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Application of pseudo-fluid approximation to evaluation of flow velocity through gravel beds

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Abstract: Flows seeping through a gravel bed are usually non-Darcian and closely related to non-linear drag. Such flows may be significantly affected by particle shape and bed configuration. In this study, a pseudo-fluid model is developed to calculate average flow velocity through gravel beds. The proposed approach is able to take into account particle shape effect using the drag coefficient associated with an isolated sediment grain and also bed configuration effect in terms of apparent viscosity. The model was then calibrated with ten series of laboratory data, which were collected using vertical columns packed with spherical and natural gravels. Finally, the model was successfully applied to estimate total flow discharges for laboratory-scale open channel flows over a gravel bed.

Keywords: apparent viscosity; drag coefficient; gravel bed; pseudo-fluid; settling velocity

Introduction

A flow system of particle-fluid mixture may be treated as a single phase characterized with apparent density and viscosity, which would yield a pseudo-fluid model. Such models have been successfully applied in the description of characteristics of various particle-fluid mixtures, for example, in studying fluidization [1] and the transport of high-concentrated sediment [2]. Cheng [3] also shows that the hindered settling velocity of sediment particles could be well estimated based on the pseudo-fluid concept.

Although the pseudo-fluid approximation usually applies for particle-fluid mixtures of which both phases are mobile, it could also be extended to flow passing through fixed solid phase. Such an attempt was recently reported by Cheng [4], who developed a pseudo-fluid approach to estimate the drag coefficient for cylinder-simulated vegetation stems presented in open channel flows. To derive the approach, an analogy was made between the channel flow through vegetation stems and the settling of a cylinder array, which provides an effective connection between the parameters used in the pseudo-fluid model and those measurable for open channel flows subject to the simulated vegetation. The result obtained by Cheng [4] shows that the relationship between drag coefficient and Reynolds number, which applies for an isolated cylinder, could be generalised for evaluation of the drag coefficient for one cylinder in an array. The present study aims to develop a similar method to calculate flow velocity through a sediment bed comprised of immobile gravels.

Flows passing through a sediment bed comprised of gravels are usually non-

Darcian, as observed in flows through other coarse materials like rockfills and waste dumps. Non-Darcian flows are closely related to nonlinear drag. Some theoretical attempts have been devoted to associate the nonlinear drag with inertial and/or turbulent effects of viscous flow [5-7]. However, the current understanding of relevant flow phenomena is limited and thus it is still challenging to theoretically describe non-Darcian flows [8]. Alternatively, the pseudo-fluid concept is implemented in this study to indirectly consider effects of non-linear drag on flow through porous sediment beds.

This paper is outlined as follows. First, a pseudo-fluid approximation is applied to develop an approach to the quantification of the average velocity of the flow through a packed bed. Then, the resulted pseudo-fluid model is calibrated using experimental data. Finally, it is shown that the model can be used to estimate the total flow discharge of laboratory-scale open channel flows over a gravel bed.

Pseudo-fluid approximation

To apply the pseudo-fluid concept, we start with the terminal velocity of a single particle settling in a stationary fluid. For this case, the effective weight of the particle is equal to the drag induced by its downward motion relative to the fluid. The drag is expressed as

$$F_D = C_D \frac{\pi D^2}{4} \frac{\rho w^2}{2} \quad (1)$$

where C_D is the drag coefficient, D is the diameter of particle, w is the settling velocity, and ρ is the density of fluid. The effective weight of the particle is

$$W = (\rho_s - \rho)g \frac{\pi D^3}{6} \quad (2)$$

where ρ_s is the particle density and g is the gravitational acceleration. Under the terminal condition, $F_D = W$, and thus with Eqs. (1) and (2),

$$C_D = \frac{4}{3} \frac{\Delta g D}{w^2} \quad (3)$$

where $\Delta = (\rho_s - \rho)/\rho$ is the relative density difference.

