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Lowering the potential barrier of a bistable energy harvester with mechanically rectified motion of an auxiliary magnet oscillator

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In this study, we propose a bistable energy harvester with an auxiliary magnet oscillator (BEH-O), whose motion is mechanically rectified to enhance its broadband energy harvesting performance. The design of the proposed BEH-O system is based on local modification of the double-well potential. The auxiliary oscillator with the mechanical rectifier is designed in such a manner that its rectified half-sine motion tends to reduce the saddle barrier of restoring potential, leading to a possibility of an easier escape from the potential well, while keeping other parts of the potential, thereby maintaining high-energy orbital motion. Consequently, the lower bound of the operating frequency band of the BEH-O is reduced, when compared to its conventional counterpart, i.e., conventional bistable energy harvester (CBEH), while its upper bound remains unchanged. Such a broader frequency band of the BEH-O can be directly compared with that of the CBEH, as there is little difference in the high-energy orbital motion between the two. This beneficial effect of the BEH-O system is theoretically and experimentally supported by bifurcation analyses and frequency response analyses. *Published by AIP Publishing.* <https://doi.org/10.1063/1.4994111>

A bistable electromechanical oscillator has been considered as a promising structure of a vibration energy harvester for broadband energy harvesting.^{1,2} The main characteristic of a bistable oscillator is the double-well configuration of the potential energy function. The two potential wells of the bistable oscillator are separated by a potential saddle barrier. This potential energy function produces two distinct dynamic motions of a bistable energy harvester (BEH), intrawell and interwell motions, depending on the excitation intensity.^{1,2} When the ambient excitation is weak, the oscillation of a BEH exhibits small-amplitude intrawell motion or low-energy orbit motion, confined to either of the two potential wells by a potential saddle barrier. On the other hand, when the excitation is sufficiently strong for inducing the potential well escape phenomenon,^{1,2} the BEH achieves large-amplitude interwell motion or high-energy orbit motion, oscillating across the two potential wells in a broad frequency band. From this interwell motion, the BEHs extract electrical energy, using energy conversion mechanisms such as piezoelectric, electrostatic, and electromagnetic conversions,^{2,3} which outperform linear and nonlinear monostable energy harvesters³⁻⁹ in terms of broadband energy harvesting.

In the past decade, a variety of bistable energy harvesters (BEHs) were developed.¹⁰⁻²⁰ Refer to Ref. 2 for more information on various types of BEHs and their applications. As the performance of BEHs is contingent on the occurrence of high-energy orbital motion, the majority of theoretical and experimental research studies were focused mainly on examining and characterizing the dependence of the frequency

bandwidth of the interwell motion on the type and intensity of ambient vibration sources (e.g., swept sine,^{7,12} impulse,¹⁰ and Gaussian white noise^{14,15}) and design parameters affecting the potential well configuration (e.g., magnetic field or structural stiffness^{18,19}). Recent research efforts were devoted to either further enhancing the operating frequency bandwidth of a BEH or reducing the threshold excitation intensity required for triggering interwell motion. Multistable (tri- or quad-stable) energy harvesters²¹⁻²⁶ were proposed to lower potential saddle barriers and were shown to be favorable for energy harvesting, particularly from low-intensity ambient vibration. Multimodal or multiple degree-of-freedom (DOF) systems of BEHs¹⁸⁻²⁰ were reported to be suitable for multidirectional vibration sources, and their high-order resonances provided additional operating frequency bands for energy harvesting.

However, most of the energy harvesting systems mentioned above¹⁸⁻²⁵ inevitably produce distortion of high-energy orbital motion and/or undesirable changes of its frequency band, which adversely affect the efficiency of energy harvesting and/or make evaluating the performance of the energy harvesting system more difficult. This study proposes a BEH system with an auxiliary magnet oscillator (referred to as a BEH-O in this study) to achieve energy harvesting performance in a broader frequency band than conventional BEHs (CBEHs). In particular, the motion of the auxiliary oscillator in the proposed BEH-O system is mechanically rectified to prevent any severe distortion of high-energy orbital motion or large changes in its frequency band.

