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On the internal vorticity and density structures of miscible thermals

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Miscible thermals are formed by instantaneously releasing a finite volume of buoyant fluid into stagnant ambient. Their subsequent motion is then driven by the buoyancy convection. The gross characteristics (e.g. overall size and velocity) of a thermal have been well studied and reported to be self-similar. However, there have been few studies concerning the internal structure. Here, turbulent miscible thermals (with initial density excess of 5% and Reynolds number around 2100) have been investigated experimentally through a large number of realizations. The vorticity and density fields were quantified separately by particle image velocimetry (PIV) and planar laser-induced fluorescence (PLIF) techniques. Ensemble-averaged data of the transient development of the miscible thermals are presented. Major outcomes include: (i) validating Turner's assumption of constant circulation within a buoyant vortex ring; (ii) measuring the vorticity and density distributions within the miscible thermal; (iii) quantifying the effect of baroclinicity on the generation and destruction of vorticity within the thermal; and (iv) identifying the significantly slower decay rate of the peak density as compared to the mean.

Key words: convection, plumes/thermals

1. Introduction

A miscible thermal is formed when a finite amount of positively buoyant fluid is released from rest, and then moves as an isolated fluid volume by its own buoyancy. The thermal can also be considered as a special case of a buoyant vortex ring (Turner 1957) due to its vortex ring shape, but with its initial buoyancy and total circulation directly linked to each other. Here, a thermal may be referred to as a buoyant vortex

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ring when the internal structure is being examined. Investigations of miscible thermals were first initiated by meteorologists interested in the convection of cumulus clouds (Scorer 1957). Miscible thermals can also be observed during the explosion of nuclear bombs as mushroom clouds, and in convection in the Earth's mantle (Whittaker & Lister 2008). During the past few decades, the gross characteristics of miscible thermals have been well studied, and found to follow relationships based on the assumption of constant total buoyancy excess (Scorer 1957; Turner 1973) as:

$$r \propto (z - z_0), \quad r \propto (t - t_0)^{1/2}, \quad (1.1)$$

where r is the bulk radius of the thermal, and z and t are the distance and time travelled downstream from the physical references, respectively. As the radius grows linearly with the travel distance, the thermal develops along a conical path which can be assumed to originate from a virtual point source (included in (1.1) where z_0 and t_0 are the virtual distance and time origin, respectively). Experimental observations consistently suggest that, as the thermal travels downstream, buoyancy loss to the wake is negligible (Scorer 1957; Richards 1961; Thompson & Snyder 2000). Therefore, the total buoyancy excess is conserved and equal to the initial value.

$$B_0 = \left(\frac{\Delta\rho_0}{\rho_a} \right) V_0 g_z, \quad (1.2)$$

where $\Delta\rho_0$ is the initial density excess, ρ_a is the ambient density, V_0 is the initial volume of buoyant fluid, and g_z is the gravitational acceleration. Equation (1.2) justifies the assumption made for (1.1).

Few studies have explored the internal structures of thermals or buoyant vortex rings. Bond & Johari (2005) investigated the near-source development of buoyant vortex rings generated with various initial aspect ratios. They also measured the vorticity distribution in the developing stage of a buoyant vortex ring, and compared buoyant and non-buoyant vortex rings (Bond & Johari 2010). Hart (2008) measured the concentration field of miscible thermals indirectly based on an integrated view (illuminating from behind using a fluorescent light bank), and suggested that 11 trials were enough to yield representative ensemble-averaged data.

The present study directly quantifies the internal density and vorticity fields within a fully developed miscible thermal. Experiments were conducted with planar laser-induced fluorescence (PLIF) and particle image velocimetry (PIV) techniques. In the following, the experimental methods and rationale for data selection during ensemble averaging are first described; the experimental results on the internal structures are then discussed; and a summary is given at the end.

