Reinforced high strength concrete beams under shear

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2019

Lim, D. T. Y. (2019). Reinforced high strength concrete beams under shear. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/90124>

<https://doi.org/10.32657/10220/48381>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**REINFORCED HIGH STRENGTH
CONCRETE BEAMS UNDER SHEAR**

**LIM TZE YANG DARREN
SCHOOL OF CIVIL AND ENVIRONMENTAL
ENGINEERING**

2019

REINFORCED HIGH STRENGTH CONCRETE BEAMS UNDER SHEAR

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School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of

Doctor of Philosophy

2019

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

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Part of the materials in Chapter 3 on the material properties of the concrete used in all the specimens are published as *Divsholi, B. S., Lim, T. Y. D., and Teng, S. (2014). "Durability Properties and Microstructure of Ground Granulated Blast Furnace Slag Cement Concrete." International Journal of Concrete Structures and Materials. V. 8, No. 2, pp. 157-164.*

The contributions of the co-authors are as follows:

- Associate Professor Susanto Teng commented and edited on the manuscript drafts.
- Dr. Divsholi provided the initial directions to the paper structure and designed the specimens test regime required for the paper.
- Dr. Divsholi and I prepared the manuscript drafts.
- I performed all the laboratory work at the Protective Engineering laboratory in School of Civil and Environmental Engineering.
- I analysed all the data with help from Dr. Divsholi.

Part of the materials in Chapter 3 on the material properties of the concrete used in all the specimens are published as *Lim, T. Y. D, Teng, S, Bahador, S. D, and Gjrv. O. E. (2016). "Durability of Very-High-Strength Concrete with Supplementary Cementitious Materials for Marine Environments." ACI Journal, V. 113, No. 1, pp. 95-104.*

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- Associate Professor Susanto Teng provided the initial project direction and edited on the manuscript drafts.
- I designed the specimens test regime required for the paper, with guidance from Associate Professor Susanto Teng.
- I performed all the laboratory work at the Protective Engineering laboratory in School of Civil and Environmental Engineering.
- I analysed all the data and I prepared the manuscript drafts.
- Dr. Divsholi and Professor Gjörv commented on the revised manuscript draft.

Materials in Chapters 4 and 5 on the experimental results of high strength concrete beams, with and without shear reinforcement, and the two proposed analytical methods in Chapters 6 and 7 in this study are in preparation to be submitted in 2 journal papers.

10 March 2019

Date

Lim Tze Yang Darren

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisor, Associate Professor Susanto Teng, for the opportunity to undertake my studies under his supervision. I am especially thankful to my co-supervisor Professor Tan Kang Hai, who has been helpful in providing guidance and valuable comments on my research work. I am grateful for both professors' patient guidance, kind advice, and selfless sharing of knowledge throughout the period of my research. Their teachings will undoubtedly influence my thought processes, future career, and personal life positively in the years ahead.

My sincere appreciation goes toward the School of Civil and Environmental Engineering, Nanyang Technological University, for the scholarship, research facilities, and resources. Likewise, the grants from National Research Foundation of Singapore — this research work is part of the NRF-CRP research programme “Underwater Infrastructure and Underwater City of the Future” — has contributed significantly to the culmination of this research effort.

I would like to thank my research teammates and friends, Dr Liu Yu, Dr Sabet, Dr Soerya, Dr Kondrai, Dr Jimmy, Dr Khatthanam, Mr Hidayat, Mr Hieu and Mr Shengxin for their generous help with the experimental and technical works. I would also like to express my appreciation to the laboratory technicians, especially Mr Subas, Mr David, Mr Jeffery, and Mr Chan, for their assistance in carrying out the experiments. I would also like to thank my friends for their support.

I am thankful to my family: my parents, my brother, sister-in-law, and my sister, for their unconditional support during high and low moments throughout the period of my research. Especially thankful to my brother who has shown me the courage to fight through difficult times.

My final thanks go to my beloved wife for her support and encouragement. Her support and understanding enabled me to complete my research work with much less difficulty.

A heartfelt thank you to everyone who has helped in one way or another in this journey.

ABSTRACT

This thesis presents a discussion on the shear behaviour of reinforced high strength concrete beams and new proposed methods to predict the shear strength of reinforced normal and high strength concrete beams. High strength concrete is an advanced construction material with increased tensile strength and modulus of elasticity compared to normal strength concrete. Concrete structural elements with higher tensile strength for concrete material could delay the formation of the first crack and increase the cracking load.

An experimental programme consisting of eleven full-scale beam specimens made of high strength concrete was conducted. The objectives of the experimental programme were: (1) to investigate the influence of effective depth on large high strength concrete beams; (2) to investigate the influence of maximum aggregate size in high strength concrete beams; (3) to study the influence of shear reinforcement ratio on size effect in high strength concrete beams; (4) to find out the minimum shear reinforcement ratio required to eliminate size effect on the shear strength, should shear reinforcement be able to do so; (5) to evaluate the efficacy of existing codes, such as ACI 318-14 (2014) and Eurocode 2 (2004), in predicting the shear strength of high strength concrete beams with varying depths; and (6) to propose new methods to predict the shear strength of large sized high strength concrete beams with and without shear reinforcement.

Three parameters were studied in the eleven beam specimens to achieve the above-mentioned objectives. These three parameters were: (1) the depth of the beam specimens; (2) the maximum aggregate size; and (3) the shear reinforcement ratio. The depths of the beam specimens ranged from 450 to 1800 mm. The maximum aggregate sizes used in the concrete material varied between 10 and 20 mm. The shear reinforcement ratio provided in the two shear spans of each beam specimen was different. Four shear reinforcement ratios were provided for each beam depth (0%, 0.13%, 0.20%, and 0.30%). Shear spans with shear reinforcement ratios of 0% and 0.13% formed one beam specimen, and shear spans with shear reinforcement ratios of 0.20% and 0.30% formed another beam specimen. The shear span with lesser shear reinforcement was weaker and failed first. This failed shear span was strengthened to resume the test until failure of the other shear span. This arrangement enabled the collection of data from twenty-two shear span specimens in the eleven beam specimens.

The specimens with no shear reinforcement and depths of 1350 and 1800 mm were among the largest reinforced high strength concrete beams tested to date. Together with the results of smaller specimens (with no shear reinforcement), it was shown that effective depth can influence the shear strength of high strength concrete beams in the same manner as normal strength concrete beams. The

results from this study indicated that aggregate size plays a less important role in high strength concrete compared to normal strength concrete. The experimental results of the specimens with shear reinforcement showed that while shear reinforcement increases the shear strength of the specimen, it does not eliminate size effect between specimens of different depths. Yet, shear reinforcement could mitigate the influence of size effect between two specimens of different depths.

The experimental results were used to evaluate the accuracy of the shear strength calculated from code equations in ACI 318-14 and Eurocode 2. Two new approaches to predict shear strength of beams with shear reinforcement were proposed. The first approach is an analytical truss method, based on softened truss model, with proposed inclusion of size effect and dowel action. These two elements were not considered in Hsu and Mo's method (2010). Without considering size effect, the predicted shear strength from the truss model would be un-conservative for large-size concrete beams. Without considering the shear strength contribution from longitudinal reinforcement, the predicted shear strength would be very conservative for concrete beams with high amount of longitudinal reinforcement. The second approach is a simplified truss method that predicts shear strength in the form of $V_n = V_c + V_s$. The motivation to develop a second approach is to obtain the inclination angle of the diagonal truss similar to that obtained from the first approach, but with shorter computation procedures. Using the inclination angle of the diagonal truss, the shear strength contribution from the shear reinforcement can be obtained. In addition, an empirical equation is proposed as part of second approach to calculate the shear strength contribution from concrete, and added to the contribution from shear reinforcement to obtain the total shear strength. The novelty of the simplified truss method (second approach) is the ability to obtain both the inclination angle of the diagonal truss and shear strength of the beam without going through the iterative procedure of the first approach.

The two proposed approaches were verified using the experimental results and existing published test data. The results and subsequent discussions indicated that the two approaches for predicting shear strength of a shear reinforced beam can provide reasonable accurate predictions, and at the same time is simple to apply.

Keywords: high strength concrete; shear; reinforced concrete beam; size effect; shear reinforcement; aggregate size.

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NOTATIONS

Greek symbols

α_c	= a coefficient to consider the state of stress in the compression chord
α_w	= a coefficient considering the confinement effect to the concrete strut's strength from the shear reinforcement, in relation to the spacing between the shear reinforcement bars
β	= efficiency factor of concrete
ε'_c	= strain corresponding peak stress in a standard cylinder test
ε_d	= principal compressive strain
ε_o	= strain of concrete at maximum compressive stress;
ε_r	= principal strain of concrete in r -axis
ε_s	= average steel strain
ε	= normal strain
ε_y'	= average yield strain of steel bars
γ	= average shear strain
λ	= modification factor for lightweight concrete
v_1	= strength reduction factor for concrete cracked in shear
θ	= angle of inclination
ρ, ρ_l	= longitudinal reinforcement ratio
ρ_y, ρ_z	= vertical web shear reinforcement ratio
ρ_{vv}	= vertical web shear reinforcement ratio
ρ_x, ρ_{vh}	= horizontal web reinforcement ratio
σ	= applied normal stress
σ_r	= principal stress of concrete in r -axis
σ_d	= principal stress of concrete in d -axis
σ_2^c	= normal stresses of concrete in 2-direction
σ_1^c	= normal stresses of concrete in 1-direction
τ	= average shear stress
τ_{12}^c	= shear stresses of concrete in 1-2 coordinate
ω_w	= a coefficient considering the confinement effect to the concrete strut's strength from the shear reinforcement, in relation to the amount of vertical and horizontal shear reinforcement

ζ = softening factor of concrete

Alphabetic symbols

A_v = area of shear reinforcement within the spacing, s
 $A_{v,min}$ = minimum area of shear reinforcement
 C = compression component representing the diagonal truss
 D_u = shear strength contribution from dowel action
 E_c = concrete's modulus of elasticity
 E_s = modulus of elasticity for steel
 F_t = tension force in the tension chord
 F_c = compression force in the compression chord
 L = overall beam length
 M_u = factored design bending moment at the critical section
 T = tensile force in the longitudinal reinforcement
 V = applied load
 V_c = nominal shear strength contribution from concrete
 V_n = nominal shear strength
 $V_{Rd,c}$ = shear capacity of a member with no shear reinforcement
 $V_{Rd,max}$ = shear capacity of the compression struts before crushing
 $V_{Rd,s}$ = shear capacity provided by the yielding shear reinforcement
 V_s = nominal shear strength contribution from shear reinforcement
 $V_{tension}$ = shear strength contribution of the tension component
 V_{tie} = shear strength contribution of the steel component
 V_u = factored design shear force at the critical section
 V_{test} = experimental shear strength
 a = shear span
 $a_{g,eff}$ = effective aggregate size
 a/d = shear-span-to-effective-depth ratio
 b = width of base of the beam
 b_n = net width of beam
 b_w = web width of the beam
 d = effective depth of the beam
 d_a = maximum aggregate size

d_b	= equivalent effective bar diameter
d_o	= empirical transitional size parameter
f'_c, f_{ck}	= concrete cylinder compressive strength
f_{cr}	= cracking stress of concrete
f_y	= average stress in the vertical web reinforcement
f_x	= average stress in the longitudinal web reinforcement.
f_{yt}, f_{yk}	= yield strength of the shear reinforcement
f_{cd}	= design compressive strength
f_s	= average stress in reinforcement
f_x	= normal stress in the x -direction
f_y	= yield stress of steel bars
f_y'	= average yield stress of steel bars
f_z	= normal stress in the z -direction
f_1	= principal tensile stress
f_2	= principal compressive stress
$f_{2,max}$	= maximum allowable principal compressive stress
h	= overall beam height
jd	= flexural lever arm
l'	= distance between loading points
f_{cr}	= cracking stress of concrete
s	= spacing of the shear reinforcing bars
v	= shear stress at a cross section
v_{ci}	= shear stresses transmitted between the crack interfaces
w	= crack width
s_m	= average spacing of the diagonal cracks
s_{xe}	= equivalent crack spacing
z	= inner lever arm

LIST OF PUBLICATIONS

Teng, S., Lim, T. Y. D., and Divsholi, B. S. (2013). “Durability and Mechanical Properties of High Strength Concrete Incorporating Ultra Fine Ground Granulated Blast-Furnace Slag.” *Construction and Building Materials*, V. 40, pp. 875–81.

Divsholi, B. S., Lim, T. Y. D., and Teng, S. (2014). “Durability Properties and Microstructure of Ground Granulated Blast Furnace Slag Cement Concrete.” *International Journal of Concrete Structures and Materials*. V. 8, No. 2, pp. 157-164.

Teng, S., Divsholi, B. S., Lim, T. Y. D., and Gjrv, O. E. (2014). “Concrete with Very High Resistance to Chloride Ingress.” *ACI Concrete International*, V. 36, No. 5, pp. 30-36.

Lim, T. Y. D, Teng. S, Bahador, S. D, and Gjrv. O. E. (2016). “Durability of Very-High-Strength Concrete with Supplementary Cementitious Materials for Marine Environments.” *ACI Journal*, V. 113, No. 1, pp. 95-104.

Materials in Chapters 4 and 5 on the experimental results of high strength concrete beams, with and without shear reinforcement, and the two proposed analytical methods in Chapters 6 and 7 in this study are in preparation to be submitted in 2 journal papers.

Chapter 1

INTRODUCTION

1.1. Background

Reinforced concrete beam is one of the most widely used structural elements in the construction industry. In practice, designing a reinforced concrete beam to fail in flexure is generally preferred to the beam failing in shear mode. This is because a flexural failure is of a more ductile nature compared to the sudden and brittle effect of shear failure. While a structure is not typically designed to fail in shear, incidents of a structure collapsing unexpectedly in shear does occur. One prominent incident is the structural failure of concrete beams in the Wilkins Air Force Depot, Shelby Ohio, in 1955 which raised concerns on the knowledge of shear behaviour in beams. Reports compiling existing and updated knowledge on the shear behaviour of concrete beams were published by various ACI-ASCE Committee over the years: ACI-ASCE Committee 326 “Shear and Diagonal Tension” (ACI-ASCE 326, 1962); ACI-ASCE 426 Committee “The Shear Strength of Reinforced Concrete Members” (ACI-ASCE Committee 426, 1973); and ACI-ASCE Committee 445 “Recent Approaches to Shear Design of Structural Concrete” (ACI-ASCE 445, 1999).

Shear behaviour of a reinforced concrete beam can be influenced by several parameters such as the effective depth of the beam, the amount of flexural reinforcement, concrete compressive strength, and the shear-span-to-effective-depth (a/d) ratio. Aggregate size appears to influence shear strength but it is unclear if the use of larger aggregates would always lead to an increase in shear strength and vice versa. Experimental research had been conducted to investigate the influence of each of the above stated parameters on shear behaviour. Most of the experiments were conducted using specimens made of normal strength concrete; significantly fewer research studies had specimens made of high strength concrete ($f'_c > 70$ MPa).

In recent years, high strength concrete has become a popular choice of construction material in the industry, especially for vertical elements to carry vertical loads in tall buildings and bridge decks where it requires high durability performance. High strength concrete can be obtained in two ways: 1) by reducing the amount of water in a concrete mix; or 2) use of supplementary cementitious materials as a partial replacement of cement. Reducing the

amount of water in the concrete mix increases the compressive strength of the concrete, yet it may not reduce the porosity significantly. A concrete made in this manner has high compressive strength but its durability properties may not be significantly improved compared to a normal strength concrete (Lim *et al.*, 2016). Adding supplementary cementitious materials such as fly ash and ground granulated blast-furnace slag into a concrete mix increases the amount of pozzolanic reactions, hence, increasing the amount of calcium silicate hydrate (C-S-H). As a result, the volume and connectivity of the pores in the concrete matrix are reduced as additional C-S-H fill up the pore spaces in the matrix (Nakamura *et al.*, 1992; Luo *et al.*, 2003; Binici *et al.*, 2007; and Vejmelkova *et al.*, 2009). The durability properties of the concrete, measured in terms of chloride diffusivity and electrical resistivity, were also observed to be better than normal strength plain concrete with the inclusion of supplementary cementitious materials (Lim *et al.*, 2016).

Past experiments have shown that certain behaviours of high strength concrete beams differ from those of normal strength concrete beams. For example, the crack surface observed in a high strength concrete beam is smoother than that observed in a normal strength concrete beam (Elzanaty *et al.*, 1986; Kong *et al.*, 1994; and Janaka Perera and Mutsuyoshi, 2013). A smoother crack surface leads to a reduction in aggregate interlock (Khuntia and Stojadinovic, 2001). A proposed model by Sherwood *et al.* (2007), that presumes the aggregate interlock is a contributor to concrete shear strength, observed that there is no contribution from the aggregate interlock when the concrete strength is greater than 70 MPa.

1.2. Motivation

Land scarcity in Singapore has seen the construction of increasingly tall buildings as well as the continual reclamation of land outwards into the sea. Land reclamation in Singapore has generally been conducted using the traditional method of landfill. A recent NRF-CRP Project “Underwater Infrastructure and Underwater City of Future” proposed an alternative — creating large cylindrical-like concrete structures and placing them in the sea as a means of land reclamation. These structures would create space in and above the cylindrical-like structures. The service periods of these structures were estimated to last for at least a few hundred years. For a structure to last a few centuries, there must be negligible corrosion to the reinforcement steels when the structure is in contact with seawater. A high strength concrete with good mechanical and durability properties is thus required. Within the structure, several different

types of structural elements such as wall, slab, and beam were required. Various behaviours of each structural element constructed using high strength concrete were required to be investigated. This research emphasised on the shear behaviour of slender high strength concrete beams with and without shear reinforcement.

In a database presented by Reineck (2013), compiling experimental slender beams without shear reinforcement, 82 percent of the recorded data had a depth smaller than 450 mm. Several of these specimens were part of research programmes that focussed on the influence of size effect on shear strength. Two observations were made from a review on the research programmes: 1) Most experimental programmes tested specimens with a depth smaller than 450 mm. To conclude the influence of size effect on reinforced concrete beams with depths between 200 mm and 2000 mm requires a combination of results from various research programmes. However, different experimental specimens often had different compressive strengths, longitudinal reinforcement ratios, maximum aggregate size or geometries. Drawing conclusions using the data from various sources with different backgrounds would include many uncertainties and might not be fully conclusive; and 2) Most experimental programmes tested specimens with concrete compressive strength lower than 70 MPa. Several research programmes among others, such as Bhal (1968); Taylor (1972); Chana (1981); Niwa *et al.* (1987); Shioya *et al.* (1989); Shioya (1989); Walraven and Lehwalter (1994); Kim and Park (1994); Kawano and Watanabe (1997); Collins and Kuchma (1999); Tan and Lu (1999); Tan *et al.* (1999); Fujita *et al.* (2002); Bentz and Buckley (2005); Tan and Cheng (2006); Sherwood *et al.* (2007); and Syroka-Korol and Tejchman (2013) had conducted size effect tests with normal strength concrete beams and had the beam depth as its only variable. Yet, only the test programmes conducted by Matsui *et al.* (1995), Collins and Kuchma (1999), Fujita *et al.* (2002), Sherwood *et al.* (2007), and Sneed and Ramirez (2008 and 2010) used high strength concrete specimens. The number of experimental specimens made of high strength concrete is considerably fewer compared to that of normal strength concrete. It was discussed that the guidelines in the ACI 318 code might require revisions to be used for high strength concrete beams (Lee and Hwang, 2010).

Considering these observations, a set of life-sized beam specimens with similar concrete compressive strength, longitudinal reinforcement ratio, beam width and a/d ratio were necessary to investigate the influence of effective depth in high strength concrete beams under shear. The depth of the specimens used in this study varied between 450 mm and 1800 mm.

These specimens allowed an independent parametric study on the effect of different parameters on shear behaviour in slender high strength concrete beams, and could be added into the database for the validation of proposed design provisions.

1.3. Objective and Scope

The objectives of this study are (1) to investigate the influence of effective depth on large high strength concrete beams; (2) to investigate the influence of maximum aggregate size in high strength concrete beams; (3) to study the influence of shear reinforcement ratio on size effect in high strength concrete beams; (4) to find out the minimum shear reinforcement ratio to eliminate the size effect on shear strength of high strength concrete beams; (5) to study the efficacy of existing codes, such as ACI 318-14 (2014) and Eurocode 2 (2004), to predict shear strength of high strength concrete beams with varying depths; and (6) to propose new methods to predict the shear strength of large-sized high strength concrete beams with and without shear reinforcement.

To achieve these objectives, an experimental programme to test eleven life-sized beam specimens under static loading was proposed. Every shear span in each beam specimen has different configurations in either the maximum aggregate size, shear reinforcement ratio, or effective depth of the beam. Thus, each beam has different shear reinforcement ratio in each shear span. When the weaker shear span of each beam failed in shear, the shear span was strengthened with steel rods and plates. The test was then restarted in the same manner until the other shear span failed. Therefore, the eleven specimens in this experimental programme contributed twenty-two test data in total. The findings from the tests would aid engineers in designing large structural beams, as well as provide useful information for fellow researchers.

Three methods to predict shear strength were proposed in this study: one empirical equation for beams with no shear reinforcement, and two methods for beams with shear reinforcement. The empirical equation for beams with no shear reinforcement considered influential parameters such as effective depth, concrete compressive strength, longitudinal reinforcement ratio, and shear-span-to-effective-depth (a/d) ratio. The two methods to predict shear strength of beams with shear reinforcement included one analytical truss model and a simplified truss method. The analytical truss model was based on the softened truss model by Hsu and Mo (2010), with proposed additions of size effect and dowel action to the truss model.

The simplified method consisted of an empirical equation to estimate the inclination angle of the diagonal trusses in the beam at close to failure. Using the predicted diagonal truss inclination, the contribution from the shear reinforcement towards shear strength can be obtained. The concrete contribution to shear strength of the beam with shear reinforcement was predicted by the empirical equation that was proposed for beams with no shear reinforcement. Together, the contributions from concrete and shear reinforcement makes up the shear strength of the beam.

The current research work is limited to shear strength and behaviour of life-sized high strength concrete beams subjected to monotonic loads. The eleven life-sized concrete beams were tested to failure through applied loads by either hydraulic jacks or actuator. The test results provide information towards understanding the influences of effective depth, aggregate size, and shear reinforcement ratio on the shear strength of high strength concrete beams. The a/d ratio was kept constant in all the beam specimens. While varying the a/d ratio allows the investigation of different shear failure modes, the scope of the current experimental programme is limited to investigating the influence of effective depth, shear reinforcement and aggregate size. Therefore, the a/d ratio was kept constant to minimise any influence. The flexural behaviour of the studied beams lies outside the scope of this research. The influence of compressive strength, longitudinal reinforcement ratio, a/d ratio, pre-stressing force, and cyclic loading is also outside the scope of current research work. Numerical works such as finite element model and analysis are similarly excluded from the study.

1.4. Originality and Contribution

The contribution of this experimental programme lies in the testing of full-scale large size high-strength concrete beams under shear. The results from the five concrete beams with effective depths greater than 1000 mm allow the investigation of the influence of effective depth and shear reinforcement on shear strength of large high strength concrete beams under shear. Together with the smaller size beams, the concrete beams in the experimental programme in this study provide a full set of data with overall depth ranging from 450 to 1800 mm. Testing of two sets of high strength concrete beams with different aggregate sizes also show the influence of aggregate size on shear strength.

The contribution from analytical work to predict shear strength stems from proposed inclusion of dowel action and size effect to the truss model for beams with shear links in the first approach, and a proposed simplified truss formula in the form of $V_n = V_c + V_s$ as a second approach. The second approach is similar to the shear equations in ACI 318-14 for predicting shear strength of beams with shear reinforcement.

The first approach is a truss model based on Hsu and Mo's method (2010) that considers equilibrium conditions, compatibility conditions, and constitutive laws of concrete and steel materials. The truss model considers the compression field action in the web panel to compute the shear strength of a beam. The eleven governing equations from the equilibrium conditions, compatibility conditions and constitutive laws of materials with fourteen variables were solved by an iterative process, in which it requires one pre-determined value for a variable and assumed values for two other variables. The value of the applied normal stress in the z -direction (σ_z) was taken as zero, and initial values for average shear stress (τ_{xz}) and strain in the r -direction (ε_r) were assumed. A solution is obtained only when the calculated values for the two variables (τ_{xz} and ε_r) match the initially assumed values. The average shear stress is multiplied by the width and the depth of the web panel to obtain the shear strength of the beam. The truss model does not consider size effect in its calculations as well as any shear strength contribution from longitudinal reinforcement because they are placed outside the web panel. Without considering size effect, the predicted shear strength from the truss model would be unconservative for large-size concrete beams. Without considering the shear strength contribution from longitudinal reinforcement, the predicted shear strength would be very conservative for concrete beams with high amount of longitudinal reinforcement.

The author has considered these two effects in the first approach. By considering size effect in the truss model, the truss model can predict shear strength more accurately with lower predicted shear strengths for beams with increasing effective depths. The approach would be more conservative when predicting shear strength of large-sized beams. Considering the contribution from longitudinal reinforcement as an additional component in the total shear strength, the predicted shear strengths will be less conservative especially for beams with high amount of longitudinal reinforcement. It will allow the engineers to obtain a more economical beam design with a thinner web width or require lower shear reinforcement ratio.

The proposed truss model includes the size effect when calculating the depth of the web panel. A size effect term considering the influence of effective depth, shear reinforcement ratio and yield strength of shear reinforcement was proposed. The size effect term is added as a coefficient to the calculation of the depth of the web panel. The inclusion of dowel action is to consider contribution from longitudinal reinforcement towards shear strength. The total area of the longitudinal reinforcement is converted into one equivalent effective bar diameter to calculate dowel action. The calculated dowel action is added onto the shear strength contribution from the web panel to obtain total shear strength of the beam. A database of 189 beams' data which includes shear reinforcement was used to validate the result of the truss model with the proposed inclusions. A test-to-predicted shear strength ratio of 1.13 with a coefficient of variation of 0.16 was obtained for the truss model.

The results from the truss model in the first approach includes not just the shear strength of the beam, but also the full stress-strain relationship of the web element. The solution procedure of the truss model requires numerous iterations and is time-consuming. Among all the results obtained from the truss model, the main variables most useful to engineers designing a reinforced concrete beam would be the shear strength of the beam and the inclination angle of the diagonal truss. Instead of going through the iterative process in the first approach and obtain these two parameters at the end of the process, an approach with shorter computation procedure to obtain the two parameters (that are obtained in approach one) would benefit the engineers in their design work. The motivation to develop a second approach is to obtain the inclination angle of the diagonal truss similar to that obtained from the first approach, but with shorter computation procedures. Using the inclination angle of the diagonal truss, the shear strength contribution from the shear reinforcement can be obtained. In addition, an empirical equation is proposed as part of second approach to calculate the shear strength contribution from concrete, and added to the contribution from shear reinforcement to obtain the total shear strength.

The second proposed approach is the simplified truss formula in the form of $V_n = V_c + V_s$. This is to allow the engineer to directly obtain predictions of shear strength and inclination angle of the diagonal truss, without going through the iterative process of the first approach. The proposed simplified truss formula includes two parameters – contribution from concrete (V_c) and contribution from shear reinforcement (V_s). An empirical expression (V_c) consisting of compressive strength, longitudinal reinforcement ratio, shear-span-to-effective-depth ratio

and size effect was proposed to predict the shear strength of beams with no shear reinforcement. A database of 980 beams' data without shear reinforcement was used to validate the results of the proposed empirical expression (V_c), with a test-to-predicted shear strength ratio of 1.14 and a coefficient of variation of 0.14 obtained.

The calculation of shear strength contribution from shear reinforcement (V_s) in the second approach is similar to the equation in Eurocode 2. The inclination angle of the diagonal truss is required to determine the contribution of shear reinforcement towards shear strength. In the second approach, an empirical equation was proposed to predict the inclination angle of the diagonal truss. The empirical equation considers the influence of shear-span-to-effective-depth ratio, shear reinforcement ratio, yield strength of shear reinforcement and spacing between shear reinforcement with the objective to obtain inclination angles that would match those inclination angles obtained from the truss model in the first approach. The contribution from shear reinforcement (V_s) is calculated using the inclination angles obtained from the empirical equation. Shear strength contribution from shear reinforcement (V_s) is added to the concrete contribution (V_c) to determine the total shear strength (V_n). In this second approach, engineers can directly obtain the inclination angle of the diagonal truss from the empirical equation without going through the iterative process of the truss model.

189 beams' data was used to validate the results from this simplified truss formula ($V_n = V_c + V_s$), with a test-to-predicted shear strength ratio of 1.08 and a coefficient of variation of 0.15 obtained. The simplified truss formula would be useful to engineers who can obtain both the shear strength of the beam and inclination angle of the diagonal truss in a short calculation procedure.

1.5. Thesis organisation

The thesis is separated into eight chapters as described below:

Chapter 1 gives a general introduction, providing the scope of work and the objectives of this study. **Chapter 2** provides a background study on high strength concrete material and some discussions on past experimental programmes investigating the influence of effective depth on shear strength. A brief discussion on the mechanisms of shear strength for reinforced concrete beams with or without shear reinforcement is also presented. The provisions and

guidelines for codes such as ACI 318-14 (2014) and Eurocode 2 (2004) are presented in this chapter as well, together with specific models from past research that are relevant to this present study.

Chapter 3 details the objectives for the experimental programme and the design of beam specimens. The details for the instrumentation and different test setups of specimens in different series are also described. **Chapter 4** presents the test results and observations for the seven shear spans with no shear reinforcement among the eleven beams tested. The influences of effective depth and maximum aggregate size on the shear strength of the specimens are highlighted in this chapter. **Chapter 5** presents the test results and observations for the remaining fifteen shear spans that were provided with shear reinforcement. The influence of shear reinforcement on shear strength and on size effect in high strength concrete beams are highlighted in this chapter.

Chapter 6 proposes an empirical method for calculating the shear strength of a reinforced concrete beam with no shear reinforcement. The proposed equation considers various influential parameters on the shear strength of concrete beams. This equation is validated by the current and existing test results collected in a database. The test-to-predicted shear strengths (V_{test}/V_{pred}) ratios calculated using the proposed empirical equation are also compared with other proposed methods and codes in this chapter. **Chapter 7** presents two proposed models for reinforced concrete beams with shear reinforcement. The framework of this chapter is similar to **Chapter 6**. **Chapter 8** presents the key conclusions of this thesis. Suggestions for future works are also provided in this chapter. Photographs and complete test data from the experiments conducted in this study, Excel VBA codes, and databases of reinforced concrete beams with and without shear reinforcement, are provided in the **Appendices**.

Chapter 2

LITERATURE REVIEW

2.1. Overview

An introduction on high strength concrete is presented in this chapter. The mechanics of the shear strength of a reinforced concrete beam with and without shear reinforcement are discussed. A brief discussion of several prominent research programmes investigating the influence of effective depth on the shear strength of a reinforced concrete beam is also presented. Details of code provisions from ACI 318-14 (2014), Eurocode 2 (2004), and relevant methods proposed in other research to predict the shear strength of a reinforced concrete beam are also discussed in the chapter.

2.2. Shear Strength of Reinforced Concrete Beam

There must be a formation of diagonal crack before a beam could fail in shear (ACI-ASCE 426, 1973). Vertical flexural cracks at the mid-span of the beam are the first cracks to form when a beam is loaded. The width of the flexural cracks widens and the cracks propagate upwards as the applied load on the beam is increased. Upon a certain applied load, the propagation of the vertical flexural cracks slow down and new flexural cracks starts to form in both shear spans of the beam. In a beam with no shear reinforcement, one diagonal crack would start to form from the tip of one of the flexural cracks in the shear spans. The inclination of this crack would change after it propagates past the mid-depth of the beam and propagates towards the loading point of the beam. This change in inclination is taken to be the point where the diagonal crack is formed. Failure of the beam would occur soon after the formation of the diagonal crack.

The formation and widening of the diagonal crack would activate the shear resistance mechanism of the reinforced concrete beam. The shear resistance of a reinforced concrete beam with no shear reinforcement is provided by three components as shown in **Figure 2-1**: (1) V_{cz} , shear resistance provided by the compression zone; (2) V_{ay} , vertical component of the shear friction and aggregate interlock between two surfaces of the cracked concrete; and (3) V_d , dowel action forces contributed by the longitudinal tension reinforcement (MacGregor *et al.*, 2005).

Figure 2-1—Internal forces in a cracked beam with no shear reinforcement providing shear resistance (MacGregor *et al.*, 2005).

The V_{cz} component in a reinforced concrete beam depends on the compressive strength of the concrete and the depth of the compression zone. The V_a component is dependent on the type of aggregate used in the concrete, and the surface roughness of the aggregate. In normal strength concretes where cracks propagate around the aggregates, the contribution of aggregate interlock is provided by the aggregates that protrude out of the crack surface. These protruding aggregates create rough crack surfaces, resulting in interlock resistance against slip and to transfer shear. On the contrary, the cracks propagate through the aggregates in high strength concrete resulting in smooth crack surfaces. The shear transfer capability in smooth crack surfaces is provided by the friction between the crack surfaces. Smoother crack surfaces in high strength concrete beams result in reduced contribution from the V_a component compared to a normal strength concrete beam (Elzanaty *et al.*, 1986; Johnson and Ramirez, 1989; and Angelakos *et al.*, 2001). The contribution from the V_d component could be influenced by the tensile strength of the concrete cover and the amount of the longitudinal reinforcement provided in the beam. The distribution and layout of the longitudinal reinforcement could also influence the amount of dowel action.

As the width of the diagonal crack widens, the contribution from the V_a component reduces. This then increases the contributions from the other two shear resistance components, V_{cz} and V_d . Further load increments would result in splitting cracks along the levels of longitudinal reinforcements, causing the loss of bond between the longitudinal reinforcing bars and the concrete. The loss of bond between the bars and concrete would reduce the contribution from V_d . With the reduction in contribution from both V_d and V_a components, the shear resistance is solely provided by the compression zone above the diagonal crack. The

compression zone crushes and causes failure when the compression zone is not able to provide the required shear component to resist the shear force (MacGregor *et al.*, 2005).

Figure 2-2 presents the shear resistance of a reinforced concrete beam with shear reinforcement. There is an additional component to the shear resistance, V_s , provided by the web reinforcement.

Figure 2-2—Internal forces in a cracked beam with shear reinforcement providing shear resistance (MacGregor *et al.*, 2005).

The behaviour of a reinforced concrete beam with shear reinforcement is the same as a reinforced concrete beam with no shear reinforcement before the diagonal shear crack appears. Before the diagonal shear crack appears, the stress in the shear reinforcing bars would be small. The steel bars therefore have no contribution to the shear resistance before a diagonal crack crosses it. The shear resistance provided by the steel reinforcing bar is labelled V_s in **Figure 2-2**. The web reinforcement in the concrete beam do not prevent the formation of diagonal cracks. The web reinforcement could delay the widening of crack widths, hence slowing the loss of aggregate interlock or friction between the crack surfaces. The web reinforcement ideally increases the shear resistance of the reinforced concrete beam until the full flexural capacity of the beam. The shear failure of a reinforced concrete beam with web reinforcement would be less brittle compared to that of a beam with no shear reinforcement.

2.3. High Strength Concrete

Interest in high strength concrete has grown rapidly in recent years due to advancing concrete technologies. It has since been used in several building structures, such as the Two Union Square (1988) and Pacific First Centre (1989) in Seattle, and the CapitaGreen building (2014)

in Singapore. Good mechanical and durability properties of high strength concrete have resulted in its usage for columns in tall buildings, offshore structures, and long-span bridges.

The term “high strength concrete” is commonly used interchangeably with the term “high performance concrete”. However, it should be noted that these two terms refer to concretes with different properties. High strength concrete refers to a concrete with high compressive strength and reasonable durability properties. Compressive strength of at least 80 MPa is achieved at twenty-eight days before the concrete is classified as a high strength concrete. The increase in compressive strength does not always translate to a significant improvement in the durability aspect of the concrete. Reducing the water to cementitious material (w/c) ratio is the main condition in producing a high strength concrete. The amount of free water content is reduced, and hence, the porosity, inhomogeneity, and micro-cracks in the hydrated cement paste and transition zone are reduced, thereby increasing the compressive strength of the concrete.

A high performance concrete is different from a high strength concrete due to its high durability properties in addition to high compressive strength. Rougeron and Aïtcin (1994) described the process to obtain an ideal mix proportion for high performance concrete as “more of an art than science, but generally the water-to-binder (w/b) ratio must be low”. With a low w/b ratio, the amount of water available for hydration purpose is reduced. Chemical admixture, such as high range superplasticizer, is added to create a reasonable workability of the concrete, or even a flowable concrete. The durability properties of the concrete are enhanced using supplementary cementitious materials in the concrete, such as ground granulated blast-furnace slag and silica fume. These supplementary cementitious materials react with water to create secondary hydration products that reduces the amount of capillary pores and pores connectivity. This leads to increased resistance against corrosion (Teng *et al.*, 2013 and 2014; and Divsholi *et al.*, 2014). Essentially, a high performance concrete could be termed as a high strength concrete, but not vice versa. Extensive research has been conducted on the properties of high strength concrete as a material (Rols *et al.*, 1999; Hooton *et al.*, 1999; Smith *et al.*, 2004; Binici *et al.*, 2007; Divoshli *et al.* 2014; and Lim *et al.*, 2016).

A high strength concrete performs better than a normal strength concrete in most aspects of mechanical and durability properties. The tensile strength generally increases as the compressive strength of the concrete increases. However, the ratio between the tensile strength

and the compressive strength could also reduce as the compressive strength increases (Arioglu *et al.*, 2006). An experimental programme by Angelakos (1999) tested five reinforced concrete specimens with compressive strength between 20 to 80 MPa. Angelakos (1999) observed that there was no significant increase in shear strength as compressive strength increased. The experiments by Elzanaty *et al.* (1986), on the other hand, showed an increase in the shear strength of the specimens as compressive strength increased. However, the ratio of test-to-predicted shear strength for the specimens reduced as the compressive strength increased. The high strength concretes exhibited smoother crack surfaces compared to normal strength concretes. This reduced roughness in crack surface was reported to have resulted in reduced contribution from the aggregate interlock. Lubell *et al.* (2004) suggested the usage of an effective aggregate size, when considering the amount of shear transfer in a reinforced concrete beam, to account for the smooth crack surface. For concrete strengths greater than 60 MPa, the effective aggregate size was reduced linearly from the aggregate size until zero in concretes with compressive strength of 70 MPa (Sherwood *et al.*, 2007). Yoon *et al.* (1996) conducted an experimental programme on reinforced concrete specimens with shear reinforcement. The results showed that the additional shear strength carried by the specimens, after the first diagonal crack appeared, reduced as the compressive strength of the specimen increased.

The improvement to the shear strength because of an increase in compressive strength did not seem to be significant or obvious in all experimental programmes. However, the benefit of high strength concrete lies in its increased tensile strength. The increased tensile strength, compared to normal strength concretes, could delay the appearance of first crack. This would be useful in structures sensitive to cracks or where durability requirements are high. The delay of the first crack could improve the resistance of the concrete against corrosion to its reinforcement.

Most equations in codes were proposed based on concretes with lower strengths. The behaviour of structural elements made of high strength concrete should thus be examined against these equations, to evaluate the equations' performance on high strength concrete elements.

2.4. Size Effect in Shear

Size effect has always been a topic of interest throughout the world in experiments among others by: Leonhardt and Walther (1964); Kani (1966, 1967); Bhal (1968); Taylor (1972); Okamura and Higai (1980); Chana (1981); Bažant and Kim (1985); Iguro *et al.* (1985); Bažant and Sun (1987); Niwa *et al.* (1987); Walraven and Lehwalter (1987); Shioya *et al.* (1989); Shioya (1989); Bažant and Kazemi (1991); Tan *et al.* (1995, 1999); Kawano and Watanabe (1997); Ghannoum (1998); Collins and Kuchma (1999); Tan and Lu (1999); Tan *et al.* (1999); Angelakos *et al.* (2001); Fujita *et al.* (2002); Lubell *et al.* (2004); Lubell (2006); Tan and Cheng (2006); Sherwood *et al.* (2007); and Syroka-Korol, and Tejchman (2013). The influence of size effect on shear strength has generally been accepted and has been introduced into code provisions (Model Code 1990, 1993; Japanese Code, 2007; BS 8110, 1997). More recently, the Eurocode 2 (2004) included a size effect term into its shear strength equation to reduce the shear strength when the effective depth of the beam is greater than 200 mm. The equations in Eurocode 2 considers size effect in the form of ($k = 1 + \sqrt{200/d} \leq 2$). The Model Code 2010 (2010) introduced a new size effect term in the form of $1/(1 + d/d_0)$ to replace the existing term in Model Code 1990. This size effect term, however, was commented by Yu *et al.* (2016) to be unrealistic. The size effect term in ACI 446 (2007), based on the works of Bažant, was suggested to be in the form of $1/\sqrt{1 + d/d_0}$.

A database of 980 slender reinforced concrete beams with no shear reinforcement (shear-span-to-effective-depth (a/d) ratio > 2.5) was compiled, and is presented in the Appendix. Among the data, only thirty-three data have effective depths larger than 1000 mm. The details of these thirty-three data are listed in **Table 2-1**.

Table 2-1—Details of previous experimental data with effective depth greater than 1000 mm.

Authors	Year	Beam label	b_w (mm)	d (mm)	a/d ratio	ρ (%)	f_c' (MPa)	d_a (mm)
Kani	1967	3043.00	154	1092	3.00	2.71	27	19
Kani	1967	3044.00	152	1097	3.98	2.72	30	19
Kani	1967	3045.00	155	1092	5.00	2.70	28	19
Kani	1969	3061.00	154	1091	3.10	0.80	27	19
Bhal	1968	B4	240	1200	3.00	1.26	25	30
Niwa, Yamada <i>et al.</i>	1987	1.00	600	2000	3.00	0.28	27	25
Niwa, Yamada <i>et al.</i>	1987	2.00	600	2000	3.00	0.14	26	25
Niwa, Yamada <i>et al.</i>	1987	3.00	300	1000	3.00	0.14	25	25
Shioya <i>et al.</i>	1989	S3: No. 4	500	1000	3.00	0.40	27	10
Shioya <i>et al.</i>	1989	S3: No. 5	500	1000	3.00	0.40	22	25
Shioya <i>et al.</i>	1989	S3: No. 6	1000	2000	3.00	0.40	29	25
Shioya <i>et al.</i>	1989	S3: No. 7	1500	3000	3.00	0.41	24	25
Shioya <i>et al.</i>	1989	S4: No. 6	500	1000	3.00	0.40	28	5
Kawano & Wantanabe	1997	A4A	600	2000	3.00	1.20	22	40
Kawano & Wantanabe	1997	A4B	600	2000	3.00	1.20	23	40
Yoshida	2000	YB2000/0	300	1890	2.86	0.74	34	10
Cao	2001	SB2003/0	300	1925	2.81	0.36	31	10
Cao	2001	SB2012/0	300	1845	2.93	1.52	28	10
Fujita <i>et al.</i>	2002	L-100-3	350	1000	3.00	1.36	36	20
Fujita <i>et al.</i>	2002	M-100-3	350	1000	3.00	1.36	59	20
Fujita <i>et al.</i>	2002	U-100-3	350	1000	3.00	1.36	103	20
Higgins <i>et al.</i>	2004	37T	356	1151	2.91	0.74	32	19
Sherwood <i>et al.</i>	2007	L-10N1	300	1400	2.89	0.83	38	10
Sherwood <i>et al.</i>	2007	L-10N2	300	1400	2.89	0.83	40	10
Sherwood <i>et al.</i>	2007	L-10H	300	1400	2.89	0.83	74	10
Sherwood <i>et al.</i>	2007	L-20N1	300	1400	2.89	0.83	31	20
Sherwood <i>et al.</i>	2007	L-20N2	300	1400	2.89	0.83	33	20
Sherwood <i>et al.</i>	2007	L-40N1	300	1400	2.89	0.83	28	40
Sherwood <i>et al.</i>	2007	L-40N2	300	1400	2.89	0.83	29	40
Sherwood <i>et al.</i>	2007	L-50N1	300	1400	2.89	0.83	41	50
Sherwood <i>et al.</i>	2007	L-50N2	300	1400	2.89	0.83	40	50
Sherwood <i>et al.</i>	2007	L-50N2R	300	1400	2.89	0.83	40	50
Collins <i>et al.</i>	2015	Specimen 1	250	3840	3.12	0.65	40	14

Collins *et al.* (2015) tested the largest specimen with an effective depth of 3840 mm. These thirty-three data, with effective depths larger than 1000 mm, represent only 3% of all

the data. 90% of the data have an effective depth of 500 mm or less. While extensive research has been conducted on size effect, there were few large-sized specimens investigated. As such, more large-sized specimens are required to allow better understanding of the influence of effective depth on large-sized specimens with varying depths.

The following sections present brief discussions of eight representative research programmes that investigated size effect in recent years.

2.4.1. Kani (1967)

The experimental programme conducted by Kani (1967) was one of the pioneer programmes to investigate the influence of effective depth on the shear strength of a concrete beam. The programme tested many specimens that had no shear reinforcement, with varying effective depths and shear-span-to-effective-depth (a/d) ratios, to investigate the influence of these two parameters. The details of specimens with a/d ratio between 2.5 and 6 used to examine the influence of depth on the shear strength are presented in **Table 2-2**. The results of specimens, with varying depths and a/d ratios, are presented in **Figure 2-3**.

Table 2-2—Summary of experimental specimens from Kani (1967).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
48	152	151	133	25	2.82	5.1	27
52	152	152	140	25	2.67	3.9	29
55	152	150	135	25	2.80	3.0	33
63	610	154	543	26	2.77	4.0	93
74	610	152	523	27	2.84	3.1	108
75	610	152	524	27	2.84	3.1	108
96	305	153	275	25	2.76	3.9	56
97	305	152	276	27	2.68	3.0	63
3043	1219	154	1092	27	2.71	3.0	165
3044	1219	152	1097	30	2.72	4.0	159
3045	1219	155	1092	28	2.70	5.0	152

Figure 2-3—Results of shear stress at failure against different a/d ratios and varying beam depth (Kani, 1967).

The depths of these specimens varied between 152 to 1219 mm. The results in **Figure 2-3** clearly show that, the shear stress at failure of specimens with a/d ratios greater than 2.0 reduced, as the effective depths of the specimens increased. Kani (1967) commented that the results showed that there is no critical beam depth that was previously suggested by Rusch *et al.* (1962). Rusch *et al.* (1962) had suggested that any change in beam depth beyond the critical beam depth would not affect the strength of the specimen. Following Kani's experimental programme (1967), the influence of effective depth on shear strength of reinforced concrete beams became a hot topic for research. The influence of effective depth is subsequently termed as size effect.

2.4.2. Shioya (1989)

One objective of the experimental programmes conducted by Iguro *et al.* (1985), Shioya (1989), and Shioya *et al.* (1989) was to examine the influence of size effect on reinforced concrete beams with depths greater than 1000 mm. Prior to them, the largest effective depth of any specimen was 1080 mm. The experimental programme by Shioya (1989) consisted of specimens with effective depth varying from 100 to 3000 mm. The details of the specimens are presented in **Table 2-3**. Apart from size effect, the influence of aggregate size on shear

strength was also investigated. The aggregate sizes used in the specimens ranged between 1 to 25 mm. The results for all the specimens are presented in **Figure 2-4**.

Table 2-3— Summary of experimental specimens from Shioya (1989).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
S3: No. 3	655	300	600	21	0.43	3.0	105
S3: No. 4	1200	500	1000	27	0.40	3.0	229
S3: No. 5	1200	500	1000	22	0.40	3.0	251
S3: No. 6	2100	1000	2000	29	0.40	3.0	840
S3: No. 7	3140	1500	3000	24	0.41	3.0	1575
S4: No. 1	120	158	100	21	0.41	3.0	13
S4: No. 2	220	158	200	29	0.41	3.0	28
S4: No. 3	220	158	200	22	0.41	3.0	29
S4: No. 4	655	300	600	27	0.43	3.0	86
S4: No. 5	655	300	600	21	0.43	3.0	132
S4: No. 6	1200	500	1000	28	0.40	3.0	187

Figure 2-4—Results of shear strength with varying beam depth and aggregate size (Shioya et al., 1989).

Figure 2-4 shows that the shear strength of a specimen decreased either when the depth of the specimen increased or when the maximum aggregate size used in the concrete reduced. The experimental shear strength of the largest specimen was less than 50% of the predicted

shear strength calculated using the equations in ACI 318-14 (2014). The results showed that size effect could influence the shear strength of a reinforced concrete beam with an effective depth greater than 1000 mm. The observation from Shioya *et al.* (1989) was the size effect is a result of a combined effect from the loss of shear transfer due to reduced aggregate interlock and the presence of size effect in the concrete flexural tensile strength. The results from this set of tests certainly showed the influence of size effect and the concrete strength of all the specimens were less than 30 MPa. There had been no test series conducted with high strength concrete specimens of such sizes.

2.4.3. Collins and Kuchma (1999)

A total of twenty-two specimens were tested in the experimental programme by Collins and Kuchma (1999). Of these twenty-two specimens, four normal strength concrete and three high strength concrete specimens were used to investigate the influence of size effect in both normal and high strength concrete beams. The effective depth of the specimens varied between 225 to 925 mm. The normal strength concrete beam series had an additional specimen with effective depth of 110 mm. The details of these seven specimens are presented in **Table 2-4**.

Table 2-4—Summary of experimental specimens from Collins and Kuchma (1999).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
BN100	1000	300	925	37	0.76	2.9	192
BN50	500	300	450	37	0.81	3.0	132
BN25	250	300	225	37	0.89	3.0	73
BN12	125	300	110	37	0.91	3.1	40
BH100	1000	300	925	99	0.76	2.9	193
BH50	500	300	450	99	0.81	3.0	132
BH25	250	300	225	99	0.89	3.0	85

The results from the experiments are presented in **Figure 2-5**. **Figure 2-5** shows that the shear stress at failure (in terms of MPa) reduced as the effective depth of the specimen increased.

Figure 2-5—Results of shear stress at failure with varying beam depth (Collins and Kuchma, 1999).

Considering the shear stress at failure over the square root of compressive strength of the specimen ($\sqrt{f'_c}$), it can be seen that this ratio for the normal strength concrete specimens was higher than that for the high strength concrete specimens. Comparing the test shear strengths against the predicted shear strengths from the equations in ACI 318-14 ($V_{test}/V_{predicted}$), five out of the seven specimens' experimental shear strengths were lower than predicted. The lowest $V_{test}/V_{predicted}$ ratio was recorded for the high strength concrete specimen with the largest effective depth. The two normal strength concrete specimens with the smallest effective depths had conservative results against the predicted values. Collins and Kuchma (1999) thus commented that size effect could affect the shear strength of high strength concrete specimens more than normal strength concrete specimens, and that high strength concrete specimens could fail at unexpectedly low values of shear. While the results showed that size effect influence shear strength of high strength concrete beams, the largest effective depth of the specimens was 925 mm. The influence of size effect on specimens with effective depths greater than 1000 mm was not examined.

2.4.4. Fujita *et al.* (2002)

The experimental programme conducted by Fujita *et al.* (2002) consisted of twenty-seven specimens to investigate the influence of size effect on shear strength of the concrete beam. The effective depths were varied between 250 and 1000 mm. The details of these nine specimens are presented in **Table 2-5**. The compressive strengths of these nine specimens are

grouped into three categories (30 to 40 MPa, 50 to 70 MPa, and 85 to 105 MPa), and the results from the experiments are presented in **Figure 2-6**.

Table 2-5—Summary of experimental specimens from Fujita et al. (2002).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
L-25-3	300	150	250	37.8	1.53	3.0	43
L-25-3	300	150	250	36.4	1.53	3.0	56
L-25-3	300	150	250	36.4	1.53	3.0	56
L-50-3	600	150	500	37.4	1.53	3.0	116
L-50-3	600	150	500	36.4	1.53	3.0	79
L-50-3	600	150	500	36.4	1.53	3.0	87
L-100-3	1130	350	1000	35.7	1.36	3.0	319
L-100-3	1130	350	1000	34.7	1.36	3.0	298
L-100-3	1130	350	1000	33.7	1.36	3.0	333
M-25-3	300	150	250	68.8	1.53	3.0	51
M-25-3	300	150	250	51.9	1.53	3.0	65
M-25-3	300	150	250	51.9	1.53	3.0	54
M-50-3	600	150	500	69.4	1.53	3.0	87
M-50-3	600	150	500	51.9	1.53	3.0	108
M-50-3	600	150	500	51.9	1.53	3.0	113
M-100-3	1130	350	1000	59.2	1.36	3.0	315
M-100-3	1130	350	1000	53.7	1.36	3.0	340
M-100-3	1130	350	1000	53.0	1.36	3.0	340
U-25-3	300	150	250	101	1.53	3.0	40
U-25-3	300	150	250	92.9	1.53	3.0	56
U-25-3	300	150	250	92.9	1.53	3.0	47
U-50-3	600	150	500	102	1.53	3.0	86
U-50-3	600	150	500	92.9	1.53	3.0	83
U-50-3	600	150	500	92.9	1.53	3.0	98
U-100-3	1130	350	1000	103	1.36	3.0	280
U-100-3	1130	350	1000	89.8	1.36	3.0	263
U-100-3	1130	350	1000	92.1	1.36	3.0	280

Figure 2-6—Results of shear stress at failure with varying effective depth (Fujita *et al.*, 2002).

The specimens tested by Fujita *et al.* (2002) also enabled the investigation of influence of compressive strength on shear strength. The test results for specimens with compressive strength greater than 80 MPa showed that the shear strength (in terms of MPa) tended to reduce as the compressive strength increased. The test results in **Figure 2-6** for all the specimens showed a reduction in shear strength as the effective depth of the specimen increased. Fujita *et al.* (2002) commented that the compressive strength would affect the influence of size effect on shear strength of a reinforced concrete beam. The influence of size effect would affect specimens made of high strength concrete more significantly than specimens of normal strength concrete. The influence of effective depth on the shear strength of a reinforced concrete beam could be taken to be a function of minus $\frac{1}{4}$ power for low to normal strength concrete. For high strength concrete, the function of minus $\frac{1}{2}$ power is more applicable.

2.4.5. Sneed and Ramirez (2010)

The experimental programme reported by Sneed and Ramirez (2010) consisted of eight high strength concrete specimens with varying depths of 305 to 915 mm. The compressive strength of the specimens was about 70 MPa. Four specimens were categorised into series 1 to investigate the influence of depth with a constant width, and another four specimens were sorted into series 2, investigating the influence of depth with a constant width-to-height ratio. The summary of the eight specimens are presented in **Table 2-6**. The experimental shear strengths of the specimens compared against the effective depths are presented in **Figure 2-7**.

Table 2-6—Summary of experimental specimens from Sneed and Ramirez (2010).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
1-1	305	305	232	66	1.20	3.0	131
1-2	611	306	530	66	1.25	3.0	140
1-3	762	305	681	65	1.24	3.0	148
1-4	915	306	822	75	1.30	3.0	168
2-1	305	203	233	69	1.26	3.0	57
2-2	608	408	529	65	1.20	3.0	156
2-3	762	508	684	68	1.30	3.0	262
2-4	915	613	822	73	1.30	3.0	353

Figure 2-7—Results of shear stress at failure with varying effective depth (Sneed and Ramirez, 2010).

The results in **Figure 2-7** show that the smallest specimen obtained the greatest shear stress at failure. For specimens with effective depths between 610 to 935 mm, the shear strength of the specimen reduced almost linearly as the effective depth increased. The shear stress at failure (in terms of MPa) of the smallest specimens was significantly greater than the other specimens. As a result, the reducing trend in shear strength is no longer linear. Sneed and Ramirez (2010) commented that the reduction in shear strength of a specimen could also be influenced by other parameters that affect the shear behaviour and the mode of shear transfer in a specimen. The reduced shear strength in specimens with larger effective depth was not entirely due to the influence of size effect. From the results of the specimens with effective depths between 500 and 900 mm, it seems that the width-to-height ratio would have some influence on size effect. The influence of size effect on shear strength of the specimens in series 2 were not as significant as those in series 1.

2.4.6. Syroka-Korol and Tejchman (2013)

Syroka-Korol and Tejchman tested nine normal strength concrete specimens with varying depths from 220 to 780 mm. The details of the nine specimens are presented in **Table 2-7**, and the results presented in **Figure 2-8**.

Table 2-7—Summary of experimental specimens from Syroka-Korol and Tejchman (2013).

Specimen label	h , mm	b_w , mm	d , mm	f'_c , MPa	ρ_l , percent	a/d	V_{test} , kN
SL20.1	220	200	180	35	1.00	3.0	48
SL20.2	220	200	180	35	1.00	3.0	44
SL20.3	220	200	180	35	1.00	3.0	46
SL40.1	400	200	360	35	1.00	3.0	80
SL40.2	400	200	360	35	1.00	3.0	86
SL40.3	400	200	360	35	1.00	3.0	91
SL80.1	780	200	720	35	1.00	3.0	132
SL80.2	780	200	720	35	1.00	3.0	116
SL80.3	780	200	720	35	1.00	3.0	148

Figure 2-8—Nominal shear strength of the specimens (Syroka-Korol and Tejchman, 2013).

The crack widths on the specimen were observed to have been reduced and the spacing between the flexural cracks were increased, as the depth of the beam increased. The results were compared against the size effect law proposed by Bažant and Planas (1998), and the results agree well with the size effect law.

2.4.7. Summary of Size Effect in Shear

Many experimental programmes have been conducted to investigate the influence of size effect on the shear strength of reinforced concrete beams. The specimens in these programmes had the effective depth as the only variable to investigate the influence of size effect. These specimens are categorised according to the compressive strength and maximum aggregate size used in the concrete, to understand the influence of size effect on each category. Normal strength concrete specimens with aggregate size smaller than 15 mm are grouped and presented in **Figure 2-9**. These data were collected from research programmes by Chana (1981), Shioya *et al.* (1989), Collins and Kuchma (1999), Bentz and Buckley (2005), and Sherwood *et al.* (2007).

Figure 2-9—Influence of effective depth on normal strength concrete specimens with aggregate size smaller than 15mm.

In the left graph in **Figure 2-9**, the failure load for each specimen was normalised by $(b_w d \sqrt{f'_c})$. In the graph on the right, the failure load was normalised by $(b_w d^3 \sqrt{f'_c})$. These two graphs are required for comparison with the equations in ACI 318-14 (2014) and Eurocode 2 (EC 2) (2004). The trend line for each series of data collected from the same source is represented by a dotted line. The trend line for all the data is represented by a red solid line in the same graph. The trend line for size effect influence according to ACI 446, based on the work of Bažant *et al.* (2007), is given as a brown solid line. The trend line for ACI 318-14 is represented by a green solid line. As for EC 2, the trend line for the influence of size effect is represented by a blue solid line. The graphs show a definite reduction in the shear strength as

the effective depth of the specimen increased. The shape of the trend line for all the data is similar to that per ACI 446.

Figure 2-10 shows the results of the normal strength concrete specimens with aggregate sizes between 15 and 30 mm (Kani, 1966, 1967; Bhal, 1968; Taylor, 1972; Niwa *et al.*, 1987; Walraven and Lehwalter, 1987; Shioya *et al.*, 1989; Shioya, 1989; Bažant and Kazemi, 1991; Kim and Park, 1994; Kawano and Watanabe, 1997; Fujita *et al.*, 2002; Sherwood *et al.*, 2007; and Syroka-Korol, and Tejchman, 2013). The red solid line, taken from **Figure 2-9**, represents the trend line for the specimens with aggregate sizes smaller than 15 mm.

Figure 2-10—Influence of effective depth on normal strength concrete specimens with aggregate sizes between 15 and 30 mm.

Both **Figures 2-9** and **2-10** clearly show the influence of size effect on the shear strength of beams. The two solid trend lines in **Figure 2-10** show that aggregate size might influence the magnitude of the normalized shear stress marginally, but the shape of the size effect trend line remained unchanged. The recommendation in ACI 446 seemed to be able to predict the influence of size effect very well compared to EC 2.

Among research programmes that investigated size effect, only programmes by Matsui *et al.* (1995), Collins and Kuchma (1999), Fujita *et al.* (2002), Sherwood *et al.* (2007), and Sneed and Ramirez (2010) investigated size effect using high strength concrete specimens. The results of the specimens in each programme are presented in **Figure 2-11**.

Figure 2-11—Influence of effective depth on high strength concrete specimens.

Figure 2-11 shows a definite reduction in the shear strength as the effective depth of the specimen increased. The black solid line represents the trend line for all the data. For a specimen with a depth greater than 1200 mm, size effect could reduce the shear strength of the beam by almost 50% when compared to the predicted shear strength from the equations in ACI 318-14. The shape of the trend line for all the data with aggregate size of 10 and 20 mm is similar to the trend line per ACI 446. It can thus be deduced that size effect could influence the shear strength of reinforced concrete beam made of normal or high strength concrete. However, there is only one datum with an effective depth greater than 1200 mm in **Figure 2-11**. As such, high strength concrete specimens with effective depth larger than 1200 mm are needed to compile a comprehensive set of experimental data, in an effort to further clarify the factors that influence the shear strength of concrete beams.

The influence on shear strength due to size effect was reported to be eliminated if shear reinforcement is provided in the beam. (Lubell *et al.*, 2004; and Collins *et al.*, 2008) However, Yu and Bažant (2011) reported from their observations that shear reinforcement might be able to mitigate the influence of size effect on shear strength, but not eliminate it. The shear reinforcement's influence on size effect is another area to be examined on with the specimens in this study.

2.5. Code Provision

2.5.1. ACI 318-14 (2014)

The approach for the ACI code includes both the contributions from the concrete (V_c) and shear reinforcement (V_s) as part of the nominal shear capacity (V_n). This is expressed as Equation (2.1).

$$V_n = V_c + V_s \quad (2.1)$$

where:

V_n = nominal shear strength;

V_c = nominal shear strength contribution from concrete; and

V_s = nominal shear strength contribution from shear reinforcement.

2.5.1.1. Members not requiring shear reinforcement

The concrete's contribution to the shear capacity is considered to be the same for reinforced concrete beams with or without shear reinforcement. The general equation to determine the concrete's contribution is calculated using Equation (2.2). The nominal shear strength is taken to be proportional to the tensile strength of the concrete, and is taken to be the square root of concrete strength. Detailed calculations of the concrete's contribution could be made using Equations (2.3) to (2.5). The lowest value calculated from Equations (2.3) to (2.5) is taken to be the shear strength provided by the concrete. For most design cases, the second expression in Equations (2.3) and (2.4) will be close to $0.01\lambda\sqrt{f'_c}$, hence, the general conventional equation of Equation (2.2) can be used for ease of calculation. In cases where the concrete member is designed with high amount of longitudinal reinforcement ratios, Equation (2.2) can be very conservative. In this case, Equations (2.3) and (2.4) can be used to better account for the high longitudinal reinforcement ratios as shear strength increases with an increase in longitudinal reinforcement ratio. However, the predicted shear strength is limited by Equation (2.5).

$$V_c = 0.17\lambda\sqrt{f'_c}b_wd \quad (2.2)$$

$$V_c = 0.16\lambda\sqrt{f'_c}b_wd + 17\rho\frac{V_u d}{M_u}b_wd \quad (2.3)$$

$$V_c = 0.16\lambda\sqrt{f'_c}b_wd + 17\rho b_wd \quad (2.4)$$

$$V_c = 0.29\lambda\sqrt{f'_c}b_wd \quad (2.5)$$

where:

λ = a modification factor for lightweight concrete;

b_w = web width of the beam;

d = effective depth of the beam;

f'_c = concrete cylinder compressive strength, and the value of $(f'_c)^{0.5}$ is limited to 8.3 MPa;

M_u, V_u = factored design bending moment and shear force occurring simultaneously at the

critical section, the term $\frac{V_u d}{M_u}$ should not be greater than 1; and

ρ = longitudinal reinforcement ratio, A_s/b_wd .

2.5.1.2. Members requiring shear reinforcement

When vertical shear reinforcement bars are provided in the beam, the shear strength provided by the shear reinforcement (V_s) is calculated by using Equation (2.6). Equation (2.6) considers the tensile capacity of the vertical shear reinforcement in a 45 degrees truss model.

$$V_s = \frac{A_v f_{yt} d}{s} \quad (2.6)$$

where:

s = spacing of the shear reinforcing bars;

A_v = area of shear reinforcement within the spacing, s ; and

f_{yt} = yield strength of the shear reinforcement.

In cases where no shear reinforcement is required in the design calculation, the provision required a minimum amount of shear reinforcement to be provided if the factored shear force exceeds half of the shear strength from the concrete and/or in beams deeper than 250 mm. The minimum amount of shear reinforcement is calculated using Equation (2.7). Shear reinforcement is required to prevent sudden failure of the beam from the diagonal cracks in the shear span. The required amount of shear reinforcement in a beam calculated from Equation (2.6) should also be greater than that calculated by Equation (2.7).

$$A_{v,min} = 0.062 \sqrt{f_c} \frac{b_w s}{f_{yt}} \geq 0.35 \frac{b_w s}{f_{yt}} \quad (2.7)$$

where:

$A_{v,min}$ = minimum area of shear reinforcement.

2.5.2. Eurocode 2 (2004)

2.5.2.1. Members not requiring shear reinforcement

The shear capacity of a reinforced concrete beam with no shear reinforcement ($V_{Rd,c}$) is calculated using Equation (2.8).

$$V_{Rd,c} = \left[0.18k(100\rho f_{ck})^{\frac{1}{3}} \right] b_w d \quad (2.8)$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2 \quad (2.9)$$

where:

$V_{Rd,c}$ = design shear resistance of a member with no shear reinforcement;

b_w = web width of the beam;

d = effective depth of the beam;

f_{ck} = concrete cylinder compressive strength, and the value of f_{ck} is limited to 90 MPa; and

ρ = longitudinal reinforcement ratio ($A_s/b_w d$), and is limited to 0.02.

The term k — given in Equation (2.9) — represents a size effect factor considering the reduction in shear strength as the depth of the beam increases. The shear capacity of the beam calculated by Equation (2.8) should always be greater than the minimum capacity calculated using Equation (2.10).

$$V_{Rd,c} = v_{min} b_w d \quad (2.10)$$

$$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} \quad (2.11)$$

2.5.2.2. Members requiring shear reinforcement

The design of a reinforced concrete beam with shear reinforcement is based on a variable truss model, where the inclination (θ) between the strut and the longitudinal axis of the beam may be selected by the designer. The code considers all shear to be taken by the shear reinforcement, and neglects the contribution of the concrete's contribution. Equation (2.12) calculates the shear capacity provided by the shear reinforcement ($V_{Rd,s}$). Equation (2.13) calculates the maximum diagonal truss strength ($V_{Rd,max}$). The smaller value of these two equations gives the shear capacity of the beam with vertical shear reinforcement (V_{Rd}).

$$V_{Rd,s} = \frac{A_v}{s} z f_{ywd} \cot \theta \quad (2.12)$$

$$V_{Rd,max} = \alpha_c b_w z v f_{cd} / (\cot \theta + \tan \theta) \quad (2.13)$$

where:

$V_{Rd,s}$ = shear capacity provided by the yielding shear reinforcement;

$V_{Rd,max}$ = shear capacity of the compression struts before crushing;

z = inner lever arm, taken to be $0.9d$;

A_v = area of shear reinforcement;

s = spacing of the shear reinforcement;

f_{ywd} = design yield stress of the shear reinforcement;

f_{cd} = design compressive strength;

v = strength reduction factor for concrete cracked in shear; and

α_c = a coefficient to consider the state of stress in the compression chord, taken to be 1 for non-prestressed structures.

The value of $\cot \theta$ is restricted to between 1 and 2.5. This means that θ shall be chosen between 22° and 45° . A smaller value of θ used in Equation (2.12) would increase the shear capacity provided by the shear reinforcement. However, the same value of θ used in Equation (2.13) would reduce the strut strength of the beam, as opposed to using 45° for θ . Equations (2.12) and (2.13) could be used together by equating both equations to obtain an optimal θ with the highest values of $V_{Rd,max}$ and $V_{Rd,s}$. However, the calculated θ should still lie within the

limits of 22° and 45° . The term v in Equation (2.13) considers the reduction of truss strength when the concrete cracks along the truss, and is given by Equation (2.14).

$$v = 0.6 \quad \text{for } f_{ck} < 60 \text{ MPa} \quad (2.14a)$$

$$v = 0.9 - \frac{f_{ck}}{200} > 0.5 \quad \text{for } f_{ck} \geq 60 \text{ MPa} \quad (2.14b)$$

In reinforced concrete beams with very low shear reinforcement ratio, the larger values of $V_{Rd,s}$ and $V_{Rd,c}$ should be taken as the shear capacity of the beam. The amount of shear reinforcement provided in the beam should not be less than the ratio calculated using Equation (2.15).

$$\rho_{w,\min} = \frac{0.08\sqrt{f_{ck}}}{f_{yk}} \quad (2.15)$$

where:

f_{yk} = yield stress of the shear reinforcement.

When the shear calculation shows that shear reinforcement is not required in the beam, the minimum amount of shear reinforcement, which is calculated by Equation (2.15), should still be provided.

2.6. Models for Analysing Shear Strength of Reinforced Concrete Beams

2.6.1. 45° truss model

The classical truss analogy was introduced by Ritter (Ritter, 1899) and M\"orsch (M\"orsch, 1909). It describes a cracked reinforced concrete beam as a truss model with longitudinal chords at the top and bottom, as well as diagonal, and vertical members. The longitudinal chords represent the compressive and tensile forces, C and T , at the top and bottom of the beam. The diagonal member represents the concrete compression struts, whereas the vertical member represents the vertical reinforcement bars. The truss model ignores all the shear transfer components in the concrete, i.e. V_a , V_{cz} , and V_d . The truss model includes selecting a truss system that is in equilibrium with the applied load and support conditions. The inclination of

the diagonal compression struts was assumed to be 45° and are assumed to extend across more than one vertical reinforcement bar. This assumption forms the basis for the equation that calculates the required amount of shear reinforcement, shown in Equation (2.6).

The 45° truss model was observed to be very conservative, especially for reinforced concrete beams with low amount of vertical shear reinforcement. The 45° truss model in the ACI code provision includes the addition of an empirical value of shear capacity to the truss model. This empirical shear capacity is taken to be the amount of shear carried by the concrete before the diagonal cracks appeared, which is also known as concrete contribution (V_c).

2.6.2. Variable-angle truss model

A variable truss model is similar to the 45° truss model but with the inclination of the compression strut at varied angles. The equilibrium equations for this model is shown in **Figure 2-12**. Taking the equilibrium of vertical forces, the compressive stress could be presented as Equation (2.16).

Figure 2-12—Equilibrium equations for variable-angle truss model.

$$\sigma_d = \frac{V}{b_w jd} \left(\frac{1}{\sin \theta \cos \theta} \right) = v(\cot \theta + \tan \theta) \quad (2.16)$$

where:

V = applied load;

v = shear stress at a cross section;

b_w = web width of the beam;

jd = flexural lever arm, in which j is taken to be 0.9 and d is the effective depth of the beam;

θ = inclination of concrete compressive strut; and

σ_d = principal compressive stress.

According to the model, the vertical component of the compression force must be carried by vertical shear reinforcement bars within a distance of $jd \cot \theta$. The following equations could be obtained:

$$\frac{A_v f_y}{s} = \frac{V}{jd \cot \theta} = \frac{v b_w jd}{jd \cot \theta} \quad (2.17)$$

$$v = \frac{A_v f_y}{b_w s} \cot \theta \quad (2.18)$$

where:

A_v = area of shear reinforcement;

s = spacing of the shear reinforcement; and

f_y = yield stress of the shear reinforcement.

Taking the equilibrium of horizontal forces, the horizontal component of the compressive stress is balanced by the tension in the longitudinal reinforcement. This is shown in Equation (2.19).

$$T = V \cot \theta \quad (2.19)$$

where:

T = tensile force in the longitudinal reinforcement.

There are four unknowns (σ_d , T , f_y and θ) in Equations (2.16), (2.18) and (2.19). To solve the stresses in the beam, the inclination of the compressive strut (θ) has to be solved. One way of obtaining the inclination of the diagonal struts is to apply the plasticity methods. Plasticity methods, considering the mechanism of failure, assumed either the compressive stress at failure or the yielding of shear or longitudinal reinforcement. Using an initial assumption, V and θ could be obtained by solving simultaneous equations.

2.6.3. Compression field theory

Collins and Vecchio developed the compression field theory by applying Wagner's tension field theory onto reinforced concrete (Collins, 1978). Wagner's tension field theory considered that after buckling, the shear in a metal girder would be carried by diagonal tensions in the thin webs of the metal girder. The web would not carry any compression stresses. Wagner suggested that the diagonal tensile stresses would coincide with the direction of the principal tensile stress. Applying the theory to reinforced concrete, Collins and Vecchio assumed that a concrete would not carry any tension stresses, and that the shear would be taken by diagonal compression stress field. The direction of the diagonal compressive stresses would coincide with the direction of the principal compressive stress. With this assumption, Equation (2.20) was derived.

$$\tan^2 \theta = \frac{\varepsilon_x - \varepsilon_d}{\varepsilon_y - \varepsilon_d} \quad (2.20)$$

where:

ε_x = normal strain in the x -direction;

ε_y = normal strain in the y -direction; and

ε_d = principal compressive strain.

Vecchio and Collins built a unique “shear rig” that allowed the testing of concrete panels under the condition of pure shear. From these pure shear tests, the relationship of softened stress-strain curve of concrete in compression was developed (Vecchio and Collins, 1982 and 1986). This relationship is shown in Equations (2.21) and (2.22).

$$f_2 = f_{2,\max} \left[2 \left(\frac{\varepsilon_2}{\varepsilon_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon_c} \right)^2 \right] \quad (2.21)$$

$$f_{2,\max} = \frac{f_c'}{0.8 + 170\varepsilon_1} \leq 1.0 \quad (2.22)$$

where:

ε_c = strain corresponding peak stress in a standard cylinder test;

f_2 = principal compressive stress;

$f_{2,\max}$ = maximum allowable principal compressive stress;

- ε_2 = principal compressive strain;
 ε_1 = principal tensile strain; and
 f'_c = concrete compressive strength recorded in a standard cylinder test.

The compression field theory adopted the equilibrium conditions of the variable-angle truss model, the compatibility conditions, and the appropriate stress-strain relationship for concrete and reinforcement to predict the shear behaviour for reinforced concrete elements at any load level. As the contribution of the concrete tension in a cracked concrete is neglected in the compression field theory, the predictions from this model are always conservative.

2.6.4. Modified compression field theory

The compression field theory considers the principal tensile stress (f_t) to be zero in a cracked concrete. However, in modified compression field theory, the principal tensile stress is assumed to be non-zero and added into the theory (Vecchio and Collins, 1986 and 1988; Collins *et al.*, 1996; and Collins and Mitchell, 1997). From **Figure 2-13**, the equilibrium equations (2.23) to (2.25) for the modified compression field theory could be derived.

Figure 2-13—Equilibrium equations for modified compression field theory.

$$f_z = \rho_z f_{sz} + f_1 \cos^2 \theta - f_2 \sin^2 \theta \quad (2.23)$$

$$f_x = \rho_x f_{sx} + f_1 \sin^2 \theta - f_2 \cos^2 \theta \quad (2.24)$$

$$v = \frac{(f_1 + f_2)}{(\cot \theta + \tan \theta)} \quad (2.25)$$

where:

- f_z = normal stress in the z -direction;
- f_x = normal stress in the x -direction;
- f_2 = principal compressive stress;
- f_1 = principal tensile stress;
- θ = inclination of concrete compressive strut;
- ρ_z = vertical web reinforcement ratio;
- ρ_x = horizontal web reinforcement ratio;
- f_{sz} = average stress of vertical web reinforcement;
- f_{sx} = average stress of horizontal web reinforcement; and
- v = shear stress at a cross section.

The stress-strain relationships for concrete in compression used in the modified compression field theory are the same as that developed in the compression field theory and are given as Equations (2.21) and (2.22).

$$f_2 = f_{2,\max} \left[2 \left(\frac{\varepsilon_2}{\varepsilon_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon_c} \right)^2 \right] \quad (2.21)$$

$$f_{2,\max} = \frac{f'_c}{0.8 + 170\varepsilon_1} \leq 1.0 \quad (2.22)$$

where:

- ε_c = strain corresponding peak stress in a standard cylinder test;
- f_2 = principal compressive stress;
- $f_{2,\max}$ = maximum allowable principal compressive stress;
- ε_2 = principal compressive strain;
- ε_1 = principal tensile strain; and
- f'_c = concrete compressive strength recorded in a standard cylinder test.

The stress-strain relationships for concrete in tension, developed by Vecchio and Collins, are given as Equations (2.26) and (2.27).

$$f_1 = E_c \varepsilon_1 \quad \text{if } \varepsilon_1 \leq \varepsilon_{cr} \quad (2.26)$$

$$f_1 = \frac{0.33\sqrt{f'_c}}{1 + \sqrt{500\varepsilon_1}} \quad \text{if } \varepsilon_1 > \varepsilon_{cr} \quad (2.27)$$

where:

E_c = concrete's modulus of elasticity.

The compatibility equations derived from the Mohr's circle principle of strain are given as Equations (2.28) to (2.30).

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_x + \varepsilon_z \quad (2.28)$$

$$\gamma_{xz} = \frac{2(\varepsilon_x - \varepsilon_2)}{\tan \theta} \quad (2.29)$$

$$\tan^2 \theta = \frac{(\varepsilon_x - \varepsilon_2)}{(\varepsilon_z - \varepsilon_2)} \quad (2.30)$$

where:

ε_y = normal strain in the vertical axis;

ε_x = normal strain in the horizontal axis; and

γ_{xz} = shear strain.

The stress-strain relationships for the reinforcement are given in Equations (2.31) to (2.32).

$$f_{sz} = E_s \varepsilon_z \quad \text{for } 0 < \varepsilon_z < \frac{f_{yz}}{E_s} \quad (2.31a)$$

$$f_{sz} = f_{yz} \quad \text{for } \varepsilon_z > \frac{f_{yz}}{E_s} \quad (2.31b)$$

$$f_{sx} = E_s \varepsilon_x \quad \text{for } 0 < \varepsilon_x < \frac{f_{yx}}{E_s} \quad (2.32a)$$

$$f_x = f_{yx} \quad \text{for } \varepsilon_z < \frac{f_{yx}}{E_s} \quad (2.32b)$$

where:

E_s = modulus of elasticity for steel;

f_{yx} = yield strength of horizontal web reinforcement; and

f_{yz} = yield strength of vertical web reinforcement.

