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A Stochastic Simulation Algorithm for Bayesian Model Updating Of Linear Structural Dynamic System with Non-Classical Damping

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ABSTRACT: Model updating using measured system dynamic response has a wide range of applications in structural health monitoring and control, response prediction, reliability and risk assessment. In this paper, we are interested in model updating of a linear structural dynamic system with non-classical damping based on incomplete modal data including modal frequencies, damping ratios and partial complex mode shapes of some of the dominant modes. To quantify the uncertainties and plausibility of the model parameters, a Bayesian approach is considered in which the probability distribution of the model parameters needs to be updated. A new stochastic simulation algorithm is proposed, which allows for an efficient update of the probability distribution of the model parameters. The effectiveness and efficiency of the proposed method are illustrated by a numerical example involving linear structural dynamic system with complex modes.

1 INTRODUCTION

The need of model updating is usually motivated by the desire to improve the accuracy of prediction of the system response and control, structural health monitoring, or reliability and risk assessment. There always exist modeling errors and uncertainties associated with the process of constructing a mathematical model of a system arising either because of simplification in the modeling, or the imperfection or lack of accurate information in the modeling of physical problem. These uncertainties in the modeling process can cause the predicted system response to be different from the true system response. If accurate experimental data are available then this data could be used to update the uncertain parameters of the model.

The usual approach to update a linear dynamic system is to first identify its modal properties (especially when the data are from ambient vibration) and then use those to update the modeling parameters. There are several ambient or forced vibration based modal identification techniques available (Brincker et al., 2000, Peeters and De Roeck, 2001, Katafygiotis and Yuen, 2001, Yuen and Katafygiotis, 2001, ang et al., 2003, Yang et al., 2003, Parloo, 2003) that provide optimal estimates of the modal parameters. Model updating techniques can be grouped into two types: probabilistic and deterministic. Probabilistic techniques, particularly the Bayesian approach, provide estimates of the optimal parameters along with their probability density func-

tion (PDF) that can be used to describe their uncertainty while the outcome of deterministic techniques is usually a unique set or a finite number of sets of parameters. Several researchers (Mottershead and Friswell, 1993, Modak et al., 2002, Jaishi and Ren, 2005, Papadimitriou et al., 2012) have presented works on updating of the Finite element models, based on the experimental modal data. However, there are relatively few papers in model updating literature in which probabilistic model updating is considered (Beck and Katafygiotis, 1998, Vanik et al., 2000, Yuen and Beck, 2003, Beck and Au, 2002, Ching et al., 2006, Papadimitriou and Papadioti, 2012). Ching *et al.* (2006) proposed a new Gibbs sampling based simulation approach for model updating of linear dynamic systems with classical damping. However, in their algorithm parameters defining the damping matrix were not considered.

In this paper, a new stochastic simulation algorithm based on Gibbs sampler is proposed for Bayesian model updating of a linear dynamic system with non-classical damping based on incomplete complex modal data, namely modal frequencies, damping ratios, and partial complex mode shapes of some of the dominant modes. In the proposed algorithm, the damping matrix in the identification model is represented as a sum of individual substructures in the case of viscous damping, in terms of mass and stiffness matrices in the case of Rayleigh damping or a combination of the formers. Solving Bayesian model updating problem in a high-dimensional parameter space typically experiences convergence is-

sues. The proposed method is robust to the dimension of the problem. Finally, to demonstrate the effectiveness and accuracy of the proposed method, a numerical example is presented.

