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Cheng, Nian-Sheng; Nguyen, Hoai Thanh

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Hydraulic Radius for Evaluating Resistance Induced by Simulated Emergent

Vegetation in Open Channel Flows

Nian-Sheng Cheng¹ and Hoai Thanh Nguyen²

¹Associate Professor, School of Civil and Environmental Engineering, Nanyang

Technological University, Nanyang Avenue, Singapore 639798.

Email: cnscheng@ntu.edu.sg

²Research Student, School of Civil and Environmental Engineering, Nanyang

Technological University, Nanyang Avenue, Singapore 639798.

Email: C070041@e.ntu.edu.sg

Abstract

The resistance induced by simulated emergent vegetation in open channel flows have been interpreted differently in the literature, largely due to inconsistent uses of velocity and length scales in the definition of friction factor or drag coefficient and Reynolds number. By drawing analogies between pipe flows and vegetated channel flows, this study proposes a new friction function with the Reynolds number that is redefined using a vegetation-related hydraulic radius. The new relation is useful for consolidating various experimental data across a wide range of vegetation density. The results clearly show a monotonic decrease of the drag coefficient with the new Reynolds number, which is qualitatively comparable to other drag coefficient

relations for non-vegetated flows. This study also proposed a procedure for correcting sidewall and bed effects in the evaluation of vegetation drag.

Keywords: vegetation, open channel flow, drag coefficient, resistance, friction, Reynolds number, hydraulic radius, Manning coefficient

Introduction

The presence of emergent vegetation in wetlands and rivers has impacts on physical and biological processes in aquatic environments. The vegetation-induced drag reduces flow discharge in channels, and also helps in flood attenuation and sediment deposition. Open channel flows, once vegetated, are not only resisted by boundary shear but also the drag induced by stems and foliage. Therefore, the resistance to vegetated open channel flows generally depends on channel geometry, vegetation configuration and surface characteristics of both channel boundary and vegetation. In spite of the fact that the subject has been studied for decades, difficulties still exist in the evaluation of vegetation resistance, either using conventional formulas like Manning equation or others developed recently, because of the limited understanding of complex physics inherent in relevant flow phenomena (e.g. Yen 2002; Zima and Ackermann 2002).

For example, the vegetation drag and its relevant Reynolds number have been defined in several different forms, as reviewed in the subsequent section, and

some definitions could be misleading because of improper use of length and velocity scales. As a result, conclusions drawn by different researchers are often inconsistent, and a general formula, similar to Manning equation developed for regular open channel flows, is not available at present for evaluating resistance to vegetated open channel flows, even for the emergent case that is relatively simple.

In this study, we attempt to compare the resistance to vegetated open channel flows with that to pipe flows based on the concept of hydraulic radius. A vegetation-related hydraulic radius is then proposed to redefine the Reynolds number for vegetated open channel flows. Finally, it is shown that using the new Reynolds number, we are able to collapse drag coefficients across a wide range of stem density. For simplicity, all considerations here are limited to the case of emergent rigid vegetation simulated with circular cylinders.

Previous Considerations

The vegetation-induced drag F_D can be described using the drag coefficient C_D ,

$$F_D = C_D A \frac{\rho V_a^2}{2} \quad (1)$$

where A is the frontal area of the vegetation stem, ρ is the fluid density and V_a is the average velocity approaching the stem. Without considering effects of vegetation configuration, several researchers simply replaced V_a with the bulk flow velocity V , the latter being the flow discharge divided by the total cross sectional area (e.g. Ishikawa et al. 2000; Lee et al. 2004; Wu et al. 1999). The use of V in place of V_a applies only for vegetation of low density. Noting that in the case of emergent

1 vegetation, the longitudinal velocity does not vary much from the channel bed to the
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3 free surface (e.g. Liu et al. 2008), the depth-averaged pore velocity through the
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5 vegetation $V_v [= Q/(Bh)/(1-\lambda) = V/(1-\lambda)]$ is close to the actual approach velocity,
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7 where Q is the flow discharge, B is the channel width, h is the flow depth and λ is the
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9 vegetation density defined as the average volume fraction occupied by vegetation.
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11 Therefore, V_v can be used to be a good approximation of V_a (Kothyari et al. 2009;
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13 Tanino and Nepf 2008).
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20 The drag coefficient is generally a function of Reynolds number. However, the
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22 definition of Reynolds number varies in the literature, which involves various length
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24 and velocity scales. By ignoring variations in the characteristics of vegetation,
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26 several studies simply used flow depth in the definition of Reynolds numbers (e.g.
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28 Wu et al. 1999). Lee et al (2004) stated that other equivalent Reynolds numbers
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30 could be also formed using either stem diameter (d) or spacing (s). Table 1 provides
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32 examples of the Reynolds numbers used in some recent studies.
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38 The diversity in the definition of the drag coefficient and Reynolds number
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40 has resulted in a mixture of interpretations of the resistance results. For example,
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42 Wu et al. (1999) conducted experiments with rubberized horsehair mattress
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44 simulating bush-type vegetation, showing that the drag coefficient (defined as
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46 $2gS/V^2$, where S is the energy slope and g is the gravitational acceleration) was
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48 proportional to the Reynolds number (Vh/v) in the power form, $(Vh/v)^{-k}$, where v is
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50 the kinematic viscosity of fluid and k is a constant. They also reported that k was
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52 somehow independent of the vegetation density, but different k-value must be
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specified for particular data source. Such k-variations are understandable by noting that Wu et al.'s data analysis did not involve the vegetation density, which is an important variable for characterizing the vegetation configuration but unfortunately was not measured in their experiments. Similar observations were also presented by Tsihrintzis (2001), who compared datasets provided by seven different studies and concluded that the k-value appears almost constant for each individual study but varies from 0.22 to 1.33.