The above stated calculations considered the average stresses and strains in the concrete. However, the modified compression field theory states that the local stresses at the crack locations could differ from the calculated average values. The local stresses at a crack location and the calculated average stresses would govern the failure of the concrete member. Considering that both calculated average stresses and the local stresses at a crack location must be equated, the equilibrium equation in the vertical direction is given in Equation (2.33).

$$A_v f_{sz} \left(\frac{jd}{s \tan \theta} \right) + f_1 \frac{b_w jd}{\sin \theta} \cos \theta = A_v f_{yz} \left(\frac{jd}{s \tan \theta} \right) + v_{ci} b_w jd \quad (2.33)$$

In order to maintain the equilibrium in Equation (2.33), the value of f_1 is limited to Equation (2.34).

$$f_1 = v_{ci} \tan \theta + \left(\frac{A_v}{b_w s} \right) (f_{yz} - f_{sz}) \quad (2.34)$$

The term v_{ci} in Equation (2.34) represents the amount of shear stresses transmitted between the crack interfaces. The expression of v_{ci} , presented as Equation (2.35), was proposed by Vecchio and Collins (Vecchio and Collins, 1986) based on previous works of Walraven (Walraven, 1978 and 1981).

$$v_{ci} = \left(\frac{0.18 \sqrt{f'_c}}{0.3 + \frac{24w}{a+16}} \right) \quad (2.35)$$

The term, w , in Equation (2.35) represents the crack width and could be calculated by Equation (2.36).

$$w = \varepsilon_1 s_{m\theta} \quad (2.36)$$

The term $s_{m\theta}$ in Equation (2.36) refers to the average spacing of the diagonal cracks, and is given as Equation (2.37).

$$s_{m\theta} = \left(\frac{1}{\frac{\sin \theta}{s_{mx}} + \frac{\cos \theta}{s_{mz}}} \right) \quad (2.37)$$

The terms s_{mx} and s_{mz} in Equation (2.37) represent the average crack spacings control by the x -direction reinforcement and the z -direction reinforcement, respectively. The values of these two terms can be estimated using formulas given by Model Code 1990 (1990). For simplification, the vertical distance between the reinforcing bars in the x -direction is taken as s_{mx} , and the horizontal spacing between the vertical bars is taken as s_{mz} (Bentz *et al.* 2006). This method was simplified into shear equations for Canadian Standards Association (CSA) in the 1994 and 2004 editions (Bentz and Collins, 2006).

2.6.5. Rotating angle softened truss model

The softened truss model was proposed by Hsu and his team (Hsu and Mo, 1985; and Belarbi and Hsu, 1991, 1994 and 1995). It includes the softened constitutive laws for the concrete with equilibrium equations, and compatibility equations to obtain the load-deformation curve for a concrete element. The softened constitutive laws for concrete in compression, shown in **Figure 2-14**, are expressed as Equation (2.38).

Figure 2-14—Softened stress-strain curve of concrete in compression.

$$\sigma_d = \zeta f'_c \left[2 \left(\frac{\varepsilon_d}{\zeta \varepsilon_o} \right) - \left(\frac{\varepsilon_d}{\zeta \varepsilon_o} \right)^2 \right] \quad \text{for } (\varepsilon_d / \zeta \varepsilon_o) \leq 1 \quad (2.38a)$$

$$\sigma_d = \zeta f'_c \left[1 - \left(\frac{\frac{\varepsilon_d}{\zeta \varepsilon_o} - 1}{\frac{2}{\zeta} - 1} \right)^2 \right] \quad \text{for } (\varepsilon_d / \zeta \varepsilon_o) > 1 \quad (2.38b)$$

where:

- σ_d = principal stress of concrete in d -axis;
- ζ = softening factor of concrete;
- f'_c = concrete cylinder compressive strength;
- ε_d = principal strain of concrete in d -axis;
- ε_r = principal strain of concrete in r -axis; and
- ε_o = strain of concrete at maximum compressive stress.

The term ζ in Equation (2.38) represents a softening factor, which is a function of the principal tensile strain and the compressive strength. The softening factor is presented in Equation (2.39).

$$\zeta = \left(\frac{5.8}{\sqrt{f'_c}} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \varepsilon_r}} \right) \quad (2.39)$$

The softened constitutive laws for concrete in tension, shown in **Figure 2-14**, are expressed as Equation (2.40). The stress-strain relationship for the concrete in tension before the formation of cracks is assumed to be linear. After cracks have appeared in the concrete,

the tensile stress is reduced. The concrete cracking strain is taken as 0.008% (Hsu and Mo, 2010).

$$\sigma_r = E_c \varepsilon_r \quad \text{for } \varepsilon_r < 0.00008 \quad (2.40a)$$

$$\sigma_r = f_{cr} \left(\frac{0.00008}{\varepsilon_r} \right)^{0.4} \quad \text{for } \varepsilon_r \geq 0.00008 \quad (2.40b)$$

where:

- σ_r = principal stress of concrete in r -axis;
- E_c = modulus of elasticity of concrete; and
- f_{cr} = cracking stress of concrete ($= 0.31 \sqrt{f'_c}$).

The constitutive equations for the steel before and after yielding are presented in Equations (2.41) and (2.42), respectively (Hsu and Mo, 2010).

$$f_s = E_s \varepsilon_s \quad \text{for } \varepsilon_s \leq \varepsilon'_y \quad (2.41a) \text{ or } (2.42a)$$

$$f_s = (0.91 - 2B)f_y + (0.02 + 0.25B)E_s \varepsilon_s \quad \text{for } \varepsilon_s > \varepsilon'_y \quad (2.41b) \text{ or } (2.42b)$$

where:

$$\varepsilon'_y = \frac{f'_y}{E_s} \quad (2.41c) \text{ or } (2.42c)$$

$$f'_y = (0.93 - 2B)f_y \quad (2.41d) \text{ or } (2.42d)$$

$$B = \frac{1}{\rho} \left(\frac{f_{cr}}{f_y} \right)^{1.5} \quad (2.41e) \text{ or } (2.42e)$$

$$f_{cr} = 0.31 \sqrt{f'_c} \text{ and } \rho \geq 0.15\% \quad (2.41f) \text{ or } (2.42f)$$

E_s = modulus of elasticity of steel;

f_s = average stress in steel bars, f_s becomes f_{sx} or f_{sy} when referred to horizontal or vertical web steel respectively;

f_y = yield stress of steel bars, f_y becomes f_{yx} or f_{yy} when referred to horizontal or vertical web steel respectively;

f'_y = average yield stress of steel bars, f'_y becomes f'_{yx} or f'_{yy} when referred to horizontal

or vertical web steel respectively;

ε_s = average steel strain, ε_s becomes ε_x or ε_y when referred to horizontal or vertical web steel respectively; and

ε_y' = average yield strain of steel bars.

The equilibrium conditions for the softened truss model shown in Equations (2.43) to (2.45) represent the equilibrium of vertical and horizontal forces (in a neutral direction) in **Figure 2-15**. For a concrete element under loading, the inclination of the first crack with respect to the longitudinal axis (x -axis in **Figure 2-15**) of the concrete element depends on the direction of the principal tensile stresses distribution before the concrete cracks. The inclination of the subsequent cracks would follow the direction of the principal tensile stresses (represented as the d - r coordinate in **Figure 2-15**). The direction of the principal tensile stresses is in turn dependent on the amount of horizontal and vertical web reinforcement in the concrete element. If both reinforcement amounts are equal, the inclination of the subsequent cracks would be the same as that of the first crack. In most concrete elements, the amounts of horizontal and vertical web reinforcement would not be the same. Therefore, under increasing load, the direction of the tensile stresses would change, and so does the angle between the d -axis and x -axis. This angle would be termed as the rotating angle. As the crack angle coincides with the principal stress direction, the shear stress along the crack is zero.

Figure 2-15—Equilibrium equations for rotating angle softened truss model.

$$\sigma_y = \sigma_d \sin^2 \theta + \sigma_r \cos^2 \theta + \rho_y f_y \quad (2.43)$$

$$\sigma_x = \sigma_d \cos^2 \theta + \sigma_r \sin^2 \theta + \rho_x f_x \quad (2.44)$$

$$\tau_{xy} = (\sigma_r - \sigma_d) \sin \theta \cos \theta \quad (2.45)$$

where:

- θ = angle of inclination of the d -axis with respect to x -axis;
- σ_y = applied normal stress in the y -axis;
- σ_x = applied normal stress in x -axis;
- τ_{xy} = average shear stress in x - y coordinate system;
- ρ_y = vertical web reinforcement ratio;
- ρ_x = horizontal web reinforcement ratio;
- f_y = average stress in the vertical web reinforcement; and
- f_x = average stress in the horizontal web reinforcement.

The compatibility equations for the softened truss model are presented as Equations (2.46) to (2.48).

$$\varepsilon_y = \varepsilon_d \sin^2 \theta + \varepsilon_r \cos^2 \theta \quad (2.46)$$

$$\varepsilon_x = \varepsilon_d \cos^2 \theta + \varepsilon_r \sin^2 \theta \quad (2.47)$$

$$\frac{\gamma_{xy}}{2} = (\varepsilon_r - \varepsilon_d) \sin \theta \cos \theta \quad (2.48)$$

where:

- ε_y = normal strain in the y -axis;
- ε_x = normal strain in the x -axis; and
- γ_{xy} = average shear strain in the x - y coordinate.

The stresses used in the above equations for the model are average stresses. Likewise, the strain values are average strains. There are eleven equations [(2.38) to (2.48)] expressing equilibrium equations, compatibility, and constitutive law relationships in this model. However, there are 14 unknowns in these equations (σ_d , σ_r , τ_{xy} , ε_d , ε_r , γ_{xy} , θ , ε_x , ε_y , σ_x , σ_y , f_x , f_y , and ζ). In such a case, two unknowns must be given, with a third variable being selected in order to solve the remaining 11 unknowns by iterations. Solving all 14 unknowns in the model would provide the load-deformation relationship in a concrete element at any load level. The

rotating angle softened truss model, however, is unable to predict the shear capacity contribution by the concrete in an element. This contribution from the concrete component could only be accounted for in the fixed angle softened truss model.

2.6.6. Fixed angle softened truss model

Different from the rotating angle softening truss model, the fixed angle softened truss model assumes a fixed angle in the model. The initial angle remains the same after the first crack; hence, the concrete shear stress (τ_{12}^c) is not zero (Hsu and Mo, 2010). The stress equilibrium equations are given in Equations (2.43) to (2.45).

Figure 2-16—Equilibrium equations for fixed angle softened truss model.

$$\sigma_y = \sigma_2^c \sin^2 \theta + \sigma_1^c \cos^2 \theta + \rho_y f_y - \tau_{12}^c 2 \sin \theta \cos \theta \quad (2.43)$$

$$\sigma_x = \sigma_2^c \cos^2 \theta + \sigma_1^c \sin^2 \theta + \rho_x f_x + \tau_{12}^c 2 \sin \theta \cos \theta \quad (2.44)$$

$$\tau_{xy} = (\sigma_1^c - \sigma_2^c) \sin \theta \cos \theta + \tau_{12}^c (\cos^2 \theta - \sin^2 \theta) \quad (2.45)$$

where:

- σ_2^c = normal stresses of concrete in 2-direction;
- σ_1^c = normal stresses of concrete in 1-direction;
- τ_{12}^c = shear stresses of concrete in 1-2 coordinate;
- σ_y = applied normal stress in the y-axis;

- σ_x = applied normal stress in x -axis;
 τ_{xy} = average shear stress in x - y coordinate system;
 θ = angle of inclination between the 2-axis and x -axis;
 ρ_y = vertical shear reinforcement ratio;
 ρ_l = horizontal web reinforcement ratio;
 f_y = average stress in the vertical reinforcement; and
 f_x = average stress in the horizontal web reinforcement.

The compatibility equations for the fixed-angle softened truss model are presented in Equations (2.46) to (2.48).

$$\varepsilon_y = \varepsilon_2 \sin^2 \theta + \varepsilon_1 \cos^2 \theta - \gamma_{12} \sin \theta \cos \theta \quad (2.46)$$

$$\varepsilon_x = \varepsilon_2 \cos^2 \theta + \varepsilon_1 \sin^2 \theta + \gamma_{12} \sin \theta \cos \theta \quad (2.47)$$

$$\gamma_{xy} = 2(\varepsilon_1 - \varepsilon_2) \sin \theta \cos \theta + \gamma_{12} (\cos^2 \theta - \sin^2 \theta) \quad (2.48)$$

where:

- ε_2 = normal strain in the 2-axis;
 ε_1 = normal strain in the 1-axis;
 γ_{12} = shear strain in the 1-2 coordinate; and
 ε_y = normal strain in the y -axis;
 ε_x = normal strain in the x -axis; and
 γ_{xy} = average shear strain in the x - y coordinate.

The constitutive laws used in the fixed angle theory are similar to those used in the rotating angle theory. The stresses and strains (in a neutral direction) in the d - r coordinates used in the rotating angle theory are changed into the 1-2 coordinates, as presented in **Figure 2-16**. The expression to calculate the softening factor has an additional term to account for the deviation angle. The deviation angle refers to the difference between the angles of the d - r coordinates (principal concrete stress coordinate) and the angles of the 1-2 coordinates (applied stresses coordinate). The deviation angle considers how much the d - r coordinates have deviated from the 1-2 coordinates. The constitutive law equations for the fixed angle theory are presented in Equations (2.49) to (2.53).

$$\sigma_2^c = \zeta f_c' \left[2 \left(\frac{\varepsilon_2}{\xi \varepsilon_o} \right) - \left(\frac{\varepsilon_2}{\xi \varepsilon_o} \right)^2 \right] \quad \text{for } (\varepsilon_2 / \zeta \varepsilon_o) \leq 1 \quad (2.49a)$$

$$\sigma_2^c = \zeta f_c' \left[1 - \left(\frac{\frac{\varepsilon_2}{\xi \varepsilon_o} - 1}{\frac{2}{\xi} - 1} \right)^2 \right] \quad \text{for } (\varepsilon_2 / \zeta \varepsilon_o) > 1 \quad (2.49b)$$

$$\zeta = \left(\frac{5.8}{\sqrt{f_c'}} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \varepsilon_1}} \right) \left(1 - \frac{|\beta|}{24} \right) \quad (2.50)$$

$$\beta = \frac{1}{2} \tan^{-1} \left[\frac{\gamma_{12}}{(\varepsilon_1 - \varepsilon_2)} \right] \quad (2.51)$$

$$\sigma_1^c = E_c \varepsilon_1 \quad \text{for } \varepsilon_1 < 0.00008 \quad (2.52a)$$

$$\sigma_1^c = f_{cr} \left(\frac{0.00008}{\varepsilon_1} \right)^{0.4} \quad \text{for } \varepsilon_1 \geq 0.00008 \quad (2.52b)$$

$$f_s = E_s \varepsilon_s \quad \text{for } \varepsilon_s \leq \varepsilon_y' \quad (2.53a) \text{ or } (2.54a)$$

$$f_s = (0.91 - 2B) f_y + (0.02 + 0.25B) E_s \varepsilon_s \quad \text{for } \varepsilon_s > \varepsilon_y' \quad (2.53b) \text{ or } (2.54b)$$

where:

$$\varepsilon_y' = \frac{f_y'}{E_s} \quad (2.53c) \text{ or } (2.54c)$$

$$f_y' = (0.93 - 2B) f_y \quad (2.53d) \text{ or } (2.54d)$$

$$B = \frac{1}{\rho} \left(\frac{f_{cr}}{f_y} \right)^{1.5} \quad (2.53e) \text{ or } (2.54e)$$

$$f_{cr} = 0.31 \sqrt{f_c'} \text{ and } \rho \geq 0.15\% \quad (2.53f) \text{ or } (2.54f)$$

ζ = softening factor of concrete;

f_c' = concrete cylinder compressive strength;

ε_o = strain of concrete at maximum compressive stress;

E_c = modulus of elasticity of concrete;

f_{cr} = cracking stress of concrete ($= 0.31 \sqrt{f_c'}$);

E_s = modulus of elasticity of steel;

f_s = average stress in steel bars, f_s becomes f_{sx} or f_{sy} when referred to horizontal or vertical web steel respectively;

- f_y = yield stress of steel bars, f_y becomes f_{yx} or f_{yy} when referred to horizontal or vertical web steel respectively;
- f_y' = average yield stress of steel bars, f_y' becomes f_{yx}' or f_{yy}' when referred to horizontal or vertical web steel respectively;
- ε_s = average steel strain, ε_s becomes ε_x or ε_y when referred to horizontal or vertical web steel respectively; and
- ε_y' = average yield strain of steel bars.

One new unknown is introduced in the fixed angle theory, τ_{12}^c , which represents the shear stress of the concrete. The shear stress of the concrete can be calculated using Equation (2.55).

$$\tau_{12}^c = \frac{\sigma_1^c - \sigma_2^c}{2(\varepsilon_1 - \varepsilon_2)} \gamma_{12} \quad (2.55)$$

The shear resistance of a concrete element, derived by Pang and Hsu (Pang and Hsu, 1995 and 1996) from the three equilibrium equations, is given in Equation (2.56).

$$\tau_{xy} = \frac{(\tau_{12}^c)^2 + (\sigma_1^c)^2 + (\rho_x f_{yx}' + \rho_y f_{yy}') \sigma_1^c}{2\sqrt{\rho_x f_{yx}' \rho_y f_{yy}'}} + \sqrt{\rho_x f_{yx}' \rho_y f_{yy}'} \quad (2.56)$$

The shear resistance is expressed as the sum of the concrete contribution in the first term and the contribution of the steel in the second term. The concrete contribution from this model is contributed by the shear stress present in the crack concrete (τ_{12}^c). The introduction of this component is the main difference between the rotating angle theory and the fixed angle theory. However, if a concrete element does not consist of any web reinforcement, Equation (2.56) cannot be solved and the concrete contribution for the concrete element cannot be predicted.

The iterative procedure of solving all the parameters in the fixed angle theory is similar to that of the rotating angle theory. The values for two unknowns have to be given. Typically, a value for ε_2 is then selected, and the values for ε_1 and γ_{12} are assumed to solve for all the other

parametric values in relation to the selected ε_2 value. The shear stress-strain curve could then be completed by selecting different values of ε_2 (Hsu and Mo, 2010).

2.6.7. Strut-and-tie models

Strut-and-tie models are typically applied to deep beams (shear-span-to-effective-depth ratio ≤ 2.5) or corbels where the stresses in the region between the loading point and the support are disturbed. In these disturbed regions, compatibility conditions are difficult to be applied and be complied with. In using strut-and-tie models, the forces in the disturbed region are determined by the external forces and satisfying equilibrium equations. The strain compatibility conditions are not accounted for in this model. The model represents the complex structural behaviour in a structure with simplified truss model. An illustration of a strut-and-tie model by ACI 318-14 (2014) is presented in **Figure 2-17**. The concrete component carrying compression forces only is termed as the strut. The tension forces are carried by the reinforcing steel bars, termed as the tie in **Figure 2-17**. Struts and ties are connected at the nodal zones of the model.

Figure 2-17—Description of a strut-and-tie model (ACI 318-14, 2014).

A diagonal strut is typically connected directly between the loading point to the support in a deep beam loaded with a single concentrated force, by a diagonal shear crack in the beam. In slender beams, the diagonal shear crack does not connect directly between the support and loading point because the distance between these two points is too far for a direct strut to form, and the inclination of the strut would have been too shallow. Hence, strut-and-tie models are generally considered unsuitable for slender beams. The failure criteria of a strut-and-tie model

is controlled by either the yielding of reinforcing bars, crushing of the strut, or crushing of the nodal zone. The amount of force carried in each element (strut, tie, and node) is determined by equilibrium equations, and is compared against the strength capacity of each element. The forces meeting at a node (between the ties and struts) must satisfy equilibrium conditions.

The strength of a strut is governed by the compressive strength of the concrete, and the condition of transverse tension acting across the strut. The strength of a tie is governed by the yield strength of the reinforcing bars. The strength of a nodal zone is controlled by the type of the node, which is in turn dependent on the number of struts and ties connecting at the node. A node connected by three or more struts is termed as a CCC node, which is more commonly found at the loading point of a structural member. A node interconnecting two or more struts and a tie is known as CCT node. A CTT node is a node connecting one strut and two or more ties, whereas a TTT node connects ties only at the intersection. Different strength coefficients are used to represent the type of node and to calculate the strength of the node.

In research programmes that sought to develop the strut-and-tie model, different methods to determine the effective stress limit of nodal zone were proposed among others by Schlaich *et al.*, 1987; Bergmeister *et al.*, 1991; and MacGregor, 1997. Proposed compressive stress limit for concrete struts were suggested by Schlaich *et al.*, 1987; Alshegeir and Ramirez, 1990; MacGregor, 1997; Foster and Malik, 2002; and Zhang and Tan, 2007 among others. The proposed values for compressive stress limit for concrete struts could vary depending on the stress conditions, concrete confinement by reinforcement, and the cracking around the strut (ACI-ASCE 445, 1999).

Apart from the above proposals, Tan and Cheng proposed an efficiency factor that considers the influence of size effect on the strut strength (Tan and Cheng, 2006). The proposed efficiency factor is a function of two terms, of which one considers strut geometry and the other considers different boundary conditions. Al-Nahlawi and Wight (1992) proposed a strut-and-tie model for beams with no shear reinforcement that considered a tension tie carrying the tension forces. The tension tie fails when the tensile strength of the concrete equals the tensile stress acting on the concrete. The use of strut-and-tie model for design of deep beams and corbels has generally been accepted. It was included in the ACI 318 code provision as part of Appendix A initially, and was re-introduced as Chapter 23 in the 2014 edition. In Eurocode 2,

the strut-and-tie model is included as part of Chapter 6 under the section “Design with Strut and Tie Models”.

2.6.8. Different perspectives for shear models

Apart from the traditional models, there are many other proposed models based on different assumptions or empirical equations to predict the shear strength of a reinforced concrete beam. An example, among others, include a semi-empirical equation for shear strength predictions proposed by Bažant and Yu (2005a, 2005b) after reviewing previous experimental work from various research. This equation consists of a size effect term based on previous works and was derived from fracture mechanics. Li and Tran (2012) proposed a semi-analytical model based on the variable angle truss model. The inclination of the diagonal strut is obtained theoretically for the model, and is used to calculate the shear strength of a reinforced concrete beam. Russo *et al.* (2004 and 2006) proposed a relationship between the internal lever arm and the distance from the support with a second order differential equation. The relationship was developed via mechanical analysis into a shear strength expression, as a function of influential parameters, such as longitudinal reinforcement ratio, yield strength of steel, shear-span-to-effective-depth (a/d) ratio, compressive strength, and size effect. The expression was calibrated with published experimental results to obtain a final expression that can predict the experimental shear strengths with the lowest coefficient of variation.

Zararis (2003) commented that the ability of the compression chord to carry shear is the most influential parameter to the shear strength of a reinforced concrete beam with no shear reinforcement. The theory of splitting tensile strength was applied on the uncracked compression zone. A method relating the shear strength of reinforced concrete beam to a ratio between depth of neutral axis and the effective depth, and the splitting tensile strength of the concrete was developed (Zararis and Papadakis, 2001; and Zararis, 2003). A size effect term was also proposed in the method as a function of effective depth and a/d ratio.

Tureyen and Frosch (2002) developed an empirical equation relating the flexural-compression zone to the shear strength of a reinforced concrete beam with a/d ratio greater than 2.7. The shear strength increased as the depth of the neutral axis increased. The form of expression for the proposed equation is similar to that of the simplified shear equation in ACI 318-14 (2014). Both equations do not consider size effect. Another proposed method by Kuo

et al. (2014) considers the flexural-compression zone (proposed by Tureyen and Frosch (2002)) and the arch action to be influential to shear strength. The concept of arch action, developed based on previous experimental work on prestressed concrete beams, is a function of the a/d ratio to the power of -0.7 (Laskar *et al.*, 2010; and Hsu *et al.*, 2010).

A team led by Muttoni proposed a shear model based on the critical shear crack theory (Muttoni and Fernández Ruiz, 2008; Vaz Rodrigues *et al.*, 2010; and Fernández Ruiz *et al.*, 2015). In the model, the shear strength is taken as a function of the critical shear crack width and the maximum aggregate size. The formation of the critical shear crack was assumed in the model to lead to a new arching action mechanism to carry the shear. The arching action could develop in two ways: 1) the formation of an elbow-shaped strut to avoid the formed cracks; or 2) a direct strut is formed by the aggregate interlock between shear cracks. This method was simplified and is adopted as the shear equation in the Swiss code (Swiss Code, 2003). Apart from those discussed, several others research programmes such as Ramirez and Breen (1991); Kim and Park (1996); Park *et al.* (2006); Lee and Kim (2008); and Sneed and Ramirez (2008 and 2010) among others had looked into various other methods to analyse shear behaviour and developed models to predict accurate shear capacity of the beam.

2.7. Summary of Chapter 2

This chapter presented an introduction on high strength concrete and its influence on the shear strength of a reinforced concrete beam. Several past experimental programmes that investigated the influence of size effect on shear strength are presented. The evidence of size effect reducing the shear strength as the effective depth of a beam increases is shown clearly in the discussed experimental programmes. However, there is an obvious lack of high strength concrete specimens with large effective depths to confirm the influence of size effect on high strength concrete beams. Several proposed methods and different concepts to calculate the shear strength were discussed in this chapter. The equations in ACI 318-14 (2014) and Eurocode 2 (2004) were presented.

The experimental programme discussed in Chapter 3 would consider the lack of large sized high strength concrete beams in the published literature. The proposed methods in Chapters 6 and 7 would consider the influences of high strength concrete and effective depth discussed in this chapter as well as other influential parameters to shear strength of reinforced

concrete beam. The experimental programme and proposed methods aim to contribute to the existing database of large-sized high strength concrete beams and propose simple and accurate methods predicting shear strength.

Chapter 3

EXPERIMENTAL PROGRAMME

3.1. Significance of Experiments

Many experiments had been conducted to investigate the influence of effective depth on shear strength of normal strength concrete slender beams by using the beam depth as the only variable while keeping other properties constant, as discussed in Chapter 2. The results from these experiments show that shear strength of normal strength concrete beams reduce as effective depth increases. The influence of size effect has been widely recognised and is considered in different codes and guidelines by introducing a shear strength reduction factor (Model Code, 1990; Japanese Code, 1991; BS8110, 1997; Eurocode 2, 2004; and Model Code, 2010). Compared to normal strength concrete, there have been few experimental programmes that studied high strength concrete beams with varying beam depths.

This experimental programme included eleven (11) slender high strength concrete beams. The depths of the beams were varied from 450 mm to 1800 mm. Two maximum aggregate sizes of 20 mm and 10 mm were used, and the shear reinforcement ratio, $\rho_{vv} = A_{sv}/(b_w s)$, was designed to be 0%, 0.13%, 0.20%, and 0.30%, respectively. Other influential variables such as the longitudinal reinforcement ratio, concrete compressive strength, and shear-span-to-effective-depth ratio were kept constant for the 11 beams to avoid influences from these variables. The objectives of this experimental programme include: (1) to investigate the influence of effective depth on shear strength of large-sized high strength concrete beams; (2) to investigate the effect of maximum aggregate size on shear strength of high strength concrete beams; (3) to study the influence of shear reinforcement ratio on size effect in high strength concrete beams; and (4) to figure out the minimum requirement of shear reinforcement ratio to eliminate size effect on shear strength of high strength concrete beam, provided that shear reinforcement is able to eliminate size effect.

3.2. Details of Test Specimens

The eleven beam specimens tested in this experiment were categorised in four main series according to their overall depths (h), namely the 450-series; 900-series; 1350-series; and 1800-series. The main properties of the beams are listed in **Table 3-1**.

In general, two types of specimens were tested in this study: (1) Type 1 specimens, which had no shear reinforcement in one of the shear spans, and was provided with a minimum amount of shear reinforcement required by ACI 318-14 in the other shear span ($\rho_{vv,min} = 0.062(f'_c)^{0.5}/f_{yv} = 0.13\%$); and (2) Type 2 specimens, which had shear reinforcement of 0.20% in one shear span and 0.30% in the other. With this arrangement of one shear span having less amount of shear reinforcement than the other shear span, the weaker shear span will fail first. The failed weaker shear span will then be strengthened, so that the test can be restarted to cause failure of the other shear span. This allowed two sets of data to be collected from each beam specimen. Details of the reinforcements for each specimen in different series are presented in **Table 3-2**.

Three beam specimens were tested in each of the first three series (the 450, 900 and 1350-series). The three specimens in each series included two Type 1 specimens and one Type 2 specimen. These nine specimens had a rectangular cross-section with a beam width of 265 mm. For the two Type 1 specimens in each series, the maximum size of the aggregate used in the concrete was different. Maximum aggregate sizes of 10 and 20 mm were used to investigate the effect of maximum aggregate size. Details of the rectangular cross-sections are presented in **Figure 3-1**.

Table 3-1—Properties of beam specimens.

Series	Beam ID	Shear Span ID	b , (mm)	b_w , (mm)	d , (mm)	L , (mm)	l' , (mm)	a , (mm)	a/d	f'_c , (MPa)	ρ_{vv} (%)	ρ_{vh} (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
450-series	450-series (Type 1-20)	1V0-450-20	265	265	403	4000	600	1200	3.0	102	-	-
		1V1-450-20	265	265	403	4000	600	1200	3.0	102	0.13	-
	450-series (Type 1-10)	1V0-450-10	265	265	403	4000	600	1200	3.0	108	-	-
		1V1-450-10	265	265	403	4000	600	1200	3.0	108	0.13	-
	450-series (Type 2-20)	2V2-450-20	265	265	403	4000	600	1200	3.0	102	0.20	-
		2V3-450-20	265	265	403	4000	600	1200	3.0	102	0.30	-
900-series	900-series (Type 1-20)	1V0-900-20	265	265	808	7000	1200	2400	3.0	103	-	-
		1V1-900-20	265	265	808	7000	1200	2400	3.0	103	0.13	-
	900-series (Type 1-10)	1V0-900-10	265	265	808	7000	1200	2400	3.0	108	-	-
		1V1-900-10	265	265	808	7000	1200	2400	3.0	108	0.13	-
	900-series (Type 2-20)	2V2-900-20	265	265	808	7000	1200	2400	3.0	103	0.20	-
		2V3-900-20	265	265	808	7000	1200	2400	3.0	103	0.30	-
1350-series	1350-series (Type 1-20)	1V0-1350-20	265	265	1232	10300	1800	3650	3.0	98	-	0.10
		1V1-1350-20	265	265	1232	10300	1800	3650	3.0	98	0.13	0.10
	1350-series (Type 1-10)	1V0-1350-10	265	265	1232	10300	1800	3650	3.0	108	-	0.10
		1V1-1350-10	265	265	1232	10300	1800	3650	3.0	108	0.13	0.10
	1350-series (Type 2-20)	2V2-1350-20	265	265	1232	10300	1800	3650	3.0	97	0.20	0.10
		2V3-1350-20	265	265	1232	10300	1800	3650	3.0	97	0.30	0.10
1800-series	1800-series (Type 1-20)	1V0-1800-20	480	200	1670	13400	2400	4900	3.0	107	-	-
		1V1-1800-20	480	200	1670	13400	2400	4900	3.0	107	0.13	-
	1800-series (Type 2-20)	2V2-1800-20	480	200	1670	13400	2400	4900	3.0	107	0.20	-
		2V3-1800-20	480	200	1670	13400	2400	4900	3.0	107	0.30	-

Note:

(b_w = width of web, b = width of base, d = effective depth, L = overall beam length, l' = distance between loading points, a = shear span, a/d = shear-span-to-effective-depth ratio, f'_c = specified (cylinder) compressive strength of concrete, $\rho_{vv} = A_v/(b_w s)$, $\rho_{vh} = A_{vh}/(b_w d)$).

Table 3-2—Reinforcement details of beam specimens.

Beam series	Longitudinal reinforcement	Shear reinforcement	Skin reinforcement
450-series	2T32 + T25	- (0%) T6@170 (0.13%) T6@110 (0.20%) T6@75 (0.30%)	-
900-series	4T32 + 2T25	- (0%) T10@480 (0.13%) T10@300 (0.20%) T10@210 (0.30%)	-
1350-series	6T32 + 3T25	- (0%) T10@480 (0.13%) T10@300 (0.20%) T10@210 (0.30%)	2T10 @ 250
1800-series	6T32 + 6T20	- (0%) T10@600 (0.13%) T10@390 (0.20%) T10@260 (0.30%)	-

Note:

2T32 means 2 deformed bars (T) of 32 mm diameter.

2T10@250 indicates 2 deformed bars (T) of 10 mm diameter at a spacing of 250 mm.

The 1800-series consisted of two beam specimens, only the Type 1 and Type 2 specimens with maximum aggregate size of 20 mm were tested. The front view and the sectional view at various locations of the specimens in 1800-series are shown in **Figure 3-2**. The web thickness (200 mm) was designed thinner than those of other specimens (265 mm) to accommodate the capacity of the testing facilities such as hydraulic jacks and testing frames in the laboratory. The bases (bottom section) of the two specimens in the 1800-series were enlarged to 480 mm, forming a bulb shape (**Figure 3-2 (b)**), to allow enough space for the same longitudinal reinforcement ratio as the other specimens. The width of the sections near the loading points were also enlarged to 480 mm in these two beams to prevent local crushing of the nodal zones.

Note: 1V0-450-20 indicates: 1 = Type 1 specimen; V0 = no shear reinforcement; 450 = overall height in mm; and 20 = maximum aggregate size in mm.

Figure 3-1—Cross-sections of rectangular beam specimens in the 450-series, 900-series, and 1350-series.

Figure 3-2—(a) Front view of the specimens in 1800-series (b) Sectional view of specimens in 1800-series.

Shear span (a) is defined as the distance between the centre of the bearing support plate to the centre of the loading plate. A shear-span-to-effective-depth ratio (a/d) of 3 was chosen for all the beams, so that the beam specimens would not display deep beam behaviour. A comparison of past experimental results showed that beams with a/d ratio less than 2.7 can exhibit deep beam behaviour; and this will be discussed further in Chapter 6. The a/d ratio was kept constant at 3 for all the beams so that the influence of a/d ratio to shear strength was normalised in the investigations. The longitudinal reinforcement ratio was kept constant at 2% in the shear spans, and 3% between loading points in all beams to ensure shear failure mode.

The length required for the longitudinal tension and compression reinforcement for the specimens in the 1800-series was longer than the maximum length of commercial steel reinforcing bars (12000 mm). Therefore, bar lapping was needed for both the longitudinal tension and compression reinforcement in the beams. The arrangement of bar lapping is shown in **Figure 3-3**. All bars are lapped in the middle section of the specimen between the two loading points, to avoid any additional influences in the shear spans of specimens.

Each shear span was notated individually, representing the main variables of the shear span. For instance, the notation 2V2-450-20 represents one shear span of a Type 2 specimen in the 450-series cast with 20 mm aggregates, while 2V3-450-20 stands for the other shear span of the same beam. The only difference between 2V2-450-20 and 2V3-450-20 was the shear reinforcement ratio of 0.20% and 0.30%, respectively, in the two shear spans. Specifically, ‘V0’ and ‘V1’ refer to shear span with no shear reinforcement and with shear reinforcement ratio of 0.13%, respectively.

Figure 3-3—Detailed arrangements of bar lapping for specimens in 1800-series. (From top to bottom: first layer of longitudinal tension reinforcement nearest to the base of the beam; second layer of longitudinal tension reinforcement from the base of the beam; third layer of longitudinal tension reinforcement from the base of the beam; and longitudinal compression reinforcement at the top of the beam.)

3.3. Materials Properties

3.3.1. Concrete properties

The target cylinder compressive strength of concrete was 100 MPa at 28 days after the day of casting. Ground Granulated Blast-furnace Slag (GGBS), silica fume, and high range water reducing superplasticizer admixture were added to the concrete mix to achieve the target compressive strength and workability. The added supplementary cementitious materials (GGBS and silica fume) are green materials which helped to improve the durability properties of the hardened concrete. Previous research work was conducted to determine the mix design proportions required to achieve the various targets. More details and results of the concrete mix were reported in other publications. (Teng *et al.*, 2013; Divsholi *et al.* 2014; and Lim *et al.*, 2016). The slump flow of concrete mix was targeted to be around 650 to 700 mm to achieve a proper flowability for casting. The mix design proportion of the concrete is presented in **Table 3-3**.

Table 3-3—Mix design proportion for 1m³ of concrete.

Water-to-binder ratio (w/b)	0.25
Cement (kg)	325
Silica fume (kg)	65
Normal GGBS (kg)	260
Coarse aggregate (kg)	762
Fine aggregate (kg)	762

The eleven specimens were cast in six batches using high strength concrete supplied by a local ready-mixed concrete company. The mix design proportion was provided to the company prior to batching. General materials such as cement, coarse and fine aggregates, were provided by the ready-mixed concrete supplier. Silica fume, GGBS, and superplasticizer were collected from individual suppliers, and sent to the ready-mixed plant before the day of batching. Each beam was compacted continuously with poker vibrators during casting to ensure good compaction of concrete around steel reinforcement. The specimens were covered with moist haversack to reduce loss of moisture from the specimens to the surrounding environment. Specimens were sprinkled with water at a certain time interval to avoid excessive shrinkage of specimens. Formwork was removed after one week, and the specimens were left to cure for at least 28 days before testing.

Small concrete specimens, such as cubic (100x100 mm) and cylindrical (150 mm diameter x 300 mm height) specimens, were cast to determine the compressive strength and the modulus of elasticity of the concrete. These concrete specimens were cured under the same conditions as the beams. The compressive strength and modulus of elasticity tests were conducted according to BS EN 12390-3 (2009), and BS 1881-121 (1983) respectively. The average results for these companion specimens are presented in **Table 3-4**.

Table 3-4—Concrete compressive strength and modulus of elasticity.

Shear span ID	Compressive strength, MPa	Modulus of elasticity, GPa	Shear span ID	Compressive strength, MPa	Modulus of elasticity, GPa
1V0-450-20	102	38	2V3-900-20	103	38
1V1-450-20			1V0-1350-20	98	37
1V0-450-10	108	37	1V1-1350-20		
1V1-450-10			1V0-1350-10	108	35
2V2-450-20	102	38	1V1-1350-10		
2V3-450-20			2V2-1350-20	97	34
1V0-900-20	103	38	2V3-1350-20		
1V1-900-20			1V0-1800-20	107	37
1V0-900-10	108	40	1V1-1800-20		
1V1-900-10			2V2-1800-20	107	39
2V2-900-20	103	38	2V3-1800-20		

3.3.2. Reinforcing steel properties

Steel bars of 20 mm (T20), 25 mm (T25), and 32 mm (T32) were used in different combinations as longitudinal tension reinforcement. Furthermore, 6 mm steel bars (T6) were used as shear reinforcement for specimens in the 450-series, and bars of 10 mm (T10) were used in other specimens. Tensile tests for steel bars were carried out in the laboratory using a universal testing machine following the ASTM A184 standard (2011). The average values of yield strength and tensile strength for different bar sizes are presented in **Table 3-5**.

Table 3-5—Properties of steel reinforcing bars.

Bar ID	Steel type	Yield strength (MPa)	Yield strain (%)	Tensile strength (MPa)	Modulus of elasticity (GPa)
T6	Deformed	614	0.34	643	184
T10		596	0.34	676	178
T13		561	0.30	658	185
T20		568	0.34	657	164
T25		543	0.33	641	164
T32		545	0.34	643	178

3.4. Instrumentations

Instrumentation was planned to record vertical deflections of all beams, and strains in shear and longitudinal tension reinforcing bars. Vertical deflections of the beam specimens were recorded using Linear Voltage Differential Transducers (LVDTs). Seven LVDTs were placed under each beam along the longitudinal centre line of the beam. Two were placed on each side of each support, one at the quarter-span and another at the mid-span of the beam. Typical locations of LVDTs are shown in **Figure 3-4**.

Figure 3-4—Typical locations for LVDTs.

The effective mid-span and quarter-span deflections of the beam are obtained by subtracting the average deflections of the four LVDTs near the supports from the measured

deflections at the mid-span and quarter-span. Four LVDTs were placed at different depths of the specimens for the 1800-series to measure the out-of-plane deflections of the specimens. These LVDTs were used to ensure the out-of-plane deflections were not too significant during the test. **Figure 3-5** shows examples of LVDTs' placement to measure the vertical deflection and out-of-plane displacement.

Figure 3-5—Example of LVDT placement for (left) Measurement of vertical deflections; and (right) Measurement of out-of-plane displacement in Specimen 1V0-1800-20.

Strains in reinforcement were measured using steel strain gauges attached on the reinforcing bars. Strain gauges were attached to the centre bar at each layer of the longitudinal reinforcement at the mid-span, quarter-span, and also at 150 mm away from the centre of the supports. For shear reinforcement, strain gauges were attached at mid-height of the bars. Additional strain gauges were attached at different positions along the shear reinforcing bars in the specimens of the 1800-series to obtain strain readings on the reinforcement closest to the location of the diagonal crack. The highest values recorded by different strain gauges on each reinforcing bar in the specimen were taken for analysis. The locations of strain gauges on shear reinforcement are presented in **Figures 3-6 to 3-9**.

A camera was set on a time-lapsed mode to automatically capture photos at intervals for beam tests in the 1350-series and 1800-series. This allowed real-time recording of crack propagation in these specimens. The time-lapsed photographs of crack propagations in these beam specimens are provided in Appendix A.

Figure 3-6—Strain gauge locations in specimens from the 450-series.

Figure 3-7—Strain gauge locations in specimens from the 900-series.

Figure 3-8—Strain gauge locations in specimens from the 1350-series.

Figure 3-9—Strain gauge locations in specimens from the 1800-series.

3.5. Experimental Setup and Loading Procedure

The typical experimental set-ups are shown in **Figures 3-10** and **3-11**. The set-up in **Figure 3-10** was used for specimens in the 450-series with lower failure load. One single 1000 kN-capacity actuator was used over a steel spreader beam, distributing the one-point load from the actuator into two point loads acting on the beam. Rocker bearings were placed under the two ends of the spreader beam. Under each rocker bearing, a bearing plate of 265 mm wide by 300 mm long by 50 mm high was placed in contact with the top of the beam. The beams were supported at the bottom.

Figure 3-10—(left) Drawing of test setup for specimens in 450-series using actuator; (right) Photograph of a specimen in the test setup.

An actuator was used for the specimens in the 450-series because the predicted failure loads were lower, and it provided better control of load increments than hydraulic jacks. Two individual hydraulic jacks were used to create the two-point loading conditions for other specimens. Hydraulic jacks were used for these specimens because the specimens were too large to be placed in the testing frame with the actuator. **Figure 3-11** shows the test setup of the remaining specimens. Two 100-ton load cells were used for the specimens in the 900-series, and 200-ton load cells were used for the specimens in the 1350-series and 1800-series. Other than the applied loading, the set-up configurations for all the specimens were the same.

All specimens were white-washed before testing to allow easy observation of cracks. All the readings from LVDTs, strain gauges, and load cells were initialised before the test, to eliminate the effects from dead load of the beams. Each beam was loaded with a load increment of 20 kN until failure. Therefore, the increment was 10 kN in each load cell. The intention of the experimental programme was to obtain the maximum shear strength of each beam specimen. It was not planned to collect data of the post-peak behaviour of the beam because shear failure is sudden and the shear capacity of the beam will drop immediately after the applied load reaches the maximum shear capacity of the beam. Therefore, a load increment technique was adopted during the test. The beams were checked after every 20kN increment for formation or propagation of cracks. During the tests, cracks were traced, and marked next to the new crack lines indicating the respective load stages. In addition, a crack width detector with an accuracy of 0.02 mm was used to measure crack width.

Figure 3-11—(a) Drawing of test setup for specimens in 900-series, 1350-series and 1800-series using hydraulic jacks; (b) Photograph of a specimen from 1350-series in the test setup; and (c) Photograph of a specimen from 1800-series in the test setup.

3.6. Strengthening of Failed Shear Span

When failure took place on the weaker side of each beam (the shear span with lower shear reinforcement ratio), the applied loads were removed. The failed shear span was strengthened using a steel clamping unit as shown in **Figure 3-12**. The steel clamping unit consisted of two steel plates which were placed on the top and bottom faces of the beam and constrained by four steel rods and nuts. The details of steel plates used for the clamping unit are shown in **Figure 3-13**. After the strengthening unit was installed, the test was restarted under the same loading condition until the other shear span failed.

Figure 3-12—(left) Drawing of test setup for specimens in 900, 1350 and 1800 series using hydraulic jacks with the strengthening set; (right) Photograph of a specimen from 1350-series strengthened in the weaker shear span.

Figure 3-13—Details of steel plate used to strengthen failed shear span.

3.7. Safety Measure of the Test Setup

Some of the beam specimens tested in this study were large, such as specimens in the 900-series, 1350-series, and 1800-series. Necessary precautions were taken to ensure safe and accident-free experiments. Thus, four steel A-frames were constructed and installed on both front and rear faces of each beam. These A-frames were secured to the strong floor during the tests for the specimens in the 900-series, 1350-series and 1800-series. These frames were designed to resist the accidental impact from a beam if it fell out of plane during the test. In addition, chain slings and polyester slings were wrapped around the beam specimen and connected to a ceiling crane. The slings were wrapped loosely so as not to interfere with the test, and would only be activated in the event of out-of-plane falling of the beam.

3.8. Summary of Chapter 3

This chapter described the test programme of eleven beam specimens made of high strength concrete. The main variables among the specimens were: (1) beam depth (ranging from 450 to 1800 mm); (2) the maximum aggregate size used in concrete (10 and 20 mm) and; (3) shear reinforcement ratio in the shear span (0%, 0.13%, 0.20%, and 0.30%). The objectives of this experimental programme include: (1) an investigation of the influence of effective depth on shear strength of large high strength concrete beams; (2) an investigation on the effect of maximum aggregate size on the shear strength of large high strength concrete beams; (3) an investigation of shear reinforcement ratio on size effect of high strength concrete beams; and (4) a study to seek the minimum requirement of shear reinforcement ratio to eliminate size effect. The experimental results from this study will be combined with existing data from literature to investigate the above stated objectives.

Chapter 4

RESULTS AND DISCUSSIONS FOR BEAMS WITH NO SHEAR REINFORCEMENT

4.1. Introduction

This chapter presents the experimental results of reinforced concrete shear spans with no shear reinforcement. There were two shear spans (specimens) in a beam, each containing different amounts of shear reinforcement. For example, 1V0-450-10 and 1V1-450-10 formed one beam specimen. The results of Specimen 1V0-450-10 with no shear reinforcement is discussed in this chapter, whereas the result of Specimen 1V1-450-10 with 0.13% shear reinforcement will be discussed in Chapter 5. There were seven (7) specimens with no shear reinforcement, of which the depths varied from 450 to 1800 mm, and concrete compressive strength between 95 to 110 MPa. The main properties of these seven specimens have been presented in **Table 3-1**. The material properties of the reinforcing steel and concrete used for the specimens are listed in **Tables 3-2** and **3-4** respectively. The results show that the effective depth of the specimen influenced the shear strength of the specimen. The maximum aggregate size used in the high strength concrete also influenced the shear strength of the specimen. However, this observation regarding the influence of aggregate size differs from the observations made from other researchers using normal strength concrete.

4.2. Experimental Results

4.2.1. General Behaviour

All the specimens had failed in brittle shear failure accompanied by loud and sudden sounds. The crack surfaces observed on all the specimens were relatively smooth, as shown in **Figure 4-1**. A smooth crack surface is typical for high strength concrete beams, unlike that in normal strength concrete where the crack surface is rough (Elzanaty *et al.*, 1986; Kong *et al.*, 1994; Khuntia and Stojadinovic, 2001; and Janaka Perera and Mutsuyoshi, 2013). In high strength concrete, the strength of the hardened cement is stronger than the aggregate. The crack is thus forced to cut through the aggregate instead of developing around the aggregate; hence a smooth crack surface is created.

Figure 4-1—Observed smooth crack surface in beam specimens.

4.2.2. Crack Patterns

The crack patterns of the specimens after failure are shown in **Figure 4-2**. The first crack to form on the beam during the test was a flexural crack near the mid-span of the beam. The load stage which the first flexural crack was observed in each specimen is listed in **Table 4-1**.

Table 4-1—Measured ultimate loads.

Specimen ID	Measured loads, kN			
	Flexural cracking load	Diagonal cracking load	Total load	V_{test}
(1)	(2)	(3)	(4)	(5)
1V0-450-20	95	220	268	142
1V0-450-10	90	150	320	168
1V0-900-20	110	280	340	194
1V0-900-10	80	280	520	284
1V0-1350-20	110	560	666	384
1V0-1350-10	200	420	900	501
1V0-1800-20	240	540	920	545

Note:

-Failure mode is diagonal tension for all specimens.

- V_{test} is the shear force (or half of the total load) and is inclusive of self-weight of the beam.

The first flexural crack was observed at around the same load stages for all specimens with the same depth, except for Specimen 1V0-1350-10. The first flexural crack was observed considerably later than Specimen 1V0-1350-20, and with a wider crack width. It was likely that the author missed the first flexural crack. More flexural cracks around the mid-span were observed as the load stage increased. At about 30-50% of the failure load, flexural cracks were

observed in both shear spans of the beam. One of the flexural cracks on the shear span would propagate further upwards and develop into a diagonal shear crack. The diagonal crack was taken to have formed when the crack changed its inclination with respect to the longitudinal axis of the beam and propagated towards the loading point. The point where the crack changed its inclination was about two-third depth of the beam (measured from the soffit).

All the specimens failed suddenly after one diagonal shear crack appeared. As one end of the diagonal shear crack propagated towards the loading point, horizontal splitting cracks were observed at the level of the longitudinal reinforcement. These cracks appeared near the diagonal cracks and propagated along the longitudinal reinforcement towards the support. More horizontal cracks were also observed in the specimens with greater beam depth. When the beam failed, the diagonal shear crack did not connect the loading point and the support like how it usually does in a deep beam. Most of the diagonal shear cracks in the large specimens tested in this study were initiated at a distance approximately d away from the support and were connected to the support by horizontal cracks along the longitudinal reinforcement.

The load stages at which the crack was observed to have changed its inclination in each specimen are listed in **Table 4-1**. The recorded load stages for those specimens with aggregate size of 20 mm in the 450, 900 and 1350 series were at about 80 percent of its failure load. In Specimen 1V0-1800-20, the change in the crack inclination was observed at about 60 percent of the failure load. The wider base of Specimen 1V0-1800-20 seemed to increase the shear strength of the specimen. For the specimens with maximum aggregate size of 10 mm, the change in crack inclination was observed at about 50 percent of the specimens' failure load.

(a)

(b)

(c)

(d)

Figure 4-2—Crack patterns at failure for Specimens: (a) 1V0-450-20; (b) 1V0-450-10; (c) 1V0-900-20; (d) 1V0-900-10; (e) 1V0-1350-20; (f) 1V0-1350-10; and (g) 1V0-1800-20 (Not to scale).

The inclination of the diagonal shear crack on all the specimens with no shear reinforcement are shown in **Figure 4-3**. The inclination of the crack was very subjective to one's perceptions on which part of the crack to measure. In this study, the inclination of the crack was measured between the point where the diagonal crack meets the compression bars at the top of the beam and the location where the horizontal cracks formed at the level of the longitudinal reinforcement. From **Figure 4-3**, except for Specimen 1V0-1350-20, it seems that the inclination of the diagonal crack reduced as the depth of the specimens increased. However, there were not much difference between the inclinations observed on the specimens in the 900-series and 1350-series. Therefore, it is inconclusive that the effective depth influences the inclination of the diagonal crack.

Figure 4-3—Inclination of crack in specimens with no shear reinforcement: (a) 1V0-450-20; (b) 1V0-450-10; (c) 1V0-900-20; (d) 1V0-900-10; (e) 1V0-1350-20; (f) 1V0-1350-10; and (g) 1V0-1800-20.

4.2.3. Crack Widths

The development of flexural crack widths at mid span in all the beams are plotted against the shear stress ($V_{test}/(b_w d \sqrt{f'_c})$) at different load stages in **Figure 4-4**. Results of 1V0-450-10 and 1V1-450-10 came from one beam specimen, so it is labelled in **Figure 4-4** as “450-series (Type 1-10)”, representing the Type 1 specimen in 450-series with a 10 mm maximum aggregate size. The crack widths were measured using a crack width detector with an accuracy of 0.02 mm. At every load stage, the width of all the cracks were measured and the measurements of the widest section of the crack was recorded. The recorded measurements in **Figure 4-4** show the widest flexural crack width at every load stage. Except for “450-series (Type 1-10)”, the growth rate of flexural cracks in the other beams were very similar. The measurements for flexural cracks were taken until no further widening of the crack width was observed and the largest measured flexural crack width was 0.20 mm.

Figure 4-4—Development of flexural crack widths.

4.2.4. Ultimate Failure Load

As discussed in Chapter 3, Specimen 1V0-1800-20 had a bulb cross section with a smaller web width and a greater base width compared to the other specimens. Using either the web width or base width in the analysis would not provide an accurate representation of the shear stress carried by the specimen. Therefore, two methods were used to estimate an approximate effective width for the specimen: 1) finite element analysis by SAP2000 comparing the stress distribution at the web, and 2) calculating the equivalent width using the second moment of area. A rectangular cross-section with a width of 280 mm gave the closest results to the bulb cross-section in both methods.

The calculated self-weight of the beam specimen in this study included both the concrete component and the amount of steel bars provided in the specimen. In addition, the weight of the test equipment placed on top of the beam was added into ultimate failure load. The ultimate failure load (V_{test}) was obtained by averaging the failure load from the two point loads, and included both the self-weight of the specimen and weight of the test equipment on top of the beam. The horizontal web bars in the specimens of 1350-series were placed by mistake. The contributions from these horizontal web bars to shear strength were not significant, estimated to be less than 10% of the failure load. The values of V_{test} for all the specimens with no vertical shear reinforcement are listed in **Table 4-1**.

The value for V_{test} increased as the depth of the beam increased, regardless of the maximum size of aggregate. Specimen 1V0-900-20 failed at an abnormally low load with no clear explanation. The failure load of Specimen 1V0-900-20 was higher than that of Specimen 1V0-450-20, however, an increase of 26 kN could be considered too little despite Specimen 1V0-900-20 having twice the depth of the latter specimen. The low failure load of Specimen 1V0-900-20 could be due to fabrication errors. Insufficient wetting on the concrete specimens might have led to formation of shrinkage cracks which could affect the strength of the specimen. Another reason could be the concrete material was not so consistent and the concrete used for the specimen was weaker than the test cube specimens.

The normalized shear stresses at failure ($V_{test}/b_w d \sqrt{f'_c}$) for all the specimens are plotted against its effective depth in **Figure 4-5**.

Figure 4-5—Normalized shear stress at failure for all the specimens with no shear reinforcement.

Apart from the results of the two specimens in the 900-series, which obtained lower shear stress at failure compared to the specimens with larger effective depths for no obvious reasons, the results in **Figure 4-5** show that the shear stress at failure reduced in specimens with greater effective depths. Comparing the shear strength of the largest and the smallest rectangular specimen, the shear strength of specimens in 1350-series was about 10% lower than those in the 450-series. Apart from the effective depth influencing the shear strength, the size of the aggregates was observed to have affected the shear strengths of the specimens in this study as well. The shear strengths of all the specimens with the maximum aggregate size of 10 mm were greater than those with the maximum aggregate size of 20 mm by approximately 20%.

4.2.5. Deflection

Figure 4-6 shows the load-to-mid-span deflection curves of the specimens in 450-series, 900-series, 1350-series, and 1800-series. The load stages are plotted in the y-axis and the effective mid-span deflections are plotted in the x-axis. The effective mid-span deflections of the beam were normalised by subtracting the average deflections of the four Linear Variable Differential Transformers (LVDTs) near the supports from the measured value at mid-span. The LVDTs were removed from the test setup at several load stages before the failure of the weaker shear span to prevent damage to the LVDTs during the sudden failure of the shear span. The label for each curve in **Figure 4-6** indicates the series, the type of specimen, and maximum aggregate size of the specimen. For example, the label “1800-series (Type 1-20)” refers to Type 1 specimen with aggregate size of 20 mm in 1800-series. The deflections for Type 1-10

specimens were moved to start at 4 mm in **Figure 4-6** to show clear separation from Type 1-20 specimens and allow easy comparison.

Before the first flexural crack, the relationship between the load and deflection is linear. After the first crack is formed, the slope of the load-deflection curve become gentler. The slope of the curve become increasingly gentler as the test proceeded until failure of the weaker shear span. Except for 1800-series (Type 1-20), the slopes of the load-deflection curves for specimens with greater effective depths are slightly steeper than that of the specimens with smaller effective depths. The specimen 1800-series (Type 1-10) with depth of 1800 mm and aggregate size of 10 mm was not tested in the study.

Figure 4-6—Mid-span deflection in various beam specimens: (a) 450-series; (b) 900-series; (c) 1350-series; (d) 1800-series; and (e) combined.

Figure 4-7 shows the vertical deflections along the beam specimens with respect to various load stages during the test. The deflection values are plotted in the y-axis of **Figure 4-7** and the positions of the LVDTs are plotted in the x-axis. The labels “1” and “5” represent the LVDTs at the two supports on each end of the beam specimen. The labels “2”, “3”, and “4” refer to the LVDTs at a distance of $L/4$, $L/2$, and $3L/4$ away from the support nearer to the weaker shear span of the beam respectively. That means labels “1” and “2” in **Figure 4-7** represents the deflection in the weaker shear span in each beam specimen, whereas labels “4” and “5” represents the stronger shear span. All the deflection values at the quarter-span and mid-span were normalised by the deflection values at the supports. Normalisation of deflection values is to obtain the net deflection values at the quarter-span and mid-span due to the applied loadings. All the recorded deflection values from the tests are provided in Appendix B. **Figure 4-7** shows that the deflection at the weaker shear span in each beam tends to be larger than that in the stronger shear span. For specimen 450 series (Type 1-10), the shape of the deflection along the beam was unexpected. The deflection was unsymmetrical since the beginning of the test, unlike the other specimens. There was no apparent reason for the unsymmetrical deflection.

Figure 4-7—Measured deflection at different locations along the beam specimen. Beam specimens of: (a) 450-series (Type1-20); (b) 450-series (Type1-10); (c) 900-series (Type1-20);

(d) 900-series (Type1-10); (e) 1350-series (Type1-20); (f) 1350-series (Type1-10); and (g) 1800-series (Type1-20).

4.2.6. Strains in Flexural Reinforcement

The strain readings recorded at failure in the longitudinal reinforcing bars on all the specimens with no shear reinforcement are presented in **Figure 4-8**. The strain readings are plotted on the y-axis and the distance of the strain gauge away from the centre of the beam is plotted on the x-axis. The centre of the beam is labelled as “0” in **Figure 4-8**; and the value “1000” means that the strain gauge was placed 1000 mm to the right from the centre of the beam. A negative value means the strain gauge was on the left side of the beam. The strain readings reflected the values recorded along the longitudinal reinforcing bars at the failure load of the specimen with no shear reinforcement. Generally, the strain readings from the strain gauge near the centre of the shear span were the highest when the shear span failed. The strain readings showed that the longitudinal reinforcing bars did not yield in any of the beam specimens at the failure load.

Figure 4-8—Strains recorded in longitudinal reinforcing bars: (a) 450-series; (b) 900-series; (c) 1350-series; and (d) 1800-series.

4.3. Discussions

4.3.1. Influence of Depth on Shear Strength

The normalised shear strength for all the specimens with no shear reinforcement are presented in **Figure 4-9**. The normalised shear strength is normalised as $V_{test}/bd\sqrt{f'_c}$ to compare with the equations in ACI 318 (2014), and as $V_{test}/bd^3\sqrt{f'_c}$ to compare with Eurocode 2 (EC 2) (2004). These results are plotted together with existing high strength concrete beam data investigating size effect (Matsui *et al.*, 1995; Collins and Kuchma, 1999; Fujita *et al.* 2002; Sherwood *et al.*, 2007; and; Sneed and Ramirez, 2010). The trend line for all the existing data is represented by a black solid curve in **Figure 4-9**; those data including the results from this study are represented by a red solid curve. The trend line for size effect influence according to ACI 446, based on the work of Bažant *et al.* (2007), is given as brown solid line. That of ACI 318-14 (as green solid line) and EC 2 (as blue solid line) are also shown for comparison in **Figure 4-9**.

Figure 4-9—Influence of depth on high strength concrete with results from this study.

Figure 4-9 shows that the results from this study fit in well into the trend with the collected data from other researchers. The shapes of the trend lines for the two sets of data (with and without the results from this study) are similar to the trend line per ACI 446. In fact, it seems that the ACI 446 predicts the influence of effective depth very well compared to EC 2. The shapes of the trend lines are also similar to the trend line representing normal strength concrete beam data shown in **Figure 2-10**. This confirms that effective depth influences the shear strength of high strength concrete beams in the same manner as in normal strength concrete beams. The size effect curve shown in **Figure 4-9** seems to flatten out in beams with

greater effective depth. Thus, the influence of size effect on two reinforced concrete beams with effective depths of 800 mm and 1600 mm would not be as significant as that on two beams with smaller effective depths, such as 200 mm and 600 mm.

It was observed that the diagonal cracking strength (*Diagonal cracking load divided by width and effective depth* = V_{cr}/bd) of those specimens with the same aggregate size and cross-section is uniform, even though the depth was varied. It shows that the diagonal cracking strength of a beam is not dependent on the depth. The results of this study showed that the ratio of diagonal cracking load over the failure load increases slightly as the beam depth increases. This meant that larger-sized beams carried less load than a smaller-sized beam after the formation of the diagonal crack. It seems that the size effect in beams is caused by the difference in load carried by the beam after the diagonal crack is formed (or reserve capacity). With a constant shear-span-to-effective-depth (a/d) ratio, the beams will have the same inclination angle of the compression strut. The difference in effective depth will result in a difference in size of the strut. A beam with a larger effective depth would have a longer and slenderer strut compared to a stockier strut with greater capacity in a beam with smaller effective depth (Tan *et al.*, 2005). Therefore, there will be a reduction in the reserve capacity of a larger-sized beam compared to a smaller-sized beam.

4.3.2. Influence of Aggregate Size on Shear Strength

Figure 4-5 shows that the shear strength of the specimens with maximum aggregate size of 10 mm were greater than that of the specimens with maximum aggregate size of 20 mm. This influence from the aggregate size differs from the observations made in other research using normal strength concrete (Taylor, 1972; Chana, 1981; Shioya *et al.*, 1989a, 1989b; and Sherwood *et al.*, 2006 and 2007). The results in those research generally show that larger aggregates would lead to higher shear strength in normal strength concrete beams.

In normal strength concrete, the strength of the hardened cement is weaker than the aggregate. Cracks developing in normal strength concrete would propagate through the hardened cement and circumvent the aggregate. A larger aggregate would create more areas of friction within the crack, resulting in higher aggregate interlock, and shear strength of the beam. In high strength concrete, the hardened cement would be stronger than the aggregate, thus forcing a crack to propagate straight through the aggregate instead of its surrounding area.

Compared to small-sized aggregate, large-sized aggregate results in more critical interfacial transition zones and is susceptible to fracture at a lower load. Once fractured, the strength of the aggregate could be reduced significantly (Walraven and Stroband, 1994). Hence, larger-size aggregate would lead to higher shear strength in normal strength concrete but might reduce the shear strength in high strength concrete.

4.3.3. Influence of Aggregate Size on Size Effect

The maximum aggregate sizes used in the concretes for the specimens involved only two sizes to understand the influence of aggregate size on size effect. The results from this study are plotted together with existing beam data from research varying the maximum aggregate sizes and effective depths in **Figure 4-10** (Taylor, 1972; Shioya *et al.*, 1989a, 1989b; and Sherwood *et al.*, 2007). The normalised shear strength ($V_{test}/bd\sqrt{f'_c}$) is plotted in the y-axis, and the effective depth of the beam is plotted in the x-axis in **Figure 4-10**.

Figure 4-10—Influence of maximum aggregate size on size effect.

The three bold lines in **Figure 4-10** are linear trendline plotted based on the beams' data with similar aggregate size (10 mm, 20 mm and 40 mm). All three bold trendlines show a similar downward trend in shear strength as the beam effective depth increases, with the trendline for beams with 40 mm aggregate size displaying a slightly larger gradient than the other two trendlines. This demonstrates that size effect exists regardless of the maximum aggregate size used in the concrete. Most of the beams' data collected in **Figure 4-10** are made of normal

strength concrete. The cracks development in a normal strength concrete propagates through the cement matrix but circumventing the aggregates which are of higher strength. Compared to high strength concrete, cracks in normal strength concrete develops in a more tortuous manner. In normal strength concrete, larger-size aggregates would create longer crack paths than smaller-size aggregates. However, in high strength concrete, the size of aggregates does not seem to matter as much because the cracks develop through the aggregates. Therefore, aggregate size plays a less important role in high strength concrete than normal strength concrete.

4.4. Comparison of Methods with Test Results

The experimental and predicted shear strengths of the seven specimens using equations from ACI 318-14 (2014) and Eurocode 2 (EC 2) (2004) are listed in **Table 4-2**. The experimental shear strengths of the seven specimens without shear reinforcement will be compared against the predicted shear strengths from equations proposed by Bažant and Yu (2005a, 2005b) and Collins *et al.* (2008) and discussed in Chapter 6. All the load factors and material safety factors were set as 1.0. The compressive strength and flexural reinforcement limits per the provisions in ACI 318-14 and EC 2 were applied in the calculations. Both columns (8) and (9) show the comparison between experimental shear strength and predicted shear strength calculated from the equations in EC 2. A compressive strength limit of 90 MPa was applied in the EC 2 equations to obtain the predicted shear strengths used for comparison in column (8). For column (9), the compressive strength limit of 50 MPa is used for the calculations. This compressive strength limit of 50 MPa was provided in the Singapore National Annex to Eurocode 2 (2008). The observed yield strengths of the steel were used in the calculations. In column (6) of **Table 4-2**, the ultimate failure load (V_{test}) included the self-weight of the specimen.

Table 4-2—Measured and predicted ultimate loads.

Specimen label	b_w , mm	d , mm	f'_c , MPa	ρ_l (%)	V_{test} (kN)	Test-to-predicted shear strength ratios, $V_{test}/V_{predicted}$		
						ACI 318 -14 method	Eurocode 2 method ¹	Eurocode 2 method ²
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1V0-450-20	265	403	102	2	142	0.94	0.77	0.93
1V0-450-10	265	403	108	2	168	1.11	0.91	1.10
1V0-900-20	265	808	103	2	194	0.64	0.59	0.72
1V0-900-10	265	808	108	2	284	0.94	0.87	1.06
1V0-1350-20	265	1232	98	2	384	0.83	0.82	1.00
1V0-1350-10	265	1232	108	2	501	1.09	1.08	1.31
1V0-1800-20	200	1670	107	2	545	1.16	1.19	1.45
All seven specimens					Average	0.96	0.89	1.08
					COV	0.19	0.22	0.22
Six specimens with rectangular cross-sections					Average	0.93	0.84	1.02
					COV	0.19	0.19	0.19

Note:

-COV = Coefficient of Variation.

-Eurocode 2 method¹ meant the compressive strength limit of 90 MPa was applied in the equations.

-Eurocode 2 method² meant the compressive strength limit of 50 MPa was applied in the equations.

-Failure mode is diagonal tension for all the specimens.

- V_{test} is the shear force (or half of the total load) and is inclusive of self-weight of the beam.

4.4.1. ACI 318-14

The average ratio of experimental over predicted shear strength values ($V_{test}/V_{predicted}$) for all seven specimens in this study is 0.96 with a Coefficient of Variation (COV) of 0.19. If only the rectangular specimens were considered (450-series, 900-series, and 1350-series), the average ratio of $V_{test}/V_{predicted}$ reduces to 0.93 with the same COV of 0.19. Among the predictions for the rectangular specimens, only the predictions for Specimens 1V0-450-10 and 1V0-1350-10 were conservative. The inconsistency in the predictions from ACI 318-14 was because the equations did not consider the influence of size effect. The results from this study and previous research have shown the influence of size effect on shear strength. Without the consideration of size effect in the equation, the predictions from ACI 318-14 can increasingly over-estimate the shear strength as the effective depth increases. If the upper limit for compressive strength in ACI 318-14 was not applied in the calculations, the predicted shear strength would have been even more un-conservative.

4.4.2. Eurocode 2

Column (8) in **Table 4-2** shows the ratios of $V_{test}/V_{predicted}$ for the equation in EC 2 with the compressive strength limit of 90 MPa. It shows that the predictions from EC 2 were very un-conservative. Except for Specimen 1V0-1800-20, the shear strength predictions from EC 2 were un-conservative for all the rectangular specimens. The widened base of Specimen 1V0-1800-20 is believed to have increased the failure load of the specimen. This would explain why both the predictions from ACI 318-14 and EC 2 for this specimen was conservative as compared to the un-conservative predictions for most of the rectangular specimens.

Considering only the rectangular specimens, the predictions from EC 2 for the six specimens obtained a COV similar to that for ACI 318-14. However, the average $V_{test}/V_{predicted}$ ratio for the shear strength predictions from EC 2 was lower, at 0.84, compared to 0.93 for ACI 318-14. The equations in EC 2 considered the influence of size effect. However, it seemed that the predictions from these equations were still not accurate enough. **Figure 4-9** also shows that the predicted influence from size effect in EC 2 was not as accurate as that considered in ACI 446. Column (9) shows the $V_{test}/V_{predicted}$ ratios for all the specimens considering the compressive strength limit of 50 MPa. While the COV of the results did not improve, the predictions were more conservative. Only the predictions for the two smallest specimens with 20 mm aggregates (Specimens 1V0-450-20 and 1V0-900-20) were un-conservative.

4.5. Summary of Chapter 4

Based on the results from all the specimens presented in this chapter and the accompanying discussions, the following conclusions can be made:

1. The crack surfaces observed in high strength concrete beams were relatively smooth, unlike the uneven surface observed in normal strength concrete beams.
2. The ultimate failure load for the specimens increased with effective depth. However, the normalised shear strength of the specimen ($V_{test}/bd\sqrt{f'_c}$) reduced as the effective depth increased.

3. Reviewed alongside experimental data from Matsui *et al.* (1995), Collins and Kuchma (1999), Fujita *et al.* (2002), Sherwood *et al.* (2007), and Sneed and Ramirez (2010), the experimental results from this study showed that the influence of effective depth reduced the shear strength of high strength concrete beams in the same manner as in normal strength concrete beams. The difference in shear strength between the largest and smallest rectangular specimen in this experimental programme (450 and 1350-series) without shear reinforcement was about 10%.
4. The specimens with smaller maximum aggregate size tested in this study had approximately 20% higher shear strength than those with larger maximum aggregate size.
5. **Figure 4-10** showed that, despite the different aggregate sizes used in the experimental beams, the size effect trend in the reinforced concrete beams did not change.
6. The ACI 318-14 and Eurocode 2 methods might not be safe for beams with effective depths greater than 400 mm, even though the equations in Eurocode 2 method consider size effect. The results in **Table 4-2** show that the provisions in ACI 318-14 should consider introducing size effect into its equation to ensure safe design of large reinforced concrete beams.

Chapter 5

RESULTS AND DISCUSSIONS FOR BEAMS WITH SHEAR REINFORCEMENT

5.1. Introduction

This chapter presents the experimental results of the reinforced concrete shear spans with shear reinforcement. As aforementioned in Section 3.2, the shear span (specimen) in each beam specimen contained different amounts of shear reinforcement. For example, Specimens 1V0-450-10 and 1V1-450-10 formed one beam specimen; the results of Specimen 1V0-450-10 have been discussed in Chapter 4. The results of Specimen 1V1-450-10, reviewed alongside that of other specimens containing shear reinforcement, will be discussed in this chapter. There were fifteen (15) specimens with different shear reinforcement ratios varying between 0.13% and 0.30%. The compressive strengths of these specimens were between 95 to 110 MPa, and the beam depths were between 450 to 1800 mm.

Specimens 2V3-1350-20 and 2V3-1800-20 did not fail during the test. The estimated results for these two specimens, together with the results for other specimens, show that providing shear reinforcement increases the shear strength and mitigates the influence of size effect. The ultimate shear strength of these fifteen specimens will be compared against the predicted shear strength from the provisions in ACI 318-14 (2014) and Eurocode 2 (2004), and discussed in this chapter.

5.2. Experimental Results

The main properties of the fifteen (15) specimens discussed in this chapter have been presented in **Table 3-1**. The material properties of the reinforcing steel and concrete used for the specimens are listed in **Tables 3-2** and **3-4** respectively.

5.2.1. General Behaviour

All fifteen (15) specimens failed in shear, and the failures were brittle and loud. However, compared to the specimens with no shear reinforcement, failures of these specimens with shear reinforcement were more ductile and less loud. The drawings of the crack patterns on these specimens after the whole test are shown in **Figure 5-1**. The crack surfaces observed in all the

specimens were relatively smooth, similar with those observed in the specimens with no shear reinforcement. This shows that shear reinforcement does not influence the condition of crack surface. The first crack to form on a beam was a flexural crack near the mid-span of the beam. As the applied load increased, more flexural cracks were observed in the high moment regions and flexural cracks formed in both shear span regions. Several flexural cracks in the shear spans propagated upwards until about two-third depth (from the bottom of the beam), then changed its inclination and became diagonal shear cracks propagating towards the loading point.

There were several diagonal shear cracks in the specimens with shear reinforcement, instead of a single diagonal shear crack at the failure of the specimens with no shear reinforcement. The number of diagonal cracks observed in the specimens increased with a higher amount of shear reinforcement in the specimen. The widening of these cracks was also more gradual compared to the cracks in specimens with no shear reinforcement.

Figure 5-1— Crack patterns at failure (Not to scale).

As the diagonal cracks propagated towards the loading point, horizontal cracks were observed to form at the level of the longitudinal reinforcement. These cracks appeared near the location of the diagonal cracks and propagated towards the supports. Similar to the specimens with no shear reinforcement, more horizontal cracks were observed in those specimens with larger beam depths. As the cracks propagated in two directions — towards the loading point and the support — shear failure of the specimen took place suddenly when one of the diagonal cracks widened significantly.

The load stages for each specimen when the diagonal shear crack first formed are listed in **Table 5-1**. The load stages at which the first diagonal crack in the specimens of the 450, 900 and 1350 series started to propagate towards the loading point were about 20-40 percent of the failure load. Comparatively, the load stages recorded for similar specimens with no shear reinforcement was at about 80 percent of the failure load of the specimen. The presence of shear reinforcement did not influence the formation of the first diagonal crack significantly. However, the presence of shear reinforcement increased the failure load of the specimen. That is evident in which after the diagonal crack started to propagate towards the loading point, the specimen could still carry much more shear load compared to a specimen with no shear reinforcement.

The inclinations of the main diagonal shear crack on all the specimens with shear reinforcement are shown in **Figure 5-2**. No actual failure took place for Specimens 2V3-1350-20 and 2V3-1800-20 during the test. Hence, the inclination of the main diagonal shear crack was not marked in **Figure 5-2**. The effective depths of the specimens did not seem to influence the inclination of the main diagonal crack. The observed diagonal crack inclinations in all the specimens were between 28° and 37° . It did seem that, in general, the inclinations of the diagonal cracks in the specimens with higher shear reinforcement ratio were steeper than those observed in the specimens with less shear reinforcement. However, the main diagonal crack which widened suddenly at failure could be a combination of several cracks, which could then affect the measurement of the critical diagonal crack. The measured inclinations should not be used to check the accuracy of any methods. The measured inclinations are subjective, depending on one's perspective on which part of the crack to measure. Tompos and Frosch (2002) calculated the development length of the shear reinforcement from the top of the beam and measured the angle between that point (where the crack cuts across) and the point where the crack cuts the longitudinal reinforcement bars. The measured angle is taken as the

inclination of the diagonal crack. Sherwood *et al.* (2007) measured the angle of the crack development between the tip of the flexural crack and the point where the crack changed its inclination (at around mid-depth of the beam) to propagate towards the loading point. This angle was taken as the inclination of the diagonal crack. The inclinations of diagonal cracks for all the specimens in this experimental programme were taken to be the angle between the points where the horizontal cracks appear and where the diagonal crack meets the compression bars at the top of the beam.

The crack spacing measured between two diagonal cracks are marked out in **Figure 5-2** and the measured spacing for each specimen are listed in **Table 5-1**. The crack spacings were smaller in specimens with higher shear reinforcement ratio. The measurement of the spacing between the diagonal cracks is also subjective to one's perspective on which part of the cracks to make the measurement. In this study, the measurement was taken as the largest spacing between two inclined major diagonal cracks.

Figure 5-2— Mid-span deflection in various beam specimens: (a) 1V1-450-20; (b) 1V1-450-10; (c) 2V3-450-20; (d) 2V2-450-20; (e) 1V1-900-20; (f) 1V1-900-10; (g) 2V2-900-20; (h) 2V3-900-20; (i) 1V1-1350-20; (j) 1V1-1350-10; (k) 2V2-1350-20; (l) 2V3-1350-20; (m) 1V1-1800-20; (n) 2V2-1800-20; and (o) 2V3-1800-20.

5.2.2. Crack Widths

The development of flexural crack widths on all the beams are plotted against the shear stress $[V_{test}/(b_w d \sqrt{f'_c})]$ at different load stages in **Figure 5-3**. The crack widths were measured using a crack width detector with an accuracy of 0.02 mm. At every load stage, the width of all the cracks were measured and the measurements of the widest section of the crack was recorded. The recorded measurements show the widest shear and flexural crack width at every load stage. The measured crack widths of the flexural cracks around the mid-span of the beam were smaller than 0.3 mm in all the tested beams. The largest flexural crack width of 0.26 mm was observed on Type 2 specimen in 1350-series, containing Specimens 2V2-1350-20 and 2V3-1350-20. The maximum crack widths observed in the other beams were smaller than 0.20 mm. This indicated that the amount of shear reinforcement in a beam would not influence the propagation of a flexural crack.

