Measurement errors induced by retardance deviation in a rotatable retarder fixed polarizer Stokes polarimeter

Dong, Hui; Shum, Ping; Gong, Yandong; Sun, Qizhen

2012

Dong, H., Shum, P., Gong, Y., & Sun, Q. Measurement errors induced by retardance deviation in a rotatable retarder fixed polarizer Stokes polarimeter. *Optical Engineering*, 51(3), 033001.

<https://hdl.handle.net/10356/101565>

<https://doi.org/10.1117/1.OE.51.3.033001>

© 2012 Society of Photo-Optical Instrumentation Engineers. This paper was published in *Optical Engineering* and is made available as an electronic reprint (preprint) with permission of Society of Photo-Optical Instrumentation Engineers. The paper can be found at the following official DOI: [<http://dx.doi.org/10.1117/1.OE.51.3.033001>]. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law.

Downloaded on 26 Apr 2023 20:17:17 SGT

Optical Engineering

SPIEDigitalLibrary.org/oe

Measurement errors induced by retardance deviation in a rotatable retarder fixed polarizer Stokes polarimeter

Hui Dong
Ping Shum
Yandong Gong
Qizhen Sun

Measurement errors induced by retardance deviation in a rotatable retarder fixed polarizer Stokes polarimeter

Hui Dong

Institute for Infocomm Research
RF & Optical Department
1 Fusionopolis Way, #21-01 Connexis
Singapore 138632, Singapore
E-mail: HDong@i2r.a-star.edu.sg

Ping Shum

Nanyang Technological University
OPTIMUS-Photonics Centre of Excellence, 4th
Storey
XFrontier Block, Research Techno Plaza, 50
Nanyang Drive
Singapore 637553, Singapore

Yandong Gong

Institute for Infocomm Research
RF & Optical Department
1 Fusionopolis Way, #21-01 Connexis
Singapore 138632, Singapore

Qizhen Sun

Huazhong University of Science and Technology
School of Optoelectronic Science and Engineering
Wuhan 430074, China

Abstract. The retardance of an achromatic retarder may vary in the working frequency range and therefore deviate from its nominal value. In a rotatable retarder and fixed polarizer (RRFP) Stokes polarimeter, the retardance deviation of the retarder will result in significant measurement errors of the Stokes vector. The relationship between the retardance error and the measurement errors is theoretically derived in a RRFP Stokes polarimeter. Theoretical results show that the resultant measurement errors of the Stokes vector are determined by the retardance of the retarder, the retardance error and the Stokes vector under test, but are independent of the angular orientations of the retarder and the intensity measurements. Such measurement errors can not be reduced by choosing a set of optimum angular orientations or increasing the measurement number. To suppress such measurement errors in a complete RRFP Stokes polarimeter, the retarder with retardance from 110-deg to 135-deg is recommended. Simulation results in two measurement scenarios confirm the validity of the theoretical findings. © 2012 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.OE.51.3.033001]

Subject terms: Stokes polarimeter; achromatic retarder; Stokes vector; terahertz.

Paper 111159 received Sep. 19, 2011; revised manuscript received Dec. 16, 2011; accepted for publication Jan. 5, 2012; published online Mar. 8, 2012.

1 Introduction

Measurement of the state of polarization (SOP) of an electromagnetic (EM) wave plays an important role in many applications, such as polarization mode dispersion measurements in optical fibers^{1,2} and birefringence and polarization-dependent loss measurements in terahertz (THz) spectroscopy.^{3,4} The SOP of an EM wave is completely depicted by the four Stokes parameters, and a Stokes polarimeter is just an optoelectronic equipment for measuring Stokes parameters.⁵ A type of commonly used Stokes polarimeter is constructed with a rotatable retarder (also termed waveplate), a fixed polarizer (RRFP) and a photodetector^{6,7} as illustrated in Fig. 1. By means of its simple and reliable structure, a RRFP Stokes polarimeter can be easily integrated in many optical systems, such as a polarization optical time domain reflectometry (POTDR) system or a terahertz frequency domain spectroscopy (THz-FDS) system.^{8,9}