It is noted that C_D generally varies with Reynolds number Re defined as wD/ν , where ν is the kinematic viscosity of fluid. When the settling occurs in the Stokes regime, e.g. for $Re < 1$, C_D is linearly proportional to $1/Re$. In the inertial regime, e.g. for $Re > 1000$, the viscous effect is insignificant and C_D can be approximated as a constant. In between the two regimes, the dependence of C_D on Re is complex. In the literature, many empirical formulas have been proposed to describe the relationship of C_D and Re in a wide range of Re [9-12]. However, for simplicity, the variation of C_D with Re can be approximated using the following three-parameter formula,

$$(C_D)^m = \left(\frac{M}{Re} \right)^m + (N)^m \quad (4)$$

where M and N are constants and m is an exponent, all varying largely with particle shape. For example, for natural sediment grains, $M = 32$, $N = 1$ and $m = 2/3$, as proposed by Cheng [11]. For spherical particles, it can be shown that by taking $M = 24$, $N = 0.4$ and $m = 0.6$, Eq. (4) provides a good representation of classical data [12].

By noting

$$\frac{4}{3} C_D \text{Re}^2 = \frac{\Delta g D^3}{\nu^2} = D_*^3 \quad (5)$$

where

$$D_* = \left(\frac{\Delta g}{\nu^2} \right)^{1/3} D \quad (6)$$

is the dimensionless diameter, Eq. (4) can be rewritten to be

$$C_D = \frac{4}{3} D_*^3 \left(\sqrt{\frac{1}{4} \left(\frac{M}{N} \right)^{2m} + \left(\frac{4}{3} \right)^m \frac{D_*^{3m}}{N^m}} - \frac{1}{2} \left(\frac{M}{N} \right)^m \right)^{-2/m} \quad (7)$$

Eq. (7) shows that for a grain of particular shape with known values of M , N and n , C_D can be calculated with given Δ , D and ν . In the subsequent analysis, Eq. (7) will be used to develop a pseudo-fluid model. However, it is noted that Δ is not a parameter physically applicable for a packed bed, but it can be expressed as a function of the hydraulic gradient and porosity, as shown later.

Drag exerting on a grain in sediment bed

Now consider a sediment bed comprised of uniform grains. It is assumed that the bed in the streamwise direction is long enough so that the flow through the bed can be considered fully developed. Two scenarios are compared here, as sketched in Fig.

1. The first is a sediment bed applied with an upward flow, of which the hydraulic gradient is S , the cross-sectional average velocity is V , and the average velocity through the pores is V_s ($=V/\varepsilon$, where ε is the porosity). For this scenario, if the drag coefficient is denoted by C_{Ds} , the average drag acting on a grain in the bed is

$$F_{Ds} = C_{Ds} \frac{\pi D^2}{4} \frac{\rho V_s^2}{2} \quad (8)$$

where subscript 's' is used to denote the parameters related to the sediment bed.

Furthermore, a unit volume is selected in the sediment bed. In this volume, the total number of the grains is $n = (1-\varepsilon)/(\pi D^3/6)$, and the total seepage force is $\rho g S$ [13, 14]. Then, the average drag acting on a grain in the sediment bed is

$$F_{Ds} = \frac{\rho g S}{n} = \frac{\pi D^3}{6} \frac{\rho g S}{1-\varepsilon} \quad (9)$$

Here, it is assumed that wall friction is negligible in comparison with the total drag related to all grains. In other words, the hydraulic gradient S is solely associated with the energy loss caused by the grains. With Eqs. (8) and (9),

$$C_{Ds} = \frac{4}{3(1-\varepsilon)} \frac{g D S}{V_s^2} \quad (10)$$

The second scenario concerns the settling of the same grains that are packed in the same configuration as in the first scenario [see Fig. 1(b)]. It is assumed that the settling occurs in a stationary fluid, and the settling velocity of the grains relative to the fluid is set at V_s , the same velocity as that observed in the first scenario. Therefore, in terms of the grain size, porosity, relative flow velocity, fluid viscosity and induced drag, both scenarios could be considered equivalent.

When the settling is steady, the drag induced by each grain is equal to its effective weight,

$$F_{Ds} = \frac{\pi D^3}{6} (\rho_s - \rho) g = \frac{\pi D^3}{6} \Delta \rho g \quad (11)$$

Eq. (11) shows that the drag is proportional to the density difference. It means that the density difference serves as the driving force that makes possible the settling of

the packed bed. In comparison, in the first scenario, the driving force originates from the pressure drop quantified using the hydraulic gradient [see Eq. (9)].

