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Improvements to time-series TR-PIV algorithms using historical displacement and displacement variation information

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Abstract

Improvements to two widely used particle-image velocimetry (PIV) algorithms, e.g., multi-grid and iterative image deformation cross-correlations, are proposed here to reduce the computational costs associated with time-resolved PIV (TR-PIV) data-processing. TR-PIV typically involves capturing significant time-series particle-image datasets across to allow statistically meaningful temporal and spectral analyses; hence considerable computational cost-savings can be realized. The improvements involve using the historical particle displacement field and its variation to determine the required window offsets and image deformations in the above-mentioned algorithms respectively. In this case, cross-correlation based on the smallest interrogation window size can be used directly instead of multi-pass cross-correlations based on decreasing interrogation window sizes. To evaluate their efficacy, the proposed improvements were implemented and evaluated using synthetic PIV images of a Rankine vortex flow, numerical solutions for a square cylinder wake flow, as well as actual experimental time-series TR-PIV measurements. Comparisons show that the proposed improvements save up to 50% computational time while maintaining relatively similar measurement accuracy levels as conventional algorithms. In particular, the new algorithms successfully resolve unsteady flow fields where particle displacements vary by more than 20% between successive particle-images, where error propagations associated with large displacement variations are mitigated by employing suitable recalculation thresholds.

Keywords

Time-resolved PIV, Multi-grid cross-correlation, Iterative image deformation cross-correlation

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1. Introduction

Due to rapid developments in high-speed digital camera, laser and computer technologies, high-fidelity measurements of spatio-temporal variations of unsteady flows using time-resolved particle image velocimetry (TR-PIV) is now gaining foothold in many experimental fluid flow studies.

TR-PIV technique, which typically integrates high repetition-rate pulsed or high-powered continuous wave (CW) laser systems with high frame-rate CMOS cameras, is well suited to resolve the instantaneous flow structures and dynamics of unsteady flows. This enables accurate capturing of time-series flow field information for subsequent data analyses into important flow features, such as Lagrangian Coherent Structures [1-5]. In particular, TR-PIV systems are commonly used to capture instantaneous flow fields at high operating frequencies under prolonged time-durations, resulting in significant numbers of particle-images for subsequent analysis. At present, particle-images deriving from TR-PIV experiments are commonly processed using mainstream iterative cross-correlation methods [6, 7], which were originally developed for single-frame/double-exposure recordings. On the other hand, one particularly suitable method to analyse TR-PIV time-series particle-image sequences is the multi-frame approach [8], which aims to increase measurement accuracy and dynamic velocity range by applying the optimal time interval to each interrogation window locally. As some sub-image areas are correlated using multiple time-intervals, this multi-frame technique can be used to reduce computational time at the cost of reduced data temporal resolution. Shi and Chen [9] later proposed a Bootstrap filter-based particle tracking algorithm for equally sampled image sequences. To reduce measurement errors caused by velocity variations between frames, a nonlinear dynamic model which takes particle accelerations into account is used in conjunction with Bootstrap filter to track the particles through an image-sequence. This method, however, is time-consuming due to its particle tracking nature and iterative prediction-update procedure.

Nonetheless, the tremendous amount of particle-image datasets resulting from TR-PIV experiments continues to drive the need for alternative processing approaches and algorithms that are both fast and accurate. While significant increases in raw computing power have been able to mitigate this need somewhat over the past two decades, it does not represent an efficient and appropriate long-term approach, particularly when energy consumption levels of computing resources are taken into account. In view of these challenges, the motivation of the present study is to propose simple but yet effective improvements to conventional multi-grid and iterative image deformation cross-correlation algorithms that allow rapid processing of a large number of TR-PIV particle-image sequences, while maintaining similar measurement accuracy levels as before. In the proposed improvements, historical particle displacement and its variation information are used to either determine the interrogation window offset for multi-grid analysis or image deformation information for image-deformation analysis. Here, the historical displacement is defined as the displacement vector map (in pixels) determined at the previous time step (i.e. $d(t-1)$), and the

historical displacement variation is the difference between two adjacent displacement vector map (i.e. $d(t-1)-d(t-2)$). To further understand the relative impact upon the computational efficiency and measurement accuracy made by historical particle displacement and its variation information, a distinction is further drawn between two possible improvements: using historical particle displacement information alone or using it together with its variations.

For the sake of clarity, multi-grid and iterative image deformation methods incorporating these proposed improvements specially constructed with TR-PIV in mind will be termed as “TR-multi-grid” and “TR-image-deformation” respectively. Correspondingly, conventional algorithms will be simply known as “multi-grid” and “image-deformation” respectively. The next section will elaborate upon the principles of the proposed TR-multi-grid and TR-image-deformation methods, while Section 3 will validate the new algorithms and demonstrate their robustness through the use of synthetic TR-PIV image series with different displacement variation between consecutive image-frames. On the other hand, Section 4 will test the algorithms on experimentally acquired TR-PIV image series and Section 5 will conclude with some insights into the proposed algorithms from the present work.

