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Zhao, Kuifeng; Cheng, Nian-Sheng; Huang, Zhenhua

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Experimental study of free-surface fluctuations in open channel flow in the presence of periodic cylinder arrays

KUIFENG ZHAO, PhD Student, *School of Civil and Environmental Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798. Tel.: +65 6790 6619; fax: +65 6790 6620; Email: zhao0142@e.ntu.edu.sg (author for correspondence)*

NIAN-SHENG CHENG, Associate Professor, *School of Civil and Environmental Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798. Email: cnscheng@ntu.edu.sg*

ZHENHUA HUANG, Assistant Professor, *School of Civil and Environmental Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798. Email: zhhuang@ntu.edu.sg;*
Assistant Professor, *School of Ocean and Earth Science and Technology (SOEST) University of Hawaii at Manoa, Honolulu, HI, USA. Email: zhenhua@hawaii.edu*

ABSTRACT

The presence of periodic cylinder arrays in a laboratory-scale open channel flow may induce significant fluctuations in the free surface elevation. Experimental observations show four types of free surface patterns, which include plane surface, transversely oscillating surface, longitudinally oscillating surface, and locally disturbed surface. The transversely oscillating surface appears to be standing waves confined between the two sidewalls in the flume and is well described by velocity potential equations. By considering the effect of the cylinder density, a modified dispersion equation is used to compute the frequency of the standing waves. The computed results agree well with the measured dominant frequency especially for cases with higher cylinder density, in comparison with those calculated using the equation without considering the effect of cylinder density. The data analysis also shows that the fluctuation in the free-surface elevation varies with flow conditions, and the maximum fluctuation occurs when the cylinder diameter based Reynolds number varies in the range of 600–800 or the cylinder diameter based Froude number varies in the range of 0.24 – 0.32.

Keywords: emergent vegetation, free-surface fluctuation, frequency, open channel flow, standing waves

1 Introduction

In conducting laboratory studies of vegetated open channel flows, rigid cylindrical rods are often employed to simulate vegetation stems (e.g., Stone and Shen 2002; James *et al.* 2004; Liu *et al.* 2008; Tanino and Nepf 2008; Cheng and Nguyen 2011). Such experimental setups, though facilitating

investigations of vegetation-related resistance (Cheng and Nguyen 2011) and sediment transport (Tollner *et al.* 1982; Jordanova and James 2003; Kothiyari *et al.* 2009), introduce free-surface fluctuations for emergent vegetation cases. This wave phenomenon has been mentioned in several previous studies (e.g., Zima and Ackermann 2002; Ghomeshi *et al.* 2007; Tanino and Nepf 2008; Tinoco and Cowen 2013), but the understanding of it is still limited at present.

Zima and Ackermann (2002) measured the amplitude of transverse surface waves triggered by the flow through rigid cylinders, and observed that resonance occurred when the wave frequency calculated using the small amplitude wave theory was equal to the frequency of the vortex shedding. However, Zima and Ackermann (2002) did not experimentally measure the frequency of vortex shedding or surface fluctuation. Ghomeshi *et al.* (2007) studied the wave amplitudes generated by vortex shedding in open channel flow with rigid cylinder arrays. They used equations from Žukauskas *et al.* (1988) to calculate Strouhal (S) number for both staggered and in-line bundles and obtained $S \approx 0.2$ for all experimental conditions. However, Ghomeshi *et al.* (2007) did not measure the frequency of waves either. To estimate wave amplitude, both Zima and Ackermann (2002) and Ghomeshi *et al.* (2007) derived working equations from dimensional analysis only and neither of them experimentally measured instantaneous free-surface fluctuation. Tinoco and Cowen (2013) applied small amplitude wave theory to obtain the frequency of lateral waves and concluded that the lateral standing wave occurs when the frequency is equal to the vortex shedding frequency. However, their application of small amplitude wave theory did not consider the effect of cylinder density and the calculated frequencies were verified with only two experimental measurements.

From the literature review, it follows that the previous studies have focused on prediction of amplitudes, and the theoretical analysis of dominant frequencies has not been extensively verified by experimental data. In this study, experiments were first conducted to investigate the surface wave phenomena generated by rigid staggered cylinders in open channel flows. Four types of surface patterns were classified based on the observations. Various parameters that contribute to the patterns were briefly discussed. Then, an analysis is carried out to estimate the dominant frequency of transversely oscillating surfaces which is associated with the maximum amplitude of surface fluctuations. The result shows that the dispersion equation, when modified by considering the effect of cylinder density, yields good agreement with the measured frequency of surface fluctuation. At the end, the discussion section presents possible effects of different parameters on the wave mode.