Figure 5-3—Development of flexural crack widths.

The developments of shear crack widths on all the beams are plotted against the shear stress $[V_{test}/(b_w d \sqrt{f'_c})]$ at different load stages in **Figure 5-4**. Like the measurements of flexural cracks, the width of all the shear cracks were measured at every load stage. The largest width measured was recorded and shown in **Figure 5-4**. The crack width development in the specimens of 450, 900, 1350, and 1800-series indicated that the maximum crack width observed on the specimen was smaller as the amount of shear reinforcement in the specimen increased. Higher shear reinforcement ratio in the specimen meant that there were more reinforcing bars in the specimen if the size of the bars used in all the specimens were the same. As the shear reinforcement ratio of the specimen increased, the widest crack width measured on the specimens generally reduced. The shear reinforcing bar restrained the growth of shear cracks. Additional vertical reinforcing bars were able to control widening of the crack width, hence allowing the specimen to carry more load compared to a specimen with lesser amount of shear reinforcement. It was observed that, at any measured crack width interval, the specimen with the smallest beam depth always carried the greatest amount of shear stress regardless of the shear reinforcement ratio in the specimen.

ACI 224R-01(2001) indicates that a crack width of 0.30 mm is the maximum reasonable crack width for a reinforced concrete beam in a humid environment and under service loads. With a crack width of 0.30 mm, the shear stress carried by Specimen 1V1-450-10, which had a shear reinforcement ratio of 0.13%, was about 1.4 times greater than that recorded for Specimen 1V0-450-10, which had no shear reinforcement. The difference in the shear stress for the specimen with 0.13% shear reinforcement ratio and that for the specimen with no shear reinforcement got smaller as the beam depth increased. Comparing Specimen 1V0-1800-20 with 1V1-1800-20, the difference in the shear stress for the two specimens was negligible. Even with a shear reinforcement ratio of 0.30%, the recorded shear stress for Specimen 2V3-1800-20 was not much greater than that of the specimen with no shear reinforcement (1V0-1800-20). This indicated that as the beam depth increases, more shear reinforcement is required to increase the amount of load the beam can carry before the crack width widens to 0.3 mm.

Figure 5-4—Development of shear crack widths.

5.2.3. Adjustment to Failure Load

5.2.3.1. Non-failure of Specimens 2V3-1350-20 and 2V3-1800-20

Specimens 2V3-1350-20 and 2V3-1800-20 did not fail during the test. The testing procedure was discussed in Section 3-5: the weaker shear span of the beam was strengthened by steel rods after it failed, so that the test could continue to be carried out until failure of the other shear span occurred. With strengthening of the weaker shear span, failure was able to take place in all the stronger shear spans in the beams except for two — Specimens 2V3-1350-20 and 2V3-1800-20. The damage to the weaker shear span with these two specimens were too severe; upon re-testing, concrete crushing was observed in the same shear spans (2V2-1350-20 and 2V2-1800-20) after strengthening. The failure loads of these two specimens with no failure, presented in **Table 5-1**, were estimated. The strain readings recorded on the shear reinforcement bars in Specimens 2V3-1350-20 and 2V3-1800-20 showed that the last recorded load stages were very close to the failure loads of the specimens. The additional loads that Specimens 2V3-1350-20 and 2V3-1800-20 can carry, after the test was stopped, was estimated based on the amount of load that Specimens 2V2-1350-20 and 2V2-1800-20 carried beyond similar strain readings, respectively. This was done because the specimens have similar material properties and were expected to display similar behaviour. The estimated additional loadings were added to the last recorded load stages of Specimens 2V3-1350-20 and 2V3-1800-20. The steps to estimate the failure loads of Specimens 2V3-1350-20 and 2V3-1800-20 are listed as follows:

1. Obtain the last known load stage for the specimen just before the test (for 2V3-1350-20 and 2V3-1800-20) was stopped.
2. Check the strain readings on the shear reinforcement bars in the specimen (2V3-1350-20 and 2V3-1800-20) at the last known load stage.
3. Compare the strain readings collected in (2) against the strain readings in Specimens 2V2-1350-20 and 2V2-1800-20 and obtain the equivalent load stages of Specimens 2V2-1350-20 and 2V2-1800-20 at these strain readings.
4. Obtain the load difference between the final failure load and the load stages of Specimens 2V2-1350-20 and 2V2-1800-20 (load stages obtained in (3)), in terms of percentage.

5. Add the same percentage of load to the last known load stage in (1) to estimate the failure loads of Specimens 2V3-1350-20 and 2V3-1800-20.

5.2.4. Ultimate Failure Load

The ultimate shear strengths for all the specimens with shear reinforcement are listed in **Table 5-1**. The ultimate failure load (V_{test}) for each specimen was obtained by averaging the two point loads (P) and includes self-weight of the specimen. The ultimate failure loads of all specimens with no shear reinforcement for each series are listed in column (10) of **Table 5-1** as $V_{c,test}$. The difference between V_{test} and $V_{c,test}$ is represented by $V_{s,test}$, the shear strength contribution from the shear reinforcement in the specimen, listed in column (11) of **Table 5-1**. Column (11) shows that providing greater amount of shear reinforcement in the specimen would lead to greater failure load of the specimen. The shear reinforcement ensured that the specimens would not collapse after the first diagonal crack.

Table 5-1—Properties of beam specimens.

Specimen ID	d , mm	f'_c , MPa	ρ_{vv} (%)	Measured crack spacing (mm)	Measured loads, (kN)					
					Flexural cracking load	Diagonal cracking Load	Failure load, P	Ultimate failure load V_{test}	$V_{c,test}$	$V_{s,test} =$ ($V_{test} -$ $V_{c,test}$)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1V1-450-20	403	102	0.13	148	95	220	476	246	142	104
1V1-450-10	403	108	0.13	140	90	190	576	296	168	128
2V2-450-20	403	102	0.20	130	80	180	594	305	142	163
2V3-450-20	403	102	0.30	145	80	140	640	328	142	186
1V1-900-20	808	103	0.13	330	110	240	780	414	194	220
1V1-900-10	808	108	0.13	280	80	280	820	434	284	150
2V2-900-20	808	103	0.20	260	100	340	1060	554	194	360
2V3-900-20	808	103	0.30	190	100	340	1450	749	194	555
1V1-1350-20	1232	98	0.13	390	110	360	1500	801	384	417
1V1-1350-10	1232	108	0.13	340	200	420	1750	926	501	425
2V2-1350-20	1232	97	0.20	300	100	380	1828	965	384	581
2V3-1350-20	1232	97	0.30	250	100	380	2340	1221	384	837
1V1-1800-20	1670	107	0.13	550	240	620	1680	925	545	380
2V2-1800-20	1670	107	0.20	400	220	660	1926	1048	545	503
2V3-1800-20	1670	107	0.30	280	220	620	2400	1285	545	740

Note:

-Failure mode is diagonal tension for all the specimens.

$-V_{test}$ is the shear force (or half of the total load) and is inclusive of self-weight of the beam.

$-V_{c,test}$ is the shear force of a similar specimen in the same series with no shear reinforcement.

5.2.5. Deflection

Figure 5-5 shows the load to mid-span deflection curves of the specimens in 450, 900, 1350, and 1800-series. The total applied load at the load stages are plotted in the y-axis, and the effective mid-span deflections are plotted in the x-axis. The label for each data in **Figure 5-5** indicates the series, type of specimen, and maximum aggregate size of the specimen. For example, the label “1800-series (Type 2-20)” refers to Type 2 specimen with aggregate size of 20 mm in the 1800-series.

Figure 5-5— Mid-span deflection in various beam specimens: (a) 450-series; (b) 900-series; (c) 1350-series; and (d) 1800-series.

As discussed in Section 4.2.6, the relationship between the load and the mid-span deflection was linear for specimens with no shear reinforcement in one of the shear span before the first flexural crack formed. For specimens with shear reinforcement in both shear spans, (i.e. 450-series (Type 2-20), 900-series (Type 2-20), 1350-series (Type 2-20), 1800-series (Type 2-20)), the relationship for load-mid span deflection was linear as well. After the first

flexural crack, the slope of the load-deflection curve changed and became gentler compared to the initial stages of the test. The load-deflection curves of specimens with shear reinforcement are similar to those of specimens with no shear reinforcement. The presence of shear reinforcement allowed the beams to deflect more and carry higher amount of shear load before failure.

Figure 5-6 shows the deflection along the beam specimen with respect to the various load stages during the test. The deflection values are plotted in the y-axis of **Figure 5-6** and the positions of the LVDTs are plotted in the x-axis. The labels “1” and “5” represent the LVDTs at the two supports on each end of the beam specimen. The labels “2”, “3”, and “4” refer to the LVDTs at a distance of $L/4$, $L/2$, and $3L/4$ away from the support at the weaker section of the beam respectively. All the deflection values at the quarter-span and mid-span were subtracted by the deflection values at the supports. Similar in all the beam specimens, the deflection at the weaker shear span in each beam tended to be greater than that at the stronger shear span when it was close to failure.

Figure 5-6—Measured deflection at different locations along the beam specimen. Beam specimens of: (a) 450-series (Type2-20); (b) 900-series (Type2-20); (c) 1350-series (Type2-20); and (d) 1800-series (Type2-20).

5.2.6. Strains in Flexural Reinforcement

The strain readings recorded on the longitudinal reinforcement in the beam specimens with shear reinforcement at failure are presented in **Figure 5-7**. **Figure 5-7 (a) to (d)** represents the specimens from 450-series, 900-series, 1350-series, and 1800-series, respectively. The strain readings are plotted on the y-axis, and the distance of the strain gauge away from the centre of the beam is plotted on the x-axis. The centre of the beam is plotted on the x -axis. The centre of the beam is labelled as “0” on the x -axis in **Figure 5-7**. The value “1000” indicates that the strain gauge was placed 1000 mm to the right from the centre of the beam. A negative value means that the strain gauge was on the left side of the beam.

Figure 5-7—Strains recorded on longitudinal reinforcement in all specimens: (a) 450-series; (b) 900-series; (c) 1350-series; and (d) 1800-series.

In **Figure 5-7**, the strain gauges on the longitudinal reinforcing bars at failure to the left side of the beam recorded zero strain readings because the left side of the beam contained the weaker shear span, and most strain gauges with its connecting wires were damaged when the shear span failed. Hence, the strain readings on the longitudinal reinforcing bars were recorded as zero at the failure of the stronger shear span in the beam. The strain readings near the centre of the shear span were the highest when the shear span failed. The strain readings showed that the longitudinal reinforcing bars did not yield in all the specimens at failure load. For Specimens 2V2-450-20 and 2V3-900-20, the strain readings show that the reinforcing bars were very close to yielding.

5.2.7. Strains in Shear Reinforcement

The strain distribution on the shear reinforcing bars in all specimens are shown in **Figure 5-8**. The x -axis shows the strain gauges labels on the shear reinforcing bar and the y -axis indicates the recorded strain readings. The strain readings recorded at five different load stages (at $1/5$, $2/5$, $3/5$, $4/5$ of V_{test} and at V_{test}) are shown in the graph. The red solid line in each graph indicates the yield strain of the steel bar. The location of each reinforcing bar with respect to the shear cracks on the is shown in each graph as well. The strain gauges on the specimens in 450-series, 900-series, and 1350-series were placed at the mid-depth of the specimen. If the shear crack crosses the shear reinforcement far away from the position of the strain gauge, the recorded strain reading could be small. The specimens in 1800-series had several strain gauges attached at different locations on each shear reinforcing bar. The highest strain reading recorded among the strain gauges was used to represent the strain on each reinforcing bar.

The shear reinforcing bars did not have any contribution before the formation of diagonal shear cracks. The strain readings on the shear reinforcing bars were negligible at low load levels (“x” marker representing $1/5 V_{test}$ in **Figure 5-8**). After a shear crack crossed a shear reinforcing bar, the reinforcing bar was activated to carry shear load. The strain readings on the shear reinforcing bar started to increase rapidly (shown by the “◊” and “○” markers, representing $3/5 V_{test}$ and $4/5 V_{test}$, respectively). The strain distribution showed that those shear reinforcing bars crossed by the critical diagonal crack reached their yield strain near the maximum load stage or at failure load (V_{test}).

(a)

(b)

Figure 5-8— Strain distribution recorded on shear reinforcement in all specimens: (a) 450-series; (b) 900-series; (c) 1350-series; and (d) 1800-series.

Several reinforcing bars placed close to the loading point and registering small strain readings could be due to either a damaged strain gauge from fabrication of the beam specimen, or that the shear crack is far away from the strain gauge. It was assumed that if the strain readings showed that a reinforcing bar had yielded, those shear reinforcements placed between this reinforcing bar and the loading point would have also yielded and contributed to the shear strength of the specimen.

The strain readings on the reinforcing bars near to the supports were observed to be very small in all the specimens. There was also no observed shear crack to cross those shear reinforcing bars with strain gauges. This meant that there is no direct strut connecting the loading point and the support. Also, not all the shear reinforcing bars provided in the specimen would contribute to the shear strength of the specimen.

5.3. Discussions

5.3.1. Shear Strength Contribution from Concrete Component

Strain readings on the shear reinforcing bars were recorded at different load stages of the test. These strain readings were multiplied with tensile modulus of steel and the area of the reinforcing bar to obtain the actual load carried by the reinforcing bar. The strain readings on different bars can vary because of the distance between the strain gauge and the position where a major shear crack crosses the reinforcing bar. The location of the strain gauge might not have been placed at the location at which the shear crack crossed the shear reinforcement. The reading recorded at the strain gauge can be very small if the strain gauge is placed far away from the crack. However, a shear reinforcement bar would still have carried the shear load once the shear crack crosses it. Therefore, the highest strain value recorded by any of the strain gauges on the shear reinforcement bar that was crossed by the shear crack was used to calculate and estimate the load carried by the reinforcement bar. If the highest strain value exceeds the yield strain value, the yield strain value is used to estimate the load carried by the reinforcement bar. The calculated load carried by the reinforcing bar was multiplied by the number of shear reinforcing bars that was observed to have been crossed by the shear crack, to obtain the estimated load carried by the shear reinforcement in each specimen. The estimated amount of load carried by the shear reinforcement ($V_{s,cal}$) in each specimen is presented in **Figure 5-9**. The difference between the amount of load carried by the shear reinforcement ($V_{s,cal}$) and the

applied load was referred to as the amount of load carried by the concrete component ($V_{c,cal}$). The applied load was taken to be the average of the two point loads. The two point loads for those specimens in 900, 1350 and 1800-series were provided by two individual hydraulic jacks. Due to the difference in the amount of oil supplied to the two hydraulic jacks through the valves and pipes, there was a difference in the two applied loads. Efforts were taken to make sure that the applied loads from both hydraulic jacks were as close as possible, and the difference between the applied loads was less than 5%. The values of both the applied load and the load taken by the concrete component for each specimen are plotted in **Figure 5-9**.

In all specimens, the strain readings recorded in the shear reinforcements were very small at the initial stages of the test. This represented small values of load carried by the shear reinforcement, as shown in **Figure 5-9**. The amount of load carried by the concrete component ($V_{c,cal}$), represented by red markers, was almost the same as the applied load which was represented by black markers. Upon reaching a certain load stage, the value of the load carried by the shear reinforcement started to increase (shown by the blue markers). This indicated that a diagonal shear crack has formed, and has activated the shear reinforcing bars. As the applied load increased, the value of $V_{s,cal}$ increased, indicating increased load carried by the shear reinforcing bars. As the value of $V_{s,cal}$ increased, the value of $V_{c,cal}$ dropped. However, even though the value of $V_{c,cal}$ began dropping with every increase in the applied load, the value of $V_{c,cal}$ would not decrease to zero. Instead, the value of $V_{c,cal}$ will maintain at about 25 - 35% of the applied load. This observation was noted in all the tested specimens. When the strain readings in the shear reinforcing bars showed that the bar had reached yield or become erratic, the yield strain of the reinforcing bar was used to calculate the maximum load carried by the shear reinforcing bar. **Figure 5-9** shows that the concrete component provided a certain amount of contribution towards the shear strength until the steel reinforcement yielded or, the concrete itself was crushed.

Figure 5-9— Calculated load taken by the concrete component in each specimen.

5.3.2. Influence of Shear Reinforcement on Ultimate Failure Load

The shear strengths $[V_{test}/(b_w d \sqrt{f'_c})]$ of all the specimens with maximum aggregate size of 20 mm are plotted in **Figure 5-10**. The shear strengths of the specimens are indicated in the y-axis of **Figure 5-10** and the x-axis shows the shear reinforcement ratio in the specimen. **Figure 5-10** generally shows a linear increasing trend in the shear strength of the specimens with an increase in shear reinforcement ratio. The shear strengths of the specimens with shear reinforcement ratio of 0.3% were about 2.8 times of the specimens with no shear reinforcements. Between the specimens with shear reinforcement ratios of 0.13% and 0.3%, the difference in shear strengths was about 1.5 times. The scatter between the results widens as the shear reinforcement ratio increases. The contribution from the shear reinforcement did not seem to have been influenced by the beam depth.

Figure 5-10—Shear strength of all specimens with maximum aggregate size of 20 mm.

The load at which the first diagonal crack was formed (V_{cr}) in each specimen are plotted in **Figure 5-11**. The load stage when the first diagonal crack was formed referred to the total applied load from the two point loads. The value of V_{cr} was taken to be half of the total applied load at the particular load stage. The ultimate failure loads of the specimens are also plotted in **Figure 5-11**. The average loads for the first diagonal crack and the ultimate failure load among all the specimens are represented by the black dashed and black solid lines respectively. **Figure 5-11 (a), (b), (c), and (d)** show the corresponding results for the specimens in the 450, 900, 1350, and 1800-series. The difference between the ultimate failure and the load at first diagonal crack represents the post-cracking reserve strength of a beam. The post-cracking reserve strength indicates the available strength of the beam after the occurrences of major diagonal cracks.

A representative load of $1.5V_{cr}$ is represented by the red dashed line in **Figure 5-11**. The load of $1.5V_{cr}$ meant 1.5 times the first diagonal cracking load. It can be seen from the graph that the ultimate failure load of the specimens with no shear reinforcement was very close to $1.5V_{cr}$. **Figure 5-11** shows that after providing a shear reinforcement ratio of 0.13%, the specimen was able to carry at least twice the load (compared to $1.5V_{cr}$). It shows that shear reinforcement is required to ensure a beam has sufficient post-cracking reserve strength after the first diagonal crack. All the specimens have an average shear strength three times of the $1.5V_{cr}$ when the shear reinforcement ratio is 0.3%. The post-cracking reserve strength of the specimen increased as the shear reinforcement ratio increased. However, the post-cracking reserve strength does not increase with the increase in effective depth of the specimen.

Figure 5-11—Post-cracking reserve strength of all specimens: (a) 450-series; (b) 900-series; (c) 1350-series; and (d) 1800-series.

Figure 5-1 shows that the inclinations of shear cracks in every specimen were quite different. This meant that the shear cracks could have crossed the shear reinforcing bars at different locations and the number of shear reinforcing bars crossed and activated would vary. The number of shear reinforcing bars activated would affect the shear strength of the specimen. The number of shear reinforcing bars visually observed to have been crossed by the shear crack and taken to have activated to contribute to the shear strength in each specimen are listed in column (2) of **Table 5-2**.

Table 5-2—Predicted shear strength contribution from shear reinforcement.

Specimen ID	Number of shear reinforcing bars activated, N	Shear strength contribution from shear reinforcing bars ($V_{s,predicted} = N \cdot A_v f_y$)	$V_{s,predicted} / V_{s,test}$	d/s
(1)	(2)	(3)	(4)	(5)
1V1-450-20	3	104	1.00	2.4
1V1-450-10	4	139	1.09	2.4
2V2-450-20	5	174	1.07	3.7
2V3-450-20	5	174	0.94	5.4
1V1-900-20	2	187	0.85	1.7
1V1-900-10	2	187	1.25	1.7
2V2-900-20	4	374	1.04	2.7
2V3-900-20	6	562	1.01	3.8
1V1-1350-20	4	374	0.90	2.6
1V1-1350-10	4	374	0.88	2.6
2V2-1350-20	6	562	0.97	4.1
2V3-1350-20	-	-	-	5.9
1V1-1800-20	4	374	0.99	2.8
2V2-1800-20	5	468	0.93	4.3
2V3-1800-20	-	-	-	6.4

Some of the shear reinforcing bars close to the loading point were crossed by the shear crack near to the top of the bar. It is unlikely that there had been enough anchorage length developed for the bar to effectively contribute to shear strength. These bars were therefore considered not to have contributed to the shear strength of the specimen. The predicted shear strength contribution was calculated by multiplying the number of shear reinforcement with its cross-sectional area and yield strength of the bar ($V_{s,predicted} = N \cdot A_v f_y$). The assumption taken was those reinforcing bars crossed by the diagonal shear crack and had enough anchorage length would have yielded at the failure load.

The values of $V_{s,test}$ used in column (4) of **Table 5-2** were obtained by taking the difference in ultimate failure load between the specimens with shear reinforcement and the specimen with the same depth and aggregate size but had no shear reinforcement. The comparison between the $V_{s,predicted}$ and $V_{s,test}$ showed that the predicted shear strength calculated using the assumed effective reinforcing bars compared fairly accurately to the values of $V_{s,test}$. The term d/s in column (5) represents the number of shear reinforcing bars contributing to the shear strength assumed by the equations in ACI 318-14 (2014). The term d/s represents the

effective depth of the beam divided by the spacing between the vertical shear reinforcing bars. For most of the specimens, the number of shear reinforcing bars assumed by ACI 318-14 to contribute to shear strength (d/s) was smaller than the observed number of shear reinforcing bars crossed by the shear cracks. This meant that the shear strength contribution from shear reinforcement predicted by ACI 318-14 would always likely to be conservative compared to the actual contribution from the shear reinforcing bars.

5.3.3. Influence of Shear Reinforcement on Size Effect

The results of the specimens with no shear reinforcement that have been discussed in Chapter 4 showed that the difference in effective depth influenced the shear strength of reinforced concrete beam by reducing its shear strength as the effective depth of the beam increased. The influence of size effect was considered in the equation of Eurocode 2 (2004) for reinforced beams with no shear reinforcement. However, for reinforced concrete beams with shear reinforcement, the equations in both ACI 318-14 (2014) and Eurocode 2 (2004) do not consist of a size effect term. This indicates that both codes assume that providing the minimum amount of shear reinforcement in the beam can eliminate the influence of size effect. The influence of size effect is eliminated if, the shear strength (in terms of MPa) of a larger beam equals to, or is greater than that of a smaller beam with the same shear reinforcement ratio.

The influence of shear reinforcement on size effect was investigated in this study using the results of specimens with similar shear reinforcement ratios, albeit different effective depths. The results of the specimens with minimum amount of shear reinforcement required by ACI 318-14 (0.13%) are presented in **Figure 5-12**. Apart from this study, data from other research programmes with specimens having the same shear reinforcement ratio but different effective depths are also presented in **Figure 5-12** (Bhal, 1968; and Sherwood *et al.*, 2007).

Figure 5-12—Influence of shear reinforcement on size effect.

The data consist of different longitudinal reinforcement ratios and compressive strengths. Therefore, the shear strengths of the data were normalised by both the compressive strength and longitudinal reinforcement ratio to the power of $1/3$ ($V_{test}/bd^3 \sqrt[3]{f_c^3 \rho}$) to eliminate the effect of these parameters. The effective depths from the data are plotted on the x -axis and the normalised shear strengths are plotted on the y -axis of **Figure 5-12**. The markers with no fill represent specimens with no shear reinforcement. The filled markers represent specimens with shear reinforcement. A linear trend line of each series of data is represented by the dashed line. Linear interpolation equations are also plotted based on all the data with and without shear reinforcement, represented by the blue and red solid lines, respectively. As shown in **Figure 5-12**, the scatter of the results for data with shear reinforcement is wider compared to the specimens with no shear reinforcement. This was probably due to the different shear reinforcement ratios used in each series. The normalised shear strengths of the specimens with shear reinforcement are greater than that of the specimens with no shear reinforcement. However, the shear strength of these data with shear reinforcement exhibits a downward trend as the effective depth of the beam increases. The linear interpolation equations for both sets of data (with and without shear reinforcement) are similar. This meant that providing minimum amount of shear reinforcement in the reinforced concrete beam was not able to eliminate size effect in beams with different effective depths.

The normalised shear strengths [$V_{test}/(bd \sqrt[3]{f_c})$] for all the specimens in this study with different shear reinforcement ratios are presented in **Figure 5-13**, which shows the influence of different shear reinforcement ratios on size effect. It seemed that providing a shear

reinforcement ratio of 0.30% in the specimens with different effective depths would not eliminate size effect. The linear trend line for the specimens still exhibits a downward trend for shear strength. The difference in shear strength between the largest and smallest specimens with the same shear reinforcement ratio is 10%. However, the influence of shear reinforcement seemed to mitigate size effect instead of eliminating it. For example, the shear strength of the specimen with no shear reinforcement in 1800-series was lower than the specimen with no shear reinforcement in 450-series. Providing a shear reinforcement ratio of 0.13%, the shear strength of the larger specimen would be comparable to the smaller specimen with no shear reinforcement. Likewise, increasing the shear reinforcement ratio (to 0.20%) in a larger specimen with a depth of 1800 mm would increase the shear strength of the specimen to a level comparable to the smaller specimen with a depth of 450 mm and smaller shear reinforcement ratio of 0.13%.

Figure 5-13—Influence of shear reinforcement on shear strength of the specimens.

5.4. Comparison of Methods with Test Results

The experimental and predicted shear strengths of the fifteen specimens using the equations from ACI 318-14 (2014) and Eurocode 2 (EC 2) (2004) are listed in **Table 5-3**. The experimental shear strengths of the fifteen specimens with shear reinforcement will be compared against the predicted shear strengths from equations proposed by Bažant and Yu (2005a, 2005b) and Collins *et al.* (2008) and discussed in Chapter 7. All the load factors and material partial safety factors were set as 1.0. The compressive strength and flexural reinforcement limits per the provisions in ACI 318-14 and EC 2 were applied in the calculations as well. The observed yield strengths of the steel were used in the calculations. The ultimate failure load (V_{test}) in column (7) of **Table 5-3** included the self-weight of the specimen.

Table 5-3—Measured and predicted ultimate loads.

Specimen ID	b_w , mm	d , mm	f'_c , MPa	ρ_l (%)	ρ_{vv} (%)	V_{test} (kN)	Test-to-predicted shear strength ratios, $V_{test}/V_{predicted}$	
							ACI 318-14 method	Eurocode 2 method
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1V1-450-20	265	403	102	2	0.13	246	1.04	1.30
1V1-450-10	265	403	108	2	0.13	296	1.25	1.56
2V2-450-20	265	403	102	2	0.20	305	1.08	1.04
2V3-450-20	265	403	102	2	0.30	328	0.94	0.75
1V1-900-20	265	808	103	2	0.13	414	0.88	1.12
1V1-900-10	265	808	108	2	0.13	434	0.93	1.17
2V2-900-20	265	808	103	2	0.20	554	0.99	0.97
2V3-900-20	265	808	103	2	0.30	749	1.09	0.88
1V1-1350-20	265	1232	98	2	0.13	801	1.12	1.42
1V1-1350-10	265	1232	108	2	0.13	926	1.30	1.64
2V2-1350-20	265	1232	97	2	0.20	965	1.14	1.11
2V3-1350-20	265	1232	97	2	0.30	1221	1.17	0.94
1V1-1800-20	200	1670	107	2	0.13	925	1.27	1.60
2V2-1800-20	200	1670	107	2	0.20	1048	1.21	1.18
2V3-1800-20	200	1670	107	2	0.30	1285	1.20	0.97
All fifteen (15) specimens						Average	1.11	1.18
						COV	0.12	0.23
Specimens with rectangular cross-sections						Average	1.08	1.16
						COV	0.12	0.24

Note:

-COV = Coefficient of variation.

-The values of V_{test} used for the specimens in 1350-series were the actual failure loads of the specimens.

5.4.1. ACI 318-14

The average ratios of test-to-predicted shear strength ($V_{test}/V_{predicted}$), presented in column (8) of **Table 5-3**, showed good predictions and with low Coefficient of Variation (COV). Generally, the predictions from ACI 318-14 were conservative as the ratio of $V_{test}/V_{predicted}$ was greater than 1.0.

The predicted shear strengths from the equation of ACI 318-14 were based on the contributions from the concrete component and shear reinforcement component. As discussed in Section 4.4.1, the predictions from the equation of ACI 318-14 for the rectangular specimens with no shear reinforcement were un-conservative. **Table 5-2** shows that the number of shear reinforcing bars assumed by ACI 318-14 (given by d/s) to contribute to the shear strength of

the specimen was smaller than or equal to the number of bars that was observed to be crossed by the shear crack. Therefore, the shear strength contribution from the reinforcing bars predicted by the equation of ACI 318-14 was less than the actual contribution from the shear reinforcing bars. It seemed that the combination of un-conservative predictions for the concrete contribution and conservative predictions for the shear reinforcement contribution led to coincidental good predictions of the shear strength for the fifteen specimens in this study.

5.4.2. Eurocode 2

The optimum inclination of the diagonal strut was obtained by equating Equations (2.12) and (2.13), $V_{Rd,s} = V_{rd,max}$, to maximise the allowable shear strength of the reinforced concrete beam. If the calculated inclination falls outside of the limiting angles of 22^0 and 45^0 , the limiting angle is used to calculate the predicted shear strength of the beam. The ratio of $V_{test}/V_{predicted}$ from EC 2, presented in column (9) of **Table 5-3**, shows fluctuations in the shear strength between conservative and un-conservative predictions. The lowest ratio was 0.75 for Specimen 2V3-450-20, while the highest was 1.64 for Specimen 1V1-1350-10. The average prediction for all fifteen specimens was 1.18 with a COV of 0.23.

Most of the predicted shear strengths were conservative because the equations in EC 2 do not consider concrete contribution in specimens with shear reinforcement. The conservatism of shear strength prediction was the highest in specimens with low shear reinforcement ratio. As the shear reinforcement ratio increased, the predictions became less conservative. For specimens with shear reinforcement ratio of 0.30%, the predicted shear strength became un-conservative.

The results from this study showed that the concrete contribution to shear strength continues even as shear reinforcement starts to carry the load. Using the optimal inclination obtained from equating $V_{Rd,s}$ and $V_{rd,max}$, the shear strength predictions became un-conservative as the shear reinforcement ratio increases. An inclination angle of 45 degrees would be used in the equations to ensure conservative shear strength predictions. Neglecting the concrete contribution, the beam has to be designed with more shear reinforcement to carry the necessary load. This could lead to uneconomical design of reinforced concrete beams.

5.5. Summary of Chapter 5

Based on the results from the fifteen specimens with shear reinforcement presented in this chapter, the following conclusions could be made:

1. The shear strength of a reinforced concrete beam could be increased with the provision of shear reinforcement. The shear strengths of the specimens with shear reinforcement ratio of 0.3% were about 2.8 times of the specimens with no shear reinforcements. Between the specimens with shear reinforcement ratios of 0.13% and 0.3%, the difference in shear strengths was about 1.5 times.
2. The amount of load carried by the beam after the occurrence of first diagonal crack, referred as the post-cracking reserve strength of the beam, increases as the shear reinforcement ratio increases. Shear reinforcement ratio of at least 0.13% could provide a post-cracking reserve strength about twice the load at the first diagonal cracking load. All the specimens with shear reinforcement ratio of 0.3% have an average shear strength three times of the $1.5V_{cr}$. The post-cracking reserve strength does not increase with the increase in effective depth of the specimen.
3. The shear strength of a reinforced concrete beam with shear reinforcement was provided by both the concrete and the shear reinforcement. The results showed that the concrete contribution to shear strength did not drop to zero when the shear reinforcing bars started to carry the shear load, instead, it maintained at 25-25% of the applied load.
4. Providing shear reinforcement in the specimen could only mitigate the size effect between two specimens of different effective depths. A shear reinforcement ratio up to 0.30% still could not eliminate the size effect. The difference in shear strength between the largest and smallest specimens with the same shear reinforcement ratio is 10%.
5. The equation from ACI 318-14 (2014) could predict the shear strength of the specimen well and with a low COV. The equations from Eurocode 2 (EC 2) (2004)

neglect the concrete contribution. Using the optimal inclination, obtained from equating $V_{Rd,s}$ and $V_{rd,max}$, it could result in increasingly un-conservative shear strength predictions as the shear reinforcement ratio increases.

Chapter 6

PROPOSED SHEAR METHOD FOR REINFORCED CONCRETE BEAMS WITH NO SHEAR REINFORCEMENT

6.1. Introduction

The simplified equation in ACI 318-14 (2014) (expressed as Equation (2.2)) to calculate shear strength of a reinforced concrete beam with no shear reinforcement is simple and straightforward. However, the equation considers only the tensile strength (taken as square root of compressive strength) and does not consider size effect. This often leads to inaccurate shear strength predictions, especially for beams with large effective depths. The equation in Eurocode 2 (2004) considers size effect, longitudinal reinforcement ratio and compressive strength. The equation is expressed as Equation (2.8). The results and discussions in Chapter 4 show that the size effect term in Eurocode 2 does not predict well, especially compared to the size effect term presented in ACI 446 (2007). There is a need to propose a new equation that is accurate and considers the important parameters that influence shear strength and at the same time, it must be simple and straightforward like the Equation (2.2) from ACI 318-14.

This chapter presents a simple and reliable empirical equation to predict the shear strength of a reinforced concrete beam with no shear reinforcement. The proposed expression considers important parameters that influence the shear strength of a beam. The experimental results presented in Chapter 4 are combined with existing published data to create a database of 980 beams' data. These data will be used to evaluate the accuracy of the proposed equation vis-à-vis shear equations from ACI 318-14, Eurocode 2, and some proposed methods by researchers.

6.2. Derivation of Proposed Equation

The proposed equation considers parameters such as the effective depth, compressive strength, longitudinal reinforcement ratio, and shear-span-to-effective-depth (a/d) ratio. The influence of these parameters on shear strength of a reinforced concrete beam are discussed in the following sections, to justify its inclusion in the proposed equation. The 980 data collected in

a database (DB-1) will be used to calibrate the proposed expression and to obtain a suitable relationship for each variable to illustrate the influence of that particular parameter.

6.2.1. Influence of Effective Depth

The influence of effective depth (size effect) on the shear strength of a reinforced concrete beam with no shear reinforcement has been discussed in Chapter 2. Results from published experimental programmes that had effective depth as its only variable were collected. The results and discussion have shown that ultimate shear strength reduces as effective depth increases. A size effect influence trend line from ACI 446, based on the work of Bažant *et al.* (2007), was able to estimate the phenomenon of reduction in shear strength as effective depth increased. The results from the new high strength concrete beams discussed in Chapter 4 and high strength concrete beam data collected from other researchers fitted well into the size effect trend.

The influence of size effect on shear strength is considered in code equations (Model code 1990, 1993; Japanese Code, 1991; BS8110, 1997; Eurocode 2, 2004; and Model code 2010, 2012). For example, the size effect term in Eurocode 2 represents a strength reduction factor for reinforced concrete beams when effective depth of the beam is greater than 200 mm. The influence of effective depth on shear strength of a reinforced concrete beam is widely recognised. Hence, the influence of effective depth must be included in the proposed equation.

6.2.2. Influence of Compressive Strength

There have been several test programmes involving specimens made of high strength concrete beams but with no shear reinforcement. In these research programmes, compressive strength of the specimens was the only variable to investigate the influence of compressive strength on shear strength.

Thorenfeldt and Drangsholt (1990) tested reinforced concrete beam specimens with compressive strength varying between 50 to 100 MPa. Diagonal cracking strengths of all the specimens were similar even though their compressive strengths were varied. However, ultimate shear stress (in terms of MPa) of specimens with compressive strength greater than 80 MPa was relatively lower compared to that of specimens with lower compressive strength.

Cladera and Mari (2005) conducted an experimental study on reinforced concrete beams with compressive strength between 50 to 90 MPa. Brittleness of failure mode increased with compressive strength of the specimen. Ultimate shear strength of the specimen also increased as compressive strength increased. Sherwood *et al.* (2007) shared the same observations in their experimental programme.

Janaka Perera and Mutsuyoshi (2013) tested beam specimens with compressive strength varying from 36 to 185 MPa. The diagonal cracking shear strength of the specimen with compressive strength of 114 MPa was about 10% higher than that of the specimens with compressive strength of 36 MPa. This increase in diagonal shear strength is negligible considering the three-fold increase in compressive strength. Arioglu *et al.* (2006) conducted regression analysis evaluating the relationship of splitting tensile strength and compressive strength between 4 and 120 MPa. A ratio of 10% between the splitting tensile strength and compressive strength was observed at low compressive strength. This ratio reduced until 5% as compressive strength increased beyond 80 MPa. The results showed that an increase in compressive strength does not translate to a linear increase in tensile strength.

Results from several experimental programmes where the specimens had similar properties in its effective depth, maximum aggregate size, and longitudinal reinforcement ratio were collected and presented in **Figure 6-1**. The effective depths of these data ranged between 150 and 300 mm, the maximum aggregate size was about 20 mm, and the longitudinal reinforcement ratios were between 2 and 3.5% (Thorenfeldt and Drangsholt, 1990; Konig *et al.*, 1993; Hallgren, 1994; Xie *et al.*, 1994; Matsui *et al.*, 1995; Fujita *et al.*, 2002; and Janaka Perera and Mutsuyoshi, 2013).

The shear strength of the specimen (V/bd) is plotted on the y-axis and the compressive strength on the x-axis. The trend line for each individual experimental series is represented by a dotted line and the trend line for all data is represented by a solid line. The trend lines show that the shear strength of a beam generally increases as concrete compressive strength increases. However, increase in shear strength reduces as compressive strength exceeds 60 MPa.

Figure 6-1—Influence of compressive strength on shear strength of reinforced concrete beams with no shear reinforcement.

There were several other test programmes that had investigated the influence of compressive strength, albeit with specimens of different properties as those presented in **Figure 6-1** (Mphonde and Frantz, 1985; Elzanaty *et al.*, 1986; Yoon *et al.*, 1996; Collins and Kuchma, 1999; and Angelakos *et al.*, 2001). The observations from these experimental programmes generally agree with the observations presented in **Figure 6-1**. The discussion in this section shows that compressive strength can influence the shear strength of a reinforced concrete beam and should be considered in the proposed equation.

6.2.3. Influence of Shear-Span-to-Effective-Depth Ratio

The shear-span-to-effective-depth (a/d) ratio can determine the behaviour of a reinforced concrete beam and the type of failure of the beam. Fenwick and Paulay (1968) showed that for beams with small a/d ratios, load transfer can take place via arch action as a second transfer mechanism after the beam action had broken down. Therefore, shear strength of beams with a/d ratio less than 2.5 were generally observed to be higher than that of beams with greater a/d ratio (Collins *et al.*, 2008). The failure of reinforced concrete beams (with no shear reinforcement) with a/d ratio greater than 2.5 is usually initiated by an inclined shear crack, followed by a sudden and brittle failure. For beams with a/d ratio greater than 6, the beam typically failed in flexure.

Figure 6-2 shows a collection of data with comparable properties in compressive strength, longitudinal reinforcement ratio, and effective depth to illustrate the influence of a/d ratio on shear strength (Moody *et al.*, 1954; de Cossio and Seiss, 1960; Leonhardt and Walther, 1962; Krefeld and Thurston, 1966; Cederwall *et al.*, 1974; Kani *et al.*, 1979; Nakazawa, 1980; Hamadi and Regan, 1980; and Mansur *et al.*, 1986). These experimental programmes consisted of specimens with varying a/d ratios among the specimens. The compressive strengths of these data ranged between 20 and 40 MPa, longitudinal reinforcement ratios were between 1 and 2%, and the effective depths of the beams were between 200 and 370 mm. Shear strength of the specimen $[V/(bd\sqrt{f'_c})]$ is plotted on the y-axis and a/d ratio of the specimen is plotted on the x-axis.

Figure 6-2—Influence of shear-span-to-effective-depth ratio on the shear strength of reinforced concrete beams with no shear reinforcement.

The trend line for each individual experimental programme is represented by a dashed line and the trend line for all the data is represented by a solid line in **Figure 6-2**. Although a deep beam is generally classified to have an a/d ratio less than 2.5, however, the trend lines in **Figure 6-2** show that the shear strength of specimens with a/d ratio less than 2.7 is significantly higher than that of specimens with a/d ratio greater than 2.7. This means that the beams with a/d ratio less than 2.7 can still exhibit deep beam behaviour and will behave somewhat differently from beams with larger a/d ratio. It was observed that there are differences in the shear strengths of specimens with a/d ratio between 2.8 and 6. The shear strength (in terms of MPa) reduces as the a/d ratio increases, therefore, a/d ratio should be considered as a parameter

in the proposed equation. The differences in shear strength between beams with higher a/d ratios, however, are not as significant as those observed between beams with a/d ratio in the region around 2.7.

Rommel (1991), Matsui *et al.* (1995), and Islam *et al.* (1998) tested high strength concrete beams with compressive strengths up to 85 MPa, 125 MPa, and 85 MPa, respectively, to investigate the shear strength of beams with a/d ratio between 3 and 4. The results from these three experimental programmes showed that the shear strength reduced as the a/d ratio increased, similar to the trend presented in **Figure 6-2**.

6.2.4. Influence of Longitudinal Reinforcement Ratio

Cao (2001) tested two-metre deep reinforced concrete beams and reported that the difference between the predicted shear strength from ACI 318-14 and the test results of lightly reinforced concrete beams could vary up to two times. This difference in predicted and experimental strengths was mitigated when more longitudinal reinforcement was placed in the beam. Collins and Kuchma (1999) reported that placing longitudinal reinforcing bars distributed along the beam depth would increase the shear strength, and size effect could be influenced by the longitudinal reinforcement ratio.

Krefeld and Thurston (1966) tested reinforced concrete beams in a few series with various a/d ratios. In each series, the specimens had different longitudinal reinforcement ratios up to 4.5%. The test results showed that the shear strength of the specimens increased as the longitudinal reinforcement ratio increased, regardless of a/d ratio. However, the increase in shear strength for specimens with higher longitudinal reinforcement ratio was not as significant as the increase in specimens with lower reinforcement ratio. Test results from the experimental programme by Bresler and Scordelis (1963), using specimens with a/d ratio of 3.8, showed similar diminishing influence on shear strength in specimens with higher reinforcement ratio. König *et al.* (1993), Hallgren (1994), and Scholtz (1994) tested high strength concrete specimens with different longitudinal reinforcement ratios and the test results showed similar increase in shear strength in specimens with higher reinforcement ratio.

Experimental data with similar properties in beam depth and a/d ratio from past research programmes investigating the influence of longitudinal reinforcement are presented in **Figure**

6-3. The shear strength of the specimen [$V/(bd\sqrt{f'_c})$] is plotted on the y-axis and the longitudinal reinforcement ratio of the specimen is plotted on the x-axis. The effective depths of the data were between 200 and 370 mm and the a/d ratio for all the data was about three (Mattock, 1969; Ahmad and Lue, 1987; Konig *et al.*, 1993; Kim and Park, 1994; Scholtz, 1994; Adebar and Collins, 1996; Tureyen and Frosch, 2002; and El-Sayed, 2005). The compressive strengths for the specimens ranged between 40 and 100 MPa. All the trend lines in **Figure 6-3** show that the shear strength of a reinforced concrete beam increased with increasing amount of longitudinal reinforcement. **Figure 6-3** also shows that the influence of longitudinal reinforcement ratio on shear strength of concrete did not seem to be affected by the compressive strength of concrete.

Figure 6-3—Influence of longitudinal reinforcement ratio on the shear strength of reinforced concrete beams with no shear reinforcement.

6.3. The Proposed Equation

The influence of the important parameters on shear strength of a reinforced concrete beam have been discussed in the earlier sections. The parametric shear strength expression to consider the influence of each parameter is proposed as Equation (6.1).

$$V_c = k_1(\rho_l)^A \left(\frac{a}{d}\right)^B f'_c{}^C s_z^D b_w^E d^F \quad (6.1)$$

where

- b_w = the width of the beam;
 d = the effective depth of the beam;
 a = the shear span of the beam;
 s_z = size effect term;
 f'_c = the compressive strength of the concrete; and
 ρ_l = the longitudinal reinforcement ratio.

The value of parameter k_l is a positive constant which will be a multiplying factor chosen to ensure the computed shear strength would have around 15% safety factor on the conservative side. The values A , B , and C represent the exponents for each influential shear strength parameter. The values of A , B , and C were iteratively changed, to achieve the best accuracy and consistency in prediction of shear strength. The accuracy and consistency is obtained by lowering the Coefficient of Variation (COV) relevant to the $V_{test}/V_{predicted}$ ratios. The parameter s_z represents the size effect term in the proposed equation to account for the reduction of nominal shear strength (in terms of MPa) at failure as the depth of the beam increases.

Experimental programmes focused only on the influence of individual parameter such as longitudinal reinforcement ratio, a/d ratio and compressive strength were sorted out from the database of 980 specimens. The data in these programmes were used to observe the dependence of shear strength on each parameter by best-fitting the data using the least sum of squares method. The values of $A=0.4$, $B=-0.3$, and $C=0.25$ represent the dependence of shear strength on longitudinal reinforcement ratio, a/d ratio, and compressive strength, respectively. Putting these values into the parametric shear strength expression, Equation (6-1), the expression predicts the shear strength of the 980 specimens with the lowest COV. COV was chosen as the optimisation target to minimise the variability from the mean ratio of test shear strengths against predicted shear strengths. In **Table 6-1**, the performance of the Equation (6-1) was compared against that from other expressions if the values of A , B , and C were changed. That means, in columns (2) to (6) of **Table 6-1**, the exponent value of compressive strength was varied between 0.25 to 0.5 while the exponent values of longitudinal reinforcement ratio and a/d ratio is kept constant at 0.4 and -0.3, respectively. This is to show the performance of various exponent values of compressive strength and ensure that the exponent results in the lowest COV. This process was repeated for longitudinal reinforcement ratio and a/d ratio.

Table 6-1 shows that the chosen exponent values for compressive strength (column (3)), longitudinal reinforcement ratio (column (8)), and a/d ratio (column (11)) results in the the lowest COV value. In addition to achieving the lowest COV, the influence of the parameters (compressive strength, longitudinal reinforcement ratio, and a/d ratio) with the chosen exponent is plotted (as red dashed line) to compare against the collected literature data in **Figure 6-4**. These collected literature data are shown previously in **Figures 6-1 to 6-3**; and **Figure 6-4 (a), (b), and (c)** presents the comparison for parameters f'_c , ρ_l , and a/d ratio respectively. It can be seen from each figure that the trendline for each parameter with the chosen exponent value matches the trendline for the collected literature data well, and better than other exponent values.

Size effect term of various forms and different exponent values are adopted in various methods to predict the influence of size effect. For example, the equations in Eurocode 2 (2004) adopted a size effect term in the form of $1 + \sqrt{(200/d)}$. The size effect term recommended in ACI 446 (2007) is $1/\sqrt{(1 + d/d_0)}$. The size effect term in Model Code 2010 (2010) is in the form of $1/(1 + d/d_0)$. British Standard Code (1997) and Japanese Code (2007) adopted different forms of size effect terms with the same exponent of $-1/4$, $\sqrt[4]{(400/d)}$ and $\sqrt[4]{(1000/d)}$ respectively. **Figure 6-4 (d)** shows a comparison of size effect terms in the form of power law with different exponent values and a size effect term in the form of $1/(1 + \sqrt{d/d_0})$ against the 980 data compiled in the database (DB-1). The comparison in **Figure 6-4 (d)** shows that the size effect terms of $1/(1 + \sqrt{d/d_0})$ and $d^{-0.3}$ are able to match the shear strength trend with respect to different effective depths of the beams.

In **Table 6-1**, the results in column (17) showed that the proposed equation with the size effect term of $1/(1 + \sqrt{d/d_0})$ is able to obtain a marginally lower COV compared to the size effect term of $d^{-0.3}$. In addition, the size effect term to the power of -0.5 is the same as the exponent of the proposed size effect term by Bažant and Yu (2005a, 2005b), derived from fracture mechanics. Therefore, the size effect term in the form of $1/(1 + \sqrt{d/d_0})$ with $d_0=250$ is used in the proposed empirical method. The final proposed empirical equation for shear strength with all the parameters is presented as Equation (6.2).

$$V_c = 7(\rho_l)^{0.4} \left(\frac{d}{a}\right)^{0.3} f'_c{}^{0.25} \frac{1}{1 + \left(\frac{d}{250}\right)^{0.5}} b_w d \quad (6.2)$$

Figure 6-4—Comparison of different exponent values against data in the database (a) compressive strength, (b) longitudinal reinforcement ratio, (c) a/d ratio, and (d) effective depth.

Table 6-1—Comparison of $V_{test}/V_{predicted}$ ratios with different exponent values for proposed equation.

	Compressive strength, f'_c					Longitudinal reinforcement ratio, ρ			Shear-span-to-effective-depth ratio, a/d			Effective depth, d					
	f'_c ^ (Exponent value)					ρ ^ (Exponent value)			(a/d) ^ (Exponent value)			d ^ (Exponent value)				$1/(1+(d/d_o)^{0.5})$ (Exponent value)	
Exponent value	0.20	0.25	0.33	0.40	0.50	0.30	0.40	0.50	-0.2	-0.3	-0.4	-0.25	-0.3	-0.4	-0.5	0.5	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	
Average $V_{test}/V_{pred} =$	1.19	1.14	1.10	1.18	1.11	1.14	1.14	1.09	1.09	1.14	1.13	1.12	1.15	1.13	1.10	1.14	
STD =	0.17	0.16	0.16	0.19	0.20	0.18	0.16	0.17	0.16	0.16	0.16	0.16	0.17	0.20	0.24	0.16	
COV =	0.15	0.14	0.15	0.16	0.18	0.16	0.14	0.16	0.15	0.14	0.14	0.15	0.15	0.17	0.22	0.14	

Note:

-STD = Standard Deviation.

-COV = Coefficient of Variation.

6.4. Comparison with Experimental Results

The experimental shear strengths of the seven new specimens discussed in Chapter 4 are listed with the predicted shear strengths using the equations from ACI 318-14 (2014) and Eurocode 2 (2004) in **Table 6-3**. The simplified equation in ACI, given as Equation (2.2), was used to predict the shear strength. The predicted shear strengths from the proposed equation (Equation (6.2)) are also presented in **Table 6-3**. Shear strengths for the seven new specimens predicted by the shear equations proposed by Bažant and Yu (2005a, 2005b) and Collins *et al.* (2008) are also shown in **Table 6-3** for comparison. The two proposed equations are presented as Equations (6.3) and (6.7), respectively.

$$V_c = \mu \rho^{\frac{3}{8}} b_w d \left(1 + \frac{d}{a}\right) \sqrt{\frac{f'_c}{1 + \frac{d}{d_0}}} \quad (\text{by Bažant and Yu, 2005}) \quad (6.3)$$

$$d_0 = k f_c'^{\frac{2}{3}} \quad (6.4)$$

$$k = 693.76 \sqrt{d_a} \quad (6.5)$$

$$\mu = 1.10 \quad (6.6)$$

where:

d_a = maximum aggregate size;

μ = coefficient to adjust the predictions to achieve a best fit for all the data in the database;

b_w = width of the beam;

d = effective depth of the beam;

a = shear span of the beam;

ρ = longitudinal reinforcement ratio;

f'_c = compressive strength of the concrete; and

d_o = empirical transitional size parameter.

$$\frac{V_c}{b_w d} = \left[\sqrt{1 + \frac{7.02 m \sqrt{f'_c}}{(1000 + s_{xe})}} \right] - 1 \left[\frac{133.3}{m} \right] \quad (\text{by Collins } et al., 2008) \quad (6.7)$$

$$s_{xe} = \frac{31.5d}{(16 + a_{g,eff})} \quad (6.8)$$

$$m = \frac{1}{\rho_w} \left(1.11 \frac{M}{Vd} + 1 \right) \quad (6.9)$$

where:

s_{xe} = equivalent crack spacing;

b_w = width of the beam;

d = effective depth of the beam;

ρ_w = longitudinal reinforcement ratio;

f'_c = compressive strength of the concrete;

M, V = moment and shear force respectively at the critical section; and

$a_{g,eff}$ = effective aggregate size taken as the maximum aggregate size when the compressive strength is less than 60 MPa, and is reduced linearly until zero for concretes with compressive strength of 70 MPa.

Table 6-2—Measured and predicted ultimate loads.

Shear span ID	Measured loads, kN		Test-to-predicted shear strength ratios, $V_{test}/V_{predicted}$					
	Total load	V_{test}	EC 2 method ¹	EC 2 method ²	ACI method	Bažant and Yu	Collins et al.	Proposed Method
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1V0-450-20	268	142	0.77	0.93	0.94	0.76	1.07	0.90
1V0-450-10	320	168	0.91	1.10	1.11	1.01	1.27	1.05
1V0-900-20	340	194	0.59	0.72	0.64	0.68	0.95	0.75
1V0-900-10	520	284	0.87	1.06	0.94	1.15	1.39	1.09
1V0-1350-20	666	384	0.82	1.00	0.83	1.07	1.52	1.14
1V0-1350-10	900	501	1.08	1.31	1.09	1.61	1.98	1.45
1V0-1800-20	920	545	1.19	1.45	1.16	1.67	2.49	1.72
All seven data		Average	0.89	1.08	0.96	1.14	1.52	1.16
		COV	0.22	0.22	0.19	0.34	0.36	0.28
Six specimens with rectangular cross-sections		Average	0.84	1.02	0.93	1.05	1.36	1.06
		COV	0.19	0.19	0.19	0.32	0.27	0.22

Note:

- EC 2 method¹ and EC 2 method² refers to Eurocode 2 method with compressive strength limit of 90 and 50 MPa respectively.
- ACI method refers to ACI 318-14 method.
- Failure mode is diagonal tension for all the specimens.
- V_{test} is the shear force (or half of the total load) and is inclusive of self-weight of the beam.
- COV = Coefficient of Variation.

The ratio of experimental over predicted shear strengths ($V_{test}/V_{predicted}$) from the equations of ACI 318-14 (ACI), Eurocode 2 (EC 2), Bažant and Yu, Collins *et al.*, and author's proposed method are presented in columns (4) to (9) in **Table 6-3**. The performance of the EC 2 equation with a compressive strength limit of 90 MPa is presented in column (4). In the Singapore National Annex for Eurocode 2, the compressive strength limit to be used for shear strength equation is set as 50 MPa for specimens with compressive strength greater than 50 MPa. The results of the EC 2 equation with this limit of 50 MPa is presented in column (5).

The limit of 50 MPa in the EC 2 equations meant that the shear strength predictions were lower than those obtained using the limit of 90 MPa. It can be seen from columns (4) and (5) in **Table 6-3** that average $V_{test}/V_{predicted}$ ratios for EC 2 with the 50 MPa limit showed more conservative results. Together with ACI method, the EC 2 method obtained the lowest COV for the six specimens with rectangular cross sections among all the methods in **Table 6-3**. A strength limit similar to that specified in Singapore National Annex for EC 2 would be useful to provide conservative shear strength predictions for high strength concrete beams.

The equation in ACI performed unexpectedly well for all the specimens tested in this study compared to the other methods, even though the ACI equation does not consider size effect. The Specimen 1V0-1350-10 failed at an unexpectedly high load, therefore, the prediction from ACI for the specimen without considering size effect turned out to be very close to the failure load and was conservative. This explains the conservative prediction for 1V0-1350-10, compared to the un-conservative prediction for Specimen 1V0-1350-20. The unexpected higher failure load might be attributed to the influence of aggregate size as the maximum aggregate size for Specimens 1V0-1350-10 and 1V0-1350-20 was different.

The base of Specimen 1V0-1800-20 was enlarged to accommodate the longitudinal reinforcement bars and are shown in **Figure 6-5**. The enlarged base inevitably increased the shear strength of the specimen. This increased failure load was reflected by the high $V_{test}/V_{predicted}$ ratio for the predictions from the three methods (Bažant and Yu, Collins *et al.*,

and author) in **Table 6-3**. These three methods were not able to predict the shear strength contribution from the enlarged base.

Comparing the three non-code equations (Bažant and Yu, Collins *et al.*, and author), the main difference between these methods is the consideration of aggregate size in Bažant and Yu's and Collins *et al.*'s equations. Discussed earlier, the test results showed uncertainty on the influence of aggregate size on the shear strength of beams with various compressive strength. Therefore, it seemed like the consideration of aggregate size in Bažant and Yu's and Collins *et al.*'s equations reduced the accuracy of the two methods predicting the shear strength of the beams in this study. Apart from comparing with the specimens from this study, the five methods were also evaluated with the database of 980 beams with no shear reinforcement in the following sections.

Figure 6-5—(a) Front view and (b) Cross-sectional view of Specimen IV0-1800-20.

6.5. Comparison with Beam Database

The ratios of experimental over predicted shear strength values ($V_{test}/V_{predicted}$) for 980 beams in a collected database (DB-1) are presented in **Table 6-4**. Most of the data were collected from a database presented by Collins *et al.* (2008). All the data are arranged correspondingly to five parameters (effective depth, longitudinal reinforcement ratio, aggregate size, shear-span-to-effective-depth ratio, and compressive strength) to study the influence of each parameter on the predictions. For each parameter, the data are divided into 3 to 4 different divisions as shown in **Table 6-4** and **Figure 6-6**. The divisions in **Table 6-4** correspond to the divisions shown in **Figure 6-6**. Using all 980 data, the performance of the author's proposed method obtained the lowest COV of all the methods with a conservative average $V_{test}/V_{predicted}$ ratio greater than one (1.14). The coefficient in the author's proposed equation was specifically

chosen to obtain a safety factor of around 15% for all the shear strengths. Comparing the author's proposed method with the Bažant and Yu's method, which has the next lowest COV value, the advantage of the author's proposed method is the use of fewer parameters in the equation. This reduced the computation effort required for its use in design, and yet could obtain a high level of accuracy.

Table 6-3—Comparison of experimental and calculated failure load by all methods.

	Division	All	Effective depth, d			Longitudinal reinforcement ratio, ρ_l				Aggregate size, d_a		
			[A]	[B]	[C]	[A]	[B]	[C]	[D]	[A]	[B]	[C]
			$d < 500$	$500 \leq d < 1000$	$1000 \leq d$	$\rho_l < 1$	$1 \leq \rho_l < 3$	$3 \leq \rho_l < 5$	$5 \leq \rho_l$	$d_a < 15$	$15 \leq d_a < 25$	$25 \leq d_a$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Nos. of data		980	840	98	42	145	676	112	47	321	386	273
V_{test}/V_{pred} ACI 318-14	Min =	0.33	0.50	0.42	0.33	0.33	0.37	1.17	1.40	0.33	0.50	0.37
	Max =	3.95	3.95	1.55	1.16	1.71	2.73	3.05	3.95	2.80	3.95	2.69
	Average =	1.41	1.51	0.88	0.66	0.86	1.41	1.74	2.48	1.35	1.47	1.41
	COV =	0.36	0.30	0.31	0.33	0.28	0.27	0.21	0.24	0.36	0.38	0.31
	Average =		0.31			0.26				0.35		
V_{test}/V_{pred} Eurocode 2 ¹	Min =	0.54	0.61	0.54	0.55	0.55	0.54	0.83	1.06	0.54	0.58	0.65
	Max =	2.89	2.89	1.37	1.19	1.31	1.85	2.13	2.89	2.12	2.89	1.69
	Average =	1.08	1.12	0.90	0.82	0.93	1.05	1.23	1.75	1.05	1.13	1.06
	COV =	0.25	0.24	0.21	0.17	0.17	0.19	0.18	0.24	0.24	0.29	0.19
	Average =		0.23			0.19				0.24		
V_{test}/V_{pred} Eurocode 2 ²	Min =	0.55	0.68	0.62	0.55	0.55	0.62	0.83	1.15	0.55	0.68	0.65
	Max =	2.89	2.89	1.42	1.45	1.31	1.92	2.13	2.89	2.34	2.89	1.69
	Average =	1.11	1.14	0.94	0.85	0.94	1.07	1.29	1.77	1.07	1.17	1.06
	COV =	0.25	0.24	0.18	0.19	0.16	0.19	0.19	0.24	0.24	0.28	0.19
	Average =		0.23			0.18				0.24		
V_{test}/V_{pred} Bažant and Yu	Min =	0.57	0.57	0.68	0.74	0.76	0.57	0.68	0.80	0.71	0.57	0.71
	Max =	1.72	1.72	1.41	1.67	1.72	1.67	1.52	1.69	1.63	1.69	1.72
	Average =	1.03	1.03	1.00	1.04	1.03	1.02	1.01	1.13	1.07	1.02	0.99
	COV =	0.16	0.15	0.17	0.18	0.15	0.15	0.16	0.19	0.16	0.15	0.14
	Average =		0.16			0.15				0.15		
V_{test}/V_{pred} Collins <i>et al.</i>	Min =	0.70	0.80	0.70	0.76	0.76	0.70	0.89	1.06	0.70	0.83	0.80
	Max =	2.49	2.26	1.65	2.49	1.58	2.49	2.04	2.26	1.98	2.49	1.60
	Average =	1.20	1.22	1.06	1.07	1.09	1.19	1.27	1.52	1.21	1.24	1.13
	COV =	0.18	0.17	0.15	0.28	0.14	0.16	0.17	0.20	0.18	0.19	0.13
	Average =		0.17			0.16				0.17		
V_{test}/V_{pred} Proposed	Min =	0.71	0.71	0.72	0.81	0.76	0.71	0.79	0.92	0.72	0.71	0.76
	Max =	1.89	1.89	1.63	1.72	1.63	1.78	1.69	1.89	1.78	1.89	1.53
	Average =	1.14	1.14	1.11	1.15	1.16	1.13	1.14	1.26	1.10	1.17	1.15
	COV =	0.14	0.14	0.17	0.15	0.13	0.14	0.15	0.18	0.15	0.14	0.13
	Average =		0.14			0.14				0.14		

Note:

-Divisions A, B, C and D correspond to the zones in Figure 6-6.

-Min = Minimum V_{test}/V_{pred} ratio.

-Max = Maximum V_{test}/V_{pred} ratio.

-COV = Coefficient of Variation.

- EC 2 method¹ and EC 2 method² refers to Eurocode 2 method with compressive strength limit of 90 and 50 MPa respectively.

Table 6-3 (cont.)—Comparison of experimental and calculated failure load by all methods.

	Division	Shear-span-to-depth ratio, a/d			Compressive strength, f'_c			
		[A]	[B]	[C]	[A]	[B]	[C]	[D]
		$2.5 \leq a/d < 2.8$	$2.8 \leq a/d < 4.5$	$4.5 \leq a/d < 6$	$f'_c < 30$	$30 \leq f'_c < 60$	$60 \leq f'_c < 90$	$90 \leq f'_c$
(1)	(2)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Nos. of data		68	796	116	440	411	73	56
V_{test}/V_{pred} ACI 318-14	Min =	0.68	0.33	0.61	0.37	0.33	0.40	0.42
	Max =	3.95	3.95	2.22	3.95	3.95	2.80	2.27
	Average =	1.71	1.39	1.43	1.53	1.36	1.28	1.12
	COV =	0.33	0.37	0.21	0.35	0.33	0.42	0.38
	Average =	0.35			0.35			
V_{test}/V_{pred} Eurocode 2 ¹	Min =	0.77	0.54	0.68	0.64	0.55	0.54	0.59
	Max =	2.66	2.89	1.62	2.70	2.89	2.12	1.58
	Average =	1.23	1.08	1.01	1.11	1.09	1.05	0.92
	COV =	0.27	0.25	0.17	0.25	0.23	0.32	0.26
	Average =	0.24			0.25			
V_{test}/V_{pred} Eurocode 2 ²	Min =	0.77	0.55	0.68	0.64	0.55	0.62	0.71
	Max =	2.66	2.89	1.62	2.70	2.89	2.34	1.92
	Average =	1.25	1.11	1.01	1.11	1.09	1.18	1.12
	COV =	0.28	0.25	0.16	0.25	0.23	0.32	0.26
	Average =	0.24			0.25			
V_{test}/V_{pred} Bažant and Yu	Min =	0.75	0.57	0.75	0.71	0.75	0.71	0.57
	Max =	1.66	1.72	1.22	1.72	1.69	1.56	1.67
	Average =	1.08	1.03	0.97	1.04	1.03	0.99	0.92
	COV =	0.19	0.16	0.11	0.15	0.14	0.18	0.22
	Average =	0.15			0.15			
V_{test}/V_{pred} Collins <i>et al.</i>	Min =	0.87	0.70	0.92	0.76	0.84	0.70	0.83
	Max =	2.23	2.49	1.51	2.23	2.26	2.04	2.49
	Average =	1.27	1.20	1.19	1.21	1.18	1.23	1.30
	COV =	0.22	0.18	0.10	0.17	0.16	0.23	0.23
	Average =	0.17			0.17			
V_{test}/V_{pred} Proposed	Min =	0.72	0.71	0.89	0.72	0.84	0.72	0.71
	Max =	1.78	1.89	1.43	1.77	1.89	1.78	1.72
	Average =	1.15	1.14	1.12	1.14	1.17	1.10	1.05
	COV =	0.20	0.14	0.11	0.14	0.13	0.20	0.19
	Average =	0.14			0.14			

Note:

-Divisions A, B, C and D correspond to the zones in Figure 6-6.

-Min = Minimum V_{test}/V_{pred} ratio.

-Max = Maximum V_{test}/V_{pred} ratio.

-COV = Coefficient of Variation.

- EC 2 method¹ and EC 2 method² refers to Eurocode 2 method with compressive strength limit of 90 and 50 MPa respectively.

(a)

(b)

(c)

(d)

Figure 6-6—Failure load predictions of 980 test data with respect to influence from (a) Effective depth; (b) Longitudinal reinforcement ratio; (c) Aggregate size; (d) Shear-span-to-effective-depth ratio; and (e) Compressive strength.

6.5.1. Influence of Effective Depth

Figure 6-6 (a) shows the performance of the five equations in predicting the shear strength of specimens with respect to effective depth. The ratios of the experimental to predicted shear strength ($V_{test}/V_{predicted}$) are plotted in the y-axis, with effective depth in the x-axis. The horizontal dashed line in each zone represents the average value of the $V_{test}/V_{predicted}$ for that

zone. **Figure 6-6 (b), (c), (d), and (e)** show the predictions with respect to longitudinal reinforcement ratio, aggregate size, a/d ratio, and compressive strength, respectively.

The leftmost graph in **Figure 6-6 (a)** shows the performance of the ACI equation with respect to effective depth. The graph shows a trend of reduced $V_{test}/V_{predicted}$ ratios as the effective depth increases. Comparing the average ratios in columns (4) to (6) of **Table 6-4**, the average value of $V_{test}/V_{predicted}$ dropped from 1.51 to 0.66 as effective depth increases from 500 mm (Division A) to more than 1000 mm (Division C). The large difference in predictions of shear strength was caused by the omission of size effect term in the ACI equations. Without a size effect term, the design equation in ACI method would predict the same nominal shear strength (in terms of MPa) for beams with the same compressive strength even if the effective depths are different, as shown in **Figure 6-7**. **Figure 6-7** presents the influence of effective depth on high strength concrete beams, and was previously shown and discussed in Chapter 2 (**Figure 2-11**). The test results show that the shear strength (in terms of MPa) of a larger beam will be lower than that of a smaller beam. This resulted in over-estimated shear strength prediction by the ACI method, therefore an average $V_{test}/V_{predicted}$ ratio of 0.66 for beams with effective depth more than 1000 mm. The design equations from the other four methods includes a size effect term, and therefore the results did not reflect a significant drop in the $V_{test}/V_{predicted}$ ratios as per the ACI method. The combined COV average (the COV in each division multiply by the number of data in that division, summation of all the divisions and then divided by the total number of data) for the ACI method was the greatest amongst the five methods.

Although the design equation in EC 2 considers the influence of size effect, **Figure 6-7** shows that the size effect term do not fully reflect the influence of size effect. This resulted in the average $V_{test}/V_{predicted}$ ratio for specimens with effective depth larger than 1000 mm being un-conservative, at 0.82. These predictions were considerably more consistent than that from the ACI method, evident by the much lower combined COV across three divisions for the EC 2 method. The results show that the influence of effective depth on shear strength is significant, and a size effect term is necessary to provide conservative shear strength predictions for large-sized beams.

Figure 6-7—Influence of effective depth on high strength concrete specimens.

Most of the predictions from Collins *et al.*'s method were conservative throughout the three divisions. The combined average COV for Collins *et al.*'s method was significantly lower than that for the EC 2 method, viz 0.17 and 0.23, respectively. The size effect term in Bažant and Yu's method was derived from fracture mechanics and was recommended for predicting the influence of size effect by ACI 446 (2007). The trendline notated as ACI 446 in **Figure 6-7** shows the performance of the size effect term by Bažant and Yu and it can be seen that the size effect term from Bažant and Yu is able to predict the size effect well. This was also reflected in the results in **Table 6-4** where the predictions for all three divisions had low COV.

6.5.2. Influence of Longitudinal Reinforcement Ratio

The EC 2 method limits the longitudinal reinforcement ratio in its shear strength calculation to two percent. It can be seen in columns (9) and (10) of **Table 6-4** and **Figure 6-6 (b)** that the average $V_{test}/V_{predicted}$ ratios for specimens with longitudinal reinforcement ratio greater than 2% are high; 1.23 for specimens with longitudinal reinforcement ratio between 3 and 5% (Division C), and 1.75 for specimens with longitudinal reinforcement ratio greater than 5% (Division D). Most of the data with longitudinal reinforcement ratio greater than 5% were T-beams. The failure load of T-beams are higher, hence explaining why the predictions for the data in this division are relatively low compared to the experimental results. This observation is similarly present in the predictions from other methods for this division. The average $V_{test}/V_{predicted}$ ratio of EC 2 for specimens with lower reinforcement ratios are close to unity, 0.93 for specimens with reinforcement ratios less than 1% and 1.05 for specimens with

reinforcement ratios between 1 and 3%. It can be seen that the limit of longitudinal reinforcement ratio for EC 2 design equation is meant to allow conservatism in the beam design.

The ACI method is the only method out of the five not to consider the influence of longitudinal reinforcement ratio on shear strength. The results in **Table 6-4** showed two spectrums of shear strength predictions. The average $V_{test}/V_{predicted}$ ratio was un-conservative at 0.85 for specimens with longitudinal reinforcement ratio less than one (Division A). For specimens with longitudinal reinforcement ratio greater than one, the average $V_{test}/V_{predicted}$ ratio became conservative, and this conservatism increased as the longitudinal reinforcement ratio increased. This suggest that the ACI method does not predict well for beams with low longitudinal reinforcement ratio ($\rho_l < 1\%$) and also beams with high longitudinal reinforcement ratio ($\rho_l > 5\%$). The shear strength predictions from the other three methods (Bažant and Yu's, Collins et al.'s, and author's) are conservative regardless the reinforcement ratio of the beam. The combined COV average for these three methods were low: 0.15, 0.16 and 0.14 for Bažant and Yu's, Collins et al.'s, and author's method, respectively.

6.5.3. Influence of Aggregate Size

Figure 6-6 (c) shows the influence of aggregate size on the predictions from the five methods. Out of the five methods, only Bažant and Yu's and Collins *et al.*'s methods consider the influence of aggregate size in its equation. The aggregate size is considered in Bažant and Yu's method as the term d_a to calculate the empirical transitional size parameter as part of the size effect term. In Collins *et al.*'s method, the influence of aggregate size is considered as the term $a_{g,eff}$, taken as the maximum aggregate size for concrete with compressive strength less than 60 MPa, and then reduced to zero when the compressive strength is 70 MPa. The results in **Table 6-4** show that both methods seemed to predict better as the aggregate size increased, reflected by the reduction in $V_{test}/V_{predicted}$ ratio and COV in each division as the aggregate size increased.

The author's proposed method did not include an aggregate size parameter, yet, it was able to achieve consistency of the results similar to that of Bažant and Yu's method. The influence of aggregate size on shear strength is shown in Chapter 4 to be unclear based on the results from current study and existing literature data. The confusion on the aggregate size's

influence on shear strength might explain why there is a lack of significant improvement in the predictions from Bažant and Yu's and Collins *et al.*'s methods with the inclusion of aggregate size term.

6.5.4. Influence of Shear-Span-to-Effective-Depth Ratio

Figure 6-6 (d) shows the predictions from the various methods influenced by the shear-span-to-effective-depth (a/d) ratio. As discussed in section 6.2.3, beams with a/d ratio between 2.5 and 2.8 (Division A) could still display deep beam behaviour. This is reflected in conservatism of the predictions from the various methods for the beams in this division. The ACI method registered the largest average $V_{test}/V_{predicted}$ ratio of 1.71 for beams with a/d ratio between 2.5 and 2.8 (Division A). In fact, the predictions from ACI method for all three divisions were very conservative, with all three $V_{test}/V_{predicted}$ ratios exceeding 1.35.

The design equations in EC 2 method did not consider the influence of a/d ratio. Although the average predictions from EC 2 method were conservative, the failure load of a beam with a/d ratio greater than 2.8 can be less than 70% of the predicted shear strength. The over-estimation of shear strength may create concern with using EC 2 method for predicting the shear strength of beams with large a/d ratios. The other three methods (Bažant and Yu's, Collins *et al.*'s, and author's) considers the influence of a/d ratio in their equations, and obtained a low COV in all the divisions (0.14 to 0.17). These results show that the a/d ratio plays a significant part in influencing the shear strength of a beam.

6.5.5. Influence of Compressive Strength

The ACI method limits the compressive strength used in its equation to 70 MPa. It assumes that the contribution from concrete towards shear strength do not increase once the concrete compressive strength goes beyond 70 MPa. However, this limitation in the ACI method made the design equation gave poor predictions for high strength concrete specimens: average $V_{test}/V_{predicted}$ ratio of 1.28 with a COV of 0.42 for beams with compressive strength between 60 and 90 MPa, and an average $V_{test}/V_{predicted}$ ratio of 1.12 with a COV of 0.38 for beams with compressive strength greater than 90 MPa. This shows that the concrete contribution towards shear strength would continue to increase as the concrete compressive strength increases, but it might not be at a linear rate of increase.

The EC 2 method limits the compressive strength used to 90 MPa, directly influencing the predictions for beams in Division D. The average $V_{test}/V_{predicted}$ ratio for EC 2 method in Division D was 0.92, which was slightly un-conservative. As such, if the compressive strength limit is removed, the over-estimation of shear strength from EC 2 would be higher. Under the Singapore National Annex for Eurocode 2 where the compressive strength limit is set at 50 MPa, the average $V_{test}/V_{predicted}$ ratios for Divisions C and D were 1.18 and 1.12, respectively. It seems that a low compressive strength limit of 50 MPa would be more suitable for the EC 2 design equations to predict the shear strength of high strength concrete beams so as to prevent over-estimation of shear strength.

There is a consideration in Collins *et al.*'s method to reduce the influence of aggregate size when the concrete compressive strength is greater than 60 MPa, and then eliminate the influence of aggregate size when the compressive strength is greater than 70 MPa. This consideration is based on the crack propagating directly through the aggregate in high strength concrete, rather than around the aggregate in normal strength concrete. The assumption is based on the crack passing through the aggregate will result in the loss of aggregate interlock, and hence a reduction in shear strength. This consideration resulted in low shear strength predictions, reflected by the high $V_{test}/V_{predicted}$ ratio in columns (19) and (20). Moreover, it does not seem to result in better shear strength predictions in terms of low COV obtained by both Bažant and Yu's and author's methods.

6.6. Adaptation of Proposed Method

The discussion in Section 6.5 has shown that the author's proposed equation can predict the shear strengths of beams with no shear reinforcement. However, in reality, all beams will be designed with shear reinforcement in accordance to the design codes. Hence, there will be limited applications of the proposed method on its own for practical beam design. Chapter 7 will present two proposed methods to predict the shear strength of beams with shear reinforcement. The first method would be an analytical truss method considering size effect and dowel action. The second method will be a simplified design method taking into account the contribution from both concrete and shear reinforcement ($V_n = V_c + V_s$), a consideration

similar to ACI 318-14. Equation (6.2) will be used to predict the shear strength contribution of the concrete portion in the proposed second method.

6.7. Summary of Chapter 6

Based on the discussion on the effect on shear strength of a reinforced concrete beam from various influential parameters, an empirical equation was proposed. The proposed equation considered the influence of compressive strength, shear-span-to-effective-depth ratio, longitudinal reinforcement ratio, and effective depth in its calculation for shear strength. A database of 980 data was collected and was used to evaluate the accuracy of the proposed equation. Comparing the experimental and predicted shear strength for all 980 data, the proposed method obtained an average of 1.14 with a COV of 0.14. This result was compared with the predictions from the design equations of ACI 318-14 (2014), Eurocode 2 (2004) as well as proposed methods by Bažant and Yu (2005a, 2005b), and Collins *et al.* (2008). The data from the database were classified into various divisions for each of the important parameters and the influence of each parameter was discussed. From the discussions, the proposed method was shown to be able to achieve good results and low COV in every division.

The strength of the proposed empirical equation lies in the simplicity of the equation and its reliability to predict the shear strength of reinforced concrete beams with no shear reinforcement. Although the number of variables considered in the proposed equation is more than that considered in the ACI 318-14 design equation (Equation (2.2)), the performance of the proposed method is significantly better. This improvement in performance would compensate for the longer calculation process. A limitation of the proposed method would be the proposed equation not considering the effect of aggregate size towards shear strength. The effect of aggregate size was considered in the equation proposed by Bažant and Yu (2005a, 2005b) and Collins *et al.* (2008). However, the influence of aggregate size on shear strength was not clearly shown by the data collected from existing literature and the results from this study, hence the exclusion of aggregate size in the proposed equation. The proposed equation can be improved further to achieve better performance when more data is available to clarify the influence of aggregate size on shear strength.