2 BAYESIAN MODEL UPDATING

Bayesian model updating approach provides a robust and rigorous framework to characterize modeling uncertainties. Given the data D and the prior PDF $p(\boldsymbol{\theta})$ of the uncertain system parameters $\boldsymbol{\theta}$, by applying the Bayes' theorem the posterior PDF $p(\boldsymbol{\theta}|D)$ can be written as (Beck and Katafygiotis, 1998):

$$p(\boldsymbol{\theta} | D) = \frac{p(D | \boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(D | \boldsymbol{\theta})p(\boldsymbol{\theta})d\boldsymbol{\theta}} \quad (1)$$

It is not feasible to compute the integral in the denominator of Equation (1) in the case of high dimensionality, thus $p(\boldsymbol{\theta}|D)$ is often not known explicitly a priori and only known up to a normalizing constant. Beck and Katafygiotis (1998) proposed Laplace's method of asymptotic approximation to obtain the distribution of the updated parameters. However, the accuracy of such an approximation is questionable when either the amount of data is not sufficiently large or the chosen class of models turns out to be unidentifiable based on the available data. Also, the approach is computationally challenging, especially, in a high-dimensional parameter space or when the model class is not globally identifiable.

Let $D \equiv \{\hat{\omega}_{m,s}, \hat{\zeta}_{m,s}, \hat{\psi}_{m,s} : m=1...M, s=1...S\}$ be the experimental modal data from the structural system, consisting of modal frequencies $\hat{\omega}_{m,s} \in R$, damping ratios $\hat{\zeta}_{m,s} \in R$, and mode shape components $\hat{\psi}_{m,s} \in C^{N_o}$ where N_o is the number of measured DOF, M is the number of observed modes, and S is the number of modal data sets available. The objective of the proposed Gibbs sampling based Bayesian model updating approach is to estimate the optimal system parameters along with their PDF, based on the experimental modal data D .

3 THE PROPOSED APPROACH

In state-space, the equation of motion of a general 2nd-order time invariant linear dynamical system can be expressed by a first order differential equation as follows:

$$\dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t) \quad (2)$$

$$\mathbf{X}(0) = \mathbf{X}_0 \quad (3)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad (4)$$

where $\mathbf{X}(t)$ and $\mathbf{U}(t)$ denote the state and excitation vectors at time t , respectively, and $\mathbf{X}_0$ denotes the initial conditions. The system matrix $\mathbf{A}$ is a function of mass, damping, and stiffness matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$. Complex eigenvalues λ_m and eigenvector Ψ_m for $m=1,2,...,M$, can be obtained from the solution of the eigenvalue problem corresponding to the system matrix $\mathbf{A}$ as:

$$\mathbf{A}\Psi_m = \lambda_m \Psi_m \quad (5)$$

$$\Psi_m = \begin{Bmatrix} \psi_m \\ \lambda_m \psi_m \end{Bmatrix} \quad (6)$$

$$\lambda_m = -\zeta_m \omega_m + i \omega_m \sqrt{1 - \zeta_m^2} \quad (7)$$

The eigenvalues and eigenvectors occur in complex conjugate pairs. Using Equations (4)-(5) the following relationship between modal properties and dynamic model parameters can be obtained:

$$\lambda_m^2 \mathbf{M} \psi_m + \lambda_m \mathbf{C} \psi_m + \mathbf{K} \psi_m = 0 \quad (8)$$

Replacing system eigenvalues λ_m experimentally obtained eigenvalues $\hat{\lambda}_{m,s}$ gives:

$$\hat{\lambda}_{m,s}^2 \mathbf{M} \psi_m + \hat{\lambda}_{m,s} \mathbf{C} \psi_m + \mathbf{K} \psi_m = \boldsymbol{\varepsilon}_{m,s} \quad (9)$$

where the system mode shape ψ_m is related to the observed mode shape $\hat{\psi}_{m,s}$ through a selection matrix Γ that picks the observed DOF from the system mode shape.