2 Experiments

Experiments were conducted in a laboratory flume (12 m long and 0.3 m wide) to measure the water surface fluctuation. Flowrates were recorded using a flow meter accurate to 0.01 L s^{-1} . Rigid Perspex cylinders (110 mm in length and 8.3 mm in diameter) were fixed into precisely machined holes on five Perspex plates (1.2 m long, 0.3 m wide and 0.01 m thick). The zone of cylinder array covered a

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length of 6.0 m. To investigate the effect of cylinder density on the flow, three different inter-cylinder spacings (0.03, 0.06 and 0.09 m) were applied to arrange the cylinders in a staggered pattern. The configurations were labelled as C30, C60 and C90, respectively. Here, the cylinder density (λ) was taken to be the fraction of the bed area covered by rigid cylinders. The λ -values calculated for the three inter-cylinder spacings were 12.0%, 3.0% and 1.3%, respectively. For each test, a uniform flow was achieved by adjusting the bed slope, tailgate and flowrate. The flow depth ranged from 6.2 cm to 9.7 cm, which were all less than the cylinder height.

Six wave gauges of the resistance type were used for measuring free-surface elevations. Three of them were installed at the cross section in the centre of the cylinder array. Each probe of the wave gauge consisted of two parallel metal rods which were able to conduct electricity when submerged in water. The two rods were 1.0 cm apart and formed a plane that was perpendicular to the sidewall. The output voltage signal was amplified by an amplifier, and the optimized voltage signal was recorded using software DEWESoft 7.0. The centerlines of the three probes were located at distances of 1.9, 11.6, and 26.3 cm from one sidewall of the flume (see Fig. 1a). A plan view is shown in Fig. 1b to illustrate the relative position of the probes to the surrounding cylinders for configuration C30. The other three wave gauges (see Probe D, E, F in Fig. 1c) were installed at the position 31.5, 62.7, and 91.5 cm downstream of cross section ABC. The probes were calibrated in still water before pump was turned on. In each run, the voltage signal was measured by the probes and amplified signal was converted to the surface displacement based on the calibration. DEWESoft data acquisition device was used to take recordings at a frequency of 100 samples per second for 375 seconds. A sample of recorded time series of the surface displacement (η) normalized by diameter (d) is shown in Fig. 2 for five runs. Figure 2(a, b, c) show the recordings from Probes A, B and C for wave modes $n = 1, 2$ and 3. The variation of surface displacement in the lateral direction is clearly demonstrated. Figure 2(d, e) are the recordings of surface fluctuation for Run 8Q7 and Run 10Q1 respectively. In each run, all the local peak values were identified, the average of them was taken as the amplitude, and the standard deviation was taken as its error.

By applying the `fft` function available in software Matlab to the recorded surface displacements, the Fourier transform (P) of the data series was obtained. Then the spectra (A_{FFT}) was obtained as $A_{FFT} = 2|P|/N$, where N is total number of samples. Dominant frequency (f_h) was determined as the frequency corresponding to the peak value in the spectra. This procedure is further explained in the following (Whitford *et al.* 2001). First, the surface fluctuation (η) is expressed as a finite Fourier series,

$$\eta = \sum_{q=1}^{N/2} A_q \cos(q\omega_1 t) + \sum_{q=1}^{N/2-1} B_q \sin(q\omega_1 t) \tag{1}$$

where N is the total number of samples, A_q and B_q are the Fourier coefficients, q is a counting index, $\omega_1 (= 2\pi / T_R)$ is the fundamental frequency, T_R is the length of recorded sample, and t is the time.

The coefficients A_q and B_q are calculated as

$$A_q = \frac{2}{N} \sum_{i=1}^N \eta_i \cos\left(\frac{2\pi qi}{N}\right), q = 1, 2, 3, \dots, N/2-1 \quad (2)$$

$$A_{N/2} = \frac{1}{N} \sum_{i=1}^N \eta_i \cos(i\pi) \quad (3)$$

$$B_q = \frac{2}{N} \sum_{i=1}^N \eta_i \sin\left(\frac{2\pi qi}{N}\right), q = 1, 2, 3, \dots, N/2-1 \quad (4)$$

where i is an integer from 1 to N , and η_i is the i^{th} instantaneous surface displacement. The spectra at each frequency was calculated as

$$A_{FFT} = \sqrt{A_q^2 + B_q^2}, q = 1, 2, 3, \dots, N/2-1 \quad (5)$$

Illustrated in Fig. 3 are the sample amplitude spectra for the runs as shown in Fig. 2. Then the dominant frequency (f_h) was obtained from the frequency corresponding to the maximum value of the A_{FFT} . The dominant frequencies for Run 4Q1, Run 7Q6, Run 8Q3, and Run 8Q7 are 1.30, 2.15, 2.66 and 3.22 Hz, respectively. The spectra for Run 10Q1 did not show clear peak spectra value, thus no dominant frequency could be identified.

