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Shear Strength Model for Reinforced Concrete Columns with Low Transverse Reinforcement Ratios

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Abstract: This paper introduces an equation developed based on the strut-and-tie analogy to predict the shear strength of reinforced concrete columns with low transverse reinforcement ratios. The validity and applicability of the proposed equation are evaluated by comparison with available experimental data. The proposed equation includes the contributions from concrete and transverse reinforcement through the truss action, and axial load through the strut action. A reinforced concrete column with a low transverse reinforcement ratio, commonly found in existing structures in Singapore and other parts of the world was tested to validate the assumptions made during the development of the proposed equation. The column specimen was tested to failure under the combination of a constant axial load of $0.30 f'_c A_g$ and quasi-static cyclic loadings to simulate earthquake actions. The analytical results revealed that the proposed equation is capable of predicting the shear strength of reinforced concrete columns with low transverse reinforcement ratios subjected to reversed cyclic loadings to a satisfactory level of accuracy

Key words: reinforced concrete columns, strut-and-tie, seismic, shear strength.

1. INTRODUCTION

The strut-and-tie analogy is a discrete modeling of actual stress fields in reinforced concrete members. The complex stress fields within structural components resulting from applied external forces are simplified into discrete compressive and tensile force paths. The analogy utilizes the general idea of concrete in compression and steel reinforcement in tension. The longitudinal reinforcement in a beam or column represents the tensile chord of a truss while the concrete in the flexural compression zone is considered as part of the longitudinal compressive chord. The transverse reinforcement serves as ties holding the longitudinal chords together. The diagonal concrete compression struts, which discretely simulate the concrete compressive stress field, are connected to the ties and longitudinal chords at rigid nodes to attain static equilibrium within the truss. This truss model provides

a convenient means of analyzing the strength of reinforced concrete because it provides a visible representation of the failure mechanism. Many researchers have made significant contributions into the development of truss models of reinforced concrete beams subjected to shear and flexure. However, there is limited effort focused on the utilization of truss models to capture the shear strength of columns with low transverse reinforcement ratios. The objective of this paper is to propose a strut-and-tie model which is capable of predicting the shear strength of columns with low transverse reinforcement ratios.

The paper reported herein comprises two parts. The first part presents the derivation of the equation used to estimate the shear strength of reinforced concrete columns with low transverse reinforcement ratios. The validity and applicability of the proposed equation are evaluated by comparison with available experimental

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data. The second part examines the assumptions made during the development of the proposed equation by checking the capability of the model to predict the experimental results obtained from the test of a reinforced concrete column with low transverse reinforcement ratio.

2. PREVIOUS DESIGN EQUATIONS FOR SHEAR STRENGTH OF COLUMNS

2.1. ACI 318 (2008) Code Provisions

According to ACI 318 (2008), the shear strength of reinforced concrete columns are calculated as:

$$V_n = V_c + V_s \quad (1)$$

where

$$V_c = 0.166\sqrt{f'_c} \left(1 + \frac{P}{13.8A_g} \right) bd \quad (\text{MPa})$$

$$V_c = 2\sqrt{f'_c} \left(1 + \frac{P}{2000A_g} \right) bd \quad (\text{psi}) \quad (2)$$

The contribution of truss mechanism is taken as:

$$V_s = \frac{A_v f_y d}{s} \quad (3)$$

2.2. Sezen and Moehle (2004)'s Equation

Sezen and Moehle (2004) developed a shear strength model, which applies to columns with light transverse reinforcement accounting for apparent strength degradation associated with flexural yielding. The shear strength based on Sezen and Moehle (2004)'s model is defined as:

$$V_n = V_c + V_s = k \frac{A_v f_y d}{s} + k \left(\frac{0.5\sqrt{f'_c}}{a/d} \sqrt{1 + \frac{P}{0.5\sqrt{f'_c} A_g}} \right) 0.8A_g \quad (\text{MPa}) \quad (4)$$

$$V_n = V_c + V_s = k \frac{A_v f_y d}{s} + k \left(\frac{6\sqrt{f'_c}}{a/d} \sqrt{1 + \frac{P}{6\sqrt{f'_c} A_g}} \right) 0.8A_g \quad (\text{psi}) \quad (5)$$

where the parameter k is taken as 1 for displacement ductility less than 2, as 0.7 for displacement ductility more than 6 and varies linearly for intermediate displacement ductility.

