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LaIF: A Lane-Level Self-Positioning Scheme for Vehicles in GNSS-Denied Environments

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Abstract—Vehicle self-positioning is of significant importance for intelligent transportation applications. However, accurate positioning (e.g., with lane-level accuracy) is very difficult to obtain due to the lack of measurements with high confidence, especially in an environment without full access to a global navigation satellite system (GNSS). In this paper, a novel information fusion algorithm based on a particle filter is proposed to achieve lane-level tracking accuracy under a GNSS-denied environment. We consider the use of both coarse scale and fine scale signal measurements for positioning. Time-of-arrival measurements using the radio frequency signals from known transmitters or roadside units, and acceleration or gyroscope measurements from an inertial measurement unit (IMU) allow us to form a coarse estimate of the vehicle position using an extended Kalman filter. Subsequently, fine scale measurements including lane-change detection, radar ranging from the known obstacles (e.g., guardrails) and information from a high resolution digital map are incorporated to refine the position estimates. A probabilistic model is introduced to characterize the lane changing behaviors and a multi-hypothesis model is formulated for the radar range measurements to robustly weigh the particles and refine the tracking results. Moreover, a decision fusion mechanism is proposed to achieve a higher reliability in the lane-change detection as compared to each individual detector using IMU and visual (if available) information. The posterior Cramér-Rao lower bound is also derived to provide a theoretical performance guideline. The performance of the proposed tracking framework is verified by simulations and real measured IMU data in a four-lane highway.

Index Terms—Vehicle localization, GNSS-denied, lane-level accuracy, extended Kalman filter, particle filter, information fusion, inertial navigation systems.

I. INTRODUCTION

Localizing and tracking a vehicle using a global navigation satellite system (GNSS) or various commercial products with built-in GNSS technology has been widely adopted [1], [2]. However, GNSS-based localization schemes require lines of sight (LOS) to at least four navigation satellites and accessibility of such a number of navigation satellites is not guaranteed in many areas, especially in a highly built-up environment due to the urban canyon effect [3]. Furthermore, the root mean

square error (RMSE) of the pure GNSS localization in an urban city is usually larger than 5 m. Hence, the localization service can only support navigation applications with a road-level accuracy. In this paper, our aim is to achieve lane-level accuracy such that the location RMSE is not more than 1.5 m.

Vehicle-to-everything (V2X) communication technology enables cars to communicate among each other and with surrounding infrastructure access points like roadside units (RSUs). It has drawn extensive attention due to its broad application prospects including advanced driver assistance systems, enhanced collision avoidance and electronic toll fee collection. The position of the vehicle is one of the most important information transmitted in the V2X communication network and vehicle localization and tracking plays a critical role in all these V2X applications [4]–[6]. Dedicated short-range communication (DSRC) technology using the IEEE 802.11p standard has been adopted for communication between the vehicle on-board unit (OBU) and the RSUs. Such a communication scheme makes localization using radio frequency (RF) signals possible [6]–[9]. In [5], the difference in time-of-arrival (TOA) measurements of consecutively received messages along the trajectory of a moving vehicle, referred to as virtual time difference of arrival (V-TDOA) [10], [11], has been used for vehicle localization. Nonetheless, a major challenge in V-TDOA is to separate the time offset caused by the drift of the local oscillator (LO) of the receiver from the time offset due to the movement of the receiver. For terrestrial vehicles, the time offsets due both to LO drift and the vehicle movement will be in the order of tens of nanoseconds, and simply ignoring the LO drift leads to large estimation errors. Utilizing the TOAs of a periodic message sent from the transmitter, the authors in [12] designed an adaptive filtering framework to simultaneously estimate both the LO drift and the transmitter location in cellular networks.

Inertial navigation systems (INS) are self-contained dead reckoning (DR) systems that provide dynamic information such as accelerations and angular velocities directly from inertial measurement unit (IMU) signals. However, the errors due to bias, scale factors, and nonlinearity in the sensor readings result in accumulation of navigation errors with time. To bound the accumulation of errors, INS is coupled with GNSS [2], [13], which is not applicable in GNSS-denied environments. Nevertheless, vehicular constraints can be exploited to mitigate the growth of the positioning error when a GNSS signal is not available [14].

The pros and cons of various sensor measurements and different tracking algorithms such as Kalman filter (KF) and

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particle filter (PF) are presented in [15]–[19]. An extended Kalman filter (EKF) is recently employed to fuse the information from RF signals, IMU and map to track a vehicle in a GNSS-denied environment within the V2X communication framework [5]. Nonetheless, the reported tracking accuracy did not meet the requirement of lane-level precision. The reason is that in principle, none of these measurements is able to provide location information to discriminate between different lanes of the same road.

In this work, we propose a novel lane-level accuracy information fusion (LaIF) scheme to localize and track a vehicle. To the best of our knowledge, our fusion algorithm incorporating EKF, multi-hypothesis modeling and PF is a novel localization and tracking scheme with lane-level accuracy that fuses DSRC signals, IMU, radar and map information. The measurements employed for tracking are divided into two categories: *i)* coarse scale measurements; and *ii)* fine scale measurements. The former includes TOA from the RF signals and vehicle dynamics from IMU signals which are able to provide global information of the vehicle location, i.e., which road segment the vehicle is located in. The latter category contains the measurements such as driving feature detection, radar ranging, and high resolution map information. The driving features, which includes detection of lane changing, curvy road turning and U-turning can be extracted from the IMU signals (i.e., acceleration and angular velocity) [20], [21] and visual information (e.g., camera) [22]–[24].

We further propose a decision fusion mechanism to be applied to the outputs of multiple maneuver detectors to achieve a more reliable lane-change detection as compared to each individual detector. Side-scan radars are deployed at the two sides of the vehicle to provide range measurements with respect to the road guardrails and building boundaries along the vehicle trajectory. High resolution map information is employed to provide the reference points for the radar measurements and impose constraints on the vehicle positioning via map matching [25]. An illustration of complete framework of the LaIF scheme is shown in Fig. 1. Further, a particle filtering approach is developed to fuse the above-mentioned measurements and track the vehicle. In practice, the radar range measurements are susceptible to different obstacles. For example, other vehicles may pass by and block the radar signals. In such cases, the range measurements are in fact with respect to vehicles on other lanes rather than guardrails. To reduce the tracking error, a multi-hypothesis likelihood model is proposed to incorporate the possibility of ranging from other vehicles or obstacles.

The posterior Cramér-Rao lower bound (PCRLB) is also derived to provide a guideline of the potential tracking performance by incorporating different measurement information. Simulations are provided to demonstrate the derived PCRLB. The performance of the proposed LaIF tracking algorithm is also studied by simulations and verified by real measured data.

It is worth mentioning that the developed fusion scheme can easily incorporate GNSS information if it is available. The rest of the paper is organized as follows. The coarse scale measurements and the EKF fusion steps are introduced in Section II. In Section III, the fine scale measurements

Fig. 1: Framework of the LaIF scheme for vehicle tracking in the V2X communication system.

together with the decision fusion mechanism and the PF tracking algorithm are described. PCRLB performance analysis is derived in Section IV. The performance evaluation of the proposed LaIF algorithm is discussed in Section V through presenting simulations and experimental results. Finally, some conclusions are given in Section VI.

Throughout this work, the navigation frame is the coordinate frame with respect to which the location of the vehicle needs to be estimated, whereas the body frame is attached to the vehicle and is aligned with the axes of the on-board IMU. The IMU is usually placed at the center of the rear or front axle of the vehicle. Note that the superscript $(\cdot)^n$ denotes a term in the navigation frame and $(\cdot)^b$ denotes a term in the body frame. We use boldfaced characters to denote vectors and matrices. The superscript $(\cdot)^T$ denotes matrix transpose. We use $\mathbb{R}$ to denote the set of real numbers, and $\|\cdot\|$ is the Euclidean norm, while $\|\cdot\|_p$ denotes p -norm. The symbols $\mathbf{0}_{n \times m}$ and $\mathbf{I}_n$ represent $n \times m$ zero matrix and $n \times n$ identity matrix, respectively. We use $p(x | y)$ to denote the probability density function of the random variable x conditioned on y . $\mathbb{E}\{\cdot\}$ is the expectation operation.

II. COARSE SCALE MEASUREMENT INFORMATION FUSION

In this section, we present a model for the vehicle dynamics, and the coarse scale measurements used in our scheme. We also propose an EKF to perform coarse estimation of the vehicle position.

A. Model for Vehicle Dynamics

At processing time step k , the vehicle state is defined as

$$\mathbf{x}_k \triangleq [(\mathbf{p}_k^n)^T, (\mathbf{v}_k^n)^T, (\mathbf{a}_k^n)^T, (\boldsymbol{\Psi}_k)^T, (\mathbf{a}_k^b)^T, (\mathbf{w}_k^b)^T, (\mathbf{b}_{a,k})^T, (\mathbf{b}_{w,k})^T, \tau_k, \alpha_k]^T \in \mathbb{R}^{26}, \quad (1)$$

where

- $\mathbf{p}_k^n = [p_k^{nx}, p_k^{ny}, p_k^{nz}]^T \in \mathbb{R}^3$ is the position of the vehicle in the navigation frame,
- $\mathbf{v}_k^n = [v_k^{nx}, v_k^{ny}, v_k^{nz}]^T \in \mathbb{R}^3$ is the velocity of the vehicle in the navigation frame,

- $\mathbf{a}_k^n = [a_k^{nx}, a_k^{ny}, a_k^{nz}]^T \in \mathbb{R}^3$ is the acceleration of the vehicle in the navigation frame,
- $\Psi_k = [\phi_k, \theta_k, \psi_k]^T \in \mathbb{R}^3$ consists of the three Euler angles corresponding to roll, pitch, and yaw describing the orientation of the body frame with respect to the navigation frame,
- $\mathbf{a}_k^b = [a_k^{bx}, a_k^{by}, a_k^{bz}]^T \in \mathbb{R}^3$ represents the acceleration in body frame,
- $\mathbf{w}_k^b = [w_k^{bx}, w_k^{by}, w_k^{bz}]^T \in \mathbb{R}^3$ is the angular velocity, from body frame to navigation frame, resolved in the body frame,
- $\mathbf{b}_{a,k} \in \mathbb{R}^3$ and $\mathbf{b}_{w,k} \in \mathbb{R}^3$ are the biases of the accelerometer and the gyroscope measurements,
- τ_k is the local time at RSU, and
- α_k is the parameter characterizing the TOA drift.