Both Ishikawa et al. (2000) and Kothyari et al. (2009) used strain gauge to directly measure the drag force, F_D , exerted on a typical cylindrical stem placed in the middle of vegetation zone. By plotting the V-based drag coefficient $[2F_D/(\rho d h V^2)]$ against the Reynolds number (Vd/ν) , Ishikawa et al. (2000) observed significant changes in the drag coefficient but unclear dependence of it on the Reynolds number. In comparison, Kothyari et al.'s results indicate that the V_v -based drag coefficient $[2F_D/(\rho d h V_v^2)]$ slightly varies with the Reynolds number $(V_v d/\nu)$, but increases clearly with increasing vegetation density. It should be noted that Kothyari et al.'s experiments were conducted with a very short vegetation zone, which was 0.5 m wide and 1.8 m long. The short vegetation zone may not allow the vegetated flow to be fully developed, in particular, for the case of low vegetation density. We notice that the drag coefficients derived from Kothyari et al.'s experiments are generally overestimated in comparison to others observed in fully developed flows.

Tanino and Nepf (2008) performed experiments in open channels with relatively dense, randomly distributed circular cylinders, and estimated the drag

force from measured water surface slopes. They stated that the normalized drag, i.e. the ratio of the mean drag per unit cylinder length to the product of the viscosity and pore velocity, has a linear dependence on the Reynolds number. This result seems to be consistent with Ergun equation developed for flows through packed beds (Ergun 1952). However, by performing linear regression analysis, they also observed that both the gradient and intercept of the linear relation, though taken as constant in Ergun equation, vary clearly with the vegetation density.

From the above examples, it follows that with different length and velocity scales, similar sets of vegetation drag data could be interpreted with inconsistent conclusions. Therefore, an immediate issue raised here is how to properly scale vegetated open channel flows by taking into account vegetation characteristics.

Length Scale for Characterizing Vegetated Channels

To select a reasonable length scale for characterizing the resistance induced by emergent vegetation to open channel flow, we first make a qualitative comparison of resistance between vegetated open channel and pipe flows. It is well-known that the pipe friction factor f can be estimated from known Reynolds number and relative roughness height, e.g. using the Moody diagram or Colebrook-type function,

$$f = f[Re, k_s / (4r)] \quad (2)$$

where $f = 8(u_*^2/V^2)$, $Re = 4Vr/\nu$, k_s is the equivalent roughness height, r is the hydraulic radius, $u_* (= \sqrt{grS})$ is the shear velocity, V is the cross-sectional average velocity and S is the energy slope. In terms of length scales, Eq. (2) also presents a

relationship among v/V , v/u_* , k_s , and r . Here, both v/u_* and k_s are boundary-related length scales, describing roughness effects in the sense of hydrodynamics. On the other hand, the hydraulic radius r serves as a measure of the dimension of flow domain or geometry. If the boundary is hydrodynamically smooth, then v/V is related only to v/u_* and r , or in the dimensionless form, V/u_* is related only to Re . Such relationships, though developed for pipe flows, have been successfully extended to open channel flows through the concept of the hydraulic radius.

In the following, we aim to develop a similar friction relationship for open channel flows in the presence of emergent vegetation. Therefore, the hydraulic radius needs to be redefined by taking into account the size and configuration of stems as well as the main channel geometry.

Vegetation-related Hydraulic Radius, r_v

As shown in Fig. 1, we consider emergent vegetation simulated with staggered rigid cylinders. For this case, the effective vegetation height is the same as the flow depth h , and the configuration of vegetation is solely governed by the stem diameter d and density λ . It is also assumed that the form drag induced by the vegetation is much more important than its skin friction.

For open channel flows without vegetation, the hydraulic radius is associated only with the geometry of cross section, being taken as the ratio of the cross-section area to the wetted perimeter. For vegetated flows without sidewall and bed effects, we consider the volume of the vegetation zone that measures $B \times h \times L$, where L is

the length of vegetation zone. Here, since the fraction of the stem-occupied bed area is the same as λ , the number of cylinders is $N = BL\lambda/(\pi d^2/4)$. Similar to the regular hydraulic radius, we may define the hydraulic radius as the ratio of the volume occupied by water to the wetted surface area of all cylinders. Such approaches were applied previously to quantify flow resistance associated with porous media (Cheng 2003; Cheng et al. 2008). However, in terms of vegetation-induced form drag, it is not the total wetted surface area that should be taken into account. Instead, we only need to consider the frontal area of the stem, which is the area of the stem projected on a plane normal to the streamwise direction. This yields an effective wetted area equal to Nhd , and then the vegetation-related hydraulic radius can be reasonably defined as

$$r_v = \frac{(1-\lambda)BhL}{Nhd} = \frac{\pi}{4} \frac{1-\lambda}{\lambda} d \quad (3)$$

It is noted that r_v defined above is comparable to the concentration length proposed by James et al. (2008). In addition, r_v can be related to the overall hydraulic radius for a typical vegetated cross section, which is discussed in Appendix.