2.3. Priestley *et al.* (1994)'s Equation

Priestley *et al.* (1994) proposed an additive shear strength equation:

$$V = V_c + V_s + V_a \quad (6)$$

where

$$V_c = k\sqrt{f'_c} A_e \quad (7)$$

k depends on the displacement ductility factor μ_Δ , which reduces from 0.29 (3.49) in MPa (psi) units for $\mu_\Delta \leq 2.0$ to 0.1 (1.2) in MPa (psi) units for $\mu_\Delta \geq 4.0$; and A_e is taken as $0.8 A_g$. The shear strength contribution by truss mechanism is given by:

$$V_s = \frac{A_v f_y h_c}{s} \cot \theta \quad (8)$$

where h_c = the core dimension measured center-to-center of the peripheral transverse reinforcements; and θ = the angle of truss mechanism, taken as 30 degrees.

The shear strength enhancement by axial load is given by:

$$V_a = P \tan \alpha = \frac{k_1 P (h - x)}{L} \quad (9)$$

where L = column height; h = section height; x = compression zone depth, determined from flexural analysis; and $k_1 = 1.0$ and 0.5 for double and single column curvature respectively.

3. PROPOSED SHEAR STRENGTH MODEL

The concept of superposition of both truss and strut actions in developing the shear strength model for reinforced concrete columns has been previously proposed by Watanabe and Ichinose (1991); and Priestley *et al.* (1994). The truss action transfers shear forces through the transverse reinforcement which act as tension members and concrete struts running parallel to the diagonal cracks act as compression members. The strut action, on the other hand, transfers shear forces directly through struts forming between centers of flexural compression at the top and bottom of the column. This shear force transfer mechanism concept is applied herein to develop the new shear strength equation.

3.1. Truss Mechanism

Dissimilar to Priestley *et al.* (1994)'s shear strength model, in which the concrete contribution was considered independently based on the tensile stress and strain within transverse reinforcement, this paper employs the tensile strain of transverse reinforcement as an indirect parameter which incorporates the concrete contribution into the shear strength of reinforced concrete columns.

3.1.1. Concrete contribution

Shear carried by concrete has long been recognized as an important portion of the shear strength of a reinforced concrete member. Some research has tried to use other parameters to represent this concrete contribution. But amongst all these parameters, transverse tensile stress and strain have prevailed (Vecchio 1986). In this paper, the concrete contribution is assumed as the amount of force transferred across cracks, as shown in Figure 1. Transverse tensile stress and strain were used to indirectly incorporate this amount of force transferred across cracks into the shear strength of reinforced concrete columns through the compatibility conditions. By assuming a uniform distribution of transverse reinforcement along cracks and that the tensile strain in the transverse direction is equal to the strain in the transverse reinforcement, the tensile strain in the transverse direction can be calculated as:

Figure 1. Local stresses and strains at a crack

$$\epsilon_x = \frac{V_s}{E_s \left(A_v \left(\frac{jd \cot \theta}{s} \right) \right)} = \frac{V_s s}{A_v E_s jd \cot \theta} \quad (10)$$

The principal stress directions are the direction of inclined strut, the angle θ measured from its longitudinal direction to the direction perpendicular to it. At this stage, the element has a compressive stress along the strut direction and a tensile stress perpendicular to it. However, the directions of the principal strains deviate from the principal stress directions. Vecchio and Collins (1986) have summarized a number of experimental data and found that the direction of the principal strains only differed from the principal stresses by $\pm 10^\circ$. Therefore, it is reasonable to assume that the principal stress and strain directions for an infinitesimal element of concrete coincide with each other. The principal strain in the compressive direction is readily determined by the stress and geometrical condition of a strut as illustrated in Figure 2, thus,

$$\begin{aligned} \epsilon_1 &= -\frac{(V_s/\sin \theta)}{E_c (c_{strut} b)} = -\frac{(V_s/\sin \theta)}{E_c (jd \cos \theta b)} \\ &= -\frac{V_s}{jdb E_c \sin \theta \cos \theta} \end{aligned} \quad (11)$$

with the known values of θ , ϵ_x , and ϵ_1 , a Mohr's circle can then be constructed as shown in Figure 3 to calculate the tensile strain ϵ_2 , given below:

Figure 2. Truss mechanism

Figure 3. Compatible strain conditions in a reinforced concrete element

$$\varepsilon_2 = \frac{2(\varepsilon_x - \varepsilon_1)}{|\cos 2\theta| + 1} - \varepsilon_1 \quad (12)$$

This equation takes into consideration that θ may be more than 45° .

