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A Stochastic Simulation Algorithm for Updating Robust Reliability of Nonlinear Structural Dynamic Systems Based on Incomplete Modal Data

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ABSTRACT

In this paper, we are interested in using the incomplete modal data identified from the system to update the robust failure probability that any particular response of a nonlinear structural dynamic system exceeds a specified threshold during the time when it is subjected to future stochastic dynamic excitation. Uncertainties from structural modeling, the modeling of the stochastic excitation that the structure will experience during its lifetime and any other uncertainties can all be taken into account. A new efficient approach based on stochastic simulation methods, which is very robust to the number of random variables and uncertain model parameters, is proposed to update the robust reliability of the structure. The efficiency and effectiveness of the proposed approach are illustrated by a numerical example involving a nonlinear structural dynamic model of a building.

INTRODUCTION

Reliability considering model uncertainties in addition to the modeling of the uncertain excitation is termed as ‘robust reliability’ (Papadimitriou et al., 2001). Due to space limitations, readers can refer to the journal version of this paper for more detailed reviews of other existing approaches. One common shortcoming for all the existing approaches is that they all experience problems handling the problem involving a huge number of uncertain model parameters whose joint PDF is updated by high-dimensional vibration data. In this paper, an efficient stochastic simulation method is presented for computing the updated robust failure probability (or its complement robust reliability) of a dynamic system using system data when the system is subjected to future stochastic excitation. The updating is based on incomplete modal data including modal frequencies, damping ratios and partial mode shapes of some of the dominant modes.

Let $\boldsymbol{\theta}_s \subset \mathbb{R}^n$ denote the uncertain model parameter vector which is related to the linear structural dynamic behavior, specified by a model class $\mathcal{M}$, whose PDF is to be updated by vibration data collected from the dynamic system of interest. Let $\boldsymbol{\theta}_{nl} \subset \mathbb{R}^s$ denote the uncertain model parameter vector whose PDF is not updated by

the vibration data (e.g., those related to the nonlinear structural dynamic behavior, those related to stochastic input excitation model etc.). Let $p(\boldsymbol{\theta}_s | \mathcal{M})$ denote the prior PDF of $\boldsymbol{\theta}_s \in \mathbb{R}^n$ and $\mathbf{Y}(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z}) \in \mathbb{R}$ denote any output (performance) quantity of interest specified by $\boldsymbol{\theta}_s$ and $\boldsymbol{\theta}_{nl}$, and future dynamic input specified by a stochastic input model $\mathcal{U}$, which can be expressed in terms of independently and identically distributed (i.i.d) standard normal random variables $\mathbf{Z} \in \mathbb{R}^N$. The updated robust failure probability is given by the following multi-dimensional integral with respect to $\boldsymbol{\theta}_s$, $\boldsymbol{\theta}_{nl}$ and $\mathbf{Z}$:

$$P(F | D, \mathcal{M}, \mathcal{U}) = \int I_F(\mathbf{Y}(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z})) p(\boldsymbol{\theta}_s | D, \mathcal{M}) p(\boldsymbol{\theta}_{nl}) p(\mathbf{Z} | \mathcal{U}) d\mathbf{Z} d\boldsymbol{\theta}_s d\boldsymbol{\theta}_{nl} \quad (1)$$

where F denotes failure, and I_F is an indicator function that is equal to 1 if $\mathbf{Y}$ exceeds the specified threshold and otherwise is equal to 0. Given the measurement data D from the system which behaves linearly when the measurement data are taken, the posterior (updated) PDF of $\boldsymbol{\theta}_s$ is given by Bayes' theorem:

$$p(\boldsymbol{\theta}_s | D, \mathcal{M}) = \frac{p(D | \boldsymbol{\theta}_s, \mathcal{M}) p(\boldsymbol{\theta}_s | \mathcal{M})}{p(D | \mathcal{M})} \quad (2)$$

Let $D \equiv \{\hat{\omega}_{m,s}, \hat{\zeta}_{m,s}, \hat{\psi}_{m,s} : m=1 \dots M, s=1 \dots S\}$ be the experimentally obtained modal data from a linear structural dynamic system, consisting of modal frequencies $\hat{\omega}_{m,s} \in \mathbb{R}^+$, damping ratios $\hat{\zeta}_{m,s} \in \mathbb{R}^+$, and complex mode shape components $\hat{\psi}_{m,s} \in \mathbb{C}^{N_o}$, where N_o is the number of measured DOFs, M is the number of observed modes and S is the number of modal data sets available. For clarity in presentation, the conditioning on $\mathcal{M}$ and $\mathcal{U}$ will be left implicit in the rest of the paper.

