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2011

Tang, L., & Yang, Y. (2011). Analysis of synchronized charge extraction for piezoelectric energy harvesting. *Smart materials and structures*, 20(8), 085022-.

<https://hdl.handle.net/10356/100080>

<https://doi.org/10.1088/0964-1726/20/8/085022>

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[DOI:<http://dx.doi.org/10.1088/0964-1726/20/8/085022>].

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Analysis of Synchronized Charge Extraction for Piezoelectric Energy Harvesting

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ABSTRACT

In the past few years, various power conditioning circuits have been proposed to improve the efficiency of piezoelectric energy harvesting, among which the synchronized charge extraction (SCE) technique has been enthusiastically pursued. In the literature, the SCE technique is investigated based on the uncoupled or in-phase assumptions. The uncoupled assumption is only valid for weak electromechanical coupling and the in-phase assumption is not applicable for energy harvesting at off-resonance. In this paper, we derive an accurate analytical solution for the piezoelectric energy harvesting systems with the SCE technique. Based on this solution, we investigate the applicability of the SCE technique for different cases, i.e., the piezoelectric energy harvester (PEH) with various degrees of electromechanical coupling and the PEH excited at various frequencies. Circuit simulation is also conducted with an accurate circuit model derived for PEHs and the results validate the analytical outcomes. Both the accurate analytical solution and the circuit simulation show that the SCE technique cannot improve or even reduces the power output at resonance if the coupling of the PEH is not negligible. The SCE technique is found capable of significantly boosting the efficiency of energy harvesting only for the PEH vibrating at off-resonance frequencies or with weak coupling.

Keywords: piezoelectric energy harvesting; synchronized charge extraction; electromechanical coupling; vibration.

1 INTRODUCTION

The development of wireless sensor network (WSN) technologies has aroused a great many applications in civil structure health monitoring, military tracking devices, offshore climate observation systems, etc. However, using batteries to provide power for wireless communication nodes is a practical constraint because of their relative large size compared with the electronics as well as their replacement cost and limited lifespan. The abundant vibration energy in the environment has been realized by researchers as a potential power source, which can be converted to electricity as power supply via electrostatic [1], electromagnetic [2] or piezoelectric transductions [3]. The piezoelectric transduction surpasses the others because of its highest energy density [1] and no reliance on external magnetic field (required in electromagnetic transduction) or initial voltage (required in electrostatic transduction).

In the past few years, considerable research efforts have been devoted into improving the efficiency of PEHs. Some researchers have developed alternate structural configurations to improve their capability of power generation [4, 5]. Another concern of increasing the functionality of PEHs involves developing broadband vibration energy harvesting techniques, e.g., resonance frequency tuning [6, 7], multi-frequency energy converters [8, 9] and nonlinear oscillation techniques [10, 11], as summarized in the recent review article [12]. Besides, from the

perspective of circuit optimization, various power conditioning circuit techniques have been proposed to improve power transfer from PEHs to electrical loads or energy storage media. These circuit interfaces include the impedance adaptation [13], synchronized switching harvesting on an inductor (SSHI) [14-17] and synchronized charge extraction (SCE) [17-19]. Among them, the SCE technique has been enthusiastically pursued since no effort of load impedance matching is required [19]. When investigating the capability of these circuit interfaces, some assumptions have been applied by researchers to simplify the formulation procedures. Ottman *et al* investigated the impedance adaptation technique under the uncoupled assumption [13]. However, the uncoupled assumption is only valid for the PEH with weak electromechanical coupling. Wickenheiser *et al* [21] modeled the effects of electromechanical coupling on energy storage through piezoelectric energy harvesting under the in-phase assumption. In [14, 17-20], the efficiency improvement of the SCE and SSHI techniques was achieved by the in-phase analyses. However, the in-phase assumption can only apply for the analysis of the PEH vibrating at resonance. To date, accurate analytical solutions have been derived to predict the performance of standard circuits [22] and SSHI techniques [15]. However, no accurate analytical solution is available in the literature for the SCE technique. It is thus valuable to derive such an accurate solution to further investigate the applicability of the SCE circuit.

On the other hand, electronic simulators such as SPICE are powerful tools for circuit

design and optimization. If a PEH can be represented by certain circuit model, the whole energy harvesting system can be modeled in an electronic simulator to predict its performance. Under the uncoupled assumption, a circuit model of a PEH has been established as an ideal voltage source in series or an ideal current source in parallel with the internal capacitance of the transducer [13], but this model ignored damping effect on the dynamics of a PEH induced by the energy harvesting process. The accurate circuit model of a PEH should account for the backward electromechanical coupling effect, which can be derived by the approach proposed in [23]. With the derived circuit model, the power output from a PEH with different circuit interfaces can be predicted by system-level circuit simulation.

In this paper, we present an accurate analytical solution and a circuit simulation method to predict the performance of piezoelectric energy harvesting systems with the SCE technique. In Section 2, the solutions based on the uncoupled and in-phase assumptions are first presented, followed by the derivation of the accurate analytical solution. The performance of SCE circuit is then compared with that of a standard circuit by different analytical solutions to investigate the applicability of the SCE technique for efficiency improvement. When the PEH is excited at open circuit resonance frequency, the accurate solution shows that the power with the SCE circuit fails to increase by four times of the optimal power from a standard circuit, unless the electromechanical coupling is weak. The efficiency of the SCE can be dramatically lower than that of the standard circuit if the coupling of the PEH is

strong. When the PEH vibrates at off-resonance frequency, the accurate solution predicts that the power with the SCE circuit can be significantly improved as compared with the power with the standard circuit. Circuit simulation is conducted in Section 3. An accurate circuit model of PEHs is presented based on the approach in [23] and validated by the experimental results. Subsequently, the whole energy harvesting system is modeled in an electronic simulator. The SCE circuit is simulated and compared with the analytical solution for different cases, i.e., the PEH with various degrees of coupling and the PEH excited at resonance or at off-resonance frequencies. Summary remarks are given in Section 4.

