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Time-domain characterization of the acoustic damping of a perforated liner with bias flow

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Combustion instabilities are caused by the interaction of unsteady heat releases and acoustic waves. To mitigate combustion instabilities, perforated liners, typically subjected to a low Mach number bias flow (a cooling flow through perforated holes), are fitted along the bounding walls of a combustor. They dissipate the acoustic waves by generating vorticity at the rims of perforated apertures. To investigate the absorption of plane waves by a perforated liner with bias flow, a time-domain numerical model of a cylindrical lined duct is developed. The liners' damping mechanism is characterized by using a time-domain "compliance." The development of such time-domain compliance is based on simplified or unsimplified Rayleigh conductivity. Numerical simulations of two different configurations of lined duct systems are performed by combining a 1D acoustic wave model with the compliance model. Comparison is then made between the results from the present models, and those from the experiment and the frequency-domain model of previous investigation [Eldredge and Dowling, *J. Fluid Mech.* **485**, 307–335(2003)]. Good agreement is observed. This confirms that the present model can be used to simulate the propagation and dissipation of acoustic plane waves in a lined duct in real-time.

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I. INTRODUCTION

In order to meet more stringent requirements on emissions, combustors in ground based gas turbines and aero-engines tend to operate under lean premixed conditions to achieve low NO_x. However, under lean premixed conditions, combustors are more susceptible to combustion instabilities.¹ Combustion instabilities are generated by an interaction between acoustic waves and unsteady combustion. Unsteady heat release is an efficient acoustic source and generates acoustic waves. These pressure waves propagate within the combustor and partially reflect from boundaries to arrive back at the combustion zone, where they cause more unsteady heat release. Under certain conditions, this feedback can result in large and damaging self-excited oscillations.

To suppress combustion instabilities, the coupling between unsteady heat release and the pressure perturbation must somehow be interrupted.² Perforated liners^{3–5} are widely used as acoustic dampers to dissipate acoustic waves. They are usually metal sheets, frequently arranged in layers, which have tiny perforated holes in them. In practice, a cooling air flow through the perforated holes (i.e., a bias flow) is needed to prevent the liners from being damaged under high temperature. Remarkably, the bias flow is found to increase liners damping performance. The main mechanism of acoustic damping involves vortex shedding generated over the rims of the perforated holes.

The majority of modelling work of perforated liners has been carried out in frequency domain. Howe⁶ modelled the

acoustic energy dissipated by the periodic shedding of vorticity for a single orifice in a high Reynolds-number flow using a Rayleigh conductivity. Following Howe's research, Hughes and Dowling⁷ studied the acoustic damping of screens with regular array of slits and circular perforations with mean bias flow, showing that all impinging sound could be absorbed at a particular frequency in theory. Jing and Sun⁵ experimentally investigated the effect of the screen thickness and the bias flow rate, showing that an appropriate bias flow rate can significantly increase damping and that the screen thickness is crucial. Eldredge and Dowling³ recently developed a 1D duct model in frequency domain to simulate the absorption of axial plane wave by a double-liner with a bias flow. The damping mechanism of vortex shedding was embodied using a homogeneous liner compliance adapted from the Rayleigh conductivity. In addition, the effect of liners' thickness was considered by the compliance model. Their numerical results are found to agree very well with their experimental ones.

In comparison with frequency-domain modeling, time-domain modeling is more challenging. This is most likely due to the fact that either the perforated holes are too tiny to accurately simulate or higher computational cost is involved with the studies of entire liners. However, when air flow through perforated holes do not interact significantly, a single hole with its vicinity can be analyzed, which behaves like a Helmholtz resonator.⁸ To investigate the acoustic damping mechanism of perforated liners, Tam *et al.*⁹ carried out direct numerical simulation (DNS) of a single aperture. It is showed that vortex shedding is the dominant mechanism of absorption for incident waves of high amplitude.

Recently, Mendez and Eldredge¹⁰ conducted compressible large-eddy simulations in time domain to study the flow

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through a single perforated hole or multiple holes. Reichert and Biringen¹¹ proposed a time-domain approach which introduced a source term to the momentum equations, and assessed the effect of a bias flow numerically. They found that with an optimum bias flow rate, the liners damping efficiency was significantly improved over that with no bias flow. Sbardella *et al.*¹² proposed a liner model, which combines a frequency-independent resistive part and a reactive part. The liner model is then coupled with the acoustic wave model in the duct. Ozyoruk *et al.*¹³ proposed a time-domain impedance to describe the damping of a constant depth ceramic tubular liner, which was attached to the NASA Langley flow-impedance tube. The time-domain impedance was converted from the frequency-domain one¹⁴ by using z-transform method. Excellent agreement between their numerical results and the experimental data was observed. They also found that applying z-transform technique resulted in much less computational cost.

In this work, a 1D numerical time-domain model was developed to simulate the acoustic waves propagation and dissipation along a cylindrical lined duct. To characterize the liner's damping, a time-domain liner model based on the homogeneous compliance model derived by Eldredge and Dowling³ was developed. The compliance model is associated with frequency-domain Rayleigh conductivity.⁶ It was firstly formulated in a simplified rational form in z-domain, and then converted into the time-domain one. For different flow conditions, curve-fitting was involved in estimating the compliance. To develop a more general time-domain compliance model, a simplified Rayleigh conductivity as proposed by Luong *et al.*¹⁵ was used to replace the unsimplified one in our compliance model. In Sec. II, the model equations of a cylindrical lined duct with a single-liner surrounded by a large annular cavity were developed. Emphasis is being placed on characterizing the liners damping over a broad frequency range by simply setting the incident wave consisting of multiple tones (non-harmonic). The numerical results by using either the simplified compliance model or unsimplified one were then compared with those from the frequency-domain model of Eldredge and Dowling.³ To solve the partial differential equations (PDEs) of the system, method of lines (MOL)¹⁶ was applied. Further validation of the model was conducted by investigating the acoustic damping of a lined duct with a double-liner encased by a large annular cavity, as typically found within a combustion system. This is described in Sec. III. Comparison was then made between the numerical results and the experimental ones.³ Finally, in Sec. IV, transmission loss analysis of perforated liners are conducted.