The BEH-O, shown in Fig. 1, is composed of a bimorph piezoelectric beam oscillator, with a permanent magnet at its free end, and an external magnet oscillator. The two identical permanent magnets face each other with the same polarities,

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and thus, a repulsive magnetic interaction acts as an axial force on the beam and external magnet oscillators, imposing bistability on the system. Unlike CBEHs,^{10–20} the external magnet of the BEH-O is not fixed in space but movable. As the tip magnet approaches its trivial position [at $w = 0$ in Fig. 1(a)], it drives the external magnet away from the equilibrium state to the right by magnetic repulsion. This motion of the external magnet oscillator results in weak magnetic repulsion around the trivial position of the bimorph oscillator (when compared with the CBEH with a fixed external magnet), reducing the potential saddle barrier, thus enabling easier occurrence of high-energy orbital motion of the bimorph oscillator in a broader frequency band. Conversely, as the tip magnet moves far away from the trivial position after the potential well escape phenomenon occurs, the external magnet tends to return to the equilibrium position. A mechanical rectifier prevents the external magnet from moving past the equilibrium state to the left; therefore, the distortion of high-energy orbital motion can be minimized. Thus, the design of the BEH-O provides a broader operating frequency band in comparison to CBEHs, without deteriorating the efficiency of energy harvesting. In this study, the geometric and material properties of the BEH-O were chosen arbitrarily, within realistic ranges, to demonstrate the operation mechanism of the proposed BEH-O system. The detailed parameter values are listed in Table S1 in the [supplementary material](#). A simple comparison of numerical and experimental results is also given as Fig. S1 ([supplementary material](#)) to support the validity of these parameter values.

A mathematical model of the bimorph piezoelectric beam is established on the basis of Euler–Bernoulli beam theory and linear piezoelectricity.¹⁷ The external magnet oscillator is modeled as a simple mass-damper-spring system. The magnetic charge model^{21,23} is employed to describe the two permanent magnets, and the resulting magnetic coupling forces are considered as external forces exerted at the tip of the beam and the center of mass of the external magnet oscillator in the model. The governing equation of the beam

is discretized using the Galerkin projection method. After a series of discretization process,^{21,23} the magneto-electro-mechanical oscillator model of the BEH-O can be expressed as

$$\ddot{w} + 2\zeta_b\omega_b\dot{w} + \omega_b^2w - \kappa V - \gamma_b F_b = -f \sin \Omega t, \quad (1a)$$

$$\dot{V} + 2R^{-1}C^{-1}V + 2C^{-1}\kappa\dot{w} = 0, \quad (1b)$$

$$\ddot{x} + 2\zeta_e\omega_e\dot{x} + \omega_e^2x + \gamma_e(F_e + F_c) = 0, \quad (1c)$$

where w and x are the tip deflection of the beam and the displacement of the external magnet oscillator, respectively; ζ and ω are the damping ratio and the natural frequency, respectively, and their subscripts b and e denote the quantities for the beam and external magnet oscillators, respectively; V is the output voltage across the electrical resistance R ; κ and C are the electromechanical coupling constant and the equivalent capacitance, respectively; F_b and F_e are the magnetic coupling forces; and f and Ω are the amplitude and frequency of the base acceleration, respectively. The magnetic coupling forces are expressed in the following forms:

$$F_b = \frac{\mu Q Q'}{4\pi} \sum_{j,k=1,2} \frac{(-1)^{j+k} \alpha^{(k)2} w}{((x+d+\beta^{(j,k)})^2 + \alpha^{(k)2} w^2)^{3/2}}, \quad (2a)$$

$$F_e = \frac{\mu Q Q'}{4\pi} \sum_{j,k=1,2} \frac{(-1)^{j+k} (x+d+\beta^{(j,k)})}{((x+d+\beta^{(j,k)})^2 + \alpha^{(k)2} w^2)^{3/2}}, \quad (2b)$$

where $\alpha^{(k)} = 1 + (l_t + (-1)^k l_m)\eta$; $\beta^{(j,k)} = (-1)^j l_m - (-1)^k l_m$; μ is the permeability of free space; Q and Q' are the magnetic surface charges of the tip magnet and external magnet, respectively; l_m is the half-length of the magnet; l_t is the distance from the tip of the beam to the mass center of the tip magnet; and η is a constant coefficient. Furthermore, to account for the operation of the mechanical rectifier during the oscillation of the system, the following piecewise function F_c that includes contact restoring and damping forces was introduced into Eq. (1c):

$$F_c = \gamma_e^{-1} (2\zeta_e\omega_e\dot{x} + \omega_e^2(x - \bar{x}))H(-x + \bar{x}), \quad (3)$$

where ζ_e and ω_e are the damping ratio and natural frequency involved during the contact of the mechanical rectifier and the external magnet, respectively; $\bar{x}$ is the equilibrium position of the external magnet; and H denotes the Heaviside function. The mathematical model for a CBEH to be compared with the present BEH-O can be obtained as a limiting case of Eqs. (1a)–(1c), when the displacement of the external magnet oscillator is fixed at its equilibrium position, i.e., $x \rightarrow \bar{x}$.