2. Experiments

2.1. Experimental setup

Experiments were done in a glass tank with dimensions of 2.85 m (length) by 0.85 m (width) by 1.0 m (height). A brine and fluorescent (Rhodamine B) dye mixture was initially held inside a fully submerged cylindrical tube (inner diameter of $D_0 = 1.8$ cm and length of $L_0 = 3.6$ cm), with the top and bottom sealed by latex sheets. Ambient water density was $\rho_a = 0.9982$ g cm⁻³ and the initial density excess was $\Delta\rho_0/\rho_a = 5\%$, which led to $V_0 = 9.16$ cm³ and $B_0 = 449.3$ cm⁴ s⁻². Prior to a run, a motor-driven needle (following Lundgren, Yao & Mansour 1992) was gradually moved downward to pierce the top seal, and then positioned just touching the bottom

seal. A lapse period of 15 min allowed the induced turbulence to subside, and then the experiment began by bursting the bottom membrane.

Separate PLIF and PIV measurements of density and velocity fields were performed. During the PLIF measurements, images were captured by a CCD camera at 15 Hz with a resolution of 1600×1186 pixels, corresponding to a spatial resolution of $0.273 \text{ mm pixel}^{-1}$. Data were extracted from the region of $8D_0$ to $14D_0$ from the bottom of the release tube in the vertical direction. Flow within the miscible thermal was measured separately using PIV. The same release mechanism and camera were adopted, and synchronized with a pulse laser. Image pairs were recorded at 15 Hz and analysed by cross-correlation with an interrogation area of 32 by 32 pixels and 50 % overlap, which produced one vector in an area of 3.76 by 3.76 mm. Vorticity data were extracted from the region of $6D_0$ to $12D_0$. Within the whole region of interest ($6D_0$ – $14D_0$), the refractive index mismatch between the thermal region and ambient water was estimated to vary from 0.0012 to 0.0004 for the peak value and from 0.0008 to 0.0001 for the mean value. To collect sufficient data for ensemble averaging, PLIF and PIV experiments with identical release conditions were conducted 100 and 25 times, respectively.

2.2. Variability and data selection

Miscible thermals are essentially buoyancy-induced natural convection, which typically displays great variability due to the unsteady nature and coupling effects between the flow and buoyancy. Starting from rest, fluid velocity is initiated by the buoyant force, and the density distribution is modified as soon as the fluid moves (Batchelor 1954). Therefore, a small disturbance, either resulting from turbulent fluctuation or asymmetry at release, may introduce variations to the main flow structure during descent.

To quantify the variations, the shape of the thermal region was monitored by a video camera from the bottom of the glass tank in addition to cross-sectional PLIF. Ambient illumination was provided by placing a spotlight in line with the laser sheet so that the thermal region could be distinguished due to the red dye. The edge of the dyed region was detected during post-processing by the Otsu (1979) method. Experiments affected by extremely large release disturbances (e.g. uneven rupture of the latex sheet) were normally characterized by extreme asymmetry and/or significant drift from the vertical path during descent. The variability of individual runs was quantified by extracting the gross characteristics from the bottom view: (i) the shift of centre of mass perpendicular to the laser sheet (y_b), which indicated the straightness of the travel path; and (ii) the ratio (η_b) between the maximum widths along and perpendicular to the laser plane, which indicated symmetry. Since the scaling factor changed as the thermal moved towards the video camera, y_b was normalized at each frame by the corresponding equivalent radius, r_{eq} (radius of a circle with the same total area as the dyed region).

The variability for a miscible thermal had been found to be quite large in previous studies (Scorer 1957; Richards 1961). Hart (2008) reported that the variation of maximum radius reached an average of $\pm 9\%$ and a maximum of $\pm 19\%$ for 11 trials with the same release conditions. Therefore, $\pm 20\%$ was adopted in the present study as the maximum variation that could be generated by turbulent variability. An experimental run was selected for the ensemble average if both y_b/r_{eq} and η_b varied within $\pm 20\%$ from the mean over 100 runs. Finally, 84 out of the total of 100 runs were selected for ensemble-averaging of the PLIF measurements. Assuming the same percentage, 21 out of 25 runs from the PIV measurements were selected. Although it was not possible to completely differentiate the effects of release disturbances from

turbulent fluctuations, this approach represented our best effort to eliminate variations caused by the release mechanism.