Many researchers including Walraven (1981) have concentrated on the experimental relationships between the shear carried by concrete v_c and the tensile strain ε_2 . Vecchio and Collins (1986) derived the equation for the limiting value of shear stress transferred across the crack; the equation further used by Walraven (1981) in his study is given below:

$$v_c = \frac{0.18\sqrt{f'_c}}{0.31 + \frac{24w}{a' + 16}} \quad (\text{MPa})$$

$$v_c = \frac{2.16\sqrt{f'_c}}{0.31 + \frac{24w}{a' + 0.63}} \quad (\text{psi}) \quad (13)$$

The average crack width w can be taken as:

$$w = \varepsilon_2 s_\theta \quad (14)$$

where

$$s_\theta = \frac{1}{\frac{\sin \theta}{s_{mx}} + \frac{\cos \theta}{s_{my}}} \quad (15)$$

and where s_{mx} and s_{my} are the indicators of the crack control characteristics of the transverse and longitudinal shear reinforcement, respectively. According to the provision of the CEB-FIP Code (1978):

$$s_{mx} = 2 \left(c_x + \frac{s}{10} \right) + 0.25k_1 \frac{d_{bx}}{\rho_v} \quad (16)$$

$$s_{my} = 2 \left(c_y + \frac{s_y}{10} \right) + 0.25k_1 \frac{d_{by}}{\rho_s} \quad (17)$$

where k_1 is taken as 0.4 for deformed reinforced bars and 0.8 for plain reinforcing bars.

The calculated v_c from Eqn 13 is the shear stress transferred at the crack surface. Hence, the shear strength contributed from concrete is:

$$V_c = \frac{jdb}{\sin \theta} v_c \sin \theta = jdbv_c \quad (18)$$

3.1.2. Transverse reinforcement contribution

Additional contribution to the truss mechanism from transverse reinforcement can be defined as (ACI 2008):

$$V_s = \cot \theta A_v f_{yt} \frac{d}{s} \quad (19)$$

The shear force carried by the truss mechanism is assumed to reach its maximum value when the transverse reinforcement yields. The yield strain of transverse reinforcement can be reasonably taken as 0.002. Hence, the maximum shear force carried by the truss mechanism is given by:

$$V_T = V_c + V_s = jdb(v_c)_{\varepsilon_x = 0.002} + \cot \theta A_v f_{yt} \left(\frac{d}{s} \right) \quad (20)$$

If the inclination of compression strut θ and flexural lever arm jd are assumed as 45° and $0.8d$ respectively, Eqn 20 becomes:

$$V_c + V_s = 0.8db(v_c)_{\varepsilon_x = 0.002} + A_v f_{yt} \left(\frac{d}{s} \right) \quad (21)$$

3.2. Strut Mechanism

There are similarities to the strut action of Priestley *et al.* (1994)'s shear strength model, in which the beneficial effects of axial load on shear strength were considered in the proposed model through the strut action; although in this model, ultimate compressive stress of the direct strut was limited to cater for skew cracks along the columns. The maximum shear force applied to the strut mechanism is given as (Priestley 1994):

$$V_{a1} = P \tan \alpha \quad (22)$$

As shown in Figure 4, the shear strength of the direct strut is calculated as:

$$V_{a2} = C_u \sin \alpha \quad (23)$$

Figure 4. Strut mechanism

where

$$C_u = Wf_u \quad (24)$$

Following Schaich *et al.* (1987) and Schlaich and Schafer (1991)'s suggestions, the ultimate compressive strength of the direct strut f_u of $0.4f'_c$ was chosen to cater for skew cracks with extraordinary crack width. While the effective depth, W , was calculated as:

$$W = c \cos \alpha \quad (25)$$

where the neutral axis depth c could be estimated following Paulay and Priestley (1992)'s suggestion

$$c = \left(0.25 + 0.85 \frac{P}{A_g f_c} \right) h \quad (26)$$

Considering the geometrical condition, the direct strut angle α is given as:

$$\alpha = \arctan\left(\frac{h-c}{L}\right) \quad (27)$$

By substituting the Eqns 24, 25 and 26 into Eqn 23, the shear strength of the direct strut becomes:

$$V_{a2} = 0.2f'_c \left(0.25 + 0.85 \frac{P}{A_g f'_c} \right) h \sin(2\alpha) \quad (28)$$