With the vegetation-related hydraulic radius, the Colebrook-type resistance relation for open channel flows subject to emergent vegetation is proposed here,

$$f_v = f(\text{Re}_v) \quad (4)$$

where f_v is the vegetation friction factor defined as

$$f_v = \frac{8gr_v S}{V_v^2} \quad (5)$$

and Re_v is the vegetation Reynolds number defined as

$$Re_v = \frac{V_v r_v}{\nu} \quad (6)$$

In comparison with the previous studies (see Table 1), the use of r_v in the Reynolds number is novel. As shown subsequently, r_v performs much better than other length scales, such as h , d and s , for collapsing drag coefficient data from disparate sources.

In the above consideration, effects of the channel boundary have not been included. In natural situations and laboratory studies, vegetated flows are usually subject to boundary roughness, such as channel beds covered by sediment grains and even bed forms. Although some experimental observations have shown that vegetation-induced drag could be dominant in comparison to boundary shear, it is generally necessary to correct boundary effects so that vegetation-induced drag can be singled out. Such a procedure called sidewall and bed corrections is introduced later in this paper for modifying r_v , and thus the resistance relation given in Eq. (4).

Drag coefficient

Similar to the friction factor, drag coefficient is also a useful parameter for quantifying vegetation-induced drag. For each cylindrical stem, the drag force in the streamwise direction is defined as (e.g. Kothyari et al. 2009; Tanino and Nepf 2008)

$$F_D = C_{Dv} \rho h d \frac{V_v^2}{2} \quad (7)$$

where C_{Dv} is the drag coefficient, the product of h and d measures the size of the frontal area and V_v is the average pore velocity approaching the stem. The total drag per unit bed area is

$$\frac{4\lambda}{\pi d^2} F_D = \frac{4\lambda}{\pi d^2} C_{Dv} \rho h d \frac{V_v^2}{2} = C_{Dv} \frac{2\lambda \rho h V_v^2}{\pi d} \quad (8)$$

which is equivalent to the streamwise component of the gravitational force for the condition of uniform flows, i.e.

$$C_{Dv} \frac{2\lambda \rho h V_v^2}{\pi d} = (1 - \lambda) \rho g h S \quad (9)$$

Here, the shear forces induced by sidewalls and bed are considered negligible. Otherwise, sidewall and bed corrections are applied in the way as described subsequently. From Eq. (9),

$$C_{Dv} = \frac{1 - \lambda}{2\lambda V_v^2} g S \pi d = 2 \frac{g r_v S}{V_v^2} \quad (10)$$

It is noted that the definition of the drag coefficient as given by Eq. (10) has also been proposed previously (James et al. 2008; Tanino and Nepf 2008). Furthermore, by comparing Eq. (10) with Eq. (5), one gets

$$C_{Dv} = \frac{1}{4} f_v \quad (11)$$

Therefore, from Eqs. (4) and (11), it follows that C_{Dv} would generally vary with Re_v .

To justify the suitability of C_{Dv} and Re_v for the description of resistance to vegetated open channel flows, laboratory experiments were conducted in this study to facilitate data analysis.

Laboratory Experiments

The experiments were carried out in a tilting rectangular flume, 12 m long, 0.3 m wide, and 0.45 m deep, with glass sidewalls and a steel bottom. Flow discharges

were measured as an average of readings taken from a built-in electromagnetic flowmeter. The flowmeter reading varied and its standard deviation was about 0.1-7.5%. The channel slopes were calculated from longitudinal flow depth variations, which were measured using a point gauge accurate to 0.1 mm while water in the flume remained stationary. In the middle of the flume was the vegetation zone, 9.6 m long and 0.3 m wide, which was simulated with arrays of rigid, circular cylindrical rods. Three sizes of rods with diameters 3.2, 6.6 and 8.3 mm were planted at a false floor along the channel with two kinds of spacing (see Fig. 1). The resulted vegetation density varied from 0.0043 to 0.1189. Table 2 summarizes the data collected from 143 runs of experiments. For each run, the flow depth was measured at five stations to ensure the achievement of uniform flow. The streamwise flow velocity among the rods in the centre of the vegetation zone was also measured at the midpoint between two adjacent rods (as shown in Fig. 1) using an electromagnetic current meter. Almost uniform velocity distributions in the vertical direction were observed for all cases. As the depth-averaged velocity computed based on such measurements may not represent the average pore velocity, the values of V_v used for the following analysis were all calculated from the measured flow discharges, i.e. $V_v = Q/(Bh)/(1-\lambda)$. An uncertainty analysis shows that the relative root mean square deviations of Re_v and C_{Dv} that were calculated from the collected data are 7.6% and 15.1%, respectively, whereas the maximum deviations are 15.2% and 30.2%, respectively, at the 95% confidence level.

