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PICK-UP PROBABILITY FOR SEDIMENT ENTRAINMENT

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ABSTRACT

The study presents a theoretical derivation of a new pick-up probability formulation for sediment transport. Sand particles are assumed to be subjected to a hydraulically rough flow. The derived equation compares well with the experimental data found in published literature. Using the proposed relation, the Shields criterion for the definition of the threshold condition for sediment transport is equivalent to 0.6% of the pick-up probability.

INTRODUCTION

Different sediment transport theories have been developed by using either a deterministic or a probabilistic model. The earlier application of the stochastic concept can be attributed to Lane and Kalinske (1939) and Einstein (1942). With this approach, a pick-up probability may be used to derive the bed load function. Many studies can be found in the literature to date on pick-up functions or erosion rate, and a succinct review can be found in van Rijn (1984). However, much fewer works have been directed towards solving the problem using the pick-up probability. Up to now, it appears that only Einstein (1950), Engelund and Fredsoe (1976), and Fredsoe and Deigaard (1992) have proposed concrete formulations for calculating the pick-up probability.

The definition of the pick-up probability can be attributed to Einstein (1950), who stated that this probability refers to “the probability of the dynamic lift on the particle is

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larger than its weight (under water)”. Based on this definition, he theoretically derived the following formula to evaluate the pick-up probability, P

$$P = 1 - \int_{-0.143/\theta-2}^{0.143/\theta-2} \frac{1}{\sqrt{\pi}} \exp(-t^2) dt \quad (1)$$

where t = variable of integration; $\theta = u_*^2/(\Delta g d)$ = dimensionless shear stress; u_* = shear velocity; $\Delta = (\rho_s - \rho)/\rho$; ρ_s = density of particle; ρ = density of fluid; d = diameter of particle and g = gravitational acceleration.

When considering the motion of the individual particles of bed load, Engelund and Fredsoe (1976) related the pick-up probability to both the dimensionless bed load rate and dimensionless shear stress. By empirically fitting the experimental data on bed load transport by Luque (1974) and the Fort Collins data as cited in Guy et al (1964), they found that the dependence of the probability on the dimensionless bed load rate can be ignored, and the relationship can simply be expressed as

$$P = \frac{\theta - \theta_c}{0.2668} \quad (2)$$

where $\theta_c = 0.05$. However, (2) is a linear equation relating P with θ and it does not adequately describe the experimental data for high probabilities (or high θ s). In order to overcome this problem, Engelund and Fredsoe imposed a boundary of $P \rightarrow$ unity at $\theta \rightarrow 0$.

With this additional boundary condition, (2) is modified to

$$P = \left[1 + \left(\frac{0.2668}{\theta - \theta_c} \right)^4 \right]^{-0.25} \quad (3)$$

More recently, Fredsoe and Deigaard (1992) re-examined the same two experimental data sets discussed earlier, and proposed a new empirical equation in the form

$$P = \left[1 + \left(\frac{0.419}{\theta - 0.045} \right)^4 \right]^{-0.25} \quad (4)$$

Note that (4) is identical to (3) except for different constants. The equation has a singularity at $\theta = 0.045$, it only gives meaningful results for $\theta > 0.045$.

The objective of this study is to theoretically explore the relationship between the pick-up probability and flow condition. By adopting the Einstein's definition of the pick-up probability, a detailed theoretical derivation is conducted to yield a new expression of pick-up probability.

PICK-UP PROBABILITY

The initial motion of a sediment particle occurs when the ratio of the hydrodynamic force to the submerged weight force acting on the particle exceeds a certain limit determined by the geometry of the particle and the particle Reynolds number. Different critical relations for the incipient motion can be derived based on the mode of transport of the particles: saltating, rolling or sliding. Many researchers, such as Einstein (1950) and Yalin (1977), simply expressed that the threshold condition of a particle will occur when

$$F_L > W \tag{5}$$

where F_L = instantaneous lift force acting on a particle; and W = submerged weight force of the particle. A comprehensive analysis associated with (5) was provided by Yalin (1977).

The effective weight of a particle is a constant for a given diameter while the lift force varies depending on the near-bed turbulent flow and the relative position of the particles. The particle can only move provided that the instantaneous lift force acting on the particle exceeds its effective weight force. Hence, the pick-up probability for a bed particle can be defined as that of the instantaneous lift force greater than the effective weight force of the particle:

$$P = P(F_L > W) \tag{6}$$

The pick-up probability can also be considered as the fraction of the time when the lift force is greater than the effective weight for a given time interval, or the percentage of the number

of particles in motion on a fixed area of bed surface (Einstein, 1942; 1950).