3 Results and analysis

Table 1 summarizes the flow conditions and experimental results. The average pore velocity was calculated as $V_V = Q/Bh(1-\lambda)$, where Q is the flowrate, B is the channel width and h is the time-mean flow depth. The 6th column shows the bed slope in the range of 0.0004 - 0.0102. The following four columns include the amplitude (A), the dominant frequency (f_h), the surface patterns that were observed by video recording from one side of the flume, and the transverse wave mode (n) recorded. The labelling of surface patterns is discussed in Section 3.1. The last three columns are the cylinder diameter based Reynolds number ($R_d = V_V d / \nu$, where d is cylinder diameter and ν is kinematic viscosity of water), the cylinder diameter based Froude number ($F_d = V_V / \sqrt{gd}$, where g is the gravitational acceleration), and the flow depth based Froude number ($F_h = V_V / \sqrt{gh}$). The range of R_d in this study is from 278-2402, which is well within the range of cylinder Reynolds number 40 - 300,000 for vortex shedding to occur (Zima and Ackermann 2002). The value of F_d is in the range of 0.11 - 0.95. It should be mentioned that as both d and ν in this study are constant, both R_d and F_d only represent the variation in V_V . The channel Froude number (F_h) was less than unity in all runs.

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3.1 Four types of free-surface fluctuations

Based on the experimental observation, four types of free-surface fluctuating patterns could be identified. They are (a) plane surface without significant fluctuations, (b) transversely oscillating surface, (c) longitudinally oscillating surface, and (d) locally disturbed surface. All the patterns are sketched in Fig. 4. To summarize the observed surface patterns, different labelling are used in Table 1. In particular, the labelling of surface patterns b(c) indicates that in that run pattern (b) and (c) coexisted, and pattern (b) was visually dominant. Otherwise the labelling of c(b) applies.

It seems that each surface pattern occurred in a particular range of the cylinder Reynolds number (R_d). For example, at low Reynolds number ($R_d < 340$), the water surface was stable and almost flat (Fig. 4a). When the value of R_d is small, vortex shedding frequency is small and the transverse velocity component is small. The resulted displacement of fluid element is insufficient to produce measurable oscillation of surface. As R_d increases, the frequency of vortex shedding increases. If the frequency approaches the wave frequency dictated by the channel width and flow depth, resonance will occur (Zima and Ackermann 2002; Tinoco and Cowen 2013) and the amplitude of oscillation will be amplified. In the present study, the water surface oscillated in both transverse (Fig. 4b) and longitudinal direction (Fig. 4c) for $R_d \approx 340 - 1770$. As for the oscillation in the longitudinal direction, F_h may play an important role. The flow was subcritical for all runs. In subcritical flow, a small disturbance can propagate upstream (Jain 2000). As the flow exits from the cylinder array to the downstream reach where no cylinders were planted, the average velocity changes from $Q/Bh(1-\lambda)$ to Q/Bh . Since Q , B , and h are constants for each run, the velocity suddenly decreases after water flows into the reach without cylinders. This may cause disturbance that propagates upstream. For $R_d > 1918$, pattern (d) occurred with the water surface being deformed locally around each cylinder (Fig. 4d). It is worth noting that F_d is ≥ 0.8 for the pattern (d) in this study. Moreover, the FFT analysis of surface fluctuation showed no dominant frequency for pattern (d). This suggests that the effect of R_d decreases as F_d approaches 1.

3.2 Variation of free-surface fluctuation with Reynolds number

To quantify the variation in the free surface elevation, we may use either the amplitude (A) of the free-surface fluctuation or the root-mean-square value (h_{rms}) of the flow depth. These values could be further normalized with the cylinder diameter. The dimensionless amplitude (A/d) for each run was obtained as follows. First, for each probe recording, the average peak value was calculated. Then, from the three values associated with the three probes in the transverse direction, the highest value was selected as the amplitude of fluctuation. In Fig. 5, the variation of A/d (denoted by solid symbols with error bar) is plotted against F_d and R_d for the range $F_d = 0.11 - 0.95$ and $R_d = 278 - 2402$. For comparison, the normalized RMS value of the flow depth (h_{rms}/d , denoted by open symbols) is also plotted in Fig. 5. The values of h_{rms}/d are slightly lower than those of A/d , but both follow a similar

trend. The range of R_d covered the four types of surface patterns. With increasing F_d and R_d , A/d first increases until a peak value occurring at $F_d = 0.24 - 0.32$ (or $R_d = 600 - 800$), which corresponds to a standing wave pattern with the wave mode of $n = 2$. Then A/d starts to decrease, and when $F_d > 0.8$ (or $R_d > 2000$), A/d tends to be a constant around 0.08.

3.3 Standing waves

Among the four surface patterns sketched in Fig. 4, pattern (b) could be theoretically described using the theory of standing waves. This is explored as follows.

Consider water flows through staggered cylinders with an average pore velocity V_V . The Cartesian coordinates are chosen as follows: x is the streamwise direction, y is the transverse direction with $y = 0$ at one sidewall, and z is directed upward with $z = 0$ at the mean free-surface elevation. The surface variation, $\eta(y, t)$, associated with a standing wave can be described by

$$\eta(y, t) = \frac{A_{pp}}{2} \cos ky \cos \omega t \quad (6)$$

where A_{pp} is the height of standing waves, k is the wave number, ω is the angular frequency, and t is the time.