II. MODEL OF A LINED DUCT WITH A SINGLE-LAYER LINER

The acoustic damping of perforated liners is generally characterized by power absorption, which describes the fraction of incident waves being absorbed. Following the work of Eldredge and Dowling,³ power absorption is defined as

$$\Delta = 1 - \frac{\|H_d^+(L, t)\|^2 + \|H_u^-(0, t)\|^2}{\|H_u^+(0, t)\|^2 + \|H_d^-(L, t)\|^2}, \quad (1)$$

FIG. 1. Schematic of acoustic waves and flow quantities in a cylindrical duct with a single-liner attached.

where $H_u^+(0, t)$ and $H_u^-(0, t)$ denote the incident and reflected enthalpy fluctuations upstream the lined section at $x = 0$. $H_d^+(L, t)$ and $H_d^-(L, t)$ are downstream ones at $x = L$, as shown schematically in Fig. 1. A single-layer liner was encased in a large annular cavity formed by a large rigid cylinder and attached to a cylindrical duct. An imposed mean flow $\bar{u}_u$ pass through the upstream duct. A steady supply of air is fed into the cavity to produce bias flow through the liners, and $\bar{v}_1$ denote the homogeneous bias flow velocity through the liner. The geometry of the lined duct is summarized in Table I. Here L is the liner length, D_1 , C_1 , A_1 and σ_1 are diameters, circumferences, cross-sectional areas and open-area ratio of the liner, respectively. $\bar{M}_{h,1}$ is the bias flow Mach number through the liner holes.

A. Governing equations

According to Eq. (1), acoustic plane waves either side of the lined section need to be characterized so that the power absorption is estimated. This means that the lined duct can be divided into 3 regions (i.e., upstream $x \leq 0$, lined segment $0 < x < L$ and downstream region $x \geq L$), and they need to be coupled in terms of flow properties. Thus the stagnation enthalpy and velocity perturbations need to match at $x = 0$ and $x = L$:

$$\begin{aligned} H'(t, 0) &= H_u^+(t, 0) + H_u^-(t, 0), \\ u'(t, 0) &= \frac{H_u^+(t, 0)}{\bar{c} + \bar{u}(0)} - \frac{H_u^-(t, 0)}{\bar{c} - \bar{u}(0)}, \end{aligned} \quad (2a)$$

$$\begin{aligned} H'(t, L) &= H_d^+(t, L) + H_d^-(t, L), \\ u'(t, L) &= \frac{H_d^+(t, L)}{\bar{c} + \bar{u}(L)} - \frac{H_d^-(t, L)}{\bar{c} - \bar{u}(L)}, \end{aligned} \quad (2b)$$

TABLE I. Parameters used in the numerical model.

Parameter	Value (unit)	Parameter	Value (unit)
A_1	0.0128 (m ²)	R	0.0635 (m)
$\mathcal{R}_1$	0.375×10^{-3} (m)	A_2	0.0181 (m ²)
C_1	0.399 (m)	$\mathcal{R}_2$	1.35×10^{-3} (m)
σ_1	0.04	C_2	0.478 (m)
$\bar{c}$	343 (m/s)	σ_2	0.02
d_1	1.493 (m)	d_2	0.457 (m)
ϵ_1	3×10^{-3} (m)	ϵ_2	3×10^{-3} (m)
L	0.178 (m)	v	0.167
L_d	0.8 (m)	μ	0.039 (m)
D_1	0.127 (m)	D_2	0.152 (m)

where the fluctuating stagnation enthalpy, $H'(t, x) = p'(t, x)/\bar{\rho} + \bar{u}(x)u'(t, x)$. p' is the fluctuating pressure and u' is the fluctuating axial velocity.

In the lined region $0 < x < L$, the instantaneous flow properties consisting of both mean and fluctuating parts are given by

$$\begin{aligned} H(t, x) &= \bar{H}(x) + H'(t, x), \\ u(t, x) &= \bar{u}(x) + u'(t, x), \\ \rho(x, t) &= \bar{\rho} + \rho'(t, x). \end{aligned} \quad (3)$$

The axial mean flow velocity $\bar{u}(x)$ in the duct is varied with along the liner due to the presence of bias flow, and can be expressed as

$$\bar{\rho}A_1\bar{u}(x) = \bar{\rho}A_1\bar{u}_u + \bar{\rho}C_1\bar{v}_1x, \quad (4)$$

where $\bar{v}_1$ the mean bias flow velocity through the liner and $\bar{\rho}$ is the mean density. Since both the mean axial pipe and bias flow are small with the same stagnation temperature associated with them, the mean density $\bar{\rho}$ and mean sound speed $\bar{c}$ is uniform in the duct. In addition, the density fluctuation ρ' can be written in terms of H' as

$$\rho'(t, x) = \frac{\bar{\rho}H'(t, x)}{\bar{c}^2} - \frac{\bar{\rho}\bar{u}u'(t, x)}{\bar{c}^2}. \quad (5)$$

The equation of mass conservation over an annular control volume of length Δx in the lined section is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = \frac{C_1}{A_1} \rho v_1, \quad (6)$$

where ρ , u , and v are the instantaneous density, axial flow velocity and bias flow velocity respectively. The motion equation of the air flow through the duct, i.e., the momentum equation, can be written as:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = 0, \quad (7)$$

where p is the instantaneous pressure.