2 ANALYTICAL SOLUTIONS

In the literature, a PEH is usually designed as a unimorph cantilever, a bimorph cantilever or a cantilevered beam bonded with piezoelectric transducers. The distributed parameter model of a cantilevered PEH can be depicted by the following modal governing equations,

$$\ddot{u}_r + 2\zeta_r \omega_{nr} \dot{u}_r + \omega_{nr}^2 u_r + \chi_r V = -f_r \ddot{u}_g \quad (1a)$$

$$I + C^S \dot{V} - \sum_{r=1}^{\infty} \chi_r \dot{u}_r = 0 \quad (1b)$$

where the subscript $()_r$ represents the r -th vibration mode; u_r , ω_{nr} , ζ_r and χ_r are the modal coordinate, natural frequency, damping ratio and modal electromechanical coupling coefficient of the r -th mode, respectively; $-f_r \ddot{u}_g$ is the modal force induced by base excitation $\ddot{u}_g$; V and I are the voltage across the PEH and the

current output from the PEH, respectively; and C^S is the clamped capacitance of the PEH. In the frequency range dominated by the first vibration mode, the system can be simplified as a single-degree-of-freedom (SDOF) system as,

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2u + \chi V = -f\ddot{u}_g \quad (2a)$$

$$I + C^S\dot{V} - \chi\dot{u} = 0 \quad (2b)$$

Equation (2) is mass normalized. If we set mass $M=1$, damping $\eta=2\zeta\omega_n$, stiffness $K=\omega_n^2$, coupling coefficient $\theta=\chi$ and excitation force $F=-f\ddot{u}_g$, we obtain the similar form of the electromechanical equations of the SDOF model given by other researchers [15, 22] as,

$$M\ddot{u} + \eta\dot{u} + Ku + \theta V = F \quad (3a)$$

$$I + C^S\dot{V} - \theta\dot{u} = 0 \quad (3b)$$

For the convenience of comparison with the previous work in the literature, the notations in equation (3) will be used in the later theoretical derivation.

A standard energy harvesting circuit comprises an AC-DC rectifier, a filtering capacitor C_R and an electrical load R_l in parallel, as shown in figure 1(a). In order to ensure a nearly constant DC voltage V_{DC} [22], C_R should be large enough such that the time constant R_lC_R is much larger than the oscillating period of the harvester. At the steady state, the rectifier will be blocked when $V < V_{DC}$ and will conduct and transfer energy when V reaches V_{DC} . The waveform of the voltage V across the PEH is shown in figure 1(b) [18].

The main components of the SCE circuit comprise an inductor L_{SCE} , a diode D_{SCE} and a switch S , as shown in figure 2(a). The switch S remains open in most time of one vibration period except for the two instances (t_1 and t_2 in figure 2(b)) when the deflection of the PEH reaches maximum. Once the PEH bends to its maximum deflection, S will be closed for a short time for energy transfer. Hence, the voltage across the PEH at these instances quickly drops to zero. The power consumption for the control circuit to turn on and off the switch can be compensated from less than 5% of the harvested power [17], thus it is not taken into consideration in this paper. At the steady state, the voltage waveform is assumed to have the profile shown in figure 2(b) [18]. This assumed profile will be verified in later circuit simulation.

Figure 1 (a) A standard energy harvesting circuit; (b) corresponding voltage across PEH and displacement waveform (source: [18], copyright: SAGE Publications)

Figure 2 (a) An SCE energy harvesting circuit; (b) corresponding voltage across

PEH and displacement waveform (source: [18], copyright: SAGE Publications)

2.1 SCE Circuit Analysis

To simplify the derivation procedure, the analysis of piezoelectric energy harvesting is usually carried out based on some assumptions. In this section, the power extracted from a PEH using the SCE technique under various assumptions will be summarized and an accurate analytical solution will be derived.

2.1.1 Solution Based on Uncoupled Assumption

The uncoupled assumption implies that the electromechanical coupling is extremely weak in the system, i.e., the backward coupling force will not affect the vibration of the cantilevered PEH. Hence, the coupling term in the governing equation (3a) can be dropped out as

$$M\ddot{u} + \eta\dot{u} + Ku = F \quad (4)$$

Referring to [22], the following parameters are introduced to simplify the analysis,

$$\omega_n = \sqrt{\frac{K}{M}}, \quad k_e^2 = \frac{\theta^2}{KC^S}, \quad \zeta = \frac{\eta}{2\sqrt{KM}}, \quad \Omega = \frac{\omega}{\omega_n}, \quad r = C^S \omega_n R_l \quad (5)$$

where k_e is the alternative electromechanical coupling coefficient of the PEH; and Ω and r are the normalized frequency and electrical resistance, respectively. For the open circuit condition, from equations (4) and (3b), we can easily obtain the magnitude of displacement u_{moc} and voltage V_{moc} as,

$$u_{moc} = \frac{C^S}{\theta} V_{moc} = \left(\frac{F_m}{K} \right) \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (6)$$

$$V_{moc} = \left(\frac{F_m}{\theta} \right) k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (7)$$

where F_m is the magnitude of the base excitation force. With the uncoupled assumption, the vibration amplitude will not be affected by the electrical load (or energy extraction procedure). Hence, for any circuit interface attached to the PEH, the magnitude of vibration u_m remains constant, i.e., $u_m = u_{moc}$. The charge accumulated from t_1^+ to t_2^- (the switch S is open in this duration.) on the piezoelectric transducer to be extracted at t_2 is

$$Q = \int_{t_1^+}^{t_2^-} \theta \dot{u} dt = \int_{t_1^+}^{t_2^-} C^S \dot{V} dt \quad (8)$$

Based on the profile of u and V shown in figure 2(b), the magnitude of voltage across the PEH V_m can be obtained from equation (8) as

$$V_m = 2 \frac{\theta}{C^S} u_m \quad (9)$$

It should be mentioned that equations (8) and (9) are independent of the uncoupled assumption and hence valid for the in-phase and accurate solutions presented in the later sections. Considering $u_m = u_{moc}$ and equations (6) and (7), equation (9) gives

$$V_m = 2V_{moc} = \left(\frac{F_m}{\theta} \right) 2k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \quad (10)$$

Obviously, the magnitude of voltage V_m is double of the magnitude of the open circuit voltage V_{moc} . The average power during the half period of vibration can be calculated by

$$P_{ave} = \frac{\frac{1}{2}C^S V_m^2}{\frac{T}{2}} = \frac{2\omega C^S}{\pi} V_{moc}^2 = \left(\frac{F_m^2}{M\omega_n} \right) \frac{2k_e^2 \Omega}{\pi} \frac{1}{(1-\Omega^2)^2 + (2\zeta\Omega)^2} \quad (11)$$