Note that the three Euler angles ψ_k (yaw), θ_k (pitch) and ϕ_k (roll) follow the sequence of b_z , b_y and b_x in terms of the order of the rotation. The orientation of body frame with respect to the navigation frame is described by the rotation matrix

$$\mathbf{C}_k^{b \rightarrow n} = \begin{bmatrix} \cos\theta_k \cos\psi_k & \sin\phi_k \sin\theta_k \cos\psi_k - \cos\phi_k \sin\psi_k & \cos\phi_k \sin\theta_k \cos\psi_k + \sin\phi_k \sin\psi_k \\ \cos\theta_k \sin\psi_k & \sin\phi_k \sin\theta_k \sin\psi_k + \cos\phi_k \cos\psi_k & \sin\phi_k \sin\theta_k \sin\psi_k - \cos\phi_k \cos\psi_k \\ -\sin\theta_k & \sin\phi_k \cos\theta_k & \cos\phi_k \cos\theta_k \end{bmatrix}. \quad (2)$$

We assume that the rotation rate of the earth is negligible. The gravity vector $\mathbf{g} = [0, 0, g]^T$ is constant and parallel to the z -axis of the navigation frame, and the acceleration measured in the body frame, $\mathbf{a}_k^b$, is related to the acceleration in the navigation frame, $\mathbf{a}_k^n$, by

$$\mathbf{a}_k^n = (\mathbf{C}_k^{b \rightarrow n})^T \mathbf{a}_k^b + \mathbf{g}, \quad (3)$$

To develop an algorithm for the sequential estimation of the state $\mathbf{x}_k$, the dynamic models for every element of $\mathbf{x}_k$ and the models of the measurements available for state estimation need to be defined. The dynamic models adopted in this work are

$$\mathbf{p}_k^n = \mathbf{p}_{k-1}^n + \Delta t \mathbf{v}_{k-1}^n + \frac{1}{2} \Delta t^2 \mathbf{a}_{k-1}^n + \mathbf{n}_{p,k}^n; \quad (4)$$

$$\mathbf{v}_k^n = \mathbf{v}_{k-1}^n + \Delta t \mathbf{a}_{k-1}^n + \mathbf{n}_{v,k}^n; \quad (5)$$

$$\mathbf{a}_k^n = (\mathbf{C}_{k-1}^{b \rightarrow n})^T \mathbf{a}_{k-1}^b + \mathbf{g} + \mathbf{n}_{a,k}^n; \quad (6)$$

$$\Psi_k = \Psi_{k-1} + \Delta t \mathbf{A}(\Psi_{k-1}) \mathbf{w}_{k-1}^b + \mathbf{n}_{\Psi,k}, \quad (7)$$

where

$$\mathbf{A}(\Psi_k) = \begin{bmatrix} 1 & \sin\phi_k \tan\theta_k & \cos\phi_k \tan\theta_k \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \frac{\sin\phi_k}{\cos\theta_k} & \frac{\cos\phi_k}{\cos\theta_k} \end{bmatrix}. \quad (8)$$

The random walk models are employed to characterize the noise in the accelerometer and gyroscope measurements, given as

$$\mathbf{a}_k^b = \mathbf{a}_{k-1}^b + \mathbf{n}_{a,k}^b; \quad (9)$$

$$\mathbf{w}_k^b = \mathbf{w}_{k-1}^b + \mathbf{n}_{w,k}^b. \quad (10)$$

The autoregressive models of order 1 are used to model the biases of the accelerometer and gyroscope measurements, addressed as

$$\mathbf{b}_{a,k} = \text{diag}\{\beta\} \mathbf{b}_{a,k-1} + \mathbf{n}_{b_{a,k}}; \quad (11)$$

$$\mathbf{b}_{w,k} = \text{diag}\{\beta\} \mathbf{b}_{w,k-1} + \mathbf{n}_{b_{w,k}}. \quad (12)$$

Following [5], $\beta = [0.9999, 0.9999, 0.9999]^T$ is used.

Since the transmitter and the receiver are not synchronized, the filter should simultaneously estimate the local time and the LO drift to compensate TOA errors. Following equations are now in order

$$\tau_k = \tau_{k-1} + \Delta t \alpha_{k-1} + n_{\tau,k}; \quad (13)$$

$$\alpha_k = \alpha_{k-1} + n_{\alpha,k}. \quad (14)$$

Note that all noise terms in the vehicle dynamic and TOA models are assumed to be zero-mean Gaussian processes. The variances are set according to each state term.

B. Models for Coarse Scale Measurements

1) *IMU Measurements:* IMU measures the body frame accelerations and angular velocities. The measurement models which relate the IMU sensed quantities with the state vector (1) are given by

$$\mathbf{z}_{a,k} = \mathbf{a}_k^b + \mathbf{b}_{a,k} + \mathbf{u}_{a,k}; \quad (15)$$

$$\mathbf{z}_{w,k} = \mathbf{w}_k^b + \mathbf{b}_{w,k} + \mathbf{u}_{w,k}, \quad (16)$$

where $\mathbf{u}_{a,k}$ and $\mathbf{u}_{w,k}$ are zero-mean Gaussian processes characterizing the IMU measurement noise. We can write the IMU measurement function as

$$\mathbf{z}_k^{\text{IMU}} = \mathbf{h}^{\text{IMU}}(\mathbf{x}_k) + \mathbf{u}_k^{\text{IMU}}, \quad (17)$$

where $\mathbf{h}^{\text{IMU}}(\cdot)$ is given by (15) and (16), and $\mathbf{z}_k^{\text{IMU}} = [(\mathbf{z}_{a,k})^T, (\mathbf{z}_{w,k})^T]^T$, $\mathbf{u}_k^{\text{IMU}} = [(\mathbf{u}_{a,k})^T, (\mathbf{u}_{w,k})^T]^T$.

2) *TOA Measurements:* Since the locations of RSUs are supposed to be fixed and preknown, let $\mathbf{p}_i^{\text{RSU}}$, for $i = 1, \dots, I$, be the position of the i -th RSU. TOA measurements obtained from all accessible RSUs have the relationship with the vehicle's position and the local time such that

$$\mathbf{z}_k^{\text{TOA}} = \mathbf{h}^{\text{TOA}}(\mathbf{x}_k) + \mathbf{u}_k^{\text{TOA}} = [z_{1,k}^{\text{TOA}}, \dots, z_{i,k}^{\text{TOA}}, \dots, z_{I,k}^{\text{TOA}}]^T; \quad (18)$$

$$z_{i,k}^{\text{TOA}} = \tau_k + \frac{\|\mathbf{p}_k^n - \mathbf{p}_i^{\text{RSU}}\|}{c} + u_{i,k}^{\text{TOA}},$$

where c is the speed of the light, τ_k is the local time and $u_{i,k}^{\text{TOA}}$ is a zero-mean Gaussian process characterizing the TOA measurement noise.

3) *Pseudo Measurements from Vehicle Constraints:* In practice, the velocity of the vehicle in the plane perpendicular to the forward direction can usually be assumed to be zero. In addition, there is no side slip along the direction of the rear axle under normal situations. Hence, pseudo measurements of vehicle velocities along y - and z -axis (in body frame) can be formed in a straightforward manner, i.e., $v_k^{by} = 0$ and $v_k^{bz} = 0$. Such pseudo measurements can be regarded as constraints to limit the growth of the velocity errors due to incorrect

estimates of $\mathbf{a}_k^b$ and $\mathbf{w}_k^b$. The measurement models are given by

$$\begin{aligned} \mathbf{z}_k^{\text{VC}} &= \mathbf{h}^{\text{VC}}(\mathbf{x}_k) + \mathbf{u}_k^{\text{VC}} \\ &= \begin{bmatrix} v_k^{\text{nx}}(\sin\phi_k \sin\theta_k \cos\psi_k - \cos\phi_k \sin\psi_k) \\ + v_k^{\text{ny}}(\sin\phi_k \sin\theta_k \sin\psi_k + \cos\phi_k \cos\psi_k) \\ + v_k^{\text{nz}} \sin\phi_k \cos\theta_k \\ v_k^{\text{nx}}(\cos\phi_k \sin\theta_k \cos\psi_k + \sin\phi_k \sin\psi_k) \\ + v_k^{\text{ny}}(\cos\phi_k \sin\theta_k \sin\psi_k - \sin\phi_k \cos\psi_k) \\ + v_k^{\text{nz}} \cos\phi_k \cos\theta_k \end{bmatrix} \\ &\quad + \begin{bmatrix} u_{v_y,k} \\ u_{v_z,k} \end{bmatrix}, \end{aligned} \quad (19)$$

where $\mathbf{z}_k^{\text{VC}} = \mathbf{0}_2$, and $u_{v_y,k}$ and $u_{v_z,k}$ are zero-mean Gaussian processes to model the perturbations on the pseudo measurements.

C. Coarse Estimation of Vehicle Position

Writing the vehicle dynamic model and measurement models together, a state-space model can be formulated. The state transition model is given by

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}) + \mathbf{D}\mathbf{g} + \mathbf{n}_k, \quad (20)$$

where $\mathbf{f}(\mathbf{x}_{k-1})$, $\mathbf{D}$ and $\mathbf{n}_k$ are defined, respectively, in (21), (22) and (23), shown at the bottom of the page. The complete coarse scale measurement model is given by

$$\mathbf{z}_k^{\text{CS}} = \mathbf{h}^{\text{CS}}(\mathbf{x}_k) + \mathbf{u}_k^{\text{CS}}, \quad (24)$$

where

$$\mathbf{z}_k^{\text{CS}} = \begin{bmatrix} \mathbf{z}_k^{\text{IMU}} \\ \mathbf{z}_k^{\text{TOA}} \\ \mathbf{z}_k^{\text{VC}} \end{bmatrix}; \mathbf{h}^{\text{CS}}(\mathbf{x}_k) = \begin{bmatrix} \mathbf{h}^{\text{IMU}}(\mathbf{x}_k) \\ \mathbf{h}^{\text{TOA}}(\mathbf{x}_k) \\ \mathbf{h}^{\text{VC}}(\mathbf{x}_k) \end{bmatrix}; \mathbf{u}_k^{\text{CS}} = \begin{bmatrix} \mathbf{u}_k^{\text{IMU}} \\ \mathbf{u}_k^{\text{TOA}} \\ \mathbf{u}_k^{\text{VC}} \end{bmatrix}. \quad (25)$$

The state noise and the measurement noise are assumed to be zero-mean Gaussian processes with covariance matrices Σ_n and Σ_u^{CS} , i.e., $\mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \Sigma_n)$ and $\mathbf{u}_k^{\text{CS}} \sim \mathcal{N}(\mathbf{0}, \Sigma_u^{\text{CS}})$. The entries of diagonal elements are respectively the variance of each state component.

For sufficiently small sampling intervals, Δt , the state transition model (20) can be linearized. Using the above state-space models (20) and (24), an EKF can be formulated to estimate the state in a straightforward manner. The detailed steps are

$$\mathbf{x}_{k|k-1} = \mathbf{f}(\mathbf{x}_{k-1}); \quad (26)$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1} \mathbf{F}_{k-1}^T + \Sigma_n; \quad (27)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \Sigma_u^{\text{CS}})^{-1}; \quad (28)$$

$$\mathbf{x}_k = \mathbf{x}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k^{\text{CS}} - \mathbf{H}_k \mathbf{x}_{k|k-1}); \quad (29)$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1}, \quad (30)$$

where $\mathbf{F}_{k-1}$ and $\mathbf{H}_k$ are the Jacobian matrices of the state dynamic model and measurement model separately. It is worth mentioning that the state dynamics (20) can be applied to model arbitrary motion trajectories and variable speeds. However, the measurements due to IMU, RSU and vehicle constraints can only provide the position information on a coarse scale. Hence, let define $\mathbf{H}_k := \mathbf{H}_k^{\text{CS}}$ given in Appendix A.

III. FINE SCALE MEASUREMENT INFORMATION FUSION

In this section, fine scale measurements such as lane-change detection, map data and radar range measurements that are able to provide the position information with lane-level resolution are introduced and fused to further refine our coarse position estimate from Section II-C.

