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Threshold optimization of cooperative spectrum sensing in cognitive radio networks

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[1] We consider the threshold optimization problem of cooperative spectrum sensing for cognitive radio (CR) and investigate the threshold optimization algorithm for both single-channel and multichannel cooperative spectrum sensing. In order to obtain the optimal threshold for single-channel cooperative spectrum sensing, we deploy the fusion rules AND Logic, OR Logic, and K-OUT-N Logic. Moreover, an iterative optimization algorithm is proposed to obtain optimal CRs in cooperative spectrum sensing and their optimal thresholds. In multichannel cooperative spectrum sensing, two threshold optimization methods—namely nonrestrained multichannel threshold optimization (NRMTO) and restrained multichannel threshold optimization (RMTO)—have been proposed in order to decrease the total error detection probability of all the subchannels. The simulation results show that in single-channel cooperative spectrum sensing the proposed algorithm outperforms traditional cooperative spectrum sensing with the uniform threshold if the SNR is different, while decreasing the detection performance slightly if the SNR is identical. The results also indicate that the NRMTO can achieve the minimal total error detection probability of multichannel cooperative spectrum sensing, while the RMTO can guarantee the detection performance of each subchannel but with a higher total error detection probability.

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1. Introduction

[2] Wireless networks today are characterized by a fixed spectrum allocation strategy. With the rapid increasing demand for wireless communications and the limited availability of frequency resources, the Federal Communications Commission (FCC) has decided to make a paradigm shift by allowing more unlicensed users to transmit their signals in the bands licensed to primary users (PUs) so as to efficiently improve spectrum utilization [Politis, 2009; Weiss and Jondral, 2004]. The motivating factor behind this decision is a measurement by FCC which has shown that 70% of the allocated spectrum in the United States has not been well utilized [Krenik and Batra, 2005]. Cognitive radio (CR) is proposed as an efficient spectrum sharing technique for unlicensed users [Akyildiz et al., 2006].

[3] CR can smartly sense and adapt for the changing environment by altering its transmitting parameters, such as

modulation, frequency, and frame format [Mitola, 1999]. The main challenge is that CR should sense the PU signal exactly in order to avoid disturbing the PU [Cabric et al., 2006]. Energy detection is widely used by CR due to its simple implementation and unnecessary signal knowledge; however, its performance may decrease when the signal is in the shadowing and fading [Zhang et al., 2009]. Cooperative spectrum sensing is proposed to solve this problem, where each CR senses the spectrum by energy detection independently, and the fusion center combines the detection results from all the CRs in order to obtain the final decision on the presence of the PU [Liu and Tan, 2012]. If the local detection results of the CRs are binary decisions (0/1), they are combined by hard-decision rules such as AND Logic, OR Logic, and K-OUT-N Logic; if the local detection results are the energy statistics, they are combined by soft decision [Letaief and Zhang, 2009].

[4] An iterative threshold selection scheme for cooperative spectrum sensing with OR Logic was proposed in Teo et al. [2010], which significantly outperformed the conventional spectrum sensing with the uniform threshold in terms of the error detection probability (mean of false alarm probability and misdetection probability); however, the cooperative spectrum sensing with AND Logic and K-OUT-N Logic was not considered. A fast and accurate threshold searching method was proposed in Liu et al. [2010], which minimizes the error detection probability if the SNR was identical; however, its performance might decrease if the SNR was different. The optimization algorithms in Teo et al. [2010] and Liu et al. [2010] were both about single-channel

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cooperative spectrum sensing. The optimal multichannel cooperative spectrum sensing was investigated in *Quan et al.* [2009] and *Michele and Maurizio* [2011], maximizing the total throughput of all the subchannels; however, how to decrease the multichannel error detection probability was still a problem.

[5] In this paper, the detection threshold is optimized in order to minimize both the error detection probabilities of single-channel and multichannel cooperative spectrum sensing. In single-channel cooperative spectrum sensing, the iterative optimal thresholds with AND Logic, OR Logic, and K-OUT-N Logic are respectively proposed, which can greatly decrease the error detection probability if the SNR is different. In multichannel cooperative spectrum sensing, the nonrestrained multichannel threshold optimization (NRMTO) and the restrained multichannel threshold optimization (RMTO) are proposed. The NRMTO can achieve the minimal total error detection probability, while the RMTO can guarantee the false alarm and misdetection probabilities of each subchannel but with a higher total error detection probability.

[6] The rest of this paper is structured as follows. In section 2, the energy detection and its optimal threshold are analyzed. In section 3, single-channel cooperative spectrum sensing is described, and the iterative optimal thresholds with different hard fusion rules are described. In section 4, multichannel-channel cooperative spectrum sensing with soft decision is analyzed, and the two threshold optimization methods, namely NRMTO and RMTO, are proposed. The simulation results and relevant discussions are given in section 5. Finally, we conclude the work in section 6.

2. Energy Detection

[7] Many spectrum sensing methods have been proposed for CR, such as matching filter detection, cyclic feature detection, and energy detection [Letaief and Zhang, 2009]. Energy detection is widely used when CR cannot obtain any signal knowledge.

2.1. Received Signal Model

[8] Considering a CR network consisting of N users and one fusion center, the received signal of CR_i is given by the binary assumption in equation (1), where the hypotheses H_0 and H_1 denote the absence and presence of the PU, respectively.

$$\begin{aligned} H_0 : x_i(m) &= v(m), & i &= 1, 2, \dots, N \\ H_1 : x_i(m) &= h_i s(m) + v(m), & m &= 1, 2, \dots, M \end{aligned} \quad (1)$$

where $x_i(m)$ is the received signal of CR_i , M is the sampling number, $s(m)$ is the PU signal with zero mean and variance σ_s^2 , $v(m)$ is Gaussian noise with zero mean and variance σ_v^2 , and h_i is the channel gain between PU and CR_i . We also assume that signals $s(m)$ and $v(m)$ are completely independent.

