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Dynamical Systems: A Tool for Analysis and Design of Silicon Half Center Oscillators

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Abstract—In this paper, a new oscillator circuit for central pattern generator systems has been proposed. Switched-capacitor technique which enables controllable, more accurate and stable resistance has been used. Oscillation frequency can be easily controlled by the frequency of switching which is very useful for global change of oscillation frequency in an array of oscillators. SPICE simulation and dynamical systems analysis have shown that when used as a single oscillator, the proposed circuit is able to produce a phase response curve (PRC) close to the one of a lamprey central pattern generator (CPG) system. Applying averaging theory to the system of coupled oscillators, we obtain averaged H and G functions for unidirectional and bidirectional coupling cases. Analysis of these functions shows our circuit's superior capability to achieve fast entrained oscillation with sensory feedback and reach equilibrium even with high frequency mismatch.

I. INTRODUCTION

A central pattern generator is a neural circuit that has been found to be the basis for generating rhythmic patterns of breathing, swimming, walking, etc. in all vertebrates. This circuit helps in coordinating the rhythms at the low level and alleviates the need for higher control centers to generate high dimensional signals to micro-manage the muscle activation patterns. Typically the CPG is comprised of a chain of oscillatory units interconnected for coordinated activity along the chain. Each unit oscillator is generally modeled as a half-center oscillator (HCO) or a pair of mutually inhibitory coupled oscillators. The mutual inhibition guarantees that when one oscillator is active, the other is silent. This is necessary to guarantee stable locomotory gaits (e.g., the left and right side muscles of a lamprey will not contract simultaneously or the left and right feet of a biped will not move forward at the same time). Our ultimate goal is to design a fish-like autonomous underwater vehicle (AUV) that can use bio-inspired fin for swimming. Hence, lamprey CPG system was chosen as the focus of this study.

Several “neuromorphic” silicon implementations of these circuits have been attempted in the past with varying degrees of success. A good implementation of an artificial CPG is extremely beneficial in robotics [1][2] for quick and robust control of locomotion in uncertain conditions and also as a possible tool in prosthesis [3] for restoring some degree of locomotion in patients who have impairments in motor control. Some implementations like [4] have tried to mimic

biology in great details by implementing multiple conductance neuron based oscillators; such systems, though invaluable for studying biology, are not so easy to use in practical applications because of their large parameter set and sensitivity to variations. We rather concentrate on implementations that use some form of abstraction of biology (in particular, the CPG of the lamprey system) that may result in a practical, usable system.

In this paper, an oscillator circuit based on charging and discharging of a capacitor is presented, and dynamical system's analysis is performed on the proposed circuit together with the Integrate-and-Fire (I&F) oscillator circuit used in pacemaker mode [2], and the oscillator proposed in [5] as candidates for a silicon HCO implementation and assess the circuit's effectiveness as a robust locomotion controller.

II. PROPOSED OSCILLATOR CIRCUIT

A. Biological HCO vs. Silicon Oscillators

The concept of PRC from dynamical systems is used to analyze each type of oscillator to verify if a single oscillator can serve to model a whole HCO. Being able to use just one oscillator reduces the power consumption. PRC of one dynamical neural system can be measured by stimulating the neuron circuit at different phases. The output is defined as a function that maps the change of phase of the oscillator to the phase at which it is perturbed.

$$PRC(\phi) = \{\varphi_{new} - \varphi_{old}\} \quad (1)$$

The PRC of a lamprey CPG system, as reported in [6], can have either positive or negative values depending on different stimulating phases. The phase shift also increases or decreases gradually with the stimulating phases which means that the lamprey HCO is able to produce various levels of phase shifts with respect to a forcing oscillator. A single oscillator can model a whole HCO of a lamprey CPG system only when it is able to produce a PRC similar to the lamprey's. The PRCs of the two benchmarking oscillators whose circuit diagram are shown in Fig. 1(a) and (b) have been simulated and plotted in Fig. 2.

It can be observed that the oscillator in [5] has only positive phase shifts. Both PRCs are very close to square waveform, implying that both oscillators can only produce two levels of phase shifts. Neither oscillator can be considered as a good candidate to model a whole HCO.