$$\hat{\psi}_{m,s} = \Gamma \psi_m + \mathbf{e}_{m,s} \quad (10)$$

In the above equation $\boldsymbol{\varepsilon}_{m,s}$ and $\mathbf{e}_{m,s}$ are the complex random vectors representing the model prediction errors, i.e., the errors between the response of the structure under consideration and that of the assumed model. The mass and stiffness matrices in Equation (9) are represented as a linear sum of contribution of the corresponding mass, and stiffness matrices from the individual prescribed substructures:

$$\mathbf{M}(\alpha) = \mathbf{M}_0 + \sum_{i=1}^{N_\alpha} \alpha_i \mathbf{M}_i \quad (11)$$

$$\mathbf{K}(\eta) = \mathbf{K}_0 + \sum_{i=1}^{N_\eta} \eta_i \mathbf{K}_i \quad (12)$$

The damping matrix, in general, can be represented in terms of mass and stiffness matrix (as in the case

of Rayleigh damping), and contribution from other damping sources (as in the case of viscous damping):

$$\mathbf{C}(\boldsymbol{\beta}, \mathbf{a}) = \mathbf{C}_0 + \sum_{i=1}^{N_\beta} \beta_i \mathbf{C}_i + a_0 \mathbf{M}(\boldsymbol{\alpha}) + a_1 \mathbf{K}(\boldsymbol{\eta}) \quad (13)$$

In Equations (11)-(13), $\boldsymbol{\alpha}$ and $\boldsymbol{\eta}$ are the mass and stiffness contribution parameters, and $\boldsymbol{\beta}$ are the damping contribution parameters for non-classical damping and $\mathbf{a}$ are the Rayleigh damping coefficients. Other parameters which are unknown in Equations (9)-(10) and need to be updated are the system mode shapes ψ_m , $m=1,2,\dots,M$ and the parameters defining the probabilistic models of the model prediction errors. By equating the real and imaginary parts on both sides of Equations (9)-(10) respectively, the following equations are obtained:

$$\text{Re}(\hat{\lambda}_{m,s}^2 \mathbf{M}(\boldsymbol{\alpha}) \psi_m + \hat{\lambda}_{m,s} \mathbf{C}(\boldsymbol{\beta}) \psi_m + \mathbf{K}(\boldsymbol{\eta}) \psi_m) = \text{Re}(\boldsymbol{\epsilon}_{m,s}) \quad (14)$$

$$\text{Im}(\hat{\lambda}_{m,s}^2 \mathbf{M}(\boldsymbol{\alpha}) \psi_m + \hat{\lambda}_{m,s} \mathbf{C}(\boldsymbol{\beta}) \psi_m + \mathbf{K}(\boldsymbol{\eta}) \psi_m) = \text{Im}(\boldsymbol{\epsilon}_{m,s}) \quad (15)$$

$$\text{Re}(\hat{\psi}_{m,s} - \Gamma \psi_m) = \text{Re}(\mathbf{e}_{m,s}) \quad (16)$$

$$\text{Im}(\hat{\psi}_{m,s} - \Gamma \psi_m) = \text{Im}(\mathbf{e}_{m,s}) \quad (17)$$

Based on the Principle of Maximum Entropy (Jaynes, 1978), the PDFs for vectors $\text{Re}(\boldsymbol{\epsilon}_{m,s})$, $\text{Im}(\boldsymbol{\epsilon}_{m,s})$, $\text{Re}(\mathbf{e}_{m,s})$, $\text{Im}(\mathbf{e}_{m,s})$ are taken to be Gaussian. For illustration, their means are assumed to be equal to zero and covariance matrices equal to scaled versions of the identity matrix $\mathbf{I}$ of appropriate order, respectively.

$$\text{Re}(\boldsymbol{\epsilon}_{m,s}) \sim N(0, \sigma_{\text{Re},m}^2 \mathbf{I}) \quad (18)$$

$$\text{Im}(\boldsymbol{\epsilon}_{m,s}) \sim N(0, \sigma_{\text{Im},m}^2 \mathbf{I}) \quad (19)$$

$$\text{Re}(\mathbf{e}_{m,s}) \sim N(0, \delta_{\text{Re},m}^2 \mathbf{I}) \quad (20)$$

$$\text{Im}(\mathbf{e}_{m,s}) \sim N(0, \delta_{\text{Im},m}^2 \mathbf{I}) \quad (21)$$