Conventionally, F_L is related to the representative velocity near the bed in the form

$$F_L = C_L \frac{\pi d^2}{4} \frac{\rho u_b^2}{2} \quad (7)$$

where C_L = lift coefficient and u_b = instantaneous velocity approaching the particle on the bed. The lift coefficient is generally considered as an unknown function of the particle Reynolds number, $u_* d / \nu$ where u_* = shear velocity; and ν = viscosity of fluid. For a fully turbulent flow, C_L is a constant and is independent on the particle Reynolds number. The submerged weight force of a particle is equal to

$$W = (\rho_s - \rho) g \frac{\pi d^3}{6} \quad (8)$$

Substituting (7) and (8) into (6) yields

$$P = P(u_b^2 > B) = P(u_b > B) + P(u_b < -B) \quad (9)$$

where

$$B = \sqrt{\frac{4 \Delta g d}{3 C_L}} \quad (10)$$

Fig. 1 illustrates the probability in (9). Assuming that u_b obeys the normal or Gaussian distribution, its density function can be expressed as

$$f(u_b) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(u_b - \bar{u}_b)^2}{2\sigma^2}\right] \quad (11)$$

where \bar{u}_b = time-mean value of the instantaneous velocity u_b ; and the longitudinal intensity

of turbulence $\sigma = \sqrt{\overline{(u_b - \bar{u}_b)^2}}$. Using (11), one can rewrite (9) as

$$\begin{aligned} P &= 1 - \int_{-B}^B f(u_b) du_b \\ &= 1 - \int_{-B}^B \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\frac{u_b - \bar{u}_b}{\sigma})^2}{2}\right] d\left(\frac{u_b - \bar{u}_b}{\sigma}\right) \end{aligned} \quad (12)$$

Using the approximation analysis of the error function by Guo (1990), we get

$$\frac{1}{\sqrt{2\pi}} \int_0^x \exp\left(-\frac{t^2}{2}\right) dt \approx 0.5 \frac{x}{|x|} \sqrt{1 - \exp\left(-\frac{2x^2}{\pi}\right)} \quad \text{for } x \neq 0 \quad (13)$$

The error associated with using the approximation analysis in (13) is less than 0.7%. Using (13), (12) can be approximated as

$$P = 1 - 0.5 \frac{B - \bar{u}_b}{|B - \bar{u}_b|} \sqrt{1 - \exp\left[-\frac{2(B - \bar{u}_b)^2}{\pi\sigma^2}\right]} - 0.5 \sqrt{1 - \exp\left[-\frac{2(B + \bar{u}_b)^2}{\pi\sigma^2}\right]} \quad (14)$$

Mean Approach Velocity

The approach velocity in (14) can be calculated by using the universal logarithmic profile:

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln\left(\frac{y}{y_o}\right) \quad (15)$$

where \bar{u} = velocity at a height y above the theoretical level; κ = von Karman constant = 0.4 in clear water; y_o = zero-velocity level, and is assumed to be = $0.033 k_s$; and k_s = representative roughness height of the bed surface.

According to Hinze (1975) and van Rijn (1984), the origin of the velocity profile is located at a distance of $0.25d$ below the top of the bed particles. At the beginning of motion, a particle may be assumed to be lying on a bed consisting of closely packed particles with identical size. The most stable situation is that when the particles rest in an interstice formed by the top layer particles. This schematization results in an initial position at $y_b = 0.6d$. Using (15) and assuming that $k_s = 2d$, we get $\bar{u}_b = 5.52u_*$.

Near-Bed Turbulence Intensity

With reference to the measurements in a rough boundary open-channel flow conducted by Kironoto and Graf (1994), the dimensionless near bed turbulence intensity σ/u_* is approximately 2.0.