The addition of ambient light introduced additional excitation to the dye tracer. To test this effect on PLIF measurements, calibration images were captured both with and without ambient light for the same calibration condition. Least-square fitting was carried out to relate the grey-scale values to the dye concentrations in a pixel-by-pixel manner using nine different points (a 3×3 matrix) in the region of interest. R^2 -values were found to be 0.987 ± 0.012 without the spotlight, and 0.969 ± 0.022 with the spotlight. Therefore, the ambient light slightly amplified the variation for PLIF calibration. However, the linear correlation was fortunately maintained.

3. Results and discussion

The ensemble averaging was performed on a temporal basis in the following manner. In order to focus on the internal structure within the miscible thermal, the locations of the vortex cores were first identified by tracing the peak vorticity/density. The radius of the vortex ring (r_{Ring}) was then defined as the half-distance between the two vortex cores, and the centre of the head vortex ring as the mid-point in between. Data from different runs taken at the same time following release were first normalized by their respective r_{Ring} , and then superimposed so that their centres coincided. The averaged data were finally calculated by averaging the results from all runs. The physical references were set at the time of release ($t = 0$) and the bottom of the release tube ($z = 0$).

3.1. Constant circulation

Assuming constant circulation (Γ_0) and buoyancy excess (B_0), Turner (1957) derived a growth relationship for a Boussinesq buoyant vortex ring as

$$r^2 = \frac{B_0}{\pi\Gamma_0}t \quad (3.1)$$

which agrees with (1.1). Equation (3.1) suggests that the growth rate of r^2 is constant and dependent on the ratio of total buoyancy excess and circulation. However, no direct measurements of the circulation have yet been reported to support this assumption. A recent experimental study by Bond & Johari (2005) on the near-source development of miscible thermals found that the total circulation of a buoyant vortex ring approached a constant towards the very end of their measurement region, which was up to $7D_0$ downstream. With the present PIV data that covered a region further downstream ($6D_0$ – $12D_0$), Turner's assumption can now be fully validated.

An extra PIV experiment with simultaneous planar flow visualization was first performed to check if the buoyancy- and vorticity-containing regions travelled together. In figure 1(a), an instantaneous vorticity map and the corresponding edge of the buoyancy-containing region are plotted together. The buoyant fluid is mainly inside the head vortex region, whereas a significant amount of vorticity exists in the tail region (which was generated by boundary shear when the induced flow passed through the release tube). A moving control volume was defined as the head vortex region. As observed by Scorer (1957) and the present study, the height of a thermal was ~ 0.7 times the maximum width ($2r_{max}$). Counting from the level of the vortex core (z_{core}), the head vortex region could be well covered and distinguished from the trailing vortices when it was defined from $z_{core} - 0.6r_{max}$ to $z_{core} + 0.8r_{max}$.

FIGURE 1. (a) Instantaneous vorticity map and detected edge of buoyancy-containing region. (b) Average circulation within the miscible thermal; T = total domain, H = head vortex, P = positive (clockwise), N = negative (counter-clockwise). $T_{TM} = B_0^{1/6}/g_z^{2/3}$ as defined in (3.5).

In figure 1(b), the averaged circulations within both the total domain and the head vortex region are plotted with their respective standard deviations as error bars. Results are normalized, and the data from Bond & Johari (2005) with the same initial aspect ratio (i.e. $L_0/D_0 = 2$) but slightly different total buoyancy excess are plotted alongside. As suggested by Bond & Johari (2005), a miscible thermal could be considered as fully developed once the circulation reached a constant. As the present study focused more on the developed region, the circulation within the head vortex was indeed constant. The circulation within the total domain reached a higher maximum at a later time because the induced downstream flow was still generating circulation when passing the tube wall, and then it started to gradually decay due to the viscous dissipation in the non-buoyant trailing vortices. By extending the work of Bond & Johari (2005), the present study verifies Turner (1957)'s assumption.