The variance parameters $\delta_{\text{Re},m}^2$ and $\delta_{\text{Im},m}^2$ are assumed to be known or are directly estimated from the sample variance of the experimental modal data:

$$\hat{\delta}_{\text{Re},m}^2 = \frac{1}{SN_o} \sum_{j=1}^S |\text{Re}(\hat{\psi}_{m,s} - \bar{\psi}_m)|^2 \quad (22)$$

$$\hat{\delta}_{\text{Im},m}^2 = \frac{1}{SN_o} \sum_{j=1}^S |\text{Im}(\hat{\psi}_{m,s} - \bar{\psi}_m)|^2 \quad (23)$$

where $\bar{\psi}_m$ is the averaged mode shape for m -th mode. The variance parameters $\sigma_{\text{Re},m}^2$, $\sigma_{\text{Im},m}^2$ are left for updating. In total, the parameters to be updated are the contribution parameters $[\boldsymbol{\alpha}^T, \boldsymbol{\beta}^T, \boldsymbol{\eta}^T]^T$, mode shapes $[\text{Re}(\psi_1), \text{Im}(\psi_1), \dots, \text{Re}(\psi_M), \text{Im}(\psi_M)]$ and prediction error variance $[\sigma_{\text{Re},1}^2, \sigma_{\text{Im},1}^2, \dots, \sigma_{\text{Re},M}^2, \sigma_{\text{Im},M}^2]$.

3.1. The proposed Gibbs sampling based algorithm

The Gibbs Sampler (Geman and Geman, 1984) is one type of Markov chain Monte Carlo (MCMC) algorithms that allow sampling from an arbitrary multivariate PDF if sample simulation according to the PDFs of each group of uncertain parameters conditioned on all the others groups is possible. Usually some initial portion of Markov chain samples are discarded before the stationary stage is reached. After the burn-in period, the Markov chain samples obtained are distributed as according to the target PDF.

Similar in spirit to how Ching et al. (2006) derived their proposed algorithm, our proposed algorithm is derived as shown as follows.

In the proposed Gibbs sampling based algorithm, four groups of parameters are considered:

$$\boldsymbol{\theta}_1 = [\boldsymbol{\alpha}^T, \boldsymbol{\eta}^T, \boldsymbol{\beta}^T]^T,$$

$$\boldsymbol{\theta}_2 = \mathbf{a},$$

$$\boldsymbol{\theta}_3 = [\text{Re}(\psi_1), \text{Im}(\psi_1), \dots, \text{Re}(\psi_M), \text{Im}(\psi_M)],$$

$$\boldsymbol{\theta}_4 = [\sigma_{\text{Re},1}^2, \sigma_{\text{Im},1}^2, \dots, \sigma_{\text{Re},M}^2, \sigma_{\text{Im},M}^2]$$

It will be more convenient to choose Bayesian conjugate priors which will allow exact sampling from the full conditional PDFs $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D)$, $p(\boldsymbol{\theta}_2 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_3, \boldsymbol{\theta}_4, D)$, $p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_4, D)$ and $p(\boldsymbol{\theta}_4 | \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3, D)$. Thus, the initial PDF for the system parameters $\boldsymbol{\theta}_1$ is taken to be the product of independent Gaussian PDFs, $\boldsymbol{\theta}_1 \sim N(\boldsymbol{\theta}_1^{(0)}, \mathbf{P}_1^{(0)})$ with mean $\boldsymbol{\theta}_1^{(0)}$ and diagonal covariance matrix $\mathbf{P}_1^{(0)}$ to express the initial uncertainties. Similarly, the prior PDF for $\boldsymbol{\theta}_2$ is taken to be Gaussian PDF, i.e., $\boldsymbol{\theta}_2 \sim N(\boldsymbol{\theta}_2^{(0)}, \mathbf{P}_2^{(0)})$. The initial PDF for the system mode shapes $\boldsymbol{\theta}_3$ is taken to be the product of either independent Gaussian PDFs in case any prior information is available, or independent uniform PDFs in case of no prior information (as for the case with unknown components of the mode shapes). The initial PDF for prediction error variances $\boldsymbol{\theta}_4$ is taken to be the product of independent inverse gamma PDFs, $IG(\rho_0, \kappa_0)$ with prespecified parameters ρ_0 and κ_0 .