2. Working principles of the proposed TR-PIV algorithms

In conventional multi-grid and iterative image deformation methods where no prior flow field knowledge is available, cross-correlation analysis has to begin with a relatively large interrogation window size to provide an initial guidance for the required window-offset or image-deformation at the next iteration steps. In order to satisfy the requirement that particle-shift between two successive particle-image frames should not exceed one-quarter of the interrogation window size in both directions to prevent significant particle “drop-outs” [18], as well as to achieve the desired spatial resolutions, iterative multi-grid and window deformation cross-correlation algorithms typically require at least three iterations to reach the smallest interrogation window size [19]. Figure 1 shows schematically how an interrogation window size varies from a coarse one to a fine one during a typical iterative multi-grid calculation. It is well known that TR-PIV measurements provide time-series particle-images captured at high frequencies. As such, it can be reasonably assumed that the particle displacement variations between two successive time-steps are not significant. In that case, the particle displacement field and its variations determined at the previous time-step can theoretically be used to predict the required window-offset or image-deformation at the current time-step in iterative multi-grid or image deformation methods respectively. Hence, it is possible then, that TR-multi-grid and TR-image-deformation algorithms can use the smallest possible interrogation window size for cross-correlation analysis directly at the current time-step. This mitigates the need to use significantly larger initial interrogation window sizes that will have to be repeatedly reduced to smaller ones under multi-pass schemes. Hence, the proposed improvements can potentially save substantial computational time, as

Fig. 1 Concept of a) multi-grid cross correlation method, b) Iterative image deformation cross correlation method [7]

Fig. 2 Concept of the fast TR-PIV algorithm (illustrated with multi-grid cross-correlation method, flow move towards upper right corner)

compared to conventional iterative methods, while arguably not suffer from deteriorations in the measurement accuracy levels.

The detailed working principle of the TR-multi-grid method is illustrated in Fig. 2. As the general working principle of TR-image-deformation method is almost similar (i.e. the only difference is that the TR-image-deformation algorithm will determine the direct image-deformation information instead of direct window-offset information), its working principle will not be presented here for the sake of brevity. Returning to the TR-multi-grid method working principle, only regular time-series TR-PIV particle-images captured at fixed time-intervals are considered in the explanation here. Note that this methodology is equally valid for double-frame PIV systems operating at significantly higher repetition rates such that the flow fields are reasonably resolved. Their particle-image processing can be improved in a similar manner as the displacement variation between successive particle-image frames will likely to be small at high repetition rates.

The detailed step-by-step procedures of the improved TR-PIV algorithm are outlined as follow:

Step 1 Process the first three particle-images of the selected TR-PIV particle-image sequence using traditional multi-grid (for TR-multi-grid method) or iterative image deformation (for TR-image-deformation method) to obtain two initial displacement fields [i.e. $d(t1)$ and $d(t2)$ respectively]. Subsequently, the initial displacement fields are filtered with a 3-point by 3-point median local filter, where information from their nearest neighbours is used.

Step 2 Calculate the initial displacement variation [i.e. $d'(t1) = d(t2) - d(t1)$] from the two displacement fields and smooth it using moving-average method.

Step 3 At time-step, t , particle-image frames $I(t)$ and $I(t+1)$ are segmented into the smallest appropriately-sized interrogation windows. Before subjecting the particle-images to cross-correlation analysis, the interrogation windows are offset (for TR-multi-grid method) or deformed (for TR-image-deformation method) according to the displacement field and displacement variation at the previous time-step (i.e. $d(t-1)$ and $d'(t-1)$ respectively) according to Equation 1 below:

$$D(i, t) = d(i, t - 1) + d'(i, t - 1) \quad (1)$$

where,

$D(i, t)$ is the predicted displacement of the i -th interrogation window at time-step t ;

$d(i; t-1)$ is the calculated displacement of the i -th interrogation window at time-step $t-1$;

$d'(i; t-1)$ is the displacement variation of the i -th interrogation window between time-step $t-1$ and $t-2$;

Step 4 For each interrogation window, the ratio of its highest to second highest correlation peak is used to evaluate the correlation quality. A displacement vector is considered to be valid only if the amplitude of its highest correlation peak is at least 50% larger than the amplitude of the second highest correlation peak. Otherwise, **conventional** iterative cross-correlation (from coarse-to-fine interrogation window sizes) will be performed for that **particular image sub-area**.

Step 5 Update displacement variation according to

$$d'(i, t) = d(i, t) - d(i, t - 1) \quad (2)$$

Step 6 Repeat Steps 3-5 until all the successive TR-PIV particle-images are processed.