In the experiments, the probes installed at the different locations recorded different magnitudes of the free-surface oscillation. The highest magnitude of fluctuation recorded with the three probes was taken as the height of standing waves. Given the height of standing wave, dominant frequency and transverse wave mode of a standing wave, Eq. (6) could be used to predict the surface displacement profiles. Figure 6 shows the surface displacement calculated using Eq. (6), in comparison with the measurements for wave mode (a) $n = 1$, (b) $n = 2$ and (c) $n = 3$. The data were taken at four typical times, i.e. $t = 0$ ($\circ$), $t = T/6$ (Δ), $t = T/3$ ($\blacktriangle$), and $t = T/2$ ($\bullet$), where T is the wave period. The error bar at the four typical times was calculated by taking three steps. First, find all peak values in each recording and take their respective time t_p as $t = 0$. Then, find the values at $t_p + T/6$, $t_p + T/3$ and $t_p + T/2$. Finally, find the standard deviations of the values at $t = 0$, $t = T/6$, $t = T/3$ and $t = T/2$. Figure 6 shows that the calculated results agree well with the measured free-surface variations. Figure 6 also suggests that the observed surface profiles were not influenced by individual cylinder but rather confined by the sidewalls.

The velocity potential (φ) corresponding to Eq. (6) is expressed as (Dean and Dalrymple 1991)

$$\varphi = \frac{A_{pp} g}{2\omega} \frac{\cosh k(h+z)}{\cosh kh} \cos ky \sin \omega t \quad (7)$$

Because no flow passes through the sidewalls at $y = 0$ and $y = B$, the standing wave solution is subjected to the boundary condition

$$v_{y=0,B} = \frac{\partial \phi}{\partial y} \bigg|_{y=0,B} = 0 \Rightarrow \frac{A_{PP} g k}{2\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin \omega t \sin ky \bigg|_{y=0,B} = 0 \quad (8)$$

At $y = 0$, the zero-flow condition is unconditionally satisfied. To satisfy the zero-flow condition at $y = B$, $\sin(kB) = 0$ is required, which implies

$$k = n\pi/B \quad (9)$$

where the wave mode n is an integer greater than zero. When $n = 1$, the wave has two antinodes at the wall where maximum displacements occur, and one node in the middle of channel (i.e., $y = B/2$) where the surface elevation remains stationary (see Fig. 6a). This is the fundamental mode of the transverse wave, and its wave length is twice the channel width. At each node, $\eta = 0$ and $\cos(ky) = 0$. Hence the location of nodes y_n is

$$y_n = \frac{2j-1}{2n} B \quad (10)$$

where j is an integer varying from 1 to n .

The angular frequency (ω) may be calculated using the following linear dispersion relationship for small-amplitude waves (Dean and Dalrymple 1991),

$$\omega^2 = gk \tanh kh \quad (11)$$

Noting that frequency $f = \omega/2\pi$, combining Eq. (9) and Eq. (11) yields the wave frequency,

$$f_h = \frac{1}{2\pi} \cdot \sqrt{g \frac{n\pi}{B} \tanh\left(\frac{n\pi}{B} h\right)} \quad (12)$$

However, Eq. (12) is applicable only when the effect of the cylinder density is negligible. To consider the cylinder effect, we may treat the cylinder array as a porous medium with porosity $= 1 - \lambda$. Then the dispersion equation [Eq. (11)] can be modified as (Yu and Chwang 1994)

$$S\omega^2 = gk \tanh(kh) \quad (13)$$

where $S [= 1 + C_A \lambda / (1 - \lambda)]$ is an inertial coefficient of the porous medium, and C_A is the added mass which depends on the shape of the object. In Eq. (13), the damping effect on the wave number is not considered by noting the short distance between the two sidewalls. For a circular cylinder, the value of C_A is 1 (Dean and Dalrymple 1991). Thus we can get a modified frequency equation as

$$f_h = \frac{1}{2\pi} \cdot \sqrt{\frac{g \frac{n\pi}{B} \tanh\left(\frac{n\pi}{B} h\right)}{S}} \quad (14)$$

Figure 7 shows comparisons of the frequencies calculated using Eq. (12) and Eq. (14) with those obtained by applying the FFT method to the measured free-surface fluctuations. Figure 7(a, b, c)

correspond to $n = 1, 2$ and 3 respectively. It can be seen that the frequencies predicted using the modified equation [Eq. (14)] for $n = 1$ and 2 show better agreement with the measurements than those predicted with Eq. (12). However, neither Eq. (12) nor Eq. (14) predicts the frequency correctly for $n = 3$. This is because when $n = 3$, the longitudinal free-surface oscillation (see Fig. 4c) also becomes observable.