A. Lane-change Detection

Let $\ell_k \in \{1, 2, \dots, L\}$ be a variable indicating the index of the lane that the vehicle is in at time step k for an L -lane highway. In [26], a predefined confusion matrix is considered, in which the detected event, e_k , can only be one of no lane-change (e^0), left lane-change (e^l) and right lane-change (e^r) maneuvers. Even so, there may exist some other scenarios that should be taken into account, such as a maneuver with more than one lane-change, or when the previous lane has been detected wrongly which must be corrected as soon as possible. Therefore, in this work, we define a decision fusion scheme, as shown in Fig. 2, which fuses detection results from different lane-change detectors, called maneuver recognition modules

$$\mathbf{f}(\mathbf{x}_{k-1}) = \begin{bmatrix} \mathbf{I}_3 & \Delta t \mathbf{I}_3 & \frac{1}{2} \Delta t^2 \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \Delta t \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \Delta t \mathbf{A}(\Psi_{k-1}) & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \text{diag}\{\beta\} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \text{diag}\{\beta\} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & 1 & \Delta t \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & 0 & 1 \end{bmatrix} \mathbf{x}_{k-1}; \quad (21)$$

$$\mathbf{n}_k = [(\mathbf{n}_{p,k}^n)^T \quad (\mathbf{n}_{v,k}^n)^T \quad (\mathbf{n}_{a,k}^n)^T \quad (\mathbf{n}_{\Psi,k}^n)^T \quad (\mathbf{n}_{a,k}^b)^T \quad (\mathbf{n}_{w,k}^b)^T \quad (\mathbf{n}_{b_a,k}^b)^T \quad (\mathbf{n}_{b_w,k}^b)^T \quad n_{\tau,k} \quad n_{\alpha,k}]^T; \quad (22)$$

$$\mathbf{D} = [\mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{I}_3 \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 1} \quad \mathbf{0}_{3 \times 1}]^T. \quad (23)$$

(a) Flowchart of the lane-change detection scheme.

(b) Decision fusion.

Fig. 2: A lane-change detection scheme with monitoring detected events by MRMs and applying decision fusion.

(MRMs), and dynamically updates a confusion vector, $\mathbf{P}_k^{\text{MRM}}$, based on the reliabilities of the detectors in the current environmental conditions (e.g., weather condition and road type), outputs of the MRMs (i.e., e_k^G , e_k^A and e_k^C from gyro-, accelerometer- and camera-based detectors, respectively) and all possible lane-changes with respect to lane index the vehicle is currently supposed to be in. Consequently, the drawn events based on this dynamic confusion vector, e_k^P , would be chosen from a more complete set of events which includes all possible lane-change cases, although the decision fusion inputs (the detected events by MRMs) can only be from three detectable cases $\{e^0, e^l, e^r\}$. Subsequently, drawn events (called particles) will be evaluated, as will be explained later in Section III-D. As the final result, this decision fusion mechanism provides a final predicted event, e_k^F , with the help of particle filtering with higher reliability as compared with a simple maneuver detection scenario that uses a predefined confusion matrix.

In the proposed mechanism, shown in Fig. 2, it is supposed that there exist three MRMs which are monitoring received data from different sensors and detecting lane-change events e_k^G , e_k^A and e_k^C by using, respectively, yaw rate (from IMU's gyroscope) [20], lateral acceleration (from IMU's accelerometer) [21] and visual lane detection (from camera) [24]. The outputs are from the set of $\{-1, 0, 1\}$ (i.e., left, no and right lane-change).

From Fig. 2a, the system monitors outputs of different MRMs, continuously. Once one of them alerts a lane-change maneuver (i.e., $\text{OR}(|e_k^G|, |e_k^A|, |e_k^C|) \neq 0$), system will buffer detected events by all MRMs for a period of time, since MRMs may detect the same maneuver with different delays. This buffering period should not be too long due to higher detection latency and should not be too short due to detecting the same lane-change maneuver more than once. After buffering, the system chooses the most frequent detected maneuver in buffering period, just in case of more than one detection by an MRM, as new e_k^G , e_k^A and e_k^C and feed them to the decision fusion section.

Note that the previous lane index is not necessarily the lane index at the previous time step (i.e., ℓ_{k-1}) due to the above-mentioned buffering process in which the previous lane index must be kept as the lane index prior to the detection of the first alerting MRM. Hence, let define ℓ_p as the previous lane index where $p < k$ to be kept equal to the time index of the beginning of the most recent buffering process till the end of that buffering period, and it is $p = k - 1$, otherwise.

Therefore, considering the Markovian process assumption for the lane-change model as in [26], the dynamics of the lane-change model for i -th particle, $\ell_k^{(i)}$, and the final one, ℓ_k , can be updated as

$$p(\ell_k^{(i)} | e_k^{P(i)}, e_{1:k-1}^F, \ell_{0:k-1}) = p(\ell_k^{(i)} | e_k^{P(i)}, \ell_p),$$

$$p(\ell_k | e_{1:k}^F, \ell_{0:k-1}) = p(\ell_k | e_k^F, \ell_p), \forall k > 0. \quad (31)$$

As shown in Fig. 2b, the proposed decision fusion block has three main modules as follows:

- *Detection evaluator*: Considering the previous lane index, ℓ_p , and the number of lanes (obtained from a digital map), L , the system evaluates the possibility of detected events and constructs a matrix $\mathbf{M}_k^{\text{MRM}}$ shown in (32) at the top of the next page, in which $\delta_{(\cdot)}$ is the Kronecker delta function.
- *Sensor evaluator*: This module evaluates the reliability of the sensors based on the environmental information and derives a reliability weight vector $\mathbf{R}_k^{\text{MRM}} = [R_k^{\text{gMRM}}, R_k^{\text{aMRM}}, R_k^{\text{cMRM}}]$, where R_k^{gMRM} , R_k^{aMRM} and R_k^{cMRM} are, respectively, reliability weights of gyro-, accelerometer- and camera-based MRMs at time step k . For example, if the distance to another vehicle in front of the host vehicle is less than a certain range, the lane detection by camera information may not be reliable, or the reliability of camera information varies in different weather and brightness conditions. In a curvy road, the gyro-based MRM proposed in [20], does not perform well due to an initial angular velocity, while it has better performance than the accelerometer-based

$$\mathbf{M}_k^{\text{MRM}} = \begin{bmatrix} \delta_{e_k+1}^G [1 - \delta_{\ell_p-1}] & \delta_{e_k}^G + \delta_{e_k+1}^G \delta_{\ell_p-1} + \delta_{e_k-1}^G \delta_{\ell_p-L} & \delta_{e_k-1}^G [1 - \delta_{\ell_p-L}] \\ \delta_{e_k+1}^A [1 - \delta_{\ell_p-1}] & \delta_{e_k}^A + \delta_{e_k+1}^A \delta_{\ell_p-1} + \delta_{e_k-1}^A \delta_{\ell_p-L} & \delta_{e_k-1}^A [1 - \delta_{\ell_p-L}] \\ \delta_{e_k+1}^C [1 - \delta_{\ell_p-1}] & \delta_{e_k}^C + \delta_{e_k+1}^C \delta_{\ell_p-1} + \delta_{e_k-1}^C \delta_{\ell_p-L} & \delta_{e_k-1}^C [1 - \delta_{\ell_p-L}] \end{bmatrix}. \quad (32)$$

MRM proposed in [21], when the road is straight. The performance of camera-based lane detection [24] differs by changing the road curvature as well. It is worth noting that there may exist many other situations and sensors which can be added to this fusion scheme.

- *Confusion vector generator*: Finally, a confusion vector is generated by multiplying detection matrix $\mathbf{M}_k^{\text{MRM}}$ by reliability vector $\mathbf{R}_k^{\text{MRM}}$ as $\mathbf{P}_k^{\text{MRM}} = \mathbf{R}_k^{\text{MRM}} * \mathbf{M}_k^{\text{MRM}}$. Obtained confusion vector, $\mathbf{P}_k^{\text{MRM}}$, consists of three elements defining probabilities of possible left, no and right lane-change maneuvers. If $\|\mathbf{P}_k^{\text{MRM}}\|_1 < 1$, the module expands $\mathbf{P}_k^{\text{MRM}}$ by assigning the remain probability to all possible lane-change scenarios uniformly (considering the current lane index and total number of the lane). That is $\mathbf{P}_k^{\text{MRM}}$ may have more elements after expansion in which other possible lane-changes are now taken into account as well, although with a small probability. Note that, by setting required parameters in the sensor evaluator properly, $\|\mathbf{P}_k^{\text{MRM}}\|_1$ can be made to be less than one. Then, we would be able to consider all possible lane-change scenarios (i.e., including more than one lane-change) as explained.

In the next step, we use the confusion vector to draw the required number of events (called particles), e_k^P , evaluate them and find the most reliable lane-change maneuver as e_k^F .

B. Incorporating Map Information

After obtaining the predicted lane label, ℓ_k , and a coarse estimation of the vehicle state, $\mathbf{x}_k$, map matching can be applied to correct the coarse position estimates to the corresponding lane center [5], [26], where a road link is selected and its heading is used to sequentially update the positions. However, in this work, rather than updating the estimated position of the vehicle by direct projection onto the lane center, we consider map matching as a kind of measurement where distance between the projected point, $\hat{\mathbf{p}}_k^n$, and the estimated position, $\mathbf{p}_k^n$, is expected to be zero in a 2D map, i.e., $\mathbf{z}_k^{\text{MAP}} = \mathbf{0}_2$, due to the assumption of driving around center of the lane, as follows:

$$\begin{aligned} \mathbf{z}_k^{\text{MAP}} &= \mathbf{h}^{\text{MAP}}(\mathbf{x}_k, \ell_k) + \mathbf{u}_k^{\text{MAP}} \\ &= \begin{bmatrix} p_k^{\text{nx}} - \hat{p}_k^{\text{nx}} \\ p_k^{\text{ny}} - \hat{p}_k^{\text{ny}} \end{bmatrix} + \begin{bmatrix} u_{x,k}^{\text{MAP}} \\ u_{y,k}^{\text{MAP}} \end{bmatrix}, \end{aligned} \quad (33)$$

where $u_{x,k}^{\text{MAP}}$ and $u_{y,k}^{\text{MAP}}$ are zero-mean Gaussian processes, i.e., $\mathbf{u}_k^{\text{MAP}} \sim \mathcal{N}(\mathbf{0}, \Sigma_u^{\text{MAP}})$, and

$$\hat{\mathbf{p}}_k^n = \mathbf{f}_{\ell_k}^{\text{MAP}}(\mathbf{p}_k^n) \quad (34)$$

follows map matching steps explained in [5], [26] to find the closest point at the center of the predicted lane, ℓ_k . We then define a hypothesis model to evaluate the particles by obtaining their likelihoods, $p(\mathbf{z}_k^{\text{MAP}} | \mathbf{x}_k)$.

C. Multi-Hypothesis Model for Radar Measurements

The car-borne radar is able to measure the range between the vehicle and a reference point. In practice, the reference point is usually not readily available. In our work, the positions of the road guardrails are accessible from the digital map. The reference points are obtained as the points on the guardrails that are closest to the vehicle. Detailed steps of computing reference points are the same as those in map matching. Assume that M radars are deployed in the vehicle and let $\mathbf{p}_{m,k}^R$ and $\mathbf{p}_{m,k}^O$, for $m = 1, \dots, M$, be positions of the radar and its corresponding obstacle's reference point computed for the m -th radar range measurement. The measurement model can be addressed as

$$\begin{aligned} z_{m,k}^{\text{RAD}} &= h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{m,k}^O, \mathbf{D}_v) + u_{m,k}^{\text{RAD}} \\ &= \|\mathbf{p}_{m,k}^R - \mathbf{p}_{m,k}^O\| + u_{m,k}^{\text{RAD}} \\ &= \|\mathbf{p}_k^n - \mathbf{p}_{m,k}^O\| - d_m + u_{m,k}^{\text{RAD}}, \quad \forall m = 1, \dots, M, \end{aligned} \quad (35)$$

where $\mathbf{D}_v = [L_v, W_v]^T$ is the vector of vehicle dimensions (to obtain positions of the deployed radars with respect to the vehicle's position) and $u_{m,k}^{\text{RAD}} \sim \mathcal{N}(\mathbf{0}, \sigma_u^{\text{RAD}})$ is the noise following a zero-mean Gaussian process. Note that $\mathbf{p}_{m,k}^R$ is related to the vehicle's position, $\mathbf{p}_k^n$, heading of the vehicle, ψ_k , and its dimensions, $\mathbf{D}_v$. Hence, d_m is half-width of vehicle, $d_m = W_v/2$, for left- and right-side radars, and it is half-length of vehicle, $d_m = L_v/2$, for front- and rear-side radars.

