Effect of polishing disc compliance on the pressure distribution and material removal

Arunachalam, Adhithya Plato Sidharth

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**EFFECT OF POLISHING DISC COMPLIANCE ON THE
PRESSURE DISTRIBUTION AND MATERIAL REMOVAL**

ARUNACHALAM ADHITHYA PLATO SIDHARTH

SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING

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PRESSURE DISTRIBUTION AND MATERIAL REMOVAL**

ARUNACHALAM ADHITHYA PLATO SIDHARTH

SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING

A thesis submitted to the Nanyang Technological University in partial
fulfilment of the requirement for the degree of Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Arunachalam Adhithya Plato Sidharth

Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

5 April 2019

.....
Date

.....
Dr Sridhar Idapalapati

Assoc Prof Sridhar Idapalapati

Authorship Attribution Statement

This thesis contains material from 3 paper(s) published in the following peer-reviewed journals and 1 conference proceeding in which I am listed as an author.

Chapter 3 is published as Adhithya Plato Sidharth A, Sridhar I, Sathyan S (2015) Multi-criteria decision making techniques for compliant polishing tool selection. Int J Adv Manuf Technol 79 (1-4):519-530.

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- A/Prof Sridhar Idapalapati provided the initial project direction and edited the manuscript drafts.
- I prepared the manuscript drafts. The manuscript was revised by Associate Professor Sathyan S.
- The algorithms were implemented in MATLAB® by me.

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- A/Prof Sridhar Idapalapati provided the initial project direction, suggestions in numerical simulations and edited the manuscript drafts.
- All experimentation like machining trials and measurements was conducted by me.
- The numerical simulations were carried out by me using the commercial FEA software ABAQUS ® in NTU.

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- A/Prof Sridhar Idapalapati provided the initial project direction, suggestions in validations and edited the manuscript drafts.
- Dr Takashi Sato provided suggestion on measurement techniques and edited the manuscript.
- All experimentation like profilometer measurements was conducted by me at NTU.
- The algorithms for simulating the 3D topography was implemented in MATLAB by me and the simulations were conducted at NTU.

5th April 2019
.....
Date

.....
Arunachalam Adhithya Plato Sidharth

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Abstract

Polishing is a key manufacturing process which has become inevitable in many industries such as shipbuilding, aerospace, automobile and bio-medical sectors. The compliant polishing tools (CPTs) are commonly used in industries, and there are different variants of the compliant polishing tools such as disc, wheel or belt types. Initially in this dissertation, in order to reduce the ambiguity involved in tool selection, two multi-criteria decision-making methods (MCDMs) methods are considered for a case study involving the selection of optimal polishing tools at different polishing stages. The CPTs are usually preferred to the rigid tools for the reasons like controlled material removal, better conformability to the polishing surface and better blending of finished and unfinished surfaces. Although these CPTs are being used commonly in industries for a long time, the material removal behaviour of these tools has not been investigated intensively.

To study the underlying relationship between the compliance and contact pressure which influences the material removal profile, contact pressure distribution experiments are carried out for tools with different compliance using pressure films. The static pressure distribution showed variations in the contact area and pressure that could be correlated to material removal profile. Polishing experiments are carried out using a robotic polishing setup with force control, at different load values for both ‘Hard’ (low compliance) and ‘Soft’ (high compliance) rubber backing polishing tools. In order to predict the material removal profile, the finite element analysis simulation is carried out using ‘dynamic-implicit’ in ABAQUS® for the grainless disc, followed by executing the UMESHMOTION FORTRAN subroutine in ‘General-static’.

In order to model the material removal profile in compliant tools with coated abrasive discs along with grains, the first step is to model the three-dimensional surface topography of

the abrasive disc. Hence, an abrasive disc with prismatic shaped abrasive grains is considered. Initially, abrasive discs with grit sizes 60 and 120 grit sizes are scanned to obtain the peak height and spatial distribution, which are found to follow a normal distribution. In this modelling technique, the protrusion height is determined not only by the position of the grain in the resin but also by the orientation of the prismatic grains. Using MATLAB® code, iterations are carried by randomly orienting the grains in the three mutually perpendicular axes, till the required statistical parameters are achieved. Later, the surface fitting is performed for the randomly distributed grains, and different 3D surface parameters such as amplitude, functional, spatial and hybrid parameters of the simulated surface are compared with the measured surface. Using the same methodology three-dimensional surface topology is simulated for the irregularly shaped grains too.

To model the material removal profile considering the grains, two different approaches are proposed. In the first approach, using the grain distribution information, the abrasive disc with randomly oriented grains is modelled using python script in ABAQUS®. Using the Johnson-Cook damage model, the simulated material removal profile for three rotations are considered, and the effect of compliance on the material removal is discussed. In the second approach, a semi-analytical model is proposed to simulate the material removal profile. The randomly oriented grains are modelled as conically shaped grains with spherical tips. Using the average dynamic contact pressure distribution from the FEA results, the normal force acting on top of each grain is obtained, and the equivalent indentation depth is computed. Later, using relevant scaling, the complete material removal profile is obtained and compared with the experimental measurements for the hard backing pad.

From the above-mentioned studies, it is evident that tool compliance plays a predominant role in material removal. Presently different tools are used for different compliance, and each time the tool is changed according to required material removal. A novel

active compliant tool is designed to achieve the variable compliance within a single tool. Initially, different designs are proposed, and finally, the rubber tool design with retractable stiffeners is considered based on the stiffeners position in the disk (completely retracted to completely inserted), a range of compliance can be achieved in an analogue sense. Experimental trials are carried out to test the tool at three different positions of the stiffeners, and the material removal profile for different cases is compared. The material removal depth changed with respect to the stiffener positions where maximum material removal depth (with lower contact area) is noticed for the completely inserted stiffeners condition and vice-versa for completely retracted compliant state. Hence, this tool could be considered for high material removal operations like in case of weld seam and also during the blending operations where lesser material removal is needed.

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Chapter 1 Introduction

1.1 Background and Motivation

Abrasive finishing is a key process during the final stages of manufacturing a component, usually carried out after conventional machining operations such as milling, turning etc. Many industries such as aerospace, automobile, bio-medical and other manufacturing sectors which rely on polished dies demand a mirror-like surface finish with minimal number of internal defects and lesser deviations from the desired geometric profile [1,2]. Polishing is a kind of finishing process carried out to (i) reduce surface roughness, (ii) remove undesirable materials like weld seams, burrs, (iii) improve fatigue life, (iv) and add aesthetic sense. In a nutshell, polishing is an important process for parts which largely requires dimensional accuracy and good surface finish [3-5]. These desired characteristics such as high precision and better surface finish were initially obtained by manual and labour-intensive methods. The outcome of the polished surface depends on the skill and experience of the operator. Apart from the end characteristics, the cost and time also come into picture which led to the enhanced technology of robotic polishing.

1.1.1 Robotic polishing

Robots are playing a crucial role in the Industry 4.0 framework of future manufacturing where automation and data exchange are key points by exploiting Internet of Things (IoT). Many countries like United States, United Kingdom, Japan, China, Canada and Singapore are adapting fast to robots as they are 15% cheaper per hour than employing humans and also improves productivity considerably [6]. The robots are widely used in industrial applications such as welding, painting, masking and finishing operations. The common finishing processes generally carried out by industrial robots include machining operations such as deburring, grinding, and polishing. Robotic polishing is one such technique where the process time

reduces by 10 times compared to manual process. It helps to overcome another crucial problem in manual polishing which is the lack of control over the material removal, thereby helping to achieve the desired dimensional accuracy [7]. Hence, automated robotic polishing is suggested to eliminate the problems of manual polishing with reduced process time, cost and improved quality consistency by controlling the tool end effectors [8,9]. The major difference between robotic polishing process and other robotic processes such as grinding and cutting is that the polishing process is performed by force control and not by conventional position control [10]. Even the robotic polishing has its own disadvantages. Robot has lesser stiffness compared to the conventional CNC machines, especially while using robots with lesser payload which are commonly used for polishing operations. Usually, when the tool traces the programmed path on the workpiece during polishing, even a slight deviation in the path can cause high contact force which results in undesired material removal [11]. This is overcome by using force-controlled machining and using flexible polishing tools. In the active force-controlled robot, a force sensor is used to measure the contact force so that the programmed path trajectory or feed velocity is compensated accordingly so that the desired force is maintained in real-time.

1.1.2 Compliant polishing tool

Generally, abrasive tools are classified into bonded and coated abrasives. Coated abrasives are more commonly used in robotic finishing applications. The coated abrasives consist of abrasives glued to the backing materials such as paper or cloth. Since these coated abrasives cannot be directly used for machining, an additional support is provided to these coated abrasives by backing layer like rubber or felt. This arrangement makes the compliant polishing tool more flexible and easy for handling. There are many variants of compliant polishing tools like flap wheel, flap disc, fiber discs, belt type. They can be further categorized based on the type of backing material being used. The mechanical property of these backing materials decides the compliance of the polishing tools. The basic purpose of using such

compliant tools is to enhance (i) polishing the complex shaped workpieces (ii) the flexibility in the tool will result in better conformability of the tool with the workpiece, thereby exerting lesser force, which leads to controlled material removed and a finer finish.

1.1.3 Research purpose

Although these compliant tools are being used for a long time in industries, still there is a lack of understanding concerning the mechanics or physics of polishing. Although the finishing process involves lesser material removal compared to conventional grinding, the final form and dimensional accuracy of the workpiece is dependent on the material removal during polishing. Such profile error becomes more when a constant force is used to polish surfaces with complex geometry. Although the tool is compliant, due to irregular pressure distribution caused by the geometry, material removal will change causing an error in the desired profile. These machining errors could be crucial in applications such as turbine blades where impeller efficiency and pressure ratio are reduced considerably. To overcome such situations, it is important to have a deeper understanding of the underlying mechanics of these compliant polishing tools.

Due to lack of detailed material removal models, the polishing with the compliant tool is usually carried out by trial and error. Based on the operator's experience the compliant tool with specific backing material stiffness, spindle speed, force, feed rate etc., are selected. Such trial and error methods of process variable selection would result in more rework thereby giving way to machining errors. In order to reduce such machining errors, an efficient model to predict the material removal profile while using compliant tools is necessary. Such models will be helpful to industries in cutting costs due to rework and maintaining dimensional accuracy.

Presently, the conventional tools are passive and according to the part to be processed, a polishing tool with relevant backing pad compliance must be selected. This selection process

requires more time. But in the case of an active compliant tool, one tool can adapt well to the most intricate shapes. Additionally, the tool stiffness can be increased for higher material removal rate scenarios such as in the case of weld seam removal. In the future, where robotic processes would be replacing human processes, these kinds of tools with a wide range of compliance would be preferred especially by the aerospace and automobile industries.

1.2 Project Objective

The major objectives of the work are as follows:

- To investigate the effect of backing pad compliance on pressure distribution and material removal profile in the disc type compliant polishing tool,
- To establish finite element and semi-analytical models to predict the compliance effect on pressure distribution and material removal profile, and
- To develop an active compliant polishing tool where the material removal can be varied by changing the tool compliance.

1.3 Project Scope

- To study the pressure distribution and material removal variations caused by tool compliance and force applied by conducting experimental trials in a robotic polishing environment.
- To simulate the pressure distribution and material removal through finite element models and compare with experimental values.
- To model the three-dimensional abrasive disc surface topography by considering the random orientation and distribution of the grains for both regular and irregular shaped grains.

- To develop both semi-analytical and numerical models to predict material removal for the modelled three-dimensional abrasive disc compliant polishing tools along with the backing pads.
- To design and fabricate a novel tool in which material removal depth can be controlled by adjusting the tool compliance. Also, to conduct preliminary experimental trials to demonstrate the working of the tool according to the design.

Due to time limitation, the following aspects or parameters are not included in the scope of the dissertation:

- Since there are many varieties of compliant tools, the developed models for material removal or pressure distribution will be limited to a disc compliant tool.
- The primary focus of the study is to investigate the effect of compliance on the pressure distribution and material removal and derive relevant models to predict the same. Hence, the study is confined to flat aluminium (Al 6061-T6) work coupons.
- The effect of variations in parameters such as tilt angle, coolant, dwell time or feed are not considered in this study.
- The experimental trials are conducted for the active compliant tool by manually adjusting the stiffener positions instead of the closed-loop control which includes actuator mechanisms and relevant feedback system.

1.4 Thesis Layout

The research work in this thesis is arranged into the following chapters as explained below. Chapter 2 presents a comprehensive review of the abrasive polishing process and the compliant polishing tools used so far. It also includes an extensive review of the numerical and analytical models used to predict the material removal in the abrasive machining process. In

addition to it, the importance of an active compliant tool and the existing active compliant tools designs that are reported are also discussed. The existing research gaps and the importance of the present problem are highlighted in this chapter. In Chapter 3, the multi-criteria decision-making methods (MCDMs) are implemented for compliant polishing tools selection. The tool selection process in finishing a concave mould at different polishing stages is demonstrated. In Chapter 4, the details regarding the experimental trials and the finite element model used for pressure distribution and material removal modelling using Archard's abrasive wear model is discussed. The results from the experimental trials are presented and compared with the numerical simulations. Before modelling the material removal profile with grains-workpiece interaction, it is essential to model three-dimensional abrasive surface topography considering the random orientation and placement of the grains. A detailed procedure is discussed in Chapter 5 for both regular and irregular shaped grains. From the obtained grain distribution information, numerical material removal model using Johnson-Cook damage model and the algorithm for predicting material removal model using semi-analytical technique is discussed in Chapter 6.

In Chapter 7, a novel active compliant polishing tool design is discussed. The initially considered designs are highlighted. Also, a detailed design showing the working of the tool is presented. The fabrication, assembly, experimental setup, machining trial, and the corresponding material removal profile values are discussed in detail. In addition, the FEA model to understand the working of the tool is elaborated.

Chapter 2 Literature review

This chapter delineates different polishing processes, types of compliant polishing tools in section 2.1. The contact pressure distribution studies carried out in different types of compliant polishing tools so far is discussed in section 2.2. The material removal modelling based on the contact pressure distribution is a key aspect of this study and the literature pertaining to numerical material removal modelling methodology using Archard wear equation in different applications are discussed in section 2.3. In order to model material removal considering the grain-workpiece interaction, the three-dimensional topography modelling techniques employed by researchers to study the random distribution and orientation of the abrasive grains in different tools are discussed in section 2.4. Later, the numerical and analytical models to predict material removal depth using grain-workpiece interactions is discussed in section 2.5 and 2.6 respectively. The development of a novel active compliant polishing tool is a key aspect of this study and the existing active compliant tool using gasbag polishing, and magnetorheological fluid is highlighted in section 2.7.

2.1 Polishing processes

According to the energy source, basic principles, construction, the polishing processes are segregated as shown in Figure 2.1. Under the coated abrasives, very few studies have been done in the compliant polishing tools, and the detailed groupings of compliant tools are shown in Figure 2.1.

Figure 2.1 Different types of polishing processes [12]

According to the backing pad, the compliant tools are further classified as fibre discs, rubber discs, wool pad, flap discs, flap wheel and gas bag or ballonet polishing. Laser polishing is mostly used for precise polishing of micromachined components obtained from micro-milling, micrometal injection molding (μ MIM) and microelectrical-discharge-machining (μ EDM). The other polishing methods include abrasive flow machining, and chemical process assisted polishing such as CMP and electrolytic polishing. Except for the laser polishing process, all other processes use abrasives in one way or the other. The nature of the abrasives depends on the process which in turn depends on the product's end surface requirement.

As the main focus of this paper is on metallic polishing in the aerospace and automotive industries, we would discuss here more about the abrasive machining process used. Abrasive machining is defined as a material removal machining process which employs abrasive cutting tools. These tools are generally classified into three major types according to the constraints in the abrasive grain.

(i) Bonded abrasive tools: abrasive grains are closely packed into different shapes, the most common is the abrasive grinding wheel. Grains are held together by a bonding material. Bonded abrasives compose of tiny abrasive particles suspended in a ceramic or polymer matrix, represented as vitrified or resinoid respectively. Grinding, honing, superfinishing are processes that fall under bonded abrasive tools.

(ii) Free abrasives: In this type, the abrasive grains are not restrained, but they are free to move. Usually, they are introduced either in oil-based fluids (lapping, ultrasonic machining), or in water (abrasive water jet cutting) or air (abrasive jet machining), or contained in a semisoft binder (buffing).

(iii) Coated abrasive tools: abrasive grains are glued onto a flexible substrate usually which is cloth, paper type materials. Coated abrasives are available in sheets, rolls, endless belts.

Considering the previous works, bonded abrasive tools and free abrasives are explored, and the effect of different machining parameters on the process has been studied. But the compliant tools remain an unexplored domain. This study mainly concentrates on the coated abrasive tools, whose details are as given below.

2.1.1 Coated Abrasive Polishing

The main purposes for using compliant coated abrasive tools are as follows,

- The compliant tool helps to minimize the material removal rate
- It can conform to non-planar surface contours – this shall keep the original required surface form during the finishing process
- It can blend finished and unfinished surfaces – since many times the surface finishing can be selectively applied in some regions of a surface [13]

Generally, coated abrasive tools are also referred to as flexible backed abrasive, which consist of three main components viz. (i) backing sheet (ii) adhesive bond and (iii) abrasive. The abrasive grains are fixed to the backing material using base coat as shown in Figure 2.2(a). Electrostatic deposition is used to apply the abrasive to the backing which is then propelled into the resin by an electrostatic field. The orientation of the abrasive grains on the backing is controlled by the passage of the electric field. In order to fix strongly the abrasive grains, another coating called size coat is used. In some coated abrasive special coatings are made for cooling effects during polishing. Coated abrasives are described in terms of their grit size, shape, type of base and adhesive glue [14,13].

(a)

(b)

Figure 2.2 (a) Typical elements of coated abrasives [14] (b) Coated abrasives [15]

2.1.2 Types of coated abrasive polishing tools

A similar construction is seen in the bonded abrasive used for grinding purposes, but the difference between them is the lack of flexible backing material. The integrated backing

(paper, cloth) as shown in Figure 2.2(a) is different from the detachable external backing pad. As shown in Figure 2.2(b), there is a wide range of coated abrasive products available in the market which can be used for finishing workpiece with different contours. Based on the shape of the tools, they can be classified as disc, wheel, belt etc., which are widely used in industries for a wide range of workpiece geometries, and there are other unconventional tools like polystar type, overlap slotted discs which are mostly used for finishing slots, grooves and internal surfaces.

Figure 2.3 (a) Fibre discs (b) Rubber backing pad with different compliance [15]

The disc type can be classified into general fibre disc, flap discs, slotted disc etc. The general fibre (Figure 2.3 (a)) disc usually resembles the conventional coated abrasive cloth/paper which has a thread on the top. This is in turn attached to different kind of backing pads made of rubber with different hardness. Usually, these rubber pads are classified as ‘soft’, ‘medium’ and ‘hard’ according to their relative hardness values. Depending upon the user requirement, the combination of fibre discs and abrasive backing pads are chosen.

Flap discs have abrasive flaps arranged one below the other alternatively so that once the bottom flaps are worn out, the top flap would participate in finishing. It comes as single piece, where the flaps are attached to the external backing pad, unlike the ordinary fibre discs.

Used in applications such as weld grinding, deburring, rust removal, edge grinding, and weld seam blending.

Figure 2.4 (a) Flap discs [15] and (b) flap wheel [15]

Flap wheels are more like fan type as shown in Figure 2.4, where each blade is made of flexible abrasive flaps. Each of the flaps is attached to a central hub, which in turn is connected to the spindle. It is efficient in penetrating into slots, grooves, tubing, inside of holes and blending corners on curved surfaces. There are different varieties of flap wheels, where foam-based flaps, non-woven type flaps, etc. are available [16,17]. Belt type finishing is one of the common polishing used in industries, robot or CNC assisted to polish complex 3D parts [18,19]. The belt consists of coated abrasives and is attached around at least two rotating wheels. In a few cases, there are more than two wheels. The part to be polished is pressed against the contact wheel in some cases the vice-versa. It depends upon the size of the workpiece, whether the tool or workpiece has to be displaced or feed-controlled for polishing. The hardness of the rubber wheels used would affect the material removal rate. There are also other types of finishing tools like gas bag polishing tool, elastic abrasives which are seldom used in the manufacturing lines as they are not suitable for mass production.

Out of all these tools, this study focuses on disc with external backing pad, because this type of finishing tool is commonly used for both manual and robotic polishing of common industrial parts because the flexibility and polishing area is high compared to other tools.

2.2 Contact pressure distribution in compliant polishing tools

In order to understand the material removal profile in compliant polishing tools, researchers studied the contact pressure distribution using (i) Analytical models - Hertzian contact (ii) numerical simulation (iii) experimental technique.

Hertzian contact models are considered for contact problems involving elastic, frictionless and small strain. But in the case of polishing tools, the backing materials commonly made of rubber, tend to exhibit non-linear behaviour and involves large deformation. Also, the friction involved in the contact region is high. In spite of these shortcomings, the Hertzian contact model was used by the researchers to approximately estimate the contact pressure distribution. Zhang et al. [20] investigated the material removal by studying the contact pressure distribution in cylindrical and spherical shaped compliant tools. The contact area is considered elliptical in shape, and the contact pressure value is obtained from the existing formula, where the tool radius of curvature, workpiece and the Youngs' moduli of these materials are given as input along with the force applied. A similar approach to model the contact stress in polishing tool was implemented by few researchers [21-23]. Using the obtained pressure model, the tool speed and torque model was calculated, and the pressure distribution with respect to the part geometry was investigated.

(a)

(b)

Figure 2.5 (a) Schematic diagram of belt finishing (b) Ellipsoid distribution of contact pressure[24]

For the belt finishing process as shown in Figure 2.5(a), Wang et al. [24] employed the Hertzian contact theory to model the contact pressure distribution as given by Eq. 2.1 to Eq. 2.5 between the wheel and the material to be polished. Similar to the earlier studies, the frictional forces are ignored. Later this pressure distribution values were used for material removal rate calculations. Savio et al. [25] studied the effect of the spherical tool on the mould by modelling pressure distribution using Hertz theory. The assumption of a frictionless process is justified by the presence of lubricant.

$$p(x, y) = -p_o \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2} \quad (2.1)$$

$$p_o = \frac{1}{\pi} \left(\frac{6 F_n E^2}{R_e^2} \right)^{\frac{1}{3}} \quad (2.2)$$

where, contact radii a , b are given by

$$a = \left(\frac{3k^2 \varepsilon F_n}{\pi E(A+B)} \right)^{\frac{1}{3}} \quad (2.3)$$

$$b = \left(\frac{3\varepsilon F_n}{\pi k E (A + B)} \right)^{\frac{1}{3}} \quad (2.4)$$

where $p(x,y)$ is the pressure of the ellipse center, a is the long half axes of the elliptical contact area, b is the short half axes of the elliptical contact area, ε is the elliptic integral, E is the contact modulus, k is approximately the ratio of a/b , F_n is the normal force and effective radius of curvature R_e

$$R_e = \left(AB \frac{A+B}{2} \right)^{\frac{1}{3}} \quad (2.5)$$

where A and B relative principle radii curvatures of elliptical contact which could be derived from major and minor radii of curvature [26].

Another technique to predict contact pressure distribution in polishing tools and workpiece is by using finite element based numerical analysis (FEA) . Since the backing materials are usually hyperelastic in nature, they undergo highly non-linear deformation which could not be captured well in the existing Hertzian contact models. Hence, by creating finite element model with relevant material properties and contact conditions between the tool and workpiece, the contact pressure distribution could be predicted. Sun et al. [26] developed FEA models to simulate various contact conditions of the serrated polishing wheels and aluminium plate. The shortcomings of the Hertzian model in predicting the contact pressure distribution in the case of serrated wheels was observed in this study. The maximum contact pressure was observed in the case of the wheel contacting the convex surface showed higher contact pressure and lower contact area.

The hyperelastic material model was used for the flat and serrated wheel FE models, and the contact pressure distribution is shown in Figure 2.6 (a). Umer et al. [27] created FEA models to investigate the compliance effect of glass fibre backing pad. Since the glass fibre

disc is considerably stiffer compared to other flexible tools, the disc was assigned elastic material property. The load-displacement and contact pressure distribution as shown in Figure 2.6 (b) were obtained from simulations and compared with the experimental values.

Figure 2.6 Finite element models for (a) serrated wheels and aluminium block (b) Glass fibre backing polishing tool pressure distribution [27]

Apart from the Hertzian model, there were few more models using coefficients obtained from experimental trials to predict the contact pressure distribution. Feng et al. [28] established a relation between the displacement of the polishing disk and the force. Then, using the coefficients from the experiments for different tilt angle and disk material, the contact pressure distribution was obtained. Xavier et al. [29] modelled the contact area of the tool by considering the tool displacement and force values. Using this relation, the approximate contact area was calculated and used for the trochoidal polishing path in 5 axis milling machine.

The experimental method to study the pressure distribution is usually done by either dynamic pressure sensors or by using pressure films. There are only a few studies that used experiments to study the contact pressure distribution, that makes this topic more explorable.

The experimental contact pressure was obtained by stacking the pressure films of various pressure ranges together.

Each pressure film assembly comprises an A-film and a C-film, which are used together for the colour to develop. The A-film has a polyester base and micro-encapsulated colour forming layer, while the C-film has a colour developing layer [27]. With the application of pressure, the microcapsules on the A-film break once it comes into contact with the C-film, as indicated in Figure 2.7. The colour forming material reacts with the colour developing material, and red patches appear on the C-film. To obtain the momentary pressure, the measured pressure holding time should be at least 5 seconds. The different pressure film nomenclature and the measurable pressure range is listed in Table 2.1.

Figure 2.7 A-film and C-film structure [30]

Table 2.1 Fujifilm Pressure films [30]

Film Type	Abbreviation	Pressure Range (MPa)
Extreme Low Pressure	LLLLW (4LW)	0.05-0.2
Ultra Super Low Pressure	LLLW (3LW)	0.2-0.6
Super Low Pressure	LLW (2LW)	0.5-2.5
Low Pressure	LW	2.5-10

The pressure film can then be analyzed by converting the intensity from the RGB scale to greyscale using MATLAB code or special software like Fujifilm pressure mapping system.

Umer et al. [27] studied the pressure distribution in glass fibre disc polishing tool. The contact pressure distribution was obtained by stacking these pressure films together, and the pressure values are obtained. FEA model was also created and the simulated results matched with the experimental measurements. Similarly, pressure films were used by Sun et al. [26] to investigate the contact pressure distribution between serrated and flat wheels on the different geometry of aluminium coupons. The dynamic pressure sensor was used by Sun et al. [31] to capture the varying dynamic pressure with respect to time. Later, these values were utilized to compute the material removal as discussed in the following section.

2.3 Material removal models using Archard wear equation

In the case of a traditionally used hard tool like a grinding wheel, the calculation of material removal is straightforward. Since the depth of cut is one of the machining input parameters, the total volume of material removal for single pass is calculated using the Eq.2.6

$$MRR = b_w * a_e * v_w \quad (2.6)$$

where MRR - material removal rate, b_w - ground width, a_e -depth of cut and v_w - table speed (i.e feed rate).