Chapter 7

PROPOSED SHEAR METHODS FOR REINFORCED CONCRETE BEAMS WITH SHEAR REINFORCEMENT

7.1. Introduction

This chapter presents two proposed method to predict shear strength of a beam with shear reinforcement: 1) an analytical truss model and 2) a simplified truss model. The equations governing the analytical truss model, based on rotating softened truss model by Hsu and Mo (2010), are discussed in Sections 7.2.1 to 7.2.4. The analytical truss model considers the compression field action in the web panel of the beam to obtain shear strength. The author's contribution to the proposed analytical truss model is to propose the addition of dowel action and consideration of size effect when predicting the shear strength of the beam. Longitudinal reinforcing bars placed near the soffit of a reinforced concrete beam were outside the web panel of the beam and hence, the contribution from longitudinal reinforcing bars were not taken into consideration in the analytical truss model by Hsu and Mo (2010). However, the longitudinal reinforcing bars contribute to the shear strength in the form of dowel action. Without considering the shear strength contribution from longitudinal reinforcement, the predicted shear strength would be very conservative for concrete beams with high amount of longitudinal reinforcement. Therefore, dowel force resulting from dowel action from the longitudinal reinforcing bars is proposed in this chapter as an additional component to the shear strength of the reinforced concrete beam.

Size effect was also not considered in analytical truss model by Hsu and Mo (2010). Without considering size effect, the predicted shear strength from the truss model would be unconservative for large-sized concrete beams. The proposed truss model includes a size effect term proposed in Section 7.2.7 to consider the size effect when calculating the depth of the web panel that contributes to shear strength.

The analytical truss model requires initial assumptions of certain variables and an iteration process to solve fourteen variables in eleven governing equations. Although current computational technology can complete the solution process in a relatively short period of time, computation by hand would require far more effort and is time consuming. The results from

the truss model in the first approach includes not just the shear strength of the beam, but also the full stress-strain relationship of the web element.

A second approach, a simplified truss model designed as a sectional method, is proposed in this chapter to predict the shear strength of a reinforced concrete beam. The motivation to develop a second approach is to obtain the inclination angle of the diagonal truss similar to that obtained from the first approach (truss model), and then obtain the total shear strength of the beam, without going through the iterative process in the first approach.

In the simplified truss model (second approach), an empirical equation is proposed to estimate the inclination angles of the diagonal trusses. The equation considers main parameters such as the shear reinforcement ratio, yield strength of shear reinforcement, shear-span-to-effective-depth ratio and spacing of vertical shear reinforcement. The objective of this equation is to estimate the inclination of diagonal trusses that are similar to those calculated using the analytical truss model. This equation reduces the effort of computing, yet is able to obtain the inclinations of the diagonal trusses. The inclinations of the diagonal trusses can then be used to determine the shear strength of the reinforced concrete beam.

The simplified truss method will consider the contribution from both concrete and shear reinforcement, similar to ACI 318-14 in the form of $V_n = V_c + V_s$. The empirical equation proposed in Chapter 6 - Equation (6.2) is used to predict the concrete's contribution to shear strength (V_c) as part of the simplified method. The simplified model allows an engineer to perform analysis at any section to obtain the inclination angles of the diagonal trusses. Using the inclination angles of the diagonal trusses, the steel reinforcement's contribution to shear strength can be calculated, and subsequently shear strength of the beam can be obtained. This method is similar to the current ACI 318-14 (2014) or Eurocode 2 (2004) methods in its application. The novelty of the simplified truss method is the ability to obtain both the diagonal truss inclination and shear strength of the beam that are not available if the code methods are used instead.

The new experimental results presented in Chapter 5 were combined with existing published data to form a database of 189 reinforced concrete beams with shear reinforcement. These 189 data were selected because beam details such as effective depth, concrete

compressive strength, shear and longitudinal reinforcement ratios, as well as spacing between the shear reinforcing bars are provided in the published literature. The results of the 189 data were used to evaluate the accuracy of the methods of ACI 318-14 (2014), Eurocode 2 (2004), the proposed analytical truss model, simplified truss method, as well as methods proposed by other researchers.

7.2. Truss Model for Predicting Shear Strength of Reinforced Concrete Beam

The analytical truss model based on the softened truss model (Hsu and Mo, 2010) is described in Sections 7.2.1 to 7.2.4. The model considers plane stress equations of equilibrium, compatibility, and constitutive laws for concrete and reinforcing steel materials.

7.2.1. Equilibrium Conditions

A typical reinforced concrete beam with shear reinforcement is shown in **Figure 7-1**. External shear force (V) is applied on the loading point at the top of the beam. To obtain equilibrium for external forces, vertical force at the support must be the same as the applied force at the top of the beam.

Figure 7-1—Typical reinforced concrete beam with its force components and shear reinforcement.

Shear force is assumed to be carried by a series of diagonal concrete trusses and vertical shear reinforcement between the loading point and the support. The diagonal trusses are represented in **Figure 7-1** by dashed lines. The vertical shear reinforcing bars are placed across the entire span between the loading point and support, represented by the solid black line as a vertical tie in **Figure 7-1**. The stresses (in a neutral direction) in the diagonal concrete trusses, vertical and horizontal shear reinforcement form the element ABCD in Panel 1 and are illustrated in **Figure 7-2**. The r - d axis are assumed to be the principal directions.

Figure 7-2—Stresses in element ABCD in Panel 1 in **Figure 7-1**, assuming that the principal directions r - d coincide with the direction of the cracks.

The vertical and horizontal web reinforcing bars are assumed to take only axial forces. As the concrete cracks, the principal stress directions of the concrete are assumed to coincide with the direction of the cracks. Therefore, shear stresses τ_{rd} and τ_{dr} in the principal major and minor axes are zero. The stress transformations from the r - d coordinate system into the x - z coordinate system are performed using the triangle wedges D - p - n and C - o - m for the z - and x -axes, respectively. These two triangle wedges are shown in **Figure 7-3** with all the stress components.

Figure 7-3—(a) Triangle wedge D-p-n; (b) Triangle wedge C-o-m.

In **Figure 7-3 (a)**, the thickness and length of $p-n$ are assumed to be one unit length. Thus, the area of the $p-n$ side (length multiplied by the thickness) is one-unit. Representing the angle between $D-n$ and $p-n$ as θ , the areas of $D-p$ and $D-n$ are $\sin \theta$ and $\cos \theta$, respectively. Taking equilibrium in the z -direction in **Figure 7-3 (a)**, vertical force equilibrium requires

$$\sigma_z = \sigma_d \sin^2 \theta + \sigma_r \cos^2 \theta + \rho_z f_{sz} \quad (7.1)$$

where:

- σ_z = applied normal stress in the vertical z -axis, positive for tension;
- σ_d = principal stress of concrete in d -axis, positive for tension;
- σ_r = principal stress of concrete in r -axis, positive for tension;
- θ = inclination of diagonal concrete strut (d -axis) with respect to horizontal axis;
- ρ_z = vertical shear reinforcement ratio; and
- f_{sz} = average stress in the vertical reinforcement.

Taking equilibrium in the x -direction in **Figure 7-3 (b)**, horizontal force equilibrium results in Equation (7.2). The term ρ_x in Equation (7.2) represents the steel ratio of the horizontal steel reinforcing bars in the web panel. The flexural longitudinal reinforcement placed near the base of the beam are not considered in the calculation of ρ_x . The shear stress (τ_{xz}) in the triangle wedge $C-o-m$ can be derived from Equation (7.3) by equilibrating all the forces in the z -direction.

$$\sigma_x = \sigma_d \cos^2 \theta + \sigma_r \sin^2 \theta + \rho_x f_{sx} \quad (7.2)$$

$$\tau_{xz} = (\sigma_d - \sigma_r) \sin \theta \cos \theta \quad (7.3)$$

where:

σ_x = applied normal stress in horizontal x -axis, positive for tension;

τ_{xz} = average shear stress in x - z coordinate system;

ρ_x = horizontal shear reinforcement ratio; and

f_{sx} = average stress in the horizontal reinforcement.

7.2.2. Compatibility Conditions

The principal strains in the r - d coordinates are transformed into the x - z coordinates and the transformation equations are similar to those for the stresses. σ_z corresponds to ε_z , σ_x to ε_x , and τ_{xy} corresponding to $\gamma_{xy}/2$. The compatibility equations are presented in Equations (7.4), (7.5), and (7.6).

$$\varepsilon_z = \varepsilon_d \sin^2 \theta + \varepsilon_r \cos^2 \theta \quad (7.4)$$

$$\varepsilon_x = \varepsilon_d \cos^2 \theta + \varepsilon_r \sin^2 \theta \quad (7.5)$$

$$\frac{\gamma_{xz}}{2} = (\varepsilon_d - \varepsilon_r) \sin \theta \cos \theta \quad (7.6)$$

where:

ε_z = normal strain in the vertical z -axis, positive for tension;

ε_x = normal strain in the horizontal x -axis, positive for tension;

θ = inclination of diagonal concrete strut (d -axis) with respect to horizontal axis;

γ_{xz} = shear strain in x - z coordinate system;

ε_d = principal strain in d -axis, positive for tension; and

ε_r = principal strain in r -axis, positive for tension.

7.2.3. Constitutive Law of Materials

7.2.3.1. Constitutive law of steel

The average stress-strain relationship for the steel bars in both vertical and horizontal directions is assumed to be a bilinear model as shown in **Figure 7-4**.

Figure 7-4—Stress-strain relationship for steel reinforcement.

The stress-strain equations for the horizontal and vertical steel reinforcement are given as Equations (7.7) and (7.8).

$$f_{sz} = E_s \varepsilon_z \quad \text{for} \quad \varepsilon_z < \frac{f_{yz}}{E_s} \quad (7.7a)$$

$$f_{sz} = f_{yz} \quad \text{for} \quad \varepsilon_z \geq \frac{f_{yz}}{E_s} \quad (7.7b)$$

$$f_{sx} = E_s \varepsilon_x \quad \text{for} \quad \varepsilon_x < \frac{f_{yx}}{E_s} \quad (7.8a)$$

$$f_{sx} = f_{yx} \quad \text{for} \quad \varepsilon_x \geq \frac{f_{yx}}{E_s} \quad (7.8b)$$

where:

- f_{sz} = average stress in the vertical reinforcement;
- f_{sx} = average stress in the horizontal reinforcement;
- f_{yz} = yield strength of vertical reinforcement;

- f_{yx} = yield strength of horizontal reinforcement;
 E_s = modulus of elasticity of steel bars;
 ε_z = average strain in the vertical reinforcement; and
 ε_x = average strain in the horizontal reinforcement.

7.2.3.2. Constitutive law of concrete

Various researchers have suggested different stress-strain relationships for concrete in tension after the concrete has cracked. The concrete cracking strain is taken as 0.008% (Hsu and Mo, 2010). There are various relationships because the post-peak relationship for concrete in tension is dependent on the type of test machine used and the properties of the concrete itself (Gopalaratnam and Shah, 1985). The relationship suggested by Belarbi and Hsu (1994) and Pang and Hsu (1995) is presented in Equation (7.9) while the relationship suggested by Bentz and Collins (2006) is presented in Equation (7.10).

Chandra *et al.* (2018) proposed a relationship based on observations from experimental data of stress-strain behaviour of concrete in tension. Assuming the peak tensile strength to be 10% of its compressive strength and the residual tensile strength to be 20% of its peak tensile strength, the residual tensile strength is taken to be 2% of compressive strength. In the analytical truss model discussed in this chapter, the stress-strain relationship for concrete in tension before the crack formation is assumed to be linear, and is presented by Equation (7.12a). After the concrete has cracked, the tensile stress is reduced and is assumed to be 2% of compressive strength, presented as Equation (7.12b).

$$\sigma_r = f_{cr} \left(\frac{0.00008}{\varepsilon_r} \right)^{0.4} \quad \text{for} \quad \varepsilon_r \geq 0.00008 \quad (7.9)$$

$$\sigma_r = \frac{f_{cr}}{\sqrt{1 + 500\varepsilon_r}} \quad \text{for} \quad \varepsilon_r \geq 0.00008 \quad (7.10)$$

$$f_{cr} = 0.33\sqrt{f'_c} \quad (7.11)$$

$$\sigma_r = E_c \varepsilon_r \quad \text{for} \quad \varepsilon_r < 0.00008 \quad (7.12a)$$

$$\sigma_r = 0.02f'_c \quad \text{for} \quad \varepsilon_r \geq 0.00008 \quad (7.12b)$$

$$E_c = 3320\sqrt{f'_c} + 6900(MPa) \quad (7.13)$$

where:

ε_r = principal strain in r -axis, positive for tension;

E_c = modulus of elasticity of concrete according to ACI 363 (1992); and

f_{cr} = cracking stress of concrete.

The stress-strain relationship for concrete in compression, presented in **Figure 7-5**, adopts the model proposed by Belarbi and Hsu (1994), Belarbi and Hsu (1995), Pang and Hsu (1995), and Zhang and Hsu (1998). The maximum principal stress ($\sigma_{d,max}$) for concrete in compression is given by Equation (7.14). The stress-strain relationships presented in Equation (7.15) consider the softening behaviour of a cracked concrete due to tensile strains in the direction perpendicular to the compression. The value of σ_d is limited to the value of $\sigma_{d,max}$. The softening factor (ζ) in Equation (7.15) is defined in Equation (7.16). The strain of the concrete at maximum compressive stress is given by Equation (7.17), as suggested by Foster and Gilbert (1996).

Figure 7-5—Compressive stress-strain relationship for concrete.

$$\sigma_{d,max} = -\zeta f'_c \quad (7.14)$$

$$\sigma_d = \sigma_{d,max} \left[2 \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right) - \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right)^2 \right] \quad \text{for} \quad (\varepsilon_d / \zeta \varepsilon_0) \leq 1 \quad (7.15a)$$

$$\sigma_d = \sigma_{d,max} \left[\frac{\frac{\varepsilon_d}{\zeta \varepsilon_0} - 1}{\frac{2}{\zeta} - 1} \right] \quad \text{for} \quad (\varepsilon_d / \zeta \varepsilon_0) > 1 \quad (7.15b)$$

$$\zeta = \min \left(\frac{5.8}{\sqrt{f'_c}}, 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \varepsilon_r}} \right) \quad (7.16)$$

$$\varepsilon_0 = -(0.002 + 0.001 \frac{f'_c - 20}{80}) \quad (7.17)$$

where:

σ_d = principal stress of concrete in d -axis, positive for tension;

$\sigma_{d,max}$ = maximum principal stress of concrete in d -axis;

ζ = softening factor of concrete;

f'_c = concrete cylinder compressive strength;

ε_d = principal strain of concrete in d -axis, positive for tension;

ε_r = principal strain of concrete in r -axis, positive for tension; and

ε_o = strain of concrete at maximum compressive stress.

7.2.4. Summary of Governing Equations

There are eleven governing equations from the equilibrium, compatibility, and constitutive laws for the materials in the analytical truss model by Hsu and Mo (2010). These eleven equations are summarised below:

$$\sigma_z = \sigma_d \sin^2 \theta + \sigma_r \cos^2 \theta + \rho_z f_{sz} \quad (7.1)$$

$$\sigma_x = \sigma_d \cos^2 \theta + \sigma_r \sin^2 \theta + \rho_x f_{sx} \quad (7.2)$$

$$\tau_{xz} = (\sigma_d - \sigma_r) \sin \theta \cos \theta \quad (7.3)$$

$$\varepsilon_z = \varepsilon_d \sin^2 \theta + \varepsilon_r \cos^2 \theta \quad (7.4)$$

$$\varepsilon_x = \varepsilon_d \cos^2 \theta + \varepsilon_r \sin^2 \theta \quad (7.5)$$

$$\frac{\gamma_{xz}}{2} = (\varepsilon_d - \varepsilon_r) \sin \theta \cos \theta \quad (7.6)$$

$$f_{sz} = E_s \varepsilon_z \quad \text{for} \quad \varepsilon_z < \frac{f_{yz}}{E_s} \quad (7.7a)$$

$$f_{sz} = f_{yz} \quad \text{for} \quad \varepsilon_z \geq \frac{f_{yz}}{E_s} \quad (7.7b)$$

$$f_{sx} = E_s \varepsilon_x \quad \text{for} \quad \varepsilon_x < \frac{f_{yx}}{E_s} \quad (7.8a)$$

$$f_{sx} = f_{yx} \quad \text{for} \quad \varepsilon_x \geq \frac{f_{yx}}{E_s} \quad (7.8b)$$

$$\sigma_r = E_c \varepsilon_r \quad \text{for} \quad \varepsilon_r < 0.00008 \quad (7.12a)$$

$$\sigma_r = 0.02 f'_c \quad \text{for} \quad \varepsilon_r \geq 0.00008 \quad (7.12b)$$

$$\sigma_d = -\zeta f'_c \left[2 \left(\frac{\varepsilon_d}{\xi \varepsilon_o} \right) - \left(\frac{\varepsilon_d}{\xi \varepsilon_o} \right)^2 \right] \quad \text{for} \quad (\varepsilon_d / \zeta \varepsilon_o) \leq 1 \quad (7.15a)$$

$$\sigma_d = -\zeta f'_c \left[1 - \left(\frac{\frac{\varepsilon_d}{\xi \varepsilon_o} - 1}{\frac{2}{\xi} - 1} \right)^2 \right] \quad \text{for} \quad (\varepsilon_d / \zeta \varepsilon_o) > 1 \quad (7.15b)$$

$$\zeta = \min \left(\frac{5.8}{\sqrt{f'_c}}, 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \varepsilon_r}} \right) \quad (7.16)$$

7.2.5. Dowel Action

The empirical expression by Baumann and Rusch (1970), shown in Equation (7.18), is used in this model to consider the contribution of dowel action from longitudinal reinforcing bars. These longitudinal reinforcing bars were not considered in the plane stress equations as discussed in Section 7.2.1 to 7.2.4. The contribution of dowel action is a proposed addition to the analytical truss model by Hsu and Mo (2010). The total area of longitudinal reinforcing bars provided in the beam is converted into one equivalent effective bar diameter, expressed as d_b . The net width of the beam (b_n) is taken to be the width of beam minus the equivalent effective bar diameter, as shown in **Figure 7-6**.

$$D_u = 1.64 b_n d_b \sqrt[3]{f'_c} \quad (7.18)$$

where:

D_u = shear strength contribution from dowel action;

d_b = equivalent effective bar diameter;

f'_c = concrete cylinder compressive strength; and

b_n = net width of beam.

Figure 7-6 —Symbols for Equation (7.18).

7.2.6. Effect of Applied Moment

The current ACI method (2014) considers the effect of applied moment in its equation when calculating the shear strength of the beam. **Figure 7-7** shows the failure patterns of beam specimens from two different research programmes, viz Yoshida (1998) and Tompos and Frosch (2002). In each research programme, the specimens had the same compressive strength, shear reinforcement ratio, longitudinal reinforcement ratio, and effective depth. **Figure 7-7** shows that the location at which failure takes place; it could be closer to the loading point or to the support. This depends whether applied moment plays a role in the location of failure, and shear strength of the beam. In the proposed analytical truss model, the effect of applied moment on the shear strength of the beam was not considered.

Figure 7-7—Failure patterns of specimens from previous research: (a) Tompos and Frosch, 2002; and (b) Yoshida, 1998.

7.2.7. Size Effect

The experimental results from Chapter 5 showed that providing shear reinforcement in the specimen could mitigate the size effect, but not eliminate it. Therefore, a size effect term is proposed, as Equation (7.19), to be included in the proposed analytical truss model. Size effect is still evident in shear reinforced beams, albeit with less influence compared to non-shear reinforced beams. Equation (7.19) consists of two terms; the first term is the size effect term for beams with no shear reinforcement (Equation (6.2)), and the second term represents a reduction in the influence of size effect due to inclusion of shear reinforcement. The reduction in size effect influence is directly related to presence of shear reinforcement, and hence, the empirical term α in Equation (7.19) is proposed in the form of $(A \cdot \rho_z^B \cdot f_{yz}^C)$. The values A , B and C were iteratively changed to achieve the lowest Coefficient of Variation (COV) relevant to the mean test-to-predicted shear strength ($V_{test}/V_{predicted}$) ratios. Initial values for A , B and C were used in Equation (7.20) to represent the influence of shear reinforcement on size effect. The size effect term, Equation (7.20), was placed in Equation (7.24) to calculate the depth of the web panel that contributes to shear strength. The depth of web panel, calculated from Equation (7.24), was used in Equation (7.25) to calculate the web shear strength. The calculated web shear strength was then added to the dowel action, calculated using Equation (7.18), to obtain the predicted total shear strength of the beam. The predicted total shear strength of 189 specimens in the database was compared with the test shear strengths and obtain the $V_{test}/V_{predicted}$ ratios as well as COV. Different values of A , B and C were used in Equation (7.19), and the procedure was repeated until the lowest COV relevant to the $V_{test}/V_{predicted}$ ratios was obtained. In **Table 7-1**, the performance of the proposed truss model was compared against that when other values of A , B , and C were used. That means, in columns (6) to (10) of **Table 7-1**, the value of B was varied between 0.25 to 1.35 while the values of A and C were

kept constant at 0.4 and 1, respectively. As shown in **Table 7-1**, the values of $A=0.4$, $B=1$, and $C=1$ represent the coefficients that results in the lowest COV, and therefore these values were chosen. The final form of the size effect term used in the proposed analytical truss model is presented as Equation (7.20).

Table 7-1—Results of $V_{test}/V_{predicted}$ ratios with different exponent values for proposed size effect term.

					Shear reinforcement ratio, ρ_z					Yield Strength of shear reinforcement, f_{yz}				
Division	Coefficient values (A)				Exponent values (B)					Exponent values (C)				
	0.20	0.30	0.40	0.50	0.25	0.50	0.75	1.00	1.25	0.25	0.50	0.75	1.00	1.25
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Average $V_{test}/V_{pred} =$	1.33	1.22	1.13	1.07	0.84	0.84	0.84	1.13	1.49	1.67	1.64	1.50	1.13	0.86
STD =	0.23	0.20	0.19	0.18	0.21	0.21	0.21	0.19	0.31	0.45	0.42	0.32	0.19	0.21
COV =	0.18	0.16	0.16	0.17	0.25	0.25	0.25	0.16	0.20	0.27	0.26	0.21	0.16	0.24

Note:

-STD = Standard Deviation.

-COV = Coefficient of Variation.

$$s_z = \frac{1}{1 + \left(\frac{d}{250}\right)^{0.5}} + \alpha \quad (7.19)$$

$$s_z = \frac{1}{1 + \left(\frac{d}{250}\right)^{0.5}} + 0.4\rho_z f_{yz} \quad (7.20)$$

where:

d = the effective depth of the beam;

ρ_z = shear reinforcement ratio; and

f_{yz} = yield strength of shear reinforcement.

7.2.8. Solution Procedure

The eleven governing equations listed in Section 7.2.4 contain fourteen unknown variables, namely σ_z , σ_x , σ_r , σ_d , τ_{xz} , γ_{xz} , f_{sz} , f_{sx} , ϵ_x , ϵ_z , ϵ_d , ϵ_r , ζ , and θ . It is necessary to either determine or assume the values of any three variables to obtain the solutions for the other variables. The applied normal stress in the z -direction (σ_z) is usually zero in typical reinforced concrete beams,

hence the value of σ_z is taken to be zero. The values for the other variables cannot be obtained directly. As such, the values of average shear stress (τ_{xz}) and strain value in the r -axis (ε_r) are assumed in the first iteration. The solution for all the other eleven variables could be obtained only when the assumed values for τ_{xz} and ε_r are matched by the iterated values for these two variables within a specified tolerance.

An initial value for the strain in the d -direction (ε_d) is selected. Taking the assumed values for τ_{xz} and ε_r , the value for the principal stress of concrete in r -axis (σ_r) is calculated using Equation (7.12). The softening factor of concrete is calculated using Equation (7.16) with the assumed value of ε_r and the compressive strength of the concrete. Taking into account the softening factor, the principal stress of the concrete in compression (σ_d) is then calculated using Equation (7.15). The average strain in the vertical reinforcing bars (ε_z) is calculated using Equation (7.22), assuming that the vertical reinforcement did not yield ($f_{sz}=E_s\varepsilon_z$). The expression for Equation (7.22) is obtained by re-arranging Equations (7.1) and (7.4). Equation (7.1) is re-arranged by placing $\sin^2\theta$ on the left side of the equation, presented as Equation (7.1a).

$$\sin^2\theta = \frac{-\sigma_z + \sigma_r + \rho_z f_{sz}}{-\sigma_d + \sigma_r} \quad (7.1a)$$

Equation (7.4) is re-arranged by converting the term $\cos^2\theta$ into $\sin^2\theta$, as presented in Equation (7.4a). The expression for Equation (7.1a) is then substituted into Equation (7.4a) to obtain Equation (7.21). Substituting $f_{sz}=E_s\varepsilon_z$ into Equation (7.21) gives Equation (7.22) to calculate the value of ε_z .

$$\varepsilon_z = \varepsilon_r + (\varepsilon_d - \varepsilon_r) \sin^2\theta \quad (7.4a)$$

$$\varepsilon_z = \varepsilon_r + \left(\frac{-\sigma_z + \sigma_r + \rho_z f_{sz}}{-\sigma_d + \sigma_r} \right) (\varepsilon_d - \varepsilon_r) \quad (7.21)$$

$$\varepsilon_z = \frac{\varepsilon_r (-\sigma_d + \sigma_z) + \varepsilon_d (-\sigma_z + \sigma_r)}{(-\sigma_d + \sigma_r) - \rho_z E_s (\varepsilon_d - \varepsilon_r)} \quad (7.22)$$

If the value of ε_z calculated using Equation (7.22) is less than (f_{yz}/E_s) , it means that the assumption of vertical reinforcement had not yielded is correct. The value of f_{sz} can then be calculated using Equation (7.7). If the value of ε_z calculated using Equation (7.22) is greater than (f_{yz}/E_s) , it means that the assumption of stress on the vertical reinforcement is incorrect. Consequently, the value of f_{sz} is taken to be the yield strength of the vertical reinforcement (f_{yz}) and the value of ε_z can be re-calculated using Equation (7.21). The inclination of the diagonal truss, θ , can then be calculated using Equation (7.3a), which was re-arranged using Equation (7.3).

$$\theta = 0.5 * \sin^{-1} \left(\frac{2\tau_{xz}}{\sigma_d - \sigma_r} \right) \quad (7.3a)$$

Using the calculated inclination of the diagonal truss, the value of average strain in the horizontal reinforcement (ε_x) can be determined using Equation (7.5). With the value of ε_x , the average stress of the horizontal reinforcement (f_{sx}) can be calculated using Equation (7.8). Subsequently, the value for applied normal stress in the x -axis (σ_x) can be calculated using Equation (7.2).

The strain value in the r -axis (ε_r) is then calculated using Equation (7.23) and is compared with the initial assumed value of ε_r . Equation (7.23) is obtained by adding Equations (7.4) and (7.5). If the difference between the calculated and assumed values exceeds 1%, another value for ε_r should be assumed and the iterative process begins.

$$\varepsilon_r = \varepsilon_x + \varepsilon_z - \varepsilon_d \quad (7.23)$$

The value for the average shear stress (τ_{xz}) can be calculated using Equation (7.3), and should be cross-checked with the assumed value of τ_{xz} . If the difference between the calculated and assumed values is greater than 1%, another value for ε_r should be assumed and the iterative process begins. If the value of ε_r reaches 0.2, it means that there will be no solution for τ_{xz} . New values for τ_{xz} and ε_r will be assumed and the iteration process would restart. Only when the calculated values for both τ_{xz} and ε_r are close to the assumed values, a convergent solution is obtained. The value for shear strain in the x - z coordinate system (γ_{xz}) can then be calculated

using Equation (7.6).

If the assumed values are verified, the three following failure criteria are checked to obtain the strain in the d -direction (ε_d) representing the failure of the beam: (1) the vertical reinforcement has yielded ($f_{sz} \geq f_{yz}$); (2) the horizontal reinforcement has yielded ($f_{sx} \geq f_{yx}$); and (3) the concrete strut has crushed ($\sigma_d \geq \sigma_{d, max}$). If any of the three conditions is satisfied, it indicates that the beam has failed either by yielding of reinforcement, or by concrete crushing. The ultimate solutions can be represented by the variables satisfying the failure criteria. Different values of ε_d are selected and the entire iteration process is repeated to obtain a full τ_{xz} - γ_{xz} relationship for the element. The depth of the web panel is calculated using Equation (7.24). The depth of the web panel considers the part of concrete that follows the plane stress conditions, and is not considered in the calculation of dowel action. This would ensure no area of the concrete is counted twice in the calculations. The size effect term in Equation (7.20) is also considered in the calculation for the depth of the web panel. The shear strength contribution from the dowel action is calculated using Equation (7.18), and then added onto the shear strength calculated by Equation (7.25). The shear strength of the beam element is calculated using Equation (7.25) if either the vertical or horizontal reinforcement had yielded. The shear stress is multiplied by the depth of the web panel and the web width of the beam. Failure of slender beams is mainly caused by failure of tension ties or yielding of shear reinforcement, instead of failure of the compression strut. The width of the strut in the truss model should be limited by the depth of the compression zone from the bending analysis.

$$d_e = s_z[h - 2(h - d)] \quad (7.24)$$

$$V_{tie} = \tau_{xz} b_w d_e \quad (7.25)$$

The proposed solution procedure, as shown in the flow-chart in **Figure 7-8**, is summarised below. The iterative calculations are carried out by MS-Excel programme. The Visual Basic for Applications (VBA) sub-routines codes for MS-Excel are provided in Appendix C.

Step 1: Select a value of ε_d .

Step 2: Assume a value for τ_{xz} .

Step 3: Assume a value of ε_r .

Step 4: Calculate σ_r using Equation (7.12).

Step 5: Calculate ζ using Equation (7.16).

Step 6: Calculate σ_d using Equation (7.15).

Step 7: Calculate ε_z using Equation (7.21) or (7.22).

Step 8: Calculate f_{sz} using Equation (7.7).

Step 9: Calculate the inclination of θ using Equation (7.3a).

Step 10: Calculate the value for ε_x using Equation (7.5).

Step 11: Calculate the value of f_{sx} using Equation (7.8).

Step 12: Calculate the value for σ_x using Equation (7.2).

Step 13: Calculate the values of ε_r using Equation (7.23) and check with assumed value from Step 3.

Step 14: Calculate the value of τ_{xz} using Equation (7.3) and check with assumed value from Step 2.

Step 15: Calculate the value of γ_{xz} using Equation (7.6).

Step 16: Check for failure of the beam.

Step 17: Calculate the depth of the web panel using Equations (7.20) and (7.24).

Step 18: Calculate the shear strength of the web of the beam using Equation (7.25).

Step 19: Calculate the contribution from dowel action using Equation (7.18).

Step 20: Calculate the shear strength of the beam.

Figure 7-8—Flow-chart showing the solution procedure of the truss model.

7.2.9. Example of Calculation of Shear Strength of Reinforced Concrete Beam using the Truss Model

The calculation to predict the shear strength of Specimen 1V1-450-10 using the analytical truss model is presented in this section as a worked example. The calculation procedure is presented in **Figure 7-8**. The calculation procedure to calculate shear strength requires many cycles of iterations. The iteration steps for the example below are skipped, and the final values for ε_d , τ_{xz} and ε_r that satisfy equilibrium are presented here. Information of the specimen is given as follows:

Concrete cylinder compressive strength, $f'_c = 108$ MPa;

Width of beam, $b_w = 265$ mm;

Effective depth of beam, $d = 403$ mm;

Distance from support to loading point, $a = 1200$ mm;

Shear-span-to-effective-depth ratio, $a/d = 2.97$;

Longitudinal reinforcement ratio, $\rho_l = 2$ %;

Yield strength of longitudinal reinforcement, $f_{yl} = 544$ MPa;

Shear reinforcement ratio, $\rho_v = 0.13$ %;

Yield strength of shear reinforcement, $f_{yv} = 614$ MPa;

Spacing of shear reinforcement, $s = 170$ mm; and

Experimental shear strength, $V_{test} = 296$ kN.

Step 1: Select a value of ε_d .

The value of -0.00006 was selected for ε_d .

Step 2: Assume a value for τ_{xz} .

The value of -2.95 was selected for τ_{xz} .

Step 3: Assume a value of ε_r .

The value of 0.08161 was selected for ε_r .

Step 4: Calculate σ_r using Equation (7.12b).

$$\sigma_r = 0.02 f'_c = 0.02 * 108 = 2.16$$

Step 5: Calculate ζ using Equation (7.16).

$$\zeta = \left(\frac{5.8}{\sqrt{f'_c}} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \varepsilon_r}} \right) = \left(\frac{5.8}{\sqrt{108}} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 * 0.08161}} \right) = 0.09622$$

Step 6: Calculate σ_d using Equation (7.15).

$$\sigma_{d, \max} = -\zeta f'_c = -0.096 * 108 = -10.368$$

$$\varepsilon_0 = -(0.002 + 0.001 \frac{f'_c - 20}{80}) = -(0.002 + 0.001 \frac{108 - 20}{80}) = -0.0031$$

$$(\varepsilon_d / \zeta \varepsilon_0) = -0.00006 / (0.096 * -0.0031) = 0.202$$

$$\sigma_d = -\zeta f'_c \left[2 \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right) - \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right)^2 \right] = -10.368 * \left[2(0.202) - (0.202)^2 \right] = -3.76$$

Step 7: Calculate ε_z using Equation (7.21) or (7.22).

$$\varepsilon_z = \frac{\varepsilon_r (-\sigma_d + \sigma_z) + \varepsilon_d (-\sigma_z + \sigma_r)}{(-\sigma_d + \sigma_r) - \rho_z E_s (\varepsilon_d - \varepsilon_r)} = \frac{0.08161 [-(-3.76) + 0] + -0.00006 (-0 + 2.16)}{[-(-3.76) + 2.16] - \left(\frac{0.13}{100} \right) 200000 (-0.00006 - 0.08161)} = 0.011$$

$$\varepsilon_z = \varepsilon_r + \left(\frac{-\sigma_z + \sigma_r + \rho_z f_{sz}}{-\sigma_d + \sigma_r} \right) (\varepsilon_d - \varepsilon_r) = 0.08161 + \left(\frac{-0 + 2.16 + \left(\frac{0.13}{100} \right) (614)}{-(-3.76) + 2.16} \right) (-0.00006 - 0.08161)$$

$$= 0.041$$

Step 8: Calculate f_{sz} using Equation (7.7).

$$f_{sz} = f_{yz} = 614$$

Step 9: Calculate the inclination of θ using Equation (7.3a).

$$\theta = 0.5 * \sin^{-1} \left(\frac{2 \tau_{xz}}{\sigma_d - \sigma_r} \right) = 0.5 * \sin^{-1} \left(\frac{2 * -2.95}{-3.76 - 2.16} \right) = 44.7^\circ$$

Step 10: Calculate the value for ε_x using Equation (7.5).

$$\varepsilon_x = \varepsilon_d \cos^2 \theta + \varepsilon_r \sin^2 \theta = -0.00006 \cos^2 (44.7) + 0.08161 \sin^2 (44.7) = 0.040$$

Step 11: Calculate the value of f_{sx} using Equation (7.8).

$$f_{sx} = 0$$

Step 12: Calculate the value for σ_x using Equation (7.2).

$$\sigma_x = \sigma_d \cos^2 \theta + \sigma_r \sin^2 \theta + \rho_x f_{sx} = -3.76 \cos^2(44.7) + 2.16 \sin^2(44.7) + (0) = -0.82$$

Step 13: Calculate the values of ε_r using Equation (7.23) and check with assumed value from Step 3.

$$\varepsilon_r = \varepsilon_x + \varepsilon_z - \varepsilon_d = 0.0405 + 0.0408 - 0.00006 = 0.8124$$

If the difference between the calculated value from Equation (7.23) and assumed value from Step 3 exceeds 1%, another value for ε_r should be assumed and the iterative process begins.

Step 14: Calculate the value of τ_{xz} using Equation (7.3) and check with assumed value from Step 2.

$$\tau_{xz} = (\sigma_d - \sigma_r) \sin \theta \cos \theta = (-3.76 - 2.16) \sin(44.7) \cos(44.7) = -2.95$$

If the difference between the calculated value from Equation (7.3) and assumed value from Step 2 exceeds 1%, another value for τ_{xz} should be assumed and the iterative process begins.

Step 15: Calculate the value of γ_{xz} using Equation (7.6).

$$\frac{\gamma_{xz}}{2} = (\varepsilon_d - \varepsilon_r) \sin \theta \cos \theta = (-0.00006 - 0.08161) \sin(44.7) \cos(44.7) = -0.041$$

$$\gamma_{xz} = 2 * -0.041 = -0.82$$

Step 16: Check for failure of the beam.

The value of f_{sz} calculated in Step 8 is equal to the yield strength of the steel bars. Therefore, it is taken that the steel bars have yielded and the beam has failed.

Step 17: Calculate the shear strength of the web of the beam using Equation (7.25).

$$s_z = \frac{1}{1 + \left(\frac{403}{250}\right)^{0.5}} + 0.4\left(\frac{0.13}{100} 614\right) = 0.76$$

$$d_e = s_z [h - 2(h - d)] = 0.76 * [450 - 2(450 - 403)] = 270.5$$

$$V_n = \tau_{xz} b_w d_e = 2.95 * 265 * 270.5 = 211kN$$

Step 18: Calculate the contribution from dowel action using Equation (7.18).

$$d_b = \sqrt{\frac{A_{sl}}{\left(\frac{\pi}{4}\right)}} = \sqrt{\frac{(0.02 * 265 * 403)}{\left(\frac{\pi}{4}\right)}} = 52.15$$

$$b_n = b - d_b = 265 - 52.15 = 212.85$$

$$D_u = 1.64 b_n d_b \sqrt[3]{f'_c} = 1.64 * 212.85 * 52.15 * \sqrt[3]{108} = 87kN$$

Step 19: Calculate the shear strength of the beam.

$$V_{predicted} = V_n + D_u = 211 + 87 = 298kN$$

$$V_{test} / V_{predicted} = 296 / 298 = 0.99$$

7.2.10. Comparison with Experimental Results

The results obtained from the analytical truss model for the fifteen specimens are presented in column (12) of **Table 7-2**. The predicted shear strength consists of two components — shear strength from the plane stress relationships and contribution from the dowel action. The contribution from the dowel action for each specimen is presented in column (11). The results in column (13) show the ratio of the test-to-predicted shear strength ($V_{test}/V_{predicted}$) for each specimen. The average $V_{test}/V_{predicted}$ ratio obtained from the truss model was 1.03 with a Coefficient of Variation (COV) of 0.23. The prediction results for the specimens in 1800-series (with depths of 1800mm) were very conservative because the base of the specimen was enlarged to accommodate the longitudinal reinforcement bars, and hence, increasing the shear strength of the specimen.

Table 7-2—Results of fifteen new specimens obtained from the analytical truss model.

Specimen ID	ϵ_d	ϵ_r	σ_r	σ_d	ϵ_z	ϵ_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1V1-450-20	-0.00006	0.082	2.02	-3.64	0.041	0.041	614	44.8	-2.83	85	288	0.85
1V1-450-10	-0.00006	0.081	2.14	-3.76	0.041	0.040	614	44.8	-2.95	87	298	0.99
2V2-450-20	-0.00008	0.133	2.02	-4.48	0.066	0.066	614	44.9	-3.25	85	371	0.82
2V3-450-20	-0.00012	0.148	2.02	-5.92	0.076	0.072	614	44.1	-3.97	85	460	0.71
1V1-900-20	-0.00006	0.099	2.04	-3.62	0.050	0.049	596	44.7	-2.83	109	467	0.89
1V1-900-10	-0.00006	0.116	2.14	-3.68	0.058	0.058	596	45.0	-2.91	110	479	0.91
2V2-900-20	-0.00008	0.146	2.04	-4.46	0.073	0.073	596	44.8	-3.25	109	623	0.89
2V3-900-20	-0.00011	0.149	2.04	-5.62	0.074	0.074	596	44.9	-3.83	109	835	0.90
1V1-1350-20	-0.00006	0.118	1.94	-3.48	0.059	0.059	596	45.0	-2.71	120	616	1.30
1V1-1350-10	-0.00006	0.116	2.14	-3.68	0.058	0.058	596	45.0	-2.91	124	657	1.41
2V2-1350-20	-0.00008	0.137	1.92	-4.34	0.069	0.068	596	44.8	-3.13	119	847	1.14
2V3-1350-20	-0.00013	0.149	1.92	-6.04	0.080	0.069	596	43.0	-3.97	119	1292	0.95
1V1-1800-20	-0.00006	0.117	2.12	-3.66	0.058	0.058	596	44.9	-2.89	77	601	1.54
2V2-1800-20	-0.00008	0.139	2.12	-4.58	0.070	0.068	596	44.6	-3.35	77	857	1.22
2V3-1800-20	-0.00011	0.148	2.12	-5.74	0.075	0.074	596	44.8	-3.93	77	1281	1.00
Average											1.03	
COV											0.23	

Note:

-COV = Coefficient of Variation.

7.2.11. Comparison with Beam Database

A database (DB-2) of 189 rectangular beam specimens with shear reinforcement was compiled and is presented in the Appendix. The database was used to evaluate the performance of the truss model in predicting the shear strength of the data. An average ratio for test-to-predicted shear strengths ($V_{test}/V_{predicted}$) of 1.13, together with a COV of 0.16, was obtained for the analytical truss model (**Table 7-3**). The full set of results for the various parameters determined by the analytical truss model are presented together with the database in the Appendix. The data with the lowest test shear strength to predicted shear strength ($V_{test}/V_{predicted}$) ratio had a compressive strength of 125 MPa and had a beam web width of 457 mm. Most of the data with $V_{test}/V_{predicted}$ ratio lower than 1.0 had either compressive strength higher than 60 MPa, or a beam web width greater than 300 mm. It seems that the proposed truss model could not provide conservative predictions of shear strength for high strength concrete beams or beams with larger web width. In the proposed truss model, it is assumed that higher compressive strength and larger web width would result in higher shear strength. However, past experimental results show that the shear strength did not increase linearly as the compressive strength increases. Modification to the constitutive law of materials in the truss model to consider the effect of high strength concrete can be proposed in the future study. The test-to-predicted shear strength ratios from the equations in ACI 318-14 (ACI) (2014) and Eurocode 2 (EC 2) (2004) are presented in **Table 7-3** for comparison purpose. The main difference between the proposed truss model and the code equations lies in the proposed truss model's ability to allow the user obtain the full τ_{xz} - γ_{xz} relationship for an element throughout the loading process. The methods of ACI and EC 2 allow the user to obtain only the ultimate shear strength of the beam. Comparing the mean and COV values, the results in **Table 7-3** show that the performance of the truss model is better and is an improvement over existing ACI and Eurocode procedures.

Table 7-3—Results of 189 data in database (DB-2) from the truss model compared to ACI and EC 2 method.

189 data	Test-to-predicted shear strength ratios, $V_{test}/V_{predicted}$		
	ACI method	EC 2 method	Truss model
Average	1.25	1.29	1.13
COV	0.20	0.22	0.16
Min	0.46	0.74	0.64
Max	1.88	2.25	1.60

Note:

-ACI refers to ACI 318-14 and EC 2 refers to Eurocode 2.

-COV = Coefficient of Variation.

-Min = Minimum $V_{test}/V_{predicted}$ ratio.

-Max = Maximum $V_{test}/V_{predicted}$ ratio.

7.3. Proposed Method

7.3.1. Proposed Equation to Predict Inclination of Diagonal Truss

The analytical truss model discussed in Sections 7.2.1 to 7.2.6 is a method that allows the user to obtain the full τ_{xz} - γ_{xz} relationship for an element throughout the loading process. The solution procedure requires numerous iterations and is time-consuming. To reduce computational time, a simplified truss model is proposed to predict the inclination of the diagonal trusses and shear strength of the beam. The simplified model is a sectional model which can be applied at any section and is not dependent on the next section. For example, an engineer could perform his analysis at a section d distance away from the support and obtain the diagonal truss inclination angle. The inclination angle of the diagonal truss is then be used to calculate the shear strength of the beam. The method is similar to the application of the current ACI 318-14 or Eurocode 2 method. The novelty of the simplified truss method is the ability to obtain both the diagonal truss inclination and shear strength of the beam that are not available from the methods in ACI 318-14 and Eurocode 2.

Equation (7.26) is an empirical expression in the simplified method to estimate the inclination of the diagonal trusses in a reinforced concrete beam. The inclinations calculated from Equation (7.26) would aim to be similar to those calculated from the proposed analytical truss model. The empirical expression is designed to be simple and aims for computational

efficiency. Using the calculated inclination of the diagonal truss, the shear strength of the reinforced concrete beam can then be determined.

The proposed empirical equation of Equation (7.26) is built on parameters that would influence the inclination of the diagonal truss. It is made up of two terms; the first term considers the geometrical properties of the beam, and the second term considers the steel reinforcement properties that would influence the inclination angle of the diagonal trusses. The first term in Equation (7.26) considers the inclination angle as a function of geometric properties such as the length of the shear span and the effective depth of the beam. The second term takes into account the shear reinforcement ratio, the yield strength of the shear reinforcement bars, and the spacing of the vertical shear reinforcement bars. The presence of shear reinforcement bars in the web of the beam will alter the inclination angle of the diagonal truss (Hsu and Mo, 2010). Generally, an increase in the shear reinforcement ratio will increase the inclination angle, whereas an increase in the vertical reinforcement spacing will reduce the inclination angle. This forms the basic expression of the second term in Equation (7.26).

The exponent value of the second term in Equation (7.26) was obtained via iterations with the data collected in the database (DB-2). Equation (7.26) was proposed to estimate the inclination angle of the diagonal truss that is as close as that calculated using the proposed truss model as discussed in Sections 7.2.1 to 7.2.6. The exponent values in Equation (7.26) were changed iteratively; the exponent value of 0.2 for the second term in Equation (7.26) achieved the best match with a correlation coefficient of 0.65. The correlation between the inclination angles obtained from Equation (7.26) and from the proposed truss model is shown in **Figure 7-9**. The objective to develop Equation (7.26) was to use basic geometrical parameters of a concrete beam to estimate the inclination angle of the diagonal trusses, rather than using parameters such as longitudinal strain at mid-depth of the beam.

The predicted inclination angle was used to calculate the shear strength contribution from shear reinforcement (V_s). This was added to the shear strength contribution from concrete (V_c) to predict the total shear strength of the beam (V_n) as part of the second approach. The shear strengths of 189 beams calculated using the inclination angle obtained from the truss model (first approach) was compared against the shear strengths calculated using the inclination angle obtained from Equation (7.26) (second approach). The difference between the two calculated shear strengths is insignificant. 94 percent of the 189 beams' data had a difference less than

10% between the calculated shear strengths using inclination angles from the truss model and Equation (7.26).

The lower and upper limits specified in Eurocode 2 for the inclination angle of diagonal truss are 22° or 45° , respectively. If a constant inclination angle of 45° is used to calculate the shear strength for all the beams' data and then compared against the shear strength calculated using the inclination angle obtained from the truss model (first approach), the average difference between the two shear strength values is only 8%.

$$\theta = \tan^{-1}\left(\frac{d}{a}\right) + \tan^{-1}\left(\frac{\rho_z f_{yz}}{s}\right)^{0.2} \quad (7.26)$$

where:

θ = inclination of diagonal truss (d axis) with respect to horizontal axis;

ρ_z = vertical shear reinforcement ratio;

f_{yz} = yield strength of vertical reinforcement;

d = the effective depth of the beam;

a = the shear span of the beam; and

s = spacing of vertical shear reinforcement.

Figure 7-9—Correlation between the inclination angles obtained from Equation (7.26) and proposed truss model.

The truss model notation for a shear reinforced beam is shown in **Figure 7-10**. Equation (7.26) is used to calculate the inclination angle of the diagonal truss, marked as θ in

Figure 7-10. Using Equation (7.26), the value of θ will be the same throughout the shear span of the beam. After obtaining the inclination of the diagonal truss from Equation (7.26), the shear strength of the reinforced concrete beam can be determined with respect to the inclination.

Figure 7-10—Truss model notation for shear reinforced member.

7.3.2. Shear Strength of Reinforced Concrete Beam

The shear strength contribution of the tension component is given as Equation (7.27). The experiments showed that contribution from concrete after cracking did not drop to zero. Therefore, the shear strength contribution of the tension component should include the contribution from both the shear reinforcement and concrete, in the form of $(V_s + \text{constant} * V_c)$. The conclusion discussed in Chapter 5 is valid for high strength concrete beams. For a reinforced concrete beam with regular concrete strength, the constant could be taken as 1.0. This meant that the shear strength of a reinforced concrete beam will be in the form of $V_n = V_s + V_c$ (Equation (7.27)), similar to the ACI 318-14 code (2014).

The term V_s represents the shear strength contribution from shear reinforcement. The equation in Eurocode 2 to calculate V_s is shown in Equation (7.28), considering the shear strength contributions as a function of the shear reinforcing bars cut across by a line of action of the diagonal truss with an inclination (θ). Equation (7.28) is used as part of the simplified truss method to calculate the shear strength contribution from shear reinforcement. Different from the equation used in Eurocode 2 is the value of θ that will be obtained from Equation (7.26).

$$V_n = V_s + V_c \quad (7.27)$$

$$V_s = \rho_z f_{yz} b_w z \cot \theta \quad (7.28)$$

where:

- V_s = shear strength contribution of the steel component;
- V_c = shear strength contribution of the concrete component;
- f_{yz} = yield strength of the vertical reinforcement;
- ρ_z = vertical shear reinforcement ratio;
- θ = inclination of diagonal truss (d -axis) with respect to horizontal axis;
- b_w = width of the beam; and
- z = lever arm of internal forces.

The concrete component contribution to the shear strength of the beam (V_c) is calculated using Equation (6.2), which is the proposed empirical expression discussed in Chapter 6 to predict the concrete's contribution to shear strength.

$$V_c = 7(\rho_l)^{0.4} \left(\frac{d}{a}\right)^{0.3} f_c'^{0.25} \frac{1}{1 + \left(\frac{d}{250}\right)^{0.5}} b_w d \quad (6.2)$$

where:

- V_c = shear strength contribution of the concrete component;
- ρ_l = longitudinal reinforcement ratio;
- d = effective depth of the beam;
- f_c' = concrete cylinder compressive strength; and
- a = shear span of the beam.

7.3.3. Solution Procedure

The inclination of the diagonal truss is calculated using Equation (7.26). With the inclination angle, the shear strength contribution of the tension components of the reinforced concrete beam can be calculated using Equations (7.28) and (6.2). Failure of shallow beams is typically governed by failure of tensile components in the beam instead of the compression strut. While a slender beam does not fail in compression, the width of the strut is limited by the depth of the

compression zone from the bending analysis. The width of the strut is multiplied by the effective compressive stress and the width of the beam. The proposed solution procedure for the simplified method is summarised below.

Step 1: Calculate θ using Equation (7.26).

Step 2: Calculate V_s using Equation (7.28).

Step 3: Calculate V_c using Equation (6.2).

Step 4: Calculate V_n by adding V_c to V_s .

7.3.4. Example of Calculation of Shear Strength of Reinforced Concrete Beam

A worked example is presented in this section based on the prediction for Specimen 1V1-450-10 with the proposed solution procedure. The information of the specimen is given as follows:

Concrete cylinder compressive strength, $f'_c = 108$ MPa;

Width of beam, $b_w = 265$ mm;

Effective depth of beam, $d = 403$ mm;

Distance from support to loading point, $a = 1200$ mm;

Shear-span-to-effective-depth ratio, $a/d = 2.97$;

Longitudinal reinforcement ratio, $\rho_l = 2\%$;

Yield strength of longitudinal reinforcement, $f_{yl} = 544$ MPa;

Shear reinforcement ratio, $\rho_v = 0.13\%$;

Yield strength of shear reinforcement, $f_{yv} = 614$ MPa;

Spacing of shear reinforcement, $s = 170$ mm; and

Experimental shear strength, $V_{test} = 296$ kN.

Step 1: Calculate θ using Equation (7.26).

$$\theta = \tan^{-1}\left(\frac{1}{2.97}\right) + \tan^{-1}\left(\frac{\frac{0.13}{100} \cdot 614}{170}\right)^{0.2} = 37.5^\circ$$

Step 2: Calculate V_{tie} using Equation (7.28).

$$V_s = \frac{0.13}{100} * 614 * 265 * 0.9 * 403 * \cot(37.5^\circ) = 100kN$$

Step 3: Calculate V_c using Equation (6.2).

$$V_c = 7\left(\frac{2}{100}\right)^{0.4} * 108^{0.25} * \left(\frac{403}{1200}\right)^{0.3} * \frac{1}{1 + \left(\frac{403}{250}\right)^{0.5}} * 265 * 403 = 160kN$$

Step 4: Calculate V_n .

$$V_n = V_s + V_c = 100 + 160 = 260kN$$

$$V_{test} / V_n = 296 / 260 = 1.14$$

7.3.5. Comparison of Experimental Results

The experimental shear strengths of the fifteen new specimens are listed in **Table 7-4** alongside the predicted shear strengths calculated using the equations from ACI 318-14 (ACI) (2014), Eurocode 2 (EC 2) (2004), Bažant and Yu's method (2005a, 2005b), Collins *et al.*'s method (2008), and the author's proposed method. The concept from the two proposed methods (Bažant and Yu, and Collins *et al.*) to calculate shear strength of shear reinforced concrete beams are similar, in the form of $V_n = V_s + V_c$. The respective expression for V_s and V_c for Bažant and Yu's and Collins *et al.*'s methods are presented in Equations (7.29) to (7.30) and (7.31) to (7.33), respectively.

$$V_s = \rho_z f_{yz} b_w d \quad (7.29)$$

$$V_c = \mu \rho^{\frac{3}{8}} b_w d \left(1 + \frac{d}{a}\right) \sqrt{\frac{f'_c}{1 + \frac{d}{d_0}}} \quad (7.30)$$

where:

- μ = coefficient to adjust the predictions to achieve a best fit for all the data in the database;
- b_w = width of the beam;
- d = effective depth of the beam;
- a = shear span of the beam;
- ρ = longitudinal reinforcement ratio;

- f'_c = compressive strength of the concrete;
 d_o = empirical transitional size parameter;
 f_{yz} = yield strength of the vertical reinforcement; and
 ρ_z = vertical shear reinforcement ratio.

$$V_s = \rho_z f_{yz} b_w d \quad (7.31)$$

$$V_c = \left[\sqrt{1 + \frac{7.02 m \sqrt{f'_c}}{(1000 + s_{xe})}} - 1 \right] \frac{133.3}{m} b_w d \quad (7.32)$$

$$s_{xe} = 305 \quad (7.33)$$

where:

- s_{xe} = equivalent crack spacing;
 b_w = width of the beam;
 d = effective depth of the beam;
 f'_c = compressive strength of the concrete;
 $a_{g,eff}$ = effective aggregate size;
 f_{yz} = yield strength of the vertical reinforcement; and
 ρ_z = vertical shear reinforcement ratio.

The equation to calculate the concrete contribution to shear strength of a reinforced concrete beam using these two methods has been discussed in Chapter 6. In the method by Collins' *et al.*, the value for the term s_{xe} can be taken as 305 mm if the amount of shear reinforcement provided in the reinforced concrete beam satisfies the minimum requirement in ACI 318-14 (Collins *et al.*, 2008). The equation provided by ACI 318-14 to calculate the shear strength contribution from shear reinforcement was adopted by both methods in their calculations.

The ratio of test-to-predicted shear strengths ($V_{test}/V_{predicted}$) for the five methods, including the author's proposed simplified truss method, are presented in columns (4) to (8) of

Table 7-4. The performance of the equations from ACI 318-14 (ACI) and Eurocode 2 (EC 2) for the fifteen specimens tested in this study have been discussed in Chapter 5.

Table 7-4—Measured and predicted ultimate loads.

Specimen ID	Measured loads, kN		Test-to-predicted shear strength ratios, $V_{\text{test}}/V_{\text{predicted}}$				
	Total load	V_{test}	ACI method	EC 2 method	Bažant and Yu	Collins <i>et al.</i>	Proposed Simplified Method
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1V1-450-20	476	246	1.04	1.30	0.90	0.79	0.95
1V1-450-10	576	296	1.25	1.56	1.08	0.95	1.14
2V2-450-20	594	305	1.08	1.04	0.96	0.85	1.03
2V3-450-20	640	328	0.94	0.75	0.86	0.77	0.96
1V1-900-20	780	414	0.88	1.12	0.92	0.67	0.87
1V1-900-10	820	434	0.93	1.17	0.96	0.70	0.90
2V2-900-20	1060	554	0.99	0.97	1.02	0.78	0.98
2V3-900-20	1450	749	1.09	0.88	1.12	0.89	1.11
1V1-1350-20	1500	801	1.12	1.42	1.31	0.84	1.19
1V1-1350-10	1750	926	1.30	1.64	1.50	0.98	1.36
2V2-1350-20	1828	965	1.14	1.11	1.30	0.89	1.20
2V3-1350-20	2340	1221	1.17	0.94	1.30	0.95	1.26
1V1-1800-20	1680	925	1.27	1.60	1.59	0.95	1.39
2V2-1800-20	1926	1048	1.21	1.18	1.45	0.94	1.30
2V3-1800-20	2400	1285	1.20	0.97	1.40	0.98	1.31
All fifteen data		Average	1.11	1.18	1.18	0.86	1.13
		COV	0.12	0.23	0.21	0.12	0.15
Twelve specimens with rectangular cross-sections		Average	1.08	1.16	1.10	0.84	1.08
		COV	0.12	0.24	0.19	0.12	0.14

Note:

- ACI refers to ACI 318-14, and EC 2 refers to Eurocode 2.

-COV = Coefficient of Variation.

The predictions from the ACI method for all the fifteen specimens were very consistent; the Coefficient of Variation (COV) for the ACI method is the lowest together with the method by Collins *et al.* Good results from the ACI method may be due to a result of an over-estimation of the concrete contribution, by ignoring the size effect, and under-estimating the shear reinforcement contribution. It seems that the two combinations led to coincidental good predictions of the shear strength for the fifteen specimens in this study, as discussed in Chapter 5. The predictions from EC 2 were conservative because the equations do not consider concrete

contribution. Conservatism in the EC 2 method resulted in inconsistent predictions, as shown by its COV, which is the highest among the five methods in this study. Among the three proposed methods: Bažant and Yu, Collins *et al.*, and author's, the method from Collins *et al.* has the lowest COV when considering all fifteen specimens. However, the average ratio of $V_{test}/V_{predicted}$ from this method is also the lowest, meaning un-conservative predictions. In Chapter 6, the average $V_{test}/V_{predicted}$ ratio for Collins *et al.*'s method predicting the shear strengths of 980 concrete beams with no shear reinforcement in database (DB-1) is 1.23. This shows that the method is very conservative. For beams with no shear reinforcement, the influence of size effect is considered using the s_{xe} term given by Equation (6.7). When shear reinforcement is provided, the influence of size effect is removed as the value of s_{xe} was taken to be a constant of 305 (Collins *et al.*, 2008). As a result, the predictions were un-conservative for those specimens with shear reinforcement. This meant that the substitution of a constant to represent elimination of size effect once shear reinforcement is provided might have caused the un-conservative predictions. As the experimental results in this study shows, there is still presence of size effect even if shear reinforcement is provided.

The shear strength predictions for the three specimens in 1800-series were expected to be conservative because of higher experimental shear strength due to the beams' enlarged base. Similarly, for specimens in 1350-series, the horizontal web bars increased the shear strength of the specimens, albeit not significantly. The predictions for these specimens from Bažant and Yu's and author's method achieved this expectation. The $V_{test}/V_{predicted}$ ratio for the specimens in 1350 and 1800-series were very conservative compared to the predictions for smaller specimens.

7.3.6. Comparison with Beam Database

The performance of the proposed analytical truss model and simplified truss method was evaluated using the database (DB-2) consisting of 189 rectangular beam data and the results are presented in both **Table 7-5** and **Figure 7-11**. The performance of the other four methods (ACI 318-14 (ACI) (2014), Eurocode 2 (EC 2) (2004), Bažant and Yu's method (2005a, 2005b), Collins *et al.*'s method (2008)) discussed in the previous section against these data are also presented to provide comparison. The data are arranged into several divisions in **Table 7-5** and **Figure 7-11** under three parameters — effective depth, compressive strength, and shear reinforcement ratio— to study the influence of each parameter on the predictions from each

method. The author's proposed simplified truss method and Bažant and Yu's method obtained the lowest Coefficient of Variation (COV) with good average $V_{test}/V_{predicted}$ ratios. The obtained COV from these two methods was slightly lower than that obtained for the proposed analytical truss model. The objective of Bažant and Yu's method was to determine only the ultimate shear strength of a reinforced concrete beam, similar to that of ACI and EC 2 methods. The objective of the proposed analytical truss model was to obtain the full τ_{xz} - γ_{xz} relationship for an element throughout the loading process. The ability to obtain the full τ_{xz} - γ_{xz} relationship with good $V_{test}/V_{predicted}$ ratios and low COV is the benefit of using the analytical truss model. For the proposed simplified truss method, the objectives of the method were to obtain the inclination angle of the diagonal trusses together with the ultimate shear strength of the beam. The method considers a varying inclination angle of diagonal trusses as a function of various parameters, compared to a fixed inclination angle for Bažant and Yu's method. While the solution procedure of the author's simplified truss method requires the same number of procedure steps (four steps), the simplified method can provide more information regarding the reinforced concrete beam behaviour.

Table 7-5—Comparison of experimental and calculated failure load by all methods.

	Division	All	Effective depth, d			Compressive strength, f'_c				Shear reinforcement ratio, ρ_v		
			[A]	[B]	[C]	[A]	[B]	[C]	[D]	[A]	[B]	[C]
			$d < 500$	$500 \leq d < 1000$	$1000 \leq d$	$f'_c < 30$	$30 \leq f'_c < 60$	$60 \leq f'_c < 90$	$90 \leq f'_c$	$\rho_v < 0.3$	$0.3 \leq \rho_v < 0.5$	$0.5 \leq \rho_v$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Nos. of data		189	138	40	11	20	98	50	21	136	45	8
V_{test}/V_{pred} ACI 318-14	Min =	0.46	0.86	0.69	0.46	0.46	0.61	0.79	0.78	0.46	0.94	1.22
	Max =	1.86	1.86	1.38	1.30	1.74	1.73	1.86	1.38	1.82	1.86	1.54
	Average =	1.24	1.33	1.03	0.98	1.33	1.21	1.34	1.12	1.20	1.35	1.33
	COV =	0.20	0.15	0.18	0.30	0.23	0.19	0.17	0.14	0.21	0.14	0.08
	Average =		0.17			0.19				0.19		
V_{test}/V_{pred} Eurocode 2	Min =	0.74	0.75	0.74	0.75	0.75	0.74	0.92	0.75	0.75	0.74	0.87
	Max =	2.25	2.25	1.80	1.64	1.88	1.75	2.25	1.64	2.25	1.88	1.11
	Average =	1.29	1.29	1.29	1.22	1.25	1.21	1.50	1.16	1.38	1.06	0.97
	COV =	0.22	0.22	0.19	0.25	0.22	0.19	0.17	0.23	0.18	0.22	0.08
	Average =		0.22			0.20				0.18		
V_{test}/V_{pred} Bažant and Yu	Min =	0.74	0.74	0.83	0.80	0.80	0.74	0.82	0.84	0.74	0.86	0.84
	Max =	1.59	1.43	1.43	1.59	1.19	1.42	1.43	1.59	1.59	1.43	1.26
	Average =	1.08	1.06	1.09	1.27	0.98	1.05	1.16	1.12	1.07	1.12	0.99
	COV =	0.15	0.13	0.14	0.19	0.09	0.12	0.13	0.22	0.16	0.11	0.16
	Average =		0.14			0.13				0.15		
V_{test}/V_{pred} Collins <i>et al.</i>	Min =	0.44	0.58	0.54	0.44	0.44	0.51	0.62	0.56	0.44	0.77	0.90
	Max =	1.37	1.37	1.01	0.98	1.16	1.23	1.37	1.01	1.27	1.37	1.23
	Average =	0.91	0.96	0.76	0.78	0.94	0.89	0.95	0.86	0.85	1.05	1.03
	COV =	0.19	0.16	0.15	0.27	0.19	0.21	0.17	0.14	0.19	0.12	0.11
	Average =		0.17			0.19				0.17		
V_{test}/V_{pred} Proposed Simplified Method	Min =	0.67	0.71	0.78	0.67	0.67	0.71	0.88	0.78	0.67	0.88	0.90
	Max =	1.56	1.56	1.28	1.39	1.19	1.35	1.56	1.39	1.56	1.43	1.35
	Average =	1.08	1.09	1.02	1.14	0.97	1.05	1.16	1.09	1.05	1.15	1.09
	COV =	0.15	0.14	0.13	0.22	0.13	0.14	0.13	0.16	0.15	0.11	0.15
	Average =		0.14			0.14				0.14		
V_{test}/V_{pred} Proposed Truss Model	Min =	0.64	0.64	0.67	0.83	0.83	0.79	0.86	0.64	0.67	0.64	0.87
	Max =	1.60	1.60	1.39	1.54	1.52	1.53	1.60	1.54	1.59	1.60	1.31
	Average =	1.13	1.15	1.06	1.19	1.28	1.12	1.15	0.98	1.12	1.17	1.08
	COV =	0.16	0.16	0.16	0.20	0.13	0.14	0.16	0.23	0.17	0.16	0.16
	Average =		0.16			0.15				0.16		

Note:

- ACI refers to ACI 318-14, and EC 2 refers to Eurocode 2.
- Divisions A, B, C and D correspond to the zones in Figure 7-11.
- Min = Minimum V_{test}/V_{pred} ratio.
- Max = Maximum V_{test}/V_{pred} ratio.
- COV = Coefficient of Variation.

(a)

(b)

(c)

Figure 7-11—Failure load predictions of 189 test data with influence from (a) Effective depth; (b) Compressive strength; and (c) Shear reinforcement ratio.

7.3.7. Influence of Effective Depth

Figure 7-11 (a) to (c) show the influence of three parameters — effective depth, compressive strength, and shear reinforcement ratio, respectively — on the performance of the six equations. In **Figure 7-11**, the ratios of the test-to-predicted shear strength ($V_{test}/V_{predicted}$) are plotted in the y-axis, and the parameters are plotted in the x-axis. The red horizontal dashed line plotted in each zone represents the average ratio of $V_{test}/V_{predicted}$ for that zone.

The first two graphs on the left in **Figure 7-11 (a)** show the performance of the ACI and EC 2 equations with respect to effective depth. There is an obvious reduction in the $V_{test}/V_{predicted}$ ratio for the predictions from the ACI method as effective depth increases. This represents reduced conservatism in its predictions as the effective depth increases and could prove to be very dangerous for specimens with very large effective depths. This drop in conservatism in the predictions was also observed in the predictions by ACI method for non-shear reinforced beams in Chapter 6, albeit more significant. The ACI method considers that the influence of size effect can be mitigated by providing shear reinforcement. Results in Chapter 5 have showed that providing shear reinforcement in a reinforced concrete beam does not totally eliminate the size effect. Hence, the absence of size effect in the ACI equation can still make the prediction increasingly un-conservative as effective depth increases.

Apart from the ACI and EC 2 method, the other four methods (Bažant and Yu, Collins *et al.*, author's proposed analytical truss model, and author's simplified truss method) consist of a size effect term. It can be seen in **Figure 7-11 (a)** that there was no significant reduction in the average $V_{test}/V_{predicted}$ ratios across the different zones similar to that observed in the ACI

method. In fact, the average predictions for Bažant and Yu's and the author's two methods for beams with effective depths greater than 1000 mm (Division C in **Table 7-5**) was more conservative than the beams with smaller effective depths.

7.3.8. Influence of Compressive Strength

The combined COV average for each method is obtained by multiplying the COV in each division with the number of data in that division, and then summation of all the divisions and divided by the total number of data. The combined COV average shows how consistent a method is when predicting shear strength of a beam with regards to a varying parameter. Columns (7) to (10) of **Table 7-5** shows the performance of Bažant and Yu's method as the compressive strength increases from 15 to 125 MPa. The combined COV average for Bažant and Yu's method is the lowest among all the methods, at 0.13.

The predictions from EC 2 is very conservative, with $V_{test}/V_{predicted}$ ratio up to 2.25. This level of conservatism is due to the neglect of the concrete's contribution to shear strength. The EC 2 method only considers the shear reinforcement to contribute to shear strength of the shear reinforced beam. This consideration can lead to uneconomical beam designs with a high safety factor, as shown by the conservative predictions.

7.4. Summary of Chapter 7

The contribution of the longitudinal reinforcing bars to shear strength, in the form of dowel forces, was proposed in this chapter to be added onto an analytical truss model. The influence of size effect was also proposed to be considered in the truss model. The analytical truss model, considering the compression field action in the web panel of the beam, is based on the rotating softened truss model. The objective of the proposed truss model is to be able to obtain the full τ_{xz} - γ_{xz} relationship with good $V_{test}/V_{predicted}$ ratios and low COV. A second approach, a simplified truss method, was also proposed in this chapter to predict shear strength of a shear reinforced beam in the form of $V_n = V_s + V_c$. In this second approach, engineers can directly obtain the inclination angle of the diagonal truss from the empirical equation and the shear strength of the beam without going through the iterative process of the truss model. Within the simplified truss model, a simple empirical expression was proposed to predict the inclination angle of diagonal trusses prior to failure of a reinforced concrete beam. The inclination angle obtained from the empirical expression would match those inclination angles obtained from

the truss model in the first approach. The empirical expression considers main parameters such as shear-span-to-effective-depth ratio, shear reinforcement ratio, and spacing between the shear reinforcement. With the diagonal truss inclinations determined, the shear strength contribution from the shear reinforcement could be calculated. The concrete's contribution to shear strength was calculated using Equation (6.2) proposed in Chapter 6. These contributions from the shear reinforcement and concrete form the shear strength of the beam. Together with obtaining the inclination angle of the diagonal trusses, these two components form the objectives of the simplified truss method. The performance of the two author's proposed methods was evaluated against a database consisting of 189 data points. Comparing the predicted shear strengths of reinforced concrete beams with the test data, the analytical truss model and simplified method obtained an average $V_{test}/V_{predicted}$ ratio of 1.13 and 1.08, respectively. The COV for the predictions from the two methods was 0.16 and 0.15, respectively. The performance of the two proposed methods was compared against the predictions from the equations of ACI 318-14 (2014), Eurocode 2 (2004), as well as other proposed methods such as those by Bažant and Yu (2005a, 2005b), and by Collins *et al.* (2008). From the discussions, the two proposed methods provided satisfactory and consistent predictions with low COV compared to the other methods. The simplified truss method is also simple, and represents an improvement over existing ACI and Eurocode procedures. A list of the benefits for the two proposed methods (first and second approach) are shown in **Table 7-6**.

A limitation of the simplified truss method lies in its inability to consider the contribution from horizontal web reinforcement towards the inclination angle of the diagonal trusses as well as shear strength. Beams with large depths are required to be designed with horizontal reinforcement in the web. However, the number of experimental data with such design is small. Therefore, this represents a shortfall to the method and can be improved on in the future with more experimental data.

Table 7-6— List of benefits of the two proposed methods.

First approach (Proposed truss model)	Second approach (Simplified method, $V_n = V_s + V_c$)
• The approach allows the user to obtain the full stress-strain relationship of the web element at each loading stage. • The approach considers equilibrium equations, constitutive law of materials and compatibility conditions in its iteration processes. • The approach considers size effect to reduce over-estimation of predicted shear strength in large-sized concrete beams. • The predicted total shear strength includes dowel forces (shear strength contribution from the longitudinal reinforcement) as an add-on to the shear strength contributed by the web panel. This reduces the conservatism of shear strength in beams with high longitudinal reinforcement ratio and allow a more economical design of the beam.	• The approach allows the user to obtain four deliverables in four computation steps. 1. Inclination angle of the diagonal truss 2. Shear strength contribution from concrete 3. Shear strength contribution from shear reinforcement 4. Total shear strength of the beam • The approach in the form of $V_n = V_c + V_s$ would aid the engineer in designing for shear. • The approach considers size effect when calculating shear strength contribution from concrete component to be conservative for large-sized concrete beams. • The inclination angles of the diagonal truss obtained from this approach is similar to that obtained in the first approach. This eliminates the need to go through the iterative process to obtain the inclination angle and subsequently the total shear strength.

Chapter 8

GENERAL CONCLUSIONS

8.1. Conclusions

This study was conducted to address the need to determine the shear strength of large-sized high strength concrete beams with effective depth greater than 1200 mm and compressive strength greater than 70 MPa. The objectives of the study were: (1) to investigate the influence of effective depth on large high strength concrete beams; (2) to investigate the influence of maximum aggregate size in high strength concrete beams; (3) to study the influence of shear reinforcement ratio on size effect in high strength concrete beams; (4) to find out the minimum shear reinforcement ratio required to eliminate the size effect on the shear strength; (5) to evaluate the efficacy of existing design codes, such as ACI 318-14 (2014) and Eurocode 2 (2004), in predicting the shear strength of high strength concrete beams with varying depths; and (6) to propose new methods to predict the shear strength of large-sized high strength concrete beams with and without shear reinforcement.

The study consisted of two parts to achieve these objectives: an experimental programme to test eleven life sized high strength concrete beam specimens (contributing twenty-two data), and an analytical programme to propose shear methods. Each chapter was summarised at the end, and the key conclusions are highlighted in the following sections.

8.1.1. Experimental Work

Seven high strength concrete shear spans (specimens) with no shear reinforcement were tested in the first part of the experimental programme. The studied variables among the specimens were: (1) beam depth (ranging from 450 to 1800 mm); and (2) the maximum aggregate size used in concrete (10 and 20 mm). The objectives for these seven specimens included: (1) an investigation of the influence of effective depth; (2) an investigation on the effect of maximum aggregate size; and (3) to evaluate the predictions of the studied beam shear strength using existing guidelines, such as ACI 318-14 (2014) and Eurocode 2 (2004). The following conclusions were drawn from the results of the seven specimens.

- The failures for all the specimens was brittle and sudden. The crack surfaces observed in all the specimens were relatively smooth, unlike the uneven surface observed in normal strength concrete beams.
- Reviewed alongside high strength concrete experimental data from Matsui *et al.* (1995), Collins and Kuchma (1999), Fujita *et al.* (2002), Sherwood *et al.* (2007), and Sneed and Ramirez (2010), the experimental results from this study showed that the influence of effective depth reduced the shear strength of high strength concrete beams in the same manner as in normal strength concrete beams. The difference in shear strength between the largest and smallest rectangular specimen in this experimental programme (450 and 1350-series) without shear reinforcement was about 10%.
- The specimens with smaller maximum aggregate size tested in this study had higher shear strength than those with larger maximum aggregate size by approximately 20%.
- The results from this study, together with data from other research programmes studying specimens with different aggregate sizes and depths, showed that the size effect trend in the reinforced concrete beams does not change regardless of aggregate size.
- The predictions from the provisions in both ACI 318-14 (2014) and Eurocode 2 (2004) could be unsafe for beams with effective depths greater than 500 mm. The equation in ACI 318-14 does not include the influence of size effect, which could lead to over-prediction of the shear strength. The equation in Eurocode 2 considers size effect, but it does not predict the influence of size effect as well as ACI 446.

Fifteen high strength concrete shear spans (specimens) with shear reinforcement were tested in the second part of the experimental programme. The main variables studied in these fifteen specimens were: (1) the maximum aggregate size used in the concrete material (10 and 20 mm), and (2) shear reinforcement ratio in the specimen (0%, 0.13%, 0.20%, and 0.30%). The objectives for these fifteen specimens were (1) to investigate the influence of shear reinforcement ratio on the size effect in high strength concrete beams; (2) to evaluate the accuracy of existing codes, such as ACI 318-14 (2014) and Eurocode 2 (2004), in predicting the shear strength of high strength concrete beams with shear reinforcement; and (3) to find out

the minimum shear reinforcement ratio required to eliminate the size effect on the shear strength. The following conclusions were made from the results of the fifteen specimens.

- The shear strength of a reinforced concrete beam increased with increasing amount of shear reinforcement in the beam. The shear strengths of the specimens with shear reinforcement ratio of 0.3% were about 2.8 times of the specimens with no shear reinforcements. Between the specimens with shear reinforcement ratios of 0.13% and 0.3%, the difference in shear strengths was about 1.5 times.
- The amount of load carried by the beam after the first diagonal crack, referred as the post-cracking reserve strength of the beam, increases with increasing shear reinforcement ratio. A beam with shear reinforcement ratio greater than 0.13% possesses a post-cracking reserve strength twice as much as the cracking load of the specimen. All the specimens with shear reinforcement ratio of 0.3% have an average shear strength three times of the $1.5V_{cr}$. The post-cracking reserve strength does not increase as the effective depth of the specimen increases.
- The results showed that the concrete's contribution to shear strength drops when the shear reinforcing bars were activated to carry shear load. However, the concrete's contribution did not drop to zero, instead, it maintained at about 25 - 35% of the applied load.
- The shear reinforcement in the specimens did not eliminate size effect on beam shear strength. Specimens with shear reinforcement ratio up to 0.30% still exhibited size effect. The difference in shear strength between the largest and smallest specimens with the same shear reinforcement ratio is 10%.
- The equation from ACI 318-14 predicts the shear strength of specimens with shear reinforcement well with a low Coefficient of Variation (COV). The equations from Eurocode 2 neglect the concrete's contribution, leading to over-conservative predictions of reinforced concrete beams with low shear reinforcement ratio. For specimens with high shear reinforcement ratio, the equations from Eurocode 2 could lead to un-conservative shear strength predictions.

8.1.2. Analytical Work

An empirical equation was proposed in Chapter 6 to predict the shear strength of reinforced concrete beams with no shear reinforcement. The empirical equation takes into account the influence of compressive strength, shear-span-to-effective-depth ratio, longitudinal reinforcement ratio, and effective depth of the beam. A database (DB-1) of 980 data was collected and used to evaluate the accuracy of this proposed equation. The results were compared with those from equations of ACI 318-14 (2014), Eurocode 2 (2004), Bažant and Yu (2005a, 2005b), and Collins *et al.* (2008). The performance of the proposed method was better compared to the other methods. An average experimental over predicted shear strength ratio of 1.14 with a COV of 0.14 was obtained for the proposed method.