The range measurements are susceptible to different obstacles. For example, vehicles on adjacent lanes may pass by and block the radar signals. As such, the range measurements are in fact with respect to vehicles on other lanes rather than the guardrails. To reduce the tracking error caused by the obstacles, a multi-hypothesis likelihood model is proposed to incorporate the ranging information due to obstacles. Assume that the vehicle to be tracked is located at the ℓ_k -th lane. Other than the guardrails, the ranging measurement may be due to other vehicles located at lane 1 up to lane $\ell_k - 1$ from one side and vehicles at lane $\ell_k + 1$ up to lane L from the other side. Let first two radars (i.e., $m = 1, 2$) are the two side-scan radars, and L_m indicates the set of remained lanes and guardrail at each side, i.e., $L_1 = \{0, 1, \dots, \ell_k - 1\}$ and $L_2 = \{\ell_k + 1, \dots, L, L + 1\}$ with 0 and $L + 1$ indicating the guardrails at two sides. Corresponding range measurement models due to the obstacles are

$$\begin{aligned} z_{\ell',k}^{\mathcal{H}} &= h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{\ell',k}^O, \mathbf{D}_v) + u_{\ell',k}^{\text{RAD}} \\ &= \|\mathbf{p}_k^n - \mathbf{p}_{\ell',k}^O\| - \frac{W_v}{2} + u_{\ell',k}^{\text{RAD}}, \quad \forall \ell' \in L_m. \end{aligned} \quad (36)$$

Note that here we only consider the obstacles from two sides. Hence, for $m > 2$, there is no hypothesis due to obstacles, i.e., $|L_m| = 1$. However, measurement models and hypotheses

due to obstacles can also be formulated in a similar manner if needed. The collection of hypotheses from all radars is

$$\mathcal{H}_k = \bigcup_{m=1}^M \mathcal{H}_{m,k} \triangleq \bigcup_{m=1}^M \{\mathcal{H}_{\ell',k}\}_{\ell' \in L_m}. \quad (37)$$

In the developed LaIF tracking algorithm, the prior probabilities of hypotheses of each radar are equally assigned, i.e., $p(\mathcal{H}_{\ell',k}) = 1/|L_m|$. Let $\mathbf{z}_k^{\text{RAD}} = [z_{1,k}^{\text{RAD}}, z_{2,k}^{\text{RAD}}, \dots, z_{M,k}^{\text{RAD}}]^T$. The total likelihood of radar measurement can be computed as

$$\begin{aligned} p(\mathbf{z}_k^{\text{RAD}} | \mathbf{x}_k) &= \prod_{m=1}^M \sum_{\ell' \in L_m} p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathcal{H}_{\ell',k}) p(\mathcal{H}_{\ell',k}) \\ &= \prod_{m=1}^M \frac{1}{|L_m|} \left(\sum_{\ell' \in L_m} p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathbf{p}_{\ell',k}^{\text{O}}) \right) \\ &\quad \times \prod_{m=1}^M p(z_{m,k}^{\text{RAD}} | \mathbf{x}_k, \mathbf{p}_{m,k}^{\text{O}}), \end{aligned} \quad (38)$$

in which likelihood terms can be determined by (35) and (36).

D. Particle Filtering for Lane-level Information Fusion

For the fine scale measurements, the state-space model with lane transition can be addressed as

$$\ell_k \sim p(\ell_k | e_k^{\text{F}}, \ell_p); \quad (39)$$

$$\mathbf{z}_k^{\text{FS}} = \mathbf{h}^{\text{FS}}(\mathbf{x}_k, \ell_k) + \mathbf{u}_k^{\text{FS}}; \quad (40)$$

$$p(\mathbf{z}_k^{\text{FS}} | \mathbf{x}_k) = p(\mathbf{z}_k^{\text{MAP}} | \mathbf{x}_k) p(\mathbf{z}_k^{\text{RAD}} | \mathbf{x}_k), \quad (41)$$

where (39) describes the transition process of the lane label given a lane-change event and previous lane index, (40) is the fine scale measurement process including map matching and radar ranging, i.e., $\mathbf{z}_k^{\text{FS}} = [(\mathbf{z}_k^{\text{MAP}})^T, (\mathbf{z}_k^{\text{RAD}})^T]^T$, $\mathbf{h}^{\text{FS}} = [(\mathbf{h}^{\text{MAP}})^T, (\mathbf{h}^{\text{RAD}})^T]^T$ and $\mathbf{u}_k^{\text{FS}} = [(\mathbf{u}_k^{\text{MAP}})^T, (\mathbf{u}_k^{\text{RAD}})^T]^T \sim \mathcal{N}(\mathbf{0}, \Sigma_u^{\text{FS}})$, and (41) provides total likelihood. We develop a PF approach to achieve a refined position estimation.

Let $\{\mathbf{x}_{k-1}^{(i)}, \ell_{k-1}^{(i)}, w_{k-1}^{(i)}\}_{i=1}^N$ denote the set of N particles of the states and corresponding weights at time step $k-1$. For each particle, the state $\mathbf{x}_{k-1}^{(i)}$ is propagated according to the IMU and TOA information to obtain a coarse estimation first. Such a coarse estimation is regarded as the state prediction employed for subsequent fine scale measurement information fusion. Consequently, the lane index is predicted according to the confusion vector and the previous lane index, i.e., $\ell_k^{(i)} \sim p(\ell_k^{(i)} | e_k^{\text{P}}, \ell_p)$. Further, map information is incorporated via map matching, and used together with radar measurements to evaluate the weights of the particles. Finally, the resampling scheme is performed to refine the state estimation and new particles $\{\mathbf{x}_k^{(i)}, \ell_k^{(i)}, w_k^{(i)}\}_{i=1}^N$ are obtained. The detailed PF steps are summarized in Algorithm 1. Since the fine scale measurement information are considered, the algorithm is able to differentiate the positions of vehicle on various lanes. In addition, the incorporation of the multi-hypothesis model for radar measurements makes the algorithm immune to sideway obstacles.

Algorithm 1: PF implementation of the LaIF.

Initialization: $w_0^{(1:N)} \leftarrow 1/N$; $\ell_0^{(i)} \leftarrow \mathcal{U}\{1, L\}$;
 $\mathbf{p}_0^{n,(i)} \sim \mathcal{N}(\mathbf{m}(\ell_0^{(i)}), \Sigma_{p_0})$;
 $\mathbf{x}_0^{(i)} \setminus \mathbf{p}_0^{n,(i)} \sim \mathcal{N}(\mathbf{x}_0 \setminus \mathbf{p}_0^n, \Sigma_{x_0 \setminus p_0^n})$;
 where $\mathcal{U}$ and $\mathcal{N}$ are, respectively, uniform and multivariate Gaussian distributions, and ‘\’ indicates excluding.
 // Over all time steps:
for $t \leftarrow 1$ **to** T **do**
 // Over all particles:
 for $i \leftarrow 1$ **to** N **do**
 1. coarse estimation of the state $\mathbf{x}_k^{(i)}$ according to (26)-(30);
 2. predict the lane state $\ell_k^{(i)}$ according to (31);
 3. predict the vehicle position according to map matching (34);
 4. obtain the required map measurement according to (33);
 5. generate different hypothesis and modes for radar measurements according to (36);
 6. evaluate the weight $w_k^{(i)}$ according to the likelihood (41);
 end
 7. weight normalization $w_k^{(i)} = w_k^{(i)} / \sum_{i=1}^N w_k^{(i)}$;
 8. resample $\{\mathbf{x}_k^{(i)}, \ell_k^{(i)}\}_{i=1}^N$ according the particle weights;
 9. output the estimates.
end

IV. PERFORMANCE BOUND ANALYSIS

The PCRLB provides a lower performance bound on the mean square error (MSE) matrix for sequential Bayesian estimation of random parameters. It sets a lower bound on the performance of estimating $\mathbf{x}_k$ when the new measurement $\mathbf{z}_k$ becomes available given that the past measurements up to time step k are known. Let $\hat{\mathbf{x}}_k$ denote an unbiased estimate of the state vector $\mathbf{x}_k$ and $\mathbf{z}_{1:k}$ represents the complete data set from time step 1 to k . The PCRLB on the estimation error has the form

$$\mathbb{E} \{ (\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^T | \mathbf{z}_{1:k-1} \} \geq \mathbf{I}_k^{-1}. \quad (42)$$

The Fisher information matrix (FIM), $\mathbf{I}_k$, is given by

$$\mathbf{I}_k = \mathbb{E} \{ -\Delta_{\mathbf{x}_{1:k}}^{\mathbf{x}_{1:k}} \ln p(\mathbf{x}_{1:k}, \mathbf{z}_k | \mathbf{z}_{1:k-1}) \}, \quad (43)$$

where $\Delta_{\mathbf{x}_k}^{\mathbf{x}_k}$ is the second order partial derivative, defined by

$$\nabla_{\mathbf{x}_k} = \frac{\partial}{\partial \mathbf{x}_k} = \left[\frac{\partial}{\partial x_k^1}, \dots, \frac{\partial}{\partial x_k^M} \right]^T; \quad (44)$$

$$\Delta_{\mathbf{x}_k}^{\mathbf{y}_k} = \nabla_{\mathbf{x}_k} \nabla_{\mathbf{y}_k}^T, \quad (45)$$

where $\mathbf{x}_k^j$ denotes the j -th element in vector $\mathbf{x}_k$. We omit the conditional part in the FIM $\{\cdot | \mathbf{z}_{1:k-1}\}$ when there is no ambiguity.

We assume that the second order derivatives and the expectation in (43) exist. Given the posterior distribution

Fig. 3: An illustration of the simulation setup for PCRLB study in a single carriageway.

Fig. 4: PCRLB with different numbers of TOA and their variances under GNSS-denied environment, with and without radar information.