In this paper, it is assumed that vibration data of structures are obtained when the dynamic system behaves linearly. The proposed method can tackle the case where a dynamic system may behave nonlinearly when it is subjected to future stochastic dynamic excitation. The proposed approach, which is robust to the number of uncertain parameters and random variables and the dimension of the vibration data involved in the problem, integrates a newly-developed stochastic simulation method for Bayesian model updating of a linear dynamic system using incomplete modal data proposed by the authors in (Cheung and Bansal, 2013) and a very efficient algorithm called Subset Simulation to compute small failure probabilities (Au and Beck, 2001) together with a new efficient conditional sampling algorithm proposed by the authors, referred to as Constrained Multi-group Metropolis within Gibbs algorithm.

PROPOSED APPROACH

First consider the following integral

$$P(F | D) = \int I_F(\mathbf{Y}(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z})) p(\boldsymbol{\theta}_s | D) p(\boldsymbol{\theta}_{nl}) p(\mathbf{Z}) d\mathbf{Z} d\boldsymbol{\theta}_s d\boldsymbol{\theta}_{nl} \quad (3)$$

The above integral can be approximated by the following MCS estimator:

$$P(F | D) \approx \frac{1}{N} \sum_{k=1}^N I_F(\mathbf{Y}(\boldsymbol{\theta}_s^{(k)}, \boldsymbol{\theta}_{nl}^{(k)}, \mathbf{Z}^{(k)})) \quad (4)$$

where samples $\{\mathbf{Z}^{(k)} : k = 1, \dots, N\}$ and $\{\boldsymbol{\theta}_{nl}^{(k)} : k = 1, \dots, N\}$ are distributed according to the PDFs $p(\mathbf{Z})$ and $p(\boldsymbol{\theta}_{nl})$, respectively; samples $\{\boldsymbol{\theta}_s^{(k)} : k = 1, \dots, N\}$ are distributed according to the PDF $p(\boldsymbol{\theta}_s | D)$. Samples distributed according to the PDFs $p(\mathbf{Z})$ and $p(\boldsymbol{\theta}_{nl})$ can easily be obtained. Samples distributed according to the PDF $p(\boldsymbol{\theta}_s | D)$ are simulated using the Gibbs sampling based algorithm proposed in Cheung and Bansal (2013a). Even though the samples obtained using the Gibbs sampler are not independent, the Monte Carlo estimator for independent samples in (4) can still be used. However, using (4) will be computationally expensive especially when dealing with small failure probability as the minimum number of samples N required to achieve a given c.o.v is inversely proportional to the failure probability. The Subset Simulation algorithm is adapted to compute the updated robust failure probability as follows:

$$P(F | D) = P(F_m | D) = \left[\prod_{i=1}^{m-1} P(F_{i+1} | F_i, D) \right] P(F_1 | D) \quad (5)$$

where $P(F_1 | D)$ is estimated by (4), $F_1 \supset F_2 \dots \supset F_m$ and $P(F_{i+1} | F_i, D)$ is estimated using samples $\{\boldsymbol{\theta}_s^{(i,k)}, \boldsymbol{\theta}_{nl}^{(i,k)}, \mathbf{Z}^{(i,k)}, k = 1, \dots, N\}$ distributed according to the conditional PDF $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$ as follows:

$$P(F_{i+1} | F_i, D) \approx \frac{1}{N} \sum_{k=1}^N I_{F_{i+1}}(\mathbf{Y}(\boldsymbol{\theta}_s^{(i,k)}, \boldsymbol{\theta}_{nl}^{(i,k)}, \mathbf{Z}^{(i,k)})) \quad (6)$$

Another key for making the proposed method robust to the dimension of $\boldsymbol{\theta}_s$, $\boldsymbol{\theta}_{nl}$ and $\mathbf{Z}$ space depends on how to generate samples from $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$ effectively and efficiently. The proposed method for this will be presented as follows.