Linearizing Eqs. (6) and (7) gives

$$\frac{\partial \rho'(t, x)}{\partial t} + \bar{\rho} \frac{\partial u'(t, x)}{\partial x} + \bar{u}(x) \frac{\partial \rho'(t, x)}{\partial x} = \frac{C_1}{A_1} \bar{\rho} v_1'(t, x), \quad (8)$$

$$\bar{\rho} \frac{\partial u'(t, x)}{\partial t} + \bar{\rho} \bar{u}(x) \frac{\partial u'(t, x)}{\partial x} + \frac{\partial p'(t, x)}{\partial x} = 0. \quad (9)$$

Substituting Eq. (5) into Eqs. (8) and (9) gives

$$\begin{aligned} \frac{\partial H'(t, x)}{\partial t} - \bar{u}(x) \frac{\partial u'(t, x)}{\partial t} + (\bar{c}^2 - \bar{u}^2(x)) \frac{\partial u'(t, x)}{\partial x} \\ + \bar{u}(x) \frac{\partial H'(t, x)}{\partial x} = \frac{C_1}{A_1} \bar{c}^2 v_1'(t, x), \end{aligned} \quad (10)$$

$$\frac{\partial u'(t, x)}{\partial t} + \frac{\partial H'(t, x)}{\partial x} = 0. \quad (11)$$

Manipulating Eqs. (10) and (11) leads to

$$\begin{aligned} \frac{\partial H'(t, x)}{\partial t} + 2\bar{u}(x) \frac{\partial H'(t, x)}{\partial x} + (\bar{c}^2 - \bar{u}^2(x)) \frac{\partial u'(t, x)}{\partial x} \\ = \frac{C_1}{A_1} \bar{c}^2 v_1'(t, x). \end{aligned} \quad (12)$$

If we decompose the stagnation enthalpy fluctuation $H'(t, x)$ in the lined section into the incident ζ^+ and reflected one ζ^- , then they can be shown as

$$\begin{aligned} \zeta^+(t, x) &= \frac{1}{2} [H'(t, x) + \bar{c}u'(t, x) + H'(t, x)\bar{M}(x) \\ &\quad - \bar{c}u'(t, x)\bar{M}^2(x)], \end{aligned} \quad (13a)$$

$$\begin{aligned} \zeta^-(t, x) &= \frac{1}{2} [H'(t, x) - \bar{c}u'(t, x) - H'(t, x)\bar{M}(x) \\ &\quad + \bar{c}u'(t, x)\bar{M}^2(x)]. \end{aligned} \quad (13b)$$

Note that $H'(t, x) = \zeta^+(t, x) + \zeta^-(t, x)$. $\bar{M}(x) = \bar{u}(x)/\bar{c}$ is the Mach number of axial mean flow through the duct.

Substituting Eqs. (13a) and (13b) into Eq. (12) gives rise to two coupled PDEs as

$$\begin{aligned} \frac{\partial \zeta^+(t, x)}{\partial t} &= -[\bar{c} + \bar{u}(x)] \frac{\partial \zeta^+(t, x)}{\partial x} \\ &\quad + \frac{C_1}{2A_1} \bar{c} [\bar{c} + \bar{u}(x)] v_1'(t, x), \end{aligned} \quad (14a)$$

$$\begin{aligned} \frac{\partial \zeta^-(t, x)}{\partial t} &= [\bar{c} - \bar{u}(x)] \frac{\partial \zeta^-(t, x)}{\partial x} \\ &\quad + \frac{C_1}{2A_1} \bar{c} [\bar{c} - \bar{u}(x)] v_1'(t, x). \end{aligned} \quad (14b)$$

The boundary values of ζ^+ and ζ^- at $x = 0$ and $x = L$ are

$$\begin{aligned} \zeta^+(0, t) &= H_u^+(0, t), \quad \zeta^-(0, t) = H_u^-(0, t), \\ \zeta^+(L, t) &= H_d^+(L, t), \quad \zeta^-(L, t) = H_d^-(L, t). \end{aligned} \quad (15)$$

Note that Eqs. (14a), (14b), and (15) are corresponding to the frequency-domain Eqs. (2.14) and (2.15).³ It can be seen that the time evolution of acoustics waves in the lined section can be determined by solving Eqs. (14a) and (14b) with the boundary values and the bias flow $v_1'(t, x)$.

Following the work of Eldredge and Dowling,³ we assume $H_u^+(0, t)$ is known. In addition as described in Eq. (2.16b),³ the incident wave $H_d^+(L, t)$ is coupled to reflected one $H_d^-(L, t)$ via a downstream reflection coefficient $R_d(t)$ as

$$\hat{H}_d^-(L, \omega) = \hat{R}_d(\omega) \hat{H}_d^+(L, \omega), \quad (16)$$

where $\hat{R}_d$, $\hat{H}_d^-(\omega, L)$ and $\hat{H}_d^+(\omega, L)$ are Fourier transform of $R_d(t)$, $H_d^-(t, L)$ and $H_d^+(t, L)$ respectively. The bias flow through the liner $v_1'(t, x)$ is shown to related to the difference of stagnation enthalpy fluctuations across the liner through a homogeneous compliance³ as

$$\hat{v}_1(\omega, x) = \hat{Y}_1(\omega) [\hat{\zeta}^+(\omega, x) + \hat{\zeta}^-(\omega, x) - \hat{H}_1(\omega, x)], \quad (17)$$

where $\hat{v}_1(\omega, x)$ and $\hat{H}_1(\omega, x)$ denote Fourier transform of $v'_1(t, x)$ and $H'_1(t, x)$. Here, $H'_1(t, x)$ is defined as the stagnation enthalpy fluctuation of the flow disturbances external to the inner liner.³ Its derivation is similar to $H'(t, x)$ and is neglected here for simplicity. Interested readers can refer to the work of Eldredge and Dowling³ for more detailed information. $\hat{Y}_1(\omega)$ is the frequency-domain compliance and is given as

$$\hat{Y}_1(\omega) = \frac{-1}{j\omega} \frac{\sigma_1 K_a}{(\pi \mathcal{R}_1^2 + \epsilon_1 K_a)}, \quad (18)$$

where $\mathcal{R}_1$ is the radius of the perforated aperture, σ_1 and ϵ_1 are the open area ratio and thickness of the liner. K_a is the Rayleigh conductivity, which is a function of Strouhal number, $S_l = \omega \mathcal{R}_1 \sigma_1 / \bar{v}_1$, as derived by Howe⁶ (See Appendix).