As can be seen from the above procedures, interrogation windows are offset (for TR-multi-grid method) or deformed (for TR-image-deformation method) according to the particle displacement

and its variation information determined in the previous iteration time-step. Thereafter, cross-correlation analysis between two successive particle-images will be performed directly with the smallest suitable interrogation window size, which intuitively saves significant computational time over conventional iterative multi-grid or image deformation based cross-correlation analyses were to be carried out. Take for example, to process a 1024px by 1024px particle-image pair, conventional iterative multi-grid and image deformation algorithms need to perform 5376 operations (i.e. including FFT cross-correlation, image interpolation and sub-pixel displacement detection) when the initial and final interrogation window sizes are 64px by 64px and 16px by 16px respectively with zero window-overlapping. Since almost all practical PIV image-pair interrogations involve some form of window-overlapping (typically 50%), the number of operations is actually much higher in practice. In contrast, if cross-correlation is performed directly with only a single and smallest 16px by 16px interrogation window size using the proposed algorithms here, only 4096 operations are needed, or a nearly 24% reduction in the number of calculations for a single particle-image pair.

Considering the significant number of images captured during a typical TR-PIV experiment (not to mention across multiple experiments), the reduction in computational time (or cost) by the proposed methods is substantial and impacts favourably upon the efficiency of future TR-PIV systems. For flows where the displacement difference between the two neighbouring time-steps is significant, the displacement variation calculated from previous time-steps can be used in conjunction with historical displacement field for guiding the window offsetting/deforming calculation. Furthermore, to prevent measurement error from propagating along the analysis of a selected TR-PIV particle-image sequence, a threshold can be applied to the ratio of the highest and second highest correlation peaks as laid out in the procedures. If the ratio is lower than a preset threshold (i.e. 1.5 is used in the current study according to [19]), conventional iterative cross-correlations (from coarse-to-fine interrogation window size) will be performed for affected image areas. While this will be slower than the case for which the ratio between the highest and second highest correlation peaks was to exceed the threshold level, the overall computational time will still be likely to remain significantly shorter than conventional methods, due to the small number of expected instances for which this is necessary.

3. Performance evaluations of the proposed TR-PIV algorithms

In this section, the performance of the proposed TR-PIV algorithms is evaluated against a comprehensive range of flow scenarios achieved through synthetic PIV images, numerical simulations and actual TR-PIV experiments. The intention is to evaluate the effectiveness of the improved algorithms, as well as their robustness across very different flow scenarios and datasets, and the level of computational cost-savings possible if they are implemented. For evaluations based on synthetic PIV images, a classical Rankine vortex generated from a synthetic PIV image

Algorithm	Uses conventional analysis	Uses historical displacement information	Uses displacement variation information	Uses recalculation scheme
Image-deform	Yes	No	No	No
TR-image-deform-1	No	Yes	No	No
TR-image-deform-2	No	Yes	Yes	No
TR-image-deform-3	No	No	No	Yes
Multi-grid	Yes	No	No	No
TR-multi-grid-1	No	Yes	No	No
TR-multi-grid-2	No	Yes	Yes	No
TR-multi-grid-3	No	No	No	Yes

Table 1 Specific differences in the procedures and naming conventions between the different variants of the proposed algorithms

generator will be used. On the other hand, for evaluations using numerical simulations and actual TR-PIV experiments, bluff body wake flows associated with a square cylinder immersed in a free-stream are used instead. The inclusion of both numerical simulation and experimentation datasets here is to ascertain whether the proposed algorithms are effective not only for ideal Rankine vortex synthetic PIV images, but also for more realistic bluff-body wake flows typically encountered in modern numerical simulations and experiments. Furthermore, bluff-body flows typically produce vortex-dominated flow fields in the downstream wake regions, which challenge the proposed algorithms more than the Rankine vortex flow and hence important for the assessments here. Note that the use of vortex-dominated flow scenarios here is intentional; these are well-established flow scenarios and the presence of strong vortices allows better and more accurate evaluations. The performance of the improved algorithms is assessed in terms of the resultant displacement error, valid vectors and computational time. Furthermore, to better illustrate and evaluate the efficacies of the proposed algorithms across a typical time-series TR-PIV particle-image sequence, variations in the level of valid vectors across 100 time-steps are presented for each test scenario. Note that all programming and numerical analyses were performed using Matlab R2011b on a Dell Precision T5500 workstation equipped with an Intel Xeon E5620 CPU, 8GB RAM and Windows 7 64bit operating system.

To distinguish between the different variants of the proposed algorithms, Table 1 summarizes their differences and the naming convention adopted here. As mentioned earlier, the proposed methodology are implemented for both iterative multi-grid and image-deformation algorithms, where each has three variants differentiated by the last number in their suffixes (i.e. 1, 2 and 3), as indicated in Table 1. For the variant with “1” in its suffix, only particle displacement information

Image size	1024×1024 (pixel)
Laser sheet type	Gaussian
Laser sheet thickness	1 (mm)
Image density (16x16 pixels)	7
Average particle image radius	1.5 (pixel)
Projection type	znormal
Particle image intensity distribution	Gaussian
CCD background noise distribution	Gaussian
Noise STD	5
Noise mean	5
CCD saturation level	0.8
CCD fill ratio	0.75
Image quantisation	8bits/pixel
Time interval	$\Delta t = 0.06s$

Table 2 Key parameters for generating synthetic Rankine vortex flow particle-images

was used in the modified algorithm. In contrast, for the variant with “2” in its suffix, both particle displacement and its variation information are used. As for the variant with “3” in its suffix, recalculation scheme is used instead. As benchmarks, conventional multi-grid and image-deformation algorithms are included in the evaluations as well.