By noting the equivalent drag assumed for the two scenarios, with Eqs. (8) and (11),

$$C_{Ds} = \frac{4}{3} \frac{\Delta g D}{V_s^2} \quad (12)$$

Furthermore, by comparing Eq. (12) with Eq. (10), one gets

$$\Delta = \frac{S}{1 - \varepsilon} \quad (13)$$

Eq. (13) provides an important relationship between Δ and S . It implies that the flow-induced drag for a packed bed can be indirectly evaluated by considering the same bed settling in a fluid. However, it should be mentioned that the relative density difference, Δ , serves only as a working parameter. It is not physically related to the real densities of the particle and fluid involved in the flow through the packed bed, as in the first scenario. Instead, it provides a connection between the two scenarios, which ensures that the pore velocity and thus the drag induced by the settling (in the second scenario) are the same as those caused by the pressure drop (in the first scenario). Therefore, with the above consideration, whenever Δ is involved in the pseudo-fluid model to be proposed, it will be replaced with $S/(1-\varepsilon)$.

Pseudo-fluid model

For a single grain setting in a stationary fluid, the relationship between C_D and D^* is given in Eq. (7). Based on the pseudo-fluid concept, the same relationship also applies for investigating the settling of a grain in a packed bed, provided that the apparent density and viscosity are used. Therefore, Eq. (7) is rewritten as

$$C'_D = \frac{4}{3} D'^3 \left(\sqrt{\frac{1}{4} \left(\frac{M}{N} \right)^{2m} + \left(\frac{4}{3} \right)^m \frac{D'^{3m}}{N^m}} - \frac{1}{2} \left(\frac{M}{N} \right)^m \right)^{-2/m} \quad (14)$$

where superscript “ ’ ” denotes the apparent parameters used for the grain-fluid mixture. The apparent drag coefficient C'_D and dimensionless grain diameter D'_* are defined as follows:

$$C'_D = \frac{4}{3} \frac{\Delta' d g}{v_s'^2} \quad (15)$$

$$D'_* = \left(\frac{\Delta' g}{v'^2} \right)^{1/3} D \quad (16)$$

where v' is the apparent kinematic viscosity, ρ' is the apparent density,

$$\rho' = \rho_s(1 - \epsilon) + \rho\epsilon \quad (17)$$

and Δ' is the apparent relative density difference,

$$\Delta' = \frac{\rho_s - \rho'}{\rho'} \quad (18)$$

It is noted that Eqs. (15) and (16) are similar to Eqs. (3) and (6). With Eqs. (17) and (18),

$$\Delta' = \frac{\epsilon \Delta}{1 + (1 - \epsilon) \Delta} \quad (19)$$

Then Eq. (16) is rewritten to be

$$D'_* = \left[\frac{\varepsilon \Delta}{1 + (1 - \varepsilon) \Delta} \frac{g}{v_r^2 v^2} \right]^{1/3} D \quad (20)$$

where $v_r (= v'/v)$ is the relative kinematic viscosity. With the dynamic viscosity of the fluid (μ) and that of the mixture (μ'),

$$v_r = \frac{\mu' \rho}{\mu \rho'} = \frac{\mu_r}{1 + (1 - \varepsilon) \Delta} \quad (21)$$

where $\mu_r (= \mu'/\mu)$ is the relative dynamic viscosity. Moreover, by noting that $\Delta = S/(1 - \varepsilon)$ [see Eq. (13)], Eqs. (20) can be further expressed as

$$D'_* = \left(\frac{\varepsilon(1 + S)S}{1 - \varepsilon} \frac{g}{\mu_r^2 v^2} \right)^{1/3} D \quad (22)$$

Similarly, with Eqs. (19) and (13), Eq. (15) is rewritten to be

$$C'_D = \frac{4}{3} \frac{\varepsilon}{1 - \varepsilon} \frac{S}{1 + S} \frac{Dg}{V_s^2} \quad (23)$$

