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Anchor-Aided Joint Localization and Synchronization Using SOOP: Theory and Experiments

Mei Leng, François Quitin, Wee Peng Tay, Chi Cheng, Sirajudeen Gulam Razul, and Chong Meng Samson See

Abstract—We consider the problem of tracking a receiver using signals-of-opportunity (SOOPs) from beacons and a reference anchor with known positions and velocities, and where all devices have asynchronous local clocks or oscillators. We model the clock drift at individual devices by a two-state model with unknown clock offset and clock skew, and analyze the biases introduced by clock asynchronism in the received signals. Based on an extended Kalman filter, we propose a sequential estimator to jointly track the receiver location, velocity, and its clock parameters using altitude information together with time-difference-of-arrival and frequency-difference-of-arrival measurements obtained from the SOOP samples collected by the receiver and a reference anchor. The receiver was implemented on a software defined radio testbed, and field experiments are carried out using Iridium satellites as the SOOP beacons. Experiment and simulation results demonstrate that our measurement model has a good fit, and our proposed estimator can successfully track both the receiver location, velocity, and the relative clock offset and skew with respect to the reference anchor with good accuracy.

Index Terms—geo-localization, synchronization, signal-of-opportunity, EKF, USRP

I. INTRODUCTION

A signal-of-opportunity (SOOP) refers to a public signal broadcast from an established transmitting infrastructure for non-navigation purposes. Such signals conform to well-established standards and are present in most urban areas with relatively high signal-to-noise ratio (SNR). SOOP beacons can be both land-based stations and space-based transmitters. Examples of land-based beacons include AM/FM radio signal transmission stations [1], [2], terrestrial television broadcast masts [3], [4], cellular communication base stations [5], [6], and WiFi access points [7], [8]. Space-based beacons include commercial planes [9], and various space satellites like low Earth orbit (LEO) satellites, including the Iridium satellites used for communication purposes [10]–[12]. Depending on their signal characteristics and transmitter properties, various SOOP signals have proved to be useful alternatives to the use of Global Navigation Satellite Systems (GNSS) for navigation services [13], and in applications like sensor networks to provide location information in order for the networks to

correctly interpret the sensory data [14], [15]. As most GNSS are medium earth orbit systems, their SNRs are lower than that of typical SOOP, and do not penetrate well into buildings or through other obstacles. Navigation services based on GNSS are therefore usually only available when there is a clear sky view. However, since GNSS signals are designed to provide localization information, they are more informative and typically yield better location estimates than using SOOPs alone [16]–[18]. In the case where not enough GNSS satellites are in line of sight, SOOPs can help to augment GNSS localization. In some scenarios, GNSS may be completely unavailable. For example, since GNSS techniques have become mature, it is likely that GNSS signals may be jammed or disabled by adversaries during war time. Civilian GPS signals are also known to be susceptible to spoofing attacks, and several spoofers have been successfully developed in recent years [19], [20]. Therefore, navigation using SOOPs has gained increasing interest recently [2], [6], [13], [16], [21].

Since SOOPs have not been designed for navigation purposes, and SOOP beacons act as uncooperative broadcast anchors, it may be difficult to have a priori knowledge of the signal structure and certain signal characteristics like transmit power and transmit time. Typical localization metrics that can be extracted from a SOOP are limited to received signal strength and angle-of-arrival at a single receiver, and time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA) between two receivers [22]. In [16], we proposed an estimator using differential TDOA and FDOA, where the location and velocity of every receiver are estimated using measurements from at least 5 beacons simultaneously in 2D space. In practice, the assumption of multiple beacons may not hold. For example, beacons like the Iridium satellites and GSM base stations adopt TDMA schemes, making it difficult to receive SOOPs from multiple different beacons in the same short observation period. The receiver hardware implementation is also more complex if it is required to listen to multiple SOOPs at the same time. In this paper, we consider a more practical situation where one burst from one beacon is received per time slot, and design a *sequential* estimator for the receiver using TDOA and FDOA measurements.

In general, land-based SOOP beacons are typically stationary, and their precise locations are known. However, their coverage areas are smaller than those of the space-based systems, which have near-global coverage in many instances. The signal qualities of land-based beacons are also lower due to more severe multipath effects in ground-to-ground

F. Quitin, Cheng Chi and W.P. Tay are with the Nanyang Technological University, Singapore. (e-mail: {fquitin, chengchi, wptay}@ntu.edu.sg).

M. Leng, C.M. Samson See and S.G. Razul are with Temasek Laboratories@NTU (TL@NTU), Singapore. (e-mail: {lengmei, samsonsee, ESirajudeen}@ntu.edu.sg).

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communications in an urban environment. In this paper, we use Iridium satellites as beacons in our experiments, and obtain the beacon position and velocity from the SGP4 models [23]. Moreover, we consider the case where the position and velocity of one receiver is known a priori and serves as a reference anchor to help another receiver perform state estimation using the SOOP. The use of an anchor as a localization aid also applies to the scenario in which a receiver is navigating in an urban canyon while the anchor could be a static node positioned on top of a building with good GNSS coverage, or an unmanned aerial vehicle that has access to accurate location information.

The most critical challenge for localization using TDOA and FDOA is synchronization between the two receivers. An offset between the receivers' local clocks introduces biases into TDOA measurements, for which even a microsecond offset significantly degrades the overall estimation performance. Likewise, a relative clock skew between the two receivers distorts the FDOA measurements. It shows in [24] that synchronization error is one of the major error sources even when GPS clock discipline is applied. In order to obtain reliable timing and frequency information, it is essential that receivers are time synchronized and achieve precise frequency alignment, and therefore most existing methods make this simplifying assumption (see [25], [26] and the references therein). However, clock synchronization is difficult to achieve and maintain in practice [27], [28]. Moreover, in order not to interfere with the existing operations of SOOP beacons, localization using SOOP will have no active communication with beacons, and beacons will provide no common clock reference for the receivers.

In the context of wireless sensor networks, it has been proposed to jointly estimate sensors' positions and clock drifts so that localization and synchronization can be achieved simultaneously. Following [29], this problem has received considerable attention. The paper [30] considers localization for a single receiver using signals from multiple synchronized beacons, while [31] and [18] investigate a more general case where a set of anchor nodes cooperate with each other to localize a single transmitter. The work [32] considers a similar source localization problem using TOA measurements with the transmit time being an unknown parameter. These works consider a static target and require certain prior knowledge such as the transmit signal [30], [31] and transmit time stamp [32]. The localization of mobile nodes was considered in [33], but it again requires cooperation amongst transmitters and receivers. The localization using long term evolution (LTE) signals was considered in [34], where the target receiver was required to perform pre-calibration with a static known initial position.

The use of TDOA and FDOA measurements for localization has been investigated in [35], [36], which proposed to localize a transmitter with asynchronous receivers. One crucial assumption in these works is that biases due to clock drifts are modeled as constant error terms in both TDOA and FDOA measurements. In [36], it is also assumed the receiver has prior knowledge of the transmission signal as well as its periodicity. The following natural and important questions

were left unanswered:

- Is it accurate to model clock drifts as constant terms in the TDOA and FDOA measurements?
- How can we obtain TDOA and FDOA measurements with minimum prior knowledge from SOOP beacons?
- Can we sequentially perform joint localization and synchronization using SOOP measurements?

In this paper, we answer the above three important questions through theoretical analysis and field experiments. We assume that a target receiver and a reference anchor receive signals from the same SOOP beacons, where the reference anchor has known location and velocity. (We relax this assumption in our simulations and experiments where the location and velocity of the anchor are estimated up to an unknown error.) We also assume that all devices, including the receiver, anchor, and beacons, are asynchronous. Our main contributions are the following:

- We propose an anchor-aided localization scheme using SOOPs. The proposed scheme assumes minimum prior knowledge of the SOOP beacons, and it is sufficient for both the receiver and anchor to know only the nominal transmission frequency,¹ signal bandwidth, position and velocity of each beacon. For space-based beacons like satellites, the location and velocity of the beacons can be calculated using the SGP4 model [23].
- We derive closed-form expressions for TDOA and FDOA measurements under clock asynchronism. We show that the bias introduced in the TDOA measurement is a time-varying term that depends linearly on the clock offsets and clock skews of the receiver and anchor. The bias in the FDOA measurement can be approximated as the relative clock skew between the receiver and anchor (i.e., a clock parameter that describes the speed of receiver clock drift relative to that of the anchor). Experiments using a software defined radio testbed and the Iridium satellites as SOOP beacons verifies the correctness of our measurement model.
- Using TDOA and FDOA between the receiver and anchor together with the receiver altitude information, we propose a sequential estimator to jointly track the location, velocity, and clock parameters of the receiver with respect to the anchor. Experiment and simulation results demonstrate that our proposed algorithm can track the clock parameters, with the location and velocity estimation performance approaching the Cramér-Rao bound (CRB).

We have investigated a similar problem in [37], in which the joint estimation was performed using TDOA and FDOA measurements. In this paper, we augment altitude information together with TDOA/FDOA, which results in a faster convergence rate and requires a less accurate initial guess. A rigorous analysis for TDOA/FDOA distortion due to clock drift is also provided in this paper. We derive the CRB to provide analytical insight into the effect of system parameters like receiving period and dynamic model inaccuracy.

¹The nominal transmission frequency refers to the publicized frequency a SOOP beacon is supposed to transmit at. However, due to local clock drifts, the actual transmission frequency may be different from the nominal frequency.

Fig. 1. Localization using space-based SOOP beacon b in urban environments.

The rest of this paper is organized as follows. In Section II, we present our model assumptions and propose an anchor-aided localization scheme. In Section III, we analyze the distortion caused by clock offsets and drifts to the received signal, obtain approximate closed-form expressions for TDOA and FDOA estimates, and propose a sequential estimator for jointly estimating the receiver's location, velocity, and clock parameters. In Section IV, we present empirical and simulation results, and we conclude in Section V.