Leaky I&F oscillators [7] discharge the membrane capacitor through a resistor. Though they are able to produce a PRC with gradually changing phase shifts at the positive cycle, to model a whole HCO, it requires another resistor to charge up the membrane capacitor to generate the other half cycle of PRC. Since the resistors occupy the largest area, the area overhead would be doubled. Besides, the current sources used to charge the nodes in these arrays incur huge mismatch due to V_T variation. To solve these issues, a new type of oscillator circuit has been proposed in this paper.

Figure. 1 Schematic of the two oscillator circuits: (a) I&F [2], Still06 [5].

Figure. 2 Simulated and fitted PRCs of the two benchmark oscillator circuits.

B. Proposed Oscillator Circuit

The proposed oscillator circuit has been implemented using TSMC0.35μm process and its circuit diagram is shown in Fig. 3(a). The circuit consists of an ordinary differential input transconductor, two inverters, one resistor implemented using switched-capacitor technology (Fig. 3(b)), one membrane capacitor C_{MEM} , one positive feedback capacitor C_{FB} , and a multiplexer which controls the charging and discharging activities on the membrane capacitor. The positive and negative terminals of the transconductor are connected to a reference voltage V_{REF} , and the membrane voltage V_{MEM} , respectively. The two input terminals to the multiplexer are connected to V_{UPP} , and V_{LOW} , whose selection bit is connected to the output terminal of inverters 2 and 1, respectively. There are no current sources that need to be matched in the circuit.

Depending on the initial condition of this circuit, charging or discharging of C_{MEM} towards V_{UPP} or V_{LOW} through R occurs. The positive feedback capacitor C_{FB} which provides a hysteresis effect at the sudden change of inverter 1 output sweeps the membrane potential up or down and toggles between charging and discharging of C_{MEM} . With a fixed V_{LOW} , V_{UPP} , and V_{REF} value, the oscillation period of this circuit is determined by R , C_{MEM} , and C_{FB} . Taking charging cycle as an example, the instantaneous voltage at membrane capacitor is governed by (2), where τ_0 is equal to RC_{MEM} .

$$v_{MEM}(t) = V_{UPP} \left(1 - e^{-\frac{t}{\tau_0}} \right) \quad (2)$$

The membrane potentials at the onset and end of charging are

$V_{REF} - \Delta V_{MEM}$, and V_{REF} , respectively, where $\Delta V_{MEM} = (C_{FB}/C_{MEM}) \times V_{DD}$. Therefore, the charging time could be derived as:

$$t_{ch} = RC_{MEM} \left[\ln \left(1 - \frac{V_{REF} - \Delta V_{MEM}}{V_{UPP}} \right) - \ln \left(1 - \frac{V_{REF}}{V_{UPP}} \right) \right]. \quad (3)$$

With equal differences between V_{LOW} and V_{UPP} to V_{REF} , the charging and discharging times are the same. The oscillation period of this circuit t_{osc} equals $2t_{ch}$.

Figure. 3 Circuit diagrams of the (a) proposed oscillator circuit, and the (b) proposed switched-capacitor resistor circuit.

In order to achieve a nominal oscillation frequency of 1 Hz, a very large resistance of up to 20 GΩ is required. It is not feasible to implement such a large resistance value using traditional poly resistors which takes up great silicon area. A compact oscillator circuit design is desirable for an area efficient locomotion controller. Besides, the oscillation frequency of an oscillator circuit used in a CPG is required to be tunable to suit different locomotion behaviors such as crawling, walking, and running. The simple SC resistor with two switches and one capacitor suffers greatly from charge injection effect when very small capacitor is used to realize very large resistor value. In addition, the value of the equivalent resistance is limited by the parasitic capacitances of the switches. To fulfill the oscillation frequency requirement and overcome the disadvantages of simple SC resistor circuits, a unique SC resistor circuit has been proposed and the circuit diagram is shown in Fig. 3(b). The proposed resistor circuit consists of four capacitors of equal capacitance, and seven switches controlled by two non-overlapping clocks, ϕ_1 and ϕ_2 .

