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Design, Analysis and Verification of Hexagon Split Ring Resonator based Negative Index Metamaterial

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Abstract — This paper proposes the design of a Hexagon Split Ring Resonator (HSRR) with skew angle rotation between inner and outer ring. Mathematical expressions for the resonant frequency of HSRR are formulated. The CAD model of the HSRR is analyzed for electric and magnetic fields in a finite element (FEM) method mode solver based electromagnetic simulator and its scattering parameters are obtained. The dips and crossings in the S-parameters are used to plot the refractive index versus frequency, which verifies its Negative Refractive Index Material (NRIM) property in certain frequency bands.

Keywords — CAD Electromagnetic simulation, Finite element method (FEM) solver, Hexagon Split Ring Resonator (HSRR), Metamaterials, Negative Refractive Index Material (NRIM).

I. INTRODUCTION

METAMATERIALS has witnessed a growing research interest in recent years. Metamaterials are defined as artificial materials having the ability to exhibit electromagnetic characteristics not readily found in naturally occurring material, such as, negative refractive index and artificial magnetism [1-3]. Metamaterials are often characterized in terms of their electric permittivity and magnetic permeability [4]. Either or both of these parameters may be negative. The former is known as single negative material (SNG) [5], while the latter as double negative (DNG) media or negative refractive index material (NRIM) [5]. Effective negative permittivity can be obtained in artificial plasmas for all frequencies smaller than plasma frequency of the Plasmon medium [6]. Effective negative permeability can be obtained in the well known Split-ring-resonator (SRR) structure, but only for a narrow magnetic resonant frequency band [7]. New innovative structures are being reported showing performance improvement in terms of size, bandwidth and ease of fabrication [8]. However, most of the structures incorporate Square SRR or Circular SRR, with plenty of literature available [9,10]. Rarer structures include Triangular SRR [8], Elliptical SRR [11,12] and generalized polygon SRR [13,14]. This paper presents the design, finite element method (FEM) mode solver based electromagnetic simulation [18] and analysis of a new Hexagon SRR,

exhibiting enhanced NRIM property. To the best of the knowledge of the authors, this work of designing and analyzing the HSRR is novel in itself.

The rest of the paper is organized as follows: *Section 2* presents the geometry and lays down the mathematical formulations for resonant frequency computation. *Section 3* evaluates the performance of the HSRR for varying skew rotation in terms of the electric and magnetic field, scattering parameters and refractive index. *Section 4* discusses the reasons for the variations and identifies the result of the experiments. *Section 5* concludes the paper.

II. MATHEMATICAL FORMULATION

A. HSRR Geometry

Figure 1.1 (a) shows the schematic geometry of a HSRR with dimensions indicated. Figure 1.1 (b) shows the equivalent circuit model of the HSRR, forming an L-C network. The inductance is due to the gap between the rings and the capacitance is due to the rings and the gaps in the rings itself.

Fig.1.1 (a) Figure of a HSRR; Fig.1.1 (b) Equivalent L-C Circuit

B. Resonant Frequency Computation

With a magnetic field applied along the z-axis, an electromotive force appears around the HSRR which makes the structure behaves like an L-C network [15] having resonant frequency, f_o , expressed as:

$$f_o = \frac{1}{2\pi\sqrt{2a_{eq}L_{net}C_{net}}} \quad (1)$$

where a_{eq} is the effective radius, L_{net} is the net inductance and C_{net} is the net effective capacitance of the equivalent L-C network of the HSRR.

Applying general trigonometry to Figure 1.1 (a) and assuming $g_1 = g_2 = g$, we estimate the effective 'a' (a_{eq}), as done in [9,10], by equating it to any one side of the hexagon, which falls short of 'a' by $g/6$. Thus we have a_{eq} as :

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$$a_{eq} = a - \frac{g}{6} \quad (2)$$

The net inductance, L_{net} can be analytically expressed [16] as:

$$L_{net} = 0.00508 \cdot l \left(2.303 \log_{10} \frac{4l}{\rho} - 2.636 \right) \quad (3)$$

where $l = 6 \times a_{eq}$ is the effective perimeter of the HSRR; and ρ is the width of the cross-section of the conductor ($= c$).