Data Analysis

The analysis is performed with two categories of experimental data. The first category is associated with randomly distributed cylindrical rods, as described by Tanino and Nepf (2008). The relevant data consist of those reported earlier by Tanino and Nepf (2008) and two additional sets of unpublished data for $\lambda = 0.031$ and 0.056 respectively (Tanino and Nepf 2010). The second category is related to stems arranged in staggered patterns. The data used for analysis are those collected in this study (Table 2) and also those published previously (i.e. Ferreira et al. 2009; Ishikawa et al. 2000; James et al. 2004; Liu et al. 2008; Stoesser et al. 2010) (see Table 3). In addition, two data points provided by Liu et al. (2008) for the case of stems arranged in a linear pattern are also included.

It is noticed that various flow and vegetation conditions have been employed by different investigators (see Table 3). For example, while smooth sidewalls were used in all studies, the channel bed involved was either smooth (e.g. Kothyari et al. 2009; Tanino and Nepf 2008) or sand-covered (Ferreira et al. 2009; Ishikawa et al. 2000; James et al. 2004; Liu et al. 2008). Ishikawa et al. (2000) and Kothyari et al. (2009) used strain gauge to measure the drag force directly, while the others estimated the drag from the energy slope of uniform or non-uniform flows. All these variations in the experimental setup and measurement technique imply that the data collected should be treated with care. In the next section, it is shown that the data can be unified to a certain extent by correcting boundary effects that are associated with characteristics of sidewall and bed roughness. However, possible

effects of other factors such as flow uniformity and vegetation configuration (regular or random) are not incorporated in the analysis.

Sidewall and Bed Corrections

Sidewall correction has been proposed by Vanoni and Brooks (1957) for evaluating bed-related hydraulic radius in sediment transport studies, in particular, for experiments conducted using a glass-sided flume. Following Vanoni and Brooks' idea, a similar procedure is proposed in this study to perform sidewall and bed corrections in the evaluation of vegetation-related hydraulic radius. As detailed in Appendix, being affected by sidewall and bed roughness, the vegetation-related hydraulic radius r_v should be modified to be

$$r_{vm} = r_v \left[1 - \left(\frac{f_w}{0.5B(1-\lambda)} + \frac{f_b}{h} \right) \frac{r}{f} \right] \quad (12)$$

where $r_v = \pi d(1-\lambda)/(4\lambda)$, as given by Eq. (3), f_w is the sidewall friction factor, f_b is the bed friction factor, $f = 8grS/V_v^2$, and

$$r = \left(\frac{1}{h} + \frac{1}{0.5B(1-\lambda)} + \frac{1}{r_v} \right)^{-1} \quad (13)$$

It is interesting to note that for vegetated open channel flows, the hydraulic radius r generally comprises three components, i.e. h , $0.5B(1-\lambda)$ and r_v , and it is equivalent to r_v when both sidewall and bed effects are negligible. Table 4 compares various hydraulic radii applied in different flows.

The above correction can be implemented with known B , h , d , λ , S , Q , k_{sb} , k_{sw} , and v , where k_{sb} is the bed roughness height and k_{sw} is the sidewall roughness height, of which the procedure is provided in Appendix. To justify the correction, we here apply it to the experiments conducted by Ishikawa et al. (2000), who measured both energy slope and vegetation-induced forces for flows bounded by smooth glass sidewalls and a rough channel bed covered with sand of diameter $d_{50} = 1.8$ mm. With the data provided by Ishikawa et al., the drag coefficient was first computed directly using the measured drag, i.e. $C_{Dv(exp)} = 2F_D/(\rho h d V_v^2)$ [see Eq. (7)]. Then, it was also estimated from the energy slope in two different methods, one being based on r_v , yielding that $C_{Dv(cal)} = 2g r_v S / V_v^2$ [see Eq.(10)] and the other based on r_{vm} , i.e. $C_{Dvm(cal)} = 2g r_{vm} S / V_v^2$.

In Fig. 2, both $C_{Dv(cal)}$ and $C_{Dvm(cal)}$ are compared with $C_{Dv(exp)}$. It can be observed that without any correction made, $C_{Dv(cal)}$ differs significantly from $C_{Dv(exp)}$; however using the corrected hydraulic radius r_{vm} , the calculated drag coefficient $C_{Dvm(cal)}$ appears close to the measurement. Here in making the correction, it is assumed that $k_{sb} = 2.5d_{50}$ (Qian and Wan 1999). Additional calculations performed show that the best agreement between the measured and calculated drag coefficients can be achieved by varying k_{sb} in the range of d_{50} to $6d_{50}$ for the four cases, which may imply the presence of bed forms.

Fig. 3 shows the ratio of r_{vm} to r_v computed with the data collected in this study and those given by Ishikawa et al. (2000) and James et al. (2004). It is noted that r_{vm} is slightly smaller than r_v for the experiments conducted in this study with

smooth bed and sidewalls, while the difference becomes large for the experiments conducted using sand-covered beds (Ishikawa et al. 2000; James et al. 2004). Moreover, the difference is expected to be generally negligible for vegetation of high density, e.g. $\lambda > 0.1$.

Dependence of C_{Dv} on Re_v

As discussed earlier, the previous studies (Ishikawa et al. 2000; Kothiyari et al. 2009; Tanino and Nepf 2008) seem to suggest that the dependence of drag coefficient on Reynolds number always varies with the vegetation density. However, the same data, when presented in the form of C_{Dv} against Re_v as shown in Fig. 4, demonstrate that C_{Dv} decreases monotonically with increasing Re_v . From Fig. 4, the following observations could be made. First, though scattered to some extent, all data points generally follow the same decreasing trend of C_{Dv} with increasing Re_v . Second, it seems that for the same Re_v , there is no significant difference in the drag coefficient between the randomly distributed and staggered stems.