Equation (17) indicates that the following equation in z-domain holds

$$v'_1(z, x) = Y_1(z)[H'(z, x) - H'_1(z, x)], \quad (19)$$

where $Y_1(z)$ is the compliance in z-domain, which is given in a general form¹⁷ of

$$Y_1(z) = \frac{a_0 + \sum_{l=1}^h a_l z^{-l}}{1 - \sum_{i=1}^m b_i z^{-i}}. \quad (20)$$

Here a 's and b 's are real constants. Substituting Eq. (20) into Eq. (19), taking the inverse z-transform and using the shift property, a time evolution description of v'_1 can be shown as

$$v'_1(n)(x) = \sum_{i=1}^m b_i v'_1(n-i)(x) + \left[a_0 (H'^{(n)}(x) - H'_1(n)(x)) + \sum_{l=1}^h a_l (H'^{(n-l)}(x) - H'_1(n-l)(x)) \right]. \quad (21)$$

The superscript n denotes the n^{th} time-step. It is obvious that the local relationship between v'_1 and the difference in stagnation enthalpy fluctuations across the liner, as given in Eq. (21) enables the system equations ready to solve. However, the z-domain compliance involves many unknown constants a 's and b 's, which might be associated a huge computational cost. Furthermore, to ensure convergence and stability, the poles of the compliance should stay inside the unit circle. These requirements limit the application of such compliance formulation.

B. Low-pass-filter type compliance model

In order to achieve robustness and low-cost computation, we propose a simple but robust low-pass-filter type compliance in z-domain as

$$Y_1^s(z) = \frac{a_0}{1 - b_1 z^{-1}}, \quad (22)$$

where a_0 and b_1 are the parameters to be estimated. The local relationship between v'_1 and the difference in

stagnation enthalpy fluctuations across the liner is now given as simply as

$$v'_1(n)(x) = b_1 v'_1(n-1)(x) + a_0 [H'^{(n)}(x) - H'_1(n)(x)]. \quad (23)$$

It is important to note that using the proposed compliance is associated much less computational cost than using the full general form of Eq. (21) or applying inverse Fourier transform directly to Eq. (18).

1. Compliance model with Rayleigh conductivity

A frequency-domain compliance in corresponding to Eq. (22) can be shown as

$$\hat{Y}_1^s(\omega) = \frac{1}{\mathcal{P}_0 + j\omega \mathcal{Q}_1} = \frac{\mathcal{P}_0}{\mathcal{P}_0^2 + \omega^2 \mathcal{Q}_1^2} - \frac{j\omega \mathcal{Q}_1}{\mathcal{P}_0^2 + \omega^2 \mathcal{Q}_1^2}, \quad (24)$$

where $\mathcal{P}_0$ and $\mathcal{Q}_1$ are real constant, which can be estimated by equating Eq. (18) and (24). A nonlinear least square (NLS) curve fitting procedure based on Levenberg-Marquardt method¹⁸ is used to match $\hat{Y}_1^s(\omega)$ with $\hat{Y}_1(\omega)$ for different frequency and mean bias flow velocity. Figure 2 shows the curve fitting results of $\hat{Y}_1(\omega)$ and $\hat{Y}_1^s(\omega)$ under two different flow conditions. Excellent curve fitting is observed.

With the curve fitting, $\mathcal{P}_0$ and $\mathcal{Q}_1$ can be estimated and so a_0 and b_1 in Eq. (23) as given as

$$a_0 = \frac{\Delta t}{\mathcal{P}_0 \Delta t + \mathcal{Q}_1}, \quad b_1 = \frac{\mathcal{Q}_1}{\mathcal{P}_0 \Delta t + \mathcal{Q}_1}. \quad (25)$$

Thus, Eq. (23) provides a simple and explicit expression between the bias flow v'_1 and the difference in stagnation enthalpy fluctuations across the liner. Since the liner is encased by a large annular cavity, the stagnation enthalpy fluctuation inside the large cavity is approximately zero, i.e., $H'_1(t, x) = 0$.

FIG. 2. The real and imaginary parts of frequency-domain compliance $\hat{Y}_1(\omega)$ with simplified and unsimplified Rayleigh conductivity implemented: (a and b) $\bar{v}_{h,1} = 3.09$ m/s, (c and d) $\bar{v}_{h,1} = 10.26$ m/s.

2. Compliance model with simplified Rayleigh conductivity

As illustrated in previous section, curve-fitting is needed to estimate the compliance components, i.e., a_0 and b_1 , which are related to Rayleigh conductivity.⁶ When different flow conditions are considered, this curve-fitting needs to be repeated, which is time-consuming and not economical in terms of computation. In order to develop a more general compliance model, a simplified Rayleigh conductivity as proposed by Luong *et al.*¹⁵ is then used to replace the unsimplified one^{3,6} in the compliance model, i.e., Eq. (24). The simplified Rayleigh conductivity K_a^s is derived by applying the Cummings equation¹⁹ to a perforated aperture with bias flow, and solving the linearized form of this modified Cummings equation. And it is given as

$$K_a^s = 2\mathcal{R}_i \left(\frac{\omega \mathcal{R}_i / \bar{v}_{hi}}{\omega \mathcal{R}_i / \bar{v}_{hi} - 2j/\pi\chi^2} \right), \quad (26)$$

where $\mathcal{R}_i$ denotes the aperture radius. Subscript i denotes inner or outer liner. $\bar{v}_{hi}$ is the aperture mean velocity, which is related to the mean flow through the inner or outer liner as $\bar{v}_{hi} = \bar{v}_i / \sigma_i$. The contraction ratio χ is set to 0.71. Substituting Eq. (26) into Eq. (18) and recasting into Eq. (24) leads to

$$\mathcal{P}_0 = -\frac{\bar{v}_{hi}}{\sigma_i \chi^2}, \quad (27a)$$

$$\mathcal{Q}_1 = -\frac{(\epsilon_i + \pi \mathcal{R}_i / 2)}{\sigma_i}. \quad (27b)$$

Now the compliance components a_0 and b_1 can be analytically determined by using Eq. (25). In order to evaluate its performance, such compliance model with simplified Rayleigh conductivity is compared with unsimplified one, i.e., Eq. (18) and shown in Fig. 2. Excellent agreement between the simplified and unsimplified compliance models is observed. Further evaluation is conducted later, when these compliance models are combined with the lined duct model.