Take C_1 as an example: when ϕ_1 is on, the charge stored on C_1 is $0.5C(V_1 - V_2)$. When ϕ_2 is on, the charge stored on C_1 becomes $0.4C(V_1 - V_2)$. Hence, the charge transferred from V_1 to C_1 during the next ϕ_1 cycle is $0.1C(V_1 - V_2)$. It shows that the effective charge transferred from the voltage terminal to one capacitor is only one tenth of the value in the simple SC resistor. In this way, the effective capacitance of the SC circuit is reduced without physically reducing the capacitor value. Compared to putting four capacitors in series where the effective capacitance is $0.25C$, this approach provides 2.5 times smaller capacitance value and it can be generalized to n capacitors. Hence, a much larger capacitor can be used to implement a large resistance with greatly reduced charge injection effect, making the equivalent resistance much more accurate and stable.

III. DYNAMICAL SYSTEMS ANALYSIS

A. Analysis Criteria

Dynamical systems has been used as a tool to analyze and compare the proposed oscillator circuit with the other two

benchmark oscillators. Besides PRC, H and G functions can also provide an insight into different oscillator circuits. The theory used is based on the assumption of weak coupling between oscillators. Though this assumption is restrictive and not always true for even the lamprey system, it has been shown [8] that the trend of results found from the weak coupling assumption still hold true for strong coupling.

Consider 'n' weakly coupled oscillators of the form:

$$\dot{x}_i = f_i(x_i) + \epsilon \sum_{j=1}^{j=n} g_{ij}(x_i, x_j), x_i \in R^m, \quad (4)$$

where x_i represents the state variables of the i -th oscillator and g_{ij} are the coupling functions between oscillators i and j . Applying the phase reduction method [9], one can obtain the corresponding phase model:

$$\dot{\phi}_i = 1 + \epsilon Q_i(\phi_i) \cdot \sum_{j=1}^{j=n} g_{ij}(x_i(\phi_i), x_j(\phi_{ij})). \quad (5)$$

Assuming the same nominal frequency for the oscillators, one can consider only the phase difference of each oscillator from its nominal frequency. By rewriting the above equations in terms of this phase difference, ϕ , and applying the averaging theory, (6) can be obtained:

$$\dot{\phi}_i = \epsilon \omega_i + \epsilon \sum_{j=1}^{j=n} H_{ij}(\phi_j - \phi_i), \quad (6)$$

where ω_i is the frequency deviation of the i -th oscillator from the nominal frequency.

In the case of a two coupled oscillators (e.g., HCO), (6) can be further simplified by taking the phase difference between the two oscillators as our variable of interest. Hence, the simplified equation becomes:

$$\dot{\phi}_1 - \dot{\phi}_2 = \dot{\chi} = \omega + G(\chi), \quad (7)$$

where $\omega = \omega_1 - \omega_2$ and $G(\chi) = H_2(-\chi) - H_1(\chi)$. The H function between two coupling oscillators can be used to verify if each type of oscillator could produce stable anti-phase oscillations when coupled with another similar oscillator using inhibitory connections to create a HCO. The H function is defined as:

$$H_{ij}(\chi) = \frac{1}{T} \int_0^T Q_i(t) \cdot g_{ij}(x_i(t), x_j(t + \chi)) dt, \quad (8)$$

where $\chi = \phi_j - \phi_i$ and ϕ_j and ϕ_i are the phase deviations of the two coupling oscillators from the natural oscillation; $Q_i(t)$ is the PRC function; and g_{ij} is the coupling function between two oscillators. The G function between two coupling oscillators is used to verify if each type of oscillator could produce synchronized but phase shifted oscillation when connected bidirectionally with a similar oscillator.

For each oscillator type, its H and G functions are computed for the following types of commonly used coupling functions:

Resistive coupling [9], defined as:

$$g_{ij} = g_0(v_i - v_j). \quad (9)$$

Pulsed constant current coupling [2], defined as:

$$g_{ij} = I_0 \cdot u(v_i - V_{th}). \quad (10)$$

Pulsed conductance coupling [10], defined as:

$$g_{ij} = I_0 \cdot u(v_i - V_{th}) \cdot (V_{syn} - v_j). \quad (11)$$

B. Analysis Method

The PRC of the oscillators are obtained from SPICE simulations using CADENCE. Each oscillator is perturbed throughout one period of oscillation in evenly distributed steps. Together with the unperturbed waveform, all simulation data are processed in MATLAB to compute the

PRC of each type of oscillator. The simulated PRCs are subsequently fitted with a set of equally spaced time variables using linear interpolation method. To compute the H and G functions, one period of oscillation is extracted from unperturbed waveform and fitted using the same setting as PRC fitting. The detailed algorithm will not be discussed due to space constraints.

