Adding Y_{11}^{ic} and Y_{21}^{ic} gives Y^{ic} , from which C_p^{ic} can be found out. Similarly subtracting Z_{21}^{si} from Z_{11}^{si} gives Z^{si} from which L_s^{si} can be found out. Z^{ic} and Y^{si} each has two parameters, so finding the other four parameters requires taking partial derivation of $\frac{1}{Y_{21}^{ic}}$

(for L_s^{ic} and C_s^{ic}) and $\frac{1}{Z_{21}^{si}}$ (for C_p^{si} and L_p^{si}) with respect to ω and simplifying. The

equivalent circuit parameters of interdigital capacitor are given as

$$C_p^{ic} = \frac{Y_{11}^{ic} + Y_{21}^{ic}}{j\omega} \quad (4.32)$$

$$L_s^{ic} = \frac{j}{2\omega} \left[\omega \frac{\partial \left(\frac{1}{Y_{21}^{ic}} \right)}{\partial \omega} + \frac{1}{Y_{21}^{ic}} \right] \quad (4.33)$$

$$C_s^{ic} = \frac{2}{j\omega} \left[\omega \frac{\partial \left(\frac{1}{Y_{21}^{ic}} \right)}{\partial \omega} - \frac{1}{Y_{21}^{ic}} \right]^{-1} \quad (4.34)$$

The equivalent circuit parameters of short circuited stub are given as

$$L_s^{si} = \frac{Z_{11}^{si} - Z_{21}^{si}}{j\omega} \quad (4.35)$$

$$C_p^{si} = \frac{1}{2j\omega} \left[\omega \frac{\partial \left(\frac{1}{Z_{21}^{si}} \right)}{\partial \omega} + \frac{1}{Z_{21}^{si}} \right] \quad (4.36)$$

$$L_p^{si} = \frac{2j}{\omega} \left[\omega \frac{\partial \left(\frac{1}{Z_{21}^{si}} \right)}{\partial \omega} - \frac{1}{Z_{21}^{si}} \right]^{-1} \quad (4.37)$$

The above derivation is for the ideal lossless case but in reality, the interdigital capacitor and short circuited stub have some losses associated with it. So, the equivalent circuits of the interdigital capacitor and short circuited stub in Fig.4.7 are modified as in Fig.4.9.

Figure 4.9 Equivalent circuits with losses of (a) interdigital capacitor (b) short circuited stub

There is no change in π and T networks shown in Fig.4.8 and also there is no change in the Y matrix (4.30) of π network and Z matrix of T network (4.31) but their elements Z^{ic} and Y^{ic} and Z^{si} and Y^{si} are modified. The Z^{ic} and Y^{ic} used in the Y matrix of π network in (4.30) are modified as

$$Z^{ic} = R_s^{ic} + j[\omega L_s^{ic} - \frac{1}{\omega C_s^{ic}}] \quad \text{and} \quad Y^{ic} = G_p^{ic} + j\omega C_p^{ic}$$

Z^{si} and Y^{si} used in the Z matrix of T network in (4.31) are changed as below

$$Y^{si} = G_p^{si} + j[\omega C_p^{si} - \frac{1}{\omega L_p^{si}}] \quad \text{and} \quad Z^{si} = R_s^{si} + j\omega L_s^{si}$$

Following the same procedure, the equivalent circuit parameters of interdigital capacitor are given as

$$R_s^{ic} = \Re \left(\frac{-1}{Y_{21}^{ic}} \right) \quad (4.38)$$

$$G_p^{ic} = \Re(Y_{11}^{ic} + Y_{21}^{ic}) \quad (4.39)$$

$$C_p^{ic} = \frac{1}{\omega} \Im(Y_{11}^{ic} + Y_{21}^{ic}) \quad (4.40)$$

$$L_s^{ic} = \frac{j}{2\omega} \left[\omega \frac{\partial \left(\frac{1}{Y_{21}^{ic}} \right)}{\partial \omega} + j \Im \left(\frac{1}{Y_{21}^{ic}} \right) \right] \quad (4.41)$$

$$C_s^{ic} = \frac{2}{j\omega} \left[\omega \frac{\partial \left(\frac{1}{Y_{21}^{ic}} \right)}{\partial \omega} - j \Im \left(\frac{1}{Y_{21}^{ic}} \right) \right]^{-1} \quad (4.42)$$

The equivalent circuit parameters of short circuited stub are given as

$$G_p^{si} = \Re \left(\frac{1}{Z_{21}^{si}} \right) \quad (4.43)$$

$$R_s^{si} = \Re(Z_{11}^{si} - Z_{21}^{si}) \quad (4.44)$$

$$L_s^{si} = \frac{1}{\omega} \Im(Z_{11}^{si} - Z_{21}^{si}) \quad (4.45)$$

$$C_p^{si} = \frac{1}{2j\omega} \left[\omega \frac{\partial \left(\frac{1}{Z_{21}^{si}} \right)}{\partial \omega} + j \Im \left(\frac{1}{Z_{21}^{si}} \right) \right] \quad (4.46)$$

$$L_p^{si} = \frac{2j}{\omega} \left[\omega \frac{\partial \left(\frac{1}{Z_{21}^{si}} \right)}{\partial \omega} - j \Im \left(\frac{1}{Z_{21}^{si}} \right) \right]^{-1} \quad (4.47)$$

The left and right handed parameters of series combination of interdigital capacitor and short circuited stub are related to equivalent circuit parameters by

$$L_R = L_s^{ic} \quad (4.48)$$

$$C_R = C_p^{si} + 2C_p^{ic} \quad (4.49)$$

$$L_L = L_p^{si} \quad (4.50)$$

$$C_L = C_s^{ic} \quad (4.51)$$

So, for finding the left and right handed parameters of series combination of interdigital capacitor and short circuited stub first the interdigital capacitor and short circuited stub are simulated separately and their S parameters are found out. S parameters of interdigital capacitor are converted to Y parameters to find C_p^{ic} , L_s^{ic} , and C_s^{ic} and Z parameters are found out from the S parameters of short circuited to find L_s^{si} , C_p^{si} , and L_p^{si} .

The variations of L_s^{ic} , C_s^{ic} and C_p^{ic} with respect to the length and width of the fingers and with respect to the spacing between the fingers are shown in Fig.4.10. From the Fig.4.10(a), it can be found out if the length of the fingers of interdigital capacitor increases the two parameters L_s^{ic} and C_s^{ic} increases. If the space between the fingers of interdigital capacitor increases L_s^{ic} increases but C_s^{ic} decreases (Fig.4.10(b)). But this reverses in Fig.4.10(c) where L_s^{ic} decreases and C_s^{ic} increases if the width of the fingers of interdigital capacitor increases. In all the three cases the value of C_p^{ic} increases as presented in Fig.4.10. The variations of L_s^{si} , C_p^{si} , and L_p^{si} with respect to the length of the Short circuited stub for different widths of it are shown in Fig.4.11. The value of C_p^{si} increases if both length and the width of short circuited stub increases as evident from Fig.4.11 (a). It can be seen from the Fig.4.11(b) the value of L_p^{si} increases with length and decreases with the width of the interdigital capacitor. From Fig. 4.11(c), it can be found out the value of L_s^{si} is very low so it is not included in the calculation of the final

right and left handed parameters from (4.48) to (4.51). Also, the value of L_s^{si} remains constant with length but its value increases with width of the short circuited stub.

(a)

(b)

(c)

Figure 4.10 Variations of L_s^{ic} , C_s^{ic} and C_p^{ic} with respect to (a) length of fingers of interdigital capacitor (b) space between the fingers of the interdigital capacitor (c) width of fingers of the interdigital capacitor

(a)

(b)

(c)

Figure 4.11 Variations of (a) C_p^{si} (b) L_p^{si} (c) L_s^{si} with respect to length of short circuited stub for width of short circuited stub w_s from 0.4 mm to 1.2 mm

The variations of the lossy elements R_s^{ic} and G_p^{ic} of interdigital capacitor with respect to length of the fingers, space between the fingers and width of the fingers of the interdigital capacitor are shown in Fig.4.12. For the short circuited stub, the variations of its lossy parameters G_p^{si} and R_s^{si} with respect to the length of the short circuited stub for its different width are presented in Fig.4.13. From the Fig.4.12 and Fig. 4.13 it can be found out, the values of these lossy parameters are very less and they can be

neglected. So, the DPCRLH unit cell designed using interdigital capacitor and short circuited stub will be similar to the symmetric unit cells shown in Fig.4.2. CST Microwave studio is used in the simulations above. The substrate used is Rogers RO4003C with dielectric constant of 3.55, loss tangent of 0.0027 and thickness of 1.524 mm.