From a practical point, the grinding wheels which are usually made of resin and abrasive are considered rigid as they have negligible compliance. Although they are considered rigid compared to the other polishing compliant tools, researchers have studied the effect of the wheel-grain-workpiece deflections. Hence the depth of cut (a_e) and MRR considered in Eq. 2.6 is an approximate value as there are some losses due to deflection of the grains and wheel [32]. The actual contact length which is dependent on the elastic modulus of the grinding wheel- workpiece interaction was studied by different researchers and various model for contact length was proposed.

As discussed, the hard tools used in the abrasive process like grinding the formula for depth of cut, material removal rate, chip thickness is well established, unlike the compliant tools. Usually, in compliant polishing tools surface roughness remains the primary focus, and hence the changes in material removal are often ignored. Though these compliant tools have been used for a long time, the underlying mechanism of the material removal and the relationship between process parameters and the surface finish obtained is not clear due to the complexity in the material removal mechanisms and many process parameters. Moreover, unlike grinding wheels, there are many varieties of compliant polishing tools whose material removal mechanisms might change owing to the completely different tool geometry and machining actions.

2.3.1 Archard wear model

From the wear phenomenon perspective, the compliant polishing could be considered as abrasive wear. Generally, wear phenomenon is classified into erosive, fatigue, adhesive, abrasive wear etc., and they are in turn sub-classified further [33]. Abrasive wear is said to occur when there is friction between a metal body and a harder body compared to the metal [34]. Usually, the harder particle is a grain, which causes surface scratches, cutting and plastic deformations. Most of the material removal modelling in abrasive machining are either available for three body wear or two body wear.

The two commonly used models for wear study are Archard's wear equation and Preston's equation. According to Archard wear rate is given by (Eq.2.7),

$$V = \frac{K * P * s}{H} \quad (2.7)$$

where, K , is the non-dimensional wear coefficient, $k = (K/H)$ is specific wear rate ($\text{m}^2 \text{N}^{-1}$), V the volume loss or wear volume, P the applied load, s the sliding distance, and H

the material surface hardness. K has much physical meaning, but usually, it is conceived as the proportion of the asperity contacts that could generate wear debris [35]. Preston's equation (Eq.2.8) and Archard's wear equation are similar, although Preston proposed this model before Archard, this was mainly used for glass polishing, and now it is mostly used for chemical mechanical polishing (CMP) processes. The Preston equation can be obtained by modifying Archard's equation and is given by

$$w = \int_{t_1}^{t_2} k_p * p * v * dt \quad (2.8)$$

where 'w' be the wear depth, p ($=P/A$) is the contact pressure, and v is the sliding velocity.

The material removal process or the abrasive wear when a conical abrasive particle penetrates and slides over a softer material is schematically shown in Figure 2.8.

Figure 2.8 Conical abrasive grain cutting a groove

The wear volume of soft metal ' V ' when a hard conical grain is indented and ploughed through a distance ' L ' bears a relation similar to adhesive wear equation. The final form of the abrasive wear equation is given by,

$$V = \frac{2}{\pi \cdot \tan \theta} \frac{WL}{H} \quad (2.9)$$

where θ is the mean angle of different conical grains and H is Hardness. This equation is further simplified to take the form as shown in the Eq 2.10 which is similar to Archard wear equation.

$$V = k_{ab} \frac{WL}{H} \quad (2.10)$$

But it is usually stated that wear mechanisms are not mutually exclusive, there are always two or three different wear mechanisms taking place simultaneously whose percentage contribution may vary according to the conditions. From the above discussion, it is evident that contact mechanics plays a crucial role in determining the contact pressure and area at the surface level of the work coupons. The material removal is obtained by determining the contact pressure distribution between abrasive grain and workpiece, using simple Hertzian law in the literature.

Zhang et al. [20] investigated the material removal in spherical and cylindrical shaped compliant polishing tool on concave and convex shaped geometries. The Hertzian contact model was used, and the contact area was considered elliptical in shape. The material removal profile was derived by using the Archard wear equation and the Hertzian contact model. This model was helpful to understand the effect of tool and workpiece curvature along with the machining parameters (force, spindle speed, feed etc.,) on the material removal profile. A similar study was carried out by Wang et al. [24] in belt grinding study, where the contact area between the tool and surface was modelled as an ellipse, and the material removal rate was derived using factors like the pressure distribution and the tool dwell time which is, in turn, a function of feed. Another method, where the experimental dynamic pressure data was used along with Archard wear formula to predict the material removal profile in belt grinding process by Sun et al. [31]. Different wheel types such as flat and serrated contact wheels were

taken to demonstrate the effect of dynamic pressure on the material removal profiles in both flat and convex work coupons at different load ranges.

Tsai et al. [36] used Archard wear formula to model the material removal in the compliant polishing tool in order to achieve uniform material removal. The wear coefficient was obtained by carrying out experiments for different conditions and using a linear fit. Later, this coefficient was used to estimate material removal by Archard formula. Tam et al. [37] proposed a wear index to calculate the material removal depth orthogonal to the tool path. Similarly, the pressure distribution was assumed to be Hertzian at the contact between the tool and surface. Material removal profiles were investigated through experiments conducted to polish cylindrical and concave surfaces. Feng et al. [28] developed an algorithm of cutter location data for CNC machine in order to obtain a mirror finish on curved parts using the disc-shaped polishing tool. In order to obtain the mirror finish, pressure distribution estimation and appropriate step over size calculations were done.

2.3.2 Archard wear in numerical simulation

2.3.2.1 Arbitrary Lagrangian-Eulerian (ALE) adaptive meshing

Adaptive meshing is generally used in simulations where more distortions of elements are expected. In cases where obtaining dynamic pressure distribution using an experimental method or analytical formula was difficult, FEA models were used. In ABAQUS, Arbitrary Lagrangian-Eulerian (ALE) adaptive meshing could be employed using a combination of Lagrangian and Eulerian analysis methods where the mesh is tied to the material in the former and mesh independent of the material in the later. Adaptive meshing consists of two fundamental steps in ABAQUS simulation, (1) sweeping where a new mesh is created, (2) remapping the solution variables from the old mesh transferred to the new mesh through advection.

2.3.2.2 Wear modelling using UMESHMOTION subroutine

In order to predict wear, FE analysis were carried out in commercial FEA packages like ABAQUS® and ANSYS®. Researchers implemented the wear prediction algorithm through the subroutine which was coded using FORTRAN in ABAQUS®. UMESHMOTION is a user-defined FORTRAN subroutine in ABAQUS where the motion of nodes is defined for the node set selected in the adaptive mesh constraint node set. The node set contains usually the nodes at the interacting surface of the part for which worn profile needs to be obtained.

UMESHMOTION was used to displace the surface nodes in the direction of the local normal by an amount calculated using above discussed Archard's wear equation. A node wears only when it experiences pressure while passing through the contact interface between the two bodies. Therefore, pressures acting on each particular node/point were integrated along the sliding direction, corresponding to one rotation for the computation of the local wear increment. The new contact pressure is updated by transferring the material quantities from the old location to the new location. The procedure, as depicted in Figure 2.9 is repeated until a predefined maximum sliding distance is reached.

Figure 2.9 Material wear using Archard wear equation in FEA [38]

A similar approach was carried out to predict wear in bearings, polymers, brake pad etc., In these approaches, based on the numerical contact pressure and slip values at the interface, the geometry of interest is modified continuously by Arbitrary-Lagrange Euler (ALE) meshing.

Hegadekatte et al. [39-41] implemented FE based Archard wear model using umeshmotion to predict wear in tribosystems. Bortoleto et al. [42] implemented ALE to model wear in unlubricated sliding wear occurring in pin on disc tribotests. A global wear coefficient was used in the numerical model to predict wear for different load ranges. The model predicted wear loss for different lower load ranges. Rezai et al. [38] implemented a similar approach in predicting wear in radial sliding bearings by a two-dimensional plane strain FE model. For a fixed sliding velocity, the wear depth for different normal pressure was compared between experimental and simulation. Similarly, a wear model in ABAQUS® to simulate wear using UMESHMOTION in polymer-metal contact pair was proposed by Martinez et al. [43]. In order to simulate the entire travelled distance, an accelerated wear model was used where the existing model is multiplied with a certain factor at regular intervals to replicate the entire sliding distance. Unlike pin on disc where the process is simpler to model, because of complex contact area and pressure, the material removal modelling in the compliant polishing process is more complex. But if this technique is implemented, it reduces the computational time involved in simulating extensively the entire micromachining mechanisms which require using a finer mesh in numerical simulations. When an approximate worn profile is needed after a certain number of cycles, instead of modelling the in-depth mechanisms, Archard's wear equation can be implemented by UMESHMOTION subroutine. Although there are disadvantages like (i) the worn profile will not be matching exactly the actual experimental values because of more randomness in the process and the simplifications assumed and (ii) the

output result is much dependent on the wear coefficient value which differs more with a change in contact condition.

2.4 Three-dimensional surface topography modelling

Modelling of material removal when abrasives and workpiece interact requires consideration of abrasive disc surface topography. Abrasive surface topography modelling involves, understanding the geometry of the abrasive grains, their spatial distribution in all three axes and their orientation. The abrasive machining processes are usually complex where a large number of irregular cutting edges (grains) interact with the workpiece material at considerable high speed. The better prediction of material removal profile relies on the better modelling of the abrasive surface topography which helps to investigate the micro-cutting phenomenon. It has been proved that the orientation and spatial distribution of grains have an effect on the machining parameters like force, power, surface finish etc. There is a considerable amount of work carried out in grinding wheels which forms the basis of modelling in coated abrasives. In this section, topography modelling not only pertaining to coated abrasives but also grinding wheels are discussed in detail to get an overall methodology involves abrasive disc topography modelling [44-46]. The main topography modelling techniques can be divided into two-dimensional and three-dimensional techniques. Both the models include the key information like grain size and placement distribution, protrusion height distribution, but in the case of 3D topography modelling technique, the surface formed at last is also three dimensional in nature, which makes 3D topography closer to the reality. So, the difference lies in the additional last step carried out to fit a three-dimensional surface for the gathered distribution information of the grains.

Chen and Rowe [47,48] modelled the topography by considering simple cubic unit cells . The evenly shaped spherical grains were placed at edges of this cube and later randomly

translated in the three-dimensional space of the grinding wheel by using relevant geometric transformation. The grain size was calculated from the mesh size value obtained from the grinding wheel. The basic difference between grinding wheel and coated abrasives lies in the arrangement of the grains. The grains in the grinding wheel models were usually stacked one above the other, but in the case of coated abrasives usually, only one layer of abrasives is present. Dressing phenomenon was considered, and the expected profile was modelled considering the random shape of grains after dressing. The actual topography contour and the expected dressing trace matched well. Since, dressing phenomena are not considered in coated abrasives which are usually disposable after use, the modelling of topography after dressing is not discussed in detail. A similar modelling technique discussed above was used by researchers [49-52] to first understand the grinding wheel topography and a simulation model for surface topography formed after the grinding process was obtained by considering the grain trajectory.

Koshy et al. [53] developed a stochastic approach to the model surface topography of the grinding wheel. The simulated abrasive grains were randomly distributed in the volume of a cube that represented the resin layer. In all the studies, the three major parameters considered were (i) grain size distribution (ii) protrusion height distribution, and (iii) inter-grain spacing. By obtaining the information regarding these parameters from the abrasive surface, the topography can be modelled.

The above-discussed cases are examples of two-dimensional (2D) surface topography where only information about grain shape, and its distribution are mainly focused. Whereas, in the case of three-dimensional (3D) topography modelling, it gives a complete modelled surface topography which is realistic and similar to the grinding surface.

Hegeman [54] proposed a model to simulate the grinding surface topography by arranging the three-dimensional ellipsoidal grains randomly in the two-dimensional lattice. The XY plane was initially divided into small cells whose sides are calculated from the grain

density. One grain was placed in each cell, and the grain's position in each cell is randomized. Using the protrusion height and spatial distribution of the grains, the simulated grinding wheel surface is shown in Figure 2.10.

Figure 2.10 (a) Ellipsoidal grains distributed in lattice [54] (b) Simulated surface after surface fitting [55]

Apart from the ellipsoidal grain shape, Feng et al. [56] modelled the abrasive grains in the grinding wheel as conical grain tips that are placed in a bonding material. The properties of the grinding wheel, protrusion height, the grain distribution information along with the angle and diameter information of these grains were taken into consideration. These grains were placed, and the relevant sinusoidal function was used to generate the resin layer. Another approach to model the three-dimensional grinding wheel topography considering dressing phenomena was carried out by Woodin [55]. The grains were considered as different sided polyhedrons using the data distribution from the number of faces in the grains, aspect ratio and the grain size. Later, using the protrusion height information and spatial distribution data, the three-dimensional surface was obtained by applying relevant surface fitting.

In a different approach, Nguyen et al. [57,58] proposed a procedure where the surface was generated by the transformation of a random field. The required conditions for the

transformation were discussed. The probabilistic characteristics of the two surfaces were found to be matching. Then using the trajectory equations, the surface grinding profile was obtained.

The above-mentioned modelling techniques were done mainly for grinding wheels. For coated abrasives, very few works were carried out. Wang et al. [59] proposed a stochastic simulation approach for the zirconia alumina abrasive belt. The grains were considered shuttle shaped (conical base with spherical tip). By considering the same above-mentioned parameters like protrusion height, grain distribution, conical angle and tip radius distribution parameters, the 3D topography modelling was carried out. Before extraction of the experimental parameters, it was essential to filter noises from the measured data. Hence, autocorrelation and power spectral density analysis were used to filter the noises. Moreover, (S_{al}) autocorrelation length and (S_{tr}) texture ratio was obtained from the autocorrelation function. The arrangement of the grains randomly in the 2D space is governed by a minimum distance value in order to avoid overlapping of grains. The protrusion height, spatial distributions for three grit sizes such as #80, #100 and #120 were considered in this study.

(a)

(b)

Figure 2.11 (a) Experimental protrusion height of grains (b) Distribution of cone angle of the grains for #80,100,120, and (c) Spatial distribution of the grains following normal distribution

Figure 2.11 shows the random distribution of grains and other statistical data distribution obtained in the topography modelling study carried out by Wang et al. [59]. The parameters generally considered for topography modelling, the statistical distributions that these parameters follow, the assumed grain shapes are listed in Table 2.2 below

Table 2.2 Parameters considered by various researchers for topography modelling

Reference	Model description	Grain shape	Parameters and their distribution
Koshy et al. [60]	Grinding wheel	Spherical	Protrusion height- Uniform distribution Grain spacing- Gamma distribution Grain size- Normal distribution
Cao et al. [49]	Grinding wheel	Spherical	Protrusion height- Normal distribution Grain interval – Fixed value Grain size- Fixed size
Zhi et al. [61]	Grinding wheel	Spherical and Decahedral	Protrusion height

			Grain spacing- Uniform distribution Grain size- Normal distribution
Li et al. [62]	Grinding wheel	Spherical, truncated rhombic dodecahedron	Grain density Protrusion height- Normal distribution Grain spacing- Normal distribution Grain size- Normal distribution
Hegeman [54]	Grinding wheel	Ellipsoidal	3 axes of the ellipsoids – Normal distribution Grain density Grain position in the lattice- Uniform distribution
Feng et al. [56]	Grinding wheel	Conical shaped	Protrusion height- Normal distribution Cutting edge angle of the cone- Normal distribution Grain distribution- Uniform distribution between -0.5 and +0.5
Woodin et al. [55]	Grinding wheel	Polyhedron	Protrusion height- Beta distribution Grain distribution- Beta distribution Aspect ratio-Weibull distribution Grain size-Normal distribution Number of facets- Gamma distribution
Chakrabarti and Paul [63]	Grinding wheel	Square pyramidal	Protrusion height- Normal distribution Included angle of pyramidal grain = 120°

			Grain distribution -Uniform distribution
Pengfei et al. [64]	Grinding pad	Spherical	Protrusion height- Normal distribution Grain distribution- Arranged in body-centered cell (BCC) and randomized using normal distribution Grain size- normal distribution
Zhen et al. [65]	Grinding wheel	Spherical	Grain diameter- Normal distribution Spatial distribution- Uniform distribution
Wang et al. [59]	Coated abrasive belt	Shuttle (cone with spherical top) shaped	Cone angle- Normal distribution Tip radius- Gamma distribution Protrusion height- Normal distribution Spatial distribution- Normal distribution

In the three-dimensional topography modelling the last step is fitting a surface to the grains. Feng et al. used a sinusoidal curve function to fit a surface replicating the resin bonding layer. In general, there are different surface fitting parameters such as least square [66], cubic [67] or bi harmonic interpolation [68]. Woodin [55] used cubic interpolation surface fitting in MATLAB® to obtain the final three-dimensional surface replicating the measured abrasive surface. The selection of an appropriate surface fitting option depends upon the expected three-dimensional surface parameters that need to be achieved from the simulated surface.

Once after simulating the surface, the three-dimensional surface parameters such as amplitude, functional, spatial parameters are compared. The amplitude or height parameters such as S_q (Root Mean Square), S_{sk} (Skewness), S_{ku} (kurtosis), functional parameters such as S_c (volume that the surface would support from 10% to 80%), S_{ci} (S_c relative to S_q), S_v (volume that the surface would support from 80% to 100%), S_{vi} (S_v relative to S_q), hybrid parameters such as S_{ds} (summit density), S_{sc} (average of principal curvature of summits in the sampling area), S_{dq} (RMS of surface slope) and spatial parameters such as S_{al} (autocorrelation length) and S_{tr} (surface texture ratio). A detailed explanation and usage of each of the individual parameters can be found elsewhere [69-73].

2.5 Material removal models using grain-workpiece interaction

In section 2.3.1, the methodology carried out by researchers to calculate wear depth from directly from Archard wear equation without considering the grains was discussed. That approach could be considered as a macro-scale approach where no grain-workpiece interactions were considered. In order to model the material removal by considering the grain-workpiece, the material removed by each grain was obtained using contact mechanics and including relevant tool kinematics. This method is considered as a micro model approach, where the key focus lies in the grain-workpiece interaction. This type of model was initially used in chemical mechanical planarization (CMP) process which was a free abrasive process. By taking into account, the interaction between the pad – grain and grain-workpiece, the indentation depth was calculated, and material removal profile was computed. The pressure distribution between pad and workpiece was given by Greenwood and Williamson model [74]. Xie et al. [75] developed a model to predict the material removal for a free abrasive particle interacting with the workpiece and soft pad. The grains were considered spherical, and indentation depth in the both elastic and plastic region was considered in this model. The abrasive particle size, the elastic modulus of the pad and the contact pressure played a

predominant role in wear depth. Fan et al. [76] extended the model and studied not only the effects of conventional parameters used in prediction of free- abrasive process (FAP) like normal force, angular velocity, spindle speed but also investigated the effect of tool path curvature, grain size, polishing slurry properties.

Jeng et al. [77] developed a model to predict the material removal rate in CMP. The contact area between the polishing pad – grain- wafer surface was calculated by employing a micro-contact model taking into account the elastic, elastoplastic, and plastic. The main difference between the free abrasive modelling technique and the fixed abrasive process was (i) the contact between the pad and the workpiece was not relevant in the fixed abrasive process, as there would be always abrasive between them (ii) additional parameters such as slurry concentration was not relevant.

Figure 2.12 Algorithm for material removal modelling with grains using indentation

In the case of a fixed abrasive process, which is relevant to the present study, Wang et al. [78] modelled the material removal considering grain-workpiece interaction. These types of models generally involved three main steps (i) finding pressure distribution between the tool and the workpiece based on contact theory (ii) then the contact pressure distribution was discretized into unit cell, and the indentation depth of each grain at each cell was calculated such that the contact force (contact pressure per unit cell) developed is equal to the applied force at that cell. (iii) then the calculations were carried out for the complete experimental time by considering the number of cutting grains participating in each cell and then integrated to get the profile. The flowchart shown in Figure 2.12 implemented by Wang et al. [78] describes the overall methodology adopted to model 2D material removal profile.

Wang et al. [78] considered the protrusion height distribution to follow normal distribution as shown in Figure 2.13 (a) . The grains were considered to be spherical in shape and conical. The relationship between the contact pressure and the indentation depth was computed by analyzing the interaction of the abrasive grains and the workpiece. The contact pressure profile was extracted by assuming the contact pressure distribution is Hertzian at the contact between the tool and the workpiece surface. Based on the indentation depth, the elastic and plastic regimes were differentiated as shown in Figure 2.13(b). $\delta_{\max}$ was considered as the transition point from elastic to plastic region. The theoretical model of linear removal intensity was calculated taking into account the removal volumes of all abrasive grains participating in the cutting process. It was integrated to get the material removal profile.

Figure 2.13 (a) Protrusion height distribution of grains (b) Spherical grain indentation [78]

In Figure 2.13, the terms d_{min} , d_{max} , d_m refer to the least, maximum and the average distance between the grain tip and the tool reference surface. The term ' h_0 's is the distance between the normal distribution mean and the workpiece surface, ' m ' is the maximum indentation depth. The indentation formula for various shapes such as sphere [79,80], ellipsoidal [81] or conical shaped with spherical tips were already studied by researchers. By computing the pressure distribution and employing the relevant numerical technique, the indentation depth could be calculated for a particular value of average height ' d_m '.

A similar approach was implemented by Junde et al. [82], in addition to the spherical shape assumption, cone with spherical tip was investigated. The protrusion height of these grains was considered to follow normal distribution. The polishing process was decomposed into elastic, plastic for the conic with spherical tip grains. Experimental and simulated values were validated. Wang et al [83] developed an analytical model for predicting material removal profile in large mould polishing with coated abrasives. Similar to the previous model, this is based on probabilistic theory. The material removal depth was calculated by integrating the linear removal intensity or the removal depth per unit time along the polishing path. The material removal depth for annular and circular abrasive tool for straight line and the curved

path was modelled. The influence of process parameters such as contact pressure, feed rate, spindle speed and path on the material removal depth was investigated.

2.6 Material removal studies using numerical simulation models

Earlier, material removal model to predict material removal depth using Archard wear equation and analytical equation model using statistical theory and contact model was discussed. To accurately take into account the wear behaviour, FE model can be used. FEA software to predict the material removal in cases of grinding phenomena was used widely. [84,85].

The micro scale models would involve the details of the interaction between the grains and the workpiece, including its surface finish. In this method, it is possible to simulate the real surface ploughing and cutting actions. When set failure criteria is reached, the element is deleted, and thus material removal is achieved. But the size of the element needs to be such that it represents the material removal in a practical manner. If the mesh size is big, the entire element would be deleted, giving over predicted material removal values. So, the mesh must be well refined such that the material removal is acceptable [86]. Since in polishing the material removal is in microns, the surface level needs to be meshed very fine. The problem with this method is the computational cost involved.

Single abrasive study (micro study) is used to investigate the actual material removal model as the contact between workpiece and grains are considered [87]. In addition to estimating the material removal during the finishing process, FEA software also serves as an important tool to investigate the different mechanisms. The three different mechanisms as depicted in Figure 2.14 in abrasive machining like grinding , polishing are (i) rubbing (ii) cutting and (iii) ploughing [88].

Figure 2.14 Three mechanisms in material removal process in grinding and polishing [88]

Cutting and ploughing are the most important mechanisms in abrasive machining process like grinding and polishing where they dominate the material removal rate. Rubbing is a phenomenon which takes place only in the elastic limit of the workpiece material and does not contribute much to the material removal. Since there are many grains attached to the wheel, the interaction of one grain with the material will influence the next contacting grain. In order to reduce the complexity involved in studying the entire setup, only single grain is taken to study the mechanisms involved as it is easy to relate the scratch developed to the single grain for which the height and orientations are known. There are a large number of grinding experiments with a single abrasive grain to study the mechanisms and the effect of parameters [89-91].

Ram et al. [92] developed a 2D model of an abrasive grain using elasticity theory. The stresses induced due to elastic stresses caused by impact and sliding was investigated. Based on the Hertzian contact theory, they used LS-Dyna® implicit finite element analysis to validate their experimental results.

Figure 2.15 (a) Simulated and experimental scratch surface (b) FEA von-mises stress contour of the scratch profile [93]

Another extensive 2D negative rake angle study was carried out by Ohbuchi et al. [94] which largely replicated the grains in the grinding and polishing processes. They studied the effect of cutting speed on the chip shapes. Doman [93] investigated the rubbing and ploughing phases of single-grain grinding. A single grain FE model was developed to simulate rubbing, ploughing phenomena. The scratches as shown in Figure 2.15 were generated using the element erosion technique, where elements get deleted upon attaining a failure strain.

The effect of cutting speed and different depth of cuts of a spherical grain during grinding was investigated by Anderson et al. [88], it was found that normal forces kept increasing as cutting speed increases and the tangential forces kept decreasing as it reduced. From the scratch profiles, the transitions of the cutting mechanisms were investigated. In a similar study, Anderson et al. [95] used a spherical single grain model and truncated model to study the effect of grain geometry on normal and tangential forces.

Opoz et al. [96,84,97] carried out a series of studies to simulate the rubbing and ploughing phenomena. The simulation studies were carried out by considering the grain to be spherical in shape. The key focus of the study was to model ploughing phenomena which

occurs in the plastic zone. ABAQUS/Standard® was used to simulate ploughing phenomena because the only plastic flow of the work coupon is simulated.

The material removal can be modelled by using Johnson Cook (JC) damage model which is predominantly used damage model [91,98]. Several constitutive models are used today in metal cutting simulations. More reliable results in terms of chip thickness, residual stresses, cutting forces were obtained from the JC model compared to model like Oxley, power law, Bodner-Partom etc. [99,100]. This model takes into consideration the effects of strain hardening, strain-rate (viscosity) and thermal softening. The Johnson-Cook plasticity model is given by Eq.2.11

$$\tilde{\sigma} = (A + B\bar{\epsilon}^n) \left[1 + C \ln\left(\frac{\dot{\bar{\epsilon}}}{\bar{\epsilon}_0}\right) \right] \left[1 - \left(\frac{T - T_{room}}{T_{melt} - T_{room}} \right)^m \right] \quad (2.11)$$

where A - initial yield stress (MPa), B - hardening modulus (MPa), $\bar{\epsilon}$ - equivalent plastic strain, n - work-hardening exponent, C - strain rate dependency coefficient (MPa), $\dot{\bar{\epsilon}}$ - plastic strain rate, $\bar{\epsilon}_0$ - reference strain rate, T - temperature at a given calculation instant (°C), T_{room} - room temperature in °C, T_{melt} - melting temperature in °C, m - thermal softening coefficient

Although many studies were carried out to study the mechanisms using single grain study, it is still challenging to validate the simulation and experimental values as there are many randomneses involved like grain orientation, height distribution, interactions among them, wear of grains, temperature and correlating the pressure, force at high speed with the numerical results is very challenging.

