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A New Stochastic Simulation Algorithm for Updating Robust Reliability of Linear Structural Dynamic Systems

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ABSTRACT: It is of great interest to assess during the operation of a dynamic system whether it is expected to satisfy specified performance objectives. To do this, the failure probability (or its complement, robust reliability) of the system when it is subjected to dynamic excitation is computed. The word 'failure' is used here to refer to unsatisfactory performance of the system. In this paper, we are interested in using system data to update the robust failure probability that any particular response of a linear structural dynamic system exceeds a specified threshold during the time when the system is subjected to future Gaussian dynamic excitation. Computation of the robust reliability takes into account uncertainties from structural modeling in addition to the modeling of the uncertain excitation that the structure will experience during its lifetime. The updating is based on partial modal data from the structure. By exploiting the properties of linear dynamics, a newly approach based on stochastic simulation methods is proposed, to update the robust reliability of the structure. The efficiency of the proposed approach is illustrated by a numerical example involving a linear elastic structural model of a building.

1 INTRODUCTION

The condition of the system may change from time to time during its operation and may deteriorate which may lead to a significant reduction of its reliability. Therefore, it is essential to assess the reliability of a system from time to time during its operation. In real practice, it is impossible to measure the physical properties of the system directly in space and time. The estimates of the model parameters of the mathematical model used to represent the behavior of the real structure always involve uncertainties due to limitations of the model and the presence of measurement error in the data etc. Reliability considering model uncertainties in addition to the modeling of the uncertain excitation is termed as robust reliability (2001).

In this paper the objective is to develop an efficient method for updating robust failure probability (or its compliment, robust reliability) of a linear dynamic system using system data when the system is subjected to future stochastic excitation. The word 'failure' is used here to refer to unsatisfactory performance of the system. Here the failure probability is the probability that any particular response (e.g., inter-storey drift, floor acceleration) of a linear structural dynamic system exceeds a specified threshold during the time when the system is subjected to future uncertain dynamic excitation. The updating is based on incomplete modal data including modal frequencies, damping ratios and partial mode shapes of some of the dominant modes.

Let $\boldsymbol{\theta}_s \in R^n$ denote the uncertain model parameter vector specified by a model class $\mathcal{M}$ with the prior probability distribution function (PDF) $p(\boldsymbol{\theta}_s|\mathcal{M})$ and $\mathbf{Y}(\boldsymbol{\theta}_s,\mathbf{Z}) \in R^{Ny}$ denoting any output quantity of interest specified by $\boldsymbol{\theta}_s$ and future dynamic input specified by a stochastic input model $\mathcal{U}$, which can be expressed as a linear combination of a finite number of independently and identically distributed standard normal random variables $\mathbf{Z} \in R^{Nt}$. The robust reliability or its complement the robust failure probability is given by the following multi-dimensional integral with respect to $\boldsymbol{\theta}_s$ and $\mathbf{Z}$:

$$
\begin{aligned}
&P(F \mid \mathcal{M},\mathcal{U}) \\
&= \int I_F(\mathbf{Y}(\boldsymbol{\theta}_s,\mathbf{Z}))\, p(\boldsymbol{\theta}_s \mid \mathcal{M})\, p(\mathbf{Z} \mid \mathcal{U})\, d\mathbf{Z}\, d\boldsymbol{\theta}_s
\end{aligned}
\qquad (1)
$$

where F denotes failure, I_F is an indicator function: $I_F = 1$ if $\mathbf{Y} \in F$ and otherwise $I_F = 0$. Given the measurement data D from the system, the updated (posterior) PDF of $\boldsymbol{\theta}_s$ is given by Bayes' theorem:

$$
p(\boldsymbol{\theta}_s \mid D,\mathcal{M}) = \frac{p(D \mid \boldsymbol{\theta}_s,\mathcal{M})\, p(\boldsymbol{\theta}_s \mid \mathcal{M})}{p(D \mid \mathcal{M})} \qquad (2)
$$

where $p(D|\mathcal{M})$ is the normalizing constant which makes the probability volume under the posterior PDF equal to unity, and $p(D|\boldsymbol{\theta}_s,\mathcal{M})$ is the likelihood function based on the predictive PDF of the response given by model class $\mathcal{M}$. The updated robust failure probability given D is given by replacing the prior PDF $p(\boldsymbol{\theta}_s|\mathcal{M})$ in Equation (1) with the updated PDF $p(\boldsymbol{\theta}_s|D,\mathcal{M})$:

$$P(F \mid D,\mathcal{M},\mathcal{U}) = \frac{\int I_F(\mathbf{Y}(\boldsymbol{\theta}_s,\mathbf{Z}))\,p(D \mid \boldsymbol{\theta}_s,\mathcal{M})\,p(\boldsymbol{\theta}_s \mid \mathcal{M})\,p(\mathbf{Z} \mid \mathcal{U})\,d\mathbf{Z}\,d\boldsymbol{\theta}_s}{\int p(D \mid \boldsymbol{\theta}_s,\mathcal{M})\,p(\boldsymbol{\theta}_s \mid \mathcal{M})\,d\boldsymbol{\theta}_s} \tag{3}$$