3.1 Comparisons with synthetic steady Rankine vortex flow field

Synthetic particle-images for a classical Rankine vortex are generated using EUROPIV synthetic image generator (S.I.G.) package [20] and Table 2 lists the parameters used for generating the synthetic images. As per usual practice, particle locations in the very first image are randomly generated with a uniform distribution without any bias. Subsequently, their positions in successive images are determined by imposing a pre-determined and well-defined flow field upon the particle locations in the previous image. The synthetic Rankine vortex used is described mathematically in Equation 3 below, where

$$\begin{cases} u_r = \frac{1}{2}\alpha r \\ u_\theta = \frac{\Gamma}{r}(1 - \exp(-\frac{\alpha}{4\nu}r^2)) \\ u_z = \frac{1}{2}\alpha z \end{cases} \quad (3)$$

Where,

u_r is the radial velocity of particle image.

u_θ is the tangential velocity of particle image.

u_z is the axial velocity of particle image.

r is particle image's radial distance.

$\Gamma = 160$ is the circulation.

$\alpha = 200$ and $\nu = 2$.

Parameters Γ, α, ν were configured such that the maximum in-plane displacement between two successive particle-images and the vortex radius were approximately 11px and 116px respectively.

Since TR-multi-grid and TR-image-deformation algorithms are based on the assumption that displacement variation between neighbouring time-steps is small, it is necessary to analyse the influence of displacement variation on the accuracy levels of the improved algorithms. To do that, three different sets of image-frames with small (5%), moderate (17%) and large displacement difference (25%) are generated. This is made possible by using synthetic particle-images with increasingly larger displacement variations, produced by superimposing a horizontal sine displacement function, u_x , on the Rankine vortex, where

$$u_x = \beta \sin(kt) \quad (4)$$

Where,

$k = 300$.

$\beta = 50, 200, 300$ for different cases.

One hundred continuous synthetic particle-images are generated for each set for subsequent evaluation. For the evaluation, the criterion used is the percentage of valid vectors determined by the proposed algorithms. It is defined here as the ratio of the number of vectors whose measurement error are less than 10% to the total number of measured vectors, and mathematically expressed in Equation 5 as:

$$p = \frac{N(|(d_{measure} - d_{true})/d_{true}| < 10\%)}{N_{total}} \quad (5)$$

where N_{total} is the total number of measured vectors.

Fig. 3 (a) Real displacement field of Rankine vortex (small displacement variation); (b) displacement variation (d') between $t=10$ and $t=11$ (normalized by the maximum displacement); (c) Difference (ξ) between the predicted location of interrogation window and its true location at next time step (d) Variation of measurement accuracy against time steps for small displacement variation

3.1.1 Evaluation of displacement error and valid vector number

Consider the case where the displacement variation is small at 5% of the maximum displacement, Figs. 3(a) and 3(b) show the real displacement field (i.e. d) at the first time-step and the normalized displacement variation (i.e. $d'/d_{\max}$) between the 10th and 11th time-steps respectively. Note that during the 10th and 11th time-steps, the horizontal displacement changes its direction and therefore produces the largest displacement variation. On the other hand, Fig. 3(c) shows the difference between the predicted displacement based on the particle historical displacement and the real displacement at next time-step. For the most part, the prediction error is less than 1px, which indicates that the actual measured residual displacement by the proposed algorithms is closer to 0.5px when interrogation windows are offset or deformed according to this prediction. Such a small residual displacement is not expected to pose any appreciable difficulties for the proposed algorithms. Figure 3(d) shows the variations in the percentage of valid vectors through the time-steps as determined by the conventional and proposed algorithms. Note that since the displacement variation is so small, making use of its information during the calculation produces negligible changes between TR-image-deform-1 and TR-image-deform-2 algorithms. It is the same between TR-multi-grid-1 and TR-multi-grid-2 algorithms. Hence, results for TR-image-

deform-2 and TR-multi-grid-2 algorithms are not shown in Fig. 3(d). As the displacement variation is very small in this case, no recalculation of image sub-areas using conventional iterative methods is needed. Returning to the figure, it can be discerned that TR-image-deform-1 algorithm achieved practically similar levels of valid vectors as conventional image-deformation algorithm. On the other hand, TR-multi-grid-1 algorithm performed slightly worse than conventional multi-grid algorithms. However, note that the former still attained over 97% valid vectors throughout the 100 time-steps. Considering the savings in computational time (to be discussed later), this level of valid vectors remains satisfactory for most practical flow measurement scenarios.