Lift Coefficient

Different values of lift coefficients have been reported in the literature. Einstein and El-Samni (1949) performed an experiment for Re_* ranging from 3300 to 5600, and proposed a lift coefficient of 0.178. They used the approach velocity at the level $0.35d$ above the theoretical bed level for their computation. Li et al (1983) measured the hydrodynamic forces acting on a single sphere lying on a smooth bed. Their results showed a lift coefficient C_L of 0.18 for high Reynolds number with the use of the representative velocity at the level $0.5d$ from the bed surface. A recent experimental study was conducted by Patnaik et al (1994) to measure the lift force experienced by a stationary sphere in a boundary layer flow. The lift coefficient for the sphere in smooth and rough boundaries was found to be dependent on both the Reynolds number and the relative diameter of the sphere. The latter is defined as the ratio of particle diameter to the boundary layer thickness. For small relative diameter, the lift coefficient reaches a value up to 0.4, while it is about 0.1 for a relative diameter of 0.5. Based on these results, the authors inferred that the lift force could become negative if the relative diameter is further increased. Their data also showed a slightly decreasing trend for the lift coefficient with increasing Reynolds number. The above summary shows that a wide variation of C_L value can be found in published literature. Because of the uncertainty, the present study uses several values of C_L (from 0.1 to 0.4) for computations.

It has been shown in the preceding analysis that $\bar{u}_b = 5.52u_*$ and $\sigma = 2.0u_*$ for a hydraulically rough bed. Substituting these values into (14) yields

$$P = 1 - 0.5 \frac{0.21 - \sqrt{\theta C_L}}{0.21 + \sqrt{\theta C_L}} \sqrt{1 - \exp\left[-\left(\frac{0.46}{\sqrt{\theta C_L}} - 2.2\right)^2\right]} - 0.5 \sqrt{1 - \exp\left[-\left(\frac{0.46}{\sqrt{\theta C_L}} + 2.2\right)^2\right]} \quad (16)$$

Eq. (16) infers that the pick-up probability only depends on the dimensionless shear stress if C_L is known. Fig. 2 shows a family of curves of P against θ , with C_L as the third parameter, ranging from 0.1 to 0.4.

COMPARISON WITH EXISTING FORMULAE

Fig. 3 shows that (16) developed in the present study agrees well with the experimental data stated in the earlier section when C_L is set to be 0.25. The choice of $C_L = 0.25$ is well within the range of most C_L values reported in published literature. The empirical equation by Fredsoe and Deigarrd (4) and the theoretical formula by Einstein (1) are superimposed in the figure for comparison. The results show that (1) overpredicts the pick-up probability as compared with the measured data.

The dimensionless shear stress, θ for high Re^* ($Re^* > 500$) in the Shields curve has often been cited in the literature to have a value of approximately 0.05 (Yalin, 1977). Based on the results in Fig. 3, the pick-up probability is approximately 0.6% when $\theta = 0.05$. This result infers that the critical condition calculated using the Shields criterion is equivalent to a pick-up probability of approximately 0.6%. The latter implies that 0.6% of all the particles on a given bed area is in motion under the threshold condition of sediment transport.

CONCLUSIONS

The study presents a theoretical derivation of the pick-up probability of sediment entrainment in a hydraulically rough flow. The proposed equation agrees well with available experimental data for the lift coefficient of 0.25. The pick-up probability is about 0.6% when compared to the Shields' criterion for the threshold condition of sediment transport.

APPENDIX I. REFERENCES

1. Einstein, H. A. (1942). "Formula for the transportation of bed load." *Trans., ASCE*, 107, 561-597.
2. Einstein, H. A., and El-Samni, E. A. (1949). "Hydrodynamic forces on a rough wall." *Review of Modern Physics*, 21, 520-524.
3. Einstein, H. A. (1950). "The bed load function for sediment transportation in open channel flows." Technical Bulletin 1026, U. S. Department of Agriculture.
4. Engelund, F., and Fredsoe, J. (1976). "A sediment transport model for straight alluvial channels." *Nordic Hydrology*, 7, 293-306.
5. Fredsoe, J. and Deigaard, R. (1992). "Mechanics of coastal sediment transport." World Scientific Publishing Co. Pte. Ltd.
6. Guo, J. K. (1990). "Re-study on Einstein's theory on bed material transport." *Proc. Youth Congress on Water Science*, Water Resources Press, 337-342 (in Chinese).
7. Guy, H. P., Simons, D. B., and Richardson, E. V. (1966). "Summary of alluvial channel data from flume experiments, 1956-1961." U. S. Geological Survey Professional Paper, 462-I.
8. Hinze, J. (1975). "Turbulence." McGraw-Hill Inc. 2nd Edition.
9. Kironoto, B. A., and Graf, W. H. (1994). "Turbulence characteristics in rough uniform open-channel flow." *Proc. Instn. Civ. Engrs., Wat., Marit. And Energy*, 106, Dec., 333-344.
10. Lane, E. W., and Kalinske, A. A. (1939). "The relation of suspended to bed material in rivers." *Trans. American Geophysical Union, Section of Hydrology*, 637-641.
11. Li, Z. Y., et al (1983). "Laboratory investigation on drag and lift forces acting on bed spheres." *Proc. 2nd International Symposium of River Sedimentation*, Water and Power Press, Beijing, 330-340.