There are several difficulties in evaluating the above integral. It can be expected that the dimension of the above integral is high due to a large number of random variables involved, and the failure region in $\boldsymbol{\theta}_s$ and $\mathbf{Z}$ space has complicated geometry, and thus it will be impossible to analytically evaluate the integral. It is not feasible to evaluate the integrals in the numerator and denominator of Equation (3) by simulation based methods such as Monte Carlo simulation (MCS) or importance sampling since the high-probability content region of their corresponding integrands may occupies a much smaller volume than that of the prior PDFs $p(\boldsymbol{\theta}_s|\mathcal{M})p(\mathbf{Z}|\mathrm{U})$ and $p(\boldsymbol{\theta}_s|\mathcal{M})$, respectively. Over the past few years, several methods have been presented to tackle the aforementioned difficulties in evaluating the robust reliability. Papadimitriou et al. (2001) presented Laplace's asymptotic approximation which can be computationally challenging in a high-dimensional parameter space and can be inaccurate when the Gaussian assumption is not valid for the global identifiable case. Beck and Au (2002) proposed a level-adaptive Metropolis-Hastings algorithm with a global proposal PDFs to obtain the samples from the posterior PDF and then use these samples to update the system reliability by evaluating the system reliability conditional on each of these samples. The approach will experience difficulty when the number of uncertain model parameters is large and is computationally inefficient because it requires multiple reliability analyses. Ching and Beck (2007) proposed a method to update the reliability based on combining a Kalman filter and smoother and modifying the algorithm ISEE (Au and Beck 2001a). Such an approach is only applicable to linear systems with no uncertainties in model parameters. Ching and Hsieh (2006) proposed a method based on Bayes' theorem and an analytical approximation of some of the required PDFs by maximum entropy PDFs. The method is applicable regardless of the number of uncertain model parameters but can only be applied to the case with very low-dimensional system output data. In practice, system data are of very high dimension (say of the order of hundreds or thousands). Cheung and Beck (2007) proposed a stochastic simulation method which can handle general nonlinear dynamic system and the case with high-dimensional system output data but may encounter problems if the number of uncertain model parameters is huge. For clarity in presentation, the conditioning on $\mathcal{M}$ and $\mathcal{U}$ will be left implicit in the rest of the paper.

In this paper, by exploiting the properties of linear dynamics, a new approach for computing the updated robust reliability of the system is proposed here, which integrates a newly-developed stochastic simulation algorithm based on Gibbs sampling algorithm for Bayesian model updating of a linear dynamic system(Cheung and Bansal, 2013), Subset Simulation (Au and Beck, 2001a) and a new algorithm called Constrained Metropolis-Hastings within Gibbs sampling algorithm proposed in this paper.

2 THE PROPOSED APPROACH

Let $D \equiv \{\hat{\omega}_{m,s}, \hat{\zeta}_{m,s}, \hat{\psi}_{m,s} : m = 1...M, s = 1...S\}$ be the experimentally obtained modal data from a linear structural dynamic system, consisting of modal frequencies $\hat{\omega}_{m,s} \in R$, damping ratios $\hat{\zeta}_{m,s} \in R$, and complex mode shape components $\hat{\psi}_{m,s} \in C^{N_O}$, where N_O is the number of measured DOFs, M is the number of observed modes, and S is the number of modal data sets available.

First consider the following integral:

$$P(F \mid D) = \int I_F(\mathbf{Y}(\boldsymbol{\theta}_s,\mathbf{Z}))\,p(\boldsymbol{\theta}_s \mid D)\,p(\mathbf{Z})\,d\mathbf{Z}\,d\boldsymbol{\theta}_s \tag{4}$$

The above integral can be approximated by the following estimator:

$$P(F \mid D) \approx \frac{1}{N}\sum_{k=1}^{N} I_F(\mathbf{Y}(\boldsymbol{\theta}_s^{(k)},\mathbf{Z}^{(k)})) \tag{5}$$

where samples $\{\mathbf{Z}^{(k)}:k=1,...,N\}$ are distributed according to the PDF $p(\mathbf{Z})$ and samples $\{\boldsymbol{\theta}_s^{(k)}:k=1,...,N\}$ are distributed according to the PDF $p(\boldsymbol{\theta}_s|D)$. Samples distributed according to the PDF $p(\mathbf{Z})$ can easily be obtained since it is assumed that the input is specified by a stochastic input model U. Computationally efficient Markov Chain Monte Carlo (MCMC) simulation based techniques (Ching and Chen, 2007, Beck and Au, 2002, Ching et al., 2006, Cheung and Beck, 2009) can be used to generate samples from the posterior PDF $p(\boldsymbol{\theta}_s|D)$ as in the Bayesian model updating problem of linear dynamic system given partial modal data. Even though these samples from $p(\boldsymbol{\theta}_s|D)$ are not independent, the Monte

Carlo estimator for independent samples in Equation (5) can still be used. However, using Equation (5) will be computationally expensive especially when dealing with small failure probability as the minimum number of samples N required to achieve a given coefficient of variation is inversely proportional to the failure probability. To efficiently compute smaller failure probabilities, a new approach for computing the updated robust reliability of the system is proposed as follows, which integrates a newly-developed stochastic simulation algorithm based on Gibbs sampling algorithm for Bayesian model updating of a linear dynamic system (Cheung and Bansal, 2013), Subset Simulation (Au and Beck, 2001a) and a new algorithm called Constrained Metropolis-Hastings within Gibbs sampling algorithm for efficiently simulating conditional samples.

