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Chiew, Yee-Meng; Cheng, Nian-Sheng

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MODIFIED LOGARITHMIC LAW FOR VELOCITY DISTRIBUTION SUBJECTED TO UPWARD SEEPAGE

Nian-Sheng Cheng¹ and Yee-Meng Chiew²

¹Postdoctoral Fellow, Dept. of Hydrodynamics and Water Resour., Tech. Univ. of Denmark, Lyngby 2800, Denmark

²Sr. Lect., School of Civ. and Struct. Engrg., Nanyang Technol. Univ., Nanyang Ave., Singapore 639798.

ABSTRACT

A modified logarithmic law is derived to describe the velocity distribution in open-channel flow with an upward seepage. The derived function agrees well with the experimental measurements conducted in a laboratory flume. The roughness function included in the modified logarithmic law is found to be dependent on the boundary Reynolds number and the ratio of the seepage velocity to the shear velocity. With the integration of the modified logarithmic law, the bed-shear stress subjected to upward seepage can be computed based on the depth-averaged velocity, water depth, boundary roughness, and seepage velocity. Using this method, variations of the bed-shear stress over the seepage zone are studied. The results show that the bed-shear stress decreases rapidly from the leading section of the seepage zone and then increases gradually toward the downstream end of the seepage zone.

INTRODUCTION

The well-known logarithmic law for velocity distribution has been used widely in various turbulent shear flows over a solid surface, such as boundary layer flows, pipe flows, and open-channel flows. Because the logarithmic law originally was derived only for the inner zone of turbulent shear flows, many methods have since been proposed to account for the deviation from the logarithmic law when applying it to other regions. For example, the modified Prandtl's mixing-length theory proposed by van Driest (1956) extends the logarithmic law to the viscous sublayer and the buffer layer. Another example is the wake function added to the logarithmic law by Coles (1956), which is of nearly universal character even though it was derived empirically. The addition of the wake function enables the application of the logarithmic law to the outer region of turbulent shear flow.

However, for practical purposes, researchers frequently have assumed that the logarithmic law can describe the velocity distribution throughout the entire depth in open-channel flow, for instance, in some applications in the fields of hydraulics and sediment transport. In the presence of a sediment bed, one of the problems encountered in applying the logarithmic law is the determination of the zero-velocity level and the representative roughness height of the bed surface. A succinct review of such studies can be found in Hinze (1975) and Yalin (1977). In general, the origin of the logarithmic velocity profile can be assumed to be located at a distance 0.25 times the

diameter of the uniform sediment particles below the upper surface of the particles, and the representative roughness height is, on the average, equal to two times the diameter of the particles.

In the presence of boundary mass transfer, velocity profiles have been reported to be different from the logarithmic law in the field of aerodynamics. Clarke et al. (1955) proposed a modified law of the wall, based on Prandtl's mixing-length assumption, to describe the velocity distribution with boundary injection. Similar studies also have been presented by Stevenson (1963) and Schlichting (1979). However, very few studies on the modifications of velocity profiles are related to open-channel flows subjected to bed seepage, where the permeable boundary consists of sediment particles. Among the studies that can be found in the literature is one by Willetts and Drossos (1975), who proposed an exponential law to depict the streamwise velocity over the suction (downward seepage) zone. Maclean (1991) inferred from laboratory measurements that the velocity profile over the suction zone may consist of two parts: a suction boundary layer near the bed and a logarithmic region above the suction boundary layer. He also reported that the velocity gradient in the logarithmic region reduced in the downstream direction.

The objective of this study is to consider theoretically the seepage effect on the velocity distribution in a two-dimensional open-channel flow based on the balance of turbulence kinetic energy. The derived function is compared with the velocity profiles measured in a laboratory flume, and it also is used to evaluate the bed-shear stress over the seepage zone.