(a)

(b)

(c)

Figure 4.12 Variations of R_s^{ic} and G_p^{ic} with respect to (a) length of fingers of interdigital capacitor (b) space between the fingers of the interdigital capacitor (c) width of fingers of the interdigital capacitor

(a)

(b)

Figure 4.13 Variations of G_p^{si} and R_s^{si} with respect to length of short circuited stub for width of short circuited stub w_s from 0.4 mm to 1.2 mm

4.4 DPCRLH unit cell design

The interdigital capacitor and short circuited stub are simulated separately and the extracted parameter values are substituted in (4.14) to check closing condition of the attenuation gap and the interdigital capacitor and short circuited stub are changed until the extracted values satisfies the condition in (4.14). This type of designing gives importance to the closing condition in (4.14). Another way of designing the DPCRLH unit cell is to fix the frequency at which the condition in (4.14) is satisfied and all $\beta d = 0$ and $\beta d = \pi$ frequency points are fixed according to the application requirement and then the parameter values are calculated. The interdigital capacitor and short circuited stub are designed based on these parameter values.

In this chapter, DPCRLH unit cell is designed with the parameters $L_{R1}=1.957$ nH, $C_{L1}=1.048$ pF, $L_{L1}=1.907$ nH, $C_{R1}=1.2245$ pF, $L_{L2}=1.185$ nH and $C_{R2}=1.58$ pF. The above parameters are substituted in (4.14) to check whether the unit cell with above

parameters satisfies the closing condition of attenuation gap between $\beta d = 0$ points or not.

$$\frac{1}{L_{R1}C_{L1}} = \frac{1}{(C_{R1} + C_{R2})\left(\frac{1}{L_{L1}} + \frac{1}{L_{L2}}\right)^{-1}} = 4.88 \times 10^{20} \text{ rad}^2 \quad (4.52)$$

Therefore, the unit cell parameters satisfy the closing condition of attenuation gap between two $\beta d = 0$ points in the dispersion diagram. DPCRLH unit cell is implemented in microstrip form. Rogers RO4003C substrate having dielectric constant of 3.55, loss tangent of 0.0027 and thickness of 1.524 mm is used to fabricate DPCRLH unit cell. Fig.4.14 shows the photograph of top and bottom layers of DPCRLH unit cell.

(a)

(b)

Figure 4.14 Photograph of DPCRLH unit cell

The simulated and measured S parameters response of the DPCRLH are shown in Fig.4.15. The simulated and measures responses match well. As mentioned earlier, the DPCRLH unit cell covers a wide bandwidth from 2.5 GHz to 9 GHz. The measured insertion loss varies from 0.1 dB to 0.8 dB over frequency range of 2.5 GHz to 8 GHz and it is approximately 1.5 dB from 8 GHz to 9 GHz.

The dispersion diagram obtained by substituting the DPCRLH unit cell parameters in (4.7) is compared with simulated and measured dispersion diagram and plotted in Fig.4.16. As seen from Fig.4.16 the dispersion diagram is closed and two $\beta d = 0$ points are merged to a single point at 3.39 GHz. So, there is no attenuation gap around 3.39 GHz. The dispersion diagram shows DPCRLH alternates between the left and right handed regions twice within the operational bandwidth of DPCRLH.

Figure 4.15 S parameters responses of the DPCRLH unit cell

Figure 4.16 Dispersion diagram of DPCRLH unit cell

At the point $\beta d = 0$ the wavelength becomes infinite, so there should not be any variation in electric field distribution over DPCRLH unit cell. This is shown in Fig.4.17 where it is evident, the electric field distribution at 3.39 GHz ($\beta d = 0$) is uniform over the entire unit cell. CST Microwave Studio is used to obtain the simulated

results shown in Fig.4.15 to Fig 4.17 and also used to obtain the rest of the simulations in this chapter.

Figure 4.17 Simulated electric field distribution at 3.39 GHz ($\beta d = 0$)

4.5 DPCRLH open circuit resonator

A DPCRLH open circuited resonator is shown in Fig.4.18. It is formed from symmetric DPCRLH unit cell in Fig.4.2(a) by open circuiting the ends.

Figure 4.18 DPCRLH open circuited resonator from Fig.4.2 (a)

The input impedance of the DPCRLH open circuited resonator in Fig.4.18 is given below

$$Z_{in} = \frac{2}{Y_2} // \left(Z_1 + \left(\left(Z_1 + \frac{2}{Y_2} \right) // \frac{1}{Y_1} \right) \right) \quad (4.53)$$

On simplifying (4.53),

$$Z_{in} = \frac{2(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)}{Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2} \quad (4.54)$$

At resonance

$$Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2 = 0 \quad (4.55)$$

Equation (4.55) is $4C=0$ where C is the C parameter of ABCD matrix in (4.6). Equation (4.55) can be further simplified as

$$4 \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) = 0 \quad (4.56)$$

From (4.56) it is found out; the resonant points are two of the four $\beta d = \pi$ points in the dispersion diagram which are represented in (4.56) as $(Z_1 Y_2 + 2) = 0$ and three of the four $\beta d = 0$ points in the dispersion diagram are represented as $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0$ in (4.56). So, there are total of five resonant points in this case.

Figure 4.19 Absolute value of Z_{in} of open circuited resonator versus frequency

Using DPCRLH unit cell parameters parameters $L_{R1}=1.957$ nH, $C_{L1}=1.048$ pF, $L_{L1}=1.907$ nH, $C_{R1}=1.2245$ pF, $L_{L2}=1.185$ nH and $C_{R2}=1.58$ pF, in (4.54), Z_{in} of the DPCRL open circuited resonator in Fig.4.18 can be found out. The graph of absolute value of Z_{in} versus frequency is shown in Fig.4.19. The resonant points are at frequencies as 1.57 GHz, 2.1 GHz, 3.5145 GHz, 6.15 GHz, and 7.71 GHz.

4.6 Design of DPCRLH open circuit resonator

The DPCRLH unit cell with the parameters $L_{R1}=1.957$ nH, $C_{L1}=1.048$ pF, $L_{L1}=1.907$ nH, $C_{R1}=1.2245$ pF, $L_{L2}=1.185$ nH and $C_{R2}=1.58$ pF is designed and experimentally verified in section 4.4. So, the unit cell reported in section 4.4 is open circuited to form open circuited DPCRLH resonator. Fig.4.20 shows the microstrip implementation of DPCRLH open circuited resonator along the dimensions. The photograph of the fabricated DPCRLH open circuited resonator is shown in Fig.4.21.

(a)

(b)

Figure 4.20 DPCRLH open circuit resonator(a) Top side with $L_1=7.55\text{mm}$, $L_2=6.15\text{mm}$, $W_1=0.4\text{mm}$, $W_2=0.4\text{mm}$, $L=2.4$, $W=0.4\text{mm}$, $S=0.1\text{mm}$, and $w_i=0.8\text{mm}$ (b) Bottom slot with $L_s=5.2\text{mm}$, $W_s=1.2\text{mm}$, $S_1=0.3\text{mm}$, and $S_2=0.5\text{mm}$. All via holes diameter= 0.24mm .

(a)

(b)

Figure 4.21 Photograph of the fabricated open circuited resonator

Figure 4.22 S parameters response of open circuited DPCRLH resonator

The measured and simulated S parameters match well that can be found out from Fig.4.22. The resonant points from Fig.4.19 are around 1.574 GHz, 2.05 GHz, 3.24 GHz, 6 GHz, and 9 GHz. Comparing Fig.4.19 and Fig.4.22 it is found out there is a shift in last resonant point from 7.71 GHz in the former to 9 GHz in the later. The other resonant points in Fig.4.22 approximately match with the resonant points in Fig.4.19. The reason for the shift in last resonant point is due to the non-uniformity of DPCRLH parameters over wide frequency range.

4.7 Design of Triple band filter

The open circuited DPCRLH resonator in Fig.4.20 is used to design the triple band filter. From the S parameters response in Fig.4.22 it can be seen; the first two resonant points have higher attenuation compared to the last three resonant points. So, the last three resonant points are used to design the triple band filter which are 3.3 GHz, 6 GHz, and 9 GHz. The filter is designed to have independent fractional bandwidth on all the three bands. So, in order to achieve that, coupling mechanism based on the approach reported in [139] is used here. The coupling structure of the filter is shown in Fig.4.23.