The basic idea of Subset Simulation (Au and Beck, 2001a) is to subdivide a failure event into a sequence of H partial failure events (subsets) $F_1 \supset F_2 \supset ... \supset F_H = F$. The division into subsets converts a rare event simulation problem into a problem of a sequence of more frequent events that are conditioned on failing at successively increasing threshold levels. In this paper, this is adapted to compute the updated robust failure probability as follows:

$$P(F \mid D) = P(F_m) = \left[\prod_{i=1}^{H-1} P(F_{i+1} \mid F_i, D) \right] P(F_1 \mid D) \quad (6)$$

where $P(F_1|D)$ is estimated by Equation (5) and $P(F_{i+1}|F_i,D)$ is estimated using samples $\boldsymbol{\theta}_s^{(i,k)}, \mathbf{Z}^{(i,k)}, k = 1,...,N,$ distributed according to the conditional PDF $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$ as follows:

$$P(F_{i+1} \mid F_i, D) \approx \frac{1}{N} \sum_{k=1}^{N} I_{F_{i+1}}(\mathbf{Y}(\boldsymbol{\theta}_s^{(i,k)}, \mathbf{Z}^{(i,k)})) \quad (7)$$

The intermediate thresholds c_i, where $c_1 < c_2 < \ldots < c_H$ are chosen ''adaptively'' so that $P(F_{i+1}|F_i,D)$ is approximately equal to some specified value p_0. At the i-th level, p_0N (rounded to the closest integer) samples out of N samples distributed according to $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_{i-1},D)$ are distributed according to $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$. Using each of these p_0N samples as a starting point (seed), a Markov Chain of $1/p_0$-1 (rounded to the closest integer) samples are then generated according to $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$ one after another. In total, there are p_0N (rounded to the closest integer) Markov Chains.

Markov Chain samples distributed according to the conditional PDF $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$ are generated by a new algorithm called Constrained Metropolis-Hastings within Gibbs sampling algorithm developed in this paper. For the current problem, given the most recent sample $\boldsymbol{\theta}_s^{(k)}$, $\mathbf{Z}^{(k)}$ from $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$, the next Markov chain sample $\boldsymbol{\theta}_s^{(k+1)}$, $\mathbf{Z}_s^{(k+1)}$ is simulated by Gibbs sampling: first simulating $\mathbf{Z}^{(k+1)}$ according to $p(\mathbf{Z}|\boldsymbol{\theta}_s=\boldsymbol{\theta}_s^{(k)},F_i,D) = p(\mathbf{Z}|\boldsymbol{\theta}_s=\boldsymbol{\theta}_s^{(k)},F_i)$ and then $\boldsymbol{\theta}_s^{(k+1)}$ from $p(\boldsymbol{\theta}_s|\mathbf{Z}=\mathbf{Z}^{(k+1)},F_i,D)$.

2.1 *Sampling from* $p(\mathbf{Z}|\boldsymbol{\theta}_s,F_i,D)$

Since the data D do not provide any information which can update the PDF of $\mathbf{Z}$, $p(\mathbf{Z}|\boldsymbol{\theta}_s=\boldsymbol{\theta}_s^{(k)},F_i,D) = p(\mathbf{Z}|\boldsymbol{\theta}_s=\boldsymbol{\theta}_s^{(k)},F_i)$. For a linear dynamic system subjected to future Gaussian inputs, $\mathbf{Y}(t)$ can be written as a linear function of standard normal vector $\mathbf{Z}$:

$$\mathbf{Y}(t) = \sum_{i=1}^{Nt} \mathbf{Y}^{(i)}(t) Z_i \quad (8)$$

The failure domain F_i can be expressed as a union of failure events $F^{(j)}$, j=1,..,$2N_yN_t$ and for each failure event $F^{(j)}$, the corresponding linear limit state function $g^{(j)}(\mathbf{Z})$ can be completely described by its own design point:

$$g^{(j)}(\mathbf{Z}) = \mathbf{a}^{(j)T}\mathbf{Z} + b^{(j)} \quad (9)$$

where $\mathbf{a}^{(j)}$ and $b^{(j)}$ are fixed given fixed $\boldsymbol{\theta}_s$ and some threshold. The design point $\mathbf{Z}^{(j)*}$ (defined as the point on the plane $g^{(j)}(\mathbf{Z})=0$ located closest to the origin) and its distance $\beta^{(j)}$ from origin are given by the following expressions:

$$\mathbf{Z}^{(j)*} = -\frac{b}{\|\mathbf{a}^{(j)}\|^2}\mathbf{a}^{(j)}; \quad \|\mathbf{a}^{(j)}\| = \sqrt{\mathbf{a}^{(j)T}\mathbf{a}^{(j)}} \quad (10)$$

$$\beta^{(j)} = \frac{b^{(j)}}{\|\mathbf{a}^{(j)}\|} \quad (11)$$

Samples distributed according to PDF $p(\mathbf{Z}|\boldsymbol{\theta}_s,F_i)$ can be simulated using Metropolis-Hastings algorithm with the following distribution as the proposal PDF:

$$q(\mathbf{Z} \mid \boldsymbol{\theta}_s) = \sum_{j=1}^{J} w_j p(\mathbf{Z} \mid \boldsymbol{\theta}_s, F^{(j)}) \quad (12)$$

$$w_j = \frac{P(F^{(j)})}{\sum_{j=1}^{2NyNt} P(F^{(j)})} = \frac{\Phi(-\beta^{(j)})}{\sum_{j=1}^{2NyNt} \Phi(-\beta^{(j)})} \quad (13)$$

where $\Phi(.)$ is the cumulative distribution function (CDF) of a standard normal random variable. The above PDF was proposed by Au and Beck (2001b) to be used as the importance sampling density for calculating the failure probability of linear dynamic system subjected Gaussian excitations with no uncertainty in the structural models. An important sampling density such as this was proposed by Au (2004) to be used as a proposal PDF to generate conditional failure samples using Metropolis-

Hastings algorithm. Candidate samples of **Z** from the above proposal PDF can be efficiently simulated using procedures as shown in Au and Beck (2001b) and Katafygiotis and Cheung (2006).

2.2 *Sampling* from $p(\boldsymbol{\theta}_s|\mathbf{Z},F_i,D)$

For a N_d-DOF linear dynamic system with both Rayleigh and nonclassical damping, the relationship between modal properties and dynamic model parameters can be written as (Cheung and Bansal, 2013):

$$\hat{\lambda}_{m,s}^2\mathbf{M}\psi_m + \hat{\lambda}_{m,s}\mathbf{C}\psi_m + \mathbf{K}\psi_m = \boldsymbol{\varepsilon}_{m,s} \qquad (14)$$

$$\hat{\psi}_{m,s} - \mathbf{L}\psi_m = \mathbf{e}_{m,s} \qquad (15)$$

where $\mathbf{L}\in\mathbb{R}^{NoXNd}$ is a selection matrix that selects only those DOFs where measurements are made. In the above equation $\varepsilon_{m,s}$ and $\in_{m,s}$ are the complex random vectors representing the model prediction errors, i.e., the errors between the response of the system under consideration and that of the assumed model. Based on the Principle of Maximum Entropy (Jaynes, 1978), the PDFs for vectors Re($\varepsilon_{m,s}$), Im($\varepsilon_{m,s}$), Re($e_{m,s}$) and Im($e_{m,s}$) are taken to be Gaussian. For illustration, here their means are assumed to be equal to zero and covariance matrices equal to scaled versions of the identity matrix **I** of appropriate order. The mass, damping and stiffness matrices in Equation (14) are represented as a linear sum of contribution from the individual prescribed substructures:

$$\mathbf{M}(\boldsymbol{\alpha}) = \mathbf{M}_0 + \sum_{i=1}^{N_\alpha}\alpha_i\mathbf{M}_i \qquad (16)$$

$$\mathbf{K}(\boldsymbol{\eta}) = \mathbf{K}_0 + \sum_{i=1}^{N_\eta}\eta_i\mathbf{K}_i \qquad (17)$$

$$\mathbf{C}(\boldsymbol{\beta},\mathbf{a}) = \mathbf{C}_0 + \sum_{i=1}^{N_\beta}\beta_i\mathbf{C}_i + a_0\mathbf{M}(\boldsymbol{\alpha}) + a_1\mathbf{K}(\boldsymbol{\eta}) \qquad (18)$$

The uncertain parameters to be updated are the contribution parameters $[\boldsymbol{\alpha}^T,\boldsymbol{\beta}^T,\mathbf{a}^T,\boldsymbol{\eta}^T]^T$ mode shapes $[\mathrm{Re}(\psi_1)^T,\mathrm{Im}(\psi_1)^T,...,\mathrm{Re}(\psi_M)^T,\mathrm{Im}(\psi_M)^T]^T$ and prediction error variance $[\sigma_{\mathrm{Re},1}^2,\sigma_{\mathrm{Im},1}^2,..,\sigma_{\mathrm{Re},M}^2,\sigma_{\mathrm{Im},M}^2]^T$ for Re($\boldsymbol{\varepsilon}_{m,s}$) and Im($\boldsymbol{\varepsilon}_{m,s}$). The variance parameters for Re($\mathbf{e}_{m,s}$) and Im($\mathbf{e}_{m,s}$) are assumed to be known or are directly estimated from the sample variance of the experimental modal data. Uncertain parameters are divided into four groups $\boldsymbol{\theta}=[\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4]$:

$$\boldsymbol{\theta}_1 = [\boldsymbol{\alpha}^T,\boldsymbol{\eta}^T,\boldsymbol{\beta}^T]^T$$

$$\boldsymbol{\theta}_2 = \mathbf{a}$$

$$\boldsymbol{\theta}_3 = [\mathrm{Re}(\psi_1)^T,\mathrm{Im}(\psi_1)^T,...,\mathrm{Re}(\psi_M)^T,\mathrm{Im}(\psi_M)^T]^T$$

$$\boldsymbol{\theta}_4 = [\sigma_{\mathrm{Re},1}^2,\sigma_{\mathrm{Im},1}^2,..,\sigma_{\mathrm{Re},M}^2,\sigma_{\mathrm{Im},M}^2]^T$$

Assuming Bayesian conjugate priors $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, $\boldsymbol{\theta}_3$ and $\boldsymbol{\theta}_4$, the full conditional PDFs $p(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$, $p(\boldsymbol{\theta}_2|\boldsymbol{\theta}_1,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$ and $p(\boldsymbol{\theta}_3|\boldsymbol{\theta}_1,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$ are Gaussian, and $p(\sigma_{\mathrm{Re},m}^2|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,D)$ and $p(\sigma_{\mathrm{Im},m}^2|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,D)$ for m=1,...,M are inverse gamma.

If samples from $p(\boldsymbol{\theta}|\mathbf{Z},F_i,D)$ are available, samples corresponding to $\boldsymbol{\theta}_s=[\boldsymbol{\theta}_1,\boldsymbol{\theta}_2]$ from these samples will be distributed according to the conditional PDF $p(\boldsymbol{\theta}_s|\mathbf{Z},F_i,D)$. The full conditional PDFs for the four groups of parameter vectors conditioned on F_i are equal to:

$$p(\boldsymbol{\theta}_1|F_i,\mathbf{Z},\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D) \propto I_{F_i}(\mathbf{Y}(\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2))\,p(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D) \qquad (19)$$

$$p(\boldsymbol{\theta}_2|F_i,\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D) \propto I_{F_i}(\mathbf{Y}(\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2))\,p(\boldsymbol{\theta}_2|\boldsymbol{\theta}_1,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D) \qquad (20)$$

$$p(\boldsymbol{\theta}_3|F_i,\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_4,D) \propto I_{F_i}(\mathbf{Y}(\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2))\,p(\boldsymbol{\theta}_3|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_4,D) \propto p(\boldsymbol{\theta}_3|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_4,D) \qquad (21)$$

$$p(\boldsymbol{\theta}_4|F_i,\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_4,D) \propto I_{F_i}(\mathbf{Y}(\mathbf{Z},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2))\,p(\boldsymbol{\theta}_4|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,D) \propto p(\boldsymbol{\theta}_4|\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,D) \qquad (22)$$

According the above, samples distributed according to the conditional PDF $p(\boldsymbol{\theta}|\mathbf{Z},F_i,D)$ can be obtained using Metropolis-Hastings within Gibbs sampling technique starting from a sample already distributed according to the conditional PDF $p(\boldsymbol{\theta}|\mathbf{Z},F_i,D)$. Making use of the characteristics that $p(\boldsymbol{\theta}_1|\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$ and $p(\boldsymbol{\theta}_2|\boldsymbol{\theta}_1,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$ are both Gaussian PDFs with mean and covariance matrix given by the equations presented in Cheung and Bansal (2013), samples for $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are simulated using the procedures given in the following section. Simulation of $\boldsymbol{\theta}_3$ and $\boldsymbol{\theta}_4$ neither requires Metropolis-Hastings step nor any dynamic analysis.

2.3 *Summary of steps for simulating samples according to* $p(\boldsymbol{\theta}_s,\mathbf{Z}|F_i,D)$

For level zero of the subset simulation, samples $\{\mathbf{Z}^{(k)}$:k=1,...,$N\}$ are directly simulated from the PDF $p(\mathbf{Z})$, and samples $\{\boldsymbol{\theta}^{(k)}$:$k$=1,...,$N\}$ are simulated from the PDF $p(\boldsymbol{\theta}|D)$ using the Gibbs sampling based approach proposed in Cheung and Bansal (2013) after discarding the samples from the burn-in period. For the i-th simulation level p_0N samples distributed according to $p(\boldsymbol{\theta},\mathbf{Z}|F_i,D)$ obtained from the (i-1)-th simulation level are used as seed samples to initialize Markov chains to obtain additional $N(1-p_0)$ samples also distributed according to the conditional PDF $p(\boldsymbol{\theta},\mathbf{Z}|F_i,D)$. The following steps are repeated for each seed sample:

1. Initialize a Markov chain, use a seed sample as the starting points and let k=1.
2. Sample $\mathbf{Z}^{(k+1)}$ from $p(\mathbf{Z}^{(k+1)}|\boldsymbol{\theta}_s^{(k)},F_i,D)$.
   i) Draw a candidate sample $\mathbf{Z}^c$ sample from $q(\mathbf{Z}|\boldsymbol{\theta}_s^{(k)},F^{(j)})$ presented in the previous section.
   ii) If $u<\min(1,r)$, $\mathbf{Z}^c$ is accepted as the next sample, i.e., $\mathbf{Z}^{(k+1)} = \mathbf{Z}^c$ where u is uniformly distributed between 0 and 1. Otherwise, $\mathbf{Z}^{(k+1)} = \mathbf{Z}^{(k)}$.
$$r = \frac{\sum_{j=1}^{J} I_{F^{(j)}}(\mathbf{Z}^{(k)})}{\sum_{j=1}^{J} I_{F^{(j)}}(\mathbf{Z}^{c})}$$
   Note: Impulse response function for $\boldsymbol{\theta}_s^{(k)}$ is already available.
3. Sample $\boldsymbol{\theta}^{(k+1)}$ from $p(\boldsymbol{\theta}^{(k+1)}|\mathbf{Z}^{(k)},F_i,D)$ by:
   i) Determine the mean vector $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_1(\boldsymbol{\theta}_2^{(k)},\boldsymbol{\theta}_3^{(k)},\boldsymbol{\theta}_4^{(k)})$ and covariance matrix $\Sigma_1 = \Sigma_1(\boldsymbol{\theta}_2^{(k)},\boldsymbol{\theta}_3^{(k)},\boldsymbol{\theta}_4^{(k)})$ of the Gaussian PDF $p(\boldsymbol{\theta}_1 | \boldsymbol{\theta}_2^{(k)},\boldsymbol{\theta}_3^{(k)},\boldsymbol{\theta}_4^{(k)},D)$ using the equations presented in Cheung and Bansal (2013). $\boldsymbol{\theta}_1 = \boldsymbol{\mu}_1 + \mathbf{L}_1\mathbf{w}_1$ where $\mathbf{w}_1$ is a random vector comprised of independent standard normal random variables and $\mathbf{L}_1$ is obtained from the Cholesky decomposition of $\Sigma_1 = \mathbf{L}_1\mathbf{L}_1^T$. $\mathbf{w}_1^{(k)} = \mathbf{L}_1^{-1}(\boldsymbol{\theta}_1^{(k)} - \boldsymbol{\mu}_1)$.
   ii) Generate a candidate state $\tilde{\mathbf{w}}_1$ for $\mathbf{w}_1$: For each component j=1,...,nc, simulate $\tilde{w}_{1,j}$ from proposal PDF $q_{1,j}(\tilde{w}_{1,j} | w_{1,j}^{(k)},\boldsymbol{\theta}_2,\boldsymbol{\theta}_3,\boldsymbol{\theta}_4,D)$ (assuming symmetric proposal). Compute the acceptance ratio
$$r_{1,j} = \frac{\phi(\tilde{w}_{1,j})}{\phi(w_{1,j}^{(k)})} = \exp\left(-\frac{\tilde{w}_{1,j}^2 - [w_{1,j}^{(k)}]^2}{2}\right)$$
   where $\phi(.)$ is the standard normal PDF. If $u<\min(1,r_{1,j})$, $\tilde{w}_{1,j}$ is accepted as a candidate state for $\mathbf{w}_1$ where u is uniformly distributed between 0 and 1. Otherwise, $\tilde{w}_{1,j}$ = $w_{1,j}^{(k)}$.
   iii) Transform $\tilde{\mathbf{w}}_1$ back to $\boldsymbol{\theta}_1$ space: $\tilde{\boldsymbol{\theta}}_1 = \boldsymbol{\mu}_1 + \mathbf{L}_1\tilde{\mathbf{w}}_1$.
   iv) If $\mathbf{Y}([\tilde{\boldsymbol{\theta}}_1,\boldsymbol{\theta}_2^{(k)}],\mathbf{Z}^{(k+1)}) \in F$ accept $\tilde{\boldsymbol{\theta}}_1$ as the next sample, i.e., $\boldsymbol{\theta}_1^{(k+1)} = \tilde{\boldsymbol{\theta}}_1$ otherwise reject it and take the current sample as the next sample, i.e., $\boldsymbol{\theta}_1^{(k+1)} = \boldsymbol{\theta}_1^{(k)}$.
   v) For getting $\tilde{\boldsymbol{\theta}}_2$, repeat steps i) to iii) by reversing the subscripts 1 and 2 and replacing $\boldsymbol{\theta}_1^{(k)}$ by $\boldsymbol{\theta}_1^{(k+1)}$.
   vi) If $\mathbf{Y}([\boldsymbol{\theta}_1^{(k+1)},\tilde{\boldsymbol{\theta}}_2],\mathbf{Z}^{(k+1)}) \in F$ accept $\tilde{\boldsymbol{\theta}}_2$ as the next sample, i.e., $\boldsymbol{\theta}_2^{(k+1)} = \tilde{\boldsymbol{\theta}}_2$ otherwise reject it and take the current sample as the next sample, i.e., $\boldsymbol{\theta}_2^{(k+1)} = \boldsymbol{\theta}_2^{(k)}$.
   vii) Sample $\boldsymbol{\theta}_3^{(k+1)}$ from $p(\boldsymbol{\theta}_3 | \boldsymbol{\theta}_1^{(k+1)},\boldsymbol{\theta}_2^{(k+1)},\boldsymbol{\theta}_4^{(k)},D)$ using the method presented in Cheung and Bansal (2013).
   viii) Sample $\boldsymbol{\theta}_4^{(k+1)}$ from $p(\boldsymbol{\theta}_4 | \boldsymbol{\theta}_1^{(k+1)},\boldsymbol{\theta}_2^{(k+1)},\boldsymbol{\theta}_3^{(k+1)},D)$ using the method presented in Cheung and Bansal (2013).
4. Let k=k+1 and go to step 2, until $(1/p_0-1)$ samples are obtained.