A RRFP Stokes polarimeter determines the four Stokes parameters by rotating the retarder to M ($M \geq 4$) different angular orientations and taking N ($N \geq M$) intensity measurements using the photodetector.^{6,7} In such measurements, there are three possible error sources, which are first, the random fluctuations in the intensity measurements; second, the relative angular orientation errors away from the correct angular orientations; and third, the imperfections of the retarder and the polarizer.⁶ In the literature, the measurement

errors induced by the first two error sources, especially the first one, have been investigated.^{6,7,10} It has been demonstrated that first, some specific sets of the angular orientations, such as -45 -deg, 0 -deg, 30 -deg, 60 -deg, can effectively reduce the measurement errors induced by the first two error sources;⁶ and second, that a 132 -deg retarder at angles of ± 51.7 -deg and ± 15.1 -deg can minimize the measurement errors induced by the signal-independent Gaussian additive noises in the intensity measurements.⁷ However, the retarder and the polarizer were assumed to be ideal in the abovementioned studies.

In practice, the polarizer can be considered to be ideal because high-performance polarizers are actually available in the market. For example, a free standing wire grid polarizer (G25 from Microtech instruments, inc.) has a polarization extinction ratio larger¹¹ than 10,000 (40 dB) in the frequency range from 0.1 to 4 THz, and a nanoparticle linear film polarizer (LPNIR from Thorlabs) also has a polarization extinction ratio larger than 10000 in the wavelength range¹² from 850 to 1600 nm. On the other hand, the retardance of a state-of-the-art achromatic retarder may vary in the working optical frequency range and obviously deviate from its nominal value. For example, a THz quarter-wave plate has a retardance deviation up to $\pm 3\%$ (± 2.7 -deg) in the frequency range¹³ from 0.25 to 1.75 THz, and optical achromatic quarter-wave plates and half-wave plates also have retardance deviations up to several degrees in the working wavelength ranges.¹⁴ As a result, the measurement errors induced by the retardance deviation are significant in a RRFP Stokes

Fig. 1 The structure of a RRFP Stokes polarimeter.

polarimeter. Hence, this paper intends to obtain the relationship between the retardance error and its induced measurement errors in a RRFP Stokes polarimeter. Further, based on this relationship, we can conclude that first, the induced measurement errors can not be reduced by choosing a set of optimum angular orientations or increasing the measurement number; and second, that the retarder with the retardance from 110-deg to 135-deg is recommended because it can effectively reduce such measurement errors. Simulations in two measurement scenarios are performed to verify these theoretical findings. The first two error sources are also considered in the simulations, including the angular orientation errors of the retarder, the signal-independent Gaussian noise and the signal-dependent shot noise in the intensity measurements. Even though the three error sources are included simultaneously, simulation results are in good agreement with the theoretical results.

2 Theoretical Analysis

The four Stokes parameters are usually written as a Stokes vector $\vec{S} = (s_0, s_1, s_2, s_3)^T$; where the superscript “ T ” denotes the vector or matrix transpose. In SOP measurements using a Stokes polarimeter, the Stokes vector $\vec{S}$ is related to $N(N \geq 4)$ intensity measurement results $\vec{I} = (I_0, I_1, \dots, I_N)^T$ as discussed in Ref. 7.

$$\vec{I} = \mathbf{W}\vec{S}, \quad (1)$$

where $\mathbf{W}$ is a $N \times 4$ matrix, which is termed the measurement matrix. The measurement matrix of a RRFP Stokes polarimeter⁷ is

$$\mathbf{W} = \frac{1}{2} \begin{pmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vdots \\ \vec{V}_N \end{pmatrix}, \quad (2)$$

where $\vec{V}_i = (1, \cos^2 2\theta_i + \cos \delta \sin^2 2\theta_i, \sin^2(\delta/2) \sin 4\theta_i, -\sin \delta \sin 2\theta_i)$, $i = 1, 2, \dots, N(N \geq 4)$; δ is just the retardance of the retarder; θ_i , $i = 1, 2, \dots, N(N \geq 4)$ are the angular orientations of the retarder where the intensity measurements are taken. Among these angles, there should be $M(4 \leq M \leq N)$ different ones to guarantee the pseudo inverse matrix of $\mathbf{W}$ exists.⁷ Then, the Stokes vector can be obtained from

$$\vec{S} = \mathbf{W}^+ \vec{I}, \quad (3)$$

where $\mathbf{W}^+$ is the pseudo inverse matrix of $\mathbf{W}$, which is defined by Ref. 15 as

$$\mathbf{W}^+ = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T, \quad (4)$$

where the superscript “ -1 ” denotes the inverse of a square matrix.