$p(\mathbf{x}_{1:k}, \mathbf{z}_k | \mathbf{z}_{1:k-1})$, the conditional FIM, $\mathbf{J}_k$, can be calculated recursively as [27]

$$\mathbf{J}_k = \mathbf{D}_k^{22} - \mathbf{D}_k^{21}(\mathbf{J}_{k-1} + \mathbf{D}_k^{11})^{-1}\mathbf{D}_k^{12}, \quad (46)$$

where $\mathbf{D}_k^{11}$, $\mathbf{D}_k^{12}$, and $\mathbf{D}_k^{21}$ are given by

$$\begin{aligned} \mathbf{D}_k^{11} &= \mathbb{E}\{-\Delta_{\mathbf{x}_{k-1}}^{\mathbf{x}_{k-1}} \ln p(\mathbf{x}_k | \mathbf{x}_{k-1})\} = \mathbf{F}^T \Sigma_n^{-1} \mathbf{F}; \\ \mathbf{D}_k^{12} &= \mathbb{E}\{-\Delta_{\mathbf{x}_{k-1}}^{\mathbf{x}_k} \ln p(\mathbf{x}_k | \mathbf{x}_{k-1})\} = -\mathbf{F}^T \Sigma_n^{-1}; \\ \mathbf{D}_k^{21} &= \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_{k-1}} \ln p(\mathbf{x}_k | \mathbf{x}_{k-1})\} = \{\mathbf{D}_k^{12}\}^T; \\ \mathbf{D}_k^{22} &= \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{x}_k | \mathbf{x}_{k-1})\} + \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k | \mathbf{x}_k)\} \\ &= \Sigma_n^{-1} + \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k | \mathbf{x}_k)\}. \end{aligned} \quad (47)$$

Let define

$$\tilde{\mathbf{D}}_k^{22} = \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k | \mathbf{x}_k)\}, \quad (48)$$

with the measurement vector $\mathbf{z}_k = [(\mathbf{z}_k^{\text{CS}})^T, (\mathbf{z}_k^{\text{FS}})^T]^T$. Since the different measurements are conditionally independent, the joint distribution in (48) can be decomposed as

$$p(\mathbf{z}_k | \mathbf{x}_k) = p(\mathbf{z}_k^{\text{CS}} | \mathbf{x}_k) p(\mathbf{z}_k^{\text{FS}} | \mathbf{x}_k). \quad (49)$$

Hence, $\tilde{\mathbf{D}}_k^{22}$ can be written as

$$\begin{aligned} \tilde{\mathbf{D}}_k^{22} &= \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k^{\text{CS}} | \mathbf{x}_k) p(\mathbf{z}_k^{\text{FS}} | \mathbf{x}_k)\} \\ &= (\mathbf{H}_k^{\text{CS}})^T \Sigma_u^{\text{CS}} \mathbf{H}_k^{\text{CS}} + (\mathbf{H}_k^{\text{FS}})^T \Sigma_u^{\text{FS}} \mathbf{H}_k^{\text{FS}}. \end{aligned} \quad (50)$$

Detailed expressions of $\mathbf{F}_{k-1}$, $\mathbf{H}_k^{\text{CS}}$ and $\mathbf{H}_k^{\text{FS}}$ are given in Appendix A.

Fig. 5: Comparison of PCRLB with and without radar information in different GPS variances, using two TOAs with $\sigma_{\text{TOA}} = 100$ ns. The access of GPS is set to every 5 s.

Fig. 6: An illustration of the simulation setup for PCRLB study in a highway with random unknown vehicles.

Simulations are organized to demonstrate the derived PCRLB and illustrate a potential tracking performance by incorporating different measurement information. TOA is available at a frequency of 10 Hz based on IEEE 802.11p standard.

We first consider a single carriageway inside the campus where there is only one radar ranging as the fine scale measurement. The radar and IMU measurements have the same data rate as that of the TOA. When the last term in (50) contains only the single measurement information from a radar, it can be computed as

$$\mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k^{\text{RAD}} | \mathbf{x}_k)\} = (\mathbf{H}_k^{\text{RAD}})^T \sigma_u^{\text{RAD}} \mathbf{H}_k^{\text{RAD}}. \quad (51)$$

Figure 3 gives an illustration of the simulation setup. In this demonstration, we assume that one radar is deployed on the vehicle's left side to measure the range to the guardrail, and two RSUs/TOAs are available at each time. Nevertheless, Figure 4 gives the PCRLB with different numbers of available TOAs and their different variances under a GNSS-denied environment. The smaller the variance of the TOA measurements, the lower the bound, and hence better tracking accuracy can be expected. However, this figure discloses a slight difference between the PCRLBs of different numbers of available TOAs when the variance is getting higher. We further analyze the case of two available TOAs with $\sigma_{\text{TOA}} = 100$ ns, as the worst case in Fig. 4, under the access of GPS every 5 s in Fig. 5. It shows that GPS information is able to correct the positioning results and is capable of improving the tracking accuracy.

Fig. 7: Comparison of PCRLB with and without map information over different numbers of TOAs/RSUs, TOA variances and measurement data rates of IMU and radars.

These figures also compare the PCRLB with and without radar information, where they imply that the tracking performance can be significantly enhanced with radar information.

We now consider a highway with more lanes where we have both radar and map information as the fine scale measurements as shown in Fig. 6. In this demonstration, we utilize two side-scan radars (i.e., one on each vehicle's side), and a few random vehicles are assumed on the road as unknown obstacles. Note that if multi-hypothesis model is considered, the likelihood term follows (41), and the last term in (50) can be written as

$$\begin{aligned} \mathbb{E}\{-\Delta \mathbf{x}_k \ln p(\mathbf{z}_k^{\text{FC}}|\mathbf{x}_k)\} &= (\mathbf{H}_k^{\text{FS}})^T \Sigma_u^{\text{FS}} \mathbf{H}_k^{\text{FS}} \\ &= \mathbb{E}\{-\Delta \mathbf{x}_k \ln p(\mathbf{z}_k^{\text{MAP}}|\mathbf{x}_k)\} \\ &\quad + \mathbb{E}\{-\Delta \mathbf{x}_k \ln p(\mathbf{z}_k^{\text{RAD}}|\mathbf{x}_k)\}. \end{aligned} \quad (52)$$

The first term can be obtained by $(\mathbf{H}_k^{\text{MAP}})^T \Sigma_u^{\text{MAP}} \mathbf{H}_k^{\text{MAP}}$, and for the second term we can show that

$$\begin{aligned} \mathbb{E}\{-\Delta \mathbf{x}_k \ln p(\mathbf{z}_k^{\text{RAD}}|\mathbf{x}_k)\} &\leq \sum_{m=1}^2 \frac{1}{|L_m|} \left(\sum_{\ell' \in L_m} (\mathbf{H}_{\ell',k}^{\text{RAD}})^T \sigma_u^{\text{RAD}} \mathbf{H}_{\ell',k}^{\text{RAD}} \right) \\ &\quad + \sum_{m=3}^M (\mathbf{H}_{m,k}^{\text{RAD}})^T \sigma_u^{\text{RAD}} \mathbf{H}_{m,k}^{\text{RAD}}, \end{aligned} \quad (53)$$

where $\mathbf{H}_{m,k}^{\text{RAD}}$ is simply the Hessian matrix with respect to the m -th radar which is given in Appendix A, and $\mathbf{H}_{\ell',k}^{\text{RAD}}$ is with respect to the ℓ' -th hypothesis in the cases of left- and

right-side radars (i.e., $m \in \{1, 2\}$). The proof of (53) is given in Appendix B.

From (47)-(50) and (52), $\mathbf{D}_k^{22}$ can be written as follows:

$$\begin{aligned} \mathbf{D}_k^{22} &= \Sigma_n^{-1} + (\mathbf{H}_k^{\text{CS}})^T \Sigma_u^{\text{CS}} \mathbf{H}_k^{\text{CS}} + (\mathbf{H}_k^{\text{MAP}})^T \Sigma_u^{\text{MAP}} \mathbf{H}_k^{\text{MAP}} \\ &\quad + \mathbb{E}\{-\Delta \mathbf{x}_k \ln p(\mathbf{z}_k^{\text{RAD}}|\mathbf{x}_k)\}. \end{aligned} \quad (54)$$

Let define $\hat{\mathbf{D}}_k^{22}$ by replacing the last term of (54) with the right-hand side of inequality (53) where $\hat{\mathbf{D}}_k^{22} \geq \mathbf{D}_k^{22}$ which results in $\hat{\mathbf{J}}_k = \hat{\mathbf{D}}_k^{22} - \mathbf{D}_k^{21}(\mathbf{J}_{k-1} + \mathbf{D}_k^{11})^{-1} \mathbf{D}_k^{12} \geq \mathbf{J}_k$. Therefore, since $1/\mathbf{J}_k \geq 1/\hat{\mathbf{J}}_k$ we can consider $1/\hat{\mathbf{J}}_k$ as the new PCRLB without any loss of generality, where

$$\begin{aligned} \hat{\mathbf{D}}_k^{22} &= \Sigma_n^{-1} + (\mathbf{H}_k^{\text{CS}})^T \Sigma_u^{\text{CS}} \mathbf{H}_k^{\text{CS}} + (\mathbf{H}_k^{\text{MAP}})^T \Sigma_u^{\text{MAP}} \mathbf{H}_k^{\text{MAP}} \\ &\quad + \sum_{m=1}^2 \frac{1}{|L_m|} \left(\sum_{\ell' \in L_m} (\mathbf{H}_{\ell',k}^{\text{RAD}})^T \sigma_u^{\text{RAD}} \mathbf{H}_{\ell',k}^{\text{RAD}} \right) \\ &\quad + \sum_{m=3}^M (\mathbf{H}_{m,k}^{\text{RAD}})^T \sigma_u^{\text{RAD}} \mathbf{H}_{m,k}^{\text{RAD}}. \end{aligned} \quad (55)$$

Figure 7 presents PCRLB analyses for different cases with and without map information where the number of available TOAs, their variances and the measurement data rate are different. This figure shows that map information is effective in reducing the lower bound in each case. Moreover, we can achieve a lower PCRLB when the data rate goes higher, as expected, due to the higher measurement resolution. By comparing Fig. 7a to Fig. 7b and Fig. 7c to Fig. 7d, one may see that there is no tangible difference in the achievable PCRLB over different numbers of available TOAs when the

Fig. 8: Lane-change detection performance by using IMU-based MRMs and proposed decision fusion scheme over real IMU data.

variance goes very high. Note that the map matching process has been considered in each 5 iterations, which means that it has update rates of 4 Hz and 20 Hz for the measurement rates of 20 Hz and 100 Hz, respectively. The fluctuations seen in depicted PCRLBs are due to the lower update rates of the TOA measurement and map matching as compared to the radar and IMU measurements rate. Definitely, the results would be better, i.e., close to their minimum values with negligible fluctuations, if we consider map information in each iteration at the expense of longer processing time.

V. SIMULATIONS AND EXPERIMENTAL RESULTS

In this section, we first evaluate the performance of the proposed decision fusion scheme using real IMU data, and then we verify the performance of the proposed LaIF tracking algorithm.

A. Performance Evaluation of the Decision Fusion Scheme

By collecting IMU data in different driving tests, we applied the proposed decision fusion scheme together with the IMU-based detection methods as in [20] and [21] on about 20 lane-change maneuvers. It is worth mentioning that the lane-change maneuvers have been done in different road types (i.e., straight and curvy) and with different driving behaviors (e.g., smooth or aggressive maneuver). Figure 8 shows that the proposed decision fusion can improve the detection performance of the system significantly. It is obvious that the defining optimum thresholds in the MRMs is quite challenging if we would like to take into account different driving behaviors as much as possible. If we choose them very strict, we may increase the possibility of missed detection, and if we choose them too wide, we may have higher false alarm rate. Therefore, there is a trade-off, and we need a huge data set to find the optimum thresholds. However, this figure implies that, by using the proposed decision fusion scheme, we can still achieve a good detection performance in which both missed detection and false alarm are negligible, although the MRMs are not quite optimum. Note that it can be enhanced by using other sensor information such as camera.

Fig. 9: NXP Cocoon radar deployment.

B. Performance Evaluation of the LaIF Tracking Algorithm

To have a more comprehensive analysis, we conducted two sets of driving tests with two different cars. For the first test, we have simulated the radars' range measurements to evaluate the performance of the proposed LaIF algorithm with different data rates and in diversified amounts of unknown obstacles. We then conducted another set of driving tests with real radar measurements. Through these tests, other than evaluating the algorithm with real data, we aim to analyze its performance in different functionalities of the radars and to examine its sensor sensitivity, i.e., how effective each sensor information is in the tracking performance of the LaIF.