To examine if the transverse standing waves and longitudinal oscillation were present simultaneously, Run 4Q1 ($n = 1$) was taken as an example. The recordings from Probes A, B, and C (see Fig. 2 and Fig. 6) have clearly demonstrated the presence of transverse standing waves. By setting the timing so that $t = 0$ when Probe C detects a peak value (phase = 0), the surface fluctuations could be well described by Eq. (6). Assuming that there was no oscillation in the longitudinal direction (the surface fluctuating as transverse standing wave only), the phase at the cross section of Probes D, E, and F would be the same as that of Probe C because the positions of Probes D, E, and F and Probe C are at the same side of a node in the transverse direction (a phase difference of π should be used if they are on different sides of the node). Then Eq. (6) could be used directly to calculate the surface displacement at the positions of Probes D, E and F. By substituting the y coordinates of the position of Probes D, E, and F into Eq. (6), the respective surface displacement could be calculated. The calculated values and measured values are compared as shown in Fig. 8. Significant difference in phase suggested that the assumption is not valid. This implies that the longitudinal oscillation also exists while the transverse oscillation is observed. As the wave mode becomes higher, it becomes more complicated to calculate the surface displacements. It should be mentioned that this paper basically focuses on the frequency of oscillation in the transverse direction. Moreover, the data recorded by the wave gauges installed at the four different cross sections were insufficient to provide further quantitative description of the free surface variation in the longitudinal direction. This needs further efforts in the future.

The prediction error associated with Eq. (12) is calculated as $|f_{\text{Eq (12)}} - f_h|/f_h$ and it is in the range of 0.2%-12.9%, while the error with Eq. (14), i.e. $|f_{\text{Eq (14)}} - f_h|/f_h$, is obtained as 0.1%-5.9%. The percentage difference in the calculations using Eq. (12) without considering the effect of cylinder density with reference to Eq. (14) is calculated as $|f_{\text{Eq (12)}} - f_{\text{Eq (14)}}|/f_{\text{Eq (14)}}$ and plotted against the cylinder density, as shown in Fig. 9. As the cylinder density increases, the percentage difference becomes more significant. The measurements in this study were conducted only for $\lambda < 12.0\%$; whether the modified dispersion equation holds for higher cylinder density remains to be verified with experiments.

The variation of the wave mode with R_d is shown in Fig. 10. As R_d increases to 600-800, standing waves occur with n changing from 1 to 2 and 3. The wave mode may also depend on some other parameters. This will be discussed in Section 4.

3.4 Comparison of fluctuations of the lateral flow velocity and free surface elevation

In the case of significant variations in the free surface fluctuation, it can be expected that the flow velocity may fluctuate in a similar fashion. To explore this possible connection, we made use of some experimental data collected recently by Zhao *et al.* (2013), who applied Particle Image Velocimetry (PIV) technique to measure velocity in a horizontal plane about 5.0 cm from a vegetated channel bed. It should be mentioned that the vegetation configuration employed in Zhao *et al.* (2013) is very similar to this study. With the measured flow velocity, the Strouhal number (S_v) for the lateral velocity component is calculated as follows. First, FFT analysis was applied for the transverse velocity component (v) along the wake axis at the point with the distance $1.5d$, $2d$, and $2.5d$ downstream of a cylinder. Next, the dominant frequency (f_v) was found identical for these points. Then, the Strouhal number (S_v) for the lateral velocity component was obtained as $S_v = f_v d / V_V$.

Both the Strouhal number calculated for the water surface fluctuation ($S_h = f_h d / V_V$) and that for the lateral velocity fluctuation (S_v) are shown in Fig. 11. It can be observed that S_h and S_v vary similarly. The value of S first decreases with increasing R_d when $R_d < 600$, and then increases when $R_d = 600 - 800$. It finally decreases to about 0.2 when $R_d > 1000$. In particular, it is noted that the variation has a transition at the Reynolds numbers ranging from 600 to 800. This is exactly the range where the wave mode changed from 1 to 2 and 3, and also the range when the maximum fluctuation of the flow depth occurred. For a single cylinder with uniform flow (R_d of $300 - 3 \times 10^5$), the Strouhal number is approximately 0.2 (Sumer and Fredsøe 2006). The seiche frequency (f_s) in the transverse direction in a rectangular basin could be calculated using Merian's formula ($f_s = n\sqrt{gh}/2B$). The Strouhal number ($S_s = f_s d / V_V$) calculated using the seiche frequency for 15 runs with $n = 1, 2$ and 3 is also shown in Fig. 11. When $n = 1$, the calculated S_s is close to the values of S_h and S_v . As n increases to 2 and 3, the calculated values of S_s deviate much from S_h and S_v . Merian's formula was simplified from the dispersion equation (Eq. 12) based on the assumption of shallow water waves ($kh < \pi/10 \Rightarrow h/B < 1/10n$) so that $\tanh(kh)$ could be simplified as kh . We found this assumption was not valid for the present study as $h/B > 6.2/30$ for all cases, thus the simplification as kh overestimates the value of $\tanh(kh)$. Moreover, it has been shown previously that the dispersion equation should be modified by considering the effect of cylinder density (λ). Eq. (12) would overestimate the frequency if λ is not considered (see Fig. 7). These factors caused the calculated S_s to be higher for higher n values.