When all error sources are taken into consideration,⁶ Eq. (1) becomes,

$$\vec{I} + \Delta \vec{I} = (\mathbf{W} + \Delta \mathbf{W})(\vec{S} + \Delta \vec{S}), \quad (5)$$

where $\Delta \vec{I}$, $\Delta \mathbf{W}$ and $\Delta \vec{S}$ denote the errors of the intensity measurement results, the measurement matrix and the Stokes vector, respectively. By ignoring the higher-order term $\Delta \mathbf{W} \cdot \Delta \vec{S}$, it has

$$\Delta \vec{S} = \mathbf{W}^+ (\Delta \vec{I} - \Delta \mathbf{W} \vec{S}). \quad (6)$$

Considering the means of the intensity measurement¹⁶ results $\langle \Delta \vec{I} \rangle = 0$, the mean values of $\Delta \vec{S}$ should be

$$\langle \Delta \vec{S} \rangle = -\langle \mathbf{W}^+ \Delta \mathbf{W} \rangle \vec{S}. \quad (7)$$

When $\Delta \mathbf{W}$ is dominated by the retardance error $\Delta \delta$ of the retarder, Eq. (7) becomes

$$\langle \Delta \vec{S} \rangle = -\mathbf{W}^+ \Delta \mathbf{W}_R \vec{S}, \quad (8)$$

where $\Delta \mathbf{W}_R$ is the error of the measurement matrix induced by the retardance error $\Delta \delta$. Obviously, $\langle \Delta \vec{S} \rangle$ is proportional to the Stokes vector $\vec{S}$ with the coefficient matrix $\mathbf{W}^+ \Delta \mathbf{W}_R$. When $\Delta \delta$ is small enough so that $\cos(\delta + \Delta \delta) \approx \cos \delta - \sin \delta \cdot \Delta \delta$ and $\sin(\delta + \Delta \delta) \approx \sin \delta + \cos \delta \cdot \Delta \delta$ are valid, $\Delta \mathbf{W}_R$ is calculated as

$$\Delta \mathbf{W}_R = \frac{\Delta \delta}{2} \begin{pmatrix} \Delta \vec{V}_1 \\ \Delta \vec{V}_2 \\ \vdots \\ \Delta \vec{V}_N \end{pmatrix}, \quad (9)$$

where

$$\Delta \vec{V}_i = (0, -\sin^2 2\theta_i \sin \delta, \sin 2\theta_i \cos 2\theta_i \sin \delta, -\sin 2\theta_i \cos \delta).$$

$i = 1, 2, \dots, N(N \geq 4)$. When $M = N = 4$, $\mathbf{W}^+ \Delta \mathbf{W}_R$ is simplified to $\mathbf{W}^{-1} \Delta \mathbf{W}_R$, which can be derived, by brute force inversion, expansion and simplification using a computer program based on symbolic math functions of Matlab, that

$$\mathbf{W}^{-1} \Delta \mathbf{W}_R = \mathbf{A} \Delta \delta, \quad (10)$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & -\cot(\delta/2) & 0 & 0 \\ 0 & \cot(\delta/2) & 0 & 0 \\ 0 & 0 & \cot(\delta/2) & 0 \\ 0 & 0 & 0 & \cot \delta \end{pmatrix}$$

and $\cot$ is the cotangent. Next, it will be demonstrated that $\mathbf{W}^+ \Delta \mathbf{W}_R$ is also equal to $\mathbf{A} \Delta \delta$ by using the recursive algorithm. Firstly, it is assumed that when $N - 1$ intensity measurements are taken, $\mathbf{W}_{N-1}^+ \Delta \mathbf{W}_{R,N-1} = \mathbf{A} \Delta \delta$, then from the definition of the pseudo inverse matrix shown in Eq. (4), it has

$$\mathbf{W}_{N-1}^T \Delta \mathbf{W}_{R,N-1} = \mathbf{W}_{N-1}^T \mathbf{W}_{N-1} \mathbf{A} \Delta \delta. \quad (11)$$