For the BEH-O, the tip magnet travels in the varying magnetic field generated by the movable external magnet. Thus, the repulsive magnetic forces are functions of the relative motion between the two magnets, and accordingly, the potential well configuration of the BEH-O cannot be easily evaluated. The potential energy function of the BEH-O, $U(w, x)$, forms a three-dimensional potential energy surface, on which a dynamic trajectory determines the configuration of the actual potential well experienced by the BEH-O during its oscillation. In this study, to evaluate such potential

FIG. 1. (a) Schematic of the BEH-O system and (b) its experimental prototype. In this figure, the two identical magnets are located at their static equilibrium states. A mechanical rectifier allows the external magnet to move from the equilibrium to the right but prevents it from moving past the equilibrium to the left.

well configurations, the steady-state response of the BEH-O is first obtained using numerical integration of Eqs. (1a)–(1c), and then, the dynamic path on the potential energy surface is calculated using the following equation:

$$U = \int F_{R1}(w, x)dw + F_{R2}(w, x)dx, \quad (4)$$

where F_{R1} and F_{R2} are the restoring forces acting on the bimorph oscillator and the external magnet oscillator, respectively.

Figure 2 shows (a) the interwell motion of the BEH-O, (b) the associated restoring force response, and (c) the dynamic potential well configuration, which is obtained numerically to demonstrate the operation of the auxiliary oscillator with the mechanical rectifier in the BEH-O. In Figs. 2(a)–2(c), the solid and dotted blue lines denote two portions of positive and negative velocities, respectively, for each response of the BEH-O. The results of the CBEH (red lines) are also provided for comparison. The values of ω_e , ω_c , ζ_e , and ζ_c used in this simulation are 130 rad/s, 110 rad/s, 0.15, and 0.45, respectively. Particularly, for ω_c , ζ_e , and ζ_c , except ω_e , the above values were consistently used for all of the simulations conducted in Figs. 2–4. As the mechanical rectifier is fixed at the equilibrium position of the external magnet, the motion of the external magnet takes a sort of a rectified half-sine oscillation rather than a simple harmonic oscillation, as shown in Fig. 2(a). The rectified motion of the external magnet [the green line in Fig. 2(a)] becomes magnified with relatively strong magnetic repulsion when the tip magnet [the blue line in Fig. 2(a)] passes through the region of small displacement around the trivial position (i.e., when two magnets are close to each other). However, it becomes smaller with weaker magnetic repulsion when the tip magnet passes into the region of large displacement (i.e., when two magnets are far away from each other). Accordingly, the resistance of the restoring force that the BEH-O experiences while crossing the potential saddle [areas A and B in Fig.

FIG. 2. (a) Stationary interwell motion of the BEH-O obtained numerically with a base acceleration of 0.6 g and a frequency of 11 Hz, (b) the associated restoring force responses, and (c) the resulting potential well configurations with respect to the deflection of the beam.

FIG. 3. Swept sine responses for the tip deflections of (a) BEH-O and (b) CBEH obtained with a base acceleration of 0.52 g. The forward and backward sweep responses are denoted by the blue and red colors, respectively.

2(b)] is reduced without significant distortion of interwell motion [as shown in Fig. 2(a) by the blue and red lines for the BEH-O and CBEH, respectively]. In Fig. 2(c), the resulting asymmetric potential well configuration of the BEH-O exhibits a lower potential saddle barrier when compared with that of the CBEH, which is beneficial to triggering interwell motion in a broader frequency band.