2.2. Energy Detection Model

[9] Matching filter detection must know signal characteristics such as modulation type, signal wave, and frame

structure. Cyclic feature detection is used only to detect the periodic signal such as cosine signal, where the cyclic power spectrum should be also known before detection. In addition, cyclic feature detection needs to search for the circular frequency, and therefore longer observed time and larger complexity are needed. Hence in this paper we use energy detector for single-user detection due to its unnecessary signal knowledge and low complexity. In energy detection, the detection result can be obtained by comparing the energy statistic of the PU signal with a threshold, as shown in Figure 1.

[10] In Figure 1, the input signal $x(t)$ is filtered by a band-pass filter in order to limit the noise and select the interested bandwidth, and the output noise has a band-limited flat spectral density. In order to detect the presence of the PU, the energy statistic is compared with a settled threshold, which is obtained by squaring and integrating the output sampling over the observed interval [Shent et al., 2008]. The energy statistic of CR_i is given by

$$T_i = \frac{1}{M} \sum_{m=1}^M |x_i(m)|^2 \quad i = 1, 2, \dots, N, \quad (2)$$

where M is the sampling number. Comparing T_i with the threshold λ_i , the activity of the PU can be estimated by

$$\begin{cases} H_1 : T_i \geq \lambda_i \\ H_0 : T_i < \lambda_i \end{cases} \quad (3)$$

[11] In the AWGN channel, T_i obeys the distributions as follows:

$$T_i \sim \begin{cases} \chi_M^2, & H_0 \\ \chi_M^2(2\gamma_i), & H_1 \end{cases}, \quad (4)$$

where χ_M^2 is the chi-square distribution with M degree, and $\gamma_i = h_i^2 \sigma_s^2 / \sigma_v^2$ is the received SNR.

[12] If $M \geq 100$, according to the central limit theorem, χ_M^2 approximates to obey Gaussian distribution. The mean $u_{i,j}$ and variance $\sigma_{i,j}^2$ of T_i under the hypothesis H_j for $j = 0, 1$ are respectively calculated as follows:

$$H_0 \begin{cases} u_{i,0} = \sigma_v^2 \\ \sigma_{i,0}^2 = \frac{2}{M} \sigma_v^4 \end{cases}, H_1 \begin{cases} u_{i,1} = (1 + \gamma_i) \sigma_v^2 \\ \sigma_{i,1}^2 = \frac{2}{M} (2\gamma_i + 1) \sigma_v^4 \end{cases}. \quad (5)$$

[13] The false alarm probability, the detection probability, and the misdetection probability of CR_i are respectively given by

$$\begin{cases} P_{f,i} = Q\left(\frac{\lambda_i - u_{i,H_0}}{\sigma_{i,H_0}}\right) = Q\left(\left(\frac{\lambda_i}{\sigma_v^2} - 1\right) \sqrt{\frac{M}{2}}\right) \\ P_{d,i} = Q\left(\frac{\lambda_i - u_{i,H_1}}{\sigma_{i,H_1}}\right) = Q\left(\left(\frac{\lambda_i}{\sigma_v^2} - r_i - 1\right) \sqrt{\frac{M}{4r_i + 2}}\right) \\ P_{m,i} = 1 - P_{d,i} \end{cases}, \quad (6)$$

where the function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} \exp\left(-\frac{t^2}{2}\right) dt$.

Figure 1. Energy detection model.

2.3. Threshold Optimization of Energy Detection

[14] The false alarm happens when CR falsely detects the presence of the PU, while the misdetection happens when CR mistakenly determines the absence of the PU. The false alarm and misdetection probabilities respectively reflect the spectrum utilization of CR and the interference level to PU, and therefore the goal of the threshold optimization is to decrease both the false alarm and misdetection probabilities. For describing this problem, we define the error detection probability of CR_{*i*} as follows:

$$P_{e,i} = P(H_0)P_{f,i} + P(H_1)P_{m,i}, \quad (7)$$

where $P(H_0)$ and $P(H_1)$ respectively denote the appearance probabilities of H_0 and H_1 . The conventional method [e.g., Quan *et al.*, 2008] is to maximize the detection probability (minimize the misdetection probability) subject to the constraint of the false alarm probability, which is given by

$$\lambda_i^* = \arg \max_{\lambda_i} P_{d,i}, \quad \text{subject to } P_{f,i} \leq \alpha. \quad (8)$$

[15] However, in this paper, we seek to minimize the error detection probability. Supposing $P(H_0)/P(H_1) = \eta$, the optimal threshold λ_i^* is obtained by

$$\lambda_i^* = \arg \min_{\lambda_i} (P_{e,i} = (\eta P_{f,i} + P_{m,i})(P_{H_1})). \quad (9)$$

[16] From equation (6), we have $P_{m,i} = 1 - P_{d,i}$ and derive equation (9) as follows:

$$\begin{aligned} \lambda_i^* &= \arg \max_{\lambda_i} (-P_{e,i} = (P_{d,i} - \eta P_{f,i} - 1)(P_{H_1})) \\ &= \arg \max_{\lambda_i} (P_{d,i} - \eta P_{f,i}) \end{aligned} \quad (10)$$

[17] By the Lagrange theorem, λ_i^* is obtained by

$$\left. \frac{\partial P_{d,i}}{\partial \lambda_i} \right|_{\lambda_i^*} = \eta \left. \frac{\partial P_{f,i}}{\partial \lambda_i} \right|_{\lambda_i^*}, \quad (11)$$

where we derive λ_i^* as follows:

$$\lambda_i^* = \frac{\sigma_v^2}{2} + \sigma_v^2 \sqrt{\frac{1}{4} + \frac{r_i}{2} + \frac{4r_i + 2}{Mr_i} \ln(\eta \sqrt{2r_i + 1})}. \quad (12)$$