Secondly, for N intensity measurements, there are $\mathbf{W}_N = \begin{pmatrix} \mathbf{w}_{N-1} \\ \vec{V}_N \end{pmatrix}$ and $\Delta \mathbf{W}_{R,N} = \begin{pmatrix} \Delta \mathbf{W}_{R,N-1} \\ \Delta \vec{V}_N \end{pmatrix}$, then

$$\begin{aligned} \mathbf{W}_N^T \Delta \mathbf{W}_{R,N} &= (\mathbf{W}_{N-1}^T \quad \vec{V}_N^T) \begin{pmatrix} \Delta \mathbf{W}_{R,N-1} \\ \Delta \vec{V}_N \end{pmatrix} \\ &= \mathbf{W}_{N-1}^T \Delta \mathbf{W}_{R,N-1} + \vec{V}_N^T \Delta \vec{V}_N. \end{aligned} \quad (12)$$

On the other hand, it can be derived that

$$\begin{aligned} \mathbf{W}_N^T \mathbf{W}_N \mathbf{A} \Delta \delta &= (\mathbf{W}_{N-1}^T \vec{V}_N^T) \begin{pmatrix} \mathbf{W}_{N-1} \\ \vec{V}_N \end{pmatrix} \mathbf{A} \Delta \delta \\ &= \mathbf{W}_{N-1}^T \mathbf{W}_{N-1} \mathbf{A} \Delta \delta + \vec{V}_N^T \vec{V}_N \mathbf{A} \Delta \delta. \end{aligned} \quad (13)$$

It is easy to verify that $\vec{V}_N^T \Delta \vec{V}_N = \vec{V}_N^T \vec{V}_N \mathbf{A} \Delta \delta$. Then, based on Eq. (11), it can be found that Eqs. (12) and (13) are actually equal. Hence, again from the definition of the pseudo inverse matrix in Eq. (4), we have

$$\mathbf{W}_N^+ \Delta \mathbf{W}_{R,N} = \mathbf{A} \Delta \delta. \quad (14)$$

Finally, from Eq. (8), it has

$$\begin{aligned} &\begin{pmatrix} \langle \Delta s_0 \rangle \\ \langle \Delta s_1 \rangle \\ \langle \Delta s_2 \rangle \\ \langle \Delta s_3 \rangle \end{pmatrix} \\ &= \Delta \delta \begin{pmatrix} 0 & \cot(\delta/2) & 0 & 0 \\ 0 & -\cot(\delta/2) & 0 & 0 \\ 0 & 0 & -\cot(\delta/2) & 0 \\ 0 & 0 & 0 & -\cot \delta \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}. \end{aligned} \quad (15)$$

Based on Eq. (15), several conclusions can be drawn. First, we can conclude that when a 180-deg retarder is used, the first three components of the Stokes vector, viz. the linear polarization components, will be free from the retardance error $\Delta \delta$. Of course, this is not a complete Stokes polarimeter because the last component of the Stokes vector can not be determined by such a polarimeter. Second when a 90-deg retarder is employed, the last component of the Stokes vector, viz. the circular polarization component, will be free from the retardance error $\Delta \delta$. Third, we can conclude that the measurement error of each Stokes parameter is proportional to the retardance error $\Delta \delta$. Finally, we conclude that the measurement errors are independent of the angular orientations θ_i , $i = 1, 2, \dots, N (N \geq 4)$ and the intensity measurement number N .

For a complete RRFP Stokes polarimeter, which determines all four Stokes parameters, the absolute values of two coefficients $\cot(\delta/2)$ and $\cot \delta$ should be less than 1 to attenuate the measurement errors of four Stokes parameters simultaneously. As shown in Fig. 2, the common retardance range, where both $\cot(\delta/2)$ and $\cot \delta$ belong to $[-1, 1]$, is from 90-deg to 135-deg.