3.2. Full conditional PDFs

Equations (14)-(15) are linear with respect to $\boldsymbol{\theta}_1$ given $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$ and $\boldsymbol{\theta}_4$, i.e., they can be written in the following form:

$$\mathbf{Y}_1 - \mathbf{A}_1 \boldsymbol{\theta}_1 = \mathbf{E}_1 \quad (24)$$

where $\mathbf{E}_1 \sim N(\mathbf{0}, \boldsymbol{\Sigma}_1)$; $\mathbf{Y}_1$ and $\mathbf{A}_1$ are a fixed vector and matrix respectively given $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$ and $\boldsymbol{\Sigma}_1$ is a fixed covariance matrix given $\boldsymbol{\theta}_4$. The interested reader

can refer to the journal version of this paper for the details about the entries of these vectors and matrices. Then, the full conditional PDF $p(\theta_1|\theta_2,\theta_3,\theta_4,D)$ is Gaussian whose first two moments are given by:

$$E(\theta_1 | \theta_2, \theta_3, \theta_4, D) = \left(\mathbf{P}_1^{(0)-1} + \mathbf{A}_1^T \mathbf{E}_1^{-1} \mathbf{A}_1 \right)^{-1} \left(\mathbf{P}_1^{(0)-1} \theta_1^{(0)} + \mathbf{A}_1^T \mathbf{E}_1^{-1} \mathbf{Y}_1 \right) \quad (25)$$

$$\text{Cov}(\theta_1 | \theta_2, \theta_3, \theta_4, D) = \left(\mathbf{P}_1^{(0)-1} + \mathbf{A}_1^T \mathbf{E}_1^{-1} \mathbf{A}_1 \right)^{-1} \quad (26)$$

Similar to the above, the PDF, $p(\theta_2|\theta_1,\theta_3,\theta_4,D)$ is Gaussian and its first two moments can be obtained in a similar manner as above. $p(\theta_3|\theta_1,\theta_2,\theta_4,D)$ is Gaussian and given $\theta_1, \theta_2, \theta_4$, $[\text{Re}(\psi_m)^T, \text{Im}(\psi_m)^T]^T$, $m=1,2,...,M$, are independent and $p([\text{Re}(\psi_m)^T, \text{Im}(\psi_m)^T]^T | \theta_1, \theta_2, \theta_4, D)$ is also Gaussian and its first two moments can be obtained in a similar manner as above. For the details, please refer to the journal version of this paper. $p(\theta_4|\theta_1,\theta_2,\theta_3,D)$ is the product of independent inverse gamma PDFs with its marginal PDFs given by:

$$p(\sigma_{\text{Re},m}^2 | \theta_1, \theta_2, \theta_3, D) = IG \left(\rho_0 + \frac{N_d S}{2}, \kappa_0 + \frac{1}{2} \sum_{s=1}^S \text{Re}(\epsilon_{m,s})^T \text{Re}(\epsilon_{m,s}) \right) \quad (27)$$

$$p(\sigma_{\text{Im},m}^2 | \theta_1, \theta_2, \theta_3, D) = IG \left(\rho_0 + \frac{N_d S}{2}, \kappa_0 + \frac{1}{2} \sum_{s=1}^S \text{Im}(\epsilon_{m,s})^T \text{Im}(\epsilon_{m,s}) \right) \quad (28)$$