MODIFIED LOGARITHMIC LAW

The transport equation of turbulence kinetic energy can be derived in the exact form from the Navier-Stokes equations (Hinze 1975; Rodi 1993). For a steady two-dimensional flow, a simplified form for modeling this equation can be given by

$$u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} = \frac{\tau}{\rho} \frac{\partial u}{\partial y} + \frac{\partial}{\partial y} \left(\frac{v_t}{\sigma_k} \frac{\partial k}{\partial y} \right) - \epsilon \quad (1)$$

where k = averaged turbulence kinetic energy per unit mass; ϵ = dissipation rate of the turbulence kinetic energy by the action of viscous forces; τ = Reynolds shear stress; v_t = turbulent or eddy viscosity; and σ_k = an empirical constant. Implicit in (1) is the assumption that variations of both the velocity and the kinetic energy can be considered to be predominant in the direction normal to the boundary. The first term on the right side of (1) denotes the production rate of the turbulence kinetic energy. The second term is the diffusion rate and the third term is the dissipation rate. The terms on the left side of (1) represent the convective rates.

Open-channel flow falls into the category of turbulent shear flows confined by a rigid boundary, of which the prominent structural feature is the existence of an equilibrium layer (Townsend 1976). In the equilibrium layer, the rates of production and of dissipation of turbulence kinetic energy are especially large compared with those in the other parts of the flow and the motion thus is determined mainly by the local flow conditions. Based on the experimental investigations conducted in open-channel flow over both smooth and rough beds without boundary seepage, Nakagawa et al. (1975) have shown that the turbulence production almost balances the

turbulence dissipation throughout the whole depth except for the free surface region. Fig. 1 is a semilogarithmic plot of Nakagawa et al.'s experimental data of the turbulence energy distribution over a smooth bed. The data show that the equilibrium layer can reach to a relative depth of approximately 0.6 up from the bed. Moreover, their experiments suggest that the equilibrium layer becomes much thicker for flows over a rough bed.

In the presence of boundary seepage, the characteristics of turbulent open-channel flow is likely to change considerably. However, according to Townsend (1976)'s study on the boundary layer, the main properties of flows with boundary mass transfer still are determined by the equilibrium layers near the boundary. With this consideration, the convection term [the terms on the left side of (1)] and the diffusion term [the second term on the right side of (1)] can be neglected and (1) thus can be simplified to

$$\frac{\tau}{\rho} \frac{du}{dy} = \epsilon \quad (2)$$

Furthermore, because the dissipation rate ϵ is the rate of conversion of the turbulence kinetic energy into heat, it can be related to both the characteristic timescale and the characteristic velocity scale for turbulent motion. With the use of dimensional analysis, the dissipation is expected to be of the form (characteristic turbulence velocity)²/(characteristic turbulence timescale). For a two-dimensional flow, the characteristic turbulence velocity can be taken as $\sqrt{\tau/\rho}$ and the characteristic turbulence timescale as $L/\sqrt{\tau/\rho}$, where L is an eddy length scale (Young 1989). Therefore

$$\epsilon = \frac{(\tau/\rho)^{3/2}}{L} \quad (3)$$

When the generation and dissipation of the turbulence kinetic energy are roughly in balance, L can be identified simply by the Prandtl's mixing length $l (= \kappa y)$, where κ = von Karman constant. Therefore, (3) changes to

$$\epsilon = \frac{(\tau/\rho)^{3/2}}{\kappa y} \quad (4)$$

Substituting (4) into (2) yields

$$\frac{du}{dy} = \sqrt{\frac{\tau}{\rho}} \frac{1}{\kappa y} \quad (5)$$

On the other hand, for a two-dimensional flow, the streamwise momentum equation can be expressed as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \sin \phi_b - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad (6)$$