3.2. Vorticity and density distributions

As discussed above, earlier studies have shown the gross characteristics of a miscible thermal to be self-similar. However, very limited information (Bond & Johari 2005, 2010) has been reported on the internal structures. The ensemble-averaged internal vorticity and density structures of a fully developed thermal (i.e. after reaching constant circulation) will now be presented. It will be shown that both structures keep evolving despite the self-similarity of the gross behaviour. At the same time, the vortex core region is found to develop in a relatively more self-similar manner.

To examine the internal structures, radial profiles are taken at z_{core} , and the resulting profiles are plotted in figure 2 and compared at various distances (i.e. z_{core}/D_0) from the source. As the ensemble-averaged profiles are roughly symmetrical about the centreline of release, the data are averages of the left and right cores. The horizontal axis starts from the axis of symmetry and is normalized by the ring radius (r_{Ring}). Continuing the trend in the developing stage (Bond & Johari 2010), the vorticity of a fully developed thermal is found to be confined within a thin core region as shown in figure 2(a), and the normalized core region appears to be shrinking with the travel distance. Similar results are observed for the density profiles as shown in figure 2(b).

FIGURE 2. Normalized radial profiles passing through the vortex core: $r = 0$ at the axis of symmetry: (a) vorticity and (b) density.

FIGURE 3. Normalized profiles within the vortex core region: (a) vorticity and (b) density.

However, the density profile appears more concentrated at the centre of the core and decays much faster in the radial direction. Irrotational buoyant fluid remains in the inner region, which leads to the skewness of the density profile.

Another view of self-similarity focuses on the vortex core region. The core radius (r_{core}) was defined by measuring the distance from the centre of the vortex core in the radial direction towards the outer region (as shown in figure 2) until it reached a certain threshold vorticity/density contour, and it was determined separately for the vorticity and density fields. Thresholds of 10, 20 and 30 % of the peak vorticity/density were tested and a 20 % threshold was adopted since it yielded the best agreement between the core radii of the vorticity- and density-containing regions. The same data, but only that from the vicinity of the vortex core, are plotted in figure 3. The horizontal axis is centred at the core and normalized by the core radius r_{core} .

As the miscible thermal develops, the vorticity profile (figure 3a) tends to expand outwards slightly, but the variation can be considered small. A Gaussian function

(following the format of Bond & Johari 2010),

$$\frac{\omega}{\omega_{peak}} = \exp \left\{ - \left[\frac{(r - r_{Ring})/r_{core}}{c_1} \right]^2 \right\} \quad (3.2)$$

where $c_1 = 0.767$, provides a good fit to the mean vorticity profile within the vortex core. For density profiles, a sech function was chosen to fit the average profile, as

$$\frac{\Delta\rho}{\rho_{peak}} = \text{sech} \left\{ \left[\frac{(r - r_{Ring})/r_{core}}{c_2} \right]^{c_3} \right\} \quad (3.3)$$

where $c_2 = 0.0478$ and $c_3 = 0.0705$. Although the actual density profile is a little skewed, the fitting function still provides a reasonable description within the vortex core region.

3.3. Generation of vorticity by baroclinicity

With measurements of both internal vorticity and density structure available, further understanding of the effect of internal baroclinicity on vorticity transformation can be pursued. With the Boussinesq approximation, the conservation of momentum can be written as follows:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \left(\mathbf{g} - \frac{\nabla p}{\rho_a} \right) + \mathbf{g} \tilde{\rho} + \nu \nabla^2 \mathbf{u}, \quad (3.4)$$

where $\tilde{\rho} = (\rho - \rho_a) / \rho_a$ is the non-dimensionalized density difference, p is the hydrodynamic pressure, and ν is the kinematic viscosity. As the motion of a miscible thermal is dominated by buoyancy excess (B_0), characteristic length, velocity and time scales can be defined as