Notations: We use bold faced upper-case letters to represent matrices and bold faced lower-case letters for vectors. The transpose and the conjugate transpose of $\mathbf{A}$ are denoted as $\mathbf{A}^T$ and $\mathbf{A}^H$, respectively. We use $\cdot^{(l)}$ to indicate that the parameter is specific to the l -th observation period. The operator $\mathbb{E}$ denotes mathematical expectation. The real multivariate Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\mathbf{P}$ is denoted by $\mathcal{N}(\boldsymbol{\mu}, \mathbf{P})$. The complex multivariate Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\mathbf{P}$ is denoted by $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{P})$.

II. SYSTEM MODEL

We consider the problem of localizing and tracking an on-ground receiver R_1 using the SOOP from a set $\mathcal{B}$ of space-based beacons. The receiver performs self-localization and velocity estimation with the aid of a reference anchor R_0 within communication range. We assume that the receiver and anchor receive signals from the same beacon during each TDOA and FDOA measurement throughout the whole observation period of the receiver. We also assume that the receiver either has access to its altitude measurements or moves in urban area of smooth terrain. An example scenario is shown in Figure 1, where a beacon $b \in \mathcal{B}$ has known position $\mathbf{p}_b$ and velocity $\mathbf{v}_b$, and broadcasts signals at a nominal carrier frequency f_b . We investigate a general scenario where the following assumptions hold.

Assumption 1

- (i) The receiver and anchor can differentiate signals from different beacons, and have prior knowledge of each beacon's location, velocity, and nominal transmit frequency.

- (ii) The anchor can communicate its location and velocity to the receiver.
- (iii) The clock offset between the receiver and anchor is sufficiently small so that the measurement period of any beacon signal at the receiver overlaps with that at the anchor.

Assumption 1(i) can be satisfied by allowing the receiver and anchor to first learn about their operating environment with preloaded beacon information. For example, each Iridium satellite has a known Earth coverage area at any given time, and its orbital trajectory can be predicted using the SGP4 or SDP4 model [23]. Hence, a receiver with prior knowledge of the region it is in, can first perform a spectrum scan to detect signals from different channels, and then identify the satellite by cross checking the satellites whose coverage regions currently intersect the region the receiver is in. With the satellites identified, a receiver that has the SGP4 model programmed in advance can calculate the satellites' positions and velocities at any given time.

For Assumption 1(iii), it is sufficient to coarsely synchronize the receiver and anchor to within an accuracy of a second. The local clocks of the receiver and anchor are then allowed to drift. Generally, oven-controlled crystal oscillator (OCXO) drifts with a clock skew ranging from 10^{-11} to 10^{-6} [38], and a clock offset of about 86.4 ms is accumulated in one day in the worst case.

We assume that SOOPs are ad hoc, and may not be received at regular intervals. In Section III-D, we discuss how the receiver state estimation accuracy is related to how frequently a SOOP is detected. Each time a SOOP is detected, the receiver and anchor both sample over a fixed time interval, and store the samples for TDOA and FDOA computations. We call this an observation period. An anchor-aided localization scheme can be conducted as follows:

- (i) The receiver detects available beacons and exchanges beacon information with an anchor in its communication range. To initiate the localization and velocity estimation procedure, the receiver and anchor agrees on the start time of receiving the signal via handshaking.
- (ii) After recording signal samples from the same set of beacons in each observation period, the anchor forwards its signal samples to the receiver, which utilizes them to compute TDOA and FDOA. A similar scheme was proposed in [17], which aims to localize a group of automated robots. In this paper, we focus on how to design a sequential algorithm to jointly track the receiver and estimate its clock drift parameters.

We perform field experiments using Iridium satellites as the beacons in Section IV, but our proposed algorithm is not limited to Iridium satellites. We also note that our algorithm is applicable to land-based SOOP beacons with line of sight to the receiver and anchor.

A. Distorted received signal for asynchronous receivers

We assume that each receiver conducts in total L observations sequentially. The l -th observation starts at nominal

receive time $t^{(l)}$ and lasts for $T_e^{(l)}$ second, and obviously we have $t^{(1)} < t^{(2)} < \dots < t^{(L)}$. In order to infer its location, the target receiver R_1 extracts TDOA and FDOA from signals received by itself and the anchor R_0 in each observation period. Since all receivers are asynchronous, the measurements are distorted due to the local oscillator biases, and it is important to analyze how such distortions affect the TDOA and FDOA measurements.

To begin with, we adopt the commonly used two-state clock model [39], [40] which describes the local oscillator with a clock skew and a clock offset [41]. Consider the l -th observation period. For the beacon b , let its clock skew be $\beta_b^{(l)}$ and its clock offset be $\Omega_b^{(l)}$. The clock skew $\beta_b^{(l)}$ characterizes the rate of clock drift and is typically related to the systematic and slowly varying frequency error due to aging and the environment, and it has nominal value 1. The clock offset $\Omega_b^{(l)}$ characterizes any clock drifts accumulated up to time $t^{(l)}$.

Since space-based beacons are equipped with high quality oscillators, their clock skew residual, i.e., $|\beta_b^{(l)} - 1|$, are stable from 10^{-12} to 10^{-10} , which accumulates at most 86.4 ns per day and hence can be neglected. Therefore, we model the local time for beacon b by

$$t_b(t) = t + \Omega_b^{(l)}. \quad (1)$$

Let $s_b(t)$ be the nominal baseband signal at beacon b . The transmitted passband signal $u_b(t)$ is

$$u_b(t) = s_b(t + \Omega_b^{(l)}) \exp\{i2\pi f_b(t + \Omega_b^{(l)})\}, \quad (2)$$

where $i = \sqrt{-1}$ and $t \in [t^{(l)}, t^{(l+1)}]$.

On the other hand, receivers are generally equipped with low-cost oscillators, such as OCXO and temperature compensated crystal oscillator (TCXO), which typically have the clock skew residual in parts per million (ppm, 10^{-6}), and we model the local time for the receiver j by

$$t_j(t) = t^{(l)} + \beta_j^{(l)}(t - t^{(l)}) + \Omega_j^{(l)}, \quad (3)$$

where clock skew $\beta_j^{(l)}$ and clock offset $\Omega_j^{(l)}$ are w.r.t. time $t^{(l)}$.

When two receivers have agreed to sample beacon b 's signal starting at time $t^{(l)}$ for a period of $T_e^{(l)}$ second, the actual receiving time at receiver R_j is given by $\hat{t}_j^{(l)} = t^{(l)} - \Omega_j^{(l)} / \beta_j^{(l)}$ and the actual observation lasts for $T_e^{(l)} / \beta_j^{(l)}$ second due to the local clock drift. Denoting $\mathbf{p}_j^{(l)}$ and $\mathbf{v}_j^{(l)}$ as the location and velocity of R_j at time $t^{(l)}$ in Earth Centered Earth Fixed (ECEF) coordinate system, as the signal travels from beacon b to R_j , we define the instantaneous time delay and Doppler shift at time $t^{(l)}$ as,

$$\text{time delay: } \mathcal{T}_{j,b}^{(l)} = \frac{\|\mathbf{p}_j^{(l)} - \mathbf{p}_b^{(l)}\|}{c}, \quad (4)$$

$$\text{Doppler shift: } \mathcal{D}_{j,b}^{(l)} = -\frac{f_b}{c} \left(\mathbf{v}_j^{(l)} - \mathbf{v}_b^{(l)} \right)^T \mathbf{u}_{j,b}^{(l)}, \quad (5)$$

where c is the speed of light, $\mathbf{u}_{j,b}^{(l)}$ is the direction from beacon b to R_j at time $t^{(l)}$ defined as

$$\mathbf{u}_{j,b}^{(l)} = \frac{\mathbf{p}_j^{(l)} - \mathbf{p}_b^{(l)}}{\|\mathbf{p}_j^{(l)} - \mathbf{p}_b^{(l)}\|}. \quad (6)$$

Assuming that the observation duration $T_e^{(l)}$ in the l -th period is short enough so that the relative movement between R_j and beacon b is small, we have the channel impulse response as [42],

$$h_{j,b}^{(l)}(t) \approx \delta(\gamma_{j,b}^{(l)}(t - \hat{t}_j^{(l)} - \eta_{j,b}^{(l)}) + \hat{t}_j^{(l)}), \quad (7)$$

where $\delta(\cdot)$ denotes the Dirac delta function, $\eta_{j,b}^{(l)} = \mathcal{T}_{j,b}^{(l)}$ and $\gamma_{j,b}^{(l)} = 1 + \mathcal{D}_{j,b}^{(l)} / f_b$.

Therefore, the noise-free received signal at R_j in the l -th time slot has the baseband representation

$$r_{j,b}^{(l)}(t) = \exp\{-i2\pi f_b \beta_j^{(l)} t\} \int_{\hat{t}_j^{(l)}}^{\hat{t}_j^{(l)} + T_e^{(l)} / \beta_j^{(l)}} u_b(t - \tau) h_{j,b}^{(l)}(\tau) d\tau. \quad (8)$$

Substituting (15) into (8), we can write the received baseband signal as $r_{j,b}^{(l)}(t) \propto s_b(\gamma_{j,b}^{(l)}(t - \hat{t}_j^{(l)} - \eta_{j,b}^{(l)}) + \hat{t}_j^{(l)} + \Omega_b^{(l)}) \exp\{i2\pi f_b(\gamma_{j,b}^{(l)} - \beta_j^{(l)})t\}$ for $t \in [\hat{t}_j^{(l)}, \hat{t}_j^{(l)} + T_e^{(l)} / \beta_j^{(l)}]$. Let T_s be the nominal sampling interval, the sample sequence at sensor j can be obtained as

$$r_{j,b}^{(l)}[n] = r_{j,b}^{(l)}\left(t / \beta_j^{(l)}\right) \big|_{t=nT_s}. \quad (9)$$

The scaling factor $1/\beta_j^{(l)}$ captures the effect of the free-running local oscillator at R_j . Let $\mathbf{r}_{j,b}^{(l)}$ be the corresponding sample sequence, it follows from (9) that

$$\mathbf{r}_{j,b}^{(l)} \propto s_b\left(\mathbf{n} - \eta_{j,b}^{(l)} + \Omega_b^{(l)}\right) \exp\{i2\pi f_b(\gamma_{j,b}^{(l)} - \beta_j^{(l)})\mathbf{n}\}, \quad (10)$$

where $\propto$ means approximately proportional to, and $\mathbf{n} = ([t / (\beta_j^{(l)} T_s)] T_s : t \in [\hat{t}_j^{(l)}, \hat{t}_j^{(l)} + T_e^{(l)} / \beta_j^{(l)}])$ is the time index sequence. Therefore, the received signal during the l -th observation period can be approximately characterized by its time delay, frequency offset, and clock biases at time $t^{(l)}$ when the observation duration $T_e^{(l)}$ is short enough. For the L observation periods, all noise-free received signal sample sequences are denoted as $\{\mathbf{r}_{j,b}^{(l)}\}_{l=1,\dots,L}$. Suppose that the received signal at receiver R_j , $j = 0, 1$, is corrupted by white Gaussian noise $\boldsymbol{\varpi}_j^{(l)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_j)$, we denote the received signal at R_j as $\tilde{\mathbf{r}}_{j,b}^{(l)} = \mathbf{r}_{j,b}^{(l)} + \boldsymbol{\varpi}_j^{(l)}$ for $l = 1, \dots, L$.