The net capacitance, C_{net} can be analytically extended for HSRR from the expression of a generalized polygon [13,14] :

$$C_{net} = \left\{ \frac{(3 + \beta)^2 - \left(\frac{\Delta}{a}\right)^2}{2 \cdot (3 + \beta)} \right\} \times a \cdot C_{pul} \quad (4)$$

where $\beta = \frac{C_g}{a \cdot C_{pul}}$; and C_g is the capacitance due to the gaps (splits), estimated using parallel plate approximation method, while assuming $g_1 = g_2 = g$, as:

$$C_g = \frac{\epsilon_0 \cdot \epsilon_r \cdot c \cdot h}{g} \quad (5)$$

and C_{pul} is the capacitance per unit length of the HSRR, which is estimated as:

$$C_{pul} = \epsilon_0 \cdot \left(\frac{\epsilon_r + 1}{2} \right) \cdot \frac{\mathcal{E}(\sqrt{1 - \sigma^2})}{\mathcal{E}(\sigma)} \quad (6)$$

where $\sigma = \frac{d}{d+2c}$ and $\mathcal{E}(\cdot)$ is the complete elliptical integral of the second kind, defined as $\mathcal{E}(k) = \int_0^{\pi/2} \sqrt{1 - (k \sin \theta)^2} d\theta$

The Δ in equation (4) is the measure by which the perimeter of the upper half-ring decreases and that of lower half-ring increases (for a counter-clockwise θ), and is derived as:

$$\Delta = a \cdot \sin\left\{\frac{\pi}{6}\right\} \cdot (2m + 1) - a \cdot \cos\left\{\frac{\pi}{6}\right\} \cdot \tan\left\{\frac{\pi}{6} - \psi\right\} \quad (7)$$

where m and ψ are such that

$$\theta = m \cdot \frac{2\pi}{6} + \psi \quad (8)$$

with m being some integer and $\psi < \frac{2\pi}{6}$.

Thus using equation (2), (3) and (4) in equation (1), one can determine the resonant frequency of the HSRR. This is a special case of the author's previous works [13, 14].

III. PERFORMANCE EVALUATION

The CAD model of the HSRR was designed and simulated in a commercially available FEM based EM simulation tool [18] for various skew rotation angle, $\theta = 0^\circ, 60^\circ, 100^\circ, 120^\circ$. The electric and magnetic fields variations for varying θ was obtained. The variations in the scattering parameter (S_{11} & S_{21}) were plotted against a frequency sweep. The cross-cuttings in the plots of S_{11} and S_{21} distinctly exhibit the

negative electrical permittivity and magnetic permeability of the HSRR. Equation (9) helps us compute the refractive index in terms of the reflection and transmission coefficient [11, 17].

$$n = \frac{1}{kdj} \cosh^{-1} \left\{ \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} \right\} \quad (9)$$

where n is the refractive index, kd is the electrical path length, S_{11} is the reflection coefficient and S_{21} is the transmission coefficient.

The other physical parameters in the CAD model were taken to be as tabulated in Table 1.

Table 1 Values of Physical Parameters in CAD design of HSRR

a	1.95 mm	d	0.18 mm
c	0.15 mm	g	0.11 mm
h	0.39 mm	ϵ_r	2.15

A. Analysis for $\theta = 0^\circ$

Figure 2. shows the electric field strength pattern for $\theta = 0^\circ$, with magnitudes mentioned aside.

Figure 2. Electric Field Strength Pattern for $\theta = 0^\circ$

Figure 3. shows the magnetic field strength pattern for $\theta = 0^\circ$, with magnitudes mentioned aside.