An analytical truss model, based on the softened truss model by Hsu and Mo (2010), was presented with proposed inclusions of dowel action and size effect to the truss model. The objective of the proposed truss model is to be able to obtain the full τ_{xz} - γ_{xz} relationship with good $V_{test}/V_{predicted}$ ratios and low COV. A proposed simplified truss method was also presented to calculate the shear strength of reinforced concrete beams with shear reinforcement as an alternative to the labour intensive proposed analytical truss model. The simplified truss method predicts shear strength of the beam in the form of $V_n = V_s + V_c$. The objective of the simplified truss method is to obtain the inclination angle of the diagonal trusses together with the shear strength of the beam in the same method. Within the simplified truss method, an empirical expression was proposed to predict the inclination of the diagonal truss when close to failure of the reinforced concrete beam. The predicted inclination angles aim to be close to the angles obtained from the proposed analytical truss model (first approach). With the diagonal truss inclinations determined, the shear strength contribution from the shear reinforcement could be calculated. The concrete's contribution to shear strength was calculated using Equation (6.2) proposed in Chapter 6. These contributions from the shear reinforcement and concrete form the shear strength of a concrete beam with shear reinforcement.

The performance of the two author's proposed methods was evaluated against a database consisting of 189 data points with shear reinforcement. Comparing the predicted shear strengths of reinforced concrete beams with the test data, the truss model and simplified method obtained an average $V_{test}/V_{predicted}$ ratio of 1.13 and 1.08, respectively. The COV for the predictions from the two methods was 0.16 and 0.15, respectively. This result was compared

against the prediction results from the equations of ACI 318-14 (2014), Eurocode 2 (2004), Bažant and Yu (2005a, 2005b), and Collins *et al.* (2008). From the discussions, the two proposed methods have proven to be reliable and consistent.

8.2. Final Remarks and Contributions

A total of twenty-two high strength concrete specimens were tested in this study. Among these twenty-two specimens, seven had no shear reinforcement and the rest had shear reinforcement ratio varying between 0% to 0.30%. These specimens extended the existing database with a series of high strength concrete specimens of varying depths between 450 and 1800 mm. The specimens with depths of 1350 and 1800 mm were among the largest high strength concrete beams tested to date. The experimental results have shown that the influence of effective depth could influence the shear strength of high strength concrete beams in the same manner as it does for normal strength concrete beams. The experimental results from the specimens with shear reinforcement showed that shear reinforcement does not eliminate size effect between specimens of different depths. However, the shear reinforcement in the specimen could mitigate the influence of size effect between two specimens of different effective depths.

An empirical equation was proposed to predict the shear strength of reinforced beams with no shear reinforcement. Two methods were proposed to predict the shear strength of shear reinforced concrete beam — one analytical truss model, and the other a simplified truss method. The empirical equation for non-shear reinforced beam was used as part of the simplified truss method. The performance of all the methods were evaluated with existing experimental data, and reliable performance has been achieved.

8.3. Suggestions for Future Works

The variables in the current experimental programme were limited to varying effective depths, and two sizes of aggregate. Future research on large-sized reinforced high strength concrete beams could expand its scope to include variations in the longitudinal reinforcement ratio and shear-span-to-effective-depth ratio. The shear behaviour of continuous beams, made of high

strength concrete, near the intermediate supports or as part of a framed structure could also be an area of research.

The use of steel fibre reinforced concrete has started to gain popularity in the construction industry. The tensile strength of the steel fibre reinforced concrete would be higher than that of a plain concrete. The propagation of crack widths in a steel fibre reinforced concrete beam could be better controlled by the steel fibres present in the concrete. The influence of steel fibre on size effect in beams with varying depth could be investigated. Influence of parameters, such as fibre material, length, volume fraction, and single or double-hooked fibre, could also be investigated.

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Appendix A

PHOTOGRAPHS FROM EXPERIMENTS

A.1. Post-test Photographs

Left

Right

Figure A-1—Photos of specimens in 450-series. (Top): 450-series (Type 1-20); (Middle): 450-series (Type 1-10); and (Bottom): 450-series (Type 2-20)

Note:

- The panoramic post-test photographs were created by digitally stitching photographs taken at different sections of the beam specimen. Hence, the photographs might look distorted. The crack patterns were not altered during the digital stitching.
- The shear span on the left end of the beam specimen contained less shear reinforcement, hence is the weaker shear span of the two.

Figure A-2—Photos of specimens in 900-series. (Top): 900-series (Type 1-20); (Middle): 900-series (Type 1-10); and (Bottom): 900-series (Type 2-20)

Note:

- The shear span on the left end of the beam specimen contained less shear reinforcement, hence is the weaker shear span of the two.

Left

Right

Figure A-3—Photos of specimens in 1350-series. (Top): 1350-series (Type 1-20); (Middle): 1350-series (Type 1-10); and (Bottom): 1350-series (Type 2-20)

Note: - The shear span on the left end of the beam specimen contained less shear reinforcement, hence is the weaker shear span of the two.

Left

Right

Figure A-4—Photos of specimens in 1800-series. (Top): 1800-series (Type 1-20); and (Bottom): 1800-series (Type 2-20)

Note:

- For Specimen 1800-series (Type 1-20), the shear span on the left end of the beam specimen contained less shear reinforcement, hence is the weaker shear span of the two.
- For Specimen 1800-series (Type 2-20), the shear span on the right end of the beam specimen contained less shear reinforcement, hence is the weaker shear span of the two.

A.2. Time-lapsed Photographs of Crack Development

Time-lapsed photographs of the crack development were taken for every shear span in the beam specimens of 1350 and 1800 series. Unfortunately, the photographs for Specimen 2V3-1350-20 were damaged, hence they are not shown. The time-lapsed photographs for the other shear spans are presented in **Figures A-5 to A-13**.

Specimen 1V0-1350-20

Total Load: 200 kN

Total Load: 300 kN

Total Load: 440 kN

Total Load: 600 kN

Total Load: 640 kN

Total Load: 666 kN

Figure A-5—Time-lapsed photographs of Specimen 1V0-1350-20.

Specimen 1V1-1350-20

Total Load: 200 kN

Total Load: 300 kN

Total Load: 600 kN

Total Load: 600 kN

Total Load: 1240 kN

Total Load: 1500 kN

Figure A-6—Time-lapsed photographs of Specimen 1V1-1350-20.

Specimen 1V0-1350-10

Total Load: 380 kN

Total Load: 500 kN

Total Load: 620 kN

Total Load: 740 kN

Total Load: 900 kN

Figure A-7—Time-lapsed photographs of Specimen 1V0-1350-10.

Specimen 1V1-1350-10

Total Load: 380 kN

Total Load: 860 kN

Total Load: 1020 kN

Total Load: 1500 kN

Total Load: 1750 kN

Figure A-8—Time-lapsed photographs of Specimen 1V1-1350-10.

Specimen 2V2-1350-20

Total Load: 200 kN

Total Load: 300 kN

Total Load: 620 kN

Total Load: 1180 kN

Total Load: 1500 kN

Total Load: 1828 kN

Figure A-9—Time-lapsed photographs of Specimen 2V2-1350-20.

Specimen 1V0-1800-20

Total Load: 260 kN

Total Load: 380 kN

Total Load: 580 kN

Total Load: 700 kN

Total Load: 920 kN

Figure A-10—Time-lapsed photographs of Specimen 1V0-1800-20.

Specimen 1V1-1800-20

Total Load: 1140 kN

Total Load: 1200 kN

Total Load: 1300 kN

Total Load: 1420 kN

Total Load: 1680 kN

Figure A-11—Time-lapsed photographs of Specimen 1V1-1800-20.

Specimen 2V2-1800-20

Total Load: 320 kN

Total Load: 540 kN

Total Load: 860 kN

Total Load: 1100 kN

Total Load: 1500 kN

Total Load: 1926 kN

Figure A-12—Time-lapsed photographs of Specimen 2V2-1800-20.

Specimen 2V3-1800-20

Total Load: 340 kN

Total Load: 420 kN

Total Load: 700 kN

Total Load: 1200 kN

Total Load: 1400 kN

Total Load: 1800 kN

Figure A-13—Time-lapsed photographs of Specimen 2V3-1800-20. (The test was stopped because concrete crushing was observed in the other shear span of the beam)

Appendix B

EXPERIMENTAL DATA

B.1. Linear Voltage Differential Transducers (deflection) Data

Figure B-1 shows the typical positions and labels of the Linear Voltage Differential Transducers (LVDTs) measuring the vertical deflections of the eleven reinforced high strength concrete beam specimens. Every beam specimen contained two shear spans. Specimens 1V0-450-20 and 1V1-450-20 made up one beam specimen, and was labelled as 450-series (Type 1-20). Specimens 2V2-450-20 and 2V3-450-20 made up one beam specimen, and was labelled as 450-series (Type 2-20). **Tables B-1** to **B-11** show the LVDTs data for all eleven beam specimens.

Figure B-1—Typical locations of LVDTs.

Table B-1—LVDTs data for Specimen 450-series (Type 1-20).

450-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
60	0.03	0.22	0.44	0.50	0.41	0.22	0.11
71	0.06	0.28	0.54	0.63	0.51	0.27	0.13
81	0.07	0.32	0.60	0.71	0.57	0.30	0.14
91	0.08	0.37	0.69	0.81	0.66	0.33	0.16
102	0.08	0.43	0.82	0.97	0.77	0.38	0.14
121	0.07	0.54	1.09	1.35	1.02	0.45	0.11
141	0.05	0.65	1.34	1.71	1.28	0.54	0.08
161	0.03	0.77	1.66	2.11	1.58	0.63	0.03
181	0.01	0.88	1.93	2.47	1.84	0.71	-0.02
201	-0.02	1.02	2.30	2.92	2.21	0.84	-0.08
221	-0.03	1.11	2.58	3.26	2.45	0.92	-0.11
231	-0.01	1.17	3.09	3.64	2.67	0.97	-0.15
239	0.00	1.27	3.61	4.15	3.05	1.06	-0.22
251	-0.01	1.32	3.86	4.41	3.24	1.10	-0.26
261	0.04	1.36	4.67	5.02	3.59	1.19	-0.33
268	0.03	2.22	8.21	6.65	4.07	1.28	-0.58

Table B-2—LVDTs data for Specimen 450-series (Type 1-10).

450-series (Type 1-10)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
29	2.80	3.02	2.40	1.69	1.01	0.36	0.02
39	2.80	3.02	2.46	1.76	1.06	0.39	0.03
49	2.82	3.02	2.52	1.82	1.12	0.42	0.03
60	2.84	3.02	2.58	1.89	1.19	0.46	0.03
69	2.86	3.03	2.66	1.97	1.25	0.51	0.04
79	2.87	3.07	2.75	2.08	1.34	0.55	0.04
87	2.89	3.10	2.83	2.18	1.42	0.60	0.05
99	2.90	3.13	2.92	2.30	1.50	0.65	0.06
108	2.91	3.16	2.99	2.40	1.58	0.69	0.06
120	2.91	3.21	3.11	2.57	1.69	0.75	0.06
130	2.91	3.24	3.22	2.75	1.80	0.81	0.06
136	2.91	3.28	3.32	2.92	1.90	0.86	0.06
147	2.91	3.32	3.45	3.08	2.04	0.92	-0.08
208	2.85	3.55	4.29	4.20	2.96	1.29	-0.16
247	2.74	3.73	5.05	5.09	3.65	1.54	-0.26
297	2.71	3.84	7.52	7.28	5.19	2.00	-0.55
308	2.75	3.88	8.38	7.98	5.64	2.16	-0.62
319	2.75	3.96	8.96	8.49	5.99	2.29	-0.69
321	2.75	3.97	9.03	8.55	6.04	2.31	-0.69

Table B-3—LVDTs data for Specimen 450-series (Type 2-20).

450-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
28	0.05	0.10	0.25	0.30	0.28	0.17	0.06
38	0.07	0.13	0.33	0.39	0.36	0.23	0.09
47	0.09	0.17	0.41	0.49	0.44	0.29	0.10
58	0.09	0.21	0.49	0.59	0.52	0.34	0.12
68	0.11	0.24	0.56	0.68	0.59	0.38	0.13
78	0.11	0.27	0.65	0.79	0.68	0.43	0.14
87	0.12	0.31	0.73	0.91	0.76	0.47	0.14
100	0.11	0.35	0.85	1.06	0.87	0.52	0.14
117	0.08	0.46	1.12	1.45	1.13	0.55	0.08
137	0.03	0.58	1.50	1.90	1.42	0.65	0.03
157	0.00	0.68	1.82	2.33	1.76	0.76	-0.02
198	-0.09	0.91	2.40	3.07	2.27	0.93	-0.10
297	-0.29	1.38	3.94	4.97	3.69	1.34	-0.35
397	-0.55	1.97	5.69	7.15	5.51	1.86	-0.62
496	-0.79	2.47	7.44	9.31	7.33	2.45	-0.85
516	-0.84	2.56	7.81	9.78	7.72	2.55	-0.89
536	-0.88	2.65	8.27	10.31	8.17	2.65	-0.95
556	-0.93	2.73	8.73	10.87	8.66	2.77	-0.99
576	-0.97	2.86	9.32	11.58	9.32	2.94	-1.02
584	-1.46	2.62	9.28	14.02	16.62	5.46	-1.26

Table B-4—LVDTs data for Specimen 900-series (Type 1-20).

900-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
40	0.05	0.13	0.24	0.40	0.26	0.10	0.09
53	0.14	0.15	0.35	0.52	0.32	0.17	0.07
94	0.13	0.27	0.56	0.88	0.60	0.26	0.22
107	0.26	0.32	0.72	1.03	0.67	0.30	0.15
119	0.18	0.33	0.74	1.15	0.80	0.37	0.28
134	0.20	0.37	0.86	1.32	0.92	0.42	0.31
153	0.24	0.41	1.00	1.53	1.07	0.49	0.34
171	0.26	0.46	1.22	1.87	1.27	0.57	0.36
188	0.28	0.50	1.43	2.24	1.50	0.64	0.37
218	0.37	0.80	2.05	3.07	2.00	0.62	0.25
237	0.37	0.86	2.26	3.40	2.21	0.67	0.27
258	0.37	0.94	2.44	3.79	2.45	0.72	0.28
279	0.37	1.00	2.71	4.19	2.69	0.77	0.30
301	0.38	1.06	3.00	4.64	3.00	0.83	0.22
318	0.38	1.11	3.26	5.00	3.23	0.86	0.23
336	0.38	1.15	3.70	5.54	3.68	0.92	0.19

Table B-5—LVDTs data for Specimen 900-series (Type 1-10).

900-series (Type 1-10)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
65	0.23	0.16	0.49	0.66	0.52	0.26	0.25
109	0.30	0.27	0.74	1.03	0.80	0.40	0.34
122	0.32	0.30	0.81	1.13	0.88	0.45	0.36
137	0.36	0.35	0.90	1.28	0.98	0.50	0.40
154	0.38	0.38	0.99	1.42	1.08	0.54	0.42
206	0.48	0.55	1.42	2.18	1.55	0.73	0.48
250	0.50	0.69	1.87	3.01	2.01	0.84	0.50
298	0.52	0.88	2.69	4.43	2.89	0.99	0.47
349	0.52	1.11	4.44	6.60	4.27	1.11	0.37
398	0.57	1.24	6.24	8.29	5.30	1.19	0.32
446	0.51	1.36	7.81	9.89	6.28	1.30	0.26
479	0.57	1.42	8.84	10.91	6.88	1.37	0.23
501	0.65	1.59	10.44	12.37	7.65	1.44	0.18
523	0.61	1.68	11.10	13.08	8.17	1.50	0.15

Table B-6—LVDTs data for Specimen 900-series (Type 2-20).

900-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
80	0.42	0.35	0.64	0.82	0.77	0.48	0.41
97	0.45	0.40	0.78	1.02	0.93	0.55	0.46
121	0.49	0.47	0.97	1.28	1.14	0.64	0.51
138	0.52	0.53	1.11	1.47	1.30	0.69	0.54
152	0.55	0.58	1.27	1.70	1.47	0.74	0.56
179	0.59	0.67	1.62	2.17	1.82	0.85	0.58
193	0.62	0.72	1.81	2.44	2.03	0.90	0.58
301	0.73	1.10	3.49	4.67	3.80	1.27	0.56
405	0.83	1.47	5.40	6.98	5.65	1.63	0.49
499	0.89	1.76	7.00	8.94	7.24	1.91	0.42
609	0.96	2.07	8.75	11.13	9.04	2.20	0.35
699	1.02	2.36	10.25	12.97	10.50	2.43	0.28
802	1.06	2.66	12.05	15.14	12.25	2.75	0.19
902	1.07	3.00	14.30	17.89	14.30	3.10	0.03
957	1.05	3.14	15.20	19.08	15.05	3.23	-
1003	1.03	3.25	16.10	20.18	15.34	3.35	-
1026	1.03	3.31	16.64	20.81	15.70	3.41	-
1043	1.02	3.36	17.13	21.33	16.02	3.46	-

Table B-7—LVDTs data for Specimen 1350-series (Type 1-20).

1350-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.01	0.06	0.09	0.05	0.00	0.00
21	0.00	0.03	0.12	0.17	0.10	0.00	0.01
33	0.00	0.08	0.23	0.28	0.19	0.00	0.02
40	0.00	0.09	0.27	0.33	0.23	0.00	0.02
52	0.00	0.15	0.36	0.46	0.34	0.07	0.05
58	0.03	0.15	0.40	0.51	0.36	0.07	0.05
75	0.08	0.21	0.53	0.65	0.48	0.07	0.07
82	0.11	0.23	0.59	0.72	0.53	0.10	0.09
95	0.12	0.26	0.67	0.83	0.61	0.10	0.10
104	0.14	0.28	0.71	0.90	0.65	0.11	0.11
114	0.16	0.31	0.79	1.01	0.73	0.11	0.12
121	0.18	0.33	0.86	1.11	0.79	0.14	0.13
131	0.20	0.35	0.92	1.18	0.85	0.14	0.14
143	0.21	0.37	0.99	1.29	0.92	0.15	0.15
160	0.24	0.41	1.12	1.47	1.05	0.18	0.16
184	0.27	0.46	1.31	1.71	1.23	0.22	0.18
200	0.29	0.48	1.44	1.91	1.36	0.25	0.19
217	0.29	0.54	1.77	2.43	1.68	0.31	0.19
238	0.29	0.61	2.00	2.78	1.92	0.37	0.19
258	0.29	0.66	2.21	3.09	2.13	0.41	0.19
280	0.29	0.70	2.47	3.46	2.38	0.45	0.19
286	0.29	0.77	2.75	3.89	2.75	0.50	0.19
317	0.29	0.82	3.15	4.46	3.21	0.57	0.18
341	0.29	0.89	3.76	5.18	3.67	0.63	0.19
363	0.29	0.93	4.04	5.57	3.94	0.67	0.19
386	0.29	1.01	4.43	6.03	4.27	0.71	0.19
402	0.29	1.03	4.69	6.42	4.61	0.75	0.18
442	0.29	1.13	5.20	7.12	5.14	0.83	0.18
445	0.30	1.21	5.45	7.42	5.38	0.89	0.19
465	0.29	1.21	5.71	7.73	5.60	0.90	0.19
484	0.29	1.24	5.91	8.01	5.79	0.92	0.19
509	0.30	1.29	6.22	8.42	6.09	0.95	0.18
520	0.29	1.31	6.39	8.64	6.27	0.96	0.18
545	0.28	1.36	6.73	9.08	6.59	1.01	0.19
562	0.26	1.39	6.97	9.39	6.80	1.03	0.19
585	0.24	1.43	7.39	9.93	7.21	1.07	0.19
599	0.24	1.47	7.59	10.17	7.39	1.09	0.18
629	0.24	1.53	8.04	10.78	7.87	1.14	0.18
640	0.22	1.55	8.24	11.01	8.03	1.16	0.18
665	0.22	1.61	8.77	11.61	8.46	1.20	0.18

1350-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
676	0.22	1.65	9.16	11.99	8.72	1.22	0.18

Table B-8—LVDTs data for Specimen 1350-series (Type 1-10).

1350-series (Type 1-10)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
94	0.04	0.27	0.58	0.88	0.54	0.29	0.28
165	0.10	0.48	0.98	1.59	1.01	0.53	0.47
205	0.11	0.56	1.20	1.90	1.22	0.62	0.54
230	0.11	0.60	1.34	2.14	1.34	0.70	0.58
260	0.11	0.63	1.59	2.43	1.57	0.77	0.63
275	0.11	0.65	1.78	2.81	1.71	0.81	0.64
282	0.11	0.66	1.86	3.33	1.77	0.83	0.65
295	0.11	0.67	2.04	3.65	1.90	0.86	0.66
319	0.11	0.70	2.30	4.09	2.10	0.91	0.68
338	0.11	0.72	2.47	4.43	2.30	0.95	0.69
357	0.11	0.75	2.68	4.82	2.46	0.99	0.70
361	0.11	0.75	2.73	4.91	2.49	1.00	0.71
380	0.14	0.77	3.01	5.51	2.88	1.03	0.72
407	0.14	0.80	3.57	6.53	3.40	1.08	0.72
409	0.14	0.81	3.87	6.92	3.58	1.09	0.72
407	0.14	0.81	3.88	6.93	3.58	1.09	0.72
438	0.13	0.84	4.22	7.43	3.78	1.12	0.72
464	0.14	0.86	4.50	7.98	4.18	1.16	0.72
488	0.13	0.88	4.77	8.47	4.47	1.20	0.71
499	0.13	0.89	4.89	8.68	4.61	1.22	0.72
525	0.13	0.91	5.42	9.35	4.98	1.26	0.71
547	0.13	0.93	5.75	9.84	5.23	1.30	0.72
569	0.14	0.95	6.09	10.39	5.56	1.33	0.72
589	0.14	0.97	6.57	10.93	5.82	1.36	0.71
598	0.14	0.98	6.89	11.29	6.01	1.39	0.72
616	0.14	1.00	7.55	11.94	6.30	1.42	0.71
620	0.14	1.03	8.99	12.92	6.77	1.46	0.72
636	0.14	1.04	9.60	13.42	6.99	1.46	0.72
668	0.14	1.12	10.54	14.32	7.43	1.50	0.72
684	0.14	1.17	11.13	14.93	7.76	1.54	0.72
701	0.14	1.19	11.48	15.34	7.98	1.55	0.72
734	0.14	1.22	12.14	16.22	8.53	1.61	0.72
750	0.14	1.23	12.48	16.67	8.78	1.63	0.72
760	0.14	1.26	12.75	17.01	8.97	1.65	0.71
768	0.14	1.27	12.91	17.20	9.07	1.66	0.72
777	0.13	1.28	13.14	17.47	9.21	1.67	0.72
803	0.14	1.30	13.68	18.14	9.56	1.71	0.72
828	0.11	1.33	14.31	18.92	10.05	1.75	0.71
842	0.11	1.34	14.67	19.35	10.30	1.77	0.71

1350-series (Type 1-10)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
858	0.10	1.37	15.15	19.97	10.74	1.80	0.71
886	0.07	1.40	15.84	20.92	11.48	1.82	0.70
906	0.07	1.42	16.36	21.52	11.88	1.86	0.71
902	0.06	1.42	16.38	21.53	11.89	1.86	0.70

Table B-9—LVDTs data for Specimen 1350-series (Type 2-20).

1350-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0
16	0	0.07	0.16	0.24	0.25	0.15	0.13
52	0	0.1	0.41	0.58	0.39	0.26	0.21
91	0	0.22	0.76	1.12	0.8	0.48	0.38
87	0	0.22	0.76	1.12	0.8	0.48	0.38
82	-1.29	0.22	0.74	1.1	0.8	0.48	0.38
108	-1.21	0.26	0.92	1.37	0.98	0.56	0.44
101	-1.21	0.26	0.91	1.37	0.98	0.56	0.44
118	-1.21	0.28	1.04	1.59	1.13	0.61	0.46
138	-1.09	0.36	1.27	2.01	1.38	0.73	0.54
162	-1.02	0.4	1.56	2.54	1.69	0.82	0.58
172	-1	0.43	1.69	2.78	1.82	0.87	0.6
199	-0.97	0.48	2.02	3.38	2.2	0.97	0.64
218	-0.97	0.53	2.23	3.75	2.45	1.06	0.68
241	-0.97	0.59	2.65	4.38	2.85	1.17	0.73
262	-0.97	0.64	2.85	4.73	3.09	1.25	0.76
278	-0.97	0.68	3.17	5.16	3.35	1.33	0.81
297	-0.97	0.73	3.38	5.51	3.57	1.41	0.84
319	-0.96	0.8	3.7	6.04	3.93	1.51	0.89
338	-0.96	0.83	3.9	6.37	4.16	1.58	0.91
358	-0.96	0.89	4.24	6.88	4.49	1.66	0.95
378	-0.96	0.93	4.45	7.21	4.7	1.72	0.98
400	-0.96	1	4.88	7.77	5.07	1.83	1.02
420	-0.96	1.04	5.09	8.1	5.27	1.88	1.05
437	-0.96	1.08	5.28	8.43	5.5	1.94	1.08
459	-0.96	1.13	5.57	8.91	5.84	2.03	1.1
481	-0.96	1.18	5.89	9.42	6.19	2.13	1.14
497	-0.96	1.21	6.12	9.74	6.39	2.18	1.17
519	-0.96	1.27	6.39	10.16	6.65	2.25	1.19
540	-0.96	1.31	6.64	10.56	6.9	2.32	1.21
563	-0.96	1.37	7.01	11.1	7.25	2.42	1.26
580	-0.96	1.41	7.19	11.41	7.45	2.47	1.26
600	-0.96	1.46	7.43	11.78	7.69	2.53	1.29
623	-0.96	1.5	7.73	12.24	7.98	2.61	1.31
645	-0.96	1.56	8.14	12.87	8.45	2.72	1.35
666	-0.96	1.59	8.39	13.26	8.71	2.79	1.36
682	-0.96	1.63	8.68	13.75	9.12	2.86	1.38
701	-0.96	1.66	8.88	14.09	9.33	2.91	1.37
727	-0.96	1.72	9.27	14.68	9.7	3	1.41
742	-0.96	1.75	9.45	14.97	9.89	3.04	1.43
759	-0.96	1.77	9.64	15.28	10.07	3.08	1.43

1350-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
779	-0.96	1.82	9.92	15.74	10.35	3.15	1.45
803	-0.96	1.87	10.32	16.33	10.72	3.23	1.49
820	-0.96	1.9	10.5	16.63	10.9	3.27	1.49
840	-0.96	1.93	10.74	17	11.13	3.32	1.5
862	-0.96	1.96	11.03	17.44	11.41	3.39	1.51
883	-0.96	2.02	11.39	18	11.75	3.48	1.53
901	-0.96	2.04	11.58	18.31	11.94	3.51	1.54
923	-0.96	2.08	11.84	18.71	12.18	3.57	1.55
925	-0.96	2.1	12.1	18.94	12.29	3.59	1.54
943	-0.96	2.15	12.45	19.53	12.6	3.65	1.55
962	-0.96	2.19	12.73	19.99	12.89	3.72	1.59
982	-0.96	2.22	12.92	20.32	13.08	3.76	1.59
1002	-0.96	2.26	13.14	20.68	13.31	3.81	1.59
1020	-0.96	2.29	13.37	21.06	13.55	3.86	1.59
1041	-0.96	2.34	13.7	21.62	13.9	3.95	1.6
1063	-0.96	2.37	13.94	22.02	14.15	4	1.64
1081	-0.96	2.39	14.14	22.36	14.37	4.05	1.64
1102	-0.96	2.43	14.46	22.95	14.75	4.1	1.64
1122	-0.66	2.48	14.78	23.44	15.07	4.19	1.68
1143	-0.66	2.51	14.99	23.8	15.28	4.23	1.67
1163	-0.65	2.55	15.29	24.21	15.52	4.28	1.68
1181	-0.65	2.58	15.53	24.58	15.75	4.32	1.69
1202	-0.66	2.65	15.89	25.14	16.13	4.41	1.72
1224	-0.66	2.67	16.16	25.55	16.37	4.46	1.72
1241	-0.65	2.7	16.43	25.92	16.59	4.5	1.72
1250	-0.66	2.71	16.58	26.14	16.71	4.53	1.72
1262	-0.66	2.75	16.75	26.39	16.85	4.55	1.72
1283	-0.66	2.81	17.13	26.97	17.19	4.65	1.73
1302	-0.66	2.84	17.43	27.37	17.41	4.68	1.73
1321	-0.66	2.88	17.71	27.78	17.65	4.73	1.73
1340	-0.66	2.9	18	28.19	17.89	4.77	1.73
1369	-0.66	2.97	18.58	29.09	18.42	4.88	1.77
1380	-0.66	2.98	18.73	29.31	18.55	4.9	1.77
1400	-0.66	3.01	19.07	29.78	18.83	4.96	1.77
1421	-0.66	3.04	19.45	30.3	19.14	5.02	1.77
1443	-0.66	3.1	19.93	30.97	19.55	5.11	1.8
1464	-0.67	3.13	20.31	31.45	19.83	5.16	1.8
1483	-0.66	3.17	20.72	32.03	20.16	5.21	1.8
1500	-0.66	3.2	21.02	32.44	20.42	5.26	1.8
1520	-0.66	3.27	21.57	33.25	21.07	5.4	1.8
1542	-0.66	3.31	21.99	33.89	21.4	5.46	1.81
1562	-0.66	3.34	22.29	34.34	21.67	5.51	1.81
1577	-0.66	3.4	22.58	34.79	21.98	5.56	1.81

1350-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
1606	-0.66	3.46	23.09	35.57	22.44	5.65	1.82
1622	-0.66	3.49	23.34	35.91	22.63	5.68	1.82
1640	-0.66	3.55	23.94	36.57	23	5.75	1.82
1644	-0.67	3.58	24.13	36.79	23.12	5.78	1.83
1660	-0.66	3.62	24.42	37.17	23.33	5.81	1.88
1679	-0.67	3.7	25.15	38	23.78	5.9	1.88
1702	-0.66	3.78	25.85	38.74	24.17	5.97	1.91
1721	-0.66	3.85	26.59	39.55	24.6	6.04	1.95
1741	-0.61	3.95	27.5	40.56	25.09	6.11	-
1762	-0.61	4.02	28.02	41.24	25.49	6.19	-
1779	-0.6	4.06	28.34	41.71	25.77	6.23	-
1801	-0.6	4.11	29.02	42.45	26.19	6.3	-
1804	-0.56	4.15	29.59	42.94	26.45	6.34	-
1797	-0.54	4.18	29.77	43.05	26.48	6.34	-
1797	-0.54	4.2	29.97	43.2	26.53	6.35	-
1797	-0.52	4.23	30.2	43.38	26.6	6.36	-
1828	-0.52	4.32	31.23	44.41	27.13	6.44	-

Table B-10—LVDTs data for Specimen 1800-series (Type 1-20).

1800-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	-0.01	0.00	0.00
1	-0.10	0.05	0.06	0.11	0.07	0.40	0.43
27	-0.10	0.12	0.24	0.38	0.19	0.39	0.43
47	-0.07	0.20	0.44	0.63	0.37	0.39	0.46
60	-0.04	0.26	0.55	0.79	0.49	0.39	0.50
87	0.01	0.36	0.82	1.17	0.78	0.39	0.59
103	0.04	0.42	1.00	1.42	0.97	0.39	0.63
122	0.06	0.47	1.20	1.68	1.18	0.39	0.65
141	0.08	0.53	1.44	2.04	1.46	0.39	0.66
162	0.11	0.61	1.79	2.51	1.84	0.52	0.72
180	0.12	0.65	1.96	2.76	2.02	0.57	0.72
195	0.13	0.70	2.23	3.11	2.29	0.63	0.73
217	0.14	0.76	2.54	3.55	2.64	0.71	0.73
243	0.14	0.81	2.84	3.99	2.94	0.78	0.73
262	0.14	0.86	3.18	4.41	3.26	0.82	0.74
280	0.14	0.91	3.47	4.80	3.59	0.86	0.74
301	0.14	0.95	3.74	5.16	3.90	0.91	0.74
318	0.14	0.99	3.99	5.51	4.19	0.96	0.74
342	0.14	1.05	4.32	5.97	4.55	0.99	0.74
362	0.14	1.09	4.71	6.46	4.87	1.02	0.74
379	0.14	1.14	5.04	6.89	5.19	1.07	0.74
403	0.14	1.20	5.37	7.32	5.53	1.11	0.74
419	0.14	1.24	5.68	7.76	5.83	1.16	0.75
444	0.14	1.30	6.01	8.22	6.19	1.22	0.75
463	0.14	1.34	6.32	8.63	6.48	1.26	0.75
479	0.14	1.38	6.60	9.01	6.76	1.31	0.74
500	0.14	1.42	6.87	9.37	7.03	1.35	0.74
522	0.14	1.47	7.24	9.82	7.36	1.43	0.74
541	0.14	1.50	7.62	10.26	7.66	1.48	0.75
564	0.14	1.55	8.14	10.94	8.25	1.56	0.74
578	0.14	1.57	8.60	11.38	8.59	1.60	0.74
604	0.14	1.63	9.05	11.99	9.08	1.66	0.74
622	0.14	1.64	9.30	12.33	9.34	1.69	0.74
645	0.14	1.67	9.71	12.86	9.74	1.76	0.74
659	0.14	1.69	9.95	13.17	9.98	1.80	0.74
684	0.14	1.73	11.00	14.10	10.61	1.86	0.74
707	0.14	1.73	11.38	14.56	10.96	1.90	0.74
726	0.14	1.73	11.92	15.32	11.72	1.97	0.74
742	0.14	1.78	12.15	15.60	11.95	1.99	0.74
771	0.13	1.78	12.66	16.30	12.49	2.06	0.74
784	0.12	1.79	12.88	16.60	12.73	2.09	0.74

1800-series (Type 1-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
802	0.10	1.79	13.33	17.11	13.11	2.14	0.74
819	0.10	1.82	13.59	17.45	13.37	2.16	0.74
845	0.07	1.82	14.08	18.10	13.87	2.21	0.74
860	0.07	1.86	14.31	18.40	14.11	2.22	0.74
880	0.05	1.86	14.80	19.26	14.69	2.27	0.74
901	0.01	1.85	15.10	19.68	15.03	2.30	0.74

Table B-11—LVDTs data for Specimen 1800-series (Type 2-20).

1800-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
22	0.17	0.12	0.38	0.27	0.18	0.03	0.09
62	0.29	0.16	0.49	0.66	0.39	0.02	0.15
84	0.35	0.18	0.58	0.79	0.46	0.03	0.17
101	0.41	0.21	0.69	0.95	0.55	0.05	0.19
122	0.49	0.35	0.83	1.16	0.65	0.15	0.22
144	0.56	0.40	0.99	1.40	0.81	0.19	0.25
166	0.62	0.48	1.15	1.64	0.97	0.22	0.28
181	0.67	0.51	1.24	1.81	1.07	0.24	0.29
192	0.71	0.58	1.34	1.99	1.16	0.26	0.31
217	0.79	0.67	1.56	2.37	1.40	0.31	0.34
250	0.91	0.83	1.94	3.05	1.79	0.40	0.40
272	0.94	0.87	2.20	3.51	2.07	0.45	0.40
283	0.99	0.96	2.44	3.93	2.32	0.48	0.40
305	1.03	1.02	2.64	4.29	2.52	0.52	0.40
319	1.07	1.10	2.95	4.78	2.83	0.57	0.40
299	1.07	1.10	2.95	4.78	2.83	0.58	0.40
343	1.07	1.22	3.43	5.55	3.23	0.66	0.40
367	1.10	1.33	3.79	6.21	3.64	0.69	0.40
389	1.12	1.41	4.03	6.62	3.86	0.69	0.40
396	1.12	1.44	4.23	7.02	4.07	0.70	0.40
422	1.14	1.51	4.47	7.47	4.30	0.70	0.40
448	1.17	1.56	4.75	7.95	4.56	0.70	0.40
464	1.18	1.61	4.90	8.22	4.71	0.70	0.40
485	1.20	1.65	5.16	8.68	5.06	0.70	0.40
498	1.22	1.70	5.35	9.02	5.26	0.70	0.40
525	1.24	1.78	6.07	9.81	5.69	0.71	0.40
550	1.26	1.83	6.37	10.28	5.96	0.70	0.40
563	1.27	1.86	6.57	10.61	6.16	0.71	0.40
583	1.29	1.92	6.77	10.98	6.36	0.79	0.40
611	1.31	2.04	7.11	11.55	6.70	0.80	0.40
619	1.33	2.07	7.27	11.79	6.83	0.85	0.39
647	1.36	2.14	7.85	12.54	7.24	0.94	0.39
666	1.36	2.21	8.07	12.93	7.48	0.94	0.39
685	1.38	2.25	8.36	13.41	7.79	0.96	0.40
700	1.39	2.30	8.55	13.74	7.99	0.96	0.39
729	1.39	2.35	9.18	14.58	8.55	1.06	0.39
745	1.39	2.39	9.37	14.90	8.72	1.06	0.39
762	1.41	2.46	9.80	15.37	9.00	1.06	0.39
787	1.41	2.57	10.09	15.84	9.25	1.07	0.39
800	1.41	2.60	10.39	16.35	9.61	1.11	0.39

1800-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
827	1.41	2.64	10.70	16.85	9.89	1.16	0.40
843	1.41	2.65	10.93	17.21	10.09	1.21	0.39
864	1.43	2.70	11.33	17.72	10.36	1.25	0.39
884	1.43	2.77	11.96	18.43	10.80	1.30	0.40
916	1.43	2.84	12.35	19.02	11.15	1.32	0.40
935	1.43	2.87	12.59	19.39	11.38	1.32	0.40
940	1.43	2.92	12.83	19.77	11.60	1.36	0.40
973	1.43	2.96	13.26	20.54	12.01	1.37	0.39
984	1.43	2.96	13.46	20.88	12.27	1.40	0.40
1004	1.43	3.01	13.73	21.31	12.51	1.40	0.40
1028	1.43	3.06	14.10	21.89	12.84	1.43	0.39
1039	1.43	3.09	14.37	22.27	13.08	1.43	0.39
1065	1.43	3.14	14.67	22.74	13.37	1.53	0.40
1090	1.45	3.18	15.02	23.30	13.66	1.53	0.40
1090	1.45	3.21	15.40	23.58	13.81	1.54	0.40
1121	1.45	3.27	15.96	24.37	14.33	1.59	0.40
1148	1.45	3.32	16.31	24.92	14.65	1.60	0.39
1169	1.45	3.36	16.62	25.40	14.92	1.67	0.40
1186	1.45	3.43	16.96	25.84	15.17	1.70	0.39
1206	1.45	3.45	17.31	26.32	15.44	1.71	0.39
1224	1.46	3.50	17.68	26.88	15.78	1.73	0.39
1238	1.47	3.53	17.94	27.21	15.97	1.74	0.39
1257	1.48	3.57	19.18	28.16	16.43	1.78	0.39
1283	1.48	3.62	19.87	28.87	16.82	1.78	0.39
1307	1.47	3.65	20.30	29.42	17.12	1.84	0.40
1320	1.47	3.71	20.77	30.07	17.63	1.84	0.39
1343	1.47	3.74	21.20	30.63	17.99	1.92	0.40
1365	1.47	3.77	21.64	31.16	18.27	1.95	-1.03
1385	1.47	3.82	22.08	31.71	18.61	1.97	-1.03
1403	1.49	3.89	22.59	32.39	19.12	2.02	-1.03
1428	1.49	4.01	23.62	33.33	19.68	2.09	-1.03
1416	1.50	4.05	23.97	33.86	19.88	2.13	-1.03
1447	1.50	4.05	24.50	34.53	20.25	2.13	-1.03
1472	1.50	4.07	24.94	35.14	20.61	2.17	-1.03
1483	1.50	4.09	25.17	35.46	20.79	2.17	-1.03
1499	1.50	4.11	25.47	35.89	21.05	2.17	-1.03
1535	1.50	4.22	26.24	36.90	21.65	2.25	-1.03
1543	1.50	4.26	26.47	37.22	21.88	2.25	-1.03
1565	1.50	4.28	26.97	37.87	22.26	2.25	-1.03
1575	1.50	4.32	27.24	38.20	22.47	2.25	-1.03
1609	1.51	4.40	28.00	39.17	23.05	2.31	-1.02
1619	1.52	4.46	28.55	39.99	23.74	2.41	-1.03

1800-series (Type 2-20)							
Load (kN)	Vertical deflection (mm)						
	1	2	3	4	5	6	7
1649	1.52	4.54	29.09	40.72	24.17	2.42	-1.03
1666	1.52	4.56	29.49	41.24	24.46	2.45	-1.03
1682	1.53	4.59	29.81	41.66	24.71	2.47	-1.03
1703	1.53	4.65	30.44	42.41	25.16	2.50	-1.03
1726	1.57	4.74	31.16	43.44	25.73	2.63	-1.03
1739	1.57	4.75	31.45	43.82	25.96	2.62	-1.03
1763	1.56	4.82	31.97	44.49	26.35	2.63	-1.03
1773	1.56	4.84	32.19	44.78	26.51	2.64	-1.03
1776	1.56	4.85	32.34	44.93	26.60	2.66	-1.03
1773	1.56	4.85	32.35	44.94	26.61	2.66	-1.03
1792	1.57	4.87	32.72	45.41	26.88	2.73	-1.03
1798	1.57	4.89	32.94	45.67	27.03	2.73	-1.03
1820	1.55	4.93	33.75	46.59	27.54	2.90	-1.03
1842	1.54	4.96	34.34	47.24	27.91	3.20	-1.17
1851	1.55	4.98	34.92	47.78	28.18	3.42	-1.17
1862	1.54	5.01	35.48	48.30	28.48	3.51	-1.16
1864	1.54	5.02	35.65	48.48	28.57	3.54	-1.16
1870	1.53	5.03	35.83	48.66	28.68	3.62	-1.16
1875	1.52	5.08	36.35	49.16	28.97	3.79	-1.17
1897	1.51	5.10	36.71	49.62	29.24	3.87	-1.17
1898	1.51	5.10	36.91	49.82	29.37	3.92	-1.16
1902	1.51	5.12	37.11	50.02	29.49	3.95	-1.16
1896	1.48	5.16	37.44	50.24	29.61	4.00	-1.16
1893	1.49	5.18	37.78	50.47	29.70	4.06	-1.17
1896	1.49	5.20	38.03	50.68	29.79	4.09	-1.17
1903	1.49	5.23	38.54	51.13	30.01	4.24	-1.17
1913	1.50	5.24	39.00	51.58	30.25	4.38	-1.17
1910	1.49	5.26	39.31	51.82	30.39	4.39	-1.17
1921	1.49	5.28	39.57	52.11	30.54	4.52	-1.17
1927	1.47	5.30	39.86	52.41	30.73	4.64	-1.17

B.1. Strain Gauge Data on Longitudinal Reinforcing Steel

Figure B-2 shows the typical positions of the strain gauges on the longitudinal reinforcing bars in the beam specimens. The strain readings from the various strain gauges are presented in **Tables B-12 to B-22**. In each table, the distance of the strain gauge from the centre of the beam is shown. The value “0” indicates that the strain gauge was placed at the centre of the beam; the value “1000” indicates that the strain gauge was placed 1000 mm to the right from the centre of the beam, and a negative value indicates that the strain gauge was on the left side of the beam.

Figure B-2—Typical strain gauge locations on the longitudinal reinforcing bars

Table B-12—Strain readings for Specimen 450-series (Type 1-20).

450-series (Type 1-20)					
Strains (x10 ⁻⁶)					
Distance from centre of the beam (mm)	-1500	-750	0	750	1500
Load (kN)	S	Q	C	QV	SV
0	0	0	-1	-1	0
49	7	33	50	32	9
101	16	71	289	67	18
200	34	589	708	566	39
267	1399	665	526	494	76
300	0	0	828	654	437
400	0	0	1155	972	1006
470	0	0	1387	1225	1423

Table B-13—Strain readings for Specimen 450-series (Type 1-10).

450-series (Type 1-10)					
Strains (x10 ⁻⁶)					
Distance from centre of the beam (mm)	-1500	-750	0	750	1500
Load (kN)	S	Q	C	QV	SV
0	0	0	0	0	0
99	17	52	85	48	19
199	45	496	496	447	44
297	342	1233	829	894	514
321	1151	1306	886	976	877
399	0	0	1095	1256	1283
498	0	0	1357	1625	1683
577	0	0	1557	1960	1997

Table B-14—Strain readings for Specimen 450-series (Type 2-20).

450-series (Type 2-20)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-1500	-750	0	750	1500
Load (kN)	S	Q	C	QV	SV
0	-1	-2	-2	-2	-1
98	11	73	137	123	13
198	28	629	621	776	34
297	64	1101	974	1129	135
397	645	1751	1157	1455	672
496	1080	2511	1417	1757	896
586	1366	3388	1634	2051	1168
597	0	0	1666	2049	1264
607	0	0	1695	2088	1306
627	0	0	1751	2176	1372

Table B-15—Strain readings for Specimen 900-series (Type 1-20).

900-series (Type 1-20)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-3000	-1500	0	1500	3000
Load (kN)	S	Q	C	QV	SV
0	0	0	0	0	0
107	5	50	71	47	5
195	9	136	358	103	11
301	15	717	620	585	19
336	18	804	708	669	22
395	0	0	844	905	29
518	0	0	1129	1345	64
592	0	0	1297	1635	398
709	0	0	1567	2017	1045
778	0	0	1479	0	1324

Table B-16—Strain readings for Specimen 900-series (Type 1-10).

900-series (Type 1-10)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-3000	-1500	0	1500	3000
Load	S	Q	C	QV	SV
0	0	0	0	1	-2
109	5	50	91	45	0
206	8	114	336	86	5
298	12	952	493	642	10
398	22	1642	709	931	17
501	577	2050	913	1355	24
523	640	2129	950	1406	29
601	0	0	1124	1616	51
706	0	0	1315	1881	388
800	0	0	1513	2148	620
820	0	0	1548	2223	688

Table B-17—Strain readings for Specimen 900-series (Type 2-20).

900-series (Type 2-20)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-3000	-1500	0	1500	3000
Load (kN)	S	Q	C	QV	SV
0	0	0	0	0	0
97	0	0	0	0	0
193	399	0	72	57	6
301	678	0	507	496	16
405	927	0	699	765	22
499	1147	0	996	935	42
609	1410	0	1356	1141	117
699	1618	0	1551	1322	281
802	1862	0	1744	1568	521
903	2097	0	1832	1913	729
1003	2028	0	1363	1680	563
1043	2116	0	1364	1772	617
1096	0	0	255	1826	650
1448	0	0	0	3433	1438

Table B-18—Strain readings for Specimen 1350-series (Type 1-20).

1350-series (Type 1-20)					
Strains (x10 ⁻⁶)					
Distance from centre of the beam (mm)	-4550	-2275	0	2275	4550
Load (kN)	S	Q	C	QV	SV
0	0	0	0	0	0
104	5	31	45	27	1
200	9	60	84	52	3
317	13	126	429	489	5
402	16	587	535	602	7
509	19	766	677	744	8
599	21	912	794	865	11
676	0	0	892	993	13
695	0	0	927	1046	16
807	0	0	1056	1224	20
892	0	0	1157	1357	220
995	0	0	1279	1493	484
1021	0	0	1309	1510	512
1107	0	0	1411	1588	622
1201	0	0	1524	1852	738
1300	0	0	1637	2023	986
1399	0	0	1752	2206	1163
1499	0	0	1872	2391	1339

Table B-19—Strain readings for Specimen 1350-series (Type 1-10).

1350-series (Type 1-10)					
Strains (x10 ⁻⁶)					
Distance from centre of the beam (mm)	-4550	-2275	0	2275	4550
Load (kN)	S	Q	C	QV	SV
0	1	0	2	6	0
94	0	21	47	12	4
205	2	48	85	36	10
295	4	69	388	84	15
407	7	122	528	483	19
499	9	593	665	678	24
598	14	860	784	874	28
701	750	1080	923	1046	33
803	965	1257	1051	1210	41
906	1198	1453	1179	1414	140
902	0	0	1183	1414	144
1028	0	0	1336	1619	681
1094	0	0	1410	1695	788
1204	0	0	1539	1857	984
1303	0	0	1653	1997	1153
1401	0	0	1769	2131	1296
1497	0	0	1883	2366	1510
1604	0	0	2001	2518	1683
1703	0	0	2122	2787	1910
1752	0	0	2178	2892	2040

Table B-20—Strain readings for Specimen 1350-series (Type 2-20).

1350-series (Type 2-20)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-4550	-2275	0	2275	4550
Load (kN)	S	Q	C	QV	SV
0	0	-2	-6	0	0
101	2	42	105	83	4
199	7	74	278	292	9
297	11	309	408	435	15
400	16	436	590	556	21
497	22	663	717	655	29
600	28	802	850	758	36
701	39	949	979	861	109
803	58	1093	1111	966	218
901	129	1204	1227	1064	292
1002	491	1366	1342	1168	347
1102	568	1479	1461	1394	405
1202	675	1593	1581	1492	467
1302	745	1701	1699	1621	527
1400	828	1815	1813	1796	671
1500	961	2028	1932	1927	786
1606	1098	2066	2059	2073	1077
1702	1265	2186	2171	2193	1152
1801	1505	2235	2289	2332	1247
1895	0	0	2429	2519	1405
1992	0	0	2553	2633	1508
2088	0	0	2681	2833	1624
2193	0	0	2851	3141	1754

Table B-21—Strain readings for Specimen 1800-series (Type 1-20).

1800-series (Type 1-20)					
Strains (x10 ⁻⁶)					
Distance from centre of the beam (mm)	-6100	-3050	0	3050	6100
Load (kN)	S	Q	C	QV	SV
0	0	1	0	-1	0
103	11	23	55	30	8
195	13	111	132	55	10
301	16	199	268	121	14
403	19	378	377	324	19
500	21	509	445	432	23
604	24	746	518	588	26
707	27	941	591	698	30
802	30	1067	658	803	37
901	33	1210	726	897	40
923	0	0	750	957	44
1010	0	0	0	1090	723
1094	0	0	0	1168	762
1216	0	0	0	1301	883
1306	0	0	0	1402	1069
1410	0	0	0	1504	1275
1500	0	0	0	1599	1482
1604	0	0	0	1710	1639
1665	0	0	0	1766	1717

Table B-22—Strain readings for Specimen 1800-series (Type 2-20).

1800-series (Type 2-20)					
Strains ($\times 10^{-6}$)					
Distance from centre of the beam (mm)	-6100	-3050	0	3050	6100
Load (kN)	S	Q	C	QV	SV
0	2	0	0	0	1
101	2	18	20	19	4
192	5	36	165	36	6
305	6	74	310	76	9
396	8	190	390	289	13
498	10	406	461	468	16
611	11	543	535	591	21
700	13	623	594	705	26
800	16	732	655	826	32
916	20	844	728	938	38
1004	23	991	785	1041	42
1090	26	1080	841	1139	46
1206	37	1207	917	1266	54
1307	300	1320	984	1380	57
1403	419	1426	1050	1496	100
1499	1133	1569	1115	1602	371
1609	1321	1870	1190	1740	609
1703	1447	2026	1256	1858	1089
1798	1528	2163	1322	1977	1252
1902	1627	2352	1397	2085	1414
1927	0	0	1414	2112	1458
1992	0	0	1437	2191	1589
2101	0	0	1502	2299	1697
2172	0	0	1539	2371	1767

B.2. Strain Gauge Data on Shear Reinforcing Steel

Figures B-3 to B-6 show the crack patterns of all beam specimens in the four series (450, 900, 1350, and 1800 series) in relation to the position of the shear reinforcing bars placed in the specimen. The strain readings from the various strain gauges on the shear reinforcing bars in each shear span specimen are presented in **Tables B-23 to B-37**.

Figure B-3—Crack patterns of specimens in 450-series with the marking of shear reinforcing bars.

Table B-23—Strain readings for Specimen 1V1-450-20.

1V1-450-20						
Load (kN)	Strain ($\times 10^{-6}$)					
	V1	V2	V3	V4	V5	V6
0	0	0	0	0	0	0
101	3	-2	4	-1	2	-4
200	-23	29	289	-21	14	-3
260	-27	39	1644	410	-6	9
300	-12	75	7206	1987	31	6
320	-11	117	8258	2957	65	-6
340	-8	143	4912	3636	128	-8
361	-6	159	5073	5633	228	-11
380	-4	183	5251	10111	505	-8
400	-1	-	5244	13575	763	-5
420	1	-	5249	7292	960	-1
440	6	-	5324	7187	1123	3
460	14	-	-	6896	1203	7

Table B-24—Strain readings for Specimen 1V1-450-10.

1V1-450-10						
Load (kN)	Strain ($\times 10^{-6}$)					
	V1	V2	V3	V4	V5	V6
0	0	0	0	1	0	0
99	-4	7	0	-2	-1	-7
120	-9	8	0	-2	-1	-9
199	5	784	38	-1	-1	-12
227	13	1057	49	-5	3	-11
297	19	793	1555	1763	-8	7
321	26	605	1724	3144	-3	5
327	30	601	1927	3837	-2	1
349	34	652	1957	4770	0	-1
399	49	729	2220	13552	113	-12
460	66	764	2294	12255	126	-5
498	81	737	2326	10995	130	2
546	100	748	-	10562	134	10
558	105	729	-	10500	136	11
568	108	758	-	10505	137	13
577	114	802	-	10500	-	14

Table B-25—Strain readings for Specimen 2V2-450-20.

2V2-450-20								
Load (kN)	Strain ($\times 10^{-6}$)							
	V1	V2	V3	V4	V5	V6	V7	V8
0	0	0	0	-1	-1	0	-1	-2
100	0	-7	-2	0	0	0	0	-3
120	21	-12	4	-1	-2	0	0	-4
201	22	7	-56	1	19	-1	0	-6
239	32	19	80	-21	-6	7	6	-7
300	53	40	889	177	349	-25	8	-1
355	86	46	2243	1521	2797	505	5	4
400	96	56	3424	1875	3513	640	33	-4
480	112	114	10591	2760	5114	3060	1204	-1
499	119	136	13575	2946	6008	3446	1334	-2
588	117	230	-	6259	5915	9757	1672	-2

Table B-26—Strain readings for Specimen 2V3-450-20.

2V3-450-20											
Load (kN)	Strain ($\times 10^{-6}$)										
	VV1	VV2	VV3	VV4	VV5	VV6	VV7	VV8	VV9	VV10	VV11
0	-1	-1	0	-1	0	-1	-1	0	-2	0	0
100	-2	-3	-6	-2	0	0	-1	0	-1	-1	-7
120	0	-15	-10	0	2	0	-2	-1	-1	-1	-7
201	45	8	26	486	-20	49	341	-30	-5	4	-9
260	71	21	14	1000	35	388	1364	359	297	-6	-2
300	101	30	14	1497	70	719	1735	1029	669	0	0
400	172	20	23	2812	407	1746	2643	2641	1484	1796	-19
499	612	37	64	5284	496	2635	3814	3564	1839	2678	-11
519	704	45	68	6105	506	2956	4497	3792	1899	2822	-9
588	1294	303	170	10769	567	3908	9536	5547	2087	3318	0
630	713	148	105	-3399	121	1261	4970	4122	716	1119	0

Figure B-4—Crack patterns of specimens in 900-series with the marking of shear reinforcing bars.

Table B-27—Strain readings for Specimen IV1-900-20.

1V1-900-20							
Load (kN)	Strain ($\times 10^{-6}$)						
	V1	V2	V3	V4	V5	V6	V7
0	0	0	0	0	0	0	0
107	1	0	4	3	3	0	1
153	2	0	3	7	1	-1	1
195	-2	-44	0	15	13	-2	1
301	-30	-63	-16	-5	-29	-3	11
318	-78	-63	-17	5	-15	-3	11
336	-6	-66	-20	20	21	-3	14
395	-8	-66	-25	39	120	0	11
487	16	-55	15	543	2919	-5	16
518	-5	-59	-1	184	665	1	3
592	-1	-60	8	595	3374	-2	17
642	7	-55	25	650	4035	-8	22
709	-9	-52	22	687	4097	-2	1376
778	17	-47	43	751	4019	0	2250

Table B-28—Strain readings for Specimen IV1-900-10.

1V1-900-10						
Load (kN)	Strain ($\times 10^{-6}$)					
	V1	V2	V3	V4	V5	V6
0	0	82	2	0	-1	0
109	-3	6	7	0	0	-179
167	1	4	8	0	0	-308
206	2	39	9	1	0	-340
300	-11	14	-7	13	-1	-291
317	-9	38	-7	14	-1	-308
398	-16	114	22	22	-1	-343
501	10	3187	24	3	0	-274
601	12	1502	2152	3	1	-363
662	14	1450	2392	2	-6	-340
706	21	1483	2451	2	-7	-287
800	28	1475	2622	3	-13	-244
815	34	1544	2647	3	-13	-238

Table B-29—Strain readings for Specimen 2V2-900-20.

2V2-900-20								
Load (kN)	Strain ($\times 10^{-6}$)							
	V1	V2	V3	V4	V5	V6	V7	V8
0	0	0	1	0	0	2	-17	0
94	-1	1	0	5	-5	-3	87	14
193	3	7	4	1	-4	-2	-14	65
220	-5	3	9	6	-2	0	-1	4
301	21	-17	-3	20	-1	1	1	24
405	11	1	54	-15	881	-22	-13	28
424	13	1	51	-17	1145	-24	2	36
499	12	-1	58	88	2756	3	-30	52
609	8	23	69	430	4950	23	-22	2
645	8	25	80	683	9160	39	-16	46
699	9	28	87	1314	14931	90	-10	-129
802	14	38	50	2175	9429	97	-10	-91
864	15	50	94	-	9585	104	-19	-113
903	17	68	115	-	9568	115	-16	-18
1003	-2	70	83	-	352	114	-12	-25
1043	0	74	152	-	425	122	-11	-34

Table B-30—Strain readings for Specimen 2V3-900-20.

2V3-900-20											
Load (kN)	Strain (x10 ⁻⁶)										
	VV1	VV2	VV3	VV4	VV5	VV6	VV7	VV8	VV9	VV10	VV11
0	0	0	0	2	-37	0	0	-	2	1	1
94	12	3	-7	21	7	3	5	-	5	4	3
193	73	-1	7	1	18	3	1	-	0	0	-2
301	46	-2	12	46	75	13	19	-	9	6	3
405	23	4	332	40	671	38	246	-	16	12	0
499	50	39	910	56	956	41	860	-	2	13	5
580	539	48	890	87	1652	382	2980	-	6	-10	-38
609	15	44	1481	65	1559	70	1527	-	0	7	6
699	3	50	2018	79	2023	113	2229	-	0	8	4
802	33	56	2792	72	2505	215	3089	-	21	-6	-6
870	179	19	515	64	1103	273	2199	-	8	-16	-23
903	54	64	2219	104	2794	555	3559	-	17	-8	-9
1003	139	42	824	65	1452	361	3175	-	10	-16	-21
1096	399	37	757	79	1523	372	2687	-	9	-16	-28
1160	602	49	911	88	1737	385	3024	-	7	-9	-39
1201	650	50	770	90	1785	388	3048	-	8	-9	-40
1299	945	52	608	4441	1873	528	3202	-	12	-11	-41
1395	1172	55	417	4451	1993	9859	3335	-	49	-16	-41
1448	1214	65	361	4503	2041	13692	3973	-	60	-15	-37

Figure B-5—Crack patterns of specimens in 1350-series with the marking of shear reinforcing bars.

Table B-31—Strain readings for Specimen 1V1-1350-20.

1V1-1350-20												
Load (kN)	Strain ($\times 10^{-6}$)											
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
0	0	0	-	1	-	2	0	0	0	0	0	0
104	-1	-1	-	0	-	1	0	0	0	1	0	-1
200	-2	-3	-	-2	-	3	0	0	0	2	0	-3
286	4	11	-	1	-	33	-18	3	0	1	0	-4
402	13	20	-	1	-	58	140	16	0	5	0	-6
509	5	28	-	0	-	63	647	143	0	9	1	-8
599	2	28	-	4	-	56	1146	473	0	6	2	-9
695	8	-2	-	20	-	40	1856	2452	0	3	5	-6
807	10	-17	-	27	-	37	2790	4279	0	-6	4	-3
892	10	-11	-	41	-	1079	3192	1603	0	50	4	-6
998	11	-6	-	56	-	2456	3928	1889	0	68	5	-10
1107	13	-13	-	61	-	374	4144	1940	0	66	6	-12
1201	11	-13	-	70	-	379	4271	2251	0	65	5	-13
1300	11	-10	-	81	-	390	4300	2070	0	329	5	-11
1403	13	0	-	93	-	431	4429	2257	0	561	8	-10
1499	20	231	-	90	-	447	4222	2321	0	615	8	-9

Table B-32—Strain readings for Specimen 1V1-1350-10.

1V1-1350-10												
Load (kN)	Strain ($\times 10^{-6}$)											
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12
0	0	0	0	0	1	0	4	4	3	2	0	0
94	0	-1	-2	0	-1	0	2	0	0	0	0	-2
205	-4	0	-2	0	0	-2	3	4	0	0	-1	-5
295	-9	1	-38	-10	-4	12	0	0	0	0	-4	-9
350	-18	-3	-67	-5	0	20	2	-10	0	0	-4	-11
407	-22	-9	-83	10	2	40	44	-6	7	5	-4	-12
499	-27	-12	-111	117	-20	42	44	66	-9	6	0	-15
598	-29	-14	-91	178	-22	44	54	140	-17	1	0	-17
701	-27	-10	-55	245	-7	50	63	583	-16	-3	1	-18
803	-29	0	-7	277	0	60	60	1702	4	-8	0	-17
906	-33	8	5	280	0	69	64	2050	4673	2	-18	-20
1028	-36	26	24	295	28	1283	68	2246	9641	-4	-12	-31
1063	-37	28	23	302	32	1397	67	2237	9502	-4	-12	-32
1092	-40	30	23	309	35	1410	70	2279	9388	-2	-12	-33
1204	-47	36	22	340	43	1709	113	2295	9433	1	-16	-36
1303	-55	45	29	375	49	1879	130	2437	10011	7	-17	-37
1401	-61	48	49	439	60	1925	148	2497	11502	10	-18	-39
1497	-67	44	82	562	68	1976	177	2900	10912	22	-16	-38
1604	-71	34	97	644	81	2013	224	-	10674	50	-17	-40
1703	-74	38	116	734	91	2046	282	-	10312	-	-16	-41
1752	-75	39	129	816	101	-	341	-	9961	-	-17	-42

Table B-33—Strain readings for Specimen 2V2-1350-20.

2V2-1350-20														
Load (kN)	Strain ($\times 10^{-6}$)													
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14
0	-1	0	0	1	-1	0	0	0	0	1	0	-1	1	0
101	-1	-5	-3	-5	-2	-1	0	0	0	0	0	-1	-2	-3
199	-14	-7	-7	0	2	-3	-1	-1	0	0	0	-1	-3	-6
297	-39	-8	-5	10	16	-21	13	5	0	4	0	0	-3	-10
378	-47	-6	0	0	21	-23	25	8	-9	9	4	2	-3	-11
400	-49	-8	-1	-2	22	-24	23	13	71	-25	2	7	-1	-10
497	-54	-12	17	6	28	-21	23	325	99	-10	-5	10	3	-10
600	-61	-10	16	18	32	-13	20	827	143	11	-13	4	4	-11
701	-66	-7	19	19	34	0	13	1360	272	-3	-5	-2	-5	-12
742	-81	-7	20	13	39	8	11	1550	331	-4	-3	-1	-9	-14
803	-99	-4	26	-9	47	24	14	1838	449	-1	-8	-6	-13	-16
901	-118	-1	37	-13	47	150	15	2192	760	19	-8	-12	-16	-20
1002	-130	4	54	16	280	222	19	2407	1435	2340	57	-2	-36	-24
1102	-133	5	63	24	533	230	20	2577	1918	2660	115	-5	-40	-27
1202	-133	11	79	40	743	250	21	2747	2368	2949	1455	5	-42	-30
1302	-120	18	106	80	841	326	18	2830	3041	3219	1798	8	-43	-31
1400	-96	28	134	131	895	1579	28	3014	4821	3606	2070	7	-44	-32
1463	-84	32	176	168	940	2421	46	3214	15409	12026	2191	6	-45	-33
1500	-72	39	599	217	959	2580	49	3284	16764	16217	2255	6	-45	-34
1606	-60	42	898	349	1027	9188	28	3354	17850	18500	2470	6	-45	-36
1702	-53	42	1074	443	1036	10026	31	17101	-	19176	2687	10	-43	-36
1801	-49	45	1176	456	1060	10705	3138	22428	-	20474	2839	17	-37	-38
1829	-46	47	1160	450	1064	11317	-	-	-	21506	2899	20	-34	-39

Table B-34—Strain readings for Specimen 2V3-1350-20.

2V3-1350-20																			
Load (kN)	Strain (x10 ⁻⁶)																		
	VV1	VV2	VV3	VV4	VV5	VV6	VV7	VV8	VV9	VV10	VV11	VV12	VV13	VV14	VV15	VV16	VV17	VV18	VV19
0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0
101	-3	0	-2	-4	-5	0	-2	-3	0	-3	-1	0	0	0	-1	-1	-2	-3	-3
199	2	-14	-9	-2	-8	5	-8	0	-7	0	2	0	0	1	-1	-2	-3	-6	-6
297	8	-16	-10	-26	15	17	1	17	-7	-6	0	1	2	3	-1	-3	-4	-9	-10
400	14	-19	-3	-14	36	12	11	28	23	-16	37	0	-11	-2	0	-1	-3	-10	-12
481	18	-32	0	-9	36	17	15	27	98	-1	41	-7	-16	-5	-1	2	0	-9	-13
497	18	-33	0	-9	36	17	15	28	111	0	39	-8	-15	-7	-1	2	1	-10	-13
600	19	-43	3	-10	34	34	0	28	219	5	33	-12	-9	-8	0	2	1	-12	-18
701	21	-50	3	-16	43	82	-13	34	306	1	62	12	10	429	217	-26	-15	-7	-11
803	23	-59	4	-24	43	190	-20	44	404	0	89	17	11	646	373	-27	-18	-12	-16
901	27	-68	2	-22	35	417	-17	53	630	2	129	30	15	776	469	-28	-19	-13	-19
943	28	-68	3	-23	36	669	-16	58	703	3	156	39	15	837	518	-30	-20	-14	-20
1002	30	-56	7	-24	40	928	-16	63	768	4	184	73	16	904	575	-30	-20	-14	-21
1102	32	-28	15	-26	62	1420	165	2591	1167	15	268	315	3	1030	689	-30	-22	-17	-25
1202	32	-19	40	-36	67	1517	303	3088	1354	22	446	910	-7	1156	818	-31	-25	-21	-31
1302	36	-23	451	-55	137	1673	401	3398	1399	22	596	1243	-3	1317	872	-33	-27	-25	-36
1400	42	-14	717	8	1981	1966	478	4109	1428	25	784	1568	1	1649	913	-25	-34	-29	-43
1500	45	-16	808	30	2381	2072	652	13555	1468	26	1017	1863	4	1695	970	-26	-35	-32	-48
1606	48	-16	1045	69	2830	2147	996	23314	1999	44	1142	1881	17	1749	2449	33	-46	-47	-55
1702	48	-17	1182	106	3180	2196	1050	24573	2229	43	1188	1943	16	1784	2678	57	-50	-50	-60
1801	50	-14	1299	165	3585	2240	1090	26143	2516	41	1306	2160	15	1821	2881	64	-54	-51	-62
1873	60	-14	1644	467	22929	2726	1183	-	2625	2855	3400	2211	20	1919	2782	71	-54	-51	-67
1895	60	-16	1654	483	23153	2791	1188	-	2650	2860	3406	2217	21	1929	2803	71	-55	-51	-67
1992	62	-15	1770	594	23586	2980	1219	-	2773	2939	3877	2255	24	-	2873	70	-56	-52	-69

2V3-1350-20																			
Load (kN)	Strain (x10 ⁻⁶)																		
	VV1	VV2	VV3	VV4	VV5	VV6	VV7	VV8	VV9	VV10	VV11	VV12	VV13	VV14	VV15	VV16	VV17	VV18	VV19
2088	63	-14	1915	733	23745	3150	1251	-	2835	-	4082	2321	29	-	2946	68	-57	-51	-71
2140	64	-12	1983	759	23777	3250	1279	-	2870	-	9039	2376	33	-	2981	66	-57	-51	-71
2193	64	-13	2071	794	23855	3610	1334	-	2901	-	21568	2421	37	-	2995	63	-57	-50	-73
2190	67	-9	2098	833	23489	-	1533	-	2904	-	23385	2467	42	-	2965	63	-55	-48	-72

Figure B-6—Crack patterns of specimens in 1800-series with marking of shear reinforcing bars.

Table B-35—Strain readings for Specimen 1V1-1800-20.

1V1-1800-20																				
Load (kN)	Strain ($\times 10^{-6}$)																			
	V1C	V2C	V3C	V4A	V4B	V4C	V5A	V5B	V5C	V5D	V6B	V6C	V6D	V7C	V7D	V8C	V8D	V9C	V9D	V10D
0	0	0	0	0	1	1	0	0	-1	-	0	0	0	-2	-1	1	0	0	0	0
103	8	-2	-1	19	9	2	14	4	3	-	0	2	4	-3	0	2	5	4	0	2
195	8	0	11	19	0	-1	17	8	8	-	1	1	1	-2	-1	2	5	5	-4	0
301	13	4	-21	21	-1	3	24	13	17	-	-3	-5	11	2	-1	1	6	7	-8	0
342	14	3	-24	22	-4	1	28	20	26	-	-8	-14	5	6	-3	2	8	7	-10	0
403	9	5	-31	23	-7	-2	37	30	43	-	-16	-22	11	10	6	5	12	8	-15	-1
500	8	4	-37	20	-8	-2	358	31	46	-	-15	-19	-9	19	23	6	17	11	-21	-2
604	11	3	-39	19	-7	-8	831	31	48	-	157	9	17	766	22	14	63	23	-17	-1
684	13	2	-34	20	-9	-19	1177	38	54	-	872	7	23	884	19	17	85	28	-17	-1
707	14	1	-33	21	-10	-21	1253	38	56	-	1021	6	24	915	21	16	90	30	-16	0
802	16	0	-23	23	-25	608	1686	38	52	-	2200	15	-33	1447	44	-17	2141	66	78	10
901	15	-2	-22	14	-23	962	2103	47	52	-	2588	15	-28	1516	45	-20	2674	75	93	12
917	15	-1	-23	23	-18	1054	2331	49	52	-	2579	14	-26	1546	48	-20	2827	75	100	13
923	15	0	-18	83	-8	1476	2771	49	34	-	2559	23	-17	1629	48	-6	3007	68	158	20
1071	293	28	-11	656	186	2732	2934	55	2276	-	-	37	-	2256	129	3279	2934	34	204	411
1163	312	29	-13	669	301	2894	3110	57	2312	-	-	38	-	2391	139	3318	3079	34	213	427
1216	327	29	-13	678	424	2958	3358	59	2317	-	-	39	-	2443	146	3324	3051	33	215	498
1306	356	27	-14	687	1219	3030	3510	58	2487	-	-	32	-	2532	178	3325	3125	39	222	592
1363	377	27	-15	697	1301	3053	3810	64	2602	-	-	31	-	2578	331	3355	3173	41	224	592
1410	401	26	-13	1385	1311	3060	5198	63	2686	-	-	29	-	2629	858	3410	8727	43	224	593
1500	450	25	-13	1769	1331	3079	4899	69	2810	-	-	32	-	2717	1099	3417	3316	46	220	608
1604	535	25	-11	2023	1391	2953	5031	69	3263	-	-	893	-	2696	1215	3424	2932	46	223	651
1665	638	24	-14	2163	1428	3000	5034	70	-	-	-	1168	-	3066	1229	3525	2757	45	225	652

Table B-36—Strain readings for Specimen 2V2-1800-20.

2V2-1800-20																		
Load (kN)	Strain ($\times 10^{-6}$)																	
	V1C	V2C	V3C	V4A	V4B	V4C	V5A	V5B	V5C	V6A	V6B	V6C	V7A	V7B	V7C	V7D	V8C	V8D
0	-	-	0	1	0	1	2	2	0	1	2	0	0	1	0	0	2	0
101	-	-	1	11	4	2	7	6	3	10	2	3	8	5	6	-31	1	1
305	-	-	-10	33	0	0	16	12	7	33	10	15	30	17	33	-128	0	3
389	-	-	-2	67	23	39	23	-3	-16	40	3	21	38	21	77	-223	0	1
396	-	-	1	70	76	36	21	-6	-20	40	5	20	38	22	94	-238	-1	2
498	-	-	8	81	414	33	23	-12	-33	53	10	28	48	28	223	-374	-1	2
611	-	-	13	75	825	33	16	-19	-30	80	62	49	47	79	1113	-356	58	31
700	-	-	17	71	1216	37	26	1	-33	80	77	52	52	49	1739	-347	801	46
787	-	-	18	62	1636	39	64	27	-19	93	127	38	53	757	1730	-354	2151	51
800	-	-	19	63	1797	42	65	28	271	95	125	28	53	777	1757	-348	2231	52
884	-	-	21	58	2152	44	78	32	1240	109	184	23	54	1335	1827	-329	2709	54
1004	-	-	23	72	2857	54	89	34	3072	120	196	16	54	1452	1895	188	2833	67
1090	-	-	23	72	4132	57	100	36	3637	129	230	13	55	1682	1939	223	2860	69
1169	-	-	24	57	9611	57	111	39	3611	129	231	9	81	2184	2005	244	2927	73
1206	-	-	25	51	12487	58	122	41	3411	130	237	8	81	2424	2039	268	2963	75
1307	-	-	23	86	16545	58	447	61	3244	135	285	11	78	2226	2083	338	2979	75
1403	-	-	21	823	4533	59	849	69	3375	147	326	10	74	2302	2131	445	3088	80
1499	-	-	23	2593	4238	62	865	943	3156	162	387	2876	60	1913	2768	592	3253	82
1543	-	-	22	2803	4168	62	871	1011	3388	169	410	2969	62	1965	2561	633	3288	83
1609	-	-	23	3188	4189	64	876	1099	3964	180	427	3048	61	2106	2449	715	3353	-
1703	-	-	23	3755	4290	65	887	1208	4205	194	454	11678	59	2275	2296	903	3430	-
1798	-	-	23	5155	4228	66	900	1540	1370	212	514	4314	55	2425	2240	1058	-	-
1902	-	-	24	2848	4206	68	1207	1834	1424	246	696	4136	41	2465	2131	1120	-	-
1927	-	-	18	2826	4159	66	1445	1963	1372	290	803	4162	35	2470	2134	1116	-	-

Table B-36 (cont.)—Strain readings for Specimen 2V2-1800-20.

2V2-1800-20							
Load (kN)	Strain ($\times 10^{-6}$)						
	V9C	V9D	V10C	V10D	V11C	V11D	V12C
0	0	2	1	0	3	1	1
101	2	1	0	4	1	0	0
305	2	-2	1	1	2	0	-1
389	3	-2	1	1	4	0	-2
396	4	-3	1	0	3	1	-1
498	4	-2	3	3	4	1	-2
611	-12	-24	0	24	8	5	-2
700	21	69	-18	-18	14	11	-2
787	6	100	8	72	13	27	1
800	5	101	5	74	13	29	0
884	73	119	-1	114	4	20	7
1004	243	368	3	156	4	29	8
1090	866	685	2	162	5	27	8
1169	1772	967	2135	174	-7	269	15
1206	1780	1146	2445	180	-5	1112	15
1307	1834	1348	6799	1174	13	15952	-17
1403	1893	1439	16025	2269	13	19878	-18
1499	1940	1499	6268	2217	169	5700	-17
1543	1949	1512	5654	2238	174	5612	-18
1609	1962	1520	5623	2254	174	5708	-20
1703	2028	1579	5435	2281	172	5907	-25
1798	2171	1658	5283	2318	171	6010	-29
1902	2525	1990	5129	2376	172	6177	-18
1927	2674	2071	5116	2408	175	6302	-5

Table B-37—Strain readings for Specimen 2V3-1800-20.

2V3-1800-20																		
Load (kN)	Strain (x10 ⁻⁶)																	
	VV1C	VV2C	VV3C	VV4A	VV4C	VV5A	VV5B	VV5C	VV6A	VV6B	VV6C	VV7A	VV7B	VV7C	VV8A	VV8B	VV8C	VV8D
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
101	0	3	-3	6	-3	6	1	-4	8	4	-1	10	5	-2	8	5	0	2
192	5	5	-3	15	-4	13	5	-6	14	9	-1	21	8	-3	15	8	1	3
217	-1	0	-6	17	-8	16	5	-8	16	9	-2	24	8	-4	17	10	0	2
305	-6	-7	-13	27	-15	26	7	-13	25	16	1	37	9	-8	24	13	-1	3
396	-6	-3	-3	43	-7	32	34	-28	23	-22	-36	44	-9	-23	31	16	16	69
498	-8	0	-1	61	-11	26	128	-40	26	-32	-53	54	-11	-29	43	24	29	92
611	-6	5	-1	69	-15	31	241	-45	31	-40	-57	73	-9	-18	62	403	34	81
700	-7	12	-28	70	-17	32	314	-36	34	-38	-54	100	44	-5	61	1246	35	76
800	-9	23	-36	76	-33	32	423	-29	38	-32	-54	123	60	-2	69	1806	28	163
916	-11	40	-37	86	-45	35	583	-23	48	-22	-60	134	60	-3	80	2220	28	508
1004	-11	59	-37	88	-51	36	1023	-22	52	-23	-43	145	55	-24	89	2312	25	739
1090	-9	82	-37	94	-55	38	1285	-23	68	-8	-31	153	62	-27	99	2686	25	854
1206	-7	164	-38	102	-64	38	1554	-23	91	3	-15	168	76	-30	112	3190	31	1016
1307	0	214	-38	108	-73	36	1615	-24	108	8	9	185	96	-29	122	4598	36	1125
1403	7	234	-38	110	-80	31	1642	-24	113	13	32	206	140	-25	116	12536	43	1293
1499	0	245	-34	112	-85	33	1661	-26	129	16	31	223	172	-25	121	3991	49	1548
1609	1	324	-29	105	-88	80	1679	-25	182	27	30	237	190	-23	121	3631	57	1877
1703	0	413	-24	90	-90	284	1687	-24	333	42	29	247	197	-21	120	3528	65	2206
1798	-1	526	-17	87	-78	383	1761	-28	396	45	5839	255	185	2	127	3409	50	2263
1902	-4	616	-15	83	-80	530	1793	-28	493	54	3151	257	189	4	175	3351	51	2350
2V3-1800-20																		

Load (kN)	Strain (x10 ⁻⁶)																	
	VV1C	VV2C	VV3C	VV4A	VV4C	VV5A	VV5B	VV5C	VV6A	VV6B	VV6C	VV7A	VV7B	VV7C	VV8A	VV8B	VV8C	VV8D
2024	-20	880	-42	7	-120	1252	1906	-64	659	58	1665	269	199	-27	221	3253	54	2431
2101	-15	916	-41	-4	-115	1597	1925	-62	663	76	1694	271	425	-26	249	3350	55	2485
2172	-10	952	-41	-9	-109	1993	1940	-64	669	88	1692	273	505	-26	291	3463	58	2535

Table B-37 (cont.)—Strain readings for Specimen 2V3-1800-20.