With Eq. (22), D'_* can be determined if the five parameters (ε , D , S , v and μ_r) are known. Then, C'_D can be found from Eq. (14) and finally V_s can be obtained using Eq. (23), i.e.

$$V_s = \sqrt{\frac{4}{3} \frac{\varepsilon}{1 - \varepsilon} \frac{S}{1 + S} \frac{Dg}{C'_D}} \quad (24)$$

Eqs. (14), (22) and (24) form the model system for the calculation of the flow velocity through a sediment bed. Among the five parameters, ε , D , S and v are usually available for particular bed and flow configuration. However, how to fix μ_r is not clear. In studying the particle-fluid mixture, the apparent viscosity is often expressed as a function of the particle concentration, see reviews by Poletto and Joseph [15] and Cheng and Law [16]. Cheng [4] found that for flow passing through a

fixed cylinder array, μ_r could be linearly related to the fraction of the solid phase, $(1 - \varepsilon)$,

$$\mu_r = 1 + C_\mu(1 - \varepsilon) \quad (25)$$

where C_μ is a constant to be calibrated. Eq. (25) is also used for the present study.

Model Calibration

Altogether 10 sets of experimental data were used for calibration. Five sets of data were collected in the present study and the rest was taken from a laboratory study conducted by Mints and Shubert [17]. All the datasets were derived from experiments of flows through various sediment beds, each being comprised of uniform grains packed in a vertical column. In the present study, tests were conducted with three types of glass beads ($D = 11, 16, 25$ mm), and two types of natural gravels ($D = 3.2, 13.2$ mm). Each test was conducted by packing grains in a cylinder of 90 mm in diameter and 2000 mm in length. The flow discharge was measured using a flowmeter and the pressure drop was recorded using a manometer and also a differential pressure transducer. The data reported by Mints and Shubert [17] included coal grains ($D = 0.94, 2.1, 3.5, 5.1$ mm) and quartz gravels ($D = 3.7$ mm). The test cylinder was 103.2 mm in diameter and 3000 mm in length.

To avoid possible effect of grain fluidization, all the data are filtered by applying the limitation of $S < (\rho_s/\rho - 1)(1 - \varepsilon)$ [14]. For the tests with small ratio of cylinder diameter D_c to grain diameter D , the measured pressure drop could be

affected by wall friction to certain extent. This effect was corrected by modifying the grain diameter D to D_M as follows [18, 19]:

$$D_M = \left(\frac{1}{D} + \frac{2}{3} \frac{1}{(1-\varepsilon)D_c} \right)^{-1} \quad (26)$$

From the preceding derivation, the procedure for calculating the average velocity through sediment bed is summarized as follows:

- (1) For given ε , D , S and v , use Eqs. (22) and (25) to calculate D'_* ;
- (2) Find C_D' using Eq. (14); and
- (3) Then, calculate V_s with Eq. (24).

The calculation results show that the difference between the calculated and measured flow velocity minimizes when C_{μ} is taken to be approximately 30, as shown in Fig. 2. The difference was assessed using two error parameters, one being defined as $Err_1 = (| \text{measured velocity} - \text{calculated velocity} | / \text{measured velocity})$ and the other given by $Err_2 = [\log(\text{measured velocity}) - \log(\text{calculated velocity})]$. In Fig. 2, both Err_1 and Err_2 presented are the average values calculated using the 10 datasets.

The velocity comparisons are shown in Figures 3 to 5. Fig. 3 compares the calculated velocity with the measured velocity for the case of spherical grains. The calculation was conducted with $M = 24$, $N = 0.4$ and $m = 0.6$. These three constants were obtained by comparing Eq. (7) with the classical database of the settling velocity of spheres [12]. Fig. 4 shows the comparison for the case of coal grains, in which the calculation was conducted with $M = 32$, $N = 1.5$ and $m = 2/3$. The three constants were calibrated directly using the settling velocity of the coal grains measured by Mints and Shubert [17]. Fig. 5 shows the comparison for the case of natural gravels, in which the calculated velocity was obtained with $M = 30$, $N = 1.3$

and $m = 1.5$. These three constants were also calibrated directly using the settling velocity of the gravels measured by Mints and Shubert [17]. From Figures 3 to 5, it follows that the calculations with the calibrated C_μ are generally consistent with the measurements, implying that the pseudo-fluid approximation is useful for evaluating the bulk flow seeping through gravel beds.