3.3. Summary of the proposed algorithm

- 1) Initialize samples, draw from the prior PDF or choose nominal values as starting points and let $k=1$.
- 2) Sample $\theta_1^{(k)}$ from $p(\theta_1 | \theta_2^{(k-1)}, \theta_3^{(k-1)}, \theta_4^{(k-1)}, D)$ where the first two moments are given by (25) and (26).
- 3) Sample $\theta_2^{(k)}$ from $p(\theta_2 | \theta_1^{(k)}, \theta_3^{(k-1)}, \theta_4^{(k-1)}, D)$ as described above.
- 4) Sample $\theta_3^{(k)}$ by simulating $[\text{Re}(\psi_m)^{(k)T}, \text{Im}(\psi_m)^{(k)T}]^T$ from $p([\text{Re}(\psi_m)^T, \text{Im}(\psi_m)^T]^T | \theta_1^{(k)}, \theta_2^{(k)}, \theta_4^{(k-1)}, D)$ for $m=1,...,M$, as described above
- 5) Sample $\theta_4^{(k)}$ by simulating $\sigma_{\text{Re},m}^{2(k)}$ and $\sigma_{\text{Im},m}^{2(k)}$ from $p(\sigma_{\text{Re},m}^2 | \theta_1^{(k)}, \theta_2^{(k)}, \theta_3^{(k)}, D)$ and $p(\sigma_{\text{Im},m}^2 | \theta_1^{(k)}, \theta_2^{(k)}, \theta_3^{(k)}, D)$, respectively, for $m=1,...,M$ according to (27) and (28).

- 6) Let $k=k+1$ and go to step 2, until N samples are obtained.

4 CONVERGENCE ISSUES

In general, the model updating problem for a high dimensional parameter space will typically have convergence problems due to over parameterized linear identification model or data with incomplete information. The likelihood function has singularities (i.e., nonsensical maxima), and depending on the initial values, the sampling run may converge to a local maximum. For example, local maxima where a run may get trapped involves variance parameters approaching extreme values (zero or infinity). If this happens, the results obtained may be meaningless.

To tackle the aforementioned problems, one may consider an identification model where all the variance parameters are forced to be equal. Gibbs sampling based algorithm as proposed above or asymptotic approximations to posterior distributions via conditional moment can be used to obtain the estimates for the global optimal point of the posterior PDF for the model with equal variance parameters. These estimates will provide information about the neighborhood of the global optimal point of the posterior PDF for the model with unequal variance parameters and can be used as a starting point of the Markov chain in the proposed algorithm for the model with unequal variance parameters.

5 ILLUSTRATIVE EXAMPLE

In this illustrative example, a 120-DOF four-story, two-bay by two-bay steel frame originally designed for IASC-ASCE Phase-I Simulated Structural Health Monitoring Benchmark Problem (Johnson et al., 2004) is adopted and modified. It has 2.5 m X 2.5 m plan and is 3.6 m tall. Braces in each story located on the exterior faces are removed and replaced by viscous dampers with damping coefficient 20 kN-s/m (as shown in Fig. 3). In addition classical damping with 1% damping ratio for all modes is assumed. The nominal properties of other structural members are shown in Table I. The x-direction is the strong direction and each floor has 4 slabs: 800 kg slabs at the first level, 600 kg slabs at the second and third level, and 400 kg slabs at the fourth level.

The simulated modal data consist of 10 sets of modal data ($S=10$) with the first eight translational modes ($M=8$, four in the x-direction and four in the y-direction), each with four observed DOF ($N_o=4$, corresponding to translational DOF at four levels). Noisy measured modal parameters are generated by adding random values chosen from zero-mean Gaussian distribution with standard deviation equal to 2% of the exact values.