SAMPLING FROM $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$

Markov Chain samples distributed according to the conditional PDF $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$ are generated using a new algorithm proposed in this paper, referred to as the Constrained Multi-group Metropolis and Multi-group-Metropolis within Gibbs algorithm, as described below. In this algorithm, a candidate state $[\tilde{\boldsymbol{\theta}}_s \ \tilde{\boldsymbol{\theta}}_{nl} \ \tilde{\mathbf{Z}}]$ using the current state $[\boldsymbol{\theta}_s^{(k)} \ \boldsymbol{\theta}_{nl}^{(k)} \ \mathbf{Z}^{(k)}]$ is first generated using the proposed procedures presented later in the next section. This candidate state will be accepted as the new sample $[\boldsymbol{\theta}_s^{(k+1)} \ \boldsymbol{\theta}_{nl}^{(k+1)} \ \mathbf{Z}^{(k+1)}]$ (i.e., $[\boldsymbol{\theta}_s^{(k+1)} \ \boldsymbol{\theta}_{nl}^{(k+1)} \ \mathbf{Z}^{(k+1)}] = [\tilde{\boldsymbol{\theta}}_s \ \tilde{\boldsymbol{\theta}}_{nl} \ \tilde{\mathbf{Z}}]$) distributed according to the conditional PDF $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$ if it leads to F_i and otherwise, $[\boldsymbol{\theta}_s^{(k+1)} \ \boldsymbol{\theta}_{nl}^{(k+1)} \ \mathbf{Z}^{(k+1)}] = [\boldsymbol{\theta}_s^{(k)} \ \boldsymbol{\theta}_{nl}^{(k)} \ \mathbf{Z}^{(k)}]$.

Candidate state for $\mathbf{Z}$ and $\boldsymbol{\theta}_{nl}$

Same as the original Subset Simulation, the candidate state for each of the components of the candidate state for $\mathbf{Z}$ is generated by a Metropolis-Hastings algorithm using a one-dimensional adaptive symmetric proposal PDF centered at the corresponding component of the current state. The candidate state for each of the component group of the candidate state for $\boldsymbol{\theta}_{nl}$ is generated by a Metropolis-Hastings algorithm using the proposal PDF, the construction of which is discussed in the journal version of this paper.

Candidate state for $\boldsymbol{\theta}_s$

For a N_d -DOF linear dynamic system with both Rayleigh and nonclassical damping, the relationship between modal properties and dynamic model parameters can be written as (Cheung and Bansal, 2013):

$$\hat{\lambda}_{m,s}^2 \mathbf{M} \psi_m + \hat{\lambda}_{m,s} \mathbf{C} \psi_m + \mathbf{K} \psi_m = \boldsymbol{\varepsilon}_{m,s} \quad (7)$$

$$\hat{\psi}_{m,s} - \mathbf{L} \psi_m = \mathbf{e}_{m,s} \quad (8)$$

where $\mathbf{L} \in \mathbb{R}^{No \times Nd}$ is a selection matrix that selects only those DOFs where measurements are made. In the above equation $\boldsymbol{\varepsilon}_{m,s}$ and $\mathbf{e}_{m,s}$ are the complex random vectors representing the model prediction errors, i.e. the errors between the response of the system under consideration and that of the assumed model. Based on the Principle of Maximum Entropy (Jaynes, 1978), the PDFs for vectors $\text{Re}(\boldsymbol{\varepsilon}_{m,s})$, $\text{Im}(\boldsymbol{\varepsilon}_{m,s})$, $\text{Re}(\mathbf{e}_{m,s})$ and $\text{Im}(\mathbf{e}_{m,s})$ are taken to be Gaussian. For illustration, here their means are assumed to be equal to zero and covariance matrices equal to scaled versions of the identity matrix $\mathbf{I}$ of appropriate order. The mass, damping and stiffness matrices in (7) are represented as a linear sum of contribution from the individual prescribed substructures:

$$\mathbf{M}(\boldsymbol{\alpha}) = \mathbf{M}_0 + \sum_{i=1}^{N_\alpha} \alpha_i \mathbf{M}_i \quad (9)$$

$$\mathbf{K}(\boldsymbol{\eta}) = \mathbf{K}_0 + \sum_{i=1}^{N_\eta} \eta_i \mathbf{K}_i \quad (10)$$

$$\mathbf{C}(\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{c}, \boldsymbol{\eta}) = \mathbf{C}_0 + \sum_{i=1}^{N_\beta} \beta_i \mathbf{C}_i + c_0 \mathbf{M}(\boldsymbol{\alpha}) + c_1 \mathbf{K}(\boldsymbol{\eta}) \quad (11)$$