Application for open channel flows over gravel bed

To further verify the proposed model, additional experiments were also performed in a laboratory-scale flume with simulated gravel beds. The flume was 3 m long and 0.1 m wide, with an adjustable bed slope. The flow discharge was measured using an electromagnetic flow meter, of which the readings were checked against volumetric measurements for small discharges. The channel bed was 0.039 m in thickness, comprising four identical layers of glass beads (0.11 m in diameter), as sketched in Fig. 6. The bed was prepared by packing spheres in a hexagonal lattice for each layer and nesting all layers together, which yielded an average porosity of 0.357.

For open channel flows over a gravel bed, the flow velocity above the bed is usually much larger than that through the bed. This suggests that the subsurface flow discharge would be negligible in comparison with that in the surface layer if the sediment layer is limited in thickness. Such cases, though often observed in laboratory experiments, are not in favour of the objective here, when the proposed pseudo-fluid model is employed to evaluate the flow rate through gravel bed under open channel flow conditions. To this end, the flow depth above the gravel bed was

selected to be largely in the order of the glass bead diameter, and thus the tests were performed in the low flow condition. As a result, the subsurface flow discharge through the sediment bed was generally comparable to that in the surface layer.

Altogether 76 tests were completed with the channel slope varying from 0.0052 to 0.071 and the flow depth from 0.001 to 0.024 m. Given the small flow depth about the gravel bed, the vertical velocity profile was not measured in this study. Instead, the flow velocity at the free surface was taken by observing the average speed of paper scraps placed on the flowing water surface. The experimental data are summarised in Table 1.

To calculate the average velocity of the flow passing through the packed bed and also that above the bed, it is assumed that the velocity profile consists of two segments, as shown in Fig. 7. Inside the porous sediment bed, the flow velocity is considered uniformly distributed and equal to U_p . In the surface layer, the velocity profile can be described approximately using a power law, and the average flow velocity is U_s . Here the effect of the channel bed and the transition zone at the interface are not considered as the calculation concerns bulk flow velocities only.

Using the power law, the velocity profile in the surface layer is expressed as

$$\frac{u - U_p}{u_{\max} - U_p} = \left(\frac{y}{h_s} \right)^{1/\beta} \quad (27)$$

where $1/\beta$ is an exponent. Previous studies [20, 21] show that though being taken to be a constant, β varies slightly with the hydrodynamic height of the bed roughness in comparison with the flow depth, and its value can be evaluated using the friction factor. To estimate the value of β for the flows investigated in this study, the friction factor was first evaluated with the observed flow velocity at the free surface. Then

the value of β was calculated using the empirical formulas [20, 21]. The calculations show that β may vary approximately from 2.5 to 4.5. This result is consistent with the β -value recommended by Chen [20], who reported that β varies from 3.0 to 4.0 for u/u_* in the range of 2.2 to 17, where u_* is the shear velocity. Here, the upper limit of u/u_* was computed as $u_{\max}/u_*$, and the lower limit of u/u_* was taken to be U_p/u_* , in which u_* was calculated using the flow depth above the sediment bed and channel slope, and U_p was calculated for the case of the flow depth h_s being less than 0.001 m. With the above consideration, β is taken to be 3.5 in the subsequent analysis.

Integrating Eq. (27) from $y = 0$ to $y = h_s$,

$$\frac{U_s - U_p}{u_{\max} - U_p} = \frac{\beta}{1 + \beta} \quad (28)$$

Then,

$$U_s = \frac{\beta u_{\max} + U_p}{1 + \beta} \quad (29)$$

Finally the total flow discharge can be calculated as

$$\begin{aligned} Q &= Bh_p U_p + Bh_s U_s \\ &= Bh_p U_p + \frac{Bh_s}{1 + \beta} (\beta u_{\max} + U_p) \end{aligned} \quad (30)$$

In the following, we first estimate U_p using the proposed model system, i.e. Eqs. (14), (22) and (24), with the glass bead diameter, porosity and channel slope, and then calculate Q using Eq. (30) with the measured flow velocity at the free surface. Fig. 8 shows the comparison between the calculated and measured Q . It can be seen that the flow rate is slightly overpredicted by Eq. (30), but the calculations are generally in good agreement with the measurements. This shows that the proposed pseudo-

fluid model performs reasonably well in the calculation of the flow rate passing through gravel beds in the presence of surface flows.