C. Time-domain downstream reflection coefficient

In the boundary condition Eq. (16), the reflection coefficient, R_d , appears and we now discuss its form in time domain. In the downstream region of the duct, H_d^+ propagate along the duct to the open end and partially reflects back due to the boundary condition change. The reflected wave H_d^- is related to H_d^+ via R_d , and it depends on the duct specific geometry and mean flow condition. In the case of semi-infinite downstream section, no acoustic waves are reflected and $R_d = 0$. For a finite-length pipe with an unflanged open end, which is more likely to be encountered in practice, many research works have been carried out to determine the reflection coefficient in frequency domain. Levine and Schwinger²⁰ provided an exact analytical solution of the reflection coefficient using the Wiener-Hopf method for a duct with no mean flow. Based on this solution, Eldredge and Dowling³ provided a second-order reflection coefficient for low-frequency plane waves as

$$\hat{R}_d(\omega) = -\left(1 - \frac{1}{2}(k\mathcal{R})^2\right) \exp(-j2k(L + L_d + \mu)), \quad (28)$$

where d_1 and d_2 are real constant parameters, as given in Table I. $\mathcal{R}$ is the duct radius, $k = \omega/\bar{c}$ is the wave number, μ is the end correction $\mu = 0.6133\mathcal{R}$, L_d is the length of downstream pipe. The corresponding time-domain reflection coefficient can be calculated by taking IFT to Eq. (28). Silva *et al.*²¹ also proposed to approximate the reflection coefficient by using

$$\hat{R}_d(\omega) = -\frac{1 - vjk\mathcal{R}}{1 - d_1jk\mathcal{R} + d_2(jk\mathcal{R})^2} \times \exp(-j2k(L + L_d + \mu_0)), \quad (29)$$

where v , d_1 , and d_2 are real constant parameters and $\mu_0 = 0.0779$. This expression is similar to Eq. (28). However an explicit form of $R_d(t)$ can be derived by using the residue method,²² as given as

$$R_d(t) = \begin{cases} \frac{\bar{c}}{d_2\mathcal{R}(\alpha_1 - \alpha_2)} \left[(v\alpha_2 - 1) \exp\left(\frac{-\alpha_2\bar{c}(t - \mathcal{T})}{\mathcal{R}}\right) - (v\alpha_1 - 1) \exp\left(\frac{-\alpha_1\bar{c}(t - \mathcal{T})}{\mathcal{R}}\right) \right] & t \geq \mathcal{T} \\ 0 & t < \mathcal{T} \end{cases}, \quad (30)$$

where d_1 and d_2 are real constant parameters, as given in Table I. $\alpha_{1,2} = (1/2d_2)(d_1 \pm \sqrt{d_1^2 - 4d_2})$ and $\mathcal{T} = 2(L + L_d + \mu_0)/\bar{c}$. Before we choose Eq. (28) or Eq. (29) to couple $H_d^+(t, L)$ and $H_d^-(t, L)$, it is necessary to see their differences. Figure 3 shows the comparison of the reflection coefficients estimated from Eq. (28) and Eq. (29). It can be seen that the agreement is excellent. This means either

model can be used. However, for convenience, we coupled $\zeta^-(t, L)$ with $\zeta^+(t, L)$ via Eq. (30) as

$$\begin{aligned} \zeta^-(t, L) &= R_d(t) * \zeta^+(t, L) = \int_0^t R_d(\tau) \zeta^+(t - \tau, L) d\tau \\ &= \Delta t \sum_{m=0}^{\mathcal{M}} R_d(m\Delta t) \zeta^+((\mathcal{M} - m)\Delta t, L). \end{aligned} \quad (31)$$

FIG. 3. (Color online) The (a) real and (b) imaginary part of downstream reflection coefficient $\hat{R}_d$.

In our work, $\mathcal{M}$ is set to 3×10^4 with a sampling rate of $1.0 \times 10^5 \text{ Hz}$.

D. Governing equations in discrete time

Forgoing analysis shows that the governing equations for the lined duct are PDEs. To solve these equations, method of lines (MOL)¹⁶ is used. It converts the PDEs into ODEs by discretizing them in all but one dimension and leaving time variables continuous. The spatial derivatives are always approximated algebraically by finite differences (FDs). Therefore, there is only one independent variable, time, remaining and resulting in an initial-value integration problem, which can be easily solved by well-established standard routines or ODE solvers.