Eq. (6) can be simplified for the equilibrium layer with the following assumptions (Townsend 1976): first, the pressure gradient including the streamwise component of gravity is negligible because it is not sufficient to induce significant flow acceleration; second, the variation of u with x can be ignored as compared with its variation in the normal direction. Furthermore, with the boundary condition on v , application of the equation of continuity leads to

$$v = v_s \quad (7)$$

With (7) and the aforementioned assumptions, (6) reduces to

$$v_s \frac{du}{dy} = \frac{1}{\rho} \frac{d\tau}{dy} \quad (8)$$

Integration of (8) yields

$$\tau = \tau_b + \rho u v_s \quad (9)$$

where τ_b = boundary shear stress. Substituting (9) into (5), one gets

$$\frac{du}{dy} = \frac{\sqrt{u_*^2 + u v_s}}{\kappa y} \quad (10)$$

where u_* = shear velocity = $\sqrt{\tau_b/\rho}$. Integrating (10) from $y = y_0$ to $y = y$, where y_0 is the distance at which $u = 0$, one gets

$$\frac{2}{v_s} (\sqrt{u_*^2 + u v_s} - u_*) = \frac{1}{\kappa} \ln \frac{y}{y_0} \quad (11)$$

Eq. (11) also can be arranged in the form

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y}{y_0} + \frac{v_s}{4u_*} \left(\frac{1}{\kappa} \ln \frac{y}{y_0} \right)^2 \quad (12)$$

Eq. (12) reverts to the logarithmic law in the case of zero seepage. Alternatively, (12) also can be obtained by directly applying the mixing-length theory [e.g., Schlichting (1979)].

In the absence of seepage, y_0 is dependent on the viscous sublayer thickness for a smooth boundary, whereas it is dependent on the roughness height for a rough boundary (Raudkivi 1990). When an equivalent sand roughness k_s , is used, y_0 can be expressed as

$$y_0 = k_s \exp(-\kappa B) \quad (13)$$

where B = roughness function, and its evaluation will be discussed in the subsequent section for the cases with and without seepage. Substituting (13) into (11) and (12), respectively, leads to

$$u_p^+ = \frac{1}{\kappa} \ln \frac{y}{k_s} + B \quad (14)$$

and

$$u^+ = \frac{1}{\kappa} \ln \frac{y}{k_s} + B + \frac{v_s^+}{4} \left(\frac{1}{\kappa} \ln \frac{y}{k_s} + B \right)^2 \quad (15)$$

where $u^+ = u/u_*$, $v_s^+ = v_s/u_*$, and

$$u_p^+ = \frac{2(\sqrt{1 + v_s^+ u^+} - 1)}{v_s^+} \quad (16)$$

Eq. (11) or (14) is referred to as the modified logarithmic law for velocity distribution subjected to seepage.

EXPERIMENTS

The experiments were conducted in a glass-sided horizontal flume 30 m long, 0.7 m wide, and 0.6 m deep, with a seepage zone located 16 m from the upstream end of the flume. The detail of the experimental setup can be found in Cheng (1997). The seepage zone consisted of a recess that was 2.0 m long, 0.7 m wide, and 0.4 m deep (Fig. 2). An upward seepage was applied from the bottom of the recess. A layer of sediment particles, 0.2 m thick, was placed in the upper portion of the recess. The same sediment particles as used in the seepage zone also were glued to the impermeable floor in the flume to furnish a consistent bed roughness. Two sediments with diameters = 1.95 and 5.83 mm were used. Sediment particles remained stationary and no quick condition occurred in all the tests.

The velocity measurements were conducted mainly at the middle section ($x = 1.0$ m, Fig. 2) of the seepage zone using either a two-dimensional acoustic Doppler velocimeter (ADV) or an 8 mm minipropeller current meter. The ADV system applied acoustic sensing techniques to measure flow in a remote sampling volume that was located below the tip of the probe. The acoustic sensor consisted of one transmitting transducer and two receiving transducers. The sampling volume was approximately 9 mm in height and was defined by the intersection of the transmitting and receiving beams. The mini-propeller current meter was designed to measure

velocity in the range of 4-300 cm/s. Measurements of the mean velocity obtained using the ADV and the minipropeller were found to agree well with each other.