3 ILLUSTRATIVE EXAMPLE

The linear structure system selected for this illustrative example is modeled as a 4-DOF shear building as shown in Fig. 1, with the following properties: mass m_1=60,000 kg, m_2=78,000 kg, m_3=93,000 kg, m_4=103,000 kg, spring stiffness k_1=127,800 kN/m, k_2=43,500 kN/m, k_3=60,100 kN/m, k_4=100,000 kN/m, and damping coefficient for viscous dampers c_1=1200 kN-s/m, c_2=400 kN-s/m, c_3= 600 kN-s/m, c_4=900 kN-s/m. The modal data for the updating consist of 10 sets of modal data (S=10) with the first two modal frequencies, modal damping ratios, and partial complex mode shapes (corresponding to DOFs - one, three and four, N_o=3) identified for each data set (M=2). Noisy measured modal parameters are generated by adding to the exact frequencies, damping ratios, and complex mode shapes components, random values chosen from zero-mean Gaussian distribution with standard deviation equal to 2% times the exact value.

For updating robust reliability problem, independent normal distribution is assumed for masses, spring stiffnesses and damping coefficients for viscous dampers. Masses are assumed to be known with sufficient accuracy, thus the prior PDFs for [α_1, α_2, α_3, α_4] are chosen with mean values equal to 1 and c.o.v. for each equal to 1%. The prior mean values for [β_1, β_2, β_3, β_4] and [η_1, η_2, η_3, η_4] are assumed equal to 1, with prior c.o.v. for each equal to 20%.

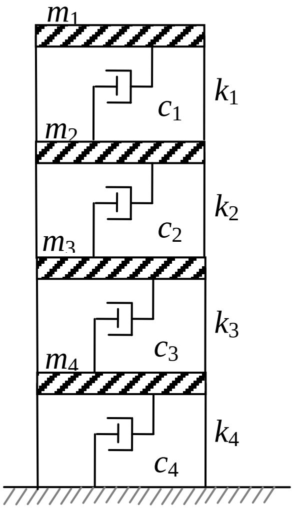

Fig. 1. The 4-DOF shear building model

Additional uncertain parameters that are considered include prediction error variances [$\sigma^2_{Re,1}$,$\sigma^2_{Im,1}$,$\sigma^2_{Re,2}$,$\sigma^2_{Im,2}$], and complete complex mode shapes [ψ_1,ψ_2] for the first two modes. Flat independent priors are taken for [ψ_1,ψ_2] and non-informative independent inverse gamma prior PDFs are taken for [$\sigma^2_{Re,1}$,$\sigma^2_{Im,1}$,$\sigma^2_{Re,2}$,$\sigma^2_{Im,2}$].

A discrete-time white noise sequence **Z** corresponding to duration of input ground motion T=20 sec and sampling interval Δt=0.02 sec is considered. The system is assumed to have zero initial conditions. Using the modal data, the updated failure probability of the linear structural dynamic system subjected to future non-stationary, non-white ground acceleration is computed using the proposed method. The frequency content of the ground acceleration is modeled by Clough-Penzien spectrums:

$$S(\omega) = S_0 \frac{\left(\omega/\omega_f\right)^4}{\left(1-\left(\omega/\omega_f\right)^2\right)^2 + \left(2\zeta_f\omega/\omega_f\right)^2} \times \frac{1+\left(2\zeta_g\omega/\omega_g\right)^2}{\left(1-\left(\omega/\omega_g\right)^2\right)^2 + \left(2\zeta_g\omega/\omega_g\right)^2}$$

where ω_g=15.7 rad/sec; ω_f=0.1ω_g; ζ_g= ζ_f =0.6; and S_0=1e^{-3} m^2/sec^3. The non-stationarity is modelled using a time envelop function $\lambda(t)$=$\alpha_1 t$e$^{-\alpha 2t}$ where α_1=0.45 sec and α_2=1/6 sec^{-1}.