The beneficial effect of axial load on shear strength in this model is defined as:

$$V_a = \min \{V_{a1}, V_{a2}\} \quad (29)$$

Then combining the Eqns 21 and 29, the shear strength of reinforced concrete columns is given as:

$$V_n = V_a + V_c + V_s$$

$$= \min \left\{ \begin{array}{l} 0.2 f_c' \left(0.25 + 0.85 \frac{P}{A_g f_c'} \right) \\ h \sin(2\alpha), P \tan \alpha \end{array} \right\} \quad (30)$$

$$+ 0.8 db(v_c)_{\varepsilon_x = 0.002} + A_v f_{yt} \left(\frac{d}{s} \right)$$

4. VERIFICATION OF THE PROPOSED SHEAR STRENGTH EQUATION

4.1. Experimental Database

Sezen and Moehle (2004) collected a database of 51 laboratory tests on reinforced concrete columns representative of columns from older reinforced buildings by applying a consistent set of criteria. All specimens were subjected to unidirectional quasi-static cyclic lateral loading and had low transverse reinforcement ratios (ρ_w) (less than 0.7%). Both yielding of longitudinal reinforcement prior to loss of lateral load capacity, and ultimate failure and deformation capacity appears to be controlled by shear mechanisms. The set of criteria applied in this paper is similar to Sezen and Moehle (2004)'s with the only exception being the lowered transverse reinforcement the lower transverse reinforcement ratios criterion was applied to ensure that the assumption of yielding of transverse reinforcement at the maximum shear force is satisfied. The database includes columns satisfying the following criteria: column aspect ratio, $1.8 \leq a/d \leq 4.0$; concrete strength, $13 \leq f'_c \leq 50$ (MPa); longitudinal and transverse reinforcement nominal yield stress, f_{yt} and f_{yl} in the range of 300–650 MPa; longitudinal reinforcement ratio, $0.01 \leq \rho_l \leq 4.0$; transverse reinforcement ratio, $0.0010 \leq \rho_w \leq 0.0031$.

4.2. Discussion of Analytical Results

The validation of the proposed equation is demonstrated by comparison with published

experimental results with respect to the maximum shear force obtained from the test results. Details of the reinforced concrete columns are shown in Table 1. These columns encompass a wide range of cross sectional sizes, material properties, and axial loads. It was found that the average ratio of the experimental to predicted shear strength by the proposed equation is 1.033 as shown in Figure 5 and Table 1, showing a good correlation between the proposed equation and experimental data. The shear strengths of columns in the database calculated based on ACI 318 (2008), Sezen and Moehle (2004), and Priestley *et al.* (1994) are also showed in Table 1. The mean ratio of the experimental to predicted strength and its coefficient of variation are 1.108 and 0.204, 1.022 and 0.171, and 0.740 and 0.128 for ACI 318 (2008), Sezen and Moehle (2004), and Priestley *et al.* (1994), respectively. Comparison of available models with experimental data indicates that Sezen and Moehle (2004) model and the proposed model produce better mean ratio of the experimental to predicted strength and its coefficient of variation than ACI 318 (2008), Sezen and Moehle (2004), and Priestley *et al.* (1994) model. Both Sezen and Moehle (2004) model and the proposed model may be suitable as an assessment tool to calculate the shear strength of reinforced concrete columns with low transverse reinforcement ratios which have similar detailing in the database.

