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Framework for gradient integration by combining radial basis functions method and least-squares method

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A framework with a combination of the radial basis functions (RBFs) method and the least-squares integration method is proposed to improve the integration process from gradient to shape. The principle of the framework is described, and the performance of the proposed method is investigated by simulation. Improvement in accuracy is verified by comparing the result with the usual RBFs-based subset-by-subset stitching method. The proposed method is accurate, automatic, easily implemented, and robust and even works with incomplete data. © 2013 Optical Society of America

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1. Introduction

Optical 3D shape metrology is widely used for inspection purposes in industrial applications [1,2]. Instead of directly measuring the dimensions from optical signals, some optical methods, such as the fringe reflection technique [3–6], indirectly obtain the dimensional results by measuring the surface gradient and then integrating the gradient data to get the shape. Consequently, a 2D integration process is necessary to nicely reconstruct the surface shape from gradient data.

There are mainly two types of integration methods in the literature. One handles gradient datasets of a small size (e.g., $40 \times 40 \times 2$), such as the radial basis functions (RBFs)-based integration method [7], which generally divides the dataset into small subsets and stitches the resultant data after integration on each subset. The other group of methods, such as the least-squares integration method, can deal with a larger dataset (e.g., $500 \times 500 \times 2$) [8–10]. However, considering the higher-order truncation error of the shape to reconstruct, the RBFs-based method uses

abundant gradient information in a surrounding area and hence will be more accurate and reliable, because the least-squares integration method builds up the relationship between neighboring pixels in one direction (x or y direction) only. If the dataset is larger than the size the algorithm can handle, a stitching process is required anyway, which can commonly be carried out along some path by adjusting the height levels of adjacent subsets. Obviously the integration error will influence the stitching process, and, as a result, the error will propagate on the whole surface.

The inputs to the gradient-to-shape integration issue are the slope data and the in-plane coordinates in both x and y directions. The general size of valid gradient dataset can be or is more than $1280 \times 960 \times 2 \approx 2 \times 10^6$ in a practical measurement, and the 2D integration method usually requires a huge memory space to optimize the integration result by considering all information in the optimizing region. The whole dataset is commonly too large to handle. In this situation, it is reasonable to divide the entire dataset into many overlapping subsets, integrate each subset individually, and stitch them together based on the overlapping regions. Based on the observation of the two types of integration methods above, this

work aims to improve the integration process by combining them. The whole integration plus stitching process should be controlled with a least-squares sense to achieve an optimized result. The rest of this article is organized as follows. Section 2 explains the principle of the proposed stitching approach with a least-squares method to control the error propagation and to increase the accuracy. Section 3 shows the feasibility of the proposed method with a simulation by using a gradient dataset with a size of $2000 \times 2000 \times 2$. Section 4 demonstrates an experiment to verify the proposed framework in a practical measurement. Section 5 discusses the merits of the new method, and Section 6 concludes the work.

2. Principle of the Framework

In the proposed method, the integration is completed as a framework with five major steps as illustrated in Fig. 1. First, the valid gradient data should be identified for integration. Second, the dataset is divided into many overlapping small pieces (subsets) due to the memory limits. Third, the RBFs-based integration method is applied to reconstruct the shape from the gradient in each subset [7]. Fourth, the least-squares integration method is adopted for stitching all the subsets together as one. Finally, the data within the area in common are merged to form a whole piece of a 3D result.

Here it must be noticed that the fourth step, the stitching process, is implemented by converting the stitching problem to another 2D integration. In the stitching problem, the height levels of the subsets

are to be adjusted. The locally expected amount of height adjustment can be easily calculated as the average height difference within the overlapping region of the two neighboring subsets as shown in Fig. 2. From another point of view, if a subset is treated as one point with a certain height, the average height difference can be considered as the slope value at the center of two neighboring points (subsets) as shown in Fig. 2(b). Therefore, the stitching of these subsets becomes equivalent to a 2D integration process with known slope values. Furthermore, it needs to note these slope values are not derivatives from analytical solutions, but first-order central differential values by taking numerical differentiation. On the other hand, the least-squares integration method is essentially good at dealing with the integration problem with numerical differential values, since there is no higher-order truncation error in this case. Additionally, the least-squares integration method is able to handle a larger dataset. For example, if the subset is 40×40 pixels and the overlapping region is 25%, a large dataset of 9000×9000 pixels needs about 301×301 subsets, which can be easily handled by the least-squares integration method. Therefore, the least-squares integration is adopted for the stitching process.

According to the grid model in Fig. 2(b), the grid coordinates (m, n) and average height differences within overlaps, which are also considered as the orthogonal slope values $p_{m,n}$ and $q_{m,n}$, are known as the inputs for stitching process. Hence, the interested height level for subsets $z_{m,n}$ have the following relationship:

$$\begin{cases} z_{m,n+1} - z_{m,n} = p_{m,n} \times (n+1 - n) = p_{m,n}, & m = 1, \dots, M, n = 1, \dots, N-1 \\ z_{m+1,n} - z_{m,n} = q_{m,n} \times (m+1 - m) = q_{m,n}, & m = 1, \dots, M-1, n = 1, \dots, N \end{cases} \quad (1)$$

which can be rewritten in terms of matrices as

$$\mathbf{DZ} = \mathbf{G}, \quad (2)$$

where

Fig. 2. Stitching problem can be considered an issue of integration with central differential values. (a) Subsets with overlaps. (b) The grid model of the equivalent integration problem.