2V3-1800-20																	
Load (kN)	Strain (x10 ⁻⁶)																
	VV9C	VV9D	VV10C	VV10D	VV11C	VV11D	VV12C	VV12D	VV13C	VV13D	VV14C	VV14D	VV15C	VV15D	VV16D	VV16C	VV17C
0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	1	0
101	1	-4	0	2	2	2	4	2	2	1	0	0	1	0	2	2	0
192	2	-8	1	4	5	4	7	4	5	4	-1	-3	2	2	4	3	0
217	3	-9	1	5	5	4	7	5	6	5	-2	-4	1	2	4	3	0
305	6	-4	2	31	8	-12	7	8	6	5	-5	-7	1	4	-5	5	0
396	47	206	2	38	5	-8	7	15	8	9	-6	-7	2	6	-10	5	0
498	86	463	-4	27	0	-19	20	7	27	34	-15	-10	-1	8	-18	5	0
611	63	426	-26	23	-17	4	23	16	47	67	-20	-2	2	41	-9	4	0
700	44	420	-16	29	-3	47	19	14	50	90	81	37	-15	10	-5	4	3
800	54	559	319	34	-2	84	42	29	53	100	1405	35	-3	38	794	0	8
916	64	815	756	39	3	172	42	1113	51	159	1784	46	2	51	1804	-1	9
1004	83	881	1036	54	5	242	47	2078	59	429	2294	60	3	67	2980	2	6
1090	106	868	1479	64	14	251	49	3272	63	768	2451	166	3	77	3363	4	7
1206	245	865	2027	73	16	265	56	3170	77	1587	2869	1093	8	84	3317	-4	6
1307	455	876	2118	78	17	272	62	3139	91	1892	3316	1187	49	85	3200	-3	8
1403	504	881	2146	83	34	275	225	2884	1605	1928	3106	1250	3201	86	3297	17	-3
1499	523	904	2160	92	42	286	297	3132	2204	2107	4016	1310	5208	89	3414	19	-3
1609	533	938	2242	92	55	299	405	3058	2077	2584	868	1389	3458	91	5240	23	-4
1703	540	1086	2501	94	61	315	506	3004	2077	2614	842	1435	1763	102	14419	60	-8
1798	558	3500	2566	132	68	349	562	3042	2084	2661	853	1531	1782	116	3592	66	-8
1902	708	3665	2801	149	76	363	623	-	2286	2700	899	1606	2068	122	3284	70	-6

2V3-1800-20																	
Load (kN)	Strain ($\times 10^{-6}$)																
	VV9C	VV9D	VV10C	VV10D	VV11C	VV11D	VV12C	VV12D	VV13C	VV13D	VV14C	VV14D	VV15C	VV15D	VV16D	VV16C	VV17C
2024	779	4431	3615	238	88	455	895	-	2388	2828	971	1797	1759	128	2319	80	-1
2101	787	4570	1329	249	96	466	960	-	2517	2873	1023	2092	1651	137	2239	84	0
2172	809	4719	1334	262	102	484	1053	-	2608	2939	1045	2427	1587	147	2086	88	2

Appendix C

MS-EXCEL SUB-ROUTINES CODES

VBA (Visual Basic for Applications) is the programming language of MS-Excel programmes. The following sections present the VBA codes of the relevant design methods evaluated in this thesis.

C.1. Analysis of Beams with no Shear Reinforcement

```
Sub ReadInput(i, bw, h, d, A, fc, Asl, rho1, fyl, da, Vtest, a_d)
    bw = Cells(i, "F").Value      'web width (in mm)
    h = Cells(i, "H").Value      'height of beam (in mm)
    d = Cells(i, "I").Value      'effective depth of beam (in mm)
    A = Cells(i, "J").Value      'length of shear span
    fc = Cells(i, "N").Value      'concrete cylinder strength (in MPa)
    Asl = rho1 * b * d            'area of longitudinal reinforcement
    rho1 = Cells(i, "M").Value    'longitudinal reinforcement ratio
    fyl = Cells(i, "P").Value     'yield strength of longitudinal reinforcement
    da = Cells(i, "O").Value      'aggregate size (in mm)
    Vtest = Cells(i, "Q").Value   'experimental failure load in kN
    a_d = Cells(i, "L").Value     'a/d ratio
End Sub
```

ACI 318-14

```
Sub ACI()
    Range("AE3:AF982").ClearContents
    For i = 3 To 982
        ReadInput i, bw, h, d, A, fc, Asl, rho1, fyl, da, Vtest, a_d
```

```
    ***Calculation
        If fc < 69 Then
            fc1 = fc
        Else
            fc1 = 69
        End If
        Vcsimplified = 0.17 * fc1 ^ 0.5 * bw * d / 1000
```

```
    ***Output
    Cells(i, "AE").Value = Vtest / Vcsimplified
    Next i
End Sub
```

Eurocode 2

```
Sub EC2()
    Range("AH3:AH982").ClearContents
    For i = 3 To 982
```

```

ReadInput i, bw, h, d, A, fc, Asl, rho1, fyl, da, Vtest, a_d

***Calculation
If (1 + (200 / d) ^ (1 / 2)) < 2 Then
k = (1 + (200 / d) ^ (1 / 2))
Else
k = 2
End If

If rho1 < 2 Then
rho = rho1
Else
rho = 2
End If

If fc < 90 Then
fc1 = fc
Else
fc1 = 90
End If

V_Rdc1 = (0.18 * k * (rho * fc1) ^ (1 / 3) * bw * d) / 1000
V_Rdc2 = (0.035 * (k ^ (3 / 2)) * (fc1 ^ (1 / 2)) * bw * d) / 1000
V_Rd_c = WorksheetFunction.Max(V_Rdc1, V_Rdc2)

If fc < 50 Then
fc50 = fc
Else
fc50 = 50
End If

V_Rdc50_1 = (0.18 * k * (rho * fc50) ^ (1 / 3) * bw * d) / 1000
V_Rdc50_2 = (0.035 * (k ^ (3 / 2)) * (fc50 ^ (1 / 2)) * bw * d) / 1000
V_Rd_c50 = WorksheetFunction.Max(V_Rdc50_1, V_Rdc50_2)

***Output
Cells(i, "AH").Value = Vtest / V_Rd_c
Cells(i, "AI").Value = Vtest / V_Rd_c50
Next i
End Sub

```

Bažant and Yu

```

Sub Bažant()
Range("AL3:AL982").ClearContents
For i = 5 To 982
ReadInput i, bw, h, d, A, fc, Asl, rho1, fyl, da, Vtest, a_d

conv1 = 145.038 'fc to psi

```

```

conv2 = 0.0393701    'mm to inch
conv3 = 0.224809    'N to pounds

mew = 13.3

If da > 0 Then
K = 3800 * (da * conv2) ^ 0.5
Else
K = 3330
End If

d0 = K * (fc * conv1) ^ (-2 / 3)

***Calculation
Vc = (mew * (rho1 / 100) ^ (3 / 8) * (1 + (d / A)) * ((fc * conv1) / (1 + ((d * conv2) /
d0))) ^ 0.5) / conv1 * bw * d / 1000

***Output
Cells(i, "AL").Value = Vtest / Vc
Next i
End Sub

```

Collins et al.

```

Sub Collins()
Range("AN3:AN982").ClearContents
For i = 3 To 982
ReadInput i, bw, h, d, A, fc, Asl, rho1, fyl, da, Vtest, a_d

If fc > 69 Then
fc1 = 69
Else
fc1 = fc
End If

If fc >= 70 Then
agg = 0
ElseIf fc <= 60 Then
agg = da
Else
agg = ((10 - (fc - 60)) / 10) * da
End If

Sxe = (35 * d) / (16 + agg)

***Calculation
prime = (A - d) / d
M = (1 / (rho1 / 100)) * ((1.11 * prime) + 1)

```


$$V_c = (133.3 / M) * (((1 + ((7.02 * M * (f_{c1} ^{0.5})) / (1000 + S_{xe}))) ^{0.5}) - 1) * b_w * d / 1000$$

***Output

Cells(i, "AN").Value = Vtest / Vc

Next i

End Sub

Authors' Proposed

Sub Proposed()

Range("AP3:AP982").ClearContents

For i = 3 To 982

ReadInput i, bw, h, d, A, fc, Asl, rho_l, f_y_l, d_a, Vtest, a_d

$$sz = 1 / (1 + (d / 250) ^{0.5})$$

$$V_n = 7 * (rho_l / 100) ^{0.4} * (d / A) ^{0.3} * f_c ^{0.25} * sz * b_w * d / 1000$$

Cells(i, "AP").Value = Vtest / Vn

Next i

End Sub

C.2. Analysis of Beams with Shear Reinforcement

Sub ReadInput(i, bw, h, d, fc, Asl, rho_l, f_y_l, d_a, Vtest, a_d, A, rho_z, f_y_z, rho_x, f_y_x, s)

bw = Cells(i, "E").Value 'web width (in mm)

b = Cells(i, "T").Value 'base width (in mm)

h = Cells(i, "O").Value 'height of beam (in mm)

d = Cells(i, "F").Value 'effective depth of beam (in mm)

fc = Cells(i, "D").Value 'concrete cylinder strength (in MPa)

Asl = rho_l / 100 * bw * d 'area of longitudinal reinforcement

rho_l = Cells(i, "I").Value 'longitudinal reinforcement ratio

f_y_l = Cells(i, "K").Value 'yield strength of longitudinal reinforcement (in MPa)

d_a = Cells(i, "H").Value 'aggregate size (in mm)

Vtest = Cells(i, "N").Value 'experimental failure load in kN

a_d = Cells(i, "G").Value 'a/d ratio

A = a_d * d 'length of shear span

rho_z = Cells(i, "L").Value 'vertical web reinforcement ratio

f_y_z = Cells(i, "M").Value 'yield strength of vertical web reinforcement (in MPa)

rho_x = Cells(i, "R").Value 'horizontal web reinforcement ratio

f_y_x = Cells(i, "S").Value 'yield strength of horizontal web reinforcement (in MPa)

s = Cells(i, "Q").Value 'spacing of shear reinforcement (mm)

End Sub

ACI 318-14

Sub ACI()

Range("AD5:AD193").ClearContents

For i = 5 To 193

ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s

***Calculation

If fc < 69 Then

fc1 = fc

Else

fc1 = 69

End If

Vcsimplified = ((0.16 * fc1 ^ 0.5 * bw * d) + (17 * (rho1/100) * bw * d)) / 1000

Vs1 = rhoz / 100 * bw * fyv * d / 1000

Vs = Application.WorksheetFunction.Min(Vs1, (2 / 3 * (fc ^ 0.5) * bw * d / 1000))

Vn = Vcsimplified + Vs

***Output

Cells(i, "AD").Value = Vtest / Vn

Next i

End Sub

Eurocode 2

Sub EC2()

Range("AN5:AN193").ClearContents

For i = 5 To 193

ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s

If (1 + (200 / d) ^ (1 / 2)) < 2 Then

K = (1 + (200 / d) ^ (1 / 2))

Else

K = 2

End If

If rho1 < 2 Then

rho = rho1

Else

rho = 2

End If

If fc < 90 Then

fc1 = fc

Else

fc1 = 90

End If

If fc < 50 Then

fc50 = fc

Else

fc50 = 50

End If

***Calculation

```

V_Rdc1 = (0.18 * K * (rho * fc1) ^ (1 / 3) * bw * d) / 1000
V_Rdc2 = (0.035 * (K ^ (3 / 2)) * (fc1 ^ (1 / 2)) * bw * d) / 1000
VRd_c = WorksheetFunction.Max(V_Rdc1, V_Rdc2)

V_Rdc50_1 = (0.18 * K * (rho * fc50) ^ (1 / 3) * bw * d) / 1000
V_Rdc50_2 = (0.035 * (K ^ (3 / 2)) * (fc50 ^ (1 / 2)) * bw * d) / 1000
VRd_c50 = WorksheetFunction.Max(V_Rdc50_1, V_Rdc50_2)

```

```

Z = 0.9 * d
V1 = 0.6 * (1 - fc / 250)
If V1 * fc / (fyv * (rhoz / 100)) < 1 Then
GoTo 5000
End If

```

```

thetha_cal = Atn(1 / (((V1 * fc / (fyv * rhoz / 100)) - 1) ^ 0.5))

```

```

If thetha_cal < (22 / 180 * 3.142) Then
thetha = 22 / 180 * 3.142
ElseIf thetha_cal > (45 / 180 * 3.142) Then
thetha = 45 / 180 * 3.142
Else
thetha = thetha_cal
End If

```

```

VRd_s = rhoz / 100 * bw * Z * fyv / Tan(thetha) / 1000
Vcmax = bw * Z * V1 * fc1 / (Tan(thetha) + (1 / Tan(thetha))) / 1000
Vcmax50 = bw * Z * V1 * fc50 / (Tan(thetha) + (1 / Tan(thetha))) / 1000
Vn = Application.WorksheetFunction.Max(VRd_c,
Application.WorksheetFunction.Min(VRd_s, Vcmax))
Vn50 = Application.WorksheetFunction.Max(VRd_c50,
Application.WorksheetFunction.Min(VRd_s, Vcmax50))

```

***Output

```

Cells(i, "AN").Value = Vtest / Vn
Cells(i, "AO").Value = Vtest / Vn50
5000
Next i
End Sub

```

Bažant and Yu

```

Sub Bažant()
Range("BH5:BH193").ClearContents
For i = 5 To 193
ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s
If bw = Empty Then GoTo 5000

conv1 = 145.038      'fc to psi

```

```

conv2 = 0.0393701    'mm to inch
conv3 = 0.224809     'N to pounds

mew = 13.3

If da > 0 Then
K = 3800 * (da * conv2) ^ 0.5
Else
K = 3330
End If

d0 = K * (fc * conv1) ^ (-2 / 3)

***Calculation
Vc = (mew * (rho1 / 100) ^ (3 / 8) * (1 + (d / A)) * ((fc * conv1) / (1 + ((d * conv2) /
d0))) ^ 0.5) / conv1 * bw * d / 1000

Vs = (rhoz / 100 * bw * fyv * d) / 1000
Vn = Vs + Vc

***Output
Cells(i, "AL").Value = Vtest / Vn
Next i
End Sub

```

Collins et al.

```

Sub Collins()
Range("BJ5:BJ193").ClearContents
For i = 5 To 193
ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s

If fc > 69 Then
fc1 = 69
Else
fc1 = fc
End If

If fc >= 70 Then
agg = 0
ElseIf fc <= 60 Then
agg = da
Else
agg = ((10 - (fc - 60)) / 10) * da
End If

Sxe1 = (35 * d) / (16 + agg)

***Calculation

```

```

vs1 = (rhoz / 100 * bw * fyv * d) / 1000
Vs = WorksheetFunction.Min(vs1, (2 / 3 * (fc1 ^ 0.5) * bw * d / 1000))

If Vs > 0 Then
    sxe = 305
Else
    sxe = sxe1
End If

prime = (A - d) / d

M = (1 / (rho1 / 100)) * ((1.11 * prime) + 1)
Vc = (133.3 / M) * (((1 + ((7.02 * M * (fc1 ^ 0.5)) / (1000 + sxe))) ^ 0.5) - 1) * bw *
d / 1000
Vn = Vc + Vs

***Output
Cells(i, "BJ").Value = Vtest / Vn
Next i
End Sub

```

Proposed model (Analytical Truss Model – First Approach)

```

Sub proposed_analytical_truss_model()
    For i = 5 To 193
        ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s

        'Caluclation of miscellaneous parameters
        Z = 0.9 * d          'lever arm of internal forces
        Ec = 3320 * fc ^ 0.5 + 6900
        Es = 200000
        sigma_z = 0

        'Step 1: Select a value for epsilon_d
        epsilon_d = -0.00001

        5100

        increase_d = 0.00001      'increment value for each step
        epsilon_d = epsilon_d - increase_d

        If epsilon_d / -0.0035 > 1 Then
            GoTo 5500
        End If

        'Step 2: Assume a value for v_xz
        Assumed_v_xz = -0.1

        'Step 3: Assume a value for epsilon_r
    
```

```

Assumed_epsilon_r = 0.0001 ' set initial value for iteration
diff1 = 1000
Do While diff1 > 0.01
5200
    increase_r = 0.00001      'increment value for each step
    Assumed_epsilon_r = Assumed_epsilon_r + increase_r

    If Assumed_epsilon_r > 0.2 Then
        inc_Assumed_v_xz = 0.02
        Assumed_v_xz = Assumed_v_xz - inc_Assumed_v_xz
        Assumed_epsilon_r = 0.00011
    End If

    If (Assumed_v_xz / -5) > 1 Then
        GoTo 5100
    End If
    If Assumed_epsilon_r > 0.15 Then Exit Do

'Step 4: Calculate sigma_r
    sigma_r = 0.02 * fc

'Step 5: Calculate softening coefficient
    If 5.8 / fc ^ 0.5 > 0.9 Then
        softening_factor = 0.9 * (1 / (1 + (400 * Assumed_epsilon_r)) ^ 0.5)
    Else
        softening_factor = 5.8 / (fc ^ 0.5) * (1 / (1 + (400 * Assumed_epsilon_r)) ^ 0.5)
    End If

'Step 6: Calculate sigma_d
    sigma_d_max = softening_factor * -fc
    epsilon_o = -(0.002 + 0.001 * (fc - 20) / 80)
    softened_epsilon_o = epsilon_o * softening_factor
    ratio_d = epsilon_d / softened_epsilon_o

    If ratio_d > 1 Then
        If ((ratio_d - 1) / (2 / softening_factor - 1)) ^ 2 > 1 Then
            sigma_d = sigma_d_max
        Else
            sigma_d = sigma_d_max * (1 - ((ratio_d - 1) / (2 / softening_factor - 1)) ^ 2)
        End If
    Else
        sigma_d = sigma_d_max * (2 * (ratio_d) - (ratio_d) ^ 2)
    End If

'Step 7: Calculate epsilon_z, taking fsz=Es*epsilon_z and fsz
    epsilon_z1 = ((Assumed_epsilon_r * (-sigma_d + sigma_z)) + (epsilon_d * (-sigma_z
+ sigma_r))) / (-sigma_d + sigma_r - rhoz / 100 * Es * (epsilon_d - Assumed_epsilon_r))

'Step 8: Calculate fsz
    If epsilon_z1 < (fyz / Es) Then

```

```

    epsilon_z = epsilon_z1
    fsz = epsilon_z * Es
    Else
    fsz = fyz
    epsilon_z = Assumed_epsilon_r + (((-sigma_z + sigma_r + rhoz / 100 * fsz) / (-
sigma_d + sigma_r)) * (epsilon_d - Assumed_epsilon_r))
    End If

'Step 9: Calculate thetha
    If (2 * Assumed_v_xz / (sigma_d - sigma_r)) > 1 Then
    GoTo 5200
    End If

    thetha = Application.WorksheetFunction.Asin(2 * Assumed_v_xz / (sigma_d -
sigma_r)) / 2
    thetha_deg = thetha / 3.142 * 180

'Step 10: Calculate epsilon_x
    epsilon_x = epsilon_d * (Cos(thetha)) ^ 2 + Assumed_epsilon_r * (Sin(thetha)) ^ 2

'Step 11: Calculate fsx

    If epsilon_x < (fyx / Es) Then
    fsx = epsilon_x * Es
    Else
    fsx = fyx
    End If

'Step 12: Calculate sigma_x
    sigma_x = sigma_d * (Cos(thetha)) ^ 2 + sigma_r * (Sin(thetha)) ^ 2 + rhox / 100 *
fsx

'Step 13: Calculate epsilon_r
    Calculated_epsilon_r = -epsilon_d + epsilon_x + epsilon_z
    diff1 = Abs(1 - (Assumed_epsilon_r / Calculated_epsilon_r))
Loop

'Step 14: Calculate v_xz
    Calculated_v_xz = (sigma_d - Calculated_sigma_r) * Sin(thetha) * Cos(thetha)
    diff2 = Abs(1 - (Calculated_v_xz / Assumed_v_xz))

    difference = Application.WorksheetFunction.Max(diff1, diff2)
    If difference < 0.01 Then
    GoTo 5300
    Else
    GoTo 5200
    End If

5300
'Step 15: Calculate gamma_xz

```

```

gamma_xz = Abs(2 * (epsilon_d - Calculated_epsilon_r) * Sin(theta) *
Cos(theta))

```

'Step 16: Checks for failure mode of beam

```

diff3 = Abs(1 - (fsz / fyz))
diff4 = Abs(1 - (sigma_d / sigma_d_max))

If diff3 = 0 Or diff4 = 0 Then
GoTo 5400
Else
GoTo 5100
End If

```

5400

'Step 17: Calculate predicted shear strength

```

If diff3 = 0 Then
sz1 = [1 / (1 + (d / 250) ^ 0.5)] + 0.4(rhoz/100*fyz)
Vpred_tie1 = -Calculated_v_xz * bw * sz1 * {h-[2*(h-d)]} / 1000
End If

If diff4 = 0 Then
Vpred_strut = -sigma_d * bw * Z * Cos(theta) * Sin(theta) / 1000
End If

If Vpred_tie1 > 0 And Vpred_strut > 0 Then
Vpred = Application.WorksheetFunction.Min(Vpred_tie1, Vpred_strut)
Else
If Vpred_tie1 = 0 Then
Vpred = Vpred_strut
ElseIf Vpred_strut = 0 Then
Vpred = Vpred_tie1
End If
End If

If Vpred = 0 Then
Vpred = 1
End If

```

'Step 18: Calculate contribution from dowel action

```

dbe = ((4 * rho_l / 100 * b * d) / 3.142) ^ 0.5
bn = b - dbe
Du = 1.64 * dbe * bn * fc ^ (1 / 3) / 1000
Vpred1 = Vpred + Du

```

```

Cells(i, "KV").Value = Vtest / Vpred1
Cells(i, "KW").Value = Vtest
Cells(i, "KX").Value = Vpred_tie1
Cells(i, "KY").Value = Vpred_strut
Cells(i, "KZ").Value = Assumed_epsilon_r

```



```

Cells(i, "LA").Value = Calculated_epsilon_r
Cells(i, "LB").Value = epsilon_d
Cells(i, "LC").Value = epsilon_x
Cells(i, "LD").Value = epsilon_z
Cells(i, "LE").Value = fsx
Cells(i, "LF").Value = fsz
Cells(i, "LG").Value = theta_deg
Cells(i, "LI").Value = Assumed_v_xz
Cells(i, "LJ").Value = Calculated_v_xz
Cells(i, "LK").Value = sigma_r
Cells(i, "LM").Value = sigma_x
Cells(i, "LN").Value = gamma_xz
Cells(i, "LO").Value = diff
Cells(i, "LP").Value = diff1
Cells(i, "LQ").Value = diff2
Cells(i, "LR").Value = diff3
Cells(i, "LS").Value = diff4
Cells(i, "LT").Value = sigma_d
Cells(i, "LV").Value = Du

```

Proposed model (Simplified Truss Method – Second Approach)

```

Sub proposed_simplified_truss_method()
    For i = 5 To 193
        ReadInput i, bw, h, d, fc, Asl, rho1, fyl, da, Vtest, a_d, A, rhoz, fyz, rhox, fyx, s

        theta_rad = (Atn(d / A)) + (Atn((rhoz / 100 * fyz / s) ^ 0.2))
        theta_deg = theta_rad / 3.142 * 180

        Vs = rhoz / 100 * bw * 0.9 * d * fyz / Tan(theta_rad) / 1000

        sz = 1 / (1 + (d / 250) ^ 0.5)

        Vc = (7 * (rho1 / 100) ^ 0.4 * (d / A) ^ 0.3 * fc ^ 0.25 * sz1) * bw * d / 1000

        Cells(i, "DN").Value = Vtest / (Vs + Vc)

    Next i
End Sub

```

Appendix D

BEAM DATABASE

D.1. Properties and Analysis Results of 980 Reinforced Concrete Beam Specimens with No Shear Reinforcement

Table D-1 presents a database of 980 reinforced concrete beam data with no shear reinforcement (DB-1) collected from existing literature as well as the specimens tested in this study. The properties of each data presented in DB-1 include the beam name, beam width (b), beam web width (b_w), height (h), effective depth (d), distance of shear span (a), type of loading (point or uniformly distributed loading), shear-span-to-effective-depth (a/d) ratio, longitudinal reinforcement ratio (ρ_l), concrete compressive strength (f'_c), aggregate size (d_a), yield strength of longitudinal reinforcement (f_y), and experimental shear strength (V_{test}). These properties are presented in columns (2) to (14) in **Table D-1** respectively.

The experimental shear strengths for each data was compared against the predicted shear strength calculated using the design equations in ACI 318-14 (2014), Eurocode 2 (2004), Bažant and Yu (2005a and 2005b), Collins et al. (2008), and the authors' proposed method (discussed in Chapter 6). The ratios of V_{test}/V_{pred} for each method are presented in columns (15) to (19) respectively. The ranges of some of the influential parameters in all the data can be summarised as follows:

- The effective depth ranges from 41 to 3840 mm.
- The shear-span-to-effective-depth (a/d) ratio ranges from 2.57 to 6.
- The concrete compressive strength ranges from 6 to 138 MPa.
- The longitudinal reinforcement ratio ranges from 0.17 to 9.28%.
- The maximum aggregate size ranges from 2 to 50 mm.

Table D-1—Properties and analysis results of 980 beam data

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Richart (1948)																			
1	501b	1829	1829	305	254	685	U1	2.70	1.38	26	25	425	578.50	1.44	1.12	1.12	1.03	1.22	1.19
2	504a	1524	1524	305	254	762	U1	3.00	1.77	25	25	510	518.10	1.58	1.12	1.12	1.05	1.27	1.20
3	504b	1524	1524	305	254	762	U1	3.00	1.77	26	25	510	557.80	1.67	1.19	1.19	1.12	1.35	1.28
Laupa, Siess, and Newmark (1953)																			
4	S3	152	152	305	265	1295	P	4.89	2.52	32	25	410	53.10	1.36	0.98	0.98	0.92	1.14	1.13
5	S4	152	152	305	263	1295	P	4.92	3.21	31	25	309	55.60	1.47	1.05	1.05	0.90	1.14	1.09
6	S5	152	152	305	262	1295	P	4.95	4.11	30	25	315	49.80	1.35	0.95	0.95	0.75	0.96	0.90
7	S11	152	152	305	267	1295	P	4.85	1.90	15	25	328	33.80	1.27	0.82	0.82	0.87	1.03	0.97
8	S13	152	152	305	262	1295	P	4.95	4.11	26	25	304	49.80	1.44	0.99	0.99	0.79	1.01	0.93
9	T-3average	102	102	203	178	765	P	4.30	3.16	24	25	359	19.60	1.30	0.83	0.83	0.71	0.89	0.79
10	T-5a	102	102	203	178	765	P	4.30	2.19	24	25	317	22.20	1.47	0.94	0.94	0.92	1.13	1.04
11	T-5b	102	102	203	178	765	P	4.30	2.19	24	25	317	22.70	1.50	0.96	0.96	0.94	1.16	1.06
12	T-5c	102	102	203	178	765	P	4.30	2.19	24	25	317	23.10	1.53	0.97	0.97	0.96	1.18	1.08
13	T-6b	102	102	203	178	765	P	4.30	1.40	22	25	331	17.30	1.21	0.85	0.85	0.88	1.06	0.99
14	T-12a	147	147	203	178	765	P	4.30	2.20	34	25	359	35.60	1.38	0.93	0.93	0.89	1.12	1.06
15	T-12b	147	147	203	178	765	P	4.30	2.20	34	25	359	33.80	1.31	0.88	0.88	0.85	1.06	1.01

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
16	T-12c	147	147	203	178	765	P	4.30	2.20	34	25	359	32.00	1.24	0.83	0.83	0.80	1.01	0.95
17	T2-Ma	152	152	305	269	915	P	3.40	1.39	30	25	331	42.30	1.11	0.89	0.89	0.87	0.99	1.03
18	T2-Mb	152	152	305	269	942	P	3.50	1.39	28	25	331	44.50	1.22	0.96	0.96	0.95	1.08	1.11
Moody, Viest, Elstner, and Hognestad (1954)																			
19	A-A1	178	178	305	262	800	P	3.06	2.17	30	25	483	60.70	1.39	0.98	0.98	0.89	1.04	1.04
20	A-A2	178	178	305	267	800	P	3.00	2.15	31	25	483	67.40	1.50	1.07	1.07	0.97	1.12	1.13
21	A-A3	178	178	305	268	800	P	2.99	2.22	31	25	483	76.30	1.69	1.20	1.20	1.08	1.25	1.25
22	A-A4	178	178	305	270	800	P	2.96	2.37	32	25	483	71.80	1.57	1.12	1.12	0.98	1.14	1.14
23	A-B1	178	178	305	267	800	P	3.00	1.62	21	25	483	56.90	1.53	1.10	1.10	1.04	1.18	1.17
24	A-B2	178	178	305	268	800	P	2.99	1.63	22	25	483	60.70	1.61	1.16	1.16	1.10	1.24	1.24
25	A-B3	178	178	305	270	800	P	2.96	1.60	19	25	483	56.30	1.57	1.12	1.12	1.07	1.19	1.18
26	A-B4	178	178	305	272	800	P	2.95	1.66	17	25	483	56.30	1.67	1.15	1.15	1.10	1.23	1.19
27	A-C1	178	178	305	268	800	P	2.99	0.81	6	25	483	20.70	1.02	0.75	0.75	0.80	0.80	0.76
28	A-C2	178	178	305	272	800	P	2.94	0.83	6	25	483	25.10	1.23	0.90	0.90	0.96	0.96	0.90
29	A-C3	178	178	305	273	800	P	2.93	0.80	7	25	483	26.00	1.20	0.91	0.91	0.95	0.96	0.92
30	A-C4	178	178	305	274	800	P	2.92	0.82	7	25	483	25.80	1.19	0.89	0.89	0.94	0.95	0.90
31	B-B1	152	152	305	268	914	P	3.41	1.89	37	25	483	58.50	1.39	1.04	1.04	1.00	1.16	1.20

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
32	B-B2	152	152	305	268	914	P	3.41	1.89	17	25	483	36.30	1.28	0.84	0.84	0.83	0.95	0.91
33	B-B3	152	152	305	268	914	P	3.41	1.89	26	25	483	53.00	1.51	1.06	1.06	1.03	1.19	1.19
34	B-B4	152	152	305	268	914	P	3.41	1.89	15	25	483	41.20	1.52	0.98	0.98	0.97	1.11	1.05
35	B-B5	152	152	305	268	914	P	3.41	1.89	31	25	483	52.70	1.37	1.00	1.00	0.96	1.11	1.13
36	B-B6	152	152	305	268	914	P	3.41	1.89	16	25	483	35.20	1.28	0.83	0.83	0.82	0.94	0.89
37	B-B7	152	152	305	268	914	P	3.41	1.89	31	25	483	51.90	1.35	0.98	0.98	0.94	1.09	1.11
38	B-B8	152	152	305	268	914	P	3.41	1.89	12	25	483	31.80	1.31	0.82	0.82	0.82	0.93	0.86
39	B-B9	152	152	305	268	914	P	3.41	1.89	41	25	483	54.10	1.22	0.93	0.93	0.89	1.03	1.08
40	B-B10	152	152	305	268	914	P	3.41	1.89	24	25	483	49.60	1.46	1.02	1.02	0.99	1.14	1.13
41	B-B11	152	152	305	268	914	P	3.41	1.89	38	25	483	60.80	1.42	1.07	1.07	1.03	1.19	1.23
42	B-B12	152	152	305	268	914	P	3.41	1.89	20	25	483	47.90	1.54	1.04	1.04	1.02	1.17	1.14
43	B-B13	152	152	305	268	914	P	3.41	1.89	38	25	483	56.30	1.32	0.99	0.99	0.95	1.11	1.15
44	B-B14	152	152	305	268	914	P	3.41	1.89	23	25	483	43.90	1.33	0.92	0.92	0.89	1.03	1.02

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
45	B-B15	152	152	305	268	914	P	3.41	1.89	37	25	483	51.90	1.23	0.92	0.92	0.88	1.02	1.06
46	B-B16	152	152	305	268	914	P	3.41	1.89	16	25	483	38.50	1.38	0.90	0.90	0.89	1.01	0.97
Moody, Viest, Elstner, and Hognestad (1955)																			
47	B40-B4	305	305	406	368	1016	P	2.76	1.85	35	6	483	157.60	1.40	1.12	1.12	1.28	1.29	1.22
48	B56-B2	305	305	406	368	1422	P	3.86	1.85	15	6	483	103.20	1.41	0.98	0.98	1.17	1.26	1.10
49	B56-E2	305	305	406	368	1422	P	3.86	0.58	15	6	483	82.70	1.13	1.15	1.15	1.45	1.47	1.40
50	B56-A4	305	305	406	375	1422	P	3.80	2.41	25	6	483	140.90	1.45	1.07	1.07	1.21	1.30	1.16
51	B56-B4	305	305	406	368	1422	P	3.86	1.85	27	6	483	125.40	1.26	0.97	0.97	1.18	1.24	1.14
52	B56-E4	305	305	406	368	1422	P	3.86	1.24	28	6	483	112.10	1.10	0.97	0.97	1.21	1.24	1.19
53	B56-A6	308	308	406	356	1422	P	4.00	3.79	40	6	483	181.10	1.54	1.22	1.22	1.20	1.30	1.16
54	B56-B6	305	305	406	372	1422	P	3.83	1.83	46	6	483	139.80	1.07	0.90	0.90	1.14	1.14	1.11
55	B70-B2	305	305	406	365	1778	P	4.87	1.86	16	6	483	93.10	1.22	0.86	0.86	1.07	1.18	1.04
56	B70-A4	305	305	406	368	1778	P	4.83	2.46	27	6	483	136.40	1.37	1.03	1.03	1.21	1.32	1.19

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
57	B70-A6	305	305	406	356	1778	P	5.00	3.83	45	6	483	182.10	1.47	1.19	1.19	1.22	1.34	1.21
58	B84-B4	305	305	406	363	2134	P	5.87	1.88	27	6	483	116.40	1.19	0.90	0.90	1.19	1.32	1.21
Ferguson (1956)																			
59	F2	101	101	210	189	610	P	3.23	2.10	29	6	483	22.20	1.26	0.83	0.83	0.91	1.00	0.89
Al-Alusi (1957)																			
60	11.00	76	330	146	127	432	P	3.40	2.63	29	6	366	17.40	1.98	1.30	1.30	1.20	1.42	1.18
61	2.00	76	330	146	127	432	P	3.40	1.47	28	6	366	14.40	1.66	1.20	1.20	1.25	1.42	1.24
62	21.00	76	330	146	127	432	P	3.40	2.74	29	6	366	24.10	2.72	1.79	1.79	1.63	1.93	1.60
63	15.00	76	330	146	127	508	P	4.00	2.67	18	6	366	17.20	2.45	1.49	1.49	1.45	1.72	1.36
64	3.00	76	330	146	127	508	P	4.00	1.49	27	6	366	14.30	1.67	1.20	1.20	1.29	1.49	1.29
65	10.00	76	330	146	127	508	P	4.00	2.70	29	6	366	14.70	1.68	1.10	1.10	1.04	1.25	1.04
66	4.00	76	330	146	127	508	P	4.00	1.50	27	6	366	13.90	1.64	1.17	1.17	1.26	1.46	1.26
67	13.00	76	330	146	127	508	P	4.00	2.72	29	6	366	17.00	1.93	1.27	1.27	1.20	1.44	1.19
68	5.00	76	330	146	127	533	P	4.20	1.48	27	6	366	14.20	1.66	1.19	1.19	1.30	1.51	1.31

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
69	18.00	76	330	146	127	572	P	4.50	2.65	27	6	366	14.00	1.65	1.07	1.07	1.05	1.27	1.05
70	7.00	76	330	146	127	572	P	4.50	2.71	25	6	366	13.50	1.63	1.05	1.05	1.02	1.24	1.01
71	24.00	76	330	146	127	572	P	4.50	2.71	29	6	366	15.40	1.76	1.15	1.15	1.12	1.36	1.12
72	16.00	76	330	146	127	572	P	4.50	2.72	28	6	366	14.80	1.70	1.11	1.11	1.08	1.31	1.08
73	17.00	76	330	146	127	699	P	5.50	2.65	30	6	366	13.80	1.55	1.02	1.02	1.03	1.29	1.07
74	8.00	76	330	146	127	699	P	5.50	2.71	27	6	366	13.40	1.57	1.02	1.02	1.02	1.28	1.05
75	19.00	76	330	146	127	699	P	5.50	4.21	31	6	366	14.00	1.54	1.02	1.02	0.87	1.12	0.89
76	25.00	76	330	146	127	699	P	5.50	2.85	26	6	366	13.50	1.61	1.04	1.04	1.03	1.29	1.05
Chang and Kesler (1958)																			
77	1A1	102	102	152	137	533	P	3.90	2.89	28	25	328	19.90	1.59	1.04	1.04	0.86	1.10	0.96
78	1C1	102	102	152	137	432	P	3.16	2.37	28	25	328	19.60	1.57	1.02	1.02	0.87	1.07	0.96
79	1C2	102	102	152	137	432	P	3.16	2.37	28	25	328	17.80	1.43	0.93	0.93	0.79	0.97	0.87
80	IIA1	102	102	152	137	432	P	3.16	1.86	18	25	328	17.00	1.70	1.05	1.05	1.00	1.17	1.02

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
81	IIA2	102	102	152	137	432	P	3.16	1.86	18	25	328	17.30	1.73	1.07	1.07	1.02	1.19	1.04
82	IIB1	102	102	152	137	533	P	3.90	2.37	18	25	328	16.60	1.66	1.00	1.00	0.93	1.13	0.97
83	IIIA1	102	102	152	137	533	P	3.90	2.37	15	25	328	17.00	1.85	1.09	1.09	1.03	1.23	1.03
84	IIIB1	102	102	152	137	432	P	3.16	1.86	15	25	328	15.30	1.67	1.01	1.01	0.97	1.12	0.96
85	IIIB2	102	102	152	137	432	P	3.16	1.86	15	25	328	15.50	1.69	1.02	1.02	0.98	1.13	0.98
86	4-21a	102	102	152	137	483	P	3.53	2.89	39	25	328	21.10	1.43	0.99	0.99	0.78	1.00	0.91
87	4-21b	102	102	152	137	483	P	3.53	1.86	39	25	328	24.60	1.67	1.18	1.18	1.07	1.34	1.26
88	4-22a	102	102	152	137	483	P	3.53	1.86	32	25	328	21.40	1.59	1.09	1.09	1.01	1.25	1.15
89	4-22b	102	102	152	137	483	P	3.53	1.86	32	25	328	23.40	1.75	1.20	1.20	1.10	1.36	1.26
90	4-23a	102	102	152	137	483	P	3.53	1.86	32	25	328	21.60	1.60	1.10	1.10	1.01	1.25	1.16
91	4-23b	102	102	152	137	483	P	3.53	1.86	32	25	328	22.50	1.67	1.14	1.14	1.06	1.30	1.21
92	5-21a	102	102	152	137	483	P	3.53	2.89	32	25	328	28.90	2.14	1.43	1.43	1.15	1.46	1.30
93	5-21b	102	102	152	137	483	P	3.53	2.89	32	25	328	27.50	2.04	1.36	1.36	1.09	1.39	1.24

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
94	5-22a	102	102	152	137	483	P	3.53	2.89	31	25	328	22.40	1.69	1.12	1.12	0.90	1.15	1.02
95	5-22b	102	102	152	137	483	P	3.53	2.89	31	25	328	25.90	1.95	1.30	1.30	1.04	1.33	1.17
96	5-23a	102	102	152	137	483	P	3.53	2.89	32	25	328	24.50	1.82	1.22	1.22	0.98	1.24	1.10
97	5-23b	102	102	152	137	483	P	3.53	2.89	32	25	328	23.40	1.74	1.16	1.16	0.93	1.19	1.05
de Cossio and Seiss (1960)																			
98	L-2	152	152	305	252	762	P	3.02	3.36	22	25	310	75.60	2.50	1.66	1.66	1.29	1.58	1.42
99	L-2A	152	152	305	252	762	P	3.02	3.36	37	25	283	80.10	2.03	1.47	1.47	1.13	1.37	1.32
100	L-3	152	152	305	252	1016	P	4.02	3.36	28	25	310	53.40	1.55	1.07	1.07	0.88	1.09	1.02
101	L-4	152	152	305	252	1270	P	5.03	3.36	26	25	303	51.20	1.55	1.05	1.05	0.91	1.15	1.07
102	L-2R	152	152	305	252	762	P	3.02	3.36	22	25	310	74.70	2.47	1.64	1.64	1.28	1.56	1.40
103	L-2aR	152	152	305	252	762	P	3.02	3.36	37	25	283	92.50	2.34	1.69	1.69	1.30	1.58	1.52
104	L-3R	152	152	305	252	1016	P	4.02	3.36	28	25	310	62.00	1.80	1.24	1.24	1.02	1.27	1.19
105	A-2	152	152	305	254	762	P	3.00	0.98	32	25	469	41.80	1.13	1.02	1.02	0.98	1.09	1.16
106	A-3	152	152	305	254	1016	P	4.00	0.98	19	25	452	34.20	1.18	0.98	0.98	1.02	1.16	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
107	A-4	152	152	305	254	1270	P	5.00	0.98	27	25	459	35.10	1.03	0.90	0.90	0.97	1.17	1.18
108	A-12	152	152	305	254	762	P	3.00	3.33	26	25	314	58.90	1.76	1.20	1.20	0.93	1.14	1.05
109	A-13	152	152	305	254	1016	P	4.00	3.33	22	25	393	46.90	1.52	1.01	1.01	0.84	1.04	0.95
110	A-14	152	152	305	254	1270	P	5.00	3.33	28	25	364	54.70	1.59	1.10	1.10	0.94	1.19	1.12
111	A-15	152	152	305	254	1524	P	6.00	3.33	25	25	332	49.40	1.51	1.02	1.02	0.91	1.18	1.10
Bower and Viest (1960)																			
112	IIB - 1	152	152	356	318	1067	P	3.36	1.53	25	6	323	45.40	1.11	0.87	0.87	1.02	1.05	0.98
113	IIB - 2	152	152	356	311	1219	P	3.92	1.56	21	6	320	40.70	1.11	0.83	0.83	1.00	1.06	0.97
114	IIB - 3	152	152	356	308	1372	P	4.46	1.58	20	6	321	36.90	1.04	0.77	0.77	0.95	1.03	0.93
115	IIA - 4a	152	152	356	305	914	P	3.00	1.59	20	25	319	37.50	1.06	0.78	0.78	0.74	0.83	0.84
116	IIA - 4b	152	152	356	292	914	P	3.13	1.59	23	25	323	42.50	1.18	0.88	0.88	0.84	0.95	0.96
117	IIA - 5	152	152	356	292	1372	P	4.70	1.59	24	25	323	40.80	1.11	0.83	0.83	0.87	1.03	1.03
118	IIA - 8	152	152	356	292	1067	P	3.65	1.59	18	25	307	36.40	1.12	0.81	0.81	0.81	0.92	0.91

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
119	IIA - 9	152	152	356	292	1219	P	4.17	1.59	21	25	328	42.30	1.22	0.90	0.90	0.92	1.06	1.06
Hanson (1961)																			
120	8A4	152	152	305	267	1321	P	4.95	1.25	21	19	611	33.80	1.07	0.84	0.84	0.92	1.08	1.05
121	8B2	152	152	305	267	1321	P	4.95	2.53	31	19	326	52.40	1.37	0.97	0.97	0.95	1.16	1.12
Leonhardt and Walther (1962)																			
122	P2	503	503	162	142	490	P	3.45	0.95	13	30	427	76.20	1.71	1.27	1.27	1.29	1.44	1.30
123	P3	502	502	162	142	490	P	3.45	1.11	13	30	427	81.10	1.83	1.29	1.29	1.30	1.46	1.31
124	P4	500	500	165	145	490	P	3.38	1.40	15	30	427	100.80	2.15	1.42	1.42	1.40	1.60	1.42
125	P5	503	503	165	145	490	P	3.38	1.86	13	30	427	100.80	2.22	1.31	1.31	1.30	1.50	1.29
126	P8	502	502	168	148	490	P	3.31	0.91	25	30	427	88.00	1.40	1.16	1.16	1.11	1.31	1.26
127	P9	500	500	166	146	490	P	3.36	1.86	25	30	427	105.80	1.71	1.12	1.12	1.04	1.27	1.16
128	P10	503	503	122	102	350	P	3.43	1.10	12	30	427	59.30	1.93	1.34	1.34	1.34	1.49	1.27
129	P11	498	498	203	183	630	P	3.44	1.11	14	30	427	101.20	1.76	1.25	1.25	1.29	1.44	1.34
130	5I	190	190	320	270	810	P	3.00	2.07	32	30	465	60.30	1.21	0.87	0.87	0.79	0.93	0.94

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
131	5r	190	190	320	270	810	P	3.00	2.07	32	30	465	76.50	1.54	1.11	1.11	1.00	1.18	1.19
132	6l	190	190	320	270	1100	P	4.07	2.07	32	30	465	60.80	1.22	0.88	0.88	0.85	1.03	1.04
133	6r	190	190	320	270	1100	P	4.07	2.07	32	30	465	68.20	1.37	0.99	0.99	0.95	1.16	1.17
134	7-1	190	190	320	278	1390	P	5.00	2.01	34	30	465	62.30	1.19	0.87	0.87	0.88	1.10	1.11
135	7-2	190	190	320	278	1390	P	5.00	2.01	34	30	465	68.20	1.30	0.95	0.95	0.96	1.20	1.21
136	8-1	190	190	320	278	1668	P	6.00	2.01	34	30	465	65.70	1.25	0.92	0.92	0.95	1.23	1.24
137	8-2	190	190	320	274	1644	P	6.00	2.04	34	30	465	65.70	1.27	0.93	0.93	0.96	1.24	1.24
138	D1/1	50	50	80	70	210	P	3.00	1.71	35	15	451	7.30	2.07	1.48	1.48	1.24	1.56	1.33
139	D1/2	50	50	80	70	210	P	3.00	1.71	35	15	451	7.20	2.04	1.46	1.46	1.23	1.54	1.31
140	D2/1	100	100	160	140	420	P	3.00	1.62	31	15	427	21.20	1.59	1.14	1.14	1.06	1.26	1.16
141	D2/2	100	100	160	140	420	P	3.00	1.62	31	15	427	23.20	1.74	1.24	1.24	1.16	1.37	1.27
142	D3/1	150	150	240	210	630	P	3.00	1.62	34	15	413	46.40	1.49	1.09	1.09	1.09	1.24	1.21
143	D3/2l	150	150	240	210	630	P	3.00	1.62	34	15	413	42.90	1.38	1.01	1.01	1.01	1.15	1.12

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
144	D3/2r	150	150	240	210	630	P	3.00	1.62	34	15	413	42.90	1.38	1.01	1.01	1.01	1.15	1.12
145	D4/1	200	200	320	280	840	P	3.00	1.68	35	15	439	74.10	1.32	1.03	1.03	1.03	1.14	1.14
146	D4/2l	200	200	320	280	840	P	3.00	1.68	35	15	439	71.30	1.27	0.99	0.99	0.99	1.09	1.10
147	D4/2r	200	200	320	280	840	P	3.00	1.68	35	15	439	71.30	1.27	0.99	0.99	0.99	1.09	1.10
148	C1	100	100	180	150	450	P	3.00	1.29	38	30	425	21.60	1.37	1.09	1.09	0.97	1.19	1.16
149	C2	150	150	330	300	900	P	3.00	1.28	38	30	425	64.70	1.37	1.20	1.20	1.11	1.27	1.38
150	C3	200	200	500	450	1350	P	3.00	1.28	38	30	425	101.50	1.07	1.03	1.03	0.97	1.05	1.21
151	C4	225	225	670	600	1800	P	3.00	1.28	38	30	425	152.10	1.07	1.08	1.08	1.06	1.11	1.31
152	EA1	190	190	320	270	750	P	2.78	1.82	22	30	439	58.40	1.42	0.99	0.99	0.90	1.04	1.03
153	EA2	190	190	320	270	750	P	2.78	1.78	22	30	490	74.60	1.82	1.27	1.27	1.16	1.34	1.33
154	E6	190	190	320	270	750	P	2.78	2.47	28	30	426	91.20	1.99	1.39	1.39	1.16	1.38	1.35
de Cossio (1962)																			
155	32-8E	321	321	102	83	332	P	4.00	1.87	23	12	418	32.80	1.50	0.97	0.97	0.93	1.15	0.94
156	32-8F	319	319	102	83	330	P	4.00	1.87	21	12	418	33.20	1.60	1.02	1.02	0.99	1.21	0.98

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
157	48-8F	479	479	102	83	331	P	4.00	1.98	28	12	408	60.70	1.70	1.11	1.11	1.05	1.32	1.09
158	48-8E	479	479	102	82	329	P	4.00	1.99	27	12	459	51.00	1.48	0.96	0.96	0.90	1.14	0.93
159	64-8F	639	639	102	83	332	P	4.00	1.88	30	12	373	71.30	1.43	0.97	0.97	0.91	1.15	0.96
160	64-8E	639	639	102	87	347	P	4.00	1.92	31	12	373	67.70	1.28	0.86	0.86	0.81	1.02	0.86
161	32-8D	320	320	102	84	336	P	4.00	1.85	21	12	367	34.10	1.62	1.04	1.04	1.00	1.23	1.00
162	32-8C	319	319	102	82	326	P	4.00	1.91	23	12	367	34.90	1.63	1.05	1.05	1.00	1.24	1.01
163	48-8D	479	479	102	84	336	P	4.00	1.94	27	12	383	65.00	1.84	1.21	1.21	1.14	1.43	1.18
164	48-8C	483	483	102	83	332	P	4.00	1.95	25	12	404	58.70	1.72	1.11	1.11	1.05	1.31	1.08
165	64-8C	640	640	102	82	328	P	4.00	1.90	29	12	383	85.20	1.79	1.19	1.19	1.12	1.41	1.17
166	64-8D	640	640	102	81	324	P	4.00	1.95	29	12	383	81.60	1.73	1.15	1.15	1.07	1.35	1.12
167	32-8A	322	322	102	81	324	P	4.00	1.90	24	12	383	37.10	1.72	1.11	1.11	1.06	1.31	1.07
168	32-8B	317	317	102	81	325	P	4.00	1.85	24	12	383	38.00	1.79	1.17	1.17	1.11	1.38	1.13
169	B32-8A	318	318	102	80	318	P	4.00	2.81	27	12	403	49.00	2.19	1.42	1.42	1.17	1.50	1.20

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
170	B32-8B	318	318	102	80	320	P	4.00	2.80	27	12	432	51.90	2.32	1.50	1.50	1.24	1.60	1.27
171	C32-8A	319	319	102	80	320	P	4.00	1.95	27	12	394	41.20	1.84	1.20	1.20	1.13	1.42	1.16
172	C32-8B	319	319	102	80	318	P	4.00	1.95	27	12	394	42.20	1.89	1.23	1.23	1.16	1.45	1.19
173	48-8B	505	505	102	82	326	P	4.00	1.98	28	12	399	65.30	1.76	1.15	1.15	1.08	1.36	1.12
174	48-8A	479	479	102	81	324	P	4.00	2.01	28	12	410	62.00	1.78	1.16	1.16	1.09	1.37	1.13
175	64-8A	640	640	102	82	327	P	4.00	1.90	29	12	397	87.00	1.83	1.22	1.22	1.14	1.44	1.19
176	64-8B	636	636	102	81	322	P	4.00	1.94	29	12	440	85.50	1.81	1.20	1.20	1.12	1.42	1.17
177	A2.1-16.8A	21	21	203	168	450	P	2.67	2.00	35	12	410	4.70	1.33	0.90	0.90	0.85	0.99	0.92
178	A2.1-16.8B	21	21	203	168	450	P	2.67	2.00	35	12	410	4.70	1.33	0.90	0.90	0.85	0.99	0.92
179	A3-12A	30	30	152	122	325	P	2.67	1.98	29	12	394	4.30	1.29	0.85	0.85	0.77	0.91	0.79
180	A3-12B	30	30	152	120	320	P	2.67	1.97	29	12	394	4.20	1.27	0.84	0.84	0.75	0.90	0.78
181	A3-12C	30	30	152	122	325	P	2.67	1.97	29	12	396	4.40	1.32	0.87	0.87	0.78	0.93	0.81
182	A4.25-17A	42	42	203	170	680	P	4.00	2.00	14	12	408	5.30	1.18	0.68	0.68	0.75	0.87	0.73
183	A4.25-17B	42	42	203	172	688	P	4.00	1.94	14	12	408	6.00	1.32	0.77	0.77	0.85	0.98	0.83

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ_t (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
184	B3-12A	29	29	152	120	480	P	4.00	2.00	29	12	394	4.80	1.51	0.99	0.99	0.98	1.20	1.04
185	B3-12B	30	30	152	120	480	P	4.00	1.97	29	12	394	4.00	1.21	0.80	0.80	0.79	0.97	0.84
186	B4.25-17A	42	42	203	170	680	P	4.00	2.00	13	12	408	5.50	1.24	0.72	0.72	0.78	0.91	0.76
187	B4.25-17B	42	42	203	170	680	P	4.00	1.94	14	12	408	5.60	1.25	0.73	0.73	0.80	0.92	0.78
188	C4.25-17A	42	42	203	170	454	P	2.67	1.96	14	12	421	6.70	1.49	0.87	0.87	0.87	0.98	0.82
189	C4.25-17B	42	42	203	170	454	P	2.67	1.96	14	12	421	5.90	1.31	0.77	0.77	0.76	0.87	0.72
190	A4.7-14.7A	46	46	178	147	393	P	2.67	2.08	28	12	436	9.60	1.57	1.03	1.03	0.94	1.10	0.98
191	A4.7-14.7B	46	46	178	147	393	P	2.67	2.08	28	12	436	9.80	1.60	1.05	1.05	0.96	1.13	1.00
192	A4.25-8.5A	42	42	102	86	230	P	2.67	1.97	30	12	397	5.60	1.67	1.11	1.11	0.95	1.15	0.97
193	A4.25-8.5B	42	42	102	86	231	P	2.67	1.97	30	12	397	5.80	1.73	1.15	1.15	0.98	1.20	1.01
194	A4.25-8.5C	42	42	102	86	230	P	2.67	1.97	30	12	397	5.80	1.73	1.15	1.15	0.98	1.20	1.01
195	A6-12A	60	60	152	121	323	P	2.67	1.94	30	12	390	10.60	1.58	1.05	1.05	0.94	1.12	0.98
196	A6-12B	60	60	152	122	326	P	2.67	1.92	30	12	390	10.50	1.55	1.04	1.04	0.93	1.10	0.97
197	A6-12C	60	60	152	120	321	P	2.67	1.94	30	12	390	10.40	1.56	1.04	1.04	0.93	1.11	0.97

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
198	A8.5-17A	87	87	203	171	456	P	2.67	1.92	35	12	392	23.50	1.57	1.08	1.08	1.03	1.19	1.11
199	A8.5-17B	85	85	203	170	454	P	2.67	1.96	35	12	392	23.90	1.64	1.12	1.12	1.07	1.23	1.15
200	A8.5-17C	86	86	203	170	454	P	2.67	1.95	34	12	390	21.00	1.45	0.99	0.99	0.94	1.08	1.01
201	A10.4-13.9A	103	103	178	139	372	P	2.67	1.97	24	12	440	21.50	1.81	1.16	1.16	1.07	1.25	1.09
202	A10.4-13.9B	103	103	178	140	374	P	2.67	1.96	24	12	440	23.80	1.99	1.27	1.27	1.18	1.38	1.21
203	A12-12A	120	120	152	121	323	P	2.67	1.95	24	12	402	27.00	2.25	1.44	1.44	1.31	1.54	1.32
204	A12-12B	120	120	152	122	326	P	2.67	1.96	29	12	402	26.00	1.93	1.28	1.28	1.15	1.37	1.20
205	A12-12C	120	120	152	121	323	P	2.67	1.95	31	12	402	23.80	1.73	1.16	1.16	1.04	1.24	1.09
206	A12-12D	120	120	152	120	320	P	2.67	1.97	31	12	402	23.70	1.74	1.16	1.16	1.04	1.24	1.09
207	A14.7-9.8A	147	147	127	99	264	P	2.67	1.96	27	12	450	22.80	1.79	1.17	1.17	1.02	1.23	1.04
208	A14.7-9.8B	146	146	127	99	263	P	2.67	1.97	27	12	450	25.80	2.04	1.33	1.33	1.16	1.39	1.18
209	B12-6A	120	120	76	62	165	P	2.67	2.88	26	12	434	16.40	2.54	1.64	1.64	1.19	1.52	1.16
210	B12-6B	120	120	76	63	168	P	2.67	2.84	26	12	434	17.90	2.73	1.76	1.76	1.29	1.64	1.25
211	A12-6A	121	121	76	60	161	P	2.67	1.44	30	12	392	17.00	2.51	1.85	1.85	1.54	1.88	1.56

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ_t (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
212	A12-6B	120	120	76	62	165	P	2.67	1.92	30	12	392	15.50	2.24	1.50	1.50	1.23	1.53	1.24
213	A12-6C	122	122	76	62	165	P	2.67	1.90	30	12	392	15.70	2.23	1.50	1.50	1.23	1.53	1.24
214	A12-6D	120	120	76	61	163	P	2.67	1.94	24	12	404	16.60	2.72	1.75	1.75	1.47	1.79	1.42
215	A12-6E	120	120	76	61	164	P	2.67	1.93	30	12	404	18.60	2.72	1.82	1.82	1.49	1.86	1.51
216	D29.4-9.8A	290	290	127	100	400	P	4.00	2.90	25	12	382	37.80	1.55	0.99	0.99	0.84	1.06	0.85
217	D29.4-9.8B	294	294	127	100	400	P	4.00	2.90	25	12	382	34.30	1.38	0.88	0.88	0.75	0.95	0.76
218	D29.4-9.8C	293	293	127	98	392	P	4.00	2.46	25	12	422	42.90	1.77	1.13	1.13	1.02	1.27	1.04
219	D29.4-9.8D	294	294	127	99	396	P	4.00	2.92	25	12	422	47.50	1.94	1.24	1.24	1.04	1.32	1.06
220	A29.4-9.8A	293	293	127	99	264	P	2.67	1.96	27	12	426	50.10	1.95	1.27	1.27	1.12	1.34	1.14
221	A29.4-9.8B	293	293	127	99	264	P	2.67	1.96	27	12	426	52.50	2.04	1.34	1.34	1.17	1.40	1.19
222	C29.4-9.8A	294	294	127	99	264	P	2.67	1.95	15	12	372	36.30	1.90	1.13	1.13	1.04	1.20	0.96
223	C29.4-9.8B	294	294	127	99	264	P	2.67	1.95	18	12	372	43.40	2.07	1.27	1.27	1.15	1.34	1.09
224	B29.4-9.8A	294	294	127	98	392	P	4.00	1.97	26	12	378	41.80	1.69	1.09	1.09	1.05	1.30	1.09
225	B29.4-9.8B	294	294	127	98	392	P	4.00	1.97	23	12	378	45.90	1.95	1.24	1.24	1.21	1.48	1.23

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
226	A.8.5-34A	85	85	406	341	910	P	2.67	1.86	28	12	408	28.60	1.10	0.83	0.83	0.83	0.90	0.88
227	A.8.5-34B	87	87	406	340	909	P	2.67	1.80	26	12	416	27.60	1.08	0.82	0.82	0.82	0.88	0.86
228	A50-25A	501	501	305	253	675	P	2.67	1.81	35	12	399	188.70	1.49	1.10	1.10	1.09	1.20	1.18
229	A50-25B	502	502	305	252	674	P	2.67	1.85	34	12	394	171.20	1.36	1.00	1.00	0.99	1.09	1.06
Ruesch, Haugli, and Mayer (1962)																			
230	X	90	90	134	111	400	P	3.60	2.65	23	30	481	14.60	1.79	1.13	1.13	0.94	1.19	1.00
231	Y	120	120	229	199	717	P	3.60	2.65	23	30	407	30.10	1.55	0.98	0.98	0.87	1.07	0.98
232	Z	180	180	302	262	947	P	3.62	2.64	24	30	412	54.70	1.39	0.94	0.94	0.82	1.00	0.95
Leonhardt and Walther (1962)																			
233	14/1	190	190	323	273	750	U2	2.75	2.05	36	30	465	85.90	1.62	1.19	1.19	1.05	1.14	1.26
234	14/2	190	190	323	273	750	U2	2.75	2.05	36	30	465	86.30	1.63	1.19	1.19	1.05	1.14	1.27
235	15/1	190	190	322	272	1000	U2	3.68	2.05	38	30	465	80.90	1.49	1.10	1.10	1.04	1.05	1.29
236	15/2	189	189	323	273	1000	U2	3.66	2.06	38	30	465	86.00	1.58	1.17	1.17	1.11	1.11	1.37
237	16/1	190	190	323	273	1250	U2	4.58	2.05	38	30	465	84.20	1.55	1.15	1.15	1.13	1.09	1.43

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
238	16/2	189	189	324	274	1250	U2	4.56	2.05	38	30	465	83.70	1.55	1.15	1.15	1.13	1.08	1.42
239	17/2	189	189	324	274	1500	U2	5.47	2.05	36	30	465	77.60	1.48	1.08	1.08	1.10	1.03	1.42
Bresler and Scordelis (1963)																			
240	0A-1	310	310	556	461	1753	P	3.80	1.81	23	19	555	166.80	1.44	1.13	1.13	1.18	1.31	1.34
241	0A-2	305	305	561	466	2210	P	4.74	2.27	24	19	555	177.90	1.51	1.16	1.16	1.20	1.38	1.39
242	IIIa-17	203	203	457	403	1524	P	3.78	2.54	29	25	505	88.10	1.17	0.90	0.90	0.82	0.94	0.97
243	IIIa-18	203	203	457	403	1524	P	3.78	2.54	25	25	505	80.70	1.16	0.87	0.87	0.79	0.91	0.92
244	Va-19	203	203	457	403	1524	P	3.78	0.93	24	25	694	63.30	0.94	0.90	0.90	0.92	1.01	1.10
245	Va-20	203	203	457	403	1524	P	3.78	0.93	26	25	694	65.90	0.94	0.91	0.91	0.93	1.02	1.12
246	VIb-21	203	203	457	403	1143	P	2.84	0.84	26	25	707	71.40	1.00	1.02	1.02	0.98	1.03	1.16
247	VIb-22	203	203	457	403	1143	P	2.84	0.84	26	25	707	62.40	0.88	0.89	0.89	0.86	0.90	1.01
248	VIb-23	203	203	457	403	1143	P	2.84	0.84	31	25	707	75.10	0.98	1.01	1.01	0.97	1.03	1.17
249	VIa-24	203	203	457	403	1524	P	3.78	0.47	26	25	696	54.50	0.76	0.94	0.94	0.99	1.09	1.21
250	VIa-25	203	203	457	403	1524	P	3.78	0.47	26	25	696	49.90	0.71	0.87	0.87	0.91	1.00	1.12

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ_l (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Kani (1964)																			
251	29.00	152	152	305	271	1221	P	4.50	1.87	25	19	350	42.90	1.24	0.87	0.87	0.92	1.08	1.05
252	30.00	152	152	305	271	1221	P	4.50	1.87	25	19	350	46.30	1.32	0.93	0.93	0.98	1.16	1.13
253	35.00	155	155	305	269	953	P	3.53	1.82	26	19	491	44.90	1.24	0.89	0.89	0.89	1.02	1.01
254	36.00	153	153	305	273	953	P	3.49	1.82	26	19	491	51.60	1.42	1.02	1.02	1.02	1.17	1.15
Krefeld and Thurston (1966)																			
255	II-11A2	152	152	381	314	915	P	2.92	3.41	30	25	400	73.40	1.65	1.21	1.21	0.92	1.10	1.06
256	II-12A2	152	152	305	238	914	P	3.85	4.50	30	25	400	64.10	1.90	1.31	1.31	0.96	1.23	1.11
257	III-18A2	152	152	381	316	915	P	2.90	2.68	19	25	370	63.20	1.76	1.20	1.20	1.01	1.18	1.11
258	III-18B2	152	152	381	316	915	P	2.90	2.68	20	25	370	72.10	1.98	1.36	1.36	1.14	1.33	1.26
259	III-18C2	152	152	381	316	915	P	2.90	2.68	23	25	370	73.40	1.89	1.33	1.33	1.11	1.29	1.24
260	III-18D2	152	152	381	316	915	P	2.90	2.68	22	25	370	60.00	1.56	1.09	1.09	0.91	1.07	1.02
261	IV-13A2	152	152	381	319	914	P	2.87	0.80	20	25	379	48.50	1.32	1.23	1.23	1.19	1.27	1.36
262	IV-14A2	152	152	305	243	914	P	3.77	1.05	21	25	379	35.10	1.23	0.99	0.99	1.02	1.16	1.16
263	IV-15A2	152	152	381	316	915	P	2.90	1.34	20	25	370	45.80	1.25	0.98	0.98	0.93	1.02	1.06

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
264	IV-15B2	152	152	381	316	915	P	2.90	1.34	21	25	370	52.00	1.40	1.11	1.11	1.05	1.15	1.19
265	IV-16A2	152	152	305	240	914	P	3.81	1.77	22	25	370	41.80	1.43	0.98	0.98	0.98	1.15	1.12
266	IV-17A2	152	152	305	243	914	P	3.77	2.09	22	25	379	44.00	1.49	0.98	0.98	0.96	1.14	1.09
267	IV-18E2	152	152	381	316	915	P	2.90	2.68	20	25	370	81.80	2.25	1.55	1.55	1.29	1.51	1.43
268	IV-19A2	152	152	305	240	914	P	3.81	3.53	21	25	370	46.30	1.64	1.07	1.07	0.87	1.07	0.96
269	IV-20A2	152	152	305	238	914	P	3.85	4.52	21	25	400	50.70	1.80	1.17	1.17	0.87	1.11	0.95
270	IV-21A2	203	203	305	238	914	P	3.85	5.01	20	25	400	76.50	2.09	1.34	1.34	0.96	1.25	1.05
271	V-2AC	152	152	305	254	1219	P	4.80	1.32	23	25	394	37.80	1.20	0.92	0.92	0.98	1.17	1.16
272	V-3AC	152	152	305	256	1219	P	4.77	1.99	21	25	379	44.00	1.46	0.96	0.96	1.01	1.21	1.16
273	V-4AC	152	152	305	254	1219	P	4.80	2.63	17	25	394	37.80	1.42	0.90	0.90	0.86	1.05	0.96
274	V-5AC	152	152	305	252	1219	P	4.83	3.35	18	25	370	41.80	1.50	0.97	0.97	0.84	1.05	0.94
275	V-6AC	152	152	305	250	1220	P	4.87	4.30	23	25	400	53.40	1.73	1.15	1.15	0.90	1.17	1.04
276	V-3CC	152	152	305	256	1523	P	5.96	1.99	21	25	379	35.60	1.19	0.78	0.78	0.85	1.06	1.01
277	V-4CC	152	152	305	254	1524	P	6.00	2.63	21	25	394	40.00	1.34	0.88	0.88	0.86	1.10	1.02

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
278	VII-6C	152	152	305	252	914	P	3.62	3.35	20	25	370	51.20	1.75	1.15	1.15	0.94	1.15	1.03
279	VIII-3AAC	152	152	305	256	915	P	3.58	1.99	35	25	379	55.60	1.43	1.03	1.03	1.00	1.17	1.19
280	VIII-4AAC	152	152	305	254	914	P	3.60	2.63	29	25	394	57.80	1.63	1.14	1.14	1.00	1.20	1.16
281	VIII-5AAC	152	152	305	252	914	P	3.62	3.35	33	25	370	56.90	1.53	1.08	1.08	0.87	1.07	1.02
282	VIII-6AAC	152	152	305	250	914	P	3.65	4.30	34	25	400	60.00	1.58	1.13	1.13	0.83	1.04	0.97
283	VIII-3AC	152	152	305	256	1219	P	4.77	1.99	32	25	379	53.40	1.43	1.01	1.01	1.04	1.27	1.27
284	VIII-4AC	152	152	305	254	1219	P	4.80	2.63	31	25	394	53.80	1.48	1.04	1.04	0.97	1.20	1.17
285	VIII-5AC	152	152	305	252	1219	P	4.83	3.35	33	25	370	54.30	1.46	1.03	1.03	0.88	1.11	1.06
286	VIII-6AC	152	152	305	250	1220	P	4.87	4.30	34	25	400	59.20	1.57	1.12	1.12	0.87	1.12	1.04
287	VIII-4CC	152	152	305	254	1524	P	6.00	2.63	38	25	394	52.50	1.29	0.94	0.94	0.90	1.17	1.15
288	IX-3AAC	152	152	305	256	915	P	3.58	1.99	13	25	379	40.50	1.72	1.05	1.05	1.07	1.22	1.11
289	IX-4AAC	152	152	305	254	914	P	3.60	2.63	13	25	394	42.70	1.81	1.10	1.10	1.01	1.19	1.05
290	IX-5AAC	152	152	305	252	914	P	3.62	3.35	15	25	370	50.30	1.97	1.23	1.23	1.02	1.25	1.09
291	IX-6AAC	152	152	305	250	914	P	3.65	4.30	13	25	400	62.30	2.63	1.61	1.61	1.23	1.56	1.27

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
292	IX-3AC	152	152	305	256	1219	P	4.77	1.99	14	25	379	36.90	1.51	0.93	0.93	0.99	1.17	1.08
293	IX-4AC	152	152	305	254	1219	P	4.80	2.63	13	25	394	40.00	1.70	1.03	1.03	1.00	1.21	1.08
294	IX-5AC	152	152	305	252	1219	P	4.83	3.35	15	25	370	43.60	1.71	1.07	1.07	0.94	1.17	1.03
295	IX-6AC	152	152	305	250	1220	P	4.87	4.30	12	25	400	40.90	1.80	1.08	1.08	0.88	1.12	0.93
296	IX-3CC	152	152	305	256	1523	P	5.96	1.99	12	25	379	31.10	1.35	0.81	0.81	0.91	1.10	1.01
297	IX-4CC	152	152	305	254	1524	P	6.00	2.63	17	25	394	35.10	1.29	0.82	0.82	0.81	1.03	0.94
298	X-*-C	203	203	533	483	1525	P	3.16	1.56	17	25	394	84.50	1.24	0.98	0.98	0.95	1.04	1.08
299	s-I-*-OCa	152	152	305	254	1524	P	6.00	2.63	36	25	394	48.50	1.24	0.89	0.89	0.86	1.11	1.08
300	s-I-*-OCb	152	152	305	254	1524	P	6.00	2.63	39	25	394	52.50	1.28	0.94	0.94	0.90	1.16	1.15
301	s-II-*-OCa	254	254	508	456	1830	P	4.02	2.22	38	25	370	146.80	1.20	1.00	1.00	0.98	1.09	1.19
302	s-II-*-OCb	254	254	508	456	1830	P	4.02	2.22	38	25	370	133.40	1.09	0.91	0.91	0.89	0.99	1.08
Baron (1966)																			
303	1.00	203	203	152	127	584	P	4.60	1.55	31	19	429	38.30	1.58	1.14	1.14	1.13	1.44	1.29
304	2.00	203	203	152	127	584	P	4.60	1.55	31	19	429	34.70	1.43	1.03	1.03	1.03	1.30	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
305	4.00	203	203	152	127	584	P	4.60	1.55	27	19	429	30.20	1.33	0.94	0.94	0.94	1.18	1.05
Kani (1966)																			
306	81.00	153	153	305	274	1628	P	5.93	2.76	28	19	343	51.20	1.37	0.96	0.96	0.94	1.18	1.12
307	83.00	156	156	305	271	814	P	3.00	2.74	27	19	343	65.00	1.73	1.21	1.21	1.04	1.22	1.15
308	84.00	151	151	305	271	1085	P	4.00	2.84	27	19	342	55.40	1.52	1.07	1.07	0.96	1.15	1.09
309	96.00	153	153	305	275	1085	P	3.94	2.76	25	19	335	56.30	1.56	1.08	1.08	0.99	1.18	1.11
310	97.00	152	152	305	276	815	P	2.95	2.68	27	19	366	62.50	1.68	1.18	1.18	1.01	1.19	1.12
311	103.00	155	155	305	274	814	P	2.97	0.74	29	19	423	38.80	0.99	0.98	0.98	0.98	1.07	1.13
312	104.00	154	154	305	269	1072	P	3.98	0.76	25	19	423	33.60	0.95	0.90	0.90	0.97	1.10	1.12
313	107.00	154	154	305	267	1356	P	5.08	0.76	27	19	422	25.70	0.71	0.68	0.68	0.76	0.92	0.92
314	116.00	152	152	305	271	815	P	3.01	0.78	26	19	384	39.30	1.09	1.04	1.04	1.04	1.14	1.19
315	121.00	152	152	305	272	815	P	2.99	1.85	20	19	330	49.00	1.55	1.06	1.06	1.03	1.16	1.12
316	122.00	150	150	305	276	1087	P	3.94	1.84	20	19	343	38.80	1.24	0.85	0.85	0.88	1.01	0.97

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
317	123.00	155	155	305	271	1085	P	4.00	1.79	15	19	346	37.80	1.35	0.89	0.89	0.94	1.07	1.00
318	124.00	154	154	305	271	1356	P	5.00	1.80	15	19	345	32.00	1.15	0.76	0.76	0.83	0.98	0.91
319	126.00	155	155	305	272	814	P	2.99	1.78	16	19	346	42.70	1.48	0.99	0.99	0.97	1.09	1.02
320	130.00	153	153	305	276	1467	P	5.32	1.79	18	19	346	40.10	1.32	0.90	0.90	0.98	1.18	1.11
321	133.00	154	154	305	273	1359	P	4.97	1.81	20	19	508	38.50	1.21	0.83	0.83	0.90	1.07	1.02
322	143.00	154	154	305	274	1085	P	3.96	0.74	18	19	428	30.20	1.00	0.91	0.91	0.98	1.10	1.10
323	152.00	149	149	305	270	815	P	3.02	0.79	20	19	384	32.50	1.07	0.97	0.97	0.98	1.05	1.08
324	153.00	152	152	305	273	815	P	2.99	0.76	20	19	384	32.80	1.05	0.96	0.96	0.97	1.04	1.07
325	164.00	156	156	305	271	1085	P	4.00	0.73	34	19	412	35.80	0.86	0.87	0.87	0.93	1.07	1.11
326	166.00	152	152	305	271	815	P	3.01	0.78	35	19	377	40.30	0.97	0.97	0.97	0.97	1.06	1.13
327	179.00	153	153	305	264	678	P	2.57	0.53	32	19	400	33.60	0.86	0.96	0.96	0.93	1.01	1.09
328	182.00	155	155	305	268	1356	P	5.05	1.80	34	19	386	48.80	1.19	0.89	0.89	0.96	1.16	1.15
329	186.00	155	155	305	272	1085	P	3.99	1.78	35	19	394	55.40	1.30	0.99	0.99	1.02	1.19	1.20

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
330	191.00	154	154	305	275	815	P	2.96	1.80	34	19	497	53.10	1.26	0.95	0.95	0.92	1.03	1.05
331	194.00	154	154	305	278	814	P	2.93	1.80	35	19	352	51.20	1.20	0.91	0.91	0.87	0.98	1.00
332	195.00	153	153	305	275	1085	P	3.94	1.82	35	19	352	47.30	1.12	0.85	0.85	0.87	1.01	1.02
333	196.00	154	154	305	269	1359	P	5.05	1.85	36	19	380	51.20	1.21	0.91	0.91	0.97	1.18	1.18
334	202.00	154	154	305	273	1628	P	5.97	2.68	34	19	377	49.90	1.20	0.87	0.87	0.86	1.08	1.04
335	208.00	157	157	305	275	1221	P	4.44	2.68	36	19	379	60.10	1.37	1.01	1.01	0.94	1.13	1.11
336	211.00	153	153	305	270	815	P	3.02	2.73	35	19	381	57.20	1.37	1.00	1.00	0.86	1.00	0.97
337	212.00	155	155	305	273	815	P	2.98	2.66	35	19	381	60.50	1.42	1.04	1.04	0.89	1.04	1.02
338	213.00	154	154	305	276	1223	P	4.44	2.66	37	19	381	57.40	1.31	0.97	0.97	0.91	1.09	1.07
339	246.00	153	153	305	274	952	P	3.47	0.51	28	19	400	25.40	0.68	0.75	0.75	0.79	0.89	0.93
340	267.00	153	153	305	269	949	P	3.53	0.52	21	19	400	24.50	0.77	0.80	0.80	0.86	0.95	0.97
341	268.00	153	153	305	275	816	P	2.96	0.49	20	19	396	27.20	0.85	0.90	0.90	0.93	0.99	1.04
342	166.00	154	154	305	274	815	P	2.97	0.76	35	19	379	38.30	0.90	0.91	0.91	0.91	0.99	1.06