To investigate the validity and applicability of the proposed equation across the range of several key parameters including axial load, aspect ratio, compressive strength of concrete and transverse reinforcement ratio, the ratio of experimental shear strength, V_u to shear strength calculated from the proposed Eqn 30 versus axial load [$P/(A_g f'_c)$], aspect ratio (a/d), transverse reinforcement index ($\rho_w f_{yt}/f'_c$) is plot in Figure 6. The good correlation between the experimental and predicted strengths across the range of axial load, aspect ratio, transverse reinforcement index indicates that the proposed model well represents the effects of these key parameters.

The effect of displacement ductility demand on the shear strength of reinforced concrete columns has been recognized and incorporated into the shear strength equations previously by some researchers [e.g., Priestley *et al.* (1994); Sezen and Moehle (2004)]. Priestley *et al.* (1994) proposed the model in which concrete contribution to shear strength reduces with increasing displacement ductility demand, whereas Sezen and Moehle (2004) suggested both concrete and steel contributions are reduced with increasing displacement ductility demand. The proposed model propounds that when the tensile strain

of transverse reinforcement increases, the concrete contribution to the shear strength decreases. Once the transverse reinforcement reaches its yield strength, the increase of displacement ductility will lead to a reduction of V_T in Eqn 20 due to constant value of V_s and reduction of V_c in Eqn 20. Hence, the proposed model could be used to qualitatively explain the effect of displacement ductility demand on the shear strength of reinforced concrete columns. In order to quantitatively investigate the effect of displacement ductility demand on the shear strength of reinforced concrete columns by the proposed model, the relationship between tensile strain of transverse reinforcement versus displacement ductility is needed. The difficulty in establishing this relationship prevents the proposed model from being able to quantitatively incorporate the effect of displacement ductility.

4.3. Uncertainties of the Proposed Model

In the proposed model, the complicated shear resisting mechanisms in reinforced concrete columns with low transverse reinforcement ratios are simplified into truss and strut mechanisms; hence, several uncertainties in the proposed model can be expected. The direct strut forming between the centers of flexural compression at the top and bottom of the columns is an imaginary stress field which helps to explain certain experimental observations. Currently, there are no physical evidences which help to explain the existence of this direct strut. In the proposed model, the effect of column axial load is incorporated through the use of the direct strut which could be one of the uncertainties. Furthermore, the assumptions of a 45° crack angle and yielding of transverse reinforcements are not always true for all cases of the specimens in the database. For simplicity, ACI 318 code (2008)'s 45° crack angle assumption is adopted in the proposed model. However, this assumption may lead to an underestimation of the contribution from the shear reinforcement. All empirical results indicate that crack angle is not a constant value. The effects of several parameters such as transverse reinforcement ratio, axial load, longitudinal reinforcement and compressive concrete strength on the crack angle are inconclusive. Further study is required to mathematically calculate the crack angle to enhance the accuracy of the proposed model. In addition, the validity of the assumption of uniform distribution of transverse reinforcements along the crack relies on the position of crack along the column. This could be an additional uncertainty in the proposed model.