2.7 Active compliant tool

The material removal rate is highly influenced by the contact pressure between the polishing tool and workpiece. Hence for a constant force and speed operating condition, the backing tool stiffness plays a crucial role in altering the contact area and maximum material removal depth. Presently in industries, according to the contour of the surface to be polished and the nature of the polishing operation such as rough or fine finishing, the compliance of the tool is considered. For example, a felt pad which is very compliant in nature, cannot be used for removing weld seams and a very stiff pad made of glass fibre reinforced polymer matrix composite cannot be used to polish intricate surfaces or blending two surfaces. Hence considerable space for storing these existing tools, and time is taken to change each time is an added problem. Hence, a single tool to replace these tools and achieve a range of compliance would be very helpful for the industries.

Jin et al. [101] developed an active compliant tool using flexible gasbag tool to change the stiffness by controlling the gas pressure. Material removal rate model is based on Preston equation which is a function of linear velocity and contact pressure. Various factors such as gasbag pressure, inclination tool angle, spindle speed, material properties of gasbag and feed were considered for simulating the material removal profile.

Figure 2.16 Gasbag polishing setup [102]

The gasbag polishing tool of Ji et al. [102] could be used in both CNC and robotic polishing system. Figure 2.16 shows the complete setup of the gasbag polishing tool. By controlling the pressure to the gasbag, the contact pressure was changed and thereby altering the material removal profile. In this study, they compared the surface roughness of the free-form mould surface polished by both the conventional and gasbag polishing tool. The following advantages were reported for the gasbag tool compared to the existing tool (i) its ability to conform the free-form contours (ii) polishing efficiency and (iii) fine finish quality (surface roughness reduction).

Figure 2.17 Modified inflatable gasbag with MR fluid [104]

The same gasbag polishing tool was modified to have a lining of the magnetorheological fluid bag [103] and thereby providing an additional control over the compliance of the tool as shown in Figure 2.17. In addition to the air pressure which inflated the gasbag to provide additional stiffness, the magnetorheological fluid was activated by an electromagnet. Apart from the earlier discussed factors like spindle speed, feed rate, tilt angle, the diameter of the tool, the internal pressure and rheological behaviour of the flexible rubber gasbag played a key role in deciding the contact pressure and material removal profile.

2.8 Research Gaps

This chapter reviewed the existing literature on compliant polishing tools and the different material removal models. Also, the need for an active compliant tool and the existing such tools are also discussed in this chapter. From the above-mentioned literature review, the key issue areas are identified as listed below.

2.8.1 Compliant Polishing tool

- There are large varieties of compliant polishing tools and selection of suitable tool for a particular scenario is time consuming, complex and highly dependent on the operator's choice. Hence, a methodology to assist in the selection of optimal tool for different stages of polishing is required to provide clarity in the selection process.
- An in-depth understanding of the material removal by the compliant polishing tools should be done by studying the contact pressure distribution due to the tool's compliance (material properties). Also, the effect of different tool compliance levels on the pressure distribution needs to be studied.

- Very few studies were carried out using the finite element model to understand the behaviour of the tool compliance and its effect on the contact pressure. Such numerical technique for a disc type compliant polishing tool with different rubber pad compliance is not studied extensively. Hence, simulation of this tool helps for a better understanding of the compliant tool-workpiece interaction.
- By investigating the contact pressure, the material removal could be predicted by establishing a relation between pressure and material removal. In order to implement this technique, from the present research techniques, either complex experimental measuring instrumentation is required to obtain the dynamic contact pressure or only static pressure is calculated using the Hertzian theory. A new methodology to predict the material removal from the dynamic contact pressure using Archard wear model by using the finite element model can be implemented. This will provide an alternative, and computationally less expensive simulation technique to predict the material removal with lesser computational cost.

2.8.2 Material removal models with grains

- In the literature, there are various models proposed to model grain distribution in abrasive tools. The surface generated using these models are mainly taking parameters such as grain protrusion height and spatial distribution along with the grain diameter distribution. These studies are mostly for grinding wheels. There are very few studies that have developed a model to simulate three-dimensional topography for the coated abrasive disc, and there is a lack of studies especially in the area of precision-shaped grains. Precision-shaped grains are being developed by various abrasive brands, and their performance is considered to be better than the conventional randomly shaped grains. Another key parameter that needs to be considered is grain orientation. In the

case of irregularly shaped grains quantifying the orientation is complex, but in the precision-shaped grains, the orientation angles play a key role. Also, it helps to match more accurately with the experimental values of three-dimensional surface parameters which are not carried out by many researchers.

- Material removal model using the grain distribution information were carried out by a few researchers. But to extract the initial pressure distribution between the tool and workpiece, they considered it to be Hertzian condition. Also, only a 2D profile was modelled. Hence, in order to study the effect of tool compliance on contact pressure distribution and in turn on the material removal profile, a three-dimensional analytical model to predict the material removal profile will be more suitable.
- Presently numerical simulation for material removal is only restricted to single grain study in the abrasive machining process. The effect of polishing tool compliance along with the complete grain distribution using finite element model has not been explored more.

2.8.3 Active compliant tool

- A novel active compliant tool to control the tool compliance while machining will be more helpful in industries. Presently, in literature, active compliance is achieved by gasbag polishing which is more complex and not suitable in an abrasive environment. Hence, a more realistic design using mechanical elements needs to be considered.

Chapter 3 Compliant polishing tool selection using Multi-criteria decision making algorithms

Several operating polishing parameters for polishing are selected based on the operators' past experience without major scientific rigour. This chapter explores the multi-criteria decision making process in selecting the most suitable compliant polishing process based on given objectives and constraints, which could be either compound or multiple in number.

Two MCDMs viz., AHP (Analytic Hierarchy Process) and fuzzy implemented VIKOR methods are considered for the case study involving the selection of optimal polishing tools at different polishing stages. Since its inception from 1980s, AHP is being widely used by decision makers. AHP method can be used alone and in combination with other decision making methods. Armillota [104] used pair wise comparison AHP to study different rapid prototyping processes. In the same study, an enhanced parameters-based approach was used to improve the efficiency of AHP and also to decrease the uncertainty. Fuzzy based AHP was implemented by Sun et al [105] to study and evaluate the three types of grinding fluids. Ayag et al [106] employed the fuzzy based AHP to evaluate different alternatives for selecting optimal machine tools. Fuzzy AHP technique is used to weigh the alternatives under multiple attributes, and then the fuzzy values are defuzzified to obtain the real number values for comparison. AHP was also used in combination with other MCDM process such as PROMETHEE and DEA also [107-109]. The AHP method is based on the pair wise comparison method, where at a time pair of attributes or alternatives is taken and compared one with the other. The algorithm for AHP could be found elsewhere [110-113].

VIKOR method is another MCDM which is a widely used process selection method in manufacturing and other industries. Anvari et al. [109] implemented VIKOR method to evaluate and rank the lean tools in manufacturing. VIKOR implementation was divided into three types viz., alternatives with common criteria, without common criteria, with integrated common, and uncommon criteria for selecting the appropriate lean tools. Chatterjee et al. [114] implemented VIKOR method and ELECTRE to select material with respect to criteria such as specific strength corrosion resistance, specific modulus etc. Similar study was carried out by in selecting an industrial robot for a given industrial application and the rankings obtained from the two methods were compared [115]. The algorithm for VIKOR method could be found elsewhere [116-120].

In this work, to overcome the uncertainty in selecting the polishing process and implementing the two MCDMs, a case study is considered. In this study, the alternatives and attributes or criteria are given weights using AHP and Fuzzy based VIKOR. Then the rankings obtained from both these methods are compared. The case study is only meant to explore the potential behind MCDMs as a decision-making tool in polishing processes. In both these case studies, linguistic variables are used to assign weights to the attributes and alternatives.

3.1 Analytical Hierarchy Process (AHP) Methodology

The general AHP procedure is briefly stated here. Initially, the objective of the problem, the criteria and alternatives involved in the problem should be stated clearly. Then, pairwise comparisons are made within the criteria (for each pair of criteria combinations), and the comparison matrix is formed. The priority vector and averaged weights are calculated from the comparison matrix based on Table 3.1. Similarly, pairwise comparisons are made within alternatives for every criterion and weights are calculated for every comparison matrix formed. Then the rankings of the alternatives are obtained from the computed overall composite

weights. Care should be taken to check the consistency of the input using the consistency ratio (CR) values calculated based on the Random Consistency Index (RI) numbers as given in Table 3.2

Table 3.1 Fundamental intensity scale for comparison matrix [121]

Intensity	Explanation
1	Two components (attributes or alternatives) are equal with respect to the objective
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme importance

Table 3.2 Random Index numbers [121]

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

CR value should be less than 0.1 to ensure the consistency of the input values. A detailed mathematical procedure for the calculations of priority vectors, CR, CI and $\lambda_{\max}$ are discussed elsewhere [111].

3.2 Fuzzy VIKOR Method

VIKOR method is another MCDM which is widely used process selection method in manufacturing and other industries. The algorithm involved fuzzy based VIKOR is discussed in the following sections.

Figure 3.1 A triangular fuzzy number

Step 1: Fuzzy set theory is used to represent uncertain information in mathematical terms. It is a convenient method to denote linguistic terms without ambiguity. The criteria comparison matrix is developed using the linguistic information from decision makers and converting them in terms of triangular fuzzy numbers using Eq. 3.1. The details of linguistic terms and the corresponding fuzzy numbers are listed in Table 3.3. The triangular function as shown in Figure 3.1 can be represented as

$$\tilde{\mu}(x) = \begin{cases} 0 & x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x \leq a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases} \quad (3.1)$$

Step 2: Calculate the aggregate fuzzy weights for each criteria from the criteria comparison matrix and form a row matrix (of dimension $1 \times n$) for aggregated importance weight criteria where ‘ n ’ is the total number of criteria considered in this study.

Table 3.3 Linguistic terms and corresponding fuzzy numbers

Linguistic terms	Fuzzy numbers
Very Low (‘VL’)	(0.0,0.0,0.1)
Low (‘L’)	(0.0,0.1,0.3)
Medium Low (‘ML’)	(0.1,0.3,0.5)
Medium (‘M’)	(0.3,0.5,0.7)
Medium High (‘MH’)	(0.5,0.7,0.9)
High (‘H’)	(0.7,0.9,1.0)
Very High (‘VH’)	(0.9,1.0,1.0)

Step 3: The DMs now form an alternative comparison matrix using the linguistic terms and corresponding fuzzy numbers. A similar 7 scale as used for the criteria matrix is used here for alternatives but with the different notation (refer to Table 3.4).[122,123]

Table 3.4 Linguistic term and corresponding fuzzy numbers for alternatives

Linguistic Term	Corresponding Fuzzy Numbers
Very Poor ('VP')	(0.0,0.0,1.0)
Poor ('P')	(0.0,1.0,3.0)
Medium Poor ('MP')	(1.0,3.0,5.0)
Fair ('F')	(3.0,5.0,7.0)
Medium Good ('MG')	(5.0,7.0,9.0)
Good ('G')	(7.0,9.0,10.0)
Very Good ('VG')	(9.0,10.0,10.0)

Step 4: Form the triangular fuzzy number decision matrix using the aggregated fuzzy ratings of 'm' alternatives with n criteria an $m \times n$ matrix is formed, where each cell represent the aggregated fuzzy number value of i^{th} alternative with respect to j^{th} criteria.

Step 5: Determine the best (aspired) values and worst fuzzy values. Assuming the j^{th} criteria representing a benefit, then the best values for setting all the criteria functions are $\{x_j^* | j = 1, 2, \dots, n\}$, and the worst values are $\{x_j^- | j = 1, 2, \dots, n\}$.

Step 6: The gap $\{S_i | j = 1, 2, \dots, m\}$ and $\{R_i | j = 1, 2, \dots, m\}$ from the L_p metric parameter with normalisation. The formula for the above calculation is listed below.

$$L_{pi} = \left\{ \sum_{j=1}^n [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)]^p \right\}^{(1/p)} \quad 1 \leq p \leq \infty \quad (3.2)$$

$$S_i = \sum_{j=1}^n [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)] \quad (3.3)$$

$$R_i = \max_j [(w_j - x_{ij}) / (x_j^* - x_j^-)] \quad (3.4)$$

where $i = 1, 2, \dots, m$

Step 7: The important part of VIKOR method is the calculation of $\{Q_i \mid j = 1, 2, \dots, m\}$

for ranking. The values are calculated by using the Eqn 3.5 as follows:

$$\tilde{Q}_i = \rho \left[\frac{(S_i - S^*)}{(S^- - S^*)} \right] + (1 - \rho) \left[\frac{(R_i - R^*)}{(R^- - R^*)} \right] \quad i = 1, 2, \dots, m \quad (3.5)$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, ρ is the weight in strategy of the maximum group utility, usually 0.5.

From the $\tilde{Q}_i$ (fuzzy values), Q_i (crisp values) are computed by using the centroid method which is one of the defuzzification methods. The detailed procedure is discussed elsewhere in detail [122].

3.3 Results and Discussion on compliant polishing tool Case Study

Consider a typical aerospace industry which requires manufacturing a convex workpiece as shown in Figure 3.2. The choice of an appropriate tool for polishing the component in each stage is critical as it would directly affect the cost and manufacturing time. Complex components in high-end industries such as aerospace, photonics, sports etc. have a combination of flat, convex and concave sections. It should be noted that during the process development, initially, the decision makers (usually 3 to 5 DMs) play a crucial role in selecting the criteria and ranking the alternatives. The decision-making methods (MCDMs) will be helpful in assisting the decision makers to choose appropriate polishing tools mathematically. The workpiece has an initial R_a of 5μ and needs to be mirror finished with R_a of 0.05μ . The desirable pre- and post-polishing attributes along with polishing stage operational parameters are given in Table 3.5.

Figure 3.2 CAD model of work-piece to be polished using compliant polishing tool

Table 3.5 Various attributes involved in coated abrasive polishing [61]

Pre-polishing	Polishing	Post-polishing
<ul style="list-style-type: none"> • Cost estimation (tool, material, other overheads etc.) Part: <ul style="list-style-type: none"> • Geometry of the part • Desired Surface Roughness • Material Properties Polishing Tool: <ul style="list-style-type: none"> • Tool type according to surface to be polished • Compliant/Rigid Tool • Tool Path • Grain Size/Shape 	<ul style="list-style-type: none"> • Pressure • Relative Speed / Spindle Torque • Polishing Time • Depth of Cut • Force • Material Removal • Type of coolant • Temperature maintained • Tilt Angle 	<ul style="list-style-type: none"> • Surface Roughness. • Surface Integrity <ul style="list-style-type: none"> • SEM analysis • Residual Strength due to polishing • EDX test for contamination check • Tool Wear • Surface Profile Error

Since the surface finish could not be obtained in a single stage, usually they are divided into multiple stages so that by varying the grit size, different compliant tool, etc. So, the attributes that changes with respect to these stages are identified separately and denoted by star

‘*’ symbol. The polishing stages are divided as discussed in Table 3.5. Figure 3.3 describes the list of alternatives and attributes considered in the case study. In a broader perspective, although there are many attributes listed in Table 3.5, the main attributes are cost, time, surface finish, surface integrity, material removal rate and compliance. Since machining parameters like pressure, relative speed, depth of cut, force and coolant are varied for the sake of surface finish and MRR of the workpiece. Hence, those attributes are not considered separately.

Figure 3.3 Attributes and alternatives used in the selection of the compliant polishing tool.

Using the AHP algorithm depicted as a flow-chart in Figure 3.4, the attribute decision matrix is formed. The relative values of the attributes are purely judgmental based on the decision makers' intuition. The criteria weightage for each attribute is shown in Figure 3.5. It is evident that more importance is given to surface integrity and surface finish. The composite weights for the competing alternative processes in each polishing stage are computed as shown in Figure 3.6. Based on the composite weights the alternatives are ranked in the order of preference. The values used in attributes comparison matrix and fuzzy ratings used can be found elsewhere [12].

Figure 3.4 AHP and fuzzy VIKOR algorithm implemented in the case study

Figure 3.5 Criteria weight for each attribute

Figure 3.6 Weight for each polishing process in the three stages

In the fuzzy VIKOR method, the fuzzy values for the attributes weights are listed in Table A. 4. The attributes and alternatives are ranked using fuzzy values corresponding to the linguistic terms given by the decision makers. The fuzzy linguistic rating for alternatives with respect to fixed and changing variables for the three stages are listed in Table A. 5 (a-c). After compiling the MATLAB® modules developed based on the fuzzy VIKOR algorithm, the fuzzy evaluation matrix is obtained as shown in Table A. 5 (d-e) and Table A. 6 for the three stages. It could be observed that (i) the dimension of the matrix is 6x6 matrix denoting the six attributes

(in rows) and six alternatives (in columns) (ii) only the ‘time’ and ‘surface integrity’ values change in each of this matrix. $\tilde{Q}_i$ and corresponding crisp number are listed in Table 3.6

Table 3.6 $\tilde{Q}_i, Q_i$ and rankings for the 6 compliant polishing tools

Alternatives	Initial polishing			Intermediate polishing			Fine polishing		
	$\tilde{Q}_i$	Q_i	Rank	$\tilde{Q}_i$	Q_i	Rank	$\tilde{Q}_i$	Q_i	Rank
A1	(1.09,0.65,-2.81)	0.1464	2	(0.01,0.8,-1.93)	0.2127	3	(0.65,0.96,-2.63)	0.3115	5
A2	(1.04,0.42,-2.2)	0.087	1	(0.03,0.88,-1.52)	0.3394	5	(0.67,0.92,-2.12)	0.3718	6
A3	(1.2,0.46,-2.13)	0.1504	3	(0.05,1,-0.26)	0.6311	6	(0.6,0.62,-1.94)	0.1914	4
A4	(1.18,0.29,-1.25)	0.1851	4	(-0.01,0,-1.01)	-0.1695	1	(0.57,0.37,-1.18)	0.1435	3
A5	(1.25,0.75,-0.58)	0.611	6	(0.01,0.23,0.11)	0.1745	2	(0.49,0,-0.63)	-0.0234	2
A6	(1.25,0.75,-1.89)	0.3933	5	(0.01,0.46,-0.54)	0.2181	4	(0.51,0.24,-1.91)	-0.0734	1

Table 3.7 Comparison of alternative rankings obtained from AHP and Fuzzy VIKOR Methods

Alternatives	Initial polishing		Intermediate polishing		Fine polishing	
	AHP	Fuzzy VIKOR	AHP	Fuzzy VIKOR	AHP	Fuzzy VIKOR
Belt polishing (A1)	1	2	6	3	5	5
Fibre disc with composite backing (A2)	2	1	5	5	6	6
Flap wheel(A3)	3	3	4	6	4	4
Flap disc with rubber backing pad(A4)	4	4	1	1	3	3
Felt backing pad(A5)	5	6	3	2	1	2
Gasbag polishing(A6)	6	5	2	4	2	1

The rankings of the alternatives obtained from both the fuzzy VIKOR and AHP are compared as shown in Table 3.7. From the Table 3.6 , Table 3.7 and Figure 3.5(b) it is noticed that, for selection of polishing tool for initial polishing by AHP method, the belt polishing is highly preferred owing to the maximum weight of 19.759, and the next preferred tool is the fibre disc polishing tool with second highest weight of 19.661. Fuzzy VIKOR method leads to a similar solution with fibre disc polishing tool as the best tool for initial polishing with least Q_i value (0.087) and belt polishing as the second preferred tool with second least Q_i value (0.1464). Thus, for initial polishing, polishing with fibre disc or belt polishing is preferred followed by other tools such as flap wheel, rubber pad, felt and gasbag. This indicates that tool with considerably higher stiffness such as fibre disc polishing tool is preferred for initial polishing compared to less stiffer backing materials like rubber or felt. In the second stage of polishing i.e, intermediate polishing, tools with rubber as packing pad is highly preferred from the AHP solution with the highest weight of 27.4768. Similar result was obtained from fuzzy VIKOR method indicating that rubber pad as the most preferred polishing tool with the least Q_i . Thus, it is evident that rubber backing pad which has comparatively lower stiffness than fibre disc and higher stiffness than felt or gas bag is suitable for intermediate polishing. By AHP method felt pad polishing is the most preferred tool with highest composite weight of 32.577 followed by gasbag polishing with 29.03. Similar results were obtained from fuzzy VIKOR where gasbag polishing tool (-0.0734) is the most preferred tool followed by felt polishing tool (-0.0234). It is evident that both felt and gasbag with relatively less stiffness are the preferred polishing tools for fine polishing. The outcome of the result suggests that compliance of the backing pad material influences the surface finish in each polishing stage apart from the usual tool parameters such as grit size, tool-workpiece contact pressure and tool velocity (rpm). From the managerial point of view, the work presented is that decision making

about choosing polishing tools can be made promptly and consistently at the conceptual stage of the process identification thereby saving on cost, time and possibly decreasing the iterations.

3.4 Summary

- In order to help in the selection of appropriate polishing tool, a scientific approach using MCDM methods such as fuzzy VIKOR and AHP are proposed in this study. Polishing scenarios at three different stages such as initial rough polishing, intermediate and fine polishing are considered.
- A case study is taken and the relevant comparison matrices for these three different polishing stages are obtained. Six alternatives and six attributes are considered in this case study and according to the given scenarios additional attributes, and alternatives can be added.
- After solving the obtained matrices through the standard algorithms followed in these two methods for the given linguistic variables, the final ranking of the alternatives (six polishing tools) are obtained using MATLAB® codes and the best possible tool for each stage of polishing is found and compared with the other method.
- Thus, these two MCDM methods help in compliant polishing tool selection and provides a better scientific approach in reducing the ambiguity involved in appropriate tool selection for the given polishing scenario.

Chapter 4 Effect of compliance on contact pressure and material removal and its numerical modelling using Archard wear

In this chapter, the effect of polishing pad compliance on the contact pressure distribution and material removal is discussed in detail. The pressure distribution values are obtained using pressure films, and the material removal studies are carried out in a robotic polishing setup. Later, the machined surface is scanned for the material removal pattern using a profilometer. Moreover, the various process parameters (polishing force, spindle speed, feed rate, grit size and tool compliance) that affect the material removal depth using disc type polishing tool is analyzed using L_8 orthogonal array in Taguchi method. Finally, a numerical three-dimensional FE model is developed for the same in order to predict the contact pressure distribution which in turn influences the material removal profile.

In order to predict the material removal profile, the FEA simulation is carried out using commercial FEA software, ABAQUS®. Using the UMESHMOTION FORTRAN subroutine option in the general static analysis, the nodes are displaced according to the pressure and slip velocity relation given by Archard's wear equation. Several researchers have employed a similar approach to predict wear rate in bearings, brake pads etc., [38,43,124]. According to the wear depth computed using Archard's formula, the geometry of interest is modified continuously by Arbitrary-Lagrange Euler (ALE) meshing algorithm. Thus, by using only the grainless pad, the material removal profile is computed. This technique helps to reduce the computational time involved in material removal rate.

4.1 Static pressure distribution

In this study, the key focus is to investigate the effect of contact pressure by the compliant polishing tool on the material removal profile. In order to investigate the contact pressure, pressure films are used. The pressure films are placed at the interface between the tool and the workpiece. Once the tool is pressed against the films, color of different intensities is formed in correlation to the contact pressure. Using the image processing software (Pressure Distribution Mapping System FPD-8010E®), the color intensities are later translated to pressure values. The films are suitable only to measure static pressure and highly sensitive to heat and rough environment. In addition, the load-displacement responses of both the hard and soft rubber tools are obtained to validate the numerical model. Films of pressure range between 0.05MPa and 10MPa are used in this study. A set of films with different pressure ranges are stacked together. Without rotation, the tool is only pressed against the aluminium block (static condition) for hard and soft 2" rubber pad, with and without abrasive disc in a load range of 10,15,20,25N . The tilt angle considered is 15°. Generally, the tilt angle is employed to flex the disc against the workpiece. In this complete study, the same tilt angle is fixed.

In this study, there are two kinds of static experiments conducted.

- Initially, the pressure distribution using only the two different rubber backing tools (hard and soft) are carried out. No abrasive disc is attached to the rubber backing pads.
- Further, to understand the effect of an abrasive disc on the static pressure distribution, abrasive pad without grains is chosen. Since the pressure films are highly sensitive and prone to get damaged by sharp objects, the grains are removed from the abrasive disc.

(a)

(b)

(c)

Figure 4.1 (a) Robotic polishing setup with pressure films and backing pad, (b) colour formation in the pressure films, and (c) backing pad attached with the grainless abrasive disc.

Figure 4.1 (a) shows the polishing setup used for the experimental static pressure study. The experimental trials with only a rubber backing pad (Figure 4.1 (b)) and in the second case, a rubber backing pad along with the grainless abrasive disc (Figure 4.1(c)). The pressure films are post-processed using Fujifilm pressure mapping software®, where for each pixel value corresponds to a pressure value. Later the pressure values of the different stacked film ranges are superimposed together to get the final pressure distribution.

4.1.1 Contact pressure distribution between rubber backing pad - aluminium block

The rubber backing pad is pressed against an aluminium workpiece (Al6061-T6), and the contact pressure map is obtained from the pressure films and plotted. From the Figure 4.2 and Figure 4.3, it is evident that although the contact length remains almost the same for the hard and soft, the contact width is higher for soft pad in comparison with the hard pad. As expected, it can be observed that as the force increases the contact area increases. It can be noticed that for the hard pad, the contact pressure is high near the initial point of contact (towards the outer circumference of the disc). But in the case of the soft pad, when the load is increased the pressure shifts toward the shaft due to the compliant nature of the tool. The contact pressure distribution formed by the disc is of crescent shape reported similarly by Umer et al. [27].

Figure 4.2 Static test of hard rubber pad for different forces

Figure 4.3 Static test of soft rubber pad for different forces

4.1.2 *Rubber pad with grainless abrasive disc*

The pressure films are made of polyester, and the presence of hard grains will damage these films by piercing. The abrasive grains are removed from the discs, and only the abrasive disc backing is considered in this study. These grainless abrasive discs are attached to the base of two different compliant backing pads. After applying the load, the grainless disc along with the rubber backing pad is made to flex against the aluminium plate where the pressure films are placed. A similar trend that is discussed in the previous case of rubber pad- aluminium plate scenario is followed in the pad with the grainless abrasive disc, as depicted in Figure 4.4. It is evident that even when an additional abrasive disc is attached, the compliance of rubber backing pad influences the contact pressure distribution.