All driving tests have been done in a trajectory along Singapore's Pan Island Expressway (PIE) close to Nanyang Technological University where a u-blox EVK-M8L GNSS receiver with built-in 3D automotive dead reckoning (ADR) technology (includes on-board sensors) was used to provide the ground-truth data when 13 ~ 17 GNSS satellites are in-view along the way. We have used 79 GHz NXP Cocoon radars for real range measurements which are deployed on both sides of the vehicle as shown in Fig. 9. The IMU data has been measured with the rate of 20 Hz by using an NXP's FRDM-KEAZ128 Development Board which includes an FRDM-FXS-MULT2-B multi-sensor expansion board. The measured data is later up-sampled to 100 Hz, through interpolation, before feeding to the algorithm for a better data resolution. Nonetheless, we will show that the positioning accuracy is still in the desired level by using the original data rate.

In order to simulate the TOA measurements, Ettus Research's USRP N210 software defined radios are used to measure the real LO drift for transmission over 5.9 GHz band DSRC technology. The measured drifts are then used to generate the required TOAs in the mentioned trajectory. Note that, the TOA data rate is set to 10 Hz based on the maximum possible transmission interval, 100 ms, as in the IEEE 802.11p standard, and it is supposed to access to maximum two RSUs (two TOA measurements) at each time. Moreover, the guardrail information along the desired trajectory, as the radar's reference points, has been obtained from the digital map released by the Land Transport Authority of Singapore [28]. By using this information together with the recorded trajectory, the required radar-measured ranges have been simulated for

(a) Normal view of the trajectory.

(b) Zoomed in view of the tracking results when passing by a vehicle on the left-side lane.

Fig. 10: Scenario 1: Tracking results with sparse unknown obstacles on adjacent lane, using 100 Hz data rate.

the first driving test when some random unknown obstacles, due to vehicles on other driving lanes, and their relative distances to the host vehicle have been taken into account. The proposed LaIF algorithm is implemented to obtain the tracking results when the number of particles is set to 50. The initial information is given from GNSS receiver at the initial point. It is worth mentioning that a GNSS-denied case is usually a temporary situation which appears along the way when it is supposed to access to GNSS information prior to that point such as entering the tunnel or turning to a highly build-up zone. Therefore, it is reasonable to consider the initial information as known.

1) *LaIF's sensitivity to unknown obstacle's density and data rate with simulated radar measurement:* Figures 10 and 11 show tracking results over different scenarios in terms of number of unknown obstacles (i.e., vehicles) on the road. It can be observed that the proposed PF approach can lock on the trajectory even in the case of unknown obstacles, when the radar ranging is no longer from the guardrails but from the surface of the vehicles which are being driven in parallel to the desired vehicle on adjacent lanes. The black dots are the points that the MRMs alert detecting lane-change maneuvers. In these figures, the first two dots are belonged to a single left lane-change event which is detected by gyro- and accelerometer-based MRMs with different delays. However, the algorithm

(a) Normal view of the trajectory.

(b) Zoomed in view of the tracking results when passing by a few vehicles in adjacent lanes during the lane-change maneuver.

Fig. 11: Scenario 2: Tracking results with dense unknown obstacles on different lanes, using 100 Hz data rate.

could deal with it properly and considered them as a single lane-change maneuver with the help of the decision fusion mechanism as explained in Section III-A.

We have also computed the RMSE of the tracking algorithm over all time steps of the trajectory. The lateral RMSEs of the proposed LaIF for these two datasets are 0.32 m and 0.43 m, separately. Although the lateral positioning accuracy is the matter of interest from the perspective of the lane-level accuracy, the total positioning accuracy is also important for some applications such as safety purposes. Therefore, we have computed the total RMSEs (due both to lateral and longitudinal errors) as 1.20 m and 1.45 m which are still in the required range. Moreover, the performance of the proposed method with the original 20 Hz data rate has been evaluated, when the lateral and total RMSEs for the scenario with sparse unknown obstacles, as shown in Fig. 12, are 0.84 m and 1.40 m, and those of the dense scenario, as depicted in Fig. 13, are 0.89 m and 1.46 m, respectively.

On the other hand, when we talk about safety applications, it is crucial to know that how fast our tracking algorithm can react to an unpredicted error (with more than 1.5 m localization error) due to erroneous measurement, such as wrong lane-change detection or incorrect radar-measured range, and bring it back to the acceptable range. Therefore, we have calculated the maximum and average error duration, T_{err} ,

(a) Normal view of the trajectory.

(b) Zoomed in views of the tracking results when passing by a vehicle on the left- and right-side lane, respectively.

Fig. 12: Scenario 3: Tracking results with sparse unknown obstacles on adjacent lanes, using 20 Hz data rate.

(a) Normal view of the trajectory.

(b) Zoomed in views of the tracking results when passing by a vehicle on the left- and right-side lane, respectively.

Fig. 13: Scenario 4: Tracking results with dense unknown obstacles on adjacent lanes, using 20 Hz data rate.

(i.e., the amount of time the localization error may go higher than 1.5 m) for all scenarios as summarized in Table I. The results imply that the proposed LaIF algorithm can react fast enough by considering the two-second rule of the safe following distance. Moreover, it is obvious that there is higher latency to correct the occurred error in the lower data rate which is quite reasonable.

To provide a deeper insight into estimation errors, cumulative distribution functions (CDFs) of total and lateral positioning errors are depicted in Fig. 14 for above-mentioned scenarios. They clarify that more than 68% of the errors are less than desired level (i.e., 1.5 m), in the both low and high data rates as well as both sparse and dense cases of the unknown obstacles. As the benchmark, an INS/GNSS based method [2], called NaveGo [29], has been applied on the recorded data whose results are presented in this figure

Scenario Number	Data rate (hertz)	Lateral RMSE (meter)	Total RMSE (meter)	Ave. T_{err} (second)	Max. T_{err} (second)
1	100	0.32	1.20	0.19	0.45
2	100	0.43	1.45	0.23	0.77
3	20	0.84	1.40	0.35	1.10
4	20	0.89	1.46	0.36	1.15

TABLE I: Performance of proposed LaIF algorithm in different scenarios.

as well. For NaveGo's GNSS input, the means of total and lateral positioning errors are $\mu_{Total}^{GNSS} = 5.4$ m and $\mu_{Lateral}^{GNSS} = 3.5$ m. Figure 14 discloses that the proposed LaIF tracking algorithm in the GNSS-denied environment can work as accurate as a GNSS-based method in terms of total positioning precision, and it outperforms the INS/GNSS scheme in terms of lateral accuracy with the same data rate.

Fig. 14: Total and lateral error CDFs in different scenarios.

Fig. 15: Tracking results in different sensor configurations.

2) LaIF's sensor sensitivity with real radar measurement:

In the next step, we conducted another set of driving tests in the same area with deployed radars on both sides of the vehicle. It is worth noting that the utilized car, traffic condition and consequently the driving behavior were different from the first set of driving tests. Here we aim to analyze the system sensitivity to sensors' functionalities, and hence the available information to the LaIF algorithm. We investigate the following cases:

- 1) Two TOAs and IMU: as the coarse estimation.
- 2) Four TOAs and IMU: to see how effective the TOA measurement is if we could access to more RSUs.
- 3) Two TOAs, IMU and map: to see the effectiveness of PF in the achieved performance with just one level of the particle evaluation by using map information.
- 4) Two TOAs, IMU, map and radar: to see how effective the radar information is to improve the performance as the second level of the particle evaluation. This part is divided into three cases:
 - a) One radar: we only consider measurements from the left-side radar.
 - b) Two radars where one of them malfunctions: the measurement from the right-side radar is not valid for half of the trajectory.
 - c) Two radars where both work well: measurements from both radars are valid all along the trajectory.

Figure 15 shows the performance of the tracking algorithm in four (out of six) cases in a part of trajectory with two lane-change maneuvers. Again, as mentioned before, the black dots are MRM alert points for detected lane-changes where the first two dots are belonged to the first lane-change and second two dots are due to the second lane-change maneuver detected by gyro- and accelerometer-based MRMs with different delays.

The detailed information about the estimation errors and reaction delays are summarized in Table II. From these results, one can see that the contribution of map information is mainly in lateral error improvement as that of the longitudinal error. Likewise, Figures 16 and 17 represent CDFs of total and lateral localization errors in different sensor cases, respectively. Figure 17 implies that more than 68 % of the lateral errors are less than the desired accuracy when PF has been utilized. Even so, it is revealed from Fig. 16 that we need at least two radars to keep the total positioning error below the desired precision, even if the measurements from one of them is not always valid. From these two figures, we can again conclude that the main contribution of the map matching is to enhance the lateral positioning accuracy. Moreover, as expected, more TOAs can improve the coarse estimation which has gains of 12 % and 20 % in probabilities of total and lateral errors being less than 1.5 m, respectively. Similarly, results with map and radar measurements are in a good agreement with those of the INS/GNSS method. Note that GNSS positioning errors of the NaveGo have the means of $\mu_{\text{Total}}^{\text{GNSS}} = 5.2 \text{ m}$ and $\mu_{\text{Lateral}}^{\text{GNSS}} = 3.3 \text{ m}$.

Fig. 16: Total error CDFs in different sensor configurations.

Utilized Sensors	Lateral RMSE (meter)	Total RMSE (meter)	Ave. T_{err} (second)	Max. T_{err} (second)
2xTOA + IMU	5.30	5.62	0.89	13.11
4xTOA + IMU	2.87	3.35	0.63	13.78
2xTOA + IMU + Map	1.07	3.57	0.37	5.45
2xTOA + IMU + Map + 1xRadar	1.03	1.74	0.20	1.12
2xTOA + IMU + Map + 2xRadar (Err)	1.12	1.54	0.27	1.18
2xTOA + IMU + Map + 2xRadar (Corr)	0.53	1.24	0.17	0.47

TABLE II: Performance of proposed LaIF algorithm in different sensor configurations.

As demonstrated in our experiments, our proposed approach can achieve comparable performance as INS/GNSS and better in some scenarios due to the larger number of sensors it has incorporated, although it has applied in GNSS-denied environments. Therefore, we can expect higher performance of the LaIF when GNSS information is available.

VI. CONCLUSIONS

In this paper, a particle filtering based information fusion scheme, called LaIF, is proposed to localize and track the vehicle in the GNSS-denied environments. To achieve a lane-level precision, a probabilistic model is introduced to characterize the lane changing behaviors and from which the predicted lane labels can be sampled. To do so, a decision fusion mechanism is also introduced to fuse the detected lane-change maneuvers by different detectors to achieve a more reliable detection output as compared to those of individual detectors, when the required confusion vector would be dynamically updated. Moreover, a multi-hypothesis model is formulated for the radar range measurements to reduce the errors due to unknown obstacles. The particles can then be evaluated both through the map matching and multi-hypothesis radar measurements obtained based on the detected lane. On the other hand, the posterior Cramér-Rao lower bound is derived to provide a guideline of the potential tracking performance by incorporating different measurement information. Finally, real IMU and radar measurements have been collected to evaluate and demonstrate the performance of the proposed LaIF algorithm. The results show that the proposed fusion scheme is able to track the vehicle accurately and is robust to different obstacles.

Fig. 17: Lateral error CDFs in different sensor configurations.

APPENDIX A

Let $\nabla(\cdot) := \frac{\partial}{\partial(\cdot)}$ denote partial derivative, and $\mathbf{J}_y(\mathbf{x}_k) := \frac{\partial}{\partial \mathbf{x}_k} \mathbf{y}(\mathbf{x}_k) := \frac{\partial \mathbf{y}_k}{\partial \mathbf{x}_k}$ denote the Jacobian matrix of $\mathbf{y}_k$ with respect to vector $\mathbf{x}_k$.