Figure 4.23 Coupling structure of the triple band filter

In the Fig.4.23, the nodes represent the resonators and the lines represent the coupling path. Each of the three band signals are given separate coupling paths to achieve independent fractional bandwidths. The resonators R_1 and R_3 in Fig.4.23 are DPCRLH open circuited resonator in Fig.4.20 which can resonate at all the three frequencies viz 3.3 GHz, 6 GHz, and 9 GHz. The other resonators in Fig.4.23 resonate at single frequency which are as follows R_{22} is stepped impedance resonator (SIR) at 3.3 GHz, R_{21} , and R_{23} are half wavelength resonators at 9 GHz and 6 GHz respectively. The stepped impedance resonator is used at 3.3 GHz because SIR is relatively much smaller than half wavelength resonator at 3.3 GHz. Because of these single frequency

resonators, independent coupling paths to three band signals can be achieved. The filter is designed using third order Chebyshev response of 0.1-dB ripple, low pass prototype values $g_0=1$, $g_1=1.0315$, $g_2=1.1474$, $g_3=1.0315$, and $g_4=1$ [140]. The fractional bandwidths are 3%, 4.7%, and 3.5% at 3.3 GHz, 6 GHz, and 9 GHz respectively.

Designing filter using Coupling coefficient and external quality factors which is used here, is a common method. The coupling coefficient between the resonators and external quality factors are calculated using fractional bandwidths and low pass prototype values as in [141].

The coupling coefficient is given as

$$M_{1,2} = \frac{FBW}{\sqrt{g_1 g_2}} \quad (4.57)$$

The external quality factors are found out as

$$Q_{e1} = \frac{g_1}{FBW}, Q_{e2} = \frac{g_3 \times g_4}{FBW} \quad (4.58)$$

where FBW is fractional bandwidth in (4.57) and (4.58).

The values of coupling coefficients and external quality factors at the three bands are found out by substituting the low pass prototype values and fractional bandwidths at the three bands. The coupling coefficients at the three bands are found out as 0.028 (3.3 GHz), 0.0432 (6 GHz), and 0.0322 (9 GHz). External quality factors on the two sides of the filter are equal, since $g_4=1$ and $g_1=g_3$ in this case. External quality factors at the three bands are found out as 34.3833 (3.3 GHz), 21.95 (6 GHz), and 29.47 (9 GHz). The methodology reported in [141] is followed to fix the positions of resonators and feed based on the calculated coupling coefficients and external quality factors.

External quality factor can be varied by adjusting the space S between feed and the resonators R_1 or R_3 (since both R_1 and R_3 are the same DPCRLH resonators) and the length of the feed L . The space S between feed and the resonators R_1 or R_3 and length of feed L are shown in Fig.4.24.

Figure 4.24 Structure to find external quality factor

Figure 4.25 Quality factor vs S -Distance between resonator R_1 or R_3 to feed in mm. Broken lines are for $L=3$, and solid lines are for $L=4$. Blue lines are for 3.3 GHz, red lines are for 6 GHz, and green lines are for 9 GHz respectively.

Figure 4.26 Final structure of the filter $L_{11}=3.6\text{mm}$, $L_{12}=5.2\text{mm}$, $W_{11}=1\text{mm}$, $L_{21}=4.2\text{mm}$, $L_{22}=4.3\text{mm}$, $L_{23}=6\text{mm}$, $W_{21}=0.9\text{mm}$, $W_{22}=2.5\text{mm}$, $L_{31}=5.08\text{mm}$, $L_{32}=5.8\text{mm}$, $W_{31}=0.9\text{mm}$.

The graph showing the variation of external quality factor with respect to distance between feed and resonators for different values of feed length L is given in Fig.4.25.

The final structure of the filter along the dimensions are shown in Fig.4.26. Comparing the figures Fig.4.23 and Fig.4.26 it can be seen how the final structure of the filter is formed from coupling structure. The photograph of the fabricated filter is shown in Fig.4.27. The S-parameter responses of the triple band filter is shown in Fig.4.28. The measured and simulated S-parameter responses match quite well except the increased insertion loss in measured responses. The measured insertion losses in the three bands are 3.06 dB, 2.71 dB, and 3.16 dB at 3.3 GHz, 6 GHz, and 9 GHz respectively. The higher insertion losses are due to fabrication errors, and connector losses, since the simulated insertion losses are 1.96 dB (3.3 GHz), 2.08 dB (6 GHz), and 2.98 dB (9 GHz).

(a)

(b)

Figure 4.27 Photograph of the fabricated filter

Figure 4.28 S parameters response of the triple band filter

The comparison of the designed triple band filter with that of the previously reported filters is given in Table 4.1. The filter has triple band while other CRLH filters in Table

2 have single or dual bands. The response of the filter is very sharp in the first two bands and it is comparable in the last band. The filter has two transmission zeros. Further the designed filter has widely spaced central frequencies which is difficult to implement using techniques mentioned in section 2.5.1 (where various triple band filter designs are explained). Also, the designed filter is planar compared to dielectric resonator filters mentioned in section 2.5.1.

Table 4.1 Comparison of the designed filter with previously reported filters

	Number of bands	Central frequencies (GHz)	Insertion loss (dB)	Fractional bandwidth (%)	Size (λ_g)
This work	3	3.3, 6, 9	3.06, 2.71, 3.16	Different 3,4,7,3.5	0.88×0.55
[45]	2	2.4, 3.3	1.3, 1.5	same	0.09×0.16
[62]	2	0.9, 2.4	<2	same	~ 0.44 $\times 0.23$
[88]	1	5	2.7	5.4	$\sim 0.28 \times$ 0.22
[90]	2	3.75, 5.25	1, 0.9	same	$> 1.03 \times 0.37$
[91]	2	2.4, 5.8	0.33, 0.74	52, 23	$\sim 0.99 \times$ 0.25

4.8 Conclusion

The symmetric unit cells of DPCRLH have been proposed and they are analyzed based on ABCD parameters. The closing condition of attenuation gap between two $\beta d = 0$ points in the dispersion diagram are reported. Self-resonant frequency of the interdigital capacitor and its effect on the response are explained. The alternate fingers of the interdigital capacitor are connected by using via holes and defected ground to

move the self-resonant frequency to a higher value. Parameter extraction of interdigital capacitor and short circuited stub are discussed. A DPCRLH unit cell is designed based on the analysis and fabricated. The measured and simulated results match with the analysis reported. The unit cell satisfies the closing condition of attenuation gap between two $\beta d = 0$ points in the dispersion diagram. DPCRLH structure alternates between left and right handed regions twice compared to CRLH structure. So, DPCRLH can be used to design several multi band components. Increasing periodicity increases the number of LH and RH regions which increases number of bands in a component.

In order to prove DPCRLH can be used to design multi band circuits. The DPCRLH unit cell designed is open circuited to form DPCRLH open circuited resonator. The input impedance of DPCRLH shows that it has five resonant points. The DPCRLH open circuited resonator with five resonant points is designed and fabricated. The measured and simulated S parameters response confirms the resonant points found out by impedance equation. The resonator has five resonant points at 1.574 GHz, 2.05 GHz, 3.24 GHz, 6 GHz, and 9 GHz.

Out of the five resonant points of DPCRLH open circuited resonator, the last three are chosen to design triple band filter. The coupling structure is designed to provide separate coupling paths to each of the three bands. So, the triple band filter has independent fractional bandwidth on all the three bands. The filter is of order three and designed using coupling coefficient and external quality factor. The triple band filter is designed at 3.3 GHz, 6 GHz and 9 GHz. The measured and simulated S parameters responses confirm this. In this chapter, the triple band filter is designed and examined as an example of the guided wave application of DPCRLH resonator. The radiated wave application of DPCRLH resonator with a different host line will be discussed in the next chapter.

Chapter 5

Coplanar waveguide based

DPCRLH unit cell and design of triple band antenna

The symmetric unit cells of DPCRLH transmission lines were introduced in chapter 4 and one of the symmetric cell is fabricated with microstrip as host line. In microstrip implementation the left handed inductances are implemented as short circuited stubs which are implemented using via holes. During fabrication the implementation of via holes is difficult. So, to avoid that DPCRLH is implemented in coplanar waveguide in this chapter. A new parametric extraction method is introduced in this chapter which is different from the parametric extraction method in chapter 4. Further DPCRLH unit cell is converted to open circuited resonator. The open circuited

resonator is converted to a triple band antenna to show the radiated wave application of DPCRLH unit cell.