III. JOINT LOCALIZATION AND SYNCHRONIZATION ALGORITHM

A. TDOA and FDOA measurements

The receiver R_1 can obtain the approximate TDOA and FDOA w.r.t. the anchor R_0 of the SOOP from beacon b by maximizing the amplitude of the cross ambiguity function $\mathcal{A}(\tau, \xi) = \sum_n \tilde{\mathbf{r}}_{1,b}^{(l)}[n] \tilde{\mathbf{r}}_{0,b}^{(l)*}[n - \tau] \exp\{-i2\pi \xi n\}$ of their respective received signals from beacon b . Let $\tau_b^{(l)}$ be the

$$\begin{aligned}\tau_b^{(l)} &= c[\beta_1^{(l)}\eta_{1,b}^{(l)} - \beta_0^{(l)}\eta_{0,b}^{(l)} + \beta_1^{(l)}\Omega_b - \beta_0^{(l)}\Omega_b + (\hat{t}_0^{(l)} + T_e^{(l)}/\beta_0^{(l)}) - (\hat{t}_1^{(l)} - T_e^{(l)}/\beta_1^{(l)})] \\ &= \beta_1^{(l)}\|\mathbf{p}_1^{(l)} - \mathbf{p}_b^{(l)}\| - \beta_0^{(l)}\|\mathbf{p}_0^{(l)} - \mathbf{p}_b^{(l)}\| + c\left[\left(\frac{1}{\beta_0^{(l)}} - \frac{1}{\beta_1^{(l)}}\right)T_e^{(l)} + \frac{\Omega_1^{(l)}}{\beta_1^{(l)}} - \frac{\Omega_0^{(l)}}{\beta_0^{(l)}} + (\beta_1^{(l)} - \beta_0^{(l)})\Omega_b^{(l)}\right] + \varpi_b^\tau, \quad (11a)\end{aligned}$$

$$\begin{aligned}\xi_b^{(l)} &= -c[(\gamma_{1,b}^{(l)} - \beta_1^{(l)})/\beta_1^{(l)} - (\gamma_{0,b}^{(l)} - \beta_0^{(l)})/\beta_0^{(l)}] \\ &= \frac{1}{\beta_1^{(l)}}(\mathbf{v}_1^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{1,b}^{(l)} - \frac{1}{\beta_0^{(l)}}(\mathbf{v}_0^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{0,b}^{(l)} + c\left(\frac{1}{\beta_0^{(l)}} - \frac{1}{\beta_1^{(l)}}\right) + \varpi_b^\xi, \quad (11b)\end{aligned}$$

estimated TDOA multiplied by the speed of light c , and $\xi_b^{(l)}$ be the estimated FDOA multiplied by $-c/f_b$ for the l -th observation period. Their expressions are shown in (11) where the clock parameters $\beta_j^{(l)}$ and $\Omega_j^{(l)}$ are the clock skew and offset of receiver R_j at time $t^{(l)}$, respectively. We also let ϖ_b^τ and ϖ_b^ξ be measurement noises, which are assumed to be zero-mean and independent Gaussian random variables for all $b \in \mathcal{B}$.

We can rewrite (11) as

$$\begin{aligned}\tau_b^{(l)} &= \|\mathbf{p}_1^{(l)} - \mathbf{p}_b^{(l)}\| - \|\mathbf{p}_0^{(l)} - \mathbf{p}_b^{(l)}\| + \left(\frac{1}{\beta_0^{(l)}} - \frac{1}{\beta_1^{(l)}}\right)cT_e^{(l)} \\ &\quad + \underbrace{\left[\frac{\Omega_1^{(l)}}{\beta_1^{(l)}} - \frac{\Omega_0^{(l)}}{\beta_0^{(l)}} + (\beta_1^{(l)} - \beta_0^{(l)})\Omega_b^{(l)}\right]}_{\triangleq \theta^{(l)}}c + \varpi_b^\tau, \quad (12a)\end{aligned}$$

$$\begin{aligned}\xi_b^{(l)} &= (\mathbf{v}_1^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{1,b}^{(l)} - (\mathbf{v}_0^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{0,b}^{(l)} \\ &\quad + \left(\frac{1}{\beta_0^{(l)}} - \frac{1}{\beta_1^{(l)}}\right)c + \varpi_b^\xi, \quad (12b)\end{aligned}$$

where we have approximate $(\beta_1^{(l)} - 1)\|\mathbf{p}_1^{(l)} - \mathbf{p}_b^{(l)}\| - (\beta_0^{(l)} - 1)\|\mathbf{p}_0^{(l)} - \mathbf{p}_b^{(l)}\| \approx 0$ and $(1/\beta_1^{(l)} - 1)(\mathbf{v}_1^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{1,b}^{(l)} - (1/\beta_0^{(l)} - 1)(\mathbf{v}_b^{(l)} - \mathbf{v}_0^{(l)})^T \mathbf{u}_{0,b}^{(l)} \approx 0$ based on the fact that the clock skew residuals, $|\beta_j^{(l)} - 1|$, $j = 0, 1$, are typically very small (several ppm), and introduce errors far smaller than the measurement noises. Let $\alpha^{(l)} = c(1/\beta_0^{(l)} - 1/\beta_1^{(l)})$. It follows from (12) that

$$\begin{aligned}\tau_b^{(l)} &\approx \|\mathbf{p}_1^{(l)} - \mathbf{p}_b^{(l)}\| - \|\mathbf{p}_0^{(l)} - \mathbf{p}_b^{(l)}\| + T_e^{(l)}\alpha^{(l)} + \theta^{(l)} + \varpi_b^\tau, \quad (13a) \\ \xi_b^{(l)} &\approx (\mathbf{v}_1^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{1,b}^{(l)} - (\mathbf{v}_0^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{0,b}^{(l)} + \alpha^{(l)} + \varpi_b^\xi. \quad (13b)\end{aligned}$$

The bias $\theta^{(l)}$ in (13a) corresponds to the clock offset accumulated up to time $t^{(l)}$. Assuming that clocks are drifting slowly and the interval between two consecutive observations is short, i.e., $\beta_j^{(l)} \approx \beta_j^{(l-1)}$, it is easy to show that $\theta^{(l)} \approx \theta^{(l-1)} + (t^{(l)} - t^{(l-1)})\alpha^{(l-1)}$, which implies that the bias in the TDOA measurement is a time-varying term that depends linearly on the relative clock skew between two receivers. This is in contrast to the models in [35], [36], which assume that the TDOA error is constant. In our model, a larger error will also be accumulated over a longer observation period.

In Section IV, our field experiments demonstrate that (13) captures the biases caused by asynchronous clocks and correctly describes the distorted TDOA and FDOA measurements. A similar measurement model was derived in [16], which proposed a batch processing method and assumes that beacons transmit simultaneously, which is equivalent to having $L = 1$ and $t^{(l)} = 0$ for all measurements in (13). In this paper, it is assumed that only one beacon is available each time. Without loss of generality, when multiple beacons are available in one time slot, it is equivalent to stacking all measurements into a vector. The target receiver then seeks to jointly estimate its state and its clock parameters relative to the anchor.

Remark 1: The exchange of received signals can be done by using a frequency mixer with a direct amplify-and-forward operation, which translates the received analog signal at the anchor to a different frequency band that does not interfere with the beacon's transmission. This requires a minimum level of processing at the anchor side. However, when spectrum is scarce, it is possible for the anchor to transmit only limited information if it knows the pulse shape and modulation scheme of the beacon. This can be done by first obtaining a bit sequence through blind demodulation, reconstructing a local version of the beacon signal with the bit sequence, and estimating time-of-arrival (TOA) and frequency-of-arrival (FOA) by cross-correlating the received signal with the reconstructed signal. The anchor then broadcasts its estimated TOA and FOA values. Since the ring alert signal from Iridium satellites uses DE-QPSK modulation and raised-cosine pulse shaping, we can apply the above local processing method. Our experimental results showed a similar performance compared to directly cross-correlating the received signals at both R_0 and R_1 [43]. The bit sequence obtained in [43] consists of 246 bits in total. Hence at most 246 bits plus the TOA and FOA information will be transmitted every 4.32 second, which significantly reduces the communication bandwidth and latency.

B. Tracking with on-ground constraint

In this paper, we consider the tracking of on-ground receivers, and we utilize the receiver's altitude $\rho_j^{(l)}$ in the l -th observation period. The altitude can be measured by a standalone barometer with accuracy of tens of meters or by combining a reference point with an altimeter to reach errors less than 5 meters [44]. When altimeters are unavailable, notice that in most urban area the elevation of terrain does not change dramatically for on-ground vehicles, we approximate the altitude measurement as a random variable whose mean

equals to the value at the starting point and whose size of variance depends on how hilly the area is.