Figure 4.4 Static pressure distribution for rubber pad along with grainless abrasive disc

4.2 Numerical simulation of contact pressure distribution

4.2.1 Tool and workpiece modelling

The beforementioned two experimental static pressure distribution experimental scenarios (i) rubber backing pad- aluminium plate interaction and (ii) rubber backing pad along with grainless disc – aluminium plate interactions are investigated using FE simulations. For this scenario, only the rubber pad along with the shaft is modelled and kept at a tilt angle of 15° with respect to the aluminium workpiece (say Model A). The internal surface of the rubber pad is tied to the shaft. The bottom surface of the rubber pad and the top surface of the workpiece is made as slave and master surfaces respectively in Model A. The shaft is modelled as a rigid body with a reference point attached to it for applying the load and boundary conditions. Once the initial validation study is performed for Model A, it is further extended by modelling an additional disc of 1.1 mm thickness and 50.8 mm diameter to represent the grainless abrasive pad. The top surface of the grainless abrasive disc is tied to the bottom surface of the rubber pad. This simulation case is referred to as Model B. The bottom surface of the pad is made to contact the top surface of the inclined workpiece as shown in Figure

4.5(b). Additional factors like grains and surface undulations on the workpiece are ignored in this study, because the focus lies in understanding the effect of compliance on pressure distribution and material removal.

Figure 4.5 (a) Rubber pad and aluminium block (Model A), (b) composite abrasive disc attached to rubber pad (Model B) and ply stack orientations

4.2.2 Material model

Hyperelastic constitutive model is used for the hard and soft rubber pads. Out of all the phenomenological models available such as Mooney Rivlin, Ogden, Yeoh, Neo-Hookean etc., the Yeoh model is selected by fitting the experimental uniaxial tensile stress-strain responses as depicted in Figure 4.6. The details regarding the Yeoh hyperelastic constitutive model can be found elsewhere [125,126]. The Al6061-T6 workpiece is modelled as a linear elastic solid with a young's modulus E of 68.9 GPa and Poisson's ratio ν of 0.3. Since in this study the contact pressure difference is the key parameter of interest, and there are no grains involved in the simulation, the stress values at the surface will be far below the yield value of Al6061-T6, and hence the workpiece can be assumed to be linearly elastic without considering the plastic material properties.

(a)

(b)

Figure 4.6 Hyperelastic models for tensile test data for (a) Hard rubber (b) Soft rubber

In order to investigate the effect of abrasive disc on the contact pressure distribution, the grains from the abrasive pad are removed as it would damage the pressure films. The grainless abrasive pad is made of polyester/cotton fabric with phenolic resin as matrix. The woven fabric is modelled as orthotropic lamina [127]. The Young's modulus values (E_1 and E_2) are obtained by performing three-point tests on the flexural samples cut in 0° and 90° directions of the disc respectively as depicted in Figure 4.7(a) and (b). The span length of these specimen is 40 mm and thickness of the samples 1.1mm. The measured material properties from the three-point bending test are as listed in Table 4.1.

Figure 4.7 (a) Woven fabric as backing material of the abrasive disc (b) Three-point bending test for the specimens cut from abrasive disc

Table 4.1 Material properties of abrasive disc

E_1 (MPa)	E_2 (MPa)	ν	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
1013.8	682.5	0.3	550	550	500

where E_1, E_2 , and are the Young's modulus in the direction of weft (E_1) and warp (E_2), ν is the poisson's ratio, G_{12} , G_{13} are the axial shear modulus and G_{23} is the out of plane modulus.

4.2.3 FE modelling

The rubber backing pads are hyperelastic incompressible in nature. Therefore they are modelled with 10-node modified hybrid, quadratic tetrahedron elements with hourglass control (C3D10MH). In order to capture the incompressible and highly non-linear deformation of the rubber material and also to model contact more accurately, C3D10MH elements are used. Mesh convergence study is done for different mesh sizes (0.5mm, 1mm, 2mm) for the rubber pad and the aluminium block. The contact pressure values at a particular node are obtained for the different mesh sizes from the simulation results. It is found that the 1mm mesh size is more suitable for both rubber pad and aluminium block based on the computational time, the pressure value and the stress singularities. The boundary condition is applied to the created reference point on the rigid shaft which is tied concentrically to the internal surface of the rubber pad.

For the boundary condition, the spatial displacement along the loading direction (U2) is left unconstrained and constraining all other degrees of freedom. The aluminium block is meshed using 3D linear brick element, with reduced integration (C3D8R) elements and the base of the aluminium block is fully constrained (made 6 dofs=0). An 8-node quadrilateral in-plane general-purpose continuum shell elements (SC8R) is used to mesh the grainless abrasive pad to capture the membrane stretching behaviour. Only one layer of mesh is used through the thickness, as each element in the through-thickness direction represent the stack of plies (0° and 90°).

4.2.4 Contact pressure distribution simulation results

4.2.4.1 Rubber backing pad-Aluminium block contact

In order to simulate this quasi-static finite strain and material non-linear problem, ‘Quasi-static analysis’ step under dynamic implicit procedure is carried out in ABAQUS®, and the numerical contact pressure results and load-displacement responses are obtained. The simulated load -displacement response results are compared with the experimental values, and there is a good agreement for both soft and hard backing pad within the material properties scatter as shown in Figure 4.8.

Figure 4.8 Experimental and numerical load versus displacement curves for hard and soft rubber pads

The load- displacement responses not only give us information about the change in stiffness of the hard and soft rubber pads, but also helps to confirm the validity of the model and the material properties assigned.

The simulated contact pressure distribution is shown in Figure 4.9 (b)(d), in comparison with the experimental results in Figure 4.9 (a) (d). The simulated pressure distribution is obtained by considering the maximum pressure values from all the time frames (from 0 to 1 second), thus replicating the pressure film behaviour. Three parameters were considered to compare the contact pressure distribution viz., (i) maximum contact length is taken as the distance between the two extreme points of contact along the length of the aluminium plate, (ii) maximum contact width is considered as the distance between initial point of contact and the point where contact pressure drops to minimum value along the width of the plate and (iii) contact area which is calculated by multiplying the number of pixels with pressure values, multiplied by the pixel area (1 mm x 1 mm in simulation and 0.125 mm x 0.125mm in experimental). The values of the above-discussed parameters for both hard and soft backing pads are listed in Table 4.2. The contact length of the soft and hard backing pad is almost the same (1.59% increase), but the contact width increased drastically by 108.7% for the soft backing pad. This can be related to the larger displacement observed in the load-displacement responses for soft backing pad compared to the hard backing pad, proving once again that compliance plays a crucial role in determining the contact area. The overall experimental contact area increased by 18.69% for the soft backing pad compared to the hard backing pad. The overall simulated contact pressure distribution values show good agreement with the experimental data as shown in Table 4.2. The deviation in the overall simulated contact area for hard rubber pad (186 mm^2) and experimental contact area (185.32 mm^2) is 0.366%. Similarly, the deviation in the soft rubber pad is 9.98%. Although the simulated and experimental contact area pattern matches, the abovementioned deviations observed in the simulated and

experimental contact area values can be attributed to the complex construction of the tool such as inbuilt cotton mesh at the bottom layer which is not considered in the simulation in order to simplify the model. Hence the load-displacement responses and the contact pressure distribution results prove that using the relevant material properties and contact conditions, the behaviour of the rubber backing pad (any backing material in a compliant tool) can be predicted and it helps to understand the overall behaviour of the tool compliance on the contact pressure distribution. Since the results are matching, this FE model can be used for simulating case ‘B’ where an additional grainless abrasive disc will be added.

Figure 4.9 Experimental pressure distribution for (a) hard (c) soft rubber tool and FE pressure distribution for (c) hard and (d) soft tool for 20N

Table 4.2 Experimental and simulated pressure distribution values for the backing pads

Contact results	Hard		Soft	
	Experimental	Simulation	Experimental	Simulation
<u>Rubber pad</u>				
Overall contact length (mm)	31.5	32.0	32.0	38.0
Overall contact width (mm)	5.75	7.0	12.0	14.0
Actual contact area (mm ²)	185.32	186	223.53	247
<u>Rubber pad with disc</u>				
Overall contact length (mm)	30.38	28.0	30.88	32.0
Overall contact width (mm)	5.13	6.0	11.75	9.5
Actual contact area (mm ²)	120.54	113	204.5	163

4.2.4.2 Rubber backing pad with grainless abrasive disc-aluminium block

Initially, the load versus displacement response curves are extracted from the simulation results for both the rubber pads with grainless abrasive discs as shown in Figure 4.10. Once again there is an excellent comparison between the experimental reading and simulated values. The response is initially linear for soft pad with disc till 2mm whereas for the hard pad the entire behaviour is non-linear.

Figure 4.10 Load versus displacement response for different pad compliance with the grainless abrasive disc.

The experimental and simulated contact pressure distribution between two types of rubber pad with the disc are shown in Figure 4.11. The contact pressure follows a similar trend to that of the backing pad without the disc. The contact pressure in the case of the soft pad is more evenly distributed, where pressure values are scattered around the tool centre and near the rim of the rubber tool. Whereas, in the case of the hard pad with the disc, the concentrated pressure regions are observed around the outer periphery of the abrasive disc, which matches with the experimental trend. The contact length and width of the pressure distributions are tabulated in Table 4.2.

Figure 4.11 (a) Experimental, (b) Numerical pressure distribution for hard pad at 20N
(c) Experimental (d) Numerical pressure distribution for soft pad at 20N

The experimental contact length of the hard and soft pad with disc is 30.38mm and 30.88mm respectively. Although, the diameter of pad (50.8mm) is 5.1 mm larger than the rubber pad diameter (45.7mm), but of the additional abrasive disc stiffness there is a reduction in contact length as mentioned. And the experimental contact width of the hard and soft pad

with disc decreased by 10.75% and 2.08% which can be related to the reduction in the displacement (as observed from load-displacement responses) compared to the earlier cases only with rubber pads. The experimental contact area of the soft pad with disc is 51.68% higher compared to that of hard pad with disc.

From Figure 4.11, for both the cases, the contact pressure near the periphery of the tool (outer portion) is comparatively higher than the experimental values. The reason for such high values can be attributed to the stress singularity caused in these regions due to the sharp edge at the bottom of the circular disc. The simulated pressure values away from the rim region match well with the experimental pressure values. From Table 4.2, it is evident that overall contact parameters computed from simulation and experimental results agree well within $\pm 20\%$, and the deviations in the results can be attributed to the facts like stress singularities and the inherent warpage of the abrasive disc which is ignored in the simulation. It must be also noted that, although, the grainless abrasive discs have higher modulus values compared to the rubber pad, yet the compliant nature of the rubber pad decides the pressure distribution.

4.3 Material removal studies

4.3.1 Process parameters

The quality of polishing is highly dependent on the process parameters considered. Therefore, it is important to study the effect of different process parameters on the material removal depth. Usually, the design of experiments (DoE) which is considered as a powerful tool to analyse the parameters' effect is carried out. Since the conventional factorial DoE method is time-consuming and complex for more number of factors, alternatives like Taguchi design is preferred. This is an experimental technique where the number of experimental trials is considerably reduced by using orthogonal arrays. Design of experiments is done by specially constructed L8 orthogonal array. The controlled factors considered in this study are polishing

pad compliance, grit size of the abrasive discs, force, feed and spindle speed. The study is conducted at two different levels as listed in Table 4.3 .

Table 4.3 Parameters and different levels used for orthogonal array

Parameters	1 (Low)	2 (High)
Pad	Soft	Hard
Grit size (Al ₂ O ₃)	#60	#120
Force	10N	20N
Feed	5mm/s	10mm/s
Spindle speed	2500 rpm	5000 rpm

Three experimental trials are carried out for each run condition, and the average of the maximum material removal depth values are listed in Table 4.4 . The material removal depth is obtained by scanning the 2D profile in the direction perpendicular to the polishing path direction. The scanning is done using Talyscan® laser profilometer, and from the scanned data the maximum depth value at different positions in the path are obtained. The material removal depth for each trial is obtained by computing the average of these values taken at different positions of the machined path. So, three machining trials are done for each combination of parameters, and later the average of these three values are listed as shown in Table 4.4.

Table 4.4 Taguchi L8 Orthogonal array

Hardness	Force	RPM	Grit	Feed	Average MR depth (μm)
1	1	1	1	1	53.181
1	1	1	2	2	40.129
1	2	2	1	1	201.083
1	2	2	2	2	148.083
2	1	2	1	2	74.886
2	1	2	2	1	184.666
2	2	1	1	2	54.325
2	2	1	2	1	106.258

The experimental material removal depth values listed in Table 4.4 are transformed into signal-to-noise (S/N) ratios. Signal-to-noise (S/N) ratio is being used in Taguchi analysis as the quality characteristics, and there are three different types of S/N ratio such as nominal is the better, smaller is the better and larger is the better. The S/N ratio larger the better is chosen, and the effect of factors on maximum material removal depth is shown in

Figure 4.12. Based on the chosen range of values for these factors, spindle speed has the largest effect on material removal depth followed by feed, force, grit size and pad compliance. Although the effect of compliance is lesser because of the range of values considered for other parameters in this study, its influence is still higher in polishing operations, which can be observed from the experimental machining trials without feed motion, where the material removal depth almost doubles for change in compliance. Hence, it is important to investigate the effect of compliance on the material removal depth.

Figure 4.12 Effect of process and material parameters on means of S/N ratios

4.3.2 Material removal depth study

In this study, 3M® ROLOC rubber pads are used to hold the abrasive discs. Two types of tool compliance such as hard and soft rubber are taken. The alumina abrasive disc of #60 grit size is used. These discs are attached to the bottom of the rubber tool which in-turn held in the spindle which is attached to the robot arm.

(a)

(b)

Figure 4.13 (a) Material removal tests with abrasive discs (b) 3D surface profile measurement using profilometer

Six-axis ABB IRB 140 robot is used in this experimental study as shown in Figure 4.13. Aluminium alloy Al6061 plates are used as to study material removal profile. Although usually titanium and nickel alloys are preferred to be polished, aluminium is considered because the key focus of the study is to develop a simulation technique to predict the material removal for the compliant tool. The spindle speed is set at 2500 RPM and the polishing force applied is 20N. Tilt angle between the tool axis and workpiece is 15°. The rotating tool is pressed against the workpiece for 3 seconds without feed motion. Later the tool is retracted, and the machined surface is measured using a laser profilometer with a spacing of 100µm (resolution).

Figure 4.14 3D material removal for (a) Hard pad with 10N loading (b) Soft pad with 10N loading (c) Hard pad with 20N loading (d) Soft pad with 20N loading (e) Hard pad with 30N loading (f) Soft pad with 30N loading

From Figure 4.14 (a), (c), (e), it can be observed that material removed near the tool's periphery is higher compared to the tool's centre region which is attributed to the beforementioned pressure distribution pattern. In the case of soft pad, the material removed is almost uniformly distributed. The material removal depth for soft pad lies in the range of 150-200 μm , whereas for the hard backing pad the material removal depth is in the range of 350 -400 μm . But as the load is increased the material removal depth reduces to 280-300 μm

for the hard pad at 30N, due to the pressure distribution. The machined area (length and width) increases for both the hard and soft backing pad polishing scenarios.

Figure 4.15 Force versus maximum material removal depth for two different compliant backing pads

The variation of maximum material removal depth with polishing force for both soft and hard backing pad is shown in Figure 4.15. It can be observed that the maximum material removal depth increases by 9.34% for hard pad at 20N (388.8 μm) in comparison with 10N (331 μm) and again drops to 297.3 μm at 30N. This can be attributed to the high contact pressure region near the point of initial contact. After 20N, due the pressure starts getting distributed and hence a downwards trend is observed in the material removal depth curve for hard pad.

Also, it is interesting to observe that for same material removal range of 360 μm (15N), and 349.2 μm (25N) using the hard pad, two different machined area of 82.5 mm^2 and 149.33 mm^2 are achieved by changing the polishing load. For the soft pad, the material removal depth decreased by 53.2% from 10N (284.8 μm) to 20N (133.3 μm) and then again increases by 78.92% for 30N (238.5 μm). The reason can be attributed to the fact that till 20N the pressure is uniformly distributed in soft backing pad and later, because of the high stiffness region near the shaft is in contact, the pressure and material removal keep increasing till 30N, and it is expected to increase further. A similar phenomenon that is observed in hard pad is also

observed in the soft pad, where for two different loading (15N and 25N) where the contact area increases by 57.62% for almost same maximum material removal depth of $198 \mu\text{m} \pm 4.24 \mu\text{m}$. Thus, using a suitable polishing load, the desired machined area and material removal depth can be obtained.

4.4 Wear model implementation using adaptive FE modelling

In order to simulate the material removal depth by the polishing operation using the compliant polishing tool, Arbitrary Lagrangian-Eulerian (ALE) adaptive mesh technique is utilised in this study. ALE combines the features of both Lagrangian and Eulerian analysis. In general, ALE is used in simulations to maintain the mesh quality by adjusting the mesh independent of material deformation and at the same time not altering the topology. The flowchart in the Figure 4.16 summarises the simulation of material removal using Archard's wear approach. Initially, the model used for dynamic simulation is validated against the experimental values under the static condition as discussed in sections 4.1 and 4.2. Then this validated 3D model is used for dynamic numerical analysis where the material removal is simulated using special UMESHMOTION FORTRAN subroutine. For implementing the ALE procedure, the aluminium part is selected as the ALE domain, and the aluminium surface nodes are selected as ALE node set. Movement of nodes replicating wear phenomena is implemented in these nodes by wear equation.

4.4.1 Procedure for predicting wear using ALE

Figure 4.16 Overall methodology for numerical simulation of material removal distribution

In the present work, the Archard abrasive wear model is used to estimate the material removal rate at each node. The volume (V) of the material removed under a normal force, ' F ', over a sliding distance ' s ' is given by Eq.4.1[40]

$$V = k \frac{F}{H} s \quad (4.1)$$

where ' H ' is the hardness of the material (N/mm²), and ' k ' is the wear coefficient. Expressing the Eq 4.1 in an infinitesimal form with respect to the sliding distance at a node ' i ', Eq 4.2 is obtained [40].

$$dh_i = \frac{k}{H} \bar{P}_i ds_i \quad (4.2)$$

where $\bar{P}_i$ is the averaged contact pressure, ' ds ' is the incremental sliding distance and ' dh ' is defined as the incremental change in the surface node position due to wear in the surface during the time period ' Δt ' (0.3 seconds) [42]. The numerical implementation of Eq 4.2 can be expressed as shown in Eq 4.3.

$$h_{i_j} = \sum_{j=1}^{j=itr} \frac{k}{H} * \bar{p}_{i_j} * \Delta s_{i_j} \quad (4.3)$$

h_{i_j} is the material removal depth computed at node ' i ' during the iteration ' j '

$\bar{p}_{i_j}$ is the averaged pressure at particular node ' i ' during the iteration ' j '

Δs_{i_j} is the sliding distance difference obtained from the final frame and initial frame within the same iteration ' j '.

Figure 4.17 Algorithm for implementing UMESHMOTION subroutine to calculate 3D material removal profile

$$itr = \frac{T}{f * \Delta t} \quad (4.4)$$

where ‘ T ’ (Total machining time), Δt is the step time, ‘ itr ’ is the number of iterations and f is the scaling factor. The justification of the scaling factor is as follows. In order to reduce the computational time, the MRR results are scaled such that the total machining time and number of revolutions can be reached using the Eq 4.3 and Eq 4.4. Similar acceleration factors were employed in earlier studies [128,43,129] governed by the justification that the wear coefficient considered being a constant.

The nodes in the contact surface between the tool and the workpiece is created as an adaptive mesh node set. Later, by using the FORTRAN user subroutine, the nodes on the contact surface are modified based on the Eq. (4.3) as shown in flowchart (Figure 4.17).

UMESHMOTION subroutine in ABAQUS® is only available in ‘General-Static’ simulation and not in explicit. Since, the model involves contact and highly non-linear deformations and inertial effects, dynamic-implicit is more suitable, but as mentioned, ALE is not presently supported in ‘dynamic-implicit’. To overcome this hindrance, the simulation is run in dynamic- implicit until step time ($\Delta t=0.3s$) in which the tool makes ‘ Δn ’ revolutions ($\Delta n=12.5$). The dynamic pressure is averaged within the revolution time of ‘ Δt ’. The average pressure and sliding distance range is calculated using python script and fed as input for the contact pressure in the subroutine for the respective nodes. In order to simulate the complete machining time ‘ T ’, the results are linearly extrapolated such that the wear depth (dh) is multiplied using scaling factor ‘ f ’ according to Eq.4.4. The simulation is again restarted in ‘General-Static’, where UMESHMOTION subroutine is applied, and the mesh is displaced according to the Archard wear depth calculation. The purpose of these steps in General-Static is only to update the mesh. The nodal displacement values are added with the nodal displacement ($U3$) from the previous step. This cycle is repeated until the set ‘ itr ’ number is

reached, and the finally displaced ALE nodes can be compared with the experimental material removal profile.

4.4.2 Numerical material removal depth distribution results

Initial pre-processing modelling steps like meshing, boundary conditions and assigning the material properties definition are carried out as discussed in section 4.2. Additionally, the rotational speed of 261.9 rad/s (2500 RPM) is given, and the ALE surface is selected as shown in Figure 4.18.

Figure 4.18 Dynamic numerical model for material removal

Using the proposed algorithm, the material removal depth at nodes in the contact surface, is computed. Totally two iterations are carried out where the duration of each iteration is given by ' f ' times ' Δt '. The average contact pressure computed using python code in the first and second iterations are shown in Figure 4.19 (a) and (b) respectively. Similarly, the sliding distance range is calculated as shown in Figure 4.19 (c) and (d). From these results, it is inferred that the assumption of having acceleration factor is justifiable as there is no considerable change (<5%) in the averaged values of the contact pressure and sliding distance. Moreover, because of the contact pressure singularities near the mesh periphery of the

composite abrasive disc, high-pressure regions are observed as shown in Figure 4.19 (a) and (b). The material removal depth obtained in these nodes are nullified using appropriate threshold limit at the end of each iteration ' j ' such that abnormal material removal depth values are avoided.

In between the two iterations, the mesh updation is done using the UMESHMOTION FORTRAN subroutine in 'General-static' and later the second iteration is restarted in Dynamic implicit, and the computational process is repeated. From the Figure 4.19 (e) and (f) show the material removal distribution obtained from the FE simulation. It can be observed that the maximum material removal depth obtained after the first iteration ' j ' is -0.266 mm and the final maximum material removal depth obtained after the last iteration is -0.366mm (negative sign indicates the material removal).

Figure 4.19 (a), (b) Average dynamic contact pressure (MPa), (c),(d) Sliding distance range (mm) and (e),(f) Material removal depth distribution (mm) for iteration 1 and iteration 2 respectively using hard pad.

Figure 4.20 Comparison between 3D (a) experimental and (b) Numerical material removal distribution and 2D material removal profiles (c) experimental and (d) numerical for hard backing pad

In Figure 4.20 (a) and (b), the experimentally measured and numerical simulated 3D material removal distribution are compared. The contact length of the considered experimental

distribution is 25.7 mm, and that of numerical distribution is 30.0 mm and whereas the contact width dimensions are 5.76 mm and 6.0 mm for experimental and simulation respectively. But the aforementioned comparison is done for a single experimental material removal distribution. Since four experimental trials are conducted, the average value of contact length and width are 25.52 ± 0.46 mm and 5.65 ± 0.21 respectively. Similarly, from the Figure 4.20 (c) and (d), for the considered experimental profile, the maximum material removal depth value (-0.388 mm) agrees with the numerical value of -0.366 mm (5.6% error), and the overall maximum material removal depth range calculated from the four experimental trials is -0.389 ± 0.007 mm ($-389 \pm 7 \mu\text{m}$) which agrees with the numerical and simulated values (5.9% error). Also, there is a shift in the location of the maximum material removal depth, this can be attributed to the randomness in the abrasive disc geometry (warp) and hence a shift in the maximum pressure position.

Figure 4.21 (a), (b) Average dynamic contact pressure (MPa), (c),(d) sliding distance range (mm) and (e),(f) material removal depth distribution (mm) for iteration 1 and iteration 2 respectively using soft pad.

Figure 4.21 (a) and (b) show the average dynamic contact pressure distribution obtained from the two iterations and Figure 4.21 (c) and (d) depicts the sliding distance parameter. Like the hard backing pad simulation results, there are contact singularity points noticed in the periphery of the disc. It is observed from Figure 4.21 (b) that the maximum pressure value is 0.295 MPa and the maximum pressure value that of the hard backing pad (Figure 4.19 (b)) is 0.457 MPa. Thus, it can be inferred that this contact pressure difference induced by the change in backing pad compliance plays a role in controlling the material removal depth. Additionally, from Figure 4.19(d) and Figure 4.21 (d) it can be observed that CSLIP values at these maximum pressure points are 1057.0 mm and 562.4 mm for hard and soft backing pads

respectively. Because of this, for the hard backing pad the maximum material removal depth is observed at the same point where the pressure is maximum, but in the case of the soft backing pad because of this reduction in CSLIP, another point where CSLIP is higher and pressure is lesser has the maximum material removal depth. Thus, confirming that the sliding distance also plays a major role in determining the material removal depth according to the Archard's wear equation.

Figure 4.22 Comparison between 3D (a) experimental and (b) Numerical material removal distribution and 2D material removal profiles (c) experimental and (d) numerical for soft backing pad

Figure 4.22 (a) and (b) shows the material removal distribution of the experimental trial and numerical simulation respectively. The mean contact length obtained from the experimental trial is 25.5 ± 0.867 mm and from the numerical simulation is 33.0 mm, whereas the experimental mean contact width is 9.43 ± 0.33 mm and the numerical result is 8.0 mm. The

mean of the experimental maximum material removal depth values is $0.133 \pm 0.009 \text{ mm}$ ($133.25 \pm 9.81 \mu\text{m}$), and the maximum material removal depth obtained through numerical simulation is 0.144 mm which agrees with the experimental measurement (8.27%). Figure 4.22 (c) and (d), show the 2D profile of the material removal depth and it can be observed that the maximum material depth matches as mentioned but due to randomness in the abrasive disc geometry such as inherent warps, the maximum material removal depth position is different from the experimental position.

4.5 Summary

In this chapter, the static pressure distribution and material removal distribution is investigated for both ‘hard’ and ‘soft’ rubber pads (two different compliance levels) by experimentally and numerically.

- From the image-processed pressure films, contact pressure distribution data is obtained. The load and tool compliance play a crucial role in deciding the contact pressure distribution. For an increase in load, irrespective of the compliance, the contact dimensions increased. For the same load, the ‘soft’ pad exhibited higher contact dimensions compared to ‘hard’ rubber pad. High pressure regions are found near the initial contact region for the hard backing pad whereas the pressure is almost uniformly distributed in the case of soft pad. For rubber pad with the grainless abrasive disc, the same trend is followed which indicates that the backing pad has a larger influence in deciding the contact pressure distribution
- From the machining trials conducted without any feed, it can be observed that the compliance plays a crucial role in deciding the maximum material removal depth and material removal distribution. The maximum material removal depth almost doubles for 15N, 20N and 25N when the backing pad compliance is changed .

- For the 20N applied load, the numerical contact pressure distribution results are compared with the experimental measurements. The overall contact length and width agrees well with the experimental results, and except for the stress singularity regions, the computed pressure matches the experimental data.
- The material removal depth distribution computed using the FE model for both hard and soft pad agrees well with the experimental material removal. The FE model is able to capture the difference in the dynamic contact pressure between the two pad compliance and relate it to the material removed using ALE. Thus, the proposed methodology will be helpful in predicting the material removal depth, once the wear coefficient and material properties of both the backing pad and abrasive discs are given as input to the model.