Written in the non-dimensional form as,

$$\hat{V}_m = \frac{V_m}{\left(\frac{F_m}{\theta} \right)} \quad \hat{V}_{moc} = \frac{V_{moc}}{\left(\frac{F_m}{\theta} \right)} \quad \hat{P}_{ave} = \frac{P_{ave}}{\left(\frac{F_m^2}{M\omega_n} \right)} \quad (12)$$

the voltage magnitude and average power finally become

$$\begin{cases} \hat{V}_m = 2\hat{V}_{moc} = 2k_e^2 \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \\ \hat{P}_{ave} = \frac{2k_e^2 \Omega}{\pi} \frac{1}{(1-\Omega^2)^2 + (2\zeta\Omega)^2} \end{cases} \quad (13)$$

2.1.2 Solution Based on In-phase Assumption

In a more accurate analysis, the coupling term θV should be kept in equation (3a).

For the open circuit condition, from equation (3b), we obtain

$$u_{moc} = \frac{C^S}{\theta} V_{moc} \quad (14)$$

Substituting it into equation (3a) gives

$$V_{moc} = \left(\frac{F_m}{\theta} \right) k_e^2 \frac{1}{\sqrt{(1+k_e^2 - \Omega^2)^2 + (2\zeta\Omega)^2}} \quad (15)$$

The charge Q accumulated from t_1^+ to t_2^- and the magnitude of voltage across the PEH V_m have the same form as equations (8) and (9), respectively. However, since the coupling term is kept, the vibration amplitude u_m is affected by the charge extraction procedure, i.e., $u_m \neq u_{moc}$ and hence $V_m \neq 2V_{moc}$. In fact, if the term θV in

equation (3a) is moved to the right-hand side, it acts as an additional excitation force induced by the electromechanical coupling. Obviously, when the coupling coefficient θ or V is not negligible, this term will affect the vibration of the PEH. Hence, we cannot simply derive the voltage magnitude V_m and the power output P_{ave} from the open circuit information (u_{moc} , V_{moc}) as we did for the uncoupled solution.

Multiplying equation (3a) by $\dot{u}$ and equation (3b) by V and integrating with respect to time t , we obtain the energy balance equations of the system as follows,

$$\int F\dot{u}dt = \frac{1}{2}M\dot{u}^2 + \frac{1}{2}Ku^2 + \int \eta\dot{u}^2dt + \int \theta V\dot{u}dt \quad (16a)$$

$$\int \theta V\dot{u}dt = \frac{1}{2}C^SV^2 + \int VI dt \quad (16b)$$

where the five terms in equation (16a) from left to right are the external input energy, kinetic energy, potential energy, mechanical dissipating energy and converted electrical energy; and the two terms at the right-hand side of equation (16b) are the energy accumulated in the PEH and energy consumed by the electrical load. At the steady state, during t_1^+ to t_2^- , there is no variation in kinetic and potential energy and no energy consumed by the load. Hence, combining equations (16a) and (16b), the energy balance during this half period of vibration (setting $t_1=0$ and $t_2=T/2$) can be expressed as

$$\int_0^{\frac{T}{2}} F\dot{u}dt = \int_0^{\frac{T}{2}} \eta\dot{u}^2dt + \frac{1}{2}C^SV_m^2 \quad (17)$$

At resonance, Lefeuvre *et al* [17] assumed that F and $\dot{u}$ are in phase, i.e., the phase between F and u is 90° ,

$$\begin{cases} F = F_m \sin(\omega t) \\ u = -u_m \cos(\omega t) \end{cases} \quad (18)$$

This is a relatively accurate assumption for a pure mechanical system. However, for an electromechanical coupling system like PEH vibrates at both resonance and off-resonance frequencies, the applicability of the in-phase assumption requires further investigation. Equation (18) is given in such a form that at $t_1=0$ and $t_2=T/2$, the displacement reaches its minimum and maximum, respectively, as shown in figure 2(b). Considering equation (9), substituting equation (18) to equation (17) and integrating over the half period of vibration,

$$F_m u_m \omega \frac{\pi}{2\omega} = \eta \omega^2 u_m^2 \frac{\pi}{2\omega} + \frac{2\theta^2}{C^S} u_m^2 \quad (19)$$

we obtain the relationship between the magnitudes of displacement u_m and force F_m as

$$u_m = \frac{F_m}{\eta\omega + \frac{4\theta^2}{C^S\pi}} \quad (20)$$

Hence, considering equation (9), the magnitude of voltage and the average power can be obtained as

$$V_m = 2 \frac{\theta}{C^S} \frac{F_m}{\eta\omega + \frac{4\theta^2}{C^S\pi}} = \frac{F_m}{\theta} 2k_e^2 \frac{1}{2\zeta\Omega + \frac{4k_e^2}{\pi}} \quad (21)$$

$$P_{ave} = \frac{\frac{1}{2} C^S V_m^2}{\frac{T}{2}} = \frac{2\theta^2}{C^S} \frac{\omega}{\pi} \frac{F_m^2}{\left(\eta\omega + \frac{4\theta^2}{C^S\pi}\right)^2} = \frac{F_m^2}{M\omega_n} \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \quad (22)$$

which can be written in the non-dimensional form as

$$\begin{cases} \hat{V}_m = 2k_e^2 \frac{1}{2\zeta\Omega + \frac{4k_e^2}{\pi}} \\ \hat{P}_{ave} = \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \end{cases} \quad (23)$$

2.1.3 Accurate Solution

Even though the result of in-phase analysis is valid for energy harvesting at resonance frequency, they cannot take into account the response of the harvester at off-resonance frequency (we will give detailed discussion later). A more accurate procedure is to set an unknown phase angle φ between the force and displacement, that is,

$$\begin{cases} F = -F_m \cos(\omega t + \varphi) \\ u = -u_m \cos(\omega t) \end{cases} \quad (24)$$

Equation (24) is given in such a form that at $t_1=0$ and $t_2=T/2$, the displacement reaches its minimum and maximum, respectively, as illustrated in figure 2(b).