It should be mentioned that the sidewall and bed corrections were applied only for the data with the information available for the relevant variables, i.e. those from Ishikawa et al. (2000), James et al. (2004) and this study. The data by Kothiyari et al. (2009) are not included in Fig. 4 by considering inconsistency related to the short vegetation zone. However, a separate analysis also shows that the use of Re_v

results in similar collapsing of their data, in spite of the small variation in the measured drag coefficient, i.e. $C_{Dv} = 0.9 - 1.9$.

For comparison purposes, we also made a companion to Fig. 4 by plotting C_{Dv} against the more commonly used Reynolds number $V_v d/v$. The result is shown in Fig. 5, indicating that the use of d rather than r_v yields significant data spread.

To empirically describe the relationship of C_{Dv} and Re_v , a best-fit function is proposed here for $Re_v = 52 - 5.6 \times 10^5$,

$$C_{Dv} = \frac{50}{Re_v^{0.43}} + 0.7 \left[1 - \exp\left(-\frac{Re_v}{15000}\right) \right] \quad (14)$$

which is superimposed on Fig. 4. In addition, by noting that an iterative procedure is needed to solve for the average pore velocity using Eq. (14), we also express C_{Dv} in the form,

$$C_{Dv} = \frac{130}{r_{v*}^{0.85}} + 0.8 \left[1 - \exp\left(-\frac{r_{v*}}{400}\right) \right] \quad (15)$$

for $r_{v*} = 24 - 5000$, where r_{v*} is the dimensionless vegetation-related hydraulic radius defined as,

$$r_{v*} = \left(\frac{gS}{v^2} \right)^{1/3} r_v \quad (16)$$

With C_{Dv} estimated from Eq. (15), the average pore velocity can be calculated as $V_v = \sqrt{2gr_v S / C_{Dv}}$ [see Eq. (10)]. When the sidewall and bed effects are significant, r_{vm} should be used in place of r_v for evaluating C_{Dv} and then V_v .

Comparison with Ergun equation

Some studies (e.g. Tanino and Nepf 2008; Zinke 2010) have attempted to explore probability of the description of vegetated flows using Ergun equation. Here we use C_{Dv} and Re_v to reformulate Ergun equation, the latter being expressed as (Ergun 1952),

$$\Delta p = 150 \frac{\lambda^2}{(1-\lambda)^3} \frac{\rho v V}{d_{50}^2} + 1.75 \frac{\lambda}{(1-\lambda)^3} \frac{\rho V^2}{d_{50}} \quad (17)$$

where Δp is the pressure drop (per unit length) through a column of packed grains, V is the superficial flow velocity (= flow discharge divided by the gross cross-sectional area of the packed bed), d_{50} is the grain diameter, and λ is the volumetric fraction of grains. To apply Ergun equation to the vegetated channel flows, we replace d_{50} with d , V/ε with V_v , and also take that $S = \Delta p/(\rho g)$, $f_v = 8gr_v S/V_v^2$ and $Re_v = r_v V_v/\nu$. As a result, Eq. (17) is revised to be

$$f_{v(Ergun)} = \frac{75\pi^2}{Re_v} + \frac{7\pi}{2} \quad (18)$$

and thus

$$C_{Dv(Ergun)} = \frac{f_{v(Ergun)}}{4} = \frac{75\pi^2}{4Re_v} + \frac{7\pi}{8} \quad (19)$$

Eq. (19) is superimposed on Fig. 4, showing that Ergun equation, if applied to vegetated open channel flows, would underestimate C_{Dv} for low Reynolds numbers and overestimate C_{Dv} for high Reynolds numbers.

Manning Coefficient

The Manning coefficient can be determined experimentally from the bulk flow velocity $V [= Q/(Bh)]$, regular open channel hydraulic radius $Bh/(B+2h)$, and energy slope S , i.e.

$$n = \frac{\sqrt{S}}{V} \left(\frac{Bh}{B+2h} \right)^{2/3} \quad (20)$$

Obviously, the n -values so obtained would generally depend on vegetation configuration for vegetated open channel flows. From Eq. (10),

$$\frac{\sqrt{S}}{V} = \sqrt{\frac{C_{Dv}}{2gr_v(1-\lambda)^2}} \quad (21)$$

Substituting Eq. (21) into (20), we get

$$n = \sqrt{\frac{C_{Dv}}{2gr_v(1-\lambda)^2}} \left(\frac{Bh}{B+2h} \right)^{2/3} \quad (22)$$

Fig. 12 shows that the Manning coefficients predicted using Eq. (22), where C_{Dv} is estimated using Eq. (15), agree well with the measurements, i.e. those determined using Eq. (20) with the data provided by Ishikawa et al. (2000), and James et al. (2004), and also those by this study. Here, r_{vm} was used in place of r_v for estimating C_{Dv} with Eq. (15) and n using Eq. (22). The average relative errors are 3.6% and 10.2% for the data by Ishikawa et al. and James et al., respectively, and 9.1% for the data collected in this study. However, it should be mentioned that the good agreement shown in Fig. 12 is inherent in the method of calculating n , by noting that the same sets of experimental data were also used in the derivation of C_{Dv} included in Eq. (22). Therefore, further efforts are needed to verify Eq. (22).