From Eq. (15), $\langle \Delta \vec{S} \rangle$ also depends on the Stokes vector $\vec{S}$. Then for a complete RRFP Stokes polarimeter, an upper bound has to be used to evaluate the overall relative measurement error of the Stokes vector, which is

Fig. 2 The relationships between $\cot(\delta/2)$, $\cot \delta$ and the retardance of the retarder δ .

$$\frac{\|\langle \Delta \vec{S} \rangle\|}{\|\vec{S}\|} \leq \|\mathbf{A}\| \Delta \delta = \Delta \delta \sqrt{3 \cot^2(\delta/2) + \cot^2 \delta}, \quad (16)$$

where $\|\langle \Delta \vec{S} \rangle\| = \sqrt{\sum_{i=0}^3 \langle \Delta s_i \rangle^2}$ and $\|\vec{S}\| = \sqrt{\sum_{i=0}^3 s_i^2}$ are 2-norms of the vectors $\langle \Delta \vec{S} \rangle$ and $\vec{S}$, respectively; $\|\mathbf{A}\| = \sqrt{\text{Tr}(\mathbf{A}^T \mathbf{A})}$ denotes the Frobenius matrix norm of $\mathbf{A}$, where “Tr” denotes the trace of a matrix.¹⁵ Equation (16) means that $\|\mathbf{A}\|$ can be used as a figure of merit to evaluate the overall relative measurement error of the Stokes vector. The relationship between $\|\mathbf{A}\|$ and δ is illustrated in Fig. 3. The minimum of $\|\mathbf{A}\|$ is achieved when $\delta \approx 124$ -deg, which means generally speaking, a retarder with a retardance around 124-deg can minimize the retardance deviation induced measurement errors in a complete RRFP Stokes polarimeter. Please note, Eq. (16) only presents an upper bound of the overall relative error of the Stokes vector. Since $\|\langle \Delta \vec{S} \rangle\|$ is dependent of the Stokes vector $\vec{S}$, the exact optimum retardance is also $\vec{S}$ -dependent.

Fig. 3 The relationship between $\|\mathbf{A}\|$ and the retardance of the retarder δ . The inset shows the “zoom in” view of the same data.

Based on Fig. 3, the retardance range from 110-deg to 136-deg can be highlighted because the figure of merit $\|A\|$ increases less than 10% from the minimum value in this range. Finally, by combining two optimum retardance ranges shown in Figs. 2 and 3, the retardance range from 110-deg to 135-deg can be recommended in a complete RRFP Stokes polarimeter.

3 Simulation

In the following, we describe simulations we performed to verify Eqs. (15) and (16) in two different measurement scenarios with the same intensity measurement number $N = 36000$. The first measurement scenario posited that the retarder rotates to four angles (−45-deg, 0-deg, 30-deg, 60-deg) which were suggested to reduce the measurement errors induced by the first two error resources.⁵ At each angle, 9000 intensity measurements are taken. Since the retarder is set at only four angles, it is reasonable and realizable that the angular orientation errors can be controlled to be small.⁷ So we take $\Delta\theta_1 = 0.25$ -deg, $\Delta\theta_2 = -0.25$ -deg, $\Delta\theta_3 = -0.25$ -deg, $\Delta\theta_4 = 0.25$ -deg in the simulations. Such a measurement scenario is applicable for the SOP measurements in POTDR or THz-FDS, where the signals under test vary rapidly with respect to time^{8,9}

The second measurement scenario is that the retarder rotates rapidly and continuously; the intensity measurements are taken at the angular orientations 0-deg, 1-deg, ..., to 359-deg for 100 circles in total. Since the angular orientation errors in this scenario are random, a Gaussian distribution with zero mean and 1-deg standard variance is used to depict these errors in the simulations.

The second measurement scenario is applicable when the signal under test is a continuous wave. In the simulations, the signal-independent Gaussian thermal noise with 2% standard variance and the signal-dependent Poisson shot noise of the photodetector are also included, which introduce the random errors in the intensity measurement results.^{7,10,16} The retardance error $\Delta\delta$ used in the simulations is 3-deg.