COMPARISON WITH EXPERIMENTAL DATA

The values of the boundary Reynolds number for all the data collected in this study are greater than approximately 70, showing that the flows considered are within the rough turbulent regime. The shear velocity is evaluated using the momentum integral equation, which was derived in Cheng (1997) for a horizontal bed in the form

$$\tau_b = \rho g h \frac{dh}{dx} \left(1 - \frac{\beta U^2}{gh} \right) - 2\rho\beta U v_s \quad (17)$$

where h = water depth; β = momentum correction factor; and U = depth-averaged velocity.

Typical velocity distributions measured for verticals at $x = 1.0$ m are plotted as u^+ against $\ln(y/y_o)$ in Figs. 3(a-d) for $v_s^+ = 0.137, 0.258, 0.472$, and 0.610 , respectively. The modified logarithmic law, in (12), and the logarithmic law of the wall, in (12) with $v_s = 0$, are superimposed in these figures for comparison. The results show good agreement between the measured data and those calculated using (12) when suitable values of B are chosen. The results show that the deviation of the measured velocity distribution from the logarithmic law of the wall increases with increasing ratio of the seepage velocity to the shear velocity v_s^+ .

When fitting the modified logarithmic law in (14) to the experimental data as shown in Fig. 4, the values of B are found to be dependent on the dimensionless seepage velocity v_s^+ . With a progressive increase of v_s^+ to 0.61, the derived B -values show a corresponding decrease roughly from 8.5 to 5.0.

ROUGHNESS FUNCTION INCLUDING SEEPAGE EFFECT

In the case of zero seepage, the roughness function B only depends on the boundary Reynolds number $k_s^+ = \rho u_* k_s / \mu$, where μ = dynamic viscosity of fluid. Fig. 5 shows the widely cited relation of B versus k_s^+ , which was derived from Nikuradse's (1933) measurements for sand-roughened pipes (Schlichting 1979). Such studies show that

$$B = 8.5 \quad (18)$$

for a completely rough regime; and

$$B = 2.5 \ln k_s^+ + 5.5 \quad (19)$$

for a completely smooth regime. Recently, Yalin (1992) proposed an empirical expression to simulate the Nikuradse's data in the form

$$B = 8.5 + (2.5 \ln k_s^+ - 3)\exp(-0.127 \ln^2 k_s^+) \quad (20)$$

Unfortunately, superimposing (20) to Fig. 5 indicates that (20) deviates from the experimental data for $k_s^+ = 7 \sim 400$. With a trial-and-error procedure, we find that Yalin's simulation can be improved by modifying (20) to the form

$$B = 8.5 + (2.5 \ln k_s^+ - 3)\exp(-0.11 \ln^{2.5} k_s^+) \quad (21)$$

Fig. 5 shows that (21) agrees much better with the experimental data.

As discussed earlier, the roughness function B is affected by the dimensionless seepage velocity v_s^+ for rough turbulent flow in the presence of seepage. Therefore, for open-channel flow subjected to seepage, B depends on both the dimensionless seepage velocity v_s^+ and the boundary Reynolds number k_s^+ . For the completely rough regime, the effect of fluid viscosity is negligible, and it follows that B is solely dependent on v_s^+ . Using the experimental data collected in this study, B can be related empirically to v_s^+ for the completely rough regime in the form (Fig. 6)

$$B = \frac{8.5}{1 + v_s^+} \quad (22)$$

Fig. 6 or (22) shows that B reverts to 8.5 when the seepage velocity vanishes. With the aid of (22), the velocity profiles with different values of dimensionless seepage velocity v_s^+ are found to collapse onto one line as shown in Fig. 7.

When studying the turbulent boundary layer on a smooth bed with suction and injection, Stevenson (1963) concluded that the constant of integration B varies very little with suction or injection. Fig. 8 is a plot of his results with u_p^+ , versus y^+ , in which $y^+ = \rho u_* y / \mu$, showing that the B -value can be expressed approximately by (19) regardless of the variations of the seepage velocity. This implies that the effect of mass transfer through the porous boundary on the roughness function can be considered insignificant for a hydraulically smooth boundary.