Conclusions

This study demonstrates that the concept of hydraulic radius is useful to unify experimental data of resistance to vegetated open channel flows for various bed and vegetation configurations. The vegetation hydraulic radius is defined by taking into account effects of vegetation size and density, and channel geometry. It serves as an important length scale in the definition of drag coefficient, friction factor and Reynolds number for open channel flows subject to emergent vegetation. From the results of data analysis, it follows that the drag coefficient decreases monotonically with the Reynolds number, independent of vegetation density. This study also shows that Ergun equation, if applied to vegetated open channel flows, would underestimate drag coefficients for low Reynolds numbers and overestimate for high Reynolds numbers. In addition, a procedure is proposed in this study for correcting sidewall and bed effects that may appear significant for both laboratory and field conditions.

Appendix: Sidewall and bed corrections for vegetated open channel flows

By noting that a sand-covered bed is generally rougher than flume walls and thus subject to higher shear stress, Vanoni and Brooks (1957) developed a procedure for determining bed-related hydraulic radius and thus average bed shear stress from known values of V , S , r , etc. In this correction, the bed and sidewall-related hydraulic radius are defined without specifying individual average velocities and energy slopes.

Despite some deficiencies, the side-wall correction procedure yields reliable estimates of the friction factor for flow over sand beds.

Here, the correction procedure is modified for determining the vegetation-related hydraulic radius in the presence of both sidewalls and channel bed. First, we consider the forces exerted on vegetated flow in the streamwise direction. The resistance forces are wall shear, bed shear and vegetation drag. With the same energy slope, the wall shear may be expressed as $\rho g S r_w p_w$, and the bed shear as $\rho g S r_b p_b$, where r_w and p_w are wall-related hydraulic radius and wetted-perimeter, respectively, and r_b and p_b are bed-related hydraulic radius and wetted-perimeter, respectively. Similarly, the vegetation drag can be taken as $\rho g S r_v p_v$, where r_v and p_v are vegetation-related hydraulic radius and wetted-perimeter, respectively. The sum of the three resistance forces is equivalent to $\rho g r S p$, which yields

$$p r = p_w r_w + p_b r_b + p_v r_v \quad (23)$$

where $p (= p_w + p_b + p_v)$ is the total wetted-perimeter and $r [= B h (1-\lambda) / p]$ is the total hydraulic radius. By defining that $f = 8 g r S / V_v^2$, $f_w = 8 g r_w S / V_v^2$, $f_b = 8 g r_b S / V_v^2$, and $f_v = 8 g r_v S / V_v^2$, Eq. (23) is rewritten to be

$$p f = p_w f_w + p_b f_b + p_v f_v \quad (24)$$

The three wetted perimeters are evaluated as follows. First, we can take that $p_w = 2h$ and on average, $p_b = (1-\lambda)B$ by noting that the channel bed is partially occupied by the vegetation stems. To evaluate p_v , it is convenient to consider a channel reach, of which the width is B and the length is unity. The number of stems

in this unit reach is $B\lambda/(\pi d^2/4)$. Then, p_v is taken to be the total frontal area of the stems, which represents the effective area that the form drag exerts on, i.e. $p_v = Bh\lambda/(\pi d/4)$. Note that p_v so obtained is an equivalent but not real wetted-perimeter.

With p_b , p_w and p_v given above, we get

$$r = \left(\frac{1}{h} + \frac{1}{0.5B(1-\lambda)} + \frac{1}{r_v} \right)^{-1} \quad (25)$$

where $r_v = \pi d(1-\lambda)/(4\lambda)$. Without vegetation or when $\lambda = 0$, r reduces to

$$r = \left(\frac{1}{0.5B} + \frac{1}{h} \right)^{-1} = \frac{Bh}{B+2h} \quad (26)$$

Furthermore, substituting the expressions of p_b , p_w and p_v into Eq. (24) and then dividing both sides by $(1-\lambda)Bh$ yields

$$f_v = r_v \left(\frac{f}{r} - \frac{f_w}{0.5B(1-\lambda)} - \frac{f_b}{h} \right) \quad (27)$$

In studying vegetated open channel flows, different investigators engaged different bed conditions. Most used smooth beds, while a few used rough and even mobile sediment beds (Ishikawa et al. 2000; James et al. 2004). Therefore, when comparing such results that are obtained for different bed conditions, it is necessary to first correct possible bed and sidewall effects. Once f_v is known, the vegetation hydraulic radius r_v is modified to r_{vm} in the way similar to that proposed by Vanoni and Brooks (1957),

$$r_{vm} = \frac{r}{f} f_v \quad (28)$$

Substituting Eq. (27) into Eq. (28),

$$r_{vm} = r_v \left[1 - \left(\frac{f_w}{0.5B(1-\lambda)} + \frac{f_b}{h} \right) \frac{r}{f} \right] \quad (29)$$

The evaluation of f_w and f_b can be made by applying the Colebrook equation, which generally requires iterations. Here, we propose an equivalent but explicit formula as follows,

$$f_w^\alpha = f_{ws}^\alpha + f_{wr}^\alpha \quad (30)$$

where

$$f_{ws} = 31 \left[\ln \left(1.3 \frac{Re}{f} \right) \right]^{-2.7} \quad (31)$$

$$f_{wr} = 11.7 \left[\ln \left(7.6 \frac{4r}{fk_{sw}} \right) \right]^{-2.5} \quad (32)$$

$$\alpha = 2 \left(\frac{4r}{fk_{sw}} \right)^{0.1} \quad (33)$$

In Eqs. (30) to (33), the Reynolds number Re is defined as $4rV_v/\nu$, k_{sw} is the equivalent wall roughness height, and subscript w indicates that the equations are proposed for finding f_w . For known bed roughness height k_{sb} , f_b is evaluated using Eqs. (30) to (33) with subscript b in place of w .