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Krefeld and Thurston (1966)																			
343	V-**-2CU	152	152	305	254	762	U2	3.00	1.32	21	25	394	45.20	1.51	1.14	1.14	1.10	1.10	1.23
344	V-**-3CU	152	152	305	256	761	U2	2.98	1.99	21	25	379	59.60	1.99	1.31	1.31	1.24	1.29	1.37
345	V-**-4CU	152	152	305	254	762	U2	3.00	2.63	21	25	394	66.30	2.23	1.46	1.46	1.25	1.35	1.38
346	V-**-5CU	152	152	305	252	762	U2	3.02	3.35	20	25	370	69.00	2.35	1.54	1.54	1.20	1.35	1.32
347	V-**-6CU	152	152	305	250	762	U2	3.04	4.30	21	25	400	65.00	2.22	1.45	1.45	1.04	1.22	1.13
348	V-**-4EU	152	152	305	254	914	U2	3.60	2.63	20	25	394	62.80	2.13	1.40	1.40	1.25	1.29	1.39
349	V-**-5EU	152	152	305	252	914	U2	3.62	3.35	19	25	370	66.70	2.33	1.51	1.51	1.24	1.34	1.36
350	V-**-6EU	152	152	305	250	914	U2	3.65	4.30	20	25	400	59.10	2.04	1.33	1.33	0.99	1.12	1.09
351	V-**-5GU	152	152	305	252	1067	U2	4.23	3.35	21	25	370	58.00	1.93	1.27	1.27	1.07	1.12	1.21
352	V-**-4JU	152	152	305	254	1219	U2	4.80	2.63	22	25	394	51.00	1.65	1.10	1.10	1.03	1.01	1.20
353	IX-**-4CU	152	152	305	254	762	U2	3.00	2.63	32	25	394	81.20	2.18	1.54	1.54	1.30	1.40	1.51
354	IX-**-5CU	152	152	305	252	762	U2	3.02	3.35	32	25	370	79.40	2.15	1.52	1.52	1.17	1.30	1.35

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
355	IX-**-6CU	152	152	305	251	762	U2	3.04	4.30	37	25	400	89.90	2.28	1.65	1.65	1.16	1.33	1.34
356	IX-**-3CU	152	152	305	256	761	U2	2.98	1.99	12	25	379	49.20	2.13	1.29	1.29	1.26	1.30	1.29
357	IX-**-4CU	152	152	305	254	762	U2	3.00	2.63	17	25	394	58.90	2.17	1.38	1.38	1.20	1.29	1.28
358	IX-**-5CU	152	152	305	252	762	U2	3.02	3.35	15	25	370	67.20	2.69	1.67	1.67	1.33	1.51	1.39
359	IX-**-6CU	152	152	305	250	762	U2	3.04	4.30	14	25	400	59.80	2.50	1.53	1.53	1.12	1.33	1.15
360	IX-**-3EU	152	152	305	256	914	U2	3.58	1.99	15	25	379	49.70	1.93	1.21	1.21	1.21	1.21	1.30
361	IX-**-4EU	152	152	305	254	914	U2	3.60	2.63	14	25	394	43.30	1.74	1.08	1.08	0.98	1.02	1.04
362	IX-**-5EU	152	152	305	252	914	U2	3.62	3.35	15	25	370	55.20	2.18	1.36	1.36	1.13	1.22	1.20
363	IX-**-6EU	152	152	305	250	914	U2	3.65	4.30	13	25	400	53.40	2.31	1.40	1.40	1.07	1.22	1.10
364	IX-**-3GU	152	152	305	256	1067	U2	4.18	1.99	14	25	379	41.90	1.72	1.06	1.06	1.11	1.06	1.19
365	IX-**-5GU	152	152	305	252	1067	U2	4.23	3.35	11	25	370	42.70	1.96	1.16	1.16	1.02	1.07	1.05
Kani (1967)																			
366	40.00	152	152	152	140	747	P	5.35	2.59	27	19	388	32.00	1.70	1.10	1.10	1.03	1.35	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
367	43.00	151	151	152	137	813	P	5.93	2.73	28	19	392	29.10	1.56	1.02	1.02	0.95	1.27	1.10
368	47.00	151	151	152	132	678	P	5.13	2.84	25	19	392	28.20	1.67	1.07	1.07	0.96	1.25	1.06
369	48.00	151	151	152	133	678	P	5.09	2.82	25	19	392	27.10	1.59	1.02	1.02	0.92	1.19	1.01
370	52.00	152	152	152	140	544	P	3.89	2.67	25	19	392	28.90	1.60	1.03	1.03	0.90	1.12	0.97
371	55.00	150	150	152	135	406	P	3.02	2.80	25	19	392	32.60	1.89	1.21	1.21	0.98	1.21	1.04
372	56.00	153	153	152	137	476	P	3.46	2.67	27	19	403	28.00	1.51	0.98	0.98	0.83	1.03	0.90
373	57.00	153	153	152	139	747	P	5.39	2.61	26	19	375	31.20	1.68	1.09	1.09	1.01	1.33	1.15
374	58.00	152	152	152	138	476	P	3.44	2.66	20	19	417	28.90	1.80	1.11	1.11	0.97	1.18	1.00
375	59.00	154	154	152	140	373	P	2.67	2.63	27	19	392	50.20	2.66	1.72	1.72	1.38	1.69	1.47
376	60.00	155	155	152	139	407	P	2.93	2.64	27	19	392	39.30	2.07	1.34	1.34	1.10	1.36	1.18
377	63.00	154	154	610	543	2170	P	4.00	2.77	26	19	352	93.20	1.28	1.03	1.03	0.97	1.10	1.11
378	71.00	155	155	610	544	1628	P	2.99	2.41	27	19	373	102.10	1.36	1.10	1.10	1.03	1.13	1.15
379	74.00	152	152	610	523	1631	P	3.12	2.84	27	19	366	107.70	1.53	1.23	1.23	1.08	1.22	1.21

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
380	75.00	152	152	610	524	1631	P	3.11	2.84	27	19	367	107.90	1.53	1.23	1.23	1.08	1.22	1.21
381	76.00	152	152	610	518	1359	P	2.63	2.87	31	19	372	114.80	1.55	1.26	1.26	1.07	1.19	1.19
382	272.00	611	611	305	271	1359	P	5.02	2.73	27	19	377	227.80	1.56	1.09	1.09	1.04	1.27	1.21
383	273.00	612	612	305	271	1087	P	4.01	2.72	27	19	377	206.20	1.40	0.98	0.98	0.90	1.07	1.02
384	274.00	612	612	305	270	815	P	3.02	2.73	27	19	377	250.20	1.71	1.19	1.19	1.02	1.20	1.14
385	3043.00	154	154	1219	1092	3277	P	3.00	2.71	27	19	376	165.10	1.11	1.01	1.01	1.02	1.11	1.12
386	3044.00	152	152	1219	1097	4364	P	3.98	2.72	30	19	376	159.10	1.03	0.95	0.95	1.04	1.12	1.16
387	3045.00	155	155	1219	1092	5461	P	5.00	2.70	28	19	381	152.40	1.00	0.91	0.91	1.03	1.14	1.18
Bhal (1967)																			
388	B1	240	240	350	297	900	P	3.03	1.26	23	30	434	70.70	1.21	0.98	0.98	0.92	1.04	1.08
389	B2	240	240	650	600	1800	P	3.00	1.26	30	30	434	119.50	0.90	0.87	0.87	0.85	0.90	1.04
390	B3	240	240	950	900	2700	P	3.00	1.26	28	30	434	166.80	0.87	0.89	0.89	0.92	0.95	1.12
391	B4	240	240	1250	1200	3600	P	3.00	1.26	25	30	434	185.20	0.75	0.80	0.80	0.86	0.89	1.05

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
392	B5	240	240	650	600	1800	P	3.00	0.63	27	30	434	106.20	0.84	1.02	1.02	1.01	1.05	1.25
393	B6	240	240	650	600	1800	P	3.00	0.63	25	30	430	114.10	0.94	1.12	1.12	1.11	1.15	1.37
394	B7	240	240	950	900	2700	P	3.00	0.63	27	30	434	139.80	0.73	0.95	0.95	1.00	1.00	1.24
395	B8	240	240	950	900	2700	P	3.00	0.63	28	30	430	127.50	0.66	0.86	0.86	0.91	0.91	1.12
Rajagopalan and Ferguson (1968)																			
396	S-1	154	154	311	259	1016	P	3.93	1.43	37	13	655	35.60	0.87	0.71	0.71	0.77	0.87	0.86
397	S-2	154	154	311	265	1016	P	3.83	0.98	33	13	655	37.40	0.94	0.85	0.85	0.94	1.05	1.05
398	S-3	152	152	311	267	1118	P	4.19	0.81	29	13	524	31.10	0.84	0.80	0.80	0.90	1.01	1.01
399	S-4	152	152	311	268	1118	P	4.17	0.63	33	13	524	28.00	0.70	0.74	0.74	0.85	0.96	0.97
400	S-5	152	152	311	262	1118	P	4.27	0.53	28	13	1779	33.60	0.94	1.02	1.02	1.17	1.34	1.33
401	S-6	151	151	311	267	1118	P	4.18	0.35	31	13	1779	27.40	0.72	0.91	0.91	1.07	1.24	1.23
402	S-7	152	152	311	268	1118	P	4.17	0.25	29	13	1779	30.00	0.81	1.14	1.14	1.35	1.58	1.56

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
403	S-9	152	152	311	262	1118	P	4.27	0.53	25	13	1779	24.50	0.72	0.77	0.77	0.89	1.01	1.00
404	S-12	153	153	311	268	1118	P	4.16	0.25	30	13	1779	24.60	0.65	0.92	0.92	1.08	1.27	1.26
405	S-13	152	152	311	265	1118	P	4.22	1.73	24	13	655	40.00	1.20	0.86	0.86	0.94	1.08	1.02
406	S-14	151	151	349	269	1119	P	4.16	0.63	30	13	524	25.00	0.66	0.69	0.69	0.78	0.89	0.89
407	S-15	761	761	311	269	1119	P	4.16	0.63	33	13	524	150.80	0.75	0.80	0.80	0.91	1.03	1.04
Taylor (1968)																			
408	1A	203	203	406	370	1118	P	3.02	1.03	29	10	350	61.80	0.90	0.85	0.85	0.93	0.96	0.98
409	2A	203	203	406	370	1118	P	3.02	1.55	33	10	350	91.60	1.25	1.05	1.05	1.14	1.18	1.19
410	1B	203	203	406	370	1118	P	3.02	1.03	29	10	350	75.60	1.10	1.04	1.04	1.14	1.17	1.19
411	2B	203	203	406	370	1118	P	3.02	1.55	33	10	350	100.50	1.37	1.15	1.15	1.25	1.30	1.30
412	3B	203	203	406	370	1118	P	3.02	1.03	32	10	350	76.10	1.06	1.02	1.02	1.12	1.14	1.18
Kani (1969)																			
413	3061.00	154	154	1202	1091	3378	P	3.10	0.80	27	19	345	97.50	0.65	0.81	0.81	0.96	0.93	1.08
Mattock (1970)																			

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
414	1.00	152	152	305	254	762	P	3.00	1.03	17	19	400	36.50	1.34	1.07	1.07	1.08	1.17	1.16
415	3.00	152	152	305	254	762	P	3.00	1.03	47	19	400	54.70	1.22	1.15	1.15	1.13	1.26	1.35
416	10.00	152	152	305	254	762	P	3.00	3.10	19	19	400	56.00	1.98	1.28	1.28	1.06	1.27	1.12
417	15.00	152	152	305	254	1372	P	5.40	1.03	26	19	400	31.10	0.93	0.79	0.79	0.89	1.08	1.06
418	18.00	152	152	305	254	1372	P	5.40	2.07	18	19	400	35.60	1.27	0.82	0.82	0.89	1.08	1.00
419	22.00	152	152	305	254	1372	P	5.40	3.10	16	19	400	40.00	1.52	0.96	0.96	0.90	1.12	0.99
420	24.00	152	152	305	254	1372	P	5.40	3.10	29	19	400	52.50	1.48	1.03	1.03	0.95	1.20	1.12
Swamy, Andriopoulos, and Adepegba (1970)																			
421	6.00	102	102	152	127	445	P	3.50	1.64	27	19	483	20.70	1.83	1.26	1.26	1.20	1.45	1.30
422	8.00	102	102	152	127	572	P	4.50	1.64	27	19	483	19.30	1.70	1.18	1.18	1.17	1.47	1.31
Smith (1970)																			
423	8/0	150	150	230	200	610	U2	3.05	2.01	28	19	500	49.40	1.83	1.20	1.20	1.17	1.22	1.29
424	10/0	150	150	230	200	763	U2	3.81	2.01	35	19	500	43.90	1.47	0.99	0.99	1.01	1.00	1.17
425	12/0	150	150	230	200	915	U2	4.58	2.01	36	19	500	46.30	1.51	1.03	1.03	1.08	1.04	1.28

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Placas and Regan (1971)																			
426	R1	152	152	305	272	914	P	3.36	0.98	26	19	621	44.90	1.25	1.10	1.10	1.12	1.25	1.28
427	R2	152	152	305	272	914	P	3.36	1.46	26	19	621	47.10	1.31	1.01	1.01	1.01	1.14	1.15
428	R3	152	152	305	272	914	P	3.36	1.46	25	19	621	44.90	1.28	0.98	0.98	0.99	1.11	1.11
429	R7	152	152	305	272	914	P	3.36	1.46	28	19	621	54.30	1.46	1.14	1.14	1.14	1.29	1.30
430	D2	152	152	305	272	914	P	3.36	1.46	30	19	621	52.50	1.36	1.07	1.07	1.08	1.21	1.23
431	T2	152	610	305	272	914	P	3.36	1.46	28	19	621	54.70	1.47	1.15	1.15	1.15	1.29	1.31
432	T18	152	610	305	254	914	P	3.60	4.16	28	19	621	74.70	2.14	1.48	1.48	1.13	1.41	1.26
Haddadin, Hong, and Mattock (1971)																			
433	C1	178	610	470	381	1619	P	4.25	3.81	26	19	517	87.40	1.49	1.11	1.11	0.92	1.11	1.04
Batson, Jenkins, and Spatney (1972)																			
434	A1	102	102	152	127	610	P	4.80	3.10	35	19	276	19.20	1.48	1.00	1.00	0.83	1.11	0.96
435	A2	102	102	152	127	610	P	4.80	3.10	35	19	276	18.80	1.45	0.98	0.98	0.82	1.08	0.94
436	A3	102	102	152	127	610	P	4.80	3.10	35	19	276	20.70	1.60	1.08	1.08	0.90	1.19	1.04

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Taylor (1972)																			
437	A1	400	400	1000	930	2800	P	3.01	1.35	29	38	420	358.40	1.06	1.08	1.08	1.07	1.15	1.36
438	A2	400	400	1000	930	2800	P	3.01	1.35	25	19	420	328.40	1.04	1.04	1.04	1.15	1.16	1.29
439	B1	200	200	500	465	1400	P	3.01	1.35	24	38	420	104.30	1.34	1.18	1.18	1.08	1.21	1.33
440	B2	200	200	500	465	1400	P	3.01	1.35	22	19	420	87.30	1.18	1.02	1.02	1.02	1.08	1.14
441	B3	200	200	500	465	1400	P	3.01	1.35	28	9	420	85.30	1.01	0.91	0.91	1.03	1.04	1.05
442	C1	100	100	250	233	700	P	3.00	1.35	23	19	420	22.50	1.19	0.89	0.89	0.88	0.98	0.97
443	C2	100	100	250	233	700	P	3.00	1.35	23	9	420	24.00	1.27	0.95	0.95	1.02	1.10	1.03
444	C3	100	100	250	233	700	P	3.00	1.35	24	9	420	27.50	1.41	1.06	1.06	1.14	1.23	1.16
445	C4	100	100	250	233	700	P	3.00	1.35	19	9	420	22.50	1.32	0.95	0.95	1.02	1.11	1.02
446	C5	100	100	250	233	700	P	3.00	1.35	20	9	420	27.00	1.53	1.12	1.12	1.20	1.29	1.20
447	C6	100	100	250	233	700	P	3.00	1.35	26	2	420	27.50	1.37	1.04	1.04	1.39	1.28	1.15
448	D1	60	60	150	139	420	P	3.02	1.35	28	2	420	11.60	1.54	1.15	1.15	1.36	1.34	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
449	D2	60	60	150	139	420	P	3.02	1.35	28	2	420	12.10	1.60	1.20	1.20	1.42	1.40	1.22
450	D3	60	60	150	139	420	P	3.02	1.35	28	2	420	10.60	1.40	1.05	1.05	1.24	1.23	1.07
451	D4	60	60	150	139	420	P	3.02	1.35	28	2	420	11.40	1.51	1.13	1.13	1.34	1.32	1.15
Aster and Koch (1974)																			
452	11.00	1000	1000	539	500	1825	P	3.65	0.46	25	30	535	267.40	0.63	0.81	0.81	0.84	0.92	1.05
453	16.00	1000	1000	794	750	2750	P	3.67	0.42	30	30	536	406.70	0.58	0.85	0.85	0.93	0.98	1.19
454	2.00	1000	1000	281	250	920	P	3.68	0.64	27	30	554	218.00	0.99	0.99	0.99	1.00	1.17	1.22
455	12.00	1000	1000	540	500	1825	P	3.65	0.65	27	30	535	330.20	0.74	0.86	0.86	0.88	0.96	1.10
456	3.00	1000	1000	289	250	920	P	3.68	0.91	27	30	535	222.50	1.00	0.89	0.89	0.89	1.04	1.08
457	8.00	1000	1000	544	500	2750	P	5.50	0.63	31	30	535	287.10	0.61	0.72	0.72	0.80	0.96	1.06
458	9.00	1000	1000	544	500	2750	P	5.50	0.63	20	30	535	260.60	0.69	0.76	0.76	0.84	0.99	1.08
459	10.00	1000	1000	544	500	2750	P	5.50	0.63	20	30	535	261.60	0.69	0.77	0.77	0.84	1.00	1.08
460	17.00	1000	1000	794	750	2750	P	3.67	0.42	29	30	535	363.50	0.53	0.77	0.77	0.85	0.89	1.08
Cederwall, Hedman, and Loeberg (1974)																			

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
461	734-34	135	135	260	234	800	P	3.42	1.07	32	19	483	41.40	1.37	1.17	1.17	1.19	1.36	1.38
462	734-40	135	135	260	234	600	P	2.56	1.08	27	19	483	40.90	1.47	1.21	1.21	1.16	1.27	1.30
Marti, Pralong, and Thurlimann (1977)																			
463	PS11	400	400	180	162	640	P	3.95	1.38	30	16	542	97.10	1.62	1.21	1.21	1.24	1.48	1.39
Reineck, Koch, and Schlaich (1978)																			
464	N8	500	500	250	226	791	P	3.50	0.79	26	16	501	101.50	1.04	0.94	0.94	1.00	1.12	1.12
465	s1_1	150	150	200	165	660	P	4.00	1.87	85	16	523	46.30	1.33	0.96	1.14	0.96	1.39	1.19
Kani, Huggins, and Wittkopp (1979)																			
466	802.00	152	152	305	276	1631	P	5.92	2.68	35	19	414	50.70	1.21	0.88	0.88	0.87	1.10	1.06
467	801.00	152	152	305	272	1087	P	4.00	2.69	35	19	439	54.30	1.31	0.95	0.95	0.88	1.04	1.02
468	803.00	153	153	305	275	816	P	2.97	2.70	35	19	429	54.70	1.29	0.94	0.94	0.81	0.94	0.92
469	4850.00	155	457	305	273	1087	P	3.99	5.49	18	19	381	70.50	2.32	1.51	1.51	1.08	1.41	1.14
470	4852.00	157	457	305	270	815	P	3.02	5.42	18	19	391	97.40	3.19	2.08	2.08	1.40	1.85	1.45
471	4853.00	155	457	305	273	1359	P	4.98	5.43	18	19	385	62.30	2.05	1.34	1.34	1.00	1.31	1.09

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
472	4857.00	153	457	305	275	953	P	3.47	5.43	18	19	348	77.00	2.57	1.67	1.67	1.16	1.52	1.22
473	4858.00	154	457	305	276	953	P	3.45	5.38	18	19	356	74.10	2.44	1.60	1.60	1.11	1.45	1.16
474	4830.00	155	457	305	272	815	P	2.99	2.69	19	19	401	64.10	2.05	1.35	1.35	1.18	1.38	1.25
475	4831.00	155	457	305	271	1087	P	4.02	2.70	19	19	407	59.20	1.90	1.25	1.25	1.16	1.39	1.27
476	4832.00	156	457	305	272	1359	P	5.00	2.67	19	19	401	55.80	1.77	1.17	1.17	1.13	1.38	1.27
477	4838.00	152	457	305	279	1359	P	4.88	2.60	18	19	328	52.30	1.71	1.12	1.12	1.10	1.33	1.22
478	4840A	153	457	305	274	953	P	3.48	2.70	18	19	343	49.20	1.65	1.07	1.07	0.97	1.14	1.03
479	4840D	152	457	305	269	953	P	3.54	2.80	21	19	463	54.30	1.72	1.15	1.15	1.02	1.21	1.11
480	4840E	153	457	305	271	953	P	3.52	2.78	21	19	477	60.00	1.88	1.25	1.25	1.11	1.32	1.21
481	4810.00	154	457	305	272	1631	P	6.00	1.82	18	19	407	47.40	1.56	1.05	1.05	1.18	1.45	1.36
482	4812.00	155	457	305	273	815	P	2.99	1.78	19	19	408	58.00	1.84	1.26	1.26	1.23	1.39	1.33
483	4813.00	154	457	305	269	1087	P	4.04	1.84	19	19	402	48.30	1.56	1.06	1.06	1.10	1.28	1.22
484	4814.00	154	457	305	271	1359	P	5.01	1.83	19	19	397	50.50	1.62	1.10	1.10	1.20	1.43	1.35

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
485	4815.00	156	457	305	271	815	P	3.01	1.81	19	19	370	53.20	1.69	1.15	1.15	1.13	1.27	1.21
486	4818.00	155	457	305	272	953	P	3.50	1.75	18	19	402	42.70	1.42	0.97	0.97	0.98	1.11	1.06
487	462.00	153	457	305	275	1631	P	5.93	8.08	30	19	391	79.60	2.03	1.45	1.45	0.94	1.32	1.10
488	463.00	153	457	305	273	1087	P	3.98	8.12	25	19	365	109.00	3.05	2.11	2.11	1.28	1.78	1.40
489	466.00	155	457	305	279	1361	P	4.89	7.89	27	19	361	77.20	2.03	1.42	1.42	0.91	1.25	1.02
490	469.00	154	457	305	271	815	P	3.01	8.15	26	19	372	139.90	3.90	2.70	2.70	1.54	2.17	1.65
491	471.00	153	457	305	269	1087	P	4.03	8.30	25	19	388	97.60	2.77	1.91	1.91	1.16	1.61	1.26
492	472.00	154	457	305	267	1082	P	4.06	8.26	26	19	386	94.50	2.64	1.83	1.83	1.11	1.54	1.22
493	473.00	154	457	305	268	1087	P	4.05	8.24	26	19	385	95.20	2.65	1.84	1.84	1.11	1.55	1.22
494	474.00	154	457	305	270	1087	P	4.03	8.18	27	19	367	90.30	2.48	1.72	1.72	1.05	1.45	1.15
495	475.00	153	457	305	269	1087	P	4.03	8.21	27	19	384	94.70	2.62	1.83	1.83	1.11	1.54	1.22
496	480.00	152	457	305	277	1224	P	4.43	8.00	27	19	376	77.40	2.07	1.46	1.46	0.91	1.25	1.01
497	481.00	152	457	305	276	1232	P	4.47	7.97	27	19	368	77.20	2.08	1.46	1.46	0.91	1.26	1.02

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
498	443.00	155	457	305	250	678	P	2.71	6.01	24	19	361	128.50	3.95	2.66	2.66	1.66	2.23	1.77
499	444.00	155	457	305	251	813	P	3.23	6.00	24	19	361	101.00	3.09	2.09	2.09	1.36	1.81	1.46
500	447.00	155	457	305	278	1628	P	5.86	5.35	26	19	381	62.70	1.67	1.17	1.17	0.89	1.18	1.03
501	453.00	154	457	305	274	815	P	2.98	5.45	26	19	374	96.10	2.63	1.83	1.83	1.21	1.57	1.31
502	454.00	153	457	305	270	1087	P	4.03	5.60	25	19	376	70.10	1.98	1.37	1.37	0.96	1.25	1.06
503	455.00	154	457	305	266	1087	P	4.08	5.60	26	19	385	78.50	2.20	1.52	1.52	1.07	1.39	1.19
504	456.00	153	457	305	268	1087	P	4.06	5.59	26	19	385	85.00	2.38	1.65	1.65	1.16	1.51	1.28
505	457.00	153	457	305	272	953	P	3.51	5.60	27	19	385	78.10	2.14	1.49	1.49	1.01	1.32	1.11
506	424.00	153	457	305	276	813	P	2.95	2.69	30	19	395	68.50	1.74	1.24	1.24	1.06	1.24	1.19
507	429.00	154	457	305	273	1359	P	4.98	2.69	27	19	412	63.60	1.70	1.19	1.19	1.14	1.40	1.33
508	430.00	153	457	305	271	953	P	3.52	2.76	26	19	402	66.30	1.83	1.28	1.28	1.13	1.34	1.26
509	404.00	154	457	305	276	813	P	2.95	1.78	26	19	417	55.60	1.50	1.09	1.09	1.05	1.18	1.17
510	405.00	154	457	305	275	1085	P	3.94	1.78	29	19	438	49.60	1.27	0.94	0.94	0.97	1.12	1.11

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
511	406.00	154	457	305	280	1356	P	4.85	1.75	27	19	438	48.70	1.28	0.94	0.94	1.01	1.20	1.18
512	407.00	154	457	305	275	1628	P	5.92	1.78	30	19	401	50.70	1.28	0.95	0.95	1.05	1.31	1.28
513	412.00	155	457	305	269	1359	P	5.05	1.82	27	19	502	52.30	1.43	1.03	1.03	1.11	1.34	1.30
514	415.00	153	457	305	270	953	P	3.52	1.85	26	19	403	55.20	1.53	1.09	1.09	1.10	1.26	1.24
515	417.00	152	457	305	274	953	P	3.48	1.78	27	19	392	55.40	1.50	1.09	1.09	1.10	1.25	1.24
516	418.00	155	457	305	275	953	P	3.46	1.71	27	19	386	48.50	1.28	0.95	0.95	0.95	1.08	1.08
517	4892.00	154	457	305	274	815	P	2.98	8.06	35	19	379	167.70	3.95	2.89	2.89	1.64	2.26	1.81
518	4893.00	154	457	305	271	1087	P	4.02	8.08	35	19	382	118.80	2.83	2.06	2.06	1.25	1.71	1.42
519	4894.00	154	457	305	271	1359	P	5.02	8.09	35	19	370	93.20	2.22	1.62	1.62	1.02	1.41	1.19
520	4897.00	208	457	305	274	1631	P	5.94	5.89	37	19	377	91.90	1.57	1.15	1.15	0.84	1.13	1.02
521	4903.00	154	457	305	273	1087	P	3.98	7.99	35	19	386	108.50	2.56	1.87	1.87	1.14	1.55	1.29
522	4904.00	155	457	305	274	1224	P	4.47	7.95	35	19	374	98.50	2.32	1.69	1.69	1.06	1.44	1.21
523	4905.00	155	457	305	273	953	P	3.49	7.89	35	19	372	127.90	3.01	2.20	2.20	1.31	1.77	1.46

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
524	4882.00	155	457	305	274	815	P	2.97	5.39	35	19	381	150.30	3.50	2.56	2.56	1.69	2.16	1.89
525	4882A	154	457	305	273	815	P	2.99	5.48	38	19	380	127.40	2.88	2.14	2.14	1.40	1.79	1.58
526	4883.00	155	457	305	271	1087	P	4.01	5.46	37	19	386	84.70	1.94	1.43	1.43	1.01	1.29	1.15
527	4883A	154	457	305	271	1087	P	4.02	5.49	35	19	350	85.60	2.05	1.49	1.49	1.05	1.35	1.20
528	4889.00	154	457	305	272	953	P	3.50	5.45	35	19	354	96.10	2.30	1.67	1.67	1.14	1.46	1.29
529	4890A	155	457	305	272	1224	P	4.50	5.41	35	19	369	84.50	2.01	1.46	1.46	1.05	1.36	1.22
530	4872.00	152	457	305	269	815	P	3.03	2.82	35	19	412	70.70	1.72	1.25	1.25	1.06	1.25	1.20
531	4872A	154	457	305	271	815	P	3.01	2.73	35	19	394	77.40	1.86	1.35	1.35	1.16	1.35	1.31
532	4872B	154	457	305	273	815	P	2.99	2.70	35	19	399	76.50	1.82	1.33	1.33	1.14	1.33	1.29
533	4873.00	153	457	305	270	1087	P	4.02	2.77	35	19	392	69.80	1.68	1.23	1.23	1.11	1.33	1.29
534	4873A	153	457	305	271	1087	P	4.01	2.74	34	19	404	71.40	1.73	1.26	1.26	1.15	1.37	1.33
535	4873B	154	457	305	274	1087	P	3.97	2.70	34	19	409	61.80	1.48	1.07	1.07	0.98	1.17	1.14
536	4878.00	153	457	305	272	953	P	3.51	2.75	35	19	404	60.70	1.44	1.05	1.05	0.93	1.10	1.07

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
537	4878A	154	457	305	278	884	P	3.18	2.68	37	19	363	68.90	1.56	1.16	1.16	1.01	1.18	1.15
538	4878B	155	457	305	279	884	P	3.17	2.65	37	19	366	74.30	1.67	1.23	1.23	1.08	1.26	1.24
539	4879.00	154	457	305	271	1359	P	5.01	2.73	35	19	401	64.50	1.53	1.12	1.12	1.06	1.30	1.27
540	4880B	155	457	305	271	988	P	3.64	2.71	36	19	403	65.40	1.53	1.12	1.12	1.01	1.19	1.16
541	4880B	153	457	305	273	988	P	3.63	2.74	36	19	403	64.30	1.52	1.11	1.11	0.99	1.17	1.14
542	4862.00	153	457	305	269	815	P	3.03	1.85	35	19	421	63.60	1.53	1.14	1.14	1.10	1.25	1.27
543	4863.00	154	457	305	273	1087	P	3.99	1.82	35	19	421	60.70	1.43	1.08	1.08	1.11	1.29	1.30
544	1867.00	156	457	305	274	1224	P	4.47	1.79	37	19	401	61.20	1.38	1.06	1.06	1.12	1.32	1.33
545	1868A	154	457	305	272	815	P	3.00	1.85	34	19	419	64.90	1.55	1.16	1.16	1.12	1.26	1.28
546	4868B	155	457	305	270	815	P	3.02	1.84	34	19	419	58.30	1.40	1.04	1.04	1.01	1.14	1.15
547	4869A	154	457	305	271	953	P	3.51	1.85	34	19	414	61.20	1.47	1.10	1.10	1.10	1.26	1.27
548	4869B	154	457	305	270	953	P	3.52	1.84	34	19	408	68.70	1.66	1.24	1.24	1.24	1.42	1.43
549	6044.00	152	762	305	275	815	P	2.97	9.28	29	19	354	134.30	3.53	2.50	2.50	1.35	1.94	1.46

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
550	6045.00	152	762	305	277	1359	P	4.90	9.19	29	19	354	79.40	2.07	1.47	1.47	0.88	1.25	1.00
551	6046.00	152	762	305	275	1087	P	3.96	9.27	27	19	346	98.50	2.68	1.87	1.87	1.08	1.54	1.19
552	6021.00	152	762	305	274	1087	P	3.97	4.50	27	19	372	84.10	2.27	1.59	1.59	1.21	1.52	1.35
553	6022.00	152	762	305	274	1631	P	5.94	4.44	27	19	364	62.90	1.70	1.19	1.19	0.97	1.27	1.15
554	6024.00	152	762	305	275	1087	P	3.95	4.44	26	19	374	82.30	2.28	1.59	1.59	1.21	1.52	1.34
555	6025.00	152	762	305	273	1359	P	4.98	4.48	26	19	372	67.20	1.88	1.30	1.30	1.03	1.32	1.18
556	6027.00	152	762	305	271	815	P	3.01	4.52	28	19	374	111.90	3.05	2.13	2.13	1.52	1.90	1.66
557	6030.00	152	762	305	271	1224	P	4.52	4.55	25	19	374	77.60	2.22	1.53	1.53	1.18	1.51	1.33
Hamadi and Regan (1980)																			
558	G1	100	100	400	370	1255	P	3.39	1.70	30	20	400	44.50	1.29	1.03	1.03	1.03	1.14	1.19
559	G2	100	100	400	372	1255	P	3.37	1.08	24	20	460	41.00	1.34	1.20	1.20	1.22	1.32	1.40
560	G4	100	100	400	372	2195	P	5.90	1.08	22	20	800	30.30	1.02	0.91	0.91	1.02	1.22	1.24
Nakazawa (1980)																			
561	19.00	150	150	300	270	810	P	3.00	1.47	24	19	358	60.10	1.80	1.36	1.36	1.33	1.48	1.48

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
562	20.00	150	150	300	270	945	P	3.50	1.47	26	19	358	45.40	1.30	1.00	1.00	1.01	1.14	1.14
564	21.00	150	150	300	270	1080	P	4.00	1.47	31	19	358	46.60	1.23	0.97	0.97	1.00	1.16	1.17
565	26.00	150	150	300	270	810	P	3.00	1.91	39	19	358	58.70	1.37	1.03	1.03	0.99	1.12	1.15
566	27.00	150	150	300	270	945	P	3.50	1.91	39	19	358	49.10	1.15	0.86	0.86	0.86	0.99	1.00
567	28.00	150	150	300	270	1080	P	4.00	1.91	34	19	358	50.30	1.25	0.92	0.92	0.95	1.11	1.11
Chana (1981)																			
568	2.1a	203	203	406	356	1068	P	3.00	1.69	49	20	414	96.00	1.11	0.97	0.97	0.95	1.02	1.12
569	2.1b	203	203	406	356	1068	P	3.00	1.69	49	20	414	97.10	1.13	0.98	0.98	0.96	1.03	1.13
570	2.2a	203	203	406	356	1068	P	3.00	1.69	42	10	414	87.40	1.10	0.93	0.93	1.01	1.05	1.06
571	2.2b	203	203	406	356	1068	P	3.00	1.69	42	10	414	94.40	1.19	1.01	1.01	1.09	1.13	1.15
572	2.3a	203	203	406	356	1068	P	3.00	1.69	45	20	414	99.40	1.20	1.03	1.03	1.01	1.09	1.18
573	2.3b	203	203	406	356	1068	P	3.00	1.69	45	20	414	96.40	1.17	1.00	1.00	0.98	1.06	1.15
574	3.1a	100	100	203	177	531	P	3.00	1.74	35	10	414	23.80	1.35	0.95	0.95	0.97	1.10	1.03

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
575	3.1b	100	100	203	177	531	P	3.00	1.74	35	10	414	23.90	1.35	0.96	0.96	0.97	1.10	1.03
576	3.2a	100	100	203	177	531	P	3.00	1.74	37	10	414	24.50	1.34	0.96	0.96	0.97	1.11	1.04
577	3.2b	100	100	203	177	531	P	3.00	1.74	37	10	414	25.50	1.40	1.00	1.00	1.01	1.15	1.08
578	3.3a	100	100	203	177	531	P	3.00	1.74	40	10	414	26.50	1.39	1.01	1.01	1.02	1.16	1.10
579	3.3b	100	100	203	177	531	P	3.00	1.74	40	10	414	23.20	1.22	0.88	0.88	0.90	1.02	0.96
580	D1	100	100	203	177	531	P	3.00	1.74	32	10	414	22.10	1.31	0.91	0.91	0.93	1.05	0.97
581	D2	100	100	203	177	531	P	3.00	1.74	32	10	414	23.40	1.37	0.96	0.96	0.97	1.10	1.02
582	D3	100	100	203	177	531	P	3.00	1.74	45	10	414	21.40	1.06	0.79	0.79	0.80	0.90	0.86
583	4.1a	60	60	127	106	318	P	3.00	1.72	31	5	414	9.80	1.63	1.14	1.14	1.12	1.27	1.09
584	4.1b	60	60	127	106	318	P	3.00	1.72	31	5	414	8.70	1.45	1.01	1.01	0.99	1.13	0.97
585	4.2a	60	60	127	106	318	P	3.00	1.72	31	5	414	9.00	1.50	1.05	1.05	1.03	1.17	1.00
586	4.2b	60	60	127	106	318	P	3.00	1.72	31	5	414	9.70	1.61	1.13	1.13	1.11	1.26	1.08
587	4.3a	60	60	127	106	318	P	3.00	1.72	52	5	414	11.70	1.50	1.14	1.16	1.11	1.27	1.14

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
588	4.3b	60	60	127	106	318	P	3.00	1.72	52	5	414	12.40	1.59	1.21	1.23	1.18	1.35	1.21
589	4.4a	60	60	127	106	318	P	3.00	1.72	52	5	414	9.60	1.23	0.94	0.95	0.91	1.04	0.94
590	4.4b	60	60	127	106	318	P	3.00	1.72	52	5	414	10.50	1.34	1.02	1.04	1.00	1.14	1.02
591	5.1a	200	200	203	170	510	P	3.00	1.80	40	10	414	47.80	1.30	0.94	0.94	0.94	1.07	1.01
592	5.1b	200	200	203	170	510	P	3.00	1.80	40	10	414	47.80	1.30	0.94	0.94	0.94	1.07	1.01
593	5.2a	200	200	203	170	510	P	3.00	1.80	40	10	414	55.00	1.51	1.08	1.08	1.09	1.24	1.17
594	5.2b	200	200	203	170	510	P	3.00	1.80	40	10	414	56.00	1.54	1.10	1.10	1.11	1.26	1.19
595	6.10	23	23	51	42	126	P	3.01	1.70	36	2	414	2.10	2.14	1.54	1.54	1.38	1.63	1.27
596	6.20	23	23	51	42	126	P	3.01	1.70	38	2	414	1.90	1.88	1.36	1.36	1.22	1.44	1.13
597	6.30	23	23	51	42	126	P	3.01	1.70	39	2	414	2.10	2.05	1.49	1.49	1.33	1.58	1.24
598	6.40	23	23	51	42	126	P	3.01	1.70	60	2	414	2.10	1.65	1.29	1.37	1.13	1.37	1.12
599	6.50	23	23	51	42	126	P	3.01	1.70	47	2	414	2.60	2.31	1.74	1.74	1.53	1.84	1.47
600	6.60	23	23	51	42	126	P	3.01	1.70	64	2	414	2.70	2.06	1.63	1.77	1.42	1.73	1.41

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
601	6.70	23	23	51	42	126	P	3.01	1.70	43	2	414	2.40	2.23	1.65	1.65	1.46	1.75	1.39
602	6.80	23	23	51	42	126	P	3.01	1.70	43	2	414	2.20	2.05	1.52	1.52	1.35	1.61	1.27
603	6.90	23	23	51	42	126	P	3.01	1.70	45	2	414	2.30	2.09	1.56	1.56	1.38	1.65	1.31
Batchelor and Kwun (1981)																			
604	4-C-1	152	152	248	226	686	P	3.03	0.17	47	19	1167	19.90	0.50	0.83	0.83	0.89	1.07	1.11
605	3-S-1	154	154	305	255	762	P	2.99	0.19	37	19	1641	28.90	0.71	1.13	1.13	1.20	1.38	1.46
Heger and McGrath (1982)																			
606	SW9-0A	914	914	224	184	597	P	3.24	0.62	49	19	603	167.60	0.84	0.89	0.89	0.90	1.06	1.09
607	SW9-0B	914	914	227	190	597	P	3.14	0.60	49	19	603	155.50	0.76	0.81	0.81	0.81	0.96	0.99
608	SW9M-0A	914	914	225	187	597	P	3.19	0.61	49	19	594	155.70	0.77	0.82	0.82	0.82	0.97	1.00
609	SW9M-0B	914	914	226	185	597	P	3.23	0.62	49	19	594	174.30	0.87	0.92	0.92	0.93	1.10	1.13
610	SW14-0A	914	914	227	191	597	P	3.13	0.93	49	19	673	197.20	0.95	0.88	0.88	0.87	1.01	1.05
611	SW14-0B	914	914	226	186	597	P	3.21	0.96	49	19	673	195.90	0.97	0.89	0.89	0.88	1.03	1.05
612	SW1B-0A	914	914	225	184	597	P	3.25	1.24	48	19	633	202.60	1.02	0.86	0.86	0.84	0.99	1.00

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
613	SW1B-0B	914	914	225	180	597	P	3.31	1.27	48	19	633	222.80	1.15	0.95	0.95	0.93	1.11	1.11
Mphonde and Frantz (1985)																			
614	AO-3-3b	152	152	337	298	1067	P	3.57	3.34	21	10	414	64.60	1.82	1.25	1.25	1.12	1.32	1.13
615	AO-3-3c	152	152	337	298	1067	P	3.57	2.33	28	10	414	66.80	1.65	1.18	1.18	1.22	1.36	1.27
616	AO-7-3a	152	152	337	298	1067	P	3.57	3.34	39	10	414	82.20	1.72	1.30	1.30	1.18	1.35	1.24
617	AO-7-3b	152	152	337	298	1067	P	3.57	3.34	43	10	414	82.80	1.65	1.27	1.27	1.16	1.31	1.22
618	AO-11-3a	152	152	337	298	1067	P	3.57	3.34	77	10	414	89.70	1.40	1.13	1.30	1.06	1.33	1.14
619	AO-11-3b	152	152	337	298	1067	P	3.57	3.34	77	10	414	89.40	1.40	1.13	1.30	1.06	1.33	1.14
620	AO-15-3a	152	152	337	298	1067	P	3.57	3.34	83	10	414	93.40	1.46	1.14	1.36	1.08	1.39	1.16
621	AO-15-3b	152	152	337	298	1067	P	3.57	3.34	96	10	414	100.00	1.56	1.19	1.45	1.12	1.48	1.20
622	AO-15-3c	152	152	337	298	1067	P	3.57	3.34	94	10	414	97.80	1.53	1.17	1.42	1.10	1.45	1.18
Elzanaty, Nilson, and Slate (1986)																			
623	F7	178	178	305	268	1073	P	4.00	0.60	21	13	434	33.70	0.91	0.91	0.91	1.03	1.14	1.13
624	F11	178	178	305	270	1080	P	4.00	1.19	21	13	434	45.30	1.22	0.97	0.97	1.06	1.19	1.15

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
625	F12	178	178	305	268	1073	P	4.00	2.44	21	13	434	55.70	1.51	1.01	1.01	1.01	1.18	1.07
626	F8	178	178	305	273	1092	P	4.00	0.93	40	13	434	46.40	0.89	0.86	0.86	0.95	1.07	1.09
627	F13	178	178	305	270	1080	P	4.00	1.19	40	13	434	46.60	0.90	0.80	0.80	0.88	0.99	1.00
628	F14	178	178	305	268	1073	P	4.00	2.44	40	13	434	65.70	1.28	0.95	0.95	0.95	1.10	1.07
629	F1	178	178	305	270	1080	P	4.00	1.19	66	13	434	58.70	0.89	0.85	0.93	0.96	1.12	1.12
630	F2	178	178	305	268	1073	P	4.00	2.44	66	13	434	67.60	1.03	0.83	0.91	0.85	1.01	0.97
631	F10	178	178	305	267	1067	P	4.00	3.21	66	13	434	79.00	1.21	0.97	1.07	0.89	1.08	1.02
632	F9	178	178	305	268	1073	P	4.00	1.63	79	13	434	63.60	0.94	0.79	0.92	0.88	1.14	1.02
633	F15	178	178	305	268	1073	P	4.00	2.44	79	13	434	68.60	1.02	0.79	0.92	0.81	1.07	0.94
634	F6	178	178	305	268	1610	P	6.00	2.50	63	13	434	61.50	0.95	0.76	0.83	0.82	1.03	0.99
Muruyama and Iwabuchi (1986)																			
635	NP1	200	200	250	250	775	P	3.10	3.18	47	19	343	87.30	1.49	1.12	1.12	0.91	1.09	1.06
636	NP2	200	200	250	250	775	P	3.10	3.18	47	19	343	112.80	1.93	1.45	1.45	1.18	1.40	1.37

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
637	NP3	200	200	250	250	775	P	3.10	3.18	49	19	343	95.20	1.60	1.21	1.21	0.98	1.17	1.15
638	NS1	200	200	250	250	750	P	3.00	2.57	46	19	343	93.20	1.63	1.22	1.22	1.06	1.24	1.23
639	NS2	200	200	250	250	750	P	3.00	2.57	47	19	343	91.20	1.57	1.18	1.18	1.03	1.20	1.20
640	NS3	200	200	250	250	750	P	3.00	3.18	49	19	343	86.30	1.45	1.10	1.10	0.89	1.05	1.03
641	NS4	200	200	250	250	750	P	3.00	3.18	51	19	343	82.40	1.36	1.04	1.04	0.84	0.99	0.98
642	NL1	200	200	250	250	1000	P	4.00	2.57	47	19	343	91.20	1.57	1.18	1.18	1.10	1.31	1.31
643	NL2	200	200	250	250	1000	P	4.00	2.57	44	19	343	80.40	1.43	1.06	1.06	0.99	1.18	1.17
644	NL3	200	200	250	250	1000	P	4.00	2.57	44	19	343	83.40	1.49	1.10	1.10	1.03	1.23	1.22
645	NL4	200	200	250	250	1000	P	4.00	3.18	48	19	343	89.30	1.52	1.14	1.14	0.98	1.19	1.17
646	NL5	200	200	250	250	1000	P	4.00	3.18	47	19	343	91.20	1.56	1.18	1.18	1.01	1.23	1.20
Mansur and Ong (1986)																			
647	A2	150	150	225	200	560	P	2.80	1.34	24	20	463	45.00	1.79	1.31	1.31	1.27	1.44	1.40
648	A3	150	150	225	200	720	P	3.60	1.34	24	20	463	38.50	1.53	1.12	1.12	1.16	1.34	1.29

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
649	A4	150	150	225	200	880	P	4.40	1.34	24	20	463	33.80	1.35	0.98	0.98	1.06	1.26	1.20
Ahmad and Lue (1987)																			
650	A1	127	127	254	203	813	P	4.00	3.93	61	13	586	57.80	1.69	1.26	1.35	1.05	1.30	1.21
651	A2	127	127	254	203	610	P	3.00	3.93	61	13	586	68.90	2.02	1.50	1.61	1.18	1.42	1.32
652	A3	127	127	254	203	549	P	2.70	3.93	61	13	586	68.90	2.02	1.50	1.61	1.15	1.38	1.28
653	A7	127	127	254	208	832	P	4.00	1.77	61	13	586	46.70	1.33	1.04	1.11	1.13	1.33	1.32
654	A8	127	127	254	208	624	P	3.00	1.77	61	13	586	48.90	1.40	1.09	1.17	1.11	1.26	1.26
655	B1	127	127	254	202	807	P	4.00	5.04	67	13	586	49.80	1.40	1.06	1.16	0.80	1.06	0.92
656	B2	127	127	254	202	605	P	3.00	5.04	67	13	586	68.90	1.93	1.46	1.61	1.04	1.36	1.17
657	B3	127	127	254	202	545	P	2.70	5.04	67	13	586	100.10	2.80	2.12	2.34	1.47	1.93	1.65
658	B7	127	127	254	208	832	P	4.00	2.25	67	13	586	44.50	1.21	0.92	1.02	0.95	1.18	1.11
659	B8	127	127	254	208	624	P	3.00	2.25	67	13	586	46.70	1.27	0.97	1.07	0.94	1.13	1.07
660	B9	127	127	254	208	562	P	2.70	2.25	67	13	586	80.10	2.18	1.66	1.83	1.56	1.87	1.78

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
661	C1	127	127	254	184	737	P	4.00	6.64	64	13	483	54.30	1.70	1.28	1.39	0.86	1.15	0.98
662	C2	127	127	254	184	552	P	3.00	6.64	64	13	483	75.60	2.37	1.78	1.94	1.12	1.50	1.25
663	C3	127	127	254	184	497	P	2.70	6.64	64	13	483	68.90	2.16	1.62	1.76	0.99	1.33	1.10
664	C7	127	127	254	207	826	P	4.00	3.26	64	13	483	45.40	1.27	0.96	1.04	0.86	1.06	0.99
665	C8	127	127	254	207	620	P	3.00	3.26	64	13	483	44.50	1.24	0.94	1.02	0.79	0.96	0.89
666	C9	127	127	254	207	558	P	2.70	3.26	64	13	483	45.40	1.27	0.96	1.04	0.78	0.95	0.88
Niwa, Yamada, Kokozawa, and Okamura (1987)																			
667	1.00	600	600	2100	2000	6000	P	3.00	0.28	27	25	999	402.00	0.38	0.72	0.72	0.99	0.91	1.17
668	2.00	600	600	2100	2000	6000	P	3.00	0.14	26	25	999	382.00	0.37	0.87	0.87	1.22	1.15	1.48
669	3.00	300	300	1100	1000	3000	P	3.00	0.14	25	25	999	102.00	0.40	0.86	0.86	1.01	1.01	1.26
Johnson and Ramirez (1989)																			
670	6.00	305	305	610	539	1670	P	3.10	2.49	56	19	525	191.50	0.92	0.83	0.87	0.81	0.84	0.92
Adebar and Collins (1989)																			
671	ST1	360	360	310	278	800	P	2.88	1.57	53	19	536	127.50	1.03	0.88	0.89	0.85	0.94	1.00

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
672	ST2	360	360	310	278	800	P	2.88	1.57	53	19	536	118.70	0.96	0.82	0.83	0.79	0.87	0.94
673	ST3	290	290	310	278	800	P	2.88	1.95	49	19	536	107.70	1.12	0.88	0.88	0.84	0.94	0.98
674	ST16	290	290	210	178	800	P	4.49	3.04	52	19	536	75.10	1.19	0.86	0.87	0.75	0.96	0.91
675	ST23	290	290	310	278	800	P	2.88	0.99	59	19	536	89.90	0.85	0.86	0.91	0.85	0.93	1.03
Shioya, Iguro, Nojiri, Akiyama, and Okada (1989)																			
676	S3: No. 3	300	300	655	600	1800	U2	3.00	0.43	21	10	880	104.80	0.75	0.98	0.98	1.17	1.00	1.22
677	S3: No. 4	500	500	1200	1000	3000	U2	3.00	0.40	27	10	740	228.80	0.52	0.79	0.79	1.07	0.87	1.09
678	S3: No. 5	500	500	1200	1000	3000	U2	3.00	0.40	22	25	740	251.40	0.63	0.94	0.94	1.04	0.89	1.26
679	S3: No. 6	1000	1000	2100	2000	6000	U2	3.00	0.40	29	25	370	840.00	0.46	0.79	0.79	1.07	0.86	1.26
680	S3: No. 7	1500	1500	3140	3000	9000	U2	3.00	0.41	24	25	400	1575.0 0	0.42	0.72	0.72	1.09	0.90	1.26
681	S4: No. 1	158	158	120	100	300	U2	3.00	0.41	21	10	880	12.80	1.05	1.11	1.11	1.10	1.08	1.11
682	S4: No. 2	158	158	220	200	600	U2	3.00	0.41	29	1	880	28.30	0.99	1.10	1.10	1.72	1.23	1.31
683	S4: No. 3	158	158	220	200	600	U2	3.00	0.41	22	25	880	29.40	1.18	1.25	1.25	1.28	1.25	1.46

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
684	S4: No. 4	300	300	655	600	1800	U2	3.00	0.43	27	3	880	86.40	0.54	0.74	0.74	1.13	0.83	0.94
685	S4: No. 5	300	300	655	600	1800	U2	3.00	0.43	21	25	880	132.20	0.94	1.24	1.24	1.27	1.13	1.53
686	S4: No. 6	500	500	1200	1000	3000	U2	3.00	0.40	28	5	740	186.50	0.41	0.64	0.64	1.00	0.76	0.88
Thorenfeldt and Drangshold (1990)																			
687	B11	150	150	250	221	663	P	3.00	1.82	54	16	500	58.10	1.40	1.08	1.11	1.06	1.21	1.24
688	B13	152	152	252	207	828	P	4.00	3.24	54	16	500	70.50	1.79	1.32	1.35	1.15	1.41	1.35
689	B14	153	153	253	207	621	P	3.00	3.24	54	16	500	82.60	2.09	1.53	1.57	1.25	1.50	1.44
690	B21	150	150	250	221	663	P	3.00	1.82	78	16	500	67.90	1.45	1.12	1.30	1.11	1.46	1.32
691	B23	150	150	250	207	828	P	4.00	3.24	78	16	500	77.80	1.78	1.31	1.51	1.14	1.61	1.38
692	B24	150	150	250	207	621	P	3.00	3.24	78	16	500	82.60	1.88	1.39	1.61	1.14	1.57	1.34
693	B43	150	150	250	207	828	P	4.00	3.24	86	16	500	86.20	1.97	1.40	1.68	1.23	1.79	1.48
694	B44	150	150	250	207	621	P	3.00	3.24	86	16	500	107.20	2.45	1.74	2.08	1.43	2.04	1.69
695	B51	150	150	250	221	663	P	3.00	1.82	98	16	500	56.20	1.20	0.88	1.07	0.86	1.21	1.03

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
696	B53	150	150	250	207	828	P	4.00	3.24	98	16	500	76.80	1.75	1.23	1.49	1.05	1.59	1.28
697	B54	150	150	250	207	621	P	3.00	3.24	98	16	500	77.70	1.77	1.24	1.51	1.00	1.48	1.19
698	B61	300	300	500	442	1326	P	3.00	1.82	78	16	500	179.80	0.96	0.86	1.00	0.93	1.16	1.05
699	B61R	300	300	500	442	1326	P	3.00	1.82	78	16	500	180.30	0.96	0.87	1.00	0.93	1.16	1.05
700	B63	300	300	500	414	1656	P	4.00	3.24	78	16	500	222.50	1.27	1.09	1.26	1.03	1.38	1.18
701	B63R	300	300	500	414	1656	P	4.00	3.24	78	16	500	229.40	1.31	1.13	1.30	1.06	1.42	1.21
702	B64	300	300	500	414	1242	P	3.00	3.24	78	16	500	280.70	1.60	1.38	1.60	1.21	1.61	1.36
Bažant and Kazemi (1991)																			
703	BK2.1	38	38	51	41	124	P	3.00	1.62	46	5	690	2.90	1.61	1.23	1.23	1.01	1.29	1.04
704	BK2.2	38	38	51	41	124	P	3.00	1.62	46	5	690	2.70	1.50	1.14	1.14	0.94	1.20	0.97
705	BK2.3	38	38	51	41	124	P	3.00	1.62	46	5	690	3.20	1.78	1.35	1.35	1.12	1.43	1.15
706	BK3.1	38	38	102	83	248	P	3.00	1.62	46	5	690	5.40	1.48	1.13	1.13	1.04	1.23	1.07
707	BK3.2	38	38	102	83	248	P	3.00	1.62	46	5	690	5.00	1.37	1.04	1.04	0.97	1.14	0.99

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
708	BK3.3	38	38	102	83	248	P	3.00	1.62	46	5	690	4.50	1.23	0.94	0.94	0.87	1.03	0.89
709	BK4.1	38	38	203	165	495	P	3.00	1.62	46	5	690	7.30	1.01	0.77	0.77	0.84	0.90	0.84
710	BK4.2	38	38	203	165	495	P	3.00	1.62	46	5	690	8.40	1.16	0.88	0.88	0.97	1.03	0.96
711	BK4.3	38	38	203	165	495	P	3.00	1.62	46	5	690	8.30	1.15	0.87	0.87	0.96	1.02	0.95
712	BK2.1_2	38	38	51	41	124	P	3.00	1.65	47	5	690	3.20	1.77	1.34	1.34	1.10	1.41	1.13
713	BK2.2_2	38	38	51	41	124	P	3.00	1.65	47	5	690	3.00	1.66	1.26	1.26	1.03	1.32	1.06
714	BK2.3_2	38	38	51	41	124	P	3.00	1.65	47	5	690	3.10	1.71	1.30	1.30	1.07	1.37	1.10
715	BK3.1_2	38	38	102	83	248	P	3.00	1.65	47	5	690	5.50	1.50	1.14	1.14	1.05	1.24	1.08
716	BK3.2_2	38	38	102	83	248	P	3.00	1.65	47	5	690	5.60	1.53	1.16	1.16	1.07	1.26	1.10
717	BK3.3_2	38	38	102	83	248	P	3.00	1.65	47	5	690	5.20	1.42	1.08	1.08	0.99	1.17	1.02
718	BK4.1_2	38	38	203	165	495	P	3.00	1.65	47	5	690	9.10	1.25	0.95	0.95	1.04	1.11	1.03
719	BK4.2_2	38	38	203	165	495	P	3.00	1.65	47	5	690	9.80	1.34	1.02	1.02	1.12	1.19	1.11
720	BK4.3_2	38	38	203	165	495	P	3.00	1.65	47	5	690	10.10	1.39	1.05	1.05	1.15	1.23	1.14

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Rommel (1991)																			
721	s1_2	150	150	200	165	505	P	3.06	1.87	85	16	523	48.00	1.37	0.99	1.19	0.94	1.31	1.14
722	s1_4	150	150	200	160	640	P	4.00	4.09	85	16	474	57.60	1.70	1.21	1.44	0.91	1.37	1.11
723	s1_5	150	150	200	160	490	P	3.06	4.09	85	16	474	60.20	1.78	1.26	1.50	0.90	1.33	1.07
Xie, Ahmad, Yu, Hino, and Chung (1994)																			
724	NNN-3	127	127	254	216	648	P	3.00	2.07	40	19	421	36.70	1.25	0.88	0.88	0.84	0.98	0.96
725	NHN-3	127	127	254	216	648	P	3.00	2.07	104	19	421	45.70	1.18	0.84	1.02	0.76	1.13	0.94
Walraven and Lehwalter (1994)																			
726	A1	200	200	150	125	375	P	3.00	0.83	23	16	440	31.50	1.55	1.31	1.31	1.26	1.44	1.33
727	A2	200	200	450	420	1260	P	3.00	0.74	23	16	440	70.60	1.03	1.08	1.08	1.12	1.16	1.25
728	A3	200	200	750	720	2160	P	3.00	0.79	23	16	440	100.80	0.85	0.97	0.97	1.07	1.06	1.18
Scholtz (1994)																			
729	A-2	200	200	400	372	1116	P	3.00	0.81	81	16	500	83.00	0.79	0.89	1.04	0.96	1.19	1.13
730	D-2	200	200	400	362	1086	P	3.00	1.94	97	16	500	121.00	1.18	0.95	1.16	0.98	1.32	1.13

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
731	D-3	200	200	400	362	1448	P	4.00	1.94	97	16	500	121.00	1.18	0.95	1.16	1.05	1.45	1.23
Kim and Park (1994)																			
732	CTL-1	170	170	300	270	810	P	3.00	1.87	54	25	477	71.10	1.24	1.00	1.02	0.93	1.06	1.14
733	CTL-2	170	170	300	270	810	P	3.00	1.87	54	25	477	71.60	1.25	1.00	1.03	0.93	1.07	1.15
734	P1.0-1	170	170	300	272	816	P	3.00	1.01	54	25	477	58.30	1.01	1.00	1.02	0.95	1.07	1.19
735	P1.0-2	170	170	300	272	816	P	3.00	1.01	54	25	477	56.40	0.98	0.96	0.99	0.92	1.04	1.15
736	P3.4-1	170	170	300	267	801	P	3.00	3.35	54	25	477	78.10	1.38	1.08	1.10	0.82	0.99	1.00
737	P3.4-2	170	170	300	267	801	P	3.00	3.35	54	25	477	78.50	1.39	1.08	1.11	0.83	0.99	1.00
738	P4.6-1	170	170	300	255	765	P	3.00	4.68	54	25	477	89.70	1.66	1.28	1.31	0.86	1.08	1.04
739	P4.6-2	170	170	300	255	765	P	3.00	4.68	54	25	477	95.40	1.77	1.36	1.40	0.92	1.15	1.10
740	A4.5-1	170	170	300	270	1215	P	4.50	1.87	54	25	477	66.60	1.16	0.93	0.95	0.95	1.15	1.20
741	A4.5-2	170	170	300	270	1215	P	4.50	1.87	54	25	477	63.80	1.12	0.89	0.91	0.91	1.10	1.15
742	A6.0-1	170	170	300	270	1620	P	6.00	1.87	54	25	477	59.20	1.04	0.83	0.85	0.88	1.13	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
743	A6.0-2	170	170	300	270	1620	P	6.00	1.87	54	25	477	61.00	1.07	0.85	0.87	0.91	1.17	1.20
744	D142-1	170	170	170	142	426	P	3.00	1.87	54	25	477	41.00	1.36	1.02	1.04	0.88	1.10	1.07
745	D142-2	170	170	170	142	426	P	3.00	1.87	54	25	477	39.30	1.31	0.97	1.00	0.84	1.05	1.03
746	D550-1	300	300	620	550	1650	P	3.00	1.88	54	25	477	226.10	1.10	1.02	1.04	1.02	1.05	1.22
747	D550-2	300	300	620	550	1650	P	3.00	1.88	54	25	477	214.50	1.04	0.97	0.99	0.96	0.99	1.16
748	D915-1	300	300	1000	915	2745	P	3.00	1.87	54	25	477	271.80	0.79	0.81	0.83	0.89	0.86	1.04
749	D915-2	300	300	1000	915	2745	P	3.00	1.87	54	25	477	332.10	0.97	0.99	1.01	1.09	1.05	1.27
Ahmad, Park, and El-Dash (1995)																			
750	B7N	102	102	204	178	659	P	3.70	1.40	44	13	413	26.69	1.31	1.04	1.04	1.08	1.27	1.23
751	B7H	102	102	204	178	659	P	3.70	1.40	78	13	413	26.87	1.05	0.86	1.00	0.90	1.20	1.07
752	B8H	102	102	204	178	659	P	3.70	1.40	81	13	413	26.87	1.05	0.85	1.00	0.89	1.20	1.06
Matsui, Kurihara, Uchida, Rokugo, and Koyangi (1995)																			
753	A1	100	100	180	150	450	P	3.00	2.65	32	19	367	27.50	1.89	1.27	1.27	1.05	1.29	1.16
754	A2	100	100	180	150	450	P	3.00	2.65	32	19	367	31.90	2.20	1.47	1.47	1.21	1.49	1.34

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
755	B1	100	100	180	150	450	P	3.00	2.65	38	19	367	29.30	1.85	1.28	1.28	1.04	1.29	1.18
756	B2	100	100	180	150	450	P	3.00	2.65	38	19	367	30.70	1.94	1.34	1.34	1.09	1.35	1.24
757	C1	100	100	180	150	450	P	3.00	2.65	49	19	367	29.60	1.66	1.19	1.19	0.97	1.20	1.13
758	C2	100	100	180	150	450	P	3.00	2.65	49	19	367	32.30	1.82	1.30	1.30	1.05	1.31	1.23
759	D1	100	100	180	150	450	P	3.00	2.65	71	19	367	33.40	1.58	1.19	1.33	0.95	1.31	1.16
760	D2	100	100	180	150	450	P	3.00	2.65	71	19	367	33.90	1.60	1.20	1.35	0.97	1.33	1.17
761	E1	100	100	180	150	450	P	3.00	2.65	83	19	367	38.30	1.81	1.29	1.53	1.03	1.51	1.27
762	E2	100	100	180	150	450	P	3.00	2.65	83	19	367	42.50	2.01	1.43	1.70	1.15	1.67	1.41
763	F1	100	100	180	150	450	P	3.00	2.65	128	19	367	34.40	1.62	1.13	1.37	0.81	1.35	1.03
764	F2	100	100	180	150	450	P	3.00	2.65	128	19	367	48.10	2.27	1.58	1.92	1.13	1.89	1.44
765	L1	100	100	260	225	675	P	3.00	2.55	125	19	403	69.70	2.19	1.57	1.91	1.26	2.00	1.56
766	L2	100	100	260	225	675	P	3.00	2.55	125	19	403	42.50	1.34	0.96	1.16	0.77	1.22	0.95
767	M1	100	100	260	225	788	P	3.50	2.55	125	19	403	43.40	1.37	0.98	1.19	0.81	1.30	1.02

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
768	M2	100	100	260	225	788	P	3.50	2.55	125	19	403	43.80	1.38	0.99	1.20	0.82	1.32	1.03
769	N1	100	100	260	225	900	P	4.00	2.55	125	19	403	41.30	1.30	0.93	1.13	0.80	1.30	1.01
770	N2	100	100	260	225	900	P	4.00	2.55	125	19	403	38.80	1.22	0.87	1.06	0.75	1.22	0.95
771	S1	100	100	360	360	1080	P	3.00	2.58	128	19	336	54.00	1.06	0.85	1.03	0.71	1.09	0.84
772	S2	100	100	360	360	1080	P	3.00	2.58	128	19	336	83.10	1.64	1.30	1.58	1.10	1.67	1.30
Yoon, Cook, and Mitchell (1996)																			
773	N1-S	375	375	750	655	2113	P	3.23	2.85	36	20	400	248.80	0.99	0.87	0.87	0.81	0.87	0.91
774	M1-S	375	375	750	655	2113	P	3.23	2.85	67	10	400	295.80	0.87	0.84	0.93	0.94	1.01	0.93
775	H1-S	375	375	750	655	2113	P	3.23	2.85	87	10	400	326.80	0.94	0.85	1.03	0.98	1.19	0.96
Konig, Grimm, and Rimmel (1997)																			
776	s1.1	300	300	200	153	570	P	3.73	1.34	90	16	660	70.10	1.08	0.86	1.04	0.84	1.23	1.05
777	s4.1	300	300	200	153	570	P	3.73	1.34	111	16	660	74.20	1.15	0.91	1.11	0.83	1.30	1.06
778	s2.3	300	300	400	348	1230	P	3.53	0.94	94	16	469	123.10	0.84	0.85	1.03	0.94	1.25	1.11
779	s3.3	300	300	800	746	2630	P	3.53	0.83	94	16	487	192.80	0.61	0.75	0.91	0.97	1.21	1.07

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
780	s1.2	300	300	200	152	570	P	3.75	2.21	91	16	517	75.80	1.18	0.82	0.99	0.75	1.12	0.93
781	s4.2	300	300	200	152	570	P	3.75	2.21	111	16	517	90.30	1.40	0.97	1.19	0.84	1.34	1.06
782	s2.2	300	300	400	348	1230	P	3.53	1.88	91	16	469	187.10	1.27	1.02	1.25	1.11	1.49	1.29
783	s3.2	300	300	800	718	2556	P	3.56	1.72	94	16	487	259.10	0.85	0.81	0.99	1.01	1.32	1.11
784	s1.3	300	300	200	146	570	P	3.90	4.22	94	16	487	98.60	1.59	1.11	1.35	0.79	1.25	0.97
785	s4.3	300	300	200	146	570	P	3.90	4.22	111	16	487	122.30	1.98	1.37	1.67	0.93	1.55	1.16
786	s2.4	300	300	400	328	1230	P	3.75	3.76	94	16	487	229.80	1.65	1.29	1.57	1.09	1.58	1.27
787	s3.4	300	300	800	690	2630	P	3.81	3.57	94	16	487	379.00	1.30	1.17	1.42	1.17	1.65	1.27
Kawano and Watanabe (1997)																			
788	A1A	105	105	330	300	900	P	3.00	1.26	25	20	400	33.50	1.26	1.03	1.03	1.01	1.11	1.14
789	A1B	105	105	330	300	900	P	3.00	1.26	25	20	400	29.50	1.11	0.91	0.91	0.89	0.97	1.00
790	A2A	176	176	570	500	1500	P	3.00	1.36	27	20	400	82.50	1.06	0.96	0.96	0.96	1.01	1.10
791	A2B	176	176	570	500	1500	P	3.00	1.36	27	20	400	101.50	1.30	1.18	1.18	1.18	1.24	1.35

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
792	A3A	350	350	1050	950	2850	P	3.00	1.22	21	20	400	216.00	0.84	0.84	0.84	0.92	0.94	1.04
793	A3B	350	350	1050	950	2850	P	3.00	1.22	21	20	400	237.50	0.93	0.93	0.93	1.01	1.04	1.14
794	A4A	600	600	2200	2000	6000	P	3.00	1.20	22	40	400	610.50	0.64	0.72	0.72	0.83	0.91	1.05
795	A4B	600	600	2200	2000	6000	P	3.00	1.20	23	40	400	560.00	0.57	0.65	0.65	0.75	0.83	0.95
Kulkarni and Shah (1998)																			
796	B4JL20-S	102	102	178	152	760	P	5.00	1.38	41	10	518	19.50	1.16	0.91	0.91	1.00	1.25	1.14
797	B3NO15-S	102	102	178	152	608	P	4.00	1.38	42	10	518	22.70	1.34	1.05	1.05	1.11	1.32	1.23
798	B3NO30-S	102	102	178	152	532	P	3.50	1.38	44	10	518	24.20	1.39	1.11	1.11	1.13	1.32	1.25
Collins and Kuchma (1999)																			
799	BH25	300	300	250	225	664	P	2.95	0.89	99	10	437	85.00	0.89	0.84	1.02	0.90	1.15	1.02
800	BH50	300	300	500	450	1313	P	2.92	0.81	99	10	486	131.90	0.69	0.78	0.95	0.94	1.09	0.98
801	BH100	300	300	1000	925	2663	P	2.88	0.76	99	10	550	193.10	0.49	0.65	0.78	0.93	1.03	0.89
802	BN12.5	300	300	125	110	326	P	2.96	0.91	37	10	458	40.10	1.18	1.05	1.05	0.98	1.15	1.06
803	BN25	300	300	250	225	664	P	2.95	0.89	37	10	437	72.90	1.04	0.96	0.96	1.03	1.11	1.11

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
804	BN50	300	300	500	450	1313	P	2.92	0.81	37	10	486	131.60	0.94	1.05	1.05	1.19	1.17	1.25
805	BN100	300	300	1000	925	2663	P	2.88	0.76	37	10	550	191.70	0.67	0.86	0.86	1.13	1.04	1.13
806	SE50A-45	169	169	500	445	1200	P	2.70	1.03	53	10	480	77.40	0.83	0.90	0.92	1.02	0.98	1.07
807	SE50A-45R	169	169	500	445	1200	P	2.70	1.03	53	10	480	91.00	0.98	1.06	1.08	1.20	1.15	1.26
808	SE50A-83	169	169	500	445	1200	P	2.70	1.03	86	10	480	82.50	0.78	0.82	0.98	0.97	1.09	1.01
809	B100	300	300	1000	925	2700	P	2.92	1.01	36	10	550	225.00	0.79	0.93	0.93	1.21	1.13	1.20
810	B100R	300	300	1000	925	2700	P	2.92	1.01	36	10	550	249.00	0.88	1.03	1.03	1.34	1.25	1.33
811	B100H	300	300	1000	925	2700	P	2.92	1.01	98	10	550	193.00	0.49	0.59	0.71	0.84	0.94	0.80
812	B100B	300	300	1000	925	2700	P	2.92	1.01	39	10	550	204.00	0.69	0.82	0.82	1.08	0.99	1.06
813	B100HE	300	300	1000	925	2700	P	2.92	1.01	98	10	550	217.00	0.55	0.66	0.80	0.94	1.06	0.90
814	B100L	300	300	1000	925	2700	P	2.92	1.01	39	10	550	223.00	0.76	0.90	0.90	1.18	1.09	1.16
815	B100L-R	300	300	1000	925	2700	P	2.92	1.01	39	10	550	235.00	0.80	0.94	0.94	1.24	1.14	1.23
816	BRL100	300	300	1000	925	2700	P	2.92	0.51	94	10	550	163.00	0.42	0.62	0.76	0.92	1.00	0.90

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Kim, Kim, and White (1999)																			
817	3CNB	127	127	229	191	572	P	3.00	1.65	38	13	461	29.40	1.16	0.85	0.85	0.85	0.97	0.94
818	4CNB	127	127	229	191	762	P	4.00	1.65	35	13	461	30.30	1.24	0.90	0.90	0.96	1.14	1.08
Yoshida (2000)																			
819	YB2000/0	300	300	2000	1890	5405	P	2.86	0.74	34	10	455	257.70	0.46	0.65	0.65	1.05	0.96	0.99
Angelakos, Bentz, and Collins (2001)																			
820	DB130	300	300	1000	925	2700	P	2.92	1.01	32	10	550	185.00	0.69	0.79	0.79	1.02	0.96	1.01
821	DB140	300	300	1000	925	2700	P	2.92	1.01	38	10	550	180.00	0.62	0.73	0.73	0.95	0.88	0.95
822	DB165	300	300	1000	925	2700	P	2.92	1.01	65	10	550	185.00	0.49	0.63	0.68	0.87	0.82	0.85
823	DB180	300	300	1000	925	2700	P	2.92	1.01	80	10	550	172.00	0.44	0.54	0.64	0.78	0.84	0.75
824	DB230	300	300	1000	890	2700	P	3.03	2.10	32	10	550	257.00	1.00	0.91	0.91	1.11	1.13	1.09
825	DB0.530	300	300	1000	925	2700	P	2.92	0.50	32	10	550	165.00	0.62	0.89	0.89	1.18	1.09	1.20
Cao (2001)																			
826	SB2003/0	300	300	2000	1925	5400	P	2.81	0.36	31	10	470	227.20	0.42	0.74	0.74	1.21	1.08	1.17

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
827	SB2012/0	300	300	2000	1845	5400	P	2.93	1.52	28	10	550	407.00	0.82	0.89	0.89	1.34	1.37	1.26
Yost, Gross, and Dinehart (2001)																			
828	1Steel1	229	229	286	227	914	P	4.03	1.54	36	25	600	60.70	1.14	0.88	0.88	0.88	1.06	1.07
829	1Steelb	229	229	286	227	914	P	4.03	1.54	36	25	600	56.30	1.06	0.81	0.81	0.82	0.98	0.99
830	1Steelc	229	229	286	227	914	P	4.03	1.54	36	25	600	58.00	1.09	0.84	0.84	0.84	1.01	1.02
Tureyen and Frosch (2002)																			
831	V-S-1	457	457	427	360	1219	P	3.39	0.96	41	19	524	179.20	1.00	1.02	1.02	1.06	1.14	1.25
832	V-S-2	457	457	427	360	1219	P	3.39	1.92	41	19	524	203.70	1.13	0.92	0.92	0.92	1.02	1.08
833	V-D-2	457	457	427	360	1219	P	3.39	0.36	44	19	745	134.80	0.73	1.04	1.04	1.12	1.24	1.37
Tompos and Frosch (2002)																			
834	V18-0	229	229	484	425	1276	P	3.00	1.00	36	19	910	95.20	0.96	0.98	0.98	0.99	1.03	1.15
Fujita, Sato, Matsumoto and Takaki (2002)																			
835	L-25-3	150	150	300	250	750	P	3.00	1.53	38	20	388	43.13	1.10	0.87	0.87	0.84	0.95	0.98
836	L-25-3	150	150	300	250	750	P	3.00	1.53	36	20	388	55.50	1.44	1.14	1.14	1.10	1.24	1.27

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
837	L-25-3	150	150	300	250	750	P	3.00	1.53	36	20	388	56.25	1.46	1.15	1.15	1.11	1.26	1.29
838	L-50-3	150	150	600	500	1500	P	3.00	1.53	37	20	388	116.25	1.49	1.37	1.37	1.38	1.44	1.60
839	L-50-3	150	150	600	500	1500	P	3.00	1.53	36	20	388	78.75	1.02	0.94	0.94	0.94	0.98	1.09
840	L-50-3	150	150	600	500	1500	P	3.00	1.53	36	20	388	87.00	1.13	1.03	1.03	1.04	1.09	1.21
841	L-100-3	350	350	1130	1000	3000	P	3.00	1.36	36	20	388	318.50	0.90	0.96	0.96	1.10	1.07	1.24
842	L-100-3	350	350	1130	1000	3000	P	3.00	1.36	35	20	388	297.50	0.85	0.90	0.90	1.03	1.01	1.16
843	L-100-3	350	350	1130	1000	3000	P	3.00	1.36	34	20	388	332.50	0.96	1.02	1.02	1.16	1.14	1.31
844	M-25-3	150	150	300	250	750	P	3.00	1.53	69	20	388	51.38	0.97	0.85	0.95	0.82	1.03	1.01
845	M-25-3	150	150	300	250	750	P	3.00	1.53	52	20	388	65.25	1.42	1.19	1.20	1.15	1.30	1.37
846	M-25-3	150	150	300	250	750	P	3.00	1.53	52	20	388	53.63	1.17	0.98	0.99	0.94	1.07	1.13
847	M-50-3	150	150	600	500	1500	P	3.00	1.53	69	20	388	87.00	0.82	0.83	0.93	0.87	1.06	1.03
848	M-50-3	150	150	600	500	1500	P	3.00	1.53	52	20	388	108.00	1.18	1.14	1.15	1.17	1.20	1.37
849	M-50-3	150	150	600	500	1500	P	3.00	1.53	52	20	388	113.25	1.23	1.20	1.21	1.23	1.26	1.44

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
850	M-100-3	350	350	1130	1000	3000	P	3.00	1.36	59	20	388	315.00	0.69	0.80	0.85	0.97	0.89	1.08
851	M-100-3	350	350	1130	1000	3000	P	3.00	1.36	54	20	388	339.50	0.78	0.89	0.91	1.06	0.99	1.19
852	M-100-3	350	350	1130	1000	3000	P	3.00	1.36	53	20	388	339.50	0.78	0.89	0.91	1.07	1.00	1.20
853	U-25-3	150	150	300	250	750	P	3.00	1.53	101	20	388	40.13	0.76	0.61	0.74	0.57	0.83	0.71
854	U-25-3	150	150	300	250	750	P	3.00	1.53	93	20	388	56.25	1.06	0.85	1.04	0.83	1.16	1.02
855	U-25-3	150	150	300	250	750	P	3.00	1.53	93	20	388	46.88	0.89	0.71	0.86	0.69	0.97	0.85
856	U-50-3	150	150	600	500	1500	P	3.00	1.53	102	20	388	86.25	0.81	0.76	0.92	0.79	1.08	0.92
857	U-50-3	150	150	600	500	1500	P	3.00	1.53	93	20	388	82.50	0.78	0.72	0.88	0.77	1.03	0.90
858	U-50-3	150	150	600	500	1500	P	3.00	1.53	93	20	388	97.50	0.92	0.86	1.04	0.91	1.22	1.07
859	U-100-3	350	350	1130	1000	3000	P	3.00	1.36	103	20	388	280.00	0.57	0.62	0.75	0.76	1.03	0.83
860	U-100-3	350	350	1130	1000	3000	P	3.00	1.36	90	20	388	262.50	0.53	0.58	0.71	0.74	0.97	0.81
861	U-100-3	350	350	1130	1000	3000	P	3.00	1.36	92	20	388	280.00	0.57	0.62	0.75	0.78	1.03	0.86
862	L-50-4	150	150	600	500	2000	P	4.00	1.53	38	20	388	70.50	0.90	0.83	0.83	0.89	0.96	1.06
863	L-50-4	150	150	600	500	2000	P	4.00	1.53	36	20	388	84.75	1.10	1.01	1.01	1.08	1.17	1.28