Table 1. Experimental verification

Specimen	Column section				Transverse reinforcement		Longitudinal reinforcement		V_u (kN)	V_{Sezen} (kN)	$V_{priestley}$ (kN)	$V_{proposed}$ (kN)	V_u	V_u	V_u
	f'_c (MPa)	$\frac{P}{A_g f'_c}$	h (mm)	b (mm)	d (mm)	a (mm)	$\frac{a}{d}$	ρ_w (%)	s (mm)	f_{yt} (MPa)	ρ_l (%)	f_{yl} (MPa)	V_u	V_u	V_u
Lynn (2001)															
3CLH18	25.6	0.09	457	457	381	1473	3.87	0.07	457	400	3.0	331	271.0	225.4	206.4
3SLH18	25.6	0.09	457	457	381	1473	3.87	0.07	457	400	3.0	331	267.0	225.4	206.4
2CLH18	33.1	0.07	457	457	381	1473	3.87	0.07	457	400	2.0	331	240.0	252.9	224.9
2SLH18	33.1	0.07	457	457	381	1473	3.87	0.07	457	400	2.0	331	231.0	252.9	224.9
2CMH18	25.7	0.28	457	457	381	1473	3.87	0.07	457	400	2.0	331	316.0	282.1	273.5
3CMH18	27.6	0.26	457	457	381	1473	3.87	0.07	457	400	3.0	331	338.0	287.6	275.8
3CMD12	27.6	0.26	457	457	381	1473	3.87	0.17	305	400	3.0	331	356.0	351.8	352.3
3SMD12	25.7	0.28	457	457	381	1473	3.87	0.17	305	400	3.0	331	378.0	342.0	346.4
Sezen (2002); Sezen and Moehle (2006)															
2CLD12	21.1	0.15	457	457	394	1473	3.74	0.17	305	476	2.5	438	315.0	317.0	285.0
2CHD12	21.1	0.61	457	457	394	1473	3.74	0.17	305	476	2.5	438	359.0	410.0	409.0
2CLD12M	21.8	0.15	457	457	394	1473	3.74	0.17	305	476	2.5	438	294.0	317.0	285.0
Lee (2006)															
S1-0.0-N	27.0	0.10	300	300	251	450	1.79	0.26	180	400	2.4	400	216	156.8	227.6
S2-0.0-N	27.0	0.10	300	300	251	450	1.79	0.21	225	400	2.4	400	200	140.9	211.9
Yalcin (Sezen 2004)															
BR-S1	45.0	0.13	550	550	482	1485	3.08	0.10	300	425	2.0	445	578	535.1	546.1
Umemura and Endo (Sezen 2004)															
205	17.7	0.22	200	200	180	600	3.33	0.28	100	324	2.0	462	71	64.9	66.8
207	17.7	0.22	200	200	180	400	2.22	0.28	100	324	2.0	462	106	64.9	83.8
214	17.7	0.55	200	200	180	600	3.33	0.14	200	324	2.0	462	83	59.3	64.4
200	32.9	0.12	200	200	180	400	2.22	0.11	120	648	1.0	379	78	69.6	89.2
231	14.8	0.26	200	200	180	400	2.22	0.13	100	524	1.0	324	51	54.5	73.2
232	13.1	0.23	200	200	180	400	2.22	0.13	100	524	1.0	324	58	52.7	71.3
233	13.9	0.24	200	200	180	400	2.22	0.13	100	524	1.0	372	69	53.5	72.2
234	13.1	0.24	200	200	180	400	2.22	0.13	100	524	1.0	372	67	52.7	71.3

(Continued)

Table 1. Experimental verification

Specimen	Column section				Transverse reinforcement		Longitudinal reinforcement				V_{ACI} (kN)	V_{Sezen} (kN)	$V_{priestley}$ (kN)	$V_{proposed}$ (kN)	$\frac{V_u}{V_{ACI}}$	$\frac{V_u}{V_{Sezen}}$	$\frac{V_u}{V_{priestley}}$	$\frac{V_u}{V_{proposed}}$			
	f'_c (MPa)	$\frac{P}{A_g f'_c}$	h (mm)	b (mm)	d (mm)	a (mm)	$\frac{a}{d}$	ρ_w (%)	s (mm)	f_{yt} (MPa)									ρ_l (%)	f_{yl} (MPa)	V_u (kN)
372	19.9	0.20	200	200	170	500	2.94	0.31	100	352	1.0	524	74	69.3	77.4	107.7	78.5	1.068	0.956	0.687	0.943
	20.4	0.20	200	200	170	500	2.94	0.31	100	352	2.0	524	88	69.8	77.8	108.2	84	1.261	1.131	0.813	1.048
452	21.9	0.45	200	200	170	500	2.94	0.31	100	317	3.0	395	110	78.6	91.4	109.9	88.3	1.399	1.203	1.001	1.246
452	21.9	0.45	200	200	170	500	2.94	0.31	100	317	4.0	395	110	78.6	91.4	109.9	89.3	1.399	1.203	1.001	1.232
43	19.6	0.10	200	200	173	500	2.89	0.28	100	558	2.0	434	74	83.0	87.7	118.4	97.1	0.891	0.844	0.625	0.762
44	19.6	0.10	200	200	173	500	2.89	0.28	100	558	2.0	434	77	83.0	87.7	118.4	97.1	0.928	0.878	0.650	0.793
45	19.6	0.20	200	200	173	500	2.89	0.28	100	558	2.0	434	82	86.7	94.8	125.0	98.5	0.946	0.865	0.656	0.832
46	19.6	0.20	200	200	173	500	2.89	0.28	100	558	2.0	434	81	86.7	94.8	125.0	98.5	0.935	0.855	0.648	0.822
62	19.6	0.10	200	200	173	500	2.89	0.28	100	476	2.0	348	58	75.2	79.8	106.8	88.8	0.771	0.727	0.570	0.653
63	19.6	0.20	200	200	173	500	2.89	0.28	100	476	2.0	348	69	78.7	86.8	110.4	83.1	0.876	0.795	0.625	0.83
64	19.6	0.20	200	200	173	500	2.89	0.28	100	476	2.0	348	69	78.7	86.8	110.4	83.1	0.876	0.795	0.625	0.83
SC01	49.3	0.30	350	350	301	850	2.82	0.13	125	393	3.2	409	357.1	320.5	309.5	408.9	300.1	1.149	1.153	0.873	1.190
	Average																	1.108	1.022	0.740	1.033
	Coefficient of variation																	0.204	0.171	0.128	0.194