Chapter 5 Three-dimensional topography modelling of coated abrasive disc surface

5.1 Overview

The material removal depth calculations in polishing inherently use the contact pressure variation over the grain distribution [76]. Hence, the first step in material removal modelling of the coated abrasive disc is grain distribution modelling. The existing topography models can be broadly classified into one, two and three-dimensional models, each giving different levels of information. Although they are similar in most of the aspects, in the case of 3D models the grain geometry, the spacing between the grains and the resultant surface is three dimensional [130]. Hegeman et al. [54] considered parameters like protrusion height, inter-grain spacing for modelling the three-dimensional surface by assuming the grains to be in ellipsoidal shape. Feng et al. [76] developed a similar 3D surface for cone-shaped grains, and additional resin layer was modelled to replicate the actual surface. In another study, the grains were modelled as 3D polygons with different sides based on the statistical information obtained from the actual grain aspect ratio, facets, size, and protrusion height [55]. Unlike the conventional coated abrasive discs where the grains are irregular in shape, uniformly shaped grains are comparatively durable and have better performance. Although the shape of the grains is fixed, yet the protrusion height is still random in nature. This could be attributed to either the placement of the grains within the resin layer of the coated abrasive disc or due to the randomness of grain orientations caused while depositing the grains on the ‘make-coat’ resin layer.

Figure 5.1 Optical image of (a) #60 grit precision shaped abrasive disc (b) #120 grit precision shaped abrasive disc

In this chapter, a three-dimensional surface topography model for #60, #120 grit size alumina coated abrasive disc with precision shaped grains (3M[®] Cubitron II), as shown in Figure 5.1 is proposed by using a stochastic process. The key parameters like protrusion height, inter-grain spacing and additionally grain orientations are considered. The simulated surface is validated with the measured surface results using the 3D surface parameters. The same model is extended to randomly shaped #60 grit size coated abrasive disc to prove the suitability of the model for all kind of abrasive shapes.

5.2 Measurement of abrasive discs

The abrasive discs whose surface need to be modelled are initially scanned using laser triangulation in Talyscan 150. Since the abrasive surface is rough and the particles are hard, using contact methods such as diamond tip probe will not be feasible. Hence non-contact laser method is used. Since three-dimensional information is required, the abrasive discs are scanned both through columns and rows for the given scanning area and intervals. The sampling intervals or spacing is a key factor to avoid information loss from the scanned profile, as having a large interval would lead to information loss about the topography and having a smaller

interval will increase the scanning time [69]. Blunt et al. [131] have proposed the optimal spacing Δs in Eq.5.1 as a function of average grain diameter.

$$\frac{d_g}{4} < \Delta s < \frac{d_g}{3} \quad (5.1)$$

The average grain diameter d_g can be expressed in terms of grit size M_g as [132]

$$d_g = 28.89 M_g^{-1.18} \quad (5.2)$$

From the Eq. 5.1-5.2 equations, the sampling interval for #60 and #120 is calculated as 50 μm , and 30 μm respectively.

$$S_{ds} = \frac{\text{Number of summits}}{(M-1)(N-1) \Delta x \Delta y} \quad (5.3)$$

where Δx and Δy are the spacing along the x-axis (mm) and y-axis (mm) respectively. M, N are total number of data points along x-axis and y-axis respectively. The number of summits extraction is done using MATAB® code. In this work, the 8 cell neighbouring technique is used, where a peak is defined as the highest data point in the surrounding eight cells. Later from the number of summits, the grain density (S_{ds}) is calculated using Eq.5.3.

(a)

(b)

Figure 5.2 (a) Measuring abrasive disc using laser gauge (b) scanned area versus grain density (S_{ds})

Selection of optimal scanning area is also necessary for topography modelling [132,44]. Scanning area of 10 mm x 10 mm is chosen in this study based on Figure 5.2 (b). Different scanning area ($5^2, 10^2, 15^2, 20^2, 25^2$ mm²) versus the grain density S_{ds} is shown in Figure 5.2 (b), and 10^2 mm² is an optimal scanning area based on the scanning time and the standard error. Figure 5.3 (a) and (b) shows the sample measured the topography of # 60 and #120 grit size. The scanning is repeated in different parts of the pad for repeatability.

Figure 5.3 Scanned topography of (a) #60 (b) #120 coated abrasive disc

5.3 Fourier filtering

Generally, any scanned abrasive surface is segregated into short, medium and long wavelength components. The longer wavelength corresponds to the waviness, the medium wavelength is attributed to the grains, and shorter wavelength is always attributed to roughness and noise. In case, if roughness is the interest, a high pass filter with 0.8 mm cutoff wavelength (λ_c) is employed to extract the high frequency content ($f=1/\lambda$). But, since our interest is in extracting medium and longer wavelength, a low pass filter is used [69]. Mathematically, a surface is assumed to be a random process that can be divided into different spatial frequencies (wavevectors) with the help of power spectral density (PSD) analysis formula as shown in Eq.5.4. In turn, PSD is the Fourier transform of the autocorrelation function (Eq.5.4) of the

signal, which contains power information without the phase across a range of wavevectors [133].

$$P(k,l) = \frac{1}{MN} |FFT|^2$$

$$= \left| \frac{1}{MN} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} z(m,n) e^{-j2\pi(\frac{k}{M}m + \frac{l}{N}n)} \right|^2 \quad (5.4)$$

where $k=0,1,2 \dots M-1$ and $l=0,1, \dots N-1$

M and N are the number of rows and columns of the scanned coated abrasive surface. If $M=N=201$, it is converted to a value nearest power of 2 i.e 256. Another way to calculate PSD is from periodogram where PSD is estimated directly from the squared FFT .

Autocorrelation function (ACF) is considered as inverse FFT of PSD function

$$R(m,n) = \frac{1}{MN} \sum_{l=0}^{N-1} \sum_{k=0}^{M-1} P(k,l) e^{j2\pi(\frac{k}{M}m + \frac{l}{N}n)} \quad (5.5)$$

$$S_{tr} = \frac{R_{min}}{R_{max}} \quad (5.6)$$

(a)

(b)

Figure 5.4. (a) Average PSD versus spatial frequency, (b) autocorrelation function (c) Autocorrelation length and texture ratio from Rmin and Rmax

Figure 5.4 (a) shows the power spectrum versus the spatial frequency (mm^{-1}) for of #60 and Figure 5.4 (b) shows the derived autocorrelation function for #60 grit. From the figure, it can be observed that the power almost tends to zero after 0.5 mm or 2 mm^{-1} , which represents the noisy data. The high frequency or longer wavelength components are removed from the measured data using 2D low pass filter using MATLAB® code.

From the Figure 5.4 (c), spatial parameters such as S_{tr} and S_{al} , can be obtained from the 2D Autocorrelation function (ACF). Autocorrelation length, S_{al} , (R_{min}) is defined as the minimum autocorrelation length taken to decay to a value of 0.2 from the peak. S_{tr} , is defined as the ratio of minimum distance to decay from 1 to 0.2 to the maximum distance taken and it is calculated using Eq.5.6. S_{tr} values for #60 and #120 grit size are 0.648 and 0.816. If the S_{tr} value is close 1, it indicates that the surface is isotropic in nature. This term helps to check if the simulated surface is isotropic or anisotropic.

Figure 5.5 Fourier filtered topography with peaks (a) #60 (b) #120 grit size

Figure 5.5 (a) and (b) show the surface topography after filtering. The summits are identified and highlighted by calculating the highest point in the surrounding eight cells. Another problem in identifying grain summits using this eight cell neighbour method is to exclude the peaks created by undulations in the bond surface. So, protrusion height is considered which is defined as the height of grain tip from the average bond surface height. The threshold height is obtained by measuring the height of the size coat from the cross-section micrograph at different locations. The measurements are made using the image processing software, Image J®. The size coat helps to keep the grains rooted to the base pad. Another reason for having protrusion height is that it neglects the grains which are not likely to participate in the material removal because of their lower height values. The threshold heights for #60 and #120 are 0.2 mm and 0.091 mm, respectively. This agrees well with the Aurich et al. [134] assumption of taking 35% of grain height as its protrusion height.

Figure 5.6 (a) Optical image of a single #60 grain (b) Optical image of a single #120 grain
(c) Cross section view of #60 coated abrasive disc

From the optical images of the grains in Figure 5.6 (a) and (b), it can be observed that the grains are equilateral triangle where each side is 0.63 mm (L_g) with 0.12 mm width (W_g) for #60 grit size and L_g is 0.3 mm and W_g is 0.09 mm for #120 grit size. From Figure 5.6 (c), the cross-section of the abrasive disc can be clearly divided into three main sections such as fibre backing, resin and size coat. It can also be observed that there are few grains which are below the threshold limit, which is possibly due to the orientation of the grains during the deposition process.

5.4 Measurement results

The key parameters such as protrusion height of the grains and inter-grain spacing are obtained from the filtered experimental surface data using a MATLAB® code. The protrusion height for both #60 and #120 grit sizes follows a normal distribution which is in agreement with the works of Feng et al.[56] and Wang et al. [135]. In order to confirm whether the distribution follows normal distribution or not, Anderson-Darling (AD) test is carried using the following equations:

$$A^2 = -ns - \frac{1}{n} \sum_{i=1}^n (2i-1) [\ln F(x_i) + \ln(1 - F(x_{n+1-k}))] \quad (5.7)$$

ns is the sample size, $F(x)$ is the *cdf function* of i^{th} value when sorted in ascending order

$$F(x_i) = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x_i - \mu}{\sqrt{2}\sigma} \right) \right] \quad (5.8)$$

If the obtained p-value is greater than 0.05 (i.e., taking 95% confidence level), then according to the null hypothesis (H_0) the data follows a normal distribution [62]. The calculated p-values from the AD test are listed in the Table 5.1.

Figure 5.7 Experimental protrusion peak height distribution (a) #60 (b) #120

Figure 5.7 shows the histogram plot of peak height distribution obtained from the scanned abrasive surfaces. In order to calculate the inter-grain spacing information, the scanned area is divided into small cells of length L_c which is given by the Eq 5.9 [54]. Next, using MATLAB code, an algorithm to compute the average distance of the grains in the surrounding eight cells from the grain in the central cell is executed, as shown in Figure 5.8. In case, if there are multiple grains in the centre cell, the grain nearest to the cell centre is chosen as the ‘pivot grain’.

Figure 5.8 Inter-grain spacing calculated using average distance from pivot cell grain

$$L_c = \frac{1}{\sqrt{S_{ds}}} \quad (5.9)$$

where L_c is the lattice length and S_{ds} is the grain density expressed as the number of grains per mm^2

Figure 5.9 Experimental inter-grain distribution for (a) # 60 (b) #120

Figure 5.9 (a) and (b) shows the inter-grain spacing distribution for the coated abrasive disc. For both the types of grits, the inter-grain spacing follows a normal distribution which agrees with the existing results [135]. It can be observed that the mean intergrain spacing for #60 grit size is 1.2 mm compared to 0.674 mm for #120 grit size. Since the larger grains are used for higher stock removal, they need more space for chip flow and cooling [136]. The mean and standard deviation of these obtained experimental distributions are tabulated in Table 5.1.

5.5 Stochastic simulation of the 3D topography

5.5.1 Grain distribution

The area to be simulated is divided into smaller cells/ lattices whose length is given by L_c obtained from the Eq.5.9. In the first step, each grain is placed at the centre of each cell and by adding random displacement to the grains in the lattices. Iterations are carried out till the spacing distance between the grains follows the normal distribution with the obtained mean and standard deviation values matching the experimental spacing distribution. At the same time, each time the minimum distance between the grains must be maintained such that no two grains overlap each other. The simulation of the random spatial distribution of the grains is achieved by a virtual shaking process given by Eq. 5.10

$$\begin{bmatrix} x_{gij} \\ y_{gij} \\ z_{gij} \end{bmatrix} = \begin{bmatrix} x_{cij} + \delta_x \\ y_{cij} + \delta_y \\ z_{cij} + \delta_z \end{bmatrix} \quad (5.10)$$

where $i=1, 2, \dots, M$ and $j=1, 2, \dots, N$ with M, N are the size of row number and column of the area respectively (scan length/resolution). Suffix 'c' indicates the position of lattice centre and 'g' transformed coordinate. δ_x , δ_y are random numbers from normal distribution δ_z is displacement replicating grain position within the resin layer which follows normal distribution measured from the optical image.

5.5.2 Grain rotation and peak height stochastic simulation

The random height distribution of the grains is not only because of the placement of grains in the resin layer, but also due to random orientation of the grains. Hence unlike other topography modelling simulations, grain orientation becomes a crucial factor in deciding the protrusion height of the grains. During the coated abrasive disc production, when the grains are being dispersed by the electrostatic process, the chance of all the grains to be upright is not possible, and each grain would be oriented randomly. In other research works, the grain orientation angles are not considered during modelling, but in this study, an orientation range of grains from $+45^\circ$ to -45° is considered. The orientation range is selected such that any grain that lies below this threshold angle might not participate in the cutting process and it will lie well below the threshold height considered earlier. Using the MATLAB code, iterations to randomly vary the orientation angle is carried out until the protrusion height distribution parameters are matched with the experimental values. Figure 5.10 shows the simulated protrusion height and inter-grain spacing distributions for #60 and #120 grit coated abrasive discs.

At first, the vertices of normal grain coordinates are taken as $[v]$, then it is pre-multiplied with the rotation matrix to obtain the final rotated coordinates as shown in Eq.5.11. The roll, pitch yaw for the grain along with the local coordinate system is represented in Figure 5.11(a). Then the grain is translated to the new position by adding the already obtained spatial coordinates in the previous steps using Eq. 5.13 [137]. Similarly, the program is executed until all the grains are simulated as shown in Figure 5.11(b).

Figure 5.10 Simulated inter- grain spacing (a) #60 (c) #120, Simulated protrusion peak height for (b)#60 (d) #120

$$Rot_{zyx}(\phi, \theta, \psi) = Rot_z(\phi) * Rot_y(\theta) * Rot_x(\psi)$$

$$= \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{pmatrix} \quad (5.11)$$

$$= \begin{pmatrix} \cos \phi \cos \theta & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \sin \phi \cos \theta & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi \\ -\sin \theta & \cos \theta \sin \psi & \cos \theta \cos \psi \end{pmatrix} \quad (5.12)$$

Modifying the product matrix to perform rotation followed by translation,

$$\begin{pmatrix} v'_x \\ v'_y \\ v'_z \\ 1 \end{pmatrix} = \begin{pmatrix} Rot_{zyx}(\phi, \theta, \psi) & \begin{matrix} t_x \\ t_y \\ t_z \end{matrix} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \\ 1 \end{pmatrix} \quad (5.13)$$

where ψ, θ, ϕ are a roll, pitch and yaw angles. $[v]$, $[t]$, $[v']$ are actual coordinate, translation vector and transformed coordinates respectively.

Figure 5.11 (a) Roll, pitch, yaw for a 3D grain, (b) simulated random grain orientation

The mean and standard deviation values of protrusion height and inter-grain spacing parameters obtained from grit #60 and #120 coated abrasive grains are listed in Table 5.1. The p -values obtained from AD normality test for each distribution is included in the table. In order to statistically compare the mean and variances of the experimental and simulation distributions, statistical testing needs to be conducted [62]. Hence, t-test and F-tests are carried out at 5% significance level, and the obtained p -values are mentioned in Table 5.1. The detailed explanation regarding t -test and F -test is discussed elsewhere[138,139]. This confirms that the means and variances of experimental parameters match with that of the simulated parameters.

Table 5.1 Statistical parameters for grit #60 and #120 coated abrasive grains

Statistical parameters	#60		#120	
	Experimental	Simulation	Experimental	Simulation
Grain protrusion height				
Mean (μ_{ph}) (mm)	0.344	0.335	0.139	0.138
Standard deviation (σ_{ph}) (mm)	0.058	0.054	0.026	0.024
p-value from AD test	0.106	0.056	0.068	0.121
p-value from t-test	0.191		0.645	
p-value from F-test	0.505		0.113	
Inter-grain spacing				
Mean (μ_{igs}) (mm)	1.158	1.186	0.674	0.683
Standard deviation (σ_{igs}) (mm)	0.105	0.119	0.074	0.068
p-value from AD test	0.22	0.633	0.193	0.3
p-value from t-test	0.131		0.142	
p-value from F-test	0.292		0.190	

Once the 3D abrasive prismatic shaped grains are distributed in compliance with the required distributions and the statistical parameters such as mean and standard deviations, surface fitting needs to be carried out to replicate the actual scanned surface. Surface fitting in this study is carried out for the simulated protruding grains using the biharmonic surface interpolation in curve fitting toolbox MATLAB®. Although there are different surface fitting methods such as LOWESS, polynomial, and interpolant methods. Interpolant method is advantageous in providing a smoother fit as the surface passes through every data point. Generally, the sum of squared error (SSE), a ‘goodness of fit’ parameter is employed to evaluate the best fit. But in the case of interpolant method, this parameter cannot be used because the SSE is always equal to zero, owing to the fact that the surface passes through all the data points. Hence, the decision of selecting the more appropriate type of surface fit depends on the factors like (i) data, (ii) the required smoothness of the curve, and importantly (iii) ‘post-fit analysis’ requirement [25]. In this study, the 3D surface parameters are the key ‘post-fit analysis’ factors. These values generated through Bi-harmonic surface interpolant method

found to be in good agreement with the actual 3D surface parameters values and hence, this method was selected. In biharmonic surface fitting, the interpolating surface is a linear combination of Greens functions centred at every grain data point. The amplitude is adjusted such that the interpolating surface passes through the simulated grain co-ordinates. A detailed explanation of biharmonic spline interpolation can be found elsewhere [68].

(a)

(b)

Figure 5.12 Simulated 3D coated abrasive surface grit # (a) 60 (b) 120

Table 5.2 Surface parameters for #60 and #120 coated abrasive

3D surface parameters	#60				#120			
	Experimental	Simulation	t-test p-value	F-test p-value	Experimental	Simulation	t-test p-value	F-test p-value
<u>Height(Amplitude) parameters</u>								
S _q	0.232±0.018	0.211±0.018	0.972*	0.015**	0.102±0.002	0.105±0.003	0.491*	0.039**
S _{sk}	1.207±0.059	1.234±0.040	0.273*	0.243*	1.154±0.019	1.151±0.026	0.391*	0.749*
S _{ku}	1.559±0.171	1.686±0.128	0.398*	0.077*	1.467±0.051	1.41±0.073	0.311*	0.059*
<u>Hybrid parameters</u>								
S _{ds}	1.128±0.118	1.234±0.284	0.016**	0.290*	2.875±0.398	2.347±0.709	0.101*	0.055*
S _{sc}	4.309±0.331	4.133±0.296	0.745*	0.226*	3.516±0.152	3.792±0.316	0.04**	0.022**
S _{dq}	0.351±0.016	0.356±0.037	0.019**	0.748*	0.23±0.005	0.226±0.008	0.17*	0.172*
<u>Functional parameters</u>								
S _c	0.072±0.004	0.073±0.009	0.013**	0.733*	0.031±0.003	0.032±0.003	0.841*	0.779*
S _{ci}	0.302±0.027	0.350±0.050	0.079*	0.016**	0.31±0.027	0.306±0.041	0.248*	0.805*
S _v	0.116±0.017	0.104±0.021	0.504*	0.210*	0.057±0.004	0.067±0.01	0.032**	0.01**
S _{vi}	0.479±0.056	0.489±0.073	0.447*	0.728*	0.564±0.039	0.642±0.086	0.028**	0.017**
<u>Spatial parameters</u>								
S _{al}	1.120±0.204	1.029±0.220	0.824*	0.351*	0.888±0.131	0.834±0.109	0.589*	0.33*
S _{tr}	0.648±0.127	0.742±0.109	0.679*	0.092*	0.816±0.16	0.797±0.111	0.29*	0.76*

Note: * denotes acceptance at 5% level of significance and ** denotes 1% level of significance

The simulated surfaces of both #60 and #120 grit sizes are shown in Figure 5.12 (a) &(b). The simulated and experimentally measured abrasive surface is compared using the three-dimensional surface parameters. This helps to quantitatively verify the similarity between the two surfaces. The significance of each parameter is discussed elsewhere [140,141]. In order to check the equality of the mean and standard deviations between the simulated and

experimentally obtained data, t -test and F -test are carried out. The sample size of 10 is considered (10 data sets each from experimental measurements and simulations). As indicated in Table 5.2, the mean and variance of simulated values match well with that of the experimental values at mostly 5% significance level and at 1% significance level in some cases.

Figure 5.13 Flowchart for the stochastic simulation of the coated abrasive disc

Figure 5.13 describes the overall methodology involved in the three-dimensional surface topography modelling. The overall structure can be divided into three blocks: (i) spatial distribution module, (ii) peak protrusion height simulation module which involves the grain rotation simulation sub-module, and (iii) surface fitting and validation module. A similar approach can be used to model the surface topography of randomly shaped grains as discussed in the following section.

5.6 Randomly shaped grains

Most of the conventional abrasive discs are irregularly shaped. The suitability of the proposed methodology to model even irregularly shaped grains is demonstrated in this section. Coated abrasive disc #60 grit size is considered for modelling.

5.6.1 Scanning and grain size extraction

The abrasive surface of 10mm x 10mm is scanned using laser probe by choosing appropriate spacing of 50 μm . The scanned surface is shown in Figure 5.14 (a). Using the Fourier analysis as discussed earlier, 2D low pass filter is applied, and noisy data / high frequency content is suppressed. The filtered surface topography is shown in Figure 5.14 (b) with summits highlighted.

Figure 5.14 (a) Scanned #60 grit abrasive surface (b) Filtered surface

In order to model the randomly distributed grains as prismatic grains with equivalent size, initially the semi-minor, semi-major and height of the irregularly shaped grains are extracted. This information is extracted using MATLAB® code using Image processing toolbox® [142].

Figure 5.15 (a) Optical image of irregularly shaped grains (b) Binary image of the grains (c) Grains enclosed with approximate elliptical boundary

Initially, the grains are separated from the coated abrasive disc using sodium hydroxide solution, and the cleaned grains are scanned using an optical microscope. Figure 5.15 shows the optical images of the irregularly shaped grains. Then this color image is converted into a binary image. Later by using the MATLAB code, the irregularly shaped grains are encapsulated into equivalent ellipses. The minor, major axes length of these ellipses are computed, and the statistical distribution of these grains are obtained. The height (third axis information) of these sample grains are measured using laser profilometry and the height information; the distribution is obtained. The minor, major and height distributions follows normal distribution as shown in Figure 5.16 (a), (b) and (c).

Figure 5.16 (a) Minor axis distribution (b) Major axis distribution (c) Height distribution

From the scanned surface, the protrusion height and inter-grain spacing are calculated as discussed in the previous section. At first, the threshold height is measured from the cross-section image. Then the peaks that are above this threshold level is considered for protrusion height distribution. The inter-grain spacing is also calculated by using the previous algorithm that was followed for precision-shaped grains. Both the protrusion height and inter-grain spacing of the grains follow normal distribution as shown in Figure 5.17.

Figure 5.17 Distribution of (a) Experimental protrusion height (b) Experimental inter-grain spacing of #60 grit

Table 5.3 Statistical parameters of #60 irregular shaped grains

Statistical parameters	#60	
	Experimental	Simulation
Grain protrusion height		
Mean (μ_{ph}) (mm)	0.344	0.335
Standard deviation (σ_{ph}) (mm)	0.058	0.054
p-value from AD test	0.106	0.056
Inter-grain spacing		
Mean (μ_{igs}) (mm)	1.158	1.186
Standard deviation (σ_{igs}) (mm)	0.105	0.119
p-value from AD test	0.22	0.633

5.6.2 Simulation of 3D surface topography

In the earlier methodology all the prismatic grains have a fixed grain dimensions, but here the individual grain dimensions are random following the discussed statistical distributions. Later these grains with random dimensions are distributed satisfying the inter-grain spacing distance, and iterations are carried out until the protrusion height distribution is matched by randomly orienting these grains.

Figure 5.18 Distribution of (a) Simulated protrusion height (mm)
(b) Simulated inter-grain spacing (mm)

The simulated protrusion height and inter-grain spacing distributions are shown in Figure 5.18 (a) and (b). The simulated protrusion height distribution and inter-grain spacing distribution follow normal distribution. The mean and standard deviation of these distributions are listed in Table 5.3. In order to test the normality of the distribution, AD test was carried out and the respective p-values are listed. The p-values are greater than 0.05, which shows that the null hypothesis that the data follows a normal distribution is accepted at 95% confidence level.

The prismatic grains of different sizes can be observed from the Figure 5.19. The grains are randomly oriented till the protrusion height distribution is matched. The vertices of the grains are highlighted. Unlike the precision-shaped grains, here the grains are sized according to the derived information regarding the minor, major axes and height distribution.

Figure 5.19 Randomly distributed prismatic grains

Figure 5.20 Simulated grain surface

Once the grains are distributed, the three-dimensional surface is obtained by employing bi-harmonic surface fitting. The simulated surface is shown in Figure 5.20. From the simulated surface, the three-dimensional surface parameters are computed using MATLAB[®] code. The comparison between experimental and simulated surface is listed in Table 5.4.

Table 5.4 3D surface parameters of simulated and experimental surfaces

Surface	S_q	S_{sk}	S_{ku}	S_{dq}	S_c	S_{ci}	S_v	S_{vi}
Experimental	0.306	1.198	1.552	0.436	0.085	0.278	0.172	0.565
Simulation	0.315	1.161	1.470	0.402	0.109	0.347	0.159	0.505

The roll (ψ), pitch (θ), yaw (ϕ) orientation angles information will be helpful to randomly distribute the grains in commercial FEA software like ABAQUS® through Python program. The scratch profile for each grain can be simulated by using Johnson -Cook damage model. The same information can also be used to model the material removal using the tribological and contact mechanics theory. Initially, from the overall pressure, the force acting on these grains is calculated and the relative indentation depth and contact force is computed. These modelling approaches using the grain distribution information will be discussed in the next chapter.

5.7 Summary

In order to model the material removal depth using grains, topography modelling is an essential step. From the model, information like grain distribution and orientation information are acquired which can be given as input to the material removal model. In this study, a modelling technique for precision and irregularly shaped grains is proposed. Recently precision-shaped grains are used owing to their higher efficiency and durability compared to the conventional irregularly shaped grains.

- The three-dimensional surface data of #60 and #120 coated abrasive discs is measured using laser profilometer by selecting suitable optimal sampling interval. The high frequency noise is removed by using 2D low pass filter. Using the MATLAB® code, the experimental peak protrusion height and inter-grain spacing distributions are obtained. Both the distributions followed normal distribution. The normality of the distribution is confirmed using Anderson-Darling (AD) test at 95% confidence level.
- In the stochastic modelling of prismatic grains, the surface area to be simulated is divided into equal cells and each cell houses a grain. Later, the spatial distribution of

grains is simulated by ‘random shaking’ of the grain such that they don’t overlap and match the experimental inter-grain spacing distribution.