The uncertain parameters to be updated are the contribution parameters $[\boldsymbol{\alpha}^T \boldsymbol{\beta}^T \mathbf{c}^T \boldsymbol{\eta}^T]^T$ mode shapes $[\text{Re}(\psi_1)^T \text{Im}(\psi_1)^T \cdots \text{Re}(\psi_M)^T \text{Im}(\psi_M)^T]^T$ and prediction error variance $[\sigma_{\text{Re},1}^2 \sigma_{\text{Im},1}^2 \cdots \sigma_{\text{Re},M}^2 \sigma_{\text{Im},M}^2]^T$ for $\text{Re}(\boldsymbol{\varepsilon}_{m,s})$ and $\text{Im}(\boldsymbol{\varepsilon}_{m,s})$. The variance parameters for $\text{Re}(\mathbf{e}_{m,s})$ and $\text{Im}(\mathbf{e}_{m,s})$ are assumed to be known or are directly

estimated from the sample variance of the experimental modal data. Uncertain parameters are divided into four groups $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T \ \boldsymbol{\theta}_2^T \ \boldsymbol{\theta}_3^T \ \boldsymbol{\theta}_4^T]^T$:

$$\boldsymbol{\theta}_1 = [\boldsymbol{\alpha}^T \ \boldsymbol{\eta}^T \ \boldsymbol{\beta}^T]^T$$

$$\boldsymbol{\theta}_2 = \mathbf{c}$$

$$\boldsymbol{\theta}_3 = [\text{Re}(\psi_1)^T \ \text{Im}(\psi_1)^T \cdots \text{Re}(\psi_M)^T \ \text{Im}(\psi_M)^T]^T$$

$$\boldsymbol{\theta}_4 = [\sigma_{\text{Re},1}^2 \ \sigma_{\text{Im},1}^2 \cdots \sigma_{\text{Re},M}^2 \ \sigma_{\text{Im},M}^2]^T$$

Assuming Bayesian conjugate priors for $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$ and $\boldsymbol{\theta}_4$, the full conditional PDFs $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D)$, $p(\boldsymbol{\theta}_2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D)$ and $p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_4, D)$ are Gaussian, and $p(\sigma_{\text{Re},m}^2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, D)$ and $p(\sigma_{\text{Im},m}^2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, D)$ for $m=1, \dots, M$ are inverse gamma.

$$p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D) \sim N(\boldsymbol{\mu}_1(\boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4), \boldsymbol{\Sigma}_1(\boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4))$$

$$p(\boldsymbol{\theta}_2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D) \sim N(\boldsymbol{\mu}_2(\boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4), \boldsymbol{\Sigma}_2(\boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4))$$

$$p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_4, D) \sim N(\boldsymbol{\mu}_3(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_4), \boldsymbol{\Sigma}_3(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_4))$$

$$p(\sigma_{\text{Re},m}^2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, D) \sim IG(\rho_{\text{Re},m}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3), \kappa_{\text{Re},m}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)) \quad m=1, \dots, M$$

$$p(\sigma_{\text{Im},m}^2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, D) \sim IG(\rho_{\text{Im},m}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3), \kappa_{\text{Im},m}(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)) \quad m=1, \dots, M$$

Due to space limitations, the readers can refer to Cheung and Bansal (2013) and its journal version for the details for the above. The candidate states for $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$ and $\boldsymbol{\theta}_4$ can be simulated one group after another using the steps given in the following summary.

Summary of steps for simulating samples according to $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$

For level zero of the proposed approach, samples $\{\mathbf{Z}^{(k)}, \boldsymbol{\theta}_{nl}^{(k)} : k=1, \dots, N\}$ are directly simulated from the PDFs $p(\mathbf{Z})$ and $p(\boldsymbol{\theta}_{nl})$, respectively and samples $\{\boldsymbol{\theta}_s^{(k)} : k=1, \dots, N\}$ are simulated from the PDF $p(\boldsymbol{\theta}_s | D)$ using a new Gibbs sampling based approach proposed in Cheung and Bansal (2013a), which is robust to the number of uncertain parameters (the required full conditional probability distributions shown in the previous section), after discarding the samples from the burn-in period. For the i -th simulation level, $p_0 N$ samples distributed according to $p(\boldsymbol{\theta}_s | F_i, D)$ are obtained from the $(i-1)$ -th simulation level, which are used as seed samples to initialize Markov chains to obtain additional $N(1-p_0)$ samples which are also distributed according to the conditional PDF $p(\boldsymbol{\theta}_s, \boldsymbol{\theta}_{nl}, \mathbf{Z} | F_i, D)$. The following steps are repeated for each seed sample:

1. Initialize a Markov chain, use a seed sample as the starting point and let $k=1$.

2. Given a current state $\mathbf{Z}^{(k)}$ where $\mathbf{Z}=[Z_1 Z_2 \cdots Z_{Nt}]^T$, a candidate state $\tilde{\mathbf{Z}}$ is generated using the Metropolis-Hastings algorithm by repeating the following steps from $j=1$ till $j=Nt$:

- i) For the j -th component, draw a pre-candidate component $\xi_{\mathbf{Z},j}^{(k+1)}$ from the proposal PDF $q_{\mathbf{Z},j}$ and compute the acceptance ratio:

$$r_{\mathbf{Z},j}^{(k+1)} = \min \left\{ 1, \frac{p(\xi_{\mathbf{Z},j}^{(k+1)}) q_{\mathbf{Z},j}(Z_j^{(k)} | \xi_{\mathbf{Z},j}^{(k+1)})}{p(Z_j^{(k)}) q_{\mathbf{Z},j}(\xi_{\mathbf{Z},j}^{(k+1)} | Z_j^{(k)})} \right\}$$

- ii) Set $\tilde{Z}_j^{(k+1)} = \begin{cases} \xi_{\mathbf{Z},j}^{(k+1)} & \text{with probability } r_{\mathbf{Z},j}^{(k+1)} \\ Z_j^{(k)} & \text{with probability } 1 - r_{\mathbf{Z},j}^{(k+1)} \end{cases}$.

3. Given a current state $\boldsymbol{\theta}_{nl}^{(k)}$ where $\boldsymbol{\theta}_{nl}=[\boldsymbol{\theta}_{nl,1}^T \boldsymbol{\theta}_{nl,2}^T \cdots \boldsymbol{\theta}_{nl,G}^T]^T$, a candidate state $\tilde{\boldsymbol{\theta}}_{nl}$ is generated using the Metropolis-Hastings algorithm by repeating (from $j=1$ till $j=G$) the steps similar to 2i) and ii) above with subscripts $\mathbf{z}$ replaced by nl , ξ replaced by ξ , $Z_j^{(k)}$ replaced by $\boldsymbol{\theta}_{nl,j}^{(k)}$, $\tilde{Z}_j^{(k+1)}$ replaced by $\tilde{\boldsymbol{\theta}}_{nl,j}^{(k+1)}$:
4. Generate a candidate state $\tilde{\boldsymbol{\theta}}_s$ by the following Constrained Multi-group Metropolis within Gibbs sampling algorithm proposed in Cheung and Bansal (2013):

- i) Determine the conditional mean vector $\boldsymbol{\mu}_1^{(k)}(\boldsymbol{\theta}_2^{(k)}, \boldsymbol{\theta}_3^{(k)}, \boldsymbol{\theta}_4^{(k)})$ and covariance matrix $\boldsymbol{\Sigma}_1^{(k)}(\boldsymbol{\theta}_2^{(k)}, \boldsymbol{\theta}_3^{(k)}, \boldsymbol{\theta}_4^{(k)})$ of the Gaussian PDF $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2^{(k)}, \boldsymbol{\theta}_3^{(k)}, \boldsymbol{\theta}_4^{(k)}, D)$. Let $\mathbf{w}_1^{(k)} = \mathbf{L}_1^{(k)-1}(\boldsymbol{\theta}_1^{(k)} - \boldsymbol{\mu}_1^{(k)})$ where $\boldsymbol{\Sigma}_1^{(k)} = \mathbf{L}_1^{(k)} \mathbf{L}_1^{(k)T}$.