[18] According to equation (7), $P(H_0)$ and $P(H_1)$ can be seen as the weights of $P_{f,i}$ and $P_{m,i}$. By equations (9) and (10), $\eta P_{f,i}$ and $P_{m,i}(P_{d,i})$ have the same optimization gradient; that is, $\nabla \eta P_{f,i} = \nabla P_{m,i}$, where we have $\nabla P_{f,i} = \nabla P_{m,i}/\eta$. Hence, with $\eta \gg 1$, $\nabla P_{f,i} \ll \nabla P_{m,i}$ where $P_{f,i}$ is primarily optimized, while with $\eta \ll 1$, $\nabla P_{f,i} \gg \nabla P_{m,i}$ where $P_{m,i}$ is primarily optimized. With $\eta \gg 1$, we have $P(H_0) \gg P(H_1)$, which denotes the higher absence probability of the PU, and CR should improve its spectrum utilization by decreasing the false alarm probability. However, with $\eta \ll 1$, we have $P(H_0) \ll P(H_1)$, which denotes the higher presence probability of the PU, and CR has to reduce the interference to PU by decreasing the misdetection probability.

3. Cooperative Spectrum Sensing

[19] The performance of energy detection can be degraded in the fading and shadowing. Cooperative spectrum sensing

can solve this problem effectively by the collaborative detection of the multiple users locating in different areas.

3.1. Periodic Cooperative Spectrum Sensing

[20] In this section, we consider a cooperative spectrum sensing scenario where N CRs can be coordinated to enhance the performance of spectrum sensing as a whole. In cooperative spectrum sensing, each CR detects the PU by energy detection independently, as shown in Figure 2. Since the detection results of CR1 and CR3 are inaccurate because of the shadowing brought by the high buildings, we let the other CRs in a better environment, such as CR2 and CRN, help CR1 and CR3 for improving their detection performance by sharing sensing information. The fusion center functions as a base station, which combines the detection results from all the CRs to get the final decision on the presence of the PU by a specific fusion rule, such as AND Logic, OR Logic, or K-OUT-N Logic.

[21] In order to avoid disturbing the PU, we let these CRs sense the PU periodically, and the CRs must sense the PU before data transmission during each period, as shown in Figure 3. The sensing period includes cooperative spectrum sensing and data transmission, while the cooperative spectrum sensing includes local sensing and sensing information transmission. Each CR detects PU by energy detection independently during the local sensing, while sending its local sensing result to the fusion center in the allocated time slot through a public control channel during the sensing information transmission. The fusion center combines all the sensing information to achieve the final decision which is then broadcasted to each CR.

3.2. Threshold Optimization of Single-Channel Cooperative Spectrum Sensing

[22] In this section, N CRs detect the single channel cooperatively, and the hard decision is adopted by the fusion center. The optimal thresholds of cooperative spectrum sensing with AND Logic, OR Logic, and K-OUT-N Logic are analyzed as follows.

3.2.1. AND Logic

[23] In AND Logic, only if all the CRs detect the presence of the PU does the fusion center determine the presence of the PU, and otherwise it determines the absence of the PU. AND Logic can improve the spectrum utilization but may increase the interference to PU. The false alarm and

Figure 2. Cooperative spectrum sensing model.

Figure 3. Periodical cooperative spectrum sensing model.

detection probabilities of the AND-Logic cooperative sensing with n CRs are respectively given by

$$\begin{cases} Q_f^{(n)} = \prod_{i=1}^n P_{f,i} \\ Q_d^{(n)} = \prod_{i=1}^n P_{d,i} \end{cases}, n = 1, 2, \dots, N, \quad (13)$$

where $Q_f^{(n)}$ and $Q_d^{(n)}$ are derived as follows:

$$\begin{cases} Q_f^{(n)} = Q_f^{(n-1)} P_{f,n} \\ Q_d^{(n)} = Q_d^{(n-1)} P_{d,n} \end{cases}, \quad (14)$$

where $Q_f^{(0)} = Q_d^{(0)} = 1$. According to equations (11) and (14), the optimal threshold λ_n^* of CR n is obtained by

$$Q_d^{(n-1)} \frac{\partial P_{d,n}}{\partial \lambda_n} \Big|_{\lambda_n^*} = \eta Q_f^{(n-1)} \frac{\partial P_{f,n}}{\partial \lambda_n} \Big|_{\lambda_n^*}, \quad (15)$$

where λ_n^* is derived as follows:

$$\lambda_n^* = \frac{\sigma_v^2}{2} + \sigma_v^2 \sqrt{\frac{1}{4} + \frac{r_n}{2} + \frac{4r_n + 2}{Mr_n} \ln \left(\eta \sqrt{2r_n + 1} \frac{Q_f^{(n-1)}}{Q_d^{(n-1)}} \right)}, n = 1, 2, \dots, N. \quad (16)$$

[24] Since $Q_f^{(n-1)} \leq Q_d^{(n-1)}$, the iterative optimal threshold in equation (16) is usually smaller than the global optimal threshold in equation (12). That is because in AND Logic, the lower detection probability of cooperative spectrum sensing can be improved with the decreasing of the threshold.