Simulations were performed for 100 different incident SOPs, which uniformly span the whole Poincaré Sphere. Because the similar conclusions can be drawn from the simulation results of different incident SOPs, only the results of the incident SOP $(1, 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^T$ are shown in this paper.

The simulation results of $\langle\Delta S_i\rangle$, $i = 0, 1, 2, 3$ in the first measurement scenario are plotted in Fig. 4 using the red dashed lines. The blue solid lines present the theoretical results calculated using Eq. (15). Obviously, the simulation

Fig. 4 Simulation results of $\langle\Delta S_i\rangle$, $i = 0, 1, 2, 3$ in the first measurement scenario. Theoretical results calculated using Eq. (15) are also plotted for comparison.

Fig. 5 Simulation result of $\|\langle \Delta \tilde{S} \rangle\|/\|\tilde{S}\|$ in the first measurement scenario. Theoretical upper bound $\|\mathbf{A}\|\Delta\delta$ is also plotted for comparison. The inset shows the “zoom in” view of the same data.

results are consistent with the theoretical results. The discrepancies between the simulation results and the theoretical results can be attributed to several factors. First, the four

angular orientation errors $\Delta\theta_i$, $i = 1, 2, 3, 4$, which also contribute to $\Delta\mathbf{W}$. Second, the higher-order term $\Delta\mathbf{W} \cdot \Delta\tilde{S}$, ignored in Eq. (6), cannot be completely neglected when $\langle \Delta s_i \rangle$, $i = 0, 1, 2, 3$ are relatively large. In Fig. 5, the simulation result of the overall relative measurement error $\|\langle \Delta \tilde{S} \rangle\|/\|\tilde{S}\|$ is illustrated using the red dashed line. The theoretical upper bound $\|\mathbf{A}\|\Delta\delta$ calculated from Eq. (16) is also plotted using the blue solid line for comparison. From the red dashed line in Fig. 5, the optimum retardance is approximately 115-deg, which is different from 124-deg determined by the theoretical upper bound.

The simulation results of $\langle \Delta s_i \rangle$, $i = 0, 1, 2, 3$ in the second measurement scenario are plotted in Fig. 6 using the red dashed lines, and the blue solid lines also present the theoretical results calculated using Eq. (15). Compared with the results shown in Fig. 4, it is evident the simulation results in Fig. 6 are more consistent with the theoretical results. This consistency is due to the practice of assuming angular orientation errors to be random variables with zero means in the second measurement scenario. In this scenario, the discrepancies between the simulation results and the theoretical results are mainly caused the higher-order term $\Delta\mathbf{W} \cdot \Delta\tilde{S}$, ignored in Eq. (6). In Fig. 7, the simulation result of the overall relative measurement error $\|\langle \Delta \tilde{S} \rangle\|/\|\tilde{S}\|$ is illustrated

Fig. 6 Simulation results of $\langle \Delta s_i \rangle$, $i = 0, 1, 2, 3$ in the second measurement scenario. Theoretical results calculated using Eq. (15) are also plotted for comparison.

Fig. 7 Simulation result of $\|\langle \Delta \tilde{S} \rangle\|/\|\tilde{S}\|$ in the second measurement scenario. Theoretical upper bound $\|A\|\Delta\delta$ is also plotted for comparison. The inset shows the “zoom in” view of the same data.

using the red dashed line. The theoretical upper bound $\|A\|\Delta\delta$ is also plotted using the blue solid line for comparison. From the red dashed line in Fig. 7, the optimum retardance is approximately 122-deg, which is slightly different from 124-deg determined by the theoretical upper bound.

From the simulation results in Figs. 5 and 7, the overall relative measurement error $\|\langle \Delta \tilde{S} \rangle\|/\|\tilde{S}\|$ depends on not only the Stokes vector under test, but also the measurement scenario. This can be attributed to the way the angular orientation errors of the retarder affect the simulation results in different ways in two measurement scenarios. In the first measurement scenario, the angular orientation errors are deterministic; in the second measurement scenario, they are random.