Considering equation (9) and substituting (24) into equation (17), we have

$$\int_0^{\frac{T}{2}} -F_m \cos(\omega t + \varphi) u_m \omega \sin(\omega t) dt = \int_0^{\frac{T}{2}} \eta u_m^2 \omega^2 \sin^2(\omega t) dt + \frac{2\theta^2}{C^S} u_m^2 \quad (25)$$

The integration over the half period gives

$$F_m \sin \varphi = \eta u_m \omega + \frac{4\theta^2}{C^S \pi} u_m \quad (26)$$

There are two unknowns, i.e., u_m and φ , in equation (26), thus we need another equation to solve for them. We follow a similar procedure in [22] to establish this

equation. Differentiating equation (3a) with respect to time t , we obtain

$$M \frac{d}{dt} \ddot{u} + \eta \frac{d}{dt} \dot{u} + K \frac{d}{dt} u + \theta \dot{V} = \frac{d}{dt} F \quad (27)$$

From t_1^+ to t_2^- , the switch S is open and the circuit is an open circuit. Hence, equation (3b) is written as

$$\dot{V} = \frac{1}{C^S} (\theta \dot{u}) \quad (28)$$

Substituting it into equation (27), we obtain

$$M \frac{d}{dt} \ddot{u} + \eta \frac{d}{dt} \dot{u} + \left(K + \frac{\theta^2}{C^S} \right) \frac{d}{dt} u = \frac{d}{dt} F \quad (29)$$

Integrating equation (29) from $t_1=0$ to $t_2=T/2$,

$$Mu_m \omega^2 \cos(\omega t) \Big|_0^{\frac{T}{2}} + \eta u_m \omega \sin(\omega t) \Big|_0^{\frac{T}{2}} - \left(K + \frac{\theta^2}{C^S} \right) u_m \cos(\omega t) \Big|_0^{\frac{T}{2}} = -F_m \cos(\omega t + \varphi) \Big|_0^{\frac{T}{2}} \quad (30)$$

we find

$$F_m \cos \varphi = \left(K - M\omega^2 + \frac{\theta^2}{C^S} \right) u_m \quad (31)$$

Hence, the sum of square of equations (26) and (31) gives

$$\left[\left(K - M\omega^2 + \frac{\theta^2}{C^S} \right) u_m \right]^2 + \left[\eta u_m \omega + \frac{4\theta^2}{C^S \pi} u_m \right]^2 = F_m^2 \quad (32)$$

Solving for u_m , we obtain

$$u_m = \frac{F_m}{\sqrt{\left(K - M\omega^2 + \frac{\theta^2}{C^S} \right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^S \pi} \right)^2}} \quad (33)$$

Using equation (9), we obtain the magnitude of voltage across the PEH as

$$\begin{aligned}
V_m &= 2 \frac{\theta}{C^s} \frac{F_m}{\sqrt{\left(K - M\omega^2 + \frac{\theta^2}{C^s}\right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^s\pi}\right)^2}} \\
&= \frac{F_m}{\theta} 2k_e^2 \frac{1}{\sqrt{\left(1 - \Omega^2 + k_e^2\right)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2}}
\end{aligned} \tag{34}$$

and the power harvested as

$$\begin{aligned}
P_{ave} &= \frac{\frac{1}{2}C^s V_m^2}{\frac{T}{2}} = 2 \frac{\theta^2}{C^s} \frac{\omega}{\pi} \frac{F_m^2}{\left(K - M\omega^2 + \frac{\theta^2}{C^s}\right)^2 + \left(\eta\omega + \frac{4\theta^2}{C^s\pi}\right)^2} \\
&= \frac{F_m^2}{M\omega_n} \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(1 - \Omega^2 + k_e^2\right)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2}
\end{aligned} \tag{35}$$

Finally, equations (34) and (35) can be written in non-dimensional forms as

$$\begin{cases} \hat{V}_m = 2k_e^2 \frac{1}{\sqrt{\left(1 - \Omega^2 + k_e^2\right)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2}} \\ \hat{P}_{ave} = \frac{2k_e^2\Omega}{\pi} \frac{1}{\left(1 - \Omega^2 + k_e^2\right)^2 + \left(2\zeta\Omega + \frac{4k_e^2}{\pi}\right)^2} \end{cases} \tag{36}$$

Equation (36) provides the accurate analytical prediction for the performance of PEHs using the SCE technique.

2.2 Standard Circuit

The performance of PEHs with the standard circuit interface under different assumptions has been investigated in the literature [17, 22]. We summarize these findings below to compare with the performance of the SCE circuit.

Solution based on uncoupled assumption

The non-dimensional power delivered on the load with a standard circuit can be found in [22],

$$\hat{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{(1 - \Omega^2)^2 + (2\zeta\Omega)^2} \quad (37)$$

The optimal load and power at the steady state can be obtained by $\partial\hat{P}/\partial r = 0$ as

$$\begin{cases} r_{opt} = \frac{\pi}{2\Omega} \\ \hat{P}_{opt} = \frac{k_e^2 \Omega}{2\pi} \frac{1}{(1 - \Omega^2)^2 + (2\zeta\Omega)^2} \end{cases} \quad (38)$$

Solution based on in-phase assumption

With the in-phase assumption, the non-dimensional power is described as [18],

$$\hat{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{\left(2\zeta\Omega + \frac{2k_e^2 r \Omega}{\left(r\Omega + \frac{\pi}{2}\right)^2}\right)^2} \quad (39)$$

The optimal load and power at the steady state are determined by two cases.

Case (1): $\frac{k_e^2}{\zeta} - 2\pi\Omega \leq 0$. The optimal load and power are,

$$\begin{cases} r_1^{opt} = \frac{\pi}{2\Omega} \\ \hat{P}_{opt} = \frac{k_e^2 \Omega}{2\pi} \frac{1}{\left(2\zeta\Omega + \frac{k_e^2}{\pi}\right)^2} \end{cases} \quad (40)$$

Case (2): $\frac{k_e^2}{\zeta} - 2\pi\Omega > 0$. The optimal load and power are,

$$\begin{cases} r_{2,3}^{opt} = \frac{\left(\frac{k_e^2}{\zeta} - \pi\Omega\right) \pm \sqrt{\left(\frac{k_e^2}{\zeta} - \pi\Omega\right)^2 - (\pi\Omega)^2}}{2\Omega^2} \\ \hat{P}_{opt} = \frac{1}{16\zeta} \end{cases} \quad (41)$$

In Case (2), there exist two optimal loads to achieve the equal optimal power at each frequency.