[25] Besides calculating the optimal threshold of CR n , we also need to confirm if it is necessary for CR n to participate in the cooperative spectrum sensing. If CR n is feasible, according to equation (7), the error detection probability of cooperative spectrum sensing with n CRs $Q_e^{(n)} = P(H_0)Q_f^{(n)} + P(H_1)Q_d^{(n)}$ must satisfy $Q_e^{(n)} < Q_e^{(n-1)}$. The decrement of error detection probability in AND Logic is defined as follows:

$$\Delta^{(n)} = Q_e^{(n-1)} - Q_e^{(n)} = P(H_0)Q_f^{(n-1)}(1 - P_{f,n}) - P(H_1)Q_d^{(n-1)}P_{m,n}. \quad (17)$$

[26] If $\Delta^{(n)} > 0$, CR n can participate in the cooperative spectrum sensing, and otherwise it is not allowed.

3.2.2. OR Logic

[27] In OR Logic, in contrast with AND Logic, if at least one CR detects the presence of the PU, the fusion center determines the presence of the PU, and otherwise it determines the absence of the PU. OR Logic can decrease the interference to the PU, but may have lower spectrum utilization. The false alarm and detection probabilities of the OR-Logic cooperative spectrum sensing with n CRs are respectively given by

$$\begin{cases} Q_f^{(n)} = 1 - \prod_{i=1}^n (1 - P_{f,i}) \\ Q_d^{(n)} = 1 - \prod_{i=1}^n (1 - P_{d,i}) \end{cases}, n = 1, 2, \dots, N, \quad (18)$$

where $Q_f^{(n)}$ and $Q_d^{(n)}$ are derived as follows:

$$\begin{cases} Q_f^{(n)} = Q_f^{(n-1)} + (1 - Q_f^{(n-1)})P_{f,n} \\ Q_d^{(n)} = Q_d^{(n-1)} + (1 - Q_d^{(n-1)})P_{d,n} \end{cases}, \quad (19)$$

where $Q_f^{(0)} = Q_d^{(0)} = 0$. Like equation (16), the iterative optimal threshold of CR n in the OR-Logic cooperative spectrum sensing is given by

$$\lambda_n^* = \frac{\sigma_v^2}{2} + \sigma_v^2 \sqrt{\frac{1}{4} + \frac{r_n}{2} + \frac{4r_n + 2}{Mr_n} \ln \left(\eta \sqrt{2r_n + 1} \frac{1 - Q_f^{(n-1)}}{1 - Q_d^{(n-1)}} \right)}, \quad n = 1, 2, \dots, N. \quad (20)$$

[28] Since $Q_f^{(n-1)} \leq Q_d^{(n-1)}$, the iterative optimal threshold in equation (20) is usually larger than the global optimal threshold in equation (12). That is because in OR Logic, the higher false alarm probability of cooperative spectrum sensing can be decreased with the increasing of the threshold. The decrement of error detection probability in OR Logic is defined as follows:

$$\Delta^{(n)} = P(H_1)Q_m^{(n-1)}P_{d,n} - P(H_0)(1 - Q_f^{(n-1)})P_{f,n}. \quad (21)$$

3.2.3. K-OUT-N Logic

[29] In K-OUT-N Logic, if at least k CRs out of the N CRs detect the presence of the PU, the fusion center determines the presence of the PU, and otherwise it determines the absence of the PU. To the AND Logic, we have $k = N$, while to the OR Logic, we have $k = 1$. The detection performance of the K-OUT-N Logic is between those of the AND Logic and OR Logic. The false alarm and detection probabilities of K-OUT-N-Logic cooperative spectrum sensing with n CRs are respectively given by

$$\begin{cases} Q_f^{(k,n)} = \sum_{j=k}^n \binom{n}{j} \prod_{i=1}^j (P_{f,i}) \prod_{i=j+1}^n (1 - P_{f,i}) \\ Q_d^{(k,n)} = \sum_{j=k}^n \binom{n}{j} \prod_{i=1}^j (P_{d,i}) \prod_{i=j+1}^n (1 - P_{d,i}) \end{cases} \text{ for } n = 1, 2, \dots, N. \quad (22)$$

$Q_f^{(k,n)}$ and $Q_d^{(k,n)}$ can be derived from equation (22) as follows:

$$\begin{cases} Q_f^{(k,n)} = Q_f^{(k-1,n-1)} P_{f,n} + Q_f^{(k,n-1)} (1 - P_{f,n}) \\ Q_d^{(k,n)} = Q_d^{(k-1,n-1)} P_{d,n} + Q_d^{(k,n-1)} (1 - P_{d,n}) \end{cases}, \quad (23)$$

where $Q_f^{(0,n)} = Q_d^{(0,n)} = 1$, and $Q_f^{(k,n)} = Q_d^{(k,n)} = 0$ if $k > n$ (see Appendix A).

[30] Like equation (16), the iterative optimal threshold of CR n in K-OUT-N-Logic cooperative spectrum sensing is given by

$$\lambda_n^* = \frac{\sigma_v^2}{2} + \sigma_v^2 \sqrt{\frac{1}{4} + \frac{r_n}{2} + \frac{4r_n + 2}{Mr_n} \ln \left(\eta \sqrt{2r_n + 1} \frac{Q_f^{(k,n-1)} - Q_f^{(k-1,n-1)}}{Q_d^{(k,n-1)} - Q_d^{(k-1,n-1)}} \right)},$$

$$n = 1, 2, \dots, N \quad (24)$$

[31] Equation (24) equals equation (16) with $k = n$ while equaling equation (20) with $k = 1$. The decrement of error detection probability in K-OUT-N Logic is defined as follows:

$$\Delta^{(n)} = P(H_1)P_{d,n} (Q_d^{(k-1,n-1)} - Q_d^{(k,n-1)}) - P(H_0)P_{f,n} (Q_f^{(k-1,n-1)} - Q_f^{(k,n-1)}) \quad (25)$$