As the altitude is generally in geocentric system and the position in (13) is in ECEF coordinate system, we in the following derive the relationship between the altitude and the position. Denoting $\mathbf{p}_j^{(l)} \triangleq [x, y, z]^T$ in (13), whose corresponding geocentric latitude and longitude are denoted as $\phi_j^{(l)}$ and $\lambda_j^{(l)}$, the relationship is given by

$$x = \left(\rho_j^{(l)} + \frac{R}{\sqrt{1 - e^2 \sin^2 \phi_j^{(l)}}} \right) \cos \phi_j^{(l)} \cos \lambda_j^{(l)}, \quad (14a)$$

$$y = \left(\rho_j^{(l)} + \frac{R}{\sqrt{1 - e^2 \sin^2 \phi_j^{(l)}}} \right) \cos \phi_j^{(l)} \sin \lambda_j^{(l)}, \quad (14b)$$

$$z = \left(\rho_j^{(l)} + \frac{R(1 - e^2)}{\sqrt{1 - e^2 \sin^2 \phi_j^{(l)}}} \right) \sin \phi_j^{(l)}, \quad (14c)$$

where Earth is modelled as an ellipsoidal with semi-major axis $R = 6378.1363$ km and eccentricity $e = 0.08181919$ using WGS84. Following a straightforward manipulation of (14), it is easy to show that

$$\rho_j^{(l)} \approx \left\| \mathbf{p}_j^{(l)} \right\| - R \sqrt{1 - e^2 \sin^2 \phi_j^{(l)}} + \varpi_j^p, \quad (15)$$

where the measurement noise $\varpi_j^p \sim \mathcal{N}(0, \sigma_p^2)$, and we have approximated $e^4 \sin^2 \phi_j^{(l)} (1 - \sin^2 \phi_j^{(l)}) R / \sqrt{1 - e^2 \sin^2 \phi_j^{(l)}} \approx 0$, since its maximum value is 0.0716 and is negligible compared with that of $\left\| \mathbf{p}_j^{(l)} \right\|$. Furthermore, when $|\phi_j^{(l)}| \leq 4^\circ$, it can be shown that $R - R \sqrt{1 - e^2 \sin^2 \phi_j^{(l)}} < 0.1$, and we can further approximate $\rho_j^{(l)} \approx \left\| \mathbf{p}_j^{(l)} \right\| - R + \varpi_j^p$.

Notice that the latitude $\phi_j^{(l)}$ in (15) can be written as a function of $\mathbf{p}_j^{(l)}$, that is,

$$\phi_j^{(l)} = \tan^{-1} \left(\frac{z + \frac{e^2}{\sqrt{1-e^2}} R \sin^3 \theta}{\sqrt{x^2 + y^2 - e^2 R \cos^3 \theta}} \right) \quad (16)$$

with $\theta = \tan^{-1}(z / \sqrt{(1 - e^2)(x^2 + y^2)})$. Therefore, the altitude measurement $\rho_j^{(l)}$ is a function depending only on its corresponding position $\mathbf{p}_j^{(l)}$, and the derivative $\partial \rho_j^{(l)} / \partial \mathbf{p}_j^{(l)}$ can also be obtained in closed-form.

C. Sequential joint localization and synchronization

We collect all unknowns and measurements in the l -th observation period into vectors $\mathbf{x}^{(l)} \triangleq [(\mathbf{p}_1^{(l)})^T, (\mathbf{v}_1^{(l)})^T, \theta^{(l)}, \alpha^{(l)}]^T$ and $\mathbf{y}^{(l)} \triangleq [\tau_b^{(l)}, \xi_b^{(l)}, \rho_1^{(l)}]^T$. In the Bayesian framework, we view the state of $\mathbf{x}^{(l)}$ as a random variable, and the tracking problem is to recursively estimate $\mathbf{x}^{(l)}$ given the measurement $\mathbf{y}^{(1:l)}$ up to time $t^{(l)}$ by maximizing the posterior probability $p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l)})$. Computing $p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l)})$ involves two steps: prediction and update, as follows [45]:

1) prediction:

$$p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l-1)}) = \int p(\mathbf{x}^{(l)} | \mathbf{x}^{(l-1)}) p(\mathbf{x}^{(l-1)} | \mathbf{y}^{(1:l-1)}) d\mathbf{x}^{(l-1)}, \quad (17)$$

2) update:

$$p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l)}) \propto p(\mathbf{y}^{(l)} | \mathbf{x}^{(l)}) p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l-1)}). \quad (18)$$

The probability density function $p(\mathbf{y}^{(l)} | \mathbf{x}^{(l)})$ describes the relationship between the measurement $\mathbf{y}^{(l)}$ and the state $\mathbf{x}^{(l)}$ at the l -th observation period, while $p(\mathbf{x}^{(l)} | \mathbf{x}^{(l-1)})$ shows how the state $\mathbf{x}^{(l)}$ evolves over time.

Following (13) and (15), we define the measurement model as

$$\mathbf{y}^{(l)} = \mathbf{g}_b(\mathbf{x}^{(l)}) + \varpi^{(l)}, \quad (19)$$

with $\mathbf{y}^{(l)} = [\tau_b^{(l)}, \xi_b^{(l)}, \rho_1^{(l)}]^T$, and

$$\mathbf{g}_b(\mathbf{x}^{(l)}) = \begin{bmatrix} \left\| \mathbf{p}_1^{(l)} - \mathbf{p}_b^{(l)} \right\| + T_e^{(l)} \alpha^{(l)} + \theta^{(l)} \\ (\mathbf{v}_1^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{1,b}^{(l)} + \alpha^{(l)} \\ \left\| \mathbf{p}_1^{(l)} \right\| \end{bmatrix} - \begin{bmatrix} \left\| \mathbf{p}_0^{(l)} - \mathbf{p}_b^{(l)} \right\| \\ (\mathbf{v}_0^{(l)} - \mathbf{v}_b^{(l)})^T \mathbf{u}_{0,b}^{(l)} \\ R \sqrt{1 - e^2 \sin^2 \phi_1^{(l)}(\mathbf{p}_1^{(l)})} \end{bmatrix}$$

Assuming the measurement noise $\varpi^{(l)}$ is a zero mean Gaussian random variable with variance $\mathbf{R}_l$, we have

$$p(\mathbf{y}^{(l)} | \mathbf{x}^{(l)}) \propto \exp \left\{ -\frac{1}{2} (\mathbf{y}^{(l)} - \mathbf{g}_b(\mathbf{x}^{(l)}))^T \mathbf{R}_l^{-1} (\mathbf{y}^{(l)} - \mathbf{g}_b(\mathbf{x}^{(l)})) \right\}.$$

The noise covariance matrix $\mathbf{R}_l$ describes the measurement accuracy.

The dynamic model on the other hand requires a certain level of prior information about the unknown state. The clock state is relatively simple due to the fact that general oscillators can be characterized with a constant clock skew for a short period. Defining the observation interval as $\Delta_l \triangleq t^{(l)} - t^{(l-1)} \geq T_e^{(l)}$, and following from the discussion after (13), we have

$$\begin{bmatrix} \theta^{(l)} \\ \alpha^{(l)} \end{bmatrix} = \begin{bmatrix} 1 & \Delta_l \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \theta^{(l-1)} \\ \alpha^{(l-1)} \end{bmatrix} + \boldsymbol{\nu}_c^{(l)}, \quad (20)$$

where $\boldsymbol{\nu}_c^{(l)}$ models the clock errors in the l -th observation period. It is well known that the clock errors can be suitably modelled by a stochastic processes characterizing the evolution of several clock error components [40], [46]. Following from [40], we consider three clock error components, including phase noise, frequency noise, and the time variation of clock skew. Each component can be readily modelled as a Wiener process, and we define σ_Ω , σ_β , and σ_d as the diffusion

coefficients which gives the intensity of each noise. We have $\nu_c^{(l)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_c^{(l)})$ with

$$\mathbf{Q}_c^{(l)} = \begin{bmatrix} \sigma_\Omega^2 \Delta_l + \sigma_\beta^2 \frac{\Delta_l^3}{3} + \sigma_d^2 \frac{\Delta_l^5}{20} & \sigma_\beta^2 \frac{\Delta_l^2}{2} + \sigma_d^2 \frac{\Delta_l^4}{8} \\ \sigma_\beta^2 \frac{\Delta_l^2}{2} + \sigma_d^2 \frac{\Delta_l^4}{8} & \sigma_\beta^2 + \sigma_d^2 \frac{\Delta_l^3}{3} \end{bmatrix}. \quad (21)$$

A common maneuvering model for target motion [47] follows from the relationship between location, velocity and acceleration, and is given by

$$\begin{bmatrix} \mathbf{p}_1^{(l)} \\ \mathbf{v}_1^{(l)} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \Delta_l \mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p}_1^{(l-1)} \\ \mathbf{v}_1^{(l-1)} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{2} \Delta_l^2 \mathbf{a}^{(l-1)} \\ \Delta_l \mathbf{a}^{(l-1)} \end{bmatrix}}_{\triangleq \nu_s^{(l-1)}}, \quad (22)$$

where $\mathbf{I}$ and $\mathbf{0}$ are the identity and zero matrices with appropriate sizes, and the control vector $\nu_s^{(l)}$ describes the change of velocity during the observation interval Δ_l , which is related to the acceleration $\mathbf{a}^{(l)}$. When the receiver is moving with small variations in velocity, it is common to model the acceleration as a continuous Wiener process with intensity $\sigma_a^{(l)}$ [48], and hence we have $\nu_s^{(l)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_s^{(l)})$ where

$$\mathbf{Q}_s^{(l)} = \sigma_a^{2(l)} \begin{bmatrix} \frac{\Delta_l^5}{20} & 0 & 0 & \frac{\Delta_l^4}{8} & 0 & 0 \\ 0 & \frac{\Delta_l^5}{20} & 0 & 0 & \frac{\Delta_l^4}{8} & 0 \\ 0 & 0 & \frac{\Delta_l^5}{20} & 0 & 0 & \frac{\Delta_l^4}{8} \\ \frac{\Delta_l^4}{8} & 0 & 0 & \frac{\Delta_l^3}{3} & 0 & 0 \\ 0 & \frac{\Delta_l^4}{8} & 0 & 0 & \frac{\Delta_l^3}{3} & 0 \\ 0 & 0 & \frac{\Delta_l^4}{8} & 0 & 0 & \frac{\Delta_l^3}{3} \end{bmatrix}. \quad (23)$$