- In the same manner, iterations are carried out by randomly orienting the grains using rotation matrix, till the protrusion height distribution matches the experimental parameters. This model, helps to acquire the pitch, roll, and yaw angles which will be helpful during material removal model using grains. The similarity of the mean and standard deviation between experimental and simulated protrusion height along with the inter-grain spacing distributions is confirmed using F -test and t -tests.
- The distributed grains are fitted using biharmonic surface fitting option. Various 3D surface parameters such as S_q , S_{sk} , S_{ku} (amplitude parameters), S_{ds} , S_{sc} , S_{dq} (hybrid parameters), S_c , S_{ci} , S_v , S_{vi} (functional parameters) and S_{tr} , S_{al} (spatial parameters) are computed from both experimental and simulated surface using MATLAB code. The validity of agreement between the means and variances of the experimental and simulated 3D surface parameter values are validated using t and F tests.
- The topography modelling approach was also implemented for irregularly shaped grains, where additionally minor, major and height data distribution of the grains are extracted. Then prismatic grains of varying dimensions matching the above distribution is simulated. The other steps are the same as followed for the uniform prismatic grains.

Chapter 6 Material removal modelling with grains

In the previous chapters, the experimental trials to investigate the effect of compliance on material removal and numerical simulation technique to predict material removal using Archard's wear equation is discussed. In Chapter 5, the three-dimensional topography modelling of both precision and irregularly shaped grains is discussed in detail and this modelling technique will be employed in this chapter to simulate three-dimensional material removal using two approaches (i) FE analysis using Johnson-Cook constitutive damage model and (ii) semi-analytical method. The difference between the Archard wear model and the models proposed in this chapter is the inclusion of grains which results in better understanding of the grain-workpiece interactions. This chapter discusses the overall methodology considered to simulate material removal using three-dimensional randomly distributed grains generated by stochastic modelling approach. Generally, in grinding simulations, the model is simplified by considering the wheel as rigid and only the grains are modelled and made to interact with the workpiece material, but in this study since the tool compliance is an important factor, the entire rubber tool is modelled and its effect on material removal is investigated.

In the second technique, a semi-analytical approach is used. Initially, the average contact pressure distribution is obtained from the FE analysis. Then, using the grain distribution information obtained from the previous Chapter 5, the randomly distributed and oriented grain is simplified and modelled as equivalent conical grains with spherical tips. Using the contact mechanics approach for indentation, the material removal depth for every grain is obtained. The simulation is carried out using MATLAB® code and the simulated material removal model is compared with the experimental result.

6.1 Numerical simulation of material removal using Johnson Cook damage model

In order to understand the effect of polishing tool compliance on the material removal depth, numerical simulation is carried out using ABAQUS ®. In the earlier chapter, the contact pressure and sliding velocity were directly converted into material removal depth using Archard's wear equation using the experimental wear coefficient value for both hard and soft polishing backing pads. But, in this proposed methodology, the randomly distributed and oriented grains are imported as three-dimensional models and their interaction with the workpiece are investigated. FE simulation is considered as an important tool to study the grain-workpiece interaction. One of the biggest challenges in this simulation technique is to arrive at an optimal point so that the simulation reflects the real time material removal and at the same time, the computational cost is minimal. There are different techniques followed in the literature to simulate chip separation such as adaptive meshing , previously defined parting line , using damage criteria, meshless method and eulerian method. The material removal technique using damage criteria is more suitable in this case, as methods like parting line and eulerian need the depth to be known before, whereas adaptive meshing and meshless methods are computationally time-consuming compared to damage criteria method. Johnson Cook damage model is the predominantly used damage model for metal cutting simulation [143]. To simulate the metal cutting process, ABAQUS/Explicit offers Johnson-Cook failure model. A detailed explanation regarding this damage model is discussed in section 2.6.

In this technique, the material is considered to be removed when the elements reach the failure condition governed by Jonson -Cook damage model. The interaction effect of each grain on the workpiece is considered. Thus, the compliance effect of the tool on material removal is obtained after few tool rotations, as carrying out simulation for the complete experimental time involves high computational time.

The three-dimensional models of the rubber pad, shaft and the abrasive disc are modelled as discussed in Chapter 4. Instead of considering the abrasive disc as composite disc, here it is simplified as elastic to reduce the computational time and to accommodate the three-dimensional grains on it. The entire abrasive disc is modelled in three-dimension (3D). Hence after 3 rotations, relevant scaling has to be done to predict the material removal after complete machining time of 3 seconds. The simulations are carried out in Abaqus /Explicit for both soft and hard polishing backing pads and to reduce the simulation time, the aforementioned assumptions are made.

6.1.1 Methodology for numerical simulation

The complete algorithm for modelling the abrasive disc with grains is shown in Figure 6.1. The position and orientation information of each grain is obtained from the stochastic three-dimensional abrasive surface modelling technique discussed earlier. Later, using the information each grain is positioned and rotated accordingly. Once all the grains are created, the abrasive disc's backing is merged with the grains and relevant partitioning is carried out. Grains and abrasive disc backing are meshed using linear tetrahedral elements C3D4. The rubber pad was meshed using three-dimensional modified hybrid tetrahedral element C3D10MH in order to capture large deformation due to hyperleastic nature. The steel shaft is considered to be rigid. The same pre-processing meshing procedure followed in section 4.2.3 is followed here. Except for the change that 'explicit' type is chosen for the elements. Also, the aluminium workpiece is meshed with $100\text{ }\mu\text{m} \times 100\text{ }\mu\text{m} \times 40\text{ }\mu\text{m}$ C3D8R elements with deletion initiated if the damage factor (D) reaches 1.

Figure 6.1 Algorithm for generating 3D randomly distributed grains in ABAQUS

6.1.2 Modelling and boundary conditions

Figure 6.2 Assembly model of rubber pad, abrasive disc and workpiece

Figure 6.2 shows the 3D assembly of rubber pad, abrasive disc and aluminium work coupon used for material removal simulation. The abrasive disc and the rubber pad base are tied, and the grains are in turn tied to the abrasive disc as depicted in Figure 6.3. The base of the aluminium plate is fixed and the tool's boundary conditions are given at the reference point on the tool shaft. Tool velocity of 261.8 rad/s (2500 RPM) is applied to the reference point.

Figure 6.3 Abrasive disc with randomly distributed grains

6.1.3 Material property and meshing

The rubber pad material property is governed by hyperelastic material property. The Yeoh's constitutive model is found to be more suitable for the rubber tensile data as detailed in section 4.2. The shaft is considered as rigid body. The abrasive disc and the grains are modelled as elastic material with Young's moduli (E) 600 MPa and 400 GPa respectively. The Poisson's ratio is considered to be 0.3. For the aluminium plate (E=68.9 GPa, $\rho=2790\text{kg/m}^3$, $\nu=0.3$), the following Johnson-Cook plastic model, damage criteria and evolution values are considered.

Johnson Cook plastic model is given by

$$\bar{\sigma} = (A + B\bar{\varepsilon}^n) \left[1 + C \ln \left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0} \right) \right] \left[1 - \left[\frac{T - T_{room}}{T_m - T_{room}} \right]^m \right] \quad (6.11)$$

Table 6.1 Johnson-Cook plastic model parameters [144]

A (MPa)	B (MPa)	C	n	m	T_m	T_{room}	$\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}$
314	114	0.002	0.42	0	-	-	1

where $\bar{\sigma}$ is the equivalent plastic stress (MPa), $\bar{\varepsilon}^p$ is the equivalent plastic strain, $\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}$ is the equivalent plastic strain rate, $\dot{\varepsilon}_0$ is the reference equivalent plastic strain rate, T is the temperature ($^{\circ}\text{C}$), T_m is the melting temperature of the work material ($^{\circ}\text{C}$) and T_{room} is the room temperature ($^{\circ}\text{C}$). A , B , C , m and n are material parameters, which are determined based on the flow stress data obtained from mechanical tests (uniaxial tensile test and Split Hopkinson Pressure Bar test) [144]. The simulation is considered as temperature independent.

6.1.4 Johnson Cook ductile fracture model

There are five Johnson-Cook damage criteria from d_1 to d_5 , since the temperature effect is not considered in this study as listed in Table 6.2 . Based on the Lagrangian model, the failed elements are deleted when accumulated damage parameter D reaches one ($D=1$) in the element. The damage parameter follows a cumulative damage law based on plastic strain is given by Eq. 6.12

$$D = \sum \frac{\Delta \varepsilon^p}{\varepsilon^f} \quad (6.12)$$

where $\Delta \varepsilon^p$ is the increment of the equivalent plastic strain during an integration cycle and ε^f the equivalent strain to fracture under current conditions of strain rate, temperature, pressure and equivalent stress, as given by Eq. 6.13

$$\varepsilon^f = \left[d_1 + d_2 \exp \left(d_3 \left(\frac{p}{q} \right) \right) \right] \left[1 + d_4 \ln \left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}^0} \right) \right] \left[1 + d_5 \left(\frac{T - T_{room}}{T_m - T_{room}} \right) \right] \quad (6.13)$$

where p is the hydrostatic pressure stress, q is the Von Mises effective stress and other parameters were defined earlier.

Table 6.2 Johnson Cook damage coefficients [144]

d1	d2	d3	d4	d5
-0.77	1.45	0.47	-	-

6.1.5 Material removal results

The simulation result for the material removed in the aluminium workpiece by the compliant polishing tool along with the 3D modelled abrasive discs is obtained after three tool rotations from ABAQUS®. The material removal is obtained by deleting the elements in the

aluminium block when it reaches the failure limit. The material removal distributions for both hard and soft pad are obtained and the COORD2 contour (co-ordinate along the depth direction: y-axis) are plotted as shown in Figure 6.4. The development in the scratch profile after each tool rotation can be observed.

Figure 6.4 Scratch profile for three rotations of compliant polishing tool with hard and soft backing pads

It can be observed that the contact dimensions viz. contact length and width of the soft is higher compared to the hard backing pad (Figure 6.4). The contact width measured from the simulated scratch profile employing hard and soft rubber backing pad are 8.45 mm and 13.6 mm (after three rotations) respectively. It is evident that the difference in the backing pad compliance is getting reflected in the material removal width.

Figure 6.5 Material removal profile for hard and soft pad after each rotation of the tool

Figure 6.5 shows the development in the material removal depth after each rotation for hard and soft pad. It can be observed that for the soft pad, the material removal depth peak points are more evenly scattered throughout the profile length, whereas for the hard pad, the material removal is mostly concentrated in the initial point of contact. The maximum material removal depth after each rotation for soft rubber backing pad are 53 μm , 72 μm and 93 μm which is almost double compared to hard backing pad (93 μm , 115 μm and 174 μm).

The numerical simulation including the random grain distributions and orientations help to simulate a more realistic material removal. Since, only three rotations are carried out, owing to the large computational time required, the complete growth of the material removal depth cannot be captured in this modelling technique. Though, this model cannot exactly predict the material removal depth after 125 rotations (i.e., the experimental trial for 3 seconds), it gives an insight on how the scratch profile develop after each rotation of the tool. Also, it sheds information on how the material removal is distributed for the different pad compliance (hard and soft). This could be helpful in developing new tool design and understanding the material removal growth because of the tool compliance. In addition, the numerical simulation is helpful in predicting the contact dimensions such as length and width.

Although the grain orientation and distribution of #60 grit alumina Cubitron II® abrasive disc are considered in the simulation, the effect of the change in the orientation or distribution in the material removal profile can be studied using this FE model. Also, the effect of different grain materials such SiC on the material removal can be studied by extending this FE model and adding damage and wear model for the abrasive grains.

6.2 Semi-analytical simulation of material removal depth

In order to predict the material removal depth considering the grain distribution and orientation effect, a semi-analytical approach is proposed in this study. The study involves three main steps viz., (i) obtaining the grain distribution/orientation information and simplifying the grains into appropriate simple shapes (ii) with the obtained pressure distribution, the indentation depth of each grain in the contact region is computed (iii) the obtained depth values are extrapolated over number of rotations to estimate the material removal rate.

In this study, the randomly oriented prismatic grains are simplified into conical grains with spherical tips as shown in Figure 6.6. The nose (r_n) of this shuttle shape grain is kept constant as the measured nose radius of uniform #60 grit alumina prismatic grains is 0.019mm. According to the orientation of the grains, the prismatic grain is mapped to a conical base and spherical tip on the top. The vertices are projected and the maximum of vertices distances from the grain centre along x and y directions is taken as radius (r_b) of the conical base. The sphere meets with the cone tangentially (x_t, y_t). So, the distance from spherical tip to the tangential point is considered as spherical region and below y_t till the base y_b is considered conical region.

Figure 6.6 Converting prismatic grain into conical grain with spherical tip

6.2.1 Conical grain with spherical tip geometry

The formulae used to model the equivalent conical grain with spherical tip is given below (Eq 6.14-6.20). The transition point from spherical to conical region (x_t, y_t) can be obtained using Eq 6.18. The cone semi-angle will be used in to calculate the plastic indentation depth of the cone.

Figure 6.7 D diagram of conical grain with spherical tip

where h – equivalent height of the tilted prismatic grain

d - distance from cone base (x_b, y_b) to the centre of sphere (x_c, y_c)

d_s - distance between cone tip (x_a, y_a) to the centre of sphere (x_c, y_c)

h_t - distance from tangential point to nose tip (transition between spherical to conical indentation)

α - Cone half- angle

$$\cos \phi = \frac{x_a}{d} \quad (6.14)$$

$$\cos \theta = \frac{r_n}{d_s} \quad (6.15)$$

$$\alpha = \theta + \phi - 90 \quad (6.16)$$

$$h_t = r_n (1 - \sin \alpha) \quad (6.17)$$

$$x_t = r_n \cos \alpha \quad (6.18)$$

$$d = h - r_n \quad (6.19)$$

Figure 6.8 shows the 2D schematic representation of height distribution of the conical grains with spherical tips indenting on the aluminium workpiece. The contact area is discretised into unit cells and in each cell with certain applied force, the grains protruding beyond the mean grain height (h_m) are considered for calculating the indentation depth. The distribution of the protrusion height is discussed in section 5.5 and it is found that it follows a normal distribution given by the Eq 6.20

Figure 6.8 2D schematic representation of the abrasive grains interacting with workpiece

$$f(h) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(h-h_m)^2}{2\sigma^2}} \quad (6.20)$$

Where σ is the standard deviation of the protrusion heights and h_m is the mean height of the grains

Figure 6.9 shows the indentation depth of the conical grain with spherical tip. In general, the overall grain-workpiece interaction can be divided into rubbing, ploughing and cutting [145]. The rubbing phenomenon occurs when the grain tip is only within the elastic limit (δ_e). The similarity between the cutting and ploughing process is that their indentation depth lies in the plastic limit and whereas difference between cutting and ploughing is that cutting involves chip formation and loss in material whereas ploughing involves only pushing aside the material. In this study, detailed separation of the two phenomena are not considered and they are assumed to be as single plastic indentation phenomena. The detailed algorithm to simulate the material removal depth is shown in Figure 6.10.

Figure 6.9 Indentation of conical grain tip spherical tip

Figure 6.10 Algorithm for material removal model using semi-analytical approach

The following assumptions are made in this modelling work

- The initial surface randomness is not considered in this study
- The strain hardening behaviour and its effect when abrasive indent multiple times is ignored in this semi-analytical modelling approach
- In this analytical work, frictionless contact is assumed between the grain and the interface, whereas a friction coefficient of 1.25 is assigned in the numerical simulations based on the friction value obtained from the machining trials.
- The material removal depth is considered to be linear and the indentation depth formed in one rotation is scaled to the experimental number of rotations and compared with the experimental 3D material removal depth.
- The effect of grain fracture, wear or the nature of abrasive used on the material removal profile is not considered in this study.

6.2.2 Dynamic pressure distribution results

The initial step in this modelling process is to obtain the average dynamic contact pressure values from the numerical simulation. The pressure values are obtained for grainless pad attached to the hard rubber pad when flexed and rotated against the aluminium workpiece. The spindle speed of 261.9 rad/s (2500 RPM) is given as velocity boundary condition and load of 20N is applied through the reference point. Details of the FE modelling are discussed in section 4.2. The contact pressure values are averaged in the given time interval and the nodal values are plotted in MATLAB® as shown in Figure 6.11. Since the cells are divided into 1mm by 1mm unit area cells, the force value can be directly obtained from the pressure values in each cell.

Figure 6.11 Averaged dynamic contact pressure distribution

From Figure 6.11, it can be observed that the average dynamic contact pressure at the periphery (e.g. at $x=15$ mm, $y=23$ mm and $x=39$ mm, $y=22$ mm) are high compared to the pressure region at the centre. This can be attributed to the contact stress singularity in the FE simulations. In the region of interest, the maximum contact pressure is 0.457 MPa at ($x=31$ mm, $y=23$ mm) as depicted in Figure 6.11.

6.2.3 Grain distribution for complete abrasive disc

From the populated grains as shown in Figure 6.12, only the grains within the 50 mm diameter is considered. The red dots indicate the grains stochastically populated. The randomly oriented grains are converted into equivalent conical grains with spherical tips as shown in Figure 6.12. As observed in the figure, the cones are with different apex angle and radius values. The grain location in the cartesian coordinates is converted into polar coordinates since the increment is given in terms of the angle ($d\phi$).

Figure 6.12 Abrasive disc modelled from total grain population

The complete three-dimensional grain distribution of the abrasive disc is shown in Figure 6.13

Figure 6.13 3D modelled conical grains with spherical tips

6.2.4 Indentation depth computation

•Elastic deformation

The elastic deformation signifies the process where the rubbing phenomenon occurs and there is no permanent dislocation of material. According to Hertz theory for the elastic contact of a plane with sphere, the contact area (A_e), the critical interference depth (δ_e) till which the contact is considered elastic and the contact load (P_e) are given by the following equations

$$A_e = \pi r_n \delta \quad (6.21)$$

$$P_e = \frac{4}{3} E^* r_n^{0.5} \delta^{1.5} \quad (6.22)$$

$$\delta_1 = \left(\frac{3\pi * 0.4 * H}{4E^*} \right)^2 r_n \quad (6.23)$$

Where R is the radius of the sphere

δ indentation depth

H Brinell hardness of the softer material (aluminium Al6061-T6)

E^* equivalent young's modulus

•Elastoplastic contact

When the interference (δ) is between δ_1 and δ_2 , elastoplastic deformation occurs. This region is composed of both elastic and plastic deformation regimes. The relation between elastoplastic area (A_{ep}), interference (δ), critical elastic (δ_1) and plastic (δ_2) limits (Eq 6.24-6.25)

$$A_{ep} = \pi R \omega \left[1 + \left(\frac{\delta - \delta_1}{\delta_2 - \delta_1} \right)^2 \left(-2 + 3 \left(\frac{\delta - \delta_1}{\delta_2 - \delta_1} \right) \right) \right] \quad (6.24)$$

$$P_{ep} = \left[H - H(1-k) \frac{\ln\left(\frac{\delta_2}{\delta}\right)}{\ln\left(\frac{\delta_2}{\delta_1}\right)} \right] \pi R \omega \left[1 + \left(\frac{\delta - \delta_1}{\delta_2 - \delta_1} \right)^2 \left(-2 + 3 \left(\frac{\delta - \delta_1}{\delta_2 - \delta_1} \right) \right) \right] \quad (6.25)$$

• Plastic contact-spherical region

When the interference depth is increased greater than δ_p , at which the contact pressure of the grain reaches the value of H , fully plastic deformation occurs. The interference required to produce fully plastic flow is given by Eq 6.26-6.28 [80]

$$\delta_2 > 25\delta_1 \quad (6.26)$$

The contact area A_{ps} and force P_{pc} is given by,

$$A_{ps} = 2\pi r_n \delta \quad (6.27)$$

$$P_{ps} = 2\pi r_n \delta H \quad (6.28)$$

• Plastic indentation-Conical region

Once the indentation depth (δ) exceeds the spherical region point (x_t, y_t), the plastic indentation formulae (Eq 6.29-6.30) for conical indentation is used.

The contact area A_{pc} and force P_{pc} is given by,

$$A_{pc} = \pi \delta^2 \tan^2\left(\frac{\alpha}{2}\right) \quad (6.29)$$

$$P_{pc} = \pi \delta^2 \tan^2\left(\frac{\alpha}{2}\right) H \quad (6.30)$$

P_T , force in each cell

$$P_T = P_e + P_{ep} + P_{ps} + P_{pc} \quad (6.31)$$

6.2.5 Material removal depth

The material removal depth is computed at each grain which is subjected to different force at every rotational increment ($d\phi$) from $-\pi$ to π . The grains within the contact pressure region (15 mm to 35 mm) are extracted each time as depicted in Figure 6.14. The dots in different colour indicates the grains at different rotational increment.

Figure 6.14 Grain distribution over the cells with contact pressure values

Accordingly, the indentation depth (ω) that satisfies the force equilibrium for the grains with random radius, apex angle is computed using nonlinear programming solver command from MATLAB. Iterations are carried out for each grain till the force values converge within the given tolerance of 10^{-6} N. Then only the grains that participate in plastic indentation are considered as elastic deformation does not contribute to the material removal. The scratch depth is interpolated in between two values of ϕ , so that a continuous scratch profile can be obtained.

Figure 6.15 (a) Scratch profile top view (b) 3D scratch profile for one rotation and (c) scaled 3D material removal depth

Figure 6.15 shows the scratch profile after one rotation and it can be observed from the figure that the maximum penetration depth is $4.2 \mu\text{m}$ (0.0042 mm). Since the material removal is considered linear in this modelling, the depth is multiplied by 125 to get the value of the depth after 3 seconds of machining. Using the maximum depth data points at each cell, a surface profile fitting is carried out using MATLAB®. Cubic interpolation function is used to fit the surface through the data points. The obtained surface represents the polished surface using the disc-shaped compliant tool

Figure 6.16 (a) Experimental machined surface material removal depth (b) Simulated material removal depth

From the Figure 6.16, it can be observed that the length and width of the machined area are 25.4 mm x 6.3 mm which matches with the simulated material removal profile dimensions of 28.5 mm x 6.55 mm. The maximum experimental material removal depth is 387 μm (0.387 mm) which agrees with the simulated maximum material removal depth of 337 μm (0.337 mm).

6.3 Summary

- The grain distribution and orientation information are obtained and converted into the three-dimensional part model in ABAQUS using the python script. According to the coordinate positions and orientation angles (ψ, θ, ϕ) all grains are tied to the abrasive disc backing into two quadrants. In order to reduce the computational time, only three_rotation of the disc are considered.
- The 3D assembly of rubber pad, abrasive disc with grains, shaft and aluminium Al6061-T6 workpiece is modelled in ABAQUS® with relevant material properties and the domain is discretized with appropriate mesh. Johnson-Cook damage

parameters based on failure strain criteria are used and the element deletion occurs once it reaches the critical failure strain value.

- Using this proposed methodology, scratch profiles after one rotation are compared for both hard and soft rubber backing pads. The overall material removal area for soft pad is higher compared to the hard pad. The scratch profiles are observed towards the tool centre in soft pad and whereas scratches are observed only in the region near the initial point of contact in hard pad.
- A semi-analytical modelling approach is proposed in this study to simulate the three-dimensional material removal depth. Using the distribution and orientation details of the prismatic grains, equivalent conically shaped grains with spherical tips are modelled.
- The average dynamic contact pressure distribution that is obtained from the numerical simulation is converted into force values. Later, the indentation depth of each grain at the respective cell is computed using MATLAB. Based on the radius, height distribution, the indentation depth is within either in elastic, elastic-plastic or plastic within spherical and conical regions.
- The indentation depth at the end of one rotation is plotted and the material removal depth is extrapolated to 125 rotations linearly to compare with the experimental machining time. Later the depth points are fitted using cubic spline interpolation using MATLAB® and the simulated machined surface is compared with the experimental surface. The overall experimental material removal length and width dimensions agree with the simulated surface. The maximum material removal depth agrees well with the simulated depth.

Chapter 7 Design and analysis of active compliant tool

As discussed in the earlier chapters, the generally used polishing backing pads are segregated into ‘hard’, ‘medium’ and ‘soft’ according to the rubber hardness which is directly related to the tool’s compliance. Presently, the conventional polishing backing pads that are used for polishing are passive in nature and according to the desired material removal rate, the backing pad is changed. For example, in the case of weld seam removal scenario, initially high material removal is required and in the later stages, lesser material removal and larger contact area is needed for blending the surfaces. Another scenario is polishing intricate parts like turbine blade roots where more compliant tool is needed to conform to the sudden change in the contours. Thus, resulting in controlled material removal and better dimensional accuracy. The current practice of changing the tool every time requires to stop the machining process, change the tool from the spindle and input the tool centre point (if the tool dimensions are different) and again positioning the tool to the desired location. These steps result in additional time loss when considered in the mass production scenario.

The solution for the aforementioned problem is an active compliant tool, that can adapt well to the most intricate convex and concave shapes by making the tool more compliant, and according to the requirements, the tool can be made stiffer for higher material removal rate operations. So, in the workpiece regions which involves intricate geometries or need rough polishing the stiffness of the tool can be changed while machining itself. This would reduce a considerable amount of time for the tool change. Although few robotic polishing setups have automatic tool changer facility, installing one such is costly and from machining point of view, it time consuming too.

In order to modify the compliance of the polishing tool and thereby change the pressure distribution and material removal rate, a novel active compliant tool is proposed. Initially different conceptual designs with different working principles are considered, and the advantages and drawbacks are delineated to down-select the design. Then the 3D CAD assembly model and component selection are discussed elaborately in the second section. Then fabrication of mould, tool and assembly are discussed in the third section. Later, in the fourth section, static pressure distribution study is carried out using pressure films and the load-displacement curves are obtained and compared with the finite element model. In the fifth section, experimental machining trials are carried out in milling machine for different compliance conditions. Three-dimensional surface measurements are made and compared against compliance change.

7.1 Conceptual designs

Initially, different designs are considered based on various methods to change compliance. Out of the different methods to change compliance, by active and passive ways to change compliance are considered. The advantages and disadvantages of each tool are considered, and an optimal design is selected. The designs are discussed below individually, and their advantages and disadvantages are discussed.

7.1.1 Models using the magnetic field for active compliance

In order to change the compliance while machining, magnetic field is considered as an option. Two models as shown in Figure 7.1 and Figure 7.2 were initially considered. The design consists of having a set of electromagnet rings separated by a short distance mounted in a flexible rubber tube, which is inserted radially on the polishing pad as shown in Figure 7.1 (a). The electromagnetic repulsion tool (Model 1) has rings wound in opposite directions such as the like poles are facing each other as shown in Figure 7.1 (b). When the pad with

electromagnets are flexed, because of the repulsive force, the bending of the main tube is restricted, and hence for the same force, the contact pressure and material removal are expected to increase. In the other design (Model 2), when the tubes filled with magnetorheological (MR) fluid is exposed to a magnetic field, the suspended iron filling in MR fluid, arrange themselves so that these tubes become stiff [146,147].