Figure 3 shows the swept sine responses for the tip deflections of (a) BEH-O and (b) CBEH (refer to Fig. S2 in the [supplementary material](#) for the output voltage responses associated with Fig. 3). A negative static equilibrium state was used as an initial condition for the numerical simulation. In this simulation, the value of ω_e was set as 103 rad/s. In Figs. 3(a) and 3(b), the maximum displacements of the periodic responses and the Poincaré sections of the chaotic

FIG. 4. Variations in (a) the lower and upper bounds of the operating frequency band and (b) the frequency bandwidth with respect to base acceleration.

motions are represented. The forward and backward sweep responses are denoted by the blue and red colors, respectively. The saddle-node bifurcations of intrawell and interwell motions are designated as points SN1 and SN2, respectively. In Fig. 3(b), the dashed line indicates the interwell motions of the CBEH that do not appear in this swept sine response but coexist with the chaotic or intrawell motion. For the forward sweep result of the BEH-O [denoted by blue color in Fig. 3(a)], the intrawell motion jumps up to the large-amplitude interwell motion through its saddle-node bifurcation [point SN1 in Fig. 3(a)]. The SN1 bifurcation occurs at a frequency where a chaotic attractor does not exist, and thereafter, the interwell motion is unique in a certain frequency band. In the case of the forward sweep, this frequency band is the only route to high-energy orbital motion without any external perturbation. The initiated interwell motion increases with the excitation frequency and reduces with the chaotic motion to the intrawell motion after its saddle-node bifurcation [point SN2 in Fig. 3(a)]. It is worth noting that the interwell motion of bistable systems typically belongs to a hardening-type frequency response.^{1–3,10–20} The branch of the interwell motion may not be continuously developed if the system loses its consistent hardening characteristics, implying that it will fail to harvest vibrational energy in a broad frequency band. This situation could occur if the mechanical rectifier was not used in the BEH-O, leading to a large distortion of interwell motion. It was observed that with the mechanical rectifier in use, the interwell motion branches of the BEH-O and CBEH are nearly the same, as shown in Fig. 3, and their SN2 bifurcations are found at nearly the same frequency as well. However, for the backward sweep of the BEH-O [denoted by the red color in Fig. 3(a)], the intrawell motion changes to chaotic motion first, through a cascade of periodic doubling bifurcations, and the chaotic motion is sustained before being terminated at its boundary crisis, where the interwell motion is initiated. The bifurcation structure for the frequency response of the BEH-O discussed above is in accordance with that of the CBEH [Fig. 3(b)]; however, there is a clear difference in between. In fact, the potential barrier-lowering effect of the BEH-O indicates that a stiffness–softening effect strengthens around the potential saddle barrier, making the two potential wells shallower. Accordingly, the intrawell resonance curve of the BEH-O and its local bifurcation points are shifted towards a lower frequency band. As shown in Figs. 3(a) and 3(b), the SN1 bifurcation of the BEH-O occurs at a lower frequency than that of the CBEH. It is obvious that the BEH-O possesses a broader frequency band of interwell motion, bounded by two bifurcation points SN1 and SN2, where intrawell motion does not coexist. Such a potential barrier-lowering effect is also beneficial for harvesting relatively weak vibration energy, as observed in Fig. 3 (also see Fig. S3 in the supplementary material).

According to a previous report,²⁷ points SN1 and SN2 are known to be the lower and upper bounds of the operating frequency band, respectively, and provide more accurate predictions than Melnikov's criteria. In this sense, the BEH-O system is better-adapted to energy harvesting from low frequency vibrations than CBEHs. Figure 4 shows variations in (a) the lower (SN1) and upper (SN2) bounds of the operating

frequency band and (b) the resulting frequency bandwidth with respect to base acceleration. The results for the BEH-O obtained with several different values of ω_e are compared with the results obtained for the CBEH. In Fig. 4(a), the lower bounds where SN1 bifurcations occur are denoted by the lines without points, and the upper bounds where SN2 bifurcations occur are denoted by the lines with points. In this figure, the frequency at every SN1 bound for both the BEH-O and CBEH tends to decrease with increasing excitation intensity. However, owing to the potential barrier-lowering effect, each SN1 bound for the BEH-O is placed, to some degree, at a lower frequency than the corresponding value for the CBEH. This frequency shift in the SN1 bound of the BEH-O against the CBEH depends on the natural frequency of the external magnet oscillator; as seen in Fig. 4(a), the smaller the value of ω_e , the greater the frequency shift. A higher compliance of the external magnet oscillator must enhance its half-sine motion and further reduce the potential saddle barrier experienced by the bimorph oscillator; therefore, the SN1 bound is shifted further towards lower frequencies with a corresponding extension of the total operating frequency band. However, an excessive motion of the external magnet oscillator may also produce a significant frequency shift of the SN2 bound, causing a significant reduction of the operating frequency band [as shown in Fig. 4(a) in the case when ω_e is 70 rad/s]. Therefore, the external magnet oscillator should be well-designed, allowing the frequency shift of the lower bound but preventing the frequency shift of the upper bound, such that the total operating frequency band of the BEH-O becomes broader than for the CBEH. The results presented in Fig. 4(b) show that the bandwidth of the proposed BEH-O increased by 3.43–4.47 Hz (or by 114%–545%) compared to that of the CBEH, depending on the intensity of base excitation.