3.2.4. Threshold Optimization Algorithm

[32] We propose an iterative optimization algorithm in order to obtain the optimal CRs in cooperative spectrum sensing and their optimal thresholds. Since the optimization processes of the above three fusion rules are similar, we propose a uniform algorithm where the array of CRs is done in descending order of their SNRs, as shown in Table 1. Since all the CRs detect the same PU signal, the SNR comparison between CR i and CR j is given as follows:

$$\frac{SNR_i}{SNR_j} = \frac{h_i^2 \sigma_s^2 / \sigma_{v,i}^2}{h_j^2 \sigma_s^2 / \sigma_{v,j}^2} = \frac{h_i^2 / \sigma_{v,i}^2}{h_j^2 / \sigma_{v,j}^2} \quad (26)$$

where the channel gains h_i and h_j , and the noise variances $\sigma_{v,i}^2$ and $\sigma_{v,j}^2$ can be obtained by channel estimation without considering the power of the PU. Here we also give an easier measurement to obtain the array of CRs by comparing their energy statistics, and the measurement of CR i is given by

$$ME_i = \frac{T_i(H_1)}{T_i(H_0)}, \quad (27)$$

Table 1. Threshold optimization algorithm

Step	Threshold Optimization Algorithm
1	Initialize $n = 1$; set up a list of all the N CRs in descending order of their SNRs; the first CR in the list has the highest received SNR, and so on.
2	Calculate the optimal threshold of CR1 λ_1^* and the corresponding $P_{f,1}$, $P_{d,1}$, and $Q_e^{(1)}$ by equation (12); set $Q_f^{(1)} = P_{f,1}$, $Q_d^{(1)} = P_{d,1}$, and $n = n + 1$.
3	According to the adopted fusion rule, calculate the optimal threshold of CR n λ_n^* and the false alarm and detection probabilities $Q_f^{(n)}$ and $Q_d^{(n)}$ by equation (16), (20), or (24).
4	Calculate $\Delta^{(n)}$ by equation (17), (21), or (25).
5	If $\Delta^{(n)} > 0$, go to step 7; otherwise go to step 6.
6	Delete CR n from the list; set $n = n - 1$ and $N = N - 1$, and go to step 7.
7	Set $n = n + 1$; if $n < N$, repeat steps 3–6; otherwise go to step 8.
8	Output λ_n^* for $n = 1, 2, \dots, N$ and $Q_e^{(N)}$.

Where $T_i(H_1)$ and $T_i(H_0)$ are respectively the energy statistics under hypotheses H_1 and H_0 . Therefore the array of the CRs can also be done by comparing ME_i for $i = 1, 2, \dots, N$.

[33] The above algorithm can be implemented effectively if the SNR is different. In fact, if the SNR is identical, the detection thresholds of the N CRs should be same, and therefore we can search the uniform threshold by enumeration in order to obtain the minimal error detection probability. If the SNR is different, the detection thresholds should also be different, and therefore in order to obtain the optimal thresholds, we have to implement the N dimension searching with NP-hard complexity. However, by the proposed algorithm, we can easily find the optimal thresholds with only N iterations. In order to obtain the optimal thresholds, we should obey the following two rules.

1. If the SNR is identical, the thresholds should be same, and we search the uniform threshold by enumeration.
2. If the SNR is different, the thresholds should be different, and we use the proposed algorithm to obtain the optimal thresholds with N iterations.

4. Multichannel Cooperative Spectrum Sensing

[34] In this section, the threshold optimization of multichannel cooperative spectrum sensing is analyzed. Unlike the optimization of single-channel cooperative spectrum sensing, in multichannel cooperative spectrum sensing, the total detection performance of all the subchannels should be considered.

4.1. Nonrestrained Multichannel Threshold Optimization

[35] Supposing that there are L subchannels in the CR network and all the subchannels are independent, the energy statistic of CR i in subchannel l is calculated by

$$T_i^l = \frac{1}{M} \sum_{m=1}^M |x_i^l(m)|^2, \quad i = 1, 2, \dots, N; l = 1, 2, \dots, L, \quad (28)$$

[36] Where $x_i^l(m)$ is the received signal of CR i in subchannel l . The energy statistic T_i^l is fused by the fusion center, and the fused energy statistic in subchannel l is calculated by

$$T_{all}^l = \frac{1}{N} \sum_{i=1}^N T_i^l. \quad (29)$$

[37] By comparing the fused energy statistic with a threshold λ_l , the PU of subchannel l is estimated to be idle with $T_{all}^l < \lambda_l$ and busy with $T_{all}^l \geq \lambda_l$, and this process is referred to as soft-decisional cooperative spectrum sensing. The false alarm probability, the detection probability, and the misdetection probability of subchannel l are respectively given by

$$\begin{cases} Q_f^l = Q\left(\left(\frac{\lambda_l}{\sigma_v^2} - 1\right)\sqrt{\frac{MN}{2}}\right) \\ Q_d^l = Q\left(\left(\frac{\lambda_l}{\sigma_v^2} - \bar{\gamma}_l - 1\right)\sqrt{\frac{MN}{4\bar{\gamma}_l + 2}}\right) \\ Q_m^l = 1 - Q_d^l \end{cases}, \quad (30)$$

where $\bar{\gamma}_l$ is the average received SNR of the N CRs in subchannel l .

[38] Unlike single-channel cooperative spectrum sensing, multichannel cooperative spectrum sensing should consider the total detection performance of all the L subchannels. Like equation (9), a NRMTO method can be given by

$$[\lambda_1^*, \lambda_2^*, \dots, \lambda_L^*] = \arg \min_{\lambda_1, \lambda_2, \dots, \lambda_L} Q_{e, total} = \sum_{l=1}^L (\eta_l Q_f^l + Q_m^l) (P_{H_{1,l}}), \quad (31)$$

[39] Where $Q_{e, total}$ is the total error detection probability of L subchannels, and $\eta_l = P(H_{0,l})/P(H_{1,l})$, where $(P(H_{0,1}))$ and $P(H_{1,l})$ are respectively the appearance probabilities of H_0 and H_1 in subchannel l .