For identification a 36-DOF model that assumes rigid floor in the x-y plane and allows rotations along x-axis and y-axis is used. Each floor has 2 in-plane DOF (1 in x-direction and 1 in y-direction), 1 rotational DOF (in the z-axis), and 6 rotational DOF (3 in x-axis and 3 in y-axis). Along the x-axis and y-axis, joints having the same x-coordinates or having the same y-coordinates are assumed to have the same amount of rotation along the x-axis and y-axis, respectively.

The mass for each story is lumped at the floor level to give four uncertain mass parameters to be updated. Masses are assumed to be known with small uncertainty thus the prior PDF for $\alpha_i : i=1,...,4$ is assumed to be independent Gaussian with mean values equal to 1 and c.o.v. equal to 1%. For each floor level, the stiffness matrix is defined using three prescribed substructure stiffness matrices, one related to 2 translation and 1 rotational DOF $\mathbf{K}_{c,i} : i=1,...,4$ and two related to 6 rotational DOFs $\mathbf{K}_{R_x,i}, \mathbf{K}_{R_y,i} : i=1,...,4$. In total there are twelve uncertain stiffness parameters to be quantified for the whole structure. The prior PDF for $\eta_{c,i} : i=1,...,4$ is assumed to be independent Gaussian with mean values equal to 1 and c.o.v. equal to 2%. The prior PDF for $\eta_{R_x,i}, \eta_{R_y,i} : i=1,...,4$ is assumed to be independent Gaussian PDFs with all mean values and c.o.v. equal to 1 and 10%, respectively. The damping coefficients of the viscous dampers at each floor are assumed to be equal but uncertain. Thus, there are 4 more uncertain parameters to be quantified $\beta_i : i=1,...,4$ and their prior PDF is taken to be a product of independent Gaussian PDFs with mean value equal to 1 and c.o.v. equal to 5%. Rayleigh damping with uncertain coefficients is also assumed for the structure. The prior PDF for the two Rayleigh damping coefficients a_0, a_1 is taken to be a product of independent Gaussian PDFs with mean values corresponding to coefficients for 1% damping ratio for 1st in x-direction and 4th mode in y-direction, and c.o.v. equal to 20%. Flat independent priors are taken for mode shapes and a product of independent inverse gamma non-informative prior PDFs are taken for prediction error variances.

$$\mathbf{M}(\mathbf{a}) = \sum_{i=1}^4 \alpha_i \mathbf{M}_i \quad (29)$$

$$\mathbf{C}(\boldsymbol{\beta}, \mathbf{a}) = \sum_{i=1}^4 \beta_i \mathbf{C}_i + a_0 \mathbf{M}(\mathbf{a}) + a_1 \mathbf{K}(\boldsymbol{\eta}) \quad (30)$$

$$\mathbf{K}(\boldsymbol{\eta}) = \sum_{i=1}^4 (\eta_{c,i} \mathbf{K}_{c,i} + \eta_{R_x,i} \mathbf{K}_{R_x,i} + \eta_{R_y,i} \mathbf{K}_{R_y,i}) \quad (31)$$

The total number of uncertain parameters to be updated is 614 (576 from 8 mode shapes, 4 mass contribution parameters, 12 stiffness contribution parameters, 4 non-classical damping contribution parameters, 2 classical damping coefficients and 16

prediction error variances). The initial point of the Markov chain for the model in which all prediction error variance parameters are allowed to be different is obtained by optimization of the model where all the prediction error variance parameters are forced to be equal (only 1 prediction error variance parameter to be updated). The starting point is simulated from the prior PDFs of the uncertain parameters.