Combining (20) and (22), our receiver state model is given by

$$\underbrace{\begin{bmatrix} \mathbf{p}_1^{(l)} \\ \mathbf{v}_1^{(l)} \\ \theta^{(l)} \\ \alpha^{(l)} \end{bmatrix}}_{\mathbf{x}^{(l)}} = \underbrace{\begin{bmatrix} \mathbf{I} & \Delta_l \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 & \Delta_l \\ \mathbf{0} & \mathbf{0} & 0 & 1 \end{bmatrix}}_{\triangleq \mathbf{H}_l} \underbrace{\begin{bmatrix} \mathbf{p}_1^{(l-1)} \\ \mathbf{v}_1^{(l-1)} \\ \theta^{(l-1)} \\ \alpha^{(l-1)} \end{bmatrix}}_{\mathbf{x}^{(l-1)}} + \nu^{(l)}, \quad (24)$$

where $\nu^{(l)} = [\nu_c^{(l)}; \nu_s^{(l)}] \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_l)$ is the process noise. We hence obtain

$$p(\mathbf{x}^{(l)} | \mathbf{x}^{(l-1)}) \propto \exp\left\{-\frac{1}{2}(\mathbf{x}^{(l)} - \mathbf{H}_l \mathbf{x}^{(l-1)})^T \mathbf{Q}_l^{-1} (\mathbf{x}^{(l)} - \mathbf{H}_l \mathbf{x}^{(l-1)})\right\}. \quad (25)$$

The process covariance matrix $\mathbf{Q}_l$ is a block diagonal matrix consisting of $\mathbf{Q}_c^{(l)}$ and $\mathbf{Q}_s^{(l)}$.

Substituting $p(\mathbf{y}^{(l)} | \mathbf{x}^{(l)})$ and $p(\mathbf{x}^{(l)} | \mathbf{x}^{(l-1)})$ into (17) and (18), we can obtain the estimation of the state $\mathbf{x}^{(l)}$ in the l -th time slot as $\hat{\mathbf{x}}^{(l)} = \max_{\mathbf{x}^{(l)}} p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l)})$. The recursive relations (17) and (18) forms the basis for the optimal Bayesian solutions. When the integration cannot be obtained in closed form for general distributions, it can be solved by numerical methods such as particle filtering [45]. As we are assuming that all distributions are Gaussian, it leads to the well known Kalman filter (KF). However, since $p(\mathbf{y}^{(l)} | \mathbf{x}^{(l)})$ is associated with a nonlinear function $\mathbf{g}_b(\mathbf{x}^{(l)})$ (cf. (19)), to obtain a closed

form solution, we first approximate $\mathbf{g}_b(\mathbf{x}^{(l)})$ by the Taylor series expansion using

$$\mathbf{G}_l = \nabla_{\mathbf{x}} \mathbf{g}_b(\mathbf{x})|_{\mathbf{x}=\mathbf{x}^{(l)}} = \begin{bmatrix} \mathbf{u}_{1,b}^{(l)} & \mathbf{w}_{1,b}^{(l)} & \mathbf{e}_1^{(l)} \\ \mathbf{0} & \mathbf{u}_{1,b}^{(l)} & \mathbf{0} \\ 1 & 0 & 0 \\ T_e^{(l)} & 1 & 0 \end{bmatrix}_{\mathbf{x}=\mathbf{x}^{(l)}}, \quad (26)$$

where $\mathbf{e}_1^{(l)} = \partial \rho_1^{(l)} / \partial \mathbf{p}_1^{(l)}$ in (15), $\mathbf{u}_{j,b}^{(l)}$ is the direction vector in (6), and $\mathbf{w}_{j,b}^{(l)}$ is the corresponding perpendicular direction vector defined as

$$\mathbf{w}_{j,b}^{(l)} = \frac{\mathbf{I} - \mathbf{u}_{j,b}^{(l)} (\mathbf{u}_{j,b}^{(l)})^T}{\|\mathbf{p}_j^{(l)} - \mathbf{p}_b^{(l)}\|} (\mathbf{v}_j^{(l)} - \mathbf{v}_b^{(l)}). \quad (27)$$

Substituting the approximation into (17) and (18) then leads to the extended Kalman filter (EKF) [49]. We have $p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l-1)}) \approx \mathcal{N}(\mathbf{m}_{l|l-1}, \mathbf{P}_{l|l-1})$ and $p(\mathbf{x}^{(l)} | \mathbf{y}^{(1:l)}) \approx \mathcal{N}(\mathbf{m}_{l|l}, \mathbf{P}_{l|l})$ with

$$\mathbf{m}_{l|l-1} = \mathbf{H}_l \mathbf{m}_{l-1|l-1}, \quad (28a)$$

$$\mathbf{P}_{l|l-1} = \mathbf{Q}_l + \mathbf{H}_l \mathbf{P}_{l-1|l-1} \mathbf{H}_l^T, \quad (28b)$$

$$\mathbf{P}_{l|l} = \mathbf{P}_{l|l-1} - \mathbf{K}_l \mathbf{G}_l \mathbf{P}_{l|l-1}, \quad (28c)$$

$$\mathbf{m}_{l|l} = \mathbf{m}_{l|l-1} + \mathbf{K}_l (\mathbf{y}^{(l)} - \mathbf{g}_b(\mathbf{m}_{l|l-1})), \quad (28d)$$

where $\mathbf{K}_l = \mathbf{P}_{l|l-1} \mathbf{G}_l^T (\mathbf{R}_l + \mathbf{G}_l \mathbf{P}_{l|l-1} \mathbf{G}_l^T)^{-1}$ represents the Kalman gain. We implement the sequential tracking and synchronization algorithm based on (28). The procedure starts with $\mathbf{m}_{0|0} = \bar{\mathbf{x}}_0$ and $\mathbf{P}_{0|0} = \bar{\mathbf{P}}_0$, where $\bar{\mathbf{x}}_0$ is the initial guess for parameters and $\bar{\mathbf{P}}_0$ describes our confidence in the initial guess. Every time the receiver R_1 obtains a new pair of TDOA and FDOA measurements, it updates $\mathbf{m}_{l|l}$ and $\mathbf{P}_{l|l}$ using (28), and obtains the estimate for the l -th observation period as $\hat{\mathbf{x}}^{(l)} = \mathbf{m}_{l|l}$.

Remark 2: If R_0 is not an anchor, one can generalize the above tracking algorithm to include $\{\mathbf{p}_0^{(l)}, \mathbf{v}_0^{(l)}\}$ as unknown parameters in each time slot. In particular, we will have $\mathbf{x}^{(l)} = [(\mathbf{p}_1^{(l)})^T, (\mathbf{v}_1^{(l)})^T, (\mathbf{p}_0^{(l)})^T, (\mathbf{v}_0^{(l)})^T, \theta^{(l)}, \alpha^{(l)}]^T$ and $\mathbf{y}^{(l)} = [\tau_b^{(l)}, \xi_b^{(l)}, \rho_1^{(l)}, \rho_0^{(l)}]^T$, and the dynamic model and the gradient matrix $\mathbf{G}_l$ can be updated accordingly. With sufficient measurements, the states of both receivers can be estimated simultaneously [16]. However, it was found that a systematic bias will be introduced into both receivers' locations when there exists only one beacon. Specifically, denoting the position bias as $\Delta \mathbf{p}$, it can be shown that $\|\mathbf{p}_j^{(l)} + \Delta \mathbf{p} - \mathbf{p}_b^{(l)}\| \approx \|\mathbf{p}_j^{(l)} - \mathbf{p}_b^{(l)}\| + \mathbf{u}_{j,b}^T \Delta \mathbf{p}$. The resulting bias term in TDOA measurements is $(\mathbf{u}_{1,b} - \mathbf{u}_{0,b})^T \Delta \mathbf{p}$, which allows an arbitrary value for the position bias $\Delta \mathbf{p}$, especially when the beacon is far away and $\mathbf{u}_{1,b} - \mathbf{u}_{0,b} \approx 0$. Therefore, in this paper, we have assumed that the receiver R_0 acts as an anchor that provides a reference point to the receiver R_1 .

D. CRB discussion

In this section, we derive the CRB for joint localization and synchronization using SOOPs by considering the parameters of interest to be $\bar{\mathbf{x}}^{(l)} = \mathbb{E}[\mathbf{x}^{(l)}]$, for $l \geq 1$. Our derivation is based on the received sample sequences $\{\tilde{\mathbf{r}}_{j,b}^{(l)}\}_{j=0,1}$, which

implies that our derived CRB applies to all estimation methods and measurement metrics [16], unlike [35] in which the CRB is based on the TDOA/FDOA measurements. For simplicity, we consider a sequential estimation where only one beacon b is available during each observation period. Our analysis can be easily extended to the case where multiple beacons are available at each observation period.