With the above consideration, the calculation of r_{vm} proceeds as follows: (1) Obtain r using Eq. (25), f ($= 8grS/V_v^2$) and Re ($= 4rV_v/\nu$) from experimental data; (2) Calculate Re/f , $4r/(fk_{sw})$, and $4r/(fk_{sb})$; (3) Obtain f_w and f_b with Eqs. (30) to (33); and (4) Calculate r_{vm} using Eq. (29).

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Notation

The following symbols are used in this paper:

A = frontal area of the vegetation stem

B = channel width

C_D = drag coefficient

C_{Dv} = drag coefficient $[= 2F_D/(\rho d h V_v^2)]$

d = stem diameter

d_{50} = median grain diameter

F_D = vegetation-induced drag

f = friction factor $(= 8grS/V_v^2)$

f_b = bed friction factor

f_v = vegetation friction factor $(= 8gr_v S/V_v^2)$

f_w	=	sidewall friction factor
f_{ws}	=	smooth-sidewall friction factor
f_{wR}	=	rough-sidewall friction factor
g	=	gravitational acceleration
h	=	flow depth
k	=	constant
k_s	=	roughness height
k_{sb}	=	bed roughness height
k_{sw}	=	sidewall roughness height
L	=	length of vegetation zone
N	=	number of stems
n	=	Manning roughness coefficient
p	=	wetted perimeter
p_b	=	bed-related wetted perimeter
p_v	=	vegetation-related wetted perimeter
p_w	=	sidewall-related wetted perimeter
Q	=	flow discharge
Re	=	Reynolds number
Re_v	=	vegetation Reynolds number ($= V_v r_v / \nu$)
r	=	hydraulic radius
r_b	=	bed-related hydraulic radius
r_v	=	vegetation-related hydraulic radius

r_v^*	=	dimensionless vegetation-related hydraulic radius
r_{vm}	=	corrected vegetation-related hydraulic radius
r_w	=	sidewall-related hydraulic radius
S	=	energy slope
s	=	stem spacing
u^*	=	shear velocity ($=\sqrt{grS}$)
V	=	average flow velocity [$=Q/(Bh)$]
V_a	=	average velocity approaching the stem
V_v	=	average pore velocity [$=V/(1-\lambda)$]
α	=	exponent
Δp	=	pressure drop through a column of packed grains
ν	=	kinematic viscosity of fluid
ρ	=	fluid density
λ	=	vegetation density or fraction of vegetation-occupied volume

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Table 1. Various Definitions of Reynolds Number for Open Channel Flows subject to Emergent Vegetation

Investigator	Reynolds number	Characteristic velocity	Characteristic length	Note
Wu et al. (1999)	$\frac{Vh}{\nu}$	bulk velocity, V	flow depth, h	$V = \frac{Q}{Bh}$ $V_v = \frac{V}{1-\lambda} \quad r_v = \frac{\pi}{4} \frac{1-\lambda}{\lambda} d$
Ishikawa et al. (2000)	$\frac{Vd}{\nu}$	bulk velocity, V	stem diameter, d	
Lee et al. (2004)	$\frac{Vh}{\nu} ; \frac{Vs}{\nu} ; \frac{Vd}{\nu}$	bulk velocity, V	flow depth, h; stem spacing, s; stem diameter, d	
Tanino and Nepf (2008)	$\frac{V_v d}{\nu}$	average pore velocity among stems, V_v	characteristic plant width or stem diameter,	
Kothyari et al. (2009)			d	
Present study	$\frac{V_v r_v}{\nu}$	average pore velocity among stems, V_v	vegetation-related hydraulic radius, r_v	