5.1 CPW DPCRLH symmetric unit cell design

DPCRLH symmetric unit cells are introduced in chapter 4. In this chapter one of the two symmetric cells introduced is implemented in CPW host line. The symmetric unit cell which is shown in Fig.4.2(a) is again displayed in Fig.5.1 for convenience.

Figure 5.1 symmetric unit cell of DPCRLH

The analysis of symmetric unit cell using ABCD parameters is already given in Chapter 4. Here the implementation of the symmetric unit cell in CPW is focused. In CPW, the series capacitor is implemented by interdigital capacitor and shunt inductor is implemented by short circuited stub as in microstrip host line. But the short circuited stub in CPW does not require via hole for its implementation, since CPW has two ground planes and the signal line in the same plane. Only the extraction procedure is different in CPW compared to microstrip host line. In this chapter, meandered short circuit stub is used instead of conventional straight short circuited stub. There are two advantages, one is the higher value of the inductance can be obtained easily and second is there is no coupled slot line mode in the meandered short circuited stub. If a straight short circuited stub is used as shown in Fig. 5.2(a), it supports the coupled slot line mode so

air bridge is needed to prevent this mode. But the meandered short circuited stub shown in Fig. 5.2(b) there is no coupled slot line mode so air bridge can be avoided.

(a)

(b)

Figure 5.2 (a) straight short circuited stub (b) Meandered short circuited stub

Even though the length of the short circuited stub is decreased by using meandering but the width is increased so to satisfy the effective homogeneous condition the interdigital capacitor is made to occupy some area under short circuited stub. The

parameter extraction method used in chapter 4 cannot be used since it requires the separate simulations of interdigital capacitor and short circuited stub. Here the L_R and part of C_R introduced by interdigital capacitor is changed by short circuited stub and also the C_R introduced by the short circuited stub is altered by interdigital capacitor. The value of L_{L1} can be extracted using the method in section 4.2 of chapter 4. L_{L2} cannot be extracted by the method in section 4.2, since the short circuited stub used to implement L_{L2} is not symmetric (but the whole unit cell is symmetric). So L_{L2} obtained by the previous method is not accurate. The rest of the parameters except L_{L1} are found out by using the following method.

The ABCD matrix of the DPCRLH symmetric unit cell shown in Fig.5.1 is given in (4.6) of chapter is again repeated here

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} & 2Z_1 + Z_1^2 Y_1 \\ Y_1 + Y_2 + Z_1 Y_1 Y_2 + \frac{Z_1 Y_2^2}{2} + \frac{Z_1^2 Y_1 Y_2^2}{4} & 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} \end{bmatrix} \quad (5.1)$$

The conversion formula from ABCD matrix to Y matrix is given as [42]

$$\begin{aligned} Y_{11} &= \frac{D}{B} & Y_{12} &= \frac{BC - AD}{B} \\ Y_{21} &= \frac{-1}{B} & Y_{22} &= \frac{A}{B} \end{aligned} \quad (5.2)$$

The Y matrix parameters of the DPCRLH symmetric unit cell can be found out by substituting (5.1) in (5.2).

$$Y_{11} = Y_{22} = \frac{1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2}}{2Z_1 + Z_1^2 Y_1} \quad (5.3)$$

$$Y_{21} = Y_{12} = \frac{-1}{2Z_1 + Z_1^2 Y_1} = \frac{-1}{Z_1(2 + Z_1 Y_1)} \quad (5.4)$$

From (5.4), Y_{21} in the ideal case becomes infinity at $Z_1 = 0$ and $(2 + Z_1 Y_1) = 0$. $Z_1 = 0$ is one of the $\beta d = 0$ points in the dispersion diagram of DPCRLH unit cell. $(Z_1 Y_1 + 2) = 0$ gives two of the four $\beta d = \pi$ points in the dispersion diagram. So, these points can be found from Y_{21} .

The conversion formula from ABCD matrix to Z matrix is given as [42]

$$\begin{aligned} Z_{11} &= \frac{A}{C} & Z_{12} &= \frac{AD - BC}{C} \\ Z_{21} &= \frac{1}{C} & Z_{22} &= \frac{D}{C} \end{aligned} \quad (5.5)$$

Substituting (5.1) in (5.5) the Z matrix parameters can be found as

$$Z_{11} = Z_{22} = \frac{1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2}}{Y_1 + Y_2 + Z_1 Y_1 Y_2 + \frac{Z_1 Y_2^2}{2} + \frac{Z_1^2 Y_1 Y_2^2}{4}} \quad (5.6)$$

$$Z_{21} = Z_{12} = \frac{1}{Y_1 + Y_2 + Z_1 Y_1 Y_2 + \frac{Z_1 Y_2^2}{2} + \frac{Z_1^2 Y_1 Y_2^2}{4}} \quad (5.7)$$

The equation (5.7) can be simplified as

$$Z_{21} = Z_{12} = \frac{1}{(Z_1 Y_2 + 2) \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right)} \quad (5.8)$$

$(Z_1 Y_2 + 2) = 0$ gives the remaining two $\beta d = \pi$ points (the two left in (5.4)) in the dispersion diagram of DPCRLH unit cell. $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0$ gives the other three $\beta d = 0$ points in the dispersion diagram. So, from (5.8) two $\beta d = \pi$ points and three $\beta d = 0$ points in the dispersion diagram of DPCRLH unit cell can be found out.

The equations representing $\beta d = \pi$ in the dispersion diagram of DPCRLH unit cell are given in (4.17) of chapter 4. They are

$$Z_1 Y_1 = -2 \text{ and } Z_1 Y_2 = -2 \quad (5.9)$$

They are simplified in (4.18) and (4.19) as

$$\omega^4 L_{R1} C_{R1} - \omega^2 \left(\frac{L_{R1}}{L_{L1}} + \frac{C_{R1}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L1}} = 0 \quad (5.10)$$

$$\omega^4 L_{R1} C_{R2} - \omega^2 \left(\frac{L_{R1}}{L_{L2}} + \frac{C_{R2}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L2}} = 0 \quad (5.11)$$

Re writing (5.10) as

$$\begin{aligned} \omega^4 L_{R1} C_{R1} - \omega^2 \left(\frac{L_{R1}}{L_{L1}} + \frac{C_{R1}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L1}} &= 0 \\ L_{R1} C_{R1} - \frac{1}{\omega^2} \left(\frac{L_{R1}}{L_{L1}} + \frac{C_{R1}}{C_{L1}} + 2 \right) + \frac{1}{\omega^4 C_{L1} L_{L1}} &= 0 \\ C_{R1} - \frac{1}{\omega^2} \left(\frac{1}{L_{L1}} + \frac{C_{R1}}{L_{R1} C_{L1}} + \frac{2}{L_{R1}} \right) + \frac{1}{\omega^4 C_{L1} L_{L1} L_{R1}} &= 0 \\ L_{L1} C_{R1} - \frac{1}{\omega^2} \left(1 + \frac{C_{R1} L_{L1}}{L_{R1} C_{L1}} + \frac{2 L_{L1}}{L_{R1}} \right) + \frac{1}{\omega^4 C_{L1} L_{R1}} &= 0 \end{aligned} \quad (5.12)$$

Substituting $L_{L1} C_{R1} = a_1$ $\frac{L_{L1}}{L_{R1}} = b_1$ in (5.12) gives

$$a_1 - \frac{1}{\omega^2} \left(1 + \frac{a_1}{L_{R1} C_{L1}} + 2b_1 \right) + \frac{1}{\omega^4 C_{L1} L_{R1}} = 0 \quad (5.13)$$

$Z_1 Y_1 = -2$ contains two of the four $\beta d = \pi$ points so the above equation is satisfied at two frequency points $\omega = \omega_{\pi 1}$ and $\omega = \omega_{\pi 2}$. Also, from (4.10) one of the $\beta d = 0$ points occurs at $\omega_0^2 = \frac{1}{L_{R1} C_{L1}}$.

$$a_1 - \frac{1}{\omega_{\pi 1}^2} (1 + a_1 \omega_0^2 + 2b_1) + \frac{\omega_0^2}{\omega_{\pi 1}^4} = 0 \quad (5.14)$$

$$a_1 - \frac{1}{\omega_{\pi 2}^2} (1 + a_1 \omega_0^2 + 2b_1) + \frac{\omega_0^2}{\omega_{\pi 2}^4} = 0 \quad (5.15)$$

Solving (5.14) and (5.15) the values of $a_1 = L_{L1}C_{R1}$ and $b_1 = \frac{L_{L1}}{L_{R1}}$ can be found out.