- ii) Generate a candidate state $\tilde{\mathbf{w}}_1^{(k+1)}$ for $\mathbf{w}_1$:

For each component $j=1, \dots, N_1$, draw a pre-candidate component $\xi_{1,j}^{(k+1)}$ from proposal PDF $q_{1,j}(\xi_{1,j} | w_{1,j}^{(k)})$ and compute the acceptance ratio:

$$r_{1,j}^{(k+1)} = \min \left\{ 1, \frac{p(\xi_{1,j}^{(k+1)}) q_{1,j}(w_{1,j}^{(k)} | \xi_{1,j}^{(k+1)}, \boldsymbol{\theta}_2^{(k)}, \boldsymbol{\theta}_3^{(k)}, \boldsymbol{\theta}_4^{(k)})}{p(w_{1,j}^{(k)}) q_{1,j}(\xi_{1,j}^{(k+1)} | w_{1,j}^{(k)}, \boldsymbol{\theta}_2^{(k)}, \boldsymbol{\theta}_3^{(k)}, \boldsymbol{\theta}_4^{(k)})} \right\}$$

$$\text{Set } \tilde{w}_{1,j}^{(k+1)} = \begin{cases} \xi_{1,j}^{(k+1)} & \text{with probability } r_{1,j}^{(k+1)} \\ w_{1,j}^{(k)} & \text{with probability } 1 - r_{1,j}^{(k+1)} \end{cases}$$

where $p(\cdot)$ is a standard normal PDF.

- iii) Transform $\tilde{\mathbf{w}}_1$ back to $\boldsymbol{\theta}_1$ space: $\tilde{\boldsymbol{\theta}}_1^{(k+1)} = \boldsymbol{\mu}_1^{(k)} + \mathbf{L}_1^{(k)} \tilde{\mathbf{w}}_1^{(k+1)}$.

- iv) For getting $\tilde{\boldsymbol{\theta}}_2^{(k+1)}$, repeat steps i) to iii) by reversing the subscripts 1 and 2 and replacing $\boldsymbol{\theta}_1^{(k)}$ by $\tilde{\boldsymbol{\theta}}_1^{(k+1)}$.

5. Set

$$[\mathbf{Z}^{(k+1)} \boldsymbol{\theta}_{nl}^{(k+1)} \boldsymbol{\theta}_1^{(k+1)} \boldsymbol{\theta}_2^{(k+1)}] = \begin{cases} [\tilde{\mathbf{Z}}^{(k+1)} \tilde{\boldsymbol{\theta}}_{nl}^{(k+1)} \tilde{\boldsymbol{\theta}}_1^{(k+1)} \tilde{\boldsymbol{\theta}}_2^{(k+1)}] & \text{if } \mathbf{Y}([\tilde{\boldsymbol{\theta}}_1^{(k+1)} \tilde{\boldsymbol{\theta}}_2^{(k+1)}], \tilde{\boldsymbol{\theta}}_{nl}^{(k+1)}, \tilde{\mathbf{Z}}^{(k+1)}) \in F \\ [\mathbf{Z}^{(k)} \boldsymbol{\theta}_{nl}^{(k)} \boldsymbol{\theta}_1^{(k)} \boldsymbol{\theta}_2^{(k)}] & \text{if } \mathbf{Y}([\tilde{\boldsymbol{\theta}}_1^{(k+1)} \tilde{\boldsymbol{\theta}}_2^{(k+1)}], \tilde{\boldsymbol{\theta}}_{nl}^{(k+1)}, \tilde{\mathbf{Z}}^{(k+1)}) \notin F \end{cases}$$

6. Sample $\theta_3^{(k+1)}$ from $p(\theta_3 | \theta_1^{(k+1)}, \theta_2^{(k+1)}, \theta_4^{(k)}, D)$.
7. Sample $\theta_4^{(k+1)}$ from $p(\theta_4 | \theta_1^{(k+1)}, \theta_2^{(k+1)}, \theta_3^{(k+1)}, D)$.
8. Let $k=k+1$ and go to step 2, until $(1/p_0-1)$ samples are obtained.

Please refer to the journal version of this paper for more details for the proposed methods and algorithms and their proof of correctness.