3.5 Locally deformed surface pattern

The water surface became disturbed locally (see Fig. 4d) when $F_d \gtrsim 0.8$. A snap of such surfaces is shown in Fig. 12, where the flow was from LHS to RHS. Figure 12(a, b) show the plan view and side view, respectively. When the locally deformed pattern occurred, no dominant frequencies were found by applying the FFT analysis to the measured fluctuations (see Run 10Q1 in Fig. 3). Figure 12a shows

that a bow wave formed on the front of each cylinder and a wake developed behind it. Figure 12b shows that a small dimple formed at a distance of $x \approx 1.5d$ downstream from the center of each cylinder, with the water surface deformed in a way different from the other three patterns. However, due to the limitation of experiments, the detailed geometry of the dimple could not be investigated.

4 Discussion

The effect of different parameters on the wave mode is discussed as follows. It is well known that the cylinder Reynolds number ($R_d = Vd/\nu$) is an important parameter to investigate hydrodynamic characteristics of uniform flow passing around a circular cylinder (Kundu and Cohen 2002; Sumer and Fredsøe 2006). It is a measure of the ratio of the inertial force to viscous force (Munson *et al.* 2010). For a single cylinder in uniform flow with R_d in the range of 300 - 3×10^5 , vortices periodically shed from the cylinder at $S \approx 0.2$ (Sumer and Fredsøe 2006), where $S = fd/V$, and V is uniform velocity. If S and d remain constant, the increase of V will result in an increase in the vortex shedding frequency (f). Fluid particles may travel different distances (i.e. maximum distance of excursion) within one period in the transverse direction. In the case that the flow around the cylinder is oscillatory (Sumer and Fredsøe 2006), the parameter Keulegan-Carpenter number ($KC = U_{\max}T/d$, where $U_{\max}$ is the amplitude of harmonically varying velocity) can be used to represent the ratio of the amplitude of the motion to the cylinder diameter. A low KC means that the orbital motion of the water particle is small relative to the cylinder diameter. If the oscillation is sinusoidal [$U = U_{\max}\cos(\omega t)$, where U is instantaneous velocity component], the maximum velocity will be $U_{\max} = a\omega$, (where a is the amplitude of the motion), and $KC = 2\pi a/d$. Perhaps, it is KC rather than R_d that affects the wave length in the transverse direction, and thus correlates with the wave mode. The calculation of KC requires the value of maximum transverse velocity. However, transverse velocity component was not measured simultaneously when free surface fluctuation was monitored. Moreover, Froude number may play a role when the wave was generated in the cylinder array. These factors together with the limitation of the present experiment make it physically difficult to identify possible correlation between wave mode and R_d or KC . This remains to be further investigated.

Through the experiments, we observed that water surface oscillations within the cylinder array were much more significant than in the zone with no cylinders. It is noted that the central part of the flow channel was occupied by the cylinder array that was 6.0 m long in the streamwise direction, but there were still two shorter flow reaches which were not occupied by any cylinder (i.e., 4.0 m from the upstream edge of the cylinder array to the flow straightener, and 1.0 m from the downstream edge of the cylinder array to the tailgate). In the two reaches with no cylinders, all waves seemed suddenly attenuated, and the water surface oscillation appeared rather smooth with negligible oscillations. The difference in the water surface oscillation within and outside of the cylinder array may imply that the significant water surface oscillation is particularly associated with the presence of

cylinders. It should be mentioned that similar observations have also been reported by Zima and Ackermann (2002). It is also interesting to speculate possible effect of channel width on the wave frequency. As the transverse wave is dictated by B , h , and S as shown in Eq. (14), we may deduce a reduced fundamental wave frequency if B increases and h and S remain constants. This means the frequency of vortex shedding required for transverse standing wave to occur reduces in a wider channel. However, we cannot confirm if the standing waves exist in field without confining walls. Future efforts should be made to extend observations to field situations.

5 Summary

In this study, laboratory experiments were conducted to investigate the free-surface fluctuations generated by an array of rigid cylinders in various open-channel flow conditions. Based on experimental observation, four types of surface patterns were identified and they are (a) plane surface without significant fluctuations, (b) transversely oscillating surface, (c) longitudinally oscillating surface, and (d) locally disturbed surface. Results of surface fluctuation from multiple wave gauges show that oscillation in the longitudinal direction was also present when transverse standing waves were observed for different wave modes. The effects of R_d , F_h , and F_d on these patterns were briefly discussed. The free-surface fluctuation reached its maximum when the mode of the standing wave changed from 1 to 2. The standing waves could be well described using the velocity potential equation [Eq. (7)]. The frequency of the standing waves can be calculated using a modified dispersion equation [Eq. (14)], if flow depth, cylinder density, channel width and wave mode are all given. The calculated results show better agreement with the measured frequencies than those calculated without considering the effect of cylinder density. A short discussion was also presented to illustrate the possible correlation between wave mode and different parameters including R_d , KC , and F_h .