Summary

This study demonstrates that the pseudo-fluid approximation is useful for calculating the average velocity of flow passing through a sediment bed. By implementing the pseudo-fluid concept, the drag coefficient derived from the settling of an isolated grain is extended for the investigation of flow passing through porous bed comprised of similar grains. The apparent viscosity involved in the pseudo-fluid model was calibrated using ten sets of experimental data. The flow through a sediment bed is largely subject to particle shape and bed configuration. The result obtained in this study implies that the particle shape effect could be effectively considered using the drag coefficient associated with a single sediment grain, while the bed configuration effect is included in terms of the apparent viscosity.

In this study, the application of the proposed model to open channel flows over a gravel bed is limited to small-scale laboratory experiments. Future efforts are needed to extend the model to large-scale experiments and even field studies.

Notation

B = channel width [m]

C_D = drag coefficient [-]

C'_D = apparent drag coefficient [-]

C_{Ds} = drag coefficient for a grain in a sediment bed [-]

C_μ = constant [-]

D = sediment grain diameter [m]

D^* = dimensionless grain diameter [-]

D'^* = apparent dimensionless grain diameter [-]

F_D = drag [kg m/s^2]

F_{Ds} = drag acting on a grain in a sediment bed [kg m/s^2]

g = gravitational acceleration [m/s^2]

h_s = flow depth in the surface layer [m]

h_p = thickness of the sediment layer [m]

M = constant [-]

m = exponent [-]

N = constant [-]

n = number of grains in the bed of unit volume [-]

Q = total flow discharge [m^3/s]

$Re = wD/\nu$ = Reynolds number [-]

S = hydraulic gradient [-]

U_s = depth-averaged flow velocity in the surface layer [m/s]

U_p = depth-averaged flow velocity in the porous sediment layer [m/s]

u = longitudinal flow velocity [m/s]

$u_{\max}$ = flow velocity at the free surface [m/s]

u^* = shear velocity [m/s]

V = cross-sectional average velocity [m/s]

$V_s = V/\varepsilon$ = average pore velocity [m/s]

W = effective weight of grain [kg m/s²]

w = settling velocity [m/s]

y = vertical distance above the sediment bed [m]

β = constant [-]

$\Delta = (\rho_s - \rho) / \rho$ = relative density difference [-]

$\Delta' = (\rho_s - \rho') / \rho'$ = apparent relative density difference [-]

ε = porosity [-]

μ = dynamic viscosity of fluid [kg/(ms)]

$\mu' =$ apparent dynamic viscosity of solid-fluid mixture [kg/(ms)]

$\mu_r = \mu' / \mu$ [-]

ν = kinematic viscosity of fluid [m²/s]

$\nu' =$ kinematic viscosity of solid-fluid mixture [m²/s]

$\nu_r = \nu' / \nu$ [-]

ρ = density of fluid [kg/m³]

ρ_s = density of solid phase [kg/m³]

$\rho' =$ apparent density of solid-fluid mixture [kg/m³]