Figure 7.1 (a) Assembly of the electromagnet setup along with slip rings (b) Arrangements of electromagnets

Figure 7.2 Magnetorheological fluid active compliant tool

The advantages of these models are (i) the compliance could be controlled continuously by adjusting the power (ii) there are no moving components in the tool, the friction is reduced. The disadvantages associated with these models are (i) the weight of the tool is increased because of the presence of electromagnets. (ii) presence of magnetic field while machining will increase the complexity (iii) in order to achieve noticeable change in compliance, the size of the tool becomes very large.

7.1.2 *Disc with bent arms*

In this model (Model 3), the pressure change is achieved by flexing the base rubber pad. The circular pad is made of rubber with arms extending out from the circumference. The arms are bent, and the ends are connected to the slots in the bearing housing as shown in Figure 7.3. This pad is flexed by pulling the arms like structure connected to the bearing housing up and down. When the rubber arm is pulled up, the pad is expected to flex more and thereby causing higher contact pressure by reducing the contact area. Likewise, when the arms are lowered, contact pressure reduction is expected. The bearing assembly is designed such that it can slide and rotate simultaneously.

Figure 7.3 (a) Complete setup with arms (b) Base pad in flexed and unflexed condition

The advantage of this method is the simplicity in design where just by flexing the pad through the arms, the compliance is controlled. The main disadvantage associated with this design is that the pressure change is dependent upon the arm displacement. Since the arms are made of rubber, they might stretch more locally instead of flexing the base pad.

7.1.3 Bevel gear compliant tool and Double pad designs

Designs using basic mechanisms such as bevel gears (Model 3) is explored. Figure 7.4 shows the bevel gear connected to cam shaped stiffeners as shown in Figure 7.4. As the bevel gear is rotated independently of the spinning shaft, the flexing of the base pad is changed as the moment of inertia of the stiffeners on the top changes. The independent rotation of the bevel gears is controlled by a small stepper motor inside the main shaft. The power to the stepper motor is controlled by a small stepper motor inside the main shaft. The power to the stepper motor is transmitted using slip rings. But the disadvantages of this models are (i) complex bevel gear mechanism (ii) housing stepper motor inside the rotating shaft and supplying power to the motor likely to be complicated wiring (iii) the tool is heavy because of additional stiffeners, stepper motor, and slip rings.

Figure 7.4 Active compliant tool with bevel gear mechanism

In the dual pad design (Model 5) instead of the single pad, there are two rubber pads, separated by a distance. When the distance between the pads is reduced, the flexing of the pad can be controlled, thereby producing a greater contact pressure. The simplified assembly is depicted in Figure 7.5.

The major disadvantages of this design are (i) stability problem as two discs are spinning (ii) in case of relative offset in speed, then the friction and heat developed between the two pads will be high.

7.1.4 Backing disc with stiffeners

In order to overcome previous shortcomings in the design, a novel design with the retractable design is proposed. The active compliant tool assembly with retractable stiffeners is shown in Figure 7.6. The rubber pad is modified to have hollow ribs where the stiffeners can be moved in and out. In this model, the movement of the stiffener is controlled by the reciprocating movement of the bearing assembly. The bearing assembly is made such that it allows both reciprocating and rotational motion simultaneously. The components inside the bearing assembly are discussed in the next section. The pressure distribution is expected to change in accordance with the stiffener position inside the hollow rib.

Figure 7.6 Active compliant tool with retractable stiffeners

Figure 7.7 3D cross-section model with stiffener at (a) inserted and (b) retracted positions

In this proposed concept, when the stiffener is in the fully inserted stage, as shown in Figure 7.7(a), the pad becomes stiffer, and as the stiffeners are retracted Figure 7.7(b), the base rubber pad becomes more compliant. Hence the contact pressure distribution is expected to be lower, leading to decreased material removal depth. The contact pressure can be further controlled by optimising the stiffener material, number of circumferential ribs, geometry of the pad etc.,

7.1.5 Selection of suitable tool

The above discussed six compliant polishing design tools are ranked based on various selection criteria. Table 7.1 provides the concept of scoring each tool and highly ranked tool is selected as the most suitable solution. Herein, ‘++’ indicates highly desirable or highly ranked and ‘0’ indicates neutral compared to the standard tool and ‘--’ indicates inferior to the standard tool. From the rankings, the Model 6 active compliant tool with retractable stiffeners has higher ++ and hence analysed in detail for practical application.

Table 7.1 Selection of the most suitable model

Functions	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Weight of the tool	--	-	+	--	--	++
Overall dimensions of the tool	--	--	--	-	+	++
Initial Cost	--	--	+	-	+	+
Compliance range	--	--	-	+	+	++
Design complexity	--	--	+	--	+	++
Friction	++	++	+	-	--	-
Total Score	-8	-7	+1	-6	0	8
Rank	6	5	2	4	3	1

7.2 Design and material selection

The detailed design and components to be used in the active compliant tool are discussed in this section. Each component and its importance are elaborated. Figure 7.8 shows the assembly of the overall tool. The change in compliance is executed by the linear actuator

which is fixed with the tool. The assembled tool must be fixed with the main spindle of the CNC machine. The end of the tool shaft is mounted to the spindle by the collet.

Figure 7.8 Full assembly of the tool

The two main components of the active compliant tool setup are the backing pad with stiffeners and the bearing assembly along with the actuating system.

Backing rubber pad

Figure 7.9 shows the backing rubber pad with hollow ribs where steel wire and nylon tubes are accommodated. In this design, there are 6 hollow ribs. The base diameter of the rubber tool is 72 mm, and the detailed sketch of the rubber pad is given in Figure A. 1. The suggested rubber material is expected to have shore A hardness in the range of 45 to 60.

Figure 7.9 Backing Disc 3D view

Also, it must be noted that the design is not completely optimized, and only proof-of-concept is explored here. Hence, the change in pressure, material removal rate is likely to be affected by the curvature and other dimensions of the tool. The base rubber pad along with the ribs are fabricated as a single part using silicone rubber. Similarly, if the material properties of the rubber pad are altered the material removal profile is subjected to change.

In order to slide the stiffeners in and out of the hollow rubber rib, a protective nylon tube is inserted into it, conforming to the internal diameter of the ribs. This nylon tube not only provides an additional stiffness but also helps to protect the underlying soft rubber being damaged by the stiffener wire. In order to avoid friction between the tube assembly and the rubber pad while sliding, lubricating oil is used. These stiffeners are connected with the stiffener carrier through steel wires which in turn is connected to the bearing assembly. The shaft is made of steel, and while moulding the shaft is inserted inside the mould and so that rubber pad and the shaft becomes one single part. Nylon and steel wires are chosen as the material for tubes and stiffener rods.

Bearing assembly

The bearing assembly is a key part through which the movement of the stiffeners is controlled. It also helps the stiffeners to rotate along with the main shaft as shown in Figure 7.10.

Figure 7.10 Cross-section of the tool assembly

Figure 7.11 Cross section of bearing assembly

As shown in Figure 7.11, the bearing assembly consists of a brass bushing and deep groove ball bearing . The flange bushing rotates along with the shaft and also slides on the main shaft. The ball bearing is provided to allow the stiffener carrier to rotate freely. This bearing is housed inside the stationary bearing housing and secured using circlips. Figure 7.12 depicts the selected bushing and bearing . The specifications of the flange bushing and the ball bearing are given in Table A. 7. The detailed dimensions of the shaft, stiffener carrier and bearing housings are given in appendix Figure A. 2 to Figure A. 4

Figure 7.12 (a) Deep groove ball bearing (b) Flange bushing [148]

7.3 Tool fabrication and assembly

In order to fabricate the rubber pad using a special mould. This mould is 3D printed using Clear resin ® in Formlabs Form 2® machine. Since, the rubber tool geometry is complex with hollow ribs, fabricating the mould through the machining process is expensive and time-consuming. The mould is 3D printed along with the plastic plugs for creating hollow ribs as shown in Figure 7.13. The BLUESIL ® RTV 147 A & B silicone resin and hardener was used in a ratio of 5:1 to prepare the base. Experimental trials were carried out to arrive at this particular resin, hardener ratio based on the quality of the tool and curing time. It was observed that this ratio resulted in Shore A hardness 45 ± 5 . Initially, the mould is cleaned with acetone,

and the plugs are placed as shown in Figure 7.13. Before pouring the resin-hardener mixture into the mould, they are mixed well in a centrifugal mixer.

Figure 7.13 Rubber pad fabrication using a mould

The resin mixture along with the mould is placed in the vacuum chamber at 30°C and 40mbar pressure for 5 minutes. Most of the air bubbles were removed because of deairing. The mould is then allowed to cure for 24 hours. After curing, the part was removed from the mould, along with the plugs.

The nylon tubes and steel wires are taken and cut as shown in Figure 7.14. Then they are placed inside the hollow ribs and fixed to the flange bushing as shown in Figure 7.14 (a). The stiffener carrier is secured to the flange bushing, and the deep groove ball bearing is placed concentric to the stiffener carrier as shown in Figure 7.14 (b). Later, this setup is secured to the bearing housing using circlips as depicted in Figure 7.14 (c)

Figure 7.14 (a) Stiffener assembled with flange bushing (b) Stiffener carried and bearing (c) Assembly with bearing housing

7.4 Experimental setup

The assembled tool is subjected to both static pressure distribution test and material removal trials. Static pressure distribution study is carried out to find the pressure distribution, contact area and the load-displacement behaviour of the tool using pressure films. From the pressure map, the compliance change in the tool can be observed. In order to check the tool compliance at different retracted positions of stiffeners, instead of using an actuator, the stiffeners are retracted manually by locking the bearing housing at three different positions viz., fully inserted (*I*), half retracted position at 7.5mm (*M*) and completely retracted position at 15mm (*R*).

7.4.1 *Static experimental setup*

The load-displacement response and the pressure distribution investigations for the tool were carried out simultaneously, using the same experimental set-up. As mentioned above, the tool is positioned at three different positions, and the change in load – displacement and contact pressure distribution are obtained for 10N and 15N. Hence, the static tool testing is carried out for 6 different conditions.

Figure 7.15 Static experimental setup in Instron 5569 UTM

The six set of experimental trials were conducted on the Instron 5569 Universal Testing Machine. The setup is fixed to the 50N load cell through a holder. The workpiece is slanted at 15° Aluminium block which is clamped to the lower compression plate, as indicated in Figure 7.15. The initial point of contact was taken as zero displacement and the tool was then given a downward displacement of 0.5 mm/sec, until it reaches 10 N or 15 N.

7.4.2 *Machining runs experimental setup*

Machining trials were carried out using the fabricated tool to check the effect of tool compliance on the material removal depth and contact area. These experiments were carried

out on MIKROTOOLS DT-110 Multi-Process Micro Machining system. A dynamometer (Kistler 9254 with 10kHz sampling rate) with data acquisition system is placed under the workpiece to monitor the force exerted by the tool on the workpiece at the time of machining. The setup for the experiment is depicted in Figure 7.16.

Figure 7.16 Experiment set-up: (a) MIKROTOOLS DT-110 Multi-Process Micro Machining system and dynamometer (b) Tool set-up

In the experimental setup, the tool was clamped in the jig, as indicated in Figure 7.16. The bearing housing was clamped on both sides by these slotted aluminium plates. This setup helped to minimize vibration and to ensure that the bearing housing was held stationary at all times during the operation of the tool. The workpiece is a 3 mm thick Aluminium rectangular plate of 152 mm x 90 mm. Since tilting the entire tool setup is complex in this case, the workpiece is tilted. It was then mounted on an Aluminium block with a 15° surface tilt. Commercially available 3M 216U paperback abrasive disc with # 120 was used as the abrasive pad for the tool. Discs of 80 mm diameter were cut out and secured to the base of the active polishing tool using adhesive tape. The experiments were carried out for two different values

of tool normal force: 10 N and 15 N. For each force value, the position of the stiffeners is denoted using the following naming convention:

I: Stiffeners retracted by 0 mm (In fully inserted position)

M: Stiffeners retracted by 7.5 mm

R: Stiffeners retracted by 15 mm (In fully retracted position)

Figure 7.17 Stiffeners in the ‘I’, ‘M’ and ‘R’ positions for material removal study

Figure 7.17 shows the different tool positions (I, M, R). The dynamometer helps to confirm that on a constant force was maintained for the 3 seconds of machining time. Since, the machine is not equipped with the force-controlled system, the z-axis displacement is fixed such that the required force is maintained during the machining. After machining the samples are cleaned and the three-dimensional surface data information is acquired using laser probe in TalyScan® profilometer from which information like maximum depth, area and volume of material removal are estimated.

7.5 Experimental results

7.5.1 Static pressure distribution

In order to understand the pressure distribution and contact area, Fujifilm® pressure film was used. The detailed discussion about the pressure film is in Contact pressure distribution in compliant polishing tools section 2.2. The setup with pressure film is shown in Figure 7.18. Initially, the films are stacked together and placed on top of the angle block. Once, the required force is reached, the tool is held at the same position and loaded for 5 seconds, after which the tool was unloaded until it reached the zero position. The holding time of 5 seconds was based on the Fujifilm recommendation for momentary pressure measurement.

Figure 7.18 Static testing setup with pressure film (b) Pressure film with developed colour

The load versus displacement response measured from this experiment is indicative of the behaviour of the tool upon application of force to the tool against an inclined surface. The experiments were repeated thrice to ensure repeatability. Further, the results obtained can be compared to the load-displacement data obtained from simulations carried out using commercial FEA software ABAQUS®. For the experiments conducted, it was observed that the load-displacement data obtained from each tool condition was repeatable. The following

graphs indicate the variation in the load-displacement curves for a variation in force and stiffener position.

Figure 7.19 Load Displacement curves obtained for 15 N force

From Figure 7.19 it is observed that for the stiffeners in the ‘I’ position, the stiffness is highest, and the load-displacement curve is almost linear. This is because that the entire stiffener wire is fully inside the rib, and the entire load is carried by the wire. For the stiffeners in the ‘M’ position, as indicated in the Figure 7.19, the variation in load with displacement is linear initially till 4 mm, and there is a sudden increase in the load. The possible reason for this sudden increase is attributed to the stiffener sharing the load. Finally, for the stiffeners in the ‘R’ position, as indicated by the dotted grey curve, there is also a linear variation of the load with displacement up to 4.5 mm, after which there is a sudden increase in load. The shift in the displacement value of the tool for which this increase in load occurs is more for the ‘R’ position than the ‘M’ position, thus confirming the stiffener position does significantly impact the stiffness of the tool.

The pressure distribution study was carried out to find the effect of stiffener position on the static pressure distribution. Three categories (2LW, 3LW, and 4LW) of Fujifilm Prescale pressure films® were used. The same set of six conditions are repeated using the tool. After the tests, the films are segregated according to the pressure range and later scanned using

pressure mapping system. The obtained matrix is later plotted using MATLAB as shown in Figure 7.20.

Figure 7.20 Pressure distribution obtained for stiffeners in (a) 'I' (b) 'M' (c) 'R' position for 10N force

From Figure 7.20 (a) to (c) it can be observed that the maximum pressures of 1.01 MPa for the 'I' position, 0.501 MPa for the 'M' position and 0.428 MPa for the 'R' position are observed. The maximum contact pressure for 'I' position is 135.98 % higher compared to 'R' position and the maximum contact pressure for 'M' position is 101.6 % higher than 'R' position. This change in pressure is attributed to the change in the stiffness by retracting the stiffeners. The area of contact is also observed to increase with an increase in stiffener retraction.

Figure 7.21 Pressure distribution obtained for stiffeners in (a) 'I' (b) 'M' (c) 'R' position for 15N force

For 15N force, the pressure distribution is as follows. Maximum pressures of 1.195 MPa for the 'I' position, 0.548 MPa for the 'M' position and 0.462 MPa for the 'R' position are observed from Figure 7.21. This pressure variation trend is similar to the trend observed for 10 N force. The increase in the maximum pressure for 'I' compared to the 'R' position is 158.66 % and increase in pressure from 'M' to 'I' position is 118.07 % The material removal is observed to decrease with an increase in the retraction of the stiffeners. The area of contact is observed to increase with an increase in stiffener retraction in response to compliant behaviour.

Since the pressure under 0.05 MPa cannot be captured using the pressure films, the actual contact area is calculated by applying normal paint beneath the tool and pressed against the paper. Later, using image processing software Image J®, the actual contact area is

calculated. The contact area versus stiffener retraction shown in Figure 7.22 as a bar chart for three different positions.

Figure 7.22 Stiffener retraction Vs. Contact area

From Figure 7.22, it can be observed that the contact area increases for both 10N and 15N cases as the stiffener is retracted. Similarly, as the load increases for the same retracted position, the contact area for 15N is more than 10N. Similar behaviour is seen in the conventional polishing tool as well. The gradual increase in contact area is more pronounced at lower loads.

7.5.2 Material removal experimental results

The experimental trials were carried out for three different stiffeners retracted positions for two different loads viz. 10N and 15N. The spindle speed was set at 1000 RPM. For each experimental trial, the 3D material removal profile was scanned using laser profilometer. The contact area, depth variation, and volume of material removal have been investigated.

Figure 7.23 Material removal profile (a) 'I' position (b) 'M' position (c) 'R' position for 10N

The machined work coupons are scanned, and the material removal profiles for different stiffener retraction positions are shown in Figure 7.23 (a) to (c). A significant variation in the depth of material removal profile was observed with a variation in the position of the stiffeners. The increase in material removal depth from the half retracted (M) position to the stiffeners in the fully inserted (I) is observed to be 41.23%. The maximum increase in material removal depth from completely retracted (R) to fully inserted (I) condition is 85.5%. The effect of retraction of the stiffeners is reflected in the machined area too. The retraction of stiffeners increases the machined area increased as the tool is more compliant.

Figure 7.24 Material removal profile for (a) 'I' position (b) 'M' position (c) 'R' position for 15N

From the experiments carried out for a tool force of 15 N, a similar variation (as in 10N case) in the depth of material removal was observed with a variation in the position of the stiffeners, as shown in Figure 7.24. The increase in material removal depth from the half retracted (M) position to the stiffeners in the fully inserted (I) is observed to be 40.16%. The maximum increase in material removal depth from completely retracted (R) to fully inserted (I) condition is 84.5%. A similar effect of retraction on the machined area as observed in 15N is also observed in 10N.

Figure 7.25 Stiffener retraction position versus maximum material removal depth

From the material removal experimental results, it can be confirmed that the stiffeners affect the material removal depth, and the variation in the maximum material removal depth with respect to stiffener position is shown in Figure 7.25. The pattern as noticed in static contact pressure is reflected in the material removal depth for the polishing runs as well. The maximum contact pressure decreases as the stiffeners retract for both 10N and 15N. The similar trend is observed for maximum depth too. It is to be noted that, the difference in contact pressure as well as material removal depth for the two load cases of 10N and 15N is marginal, and this is reflected in the close range of values for 10 and 15N for the same retracted length.

7.6 Static pressure distribution simulation

The prediction of contact pressure is an essential part in active compliant tool design. By conducting finite element analysis, it will be helpful not only to predict the contact pressure but also to arrive to an optimised design where desired contact pressure can be achieved. In this study, a finite element model for the for the assumed configuration is modelled using ABAQUS®, and the FEA result is compared with the experimental measurements.

7.6.1 FE model setup

Initially, the 3D models of the components such as pad, stiffeners and workpiece are imported into the ABAQUS software. In order to replicate the experiment that was carried out in the universal testing machine under displacement control, ‘quasi-static’ analysis is selected. Moreover, since the tool involves large deformation, dynamic implicit with quasi-static analysis is more suitable than the ‘general -static’ analysis in ABAQUS®. The components are assembled as shown in Figure 7.26 and ‘general contact’ condition is specified since contact between multiple bodies is involved. The simulation is carried out for 15N loading, as the pressure change between the 10N and 15N is almost within the difference of 0.05 MPa. In order to predict the experimental results, the stiffeners at three different positions, the stiffeners and tubes are modified by measuring the distance of the stiffener tip from the rib opening.

Figure 7.26 Meshed tool assembly with boundary conditions

In the experiment, the workpiece block is fixed, and the tool is allowed to move. Whereas to simplify the modelling, in this case, the tool is fixed, and the aluminium workpiece is given displacement. The base of the tilted aluminium block surface is coupled to a reference point, and except the loading direction, all other directions are constrained with respect to the

local axis as shown in Figure 7.26. The shaft portion of the tool is modelled as rigid body, and encastered boundary condition is assigned to its reference point. The tubes and stiffeners top surfaces are fixed too. Rubber pad and ribs being made of hyperelastic material, a quadratic hybrid-three-dimensional element with a 10-node modified tetrahedron (C3D10MH) is used. For the tube and stiffeners, 20-node brick element with reduced integration (C3D20R) is used, and for aluminium block, C3D8R 3D continuum elements are used.

In order to obtain the hyperelastic material property, tensile coupons (dog-bone shaped) were fabricated using BlueSil RTV 147. The tensile tests were carried out in accordance with the ASTM D412 procedure under displacement control at 500 mm/min in a 50N load cell. Stress-strain data from these tests were observed to be repeatable. Displacement data from the video extensometer is taken in this study. This obtained stress-strain values are given as material data input in the hyperelastic material model for the tool. Later, the suitable hyperelastic model is selected based on best fit and stability aspects as shown in *Figure 7.27*.

Figure 7.27 Different hyperelastic models for the tensile data

From the Figure 7.27, it is evident that the Yeoh hyperelastic material model is the most suitable of the considered hyperelastic models. Additionally, the stability information obtained

from the evaluation tests confirms that this model is also stable for the given working strain range. The detailed explanation regarding Yeoh hyperelastic model is discussed elsewhere [149]. The workcoupon ($E=69\text{ GPa}$, $\nu=0.33$), nylon tubes ($E=1.5\text{ GPa}$, $\nu=0.45$) and steel wire stiffeners ($E=210\text{ GPa}$, $\nu=0.3$) are considered as elastic.

7.6.2 Static pressure distribution simulation results

Figure 7.28 Cross-sectional view of the active compliant tool at (a) completely inserted, (b) 7.5 mm retracted and (c) 15 mm retracted.

Figure 7.28 shows the cross-sectional view in the initial and final position for different stiffener retraction conditions (15N). It can be observed that in the fully inserted condition, the steel wire is flexed along with the nylon tube during the loading. The stress in the stiffeners keeps decreasing as the retraction increases, indicating the role of stiffeners during flexing of the active compliant tool against the inclined plate. From the assigned reference point, the displacement and applied load values are obtained and compared with the experimental measurements.

Figure 7.29 Load Vs Displacement for three different stiffener positions (a) completely inserted (b) 7.5 mm retracted and (c) 15 mm retracted.

Figure 7.29 compares the experimental and simulated load-displacement curves for the considered three stiffener retraction positions. The trend followed in the experimental data is followed in the simulation results too. From the slope of the load-displacement, it can be inferred that fully inserted stiffener condition is the stiffest and the fully retracted ('R') condition is the least stiff. The variations in the load-displacement curves can be attributed to the random undulations and curvature in the bent stiffener wires.

Figure 7.30 Simulated contact pressure distribution for (a) 'I' position (b) 'M' position (c) 'R' stiffener positions

From the Figure 7.30, the experimental and simulated contact pressure distribution is compared. The contact pressure in ABAQUS is obtained by extracting the maximum contact pressure from the initial point of loading to the final frame. The simulated maximum contact pressure values and contact area are listed in Table 7.2. It can be observed that as the retraction position increases the maximum contact pressure keep decreasing and contact area increases reflecting the same trend followed in the experimental measurements. The result is compared with the pressure film readings as listed in Table 7.2. This illustrates that using such FE model, further optimisation studies can be possibly carried out to arrive at the desired design and material parameters.

Table 7.2 Experimental and simulation values for maximum contact pressure and contact area for static tests

Position	Maximum static Contact pressure (MPa)		Static contact area (mm ²)	
	Experimental	Simulation	Experimental	Simulation
I	1.195	1.482	292	282
M	0.548	0.544	419.701	453
R	0.462	0.412	437.377	478

7.7 Summary

- A suitable active compliant design was selected from the conceived different working principles. The 3D models for the rubber pad, stiffeners, shaft and other mechanical components such as bearings, bushing, etc were developed. A mould was designed and fabricated using 3D printing technology too. The tool was fabricated and assembled with the components.
- Static pressure distribution tests were carried out for three different stiffener retraction positions (I, M, R) in the universal testing machine using pressure films. Maximum contact pressure value was observed for ‘I’ position (fully-inserted), and the least maximum contact pressure value was observed for ‘R’ position (fully-retracted). The same trend was followed for 15N load case.
- Finite element analysis was carried out to predict the contact pressure distribution for different stiffener retraction positions. The load-displacement curves followed the similar trend as the experimental results. The maximum contact pressure and contact area values were in good agreement with the experimental values. The FE model will help to optimise the design in order to achieve better variation in contact pressure.

- From the experiments trials, it is observed that the maximum depth of material removal was found to decrease as the stiffeners were retracted from 0mm, 7.5mm to 15mm. The contact area kept increasing for the increase in stiffener retraction. The same trend was observed for both 10N and 15N. Thus, it can be concluded that a variation in the stiffness of the tool does cause a variation in the material removed during polishing

Chapter 8 Conclusion and future work

This chapter summarizes the major contributions to understand the contact pressure and material removal distribution of both commercially available polishing tools with different compliance and novel active compliant polishing tool.

8.1 Concluding remarks

8.1.1 *Multi-criteria decision making methods (MCDM) for compliant tool selection*

In order to choose the suitable compliant tool based on the part to be finished and the nature of polishing (such as initial rough or fine polishing needed), multi-criteria decision making (MCDM) algorithms are considered. To demonstrate the usefulness of employing MCDMs in compliant polishing tool selection, six different types of compliant polishing tools (six alternatives) are compared against six different attributes for the considered case study. In this study, two MCDM algorithms taken are AHP and fuzzy VIKOR. The suitability of the considered tools at different polishing stages such as initial, intermediate and final are obtained after computing the overall rankings using these two methods. For the initial polishing stage, belt polishing and fibre disc with glass fibre composite backing are considered as the best tool by AHP and VIKOR respectively. In the intermediate polishing, the fibre disc with rubber backing pads are computed to have better suitability by both the methods. In the last stage (fine polishing) felt backing pad and gasbag polishing are considered to have better suitability by AHP and VIKOR methods respectively. The overall trend suggests that the backing pad compliance of the tool selected by the MCDMs is in accordance to the amount of material removal needed in each polishing stage. For the initial polishing stage, tools with stiff backing such as glass fibre composite are considered and for fine polishing, very compliant felt backing pad is suggested by the MCDMs. Thus, it is demonstrated that these MCDMs will be helpful

in selecting suitable polishing tools thereby reducing the ambiguity and time involved in tool selection and also getting required performance targets. Also, MCDMs helps in reducing rework the that is caused by the wrong selection of tools.