[40] The optimal threshold λ_l^* for $l = 1, 2, \dots, L$ can be obtained by equation (12); however, the NRMTO cannot guarantee the necessary detection performance of each subchannel, which is important for the spectrum access of CR.

4.2. Restrained Multichannel Threshold Optimization

[41] Another multichannel threshold optimization method RMTO seeks to minimize the total error detection probability subject to the constraints on the total misdetection probability and the false alarm and misdetection probabilities of each subchannel. The RMTO method is defined as follows:

$$\begin{aligned} & \min_{\lambda_1, \lambda_2, \dots, \lambda_L} Q_{e, total} \\ & s.t. \sum_{l=1}^L Q_m^l \leq \varepsilon \\ & \quad Q_f^l \leq \alpha_l \\ & \quad Q_m^l \leq \beta_l, l = 1, 2, \dots, L \end{aligned}, \quad (32)$$

[42] Where ε , α_l , and β_l are the constraint values. From equation (32), we have $Q_d^l \geq 1 - \beta_l$, and by equation (30), we can obtain the lower and upper bounds of the threshold λ_l as follows:

$$\begin{cases} \lambda_l^{low} = \left(\sqrt{\frac{2}{MN} Q^{-1}(\alpha_l) + 1}\right) \sigma_v^2 \\ \lambda_l^{up} = \left(\sqrt{\frac{4\bar{\gamma}_l + 2}{MN} Q^{-1}(1 - \beta_l) + \bar{\gamma}_l + 1}\right) \sigma_v^2 \end{cases}, l = 1, 2, \dots, L. \quad (33)$$

[43] Substituting equation (33) into equation (32), equation (32) is rewritten as follows:

$$\begin{aligned} & \min_{\lambda_1, \lambda_2, \dots, \lambda_L} \sum_{l=1}^L \eta_l Q_f^l \\ & s.t. - \sum_{l=1}^L Q_d^l \leq \varepsilon - L \\ & \quad \lambda_l^{low} \leq \lambda_l \leq \lambda_l^{up}, l = 1, 2, \dots, L. \end{aligned} \quad (34)$$

[44] Q_f^l and Q_d^l are respectively convex and concave in λ_l if $\alpha_l \leq 0.5$ and $\beta_l \leq 0.5$ (see Appendix B).

[45] Both the functions $S(\lambda) = \sum_{l=1}^L \eta_l Q_f^l$ and $G(\lambda) = -\sum_{l=1}^L Q_d^l$, where $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_L]$, are convex in λ if $\alpha_l \leq 0.5$ and $\beta_l \leq 0.5$ for $l = 1, 2, \dots, L$ (see Appendix C).

[46] If $\alpha_l \leq 0.5$ and $\beta_l \leq 0.5$ for $l = 1, 2, \dots, L$, the optimization problem takes the form of minimizing a convex function subject to a convex constraint, and thus a local optimum is also the global optimum [Quan et al., 2009]. Efficient numerical searching algorithm such as the interior-point method [Boyd and Vandenberghe, 2003] can be used to find the optimal solution of equation (34).

[47] Alternatively, we can formulate the RMTO method into another format that minimizes the total error detection probability subject to the constraints on the total false alarm probability and the false alarm and misdetection probabilities of each subchannel [Combeau et al., 2007]. The optimization problem is given by

$$\begin{aligned} & \min_{\lambda_1, \lambda_2, \dots, \lambda_L} Q_{e, total} \\ & s.t. \sum_{l=1}^L \eta_l Q_f^l \leq \varepsilon \\ & \quad Q_f^l \leq \alpha_l \\ & \quad Q_m^l \leq \beta_l, l = 1, 2, \dots, L \end{aligned}. \quad (35)$$

[48] Like equation (34), equation (35) is rewritten as follows:

$$\begin{aligned} & \min_{\lambda_1, \lambda_2, \dots, \lambda_L} - \sum_{l=1}^L Q_d^l \\ & s.t. \sum_{l=1}^L \eta_l Q_f^l \leq \varepsilon \\ & \quad \lambda_l^{low} \leq \lambda_l \leq \lambda_l^{up}, l = 1, 2, \dots, L \end{aligned}. \quad (36)$$

[49] The existence of the global optimum can be proved by using the same techniques as proving equation (34), and therefore the interior-point method can also be used to solve equation (36).

5. Simulation

[50] In this section, numerical results are presented to verify the effectiveness of our proposed algorithms. The model setup is as follows: the number of the samplings $M = 100$, the number of cooperative CRs $N = 6$, the noise variance $\sigma_v^2 = 1$, the number of subchannels $L = 6$, the appearance probabilities of $H_{0,l}$ and $H_{1,l}$ $P(H_{0,l}) = P(H_{1,l}) = 0.5$ for $l = 1, 2, \dots, L$, and the upper bounds of the false alarm and misdetection probabilities $\alpha_l = 0.1$ and $\beta_l = 0.4$ for $l = 1, 2, \dots, L$.

5.1. Single-Channel Cooperative Spectrum Sensing

[51] Figure 4 compares the error detection probabilities of proposed cooperative spectrum sensing with optimal threshold and traditional cooperative spectrum sensing with the uniform threshold, with AND Logic, OR Logic, and K-OUT-N Logic at the identical SNR $\gamma = -10$ dB. From Figure 4, it is seen that the error detection probability of the proposed algorithm (beeline) is slightly higher than the minimal error detection probability of the traditional algorithm (curve). That is because the threshold achieved by the proposed algorithm is the local optimum, which only minimizes the error detection probability of the current

iteration, and therefore the proposed algorithm may not yield a global minimum at the end of the iterative algorithm. However, the traditional algorithm can obtain a global optimum through enumeration; compared with the traditional algorithm, the error detection probability of the proposed algorithm increases less.