Next, consider the case where the displacement variation is moderate at 17.5% of the maximum displacement. In this case, Figs. 4(a) and 4(b) show the real displacement field at the first time-step and the displacement variation between the 10th and 11th time-steps respectively. If one considers the fact that the displacement variation is now more than three times that encountered in the small displacement variation case previously, some particles have already moved out of the interrogation areas when windows are offset or deformed only according to the historical displacement. As a result, Fig. 4(c) demonstrates that locations for most of the interrogation windows will deviate more than 1px from their real location when only historical displacement is used. Clearly, such a heightened level in “loss-of-pairs” behaviour will lead to deteriorations in the measurement accuracy for the proposed algorithms. This can be readily seen in Fig. 4(e) where the proposed algorithms show reductions in the levels of valid vectors. However, a closer inspection will reveal that if displacement variation information is used in the algorithms (i.e. TR-image-deform-2 and TR-multi-grid-2), more valid vectors will typically be obtained. For instance, comparing between TR-image-deform-1 and TR-image-deform-2 algorithms, the percentage of valid vectors can be increased from just above 90% to above 96%. The case for TR-multi-grid-1 and TR-multi-grid-2 is less clear-cut. While there are improvements, they are much more limited, as depicted in the figure. Nevertheless, it still shows that when interrogation windows are offset or deformed according to both historical displacement and displacement variation information, the majority of the prediction error is restricted to less than 1px as shown in Fig. 4(d). This implies a significant reduction in the “loss-of-pairs” effect and the number of areas needed to be recalculated. As a result, the measurement accuracy is on average 4% higher than corresponding algorithms that only made use of historical displacement information.

Lastly, Fig. 5 presents the test case when displacement variation is large. Figure 5(a) shows the real displacement field while Fig. 5(b) shows the normalized displacement variation between the 10th and 11th time-steps, where the maximum displacement variation is 25%. Given such a large displacement difference, the “loss-of-pairs” effect is expected to be significantly worse than the first two cases discussed previously if historical displacement information is only used in TR-

Fig. 4 a) Real displacement field of Rankine vortex (moderate displacement variation); b) Displacement variation (d') between $t = 10$ and $t = 11$ (normalized by the maximum displacement); c) Difference between the predicted location of interrogation window and its true location at next time step (without historical displacement variation information); d) Difference between the predicted location of interrogation window and its true location at next time step (with historical displacement variation information); e) Variation of measurement accuracy against time steps for moderate displacement variation

multi-grid and TR-image-deformation algorithms. As shown in Fig. 5(c), the prediction error is larger than 2px for most of the interrogation windows. In contrast, Fig. 5(d) shows that the prediction error can mostly be kept below 1px when the current displacement is predicted by using both historical displacement and its variation information. Figure 5(e) shows the variations in the percentage of valid vector for this test case and as expected, the level of valid vectors are worse than the other two earlier cases for corresponding algorithms. In this case, the percentages of

Fig. 5 a) Real displacement field of Rankine vortex (large displacement variation); b) Displacement variation (d') between $t = 10$ and $t = 11$ (normalized by the maximum displacement); c) Difference between the predicted location of interrogation window and its true location at next time step (without historical displacement variation information); d) Difference between the predicted location of interrogation window and its true location at next time step (with historical displacement variation information); e) Variation of measurement accuracy against time steps for large displacement variation

valid vectors are generally 1-3% lower. Nonetheless, the proposed algorithms using both displacement and its variation information perform better than those only use historical displacement information. In fact, the general differences between these two methodologies are grossly similar to what is observed for the case of moderate displacement variation see earlier. When historical displacement variation is taken into account, the measurement accuracies of both

Fig. 6 Variation of percentage of valid vectors against time steps for large displacement variation test-case without recalculation threshold

the TR-multi-grid and TR-image-deformation algorithms are very similar to their conventional counterparts and 2-4% higher than algorithms that only employ historical displacement information.

For large displacement variations, there will be a significant number of image sub-areas that requires recalculation based on conventional iterative algorithms. To look at the effects of the implemented recalculation scheme, Fig.6 plots the measurement accuracy for the proposed algorithms when no recalculation threshold is applied. For the TR-image-deformation algorithm with no recalculation threshold implemented (i.e. TR-image-deform-3), the measurement accuracy decreases rapidly from 92% to less than 40% after 100 time-steps. It indicates that if the particle-images are continuously deformed according to a displacement field that is very different from its real one, the reconstructed particle-images will deviate significantly from the actual ones and lead to very rapid error propagation. Similarly, the measurement accuracy of the TR-multi-grid method (i.e. TR-multi-grid-3 in the figure) decreases from 93% to 80% over 100 time-steps when no recalculation threshold is applied. In this case, the deterioration is not as severe as TR-image-deform method without recalculation threshold, possibly due to lower levels of error propagation. However, the overall reduction in the percentage of valid vectors remains substantial and should be deemed unacceptable.