Table 2. Summary of Experimental Conditions

Model	Stem diameter d (mm)	Vegetation density λ	Energy slope S	h (cm)								
				2	3	4	5	6	7	8	9	10
				Q (l/s)								
A60	3.2	0.0043	0.00950	0.52	0.74	1.11	1.41	1.65	1.93	2.27	2.58	2.75
			0.00186	0.71	1.14	1.58	2.00	2.47	2.84		3.74	4.12
			0.00047			0.69	0.86	1.16	1.34	1.39	1.48	1.61
B60	6.6	0.0192	0.00122		0.53	0.66	0.83	1.08	1.28	1.41	1.59	1.84
			0.00196	0.49	0.68	0.96	1.26	1.49	1.81	2.07	2.22	2.51
			0.00410	0.64	0.97	1.41	1.74	2.13	2.54	2.95	3.29	3.80
			0.00811	1.06	1.40	1.97	2.58	3.19	3.75	4.29	4.84	5.45
C60	8.3	0.0298	0.00100			0.61		0.97		1.35	1.31	1.43
			0.00200		0.62	0.75		1.22		1.74	1.89	2.21
			0.00400		0.83	1.07		1.71		2.83	2.74	3.19
A30	3.2	0.0173	0.00100			0.60		0.94		1.25	1.46	1.60
			0.00200		0.63	0.86		1.30		1.68	1.93	2.12
			0.00400		0.90	1.20		1.92		2.56	2.76	2.90
B30	6.6	0.0769	0.00100		0.24	0.28		0.42		0.60	0.66	0.71
			0.00200				0.58	0.64	0.69	0.79	0.94	1.02
			0.00374				0.74	0.87	1.04	1.26	1.38	1.59
			0.00750		0.67	0.81	1.06	1.29	1.55	1.79	2.08	2.34
C30	8.3	0.1189	0.00103						0.47	0.58	0.63	0.66
			0.00197					0.46	0.54	0.62	0.68	0.82
			0.00386			0.50	0.62	0.71	0.84	0.97	1.11	1.26
			0.00808			0.64	0.84	1.03	1.18	1.39	1.64	1.80

Table 3. Summary of Data of Resistance Induced by Emergent Vegetation

Investigator	Rectangular channel				Vegetation zone			Vegetation model			Drag measurement approach
	length (m)	width (m)	bed slope	bed condition	length (m)	width (m)	density λ	shape of rod	pattern	stem diameter d (mm)	
Ishikawa et al. (2000)	15	0.3	0.1-0.02 (nearly uniform flow)	covered with sand $d_{50} = 1.8\text{mm}$	15	0.3	0.00314-0.0126	cylindrical	staggered	4; 6.4	strain gauge
James et al. (2004)	3	0.1	0.002-0.01 (uniform flow)	covered with sand $d_{50} = 2.4\text{-}4.8\text{mm}$			0.0035-0.0314	cylindrical	staggered	5	energy slope
Liu et al. (2008)	4.3	0.3	0.003 (uniform flow)	smooth; glued with sand $d_{50} = 0.7\text{mm}$	3	0.3	0.0031-0.0160	cylindrical	staggered; linear	6.35	energy slope
Tanino and Nepf (2008)			0 (nonuniform flow)	smooth	2.84	0.4	0.15-0.35	cylindrical	random	6.4	water surface slope
					6.7	0.203	0.091				
Ferreira et al. (2009)	10	0.409	0 (nonuniform flow)	covered with sand $d_{50} = 9\text{mm}$	3.1	0.409	0.022-0.038	cylindrical	random	11	water surface slope
Kothyari et al. (2009)	16	0.5	0-0.02 (nonuniform flow)	smooth	1.8	0.5	0.0022-0.0885	cylindrical	staggered	10	strain gauge
Stoesser et al. (2010)							0.0157-0.2513	cylindrical	staggered		CFD (LES)
Present study	12	0.3	0.00095-0.00811 (uniform flow)	smooth	9.6	0.3	0.0043-0.119	cylindrical	staggered	3.2; 6.6; 8.3	energy slope

Table 4. Length Scales for Characterizing Flow Geometry

Flow geometry	Geometrical dimension	Hydraulic radius
Circular pipe	pipe diameter, D	$\frac{D}{4}$
Rectangular open channel	channel width, B flow depth, h	$\left(\frac{1}{h} + \frac{1}{0.5B}\right)^{-1} = \frac{Bh}{B+2h}$
Porous media comprising grains	grain size, d_{50} pore size	$\frac{1}{6} \frac{1-\lambda}{\lambda} d_{50}$
Vegetated channel without channel boundary effects	stem diameter, d stem spacing, s	$r_v = \frac{\pi}{4} \frac{1-\lambda}{\lambda} d$
Rectangular open channel with emergent vegetation	channel width, B flow depth, h stem diameter, d stem spacing, s	$\left(\frac{1}{h} + \frac{1}{0.5B(1-\lambda)} + \frac{1}{r_v}\right)^{-1}$
Notes: For porous media, the hydraulic radius is defined by considering the total wetted area (Cheng et al. 2008).

(a) Plan view

(b) Side view

Fig. 1. Emergent vegetation simulated with circular cylindrical rods. $s = 30$ mm for cases A30, B30 and C30, and $s = 60$ mm for cases A60, B60 and C60. $d = 3.15$ mm, 6.64 mm and 8.26 mm for cases A, B and C, respectively. Point “a” indicates the midpoint between the two adjacent rods, at which velocity profiles were measured.

Fig. 2. Comparisons of drag coefficients $C_{Dv(cal)}$ (denoted by solid symbols) estimated using uncorrected hydraulic radius (r_v) and $C_{Dvm(cal)}$ (denoted by open symbols) using corrected hydraulic radius (r_{vm}) with measurements ($C_{Dv(exp)}$). The data are from Ishikawa et al. (2000).

Fig. 3. Variation of ratio of corrected to uncorrected hydraulic radius with vegetation density.

Fig. 4. Variation of C_{Dv} with Re_v .

Figure5

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Fig. 5. C_{Dv} plotted against $V_v d / v$. The data are the same as those used in Fig. 4.

Fig. 6. Comparison of predicted and measured Manning coefficients.

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