Re writing (5.11) as

$$\begin{aligned}
\omega^4 L_{R1} C_{R2} - \omega^2 \left(\frac{L_{R1}}{L_{L2}} + \frac{C_{R2}}{C_{L1}} + 2 \right) + \frac{1}{C_{L1} L_{L2}} &= 0 \\
L_{R1} C_{R2} - \frac{1}{\omega^2} \left(\frac{L_{R1}}{L_{L2}} + \frac{C_{R2}}{C_{L1}} + 2 \right) + \frac{1}{\omega^4 C_{L1} L_{L2}} &= 0 \\
C_{R2} - \frac{1}{\omega^2} \left(\frac{1}{L_{L2}} + \frac{C_{R2}}{L_{R1} C_{L1}} + \frac{2}{L_{R1}} \right) + \frac{1}{\omega^4 L_{R1} C_{L1} L_{L2}} &= 0 \\
L_{L2} C_{R2} - \frac{1}{\omega^2} \left(1 + \frac{L_{L2} C_{R2}}{L_{R1} C_{L1}} + \frac{2 L_{L2}}{L_{R1}} \right) + \frac{1}{\omega^4 L_{R1} C_{L1}} &= 0 \tag{5.16}
\end{aligned}$$

Substituting $L_{L2} C_{R2} = a_2$ $\frac{L_{L2}}{L_{R1}} = b_2$ in (5.16) results in

$$a_2 - \frac{1}{\omega^2} \left(1 + \frac{a_2}{L_{R1} C_{L1}} + 2 b_2 \right) + \frac{1}{\omega^4 L_{R1} C_{L1}} = 0 \tag{5.17}$$

As in the previous case, $Z_1 Y_2 = -2$ contains remaining two of the four $\beta d = \pi$ points so the above equation is satisfied at two frequency points $\omega = \omega_{\pi 3}$ and $\omega = \omega_{\pi 4}$. Also, from (4.10) one of the $\beta d = 0$ points occurs at $\omega_0^2 = \frac{1}{L_{R1} C_{L1}}$.

$$a_2 - \frac{1}{\omega_{\pi 3}^2} (1 + a_2 \omega_0^2 + 2 b_2) + \frac{\omega_0^2}{\omega_{\pi 3}^4} = 0 \tag{5.18}$$

$$a_2 - \frac{1}{\omega_{\pi 4}^2} (1 + a_2 \omega_0^2 + 2 b_2) + \frac{\omega_0^2}{\omega_{\pi 4}^4} = 0 \tag{5.19}$$

Solving (5.18) and (5.19), the values of $a_2 = L_{L2} C_{R2}$ and $b_2 = \frac{L_{L2}}{L_{R1}}$ can be found out.

The DPCRLH parameters can be found out using the following steps.

1. L_{L1} is found out from the extraction method in section 4.2 of chapter 4.

2. a_1 and b_1 is obtained by solving (5.14) and (5.15). From b_1 and L_{L1} (from step 1) L_{R1} is found out. Also C_{R1} can be found from a_1 and L_{L1} .
3. From (5.4), $Z_1 = 0$ which gives, $\omega_0^2 = \frac{1}{L_{R1}C_{L1}}$. Since L_{R1} is known from step 2 now C_{L1} can be found out.
4. Solving (5.18) and (5.19) gives a_2 and b_2 . L_{R1} is already found out in step 2, so using L_{R1} , L_{L2} is found out from b_2 and C_{R2} is obtained from a_2 using L_{L2} .

Thus, all the parameters of DPCRLH can be found out.

The DPCRLH symmetric unit cell in CPW is shown in Fig. 5.3. The Y_{21} and Z_{21} of the DPCRLH symmetric unit cell are shown in Fig.5.4 along with the $\beta d = 0$ and $\beta d = \pi$ frequency points. All the simulated results in this chapter are obtained by using CST Microwave Studio. From the $\beta d = 0$ and $\beta d = \pi$ frequency points ($\omega_{\pi1}$ to $\omega_{\pi4}$), equations (5.14), (5.15), (5.18) and (5.19) are solved.

Figure 5.3 CPW DPCRLH unit cell with $l=3\text{mm}$, $w=0.4\text{mm}$, $w_1=0.4\text{mm}$, $w_2=0.3\text{mm}$, $w_3=1\text{mm}$, $s=s_1=s_2=0.2\text{mm}$, $w_f=4.6\text{mm}$, and $g_f=0.33\text{mm}$.

(a)

(b)

Figure 5.4 Y_{21} real and Z_{21} real of DPCRLH symmetric unit cell in Fig.5.3

The parameters of DPCRLH can be found out by using the steps from 1 to 4 given previously. L_{L1} is found out as 3.197 nH. From step 2, L_{R1} is found out as 4 nH and C_{R1} as 0.79 pF. C_{L1} found out from step 3 as 0.72 pF. From step 4, L_{L2} and C_{R2} are calculated as 1.04 nH and 1.15 pF respectively. The DPCRLH parameters are extracted

at frequency 3.54 GHz. The DPCRLH symmetric unit cell is fabricated in Rogers RO4003C substrate with dielectric constant of 3.55, loss tangent of 0.0027 and thickness of 1.524 mm. Fig.5.5 shows the photograph of the fabricated DPCRLH symmetric unit cell on CPW. The simulated and measured S parameter response of the DPCRLH unit cell are plotted in Fig.5.6. The simulated and measured S parameters responses match well.

Figure 5.5 Photograph of the fabricated filter

Figure 5.6 S parameters response of DPCRLH unit cell in CPW

The dispersion relation of DPCRLH symmetric unit cell is obtained from (4.7) which is repeated here for convenience

$$\cos \beta d = \frac{A + D}{2} = 1 + Z_1 Y_1 + Z_1 Y_2 + \frac{Z_1^2 Y_1 Y_2}{2} \quad (5.20)$$

The dispersion diagram obtained from (5.20) by substituting the DPCRLH unit cell parameter values, is plotted along with simulated and measured dispersion diagram in Fig.5.7. The simulated and measured dispersion diagram matches well with the dispersion diagram plotted using (5.20).

Figure 5.7 Dispersion diagram of DPCRLH unit cell in CPW

5.2 DPCRLH open circuited resonator

The DPCRLH symmetric unit cell in Fig.5.1 is open circuited at both ends to give DPCRLH resonator. The DPCRLH open circuited resonator is given in Fig.5.8.

Figure 5.8 DPCRLH open circuited resonator

The input impedance of the DPCRLH open circuited resonator is already derived in Chapter 4 and it is given as

$$Z_{in} = \frac{2}{Y_2} // \left(Z_1 + \left(\left(Z_1 + \frac{2}{Y_2} \right) // \frac{1}{Y_1} \right) \right) \quad (5.20)$$

Simplifying,

$$Z_{in} = \frac{2(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)}{Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2} \quad (5.21)$$

At resonance,

$$Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2 = 0 \quad (5.22)$$

$$4 \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) = 0 \quad (5.23)$$

Substituting the parameters of DPCRLH unit cell in (5.21). The value of Z_{in} is found out. The graph of Z_{in} versus frequency is shown in Fig.5.9. The theoretical resonant points obtained from the graph of Z_{in} versus frequency are 1.54 GHz, 2.28GHz, 4.02 GHz, 5.98 GHz, and 6.97 GHz.

Figure 5.9 Absolute value of Z_{in} of open circuited resonator versus frequency

The DPCRLH unit cell (in Fig.5.3) with parameters $L_{R1}=4\text{nH}$, $C_{L1}=0.72\text{pF}$, $L_{L1}=3.197\text{nH}$, $C_{R1}=0.79\text{pF}$, $L_{L2}=1.04\text{nH}$, and $C_{R2}=1.15\text{pF}$, is open circuited to form DPCRLH open circuited resonator and it is shown in Fig.5.10.