From (21), we can obtain Jacobian matrix of the non-linear state model as defined in (52), shown at the top of the next page, where

$$\mathbf{J}_{a^n}(\Psi_k) = \frac{\partial}{\partial \Psi_k} [C_k^{b \rightarrow n} \mathbf{a}_k^b] = \begin{bmatrix} \nabla_{\phi_k} a_k^{nx} & \nabla_{\theta_k} a_k^{nx} & \nabla_{\psi_k} a_k^{nx} \\ \nabla_{\phi_k} a_k^{ny} & \nabla_{\theta_k} a_k^{ny} & \nabla_{\psi_k} a_k^{ny} \\ \nabla_{\phi_k} a_k^{nz} & \nabla_{\theta_k} a_k^{nz} & \nabla_{\psi_k} a_k^{nz} \end{bmatrix}, \quad (53)$$

with

$$\begin{aligned} \nabla_{\phi_k} a_k^{nx} &= a_k^{by} (\cos \phi_k \sin \theta_k \cos \psi_k + \sin \phi_k \sin \psi_k) \\ &\quad + a_k^{bz} (-\sin \phi_k \sin \theta_k \cos \psi_k + \cos \phi_k \sin \psi_k); \\ \nabla_{\theta_k} a_k^{nx} &= a_k^{bx} (-\sin \theta_k \cos \psi_k) + a_k^{by} (\sin \phi_k \cos \theta_k \cos \psi_k) \\ &\quad + a_k^{bz} (\cos \phi_k \cos \theta_k \cos \psi_k); \\ \nabla_{\psi_k} a_k^{nx} &= a_k^{bx} (-\cos \theta_k \sin \psi_k) \\ &\quad + a_k^{by} (-\sin \phi_k \sin \theta_k \sin \psi_k - \cos \phi_k \cos \psi_k) \\ &\quad + a_k^{bz} (-\cos \phi_k \sin \theta_k \sin \psi_k + \sin \phi_k \cos \psi_k); \\ \nabla_{\phi_k} a_k^{ny} &= a_k^{by} (\cos \phi_k \sin \theta_k \sin \psi_k - \sin \phi_k \cos \psi_k) \\ &\quad + a_k^{bz} (-\sin \phi_k \sin \theta_k \sin \psi_k - \cos \phi_k \cos \psi_k); \\ \nabla_{\theta_k} a_k^{ny} &= a_k^{bx} (-\sin \theta_k \sin \psi_k) + a_k^{by} (\sin \phi_k \cos \theta_k \sin \psi_k) \\ &\quad + a_k^{bz} (\cos \phi_k \cos \theta_k \sin \psi_k); \\ \nabla_{\psi_k} a_k^{ny} &= a_k^{bx} (\cos \theta_k \cos \psi_k) \\ &\quad + a_k^{by} (\sin \phi_k \sin \theta_k \cos \psi_k - \cos \phi_k \sin \psi_k) \\ &\quad + a_k^{bz} (\cos \phi_k \sin \theta_k \cos \psi_k + \sin \phi_k \sin \psi_k); \\ \nabla_{\phi_k} a_k^{nz} &= a_k^{by} (\cos \phi_k \cos \theta_k) + a_k^{bz} (-\sin \phi_k \cos \theta_k); \\ \nabla_{\theta_k} a_k^{nz} &= a_k^{bx} (-\cos \theta_k) + a_k^{by} (-\sin \phi_k \sin \theta_k) \\ &\quad + a_k^{bz} (-\cos \phi_k \sin \theta_k); \\ \nabla_{\psi_k} a_k^{nz} &= 0, \end{aligned} \quad (54)$$

$$\mathbf{F}_{k-1} = \mathbf{J}_f(\mathbf{x}_{k-1}) = \begin{bmatrix} \mathbf{I}_3 & \Delta t \mathbf{I}_3 & \frac{1}{2} \Delta t^2 \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \Delta t \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{J}_{a^n}(\Psi_{k-1}) & \mathbf{C}_{k-1}^{\text{nb}} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{J}_\Psi(\Psi_{k-1}) & \mathbf{0}_{3 \times 3} & \Delta t \mathbf{A}(\Psi_{k-1}) & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{I}_3 & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \text{diag}\{\beta\} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \text{diag}\{\beta\} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & 1 & \Delta t \\ \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 3} & 0 & 1 \end{bmatrix}, \quad (52)$$

and

$$\mathbf{J}_\Psi(\Psi_k) = \mathbf{I}_3 + \Delta t \frac{\partial}{\partial \Psi_k} [\mathbf{A}(\Psi_k) \mathbf{w}_k^b] = \mathbf{I}_3 + \Delta t \begin{bmatrix} \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 & \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 & \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 \\ \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 & \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 & \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 \\ \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 & \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 & \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 \end{bmatrix}, \quad (55)$$

with

$$\begin{aligned} \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 &= (w_k^{\text{by}} \cos \phi_k - w_k^{\text{bz}} \sin \phi_k) \tan \theta_k; \\ \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 &= (w_k^{\text{by}} \sin \phi_k + w_k^{\text{bz}} \cos \phi_k) \sec^2 \theta_k; \\ \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_1 &= 0; \\ \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 &= -w_k^{\text{by}} \sin \phi_k - w_k^{\text{bz}} \cos \phi_k; \\ \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 &= 0; \\ \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_2 &= 0; \\ \nabla_{\phi_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 &= (w_k^{\text{by}} \cos \phi_k - w_k^{\text{bz}} \sin \phi_k) \sec \theta_k; \\ \nabla_{\theta_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 &= (w_k^{\text{by}} \sin \phi_k + w_k^{\text{bz}} \cos \phi_k) \tan \theta_k \sec \theta_k; \\ \nabla_{\psi_k} [\mathbf{A}_k \mathbf{w}_k^b]_3 &= 0. \end{aligned} \quad (56)$$

As in (25), the coarse scale measurement model function, $\mathbf{h}^{\text{CS}}(\mathbf{x}_k) = [(\mathbf{h}^{\text{IMU}}(\mathbf{x}_k))^T, (\mathbf{h}^{\text{TOA}}(\mathbf{x}_k))^T, (\mathbf{h}^{\text{VC}}(\mathbf{x}_k))^T]^T$, includes three parts as follow:

- IMU:

$$\begin{aligned} \mathbf{h}^{\text{IMU}}(\mathbf{x}_k) &= \begin{bmatrix} \mathbf{a}_k^b + \mathbf{b}_{a,k} \\ \mathbf{w}_k^b + \mathbf{b}_{w,k} \end{bmatrix} \\ &= [\mathbf{0}_{6 \times 12} \quad \mathbf{I}_6 \quad \mathbf{I}_6 \quad \mathbf{0}_{6 \times 2}] \mathbf{x}_k. \end{aligned} \quad (57)$$

- TOA:

$$\mathbf{h}^{\text{TOA}}(\mathbf{x}_k) = \tau_k + \Delta \tau_k, \quad (58)$$

where $\Delta \tau_k = [\Delta \tau_{1,k}, \dots, \Delta \tau_{i,k}, \dots, \Delta \tau_{I,k}]^T$ with $\Delta \tau_{i,k} = \|\mathbf{p}_k^n - \mathbf{p}_i^{\text{RSU}}\|/c$ in which $\mathbf{p}_i^{\text{RSU}} = [x_i^{\text{RSU}}, y_i^{\text{RSU}}, z_i^{\text{RSU}}]^T \in \mathbb{R}^3$.

- VC:

$$\mathbf{h}^{\text{VC}}(\mathbf{x}_k) = \begin{bmatrix} v_k^{\text{by}} \\ v_k^{\text{bz}} \end{bmatrix} = [\mathbf{0}_{2 \times 3} \quad {}^y_z \mathbf{C}_k^{\text{nb}} \quad \mathbf{0}_{2 \times 20}] \mathbf{x}_k, \quad (59)$$

where from (2), we have

$${}^y_z \mathbf{C}_k^{\text{nb}} = \begin{bmatrix} \sin \phi_k \sin \theta_k \cos \psi_k - \cos \phi_k \sin \psi_k \\ \cos \phi_k \sin \theta_k \cos \psi_k + \sin \phi_k \sin \psi_k \\ \sin \phi_k \sin \theta_k \sin \psi_k + \cos \phi_k \cos \psi_k \\ \cos \phi_k \sin \theta_k \sin \psi_k - \sin \phi_k \cos \psi_k \\ \sin \phi_k \cos \theta_k \\ \cos \phi_k \cos \theta_k \end{bmatrix}. \quad (60)$$

Therefore, for Jacobian matrix of the coarse scale measurement model, $\mathbf{H}_k^{\text{CS}} = \mathbf{J}_{\mathbf{h}^{\text{CS}}}(\mathbf{x}_k) = [(\mathbf{H}_k^{\text{IMU}})^T, (\mathbf{H}_k^{\text{TOA}})^T, (\mathbf{H}_k^{\text{VC}})^T]^T$, we have

- IMU:

$$\mathbf{H}_k^{\text{IMU}} = \mathbf{J}_{\mathbf{h}^{\text{IMU}}}(\mathbf{x}_k) = [\mathbf{0}_{6 \times 12} \quad \mathbf{I}_6 \quad \mathbf{I}_6 \quad \mathbf{0}_{6 \times 2}]. \quad (61)$$

- TOA:

$$\begin{aligned} \mathbf{H}_k^{\text{TOA}} &= \mathbf{J}_{\mathbf{h}^{\text{TOA}}}(\mathbf{x}_k) \\ &= [\mathbf{J}_{\Delta \tau}(\mathbf{p}_k^n) \quad \mathbf{0}_{I \times 21} \quad \mathbf{1}_{I \times 1} \quad \mathbf{0}_{I \times 1}], \end{aligned} \quad (62)$$

where

$$\begin{aligned} \mathbf{J}_{\Delta \tau}(\mathbf{p}_k^n) &= \frac{\partial}{\partial \mathbf{p}_k^n} \Delta \tau_k \\ &= \begin{bmatrix} \nabla_{p_k^{\text{nx}}} \Delta \tau_{1,k} & \nabla_{p_k^{\text{ny}}} \Delta \tau_{1,k} & \nabla_{p_k^{\text{nz}}} \Delta \tau_{1,k} \\ \vdots & \vdots & \vdots \\ \nabla_{p_k^{\text{nx}}} \Delta \tau_{i,k} & \nabla_{p_k^{\text{ny}}} \Delta \tau_{i,k} & \nabla_{p_k^{\text{nz}}} \Delta \tau_{i,k} \\ \vdots & \vdots & \vdots \\ \nabla_{p_k^{\text{nx}}} \Delta \tau_{I,k} & \nabla_{p_k^{\text{ny}}} \Delta \tau_{I,k} & \nabla_{p_k^{\text{nz}}} \Delta \tau_{I,k} \end{bmatrix}, \end{aligned} \quad (63)$$

and

$$\begin{aligned} \nabla_{p_k^{\text{nx}}} \Delta \tau_{i,k} &= \frac{p_k^{\text{nx}} - x_i^{\text{RSU}}}{\|\mathbf{p}_k^n - \mathbf{p}_i^{\text{RSU}}\|_c}; \\ \nabla_{p_k^{\text{ny}}} \Delta \tau_{i,k} &= \frac{p_k^{\text{ny}} - y_i^{\text{RSU}}}{\|\mathbf{p}_k^n - \mathbf{p}_i^{\text{RSU}}\|_c}; \\ \nabla_{p_k^{\text{nz}}} \Delta \tau_{i,k} &= \frac{p_k^{\text{nz}} - z_i^{\text{RSU}}}{\|\mathbf{p}_k^n - \mathbf{p}_i^{\text{RSU}}\|_c}. \end{aligned} \quad (64)$$