8.1.2 Experimental and numerical analysis of contact pressure distribution

To understand the effect of polishing tool's backing pad compliance on the contact pressure distribution and material removal distribution, experimental trials are carried out. For the static pressure distribution study, Fujifilm® pressure sensitive films are used to acquire the contact area and the maximum contact pressure values for different loading (10N,15N, 20N and 25N) and different tool compliance ('hard' $\delta_h=2.3\text{N/mm}$ and 'soft' $\delta_s=1.43\text{N/mm}$). The tests are carried out for two scenarios (i) only rubber backing pad and (ii) rubber backing pad with the grainless abrasive disc. Further, three-dimensional finite element models are developed to reproduce the physical condition of the experimental trials for 20N, and the effect of compliance on the contact area and pressure values are compared with the experimental measurements. The simulated load-displacement responses agree well with the experimental trials. The overall contact length and width agrees well with the experimental results, and except for the stress singularity regions, the computed pressure matches the experimental data for both 'hard' and 'Soft' pads. Compliance plays a crucial role in determining the overall contact area and pressure distribution. In both the cases, the contact region is crescent-shaped and for the same normal load, the contact area is higher (51.68%) for the soft pad compared to the hard pad with abrasive discs.

8.1.3 Experimental and numerical analysis of material removal profile

The experimental trials with hard and soft tool for different polishing loads (10N,15N,20N,25N and 30N) are carried out in six-axis ABB IRB 140 robot at 2500 rpm without feed motion. The material removal depth for soft pad lies in the range of 150-200 μm

and its almost uniformly distributed, whereas for the hard backing pad the material removal depth is in the range of 350 -400 μm . But as the load is increased from 20N, the material removal depth reduces to 280-300 μm for the hard pad at 30N, due to the reason that force being distributed. In the case of soft pad, the material removal depth increases because the high stiffness steel shaft region increases the contact pressure for 25N and 30N. The machined area (length and width) increases for both the hard and soft backing pad polishing scenarios as the polishing load is increased.

For simulating the overall material removal depth distribution, a methodology using Archard wear equation and adaptive meshing technique is proposed using the commercial FE software ABAQUS®. The grainless abrasive pad along with rubber backing pad and aluminium workpiece is modelled. The contact pressure and sliding distance values are obtained from each node on the workpiece surface, the wear depth is computed using Archard wear formula, and the workpiece is re-meshed using UMESHMOTION subroutine. The maximum material removal depth computed using the FE model for both hard and soft pad agrees well with the experimental material removal (within 10% error). Thus, the proposed methodology will be helpful in predicting the material removal depth, without taking into account the grain distribution and thereby reducing the computational time involved in the simulation with grains. But the drawback of this model is that it needs the wear coefficient value from the experimental trials.

8.1.4 Three-dimensional abrasive topography modelling of abrasive surface

In order to simulate the material removal depth by taking the grains-aluminium workpiece interactions into consideration, the topography modelling is essential to understand the grain distribution and orientations. Using the MATLAB® code, the experimental peak protrusion height and inter-grain spacing distributions are obtained from the scanned #60 and

#120 alumina abrasive discs with prismatic grains. Both the distributions followed normal distribution. The normality of the distribution is confirmed using Anderson-Darling (AD) test at 95% confidence level. To model the spatial distribution of grains, the area is discretized and by ‘random shaking’ the grains are placed in the 3D space such that they don’t overlap. Similarly, iterations are carried out by randomly orienting the grains using rotation matrix, till the protrusion height distribution matches the experimental parameters. This model helps to acquire the pitch, roll, and yaw angles which will be helpful during material removal model using grains. The similarity of the mean and standard deviation between experimental and simulated protrusion height and inter-grain spacing distributions are confirmed using F -test and t -tests. Various 3D surface parameters such as S_q , S_{sk} , S_{ku} (amplitude parameters), S_{ds} , S_{sc} , S_{dq} (hybrid parameters), S_c , S_{ci} , S_v , S_{vi} (functional parameters) and S_{tr} , S_{al} (spatial parameters) are computed from both experimental and simulated surface using MATLAB® code. The validity of agreement between the means and variances of the experimental and simulated 3D surface parameter values are validated using t -test and F -test. The same methodology is implemented for topography modelling of irregularly shaped grains.

8.1.5 Material removal profile simulation considering grain distribution

Using the grain distribution and orientation information from the topography modelling, a 3D finite element model of the abrasive disc is developed using Python script. Johnson-Cook damage parameters based on failure strain criteria are used for the Al6061-T6 workpiece. Because of the high computational time involved only three rotations of the disc against the aluminium workpiece is considered. The scratch profile after three rotations is compared for both soft and hard backing pad. The maximum scratch depth of polishing tool with hard backing pad is almost twice the maximum material removal depth in soft pad. The maximum material removal depth for hard pad is observed near the point of initial tool contact, whereas for the soft pad, the valleys in the scratch profile is uniformly distributed. Hence, this

numerical simulation is helpful in understanding the effect of compliance and grain distribution on the material removal depth. A semi-analytical approach is proposed in this study to predict the material removal depth based on the probability statistics, the contact elastic mechanics, contact plastic mechanics and the abrasive cutting theory. The randomly distributed and oriented prismatic grains are simplified as conical grains with spherical tips with different apex angles and base radii. Based on the average dynamic pressure distribution obtained from the numerical simulation, the force per unit cell is calculated, and the relevant indentation depth is estimated. The indentation depth calculated for one rotation is scaled linearly to 125 rotations, and the material removal depth and simulated contact length and width are compared with the experimental machined area dimensions.

8.1.6 Active compliant polishing tool

Based on the experimental and numerical simulations carried out for the passive compliant polishing tool with different backing pad stiffness, it is proved that compliance plays a key role in determining the maximum material removal depth and machined area dimensions. In order to combine the existing passive rubber backing pad tools with different compliance (say 'hard', 'medium', 'soft') into a single tool, an active compliant polishing tool design is proposed, fabricated and tested. The 3D modelled tool is fabricated using silicone rubber shore A hardness of 45 ± 5 and the other components such as bearing housing, stiffeners carrier and shaft are machined and assembled. Galvanised steel wires along with nylon tubes are used as the stiffener materials inside the hollow ribs of the fabricated rubber pad. To investigate the working of the newly developed active compliant tool, load-displacement responses at three different stiffener positions (completely inserted '*I*', mid-position '*M*' at 7.5mm and completely retracted '*R*' at 15mm) are carried out. As expected, a considerable difference in the load-displacement curves is observed where '*I*' position being stiffest and '*R*' position being the least stiff in the range of 2.27N/mm and 1.04 N/mm respectively. Further, the contact

pressure distribution is studied using pressure films, and the maximum contact pressure is observed to occur in the 'I' position, and the minimum contact pressure is observed in 'R' position. Numerical simulations are carried out in commercial FEA software ABAQUS®, using the obtained material properties to simulate the load-displacement curves, and static contact pressure distributions at these considered three positions. The numerical pressure distribution results agree well with the experimental measurements and the simulated load-displacement curves trend match with the experimental ones. These numerical models will be used in future studies for optimisation of the number of hollow ribs and the stiffeners' material and rubber pad geometry to enhance the compliance range of the tool. Thus, the newly developed active compliant polishing tool can combine the functions of the different passive compliant rubber tools presently used in the market thereby reducing inventory and tool change time in a mass production environment, thereby reducing the overall product cost.

8.2 Original contributions in this research

- Finite element model to simulate the effect of backing pad compliance on contact pressure distribution is developed and compared with the experimental measurements. Also, a finite element model to predict the overall material removal distribution for different backing pad compliance by Arbitrary Lagrangian-Eulerian (ALE) adaptive mesh technique is utilised by incorporating UMESHMOTION subroutine in ABAQUS®. This is the first time such an approach is used in the numerical simulation for predicting material removal profile in disc-shaped compliant polishing tools.
- Three-dimensional topography modelling of the coated abrasive disc by stochastic approach is proposed for both prismatic shaped and irregularly shaped grains. Apart from the usual spatial distribution and protrusion height parameters considered, grains' orientation in all three axes are also taken into account. The modelled three-

dimensional abrasive surface topography using the proposed methodology is compared with experimental measurement for a variety of three-dimensional surface parameters (amplitude, functional, spatial and hybrid) and confirmed using statistical tools such as p -test and F -tests.

- A new approach is proposed to simulate material removal for compliant tool with different backing material. Python script is employed to 3D model the abrasive disc with grains with random distribution and orientations. The full-scale model with abrasive grains and aluminium workpiece is helpful in predicting the material removal distribution. Also, a semi-analytical approach is proposed to simulate the material removal depth by calculating the local indentation depth for the simplified conical grains with spherical tips.
- A novel active compliant polishing tool design is proposed, fabricated and tested for proof of concept. One such tool can provide continuous compliance change, unlike passive rubber tools which have a discrete compliance range. It can also reduce the tool change time involved in changing these passive tools. Also, by changing the stiffeners position in the active compliant tool, there is a considerable difference in the material removal depth and area.

8.3 Suggestions for future works

Further recommendations for future works related to the current scope of research are as follows:

- In this study, for the tool selection, two MCDMs (AHP and VIKOR) are considered. In future studies, instead of using linguistic variable, actual data obtained from the companies can be used to rank the polishing tool with different MCDMs.

- Presently, the contact pressure distributions and material removal experimental trials are carried out in flat coupons. These studies can be extended for other concave, concave and free-form surfaces where the utility of these compliant polishing tools is more relevant.
- In this study, the material considered is Al6061-T6, in future trials, polishing trials can be carried out in metals like titanium, Inconel and steel. The compliant tool geometry considered in this study is disc type of tools, in future, different kind of tools such as flap disc, flap wheel or other finishing tools can be considered, and ways to implement the simulation approaches for these kinds of tools can be considered.
- The effect of the tilt angle, feed motion and coolant along with the compliance effect in the material removal rate can be done studied both experimentally and numerically, as these parameters like feed and coolant are usually used in industries. Additionally, the effect of compliance on surface roughness also needs to be studied.
- The numerical Archard wear model used in this study is an approximate approach to predict the material removal depth for difference backing pad compliance. This model can be made more robust by considering a dynamic wear coefficient and updating the mesh for each revolution instead of averaging or using a scaling factor. Also, an extensive study needs to be carried out to model the woven cloth backing used in the abrasive disc which also plays a key role in determining the contact area.
- Two parallel approaches to model material removal depth in compliant abrasive polishing tool with rubber backing and abrasive disc are demonstrated in this study. The similar methodology can be implemented to other compliant polishing tool where the backing material will be different. By acquiring the material property of the backing material of these new tools and other machining parameters, the material

removal depth of the machined surface can be obtained by appropriate scaling thereby reducing the computational time. In the semi-analytical approach, the overlapping of grains and the hardening behaviour are neglected which can be included in the future studies.

- The performance of the novel active compliant tool can be improved by integrating it with an existing robotic polishing setup and providing a closed loop control system with a linear actuator for the moving of the stiffeners. The tool can be optimised in both material and the geometry perspectives such that the compliance range of the tool can be improved along with its durability.
- Presently, the experimental trials are carried out in flat workpiece for the active compliant tool. To prove its capability in polishing intricate parts, the trials can be carried out in free-form surfaces. Also, a comparative study can be done for passive and active compliant tools made of same material, so that the reduction in machining time achieved by this novel active compliant tool can be proved quantitatively.
- Numerical and analytical models to predict the material removal in the active compliant tool can be developed by taking the stiffener position as an additional parameter. This will be helpful in indicating the amount of retraction of stiffeners needed while polishing complex contours thereby reducing excess material removal.

Publications

Journals

1. Adhithya Plato Sidharth A, Sridhar I, Sathyan S (2015) Multi-criteria decision making techniques for compliant polishing tool selection. *Int J Adv Manuf Technol* 79 (1-4):519-530
2. Adhithya Plato Sidharth A, Sridhar I (2018) Three-dimensional topography modelling of regular prismatic grain coated abrasive discs. *Int J Adv Manuf Technol* 96 (9):3521-3532
3. Adhithya Plato Sidharth A, Sridhar I (2019). Material removal analysis for compliant polishing tool using adaptive meshing technique and Archard wear model. *Wear*. 418: 140-150
4. Adhithya Plato Sidharth A, Sridhar I, Sathyan S, Lim YW. Experimental and numerical investigation of a novel retractable stiffeners based active compliant polishing tool. *Int J Mach Tool Manu* (Under-review)

Conference

1. Adhithya Plato Sidharth A, Sridhar I (2017) Stochastic modelling of 3D surface topography for coated abrasive discs. Paper presented at the 20th International Symposium on Advanced in Abrasive Technology (ISAAT) Okinawa, Japan, 03– 06 Dec. 2017

Technical disclosure

1. Sridhar Idapalapati, Adhithya Plato Sidharth Arunachalam, Lim Yee Wei, Sathyan Subbiah, ‘An Active Compliant Polishing Tool’, Singapore provisional patent application number 10201806167R (filed on 18 July 2018).

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Appendix

Table A. 1 Description of attributes used in decision making.

Attributes	Symbol	Definition
Cost	C ₁	Cost refers to the tool cost
Time	C ₂	Time includes the machining time, tool change time etc
Surface Finish	C ₃	Surface finish refers to the R _a value
Surface integrity	C ₄	Properties of the work piece influenced by both physical and chemical effects after polishing process
Material removal rate	C ₅	Material removed from the workpiece in a given time which is a function of tool force, spindle velocity etc.
Compliance	C ₆	Tool deflection for the applied force on the tool. Less compliant tool results in profile inaccuracy of the machined workpiece.

Table A. 2 Criteria Comparison matrix for different compliant polishing tool attributes

Attributes	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
C ₁	1	0.33	0.143	0.143	0.33	3
C ₂	3	1	0.2	0.33	3	5
C ₃	7	5	1	0.33	7	9
C ₄	7	3	3	1	5	9
C ₅	3	0.33	0.143	0.2	1	5
C ₆	0.33	0.2	0.111	0.111	0.2	1

Table A. 3(a) Fixed attributes comparison matrix

COST (*PV – Priority vector)							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV*
A ₁	1	0.33	1	0.33	9	9	16.25
A ₂	3	1	3	1	9	9	31.94
A ₃	1	0.33	1	0.33	7	7	14.37
A ₄	3	1	3	1	7	9	31.25
A ₅	0.11	0.11	0.143	0.143	1	0.2	2.25
A ₆	0.11	0.11	0.143	0.111	5	1	3.94
SURFCAE INTEGRITY							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	3	1	0.33	0.2	0.33	7.41
A ₂	0.33	1	0.33	0.2	0.143	0.143	3.25
A ₃	1	3	1	0.333	0.2	0.33	7.42
A ₄	3	5	3	1	0.33	1	19.01
A ₅	5	7	5	3	1	0.33	30.30
A ₆	3	7	3	1	3	1	32.60
MRR							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	0.33	1	3	7	5	20.13
A ₂	3	1	3	5	9	7	42.42
A ₃	1	0.33	1	3	7	5	20.13
A ₄	0.33	0.2	0.33	1	5	3	9.72
A ₅	0.143	0.11	0.143	0.2	1	3	4.16

A ₆	0.2	0.143	0.2	0.33	0.33	1	3.43
Compliance							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	3	1	0.33	0.2	0.33	7.78
A ₂	0.33	1	0.33	0.2	0.143	0.2	3.67
A ₃	1	3	1	0.333	0.2	0.33	7.79
A ₄	3	5	3	1	0.33	1	19.13
A ₅	5	7	5	3	1	3	42.51
A ₆	3	5	3	1	0.33	1	19.13

Table A. 3(b) Attributes comparison matrix for Initial Polishing

INITIAL POLISHING							
TIME							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	1	3	3	7	7	7.98
A ₂	1	1	3	3	7	7	3.93
A ₃	0.33	0.33	1	3	5	5	3.00
A ₄	0.33	0.33	0.33	1	5	5	13.58
A ₅	0.143	0.143	0.2	0.2	1	3	25.70
A ₆	0.143	0.143	0.2	0.2	0.33	1	45.81
SURFACE FINISH							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	1	3	3	7	7	4.82
A ₂	1	1	0.33	3	7	7	2.34
A ₃	0.33	3	1	3	5	5	7.55
A ₄	0.33	0.33	0.33	1	5	5	14.33

A ₅	0.143	0.143	0.2	0.2	1	3	46.64
A ₆	0.143	0.143	0.2	0.2	0.33	1	24.33

Table A. 3(c) Attributes comparison matrix for Intermediate Polishing

INTERMEDIATE POLISHING							
TIME							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	0.33	0.33	0.2	5	3	10.96
A ₂	3	1	1	0.33	7	5	4.53
A ₃	3	1	1	0.33	7	5	10.96
A ₄	5	3	3	1	5	3	45.27
A ₅	0.2	0.143	0.143	0.2	1	3	5.42
A ₆	0.33	0.2	0.2	0.33	0.33	1	22.85
SURFACE FINISH							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	3	1	0.2	3	0.33	10.41
A ₂	0.33	1	0.33	0.143	1	0.2	21.19
A ₃	1	3	1	0.2	3	0.33	21.19
A ₄	5	7	5	1	5	3	37.67
A ₅	0.33	1	0.33	0.2	1	0.33	5.04
A ₆	3	5	3	0.33	3	1	4.50

Table A. 3(d) Attributes comparison matrix for Fine Polishing

FINE POLISHING							
TIME							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	5	3	0.33	0.2	0.143	7.98
A ₂	0.2	1	3	0.2	0.143	0.11	3.93

A ₃	0.33	0.33	1	0.2	0.143	0.143	3.00
A ₄	3	5	5	1	0.33	0.2	13.58
A ₅	5	7	7	3	1	0.33	25.70
A ₆	7	9	7	5	3	1	45.81
SURFACE FINISH							
	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	PV
A ₁	1	5	0.33	0.2	0.11	0.2	4.82
A ₂	0.2	1	0.2	0.143	0.11	0.143	2.34
A ₃	3	5	1	0.33	0.143	0.2	7.55
A ₄	5	7	3	1	0.2	0.33	14.33
A ₅	9	9	7	5	1	3	46.64
A ₆	5	7	5	3	0.33	1	24.33

Table A. 4 Importance Weight of the criteria

Attributes	D1	D2	D3
C ₁	'H'	'M'	'ML'
C ₂	'H'	'MH'	'M'
C ₃	'VH'	'VH'	'VH'
C ₄	'VH'	'VH'	'H'
C ₅	'L'	'VL'	'L'
C ₆	'VH'	'VH'	'H'

Table A. 5 (a) Fuzzy Rating of 6 Alternatives with respect to the fixed 4 attributes

COST				SURFACE INTEGRITY			
	D1	D2	D3		D1	D2	D3
A ₁	'MP'	'F'	'F'	A ₁	'F '	'F'	'MG'
A ₂	'VG'	'G'	'VG'	A ₂	'G'	'G'	'F'
A ₃	'F'	'MP'	'MP'	A ₃	'F'	'F'	'G'
A ₄	'P'	'P'	'MP'	A ₄	'VG'	'G'	'MG'
A ₅	'P'	'VP'	'VP'	A ₅	'VG'	'G'	'VG'
A ₆	'P'	'VP'	'F'	A ₆	'MG'	'F'	'G'
COMPLIANCE				MRR			
	D1	D2	D3		D1	D2	D3
A ₁	'G'	'F'	'MG'	A ₁	'G'	'F'	'MG'
A ₂	'F'	'MG'	'G'	A ₂	'VG'	'VG'	'VG'
A ₃	'MG'	'G'	'VG'	A ₃	'G'	'MG'	'F'
A ₄	'VG'	'G'	'MG'	A ₄	'F'	'MP'	'F'
A ₅	'VG'	'VG'	'G'	A ₅	'VP'	'VP'	'P'
A ₆	'MG'	'G'	'VG'	A ₆	'MG'	'F'	'MP'

Table A. 5(b) Fuzzy Rating for 6 alternatives with respect to two alternatives for three stages

INITIAL POLISHING							
TIME				SURFACE FINISH			
	D1	D2	D3		D1	D2	D3
A ₁	'VG'	'VG'	'VG'	A ₁	'VG'	'VG'	'MG'
A ₂	'G'	'VG'	'VG'	A ₂	'VG'	'VG'	'MG'
A ₃	'P'	'MP'	'P'	A ₃	'P'	'MG'	'P'
A ₄	'P'	'MP'	'MP'	A ₄	'P'	'MP'	'MP'
A ₅	'VP'	'P'	'VP'	A ₅	'VP'	'VP'	'P'
A ₆	'VP'	'VP'	'P'	A ₆	'P'	'MP'	'VP'
INTERMEDIATE POLISHING							
TIME				SURFACE FINISH			

	D1	D2	D3		D1	D2	D3
A ₁	'G'	'G'	'MG'	A ₁	'G '	'G'	'MG'
A ₂	'F'	'MP'	'F'	A ₂	'MP'	'F'	'F'
A ₃	'MP'	'MP'	'MP'	A ₃	'F'	'MP'	'MP'
A ₄	'G'	'VG'	'VG'	A ₄	'VG'	'VG'	'MG'
A ₅	'F'	'MG'	'G'	A ₅	'F'	'MG'	'F'
A ₆	'F'	'MP'	'F'	A ₆	'F'	'G'	'F'
FINE POLISHING:							
TIME				SURFACE FINISH			
	D1	D2	D3		D1	D2	D3
A ₁	'MP'	'MP'	'P'	A ₁	'MP'	'P'	'MP'
A ₂	'P'	'VP'	'P'	A ₂	'VP'	'P'	'VP'
A ₃	'MP'	'MP'	'MP'	A ₃	'MP'	'F'	'MP'
A ₄	'MP'	'F'	'MP'	A ₄	'MP'	'F'	'MP'
A ₅	'VG'	'MG'	'G'	A ₅	'VG'	'VG'	'VG'
A ₆	'MG'	'VG'	'G'	A ₆	'VG'	'VG'	'G'

Table A. 5(c) Fuzzy Evaluation matrix for 6 compliant polishing tools with respect to 6 criteria (attributes) in Initial polishing stage

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
A ₁	(2.33,4.33,6.33)	(9,10,10)	(9,10,10)	(3.67,5.67,7.33)	(5,7,8.67)	(5,7,8.67)
A ₂	(8.33,9.67,10)	(8.33,9.67,10)	(7.67,9,9.67)	(2.33,4.33,6.33)	(9,10,10)	(5,7,8.67)
A ₃	(1.67,3.67,5.67)	(0.33,1.67,3.67)	(1.67,3,5)	(1.67,3.67,5.67)	(5,7,8.67)	(7,8.67,9.67)
A ₄	(0.33,1.67,3.67)	(0.67,2.33,4.33)	(0.67,2.33,4.33)	(7.67,9,9.67)	(2.33,4.33,6.33)	(7,8.67,9.67)
A ₅	(0,0.33,1.67)	(0,0.33,1.67)	(0,0.33,1.67)	(6.33,8.33,9.67)	(0,0.33,1.67)	(8.33,9.67,10)
A ₆	(1,2,3.67)	(0,0.33,1.67)	(0.33,1.33,3)	(7.67,9,9.67)	(3,5,7)	(7,8.67,9.67)

Table A. 5(d) Fuzzy Evaluation matrix for 6 compliant polishing tools with respect to 6 criteria (attributes) in Intermediate polishing stage

	C₁	C₂	C₃	C₄	C₅	C₆
A₁	(2.33,4.33,6.33)	(6.33,8.33,9.67)	(6.33,8.33,9.67)	(3.67,5.67,7.33)	(5,7,8.67)	(5,7,8.67)
A₂	(8.33,9.67,10)	(2.33,4.33,6.33)	(2.33,4.33,6.33)	(2.33,4.33,6.33)	(9,10,10)	(5,7,8.67)
A₃	(1.67,3.67,5.67)	(1,3,5)	(1.67,3.67,5.67)	(1.67,3.67,5.67)	(5,7,8.67)	(7,8.67,9.67)
A₄	(0.33,1.67,3.67)	(8.33,9.67,10)	(7.67,9,9.67)	(7.67,9,9.67)	(2.33,4.33,6.33)	(7,8.67,9.67)
A₅	(0,0.33,1.67)	(5,7,8.67)	(3.67,5.67,7.67)	(6.33,8.33,9.67)	(0,0.33,1.67)	(8.33,9.67,10)
A₆	(1,2,3.67)	(2.33,4.33,6.33)	(4.33,6.33,8)	(7.67,9,9.67)	(3,5,7)	(7,8.67,9.67)

Table A. 5(e) Fuzzy Evaluation matrix for 6 compliant polishing tools with respect to 6 criteria (attributes) in Fine polishing stage

	C₁	C₂	C₃	C₄	C₅	C₆
A₁	(2.33,4.33,6.33)	(0.67,2.33,4.33)	(0.67,2.33,4.33)	(3.67,5.67,7.33)	(5,7,8.67)	(5,7,8.67)
A₂	(8.33,9.67,10)	(0,0.67,2.33)	(0,0.33,1.67)	(2.33,4.33,6.33)	(9,10,10)	(5,7,8.67)
A₃	(1.67,3.67,5.67)	(1,3,5)	(1.67,3.67,5.67)	(1.67,3.67,5.67)	(5,7,8.67)	(7,8.67,9.67)
A₄	(0.33,1.67,3.67)	(1.67,3.67,5.67)	(1.67,3.67,5.67)	(7.67,9,9.67)	(2.33,4.33,6.33)	(7,8.67,9.67)
A₅	(0,0.33,1.67)	(7,8.67,9.67)	(9,10,10)	(6.33,8.33,9.67)	(0,0.33,1.67)	(8.33,9.67,10)
A₆	(1,2,3.67)	(7,8.67,9.67)	(8.33,9.67,10)	(7.67,9,9.67)	(3,5,7)	(7,8.67,9.67)

Table A. 6 S^* , S^- , R^* , R^- values for the three stages

	Initial Polishing	Intermediate Polishing	Final Polishing
S^*	(-0.31,1.58,-7.77)	(-0.54,1.13,-2.62)	(0.7,3.63,3.32)
S^-	(0.81,3.1,2.85)	(0.33,3.25,6.92)	(-0.48,0.63,-5.13)
R^*	(0,0.79,2.5)	(0.09,0.49,3)	(0.17,0.57,2.5)
R^-	(0.66,1,9.5)	(0.24,1,9.5)	(0.66,1,9.5)

Figure A. 1 Rubber pad with dimensions

Figure A. 2 Shaft with dimensions

Figure A. 3 Stiffener carrier with dimensions

Figure A. 4 Bearing housing with dimensions

Table A. 7 Specifications of bearing and flange bushing

Part Description - Flange Bushing [148]	
Number	SPF1014
ID	10 mm
OD	14 mm
Length	12 mm
Flange OD	22 mm
Max Rpm	2500 RPM
Max load	3480 N
Bearing Description- Deep Groove Ball Bearing [149]	
Number	W 61806-2Z
OD	42 mm
Length	30 mm
Thickness	7 mm