Notation

- A = amplitude of surface fluctuation
- A_{FFT} = amplitude at each frequency
- A_{PP} = height of standing waves
- A_q = Fourier coefficient for amplitude spectra analysis
- B = channel width
- B_q = Fourier coefficient for amplitude spectra analysis

C_A	=	added mass, = 1 for a cylinder shape
d	=	cylinder diameter
f	=	frequency
f_h	=	dominant frequency of surface fluctuation
f_v	=	dominant frequency of transverse velocity fluctuation
f_s	=	seiche frequency
F_d	=	Froude number calculated based on cylinder diameter
F_h	=	Froude number calculated based on flow depth
g	=	gravitational acceleration
h	=	flow depth
h_{rms}	=	RMS of surface fluctuation
i	=	integer from 1 to N
j	=	an integer in the range of 1 to n
k	=	wave number
KC	=	Keulegan–Carpenter number = $U_{max}T/d$
n	=	wave mode
N	=	total number of samples
P	=	result obtained from fft
q	=	counting index for amplitude spectra analysis
Q	=	average flowrate
R_d	=	Reynolds number based on cylinder diameter = $V_r d/\nu$
S	=	inertial coefficient of the medium
S	=	Strouhal number

1		
2		
3		
4	S_h	= Strouhal number based on frequency of surface fluctuation
5		
6	S_v	= Strouhal number based on frequency of velocity fluctuation
7		
8		
9	S_s	= Strouhal number based on frequency of seiche
10		
11	t	= time
12		
13		
14	t_p	= time indicating the peak values
15		
16		
17	T	= wave period
18		
19	T_R	= total length of recorded sample
20		
21	U	= oscillating velocity
22		
23		
24	U_{max}	= amplitude of the harmonically varying velocity
25		
26		
27	v	= transverse velocity
28		
29	V	= uniform velocity
30		
31		
32	V_V	= average pore velocity, $Q/Bh(1-\lambda)$
33		
34	x	= longitudinal coordinate
35		
36		
37	y	= transverse coordinate
38		
39		
40	y_n	= location of the node
41		
42	z	= vertical direction with $z = 0$ at mean surface elevation
43		
44		
45	Δt	= phase difference
46		
47	Δx	= distance in the longitudinal direction
48		
49		
50	φ	= velocity potential
51		
52	λ	= cylinder density defined as fraction of bed area occupied by cylinders
53		
54		
55	η	= surface fluctuation
56		
57	η_i	= instantaneous surface fluctuation
58		
59		
60	ν	= kinematic viscosity of fluid

ω = angular frequency

ω_1 = fundamental frequency

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Table 1. Summary of experimental results