In the l -th observation period, we have $\mathbf{x}^{(l)} = \bar{\mathbf{x}}^{(l)} + \boldsymbol{\nu}^{(l)}$ from (24), where $\boldsymbol{\nu}^{(l)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_l)$, and $\bar{\mathbf{x}}^{(l)} = \mathbf{H}_l \bar{\mathbf{x}}^{(l-1)}$. In the non-Bayesian framework, we aim to estimate $\bar{\mathbf{x}}^{(l)}$, and in the L -th period the likelihood function for $\bar{\mathbf{x}}^{(L)}$ is given by

$$p\left(\{\tilde{\mathbf{r}}_{0,b}^{(l)}, \tilde{\mathbf{r}}_{1,b}^{(l)}, \tilde{\rho}_1^{(l)}\}_{l=1}^L; \bar{\mathbf{x}}^{(L)}\right) = \prod_{l=1}^L p(\tilde{\mathbf{r}}_{0,b}^{(l)}; \bar{\mathbf{x}}^{(l)}) p(\tilde{\mathbf{r}}_{1,b}^{(l)}; \bar{\mathbf{x}}^{(l)}) p(\tilde{\rho}_1^{(l)}; \bar{\mathbf{x}}^{(l)}),$$

where $p(\bullet; \bar{\mathbf{x}}^{(l)})$ represents the likelihood function with $\bar{\mathbf{x}}^{(l)}$ as a parameter, and for $l < L$,

$$\bar{\mathbf{x}}^{(l)} = \prod_{t=l+1}^L \mathbf{H}_t^{-1} \cdot \bar{\mathbf{x}}^{(L)}.$$

Defining $\mathbf{s}^{(l)} = [\mathbf{r}_{0,b}^{(l)}; \mathbf{r}_{1,b}^{(l)}; \rho_1^{(l)}]^T$ and $\mathbf{D}_l = \text{diag}\{\mathbf{C}_0^{(l)}, \mathbf{C}_1^{(l)}, \sigma_\rho^2\}$ with $\mathbf{C}_j$ defined as the covariance matrix of the received noise in Section II, the Fisher information matrix (FIM) for estimating $\mathbf{x}^{(L)}$ can be shown to be [49],

$$\mathbf{F}_{\bar{\mathbf{x}}}(L) = \sum_{l=1}^L \mathcal{R}e \left\{ \left(\frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right)^H \mathbf{D}_l^{-1} \left(\frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right) \right\} + \sum_{l=1}^L \left[\left(\frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right)^T \mathbf{Q}_l^{-1} \left(\frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right) \right]. \quad (29)$$

In (29), the second term captures the accumulated effect of the model error due to unknown acceleration and clock drifting rate. The first term on the other hand captures the effect of other factors like measurement noise and beacon geometry, when the receiver state model is noiseless. From the differentiation chain rule, we have

$$\frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} = \frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(l)}} \cdot \frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}}, \quad \text{and} \quad \frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} = \prod_{t=l+1}^L (\mathbf{H}_t)^{-1},$$

where $\mathbf{r}_{j,b}^{(l)}$ is given in (10) and $\mathbf{H}_t$ is given in (24). We can then rewrite (29) as

$$\mathbf{F}_{\bar{\mathbf{x}}}(L) = \sum_{l=1}^L \left(\frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right)^T \left[\tilde{\mathbf{F}}_{\bar{\mathbf{x}}}(l) + \mathbf{Q}_l^{-1} \right] \left(\frac{\partial \bar{\mathbf{x}}^{(l)}}{\partial \bar{\mathbf{x}}^{(L)}} \right), \quad (30)$$

with

$$\tilde{\mathbf{F}}_{\bar{\mathbf{x}}}(l) = \mathcal{R}e \left\{ \left(\frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(l)}} \right)^H \mathbf{D}_l^{-1} \left(\frac{\partial \mathbf{s}^{(l)}}{\partial \bar{\mathbf{x}}^{(l)}} \right) \right\}. \quad (31)$$

Note that $\tilde{\mathbf{F}}_{\bar{\mathbf{x}}}(l)$ represents the instantaneous FIM at the l -th time slot. We further map the parameter vector $\bar{\mathbf{x}}^{(l)}$ into another parameter vector $\bar{\mathbf{y}}^{(l)} = [\bar{\tau}_b^{(l)}, \bar{\xi}_b^{(l)}, \bar{\rho}_1^{(l)}]$ with elements corresponding to the noise-free measurements in (13) and (15).

Substituting $\partial \bar{\mathbf{y}}^{(l)} / \partial \bar{\mathbf{x}}^{(l)} = \mathbf{G}_l$ in (26) and again utilizing the chain rule, we have

$$\tilde{\mathbf{F}}_{\bar{\mathbf{x}}}(l) = \mathbf{G}_l^T \tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l) \mathbf{G}_l, \quad (32)$$

where $\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l)$ represents the instantaneous FIM for estimating $\bar{\tau}_b^{(l)}$ and $\bar{\xi}_b^{(l)}$ using the received sequences $\{\tilde{\mathbf{r}}_{j,b}^{(l)}\}_{j=0,1}$. Following a similar procedure in [16], it can be shown that

$$\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l) \approx \begin{bmatrix} 4\pi^2 W_b^2 \sum_j \beta_j \text{SNR}_j & 0 & 0 \\ 0 & 4\pi^2 T_b^2 \sum_j \beta_j \text{SNR}_j & 0 \\ 0 & 0 & \sigma_\rho^{-2} \end{bmatrix}, \quad (33)$$

where σ_ρ is the standard deviation of altitude measurements, SNR_j is the effective output SNR for the received signal [50], and we define the root-mean-square (rms) bandwidth W_b , and the rms integration time T_b for the transmit signal $s_b(t)$ as follows [51],

$$W_b = \left[\frac{\int |f S_b(f)|^2 df}{\int |S_b(f)|^2 df} \right]^{\frac{1}{2}}, \quad \text{and} \quad T_b = \left[\frac{\int |t s_b(t)|^2 dt}{\int |s_b(t)|^2 dt} \right]^{\frac{1}{2}},$$

with $S_b(f)$ as the Fourier transform of $s_b(t)$. The FIM $\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l)$ characterizes the estimation accuracy for TDOA/FDOA measurements.

Finally, a closed-form expression for the FIM for estimating $\bar{\mathbf{x}}^{(L)}$ is given by

$$\mathbf{F}_{\bar{\mathbf{x}}}(L) = \sum_{l=1}^L \left[\prod_{t=l+1}^L \mathbf{H}_t^{-1} \right]^T \left[\mathbf{G}_l^T \tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l) \mathbf{G}_l + \mathbf{Q}_l^{-1} \right] \left[\prod_{t=l+1}^L \mathbf{H}_t^{-1} \right]. \quad (34)$$

The estimation accuracy for $\bar{\mathbf{x}}^{(l)}$ thus depends on the measurement accuracy characterized by $\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l)$, the state dynamics characterized by $\mathbf{G}_l$, and the observation schedule characterized by $\mathbf{H}_t$. The matrix $\mathbf{G}_l$, consisting of direction vectors $\mathbf{u}_{1,b}^{(l)}$ and $\mathbf{w}_{1,b}^{(l)}$, depends on relative positions and velocities of the receivers w.r.t. the beacon at time $t^{(l)}$. It can be shown that $\mathbf{G}_l^T \tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l) \mathbf{G}_l$ is always rank deficient due to the fact that only one beacon is available at each time. A feasible solution can hence be obtained from the sequential estimator only after multiple observation periods in order to accumulate a set of spanning vectors.

Furthermore, it can be shown that $\mathbf{G}_l^T \tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l) \mathbf{G}_l$ is positive semi-definite for all l and $\mathbf{F}_{\bar{\mathbf{x}}}(L) = (\mathbf{H}_L^{-1})^T \mathbf{F}_{\bar{\mathbf{x}}}(L-1) \mathbf{H}_L^{-1} + \mathbf{G}_L^T \tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(L) \mathbf{G}_L + \mathbf{Q}_L^{-1}$. If the observation is so frequent that the observation period approaches zero, i.e., $\Delta_l \rightarrow 0$, we have $\mathbf{H}_l^{-1} = \mathbf{I}$ for all l and $\mathbf{F}_{\bar{\mathbf{x}}}(L) - \mathbf{F}_{\bar{\mathbf{x}}}(L-1) \geq 0$. It implies that the estimation performance improves as more observations are available, and it holds true even when the target is maneuvering, since every movement of the target was captured in the observation and can be tracked in time. However, this is not necessarily true when additional errors are accumulated over a long time due to the dynamic model inaccuracy.

To further investigate the effect of observation frequency, we calculate the trace of $\mathbf{F}_{\bar{\mathbf{x}}}(L)$ under simplified assumptions. Let the first diagonal element of $\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l)$ be a_l and $b_l = \text{Tr}\{\tilde{\mathbf{F}}_{\bar{\mathbf{y}}}(l)\}$.

Suppose that clock offset and clock skew are stable with $\sigma_\Omega = 0$ and $\sigma_\beta = 0$, and the diffusion of clock drifting is characterized by σ_d , we let $c_l = 48/\sigma_{\mathbf{a}^{(u)}}^2 + 16/\sigma_d^2$. It can be shown that

$$\begin{aligned} \text{Tr}\{\mathbf{F}_{\bar{\mathbf{x}}}(L)\} &\propto \sum_{l=1}^{L-1} \Lambda_l^2 (a_l + 9b_l) \\ &+ \sum_{l=1}^{L-1} c_l (20\Lambda_l^2 \Delta_l^{-5} - 15\Lambda_l \Delta_l^{-4} + 20\Delta_l^{-5} + 3\Delta_l^{-3}), \end{aligned} \quad (35)$$

where $\Lambda_l = -\sum_{t=l+1}^L \Delta_t$, and we omit terms independent of Δ_l . Assuming the observation is taken periodically with $\Delta_l = \Delta$, it can be shown from (35) that $\text{Tr}\{\mathbf{F}_{\bar{\mathbf{x}}}(L)\}$ is given by

$$\begin{aligned} &\sum_{l=1}^{L-1} (a_{L-l} + 9b_{L-l}) l^2 \Delta^2 \\ &+ \sum_{l=1}^{L-1} c_{L-l} [(20l^2 + 15l + 3) \Delta^{-3} + 20\Delta^{-5}]. \end{aligned} \quad (36)$$

The first sum in (36) represents the information from measurements, and the second sum represents the information from the dynamic state update. When the inter-observation duration Δ is fixed, both sums in (36) increase with the number of observations L . However, when we are allowed to take only a fixed L number of observations, the inter-observation interval Δ should be carefully chosen to balance the contributions from the measurement and the state dynamics. Specifically, the first sum in (36) increases with the value of Δ because a larger Δ provides a longer integration time and hence results in measurements with better accuracy. However, the second sum decreases with the value of Δ , as less frequent observations result in error accumulation in the dynamic state model. By taking the first derivative of (36) w.r.t. Δ and setting to zero, the optimal Δ can be found numerically by solving $\psi_1 \Delta^7 - \psi_2 \Delta^2 - \psi_3 = 0$, where $\psi_1 = 2 \sum_{l=1}^{L-1} (a_{L-l} + 9b_{L-l}) l^2$, $\psi_2 = 3 \sum_{l=1}^{L-1} c_{L-l} (20l^2 + 15l + 3)$, $\psi_3 = 100 \sum_{l=1}^{L-1} c_{L-l}$.