Figure 5.10 DPCRLH open circuited resonator

The simulated S parameter response of the resonator is shown in Fig.5.11. The S parameters response confirms the fact, the resonator has five resonant points. The simulated resonant points are 1.5494 GHz, 2.1968 GHz, 3.5072 GHz, 5.0438 GHz and

6.089 GHz. First two simulated resonant points matches well with the resonant points in Fig.5.9. The last three simulated resonant points are slightly shifted from the resonant points in Fig.5.9

Figure 5.11 S parameters response of the DPCRLH open circuited resonator in Fig.5.10

5.2.1 Quality factor and bandwidth

The quality factor of the open circuited DPCRLH resonator can be found out as follows. The quality factor is given as

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated/cycle}} \quad (5.24)$$

If the resonator is assumed as lossless then it can be shown [134, 142],

$$\frac{\partial B}{\partial \omega} = \frac{4(W_E + W_H)}{ee^*} \quad (5.25)$$

where B is the shunt susceptance of the resonator at the end point, W_E and W_H are the stored electric energy and stored magnetic energy of the resonator and e is the terminal voltage and e^* is the complex conjugate of e .

Substituting (5.25) in (5.24), then

$$Q = 2\pi f \frac{\frac{1}{4}ee^* \frac{\partial B}{\partial \omega}}{\text{energy dissipated/sec}} \quad (5.26)$$

If G is the shunt conductance at the terminal, then the energy dissipated per second is

$\frac{1}{2}ee^*G$. So, the quality factor in (5.26) becomes

$$Q = \frac{\omega}{2G} \frac{\partial B}{\partial \omega} \quad (5.27)$$

From (5.21), the admittance of the resonator can be found out as

$$Y_{in} = \frac{Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2}{2(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)} \quad (5.28)$$

From (5.28), the susceptance of the resonator is given as

$$B = \frac{1}{j} \frac{Z_1^2 Y_1 Y_2^2 + 4Z_1 Y_1 Y_2 + 4Y_2 + 4Y_1 + 2Z_1 Y_2^2}{2(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)}$$

$$B = \frac{1}{j} \frac{4 \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2)}{2(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)} \quad (5.29)$$

$$Nr = \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \quad (5.30)$$

$$Dr = (Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2) \quad (5.31)$$

$$\frac{\partial B}{\partial \omega} = \frac{1}{j} \frac{2(Dr) \frac{\partial}{\partial \omega}(Nr) - (Nr) \frac{\partial}{\partial \omega}(Dr)}{(Dr)^2} \quad (5.32)$$

The quality factor at the resonant points are important since they decide the bandwidth at the resonant points. At resonant points, (5.30) becomes zero. So, at resonant points (5.32) becomes

$$\frac{\partial B}{\partial \omega} = \frac{2}{j} \frac{\frac{\partial}{\partial \omega} (Nr)}{(Dr)} \quad (5.33)$$

Now,

$$\frac{\partial}{\partial \omega} (Nr) = \frac{\partial}{\partial \omega} \left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \right) \quad (5.34)$$

$$= (Z_1 Y_2 + 2) \frac{\partial}{\partial \omega} \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) + \left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) \frac{\partial}{\partial \omega} (Z_1 Y_2 + 2) \quad (5.35)$$

At $(Z_1 Y_2 + 2) = 0$ resonant points (5.35) becomes,

$$\begin{aligned} \frac{\partial}{\partial \omega} (Nr) &= \frac{\partial}{\partial \omega} \left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \right) \\ &= \left(Y_1 + Y_2 + \frac{-2Y_1}{2} \right) (Z_1 Y_2 + Z_1 Y_2) \end{aligned} \quad (5.36)$$

where $Z_1 = \frac{\partial Z_1}{\partial \omega} = \left(jL_{R1} - \frac{1}{j\omega^2 C_{L1}} \right)$ and $Y_2 = \frac{\partial Y_2}{\partial \omega} = \left(jC_{R2} - \frac{1}{j\omega^2 L_{L2}} \right)$.

Simplifying (5.36) gives

$$\frac{\partial}{\partial \omega} (Nr) = \frac{\partial}{\partial \omega} \left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \right) = Y_2 (Z_1 Y_2 + Z_1 Y_2) \quad (5.37)$$

Substituting (5.37) and (5.31) in (5.33) gives

$$\frac{\partial B}{\partial \omega} = \frac{2}{j} \frac{Y_2 (Z_1 Y_2 + Z_1 Y_2)}{(Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)} \quad (5.38)$$

Substituting $Z_1 Y_2 = -2$ in (5.38) gives

$$\begin{aligned} \frac{\partial B}{\partial \omega} &= \frac{-1}{2j} (Z_1 Y_2^2 + Z_1 Y_2 Y_2) \\ \frac{\partial B}{\partial \omega} &= \frac{1}{2j} (2Y_2 - Z_1 Y_2^2) \end{aligned} \quad (5.39)$$

Substituting (5.39) in (5.27) gives

$$Q = \frac{\omega}{2G} \left(\frac{1}{2j} (2Y_2^{\mid} - Z_1^{\mid} Y_2^2) \right) \quad (5.40)$$

Substituting the values of Y_2 , $Y_2^{\mid}$ and $Z_1^{\mid}$ in (5.40) gives

$$Q = \frac{\omega}{2G} \left(\frac{1}{2j} \left(2 \left(jC_{R2} - \frac{1}{j\omega^2 L_{L2}} \right) - \left(jL_{R1} - \frac{1}{j\omega^2 C_{L1}} \right) \left(j\omega C_{R2} + \frac{1}{j\omega L_{L2}} \right)^2 \right) \right) \quad (5.41)$$

$$Q = \frac{\omega}{4jG} \left(2 \left(jC_{R2} - \frac{1}{j\omega^2 L_{L2}} \right) - \left(jL_{R1} - \frac{1}{j\omega^2 C_{L1}} \right) \left(j\omega C_{R2} + \frac{1}{j\omega L_{L2}} \right)^2 \right) \quad (5.42)$$

The quality factor Q is inversely proportional to bandwidth, so from (5.42) it is evident, decreasing the values of L_{R1} and C_{R2} and increasing the values of C_{L1} and L_{L2} , will decrease the value Q which makes the bandwidth to increase at the two resonant points given by $(Z_1 Y_2 + 2) = 0$.

At remaining resonant points $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0$, (5.35) becomes,

$$\begin{aligned} \frac{\partial}{\partial \omega} (Nr) &= \frac{\partial}{\partial \omega} \left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \right) \\ &= (Z_1 Y_2 + 2) \left(Y_1^{\mid} + Y_2^{\mid} + \frac{Z_1^{\mid} Y_1 Y_2}{2} + \frac{Z_1 Y_1^{\mid} Y_2}{2} + \frac{Z_1 Y_1 Y_2^{\mid}}{2} \right) \end{aligned} \quad (5.43)$$

where $Z_1^{\mid} = \frac{\partial Z_1}{\partial \omega} = \left(jL_{R1} - \frac{1}{j\omega^2 C_{L1}} \right)$, $Y_1^{\mid} = \frac{\partial Y_1}{\partial \omega} = \left(jC_{R1} - \frac{1}{j\omega^2 L_{L1}} \right)$ and $Y_2^{\mid} = \frac{\partial Y_2}{\partial \omega} = \left(jC_{R2} - \frac{1}{j\omega^2 L_{L2}} \right)$

Simplifying (5.43) gives,

$$\frac{\partial}{\partial \omega} \left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) (Z_1 Y_2 + 2) \right) = \frac{Y_1^{\mid}}{2} (2 + Z_1 Y_2)^2 - Z_1^{\mid} Y_2^2 + 2Y_2^{\mid} \quad (5.44)$$

Substituting (5.44) and (5.31) in (5.33)

$$\frac{\partial B}{\partial \omega} = \frac{2 \frac{Y_1}{2} (2 + Z_1 Y_2)^2 - Z_1 |Y_2|^2 + 2Y_2}{j (Z_1^2 Y_1 Y_2 + 2Z_1 Y_2 + 2Z_1 Y_1 + 2)} \quad (5.45)$$

Simplifying (5.45) gives

$$\frac{\partial B}{\partial \omega} = \frac{1}{2j} \left(\frac{Y_1}{2} (2 + Z_1 Y_2)^2 - Z_1 |Y_2|^2 + 2Y_2 \right) \quad (5.46)$$