In the lined section $0 < x < L$, the discrete-time form of Eqs. (14a), (14b), and (23) are given as

$$\frac{\partial \zeta_j^{+(n)}}{\partial t} = -(\bar{c} + \bar{u}_j) \frac{\zeta_{j+1}^{+(n-1)} - \zeta_{j-1}^{+(n-1)}}{2\Delta x} + \frac{C_1}{2A_1} \bar{c}(\bar{c} + \bar{u}_j) v_{1j}^{(n-1)}, \quad (32a)$$

$$\frac{\partial \zeta_j^{-(n)}}{\partial t} = (\bar{c} - \bar{u}_j) \frac{\zeta_{j+1}^{-(n-1)} - \zeta_{j-1}^{-(n-1)}}{2\Delta x} + \frac{C_1}{2A_1} \bar{c}(\bar{c} - \bar{u}_j) v_{1j}^{(n-1)}, \quad (32b)$$

$$\begin{aligned} \frac{\partial v_{1j}^{(n)}}{\partial t} &= \frac{v_{1j}^{(n)} - v_{1j}^{(n-1)}}{\Delta t} \\ &= \frac{b_1 - 1}{\Delta t} v_{1j}^{(n-1)} + \frac{a_0}{\Delta t} (\zeta_j^{+(n)} + \zeta_j^{-(n)} - H_{1j}^{(n)}). \end{aligned} \quad (32c)$$

In our calculation, the flow properties are evaluated at $N = 55$ locations across the lined section spacing (i.e., $\Delta x = L/N - 1$). And the time-step increment is sufficiently small, $\Delta t = \Delta x/2\bar{c}$, to satisfy the requirement of Courant-Friedrichs-Lewy number. In addition, the spatial derivatives are discretized by using a three-point centered difference approximation, which results in second-order accuracy. Moreover, the spatial derivative of a given flow variable at the boundaries $x = 0$ and $x = L$, are approximated as

$$\frac{\partial \phi_1^{(n-1)}}{\partial x} = \frac{-3\phi_1^{(n-1)} + 4\phi_2^{(n-1)} - \phi_3^{(n-1)}}{2\Delta x}, \quad (33a)$$

$$\frac{\partial \phi_N^{(n-1)}}{\partial x} = \frac{3\phi_N^{(n-1)} - 4\phi_{N-1}^{(n-1)} + \phi_{N-2}^{(n-1)}}{2\Delta x}, \quad (33b)$$

where ϕ represents the flow variable. Now Eqs. (32a)–(32c) can be systematically solved for the cylindrical lined duct with a single-layer liner attached.

E. Numerical results

Since our numerical simulation is conducted in time-domain, we can assume that the incident wave consists of multiple tones with unit magnitude as

$$H_u^+(0, t) = \sum_{i=1}^{\mathcal{N}} 1.0 \cos(\omega_i t), \quad (34)$$

where ω_i is the tone frequency, and $\mathcal{N}$ is an integer and denotes the tones number. By choosing a large $\mathcal{N}$, the liners' damping can be estimated over a broad frequency range all at once. This is different from the frequency-domain models,^{3,7} which calculate the liners' damping one frequency at a time.

Figure 4 shows the time evolution of incident and reflected enthalpy waves at the inlet and outlet of the lined section of the cylindrical duct with a single-liner attached, as $M_{h,1} = 0.023$, $M_u = 0.0$, and $\mathcal{N} = 70$. The frequencies of the tones are illustrated in the frequency spectrum of $H_u^+(0, t)$ in Fig. 5.

The liner's damping characterized by using power absorption is shown in Fig. 6(a). Comparison is then made between the results from the compliance models with simplified or unsimplified Rayleigh conductivity implemented, and those from the frequency-model of Eldredge and Dowling.³ Good agreement is observed. In order to evaluate the models performance, it is then applied to the lined duct under another 3 different flow conditions. Figure 6(b) show the variation of power absorption with the forcing frequency, as the bias flow Mach number is increased to $\bar{M}_{h,1} = 0.041$ but $\bar{M}_u$ remains 0. It can be seen that from Figs. 6(a) and 6(b) that the general trend of the maximum damping is to increase with increased frequency. In addition, as the bias flow Mach number is increased from 0.023 to 0.041, the maximum power absorption is increased slightly.

Figures 6(c) and 6(d) show the variation of the liner's damping with forcing frequency, as there is a mean axial flow through the duct, i.e., $\bar{M}_u \neq 0$. The maximum power absorption has a similar trend as the case of $\bar{M}_u = 0$, as shown in Figs. 6(a) and 6(b). However, as the bias flow $\bar{M}_{h,1}$ is reduced from 0.03 to 0.009, the liner's damping is dramatically reduced. This reveals that the bias flow plays an important role in improving liner's damping. In general, our model, especially the one with simplified Rayleigh conductivity implemented, was shown to be able to simulate the acoustic damping of a lined duct with a single-liner attached.

FIG. 4. (Color online) Time evolution of the acoustic waves consisting of 70 different tones in the lined duct with a single-liner, as $M_{h,1} = 0.023$ and $M_u = 0.0$.

III. MODEL OF A LINED DUCT WITH A DOUBLE-LAYER LINER

A. Description of the lined duct with a double-liner

In order to further validate our model, numerical investigation of a cylindrical lined-duct with a double-liner attached is conducted. The lined duct configuration is very similar to the single-liner one, as shown schematically in Fig. 7. We assign circumference C_2 , compliance $Y_2(z)$ cross-sectional area A_2 and open area ratio σ_2 to the outer liner. Since the cavity/region between inner liner and outer liner is sufficiently shallow, the acoustic equations in this region is 1D, as in the duct. Thus mass and momentum conservation equations through an annular control volume of length Δx hold as

$$\begin{aligned} \frac{\partial H_1'(t, x)}{\partial t} = & -\bar{c}^2 \frac{\partial u_1'(t, x)}{\partial x} - \bar{c}^2 \frac{C_1}{A_2} v_1'(t, x) \\ & + \bar{c}^2 \frac{C_2}{A_2} v_2'(t, x), \end{aligned} \quad (35)$$

FIG. 5. (Color online) Frequency spectrum of $\hat{H}_u^+(0, \omega)$.