References

- [1] Gibilaro LG. Fluidization-dynamics : the formulation and applications of a predictive theory for the fluidized state. Butterworth-Heinemann, Oxford ; Boston, 2001.
- [2] Wan Z, Z Wang. Hyperconcentrated flow. A.A. Balkema, Rotterdam ; Brookfield, VT, 1994.
- [3] Cheng NS. Effect of concentration on settling velocity of sediment particles. Journal of Hydraulic Engineering-ASCE. 123 (1997) 728-31.
- [4] Cheng NS. Calculation of drag coefficient for arrays of emergent circular cylinders with pseudo-fluid model. Journal of Hydraulic Engineering-ASCE. doi: 10.1061/(ASCE)HY.1943-7900.0000722 (2012).
- [5] Mei CC, JL Auriault. The effect of weak inertia on flow through a porous-medium. Journal of Fluid Mechanics. 222 (1991) 647-63.
- [6] Skjetne E, JL Auriault. High-velocity laminar and turbulent flow in porous media. Transport in Porous Media. 36 (1999) 131-47.
- [7] Wodie JC, T Levy. Nonlinear rectification of darcy law. Comptes Rendus De L Academie Des Sciences Serie Ii. 312 (1991) 157-61.
- [8] Chen ZX, SL Lyons, G Qin. Derivation of the Forchheimer law via homogenization. Transport in Porous Media. 44 (2001) 325-35.
- [9] Garcia MH. Sedimentation engineering: processes, measurements, modeling, and practice. American Society of Civil Engineers, Reston, Va., 2008.
- [10] Qian N, Z Wan. Mechanics of sediment transport. American Society of Civil Engineers, Reston, Va., 1999.
- [11] Cheng NS. Simplified settling velocity formula for sediment particle. Journal of Hydraulic Engineering-ASCE. 123 (1997) 149-52.

- [12] Cheng NS. Comparison of formulas for drag coefficient and settling velocity of spherical particles. *Powder Technology*. 189 (2009) 395-8, doi: 10.1016/j.powtec.2008.07.006.
- [13] Cheng NS, YM Chiew. Incipient sediment motion with upward seepage. *Journal of Hydraulic Research*. 37 (1999) 665-81.
- [14] Bear J. *Dynamics of fluids in porous media*. Dover, New York, 1988.
- [15] Poletto M, DD Joseph. Effective density and viscosity of a suspension. *Journal of Rheology*. 39 (1995) 323-43, doi: 10.1122/1.550692.
- [16] Cheng NS, AWK Law. Exponential formula for computing effective viscosity. *Powder Technology*. 129 (2003) 156-60.
- [17] Mints DM, SA Shubert. *Hydraulics of granular materials*. Water Resources Press, Beijing, China (in Chinese), 1957.
- [18] Cheng NS. Wall effect on pressure drop in packed beds. *Powder Technology*. 210 (2011) 261-6, doi: 10.1016/j.powtec.2011.03.026.
- [19] Mehta D, MC Hawley. Wall effect in packed columns. *Industrial & Engineering Chemistry Process Design and Development*. 8 (1969) 280-2.
- [20] Chen CI. Unified theory on power laws for flow resistance. *Journal of Hydraulic Engineering-ASCE*. 117 (1991) 371-89.
- [21] Cheng NS. Power-law index for velocity profiles in open channel flows. *Advances in Water Resources*. 30 (2007) 1775-84, doi: 10.1016/j.advwatres.2007.02.001.

Table 1. Summary of experimental data for open channel flows over a gravel bed.