$$L_{MT} = (B_0/g_z)^{1/3}, \quad U_{MT} = B_0^{1/6} g_z^{1/3}, \quad T_{MT} = B_0^{1/6} / g_z^{2/3}, \quad (3.5)$$

where ‘MT’ stands for ‘miscible thermals’. Substituting the initial conditions, $L_{MT} = 0.771$ cm, $U_{MT} = 27.5$ cm s⁻¹, $T_{MT} = 0.028$ s, it should be noted that the various scales are independent of the release geometry. This is valid when the initial aspect ratio (L_0/D_0) of release is relatively low. In the case of a high initial aspect ratio, however, the buoyant fluid may behave like a starting plume (Bond & Johari 2005). With these characteristic scales, various quantities can be non-dimensionalized as

$$\tilde{\mathbf{u}} = \frac{\mathbf{u}}{U_{MT}}, \quad \tilde{t} = \frac{t}{T_{MT}}, \quad \tilde{\mathbf{g}} = \frac{\mathbf{g}}{g_z}, \quad \tilde{\nabla} = L_{MT} \nabla, \quad \tilde{\nabla}^2 = L_{MT}^2 \nabla^2, \quad \tilde{p} = \frac{p}{\rho_a U_{MT}^2}. \quad (3.6)$$

Substituting into (3.4) and dividing both sides by ($U_{MT}^2/L_{MT} = g_z$) yields

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla}) \tilde{\mathbf{u}} = (\tilde{\mathbf{g}} - \tilde{\nabla} \tilde{p}) + \tilde{\mathbf{g}} \tilde{\rho} + \frac{1}{Re} \tilde{\nabla}^2 \tilde{\mathbf{u}}, \quad (3.7)$$

where $Re = (U_{MT} L_{MT}) / \nu = B_0^{1/2} / \nu$ is the characteristic Reynolds number. Adopting the kinematic viscosity of water, the Reynolds number for the present study can be calculated to be around 2100, and thus the thermal should be turbulent. Based on the same definition, the Re examined by Scorer (1957) ranged from 3100 to 8100; and $Re \approx 2200$ for Bond & Johari (2005)’s experiments with $L_0/D_0 = 2$.

FIGURE 4. Effect of baroclinicity: (a) production and destruction of circulation due to baroclinicity; and (b) rate of change of the volume integral of vorticity in the head vortex region. An error bar is plotted to represent the typical experimental variation.

By taking the curl of (3.7), the non-dimensionalized vorticity equation for incompressible flow ($\nabla \cdot \mathbf{u} = 0$) can be expressed as

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} + (\tilde{\mathbf{u}} \cdot \tilde{\nabla}) \tilde{\omega} = (\tilde{\omega} \cdot \tilde{\nabla}) \tilde{\mathbf{u}} + \tilde{\nabla} \tilde{\rho} \times \tilde{\mathbf{g}} + \frac{1}{Re} \tilde{\nabla}^2 \tilde{\omega}. \quad (3.8)$$

On the right-hand-side of (3.8), the first term represents the intensification of vorticity due to vortex stretching, which should be zero for an axisymmetric vortex ring with only an azimuthal vorticity component and zero swirl. The second term represents the generation of vorticity by baroclinic torque due to the radial component of density gradient. Finally, the last term is the diffusion of vorticity due to viscosity, which should be negligible for turbulent buoyant vortex rings. Therefore, for an axisymmetric turbulent miscible thermal, the non-dimensionalized vorticity equation becomes

$$\frac{D \tilde{\omega}}{D \tilde{t}} = \tilde{\nabla} \tilde{\rho} \times \tilde{\mathbf{g}}. \quad (3.9)$$

Taking the volume integral on both sides, the above equation can be rewritten in cylindrical coordinates (r, θ, z) as

$$\int_V \frac{D \tilde{\omega}_\theta}{D \tilde{t}} d\tilde{V} = \int_V \left(\frac{\partial \tilde{\rho}}{\partial \tilde{r}} g_z \right) d\tilde{V}. \quad (3.10)$$