For the transitional regime between a completely rough and a completely smooth regime, a reasonable approach for the evaluation of B is to rewrite (21) to account for the seepage effect to the form

$$B = 8.5 - \delta + (2.5 \ln k_s^+ - 3 + \delta)\exp(-0.11 \ln^{2.5} k_s^+) \quad (23)$$

where

$$\delta = \frac{8.5 v_s^+}{1 + v_s^+} \quad (24)$$

The form of (23) is chosen deliberately so that it follows the trend of (21) in the transitional regime, and it satisfies the boundary conditions for both the completely smooth and the completely rough regime, as defined by (19) and (22), respectively. Fig. 9 shows the relationship of B and k_s^+ with a variation of v_s^+ as the third parameter according to (23). It is clear that (23) reduces to (19) and (22), respectively, for relatively small and relatively large values of k_s^+ . Using the extended roughness function expressed by (23), the modified logarithmic law [see (11) or (14)] is expected to be applicable to all regions of turbulent flows subjected to seepage, from a completely smooth regime to a completely rough regime.

APPLICATION OF MODIFIED LOGARITHMIC LAW FOR EVALUATION OF BED-SHEAR STRESS

The momentum integral equation in (17) is one approach to determine the bed-shear stress in an open-channel flow with seepage. This method is practical when accurate measurements of the water surface slope subjected to seepage are available. It has been shown that the shear velocities evaluated using this method have a good agreement with those obtained by extrapolating the Reynolds shear stress distribution to the boundary (Cheng 1997). Here, the modified logarithmic law for velocity profiles is used to evaluate the bed-shear stress in open-channel flow with seepage.

When the modified logarithmic law is used to describe the variation of the velocity distribution caused by seepage, the corresponding shear velocity can be determined by directly fitting the modified logarithmic law to the velocity measurements. However, the results of the shear velocity so obtained have been found to be very sensitive to the uncertainties involved in the measurements of the near-bed velocity. An alternative approach is to relate the shear velocity to the depth-averaged velocity. This can be achieved by integrating (12) with respect to y from y_0 to h (see Appendix I), which leads to

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{h}{y_0} - \frac{h - y_0}{h} \right) + \frac{v_s}{4\kappa^2 u_*} \left[\ln^2 \frac{h}{y_0} - 2 \left(\ln \frac{h}{y_0} - \frac{h - y_0}{h} \right) \right] \quad (25)$$

Assuming that $y_0/h \ll 1$, (25) can be simplified to

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{h}{y_0} - 1 \right) + \frac{v_s}{4\kappa^2 u_*} \left[\ln^2 \frac{h}{y_0} - 2 \left(\ln \frac{h}{y_0} - 1 \right) \right] \quad (26)$$

Because y_0 is dependent on B as expressed by (13), and thus on the shear velocity, an iterative procedure is needed when using (26) to compute the shear velocity. Table 1 contains all the experimental data and Fig. 10 is a plot of the comparison of the shear velocities evaluated using (26) and the momentum integral equation in (17). The figure shows that most of the data points fall within the $\pm 20\%$ limit lines.

Variations of Bed-Shear Stress over Seepage Zone

When (26) is used to evaluate the bed-shear stress over the entire seepage zone, even at the section near the leading edge of the zone, the streamwise variation of the bed-shear stress over the zone thus can be obtained. An example is given in Fig. 11 in which the bed shear stresses were calculated using the integrated modified logarithmic law (26), where L denotes the length of the seepage zone and τ_{b0} the bed-shear stress on the impermeable bed upstream of the seepage zone (i.e., with no seepage). The computed results show that the bed-shear stress is reduced sharply at the beginning of the seepage zone. The reduction becomes more apparent for higher seepage intensity. However, toward the downstream end of the seepage zone the bed-shear stress exhibits a gradual increase. This is because the upward seepage, as an inflow, introduces additional mass to the flow. The effect of the increased flow rate on the bed-shear stress is negligible at the beginning of the seepage zone, but it becomes significant farther away from the leading section of the seepage zone.