To validate the enhanced frequency band of the BEH-O, the voltage responses generated by the BEH-O and CBEH were experimentally obtained with several base accelerations, and the results are compared, as shown in Fig. 5. The prototype of the BEH-O shown in Fig. 1(b) was prepared first. For the bimorph oscillator, piezoelectric polyvinylidene fluoride layers with a thickness of 52 μm were entirely laminated on the upper and lower surfaces of a stainless steel substrate ($70 \times 10 \times 0.3 \text{ mm}$). The two identical neodymium magnets ($2 \times 10 \times 6 \text{ mm}$) were used for the tip and external magnets. The natural frequency of the external magnet oscillator was 103 rad/s. The prototype of the BEH-O was placed on a vibration exciter (Type 4808, B&K) connected with a closed-loop vibration controller (VR8500, Vibration Research). During forward/backward swept excitation, the base acceleration was held constant at a given magnitude using the control system. The voltage generated across a resistor of 1 M Ω was simultaneously measured using a signal acquisition system (CDAQ-9171, National Instruments). The output voltage responses obtained with a base acceleration of 0.5 g [Figs. 5(a) and 5(d)] clearly reveal that the interwell motion of the BEH-O tends to occur more easily than that of the CBEH, in agreement with the theoretical predictions [Figs. 3(a) and 3(b)]. Thus, it can be converted to high electrical voltage more efficiently. With increasing base acceleration, the lower bound frequency (at point SN1 in Fig. 5) of the operating frequency

FIG. 5. Experimental comparison of the generated output voltages from the BEH-O (first column) and the CBEH (second column), obtained when the base acceleration is given at 0.52 g (first row), 0.6 g (second row), and 0.7 g (third row).

band decreases in the experimental measurement: 9.6 Hz, 9.2 Hz, and 8.3 Hz for the BEH-O [in Figs. 5(a), 5(c), and 5(e), respectively] and 13.3 Hz, 12.7 Hz, and 12.0 Hz for the CBEH [in Figs. 5(b), 5(d), and 5(f), respectively]. The lower bound frequencies of the BEH-O and CBEH are very similar in their rate of decrease but deviate from each other by an approximately constant value of 3.6 Hz. As the upper bound frequencies (at point SN2 in Fig. 5) are indistinguishably similar to within an error of $O(10^{-2})$, this constant frequency deviation in the lower bound directly contributes to a broader operating frequency band of the proposed BEH-O, which is well-matched by the theoretical predictions displayed in Fig. 4.

It should be noted that the enhanced broadband performance of the BEH-O (discussed above with Figs. 2–5) is not guaranteed if x-directional ambient excitation exists, because it could terminate the normal operation of the BEH-O. Further studies on the nonlinear dynamics of the BEH-O under multidirectional excitation are needed for general applications.

In this study, an auxiliary magnet oscillator with a mechanical rectifier was introduced into the BEH system, resulting in the broadening of the frequency band for high-energy orbital motion without significant changes to the global dynamics of the system (e.g., an amplitude reduction of interwell motion or an overall shift of operating frequency band). Mathematical modeling and simulation provided clear and understandable insights into the beneficial effects of the mechanically rectified motion of the auxiliary oscillator in the BEH-O. It was also supported theoretically and experimentally that the half-sine oscillation of the external magnet helped the bifurcation SN1 of intrawell motion to occur at a

lower frequency, while keeping the global bifurcation SN2 of interwell motion nearly unchanged. Accordingly, the voltage output response of the BEH-O was directly comparable to that of the CBEH, which clearly demonstrated the broader operating frequency band of the BEH-O, specifically, broader by 114%–545%, depending on excitation intensity, in this study. In addition, the results and discussion presented in this study indicate that local modification of the potential well configuration could be a potential design approach for enhancing energy harvesting performance of the BEH, possibly leading to a branch of research in this area.

See [supplementary material](#) for geometric and material properties of the BEH-O and their validity; output voltage responses associated with Fig. 3; and decrease in threshold excitation intensity.

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