ILLUSTRATIVE EXAMPLE

The nonlinear structure system selected in the illustrative example is modeled as a 7-DOF shear building, the details of which can be found in Ching et al. (2004). The stiffnesses and damping parameters are given by:

$$\begin{aligned} k_{i,t} &= k_{i,0} e^{-\gamma \mu_{i,t}^\rho} \\ c_{i,t} &= \delta + \lambda \mu_{i,t} \end{aligned} \quad (12)$$

where $\mu_{i,t}$ is the maximum interstory drift ratio up to time t (for story i with story height H_i), given by:

$$\begin{aligned} \mu_{1,t} &= \max_{0 \leq k \leq t} (|x_{1,k}| / H_1) \\ \mu_{i,t} &= \max_{0 \leq k \leq t} (|x_{i,k} - x_{i-1,k}| / H_i) \end{aligned} \quad (13)$$

and where the nominal values of dynamic model parameters are given by: $\gamma = 5.0$, $\rho = 0.4$, $\delta = 0.5$ kNs/m, $\lambda = 50$ kNs/m, $k_1 = 100$ kN / m, $k_2 = 120$ kN / m, $k_3 = 100$ kN / m, $k_4 = k_5 = k_6 = k_7 = 80$ kN / m, $m_1 = 120$ kg, $m_2 = m_3 = m_4 = m_5 = m_6 = 100$ kg and $m_7 = 90$ kg.

The damage in the structure is simulated by reducing the stiffnesses of the nominal model at the fourth and fifth story by 30%. The modal data consist of 10 sets of simulated modal data obtained from the damaged model. Each set consists of the first two modal frequencies, modal damping ratios, and partial complex mode shapes (corresponding to DOFs, one, three, five and seven). Noisy measured modal parameters are generated by adding random values chosen from zero-mean Gaussian distribution with standard deviation equal to 2% of the exact values.

For identification and uncertainty quantification, the same 7-DOF system is considered with the prior PDFs of $\{m_{i,0} : i=1:7\}$, $\{k_{i,0} : i=1:7\}$ and $\{c_{i,0} : i=1:7\}$ taken to be independent Gaussian with mean equal the nominal values with c.o.v equal to 1% for mass, 10% for stiffness parameters and 20% for damping parameters. In addition, independent Gaussian PDFs are assumed for γ , ρ and λ with mean equal to optimal values and 20% c.o.v. Using the modal data, only the joint PDF of $\{m_{i,0} : i=1:7\}$, $\{k_{i,0} : i=1:7\}$ and $\{c_{i,0} : i=1:7\}$ are updated, since, it is assumed that the modal data are obtained for a structure behaving approximately linearly. No information is provided by the modal data to update γ , ρ and λ that define the nonlinear behavior of the system. Additional uncertain parameters that are considered include prediction error variances $[\sigma_{\text{Re},1}^2 \ \sigma_{\text{Im},1}^2 \ \sigma_{\text{Re},2}^2 \ \sigma_{\text{Im},2}^2]$ and complete complex

mode shapes $[\psi_1 \ \psi_2]$ for the first two modes. Flat independent priors are taken for $[\psi_1 \ \psi_2]$ and non-informative independent inverse gamma prior PDFs are taken for $[\sigma_{\text{Re},1}^2 \ \sigma_{\text{Im},1}^2 \ \sigma_{\text{Re},2}^2 \ \sigma_{\text{Im},2}^2]$ (Jeffreys' non-informative prior). The total number of uncertain parameters to be updated is 56 (24 structural parameters, 28 for the two mode shapes and 4 prediction error variances).

A discrete-time white noise sequence $\mathbf{Z}$ corresponding to the duration of future input ground motion $T = 40$ s and sampling interval $\Delta t = 0.02$ s is considered. The system is assumed to have zero initial conditions. The structural dynamic system is subjected to future non-stationary, non-white ground acceleration. The frequency content of the ground acceleration is modeled by Clough-Penzien spectrum (Clough and Penzien, 1993):

$$S(\omega) = S_0 \frac{(\omega/\omega_f)^4}{\left(1 - (\omega/\omega_f)^2\right)^2 + (2\zeta_f \omega/\omega_f)^2} \frac{1 + (2\zeta_g \omega/\omega_g)^2}{\left(1 - (\omega/\omega_g)^2\right)^2 + (2\zeta_g \omega/\omega_g)^2} \quad (14)$$

where $\omega_g = 15.7$ rad/s, $\omega_f = 0.1\omega_g$, $\zeta_g = \zeta_f = 0.6$ and $S_0 = 1e^{-3} \text{ m}^2/\text{s}^3$. The non-stationarity is modeled using a time envelop function $\lambda(t) = \alpha_1 t e^{-\alpha_2 t}$ with $\alpha_1 = 0.45$ s and $\alpha_2 = 1/6 \text{ s}^{-1}$. For illustration, failure is defined as an event where the interstory drift ratio for any story exceeds a specific threshold at any discrete time instant during the total duration of the ground acceleration.