CONCLUSIONS

A modified logarithmic law was derived in this paper to account for the effect of upward seepage on the velocity distribution in open-channel flow. The derived expression reverts to the logarithmic law in the absence of seepage. The roughness function included in the modified logarithmic law depends not only on the boundary Reynolds number but also on the dimensionless seepage velocity. An empirical expression of the roughness function was proposed to enable the application of the modified logarithmic law to various turbulent flows, from a hydraulically smooth regime to a hydraulically rough regime.

With the integration of the modified logarithmic law, the shear velocity is related to the depth-averaged velocity, water depth, boundary roughness, and seepage velocity. The relationship so obtained can serve as a convenient method to determine the bed-shear stress of an open-channel flow with seepage. The computed bed-shear stress using this method shows an apparent decrease at the beginning of the seepage zone, followed by a gradual increase toward the downstream end of the seepage zone.

APPENDIX I. INTEGRATION OF MODIFIED LOGARITHMIC LAW

The depth-averaged velocity U is defined as

$$U = \frac{1}{h} \int_{y_0}^h u \, dy \quad (27)$$

It can be obtained by integrating (12) from $y = y_0$ to $y = h$ as follows:

$$U = \frac{u_*}{\kappa h} \int_{y_0}^h \ln \frac{h}{y_0} \, dy + \frac{v_s}{4\kappa^2 h} \int_{y_0}^h \ln^2 \frac{y}{y_0} \, dy \quad (28)$$

Because

$$\int_{y_0}^h \ln \frac{y}{y_0} \, dy = h \left(\ln \frac{h}{y_0} - 1 \right) + y_0 \quad (29)$$

$$\int_{y_0}^h \ln^2 \frac{y}{y_0} \, dy = h \ln^2 \frac{h}{y_0} - 2h \left(\ln \frac{h}{y_0} - 1 \right) - 2y_0 \quad (30)$$

Eq. (28) can be changed to the form

$$U = \frac{u_*}{\kappa} \left(\ln \frac{h}{y_0} - 1 + \frac{y_0}{h} \right) + \frac{v_s}{4\kappa^2} \left[\ln^2 \frac{y}{y_0} - 2 \left(\ln \frac{h}{y_0} - 1 + \frac{y_0}{h} \right) \right] \quad (31)$$

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APPENDIX III. NOTATION

The following symbols are used in this paper:

- B = roughness function;
 c = constant of integration;
 d_{50} = median size of sediment particles;
 g = gravitational acceleration;
 h = water depth;
 h_0 = water depth at leading section of seepage zone;
 k = turbulence kinetic energy per unit mass;
 k_s = equivalent sand roughness = $2d_{50}$;
 k_s^+ = $\rho u_* k_s / \mu$ = boundary Reynolds number;
 p = pressure;
 U = depth-averaged velocity;
 u = time-mean streamwise velocity;
 $u^+ = u/u_*$;
 $u_p^+ = 2(\sqrt{1 + v_s^+ u^+} - 1)/v_s^+$;
 $u_* = \sqrt{\tau_b/\rho}$ = shear velocity;
 u_{*0} = shear velocity on impermeable bed just upstream of leading edge of seepage zone;
 v = time-mean normal velocity;
 v_s = seepage discharge divided by area of recess = seepage velocity;
 ν_t = eddy viscosity;
 $v_s^+ = v_s/u_*$;
 x = streamwise coordinate or streamwise distance from leading edge of seepage zone;
 y = normal coordinate;
 $y_0 = k_s \exp(-\kappa B)$;
 β = momentum correction factor;
 $\delta = 8.5v_s^+/(1 + v_s^+)$;
 ϵ = dissipation rate of kinetic turbulent energy;
 κ = von Karman's constant;
 μ = dynamic viscosity of fluid;
 ρ = density of fluid;
 σ_k = empirical parameter;
 τ = shear stress;
 τ_b = bed-shear stress;
 τ_{b0} = bed-shear stress on upstream impermeable bed; and
 ϕ_b = angle of bed slope.

Superscript

$+$ = dimensionless terms.