Accurate solution

The accurate solution of the power for the PEH using a standard circuit is given in [22] as,

$$\hat{P} = \frac{k_e^2 r \Omega^2}{\left(r\Omega + \frac{\pi}{2}\right)^2} \frac{1}{\left(1 - \Omega^2 + \frac{r\Omega k_e^2}{r\Omega + \frac{\pi}{2}}\right)^2 + \left(2\zeta\Omega + \frac{2r\Omega k_e^2}{\left(r\Omega + \frac{\pi}{2}\right)^2}\right)^2} \quad (42)$$

Obviously, it is difficult to derive the explicit solution for the optimal load from equation (42). The optimal load and power can be determined numerically. For the strong coupling case, Shu and Lien [22] also found two equal optimal powers. The two optimal powers are achieved at $\Omega_1^{opt} \approx \Omega_{sc}$ for $r_1^{opt} \ll 1$ and at $\Omega_2^{opt} \approx \Omega_{oc}$ for $r_2^{opt} \gg 1$, where $\Omega_{sc}=1$, $\Omega_{oc}=\sqrt{1+k_e^2}$ are the normalized short circuit and open circuit resonance frequencies, respectively. This is different from equation (41), in which, for each frequency, equal optimal power can be achieved at two optimal loads.

2.3 Discussion

We have shown various analytical solutions of the power achieved with the SCE circuit under different assumptions in Section 2.1. Considering various degree of electromechanical coupling, we will examine the applicability of the SCE technique for efficiency improvement of PEHs.

2.3.1 Weak coupling

In this case, we assume the alternative coupling coefficient $k_e=0.05$ and damping ratio $\zeta=0.04$. In the literature, k_e^2/ζ has been proposed as an indicator of coupling effect [15-16, 22] and some criterions have been suggested based on this indicator for different circuit interfaces. For the standard circuit interface, Shu and Lien [22] defined the strong coupling by $k_e^2/\zeta > 10$ for the appearance of two optimal pairs (electric load and frequency). For the SCE circuit, we propose a criterion for the extent of coupling effect, which will be discussed in Section 2.3.4. Here, $k_e^2/\zeta=0.0625$ corresponds to a weak coupling. Figures 3(a)~3(c) compare the magnitude of voltage across the PEH V_m when the SCE circuit works with the magnitude of the open circuit voltage V_{moc} . And figures 3(d)~3(f) compare the power harvested with the SCE and standard circuits from the uncoupled, in-phase and accurate solutions, respectively.

Comparison of uncoupled and accurate solutions

Firstly, since the coupling is weak, V_{moc} and V_m from the uncoupled and accurate solutions are similar, as shown in figures 3(a) and 3(c). V_m approaches two times of V_{moc} , and the power harvested with the SCE circuit increases by nearly four times of the optimal power harvested with the standard circuit at various frequencies from the accurate solution (figure 3(c) and 3(f)), similar to the uncoupled solution (figure 3(a) and 3(d)). These observations show that the SCE circuit is useful to boost the power output if the PEH has a weak electromechanical coupling. In such case, the uncoupled and accurate solutions give close predictions.

Comparison of in-phase and accurate solutions

In figures 3(b) and 3(e), V_m and the power harvested obtained by the in-phase solution are totally different from the accurate solution. However, at open circuit resonance frequency $\Omega_{oc} = \sqrt{1 + k_e^2}$, it is shown that V_m and the power with the SCE circuit obtained by the in-phase solution are exactly the same as the accurate solution. If we revisit equations (23) and (36), the above conclusion is more obvious and can be extended for any degree of coupling. Since the in-phase solution can only be applied to predict the system performance at open circuit resonance, it will not be considered in later discussion.

Figure 3 Weak coupling case: (a)~(c) Comparison of V_m with SCE circuit and V_{moc} ; (d)~(f) comparison of power harvested with SCE and standard circuits. (a)(d): uncoupled solution; (b)(e): in-phase solution; (c)(f): accurate solution.

2.3.2 Medium coupling

In this case, we fix the damping ratio $\zeta=0.04$ but change the coupling coefficient to $k_e=0.3$. Hence, $k_e^2/\zeta=2.25$ is obtained, which corresponds to a medium coupling effect according to the criterion discussed in Section 2.3.4. Since the coupling is not weak, there is a frequency shift between the short circuit and open circuit resonances in the accurate solution, as shown in figure 4(b). However, there is no difference between the Ω_{sc} and Ω_{oc} in the uncoupled solution (figure 4(a)) since the coupling term is dropped out in equation (4). Figures 4(a) and 4(b) compare V_m (the magnitude of voltage across the PEH when the SCE circuit works) and V_{moc} (the magnitude of the open circuit voltage), and figures 4(c) and 4(d) compare the power harvested with the SCE and standard circuits from the uncoupled and accurate solutions.

Comparison of uncoupled and accurate solutions

Firstly, consider the PEH vibrating at open circuit resonance frequency. From the accurate solution, V_m cannot be increased to the double of V_{moc} . Actually, it is even less than V_{moc} , as shown in figure 4(b). In terms of power harvested, the SCE circuit achieves a normalized power of 1.5204, failing to significantly outperform the standard circuit, as shown in figure 4(d). However, the uncoupled solution gives the wrong estimation that V_m with the SCE circuit still can reach two times of V_{moc} and the corresponding power is improved by four times compared to the standard circuit,

as shown in figures 4(a) and 4(c). In fact, by merely examining the performance of standard circuit predicted by the uncoupled and accurate solutions, we can see that the uncoupled solution gives wrong estimation (the optimal powers from uncoupled and accurate solutions are 2.2381 and 1.1992, respectively, as shown in figures (4c) and (4d)). It is concluded that the uncoupled solution is not applicable when the electromechanical coupling is not negligible. Hence it will not be considered in the later discussion of strong coupling.

Merits of SCE technique at off-resonance

It is observed from figure 4(d) that although the SCE circuit cannot improve the power output at resonance, the performance of PEH vibrating at off-resonance is significantly enhanced. For example, at $\Omega=0.7$, the power with the SCE and the optimal power with the standard circuit are 0.1031 and 0.032, respectively, indicating a power increase of 322% by the SCE. Such phenomenon shows the merits of the SCE circuit for broadening the useful frequency bandwidth of piezoelectric energy harvesting, despite that it does not prominently increase the maximum power output from the PEH.