4 Conclusion

For a RRFP Stokes polarimeter, the relationship between the retardance error and the measurement errors of the Stokes vector is obtained theoretically and verified by simulations. Results show that the retardance deviation induced measurement errors of the Stokes vector are governed the retardance δ of the retarder, the retardance error $\Delta\delta$ and the Stokes vector under test, but independent of the angular orientations θ_i , $i = 1, 2, \dots, N (N \geq 4)$ of the retarder and the intensity measurements. Based on this relationship, we can draw the following conclusions. First, the retardance deviation induced measurement errors of the Stokes vector can not be reduced by choosing a set of optimum angular orientations or

increasing the measurement number, which were adopted to minimize the measurement errors induced by the first two error sources.^{6,7} Second, the retarder with the retardance from 110-deg to 135-deg is recommended because it can effectively reduce such measurement errors in a complete RRFP Stokes polarimeter.

Acknowledgments

This work is supported by A-STAR of Singapore and JST of Japan joint project with A-STAR/SERC/SICP Grant No. 1021630069 and Open Fund of Key Laboratory of Information Photonics and Optical Communications (Beijing University of Posts and Telecommunications), Ministry of Education, China.

References

1. R. M. Jopson, L. E. Nelson, and H. Kogelnik, “Measurement of second-order polarization-mode dispersion vectors in optical fibers,” *IEEE Photon. Technol. Lett.* **11**(9), 1153–1155 (1999).
2. H. Dong et al., “Virtual generalized Mueller matrix method for measurement of complex polarization-mode dispersion vector in optical fibers,” *IEEE Photon. Technol. Lett.* **19**(1), 27–29 (2007).
3. H. Dong et al., “Polarization state and Mueller matrix measurements in terahertz-time domain spectroscopy,” *Opt. Commun.* **282**, 3671–3675 (2009).
4. H. Dong et al., “Measurement of polarization dependent loss in terahertz time domain spectroscopy,” presented at the 34th International Conference on Infrared, Millimeter, and Terahertz Waves, 21–25, The International Society of Infrared, Millimeter, and Terahertz Waves, Busan, Korea (Sept. 2009).
5. A. Gerrard and J. M. Burch, *Introduction to Matrix Methods in Optics*, Chap. 4, Dover Publications Inc., New York, pp. 179–239 (1994).
6. A. Ambirajan and D. C. Look Jr., “Optimum angles for a polarimeter: part I,” *Opt. Eng.* **34**(6), 1651–1655 (1995).
7. D. S. Sabatke et al., “Optimization of retardance for a complete Stokes polarimeter,” *Opt. Lett.* **25**(11), 802–804 (2000).
8. R. E. Schuh and A. S. Siddiqui, “Single channel polarimetric OTDR for measurement of backscattered SOP evolution along a fibre with twist,” *Electron. Lett.* **33**(25), 2153–2154 (1997).
9. M. Tang et al., “Dual-wavelength single-crystal double-pass KTP optical parametric oscillator and its application in terahertz wave generation,” *Opt. Lett.* **35**(10), 1698–1700 (2010).
10. F. Goudail, “Noise minimization and equalization for Stokes polarimeters in the presence of signal-dependent Poisson shot noise,” *Opt. Lett.* **34**(5), 647–649 (2009).
11. Microtech Instruments, Inc. Product Guide: THz wire grid polarizers, <http://www.mtinstruments.com/thz/polarizers/index.htm>.
12. Thorlabs, Product listing of nanoparticle linear film polarizers, http://www.thorlabs.com/NewGroupPage9.cfm?ObjectGroup_ID=752.
13. J. B. Masson and G. Gallot, “Terahertz achromatic quarter-wave plate,” *Opt. Lett.* **31**(2), 26–267 (2006).
14. Thorlabs, Product listing of mounted achromatic waveplates, http://www.thorlabs.com/NewGroupPage9.cfm?ObjectGroup_ID=854.
15. R. A. Horn, *Matrix Analysis*, Cambridge University Press, Cambridge, United Kingdom, pp. 1–30 (1985).
16. R. Q. Hui and M. O’Sullivan, *Fiber Optic Measurement Techniques*, Chap. 1, Elsevier Academic Press, Burlington, USA, pp. 32–43 (2009).

Biographies and photographs of the authors are not available.