Fig. 1. Diagram of modified benchmark structure

Table I
PROPERTIES OF STRUCTURAL MEMBERS
(Johnson et al., 2004)

Property	Column	Beam
Section type	B100×9	S75×11
Cross-section area A (m ²)	1.133e-3	1.43e-3
Moment of inertia (strong direction) I_y (m ⁴)	1.97e-6	1.22e-6
Moment of inertia (weak direction) I_x (m ⁴)	0.664e-6	0.249e-6
St. Venant torsion constant J (m ⁴)	8.01e-9	38.2e-9
Youngs modulus E (Pa)	2e11	2e11
Shear modulus G (Pa)	$E/2.6$	$E/2.6$
Mass per unit volume ρ (kg/m ³)	7,800	7,800

Table II
STATISTICAL PROPERTIES OF
PREDICTION ERROR VARIANCES

Data	X-direction	
	Real (X10 ¹²)	Imag (X10 ¹²)
Mode1	0.0040	0.0001
Mode 2	0.0663	0.0042
Mode 3	0.9519	0.0603
Mode 4	8.0559	0.6817
	Y-direction	
	Real	Imag
Mode1	0.0013	0.0000
Mode 2	0.0099	0.0019
Mode 3	0.0776	0.0089
Mode 4	0.6951	0.1355

Fig. 2 shows the posterior samples for prediction error variances (shown together for all modes). It is observed that their patterns stabilize very quickly. The posterior mean estimates of prediction error variances are shown in Table II. The prediction error variances are larger for the higher modes. Relatively large variance terms for the higher modes indicate that the errors associated have larger magnitudes and the contribution of the higher mode modal data to the final parameter estimation is relatively low.

Fig. 2. Posterior samples for prediction error variances

Table. III
STATISTICAL PROPERTIES OF THE
CONTRIBUTION PARAMETERS

Parameter	Posterior mean	Standard Deviation	c.o.v. (%)
α_1	0.8794	0.0060	0.6789
α_2	0.8700	0.0055	0.6311
α_3	0.8704	0.0056	0.6416
α_4	0.8770	0.0059	0.6729
$\eta_{c,1}$	0.8874	0.0075	0.8436
$\eta_{c,2}$	0.8755	0.0057	0.6559
$\eta_{c,3}$	0.8716	0.0057	0.6517
$\eta_{c,4}$	0.8575	0.0186	2.1675
$\eta_{Rx,1}$	0.9047	0.0115	1.2658
$\eta_{Rx,2}$	0.8756	0.0097	1.1031
$\eta_{Rx,3}$	0.8916	0.0088	0.9856
$\eta_{Rx,4}$	0.8093	0.0829	10.2381
$\eta_{Ry,1}$	0.8890	0.0086	0.9653
$\eta_{Ry,2}$	0.8918	0.0073	0.8238
$\eta_{Ry,3}$	0.8732	0.0070	0.8037
$\eta_{Ry,4}$	0.8342	0.0459	5.5037
β_1	0.9206	0.0094	1.0263
β_2	0.8788	0.0062	0.7096
β_3	0.8761	0.0065	0.7460
β_4	0.8730	0.0061	0.7003
a_0	0.8577	0.0386	4.5013
a_1	1.2572	0.0903	7.1843

Fig. 3. Posterior samples for one pair of stiffness parameters $\{\eta_{Rx,4}, \eta_{Ry,4}\}$

Table III shows the statistical properties of the contribution parameters samples obtained from the proposed Gibbs sampling based algorithm. The simulation results stabilized at around 20,000 samples (not shown here due to space limitations). The posterior mean and c.o.v. estimates of the stiffness and damping contribution parameters reported as functions of the number of simulated samples are shown in the journal version of this paper. Posterior samples for one pair of stiffness parameters $\{\eta_{Rx,4}, \eta_{Ry,4}\}$ are shown in Fig. 3.

6 CONCLUSION

A Gibbs sampling based approach for Bayesian model updating of a linear structural dynamic system based on incomplete complex modal data including modal frequencies, damping ratios and partial mode shapes of some of the dominant modes is proposed. The results from the illustrative example with 614 uncertain parameters demonstrate that the posterior PDF of the uncertain parameters is reasonable. The proposed method allows the uncertainty of the parameters, quantified by their joint PDF to be updated efficiently even if there are a huge number of uncertain parameters. The cases considering dependent errors are currently under investigation and the corresponding algorithms are under development.

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