Figure 4 Medium coupling case: (a)~(b) Comparison of V_m with SCE circuit and V_{moc} ; (c)~(d) comparison of power harvested with SCE and standard circuits. (a)(c): uncoupled solution; (b)(d): accurate solution.

2.3.3 Strong coupling

In this case, we fix the damping ratio $\zeta=0.04$ but further increase the coupling coefficient to $k_e=0.9$. Hence, $k_e^2/\zeta=20.25$ is obtained, which corresponds to a strong coupling effect. Figure 5(a) compares V_m and V_{moc} , and figure 5(b) compares the power harvested with the SCE and standard circuits from the accurate solution. With

a strong coupling of the PEH, V_m is drastically reduced to a level much smaller than V_{moc} . And it is quite uniform over a wide frequency range, as shown in figure 5(a). Interestingly, although V_m approaches nearly two times of V_{moc} at off-resonance frequency $\Omega=0.7$ (figure 5(a)), the SCE circuit does not show significant improvement in power output compared to the standard circuit, as shown in figure 5(b). Obviously, when compared with the medium coupling case, the frequency at which the SCE circuit significantly outperforms the standard circuit should be further away from the resonances (Ω_{sc} and Ω_{oc}).

Figure 5 Strong coupling case: (a) Comparison of V_m with SCE circuit and V_{moc} ; (b) comparison of power harvested with SCE and standard circuits from accurate solution.

2.3.4 Applicable region of SCE circuit

In the previous three case studies, the accurate analytical solution shows that the

applicability of the SCE circuit becomes much more limited with the increase in the degree of electromechanical coupling. We define the *applicable region* of the SCE circuit as a region where the power harvested by the SCE circuit surpasses that obtained by the standard circuit. Figures 6(a) to 6(c) show the contour plots of the power harvested by the SCE and standard circuits versus frequency Ω and coupling coefficient k_e for three given damping ratios $\zeta=0.04, 0.02$ and 0.01 , respectively. The contours with mesh indicate the applicable regions of the SCE circuit, which are projected in the Ω - k_e plane in figures 6(d)~6(f). If we define G as the ratio of power harvested by the SCE circuit to the optimal power by the standard circuit, the regions with different blue colors in figures 6(d)~6(f) represent $4>G>3$, $3>G>2$ and $2>G>1$.

It is noted that the applicable region ($G>1$) bifurcates at a critical point k_e^* (for example, $k_e^*=0.33$ in figure 6(d)). For $k_e<k_e^*$, the SCE circuit outperforms the standard circuit at any frequency. Conversely, for $k_e>k_e^*$, the standard circuit can outperform the SCE circuit especially near resonance and for larger k_e . Similarly, the region for $4>G>3$ bifurcates at a critical point k_e^{**} (for example, $k_e^{**}=0.12$ in figure 6(d)). For $k_e<k_e^{**}$, at any frequency, the SCE circuit significantly improves the performance of PEH. These critical points shift to left with the decrease of damping ratio, as shown in figures 6(d)~6(f). However, we have $(k_e^*)^2/\zeta \approx 2.7$ and $(k_e^{**})^2/\zeta \approx 0.36$ for any damping ratio. Based on these critical points, we propose a criterion to classify the coupling effect of a system using the SCE circuit as follows,

- 1) Strong coupling: $k_e^2/\zeta > 2.7$, i.e., the standard circuit can outperform the SCE

circuit around the resonance.

- 2) Weak coupling: $k_e^2/\zeta < 0.36$, i.e., the SCE circuit always significantly outperforms the standard circuit ($4 > G > 3$) at any frequency.

The values of k_e and ζ have been selected based on this criterion in the previous Sections 2.3.1~2.3.3 for case studies.

Another phenomenon observed in figures 6(a)~6(c) is that, the maximum powers achieved by the SCE and standard circuits are equal, and the SCE circuit can achieve this with a smaller coupling k_e . Besides, with the decrease of damping ratio from 0.04 to 0.02 to 0.01, the maximum power output increases from 1.562 to 3.12 to 6.22, as shown in figures 6(a)~6(c).

(a)

(d)

Figure 6 (a)~(c) Power harvested by SCE and standard circuits versus Ω and k_e ;

(d)~(f) applicable region of SCE circuit for three damping ratio $\zeta=0.04, 0.02, 0.01$.

2.3.5 Summary of analytical outcomes

In summary, the accurate solution shows that (a) for a PEH with weak electromechanical coupling, the SCE technique is able to boost the power output by nearly four times compared to the standard circuit. (b) For medium coupling case, the SCE circuit fails to significantly outperform the standard circuit for PEH vibrating at resonance. However, the performance of PEH at off-resonance is

improved prominently by the SCE circuit. This characteristic of SCE circuit can be utilized to broaden the bandwidth of piezoelectric energy harvesting. (c) With further increase of the coupling, the SCE technique dramatically undermines the power output compared to the standard circuit. Although it still can improve the performance when the PEH vibrates at off-resonance, the working frequency should be further away from the resonance.

In addition, the uncoupled solution can provide approximate predictions only when the PEH has a weak electromechanical coupling. Furthermore, for various degrees of coupling, the in-phase solution cannot provide good estimation of the performance of the SCE circuit except for the PEH vibrating at the open circuit resonance frequency.

3 CIRCUIT SIMULATION AND VALIDATION

The SPICE simulators (such as Multisim 10.0 [24]) are powerful tools for circuit design and analysis. If one can obtain the circuit model of PEH, together with the standard or SCE circuit, the whole energy harvesting system can be evaluated by circuit simulation. In this section, we first derive the accurate circuit model of PEH. Subsequently, we conduct the system-level circuit simulation to validate the analytical outcomes obtained in Section 2.

3.1 Circuit Models of PEH

Regarding the coupling term $\theta \dot{u}$ as the current source i , equation (3b) actually satisfies the Kirchoff's current law. Since the electromechanical coupling term θV in equation (3a) is neglected with the uncoupled assumption, vibration of the PEH, i.e., u and thus the current source i in equation (3b), is independent of the electrical load. For open circuit condition, equation (3b) can be written as

$$C^S \dot{V} - i = 0 \quad (43)$$

which represents a circuit with an ideal current source placed in parallel to the internal capacitance of the PEH C^S (figure 7(a)). This model, or its equivalent form, i.e., an ideal voltage source placed in series with C^S (figure 7(b)), has been frequently utilized as the circuit model of PEH by some researchers [13].