Run	λ	Q (Ls ⁻¹)	h (m)	V_V (ms ⁻¹)	Slope	A (cm)	f_h (Hz)	Surface Pattern	n	R_d	F_d	F_h
4 Q1	0.120	0.88	0.084	0.040	0.0066	0.21	1.30	b (c)	1	351	0.14	0.04
4 Q2	0.120	1.05	0.080	0.049	0.0066	0.25	2.07	b (c)	2	435	0.17	0.06
5 Q1	0.120	0.66	0.078	0.032	0.0059	0.21	1.25	b (c)	1	281	0.11	0.04
5 Q2	0.120	0.66	0.066	0.038	0.0038	0.17	1.20	b (c)	1	333	0.13	0.05
5 Q3	0.120	0.65	0.077	0.032	0.0004	0.12	1.17	b (c)	1	278	0.11	0.04
6 Q1	0.030	0.85	0.068	0.043	0.0004	0.20	1.24	b (c)	1	378	0.15	0.05
6 Q2	0.030	0.97	0.085	0.039	0.0004	N.A.	N.A.	a	N.A.	344	0.14	0.04
7 Q1	0.030	1.02	0.090	0.039	0.0013	N.A.	N.A.	a	N.A.	341	0.14	0.04
7 Q2	0.030	1.60	0.083	0.066	0.0013	0.28	1.34	b (c)	1	583	0.23	0.07
7 Q3	0.030	1.83	0.090	0.070	0.0013	0.39	1.39	b (c)	1	617	0.24	0.07
7 Q4	0.030	2.51	0.085	0.101	0.0016	0.56	2.68	c (b)	3	895	0.36	0.11
7 Q5	0.030	1.39	0.077	0.062	0.0016	0.17	1.29	b (c)	1	545	0.22	0.07
7 Q6	0.030	1.80	0.080	0.077	0.0016	0.60	2.15	b (c)	2	680	0.27	0.09
8 Q1	0.030	1.83	0.080	0.078	0.0031	0.62	2.16	b (c)	2	691	0.27	0.09
8 Q2	0.030	2.31	0.082	0.097	0.0036	0.43	2.66	c (b)	3	854	0.34	0.11
8 Q3	0.030	2.22	0.081	0.094	0.0036	0.38	2.66	c (b)	3	828	0.33	0.11
8 Q4	0.030	2.50	0.083	0.103	0.0042	0.53	2.67	c (b)	3	912	0.36	0.11
8 Q5	0.030	2.73	0.080	0.117	0.0051	0.22	2.98	c (b)	N.A.	1032	0.41	0.13
8 Q6	0.030	3.04	0.080	0.130	0.0051	0.23	3.07	c (b)	N.A.	1149	0.46	0.15
8 Q7	0.030	3.24	0.085	0.131	0.0064	0.19	3.22	c (b)	N.A.	1155	0.46	0.14
8 Q8	0.030	3.51	0.085	0.142	0.0064	0.13	3.31	c (b)	N.A.	1249	0.50	0.16
8 Q9	0.030	3.66	0.081	0.155	0.0069	0.14	3.09	c (b)	N.A.	1367	0.54	0.17
8 Q10	0.030	3.86	0.090	0.147	0.0081	0.14	3.17	c (b)	N.A.	1297	0.51	0.16
9 Q1	0.030	4.07	0.083	0.168	0.0095	0.09	3.66	c (b)	N.A.	1483	0.59	0.19
9 Q2	0.030	4.38	0.086	0.175	0.0102	0.09	3.95	c (b)	N.A.	1543	0.61	0.19
10 Q1	0.013	5.00	0.071	0.238	0.0102	0.06	N.A.	d	N.A.	2098	0.83	0.28
10 Q2	0.013	5.81	0.072	0.272	0.0102	0.06	N.A.	d	N.A.	2402	0.95	0.32
10 Q3	0.013	4.83	0.072	0.226	0.0083	0.06	N.A.	d	N.A.	1997	0.79	0.27
10 Q4	0.013	6.58	0.091	0.244	0.0083	0.06	N.A.	d	N.A.	2152	0.85	0.26
10 Q5	0.013	5.18	0.081	0.217	0.0063	0.07	N.A.	d	N.A.	1918	0.76	0.24
10 Q6	0.013	4.82	0.087	0.187	0.0050	0.08	3.88	c (b)	N.A.	1650	0.66	0.20
10 Q7	0.013	4.30	0.078	0.186	0.0050	0.08	3.70	c (b)	N.A.	1643	0.65	0.21
10 Q8	0.013	4.22	0.071	0.201	0.0044	0.08	3.94	c (b)	N.A.	1770	0.70	0.24
10 Q9	0.013	2.86	0.062	0.156	0.0044	0.14	3.16	c (b)	N.A.	1375	0.55	0.20

Fig. 1. Locations of wave gauges (a) Cross-sectional view (not to scale), (b) plan view of three probes for C30, and (c) plan view of the position of Probe D, E, F in streamwise direction

Fig. 2. Time series of normalized surface displacement recorded by Probe A (solid line), Probe B (dotted line) and Probe C (dashed line) for (a) Run 4Q1, $R_d = 351$, (b) Run 7 Q6, $R_d = 680$, (c) Run 8Q3, $R_d = 828$, (d) Run 8Q7, $R_d = 1155$, and (e) Run 10 Q1, $R_d = 2098$

Fig. 3. Amplitude spectra analysis for selected runs. The square markers indicate the dominant frequency.

Fig. 4. Observed surface patterns: (a) Pattern a, plane surface, (b) Pattern b, transversely oscillating surface, (c) Pattern c, longitudinally oscillating surface, and (d) Pattern d, locally disturbed surface

Fig. 5. Variations of A/d and h_{rms}/d . The vertical bars indicate the error of normalized amplitudes.

Fig. 6. Normalized surface displacement (η/d) from mean water level calculated with Eq. (6) comparing with experimental data recorded at $t = 0$ (○), $t = T/6$ (Δ), $t = T/3$ (▲), and $t = T/2$ (●), for (a) Run 4Q1, (b) Run 7Q6, and (c) Run 8Q3. The vertical bars indicate the error of displacement recorded by probes at the respective timing.

Fig. 7. Comparison of frequency of surface fluctuation calculated using Eq. (12) and Eq. (14) with frequency measured for wave mode (a) $n = 1$, (b) $n = 2$, and (c) $n = 3$

Fig. 8. Normalized surface fluctuation (η/d) recorded by Probes C, D, E, F for Run 4Q1, $n = 1$. Solid lines are measured values by probes, dotted lines are calculated time series based on Eq. (6) and the phase of Probe C.

Fig. 9. Difference in calculations by Eq. (12) and Eq. (14). The solid line is given by

$$|f_{\text{Eq (12)}} - f_{\text{Eq (14)}}|/f_{\text{Eq (14)}} = \sqrt{1/(1-\lambda)} - 1.$$

Fig. 10. Variation of n with R_d and F_d

Fig. 11. Variation of S with R_d and F_d

Fig. 12. Locally disturbed surface (a) plan view, and (b) side view