[52] Figure 5 compares the error detection probabilities at the different SNR $\gamma = [-7, -8, -10, -13, -17, -20]$ dB. From Figure 5, it is seen that the error detection probability of the proposed algorithm (beeline) is lower than that of the traditional algorithm with the uniform threshold (curve), and therefore the proposed algorithm is predominant if the SNR is different. That is because with different SNR, the traditional algorithm cannot obtain any global optimum; however, the local optimum obtained by the proposed algorithm is outstanding.

[53] Figure 6 shows the number of the cooperative CRs versus the different fusion rules in the proposed algorithm at different SNR. It is seen that with the increasing of the average SNR, fewer CRs are needed to hold the detection performance by the proposed algorithm, and also OR Logic needs fewer CRs than do AND Logic and K-OUT-N Logic.

[54] In the threshold optimization algorithm in Table 1, we array the CRs in descending order of their SNRs and choose the CRs with higher SNR to participate in the cooperative spectrum sensing because of their better detection performance. However, if we array the CRs in ascending order of their SNRs, we may achieve lower detection probability as shown in Figure 7, because the worse detection performance of the CR with lower SNR might decrease the total performance of cooperative spectrum sensing.

[55] Figure 8 compares the performance of the proposed algorithm that minimizes Q_e and the algorithm in *Quan et al.* [2008] that maximizes Q_d . We can see that the detection probability of the proposed algorithm is lower than that of the algorithm in *Quan et al.* [2008], as shown in Figure 8(a), while the error detection probability of the algorithm in *Quan et al.* [2008] is higher than that of the proposed algorithm, as shown in Figure 8(b). That is because the proposed algorithm can decrease both the false alarm and misdetection probabilities, while the algorithm in *Quan et al.* [2008] can only decrease the misdetection probability.

Figure 4. Error detection probability comparison at identical SNR.

Figure 5. Error detection probability comparison at different SNR.

Figure 6. Number of the cooperative CRs versus different fusion rules.

[56] Figures 9 and 10 respectively indicate the false alarm probability and the detection probability of each CR versus the different algorithms by OR Logic when the SNR is different. Compared with the algorithm with the uniform threshold, the proposed algorithm can greatly decrease the false alarm probability while slightly decreasing the detection probability. Hence the proposed algorithm can achieve better detection performance if the SNR is different.

5.2. Multichannel Cooperative Spectrum Sensing

[57] In this section, the average received SNRs of L subchannels are $\bar{\gamma} = [-3, -10, -4, -5, -6, -10]$ dB, and the proposed threshold optimization methods NRMTO and RMTO are compared and analyzed.

[58] The total error detection probability of L subchannels $Q_{e, total}$ is compared between NRMTO and RMTO in Figure 11, and it is seen that the NRMTO outperforms the RMTO. When the upper bound of the total misdetection probability $\varepsilon = 0.8$, the minimal $Q_{e, total}$ of the RMTO can be achieved, which is only 0.04 higher than that of the NRMTO.

[59] Figure 12 indicates the threshold comparison between NRMTO and RMTO at the lowest $Q_{e, total}$. We can see that the thresholds of the two methods are nearly identical, except those in channels 2 and 6. That is because the SNRs in channels 2 and 6 are lower, and the higher thresholds must be adopted by the RMTO in order to keep the false alarm probabilities of channels 2 and 6 within 0.1. Therefore in Figure 13, where the false alarm probability of each subchannel is shown, the false alarm probabilities in RMTO are all within the constraint, while those of channels 2 and 6 in NRMTO exceed the constraint. In Figure 14, where the detection probability of each subchannel is shown, though the detection probabilities of channels 2 and 6 in RMTO are lower than those in NRMTO, they are still above the constraint 0.6. Hence the detection performance of each subchannel can be guaranteed by RMTO.

6. Conclusion

[60] In this paper, we derive the optimal thresholds of the cooperative spectrum sensing with different fusion rules including AND Logic, OR Logic, and K-OUT-N Logic. An iterative optimization algorithm is proposed in order to obtain the optimal CRs in cooperative spectrum sensing and their optimal thresholds. The proposed algorithm can achieve better detection performance if the SNR is different, while slightly

decreasing the detection performance if the SNR is identical. Simulation results have shown that, among the three fusion rules, the OR Logic can achieve the lowest error detection probability with fewer cooperative CRs, while the AND Logic may yield the highest error detection probability with more cooperative CRs. The results also indicate that the proposed algorithm should choose the CRs with higher SNRs for cooperative spectrum sensing because of their better detection performance, and compared with the traditional algorithm the proposed algorithm can yield lower error detection probability. The NRMTO and RMTO methods are proposed to decrease the total error detection probability of the multichannel cooperative spectrum sensing. The NRMTO can achieve the minimal total error detection probability, while the RMTO

Figure 7. Detection probability with different array orders.

Figure 9. False alarm probability comparison of each CR.

Figure 10. Detection probability comparison of each CR.

(a) Detection probability comparison

(b) Error detection probability comparison

Figure 8. Performance comparison between the proposed algorithm and the algorithm in *Quan et al.* [2008]. (a) Detection probability comparison. (b) Error detection probability comparison.

Figure 11. Total error detection probability comparison.

Figure 12. Threshold comparison between NRMTO and RMTO.

Figure 13. False alarm probability comparison between NRMTO and RMTO.

can guarantee the detection performance of each subchannel but with a higher total error detection probability. Our future work is to find an optimal subset of the CRs in order to obtain better detection performance with fewer cooperative CRs.