Figure 7 Uncoupled circuit models of PEH

To derive the accurate circuit model of PEH, one should start from equation (3a) and keep the electromechanical coupling term θV . Analogizing the displacement u as charge q , the excitation force F as the voltage source v , and the coefficients M , η , K and θ as the inductor L , resistor R , reciprocal of capacitor C and ideal transformer

with turn ratio N , equation (3a) can be rewritten as

$$L\ddot{q} + R\dot{q} + q/C + NV = v \quad (44)$$

which satisfies the Kirchoff's voltage law and represents the constitutive equation of an LCR circuit, as shown in figure 8. Such circuit is the accurate circuit model of the PEHs. The parameters L , R , C , N and v in the model can be derived by the finite element analysis (FEA) approach proposed in [23]. It should be mentioned that equations (3a) and (44) are based on the SDOF system. Hence, the circuit model is valid only for low frequency range dominated by the first mode. As to the multi-mode circuit based on the distributed parameter system, readers can refer to [23] for details.

Before applying this circuit model (figure 8) in simulation with the SCE technique, experimental tests are carried out to validate it. Figure 9 illustrates the prototype of the fabricated PEH and the experimental apparatus. Two piezoelectric transducers, electrically connected in parallel, are bonded to each side of an aluminum substrate (the beam) by epoxy adhesive. A steel proof mass is attached at the beam tip. The beam is clamped on an electromagnetic shaker. An accelerometer is attached to the shaker to monitor the input acceleration of the base excitation. Table 1 lists the properties of the fabricated PEH prototype, which are required in FEA to derive the parameters of the circuit model of the PEH. These derived parameters are shown in figure 8.

Figure 8 Accurate circuit model of fabricated PEH prototype

Figure 9 PEH prototype and experimental apparatus

A unity root mean square (RMS) acceleration input is kept in the experiment. Hence, here and in later circuit simulations, the excitation is assumed an RMS acceleration of 1 m/s^2 . Firstly, the frequency responses of V_{oc} and I_{sc} in the range dominated by the first mode from simulation and experiment are compared in figure 10. It is notable that the derived circuit model predicts the difference between the open circuit and short circuit resonance frequencies as observed in the experiment, which is one of the prominent characteristics of the electromechanical coupling effect. Additionally, the magnitudes of V_{oc} and I_{sc} from simulation with the derived circuit model are comparable with those from the experiment. Hence, the derived circuit model is validated. It should be mentioned that the response curves from the experiment are slightly bent toward left (figure 10(a)), resulting from the

unavoidable imperfect clamping condition in the experiment [3] (softening the stiffness).

Table 1 Properties of PEH

Item	Piezoelectric Transducer	Epoxy Layer	Substrate	Proof Mass
Dimensions (mm×mm×mm)	85×28×0.2 (active volume)	85×28×0.1	178×32×1.5	18.5×32×14
Density (kg/m ³)	7750	2200	2700	7850
Modulus (GPa)	E ₁₁ =60.98	E=0.1	E=70	E=200
Poisson's ratio	0.35	0.38	0.35	0.29
Dielectric constant	$\epsilon_{33}^S/\epsilon_0=830$	—	—	—
Piezoelectric constant (10 ⁻¹² m/V)	d ₃₁ =−185	—	—	—
Rayleigh damping coefficients	$\alpha=1.7, \beta=2.25 \times 10^{-5}$ (calculated by experimentally measured 1 st and 2 nd mode damping ratio $\zeta_1=0.011, \zeta_2=0.0114$)

Figure 10 Frequency responses of V_{oc} and I_{sc} : (a) experiment and (b) simulation

3.2 System Schematic

With the derived circuit model of PEH, the entire energy harvesting system can be modeled in the circuit simulator Multisim 10.0. The overall circuit diagram consists

of the circuit model of PEH, a full-wave rectifier D , the SCE circuit, a filter capacitor C_R and an electrical load R_l , as shown in figure 11. If the SCE part is removed, the system is degraded to a standard energy harvesting circuit. In the SCE circuit, L_{SCE} is used to temporarily store the energy extracted. Each time the beam reaches the maximum deflection, the switch S , controlled by the pulse generator, will conduct and transfer the energy from the PEH to the inductor L_{SCE} . Hence, the period of pulse generator is set to be half of the excitation period, i.e., $T/2$. When S is conducted, the internal capacitance C^S and the L_{SCE} form an LC circuit. Hence, the conduction duration of S should be set to be $T_0/4$ to discharge C^S , where

$$T_0 = 2\pi\sqrt{C^S L_{SCE}} \quad (45)$$

We assume $L_{SCE} = 22.5\text{mH}$ in the following simulation to ensure $T_0 \ll T$, such that the energy can be instantly transferred from the PEH. A capacitor of $100\mu\text{F}$ is selected as the filter capacitor C_R to smoothen the voltage across the load R_l . An oscilloscope is used to measure the voltage across the PEH and the load R_l . A wattmeter is used to directly measure the power delivered on the load R_l .

Figure 11 Overall circuit diagram of piezoelectric energy harvesting system with
SCE technique

3.3 Simulation and Validation

3.3.1 SCE at resonance

The system performance with SCE and standard circuits is first investigated at resonance. With the derived parameters, $C^S=366.483nF$, $N=0.0085$ (analogy of θ) and $C=124\mu F$ (analogy of $1/K$), the alternative coupling coefficient k_e is obtained as 0.1564. With damping ratio $\zeta=0.011$, we can obtain $k_e^2/\zeta=2.22$, which corresponds to a medium degree of coupling according to the proposed criterion in Section 2.3.4. Figure 12(a) shows the transient response of the voltage across the PEH when it vibrates at the open circuit resonance frequency f_{oc} . The dashed line is the open circuit voltage magnitude V_{moc} obtained from the simulation. The dash-dotted line indicates the accurate analytical solution of V_m (the magnitude of voltage across the PEH with the SCE circuit at the steady state). Similar to the accurate analytical solution, when the PEH vibrates at the steady state, V_m cannot approach two times of V_{moc} and even smaller than V_{moc} . However, if we replace the accurate circuit model of the PEH in the system diagram (figure 11) with the uncoupled circuit model (figure 7(b)), we observe erroneous prediction that V_m approaches two times of V_{moc} , as shown in figure 13. These observations are consistent with the findings obtained from the accurate solution in Section 2.3. With different values of R_l , we keep running the simulation until the PEH enters the steady state, and then acquire the