Figure 5. Correlation of experimental and predicted shear strength based on the proposed equation

5. EXPERIMENTAL STUDY

5.1. Specimen and Test Procedure

To investigate several assumptions made within the development of the shear strength model, a large-scale reinforced concrete column with a low transverse reinforcement ratio, which satisfies the set of criteria used to establish the database, was constructed and tested. Figure 7 illustrates the schematic dimensions and detailing of the specimen. A schematic of the loading apparatus is shown in Figure 8. A reversible horizontal load was applied to the top of the column using a double-acting 1000 kN capacity long-stroke dynamic actuator which was mounted onto a reaction wall. The actuator was pinned at both ends to allow rotation during the test. The base of the column was fixed to a strong floor with four post-tensioned bolts. The axial load was applied to

Figure 6. Variation of experimental to predicted strength ratio as a function of key parameters

Figure 7. Reinforcement details of test specimen (in mm)

Figure 8. Test setup (in mm)

the column using two double-acting 1000 kN capacity dynamic actuators through a transfer beam. The typical loading procedure is illustrated in Figure 9.

5.2. Experimental Results and Discussions

Figure 10 shows the load-displacement hysteresis loops of the specimen. The hysteresis loops show the degradation of stiffness and load-carrying capacity during repeated cycles due to the cracking of the concrete and yielding of the steel reinforcement. The low attainment of stiffness and strength were attributed to the shear cracks along the specimens. Pinching was seen in the hysteresis loops of the specimen when a drift ratio of 1.0% was applied, leading to limited energy dissipation as shown in Figure 10. The specimen reached its maximum horizontal strength in the first cycle at a drift ratio of 1.0%. At the next drift ratio of 1.33%, the peak lateral load attained was only 82.3% of the maximum recorded value of the specimen. Continuous cycles caused additional damage and loss of lateral resistance. During the first push cycle

at a drift ratio of 2%, the column failed catastrophically due to the failure of its transverse reinforcements. At this stage, the applied axial load dropped suddenly from 1804 kN to 400 kN showing the brittle behavior of the specimen caused by its low transverse reinforcement ratio. The maximum shear strength obtained from the specimen was 357.1 kN, whereas the value obtained by the proposed equation was 300.5 kN.

Figure 11 illustrates the formation of the cracking patterns of the specimen. At a drift ratio of 0.25%, flexural cracks were found at the bottom and top of the column. The inclined bending-shear cracks at the bottom and top of the column, which were formed at a drift ratio of 0.67%, were believed to be the extension of these flexural cracks. Shear cracks occurred at a drift ratio of 0.67% and started to develop rapidly at drift ratio of 1.0% which continued to expand as the loading progressed. Limited new flexural cracks along the specimen were observed when a drift ratio was increased to 1.0%. Failure accompanied by gradual stiffness degradation of the column occurred due to extensive opening of the shear cracks. In development of the proposed model, the crack angle is assumed as 45° , whereas the measured crack angle at the maximum shear force state is 35° . Using the experimental crack angle, 35° to predict the shear strength based on the proposed model obtains 354.6 kN. The ratio of experimental shear strength to predicted shear strength based on experimental crack angle is 1.007. The improvement in predicting the shear strength based on experimental crack angle is obtained. This indicates the uncertainty of the proposed model when the crack angle

Figure 9. Loading procedure

Figure 10. Hysteresis loops of specimen SC01

(a) At the maximum shear force (b) At axial failure

Figure 11. Cracking patterns of specimen SC01

is assumed to be 45° . Further study is required to refine the proposed model to take into account varied crack angles, to enhance the accuracy of the proposed model.