The proposed method is used to evaluate the updated robust failure probability at different thresholds with a conditional failure probability at each level approximately equal to $p_0 = 0.1$ and with the number of samples set to $N = 500$ at each conditional level. To start with, samples of contribution parameters, mode shapes and prediction error variances are obtained using the Gibbs sampling algorithm for Bayesian model updating of a linear dynamic system. After discarding the burn-in samples, $N = 500$ samples are randomly drawn from the remaining samples for evaluating $P(F_1 | D)$. For the proposed approach, the proposal PDFs $\{q_{1,j}(\xi_{1,j} | w_{1,j}^{(k)}), j = 1, \dots, N_1\}$ and $\{q_{2,j}(\xi_{2,j} | w_{2,j}^{(k)}), j = 1, \dots, N_2\}$ are selected as follows:

$$[\sigma_{l,i,1}^2 \dots \sigma_{l,i,N_l}^2]^T = 2.25 \times \text{diag}(\mathbf{L}_l^{(k)-1} (\mathbf{\Sigma}_{l,i}) \mathbf{L}_l^{(k)-T}), \quad l = 1, 2 \quad (15)$$

where $\mathbf{\Sigma}_{1,i}$ and $\mathbf{\Sigma}_{2,i}$ are the covariance matrices estimated from $\mathbf{\theta}_1$ and $\mathbf{\theta}_2$ samples obtained from the most recent simulation level distributed according to PDFs $p(\mathbf{\theta}_1 | F_i, D)$ and $p(\mathbf{\theta}_2 | F_i, D)$, respectively, and $\text{diag}(\mathbf{A})$ denotes a vector with entries equal to the diagonal entries of a matrix $\mathbf{A}$.

Following this, using the modal data, the updated robust failure probability of the structural dynamic system subjected to future stochastic excitation is computed using the proposed method. Sample means of the updated robust failure probability estimator obtained by the proposed approach, estimated by 50 independent simulation runs is shown in Figure 1(a). Results computed using 100,000 MCS samples are also shown for comparison. The results confirm that the proposed

approach is correct showing the updated robust failure probability estimator is practically unbiased. Figure 1(b) compares the sample c.o.v of the updated robust failure probability estimator by the proposed approach and the lower limit of c.o.v of MCS estimator at a particular failure probability level using the same number of dynamic analyses as in the proposed approach. The results indicate substantial improvement in efficiency over MCS.

Figure 1. Sample mean (a) and sample c.o.v (b) of the updated robust failure probability estimator obtained by the proposed approach, estimated from 50 independent simulation runs

CONCLUSION

An efficient stochastic simulation method was presented for computing the updated robust failure probability of a structural dynamic system using incomplete modal data when the system is subjected to future stochastic excitation modeled by Gaussian noise. Uncertainties from structural modeling, the modeling of the stochastic excitation that the structure will experience and any other uncertainties can all be taken into account. The proposed approach is robust to the number of uncertain parameters and random variables and the dimension of the vibration data involved in the problem. It integrates a newly-developed stochastic simulation method for Bayesian model updating of a linear dynamic system using incomplete modal data proposed by the authors and a very efficient algorithm called Subset Simulation to compute small failure probabilities together with a new efficient conditional sampling algorithm proposed by the authors, referred to as Constrained Multi-group Metropolis within Gibbs algorithm. This later algorithm and the aforementioned algorithm for Bayesian model updating are the key contributors for the robustness in efficiency of the proposed approach in handling the case involving a huge number of the uncertain model parameters whose PDF is updated by the modal data.

Results from the illustrative example shows that the proposed method provides substantial improvement in efficiency over MCS. Additional examples

involving more nonlinear structural dynamic systems are shown in the journal version of this paper. The case involving dependent errors in Bayesian model updating is presented in another paper by the authors, which is currently under review. A new method which tackles very efficiently the case where the dynamic system behaves linearly when it is subjected to future stochastic dynamic excitation can be found in Cheung and Bansal (2013b) and the corresponding journal version.

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