Test No.	Slope	Q (m ³ /s)	h _s (m)	u _{max} (m/s)	Test No.	Slope	Q (m ³ /s)	h _s (m)	u _{max} (m/s)
1	0.0052	7.26E-04	0.0240	0.400	39	0.0210	6.10E-05	0.0020	0.159
2	0.0052	6.82E-04	0.0233	0.388	40	0.0210	5.43E-05	0.0015	0.147
3	0.0052	6.10E-04	0.0213	0.377	41	0.0400	7.12E-04	0.0130	0.622
4	0.0052	5.21E-04	0.0198	0.357	42	0.0400	6.45E-04	0.0122	0.588
5	0.0052	4.22E-04	0.0173	0.325	43	0.0400	5.82E-04	0.0115	0.562
6	0.0052	3.46E-04	0.0152	0.303	44	0.0400	4.57E-04	0.0100	0.515
7	0.0052	2.77E-04	0.0132	0.282	45	0.0400	3.64E-04	0.0085	0.467
8	0.0052	2.19E-04	0.0115	0.254	46	0.0400	2.88E-04	0.0070	0.415
9	0.0052	1.50E-04	0.0090	0.212	47	0.0400	2.39E-04	0.0062	0.392
10	0.0052	1.07E-04	0.0071	0.184	48	0.0400	1.74E-04	0.0047	0.327
11	0.0052	7.48E-05	0.0052	0.143	49	0.0400	1.36E-04	0.0038	0.255
12	0.0052	5.34E-05	0.0040	0.126	50	0.0400	1.04E-04	0.0026	0.244
13	0.0052	4.07E-05	0.0028	0.118	51	0.0400	8.51E-05	0.0020	0.192
14	0.0052	3.43E-05	0.0016	0.096	52	0.0400	6.80E-05	0.0010	0.137
15	0.0150	7.10E-04	0.0178	0.500	53	0.0560	7.06E-04	0.0118	0.683
16	0.0150	6.34E-04	0.0163	0.490	54	0.0560	6.35E-04	0.0108	0.649
17	0.0150	5.54E-04	0.0150	0.455	55	0.0560	5.41E-04	0.0095	0.581
18	0.0150	4.67E-04	0.0135	0.431	56	0.0560	4.41E-04	0.0083	0.521
19	0.0150	3.82E-04	0.0118	0.385	57	0.0560	3.51E-04	0.0070	0.495
20	0.0150	3.19E-04	0.0103	0.365	58	0.0560	3.06E-04	0.0063	0.459
21	0.0150	2.35E-04	0.0083	0.314	59	0.0560	2.52E-04	0.0050	0.424
22	0.0150	1.84E-04	0.0070	0.280	60	0.0560	1.96E-04	0.0040	0.376
23	0.0150	1.40E-04	0.0057	0.247	61	0.0560	1.56E-04	0.0032	0.346
24	0.0150	1.04E-04	0.0043	0.200	62	0.0560	1.14E-04	0.0021	0.256
25	0.0150	8.00E-05	0.0032	0.169	63	0.0560	8.43E-05	0.0010	0.147
26	0.0150	6.00E-05	0.0023	0.146	64	0.0710	7.03E-04	0.0105	0.723
27	0.0150	4.98E-05	0.0018	0.128	65	0.0710	6.49E-04	0.0100	0.694
28	0.0210	7.21E-04	0.0160	0.556	66	0.0710	6.04E-04	0.0095	0.638
29	0.0210	6.43E-04	0.0148	0.518	67	0.0710	5.49E-04	0.0086	0.622
30	0.0210	5.56E-04	0.0136	0.500	68	0.0710	5.09E-04	0.0080	0.581
31	0.0210	4.69E-04	0.0125	0.455	69	0.0710	4.51E-04	0.0072	0.539
32	0.0210	3.75E-04	0.0110	0.403	70	0.0710	4.01E-04	0.0068	0.521
33	0.0210	3.08E-04	0.0092	0.373	71	0.0710	3.55E-04	0.0062	0.492
34	0.0210	2.47E-04	0.0080	0.341	72	0.0710	2.98E-04	0.0051	0.467
35	0.0210	1.68E-04	0.0060	0.284	73	0.0710	2.43E-04	0.0040	0.439
36	0.0210	1.18E-04	0.0043	0.227	74	0.0710	1.96E-04	0.0030	0.407
37	0.0210	9.36E-05	0.0036	0.201	75	0.0710	1.54E-04	0.0025	0.356
38	0.0210	7.73E-05	0.0028	0.182	76	0.0710	1.05E-04	0.0010	0.208

Fig. 1. Two scenarios: (a) Water seeping through gravel bed; (b) Gravel bed settling in still water.

Fig. 2. Variation of average error with C_μ .

Fig. 3. Comparison of calculated and measured flow velocity for spherical grains. The unit is m/s.

Fig. 4. Comparison of calculated and measured flow velocity for coal grains. The unit is m/s.

Fig. 5. Comparison of calculated and measured flow velocity for natural gravels. The unit is m/s.

Fig. 6. Laboratory gravel bed simulated with packed glass beads.

Fig. 7. Two segments of vertical velocity profile.

Fig. 8. Comparison of calculated total flow discharges and experimental measurements in open channel flows with a gravel bed. The unit is m^3/s .