Fig. 1. Entire 3D shape result can be reconstructed from a gradient through the five-step framework.

$$\mathbf{D} = \begin{bmatrix} -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & -1 & 0 & \cdots & 0 & 1 \\ -1 & 1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & -1 & 1 \end{bmatrix},$$

$$\mathbf{Z} = \begin{bmatrix} z_{1,1} \\ z_{2,1} \\ \vdots \\ z_{M,N} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} p_{1,1} \\ p_{1,2} \\ \vdots \\ p_{M,N-1} \\ q_{1,1} \\ q_{2,1} \\ \vdots \\ q_{M-1,N} \end{bmatrix}.$$

In practical situations, Eq. (2) is usually overdetermined. Therefore, the height level of subsets $z_{m,n}$ can be optimized with a least-squares estimation. The average height level of the subset at location of (m, n) are then adjusted by subtracting $z_{m,n}$.

3. Simulation

In order to investigate the validity of the proposed framework, a simulation is conducted, which demonstrates its ability to operate in the presence of noise. The true height shown in Fig. 3(a) is defined by

$$z = 3(1-x)^2 \cdot e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) \cdot e^{-x^2-y^2} - \frac{1}{3}e^{-(x+1)^2-y^2}, \quad (3)$$

Fig. 3. True height distribution (a) is generated as the ground truth. The gradient data (b) dz/dx and (c) dz/dy are generated with additive normally distributed angular noise.

where the in-plane dimensions x and y are both limited within a range from -3 to 3 mm with a sampling of 2000×2000 points.

In a practical fringe reflectometric measurement, noise can be considered as normally distributed on the fringe phase and furthermore can be approximated as normally distributed on surface normal directions. Normally distributed angular noise with a standard deviation of 10 arcsec is added in the normal directions, which is a typical noise level in the fringe reflection technique. The associated noisy slopes in x and y directions are shown in Figs. 3(b) and 3(c). The RBFs-based integration method is used to integrate the shape within subsets with a subset size of 20×20 pixels and 25% overlaps between neighboring subsets.

The results from the RBFs-based integration method are 133×133 pieces of subset with height distributions. The following step is to stitch these small pieces together to obtain the whole surface shape. For comparison purposes, the datasets are directly stitched subset by subset by adjusting the relative height levels of two adjacent subsets. The resultant height error is shown in Fig. 4(a), from which the error propagation is easily observed. Then the proposed stitching method is applied with a result shown in Fig. 4(b). The resultant height error map indicates the error propagation is well controlled by the proposed stitching method. The superiority of the proposed method is also shown by comparing the error histograms of these two methods in Figs. 4(c) and 4(d). The standard deviation of height errors is only 3.2 nm with the proposed method compared to 9.4 nm by the subset-by-subset stitching mean.

In addition, both methods are compared under different noise levels. As indicated in Fig. 5, with the increase of the standard deviation of the angular noise, the resultant height errors are getting higher for both methods. However, the stitching with the least-squares integration method outperforms the subset-by-subset one.

4. Experiment

An experimental investigation of the feasibility of the proposed framework is carried out as well.

Fig. 4. With a comparison of results, the proposed method shows its superiority apparently. (a) Reconstructed height with subset-by-subset stitching. (b) Height error of subset-by-subset stitching. (c) Histogram of (b). (d) Reconstructed height with stitching with least-squares integration. (e) Height error of stitching with least-squares integration. (f) Histogram of (e).

A specular sample with many periodic surface structures in Fig. 6(a) is measured with the fringe reflectometric system. Multifrequency phase-shifted fringe patterns are displayed on a screen, and the sample-reflected images are captured by a CCD camera as shown in Fig. 6(b).

As an intermediate result, Fig. 6(c) shows the slope data in system predefined x and y directions. The proposed framework is then utilized to integrate the gradient data to reconstruct the 3D shape.

Fig. 5. Height errors become larger along with the noise being severer, but the stitching with least-squares integration method is superior to the subset-by-subset stitching.

Although the sample has some regions that are not specular reflective enough, which leads to many invalid gradient data, the framework is still able to successfully reconstruct the dimensional information from the incomplete dataset as shown in Fig. 6(d).

5. Discussion

The proposed method aims to improve the integration procedure. Several merits of the proposed method can be identified as follows.

1. **Accurate.** The height distribution is optimized by considering all valid subsets with a least-squares sense. The residual error is mainly due to the existence of noise on gradient data. This integration is path-independent, which keeps the residual error distribution free from its propagation along the integration path as the path-dependent methods behave.

2. **Automatic and easily implemented.** The framework is simple and easy to implement in order to automatically stitch subsets after integrating the gradient locally.

3. **Robust.** The proposed framework is quite stable and able to deal with the incomplete dataset with missing data points or subsets. If the valid gradient data only have a certain region, and there are many data unavailable inside as shown in Figs. 7(a) and 7(b), the 3D shape of the surface with “holes” is still able to be accurately reconstructed with the proposed method as shown in Figs. 7(c) and 7(d).

Fig. 6. Feasibility of the proposed framework is verified with an experiment. (a) The sample photo. (b) The captured fringe patterns under a fringe reflectometric system. (c) Slope data. (d) The reconstructed 3D shape.

Fig. 7. With the proposed method, the integration is able to be accomplished with incomplete datasets. (a) Slope in x direction. (b) Slope in y direction. (c) 3D shape. (d) Height error.

6. Conclusion

This work attempts to improve the process in the 2D integration method with subsets in order to provide a better solution. The proposed method is described by introducing its basic principle and the implementation of the stitching step. In addition, the performance of the proposed framework is investigated with both simulation and experiments. Improvement in accuracy is verified with a comparison of the subset-by-subset stitching method. The merits of the proposed method are accurate, automatic, easily implemented, and robust to stitch the datasets, even when the datasets of the subsets are incomplete.

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