Figure 7 shows the percentage of recalculated areas for the proposed algorithms for moderate and large displacement variation test-cases. Clearly, the lower the number, the better it will be in terms of reduced computational cost. To ease comparisons, the number of recalculated areas was normalized by the number of total calculated vectors. Due to the fact that combined use of historical displacement and its variation can provide more accurate information for window-offsetting or deformation (i.e. TR-image-deform-2 and TR-multi-grid-2), the number of

Fig. 7 Variation of the percentage of recalculated area against time steps: a) moderate displacement variation; b) large displacement variation

recalculated areas is significantly lower than that for algorithms that only makes use of historical displacement information (i.e. TR-image-deform-1 and TR-multi-grid-1), regardless of whether the displacement variation is moderate or large. Due to lower prediction error rates and smaller number of **image sub-areas** to be recalculated, achieving good measurement accuracy for the former technique does not entail extra computational efforts. In addition, comparing between Figs. 7(a) and 7(b), the effects of a larger displacement variation serves to increase the number of recalculated areas, when displacement variation is not taken into account by the proposed algorithms. However, that is unsurprising since a larger displacement variation will lead to more instances of “loss-of-pairs” behaviour, which will in turn require a higher number of recalculated areas. When both historical displacement and displacement variations are both incorporated into the algorithms however, increasing the displacement variation intuitively does not lead to discernible increase in the number of recalculated areas.

3.2 Comparisons with synthetic unsteady flow past square cylinder flow field

In the preceding section, evaluations of the proposed algorithms against a synthetic Rankine vortex have been performed, where the flow is steady and highly stable. Now, the proposed algorithms will be used upon synthetic images generated for the unsteady wake flow behind a square cylinder immersed in a free-stream. The pre-determined velocity fields for generating these synthetic images are calculated from CFD simulations based on Unsteady Reynolds Averaged Navier–Stokes (URANS) modelling and readers are referred to [21] for more details of the numerical simulations. **In brief, initial particle locations are randomly generated in Matlab before CFD velocity information was used to determine their subsequent locations. Thereafter,**

Fig. 8 a) Real displacement field of the square cylinder wake (CFD); b) Displacement variation between the second and third time steps

these calculated particle locations are then input into S.I.G. package to generate the actual synthetic particle-images with the intended velocity field distributions. For the simulations, free-stream velocity is maintained at $U=30\text{m/s}$ and the Reynolds number based on square cylinder width, $D=8\text{mm}$, is $Re_D=24,000$. As for the synthetic image generator settings, the particle-image size is 2048px by 2048px with a corresponding 8.7D by 8.7D field-of-view. The simulation area of interest begins at the immediate region downstream of the cylinder lee-side surface. The time interval between two consecutive particle-images is configured to be $\Delta t=0.125\text{millisecond}$, which corresponds to a theoretical camera frame-rate of 8 kHz, which is more than sufficient to emulate a TR-PIV scenario. Based on these preceding configurations, the generated flow field is presented in Fig. 8. Figure 8(a) depicts the real displacement field at $t=1$, while the displacement variation between the 2nd and 3rd time-steps is shown in Fig. 8(b). Note that the maximum displacement variation of 18% indicated in the figure is close to the “moderate displacement variation” configuration studied in the previous synthetic Rankine vortex test case. Furthermore, the alternate vortex-shedding phenomenon within the wake region also indicates a significant departure from the previous test case and offers an alternate and more rigorous evaluation as to how well the proposed algorithms will perform.

Figure 9 shows the performance results obtained after the proposed algorithms are used to analyse the test case. In Fig. 9(a) where variations in the percentage of recalculated areas required for TR-multi-grid and TR-image-deformation algorithms are shown, an average of approximately 3% is recalculated when only historical displacement information is applied. When both historical displacement and its variation information are used together (i.e. TR-image-deform-2 and TR-multi-grid-2), the percentage of recalculated areas is far below 1%. With respect to the percentage of valid vectors, Fig. 8(b) shows that using both historical displacement and its variation together can provide accurate information for window offset/deformation and improve the measurement accuracy from below 95% to approximately 97% valid vectors. Note that this is on top of the expected savings in computational time. With the help of the recalculation scheme, the

Fig. 9 Synthetic CFD TR-PIV image tests: a) Variation of the percentage of recalculated area against time steps; b) Variation of measurement accuracy against time steps

measurement accuracy for the proposed algorithms are still higher than 93% when only historical displacement information is applied (i.e. TR-image-deform-1 and TR-multi-grid-1). Hence, the effectiveness of the proposed algorithms can be seen to extend towards TR-PIV measurements of unsteady flow fields as well. This gives confidence towards further evaluations using actual experimental TR-PIV data, which will be discussed in the next section.