$$\frac{\partial u_1'(t, x)}{\partial t} + \frac{\partial H_1'(t, x)}{\partial x} = 0, \quad (36)$$

where $v_2'(t, x)$ is the inward fluctuating flow through the outer liner, related to the stagnation enthalpy difference across the outer liner by

$$v_2^{(n)}(x) = \mathcal{B}_1 v_2^{(n-1)}(x) + \mathcal{A}_0 [H_1^{(n)}(x) - H_2^{(n)}(x)], \quad (37)$$

where $\mathcal{A}_0$ and $\mathcal{B}_1$ are coefficients of the z -domain compliance for outer liner $Z_2(z)$, which is given as

$$Y_2(z) = \frac{\mathcal{A}_0}{1 - \mathcal{B}_1 z^{-1}}. \quad (38)$$

It is worth noting that Eqs. (35) and (36) are corresponding to the frequency-domain ones as given in Eq. (2.21) of Eldredge and Dowling.³ Since the outer liner in this configuration is encased by a larger annular cavity. This indicates that $H_2'(t, x) = 0$. In addition, the presence of the rigid wall at the each end of the lined section gives rise to $u_1'(t, 0) = 0$ and $u_1'(t, L) = 0$. H_1' can be computed in the same way as Eqs. (10) and (11) are obtained.

The discrete-time forms of Eqs. (35)–(37) can be shown as

$$\frac{\partial H_{1,j}^{(n)}}{\partial t} = -\bar{c}^2 \frac{u_{1,j+1}^{(n-1)} - u_{1,j-1}^{(n-1)}}{2\Delta x} - \bar{c}^2 \frac{C_1}{A_2} v_{1,j}^{(n-1)} + \bar{c}^2 \frac{C_2}{A_2} v_{2,j}^{(n-1)}, \quad (39a)$$

$$\frac{\partial u_{1,j}^{(n)}}{\partial t} = -\frac{H_{1,j+1}^{(n-1)} - H_{1,j-1}^{(n-1)}}{2\Delta x}, \quad (39b)$$

$$\begin{aligned} \frac{\partial v_{2,j}^{(n)}}{\partial t} &= \frac{v_{2,j}^{(n)} - v_{2,j}^{(n-1)}}{\Delta t} \\ &= \frac{\mathcal{B}_1 - 1}{\Delta t} v_{2,j}^{(n-1)} + \frac{\mathcal{A}_0}{\Delta t} (H_{1,j}^{(n-1)} - H_{2,j}^{(n-1)}). \end{aligned} \quad (39c)$$

FIG. 6. (Color online) The variation of power absorption with forcing frequency in a lined duct with a single-liner for: (a) $\bar{M}_{h,1} = 0.023$ and $\bar{M}_u = 0$; (b) $\bar{M}_{h,1} = 0.041$ and $\bar{M}_u = 0$; (c) $\bar{M}_{h,1} = 0.03$ and $\bar{M}_u = 0.046$; (d) $\bar{M}_{h,1} = 0.009$ and $\bar{M}_u = 0.057$.

With the boundary values at $x = 0$ and $x = L$, systematically solving Eqs. (39a)–(39c) yields the time evolution of the acoustic waves and so the acoustic damping of the double-liner.

It is important to note that the mean bias flow velocities through both liners are related due to the mass conservation as shown:

$$\bar{v}_1 = \frac{C_2 \sigma_2}{C_1 \sigma_1} \bar{v}_2, \quad (40)$$

where $\bar{v}_1$ and $\bar{v}_2$ are the mean bias flow velocity through inner and outer liner, respectively.

B. Numerical results

1. Effect of forcing frequency on liners' damping

Figure 8 shows the variation of power absorption with the forcing frequency, as $\bar{M}_u = 0$ and the bias flow Mach number $\bar{M}_{h,1}$ is set to 2 different values. It can be seen that the acoustic damping of the double-liners varied dramatically with the frequency. The maximum damping reveals that about 82% of incident waves being absorbed, whereas

the minimum damping is as little as 10%. Furthermore, with bias flow Mach number increased from 0.023 to 0.041, the damping effect of the liners are slightly reduced in general. The maximum power absorption is reduced from 82% to 73%. Moreover, our results agree very well with the experimental ones from Eldredge and Dowling. This confirms that our model is able to simulate the liners damping in time domain. It is important to note that the biggest discrepancy between the experimental result and our simulation occurs at the frequency, $f \approx 535$ Hz. This is most likely due to the fact that a pressure node is located in the lined section and the power absorption of the liners depends strongly on the pressure fluctuations across the liners.

Figure 9(a) shows the variation of power absorption with forcing frequency, as the mean pipe flow is set to $\bar{M}_u = 0.046$ and bias flow $\bar{M}_{h,1} = 0.03$. Similar damping effect is observed as in the absence of mean pipe flow. The maximum power absorption is approximately 80%. Again our results agree favorably with those from Eldredge and Dowling. Figure 9(b) illustrate the liners damping varied with the forcing frequency under the condition of a small bias flow $\bar{M}_{h,1} = 0.009$ and a large mean pipe flow $\bar{M}_u = 0.057$. It can be seen that there are several damping peaks over the frequency range of 20 to 700 Hz. However, the maximum power absorption occurs at the third peak, which is different from Fig. 8. Moreover, the power absorption peaks become more flattened. This indicates that the frequency range corresponding to the maximum damping is broadened.

2. Effect of bias flow Mach number on liners' damping

The forgoing simulations showed that the model performed well in predicting the absorption of the liners over a range of frequencies, with constant flow conditions. Now we present the results when the frequency remained unchanged,

FIG. 7. Schematic of acoustic waves and flow quantities in a cylindrical duct with a double-liner attached.

FIG. 8. (Color online) The variation of power absorption with forcing frequency in a lined duct with a double-liner for: (a) $M_{h,1} = 0.023$ and $M_u = 0$, (b) $M_{h,1} = 0.041$ and $M_u = 0$.