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No. (1)	Run (2)	U (cm/s) (3)	h (cm) (4)	v_s (cm/s) (5)	β (6)	u_{s1} (cm/s) (7)	u_{s2} (cm/s) (8)
1	M4-100	32.48	11.9	0	1.024	2.62	2.23
2	M5-101	32.43	11.9	0.069	1.018	2.10	2.02
3	M5-201	34.37	12.0	0.274	1.030	2.05	1.63
4	M6-101	37.15	10.6	0	1.019	2.59	2.61
5	M7-101	39.20	10.6	0.189	1.024	2.52	2.22
6	M7-201	36.97	7.9	0.189	1.024	2.24	2.24
7	M8-104	36.04	14.8	0.235	1.030	1.72	1.71
8	M20-101	30.45	12.8	0.453	1.041	1.18	1.06
9	M21-101	30.72	15.3	0.475	1.049	1.49	0.96
10	M22-100	31.42	15.0	0.384	1.038	1.41	1.13
11	M22-200	31.75	15.2	0.447	1.044	1.20	1.05
12	M22-300	27.13	14.5	0.446	1.048	1.21	0.84
13	M22-400	29.84	10.4	0.448	1.046	1.05	1.14
14	M22-500	25.77	10.4	0.448	1.053	0.95	0.92
15	M22-600	22.98	11.4	0.476	1.066	0.78	0.71
16	M23-100	33.98	12.7	0	1.019	2.49	2.31
17	M23-200	34.81	12.7	0.178	1.025	2.29	1.85
18	M23-300	37.17	12.7	0.333	1.031	1.43	1.65
19	M23-400	38.68	12.6	0.447	1.037	1.73	1.54
20	E3-100	20.38	11.2	0.467	1.066	0.72	0.60
21	E3-200	23.41	11.2	0.467	1.049	0.99	0.74
22	E3-300	22.25	11.2	0.467	1.054	0.76	0.69
23	E3-400	37.04	11.2	0.467	1.035	1.21	1.49
24	E3-500	38.03	9.2	0.467	1.032	1.90	1.69
25	E4-100	39.37	9.5	0.469	1.036	1.48	1.74
26	E4-200	41.01	8.6	0	1.019	2.46	2.99
27	E4-300	42.96	10.2	0	1.018	2.92	3.03
28	E5-100	42.57	10.4	0	1.019	2.96	3.00
29	E5-200	45.20	10.7	0.42	1.033	1.68	2.11
30	E6-100	44.66	10.4	0.369	1.033	3.27	2.20
31	E6-200	44.87	10.1	0	1.018	2.43	3.17
32	E6-300	29.20	10.7	0.457	1.051	0.85	1.08
33	E19-100	39.60	13.0	0	1.023	3.34	3.29
34	E-19-300	41.02	13.5	0	1.023	3.56	3.38
35	E24-100	42.91	14.2	0	1.026	3.88	3.50
36	E24-200	42.82	14.2	0	1.021	3.88	3.50
37	E24-300	44.75	14.2	0.154	1.028	3.78	3.29
38	E25-100	45.20	14.2	0.298	1.03	3.77	3.04
39	E25-200	46.06	14.2	0.467	1.033	3.77	2.82
40	Y2-100	26.58	14.9	0	1.022	2.89	2.15
41	Y2-200	28.93	14.9	0.438	1.037	2.15	1.54
42	Y3-100	27.45	14.9	0.223	1.024	2.33	1.74
43	Y3-200	31.55	13.1	0.223	1.022	2.39	2.15
44	Y3-300	37.06	13.1	0.448	1.027	2.65	2.23
45	Y9-103	44.90	14.6	0	1.025	3.91	3.64
46	Y9-205	47.26	14.7	0.471	1.025	3.71	2.87
47	Y11-109	47.56	14.4	0.475	1.025	3.39	2.90
48	Y16-110	46.57	14.4	0.219	1.025	3.84	3.29

Note: Values of u_{s1} and u_{s2} were computed using (17) and (26), respectively.
 d_{90} equals 1.95 mm for Nos. 1–32 and 5.83 mm for the others.

Table 1

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11