Equation (3.10) suggests that only the radial density gradient is responsible for the generation of baroclinic vorticity, which can be illustrated by the schematic diagram in figure 4(a). For a miscible thermal/buoyant vortex ring with two concentrated cores, the density gradient is positive in the inner region between the centreline of symmetry and the vortex core, and negative within the outer region beyond the core. Hence, the outer region of the buoyant vortex contributes to the production of the vorticity, whereas the inner region contributes to the destruction. As the newly generated vorticity is always in the outer region, the centre of rotation keeps shifting outwards and the density field rotates around the new centre. This explains the growth of the radius of buoyant vortex rings as compared to the constant radius in the non-buoyant cases.

With PIV and PLIF measurements, the two sides of (3.10) can be estimated independently. Note that vorticity in the tail region was excluded since it can be attributed to the release conditions, and the various quantities were only calculated within the head vortex region (i.e. $z_{core} - 0.6r_{max}$ to $z_{core} + 0.8r_{max}$). Starting from the right-hand side of (3.10), the rate of change of vorticity due to baroclinicity at each time frame is the cross-product of gravity and the radial density gradient (derived from PLIF measurements). Assuming axisymmetry, the value on each pixel of the two-dimensional map represents an annular volume in three dimensions. The volume integral of the rate of change of baroclinic vorticity within the head vortex is thus

$$\int_V \left(\frac{\partial \tilde{\rho}}{\partial \tilde{r}} \tilde{g}_z \right) d\tilde{V} = 2\pi \int_{\tilde{z}_{core}-0.6\tilde{r}_{max}}^{\tilde{z}_{core}+0.8\tilde{r}_{max}} \int_0^\infty \left(\frac{\partial \tilde{\rho}}{\partial \tilde{r}} \tilde{g}_z \right) \tilde{r} d\tilde{r} d\tilde{z}. \quad (3.11)$$

During the calculation, the integration in the radial direction was performed up to the boundary of the field of view, within which the thermals were well contained. As illustrated in figure 4(a), radial baroclinicity is responsible for the production and destruction of vorticity in the outer and inner regions, respectively. With (3.11), the positively and negatively generated vorticity are summed up separately and plotted in figure 4(b). The rate of baroclinic vorticity destruction is constant in the inner region, while the rate of baroclinic vorticity production in the outer region decreases linearly with time, which results in a net production of vorticity. The net production rate of the volume integral is also found by linear fitting,

$$\int_V \left(\frac{\partial \tilde{\rho}}{\partial \tilde{r}} \tilde{g}_z \right) d\tilde{V} \approx -1.45 \times 10^{-3} (\tilde{t}) + 0.457. \quad (3.12)$$

If the trends in figure 4(b) are extrapolated, the production rate will decrease further until it reaches the level of constant destruction rate, at which point the net vorticity production will become zero. It is unclear whether the buoyant vortex ring will break down before or upon reaching this point.

The left-hand side of (3.10) can be quantified based on the PIV tests, during which the vorticity field (ω_θ) was measured. The volume integral of ω_θ was calculated similarly inside the moving control volume, and the data were fitted by a second-order polynomial:

$$\begin{aligned} \int_V \tilde{\omega}_\theta d\tilde{V} &= 2\pi \int_{\tilde{z}_{core}-0.6\tilde{r}_{max}}^{\tilde{z}_{core}+0.8\tilde{r}_{max}} \int_0^\infty \tilde{\omega}_\theta \tilde{r} d\tilde{r} d\tilde{z} \\ &\approx -5.68 \times 10^{-4} (\tilde{t})^2 + 0.371 (\tilde{t}) - 8.17. \end{aligned} \quad (3.13)$$