3.3 Comparisons with unsteady flow past square cylinder TR-PIV experiments

For the sake of completeness, as well as to verify the efficacy of the proposed algorithms seen so far, they are used upon a time-series TR-PIV **particle-image sequence** captured for the wake flow produced behind a square cylinder **immersed in a free-stream**. This series of experiments are performed in a recirculating water channel. As shown in the design schematic of the water channel in Fig. 10, flow is circulated by a damped Iwaki magnetic drive centrifugal pump to avoid structural vibration of the facility. A settling chamber, a flow-straightening honeycomb, three layers of fine-screens and a contraction section are located upstream of the test-section to ensure flow homogeneity. The test-section of the water channel is constructed using Plexiglas to ensure high optical access for PIV measurements and its internal dimensions measured 150mm (W) by 200mm (H) by 1050mm (L). For this series of experiments, a square cylinder $D=15\text{mm}$ wide and $L=150\text{mm}$ in length (i.e. cylinder aspect-ratio of $L/D=10$) is placed mid-depth within the test-section, such that the cylinder spans across the entire width of the test section. During the experiments, the free-stream velocity is maintained at $U=0.15\text{m/s}$ with a resultant Reynolds number (i.e. based on cylinder width) of $Re_D=2250$. At this free-stream velocity, the free-stream turbulence intensity has been **ascertained to be approximately 1.8% using PIV in an earlier study**,

1. Pump 2. Reservoir 3. Diffuser 4. Settling section 5. Contraction 6. Test section 7. Recycle tank

Fig. 10 Schematic of the recirculating open water channel [23]

which is a reasonably low for a water channel.

As for the TR-PIV setup, the measurement area is illuminated by a 2W, 532nm wavelength, continuous-wave DPSS laser, where the laser beam is formed into a 1mm thick laser sheet by a cylindrical lens. The flow field is uniformly seeded using 5micron diameter, 1.05kg/m^3 specific gravity glass beads and light scattered by the illuminated particles is captured by an 8-bit greyscale 1280px by 1024px Mikrotrotron high-speed CMOS camera. The camera is capable of operating at a maximum capturing frame-rate of 506 frames-per-second at full resolution.

However, testing has shown that a capturing frame-rate of 42 frames-per-second with an exposure time of 6millisecond is sufficient to resolve the low Reynolds number flow field. A total of 1000 successive particle-images are captured for the evaluations here. For this series of testing, the initial and final interrogation window sizes used by multi-grid and iterative image deformation are 64px by 64px and 32px by 32px respectively. 50% interrogation window overlapping was also used. In contrast, the interrogation window size used by TR-multi-grid and TR-image-deformation algorithms is 32px by 32px with 50% interrogation window overlapping.

The historical displacement information for the present test case is presented in Fig. 11(a). On the other hand, Fig. 11(b) shows the displacement difference between the 2nd and 3rd time-steps, which shows a maximum of 10% displacement variation. Since this test case makes use of experimental data rather than synthetic PIV images, there is no exact displacement field information for direct comparisons. Instead, results calculated by conventional multi-grid and

Fig. 11 a) Displacement field of the square cylinder wake (experiment); b) Displacement variation between the 2nd and 3rd time-steps

Fig. 12 Real TR-PIV image tests: a) Variation of the percentage of recalculated area against time steps; b) Variation of measurement accuracy against time steps

iterative image deformation methods are averaged to serve as a benchmark to evaluate the performance of the proposed algorithms. Doing so will avoid any significant bias caused by either one of these conventional iterative methods. Figure 12(a) shows the percentage of recalculated areas for the proposed algorithms and it can be observed that approximately 0.5% of the total areas will need to be recalculated if only historical displacement information is used for analysis (i.e. TR-image-deform-1 and TR-multi-grid-1). In contrast, if both historical displacement and its variation information are used (i.e. TR-image-deform-2 and TR-multi-grid-2), the amount of recalculated area required decreases to about 0.1%. While it is clear that the lack of displacement variation information will require a larger number of **image sub-areas** to be recalculated, it is worthwhile to mention that other experimental issues such as poorly-contrasted particle-images, background noise and non-uniform particle distributions may also reduce particle detection levels

Algorithm type	Test cases				
	Small displacement variation (5%)	Moderate displacement variation (17%)	Large displacement variation (25%)	Square cylinder wake (CFD)	Square cylinder wake (Experimental)
Multi-grid	163.2s	180.6s	167.9s	316.2s	894.6s
Iterative-image deformation	1273.3s	1272.4s	1346.4s	2303.8s	1647.9s

Table 3 Actual computational times of conventional iterative multi-grid and image-deformation algorithms for the five test-cases here

and contribute towards the need for more recalculated areas as well. The number of valid vectors also benefits from the proposed algorithms, as shown in Fig. 11(b). Similar trends as before can be observed, where the use of both historical displacement and displacement variation leads to noticeable improvements to the overall percentage of valid vectors. For instance, when only historical displacement information is used, the percentage of valid vectors typically ranges between 93% to 95%. In contrast, when both historical displacement and displacement variation information are used together, the percentage typically increases to between 94% to 97%.