12. Luque, R. F. (1974). "Erosion and transport of bed load sediment." Delft University of Technology.
13. Patnaik, P. C., Vittal, N., and Pande, P. K. (1994) "Lift coefficient of a stationary sphere in gradient flow." Journal of Hydraulic Research, 32(3), 471-480.
14. van Rijn, L. C. (1984). "Sediment pick-up function." Journal of Hydraulic Engineering, ASCE, 110(10), 1494-1502.
15. Yalin, M. S. (1977). "Mechanics of sediment transport." Pergamon Press.

APPENDIX II. NOTATION

The following symbols are used in this paper:

- C_L = lift coefficient;
- d = diameter of a particle;
- F_L = instantaneous lift force;
- g = gravitational acceleration;
- k_s = representative roughness height of the bed surface;
- t = variable of integration;
- u_b = instantaneous velocity approaching to a bed particle;
- u_* = shear velocity;
- \bar{u} = time-mean velocity at height y above the theoretical level;
- \bar{u}_b = time-mean value of the instantaneous velocity u_b ;
- W = submerged weight force of a particle;
- y = height above theoretical level;
- y_0 = zero-velocity level;
- y_b = initial position for the particle to move;

$$\Delta = (\rho_s - \rho)/\rho;$$

$$\kappa = \text{Von Karman constant};$$

$$\nu = \text{viscosity of fluid};$$

$$\theta = u_*^2/(\Delta g d).$$

$$\rho = \text{density of fluid};$$

$$\rho_s = \text{density of sand particles; and}$$

$$\sigma = \sqrt{(u_b - \bar{u}_b)^2}.$$

CAPTION FOR FIGURES

Fig. 1 Definition sketch of pick-up probability.

Fig. 2 Pick-up probability as a function of dimensionless shear stress and lift coefficient.

Fig. 3. Comparison of pick-up probability with experimental data and the theoretical expression of Einstein (1950)