IV. RESULTS AND DISCUSSIONS

For fair comparison, the oscillators used in [2] and [5] have also been implemented in TSMC0.35 μ m process. All three circuits have been modified to oscillate at a 1 Hz frequency while the equivalent capacitance values have been normalized to be 12 pF. All circuits have been perturbed with the same set of inputs. Each perturbation is modeled using a pulsed current of 1 μ A which lasts 100 ns. A total of 50 perturbations have been applied in the 1 s cycle time.

Fig. 4 plots the simulated and fitted PRCs together with one oscillation cycle of the proposed oscillator circuit. It can be observed that the PRC has both positive and negative values and the phase shifts changes gradually with stimulating phases. The obtained PRC is very close to the PRC of the lamprey CPG system, which proves that our oscillator can be used to model the whole HCO.

Figure. 4 Simulated and fitted PRCs with one oscillation cycle of the proposed oscillator circuit.

Fig. 5(a) to 5(c) compare the three H functions (H1, H2, and H3) of the three oscillator circuits based on resistive, pulsed constant current, and pulsed constant conductance coupling functions, respectively. H function represents an oscillator's ability to respond to external sensory inputs to enhance or regard phase shifts to achieve entrained oscillation. All three H functions of our proposed oscillator have both positive and negative polarities and H2 and H3 have higher range than the other two.

Fig. 5(d) to 5(f) show the three G functions (G1, G2, and G3) of the three oscillator circuits based on resistive, pulsed constant current, and pulsed constant conductance coupling functions, respectively. The max and min values of the G function determine the tolerance of the network for the frequency mismatch among coupled oscillators. It can be observed that the amplitudes of the G functions of our proposed oscillator are at least twice larger than the other two designs for all three coupling functions. When connected in a chain, the proposed oscillator circuit could achieve synchronization with a constant phase shift over a wider range of frequency mismatch.

Table 1 summarizes the power consumption, the ranges of

Figure. 5 Dynamical systems analysis of the three circuits of (a) H1 function, (b) H2 function, (c) H3 function, (d) G1 function, (e) G2 function, and (f) G3 function.

PRC, H and G functions and the polarities of the H functions. For a fair comparison, power measurement has been done by replacing the two inverters in [5] and the first inverter in [2] with the same transconductor used in our design. It can be shown that all three circuits consume very little power which is determined by the number of transconductors used in each circuit. The power consumption of the transconductor used is 40 nW, which can be further reduced.

TABLE I. PERFORMANCE COMPARISONS OF THREE OSCILLATOR CIRCUITS

	Proposed	IF [2]	Still [5]
Power (nW)	69	49	92
PRC Range	6.0E-2	1.0E-2	3.2E-3
H Range (H1/H2/H3)	4.8E-4	5.2E-4	2.6E-6
	1.2E-3	8.6E-4	1.1E-4
	6.9E-4	4.2E-5	1.1E-4
H Polarity (H1/H2/H3)	Dual	Negative	Dual
		Dual	Positive
		Dual	Positive
G Range (G1/G2/G3)	9.4E-4	4.2E-4	7.0E-05
	1.4E-3	1.9E-04	1.0E-05
	7.8E-4	1.1E-04	6.8E-05

V. CONCLUSION

A new type of oscillator circuit based on the charging and discharging of a capacitor has been proposed. The circuit consumes similar power as the other two oscillator circuits. By adjusting the frequency of a pair of non-inverting clocks in a switched-capacitor resistor in the proposed oscillator circuit, a wide range of oscillation could be achieved to model different locomotion behaviors. Dynamical system has been used as a tool to analyze the circuit's ability to model HCO and the feasibility to couple this type of oscillators in a chain to achieve synchronization with a constant phase shift. PRC, H, and G functions of this oscillator have been analyzed and compared with two other existing oscillator designs through SPICE and MATLAB simulations. Analysis show that the

proposed oscillator can produce a PRC close to the lamprey CPG system and when used alone can model the whole HCO. H and G function analysis results prove the superior capability of our proposed oscillator circuit to achieve fast entrained oscillation with sensory feedback and reach equilibrium even with higher frequency mismatch.

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