IV. EXPERIMENTS AND SIMULATIONS

In order to evaluate the proposed measurement model (13) and the proposed algorithm (28), we have conducted extensive experiments using real life data. In this section, we present a set of experimental and simulation results to verify the correctness of our models and performance of our proposed algorithm.

A. Experiment setup

The Iridium satellites are used as SOOP beacons due to their wide coverage and easy availability. Specifically, we make use of the broadcast signal in the Iridium ring alert channel whose downlink frequency is fixed at 1626.270833 MHz. The ring alert signal is broadcast by all Iridium satellites periodically. A drawback is that the ring alert signal is broadcast only every 4.32 second, which is a relatively long period for tracking

Fig. 2. A receiver used in our experiment.

Fig. 3. Locations of receivers A and B, and the trajectory of receiver C where the discontinuity is due to driving in a tunnel.

a fast maneuvering receiver. More signals from the Iridium satellites, such as those from the downlink traffic channels and message channel, can be used to achieve better performance. In this experiment, we focused only on the ring alert signal as a proof of concept.

The receiver used is a software-defined radio (SDR) testbed. In our experiments, we used the USRP-N210 SDR with WBX daughterboard [52]. The USRP is connected to and controlled by a standard PC with large and fast storage space. Other components in one receiver set include one Iridium antenna, one low noise amplifier (LNA) ZHL-1217MLN, one standard DC power supply, and one portable battery. A complete receiver is shown in Figure 2.

Our experiment was conducted in Singapore with three sets of receivers. Two receivers, A and B, were static. Receiver A was placed on the rooftop of a building on the Nanyang Technological University (NTU) campus on the western side of Singapore, while receiver B was located near Changi Airport on the eastern side of Singapore. These two receivers are at a distance of approximately 36 km apart. The third receiver C is placed in a car which moved from Changi Airport

(a) One baseband packet at receiver *A*. (b) The amplitude of the CAF between two packets, one from *A*, another from *B* in the same observation time slot.

Fig. 4. Examples of detected packet and the cross-correlation function.

to NTU. The positions of receivers *A* and *B*, and the trajectory of receiver *C* are shown in Figure 3. A separate Trimble DSM212H GPS receiver is placed in the car together with receiver *C* to obtain GPS measurements as a benchmark for comparison.

All three USRPs in the receivers used free-running local oscillators without any form of synchronization between them. The USRPs are set to receive at the frequency 1626.27 MHz with a sampling rate of 10 MHz. To guarantee that all receivers see the same satellite, as stated in Assumption 1(iii), we pre-calibrate the laptop clock before the experiment to an accuracy of one second, and we use the laptop time to initiate the recording at each receiver. Generally, when no pre-calibrations are done, a burst match procedure can be performed using two received signals to search for common bursts from the same beacon, and the simplest method is thresholding the cross correlation coefficient between each pair of bursts in both signals.

Fig. 5. TDOA and FDOA benchmark and empirical values for the receiver pair *A* and *B*; bias is calculated by subtracting benchmark value from the estimation.

B. Validating measurement model (13)

To verify the measurement model (13), we collected data over a continuous period lasting about 48 minutes. In total 7 Iridium satellites were observed, and their trajectories are plotted in Figure 7(a). For any two receivers, their received signals are processed off-line in three steps:

- 1) Lowpass filtering: a lowpass filter with 100KHz bandwidth is used to remove signals from other bands, since the maximum Doppler shift induced in the received signal is 40KHz, and each channel has a bandwidth of 41.667KHz. An example of ring alert signal packet after lowpass filtering is shown in Figure 4(a).
- 2) Packet detection: a typical ring alert burst lasts for 7 ms with a period of 4.32 second. We divide the signal segment into non-overlapping intervals of 15 ms such that each interval cover one burst sufficiently. Within each interval, we compute the average power in a sliding window of length 7ms, and we declare a packet exists in this interval if the value is twice of the average power in the interval.
- 3) TDOA and FDOA estimation: with packets detected at both receivers, their perspective TDOA and FDOA is obtained by maximizing the amplitude of the complex ambiguity function (CAF) of the corresponding packets [50]. We declare a peak exists if the cross correlation coefficient is larger than 0.8. An example of the CAF between two packets is shown in Figure 4(b), where the TDOA and FDOA estimates correspond to the time and frequency value of the maximal point.

The benchmark positions of the receivers are obtained using GPS, and the velocity of receiver *C* is obtained by differentiating its benchmark position with time stamp from GPS. The positions and velocities of Iridium satellites are obtained using the SGP4 model [53], [23]. The benchmark TDOA and FDOA values are then computed using these values.

To validate the measurement model (13), we take the TDOA and FDOA measurements between receiver *A* and *B* as an illustrative example. In total, we detected 286 packets, which

(a) Figure: histogram of estimation errors

measurements	receivers	σ of estimation errors	σ given by CRB [50]
TDOA (ns)	A-B	415.72	304.17
	A-C	334.74	295.38
FDOA (Hz)	A-B	1.316	1.08
	A-C	23.992	1.05

(b) Table: standard deviation σ

Fig. 6. Statistics for TDOA/FDOA estimation errors; In order to compare with the benchmark value, the bias in the measurements are approximated using linear regression and have been removed.

give rise to 286 pairs of TDOA and FDOA estimates as shown in Figure 5. The bias is computed by subtracting the benchmark TDOA and FDOA values from the empirical values we have estimated. It can be seen that the biases in both TDOA and FDOA are linear, which show a good match with our measurement model (13). Using linear regression over the bias values, we obtain $\alpha = -0.012869$ km/s using the TDOA biases and $\alpha = -0.012855$ km/s using the FDOA biases. The difference between the estimated values for α is at the order of 10^{-5} and is due to the approximation in (12) as well as estimation errors.

C. TDOA and FDOA estimation errors

The histograms for TDOA and FDOA measurement errors are plotted in Figure 6(a) for two pairs of receivers, A-B and A-C. The corresponding standard deviation is given in Table 6(b). The estimation accuracies depend on quantities related to signal characteristics like effective SNR, integration time, and signal bandwidth [50]. The bandwidth and integration time are fixed for Iridium satellites as 25KHz and 7ms, respectively, while the packet SNR varies due to the wireless environment of receivers at different times. For the experiment results, the SNR values range from -3.74 dB to 43.02 dB for all three receivers. Using equations in [50] with empirical SNR values, we calculate the average standard deviation given by CRB, and the values are shown in Table 6(b). The receiver C moved in an urban canyon environment with trees and buildings around the express way, and its received signal experienced NLOS and multipath propagation, hence its corresponding estimation,

(a) trajectories of Iridiums detected by all receivers.

(b) PDOP using geometries in Figure 7(a)

Fig. 7. Receiver and beacon geometries in experiments; and the corresponding PDOP over time.

especially FDOA, suffers severe bias errors [24]. The receivers A and B have better sky view, but their received signals also contain reflections from surrounding buildings, like the lift shaft close to receiver A, which degraded the estimation performance.

When more channels from Iridium satellites are available so that we can integrate over arbitrary time duration, assuming the same satellite geometry and completely unknown process noise and height, it can be shown using (34) that it requires integrating over 0.122 seconds after 100 seconds of observation to achieve an estimation accuracy of 50 meters, and the requirement drops significantly to 7.015 ms after 332.67 seconds.

D. The effect of beacon geometry for localization

Localization error is known to be affected not only by the measurement quality but also by the beacon geometry. We

Fig. 8. The performance of the proposed algorithm for jointly estimating static receiver B 's location and relative clock parameters.

show the Iridium satellite trajectories in Figure 7(a). In a geographic coordinate system, each Iridium satellite moves in the direction from south to north, and they appear sequentially from east to west. Although only a single satellite was observed in most time slots, Iridium satellite has a speed of 7.4 km/s, and it takes about 9 minute for one satellite to move from horizon to horizon, hence their trajectories span over the sky and forms a set of “virtual” beacons in a short time duration, which lead to a feasible situation for the sequential estimation.

We further calculate the position dilution of precision using the satellite trajectories and receiver locations in Figure 7(a), and the results are shown in Figure 7(b). The position dilution of precision (PDOP) decreases to a value smaller than 1 after 331.2 seconds when altitude measurements are available, and it takes about 782.46 seconds when no altitude measurements are used.

E. Experiment results for localizing static receiver

We set the altitude measurement to be zero with standard deviation 30 meter, together with the estimated TDOA and FDOA, we apply our proposed algorithm in (28) to jointly estimate the receiver B 's location and its clock parameters, where receiver A is the anchor. The estimation results are shown in Figure 8. One important user-defined factor in the proposed algorithm is the value of the covariance matrix $\mathbf{R}_l$ for measurement noise and $\mathbf{Q}_l$ for state dynamics. In our implementation, we set $\mathbf{R}_l$ to be a diagonal matrix with its components following from the standard deviation given CRB. Specifically, we use minimal SNR value -3.74 dB, and the sigmas for TDOA and FDOA are $1.8 \mu\text{s}$ and 6.75 Hz, respectively. We set $\sigma_\Omega = 10^{-10}$, $\sigma_\beta = 10^{-9}$, and $\sigma_d = 10^{-8}$ for $\mathbf{Q}_c^{(l)}$ in (21) according to the data-sheet of USRP-N210 [52]. Since the receiver B is static, it has no acceleration,

(a) Performance for tracking relative clock parameters in one simulation run.

(b) Trace for tracking the receiver's movement in one simulation run.

(c) RMSE for tracking the receiver clock parameters

(d) RMSE for tracking the receiver state

Fig. 9. The performance of the proposed algorithm for jointly tracking a receiver moving with constant velocity and its relative clock parameters.

therefore we let $\sigma_a = 10^{-10}$ for $\mathbf{Q}_s^{(l)}$ in (23).