power delivered on R_l from the wattmeter. The magnitudes of power by the SCE and standard circuits are shown in figure 12(b). Like the accurate analytical solution, the simulation results show that the SCE circuit fails to improve the system performance by four times compared to the standard circuit, though it increases the power by 45.2%. Besides, in figure 12, both V_m and power at the steady state from simulation agree well with the accurate analytical solution, verifying the analytical derivation in Section 2.1. In addition, when the SCE circuit works, the enlarged view of voltage profile across the PEH shown at the bottom of figure 12(a) depicts the same pattern as that assumed in figure 2(b).

Figure 12 Performance of PEH at f_{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state.

Figure 13 Voltage across PEH at f_{oc} under uncoupled assumption.

3.3.2 SCE at off-resonance

In figure 10(b), it is noted that at 13Hz, the voltage response dramatically decreases, which is regarded as “*off-resonance*” status. In this case, the open circuit voltage V_{moc} is obtained as 5.832v from the simulation. The simulation predicts that V_m nearly doubles V_{moc} when SCE circuit works at the steady state, as shown in figure 14(a). Besides, the simulation predicts that the power with the SCE circuit is around 0.517mW and the optimal power with the standard circuit is 0.13724mW, as shown in figure 14(b). This means a nearly four times increase of power by using the SCE circuit. Furthermore, in figure 14(b), it is noted that the predictions by simulation are smaller than the accurate analytical solution. This is probably due to the loss in the full-wave rectifier. Figure 15 indicates that there is a voltage drop of around 0.8v after the rectifier, which is not considered in analytical solution. Since V_m at off-resonance is much smaller than that at resonance, such voltage drop is significant for V_m , which results in the loss of power. The loss of power for SCE circuit is around 15.7% while the loss of optimal power for the standard circuit is around

24.8%. This is reasonable since the SCE circuit enhances the magnitude of voltage across the PEH V_m and hence reduces the loss of voltage as well as the loss of power.

Figure 14 Performance of PEH at off-resonance (13Hz): (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state.

Figure 15 Voltage loss in rectifier

3.3.3 SCE from PEH with weak and strong coupling

In Sections 3.3.1 and 3.3.2, $N=0.0085$ is derived from the fabricated PEH, which corresponds to a medium degree of coupling. To further investigate the performance of SCE circuit for weak and strong coupling, N is set to be 0.001 and 0.04, which corresponds to $k_e^2/\zeta=0.0308$ and 49.22, respectively. The other parameters derived in figure 8 are kept unchanged.

For the weak coupling case $k_e^2/\zeta=0.0308$, when the PEH vibrates at the open circuit resonance frequency f_{oc} , V_m approaches two times of V_{moc} at the steady state, as shown in figure 16(a). This is similar to what we observed in the accurate analytical solution. In figure 16(b), nearly four times increase of power can be achieved by using the SCE circuit compared to the optimal power by using the standard circuit. For the strong coupling case $k_e^2/\zeta=49.22$, V_m and the power with the SCE circuit are dramatically smaller than those with the standard circuit, as shown in Figure 17. Overall, these simulation results are not only consistent with the analytical solution but also validate the conclusions drawn in Section 2.3. The reason for the slight discrepancy between the simulation and analytical solution for the weak coupling case (figure 16(b)) is the same as what we discussed in Section 3.3.2.

Figure 16 Performance of PEH with weak coupling at f_{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state.

Figure 17 Performance of PEH with strong coupling at f_{oc} : (a) transient response of voltage across PEH when SCE circuit works; (b) comparison of power delivered by SCE and standard circuits at steady state.

4 CONCLUSION

In this paper, we presented an investigation on the applicability of the SCE circuit for improving the efficiency of piezoelectric energy harvesting. An accurate analytical solution of the PEH with the SCE circuit has been derived and compared with the uncoupled and in-phase solutions. The applicability of the SCE circuit has been investigated for different cases, i.e., the PEH with various degrees of coupling and the PEH excited at resonance or off-resonance frequencies. With the derived accurate circuit model of the PEH, circuit modeling and simulation have also been conducted to validate the analytical solution. The main findings in this work can be concluded as follows:

- ❖ For the PEH with weak coupling, the SCE technique can enhance the power output by nearly four times compared to the standard circuit.
- ❖ For the PEH with medium coupling, the SCE technique cannot significantly improve the performance of the PEH vibrating at resonance. However, it significantly improves the performance of the PEH vibrating at off-resonance. This characteristic of SCE circuit would be useful to broaden the bandwidth for piezoelectric energy harvesting.
- ❖ For the PEH with strong coupling, the SCE technique dramatically undermines the performance of PEH. Although it still can improve the performance of the PEH vibrating at off-resonance, the working frequency should be further away from the resonance frequency.
- ❖ The uncoupled solution can provide approximate predictions only when the

PEH has a weak electromechanical coupling. Furthermore, for various degrees of coupling, the in-phase solution cannot provide accurate estimation of the performance of the SCE circuit except when the PEH vibrates at the open circuit resonance frequency.

- ❖ The applicable region of the SCE technique is given for various damping ratios. These plots further confirm that the SCE circuit is useful for prominent efficiency improvement when the PEH's coupling is weak ($k_e^2/\zeta < 0.36$). With the increase of coupling, the applicable region bifurcates for $k_e^2/\zeta > 2.7$ and the applicable frequency range decreases. Based on the applicable region and the critical values of k_e^2/ζ , a criterion is proposed to classify the coupling effect of a system using the SCE circuit interface. Besides, the maximum power with the SCE and standard circuits is equal but with the SCE circuit, the system can achieve it with a smaller coupling k_e .
- ❖ In general, the results of circuit simulation on the performance of PEH with both the SCE and standard circuits agree well with the accurate analytical solution. The simulations for the PEH with various coupling and vibrating at resonance or off-resonance frequencies validate the outcomes of theoretical analysis on the applicability of the SCE circuit.

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