$\bar{M}_u = 0.0$, and the mean liner bias flow was varied. The power absorption of the double-liner at four different frequencies is illustrated in Fig. 10. It can be seen that the general trend of the power absorption variation at each given frequency is that it increased and reached the maximum and then decreased, as the bias flow is increased. In addition, there is an optimum bias flow Mach number $\bar{M}_{h,1}$, which gives rise to the maximum liners damping. The model predicts the liner damping well at the lowest frequency, but performs poorly with increased frequency. This is most likely due to the increased excitation frequency and the presence of pressure nodes at the lined section. The double-liner is associated with using the simplified compliance model for each liner. This might also contribute to the discrepancy. Finally, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified curve-fitting. The difference between the simplified and unsimplified compliance models is negligible.

IV. TRANSMISSION LOSS

In some cases, perforated liners are implemented as acoustic dampers to prevent reflected acoustic waves in the pipe/combustor from transmitting back to the noise source (e.g., flame), thus alleviating the growth of acoustically driven instabilities. For this, the fraction of the acoustic wave energy that is transmitted downstream of the lined section (i.e., the transmission loss, β) can be used as an alternative to access the liners' damping effect. The variation of the transmission loss β , as defined as

$$\beta = 1 - \|H_u^+(0, t)\|^2 / \|H_u^-(0, t)\|^2. \quad (41)$$

The variation of transmission loss with the forcing frequency for the double-liner is also investigated as shown in Fig. 11. It can be seen that there are several transmission loss peaks, which is corresponding to maximum transmission loss of approximately 100%. The local minimum transmission loss

FIG. 9. (Color online) The variation of power absorption with forcing frequency in a lined duct with a double-liner for: (a) $M_{h,1} = 0.03$ and $M_u = 0.046$, (b) $M_{h,1} = 0.009$ and $M_u = 0.057$.

FIG. 10. (Color online) The variation of power absorption with bias flow Mach number at 4 different frequencies, as there is no mean flow through the lined duct with a double-liner (i.e., $\bar{M}_u = 0.0$).

is reduced with increased forcing frequency. In general, our model captures the main damping characteristics of the double-liner in terms of transmission loss. Furthermore, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified curve-fitting one with discrepancy less than 1%. This indicates the potential wide application of such time-domain compliance model in simulating liner's damping.

V. DISCUSSION AND CONCLUSIONS

Perforated liners are widely applied in aero-engines and gas turbine combustors to suppress combustion instabilities. Before implementing such liners, the acoustic damping of the liners need to be characterized and estimated in terms of

FIG. 11. (Color online) The variations of transmission loss with forcing frequency in a lined duct with a double-liner for $\bar{M}_u = 0.0$.

power absorption or transmission loss. For this, we developed a 1D numerical time-domain model to simulate the acoustic plane waves propagation and dissipation in a cylindrical lined duct. It performs in time-domain and involves the development of a compliance model based on the principle that unsteady vortex shedding from the perforated holes is the primary mechanism for acoustic waves damping. The compliance model with Rayleigh conductivity⁶ implemented is formulated in a simplified rational form in z -domain. However, curve fitting is involved with estimating the compliance. For different flow conditions, curve fitting needs to be repeated. These limits the application of such compliance model.

In order to make our model more general and applicable, a simplified compliance formulation is developed. It is based on the simplified Rayleigh conductivity as proposed by Luong *et al.*¹⁵ Both simplified and unsimplified compliance models are then incorporated into partial differential equations (PDEs) which describe the lined-duct system. To solve the system PDEs, method of lines (MOL) is applied. The numerical results of two lined duct configurations, single- and double-liner encased in a large cavity, are evaluated and compared with those from the experiment and the frequency-domain model of Eldredge and Dowling.³ Good agreement is observed. Furthermore, the compliance model with simplified Rayleigh conductivity implemented performs as well as the unsimplified one, with discrepancy around 1%. Since the simplified compliance does not involve curve-fitting, this indicates it has great potential to be applied in time-domain model to simulate liner's damping performance. In general, the numerical model developed is shown to be able to predict the liners' damping in time domain, of which the acoustic waves either side of the lined section need to be decomposed and characterized.²³

It is important to note that the liners' damping is estimated over a broad frequency range all at once in our simulation. This is different from the frequency-domain models, which calculate the liners' damping one frequency at a time. Furthermore, the numerical time-domain liner model can be integrated with other CFD models to simulate the flow features. For example, it can be combined with thermoacoustic model of lean premixed pre-vaporized combustors with perforated liners confined to study time evolution of flow disturbances in triggering combustion instability,^{24,25} and the interaction between flow disturbances and flame. The time-domain model can also be applied in computational aeroacoustics (CAA).⁹ Finally, time-dependent boundary conditions or variables can be easily implemented in our time-domain model. These features indicate that developing such a time-domain liner model is useful for understanding the effect of acoustic liners on the flow field and noise damping.^{26,27}

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APPENDIX A: RAYLEIGH CONDUCTIVITY

From Howe,⁶ Rayleigh conductivity is given as

$$K_a = 2a(\gamma + j\sigma), \quad (\text{A1})$$

where γ and σ are given as

$$\gamma = \frac{I_1^2(S_t) \left[1 + \frac{1}{S_t} \right] + \frac{4}{\pi^2} e^{2S_t} \cosh(S_t) K_1^2(S_t) \left[\cosh(S_t) - \frac{\sinh(S_t)}{S_t} \right]}{I_1^2(S_t) + \frac{4}{\pi^2} e^{2S_t} \cosh^2(S_t) K_1^2(S_t)}, \quad \sigma = \frac{\left[\frac{2}{\pi S_t} \right] I_1(S_t) K_1(S_t) e^{2S_t}}{I_1^2(S_t) + \frac{4}{\pi^2} e^{2S_t} \cosh^2(S_t) K_1^2(S_t)}, \quad (\text{A2})$$

where I_1 and K_1 are modified Bessel functions, S_t is the Strouhal number $S_t = \omega R / \bar{U}_c$.

APPENDIX B: PARAMETERS USED IN THE NUMERICAL MODEL

The parameters used in our time-domain simulations are shown in Table I.

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