For all experiment and simulation cases, we set the initial position to be 0.3° off to the true position in both latitude and longitude, and the deviation in altitude is 10 meters. The deviation of initial position is 4.706 km. We also set the initial velocity to be zero, and the initial clock parameter to be 10% off to the true value. The initial $\bar{\mathbf{P}}_0$ is set to be a diagonal matrix with elements $[1, 0.1, 0.01, 0.001]$ corresponding to uncertainty in position, velocity, clock offset, and clock skew, respectively.

Figure 8(a) shows the results for tracking the relative clock parameters between the two receivers A and B using our proposed algorithm in (28). In the left column, we show the empirical relative clock parameters found by subtracting the benchmark TDOA and FDOA values from the empirical values we have estimated, and the tracked relative clock parameters

$\alpha^{(l)}$ and $\theta^{(l)}$ over a period of 48 minutes. It can be seen that our algorithm can track the relative clock parameters closely. In the lower-left subfigure, the empirical clock skew is at the order of 10ns/s, which conforms to the 0.01ppm specified in the USRP-N210 data-sheet [52]. As the clock skew increases slightly over time, it results in a second order behavior in the clock offset drifting, as shown in the upper-left subfigure. The more detailed results and its comparison with CRB are shown in Figure 8(b), where the estimation error for clock parameters are calculated by subtracting the tracked relative clock parameters from their empirical values.

In Figure 8(c), we show the tracked locations of receiver B over time. It can be seen that as more Iridium satellite signals are observed, our estimates converge to the true location. At time $t = 331.46$ second, our algorithm achieves an estimation error smaller than 50 meters, and the error is constantly smaller

Fig. 10. The performance of the proposed algorithm for tracking the manoeuvring receiver C .

than 50 meters after $t = 540$ second. The final estimation error at time $t = 2887.8$ seconds was 29.1 meters. The more detailed results are shown in Figure 8(d), where we have used the CRB calculated by (29) as a benchmark (in dashed line). The estimation error for positions are calculated w.r.t. the true location, and that for the relative clock parameters are calculated w.r.t. the empirical relative clock parameters.

F. Simulation results for tracking receiver with constant velocity

In this subsection, to demonstrate the performance of our proposed algorithm (28) for tracking a mobile receiver moving with constant velocity, we conduct simulations instead of experiments because of the difficulty of finding a sufficiently long and straight path in Singapore to conduct our experiments. In the next subsection, we show experimental results for a receiver moving with non-constant velocity. In our simulations, the receiver moves with a constant velocity of $[5; 1; -6]$ m/s and zero acceleration. The value is chosen so that it describes the average velocity of a ship moving along a straight line in open waters. For the simplicity of simulations, we assume the receiver moving in a fixed direction in ECEF coordinate system. The receiver A is used as an anchor. The clock offset and skew are simulated as a Gaussian random variables with zero mean and standard deviation 10^{-8} , and their values follow the linear model in (20). The measurement noises are simulated as Gaussian random variables with zero mean and standard deviation 10^{-6} second for TDOA, 10 Hz for FDOA, and 30 meters for altitude.

The results for tracking the receiver states are shown in Figure 9, and the root-mean squared error (RMSE) is averaged over 1000 simulation runs. It can be seen that our proposed algorithm achieves satisfactory performance in jointly tracking the receiver location, velocity and its relative clock parameters.

Specifically, the estimated relative clock skews $\alpha^{(l)}$ and clock offsets $\theta^{(l)}$ have zero mean errors, as shown in Figure 9(a). The receiver position is tracked with RMSE smaller than 100 m and its velocity tracked with RMSE smaller than 0.5 m/s. In Figure 9(c) and Figure 9(d), it shows that the performance for estimating clocks parameters, position, and velocity all approach their respective CRBs.

G. Experiment results for tracking manoeuvring receiver

We set the altitude measurements to be a constant equal to the initial height 42.8 meters, and the corresponding standard deviation is set to be 30 meters. Together with TDOA and FDOA measurements, we apply the proposed algorithm (28) to track receiver C w.r.t. receiver A . The results are shown in Figure 10. As observed in our experiment results, one key factor that affects the tracking performance is the correctness of the dynamic state model (24). For the static case in Section IV-E, the dynamic model (22) is simplified with $\mathbf{v}^{(l)} = \mathbf{0}$ and the acceleration acts like a white noise term with zero mean and standard deviation 10^{-10} km/s². When the receiver is moving with constant velocity as in Section IV-F, the dynamic model (22) still provides a good match with a white-noise-like acceleration. However, when the receiver is manoeuvring, e.g., taking turns and changing directions like receiver C in our experiment, the dynamic model (22) cannot accurately characterize the movement of the receiver especially when it is taking a sharp turn. In addition, as discussed in Section IV-A, the infrequent ring alert bursts limit the number of measurements. Therefore, the estimation error accumulated from previous time slots cannot be corrected timely during the manoeuvrings, resulting in larger errors, as is clear from parts of the trace in Figure 10(b). However, if the receiver C is moving with relatively constant velocity, the algorithm is able to reduce its tracking errors, as seen in the middle part

of the trajectory in Figure 10(b), where the minimum RMSE achieved is 51.94 meter.

V. CONCLUSION

In this paper, we have investigated the problem of joint localization and synchronization using SOOP. We assume two receivers have free-running oscillators, and we assume all beacons are unsynchronized with unknown clock offsets. Considering a general scenario where an anchor exists to share the received signal and the state information with the target receiver, we analysed the biases introduced by asynchronous clocks in the received signal, we derived a closed-form expression (11) for TDOA and FDOA measurements, and we proposed a sequential algorithm to jointly estimate the receiver state and its clock parameters. Receivers were implemented on SDR testbed, and field experiments were conducted with Iridium satellites as SOOP beacons. Experimental results showed that our measurement model for TDOA and FDOA correctly describes the relationship between biases and clock parameters, and our proposed algorithm can track the receiver state and clock parameters with an accuracy less than 100 meters when the dynamic model correctly characterize the change of the receiver state. In the static case, the algorithm converges to an error smaller than 50 meters within 540 seconds, and the error was 29.1 meters after 2887 seconds. For the future work, the fusion of IMU measurements and multiple beacon sources can be investigated to improve the algorithm robustness against dynamic model mismatch.

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Mei Leng received the B.Eng. degree from University of Electronic Science and Technology of China (UESTC) in 2005 and Ph.D. degree from The University of Hong Kong in 2011. She was a research fellow at the School of Electrical and Electronic Engineering, Nanyang Technological University from 2011 to 2014. She is currently a research scientist at TL@NTU. Her current research interests include tracking and navigation algorithm design and implementation, cooperative and distributed algorithms, statistical signal processing and machine learning

with applications to wireless sensor networks and wireless communication systems.

François Quitin is an Assistant Professor at the Université libre de Bruxelles (ULB), Belgium. He received the Ph.D. degree in Electrical Engineering from the Université Libre de Bruxelles (ULB), Brussels, Belgium and from the Université catholique de Louvain (UCL), Louvain-La-Neuve, Belgium in 2011. From 2011 to 2013, he was a post-doctoral researcher at the University of California, Santa Barbara (UCSB), and from 2013 to 2015, he was a postdoctoral research fellow at Nanyang Technological University (NTU), Singapore. He is the recipient of the 2012 Alcatel-Lucent Bell Scientific Award. His research interests focus on experimental and prototyping aspects in wireless communications, taking advanced theoretical ideas all the way to practice.

Wee Peng Tay (S'06 M'08 SM'14) received the B.S. degree in Electrical Engineering and Mathematics, and the M.S. degree in Electrical Engineering from Stanford University, Stanford, CA, USA, in 2002. He received the Ph.D. degree in Electrical Engineering and Computer Science from the Massachusetts Institute of Technology, Cambridge, MA, USA, in 2008. He is currently an Assistant Professor in the School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore. His research interests include distributed detection and estimation, distributed signal processing, sensor networks, social networks, information theory, and applied probability.

Dr. Tay received the Singapore Technologies Scholarship in 1998, the Stanford University President's Award in 1999, the Frederick Emmons Terman Engineering Scholastic Award in 2002, and the Tan Chin Tuan Exchange Fellowship in 2015. He is the coauthor of the best student paper award at the Asilomar conference on Signals, Systems, and Computers in 2012. He is currently an Associate Editor for the IEEE Transactions on Signal Processing, serves on the MLSP TC of the IEEE Signal Processing Society, and is the chair of DSNIG in IEEE MMTC. He has also served as a technical program committee member for various international conferences.

Chi Cheng received the B.Eng. degree in Electrical and Electronic Engineering with First Class Honours from Nanyang Technological University, Singapore in 2011. He obtained the Master degree in Electrical and Electronic Engineering from Nanyang Technological University, Singapore in 2015. He is currently a Software Engineer at Works Applications. His current interest is building cloud-based team collaboration tool.

Sirajudeen Gulam Razul received his B.Eng and M.Eng in Electrical and Electronics Engineering from the Nanyang Technological University, Singapore in 1997 and 2000 respectively. He received his Ph.D from the University of Cambridge, UK in 2003. His Ph.D work was on Bayesian estimation using Markov Chain Monte Carlo techniques. He was a faculty member in the School of EEE, Nanyang Technological University from 2004 to 2009. He is currently a Senior Research Scientist with TL@NTU. His research interests include

Bayesian signal processing, statistical signal processing and array processing.

Chong Meng Samson See received the Diploma in Electronics and Communications Engineering (with merit) from Singapore Polytechnic in 1988 and the MSc in Digital Communication Systems and the PhD in Electrical Engineering both from Loughborough University of Technology, Loughborough, U.K., in 1991 and 1999, respectively.

Since 1992, he has been with DSO National Laboratories, Singapore, where he is now a Distinguished Member of Technical Staff and is currently leading a team in the research and development of advanced array signal processing systems and algorithms. He also holds an adjunct appointment at TL@NTU as a Principal Research Scientist.

His research interests include the area of statistical and array signal processing, communications, and bio-inspired systems. He has two issued patents on direction finding. Chong Meng is a Member of IEEE and was Associate Editor of IEEE Transactions on Signal Processing and currently member of the IEEE Sensor Array and Multichannel Signal Processing Technical Committee.