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
864	L-50-4	150	150	600	500	2000	P	4.00	1.53	36	20	388	88.50	1.15	1.05	1.05	1.13	1.22	1.34
865	M-50-4	150	150	600	500	2000	P	4.00	1.53	70	20	388	81.75	0.77	0.78	0.87	0.88	1.12	1.05
866	M-50-4	150	150	600	500	2000	P	4.00	1.53	52	20	388	90.75	0.99	0.96	0.97	1.05	1.11	1.25
867	U-50-4	150	150	600	500	2000	P	4.00	1.53	101	20	388	72.75	0.69	0.64	0.78	0.71	1.00	0.85
868	U-50-4	150	150	600	500	2000	P	4.00	1.53	93	20	388	78.75	0.74	0.69	0.84	0.79	1.09	0.94
869	U-50-4	150	150	600	500	2000	P	4.00	1.53	93	20	388	84.00	0.79	0.74	0.90	0.84	1.16	1.00
Tariq and Newhook (2003)																			
870	R-S007Na	160	160	400	346	950	P	2.75	0.72	37	20	483	58.90	1.02	1.12	1.12	1.10	1.16	1.30
871	R-S007Nb	160	160	400	346	950	P	2.75	0.72	37	20	483	63.30	1.10	1.21	1.21	1.18	1.25	1.40
872	R-S010N1	160	160	400	346	1150	P	3.32	1.08	43	20	483	66.70	1.08	1.06	1.06	1.07	1.17	1.28
873	R-S010N2	160	160	400	346	1150	P	3.32	1.08	43	20	483	62.20	1.01	0.98	0.98	1.00	1.09	1.20
874	R-S015N1	160	160	400	325	1150	P	3.54	1.54	34	20	483	69.80	1.35	1.12	1.12	1.13	1.26	1.32
875	R-S015N2	160	160	400	325	1150	P	3.54	1.54	34	20	483	70.50	1.37	1.13	1.13	1.14	1.27	1.33

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Tariq and Newhook (2003)																			
876	AT-1	2010	2010	1005	920	2700	P	2.93	0.76	64	10	460	1272.1 0	0.51	0.71	0.78	1.00	0.92	0.99
Higgins, Miller, Rosowsky, Yim, Potisuk, Daniels, Nicholas, Robelo, Lee, and Forrest (2004)																			
877	37T	356	356	1219	1151	3353	P	2.91	0.74	32	19	478	243.80	0.62	0.81	0.81	0.98	0.93	1.10
Cladera and Mari (2005)																			
878	H50/1	200	200	400	359	1080	P	3.01	2.24	50	12	500	99.70	1.16	0.95	0.95	0.97	1.02	1.04
879	H60/1	200	200	400	359	1080	P	3.01	2.24	61	12	500	108.10	1.14	0.97	1.03	0.99	1.04	1.08
880	H75/1	200	200	400	359	1080	P	3.01	2.24	69	12	500	99.90	0.99	0.86	0.95	0.89	1.03	0.96
881	H100/1	200	200	400	359	1080	P	3.01	2.24	87	12	500	117.90	1.16	0.94	1.13	0.99	1.24	1.07
Bentz and Buckley (2005)																			
882	SBB1.1	104	104	103	84	248	P	2.95	1.63	33	10	494	14.50	1.70	1.22	1.22	1.08	1.31	1.12
883	SBB1.2	105	105	103	84	248	P	2.95	1.62	33	10	494	18.50	2.15	1.55	1.55	1.37	1.66	1.42
884	SBB1.3	104	104	103	84	248	P	2.95	1.63	33	10	494	15.00	1.76	1.26	1.26	1.12	1.35	1.16
885	SBB2.1	106	106	206	168	495	P	2.95	1.60	30	10	494	28.80	1.74	1.24	1.24	1.24	1.41	1.30

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
886	SBB2.2	105	105	206	168	495	P	2.95	1.62	30	10	494	30.50	1.86	1.32	1.32	1.32	1.50	1.38
887	SBB2.3	106	106	206	166	495	P	2.98	1.62	30	10	494	29.70	1.81	1.28	1.28	1.29	1.47	1.35
888	SBB3.1	105	105	378	333	990	P	2.97	1.55	34	10	490	42.20	1.22	1.01	1.01	1.08	1.13	1.13
889	SBB3.2	101	101	378	333	990	P	2.97	1.61	34	10	490	40.60	1.22	1.00	1.00	1.06	1.12	1.11
890	SBB3.3	101	101	378	333	990	P	2.97	1.61	34	10	490	42.90	1.29	1.05	1.05	1.12	1.18	1.18
Sherwood, Lubell, Bentz, and Collins (2006)																			
891	AT-2-250A	250	250	470	437	1295	P	2.96	0.92	38	10	465	115.80	1.02	1.08	1.08	1.22	1.21	1.28
892	AT-2-250B	250	250	470	440	1295	P	2.94	0.90	39	10	465	113.30	0.98	1.05	1.05	1.19	1.17	1.25
893	AT-2-1000A	1000	1000	470	440	1295	P	2.94	0.91	39	10	465	476.00	1.02	1.09	1.09	1.24	1.22	1.30
894	AT-2-1000B	1000	1000	470	437	1295	P	2.96	0.91	38	10	465	444.80	0.97	1.04	1.04	1.18	1.16	1.24
895	AT-2-3000	3000	3000	472	440	1295	P	2.94	0.91	41	10	465	1295.3 0	0.91	0.98	0.98	1.11	1.09	1.17
896	AT-3-A	696	696	338	307	1039	P	3.38	0.93	38	10	448	239.30	1.08	1.05	1.05	1.18	1.25	1.28
897	AT-3-C	706	706	338	305	1039	P	3.41	0.93	37	10	448	260.20	1.17	1.14	1.14	1.28	1.35	1.38

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
898	AT-3-B	701	701	338	305	1039	P	3.41	0.93	38	10	448	254.80	1.14	1.12	1.12	1.26	1.32	1.36
890	AT-3-D	706	706	338	307	1039	P	3.38	0.93	37	10	448	250.20	1.11	1.09	1.09	1.22	1.29	1.32
Lubell (2006)																			
900	AW1	1170	1170	590	538	1850	P	3.44	0.79	37	10	467	585.00	0.90	1.04	1.04	1.27	1.25	1.33
901	AW4	1168	1168	590	506	1850	P	3.66	1.69	40	10	467	716.00	1.13	1.02	1.02	1.21	1.23	1.26
902	AW8	1169	1169	591	507	1850	P	3.65	1.69	39	10	467	789.00	1.25	1.12	1.12	1.33	1.36	1.39
903	AX6	703	703	338	288	1040	P	3.61	1.73	41	10	467	281.00	1.28	1.02	1.02	1.13	1.22	1.21
904	AX7	704	704	335	287	1040	P	3.62	1.04	41	10	413	249.00	1.13	1.07	1.07	1.21	1.30	1.32
905	AX8	705	705	339	289	1040	P	3.60	1.72	41	10	467	272.00	1.23	0.98	0.98	1.09	1.18	1.17
906	AY1	249	249	467	434	1300	P	3.00	0.33	41	10	900	85.00	0.73	1.09	1.09	1.30	1.29	1.41
Guadagnini, Pilakoutas, and Waldron (2006)																			
907	SB40	150	150	250	223	750	P	3.36	1.35	43	20	500	45.30	1.21	0.99	0.99	0.99	1.15	1.18
Sherwood, Bentz, and Collins (2007)																			
908	L-10N1	300	300	1510	1400	4050	P	2.89	0.83	38	10	452	265.00	0.60	0.80	0.80	1.19	1.07	1.14

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
909	L-10N2	300	300	1510	1400	4050	P	2.89	0.83	40	10	452	242.00	0.53	0.72	0.72	1.07	0.96	1.03
910	L-10H	300	300	1510	1400	4050	P	2.89	0.83	74	10	452	240.00	0.40	0.58	0.67	0.94	1.00	0.88
911	L-20N1	300	300	1510	1400	4050	P	2.89	0.83	31	20	452	265.00	0.66	0.86	0.86	1.07	1.01	1.20
912	L-20N2	300	300	1510	1400	4050	P	2.89	0.83	33	20	452	266.00	0.65	0.85	0.85	1.06	0.99	1.19
913	L-40N1	300	300	1510	1400	4050	P	2.89	0.83	28	40	452	242.00	0.64	0.81	0.81	0.88	0.91	1.13
914	L-40N2	300	300	1510	1400	4050	P	2.89	0.83	29	40	452	288.00	0.76	0.96	0.96	1.04	1.08	1.33
915	L-50N1	300	300	1510	1400	4050	P	2.89	0.83	41	50	452	272.00	0.59	0.81	0.81	0.86	0.90	1.15
916	L-50N2	300	300	1510	1400	4050	P	2.89	0.83	40	50	452	298.00	0.66	0.89	0.89	0.95	0.99	1.27
917	L-50N2R	300	300	1510	1400	4050	P	2.89	0.83	40	50	452	323.00	0.71	0.96	0.96	1.03	1.08	1.37
918	S-10N1	122	122	330	280	810	P	2.89	0.83	42	10	494	36.60	0.97	0.99	0.99	1.07	1.11	1.16
919	S-10N2	122	122	330	280	810	P	2.89	0.83	42	10	494	38.30	1.02	1.03	1.03	1.12	1.16	1.21
920	S-10H	122	122	330	280	810	P	2.89	0.83	77	10	494	37.70	0.78	0.83	0.96	0.93	1.07	1.02
921	S-20N1	122	122	330	280	810	P	2.89	0.83	39	20	494	39.10	1.08	1.08	1.08	1.06	1.16	1.26

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
922	S-20N2	122	122	330	280	810	P	2.89	0.83	38	20	494	38.20	1.07	1.06	1.06	1.05	1.14	1.24
923	S-40N1	122	122	330	280	810	P	2.89	0.83	29	40	494	41.80	1.33	1.27	1.27	1.17	1.34	1.45
924	S-40N2	122	122	330	280	810	P	2.89	0.83	29	40	494	34.90	1.11	1.06	1.06	0.97	1.12	1.21
925	S-50N1	122	122	330	280	810	P	2.89	0.83	44	50	494	38.50	1.01	1.03	1.03	0.91	1.09	1.21
926	S-50N2	122	122	330	280	810	P	2.89	0.83	44	50	494	40.60	1.06	1.08	1.08	0.96	1.15	1.27
Hassan, Seliem, Dwairi, Rizkalla, and Zia (2008)																			
927	G-2.7-32	460	460	915	865	2335	P	2.70	0.72	32	19	420	261.00	0.68	0.86	0.86	0.96	0.92	1.09
929	M-2.7-32	460	460	915	865	2335	P	2.70	0.44	32	19	690	319.00	0.83	1.25	1.25	1.41	1.34	1.63
Lubell, Bentz, and Collins (2009)																			
930	AW4	1168	1168	590	506	1852	P	3.66	1.69	40	9	467	725.00	1.14	1.03	1.03	1.25	1.26	1.28
931	AW8	1169	1169	591	507	1851	P	3.65	1.69	39	9	467	800.00	1.26	1.14	1.14	1.38	1.39	1.41
Sneed and Ramirez (2010)																			
932	1-1	305	305	306	232	698	P	3.01	1.20	66	10	431	131.00	1.34	1.24	1.36	1.35	1.50	1.49
934	1-2	306	306	611	530	1579	P	2.98	1.25	66	10	453	140.00	0.62	0.68	0.75	0.84	0.84	0.85

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
935	1-3	305	305	762	681	2029	P	2.98	1.24	65	10	474	148.00	0.52	0.59	0.65	0.77	0.74	0.76
936	1-4	306	306	914	822	2466	P	3.00	1.30	75	10	475	168.00	0.47	0.54	0.62	0.75	0.80	0.72
937	2-1	203	203	306	233	697	P	2.99	1.26	69	10	483	57.00	0.86	0.79	0.87	0.86	0.97	0.94
938	2-2	408	408	608	529	1582	P	2.99	1.20	65	10	474	156.00	0.53	0.58	0.64	0.71	0.70	0.73
939	2-3	508	508	762	684	2031	P	2.97	1.30	68	10	474	262.00	0.54	0.61	0.68	0.79	0.80	0.78
940	2-4	613	613	914	822	2466	P	3.00	1.30	73	10	475	353.00	0.50	0.57	0.65	0.79	0.84	0.76
Munikrishna, Hosny, Rizkalla, and Zia (2011)																			
941	G1-M0	610	610	711	645	2000	P	3.10	0.93	28	10	689	226.95	0.64	0.69	0.69	0.84	0.82	0.85
942	G2-M0	610	610	711	645	2000	P	3.10	0.93	28	10	689	333.75	0.94	1.02	1.02	1.23	1.20	1.25
943	G3-C0	406	406	559	457	1372	P	3.00	3.10	28	10	413	275.90	1.65	1.30	1.30	1.21	1.34	1.21
944	G3-M0	406	406	559	457	1372	P	3.00	2.22	28	10	689	280.35	1.68	1.32	1.32	1.39	1.48	1.41
Lee, Choi, and Kim (2011)																			
945	S30-1	300	300	600	520	2080	P	4.00	2.92	33	25	530	190.00	1.24	1.03	1.03	0.92	1.04	1.10
946	S30-2	300	300	600	520	2080	P	4.00	2.92	33	25	530	204.80	1.34	1.11	1.11	0.99	1.12	1.19

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
947	S40-1	300	300	450	369	1018	P	2.76	4.65	38	25	554	175.80	1.51	1.20	1.20	0.80	0.98	0.94
948	S50-1	300	300	450	369	1018	P	2.76	4.65	50	20	554	186.80	1.40	1.16	1.16	0.81	0.96	0.93
949	S80-1	300	300	450	369	1018	P	2.76	4.65	81	20	554	261.60	1.68	1.39	1.63	0.99	1.46	1.15
Syroka-Korol and Teichman (2013)																			
950	SL20.1	200	200	220	180	540	P	3.00	1.00	35	32	500	47.75	1.32	1.13	1.13	1.05	1.25	1.26
951	SL20.2	200	200	220	180	540	P	3.00	1.00	35	32	500	44.40	1.23	1.05	1.05	0.97	1.16	1.17
952	SL20.3	200	200	220	180	540	P	3.00	1.00	35	32	500	45.65	1.26	1.08	1.08	1.00	1.19	1.21
953	SL40.1	200	200	400	360	1080	P	3.00	1.00	35	32	500	80.05	1.11	1.08	1.08	1.01	1.13	1.26
954	SL40.2	200	200	400	360	1080	P	3.00	1.00	35	32	500	86.10	1.19	1.16	1.16	1.09	1.21	1.36
955	SL40.3	200	200	400	360	1080	P	3.00	1.00	35	32	500	91.00	1.26	1.23	1.23	1.15	1.28	1.43
956	SL80.1	200	200	780	720	2160	P	3.00	1.00	35	32	500	132.15	0.91	1.02	1.02	1.02	1.06	1.28
957	SL80.2	200	200	780	720	2160	P	3.00	1.00	35	32	500	116.25	0.80	0.90	0.90	0.90	0.93	1.12
958	SL80.3	200	200	780	720	2160	P	3.00	1.00	35	32	500	147.55	1.02	1.14	1.14	1.14	1.18	1.42

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Janaka Perera and Mutsuyoshi (2013)																			
959	NSC40-I	200	200	300	250	750	P	3.00	3.04	38	19	750	75.00	1.43	1.04	1.04	0.85	1.01	0.97
960	NSC40-II	200	200	300	250	875	P	3.50	3.04	38	19	750	78.00	1.49	1.08	1.08	0.92	1.10	1.06
961	NSC40-III	200	200	300	250	1000	P	4.00	3.04	36	19	750	76.50	1.50	1.08	1.08	0.94	1.14	1.09
962	HA100-III	200	200	300	250	1000	P	4.00	3.04	114	19	750	85.00	1.20	0.88	1.07	0.73	1.16	0.91
963	HA120	200	200	300	250	1000	P	4.00	3.04	138	19	750	82.50	1.17	0.86	1.04	0.68	1.13	0.84
Shuraim (2014)																			
964	KQ30-R1	200	200	400	345	1100	P	3.19	0.45	27	20	632	57.00	0.94	1.13	1.13	1.18	1.27	1.38
965	KQ30-R2	200	200	400	343	1100	P	3.21	0.74	27	20	548	60.00	0.99	1.02	1.02	1.03	1.11	1.20
966	KQ30-R3	200	200	400	322	1100	P	3.42	1.58	27	20	548	83.00	1.46	1.15	1.15	1.15	1.28	1.31
967	KQ30-R4	200	200	400	318	1100	P	3.46	2.39	27	20	534	89.00	1.59	1.15	1.15	1.07	1.23	1.21
968	KQ45-R1	200	200	400	345	1500	P	4.35	0.45	27	20	632	51.00	0.84	1.01	1.01	1.12	1.29	1.36
969	KQ45-R2	200	200	400	343	1500	P	4.37	0.74	27	20	548	56.00	0.92	0.95	0.95	1.03	1.17	1.23
970	KQ45-R3	200	200	400	322	1500	P	4.67	1.58	27	20	548	73.00	1.29	1.01	1.01	1.07	1.25	1.27

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
971	KQ45-R4	200	200	400	320	1500	P	4.69	1.97	27	20	534	76.00	1.35	0.98	0.98	1.03	1.22	1.21
Collins, Bentz, Quach, and Proestos (2015)																			
972	Large	250	250	4000	3840	12000	P	3.12	0.65	40	14	573	343.00	0.33	0.55	0.55	1.10	1.02	1.05
973	Small	175	175	300	264	825	P	3.12	0.65	45	14	458	47.40	0.90	0.99	0.99	1.05	1.14	1.21
Author (2018)																			
974	1V0-450-20	265	265	450	403	1200	P	3.00	2.00	102	20	544	142.00	0.94	0.77	0.93	0.76	1.07	0.90
975	1V0-450-10	265	265	450	403	1200	P	2.98	2.00	108	10	544	168.00	1.11	0.91	1.10	1.01	1.27	1.05
976	1V0-900-20	265	265	900	809	2400	P	2.97	2.00	103	20	544	194.00	0.64	0.59	0.72	0.68	0.95	0.75
977	1V0-900-10	265	265	900	809	2400	P	2.97	2.00	108	10	544	284.00	0.94	0.87	1.06	1.15	1.39	1.09
978	1V0-1350-20	265	265	1350	1232	3650	P	2.96	2.00	98	20	544	384.00	0.83	0.82	1.00	1.07	1.52	1.14
979	1V0-1350-10	265	265	1350	1232	3650	P	2.96	2.00	108	10	544	501.00	1.09	1.08	1.31	1.61	1.98	1.45
980	1V0-1800-20	200	480	1800	1670	4900	P	2.93	2.00	107	20	557	545.00	1.16	1.19	1.45	1.67	2.49	1.72

No.	Beam Name	b_w (mm)	b (mm)	h (mm)	d (mm)	a (mm)	Load Type	a/d ratio	ρ (%)	f'_c (MPa)	d_a (mm)	f_y (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
														ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
Average														1.41	1.08	1.11	1.03	1.20	1.14
Standard deviation														0.50	0.27	0.28	0.16	0.21	0.16
Coefficient of Variation														0.36	0.25	0.25	0.16	0.18	0.14
Minimum														0.33	0.54	0.55	0.57	0.70	0.71
Maximum														3.95	2.89	2.89	1.72	2.49	1.89

Note:

-Load type: P = point load; and U = uniformly distributed load.

-EC 2 method¹ and EC 2 method² refers to Eurocode 2 method with compressive strength limit of 90 and 50 MPa respectively.

D.2. Properties and Analysis Results of 189 Reinforced Concrete Beam Specimens with Shear Reinforcement in Database (DB-2)

Table D-2 presents a database of 189 rectangular reinforced concrete beam data with shear reinforcement (DB-2). The properties of each data presented in DB-2 include the beam name, beam web width (b_w), height (h), effective depth (d), shear-span-to-effective-depth (a/d) ratio, longitudinal reinforcement ratio (ρ_l), yield strength of longitudinal reinforcement (f_{yl}), concrete compressive strength (f'_c), vertical shear reinforcement ratio (ρ_v), yield strength of vertical shear reinforcement (f_{yv}), spacing between vertical shear reinforcing bars (s), horizontal shear reinforcement ratio (ρ_h), yield strength of horizontal shear reinforcement (f_{yh}), and experimental shear strength (V_{test}). These properties are presented in columns (3) to (15) in **Table D-2** respectively.

The experimental shear strength for each data was compared against the predicted shear strength calculated using the design equations in ACI 318-14 (2014), Eurocode 2 (2004), Bažant and Yu (2005a and 2005b), Collins et al. (2008), and the authors' proposed simplified method (second approach discussed in Chapter 7). The reduced failure loads for the specimens in 1350-series tested in this study were used for the comparison. The ratios of V_{test}/V_{pred} for each method are presented in columns (16) to (20) respectively. The ranges of some of the influential parameters in all the data can be summarised as follows:

- The effective depth ranges from 95 to 1925 mm.
- The shear-span-to-effective-depth (a/d) ratio ranges from 2.8 to 5.
- The concrete compressive strength ranges from 16 to 125 MPa.
- The longitudinal reinforcement ratio ranges from 0.36 to 5.80%.
- The vertical shear reinforcement ratio ranges from 0.06 to 0.78%.

Table D-2—Properties and analysis results of 189 beam data in database (DB-2)

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_s (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
Krefeld and Thurston (1966)																				
1	26-1	254	508	456	4.0	2.22	300	40	0.16	341	152	-	-	207	1.10	1.38	1.38	0.96	0.78	0.96
2	29a-1	254	508	456	4.0	2.22	300	39	0.11	341	229	-	-	160	0.97	1.08	1.08	0.83	0.66	0.84
3	29b-1	254	508	456	4.0	2.22	300	38	0.11	341	229	-	-	160	0.98	1.10	1.10	0.83	0.67	0.84
4	213.5-1	254	508	456	4.0	2.22	300	39	0.07	341	343	-	-	148	0.98	1.00	1.00	0.83	0.65	0.86
5	29a-2	254	508	456	4.0	2.22	300	37	0.11	372	229	-	-	217	1.28	1.49	1.49	1.09	0.88	1.08
6	29b-2	254	508	456	4.0	2.22	300	41	0.11	372	229	-	-	202	1.15	1.34	1.34	0.99	0.79	1.00
7	29c-2	254	508	456	4.0	2.22	300	24	0.11	372	229	-	-	162	1.11	1.28	1.28	0.90	0.76	0.86
8	29d-2	254	508	456	4.0	2.22	300	30	0.11	372	229	-	-	165	1.04	1.21	1.21	0.87	0.72	0.85
9	29e-2	254	508	456	4.0	2.22	300	48	0.11	372	229	-	-	206	1.11	1.30	1.30	0.98	0.77	0.99
10	29g-2	254	508	456	4.0	2.22	300	16	0.11	372	229	-	-	150	1.18	1.36	1.36	0.92	0.80	0.85
11	213.5a-2	254	508	456	4.0	2.22	300	37	0.08	372	343	-	-	162	1.05	1.11	1.11	0.89	0.70	0.90
12	218a-2	254	508	456	4.0	2.22	300	38	0.06	372	457	-	-	164	1.13	1.12	1.12	0.94	0.74	0.98
13	29-3	254	508	456	4.0	2.22	300	34	0.12	237	229	-	-	178	1.21	1.26	1.26	1.00	0.80	1.03
14	39-3	254	508	456	4.0	2.22	300	34	0.20	276	229	-	-	240	1.34	1.69	1.69	1.15	0.95	1.11

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bazant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
15	318-1	254	508	456	4.0	2.22	300	41	0.12	517	457	-	-	220	1.10	1.33	1.33	0.97	0.79	0.90
16	321-1	254	508	456	4.0	2.22	300	39	0.11	517	533	-	-	164	0.88	1.11	1.11	0.76	0.62	0.71
17	318-2	254	508	456	4.0	2.22	300	39	0.13	352	457	-	-	177	1.02	1.20	1.20	0.88	0.70	0.84
18	321-2	254	508	456	4.0	2.22	300	38	0.11	352	533	-	-	167	1.01	1.14	1.14	0.86	0.69	0.83
19	313.5-3	254	508	456	4.0	2.22	300	43	0.16	276	343	-	-	214	1.18	1.40	1.40	1.03	0.82	1.01
20	313.5-2	254	508	456	4.0	2.22	300	32	0.16	352	343	-	-	234	1.32	1.59	1.59	1.12	0.93	1.05
21	318-3	254	508	456	4.0	2.22	300	43	0.12	276	457	-	-	175	1.04	1.14	1.14	0.90	0.71	0.90
22	321-3	254	508	456	4.0	2.22	300	43	0.11	276	533	-	-	141	0.86	0.92	0.92	0.74	0.58	0.75
Swamy and Andriopoulos (1974)																				
23	C3	76	114.3	95	3.0	1.97	434	29	0.16	275	70	-	-	16	1.59	1.55	1.55	1.05	1.11	1.08
24	R3	76	114.3	95	3.0	1.97	434	29	0.38	208	70	-	-	18	1.46	1.42	1.42	1.05	1.09	1.07
25	J3	76	114.3	95	3.0	1.97	434	29	0.43	253	48	-	-	21	1.41	1.17	1.17	1.05	1.09	1.12
26	C4	76	114.3	95	4.0	1.97	434	29	0.06	283	70	-	-	14	1.74	1.36	1.36	1.13	1.13	1.15
27	Y3	76	152.4	132	3.0	3.95	434	26	0.60	222	100	-	-	29	1.31	0.97	0.97	0.89	0.99	0.91
28	O4	76	152.4	132	4.0	3.95	434	28	0.12	258	100	-	-	20	1.64	1.44	1.44	0.92	1.02	0.93
29	Z4	76	152.4	132	4.0	3.95	434	26	0.34	179	100	-	-	26	1.73	1.88	1.88	1.07	1.16	1.03

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ_l (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method ²
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
30	O5	76	152.4	132	5.0	3.95	434	28	0.12	258	100	-	-	19	1.55	1.36	1.36	0.90	0.96	0.90
Mattock and Wang (1984)																				
31	C305-D0	150	350	315	3.0	2.60	362	26	0.24	355	160	-	-	108	1.34	1.22	1.22	1.02	0.97	1.03
32	C310-D10	150	350	315	3.0	2.60	360	24	0.47	356	80	-	-	132	1.11	0.75	0.75	0.92	0.89	0.99
33	C310-D20	150	350	315	3.0	2.60	360	25	0.47	356	80	-	-	140	1.17	0.79	0.79	0.97	0.94	1.04
Mphonde and Frantz (1985)																				
34	B50-3-3	152	337	298	3.6	3.36	448	22	0.12	292	89	-	-	76	1.47	1.45	1.45	0.96	0.94	0.97
35	B50-7-3	152	337	298	3.6	3.36	448	40	0.12	292	89	-	-	94	1.46	1.47	1.47	1.01	0.93	1.07
36	B50-11-3	152	337	298	3.6	3.36	448	60	0.12	292	89	-	-	98	1.30	1.34	1.42	0.94	0.84	1.03
37	B50-15-3	152	337	298	3.6	3.36	448	83	0.12	292	89	-	-	111	1.40	1.37	1.62	0.99	0.91	1.10
38	B100-3-3	152	337	298	3.6	3.36	448	28	0.26	269	89	-	-	95	1.32	1.35	1.35	0.95	0.91	0.96
39	B100-7-3	152	337	298	3.6	3.36	448	47	0.26	269	89	-	-	121	1.43	1.71	1.71	1.06	0.98	1.11
40	B100-11-3	152	337	298	3.6	3.36	448	69	0.26	269	89	-	-	151	1.58	1.98	2.14	1.23	1.09	1.31
41	B100-15-3	152	337	298	3.6	3.36	448	82	0.26	269	89	-	-	116	1.21	1.43	1.64	0.90	0.84	0.97
42	B150-3-3	152	337	298	3.6	3.36	448	29	0.38	271	89	-	-	138	1.57	1.33	1.33	1.19	1.15	1.19
43	B150-7-3	152	337	298	3.6	3.36	448	47	0.38	271	89	-	-	133	1.34	1.28	1.28	1.04	0.97	1.08

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_s (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
44	B150-11-3	152	337	298	3.6	3.36	448	70	0.38	271	89	-	-	162	1.46	1.56	1.56	1.17	1.05	1.24
45	B150-15-3	152	337	298	3.6	3.36	448	83	0.38	271	89	-	-	150	1.36	1.44	1.44	1.05	0.98	1.12
Elzanaty, Nilson, and Slate (1986)																				
46	G4	178	304.8	266	4.0	3.30	434	63	0.17	382	191	-	-	149	1.58	1.87	2.02	1.26	1.08	1.22
47	G5	178	304.8	266	4.0	2.50	434	40	0.17	382	191	-	-	111	1.36	1.62	1.62	1.12	0.96	1.05
48	G6	178	304.8	266	4.0	2.50	434	21	0.17	382	191	-	-	78	1.16	1.14	1.14	0.91	0.82	0.81
Johnson and Ramirez (1989)																				
49	1	304	610	538	3.1	2.49	525	36	0.14	493	133	-	-	339	1.21	1.35	1.35	1.05	0.86	1.06
50	2	304	610	538	3.1	2.49	525	36	0.07	500	267	-	-	222	0.99	1.12	1.12	0.83	0.66	0.84
51	3	304	610	538	3.1	2.49	525	72	0.07	500	267	-	-	263	0.91	1.06	1.20	0.85	0.62	0.88
52	4	304	610	538	3.1	2.49	525	72	0.07	500	267	-	-	316	1.10	1.27	1.44	1.02	0.75	1.05
53	5	304	610	538	3.1	2.49	525	56	0.14	493	134	-	-	383	1.19	1.52	1.52	1.10	0.86	1.12
54	7	304	610	538	3.1	2.49	525	51	0.07	500	267	-	-	281	1.10	1.27	1.28	0.97	0.74	1.00
55	8	304	610	538	3.1	2.49	525	51	0.07	500	267	-	-	258	1.01	1.16	1.17	0.89	0.68	0.92
Roller and Russell (1990)																				

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_s (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
56	6	457	870	762	3.0	1.73	464	72	0.08	457	381	-	-	665	1.08	1.40	1.59	1.32	0.79	1.21
57	7	457	870	762	3.0	1.88	483	72	0.16	440	197	-	-	788	1.07	1.44	1.44	1.24	0.81	1.16
58	8	457	870	762	3.0	1.88	483	125	0.08	457	381	-	-	483	0.78	0.92	1.12	0.85	0.56	0.78
59	9	457	870	762	3.0	2.35	483	125	0.16	440	197	-	-	749	1.02	1.37	1.37	1.04	0.74	0.97
60	10	457	870	762	3.0	2.89	464	125	0.23	450	133	-	-	1172	1.38	1.46	1.46	1.34	1.01	1.28
Sarsam and Al-Musawi (1992)																				
61	AL2-N	180	270	235	4.0	2.23	495	40	0.09	820	150	-	-	115	1.47	1.60	1.60	1.23	1.08	1.16
62	AL2-H	180	270	235	4.0	2.23	495	75	0.09	820	150	-	-	123	1.33	1.57	1.71	1.16	0.99	1.13
63	BL2-H	180	270	235	4.0	2.82	495	76	0.09	820	150	-	-	138	1.51	1.77	1.93	1.23	1.07	1.21
64	CL2-H	180	270	235	4.0	3.51	495	70	0.09	820	150	-	-	147	1.60	1.94	2.06	1.25	1.11	1.23
Xie, Ahmad, Yu, Hino, and Chung (1994)																				
65	NNW-3	127	254	203	3.0	3.20	421	41	0.49	322	102	-	-	87	1.27	0.96	0.96	0.96	0.98	1.03
66	NHW-3	127	254	198	3.0	4.54	421	98	0.51	324	102	-	-	102	1.33	1.11	1.11	0.88	0.99	1.00
67	NHW-3a	127	254	198	3.0	4.54	421	90	0.65	323	76	-	-	108	1.23	0.92	0.92	0.87	0.94	1.00
68	NHW-3b	127	254	198	3.0	4.54	421	103	0.78	324	64	-	-	123	1.24	0.87	0.89	0.88	0.97	1.05
69	NHW-4	127	254	198	4.0	4.54	421	99	0.51	324	102	-	-	94	1.22	1.01	1.01	0.84	0.90	0.90

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
McGormley, Cleary, and Ramirez (1996)																				
70	BUS-1	203	508	419	3.3	3.03	462	42	0.34	426	203	-	-	272	1.25	0.99	0.99	1.06	0.96	1.05
71	EUS-1	203	508	419	3.3	3.03	462	43	0.34	426	203	-	-	299	1.37	1.09	1.09	1.17	1.05	1.15
72	BUH-1	203	508	419	3.3	3.03	462	46	0.34	426	203	-	-	276	1.25	1.00	1.00	1.07	0.95	1.05
73	EUH-1	203	508	419	3.3	3.03	462	44	0.34	426	203	-	-	307	1.40	1.12	1.12	1.19	1.07	1.18
74	BUIS-2	203	508	419	3.3	3.03	507	35	0.34	426	203	-	-	316	1.51	1.15	1.15	1.27	1.16	1.24
75	EUIS-2	203	508	419	3.3	3.03	507	48	0.34	426	203	-	-	312	1.39	1.13	1.13	1.20	1.06	1.18
76	BUIH-2	203	508	419	3.3	3.03	507	50	0.34	426	203	-	-	334	1.48	1.21	1.21	1.28	1.13	1.26
77	EUIH-2	203	508	419	3.3	3.03	507	51	0.34	426	203	-	-	320	1.42	1.17	1.17	1.22	1.08	1.21
78	BUH-3	203	508	419	3.3	3.03	502	53	0.34	426	203	-	-	289	1.26	1.05	1.05	1.09	0.96	1.09
79	EUH-3	203	508	419	3.3	3.03	502	55	0.34	426	203	-	-	312	1.35	1.13	1.13	1.17	1.03	1.16
80	BUIS-3	203	508	419	3.3	3.03	502	57	0.34	426	203	-	-	267	1.15	0.97	0.97	1.00	0.88	1.00
81	EUIS-3	203	508	419	3.3	3.03	502	56	0.34	426	203	-	-	267	1.15	0.97	0.97	1.00	0.88	1.00
Yoon, Cook, and Mitchell (1996)																				
82	N1-N	375	750	655	3.3	2.80	430	36	0.08	438	325	-	-	457	1.36	1.60	1.60	1.17	0.89	1.15
83	N2-S	375	750	655	3.3	2.80	430	36	0.08	438	465	-	-	363	1.08	1.27	1.27	0.93	0.71	0.90

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_s (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
84	N2-N	375	750	655	3.3	2.80	430	36	0.12	417	325	-	-	483	1.29	1.69	1.69	1.13	0.88	1.09
85	M1-N	375	750	655	3.3	2.80	430	67	0.08	438	325	-	-	405	0.95	1.15	1.27	1.02	0.63	0.92
86	M2-S	375	750	655	3.3	2.80	430	67	0.12	417	325	-	-	552	1.19	1.57	1.73	1.27	0.82	1.13
87	M2-N	375	750	655	3.3	2.80	430	67	0.16	438	230	-	-	689	1.34	1.80	1.80	1.43	0.95	1.27
88	H1-N	375	750	655	3.3	2.80	430	87	0.08	438	325	-	-	483	1.12	1.26	1.52	1.16	0.75	1.04
89	H2-S	375	750	655	3.3	2.80	430	87	0.14	429	270	-	-	598	1.21	1.56	1.82	1.25	0.84	1.11
90	H2-N	375	750	655	3.3	2.80	430	87	0.23	435	160	-	-	721	1.22	1.32	1.32	1.25	0.89	1.14
Tan, Teng, Kong, and Lu (1997)																				
91	2-2.58/2.5	110	500	443	2.8	2.58	499	55	0.48	353	300	-	-	155	1.08	0.84	0.84	0.97	0.85	0.97
92	4- 5.80/2.50	110	500	398	3.1	5.80	538	74	0.48	385	300	-	-	265	1.86	1.47	1.47	1.43	1.37	1.43
Kong, and Rangan (1998)																				
93	s5-1	250	350	292	3.0	2.80	452	89	0.16	569	100	-	-	242	1.44	1.66	1.66	1.24	1.04	1.20
94	s7-1	250	350	294	3.3	4.47	433	75	0.11	569	150	-	-	217	1.47	1.69	1.94	1.15	0.96	1.07
95	s7-2	250	350	294	3.3	4.47	433	75	0.13	569	125	-	-	205	1.31	1.60	1.75	1.04	0.88	0.98
96	s7-3	250	350	294	3.3	4.47	433	75	0.16	569	100	-	-	247	1.46	1.69	1.69	1.17	1.00	1.11
97	s7-4	250	350	294	3.3	4.47	433	75	0.20	569	80	-	-	274	1.47	1.50	1.50	1.20	1.04	1.17

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ_l (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
98	s7-5	250	350	294	3.3	4.47	433	75	0.22	569	70	-	-	304	1.54	1.46	1.46	1.27	1.11	1.25
99	s7-6	250	350	294	3.3	4.47	433	75	0.26	569	60	-	-	311	1.46	1.27	1.27	1.22	1.07	1.22
Yoshida (1998)																				
100	YB2000/9	300	2000	1890	2.9	0.74	455	34	0.08	470	2700	-	-	472	0.61	0.99	0.99	0.95	0.51	0.78
101	YB2000/4	300	2000	1890	2.9	0.74	455	36	0.07	465	590	-	-	674	0.88	1.64	1.64	1.42	0.73	1.26
102	YB2000/6	300	2000	1890	2.9	0.74	455	36	0.07	465	1350	-	-	550	0.72	1.34	1.34	1.16	0.60	0.99
Ozcebe, Ersoy, and Tankut (1999)																				
103	ACI56	150	310	310	5.0	3.46	450	58	0.14	255	120	-	-	94	1.22	1.27	1.34	0.93	0.79	0.98
104	TH56	150	310	310	5.0	3.46	450	63	0.17	255	100	-	-	104	1.25	1.37	1.48	0.98	0.82	1.03
105	TS56	150	310	310	5.0	3.46	450	61	0.24	255	70	-	-	129	1.43	1.73	1.84	1.14	0.97	1.18
106	ACI59	150	310	310	5.0	4.43	438	82	0.14	255	120	-	-	97	1.18	1.17	1.38	0.82	0.74	0.88
107	TH59	150	310	310	5.0	4.43	438	75	0.19	255	90	-	-	119	1.36	1.49	1.70	0.98	0.87	1.05
108	TS59	150	310	310	5.0	4.43	438	82	0.28	255	60	-	-	125	1.27	1.52	1.70	0.93	0.85	0.99
109	ACI36	150	310	310	3.0	2.59	450	75	0.14	255	120	-	-	105	1.28	1.31	1.50	0.99	0.87	1.09
110	TH36	150	310	310	3.0	2.59	450	75	0.17	255	100	-	-	141	1.65	1.76	2.01	1.28	1.13	1.42
111	TS36	150	310	310	3.0	2.59	450	75	0.24	255	70	-	-	156	1.66	1.94	2.23	1.32	1.17	1.46

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ_l (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
112	ACI39	150	310	310	3.0	3.07	442	73	0.14	255	120	-	-	112	1.36	1.41	1.60	1.00	0.90	1.11
113	TH39	150	310	310	3.0	3.07	442	73	0.17	255	80	-	-	143	1.67	1.80	2.04	1.24	1.12	1.38
114	TS39	150	310	310	3.0	3.07	442	73	0.28	255	60	-	-	179	1.82	2.25	2.43	1.40	1.27	1.56
Frosch (2000)																				
115	V1	457	914	851	3.0	1.00	476	37	0.08	417	371	-	-	395	0.74	1.14	1.14	0.90	0.57	0.86
116	V2	457	914	851	3.0	1.00	476	37	0.08	417	371	-	-	492	0.92	1.43	1.43	1.12	0.72	1.07
Cao (2001)																				
117	SB2003/6	300	2000	1925	2.8	0.36	436	28	0.07	483	1350	-	-	326	0.46	0.75	0.75	0.80	0.44	0.67
118	SB2012/6	300	2000	1845	2.9	1.52	436	31	0.07	483	1350	-	-	613	0.86	1.28	1.28	1.12	0.62	0.97
Angelakos, Bentz, and Collins (2001)																				
119	DB0.530M	300	1000	925	2.9	0.50	550	32	0.08	500	300	-	-	263	0.70	1.06	1.06	1.05	0.63	0.92
120	DB120M	300	1000	925	2.9	1.01	550	21	0.08	500	600	-	-	282	0.86	1.14	1.14	1.03	0.66	0.87
121	DB140M	300	1000	925	2.9	1.01	550	38	0.08	500	300	-	-	277	0.69	1.12	1.12	0.92	0.54	0.82
122	DB165M	300	1000	925	2.9	1.01	550	65	0.08	500	300	-	-	452	0.92	1.53	1.67	1.40	0.76	1.23
123	DB180M	300	1000	925	2.9	1.01	550	80	0.08	500	300	-	-	395	0.79	1.25	1.46	1.19	0.65	1.04
124	BM100	300	1000	925	2.9	0.76	550	47	0.08	500	600	-	-	342	0.79	1.38	1.38	1.18	0.67	1.01

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_s (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
Tompos and Frosch (2002)																				
125	V36-2	457	914	851	3.0	1.00	483	43	0.08	417	165	-	-	488	0.86	1.34	1.34	1.08	0.68	1.07
126	V36-3	457	914	851	3.0	1.00	483	43	0.08	417	371	-	-	512	0.90	1.41	1.41	1.13	0.71	1.08
127	V18-2	229	486	425	3.0	1.00	552	36	0.15	416	186	-	-	172	1.08	1.28	1.28	1.10	0.87	1.10
128	V18-2c	229	486	425	3.0	1.00	552	36	0.15	416	186	-	-	153	0.96	1.14	1.14	0.98	0.78	0.98
Lin and Lee (2003)																				
129	N1	240	360	300	3.0	3.53	900	32	0.47	451	140	-	-	322	1.46	0.95	0.95	1.18	1.17	1.24
130	N2	240	360	300	3.0	3.53	900	41	0.47	451	140	-	-	335	1.46	0.99	0.99	1.18	1.16	1.25
131	N3	240	360	300	3.0	3.53	900	51	0.47	451	140	-	-	351	1.47	1.04	1.04	1.20	1.16	1.28
132	N5	240	360	257	3.5	3.53	900	40	0.47	451	140	-	-	278	1.41	0.96	0.96	1.14	1.12	1.15
133	N6	240	360	300	3.0	3.53	900	42	0.47	373	140	-	-	325	1.58	1.16	1.16	1.26	1.23	1.32
134	N7	240	360	300	3.0	3.53	900	42	0.47	549	140	-	-	336	1.27	0.82	0.82	1.06	1.04	1.14
135	N8	240	360	300	3.0	3.53	900	43	0.55	451	120	-	-	360	1.40	0.91	0.91	1.16	1.14	1.26
136	N9	240	360	300	3.0	3.53	900	43	0.39	451	170	-	-	284	1.38	1.02	1.02	1.10	1.07	1.14
137	H1	240	360	300	3.0	3.53	900	31	0.47	451	140	-	-	325	1.47	0.96	0.96	1.19	1.18	1.25
138	H2	240	360	300	3.0	3.53	900	43	0.47	451	140	-	-	343	1.48	1.01	1.01	1.20	1.17	1.27

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
139	H3	240	360	300	3.0	3.53	900	53	0.47	451	140	-	-	357	1.48	1.06	1.06	1.21	1.17	1.30
140	H5	240	360	257	3.5	3.53	900	43	0.47	451	140	-	-	284	1.43	0.98	0.98	1.16	1.13	1.17
141	H6	240	360	300	3.0	3.53	900	42	0.47	373	140	-	-	318	1.56	1.14	1.14	1.23	1.21	1.30
142	H7	240	360	300	3.0	3.53	900	42	0.47	549	140	-	-	339	1.29	0.83	0.83	1.07	1.05	1.15
143	H8	240	360	300	3.0	3.53	900	41	0.55	451	120	-	-	364	1.43	0.92	0.92	1.18	1.16	1.28
144	H9	240	360	300	3.0	3.53	900	42	0.39	451	170	-	-	291	1.43	1.05	1.05	1.13	1.11	1.17
145	H10	240	360	300	3.0	3.53	900	42	0.58	373	116	-	-	361	1.54	1.04	1.04	1.26	1.23	1.35
146	H11	240	360	300	3.0	3.53	900	43	0.38	549	171	-	-	307	1.33	0.92	0.92	1.08	1.06	1.13
Cladera and Marí (2005)																				
147	H50-2	200	400	360	3.1	2.28	500	50	0.11	525	260	-	-	178	1.39	1.69	1.69	1.16	0.99	1.18
148	H50-3	200	400	360	3.1	2.29	500	50	0.24	538	210	-	-	242	1.35	1.17	1.17	1.18	1.05	1.19
149	H50-4	200	400	360	3.1	2.99	500	50	0.24	538	210	-	-	246	1.37	1.19	1.19	1.14	1.03	1.15
150	H60-2	200	400	360	3.1	2.28	500	61	0.14	534	200	-	-	180	1.20	1.50	1.50	1.04	0.88	1.08
151	H60-3	200	400	360	3.1	2.29	500	61	0.24	528	210	-	-	259	1.39	1.27	1.27	1.23	1.07	1.25
152	H60-4	200	400	360	3.1	2.99	500	61	0.24	528	210	-	-	309	1.65	1.52	1.52	1.39	1.24	1.42
153	H75-2	200	400	360	3.1	2.28	500	69	0.14	534	200	-	-	204	1.31	1.70	1.70	1.15	0.96	1.20

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
154	H75-3	200	400	360	3.1	2.29	500	69	0.24	528	210	-	-	269	1.40	1.33	1.33	1.26	1.08	1.28
155	H75-4	200	400	360	3.1	2.99	500	69	0.24	528	210	-	-	225	1.17	1.11	1.11	0.99	0.88	1.02
156	H100-2	200	400	360	3.1	2.28	500	87	0.17	533	165	-	-	226	1.35	1.55	1.55	1.15	1.01	1.21
157	H100-3	200	400	360	3.1	2.29	500	87	0.24	538	210	-	-	254	1.30	1.23	1.23	1.13	1.01	1.16
158	H100-4	200	400	360	3.1	2.99	500	87	0.24	538	210	-	-	267	1.37	1.29	1.29	1.12	1.03	1.16
Lee and Kim (2008)																				
159	L1-B	350	450	410	3.0	1.79	525	41	0.18	215	80	-	-	214	1.02	1.17	1.17	0.89	0.72	0.96
160	L2-B	350	450	400	3.0	3.21	525	41	0.18	215	80	-	-	255	1.24	1.36	1.36	0.91	0.81	0.98
161	L3-B	350	450	385	3.0	4.76	525	41	0.18	215	80	-	-	317	1.61	1.75	1.75	1.03	1.00	1.11
162	L4-B	220	320	280	3.0	0.93	550	31	0.16	215	140	-	-	92	1.16	1.47	1.47	1.09	0.90	1.12
163	L5-B	220	320	280	3.0	1.86	550	31	0.16	215	140	-	-	103	1.30	1.31	1.31	1.00	0.90	1.04
164	L6-B	220	320	260	3.0	2.79	550	31	0.16	215	140	-	-	127	1.73	1.67	1.67	1.16	1.13	1.21
165	S2-B	350	450	410	3.0	2.24	525	41	0.18	215	77	-	-	222	1.04	1.16	1.16	0.86	0.71	0.92
166	S3-B	350	450	410	4.0	2.24	525	41	0.18	215	77	-	-	231	1.09	1.21	1.21	0.94	0.75	0.97
167	S4-B	220	320	280	3.0	1.40	550	31	0.16	215	140	-	-	102	1.28	1.42	1.42	1.07	0.93	1.11
168	S5-B	220	320	280	4.0	1.40	550	31	0.16	215	140	-	-	93	1.17	1.29	1.29	1.03	0.85	1.02

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_c (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
169	S6-B	220	320	280	5.0	1.40	550	31	0.16	215	140	-	-	83	1.04	1.15	1.15	0.94	0.75	0.91
170	C1-B	350	450	410	3.0	2.24	520	20	0.13	215	110	-	-	204	1.38	1.37	1.37	1.02	0.90	1.04
Lee, Choi and Kim (2011)																				
171	S30-3	300	600	520	4.0	2.90	530	33	0.10	580	320	-	-	300	1.23	1.49	1.49	1.04	0.84	0.96
172	S30-4	300	600	520	4.0	2.90	530	33	0.10	580	320	-	-	310	1.27	1.54	1.54	1.07	0.87	0.99
173	S30-5	300	600	520	4.0	2.90	530	33	0.30	580	160	-	-	447	1.05	0.74	0.74	0.95	0.83	0.88
174	S30-6	300	600	520	4.0	2.90	530	33	0.30	580	160	-	-	447	1.05	0.74	0.74	0.95	0.83	0.88
Author (2018)																				
175	1V1-450-20	265	450	403	3.0	2.00	544	102	0.13	614	170	-	-	246	1.04	1.30	1.30	0.90	0.79	0.95
176	1V1-450-10	265	450	403	3.0	2.00	544	108	0.13	614	170	-	-	296	1.25	1.56	1.56	1.08	0.95	1.14
177	2V2-450-20	265	450	403	3.0	2.00	544	102	0.20	614	110	-	-	305	1.08	1.04	1.04	0.96	0.85	1.03
178	2V3-450-20	265	450	403	3.0	2.00	544	102	0.30	614	75	-	-	328	0.94	0.75	0.75	0.86	0.77	0.96
179	1V1-900-20	265	900	808	3.0	2.00	544	103	0.13	596	480	-	-	414	0.88	1.12	1.12	0.92	0.67	0.87
180	1V1-900-10	265	900	808	3.0	2.00	544	108	0.13	596	480	-	-	434	0.93	1.17	1.17	0.96	0.70	0.90

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_x (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bažant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
181	2V2-900-20	265	900	808	3.0	2.00	544	103	0.20	596	300	-	-	554	0.99	0.97	0.97	1.02	0.78	0.98
182	2V3-900-20	265	900	808	3.0	2.00	544	103	0.30	596	210	-	-	749	1.09	0.88	0.88	1.12	0.89	1.11
183	1V1-1350-20	265	1350	1232	3.0	2.00	544	98	0.13	596	480	0.10	596	801	1.12	1.42	1.42	1.31	0.84	1.19
184	1V1-1350-10	265	1350	1232	3.0	2.00	544	108	0.13	596	480	0.10	596	926	1.30	1.64	1.64	1.50	0.98	1.36
185	2V2-1350-20	265	1350	1232	3.0	2.00	544	97	0.20	596	300	0.10	596	965	1.14	1.11	1.11	1.30	0.89	1.20
186	2V3-1350-20	265	1350	1232	3.0	2.00	544	97	0.30	596	210	0.10	596	1221	1.17	0.94	0.94	1.30	0.95	1.26
187	1V1-1800-20	200	1800	1670	2.93	2.0	556.5	107	0.13	596.00	600	-	-	925	1.27	1.60	1.60	1.59	0.95	1.39
188	2V2-1800-20	200	1800	1670	2.93	2.0	556.5	107	0.2	596.00	390	-	-	1048	1.21	1.18	1.18	1.45	0.94	1.30
189	2V3-1800-20	200	1800	1670	2.93	2.0	556.5	107	0.3	596	260	-	-	1285	1.20	0.97	0.97	1.40	0.98	1.31

No.	Beam Name	b_w (mm)	h (mm)	d (mm)	a/d	ρ_l (%)	f_{yl} (MPa)	f'_c (MPa)	ρ_z (%)	f_{yz} (MPa)	s (mm)	ρ_x (%)	f_{yx} (MPa)	V_{test} (kN)	V_{test}/V_{pred}					
															ACI 318-14	Eurocode 2 ¹	Eurocode 2 ²	Bazant and Yu	Collins <i>et al.</i>	Proposed method 2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
Average															1.24	1.29	1.32	1.08	0.91	1.08
Standard deviation															0.24	0.28	0.31	0.16	0.17	0.16
Coefficient of Variation															0.20	0.22	0.24	0.15	0.19	0.15
Minimum															0.46	0.74	0.74	0.74	0.44	0.67
Maximum															1.86	2.25	2.43	1.59	1.37	1.56

D.3. Results of 189 Reinforced Concrete Beam Specimens with Shear Reinforcement in Database (DB-2) from Analytical Truss Model.

Table D-3 presents the results for 189 rectangular reinforced concrete beam data with shear reinforcement in DB-2, obtained from the proposed analytical truss model (first approach discussed in Chapter 7).

Table D-3—Analysis results of 189 beam data obtained from proposed analytical truss model.

No.	Beam Name	ϵ_d	ϵ_r	σ_r	σ_d	ϵ_z	ϵ_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Krefeld and Thurston (1966)													
1	26-1	-0.00018	0.147	0.79	-4.22	0.107	0.039	341	31.1	-2.22	63	209	0.99
2	29a-1	-0.00015	0.140	0.77	-3.74	0.105	0.035	341	30.1	-1.96	63	177	0.90
3	29b-1	-0.00014	0.145	0.75	-3.47	0.107	0.038	341	30.9	-1.86	62	171	0.94
4	213.5-1	-0.00010	0.139	0.77	-2.83	0.100	0.039	341	32.0	-1.62	63	149	0.99
5	29a-2	-0.00016	0.144	0.74	-3.72	0.106	0.037	372	30.7	-1.96	62	181	1.20
6	29b-2	-0.00016	0.143	0.82	-4.09	0.107	0.036	372	30.2	-2.14	64	194	1.04
7	29c-2	-0.00019	0.143	0.48	-2.71	0.103	0.041	372	32.2	-1.44	53	141	1.14
8	29d-2	-0.00018	0.144	0.60	-3.29	0.106	0.038	372	30.9	-1.72	58	163	1.02
9	2.90E-01	-0.00013	0.149	0.96	-3.97	0.107	0.042	372	32.0	-2.22	67	203	1.02
10	29g-2	-0.00021	0.145	0.31	-1.83	0.095	0.050	372	35.9	-1.02	46	109	1.38
11	213.5a-2	-0.00013	0.148	0.73	-3.25	0.110	0.038	372	30.4	-1.74	61	158	1.02
12	218a-2	-0.00004	0.146	0.74	-1.25	0.076	0.070	372	43.8	-1.00	62	114	1.44
13	29-3	-0.00012	0.146	0.68	-2.88	0.107	0.039	237	31.1	-1.58	60	147	1.21
14	39-3	-0.00019	0.144	0.68	-3.74	0.104	0.040	276	31.8	-1.98	60	191	1.26
15	318-1	-0.00020	0.143	0.80	-4.49	0.104	0.039	517	31.5	-2.36	63	228	0.97
16	321-1	-0.00019	0.142	0.77	-4.22	0.104	0.037	517	30.8	-2.20	62	207	0.79

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
17	318-2	-0.00016	0.145	0.77	-3.87	0.107	0.038	352	30.7	-2.04	63	188	0.94
18	321-2	-0.00014	0.146	0.75	-3.49	0.107	0.039	352	31.0	-1.88	62	173	0.96
19	313.5-3	-0.00015	0.139	0.85	-4.05	0.102	0.037	276	30.9	-2.16	65	198	1.08
20	313.5-2	-0.00020	0.143	0.63	-3.57	0.102	0.041	352	32.3	-1.90	58	186	1.26
21	318-3	-0.00012	0.140	0.85	-3.50	0.102	0.038	276	31.4	-1.94	65	175	1.00
22	321-3	-0.00010	0.142	0.85	-3.06	0.100	0.041	276	32.7	-1.78	65	163	0.86
Swamy and Andriopoulos (1974)													
23	C3	-0.00018	0.143	0.58	-3.20	0.104	0.039	275	31.3	-1.68	4	12	1.31
24	R3	-0.00022	0.149	0.58	-3.36	0.097	0.052	208	36.1	-1.88	4	14	1.26
25	J3	-0.00023	0.149	0.58	-3.38	0.086	0.063	253	40.5	-1.96	4	16	1.27
26	C4	-0.00010	0.140	0.57	-2.18	0.102	0.038	283	31.2	-1.22	4	9	1.51
27	Y3	-0.00026	0.130	0.52	-3.20	0.065	0.064	222	44.8	-1.86	6	23	1.24
28	O4	-0.00014	0.148	0.56	-2.69	0.108	0.039	258	31.1	-1.44	6	15	1.37
29	Z4	-0.00020	0.144	0.51	-2.94	0.097	0.047	179	34.7	-1.62	6	17	1.49
30	O5	-0.00014	0.148	0.56	-2.69	0.108	0.039	258	31.1	-1.44	6	15	1.30
Mattock and Wang (1984)													
31	C305-D0	-0.00023	0.147	0.52	-3.02	0.091	0.056	355	38.2	-1.72	21	79	1.36

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
32	C310-D10	-0.00042	0.077	0.48	-3.87	0.039	0.038	356	44.7	-2.18	21	124	1.07
33	C310-D20	-0.00042	0.077	0.49	-3.91	0.039	0.038	356	44.6	-2.20	21	125	1.12
Mphonde and Frantz (1985)													
34	B50-3-3	-0.00017	0.149	0.44	-2.36	0.107	0.042	292	32.0	-1.26	22	52	1.45
35	B50-7-3	-0.00014	0.146	0.79	-3.64	0.108	0.037	292	30.4	-1.94	27	74	1.27
36	B50-11-3	-0.00005	0.112	1.18	-2.18	0.061	0.051	292	42.4	-1.67	30	71	1.37
37	B50-15-3	-0.00005	0.128	1.64	-2.67	0.069	0.059	292	42.7	-2.15	34	86	1.29
38	B100-3-3	-0.00021	0.149	0.55	-3.17	0.099	0.050	269	35.4	-1.76	24	76	1.25
39	B100-7-3	-0.00018	0.146	0.93	-4.70	0.104	0.042	269	32.5	-2.56	28	104	1.15
40	B100-11-3	-0.00011	0.146	1.36	-4.44	0.094	0.052	269	36.5	-2.77	32	115	1.32
41	B100-15-3	-0.00006	0.148	1.62	-3.08	0.075	0.073	269	44.6	-2.35	34	104	1.11
42	B150-3-3	-0.00024	0.148	0.57	-3.33	0.087	0.060	271	39.8	-1.92	24	91	1.52
43	B150-7-3	-0.00021	0.150	0.92	-4.92	0.099	0.050	271	35.3	-2.76	28	125	1.07
44	B150-11-3	-0.00014	0.148	1.38	-5.22	0.094	0.054	271	37.1	-3.17	32	143	1.13
45	B150-15-3	-0.00011	0.143	1.64	-4.99	0.085	0.057	271	39.3	-3.25	34	148	1.01
Elzanaty, Nilson, and Slate (1986)													
46	G4	-0.00013	0.143	1.24	-4.72	0.097	0.045	382	34.3	-2.77	39	123	1.21

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
47	G5	-0.00020	0.149	0.79	-4.37	0.107	0.041	382	31.9	-2.32	30	101	1.10
48	G6	-0.00023	0.149	0.41	-2.39	0.093	0.056	382	37.9	-1.36	24	66	1.19
Johnson and Ramirez (1989)													
49	1	-0.00021	0.145	0.72	-4.11	0.103	0.042	493	32.7	-2.20	91	303	1.12
50	2	-0.00014	0.145	0.72	-3.37	0.107	0.038	500	30.7	-1.80	91	230	0.97
51	3	-0.00005	0.136	1.43	-2.45	0.074	0.063	500	42.6	-1.93	114	264	1.00
52	4	-0.00005	0.136	1.43	-2.45	0.074	0.063	500	42.6	-1.93	114	264	1.20
53	5	-0.00015	0.148	1.10	-4.74	0.103	0.045	493	33.6	-2.69	105	365	1.05
54	7	-0.00010	0.142	1.02	-3.45	0.099	0.043	500	33.5	-2.06	102	261	1.08
55	8	-0.00010	0.142	1.02	-3.45	0.099	0.043	500	33.5	-2.06	102	261	0.99
Roller and Russell (1990)													
56	6	-0.00005	0.139	1.43	-2.44	0.074	0.064	457	42.8	-1.93	221	516	1.29
57	7	-0.00011	0.139	1.43	-4.62	0.090	0.049	440	36.4	-2.89	228	786	1.00
58	8	-0.00005	0.133	2.48	-3.37	0.068	0.064	457	44.1	-2.93	274	721	0.67
59	9	-0.00006	0.142	2.48	-3.93	0.072	0.071	440	44.7	-3.21	297	916	0.82
60	10	-0.00007	0.109	2.48	-4.57	0.054	0.054	450	44.8	-3.53	319	1140	1.03
Sarsam and Al-Musawi (1992)													

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
61	AL2-N	-0.00022	0.148	0.80	-4.57	0.105	0.043	820	32.6	-2.44	28	100	1.15
62	AL2-H	-0.00011	0.149	1.49	-4.69	0.095	0.054	820	37.1	-2.97	35	122	1.01
63	BL2-H	-0.00011	0.144	1.50	-4.73	0.092	0.052	820	37.0	-2.99	38	126	1.10
64	CL2-H	-0.00012	0.147	1.39	-4.77	0.096	0.051	820	36.1	-2.93	40	126	1.17
Xie, Ahmad, Yu, Hino, and Chung (1994)													
65	NNW-3	-0.00026	0.148	0.81	-4.72	0.084	0.064	322	41.0	-2.74	17	78	1.11
66	NHW-3	-0.00011	0.148	1.95	-5.48	0.076	0.072	324	44.0	-3.71	26	105	0.97
67	NHW-3a	-0.00014	0.148	1.78	-6.04	0.075	0.073	323	44.7	-3.91	25	121	0.89
68	NHW-3b	-0.00016	0.136	2.05	-7.14	0.068	0.068	324	44.9	-4.59	26	142	0.87
69	NHW-4	-0.00011	0.147	1.96	-5.51	0.076	0.071	324	44.0	-3.73	26	106	0.89
McGormley, Cleary, and Ramirez (1996)													
70	BUS-1	-0.00026	0.147	0.84	-4.87	0.088	0.059	426	39.2	-2.80	48	238	1.14
71	EUS-1	-0.00025	0.149	0.86	-4.89	0.089	0.060	426	39.3	-2.82	48	240	1.25
72	BUH-1	-0.00025	0.149	0.90	-5.02	0.090	0.059	426	39.0	-2.90	49	246	1.12
73	EUH-1	-0.00025	0.149	0.88	-4.95	0.090	0.059	426	39.2	-2.86	48	243	1.26
74	BUIS-2	-0.00026	0.149	0.70	-4.08	0.082	0.067	426	42.1	-2.38	45	207	1.53

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
75	EUIS-2	-0.00023	0.149	0.96	-5.13	0.090	0.059	426	38.9	-2.98	50	252	1.23
76	BUIH-2	-0.00023	0.149	0.99	-5.23	0.091	0.058	426	38.7	-3.04	50	257	1.30
77	EUIH-2	-0.00023	0.149	1.00	-5.26	0.090	0.058	426	38.7	-3.06	51	258	1.24
78	BUH-3	-0.00022	0.149	1.06	-5.37	0.091	0.058	426	38.6	-3.13	52	265	1.09
79	EUH-3	-0.00021	0.149	1.09	-5.42	0.091	0.058	426	38.6	-3.17	52	268	1.16
80	BUIS-3	-0.00021	0.148	1.12	-5.52	0.090	0.057	426	38.4	-3.23	53	273	0.98
81	EUIS-3	-0.00021	0.148	1.11	-5.49	0.090	0.057	426	38.5	-3.21	52	271	0.98
Yoon, Cook, and Mitchell (1996)													
82	N1-N	-0.00014	0.148	0.71	-3.32	0.109	0.039	438	30.8	-1.78	143	337	1.35
83	N2-S	-0.00014	0.148	0.71	-3.32	0.109	0.039	438	30.8	-1.78	143	337	1.08
84	N2-N	-0.00018	0.142	0.71	-3.86	0.104	0.037	417	31.0	-2.02	143	389	1.24
85	M1-N	-0.00005	0.106	1.33	-2.36	0.058	0.048	438	42.3	-1.83	175	376	1.08
86	M2-S	-0.00005	0.122	1.33	-2.34	0.061	0.061	417	44.8	-1.83	175	399	1.38
87	M2-N	-0.00012	0.142	1.33	-4.67	0.094	0.048	438	35.5	-2.83	175	569	1.21
88	H1-N	-0.00005	0.126	1.72	-2.75	0.068	0.059	438	42.8	-2.23	191	436	1.11
89	H2-S	-0.00006	0.126	1.72	-3.23	0.067	0.059	429	43.1	-2.47	191	514	1.16
90	H2-N	-0.00009	0.149	1.72	-4.42	0.083	0.066	435	41.7	-3.05	191	692	1.04

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Tan, Teng, Kong, and Lu (1997)													
91	2-2.58/0.25	-0.00022	0.148	1.08	-5.46	0.085	0.063	353	40.6	-3.23	17	169	0.91
92	4-5.80/2.50	-0.00018	0.149	1.47	-6.10	0.083	0.065	385	41.4	-3.75	21	165	1.60
Kong, and Rangan (1998)													
93	s5-1	-0.00007	0.146	1.77	-3.69	0.075	0.071	569	44.2	-2.73	74	208	1.16
94	s7-1	-0.00006	0.147	1.48	-2.91	0.078	0.070	569	43.4	-2.19	83	177	1.23
95	s7-2	-0.00010	0.148	1.48	-4.37	0.092	0.055	569	37.7	-2.83	83	212	0.97
96	s7-3	-0.00013	0.143	1.48	-5.24	0.093	0.050	569	36.4	-3.21	83	243	1.02
97	s7-4	-0.00015	0.148	1.48	-5.66	0.094	0.054	569	37.0	-3.43	83	272	1.01
98	s7-5	-0.00016	0.146	1.48	-5.86	0.091	0.055	569	37.7	-3.55	83	292	1.04
99	s7-6	-0.00017	0.149	1.48	-5.98	0.090	0.059	569	39.1	-3.65	83	317	0.98
Yoshida (1998)													
100	YB2000/9	-0.00016	0.142	0.67	-3.41	0.105	0.036	470	30.3	-1.78	88	484	0.98
101	YB2000/4	-0.00014	0.134	0.71	-3.38	0.100	0.034	465	30.2	-1.78	90	466	1.45
102	YB2000/6	-0.00013	0.145	0.72	-3.21	0.106	0.038	465	31.0	-1.74	90	458	1.20
Ozcebe, Ersoy, and Tankut (1999)													
103	ACI56	-0.00008	0.131	1.15	-3.17	0.086	0.046	255	36.1	-2.05	30	89	1.05

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
104	TH56	-0.00005	0.148	1.25	-2.22	0.077	0.071	255	43.9	-1.73	31	83	1.25
105	TS56	-0.00012	0.147	1.21	-4.37	0.099	0.048	255	34.7	-2.61	31	118	1.10
106	ACI59	-0.00005	0.129	1.62	-2.65	0.069	0.060	255	42.7	-2.13	36	97	0.99
107	TH59	-0.00005	0.137	1.49	-2.50	0.069	0.067	255	44.4	-1.99	35	97	1.24
108	TS59	-0.00006	0.148	1.62	-3.08	0.075	0.074	255	44.7	-2.35	36	119	1.05
109	ACI36	-0.00005	0.124	1.49	-2.51	0.067	0.057	255	42.6	-1.99	30	87	1.21
110	TH36	-0.00005	0.134	1.49	-2.50	0.070	0.064	255	43.7	-1.99	30	90	1.57
111	TS36	-0.00006	0.126	1.49	-2.95	0.067	0.060	255	43.3	-2.21	30	104	1.50
112	ACI39	-0.00005	0.121	1.45	-2.47	0.066	0.056	255	42.6	-1.95	31	87	1.28
113	TH39	-0.00005	0.132	1.45	-2.46	0.068	0.063	255	43.8	-1.95	31	90	1.59
114	TS39	-0.00009	0.149	1.45	-3.98	0.090	0.059	255	39.0	-2.65	31	125	1.44
Frosch (2000)													
115	V1	-0.00015	0.139	0.72	-3.56	0.104	0.035	417	30.0	-1.86	148	477	0.83
116	V2	-0.00015	0.139	0.72	-3.56	0.104	0.035	417	30.0	-1.86	148	477	1.03
Cao (2001)													
117	SB2003/6	-0.00016	0.149	0.54	-2.81	0.110	0.039	483	30.8	-1.48	63	391	0.83

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
118	SB2012/6	-0.00015	0.144	0.61	-3.03	0.107	0.037	483	30.6	-1.60	105	432	1.42
Angelakos, Bentz, and Collins (2001)													
119	DB0.530M	-0.00017	0.138	0.63	-3.39	0.103	0.035	500	30.4	-1.76	56	281	0.93
120	DB120M	-0.00019	0.150	0.42	-2.34	0.105	0.044	500	32.9	-1.26	65	226	1.25
121	DB140M	-0.00016	0.144	0.75	-3.79	0.107	0.036	500	30.2	-1.98	79	332	0.83
122	DB165M	-0.00005	0.108	1.29	-2.31	0.057	0.051	500	43.1	-1.79	95	324	1.39
123	DB180M	-0.00005	0.134	1.58	-2.61	0.071	0.064	500	43.4	-2.09	101	369	1.07
124	BM100	-0.00013	0.147	0.93	-3.90	0.107	0.040	500	31.6	-2.16	76	352	0.97
Tompos and Frosch (2002)													
125	V36-2	-0.00013	0.142	0.85	-3.68	0.104	0.037	417	30.9	-2.00	156	509	0.96
126	V36-3	-0.00013	0.142	0.85	-3.68	0.104	0.037	417	30.9	-2.00	156	509	1.00
127	V18-2	-0.00020	0.145	0.71	-3.99	0.104	0.041	416	32.1	-2.12	37	157	1.10
128	V18-2c	-0.00020	0.145	0.71	-3.99	0.104	0.041	416	32.1	-2.12	37	157	0.97
Lin and Lee (2003)													
129	N1	-0.00042	0.082	0.64	-4.92	0.041	0.040	451	44.6	-2.78	54	254	1.27
130	N2	-0.00030	0.128	0.82	-5.10	0.065	0.063	451	44.6	-2.96	59	272	1.23
131	N3	-0.00023	0.150	1.02	-5.29	0.075	0.074	451	44.7	-3.16	63	291	1.21

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
132	N5	-0.00031	0.126	0.80	-5.07	0.063	0.062	451	44.7	-2.94	56	201	1.38
133	N6	-0.00026	0.147	0.84	-4.86	0.081	0.066	373	42.2	-2.84	59	251	1.29
134	N7	-0.00040	0.095	0.83	-6.01	0.047	0.047	549	44.8	-3.42	59	306	1.10
135	N8	-0.00036	0.103	0.85	-5.83	0.052	0.051	451	44.8	-3.34	60	300	1.20
136	N9	-0.00026	0.147	0.86	-4.93	0.081	0.066	451	42.0	-2.88	60	254	1.12
137	H1	-0.00043	0.081	0.63	-4.89	0.041	0.040	451	44.7	-2.76	54	253	1.29
138	H2	-0.00028	0.136	0.87	-5.13	0.068	0.067	451	44.7	-3.00	60	276	1.24
139	H3	-0.00023	0.148	1.04	-5.39	0.076	0.073	451	44.4	-3.21	64	296	1.21
140	H5	-0.00028	0.135	0.86	-5.14	0.068	0.067	451	44.7	-3.00	57	205	1.38
141	H6	-0.00026	0.149	0.82	-4.79	0.081	0.068	373	42.5	-2.80	59	248	1.28
142	H7	-0.00040	0.094	0.83	-6.01	0.047	0.047	549	44.8	-3.42	59	305	1.11
143	H8	-0.00038	0.100	0.81	-5.74	0.050	0.049	451	44.9	-3.28	59	295	1.23
144	H9	-0.00027	0.147	0.83	-4.83	0.081	0.067	451	42.3	-2.82	59	249	1.17
145	H10	-0.00030	0.126	0.82	-5.21	0.064	0.062	373	44.6	-3.02	59	276	1.31
146	H11	-0.00029	0.139	0.85	-5.06	0.070	0.069	549	44.9	-2.96	60	273	1.13
Cladera and Marí (2005)													
147	H50-2	-0.00015	0.146	0.99	-4.43	0.104	0.042	525	32.5	-2.46	43	150	1.18

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
148	H50-3	-0.00022	0.149	0.99	-5.18	0.094	0.055	538	37.4	-2.98	43	228	1.06
149	H50-4	-0.00022	0.149	0.99	-5.18	0.094	0.055	538	37.4	-2.98	47	232	1.06
150	H60-2	-0.00014	0.147	1.20	-4.81	0.100	0.048	534	34.7	-2.81	45	181	0.99
151	H60-3	-0.00019	0.149	1.20	-5.56	0.094	0.054	528	37.2	-3.25	46	246	1.05
152	H60-4	-0.00019	0.149	1.20	-5.56	0.094	0.054	528	37.2	-3.25	50	250	1.23
153	H75-2	-0.00012	0.148	1.36	-4.72	0.096	0.051	534	36.1	-2.89	47	187	1.09
154	H75-3	-0.00017	0.146	1.36	-5.75	0.092	0.054	528	37.4	-3.43	47	259	1.04
155	H75-4	-0.00017	0.146	1.36	-5.75	0.092	0.054	528	37.4	-3.43	52	263	0.86
156	H100-2	-0.00007	0.134	1.72	-3.66	0.069	0.065	533	44.2	-2.69	51	192	1.17
157	H100-3	-0.00013	0.144	1.72	-5.72	0.086	0.058	538	39.5	-3.65	51	278	0.91
158	H100-4	-0.00013	0.144	1.72	-5.72	0.086	0.058	538	39.5	-3.65	56	283	0.94
Lee and Kim (2008)													
159	L1-B	-0.00013	0.144	0.81	-3.55	0.104	0.039	215	31.4	-1.94	95	242	0.88
160	L2-B	-0.00013	0.144	0.81	-3.55	0.104	0.039	215	31.4	-1.94	117	258	0.99
161	L3-B	-0.00013	0.144	0.81	-3.55	0.104	0.039	215	31.4	-1.94	132	262	1.21
162	L4-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	27	79	1.15
163	L5-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	36	88	1.17

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
164	L6-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	40	85	1.50
165	S2-B	-0.00014	0.146	0.81	-3.71	0.107	0.039	215	31.0	-2.00	103	257	0.86
166	S3-B	-0.00014	0.146	0.81	-3.71	0.107	0.039	215	31.0	-2.00	103	257	0.90
167	S4-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	32	84	1.20
168	S5-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	32	84	1.10
169	S6-B	-0.00015	0.137	0.60	-3.04	0.101	0.036	215	30.6	-1.60	32	84	0.98
170	C1-B	-0.00016	0.141	0.39	-2.09	0.103	0.038	215	31.2	-1.10	81	159	1.28
Lee, Choi and Kim (2011)													
171	S30-3	-0.00020	0.144	0.66	-3.74	0.103	0.040	580	32.0	-1.98	90	257	1.17
172	S30-4	-0.00020	0.144	0.66	-3.74	0.103	0.040	580	32.0	-1.98	90	257	1.21
173	S30-5	-0.00030	0.126	0.67	-4.17	0.063	0.063	580	44.8	-2.42	90	443	1.01
174	S30-6	-0.00030	0.126	0.67	-4.17	0.063	0.063	580	44.8	-2.42	90	443	1.01
Author (2018)													
175	1V1-450-20	-0.00006	0.082	2.02	-3.64	0.041	0.041	614	44.8	-2.83	85	288	0.85
176	1V1-450-10	-0.00006	0.081	2.14	-3.76	0.041	0.040	614	44.8	-2.95	87	298	0.99
177	2V2-450-20	-0.00008	0.133	2.02	-4.48	0.066	0.066	614	44.9	-3.25	85	371	0.82
178	2V3-450-20	-0.00012	0.148	2.02	-5.92	0.076	0.072	614	44.1	-3.97	85	460	0.71

No.	Beam Name	ε_d	ε_r	σ_r	σ_d	ε_z	ε_x	f_{sz}	θ	τ_{xz}	D_u	$V_{predicted}$	$V_{test}/V_{predicted}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
179	1V1-900-20	-0.00006	0.099	2.04	-3.62	0.050	0.049	596	44.7	-2.83	109	467	0.89
180	1V1-900-10	-0.00006	0.116	2.14	-3.68	0.058	0.058	596	45.0	-2.91	110	479	0.91
181	2V2-900-20	-0.00008	0.146	2.04	-4.46	0.073	0.073	596	44.8	-3.25	109	623	0.89
182	2V3-900-20	-0.00011	0.149	2.04	-5.62	0.074	0.074	596	44.9	-3.83	109	835	0.90
183	1V1-1350-20	-0.00006	0.118	1.94	-3.48	0.059	0.059	596	45.0	-2.71	120	616	1.30
184	1V1-1350-10	-0.00006	0.116	2.14	-3.68	0.058	0.058	596	45.0	-2.91	124	657	1.41
185	2V2-1350-20	-0.00008	0.137	1.92	-4.34	0.069	0.068	596	44.8	-3.13	119	847	1.14
186	2V3-1350-20	-0.00013	0.149	1.92	-6.04	0.080	0.069	596	43.0	-3.97	119	1292	0.95
187	1V1-1800-20	-0.00006	0.117	2.12	-3.66	0.058	0.058	596	44.9	-2.89	77	601	1.54
188	2V2-1800-20	-0.00008	0.139	2.12	-4.58	0.070	0.068	596	44.6	-3.35	77	857	1.22
189	2V3-1800-20	-0.00011	0.148	2.12	-5.74	0.075	0.074	596	44.8	-3.93	77	1281	1.00
Average													1.13
Standard deviation													0.18
Coefficient of Variation													0.16
Minimum													0.67
Maximum													1.60