Substituting (5.46) in (5.27) gives

$$Q = \frac{\omega}{2G} \frac{\partial B}{\partial \omega} = \frac{\omega}{2G} \left(\frac{1}{2j} \left(\frac{Y_1}{2} (2 + Z_1 Y_2)^2 - Z_1 |Y_2|^2 + 2Y_2 \right) \right) \quad (5.47)$$

$$Q = \frac{\omega}{4jG} \left(\frac{Y_1}{2} (2 + Z_1 Y_2)^2 - Z_1 |Y_2|^2 + 2Y_2 \right) \quad (5.48)$$

$$Q = \frac{\omega}{4jG} \left(\frac{1}{2} \left(jC_{R1} - \frac{1}{j\omega^2 L_{L1}} \right) \left(2 + \left(\left(j\omega L_{R1} + \frac{1}{j\omega C_{L1}} \right) \left(j\omega C_{R2} + \frac{1}{j\omega L_{L2}} \right) \right)^2 \right) \right. \\ \left. - \left(jL_{R1} - \frac{1}{j\omega^2 C_{L1}} \right) \left(j\omega C_{R2} + \frac{1}{j\omega L_{L2}} \right)^2 + 2 \left(jC_{R2} - \frac{1}{j\omega^2 L_{L2}} \right) \right) \quad (5.49)$$

From (5.49) it can be found, by decreasing the values of L_{R1} , C_{R1} and C_{R2} and increasing the values of C_{L1} , L_{L1} and L_{L2} , will decrease the value Q which makes the bandwidth to increase at three resonant points defined by $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0$.

Also comparing, (5.40) and (5.48), it can be seen that (5.48) has additional term

$\frac{Y_1}{2} (2 + Z_1 Y_2)^2$ this gives the quality factor at $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2} \right) = 0$ points is always

higher the quality factor at $(Z_1 Y_2 + 2) = 0$. So, bandwidth at resonant points defined

by $(Z_1 Y_2 + 2) = 0$ is higher than the bandwidth at the resonant points defined by $\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2}\right) = 0$.

5.3 CPW DPCRLH triple band antenna

The triple band antenna is designed by using the last three resonant points of the open circuited resonator (shown in Fig.5.10). The antenna is matched at the three bands by using the CPW discontinuities along the feed. The feed is gap coupled to the DPCRH resonator. The final structure of the antenna is shown in Fig.5.12(a) and the photograph of the antenna is shown in Fig.5.12(b). The measured and simulated S_{11} of the antenna is shown in Fig. 5.13. The measured central frequencies of the antenna are 3.66 GHz, 5.095 GHz and 6.1188 GHz. The measured 10dB impedance bandwidth at the three bands are 1.67% (3.66 GHz band), 11.08% (5.095 GHz band) and 2% (6.1188 GHz band). The bandwidth at the first and last bands are approximately equal and also the bandwidth of the second band $((Z_1 Y_2 + 2) = 0$ resonant point) is higher than the bandwidth of the first and last bands $\left(\left(Y_1 + Y_2 + \frac{Z_1 Y_1 Y_2}{2}\right) = 0\right)$ resonant points). These results are in accordance with (5.40) and (5.48). The simulated 3-Dimensional radiation pattern at the three bands are shown in Fig.5.14. The antenna is horizontally polarized in all the three bands.

(a)

(b)

Figure 5.12 (a) Triple band antenna structure with $w_f= 4.6$ mm, $l_f= 5.6$ mm, $w_d=5$ mm, $l_d=8.2$ mm, $s_d=1.4$ mm, $w_l=1$ mm, $l_l=3$ mm, and $w_g=1.2$ mm (b) Photograph of the fabricated antenna

Figure 5.13 Simulated and measured S_{11} of the triple band antenna

(a)

(b)

(c)

Figure 5.14 Simulated 3-Dimensional radiation pattern (a) first band (b) second band (c) third band

The measured and simulated col-pol radiation pattern of the antenna in the plane $\Phi = 0$ at the three bands are shown in Fig.5.15. The measured co-polarization gain of the antenna in the three bands are 2.1 dBi (3.66 GHz), 3.19 dBi (5.0119 GHz) and 2.24 dBi (6.1188 GHz). The simulated co-polarization gain of the antenna are 2.648 dBi,

3.695 dBi and 2.66 dBi respectively at three bands. The measured radiation efficiencies are 40.26% (3.66 GHz), 62.46% (5.0119 GHz) and 43.19% (6.1188 GHz).

(a)

(b)

(c)

Figure 5.15 Measured and simulated Co-polarized radiation pattern (a) First band (b) Second band (c) Third band

Table 5.1 compares the triple band antenna designed with that of the existing designs. It can be found out from Table 5.1 the antenna designed has an additional band (the other antennas are single or dual band) and also has relatively higher gains in the three bands. The CPW DPCRLH triple band antenna is of single layer and it has higher fractional bandwidth (11.08 %) in the second band and approximately equal but lower fractional band width around (1.67% and 2%) in the other two bands. Comparing with the triple band antennas described in section 2.9, the designed antenna is of smaller size and designed with single layer and can be designed with large frequency ratios if the parameters of DPCRLH are changed.

Table 5.1 Comparison of the designed triple band antenna with the existing works

	This work	[93]	[95]	[102]	[98]	[97]
Frequency (GHz) and number of bands	Triple Band antenna 3.66 5.095 6.1188	Two dual band antennas 2.33 and 2.87 ($f_{se}<f_{sh}$) 2.41 and 3.04 ($f_{se}>f_{sh}$)	Two single wide band antennas 2.03 (symmetric) 1.5 (asymmetric)	Single wide band antenna 2.16	Very wide band antenna 1.77 to 3.77 (2.36 (ZOR) 3.3 (FPOR))	Dual band antenna 1.88 (ZOR) 3.22 (FOR)
Antenna foot print (λ_0)	0.529x0.61 (including ground planes)	0.113x0.135 ($f_{se}<f_{sh}$) 0.111x0.126 ($f_{se}>f_{sh}$)	0.097x0.053 (symmetric) 0.072x0.04 (asymmetric)	0.14x0.22	0.049x0.136	0.031x0.089
Bandwidth %	1.67 11.08 2	0.6 and 1.7 ($f_{se}<f_{sh}$) 1.2 and 0.2 ($f_{se}>f_{sh}$)	6.8 (symmetric) 4.8 (asymmetric)	15.1	58.1	15.79 (ZOR) 29.03 (FOR)
Gain (dBi)	2.1 3.19 2.24	1.08 and 1.12 ($f_{se}<f_{sh}$) 1.47 and $-1.71(f_{se}>f_{sh})$	1.35 (symmetric) -2.15 (asymmetric)	1.62	1.6 at ZOR 3.25at FOR	1.94 (ZOR) 2.95 (FOR)
Radiation Efficiency %	40.26 62.46 43.19	62 and 65 ($f_{se}<f_{sh}$) 63 and 60 ($f_{se}>f_{sh}$)	62 (symmetric) 42.5 (asymmetric)	72	73.3 at ZOR 74.5 at FOR	92.09 (ZOR) 88.02 (FOR)
Layer	single	single	Double	single	single	Double

5.4 Conclusion

CPW implementation of DPCRLH symmetric unit cell is studied. The CPW implementation does not require via holes and also DPCRLH structure can be designed in a single plane. The relation between Y and Z parameters and dispersion diagram is demonstrated. A new parameter extraction method based on the relation between Y and Z parameters and $\beta d = 0$ and $\beta d = \pi$ frequency points in the dispersion diagram is proposed. The open circuited DPCRLH resonator formed from DPCRLH unit cell is demonstrated. The open circuited resonator has five resonant points. The simulated resonant points are at 1.5494 GHz, 2.1968 GHz, 3.5072 GHz, 5.0438 GHz and 6.089 GHz. The relation between the DPCRLH unit cell parameters and quality factor at the resonant points are derived. The triple band antenna based on the open circuited DPCRLH resonator is designed, fabricated and tested. The antenna is designed at the frequencies 3.66 GHz, 5.095 GHz and 6.1188 GHz. The antenna has the co-polarization gain of 2.1 dBi, 3.19 dBi and 2.24 dBi respectively at the three bands.

Chapter 6

Conclusion and Future work

6.1 Conclusion

In this thesis, triple band antennas and a filter have been designed in three different host mediums. The metamaterial theory is used in the analysis of the open circuited resonators which are used in the design of these multi band circuits. The conclusions drawn from this thesis are as follows.