Figure 12 shows the measured strain distribution of the longitudinal reinforcements along the height of the column of the specimen. It was observed that the distribution of strain along the longitudinal reinforcements varied considerably with an increase in lateral load. With reference to this strain profile, no tensile yielding of the longitudinal bars was observed during the tests, thus indicating the dominance of shear failure behavior of the specimen. The largest tensile strain of the specimen was detected at 250 mm away from the fixed-end. The tensile strains within longitudinal bars initially increased with increasing drift ratio, apparently owing to the growth of flexural cracks at top and bottom of the column but eventually these strains began to reduce as shown in Figure 12.

Figure 12. Local strains in longitudinal bars of specimen SC01

Figure 13 shows the measured strain distribution of the transverse reinforcement in the direction parallel to the lateral load direction along the height of the column of the specimen. It was observed that the distributions of strains along the transverse reinforcement varied considerably with the increase of lateral load and increased with increasing drift ratio. With reference to this strain profile, yielding of the transverse steel bars was observed at a drift ratio of 1.33%. The largest tensile strain was detected at 240 mm away from the fixed-end. It was observed that the transverse strain suddenly increased at a drift ratio of 1.33% owing to the growth and opening of shear cracks along the column. The yielding of transverse reinforcement is assumed in the development of the proposed model. However, as shown in Figure 13, when the specimen reached its maximum shear force, no yielding of transverse reinforcement was observed. It is also noticeable that the measured strains are localized strains along transverse steel bars. Yielding of transverse steel bars may occur elsewhere, such as at the shear crack locations. The measured strain distribution of the transverse reinforcement in the direction perpendicular to the lateral load direction is shown in Figure 14. It was observed that there was a sudden increase in the strain distribution within the transverse reinforcement in the direction perpendicular to the lateral load direction, at a drift ratio of 1.82%. With reference to this strain profile, yielding of the transverse steel bars in the direction perpendicular to the lateral load direction was observed only at a drift ratio of 1.82%.

Figure 13. Local strains in steel links of specimen SC01 in the direction parallel to the lateral load direction

Figure 14. Local strains in steel links of specimen SC01 in the direction perpendicular to the lateral load direction

5. CONCLUSIONS

Based on the results of this study, the following conclusions can be drawn:

The complicated shear resisting mechanism in reinforced concrete columns with low transverse reinforcement ratios can be analyzed by the proposed equation, which was derived from the strut-and-tie model and incorporated concrete contribution. The proposed equation provides a good estimate of the shear strength of reinforced concrete columns with low transverse reinforcement ratios in the database with the average ratio of experimental to predicted shear strength of the 34 shear-critical reinforced concrete columns being 1.033. The proposed equation can be utilized to determine shear strength of reinforced concrete columns with low reinforcement ratios that exhibit shear failure behaviors.

A full-scale reinforced concrete column with a low transverse reinforcement ratio, which is commonly found in existing structures in Singapore and other parts of the world, was tested under a constant axial load, $0.30 f'_c A_g$ and quasi-static cyclic loadings simulating earthquake actions to further validate the proposed model. The experimental results show improvement in predicting the shear strength of the test column based on the experimental crack angle; the ratio of experimental shear strength to predicted shear strength based on the experimental crack angle is 1.007.

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NOTATION

f'_c	compressive strength of concrete
V_n	nominal shear strength of columns
P	applied axial load
b	width of columns
h	depth of columns
f_{yt}	yield strength of transverse reinforcement
d	distance from the extreme compression fiber to centroid of tension reinforcement
s	spacing of transverse reinforcement
A_v	total transverse reinforcement area within spacing s
θ	the inclination of compression struts
V_c	shear force carried by concrete
V_s	shear force carried by transverse reinforcement
V_a	shear force carried by strut mechanism
A_g	cross sectional area
k	parameter depends on the displacement ductility demand
a/d	aspect ratio