Figure 3. Magnetic Field Strength Pattern for $\theta = 0^\circ$

Figure 4. shows the scattering parameter plot for $\theta = 0^\circ$. Figure 4. Scattering Parameter Plot for $\theta = 0^\circ$ Figure 5. shows the refractive index plot for $\theta = 0^\circ$. Figure 5. Refractive Index Plot for $\theta = 0^\circ$

B. Analysis for $\theta = 60^\circ$

Figure 6. shows the electric field strength pattern for $\theta = 60^\circ$, with magnitudes mentioned aside. Figure 6. Electric Field Strength Pattern for $\theta = 60^\circ$ Figure 7. shows the magnetic field strength pattern for $\theta = 60^\circ$, with magnitudes mentioned aside. Figure 7. Magnetic Field Strength Pattern for $\theta = 60^\circ$ Figure 8. shows the scattering parameter plot for $\theta = 60^\circ$. Figure 8. Scattering Parameter Plot for $\theta = 60^\circ$ Figure 9. shows the refractive index plot for $\theta = 60^\circ$. Figure 9. Refractive Index Plot for $\theta = 60^\circ$

C. Analysis for $\theta = 100^\circ$

Figure 10. shows the electric field strength pattern for $\theta = 100^\circ$, with magnitudes mentioned aside.

Figure 10. Electric Field Strength Pattern for $\theta = 100^\circ$

Figure 11. shows the magnetic field strength pattern for $\theta = 100^\circ$, with magnitudes mentioned aside.

Figure 11. Magnetic Field Strength Pattern for $\theta = 100^\circ$

Figure 12. shows the scattering parameter plot for $\theta = 100^\circ$.

Figure 12. Scattering Parameter Plot for $\theta = 100^\circ$

D. Analysis for $\theta = 120^\circ$

Figure 13. shows the electric field strength pattern for $\theta = 120^\circ$, with magnitudes mentioned aside.

Figure 13. Electric Field Strength Pattern for $\theta = 120^\circ$

Figure 14. shows the magnetic field strength pattern for $\theta = 120^\circ$, with magnitudes mentioned aside.

Figure 14. Magnetic Field Strength Pattern for $\theta = 120^\circ$

Figure 15. shows the scattering parameter plot for $\theta = 120^\circ$.

Figure 15. Scattering Parameter Plot for $\theta = 120^\circ$

Figure 16. shows the refractive index plot for $\theta = 120^\circ$. Figure 16. Refractive Index Plot for $\theta = 120^\circ$

IV. RESULTS AND DISCUSSIONS

The HSSR structure was analyzed mathematically and a closed form expression for the resonant frequency was derived considering its equivalent inductance and capacitance. Evidently, its resonant frequency and bandwidth of operation were found to be a strong function of its geometrical and electrical properties. The CAD model was simulated on a commercially available electromagnetic simulator [18] based on the finite element method electromagnetic mode solver.

The electric and magnetic field strength patterns were obtained for various θ , with clear distinction between them, indicating the associated changes thereby. The scattering parameters plots shows that for small θ the HSRR has a principal resonant frequency, while for larger θ it might have a secondary resonant frequency as well. This property is useful for a designer to introduce multiple resonant bands by varying θ . The changing θ gives us two associated parameters, viz. 'm' and ' ψ ', from equation (8). The 'm' can be used for coarse adjustments of the resonant frequency, as it has a linear relation with Δ (equation (7)); while the ' ψ ' can be used for relatively finer adjustments. The cross-cuttings in the scattering parameters plot give us an indication of the NRIM property of the HSRR, which was further validated by computing the values of its refractive index using equation (9). The refractive index plots of the HSRR for different θ clearly shows the enhanced NRIM property of the HSRR.

V. CONCLUSION

A new design of the Hexagon Split Ring Resonator (HSRR) is demonstrated with associated mathematical analysis and derivation of a closed form expression of its primary resonant frequency, which is a strong function of its geometrical and electrical parameters.

Finite Element Method (FEM) mode solver based Electro-

magnetic (EM) simulation of the HSRR showed the associated electric and magnetic field strength patterns. The scattering parameter plots gives the designer a tweaking tool of varying the θ to obtain an HSRR exhibiting multiple resonant bands at frequencies of interest. The refractive index plot of the HSRR demonstrates its enhanced NRIM property. This type of HSRR can be easily incorporated with microstrip antennas to get highly directional beam patterns because of the enhanced NRIM property of HSRR.

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