Taking the time derivative of the above equation, the left-hand side of (3.10) can be expressed by the following linear relationship:

$$\int_V \left(\frac{D\tilde{\omega}_\theta}{D\tilde{t}} \right) d\tilde{V} \approx -1.14 \times 10^{-3} (\tilde{t}) + 0.371, \quad (3.14)$$

which is close to and plotted alongside (3.12) in figure 4(b). Certain variation should be expected as the viscous effect was neglected, and the results were derived based on PIV and PLIF measurements that were performed separately. The difference in decay rates may also be explained by the fact that only a portion of the buoyant force was responsible for generating vortical motion (Turner 1972). Therefore, the comparison between the total and baroclinically generated vorticity confirms quantitatively that

FIGURE 5. (a) Peak density excess at vortex core; and (b) gross characteristics.

radial baroclinicity is the major source of vorticity production within a miscible thermal.

3.4. Consideration on peak concentration

In the environmental assessment of pollutant discharge, it is important to consider the dilution of both mean and peak concentrations (Fischer *et al.* 1979), which should correlate with the mean and peak excess density in the Boussinesq range. The peak density excess can be identified from the PLIF measurements. It is plotted against time in figure 5(a) and found to follow the power-law

$$\frac{\Delta\rho_{peak}}{\rho_a} \propto (\tilde{t} - \tilde{t}_0)^{-0.84}. \quad (3.15)$$

If the bulk size of the thermal grows in a self-similar manner, and the total buoyancy excess remains constant, the mean excess density should vary temporally according to the following relationship:

$$\frac{\Delta\rho}{\rho_a} \propto V^{-1} \propto r^{-3} \propto (t - t_0)^{-3/2}. \quad (3.16)$$

In order to test the above theoretical relationship, the development of two gross characteristics (z_{core} and r_{Ring}) are examined by plotting them against time in figure 5(b). The following power-law relationships were found through linear curve fittings:

$$z_{core} \propto t^{0.47}, \quad r_{Ring} \propto t^{0.47} \quad (3.17)$$

and, within experimental uncertainties, agree well with the theoretical predictions (equation (1.1)). Therefore, the mean density excess within the thermal should vary roughly according to (3.16). Due to the self-similar gross behaviour, the virtual origins (z_0 and t_0) can be located through extrapolating the linear growth paths in space and time (r_{Ring} versus z_{core} and r_{Ring}^2 versus t , respectively) backward to the virtual point source where the radius is zero (Zhao *et al.* 2012). In our setup, it was found that $z_0 = -1.22$ cm and $t_0 = 1.11$ s (or $\tilde{z}_0 = -1.58$ and $\tilde{t}_0 = 39.8$ in non-dimensional form).

Comparing (3.15) and (3.16), the peak density excess at the vortex core is decreasing at a much slower rate than the mean, which also suggests theoretically

that self-similarity does not hold. The core region appears to be protected from erosion by the ambient fluid, and diffuses at a slower rate than the bulk region. Thus, an important finding from the present study is that the maximum pollutant concentration inside the thermal can remain high while the overall thermal is diluted.

4. Conclusions

Experiments have investigated the internal vorticity and density structures within a fully developed thermal. A large number of experimental runs with the same conditions were conducted with advanced imaging techniques, and the results were filtered to obtain representative ensemble-averaged data for the test conditions (with initial density excess of 5% and Reynolds number around 2100). The internal vorticity and density structures of the overall thermal were both found to evolve non-similarly. However, the development inside the vortex core region was nearly self-similar. The radial baroclinicity was found to be responsible for both production and destruction of vorticity inside the miscible thermal. In addition, the classic assumption of constant circulation within a buoyant vortex ring made by Turner (1957) was fully validated. The significance of the slower decay of peak concentration should also be noted for environmental assessment during pollutant discharge in the form of miscible thermals. The empirical relationships were non-dimensionalized by the only dominating parameter, total buoyancy excess. Therefore, the fitting coefficients obtained should be applicable to Boussinesq thermals in general. For full verification, more source conditions should be investigated. At the same time, the experimental results reported here can provide the basis for further theoretical or numerical studies.

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