- VC:

$$\begin{aligned} \mathbf{H}_k^{\text{VC}} &= \mathbf{J}_{\mathbf{h}^{\text{VC}}}(\mathbf{x}_k) \\ &= [\mathbf{0}_{2 \times 3} \quad {}^y_z \mathbf{C}_k^{\text{nb}} \quad \mathbf{0}_{2 \times 3} \quad \mathbf{J}_{v^b}(\Psi_k) \quad \mathbf{0}_{2 \times 14}], \end{aligned} \quad (65)$$

where

$$\mathbf{J}_{\mathbf{y}^b}(\Psi_k) = \frac{\partial}{\partial \Psi_k} [\mathbf{C}^{\text{nd-b}} \mathbf{v}_k^{\text{n}}] \\ = \begin{bmatrix} \nabla_{\phi_k} v_k^{\text{by}} & \nabla_{\theta_k} v_k^{\text{by}} & \nabla_{\psi_k} v_k^{\text{by}} \\ \nabla_{\phi_k} v_k^{\text{bz}} & \nabla_{\theta_k} v_k^{\text{bz}} & \nabla_{\psi_k} v_k^{\text{bz}} \end{bmatrix}, \quad (66)$$

and

$$\begin{aligned} \nabla_{\phi_k} v_k^{\text{by}} &= v_k^{\text{nx}} (\cos \phi_k \sin \theta_k \cos \psi_k + \sin \phi_k \sin \psi_k) \\ &\quad + v_k^{\text{ny}} (\cos \phi_k \sin \theta_k \sin \psi_k - \sin \phi_k \cos \psi_k) \\ &\quad + v_k^{\text{nz}} (\cos \phi_k \cos \theta_k); \\ \nabla_{\theta_k} v_k^{\text{by}} &= v_k^{\text{nx}} (\sin \phi_k \cos \theta_k \cos \psi_k) \\ &\quad + v_k^{\text{ny}} (\sin \phi_k \cos \theta_k \sin \psi_k) \\ &\quad + v_k^{\text{nz}} (-\sin \phi_k \sin \theta_k); \\ \nabla_{\psi_k} v_k^{\text{by}} &= v_k^{\text{nx}} (-\sin \phi_k \sin \theta_k \sin \psi_k - \cos \phi_k \cos \psi_k) \\ &\quad + v_k^{\text{ny}} (\sin \phi_k \sin \theta_k \cos \psi_k - \cos \phi_k \sin \psi_k); \\ \nabla_{\phi_k} v_k^{\text{bz}} &= v_k^{\text{nx}} (-\sin \phi_k \sin \theta_k \cos \psi_k + \cos \phi_k \sin \psi_k) \\ &\quad + v_k^{\text{ny}} (-\sin \phi_k \sin \theta_k \sin \psi_k - \cos \phi_k \cos \psi_k) \\ &\quad + v_k^{\text{nz}} (-\sin \phi_k \cos \theta_k); \\ \nabla_{\theta_k} v_k^{\text{bz}} &= v_k^{\text{nx}} (\cos \phi_k \cos \theta_k \cos \psi_k) \\ &\quad + v_k^{\text{ny}} (\cos \phi_k \cos \theta_k \sin \psi_k) \\ &\quad + v_k^{\text{nz}} (-\cos \phi_k \sin \theta_k); \\ \nabla_{\psi_k} v_k^{\text{bz}} &= v_k^{\text{nx}} (-\cos \phi_k \sin \theta_k \sin \psi_k + \sin \phi_k \cos \psi_k) \\ &\quad + v_k^{\text{ny}} (\cos \phi_k \sin \theta_k \cos \psi_k + \sin \phi_k \sin \psi_k). \end{aligned} \quad (67)$$

On the other hand, in addition to the the lane prediction, we have fine scale measurements, $\mathbf{h}^{\text{FS}}(\mathbf{x}_k, \ell_k) = [(\mathbf{h}^{\text{MAP}}(\mathbf{x}_k, \ell_k))^T, (\mathbf{h}^{\text{RAD}}(\mathbf{x}_k, \ell_k))^T]^T$ with $\mathbf{h}^{\text{RAD}}(\mathbf{x}_k, \ell_k) = [h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{1,k}^{\text{O}}, \mathbf{D}_v), \dots, h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{M,k}^{\text{O}}, \mathbf{D}_v)]^T$, which includes:

- Map: Since we provide a terrestrial positioning scheme, 2D localization is the matter of interest. Therefore, map matching measurement is given by

$$\mathbf{h}^{\text{MAP}}(\mathbf{x}_k, \ell_k) = \begin{bmatrix} p_k^{\text{nx}} - \hat{p}_k^{\text{nx}} \\ p_k^{\text{ny}} - \hat{p}_k^{\text{ny}} \end{bmatrix}. \quad (68)$$

- Radar: Considering the fact that the altitude of the obstacle reference point is the same as that of the measuring radar, for the m -th radar measurement, we have

$$h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{m,k}^{\text{O}}, \mathbf{D}_v) = \sqrt{(p_k^{\text{nx}} - p_{m,k}^{\text{Ox}})^2 + (p_k^{\text{ny}} - p_{m,k}^{\text{Oy}})^2} - d_m, \quad (69)$$

in which

$$d_m = \begin{cases} \frac{L_v}{2} & \text{front- and rear-side Radars;} \\ \frac{W_v}{2} & \text{left- and right-side Radar,} \end{cases} \quad (70)$$

where L_v and W_v are length and width of vehicle, respectively.

We can obtain the Jacobian matrix of the fine scale measurements, $\mathbf{H}_k^{\text{FS}} = \mathbf{J}_{\mathbf{h}^{\text{FS}}}(\mathbf{x}_k) = [(\mathbf{H}_k^{\text{MAP}})^T, (\mathbf{H}_k^{\text{RAD}})^T]^T$, as follows:

- Map:

$$\mathbf{H}_k^{\text{MAP}} = \mathbf{J}_{\mathbf{h}^{\text{MAP}}}(\mathbf{x}_k) = [\mathbf{I}_2 \quad \mathbf{0}_{1 \times 24}]. \quad (71)$$

- Radar:

$$\mathbf{H}_k^{\text{RAD}} = [(\mathbf{H}_{1,k}^{\text{RAD}})^T, \dots, (\mathbf{H}_{M,k}^{\text{RAD}})^T]^T \quad (72)$$

where

$$\mathbf{H}_{m,k}^{\text{RAD}} = \frac{\partial}{\partial \mathbf{x}_k} h^{\text{RAD}}(\mathbf{x}_k, \mathbf{p}_{m,k}^{\text{O}}, \mathbf{D}_v) \\ = \begin{bmatrix} \nabla_{p_k^{\text{nx}}} h_{m,k}^{\text{RAD}} & \nabla_{p_k^{\text{ny}}} h_{m,k}^{\text{RAD}} & \mathbf{0}_{1 \times 24} \end{bmatrix}, \quad (73)$$

with

$$\begin{aligned} \nabla_{p_k^{\text{nx}}} h_{m,k}^{\text{RAD}} &= \frac{p_k^{\text{nx}} - p_{m,k}^{\text{Ox}}}{\sqrt{(p_k^{\text{nx}} - p_{m,k}^{\text{Ox}})^2 + (p_k^{\text{ny}} - p_{m,k}^{\text{Oy}})^2}}; \\ \nabla_{p_k^{\text{ny}}} h_{m,k}^{\text{RAD}} &= \frac{p_k^{\text{ny}} - p_{m,k}^{\text{Oy}}}{\sqrt{(p_k^{\text{nx}} - p_{m,k}^{\text{Ox}})^2 + (p_k^{\text{ny}} - p_{m,k}^{\text{Oy}})^2}}. \end{aligned} \quad (74)$$

APPENDIX B

From (38), we can rewrite (51) for the multi-hypothesis model as follows:

$$\begin{aligned} &\mathbb{E}\{-\Delta_{\mathbf{x}_k} \ln p(z_k^{\text{RAD}} | \mathbf{x}_k)\} \\ &= \sum_{m=1}^2 \underbrace{E\left\{-\Delta_{\mathbf{x}_k} \ln \left(\frac{1}{|L_m|} \sum_{\ell' \in L_m} p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathbf{p}_{\ell',k}^{\text{O}})\right)\right\}}_{(i)} \\ &\quad + \sum_{m=3}^M \underbrace{E\{-\Delta_{\mathbf{x}_k} \ln p(z_{m,k}^{\text{RAD}} | \mathbf{x}_k, \mathbf{p}_{m,k}^{\text{O}})\}}_{(ii)}. \end{aligned} \quad (75)$$

where (ii) is equal to $(\mathbf{H}_{m,k}^{\text{RAD}})^T \Sigma_u^{\text{RAD}} \mathbf{H}_{m,k}^{\text{RAD}}$ from (51).

On the other hand, from arithmetic mean-geometric mean inequality for non-negative numbers we have

$$\frac{1}{N} \sum_{n=1}^N x_n \geq \sqrt[N]{\prod_{n=1}^N x_n}. \quad (76)$$

By taking logarithms of both sides, we obtain

$$\ln \left(\frac{1}{N} \sum_{n=1}^N x_n \right) \geq \frac{1}{N} \sum_{n=1}^N \ln x_n, \quad (77)$$

which can be achieved via Jensen inequality for the logarithmic function (as a concave function) as well.

Therefore, by applying (77) on (i) in (75) and considering its negative sign inside the expectation function, we have

$$\begin{aligned} &E\left\{-\Delta_{\mathbf{x}_k} \ln \left(\frac{1}{|L_m|} \sum_{\ell' \in L_m} p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathbf{p}_{\ell',k}^{\text{O}})\right)\right\} \leq \\ &E\left\{-\Delta_{\mathbf{x}_k} \frac{1}{|L_m|} \sum_{\ell' \in L_m} \ln p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathbf{p}_{\ell',k}^{\text{O}})\right\} \\ &= \frac{1}{|L_m|} \sum_{\ell' \in L_m} E\{-\Delta_{\mathbf{x}_k} \ln p(z_{\ell',k}^{\mathcal{H}} | \mathbf{x}_k, \mathbf{p}_{\ell',k}^{\text{O}})\} \\ &\stackrel{(iii)}{=} \frac{1}{|L_m|} \left(\sum_{\ell' \in L_m} (\mathbf{H}_{\ell',k}^{\text{RAD}})^T \Sigma_u^{\text{RAD}} \mathbf{H}_{\ell',k}^{\text{RAD}} \right) \end{aligned} \quad (78)$$

where (iii) is from (51).

Consequently, for (75), we could prove that

$$\begin{aligned} & \mathbb{E}\{-\Delta_{\mathbf{x}_k}^{\mathbf{x}_k} \ln p(\mathbf{z}_k^{\text{RAD}}|\mathbf{x}_k)\} \\ & \leq \sum_{m=1}^2 \frac{1}{|L_m|} \left(\sum_{\ell' \in L_m} (\mathbf{H}_{\ell',k}^{\text{RAD}})^T \Sigma_u^{\text{RAD}} \mathbf{H}_{\ell',k}^{\text{RAD}} \right) \\ & + \sum_{m=3}^M (\mathbf{H}_{m,k}^{\text{RAD}})^T \Sigma_u^{\text{RAD}} \mathbf{H}_{m,k}^{\text{RAD}}. \end{aligned} \quad (79)$$

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