3.4 Potential savings in computational cost

In the preceding comparisons, it has been ascertained that satisfactory levels of valid vectors can be achieved through the proposed algorithms. However, if such an improvement comes with a heavy penalty in computational cost, then the proposed algorithm may not be too useful for typical TR-PIV usage, where large numbers of particle-images are usually captured and processed. Hence, an assessment on the computational costs associated with using conventional and proposed algorithms upon the current test cases will be performed here. As a benchmark, actual computational times for conventional algorithms for the various test-cases are provided in Table 3.

Figure 13 shows the computational time for the proposed algorithms for all five test scenarios discussed earlier. As the computational time is highly dependent on hardware specifications, for the ease of comparisons, computational times of the proposed algorithms are normalised against their conventional iterative counterparts. If the non-dimensionalised computational time is less than one, it implies that the computational cost has been reduced. In that case, Fig. 13(a) demonstrates that the proposed TR-multi-grid-1 and TR-multi-grid-2 algorithms are able to produce consistently remarkable reductions in computational costs with savings of approximately 40% to 50%. Note that savings are the lowest for TR-multi-grid-1 algorithm when the test case involves large displacement variations, whereas TR-multi-grid-2 algorithm still leads to significant reductions in the computational time due to its low recalculation rate (Fig. 7(b)). For

Fig. 13 Computational times of the proposed a) TR-multi-grid algorithms and b) TR-image-deformation algorithms for the various test-cases.

the proposed TR-image-deform algorithms, the computational savings are only slightly lower and practically on par with TR-multi-grid algorithms, as shown in Fig. 13(b). The only exception is when the displacement variation is large, where using historical displacement information alone (i.e. TR-image-deform-1) leads to an intriguing doubling of the computational time instead. This interesting discrepancy will be explained later. Lastly, similar to TR-multi-grid algorithms, using both historical displacement and its variation information in TR-image-deform-2 algorithm further reduces the computational cost. In this case however, the extent of reduction is larger even though TR-image-deform-2 and TR-multi-grid-2 algorithms produce practically similar computational cost reductions.

It is quite clear that the computational time results for the large displacement variation test case present an intriguing situation. Furthermore, it is also a more unique test case which warrants further explanations. For TR-image-deform-2 algorithm results for large displacement variations shown in Fig. 13(b), despite an increase in the number of recalculated areas when compared to the moderate displacement variation case, the computational cost remains similar (reduced by 40% as noted earlier on). This is due to the fact that the recalculated areas are correlated by window-offset instead of image-deformation. If the recalculated areas were to be correlated using image-deformation, the problematic areas along with their four surrounding areas will all have to be recalculated in order to provide displacement field for image interpolation. Such a measure will significantly reduce computation efficiency and hence increase computational cost. In addition, if calculated displacements for the current time-step were to be used for the four surrounding areas

of the recalculated area, the measurement error of these displacements would propagate through the recalculation process. Returning to the observation that TR-image-deform-1 algorithm requires twice the computational cost as that for its conventional iterative counterpart for large displacement variations test case. This can be explained by the fact that the average recalculated areas by TR-image-deform-1 algorithm is about 10% (Fig. 7(b)), which is nearly 10 times more than that of the TR-image-deform-2 method.

4. Conclusions

Improvements to conventional iterative multi-grid and iterative image deformation PIV algorithms are proposed with a view to reduce computational cost of processing time-series TR-PIV data while preserving satisfactory measurement accuracies. Based on the assumption that the particle displacement does not differ much between two successive particle-images due to the high-frequency nature of TR-PIV experiments, it is proposed that both the historical displacement field and its variation be employed to determine window-offset and image-deformation information, such that cross-correlation is performed with the smallest possible interrogation window directly. This is in contrast to conventional iterative algorithms where multi-pass cross-correlations with successively smaller interrogation window sizes are used. Expected savings in computational cost are confirmed when performance of the proposed algorithms is extensively tested and compared to conventional iterative algorithms using synthetic Rankine vortex particle-images, numerical and experimental TR-PIV results of square cylinder wake flow. Comparisons show that the proposed algorithms can typically save up to 50% of the computational time while preserving satisfactory and comparable measurement accuracy as conventional iterative PIV algorithms, except when the displacement variation is too large. By incorporating both historical displacement and its variation information in the algorithms, measurement accuracy and computational time are not affected by the displacement variation. It is also revealed here that error propagation associated with large displacements can be successfully mitigated by setting appropriate thresholds that will recalculate affected image sub-areas based on iterative algorithms. Due to the relatively simple procedures of the proposed algorithms and the significant reductions in computational cost, they will be suitable for post-processing huge amounts of time-series of particle-images or to provide rapid feedback on the measured flow fields during TR-PIV experiments.

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