An extensive literature review has been done on metamaterials especially CRLH and its applications. The implementation of CRLH in different host mediums viz microstrip, CPW and SIW has been studied. The applications of CRLH in various microwave circuits have been explored. The epsilon negative antennas in CPW have been studied. The DPCRLH and multi periodic CRLH implementations in the previous works have been analyzed.

The SIW and its open circuited resonator have been analyzed using waveguide theory and epsilon negative (ENG) theory. The phase constant obtained by using both

the method of analysis have been plotted together. They match quite well. The effective epsilon extracted from the unit cell parameters are compared with the effective epsilon from waveguide theory and from simulation. The SIW open circuited resonator is designed and measured. Two slots are opened on the top of the SIW to create triple band antenna. The first band is formed by merging two modes. The second and third bands are formed by using single modes only. The central frequencies of the antenna are 3.3 GHz, 4 GHz and 4.4 GHz. The measured realized gain in the three bands are 5.81 dBi (3.3GHz), 4 dBi (4 GHz), and 2.43 dBi (4.4 GHz).

After exploring the single negative metamaterial (ENG), then the analysis of a double negative metamaterial (DPCRLH) has been done. The symmetric unit cells of the DPCRLH transmission line have been proposed. The unit cells are analyzed using ABCD parameters. The expressions relating the frequency points ($\beta d = 0$ and $\beta d = \pi$) and unit cell parameters have been derived. The closing condition of the attenuation gap between two $\beta d = 0$ points in the dispersion diagram has been derived. The unit cell satisfying this condition has a wide band response. A symmetric unit cell satisfying this condition is designed and experimentally validated. DPCRLH structure has an additional left and right handed regions compared to CRLH structure. So DPCRLH can be used in the design of multi band circuits. The open circuited DPCRLH resonator from the symmetric unit cell has been analyzed using the input impedance. There are five resonant points. A triple band filter using the last three resonant frequencies is designed and verified experimentally. The triple band filter is of order three and has different fractional bandwidths in the three bands. The filter is designed at 3.3 GHz, 6 GHz and 9 GHz. Even though the unit cell satisfying the closing condition has very wide bandwidth, designing the unit cell with that condition is difficult. The interdigital capacitor needs to be changed and the overall structure becomes complex.

Having analyzed the guided wave characteristics of DPCRLH, next the radiated wave characteristics of DPCRLH is tested using CPW host line. A new parameter extraction method using Y and Z parameters of DPCRLH unit cell has been proposed. DPCRLH symmetric unit cell in the CPW is designed and validated experimentally. The open circuited DPCRLH resonator is simulated and verified. The open circuited resonator has five resonant points. A triple band antenna using the last three resonant points of DPCRLH is designed at 3.66 GHz, 5.095 GHz and 6.1188 GHz. The antenna is matched at the three bands using the CPW discontinuities. The measured realized gain at the three bands are 2.1 dBi (3.66 GHz), 3.19 dBi (5.0119 GHz) and 2.24 dBi (6.1188 GHz). The antenna is designed with CPW as host line which has signal line and grounds in same plane. Since the ground plane at the bottom (as in microstrip) is not present the triple band antenna designed using CPW has relatively lower gain.

6.2 Future work

There are several possible ways to extend the research work reported in this thesis. Some of them are as follows.

6.2.1 Exploring the DPCRLH symmetric unit cell B and other resonators of DPCRLH symmetric unit cells

The DPCRLH symmetric unit cell A (Fig 4.2(a)) has been used in the design of triple band filter and antenna. As mentioned in chapter 4 two DPCRLH symmetric unit cells are dual of each other. So short circuited resonator using Symmetric unit cell B (Fig.4.2(b)) has same characteristics as open circuited resonator used in this thesis. So, the short circuited resonator from unit cell B can be used in the design of multi band circuits. The short circuited resonator from symmetric unit cell A and open circuited

resonator from unit cell B can also be examined. The remaining resonators and their input impedance are discussed in the following sessions.

6.2.1.1 DPCRLH short circuited resonators

The short circuited DPCRLH resonator formed from DPCRLH symmetric unit cell A is shown in Fig.6.1.

Figure 6.1 DPCRLH short circuited resonator formed from symmetric unit cell A in Fig.4.2(a)

The input impedance of DPCRLH short circuited resonator in Fig.6.1 is given as

$$Z_{in} = Z_1 // \left(Z_1 // \frac{1}{Y_1} \right) \quad (6.1)$$

$$= Z_1 // \left(\frac{Z_1}{1 + Z_1 Y_1} \right) \quad (6.2)$$

On simplifying ,

$$Z_{in} = \frac{Z_1^2}{2Z_1 + Z_1^2 Y_1} \quad (6.3)$$

Figure 6.2 DPCRLH short circuited resonator formed from Fig. 4.2(b)

Next short circuited resonator is formed from symmetric DPCRLH unit cell B in Fig.4.2(b). It is shown in Fig.6.2. The input impedance of the resonator in Fig.6.2 is given as

$$Z_{in} = \frac{Z_1}{2} // \left(\frac{1}{Y_1} // \left(Z_2 + \left(\frac{1}{Y_1} // \frac{Z_1}{2} \right) \right) \right) \quad (6.4)$$

Simplifying (6.4),

$$Z_{in} = \frac{2Z_1Z_2 + Z_1^2Z_2Y_1 + Z_1^2}{4Z_1 + 4Z_2 + 4Z_1Z_2Y_1 + 2Z_1^2Y_1 + Z_1^2Z_2Y_1^2} \quad (6.5)$$

As already mentioned the short circuited resonator from symmetric unit cell B is dual of open circuited resonator from symmetric unit cell A. This can be found out by comparing (6.5) and (5.21).

6.2.1.2 DPCRLH open circuit resonator from symmetric unit cell B.

Figure 6.3 Open circuited DPCRLH resonator formed from Fig.4.2(b)

The DPCRLH open circuit resonator formed from DPCRLH symmetric unit cell B in Fig.4.2(b). It is shown in Fig.6.3. The input impedance of the resonator in Fig.6.3 is given as

$$Z_{in} = \frac{1}{Y_1} // \left(Z_2 + \frac{1}{Y_1} \right) \quad (6.6)$$

Equation (6.6) can be simplified as

$$Z_{in} = \frac{1 + Z_2 Y_1}{2Y_1 + Z_2 Y_1^2} = \frac{1 + Z_2 Y_1}{Y_1 (2 + Z_2 Y_1)} \quad (6.7)$$

6.2.2 Double periodic single negative metamaterial

In chapter 3 single periodic single negative metamaterial is analyzed and in chapter 4 and 5 double periodic double negative metamaterial is analyzed. In the same way the design of double periodic single negative metamaterial can be tested. The initial design of double periodic effective epsilon transmission line and its S parameter response are shown in Fig.6.4.

(a)

(b)

Figure 6.4 (a) Initial design of double periodic epsilon negative transmission line (b) S parameter response of structure in (a).

6.2.3 DPCRLH in SIW

In chapter 3, the SIW is analyzed by waveguide theory and also epsilon negative theory. CRLH is created in SIW by opening interdigital slots in SIW. Even though DPCRLH in SIW was created in [127, 128], the series capacitance was created by

transverse slots which cannot offer high capacitance value. So DPCRLH design in SIW can be explored further.

Figure 6.5 DPCRLH structure in SIW host line [128]

Author's Publications

Manoj Prabhakar Mohan, A. Alphones and M. F. Karim, "Triple Band Filter Based on Double Periodic CRLH Resonator," in IEEE Microwave and Wireless Components Letters, vol. 28, no. 3, pp. 212-214, March 2018.

Manoj Prabhakar Mohan, A Alphones and MF Karim, "CPW Triple Band Antenna based on DPCRLH resonator" (submitted to IEEE Antennas and Wireless Propagation Letters).

Manoj Prabhakar Mohan, A. Alphones and F. Karim, "Triple band SIW cavity backed slot antenna," 2017 IEEE Asia Pacific Microwave Conference (APMC), Kuala Lumpur, 2017, pp. 698-701.

Manoj Prabhakar Mohan and Arokiaswami Alphones, "Double Periodic CRLH Transmission line for Wideband Performance," 2016 Asia-Pacific Microwave Conference (APMC), New Delhi, 2016, pp. 1-4.

Manoj Prabhakar Mohan, A. Alphones and M. F. Karim, " Design of SIW Resonator Using Epsilon Negative Transmission Line Analysis" (Presented in EUCAP 2019).

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