Appendix A Proof of Lemma 1

[61] We suppose that the PU is really absent and the false alarm is generated by the fusion center. If CR n has detected the presence of the PU, in order to generate the false alarm, at least $k - 1$ CRs out of the other $n - 1$ CRs should detect the

Figure 14. Detection probability comparison between NRMTO and RMTO.

presence of the PU. While, if CR n has detected the absence of the PU, in order to generate the false alarm, at least k CRs out of the other $n - 1$ CRs should detect the presence of the PU. Hence we have

$$\begin{aligned} Q_f^{(k,n)} &= \sum_{j=k}^n \binom{n}{j} \prod_{i=1}^j P_{f,i} \prod_{i=j+1}^n (1 - P_{f,i}) = \sum_{j=k-1}^{n-1} \binom{n-1}{j} \prod_{i=1}^j P_{f,i} \\ &\quad \times P_{f,n} \prod_{i=j+1}^{n-1} (1 - P_{f,i}) + \sum_{j=k}^{n-1} \binom{n-1}{j} \prod_{i=1}^j P_{f,i} \prod_{i=j+1}^{n-1} (1 - P_{f,i}) \cdot \\ &\quad \times (1 - P_{f,n}) = Q_f^{(k-1,n-1)} P_{f,n} + Q_f^{(k,n-1)} (1 - P_{f,n}) \end{aligned} \quad (A1)$$

[62] Like equation (A1), we also can prove $Q_d^{(k,n)}$ defined in equation (23).

Appendix B Proof of Lemma 2

[63] Derive the second derivatives of Q_f^l and Q_d^l from equation (30) as follows:

$$\begin{cases} \frac{\partial^2 Q_f^l}{\partial^2 \lambda_l} = \frac{(\lambda_l - \sigma_v^2)(MN)^{1.5}}{4\sqrt{\pi}\sigma_v^6} \exp\left(-\frac{(\lambda_l - \sigma_v^2)^2 MN}{4\sigma_v^4}\right) \\ \frac{\partial^2 Q_d^l}{\partial^2 \lambda_l} = \frac{(\lambda_l - (\tilde{\gamma}_l + 1)\sigma_v^2)(MN)^{1.5}}{(8\tilde{\gamma}_l + 4)\sqrt{(2\tilde{\gamma}_l + 1)\pi}\sigma_v^6} \exp\left(-\frac{(\lambda_l - (\tilde{\gamma}_l + 1)\sigma_v^2)^2 MN}{(8\tilde{\gamma}_l + 4)\sigma_v^4}\right) \end{cases} \quad (B1)$$

[64] Since $\alpha_l \leq 0.5$ and $\beta_l \leq 0.5$, by equation (33) we have

$$\sigma_v^2 \leq \lambda_l \leq (\tilde{\gamma}_l + 1)\sigma_v^2. \quad (B2)$$

[65] By substituting equation (B2) into equation (B1), we have $\partial^2 Q_f^l / \partial^2 \lambda_l \geq 0$ and $\partial^2 Q_d^l / \partial^2 \lambda_l \leq 0$. Hence Q_f^l and Q_d^l are respectively convex and concave in λ_l .

Appendix C Proof of Lemma 3

[66] We define the two vectors $\mathbf{\lambda}_a = [\lambda_{a1}, \lambda_{a2}, \dots, \lambda_{aL}]$ and $\mathbf{\lambda}_b = [\lambda_{b1}, \lambda_{b2}, \dots, \lambda_{bL}]$ that respectively satisfy $\lambda_l^{low} \leq \lambda_{al} \leq \lambda_l^{upp}$ and $\lambda_l^{low} \leq \lambda_{bl} \leq \lambda_l^{upp}$ for $l = 1, 2, \dots, L$. We also define a vector $\mathbf{\lambda}_c = [\lambda_{c1}, \lambda_{c2}, \dots, \lambda_{cL}]$ as follows:

$$\mathbf{\lambda}_c = \xi \mathbf{\lambda}_a + (1 - \xi) \mathbf{\lambda}_b, \quad (C1)$$

[67] Where $0 < \xi < 1$. We know that $\mathbf{\lambda}_c$ also satisfies $\lambda_l^{low} \leq \lambda_{cl} \leq \lambda_l^{upp}$ for $l = 1, 2, \dots, L$. By lemma 2 (see Appendix

B), we know that Q_f^l are convex in λ_{al} , λ_{bl} , and λ_{cl} , so we have

$$Q_f^l(\lambda_{cl}) = Q_f^l(\xi\lambda_{al} + (1 - \xi)\lambda_{bl}) \geq \xi Q_f^l(\lambda_{al}) + (1 - \xi)Q_f^l(\lambda_{bl}). \quad (C2)$$

[68] By equations (C1) and (C2), we can have

$$\begin{aligned} S(\mathbf{\lambda}_c) &= S(\xi\mathbf{\lambda}_a + (1 - \xi)\mathbf{\lambda}_b) \\ &= \sum_{l=1}^L \eta_l Q_f^l(\xi\lambda_{al} + (1 - \xi)\lambda_{bl}) \\ &\geq \sum_{l=1}^L \eta_l \left(\xi Q_f^l(\lambda_{al}) + (1 - \xi)Q_f^l(\lambda_{bl}) \right) \\ &= \xi \sum_{l=1}^L \eta_l Q_f^l(\lambda_{al}) + (1 - \xi) \sum_{l=1}^L \eta_l Q_f^l(\lambda_{bl}) \\ &= \xi S(\mathbf{\lambda}_a) + (1 - \xi)S(\mathbf{\lambda}_b) \end{aligned} \quad (C3)$$

which implies that $S(\lambda)$ is convex in λ . Since Q_d^l is concave, $-Q_d^l$ is convex and, like the above proof, we can also prove that $G(\lambda)$ is also convex.

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