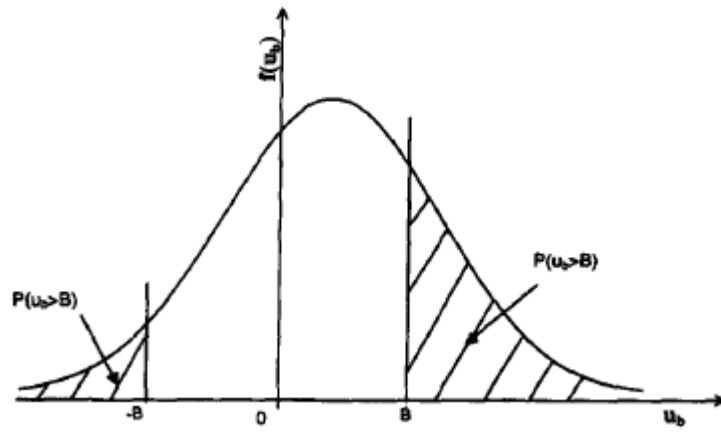


Fig. 1

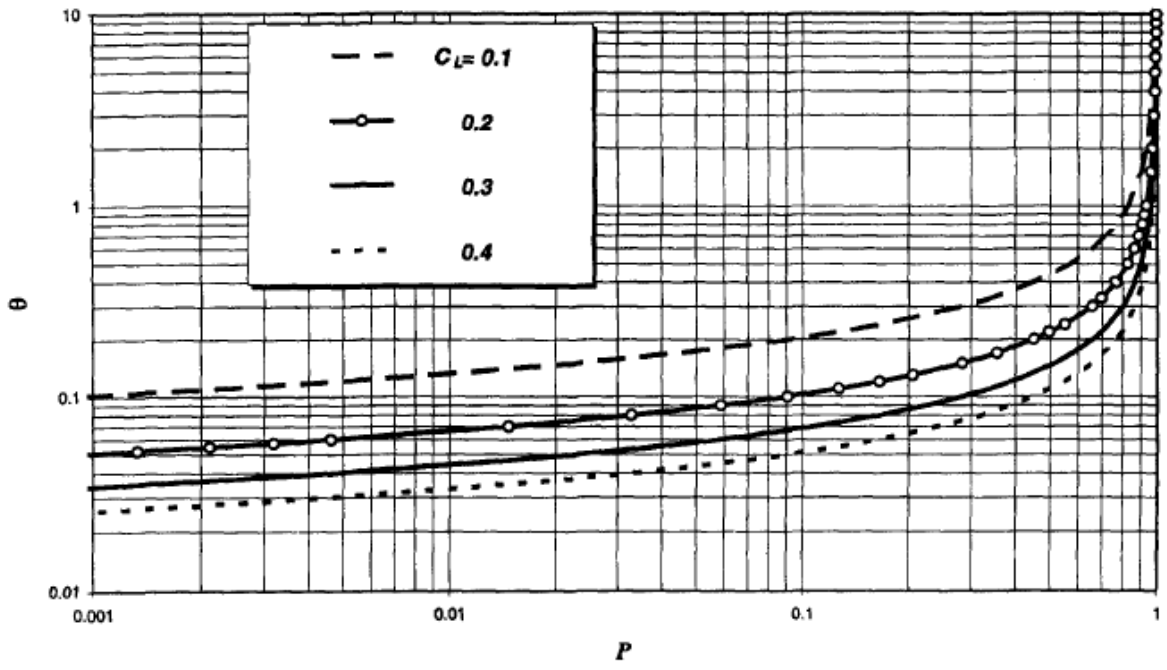


Fig. 2

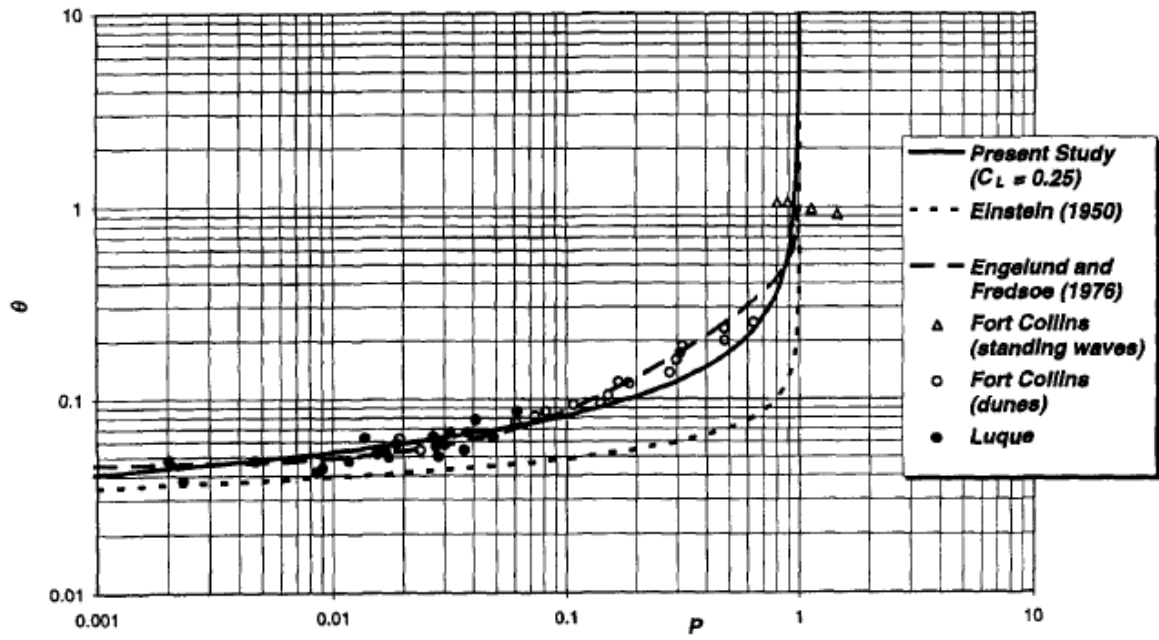


Fig. 3