For illustration, failure is defined as an event where the displacement for DOF-1 exceeds a specific threshold at any discrete time instant during the total duration of the ground acceleration. The proposed method is used to obtain the estimate of the updated robust failure probability at different thresholds with a conditional failure probability at each level approximately equal to p_0 = 0.1 and with the number of samples set to N = 500 at each conditional level.

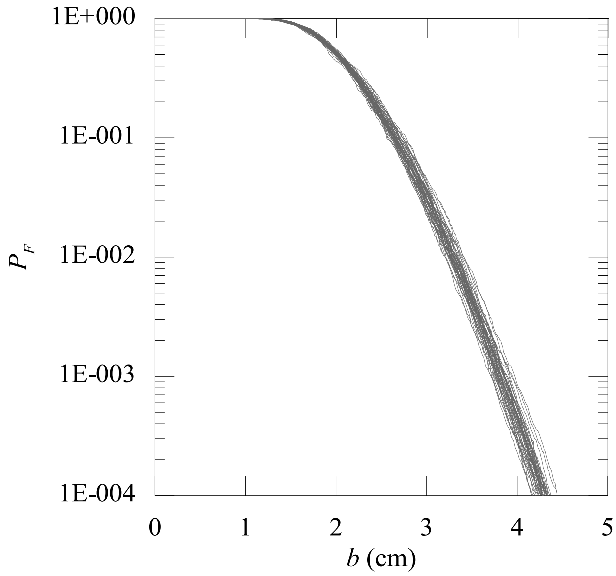

Fig. 2. Estimates of the failure probability for different threshold levels of displacements for 50 independent simulation runs

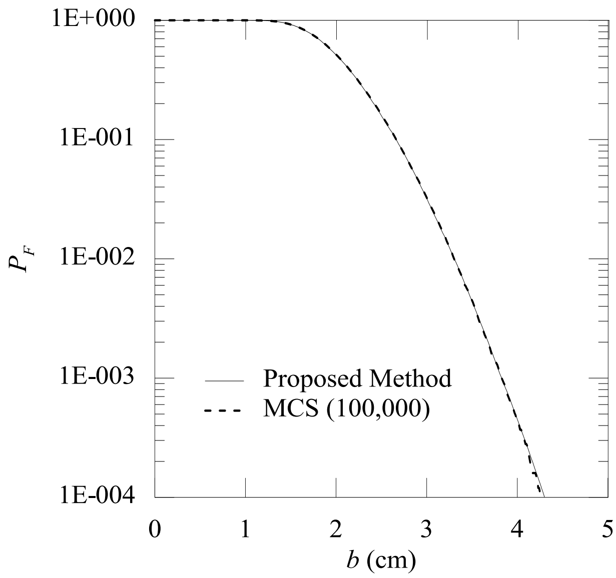

Fig. 3. Sample mean estimate of failure probability

Fig. 2 shows the estimates of the updated robust failure probability for different threshold levels of displacements from 50 independent simulation runs. Sample mean of the updated robust failure probability estimator (estimated by 50 independent simulation runs) is shown in Fig. 3. Results computed using 100,000 MCS samples are also shown for comparison. This confirms that the proposed method is correct giving a practically unbiased estimate of the updated robust failure probability. Fig. 4 compares the sample c.o.v. of the updated robust failure probability estimator and the lower limit of c.o.v. of

MCS estimator at a particular failure probability level using same number of dynamic analyses as in the proposed method. This implies that the proposed method provides substantial improvement in efficiency over MCS.

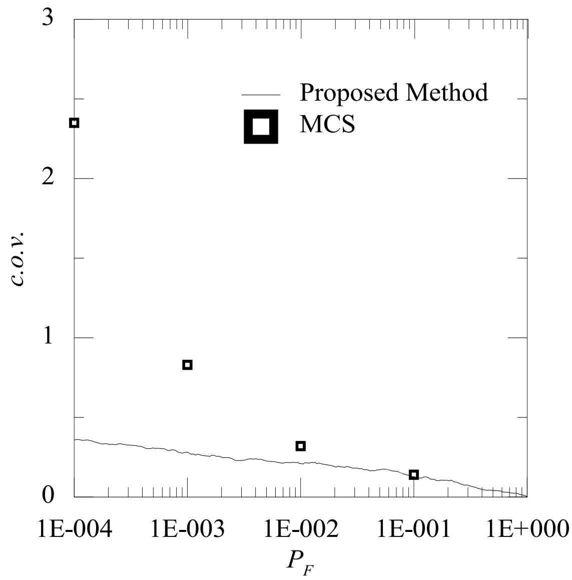

Fig. 4. Sample c.o.v. estimate of failure probability

4 CONCLUSION

By exploiting the properties of linear dynamics, a new approach based on stochastic simulation methods is proposed to update the robust failure probability that any particular response of a linear structural dynamic system exceeds a specified threshold during the time when the system is subjected to future Gaussian dynamic excitation. Results from the illustrative example shows that the proposed method provides substantial improvement in efficiency over MCS with samples from the posterior PDF in Bayesian model updating.

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